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PEARSON Education

The Pearson Guide to

Quantitative Aptitude

for **MBA Entrance Examinations**

Third Edition

- Short-cut Methods
- Solved Problems
- Practice Exercises
- Model Test Papers

Dinesh Khattar

THORPES

The Pearson Guide to

QUANTITATIVE APTITUDE

For MBA Entrance Examinations

(Third Edition)

Common Admission Test (CAT); Management Aptitude Test (MAT); Xavier Admission Test (XAT); AIMS Test for Management Admission (ATMA); Joint Management Entrance Test (JMET); Graduate Management Aptitude Test (GMAT); FMS and Other State-Level Joint Entrance Tests

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PART I

Arithmetic

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1 Numbers

INTRODUCTION

In Hindu Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called *digits* to represent any number. This is the *decimal system* where we use the numbers 0 to 9. 0 is called *insignificant digit* whereas 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant digits*.

A group of figures, denoting a number is called a *numeral*. For a given numeral, we start from extreme right as Unit's place, Ten's place, Hundred's place and so on.

Illustration 1 We represent the number 309872546 as shown below:

Ten Crore 10^8	Crores 10^7	Ten Lacs (million) 10^6	Lacs 10^5	Ten Thousand 10^4	Thousand 10^3	Hundred 10^2	Tens 10^1	Units 10^0
3	0	9	8	7	2	5	4	6

We read it as

"Thirty crores, ninety-eight lacs, seventy-two thousands five hundred and forty-six."

In this numeral:

The place value of 6 is $6 \times 1 = 6$

The place value of 4 is $4 \times 10 = 40$

The place value of 5 is $5 \times 100 = 500$

The place value of 2 is $2 \times 1000 = 2000$ and so on.

The face value of a digit in a number is the value itself wherever it may be.

Thus, the face value of 7 in the above numeral is 7. The face value of 6 in the above numeral is 6 and so on.

NUMBER SYSTEM

Natural Numbers

Counting numbers 1, 2, 3, 4, 5, ... are known as *natural numbers*.

The set of all natural numbers can be represented by

$$N = \{1, 2, 3, 4, 5, ...\}.$$

Whole Numbers

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5, ... are called *whole numbers*.

The set of whole numbers can be represented by

$$W = \{0, 1, 2, 3, 4, 5, ...\}$$

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

INTEGERS

All counting numbers and their negatives including zero are known as *integers*.

The set of integers can be represented by

$$Z \text{ or } I = \{... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$$

Positive Integers

The set $I^+ = \{1, 2, 3, 4, ...\}$ is the set of all *positive integers*. Clearly, positive integers and natural numbers are synonyms.

Negative Integers

The set $I^- = \{-1, -2, -3, ...\}$ is the set of all *negative integers*. 0 is neither positive nor negative.

Non-negative Integers

The set {0, 1, 2, 3, ... } is the set of all *non-negative integers*.

Rational Numbers

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as *rational numbers*, e.g. $\frac{4}{7}, \frac{3}{2}, -\frac{5}{8}, \frac{0}{1}, -\frac{2}{3}$, etc.

The set of all rational numbers is denoted by Q.

i.e. $Q = \{x : x = \frac{p}{q}; p, q \in I, q \neq 0\}$.

Since every natural number 'a' can be written as $\frac{a}{1}$, every natural number is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$, every integer is a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or in non-terminating repeating decimals.

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For example, $\frac{1}{5} = 0.2$, $\frac{1}{3} = 0.333\ldots$, $\frac{22}{7} = 3.1428714287$, $\frac{8}{44} = 0.181818\ldots$, etc.

The recurring decimals have been given a short notation as

$$0.333\ldots = 0.\overline{3}$$

$$4.1555\ldots = 4.0\overline{5}$$

$$0.323232\ldots = 0.\overline{32}.$$

Irrational Numbers

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as *irrational numbers*, e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, etc.

Note that the exact value of π is not $\frac{22}{7}$. $\frac{22}{7}$ is rational while π is irrational number. $\frac{22}{7}$ is approximate value of π. Similarly, 3.14 is not an exact value of it.

Real Numbers

The rational and irrational numbers combined together are called *real numbers*, e.g. $\frac{13}{21}$, $\frac{2}{5}$, $-\frac{3}{7}$, $\sqrt{3}$, $4+\sqrt{2}$, etc. are real numbers.

The set of all real numbers is denoted by R.

Note that the sum, difference or product of a rational and irrational number is irrational, e.g. $3+\sqrt{2}$, $4-\sqrt{3}$, $\frac{2}{3}-\sqrt{5}$, $4\sqrt{3}$, $-7\sqrt{5}$ are all irrational.

Even Numbers

All those numbers which are exactly divisible by 2 are called *even numbers*, e.g. 2, 6, 8, 10, etc., are even numbers.

Odd Numbers

All those numbers which are not exactly divisible by 2 are called *odd numbers*, e.g. 1, 3, 5, 7 etc., are odd numbers.

Prime Numbers

A natural number other than 1, is a *prime number* if it is divisible by 1 and itself only.

For example, each of the numbers 2, 3, 5, 7 etc., are prime numbers.

Composite Numbers

Natural numbers greater than 1 which are not prime, are known as composite numbers.

For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

Note:

1. The number 1 is neither a prime number nor a composite number.
2. 2 is the only even number which is prime.
3. Prime numbers up to 100 are:

   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e. 25 prime numbers between 1 and 100.
4. Two numbers which have only 1 as the common factor are called *co-primes* or *relatively prime* to each other, e.g. 3 and 5 are co-primes.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g. 16 and 17 are relatively prime although 16 is not a prime number.

ADDITION AND SUBTRACTION (SHORT-CUT METHODS)

The method is best illustrated with the help of following example:

Illustration 2 $54321 - (9876 + 8967 + 7689) = ?$

Step 1 Add Ist column:

$6 + 7 + 9 = 22$

To obtain 1 at unit's place add 9 to make 31. In the answer, write 9 at unit's place and carry over 3.

$$\begin{array}{r} 54321 \\ 9876 \\ 8967 \\ 7689 \\ \hline 27789 \end{array}$$

Step 2 Add 2nd column:

$3 + 7 + 6 + 8 = 24$

To obtain 2 at ten's place, add 8 to make 32. In the answer, write 8 at ten's place and carry over 3.

Step 3 Add 3rd column:

$3 + 8 + 9 + 6 = 26$

To obtain 3 at hundred's place, add 7 to make 33. In the answer, write 7 at hundred's place and carry over 3.

Step 4 Add 4th column:

$3 + 9 + 8 + 7 = 27$

To obtain 4 at thousand's place add 7 to make 34. In the answer, write 7 at thousand's place and carry over 3.

Step 5 5th column:

To obtain 5 at ten-thousand's place add 2 to it to make 5. In the answer, write 2 at the ten-thousand's place.

$\therefore$ $54321 - (9876 + 8967 + 7689) = 27789$.

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MULTIPLICATION (SHORT-CUT METHODS)

1. Multiplication of a given number by 9, 99, 999, etc., that is by $10^n - 1$

Method: Put as many zeros to the right of the multiplicant as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

Illustration 3 Multiply:

(a) 3893 by 99 (b) 4327 by 999

(c) 5863 by 9999.

Solution (a) $3893 \times 99 = 389300 - 3893 = 385407.$

(b) $4327 \times 999 = 4327000 - 4327 = 4322673$

(c) $5863 \times 9999 = 58630000 - 5863 = \mathbf{58624137.}$

2. Multiplication of a given number by 11, 101, 1001, etc., that is, by $10^n + 1$.

Method: Place n zeros to the right of the multiplicant and then add the multiplicant to the number so obtained.

Illustration 4 Multiply:

(a) 4782×11 (b) 9836×101

(c) 6538×1001.

Solution (a) $4782 \times 11 = 47820 + 4782 = 52602$

(b) $9836 \times 101 = 983600 + 9836 = 993436$

(c) $6538 \times 1001 = 6538000 + 6538 = \mathbf{6544538}$

3. Multiplication of a given number by 15, 25, 35, etc.

Method: Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2.

Illustrations 5 Multiply:

(a) 7054×15 (b) 3897×25

(c) 4563×35

Solution (a) $7054 \times 15 = \frac{1}{2}(7054 \times 30)$

$= \frac{1}{2}(211620) = \mathbf{105810.}$

(b) $3897 \times 25 = \frac{1}{2}(3897 \times 50) = \frac{1}{2}(194850)$

$= \mathbf{97425.}$

(c) $4536 \times 35 = \frac{1}{2}(4563 \times 70) = \frac{1}{2}(319410)$

$= \mathbf{159705.}$

4. Multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.

Method: Place as many zeros to the right of the multiplicant as is the power of 5 in the multiplier, then divide the number so obtained by 2 raised to the same power as is the power of 5.

Illustration 6 Multiply:

(a) 3982×5 (b) 4739×25

(c) 7894×125 (d) 4863×625

Solution (a) $3982 \times 2 = \frac{39820}{2} = \mathbf{19910.}$

(b) $4739 \times 25 = \frac{473900}{2^2} = \frac{473900}{4} = \mathbf{118475.}$

(c) $7894 \times 125 = \frac{7894000}{2^3} = \frac{7894000}{8}$

$= \mathbf{986750}$

(d) $4863 \times 625 = \frac{48630000}{2^4} = \frac{48630000}{16}$

$= \mathbf{3039375.}$

Distributive Laws

For any three numbers a, b, c, we have

(a) $a \times b + a \times c = a \times (b + c)$

(b) $a \times b - a \times c = a \times (b - c)$

Illustration 7 $438 \times 637 + 438 \times 367 = ?$

Solution $438 \times 637 + 438 \times 367 = 438 \times (637 + 367)$

$= 430 \times 1000 = \mathbf{438000.}$

Illustration 8 $674 \times 832 - 674 \times 632 = ?$

Solution $674 \times 832 - 674 \times 632 = 674 \times (832 - 632)$

$= 674 \times 200 = \mathbf{134800.}$

SQUARES (SHORT-CUT METHODS)

1. To square any number ending with 5.

Method: $(A5)^2 = A(A + 1)/25$

Illustration 9

(a) $(25)^2 = 2\,(2 + 1)/25 = 6/25 = 625$

(b) $(45)^2 = 4\,(4 + 1)/25 = 20/25 = 2025$

(c) $(85)^2 = 8\,(8 + 1)/25 = 72/25 = 7225.$

2. To square a number in which every digit is one.

Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

Illustration 10

(a) $11^2 = 121$ (b) $111^2 = 12321$

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(c) $1111^2 = 1234321$

(d) $222^2 = 2^2\,(111)^2 = 4\,(12321) = 49284$

(e) $3333^2 = 3^2\,(1111)^2 = 9\,(1234321) = 11108889$

3. To square a number which is nearer to $10x$.

Method: Use the formula:

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

Illustration 11

(a) $(97)^2 = (97 + 3)\,(97 - 3) + 3^2$
$= 9400 + 9 = 9409$

(b) $(102)^2 = (102 - 2)\,(102 + 2) + 2^2$
$= 10400 + 4 = 10404$

(c) $(994)^2 = (994 + 6)\,(994 - 6) + 6^2$
$= 988000 + 36 = 988036$

(d) $(1005)^2 = (1005 - 5)\,(1005 + 5) + 5^2$
$= 1010000 + 25 = 1010025.$

DIVISION

Division is repeated subtraction.

For example, when we divide 63289 by 43, it means 43 can be repeatedly subtracted 1471 times from 63289 and the remainder 36 is left.

```
          1471  ← Quotient
Divisor → 43 ) 63289  ← Dividend
            43
            202
            172
             308
             301
              79
              43
              36  ← Remainder
```

Dividend = (Divisor × Quotient) + Remainder

or, $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

Illustration 12 On dividing 7865321 by a certain number, the quotient is 33612 and the remainder is 113. Find the divisor.

Solution $\text{Divisor} = \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$= \dfrac{7865321 - 113}{33612} = \dfrac{7865208}{33612} = \mathbf{234.}$

Illustration 13 A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.

Solution Number = (Divisor × Quotient) + Remainder
$= (315 \times 7) + 46 = 2205 + 46 = \mathbf{2251.}$

Illustration 14 Find the least number of 5 digits which is exactly divisible by 632.

Solution The least number of 5 digits is 10000. Dividing this number by 632, the remainder is 520. So, the required number = 10000 + (632 + 520) = 10112.

```
          15
632 ) 10000
        632
        3680
        3160
         520
```

Illustration 15 Find the greatest number of 5 digits which is exactly divisible by 463.

Solution The greatest number of 5 digits is 99999. Dividing this number by 463, the remainder is 454. So, the required number = 99999 – 454 = 99545.

```
          215
463 ) 99999
        926
        739
        463
        2769
        2315
         454
```

Illustration 16 Find the number nearest to 13700 which is exactly divisible by 235.

Solution On dividing the number 13700 by 235, the remainder is 70. Therefore, the nearest number to 13700, which is exactly divisible by 235 = 13700 – 70 = 13630.

```
          58
235 ) 13700
        1175
        1950
        1880
          70
```

TESTS OF DIVISIBILITY

1. **Divisibility by 2** A number is divisible by 2 if the unit's digit is zero or divisible by 2.

   For example, 4, 12, 30, 18, 102, etc., are all divisible by 2.

2. **Divisibility by 3** A number is divisible by 3 if the sum of digits in the number is divisible by 3.

   For example, the number 3792 is divisible by 3 since 3 + 7 + 9 + 2 = 21, which is divisible by 3.

3. **Divisibility by 4** A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

   For example, the number 2616 is divisible by 4 since 16 is divisible by 4.

4. **Divisibility by 5** A number is divisible by 5 if the unit's digit in the number is 0 or 5.

   For example, 13520, 7805, 640, 745, etc., are all divisible by 5.

5. **Divisibility by 6** A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

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For example, the number 4518 is divisible by 6 since it is even and sum of its digits 4 + 5 + 1 + 8 = 18 is divisible by 3.

6. **Divisibility by 7** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

   For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16.

   Then, 44 – 16 = 28.

   Since 28 is divisible by 7, 448 is divisible by 7.

7. **Divisibility by 8** A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

   For example, the number 41784 is divisible by 8 as the number formed by last three digits, i.e. 784 is divisible by 8.

8. **Divisibility by 9** A number is divisible by 9 if the sum of its digits is divisible by 9.

   For example, the number 19044 is divisible by 9 as the sum of its digits 1 + 9 + 0 + 4 + 4 = 18 is divisible by 9.

9. **Divisibility by 10** A number is divisible by 10, if it ends in zero.

   For example, the last digit of 580 is zero, therefore, 580 is divisible by 10.

10. **Divisibility by 11** A number is divisible by 11 if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

    For example, in the number 38797, the sum of the digits at odd places is 3 + 7 + 7 = 17 and the sum of the digits at even places is 8 + 9 = 17. The difference is 17 – 17 = 0, so the number is divisible by 11.

11. **Divisibility by 12** A number is divisible by 12 if it is divisible by 3 and 4.

12. **Divisibility by 25** A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.

    For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25.

13. **Divisibility by 125** A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.

    For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

14. **Divisibility by 18** An even number satisfying the divisibility test of 9 is divisible by 18.

15. **Divisibility by 88** A number is divisible by 88 if it is divisible by 11 and 8.

SOME USEFUL SHORT-CUT METHODS

1. **Test to find whether a given number is a prime**

   Step 1 Select a least positive integer n such that $n^2 >$ given number.

   Step 2 Test the divisibility of given number by every prime number less than n.

   Step 3 The given number is prime only if it is not divisible by any of these primes.

Illustration 17 Investigate whether 571 is a prime number

Solution Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore \quad n = 24.$

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2, 571 is not a prime number.

Illustration 18 Investigate whether 923 is a prime number.

Solution Since $(30)^2 = 900 < 923$ and $(31)^2 = 961 > 923$

$\therefore \quad n = 31.$

Prime numbers less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Since 923 is not divisible by any of these primes, therefore 923 is a prime number.

2. The least number, which when divided by d_1, d_2 and d_3 leaves the remainders r_1, r_2 and r_3 respectively such that $(d_1 - r_1) = (d_2 - r_2) = (d_3 - r_3)$, is = (L.C.M. of d_1, d_2 and d_3) $- (d_1 - r_1)$ or $(d_2 - r_2)$ or $(d_3 - r_3)$.

Illustration 19 Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

Solution Here, 9 – 4 = 10 – 5 = 15 – 10 = 5.

Also, L.C.M. (9, 10, 15) = 90,

$\therefore$ the required least number = 90 – 5 = **85.**

3. A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder is $= (d_1 \times r_2 + r_1)$.

Illustration 20 A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

Solution The required remainder

$= d_1 \times r_2 + r_1 = 10 \times 7 + 5 =$ **75.**

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4. To find the number of numbers divisible by a certain integer.

The method is best illustrated with the help of following example.

Illustration 21 How many numbers up to 532 are divisible by 15?

Solution We divide 532 by 15.

$$532 = \underline{35} \times 15 + 7$$

The quotient obtained is the required number of numbers. Thus, there are 35 such numbers.

Illustration 22 How many numbers up to 300 are divisible by 5 and 7 together?

Solution L.C.M. of 5 and 7 = 35.

We divide 300 by 35.

$$300 = \underline{8} \times 35 + 20$$

Thus, there are 8 such numbers.

5. Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. The divisor is given by $r_1 + r_2 - r_3$.

Illustration 23 Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Solution The required divisor

$= 437 + 298 - 236 = \mathbf{499}$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. $7372 \times 7372 + 7372 \times 628 = ?$

(a) 58976000 (b) 58967000
(c) 5897600 (d) None of these

2. $9999 + 8888 + 777 + ? = 19700$

(a) 36 (b) 16
(c) 64 (d) 26

3. $60\ ?\ 6 \times 111 = 666666$

(a) 0 (b) 2
(c) 1 (d) 6

4. $3149 \times 1\ ?\ 5 = 425115$

(a) 3 (b) 2
(c) 4 (d) 6

5. If the two digits of the age of Mr. Manoj are reversed then the new age so obtained is the age of his wife. $\frac{1}{11}$ of the sum of their ages is equal to the difference between their ages. If Mr. Manoj is older than his wife then find the difference between their ages.

(a) Cannot be determined
(b) 8 years (c) 10 years
(d) 9 years (e) 7 years

6. If in a long division sum, the dividend is 380606 and the successive remainders from the first to the last are 434, 125 and 413, then divisor is

(a) 451 (b) 843
(c) 4215 (d) 3372

7. If $\frac{x}{y} = \frac{3}{4}$, then the value of $\left(\frac{6}{7} + \frac{y-x}{y+x}\right)$ equals

(a) $\frac{5}{7}$ (b) $1\frac{1}{7}$
(c) 1 (d) 2

8. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is

(a) 16 (b) 24
(c) 48 (d) 96

9. Which number should replace both the '*'s in $\left(\frac{*}{21}\right) \times \left(\frac{*}{189}\right) = 1$?

(a) 21 (b) 63
(c) 3969 (c) 147

10 In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder be 48, then the dividend is

(a) 240 (b) 576
(c) 4800 (d) 4848

11. What least number must be subtracted from 1294 so that the remainder when divided by 9, 11, 13 will leave in each case the same remainder 6?

(a) 0 (b) 1
(c) 2 (d) 3

12. 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is

(a) 11 (b) 13
(c) 16 (d) 17

13. $\frac{1}{4}$ of a number subtracted from $\frac{1}{3}$ of the number gives 12. The number is

(a) 144 (b) 120
(c) 72 (d) 63

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14. $\frac{4}{3}$ of a certain number is 64. Half of that number is
(a) 32 (b) 40
(c) 80 (d) 16

15. A fraction becomes 4 when 1 is added to both the numerator and denominator; and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is
(a) 2 (b) 3
(c) 7 (d) 15

16. Three numbers are in the ratio 3 : 4 : 5. The sum of the largest and the smallest equals the sum of the third and 52. The smallest number is
(a) 20 (b) 27
(c) 39 (d) 52

17. The sum of three numbers is 68. If the ratio between first and second is 2 : 3 and that between second and third is 5 : 3, then the second number is
(a) 30 (b) 20
(c) 58 (d) 48

18. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator, the fraction becomes 1. The fraction is
(a) $\frac{4}{7}$ (b) $\frac{5}{9}$
(c) $\frac{2}{3}$ (c) $\frac{10}{11}$

19. $\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is
(a) 30 (b) 60
(c) 90 (d) None of these

20. What is the sum of all prime numbers from 60 to 80?
(a) 361 (b) 341
(c) 351 (d) 349

21. The quotient arising from the division of 24446 by a certain divisor is 79 and the remainder is 35, what is the divisor?
(a) 39 (b) 309
(c) 390 (c) 3009

22. In a division sum, the quotient is 120, the divisor 456 and the remainder 333, find the dividend.
(a) 5533 (b) 50553
(c) 56053 (d) 55053

23. The quotient arising from a division of a number by 62 is 463 and the remainder is 60, what is the number?
(a) 28666 (b) 28766
(c) 28576 (c) 28676

24. A number when divided by 221 gives a remainder 43. What remainder will be obtained by dividing the same number by 17?
(a) 11 (b) 8
(c) 9 (d) 13

25. Which one of the following is the largest prime number of three digits?
(a) 997 (b) 999
(c) 991 (d) 993

26. When a certain number is multiplied by 7, the product consists entirely of fives; find the least value of such a number.
(a) 79365 (b) 78365
(c) 77365 (d) 79265

27. In a division sum, the divisor is 10 times the quotient and five times the remainder. What is the dividend, if the remainder is 46?
(a) 5636 (b) 5536
(c) 5336 (d) 5436

28 Which one of the following is the least number of four digits divisible by 71?
(a) 1006 (b) 1065
(c) 1094 (d) 1056

29. How many numbers up to 100 are dvisible by 7?
(a) 14 (b) 107
(c) 93 (d) 100

30. How many numbers up to 500 are divisible by 23?
(a) 23 (b) 27
(c) 21 (d) 19

31. How many numbers up to 200 are divisible 2 and 3 both?
(a) 35 (b) 33
(c) 29 (d) 27

32. How many numbers between 100 and 300 are divisible by 11?
(a) 11 (b) 10
(c) 12 (d) 18

33. How many numbers between 150 and 500 are divisible by 2, 3 and 7 together?
(a) 9 (b) 8
(c) 10 (d) 11

34. The number of five figures to be added to a number of four fives to obtain the least number of six figures exactly divisible by 357 is
(a) 94762 (a) 94802
(c) 94485 (d) None of these

35. The nearest figure to 58701 which is divisible by 567 is
(a) 58968 (b) 58434
(c) 58401 (d) None of these

36. The digits indicated by * in 3422213 ** so that this number is divisible by 99 are
(a) 1, 9 (b) 3, 7
(c) 4, 6 (d) 5, 5

37. The least value to be given to * so that the number 5 * 3457 is divisible by 11 is

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PA13

(a) 2 (b) 3
(c) 0 (d) 4

38. The nearest whole number to one million which is divisible by 537 is
(a) 1000106 (b) 999894
(c) 1000437 (d) 999563

39. The smallest number between 400 and 500 which is divisible by 9 is
(a) 414 (b) 405
(c) 423 (d) None of these

40. Which one of the following is the greatest number of five digits divisible by 231?
(a) 99792 (b) 99892
(c) 99692 (d) 99972

41. Find the number nearest to 16386 which is exactly divisible by 425.
(a) 16575 (b) 16375
(c) 16050 (d) 16450

42. Find the least number which must be subtracted from 9269 so that resulting number is exactly divisible by 73?
(a) 17 (b) 57
(c) 71 (d) 63

43. Find the least number which must be added to 15463 so that the resulting number is exactly divisible by 107?
(a) 52 (b) 71
(c) 55 (d) 19

44. What is the number just more than 5000 which is exactly divisible by 73?
(a) 5001 (b) 5009
(c) 5037 (d) 5027

45. The sum of two numbers is 100 and their difference is 37. The difference of their squares is
(a) 37 (b) 100
(c) 63 (d) 3700

46. The number of times 79 be subtracted from 50000, so that the remainder be 43759; is
(a) 69 (b) 79
(c) 59 (d) None of these

47. The ratio between two numbers is 3 : 4 and their sum is 420. The greater of the two numbers is
(a) 175 (b) 200
(c) 240 (d) 315

48. The difference between the squares of two consecutive numbers is 35. The numbers are
(a) 14, 15 (b) 15, 16
(c) 17, 18 (d) 18, 19

49. Three-fourths of one-fifth of a number is 60. The number is
(a) 300 (b) 400
(c) 450 (d) 1200

50. The sum of squares of two numbers is 80 and the square of their difference is 36. The product of the two numbers is
(a) 22 (b) 44
(c) 58 (d) 116

51. A number when divided by 357 gives a remainder 37. By dividing the same number by 17, the remainder would be
(a) 3 (b) 4
(c) 2 (d) None of these

52. The product of two numbers is 120. The sum of their squares is 289. The sum of the two numbers is
(a) 20 (b) 23
(c) 169 (d) None of these

53. Three numbers are in the ratio 4 : 5 : 6 and their average is 25. The largest number is
(a) 42 (b) 36
(c) 30 (d) 32

54. A number exceeds 20% of itself by 40. The number is
(a) 50 (b) 60
(c) 80 (d) 320

55. If 16% of 40% of a number is 8, the number is
(a) 200 (b) 225
(c) 125 (d) 320

56. 4767 exactly divides *** 341, the missing digits are
(a) 468 (b) 586
(c) 363 (d) None of these

57. A number when divided by a certain divisor left remainder 241, when twice the number was divided by the same divisor, the remainder was 112. Find the divisor.
(a) 370 (b) 365
(c) 380 (d) 456

58. Two numbers when divided by a certain divisor give remainders 43 and 37 respectively, when their sum is divided by the same divisor, the remainder is 13. Find the divisor.
(a) 71 (b) 67
(c) 57 (d) 77

59. Two numbers are such that the ratio between them is 3 : 5; but if each is increased by 10, the ratio between them becomes 5 : 7. The numbers are
(a) 3, 5 (b) 7, 9
(c) 13, 22 (d) 15, 25

60. Divide 50 into two parts so that the sum of their reciprocals is 1/12.
(a) 20, 30 (b) 24, 26
(c) 28, 22 (d) 36, 14

61. The sum of seven numbers is 235. The average of the first three is 23 and that of last three is 42. The fourth number is
(a) 40 (b) 126
(c) 69 (d) 195

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62. The sum of squares of two numbers is 68 and the squares of their difference is 36. The product of the two numbers is
(a) 16 (b) 32
(c) 58 (d) 104

63. What is the least value of K so that the number 6735K1 is divisible by 9?
(a) 5 (b) 7
(c) 4 (d) 3

64. For what value of K, the number 7236K2 is divisible by 8?
(a) 7 (b) 5
(c) 4 (d) 9

65. Find the least values of x and y so that the number $5x423y$ is divisible by 88.
(a) 8, 2 (b) 7, 3
(c) 9, 4 (d) 6, 5

66. 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is
(a) 13 (b) 15
(c) 17 (d) 19

67. Sum of three numbers is 132. First number is twice the second and third number is one-third of the first. Find the second number.
(a) 18 (b) 36
(c) 20 (d) 16

68. What least number must be added to 7231 so that the resulting number is exactly divisible by 5 and 9 together?
(a) 20 (b) 18
(c) 14 (d) 16

69. Find a number nearest to 9231 which is exactly divisible by 3 as well as by 11.
(a) 9240 (b) 9340
(c) 9540 (d) 9440

70. Find a nearest number to 12199 which is exactly divisible by the product of the first four prime numbers.
(a) 12181 (b) 12179
(c) 11281 (d) 11279

71. The sum of squares of two numbers is 90 and the squares of their difference is 46. The product of the two numbers is
(a) 22 (b) 24
(c) 26 (d) 28

72. If 40% of a number is 360, what will be 15% of that number?
(a) $20\frac{1}{4}$ (b) $20\frac{1}{2}$
(c) $22\frac{1}{4}$ (d) $22\frac{1}{2}$

73. The sum of the digits of a two-digit number is 8. If the digits are reversed the number is increased by 54. Find the number.
(a) 17 (b) 19
(c) 21 (d) 23

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Given Expression $= 7372 \times (7372 + 628)$
$= 7372 \times 8000$
$=$ **58976000.**

2. Ans. (a)

Solution Let $9999 + 8888 + 777 + x = 19700$
$\therefore \quad x = 19700 - 19664 =$ **36.**

3. Ans. (a)

Solution Let $x \times 111 = 666666$
$\Rightarrow \quad x = \frac{666666}{111} = 6006$
$\therefore$ Missing figure = **0**

4. Ans. (a)

Solution Let $3149 \times x = 425115$
$\Rightarrow \quad x = \frac{425115}{3149} = 135$
$\therefore$ Missing digit = **3.**

5. Ans. (d)

Solution Let the age of Mr. Manoj be $(10x + y)$ years.
$\therefore$ His wife's age $= (10y + x)$ years
Then, $(10x + y + 10y + x)\frac{1}{11} = 10x + y - 10y - x$
or, $\quad x + y = 9x - 9y, \quad$ or, $8x = 10y$
or, $\quad \frac{x}{y} = \frac{5}{4}$
$\therefore \quad x = 5$ and $y = 4$
[$\because$ any other multiple of 5 will make x of two digits]
$\therefore$ Difference $= 10x + y - 10y - x$
$= 9x - 9y = 9(x - y)$
$= 9(5 - 4) =$ **9 years.**

6. Ans. (b)

7. Ans. (c)

8. Ans. (c)

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9. Ans. (b)

Solution Let $\frac{x}{21} \times \frac{x}{189} = 1$.

Then, $x^2 = 21 \times 189 = 21 \times 21 \times 3 \times 3$

$\therefore \quad x = 21 \times 3 = \mathbf{63}$.

10. Ans. (d)

Solution Let Quotient = Q and remainder = R

Then, given $12Q = 5R$

Now, $R = 48 \Rightarrow 12Q = 5 \times 48 \Rightarrow Q = 20$

$\therefore$ Dividend = $20 \times 240 + 48 = \mathbf{4848}$.

11. Ans. (b)

Solution The number when divided by 9, 11 and 13 leaving remainder 6 = (L.C.M. of 9, 11, 13) + 6 = 1293

$\therefore$ Required number = $1294 - 1293 = \mathbf{1}$.

12. Ans. (b)

Solution Let these parts be x and $(24 - x)$.

Then, $7x + 5(24 - x) = 146 \Rightarrow x = 13$

So the first part is **13**.

13. Ans. (a)

Solution $\frac{1}{3}x - \frac{1}{4}x = 12 \Rightarrow \frac{1}{12}x = 12 \Rightarrow \boldsymbol{x = 144}$.

14. Ans. (b)

Solution $\frac{4}{5} \times x = 64 \Rightarrow x = \frac{64 \times 5}{4} = 80$

$\therefore \quad \frac{1}{2} \times x = \frac{1}{2} \times 80 = \mathbf{40}$.

15. Ans. (d)

Solution Let the required fraction be $\frac{x}{y}$

Then, $\frac{x+1}{y+1} = 4 \Rightarrow x - 4y = 3$

and, $\frac{x-1}{y-1} = 7 \Rightarrow x - 7y = -6$

Solving these equations, we get $\boldsymbol{x = 15, y = 3}$.

16. Ans. (c)

Solution Let the numbers be $3x$, $4x$ and $5x$

Then, $5x + 3x = 4x + 52 \Rightarrow 4x = 52 \Rightarrow x = 13$

$\therefore$ The smallest number = $3x = 3 \times 13 = \mathbf{39}$.

17. Ans. (a)

Solution Let the numbers be a, b, c. Then,

$$\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{5}{3} \Rightarrow \frac{a}{b} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}, \frac{b}{c} = \frac{5 \times 3}{3 \times 3} = \frac{15}{9}$$

$$\Rightarrow a : b : c = 10 : 15 : 9$$

Let the numbers be $10x$, $15x$, $9x$,

Then, $10x + 15x + 9x = 68 \Rightarrow 34x = 68 \Rightarrow x = 2$

$\therefore$ Second number = $15x = 15 \times 2 = \mathbf{30}$.

18. Ans. (c)

Solution Let the required fraction be $\frac{x}{y}$. Then,

$$\frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x - y = 1$$

and, $\frac{x+1}{y} = 1 \Rightarrow x - y = -1$

Solving $2x - y = 1$ and $x - y = -1$,

we get $x = 2$, $y = 3$

$\therefore$ The fraction is $\frac{\mathbf{2}}{\mathbf{3}}$.

19. Ans. (b)

Solution Let the number be x. Then,

$$\frac{4}{5}x - \frac{2}{3}x = 8 \Rightarrow \frac{12x - 10x}{15} = 8 \Rightarrow 2x = 120$$

or, $\boldsymbol{x = 60}$.

20. Ans. (c)

Solution $61 + 67 + 71 + 73 + 79 = \mathbf{351}$.

21. Ans. (b)

Solution Divisor = $(24446 - 35) \div 79 = \mathbf{309}$.

22. Ans. (d)

Solution Dividend = $456 \times 120 + 33 = \mathbf{55053}$.

23. Ans. (b)

Solution Number = $463 \times 62 + 60 = \mathbf{28766}$.

24. Ans. (c)

Solution Dividing 43 by 17, the remainder is **9**.

25. Ans. (a)

26. Ans. (a)

Solution

7	555555
	79365

$\therefore$ The least number is **79365**.

27. Ans. (c)

Solution Remainder = 46

Divisor = $5 \times 46 = 230$

Also $10 \times$ quotient = $5 \times$ remainder

$\therefore$ Remainder = $2 \times$ quotient

i.e., quotient = 23

Dividend = $23 \times 230 + 46 = \mathbf{5336}$.

28. Ans. (b)

Solution

$$\begin{array}{r} 71\overline{)1000}(14 \\ -71 \\ \hline 290 \\ -284 \\ \hline 6 \end{array}$$

$\therefore$ least number = $1000 - 6 + 71 = \mathbf{1065}$.

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29. Ans. (a)

Solution Quotient when 100 is divided by **7** is **14.**

30. Ans. (c)

Solution Quotient when 500 is divided by **23** is **21.**

31. Ans. (b)

Solution Quotient when 200 is divided by the L.C.M. of 2 and 3, i.e. **6** is **33.**

32. Ans. (d)

Solution Quotient when 300 and 100 are divided by 11 are 27 and 9. ∴ Between 300 and 100, there are 27 – 9, i.e. **18 numbers.**

33. Ans. (a)

Solution A number divisible by 2, 3, and 7 is divisible by their L.C.M., i.e. 42. Up to 100, there are 2 numbers divisible by 42. Up to 500, there are 11 numbers divisible by 42.

∴ Between 100 and 500, there are 11 – 2, i.e. 9 numbers divisible by **42.**

34. Ans. (a)

Solution The least no. of six figures is 100000

On dividing 100000 by 357, remainder = 40

∴ Least number of six figures which is divisible by 357

$= 100000 + (357 - 40) = 100317$

∴ Required number = 100317 – 5555 = **94762.**

35. Ans. (a)

Solution On dividing 58701 by 567

$\text{Remainder} = 300 > \frac{1}{2}(567)$

∴ Integer nearest to 58701 and divisible by 567

$= 58701 + (567 - 300)$

$= 58701 + 267 = \mathbf{58968.}$

36. Ans. (a)

Solution Let x, y be the required digits.

The number is to be divisible by 99, i.e. 9 and 11 both.

∴ Sum of digits is to be divisible by 9, i.e.

$3 + 4 + 2 + 2 + 2 + 1 + 3 + x + y = 17 + x + y$

is to be divisible by 9

and, $(y + 3 + 2 + 2 + 3) - (x + 1 + 2 + 4) = 0$

or, multiple of 11, i.e. $y - x + 3 = 0$ or multiple of 11

∴ $x = 1, y = 9.$

37. Ans. (a)

Solution Let the least value to be given to * be x

Then, $x + 4 + 7 = 5 + 3 + 5$

$x = 2.$

38. Ans. (b)

Solution On dividing 100000 by 537, remainder

$= 106 < \frac{537}{2}$

∴ Nearest whole number to one million which is divisible by 537 = 100000 – 106 = **999894.**

39. Ans. (b)

Solution On dividing 400 by 9, remainder = 4

∴ Number nearest to 400 and divisible by 9

$= 400 + (9 - 4) = \mathbf{405.}$

40. Ans. (a)

Solution Greatest number of five digits = 99,999

Dividing this by 231, the remainder = 207

∴ Required greatest number

$= 99{,}999 - 207 = \mathbf{99792.}$

41. Ans. (d)

Solution Dividing 16386 by 425, the remainder is 361 which is more than half the divisor, therefore, the required number is 16368 + (425 – 361) = **16450.**

42. Ans. (c)

Solution Divide 9269 by 73, the remainder is 71.

∴ 71 is the required least number.

43. Ans (a)

Solution Divide 15463 by 107, the remainder is 55, therefore, the number to be added = 107 – 55 = **52.**

44. Ans. (c)

Solution Dividing 5000 by 73, the remainder is 36. The number greater than 5000 is obtained by adding to 5000 the difference of divisor and the remainder.

∴ The required number = 5000 + (73 – 36) = **5037.**

45. Ans. (d)

Solution Let the numbers be a and b.

Then, $a + b = 100$ and $a - b = 37$

∴ $a^2 - b^2 = (a + b)(a - b) = 100 \times 37 = \mathbf{3700.}$

46. Ans. (b)

Solution 50000 = 79 × quotient + 43759

∴ 50000 – 43759 = 79 × quotient

or, 6241 = 79 × quotient

∴ Required number of times $= \frac{6241}{79} = \mathbf{79.}$

47. Ans. (c)

Solution Let the numbers be $3x$ and $4x$.

Then, $3x + 4x = 420 \Rightarrow 7x = 420 \Rightarrow x = 60$

∴ Greater number = 4 × 60 = **240.**

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48. Ans. (c)

Solution Let the numbers be x and $(x + 1)$. Then,

$(x + 1)^2 - x^2 = 35 \Rightarrow x^2 + 2x + 1 - x^2 = 35$

$\Rightarrow 2x = 34$

or $x = 17$

So, the numbers are **17** and **18.**

49. Ans. (b)

Solution Let the number be x. Then,

$\frac{3}{4} \times \frac{1}{5} \times x = 60 \Rightarrow 3x = 60 \times 5 \times 4$

or, $\boldsymbol{x = 400.}$

50. Ans. (a)

Solution Let the numbers be a and b. Then,

$a^2 + b^2 = 80$ and $(a - b)^2 = 36$

$(a - b)^2 = 36 \Rightarrow a^2 + b^2 - 2ab = 36$

$\Rightarrow 2ab = (a^2 + b^2) - 36 = 80 - 36 = 44$

$\Rightarrow \boldsymbol{ab = 22.}$

51. Ans. (a)

Solution Let K be the quotient. Then, number

$= 357 \times K + 37$

$= 17 \times 21K + 17 \times 2 + 3$

$= 17 \times (21K + 2) + 3$

So, required remainder = 3 and new quotient $= \boldsymbol{21K + 2.}$

52. Ans. (b)

Solution Let the numbers be a and b. Then,

$(a + b)^2 = (a^2 + b^2) + 2ab$

$= 289 + 2 \times 120 = 289 + 240 = 529$

$\therefore \quad a + b = \sqrt{529} \Rightarrow \boldsymbol{a + b = 23.}$

53. Ans. (c)

Solution Let the numbers be $4x$, $5x$ and $6x$. Then,

$\frac{4x + 5x + 6x}{3} = 25$ or $15x = 75 \Rightarrow x = 5$

$\therefore$ The largest number $= 6x = 6 \times 5 = 30.$

54. Ans. (a)

Solution Let the required number be x. Then,

$x - \frac{20}{100}x = 40$ or, $5x - x = 200$ or, $\boldsymbol{x = 50.}$

55. Ans. (c)

Solution Let $\frac{16}{100} \times \frac{40}{100} \times x = 8$

Then, $x = \frac{8 \times 100 \times 100}{16 \times 40} = \boldsymbol{125.}$

56. Ans. (b)

Solution Last digit of dividend = 1

Last digit of divisor = 7

$\therefore$ Last digit of quotient should be 3

$4767 \times 3 = 14301$

$4767 \times 20 = 95340$

$4767 \times 100 = 476700$

$\therefore \quad 4767 \times (3 + 20 + 100) = 586341$

Missing digits are **586.**

57. Ans. (a)

Solution The divisor $= r_1 + r_2 - r_3$

$= 241 + 241 - 112 = \boldsymbol{370.}$

58. Ans. (b)

Solution The divisor $= r_1 + r_2 - r_3 = 43 + 37 - 13 = \boldsymbol{67.}$

59. Ans. (d)

Solution Let the numbers be $3x$ and $5x$. Then,

$\frac{3x + 10}{5x + 10} = \frac{5}{7} \Rightarrow 7(3x + 10) = 5(5x + 10)$

$\Rightarrow x = 5$

$\therefore$ The numbers are **15** and **25.**

60. Ans. (a)

Solution Let the numbers be x and $(50 - x)$. Then,

$\frac{1}{x} + \frac{1}{50 - x} = \frac{1}{12} \Rightarrow \frac{50 - x + x}{x(50 - x)} = \frac{1}{12}$

$\Rightarrow x^2 - 50x + 600 = 0$

$\Rightarrow x = 30$ or $20.$

$\therefore$ The numbers are **20, 30.**

61. Ans. (a)

Solution $23 \times 3 + x + 42 \times 3 = 235 \Rightarrow x = 40$

$\therefore$ Fourth number $= \boldsymbol{40.}$

62. Ans. (a)

Solution Let the numbers be a and b. Then,

$a^2 + b^2 = 68$ and $(a - b)^2 = 36$

Now, $(a - b)^2 = 36 \Rightarrow a^2 + b^2 - 2ab = 36$

$\Rightarrow 68 - 2ab = 36$

$\Rightarrow 2ab = 32 \Rightarrow \boldsymbol{ab = 16.}$

63. Ans. (a)

Solution $6 + 7 + 3 + 5 + K + 1 = 22 + K$

The least number greater than 22 and divisible by 9 is 27

$\therefore \quad 27 = 22 + K \Rightarrow \boldsymbol{K = 5.}$

64. Ans. (a)

Solution The last three digits $6K2$ is divisible by 8 if K is 3 or 7 since 632 and 672 are divisible by **8.**

65. Ans. (a)

Solution Test of 8 is independent of the value of x. First, apply the test of 8. Last three digit of the given number are $23y$ which is divisible by 8 if y is 2. Substitute for y.

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The number now becomes $5x4232$.

Apply the test of 11.

Sum of the digits at odd and even places is $5 + 4 + 3$, i.e. 12 and $x + 2 + 2$, i.e. $x + 4$

$\therefore \quad x + 4 = 12 \Rightarrow x = 8$

Hence, $x = 8$ and $y = 2$.

66. Ans. (a)

Solution Let x be the first part so that the other part is $24 - x$.

$\therefore \quad 7x + 5(24 - x) = 146 \Rightarrow x = 13.$

67. Ans. (b)

Solution Let the second number be $3x$, so that the first number is $6x$ and the third one is $2x$.

$\therefore \quad 6x + 3x + 2x = 132 \Rightarrow 11x = 132 \quad$ or $\quad x = 12.$

Second number $= 3x = 3 \times 12 = \mathbf{36}$.

68. Ans. (c)

Solution Divide 7231 by 45, the remainder is 31.

$\therefore$ Required number $= 45 - 31 = \mathbf{14}$.

69. Ans. (a)

Solution A number which is divisible by 3 and 11 is also divisible by 33. Dividing 9331 by 33, the remainder is 24 which is more than half the divisor.

$$\begin{array}{r} 33\overline{)9331}(279 \\ \underline{66} \\ 263 \\ \underline{231} \\ 321 \\ \underline{297} \\ 24 \end{array}$$

Since the remainder 24 is more than half the divisor 33,

$\therefore$ The nearest number $= 9231 + (33 - 24) = \mathbf{9240}$.

70. Ans. (a)

Solution Product of first four prime numbers is

$2 \times 3 \times 5 \times 7 = 210$

Dividing 12199 by 210, we find the remainder 19, which is less than half the divisor.

$$\begin{array}{r} 210\overline{)12199}(58 \\ \underline{1050} \\ 1699 \\ \underline{1680} \\ 19 \end{array}$$

$\therefore$ The number nearer to 12199 divisible by 210 is $12199 - 18 = \mathbf{12181}$.

71. Ans. (a)

Solution Let the numbers be x and y.

According to the question:

$$x^2 + y^2 = 90 \qquad \ldots(1)$$

and, $$(x - y)^2 = 46 \qquad \ldots(2)$$

From equation (2),

$$(x - y)^2 = 46 \quad \text{or,} \quad x^2 + y^2 - 2xy = 46$$

or, $$90 - 2xy = 46 \quad \text{or,} \quad xy = \frac{90 - 46}{2} = 22$$

$\therefore$ Product of two numbers $= \mathbf{22}$.

72. Ans. (a)

Solution Let the number be x. Then, we have 40% of $x = 360$

$$\therefore \quad x = \frac{360 \times 100}{40} = 900$$

Now, 15% of $x = \frac{15}{100} \times 900 = 135$

Again, 15% of $135 = \frac{15}{100} \times 135 = \mathbf{20.25}$.

73. Ans. (a)

Solution Let the two digit number be $10x + y$

Then, we have, $x + y = 8 \qquad \ldots(1)$

and, $10y + x = 10x + y + 54$

or, $y - x = 6 \qquad \ldots(2)$

From equations (1) and (2),

the required number $= 1 \times 10 + 7 = \mathbf{17}$.

QUESTIONS FROM COMPETITIVE EXAMS

1. Let a, b, c, d be the four integers usch that $a + b + c + d = 4m + 1$, where m is a positive integer. Given m, which one of the following is necessarily true?

(a) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

(b) The minimum possible value of $a^2 + b^2 + c^2 + d$ is $4m^2 + 2m + 1$

(c) The maximum possible value of $a^2 + b^2 + c^2 + d$ is $4m^2 - 2m + 1$

(d) The maximum possible value of $a^2 + b^2 + c^2 + d$ is $4m^2 + 2m + 1$.

[Based on CAT 2003]

2. How many three-digit positive integers, with digits x, y and z in the hundred's, ten's and unit's place respectively exist such that $x < y$, $z < y$ and $x \neq 0$?

(a) 245 (b) 285

(c) 240 (d) 320

[Based on CAT 2003]

3. The number of positive integers n in the range $12 \leq n \leq 40$ such that the product $(n - 1)(n - 2) \ldots 3 \cdot 2 \cdot 1$ is not divisible by n is

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(a) 5 (b) 7
(c) 13 (d) 14

4. Let x and y be positive integers such that x is prime and y is composite. Then,
(a) $y - x$ cannot be an even integer
(b) xy cannot be an even integer
(c) $\frac{(x+y)}{x}$ cannot be an even integer
(d) None of these. **[Based on CAT 2004]**

5. If a, $a + 2$ and $a + 4$ are prime numbers, then the number of possible solutions for a is:
(a) one (b) two
(c) three (d) None of these.
[Based on CAT 2004]

6. What is the remainder when 4^{96} is divided by 6?
(a) 0 (b) 2
(c) 3 (d) 4 **[Based on CAT 2004]**

7. A student was asked to divide a number by 6 and add 12 to the quotient. He, however, first added 12 to the number and then divided it by 6, getting 112 as the answer. The correct answer should have been
(a) 122 (b) 118
(c) 114 (d) 124
[Based on MAT 2004]

8. Which of the following integers is the square of an integer for every integer n?
(a) n^2+1 (b) $n^2 + n$
(c) $n^2 + 2n$ (d) $n^2 + 2n + 1$
[Based on MAT 2004]

9. If x and y are negative, then which of the following statements is/are always true?
I. $x + y$ is positive
II. xy is positive
III. $x - y$ is positive
(a) I only (b) II only
(c) III only (d) I and III only
[Based on MAT 2004]

10. The sum of the digits of a 3 digit number is subtracted from the number. The resulting number is always
(a) Divisible by 6 (b) Not divisible by 6
(c) Divisible by 9 (d) Not divisible by 9
[Based on MAT 2004]

11. The least number that must be subtracted from each of the numbers 14, 17, 34 and 42 so that the remainders may be proportional, is
(a) 0 (b) 1
(c) 2 (d) 7
[Based on MAT 2003]

12. There are four prime numbers written in ascending order. The product of the first three is 385 and that of the last three is 1001. The first number is
(a) 5 (b) 7
(c) 11 (d) 17
[Based on MAT 2003]

13. A positive integer, which when added to 1000, gives a sum which is greater than when it is multiplied by 1000. This positive integer is
(a) 1 (b) 5
(c) 7 (d) 3
[Based on MAT 2003]

14. Of the 120 people in the room, 3/5 are women. If 2/3 of the people are married, then what is the maximum number of women in the room who could be unmarried?
(a) 40 (b) 20
(c) 30 (d) 60
[Based on MAT 2003]

15. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is
(a) 3 (b) 2
(c) 1 (d) None of these
[Based on MAT 2002]

16. A number of three digits in scale 7 when expressed in scale 9 has its digits reversed in order. The number is
(a) 248 (b) 348
(c) 148 (d) 448
[Based on MAT 2002]

17. For every positive real number

$$\left[\frac{x}{2}\right]+\left[\frac{x+1}{2}\right] = \ldots$$

where ([]) is the greatest integer function.
(a) 0 (b) 1
(c) $[x + 1]$ (d) $[x]$
[Based on MAT 2002]

18. How many five-digit multiples of 11 are there, if the five digits are 3, 4, 5, 6 and 7 in the same order?
(a) 12 (b) 13
(c) 10 (d) None of these
[Based on MAT 2002]

19. The smallest number by which 3600 can be divided to make it a perfect cube is
(a) 9 (b) 50
(c) 300 (d) 450
[Based on MAT 2002]

20. The least number having four digits which is a perfect square is
(a) 1004 (b) 1016
(c) 1036 (d) None of these
[Based on MAT 2002]

21. The remainder when 7^{84} is divided by 342 is
(a) 0 (b) 1
(c) 49 (d) 341
[Based on MAT 2001]

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PA20

21. Given $n = 1 + x$ and x is the product of four consecutive integers. Then which of the following is true?
A. n is an odd integer.
B. n is prime.
C. n is a perfect square.
(a) Both *A* and *C* are correct
(b) Both *A* and *B* are correct
(c) Only *A* is correct
(d) Only *C* is correct. **[Based on MAT 2001]**

23. A two-digit number is such that the product of the digits is 14. When 45 is added to the number, then the digits interchange their places. Find the number.
(a) 72 (b) 27
(c) 37 (d) 14
[Based on MAT 2001]

24. A player holds 13 cards of four suits of which seven are black and six are red. There are twice as many diamonds as spades and twice as many hearts as diamonds. How many clubs does he hold ?
(a) 4 (b) 5
(c) 6 (d) 7
[Based on FMS (Delhi) 2004]

25. In three coloured boxes-Red, Green and Blue, 108 balls are placed. There are twice as many in the Green and Red boxes combined as they are in the Blue box and twice as many in the Blue box as they are in the Red box. How many balls are there in the Green box?
(a) 18 (b) 36
(c) 45 (d) None of these
[Based on FMS (Delhi) 2004]

26. If $a=1^2, b=2^3, c=3^4 \ldots z=26^{27}$. In the product of all the alphabets, how many zeroes exist in the end?
(a) 100 (b) 104
(c) 80 (d) 106
[Based on FMS (Delhi) 2004]

27. The unit's digit of a two-digit number is one more than the digit at ten's place. If the number is more than five times of the sum of the digits of the number, then find the sum of all such possible numbers.
(a) 246 (b) 275
(c) 290 (d) 301
[Based on FMS (Delhi) 2004]

28. Let $20 \times 21 \times 22 \times \ldots \times 30 = A$. If A is divisible by 10^x, then find the maximum value of x.
(a) 3 (b) 4
(c) 5 (d) 6
[Based on FMS (Delhi) 2004]

29. A student was asked to find the sum of all the prime numbers between 10 to 40. He found the sum as 180. Which of the following statements is true?
(a) He missed one prime number between 10 and 20
(b) He missed one prime number between 20 and 30
(c) He added one extra prime number between 10 and 20
(d) None of these **[Based on FMS (Delhi) 2004]**

30. $\sqrt{-1}$ is not defined but it is denoted by i. Clearly, i is not a real number, so it is called imaginary number. Now find $\sum_{n=1}^{100}(i)^n$
(a) i (b) 1
(c) -1 (d) 0
[Based on FMS (Delhi) 2004]

31. $(a+b+c+d+e)/(v+w+x+y+z)=N$, where a, b, c, d, e are five consecutive even integers and v, w, x, y, z are five consecutive odd integers. If $v = a + 1$ and n represents natural numbers, then which of the following is the most suitable value of N?
(a) $(n + 4)/(n + 5)$ (b) $(n + 3)/(n + 4)$
(c) $(n + 2)/(n + 3)$ (d) $(n + 2)/(n + 2.5)$
[Based on FMS (Delhi) 2004]

32. a, b, c are even numbers and x, y, z are odd numbers. Which of the following relationships can't be justified at any cost?
(A) $\frac{a \times b}{c} = x \times y$ (B) $\frac{a \times c}{x} = yz$
(C) $\frac{xy}{z} = ab$
(a) Only B (b) Only C
(c) All the three (d) Only B and C
[Based on FMS (Delhi) 2004]

33. The number of positive integers less than or equal to 100, which are not divisible by 2, 3 or 5, is
(a) 24 (b) 26
(c) 29 (d) 32
[Based on FMS (Delhi) 2004]

34. $N + a^2 + b^2$ is a three-digit number which is divisible by 5. $a = 10x + y$ and $b = 10x + z$, where z is a prime number, and x and y are natural numbers. If $a + b = 31$, find the value of N.
(a) 565 (b) 485
(c) 505 (d) 485 or 505
[Based on FMS (Delhi) 2004]

35. Which of the following is the highest value?
(a) 12^9 (b) 10^{11}
(c) 11^{10} (4) All are same
[Based on FMS (Delhi) 2004]

36. Which of the following is a prime number?
(a) 889 (b) 997
(c) 899 (d) 1147
[Based on FMS (Delhi) 2004]

37. A cube is cut into n identical pieces. If it can be done so in only one way, then which of the following could be the value of n?

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RA1-PA18

(a) 179 (b) 203
(c) 143 (d) 267

[Based on IIT Joint Man. Ent. Test 2004]

38. A gardener has to plant trees in rows containing equal number of trees. If he plants in rows of 6, 8, 10 or 12, then five trees are left unplanted. But if he plants in rows of 13 trees each, then no tree is left. What is the number of trees that the gardener plants?
(a) 485 (b) 725
(c) 845 (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

39. Which of the following is a product of 17 and an integer?
(a) 171 (b) 1712
(c) 1724 (d) 1734

[Based on REC Tiruchirapalli 2003]

40. If a and b are two integers and $b > 0$, then there exist two integers q and r such that
(a) $b = aq + r$, where $0 < r < b$
(b) $b = rq + a$, where $0 < r < b$
(c) $a = bq + r$, where $0 < r < b$
(d) None of these **[Based on FMS (Delhi) 2003]**

41. The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 7 and the denominator is decreased by 2, we obtain 2. The rational number is
(a) $\frac{1}{4}$ (b) $\frac{5}{8}$
(c) $\frac{7}{10}$ (d) $\frac{8}{11}$

[Based on FMS (Delhi) 2003]

42. The number of natural numbers divisible by 5 between 1 and 1000 is
(a) 197 (b) 199
(c) 198 (d) 200

[Based on FMS (Delhi) 2003]

43. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is
(a) 18 (b) 24
(c) 81 (d) 42

[Based on FMS (Delhi) 2003]

44. In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 48, then what is the dividend?
(a) 240 (b) 576
(c) 4800 (d) 4848
(e) None of the above **[Based on IIFT 2003]**

45. Which of the following integers has the most divisors?
(a) 88 (b) 91
(c) 99 (d) 101

[Based on SCMHRD Ent. Exam. 2003]

46. Which of the following integers has most number of divisors?
(a) 176 (b) 182
(c) 99 (d) 101

[Based on SCMHRD Ent. Exam. 2003]

47. There is a remainder of 3 when a number is divided by 6. What will be the remainder if the square of the same number is divided by 6?
(a) 1 (b) 0
(c) 3 (d) 2

[Based on I.P. Univ. 2002]

48. In which of the following pairs of numbers it is true that their sum is 11 times their product?
(a) 1, 1/11 (b) 1, 1/10
(c) 1, 1/12 (d) 1, 10
(e) None of these **[Based on SCMHRD 2002]**

49. If m, n, o, p and q are integers, then $m(n + o)(p - q)$ must be even when which of the following is even?
(a) $m + n$ (b) $n + p$
(c) m (d) p

[Based on REC Tiruchirapalli, 2002]

50. If there are 10 positive real numbers $n_1 < n_2 < n_3 \ldots < n_{10} \ldots$. How many triplets of these numbers (n_1, n_2, n_3), (n_2, n_3, n_4)... can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number:
(a) 45 (b) 90
(c) 120 (d) 180

[Based on CAT 2002]

51. Number S is obtained by squaring the sum of digits of a two digit number D. If difference between S and D is 27, then the two digit number D is:
(a) 24 (b) 54
(c) 34 (d) 45

[Based on CAT 2002]

52. The owner of a local jewellery store hired 3 watchmen to guard his diamonds, but a thief still got in and stole some diamonds. On the way out, the thief met each watchman, one at a time. To each he gave $\frac{1}{2}$ of the diamonds he had then, and 2 more besides. He escaped with one diamond. How many did he steal originally?
(a) 40 (b) 36
(c) 25 (d) None of these

[Based on CAT 2002]

53. A rich merchant had collected many gold coins. He did not want anybody to know about him. One day, his wife asked, "How many gold coins do we have?" After pausing a moment, he replied, "Well! if I divide the coins into two unequal numbers, then 48 times the difference between the two numbers equals the difference between the squares of the two numbers. "The wife looked puzzled. Can you help the merchant's wife by finding out how many gold coins the merchant has?

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RA1-PA19

(a) 96 (b) 53
(c) 43 (d) None of these
[Based on CAT 2002]

54. A child was asked to add first few natural numbers (that is $1 + 2 + 3 + ...$) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered he had missed one number in the sequence during addition. The number he missed was:
(a) less than 10 (b) 10
(c) 15 (d) more than 15
[Based on CAT 2002]

55. When 2^{256} is divided by 17, the remainder would be:
(a) 1 (b) 16
(c) 14 (d) None of these
[Based on CAT 2002]

56. After the division of a number successively by 3,4 and 7, the remainders obtained are 2, 1 and 4 respectively. What will be the remainder if 84 divides the same number?
(a) 80 (b) 75
(c) 41 (d) 53
[Based on CAT 2002]

57. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by:
(a) 13 (b) 127
(c) 559 (d) None of these.
[Based on CAT 2002]

58. Three pieces of cakes of weights $4\frac{1}{2}$ lbs, $6\frac{3}{4}$ lbs and $7\frac{1}{5}$ lbs respectively are to be divided into parts of equal weights. Further, each part must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests that could be entertained?
(a) 54 (b) 72
(c) 20 (d) None of these
[Based on CAT 2001]

59. A red light flashes 3 times per minute and a green light flashes 5 times in two minutes at regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour?
(a) 30 (b) 24
(c) 20 (d) 60
[Based on CAT 2001]

60. Of 128 boxes of oranges, each box contains at least 120 and at most 144 oranges. The number of boxes containing the same number of oranges is at least:
(a) 5 (b) 103
(c) 6 (d) None of these
[Based on CAT 2001]

61. In a 4-digit number, the sum of the first two digits is equal to that of the last two digits. The sum of the first and last digits is equal to the third digit. Finally, the sum of the second and fourth digits is twice the sum of the other two digits. What is the third digit of the number?
(a) 5 (b) 8
(c) 1 (d) 4
[Based on CAT 2001]

62. Anita had to do a multiplication. Instead of taking 35 as one of the multipliers, she took 53. As a result, the product went up by 540. What is the new product?
(a) 1050 (b) 540
(c) 1440 (d) 1590
[Based on CAT 2001]

63. m is the smallest positive integer such that for any integer $n \leq m$, the quantity $n^3 - 7n^2 + 11n - 5$ is positive. What is the value of m?
(a) 4 (b) 5
(c) 8 (d) None of these
[Based on CAT 2001]

64. Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid their bill and noticed a bowl of mints at the front counter. Sita took 1/3 of the mints, but returned four because she had a monetary pang of guilt. Fatima then took 1/4 of what was left but returned three for similar reasons. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl?
(a) 38 (b) 31
(c) 41 (d) None of these
[Based on CAT 2001]

65. In a number system, the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system, becomes:
(a) 406 (b) 1086
(c) 213 (d) 691
[Based on CAT 2001]

66. A set of consecutive positive integers beginning with 1 is written on the blackboard. A student came along and erased one number. The average of the remaining numbers is $35\frac{7}{77}$. What was the number erased?
(a) 7 (b) 8
(3) 9 (d) None of these
[Based on CAT 2001]

67. Let D be a recurring decimal of the form $D = 0.a_1a_2a_1a_2a_1a_2 ...$, where digits a_1 and a_2 lie between 0 and 9. Further, at most one of them is zero. Which of the folloving numbers necessarily produces an integer, when multiplied by D?
(a) 18 (b) 108
(c) 198 (d) 288
[Based on CAT 2000]

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68. What is the value of the following expression?

$$\left(\frac{1}{(2^2-1)}\right)+\left(\frac{1}{(4^2-1)}\right)+\left(\frac{1}{(6^2-1)}\right)+\left(\frac{1}{(20^2-1)}\right)$$

(a) $\frac{9}{19}$ (b) $\frac{10}{19}$

(c) $\frac{10}{21}$ (d) $\frac{11}{21}$

[Based on CAT 2000]

69. Consider a sequence of seven consecutive integers. The average of the first five integers is n. The average of all the seven integers is:

(a) n (b) $n + 1$

(c) $k \times n$, where k is a function of n

(d) $n+\left(\frac{2}{7}\right)$ **[Based on CAT 2000]**

70. The sum of the place values of 3 in the number 503535 is

(a) 33.0 (b) 0.6

(c) 60 (d) 3030

[Based on MAT 2008]

71. $(999^2 - 998^2)$ is equal to

(a) 1 (b) 999

(c) 1997 (d) 998

[Based on MAT 2008]

72. The number 311311311311311311311 is

(a) divisible by 3 but not by 11

(b) divisible by 11 but not by 3

(c) divisible by both 3 and 11

(d) neither divisible by 3 nor by 11

[Based on SNAP 2007]

73. How many pairs of positive integer m, n satisfy $\frac{1}{m}+\frac{4}{n}=\frac{1}{12}$ where n is an odd integer less than 60?

(a) 7 (b) 5

(c) 3 (d) 6

(e) 4

[Based on CAT 2007]

74. Which of the following numbers is exactly divisible by 99?

(a) 114345 (b) 135792

(c) 3572404 (d) 913464

[Based on MAT 2005]

75. Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55 and the sum of the third and thrice the first is 90. The third number is

(a) 20 (b) 25

(c) 30 (d) 35

[Based on MAT 2005]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution a, b, c and d are four integers such that $a + b + c + d = 4m + 1$.

Minimum possible value of $a^2 + b^2 + c^2 + d^2$ is when a, b, c and d are as close to each other as possible. Since RHS is not the mutliple of 4 a, b, c and d can't be equal to m.

Hence the numbers may be of the form, m, m, m and $m + 1$.

$\therefore$ $a^2 + b^2 + c^2 + d^2 = \mathbf{4m^2 + 2m + 1}$.

2. Ans. (c)

Solution We have to find the number of three-digit numbers in which the digit at ten's place is greater than the digit at unit's and hundred's places. That is,

Hundred	Ten	Unit
x	y	z

$x < y > z$

The following chart shows the number of ways in which it can be formed.

Number of ways in which unit's place, i.e. x can be filled	Digit at ten's place, i.e. y	Number of ways in which unit's place i.e. y can be filled
1 (i.e.1)	2	2 (i.e. 0, 1)
2 (i.e. 1, 2)	3	3 (i.e. 0, 1, 2)
...	...	...
8 (i.e. 1, 2, 3, ..., 8)	9	9 (i.e. 0, 1, 2, ... 9)

$\therefore$ Total no. of possible 3-digit numbers

$= (1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5)$

$+ \ldots + (7 \times 8) + (8 \times 9) = \mathbf{240}$.

3. Ans. (b)

Solution Product $(n - 1)(n - 2) \ldots 3 \times 2 \times 1$ is not divisible by n if n is 4 or a prime number.

We have to find the number of primes in

$12 \le n \le 40$.

i.e. {13, 17, 19, 23, 29, 31, 37}

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$\therefore$ No. of positive integers in the range $12 \le n \le 40$ is **7.**

4. Ans. (d)

Solution Take any arbitrary value of x and y.

Let $x = 2$ (prime number)

$y = 50$ (composite number)

Going through the options,

(a), (b) and (c) are wrong because $y - x$, xy and $\frac{x+y}{x}$ are even integers for $x = 2$ and $y = 50$.

$\therefore$ None of the statements are true.

5. Ans. (a)

Solution The set of prime numbers 3, 5, 7 is the only set which satisfies the given condition.

6. Ans. (d)

Solution If 4^2 is divided by 6, remainder is 4.

If 4^3 is divided by 6, remainder is 4.

If 4^4 is divided by 6, remainder is 4.

... ...

If 4^{96} is divided by 6, remainder is **4.**

Alternative Method:

$4^6 \equiv 4 \pmod 6 \Rightarrow (4^6)^{16} = 4^{16} \pmod 6$

Also, $4^4 \equiv 4 \pmod 6 \Rightarrow (4^4)^4 \equiv 4^4 \pmod 6$

Also, $4^4 \equiv 4 \pmod 6$

$\therefore$ remainder = **4.**

7. Ans. (a)

Solution Let x be the number,

$$\therefore \quad (x + 12) \div 6 = 112 \Rightarrow \frac{x+12}{6} = 112$$

$$\Rightarrow x = 112 \times 6 - 12$$

$$\Rightarrow x = 672 - 12 = 660$$

$$\therefore \text{ Correct answer } = \frac{x}{6} + 12$$

$$= \frac{660}{6} + 12 = 110 + 12 = \mathbf{122.}$$

8. Ans. (d)

Solution $(n + 1)^2 = \boldsymbol{n^2 + 2n + 1.}$

9. Ans. (b)

10. Ans. (c)

Solution Let the three-digit number be $100x + 10y + z$.

$$\therefore \quad (100x + 10y + z) - (x + y + z)$$

$$= 99x + 9y = 9(11x + y)$$

which is always divisible by **9.**

11. Ans. (c)

Solution Let x must be subtracted from 14, 17, 24 and 42 such that

$$(14 - x)(42 - x) = (17 - x)(34 - x) \Rightarrow \boldsymbol{x = 2.}$$

12. Ans. (a)

13. Ans. (a)

14. Ans. (a)

Solution No. of women in the room $= \frac{3}{5} \times 120 = 72$

No. of married people $= \frac{2}{3} \times 120 = 80$

No. of unmarried people = 40

No. of men in the room = 48

If all the men are supposed to be married, then number of married women could be $80 - 48 = 32$.

$\therefore$ Maximum number of unmarried women could be $72 - 32 = \mathbf{40.}$

15. Ans. (d)

Solution $x = 2 + 2^{2/3} + 2^{1/3} \Rightarrow x > 2$

For $x = 2$, $x^3 - 6x^2 + 6x = -4$

For $x = 3$, $x^3 - 6x^2 + 6x = -9$

$\therefore \boldsymbol{x^3 - 6x^2 + 6x < 0.}$

16. Ans. (a)

Solution 248 in the scale of 7 is written as 503. In scale 9, it is written as 305.

17. Ans. (a)

Solution Given expression $= \frac{x}{2} + \frac{x+1}{2}$

$$= \frac{2x+1}{2} = x + \frac{1}{2} = \boldsymbol{[x].}$$

18. Ans. (a)

Solution 5 3 6 4 7 is a multiple of 11, because the difference of the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11, because

$$(5 + 6 + 7) - (3 + 4) = 11$$

$\therefore$ Total number of five-digit multiples of 11

= 3! (Permutation of 5, 6 and 7 in the odd places)

$\times$ 2! (Permutation of 3 and 4 in the even places)

$= 6 \times 2 = \mathbf{12.}$

19. Ans. (d)

Solution $\frac{3600}{450} = 8 = \boldsymbol{2^3.}$

20. Ans. (d)

Solution 1024.

21. Ans. (b)

Solution $7^3 = 343$, when divided by 342, leaves a remainder of 1

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7^4 = 2401, when divided by 342, leaves a remainder of 7.

7^5 = 16807, when divided by 342, leaves a remainder of 49.

7^6 = 117649, when divided by 342, leaves a remainder of 1.

And so on.

$\therefore$ 7^{84}, when divided by 342, will leave a remainder of **1.**

22. Ans. (a)

23. Ans. (b)

Solution Let the digits be a and b such that the number is $10a + b$.

$\therefore$ $ab = 14$ and $10a + b + 45 = 10b + a$

i.e. $9a - 9b = -45$

i.e. $a - b = -5$

$\therefore$ $(a + b)^2 = (a - b)^2 + 4ab = 81$

$\Rightarrow$ $a + b = 9$ $\Rightarrow$ $a = 2, b = 7$

$\therefore$ The number is **27.**

24. Ans. (c)

Solution No. of Spades = 1

No. of Diamonds = 2

No. of Hearts = 4,

No. of Clubs = **6.**

25. Ans. (d)

Solution No. of balls in Red Box = 18

No. of balls in Blue Box = 36

No. of balls in Green Box = **54.**

26. Ans. (d)

Solution The given product contains 5^{106} and $2x$, where $x > 106$.

$\therefore$ There will be 106 zeroes in the product, because zero will come only by multiplying **2** and **5.**

27. Ans. (c)

Solution Such numbers are **56, 67, 78** and **89.**

Sum of these numbers = 290.

28. Ans. (b)

Solution
$$20 \times 21 \times 22 \times 23 \times 24 = 5100480$$
$$25 \times 26 \times 27 \times 28 = 491400$$
$$29 \times 30 = \mathbf{870.}$$

29. Ans. (d)

Solution Sum of the prime numbers between 10 and 40 = 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 = **180.**

30. Ans. (d)

Solution $\sum_{n=1}^{100} i^n$

$= (i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + \ldots + (i^{97} + i^{98} + i^{99} + i^{100})$

$= (i - 1 - i + 1) + (i - 1 - i + 1) + \ldots$

$= 0 + 0 + \ldots + 0 = \mathbf{0.}$

31. Ans. (d)

Solution Let the five consecutive even numbers $2n$, $2n + 2$, $2n + 4$, $2n + 6$, $2n + 8$ be respectively equal to a, b, c, d and e, where n is a natural number.

Then v, w, x, y and z are equal to $2n + 1$, $2n + 3$, $2n + 5$, $2n + 7$, $2n + 9$.

$$\Rightarrow N = \frac{2n + 2n + 2 + 2n + 4 + 2n + 6 + 2n + 8}{2n + 1 + 2n + 3 + 2n + 5 + 2n + 7 + 2n + 9}$$

$$= \frac{10n + 20}{10n + 25} = \frac{n + 2}{n + 2.5}.$$

32. Ans. (d)

33. Ans. (b)

34. Ans. (b)

Solution $a = 14$, $b = 17$, $N = \mathbf{485.}$

35. Ans. (b)

36. Ans. (b)

Solution $889 = 7 \times 127$

$899 = 29 \times 31$

$1147 = \mathbf{31 \times 437.}$

37. Ans. (a)

Solution 179 is a prime number.

$203 = 7 \times 29$

$143 = 11 \times 13$

$267 = \mathbf{3 \times 89.}$

38. Ans. (c)

Solution (Multiple of 120 + 5), which is divisible by 13.

39. Ans. (d)

40. Ans. (c)

Solution Euclidean Algorithm.

41. Ans. (b)

Solution Let the rational number be $\frac{p}{q}$

$\therefore$ $q = p + 3$

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$$\therefore \quad \frac{p+7}{p+3-2} = 2 \Rightarrow p + 7 = 2p + 2$$

$$\Rightarrow p = 5$$

$$\Rightarrow \text{Given rational number} = \frac{5}{8}.$$

42. Ans. (d)

43. Ans. (b)

Solution Let the number be $10x + y$.

$$\therefore \quad xy = 8 \quad \text{...(i)}$$

$$10x + y + 18 = 10y + x$$

i.e. $$9x - 9y = -18$$

i.e. $$x - y = -2 \quad \text{...(ii)}$$

$$\therefore \quad (x+y)^2 = 4xy + (x-y)^2$$

$$= 32 + 4 = 36$$

$$\therefore \quad x + y = 6 \quad \text{...(iii)}$$

(ii) and (iii) give $x = 2, y = 4$.

Hence the number is **24.**

44. Ans. (d)

Solution Divisor = 12 × Quotient

Divisor = 5 × Remainder

Remainder = 48

$\Rightarrow$ Divisor = 240,

$\therefore$ Quotient = 20

Hence, Dividend = 240 × 20 + 48 = **4848.**

45. Ans. (a)

Solution Divisors of 88 are 2, 4, 8, 11, 22, 44.

Divisors of 91 are 7 and 13.

Divisors of 99 are **3, 9, 11, 33.**

46. Ans. (a)

Solution

176 = 2, 4, 8, 16, 11, 22, 44, 88.

182 = 2, 7, 13, 14, 26, 91

99 = 3, 9, 11, 33

101 = **101.**

47. Ans. (c)

48. Ans. (b)

Solution $1 + \frac{1}{10} = \frac{11}{10}$

$$1 \times \frac{1}{10} = \frac{1}{10}$$

$$\therefore \quad \text{Sum} = \frac{11}{10} = 11 \times \frac{1}{10} = \mathbf{11 \times Product.}$$

49. Ans. (b)

Solution $m(n + 0)(p - q)$ is even $\Rightarrow$ m must be even.

50. Ans. (c)

Solution Total possible arrangements = 10 × 9 × 8

Now three numbers can be arranged among themselves in = 3! ways = 6 ways

Given condition is satisified by only 1 out of 6 ways.

Hence, required number of arrangements

$$= \frac{10 \times 9 \times 8}{6} = \mathbf{120.}$$

51. Ans. (b)

Solution Check choices

$$54 \Rightarrow S = (5+4)^2 = 81$$

$$\Rightarrow \boldsymbol{S - D = 81 - 54 = 27.}$$

52. Ans. (b)

Solution Escaped with 1

Before 3rd watchman, he had $(1 + 2) \times 2 = 6$

Before 2nd watchman, he had $(6 + 2) \times 2 = 16$

Before 1st watchman, he had $\mathbf{(16 + 2) \times 2 = 36.}$

53. Ans. (d)

Solution Let the no. of gold coins = $x + y$

$$48(x - y) = x^2 - y^2$$

$$\Rightarrow 48(x-y) = (x-y)(x+y) \Rightarrow \boldsymbol{x + y = 48.}$$

54. Ans. (d)

Solution

$$575 = \frac{n^2 + n}{2} - x$$

$$1150 = n^2 + n - 2x$$

For $$n = 34,$$

$$40 = 2x$$

$$\boldsymbol{x = 20.}$$

55. Ans. (a)

Solution $(2^4)^{64} = (17 - 1)^{64} = 17n + (-1)^{64}$

$$= 17n + 1$$

Hence, remainder = **1.**

56. Ans. (d)

Solution $3\{4(7x + 4) + 1\} + 2 = 84x + 53$

Therefore, remainder is **53.**

57. Ans. (b)

Solution $7^{6n} - 6^{6n}$

Put $n = 1$

$$7^6 - 6^6 = (7^3 - 6^3)(7^3 + 6^3)$$

This is a multiple of $7^3 - 6^3 = \mathbf{127.}$

58. Ans. (d)

Solution $\text{HCF}\left(\frac{9}{2}, \frac{27}{4}, \frac{36}{5}\right) = \frac{\text{HCF}(9, 27, 36)}{\text{LCM}(2, 4, 5)}$

$$= \frac{9}{20} \text{lbs}$$

= weight of each piece.

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Total weight = 18.45 lbs

Maximum number of guests = $\frac{18.45 \times 20}{9}$ = **41.**

59. Ans. (a)

Solution First light blinks after 20s.

Second light blinks after 24s

They blink together after LCM of 20s and 24s = 120s = 2 min.

Hence, the number of times they blink together in an hour = **30.**

60. Ans. (a)

Solution Since he has to put minimum 120 oranges and maximum 144 oranges, i.e. 25 oranges need to be filled in 128 boxes with same number of oranges in the boxes.

Therefore, total 125 = 25 × 5 oranges could be filled in the boxes, i.e. 25 in each of the 5 boxes which would be the minimum and have the same number of oranges.

Hence, the answer is 5.

61. Ans. (a)

Solution Let the 4-digit number be *abcd*.

$a + b = c + d$ ...(i)

$b + d = 2(a + c)$ ...(ii)

$a + d = c$ ...(iii)

From (i) and (iii), $b = 2d$.

From (i) and (ii), $3b = 4c + d$

$\Rightarrow \quad 3(2d) = 4c + d \Rightarrow 5d = 4c \Rightarrow \quad c = \frac{5}{4}d$

Now d can be 4 or 8. But if $d = 8$, then $c = 10$ is not possible. So $d = 4$ which gives $\boldsymbol{c = 5}$.

62. Ans. (d)

Solution Let the number be x

Increase in product = $53x - 35x = 18x$

$\Rightarrow \quad 18x = 540 \Rightarrow x = 30$

Raised product = 53 × 30 = **1590.**

63. Ans. (d)

Solution Let $y = n^3 - 7n^2 + 11n - 5$

At $n = 1$, $y = 0$

$\therefore (n - 1)(n^2 - 6n + 5) = (n - 1)^2 (n - 5)$

Now, $(n - 1)^2$ is always positive.

Also, for $n < 5$, the expression gives a negative quantity. Therefore, the least value of n will be 6. Hence, $\boldsymbol{m = 6}$.

64. Ans. (d)

Solution Let there be x mints originally in the bowl.

Sita took $\frac{1}{3}$, but returned 4.

So now the bowl has $\frac{2}{3}x + 4$ mints

Fatima took $\frac{1}{4}$ of remainder, but returned 3.

So, the bowl has $\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3$ mints.

Eswari took half of remainder that is $\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3\right]$. She returns 2, so the bowl now has $\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3\right] + 2 = 17 \Rightarrow \boldsymbol{x = 48}$.

65. Ans. (a)

Solution The product of 44 and 11 is 484

Here, $3x^3 + 4x^2 + 1x^1 + 4 \times x^0 = 484$

$\Rightarrow \quad 3x^3 + 4x^2 + x = 480$

This equation is satisfied only when $x = 5$.

In decimal system, the number 3111 can be written as **406.**

66. Ans. (a)

Solution Let the highest number be n.

Then, $\frac{\frac{n(n+1)}{2} - x}{(n-1)} = 35\frac{7}{77} = \frac{602}{17}$,

where x is the number erased.

Hence, $n = 69$ and $x = 7$ satisfy the above conditions.

67. Ans. (c)

Solution $99 \times D = a_1 a_2$. Hence $D = \frac{a_1 a_2}{99}$. So D must be multiplied by 198 as 198 is a multiple of **99.**

68. Ans. (c)

Solution $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{19.21}$

$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots \frac{1}{2}\left(\frac{1}{19} - \frac{1}{21}\right)$

$= \frac{1}{2} - \frac{1}{42} = \frac{(21-1)}{42} = \frac{20}{42} = \boldsymbol{\frac{10}{21}}$.

69. Ans. (b)

Solution Use any 7 consecutive numbers to check the answers.

$n = \frac{(1+2+3+4+5)}{5} = 3$, average of 7 integers is

$k = \frac{(1+2+3+4+5)}{7} = 4$. So $k = n + 1$.

Alternatively, the average of the first 5 terms is the middle term which is third term, and the average of the first 7 terms is the middle term which is the fourth term. Hence, it is one more than the previous average.

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70. Ans. (d)

71. Ans. (c)

72. Ans. (d)

73. Ans. (c)

Solution $\frac{1}{m}+\frac{4}{n}=\frac{1}{12} \Rightarrow m=\frac{12n}{n-48}$

m must be greater than 48. Values of n would be $48 < n < 60$ are 49, 51, 52, 55, 57 and 59. 49, 51 and 57 give integral values of m.

74. Ans. (a)

Solution A number divisible by 99 must be divisible by 9 as well as 11.

$\therefore$ 114345 is divisible by both.

75. Ans. (c)

Solution Let the numbers be x, y and z.

$x + y = 45$, $y + z = 55$ and $3x + z = 90$

$y = 45 - x$, $z = 55 - y = 55 - (45 - x) = 10 + x$

$\therefore$ $3x + 10 + x = 90$

or, $x = 20$

$y = 45 - 20 = 25$

$z = 10 + 20 = 30$

$\therefore$ Third number is **30.**

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Out of three numbers, the first is twice the second and is half of the third. If the average of three numbers is 56, the smallest number is
(a) 24 (b) 36
(c) 40 (d) 48

2. How many numbers up to 450 are divisible by 4, 6 and 8 together?
(a) 19 (b) 18
(c) 17 (d) 16

3. The ratio of the sum and the difference of two numbers is 7 : 1. Find the ratio of those two numbers.
(a) 4 : 3 (b) 3 : 4
(c) 4 : 5 (d) 5 : 3

4. Find the greatest number that will divide 55, 127 and 175 so as to leave the same remainder in each case.
(a) 24 (b) 25
(c) 26 (d) 27

5. If the sum of a number and its square is 182, what is the number?
(a) 15 (b) 13
(c) 26 (d) 19

6. The sum of two numbers is 75 and their difference is 20. Find the difference of their squares.
(a) 1500 (b) 1300
(c) 1100 (d) 1600

7. The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 7 and the denominator is decreased by 2, we obtain 2. The rational number is
(a) $\frac{6}{5}$ (b) $\frac{8}{5}$
(c) $\frac{5}{8}$ (d) $\frac{7}{5}$

8. How many numbers between 100 and 300 are divisible by 9?
(a) 11 (b) 13
(c) 19 (d) 22

9. If one-fifth of one-third of one-half of a number is 15, find the number.
(a) 550 (b) 350
(c) 650 (d) 450

10. A number on being divided by 8 and 9 successively leaves the remainder 5 and 7, respectively. Find the remainder when the same number is divided by 72.
(a) 61 (b) 8
(c) 71 (d) 9

11. The difference between the squares of two consecutive numbers is 37. Find the numbers
(a) 17, 14 (b) 15, 16
(c) 18, 19 (d) 14, 15

12. The sum of three consecutive even numbers is 15 less than three-fourths of 60. What is the middle number?
(a) 5 (b) 10
(c) 20 (d) 15

13. The difference between a two digit number and the number obtained by interchanging the digits is 27. What are the sum and the difference of the two digits of the number?
(a) 3, 4
(b) Cannot be determined; 3
(c) 3, 6
(d) 3, 5

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Test II **[Time: 10 min]**

14. Find the number which when multiplied by 16 is increased by 225.
(a) 15 (b) 20
(c) 25 (d) 30

15. If the places of last two digits of a three digit number are interchanged, a new number greater than the original number by 54 is obtained. What is the difference between the last two digits of that number?
(a) 9 (b) 6
(c) 12 (c) 3

16. A number when divided by 899 gives a remainder 63. What remainder will be obtained by dividing the same number by 29?
(a) 5 (b) 6
(c) 7 (d) 8

17. The product of two numbers is 192 and the sum of these two numbers is 28. What is the smaller of these two numbers?
(a) 5 (b) 12
(c) 9 (d) 3

18. The difference between squares of two numbers is 256000 and sum of the numbers is 1000. The numbers are
(a) 628, 372 (b) 600, 400
(c) 640, 630 (b) None of these

19. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit at the units place is 3 more than the digit at the tens place, what is the number?
(a) 24 (b) 52
(c) 48 (d) 36

20. The largest natural number by which the product of three consecutive even natural numbers is always divisible is
(a) 48 (b) 16
(c) 96 (d) 24

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution Let the second number be x.
Then, first number = $2x$ and third number = $4x$

$\therefore \quad \frac{2x+x+4x}{3} = 56 \Rightarrow 7x = 3 \times 56$

or, $\quad x = \frac{3\times 56}{7}$

$x = 24$

So, the smallest number = **24.**

3. Ans (a)

Solution Let the two numbers be x and y. Then, we have

$\frac{x+y}{x-y} = \frac{7}{1} \Rightarrow x + y = 7x - 7y$ or, $6x = 8y$

$\therefore \quad \frac{x}{y} = \frac{8}{6} = \frac{4}{3}$

$\therefore$ **$x : y = 4 : 3$.**

5. Ans. (b)

Solution Let the number = x
Then, $x^2 + x = 182$

or, $\quad x^2 + x - 182 = 0$

or, $\quad x^2 + 14x - 13x - 182 = 0$

or, $\quad x(x + 14) - 13(x + 14)$

or, $\quad (x - 13)(x + 14) = 0$ or, **$x = 13$.**

7. Ans. (c)

Solution Let the numerator be x and the denominator $= x + 3$

According to the question,

$\frac{x+7}{x+3-2} = 2$ or, $x + 7 = 2x + 2$

$\therefore \quad x = 5$

$\therefore$ Numerator = 5 and the denominator = 5 + 3 = 8

$\therefore$ Rational number = $\frac{5}{8}$.

9. Ans (d)

Solution Let the number be x. Then, we have

$$x\left(\frac{1}{5}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = 15$$

$\therefore \quad x = 15 \times 5 \times 3 \times 2 =$ **450.**

11. Ans. (c)

Solution Let the numbers be x and $(x + 1)$.
According to the question,

$(x + 1)^2 - x^2 = 37$

or, $\quad x^2 + 1 + 2x - x^2 = 37$

or, $\quad 2x = 37 - 1 = 36$

$\therefore \quad x = 18$ and $x + 1 = 99$

$\therefore$ Numbers are **18** and **19.**

13. Ans. (b)

Solution Let the number be $10x + y$. Then, we have

$(10x + y) - (10y + x) = 27$

or, $\quad 9(x - y) = 27$

$\therefore \quad x - y = 3$

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Thus, the difference is 3, but we cannot get the sum of the two digits.

15. Ans. (b)

Solution Let the three digit number be $100x + 10y + z$

According to the question,

$(100x + 10z + y) - (100x + 10y + z) = 54$

or, $9z - 9y = 54$ **or,** $z - y = 6$.

17. Ans. (b)

Solution Let the numbers be x and y

$\therefore \quad xy = 192, x + y = 28$ ...(1)

$\therefore \quad (x - y)^2 = (x + y)^2 - 4xy$

$= 784 - 768 = 16$

$\therefore \quad x - y = 4$ ...(2)

Combining eq. (1) and (2)

$x = 16, y = 12$

$\therefore$ Smaller number = **12.**

19. Ans. (d)

Solution Let the two digit number be $10x + y$

Then, we have $\frac{10x + y}{x + y} = \frac{4}{1}$

or, $10x + y = 4x + 4y$, or, $6x = 3y$

or, $2x = y$

or, $x = y - x = 3$ (given) and $y = 6$

$\therefore$ The number is **36.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (a)	**6.** (a)	**8.** (d)	**10.** (a)	**12.** (b)	**14.** (a)	**16.** (a)	**18.** (a)	**20.** (a)

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H. C. F. and L. C. M. of Numbers

2

COMMON FACTOR

A *common factor* of two or more numbers is a number which divides each of them exactly.
For example, 4 is a common factor of 8 and 12.

HIGHEST COMMON FACTOR

Highest common factor of two or more numbers is the greatest number that divides each one of them exactly. For example, 6 is the highest common factor of 12, 18 and 24. Highest Common Factor is also called *Greatest Common Divisor* or *Greatest Common Measure*.

Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

METHODS OF FINDING H.C.F.

I. Method of Prime Factors

Step 1 Express each one of the given numbers as the product of prime factors.

[A number is said to be a *prime number* if it is exactly divisible by 1 and itself but not by any other number, e.g. 2, 3, 5, 7, etc. are prime numbers]

Step 2 Choose Common Factors.

Step 3 Find the product of these common factors. This is the required H.C.F. of given numbers.

Illustration 1 Find the H.C.F. of 70 and 90.

Solution $70 = \mathbf{2 \times 5} \times 7$

$90 = \mathbf{2 \times 5} \times 9$

Common factors are 2 and 5.

$\therefore$ H.C.F. $= 2 \times 5 = \mathbf{10}$.

Illustration 2 Find the H.C.F. of 3332, 3724 and 4508

Solution $3332 = \mathbf{2 \times 2 \times 7 \times 7} \times 17$

$3724 = \mathbf{2 \times 2 \times 7 \times 7} \times 19$

$4508 = \mathbf{2 \times 2 \times 7 \times 7} \times 23$

$\therefore$ H.C.F. $= 2 \times 2 \times 7 \times 7 = \mathbf{196}$.

Illustration 3 Find the H.C.F. of 360 and 132.

Solution $360 = 2^3 \times 3^2 \times 5$

$132 = 2^2 \times 3^1 \times 11$

$\therefore$ H.C.F. $= 2^2 \times 3^1 = \mathbf{12}$.

Illustration 4 If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y.

Solution The factors common to both x and y are 2^3, 3^5 and 5^9.

$\therefore$ H.C.F. $= \mathbf{2^3 \times 3^5 \times 5^9}$.

II. Method of Division

***A. For two numbers*:**

Step 1 Greater number is divided by the smaller one.

Step 2 Divisor of (1) is divided by its remainder.

Step 3 Divisor of (2) is divided by its remainder. This is continued until no remainder is left.

H.C.F. is the divisor of last step.

Illustration 5 Find the H.C.F. of 3556 and 3444.

$$
\begin{array}{r}
3444\overline{)\,3556\,(\,1} \\
3444 \\
\hline
112\overline{)\,3444\,(\,30} \\
3360 \\
\hline
84\overline{)\,112\,(\,1} \\
84 \\
\hline
28\overline{)\,84\,(\,3} \\
84 \\
\hline
\times
\end{array}
$$

$\therefore$ H.C.F. $= \mathbf{28}$.

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B. For more than two numbers:

Step 1 Any two numbers are chosen and their H.C.F. is obtained.

Step 2 H.C.F. of H.C.F. (of (1)) and any other number is obtained.

Step 3 H.C.F. of H.C.F. (of (2)) and any other number (not chosen earlier) is obtained.

This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.

Illustration 6 Find the H.C.F. of 13915, 9499 and 2553 by division method.

Solution

$$
\begin{array}{r|l}
9499 & 13915 \\
 & 9499 \\
\hline
4416 & 9499 \\
 & 8832 \\
\hline
667 & 4416 \\
 & 4002 \\
\hline
414 & 667 \\
 & 414 \\
\hline
253 & 414 \\
 & 253 \\
\hline
161 & 253 \\
 & 161 \\
\hline
92 & 161 \\
 & 92 \\
\hline
69 & 92 \\
 & 69 \\
\hline
23 & 69 \\
 & 69 \\
\hline
 & \times
\end{array}
\quad
\begin{array}{l}
1 \\
 \\
 \\
2 \\
 \\
 \\
6 \\
 \\
 \\
1 \\
 \\
 \\
1 \\
 \\
 \\
1 \\
 \\
 \\
1 \\
 \\
 \\
3 \\
 \\
 \\
 \\
\end{array}
$$

Now, in the next step, we will find the H.C.F. of 23 and 2553.

$$
\begin{array}{r|l}
23 & 2553 \\
 & 23 \\
\hline
25 & \\
 & 23 \\
\hline
2 & 23 \\
 & 23 \\
\hline
 & \times
\end{array}
\quad
\begin{array}{l}
111 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
\end{array}
$$

Thus, H.C.F. of 13915, 9499 and 2553 = **23.**

Illustration 7 Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm.

Solution Required length

= (H.C.F. of 700, 385, 1295) cm = **35 cm.**

COMMON MULTIPLE

A *common multiple* of two or more numbers is a number which is exactly divisible by each one of them.

For example, 32 is a common multiple of 8 and 16.

$8 \times 4 = 32$

$16 \times 2 = 32.$

LEAST COMMON MULTIPLE

The *least common multiple* of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 72, ...

Multiples of 18 are 18, 36, 54, 72, ...

Common multiples are 36, 72, ...

$\therefore$ Least common multiple, i.e. L.C.M. of 12 and 18 is 36.

METHODS OF FINDING L.C.M.

A. Method of Prime Factors

Step 1 Resolve each given number into prime factors.

Step 2 Take out all factors with highest powers that occur in given numbers.

Step 3 Find the product of these factors. This product will be the L.C.M.

Illustration 8 Find the L.C.M. of 32, 48, 60 and 320.

Solution $32 = 2^5 \times 1$

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

$320 = 2^6 \times 6$

$\therefore$ L.C.M. $= 2^6 \times 3 \times 5 =$ **960.**

B. Method of Division

Step 1 The given numbers are written in a line separated by common.

Step 2 Divide by any one of the prime numbers 2, 3, 5, 7, 11, ... which will divide at least any two of the given numbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3 Step 2 is repeated until a line of numbers (prime to each other) appears.

Step 4 Find the product of all divisors and numbers in the last line which is the required L.C.M.

PA30

Illustration 9 Find the L.C.M. of 12, 15, 20 and 54.

Solution

2	12,	15,	20,	54
2	6,	15,	10,	27
3	3,	15,	5,	27
5	1,	5,	5,	9
	1,	1,	1,	9

L.C.M. = 2 × 2 × 3 × 5 × 1 × 1 × 1 × 9 = 540.

Note: Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit.

SOME USEFUL SHORT-CUT METHODS

1. H.C.F. and L.C.M. of Decimals

Step 1 Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Step 2 Find the H.C.F./L.C.M. of these numbers without decimal.

Step 3 Put the decimal point (in the H.C.F./L.C.M. of step 2) leaving as many digits on its right as there are in each of the numbers.

Illustration 10 Find the L.C.M. of 1.2, 0.24 and 6.

Solution The given numbers can be written as 1.20, 0.24 and 6.00.

Now, ignoring the decimal we find the L.C.M. of 120, 24 and 600.

2	120,	24,	600
2	60,	12,	300
2	30,	6,	150
3	15,	3,	75
5	5,	1,	25
	1,	1,	5

∴ L.C.M. = 2 × 2 × 2 × 3 × 5 × 1 × 5 = 600

Thus, the required L.C.M. = **6.00, i.e. 6.**

Illustration 11 Find the H.C.F. of 6·16 and 13.

Solution The given numbers can be written as 6.16 and 13.00.

Now, ignoring the decimals we find the H.C.F. of 616 and 1300.

```
616 ) 1300 ( 2
      1232
      ----
        68 ) 616 ( 9
             612
             ---
               4 ) 68 ( 17
                   68
                   --
                   ×
```

∴ H.C.F. of 616 and 1300 is 4.

Thus, the required H.C.F. = **0.04.**

2. L.C.M. and H.C.F. of Fractions

$$\text{L.C.M} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in denominators}}$$

$$\text{H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in denominators}}$$

Illustration 12 Find the L.C.M. of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$.

Solution L.C.M. of numerators 2, 3 and 6 is 6.

H.C.F. of denominators 5, 10 and 25 is 5.

$$\therefore \quad \text{Required L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}} = \frac{6}{5}.$$

Illustration 13 Find the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{63}$.

Solution H.C.F. of numerators 4, 10 and 20 is 2.

L.C.M. of denominators 9, 21 and 63 is 63.

$$\therefore \quad \text{Required H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}} = \frac{2}{63}.$$

Notes:

1. If the given set of numbers includes fractions as well as whole numbers, treat whole number too as fraction with 1 in its denominator.
2. The H.C.F. of a number of fractions is always a fraction, but the L.C.M. may be a fraction or an integer.

3. Product of two numbers

= L.C.M. of the numbers × H.C.F. of the numbers

Illustration 14 The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Solution The required number

$$= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First Number}} = \frac{1260 \times 63}{315} = \mathbf{252.}$$

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4. To find the greatest number that will exactly divide x, y and z.

Required number = H.C.F. of x, y and z.

Illustration 15 Find the greatest number that will exactly divide 200 and 320.

Solution The required greatest number
= H.C.F. of 200 and 320 = **40.**

5. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.

Required number = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$.

Illustration 16 Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Solution The required greatest number
= H.C.F. of $(148 - 4)$, $(246 - 6)$ and $(623 - 11)$,
i.e. H.C.F. of 144, 240 and 612 = **12.**

6. To find the least number which is exactly divisible by x, y and z.

Required number = L.C.M. of x, y and z.

Illustration 17 What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution The required smallest number
= L.C.M. of 36, 45, 63 and 80
= **5040.**

7. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively. It is always observed that $(x - a) = (y - b) = (z - c) = k$ (say)

$\therefore$ Required number = (L.C.M. of x, y and z) $- k$.

Illustration 18 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.

Solution Since, $(36 - 25) = (48 - 37) = (64 - 53) = 11$, therefore, the required smallest number
= (L.C.M. of 36, 48 and 64) $- 11$
= $576 - 11$ = **565.**

8. To find the least number which when divided by x, y and z leaves the same remainder r in each case.

Required number = (L.C.M. of x, y and z) $+ r$.

Illustration 19 Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Solution The required smallest number
= (L.C.M. of 12, 16 and 18) + 5
= 144 + 5 = **149.**

9. To find the greatest number that will divide x, y and z leaving the same remainder in each case.

(a) When the value of remainder r is given:
Required number = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.

(b) When the value of remainder is not given:
Required number = H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$

Illustration 20 Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Solution The required greatest number
= H.C.F. of $(772 - 5)$ and $(2778 - 5)$
= H.C.F. of 767 and 2773
= **59.**

Illustration 21 Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.

Solution The required greatest number
= H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$
= H.C.F. of $|(152 - 277)|$, $|(277 - 427)|$ and $|(427 - 152)|$
= H.C.F. of 125, 275 and 150
= **25.**

10. To find the n-digit greatest number which, when divided by x, y and z,

(a) leaves no remainder (i.e. exactly divisible)

Step 1 L.C.M. of x, y and $z = L$

Step 2 $L\overline{)n\text{-digit greatest number}(}$
remainder $= R$

Step 3 Required number
= n-digit greatest number $- R$

(b) leaves remainder K in each case
Required number
= (n-digit greatest number $- R$) $+ K$.

Illustration 22 Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28 leaves 3 as a remainder in each case.

Solution L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)9999}\,(39$$
$$\underline{9828}$$
$$171$$

$\therefore$ The required number = $(9999 - 171) + 3$ = **9931.**

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Illustration 23 Find the greatest number of 4 digits which, when divided by 12, 15, 20 and 35 leaves no remainder.

Solution L.C.M. of 12, 15, 20 and 35 = 420.

$$420\overline{)9999}(23$$
$$\underline{9660}$$
$$339$$

∴ The required number = 9999 − 339 = **9663.**

11. To find the n-digit smallest number which when divided by x, y and z

(a) leaves no remainder (i.e. exactly divisible)

Step 1 L.C.M. of x, y and $z = L$

Step 2 $L\overline{)n-\text{digit smallest number}}$ (remainder = R

Step 3 Required number = n-digit smallest number + $(L - R)$.

(b) leaves remainder K in each case.

Required number = n-digit smallest number + $(L - R) + k$.

Illustration 24 Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution L.C.M. of 4, 6, 8 and 10 = 120.

$$120\overline{)1000}(8$$
$$\underline{960}$$
$$40$$

∴ The required number = 1000 + (120 − 40) = **1080.**

Illustration 25 Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Solution L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)1000}(3$$
$$\underline{756}$$
$$244$$

∴ The required number

= 1000 + (252 − 244) + 3

= **1011.**

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. What is the H.C.F. of 27, 18 and 36?
   (a) 7 (b) 11
   (c) 9 (d) None of these

2. Determine the L.C.M of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$.
   (a) $\frac{6}{5}$ (b) $\frac{11}{5}$
   (c) $\frac{9}{5}$ (d) None of these

3. What is the L.C.M. of 25, 30, 35 and 40?
   (a) 3800 (b) 4200
   (c) 4400 (d) None of these

4. What is the greatest number which divides 852, 1065 and 1491 exactly?
   (a) 193 (b) 183
   (c) 223 (d) 213

5. What is the H.C.F. of $\frac{4}{9}$, $\frac{10}{21}$ and $\frac{20}{63}$?
   (a) $\frac{4}{189}$ (b) $\frac{6}{63}$
   (c) $\frac{2}{63}$ (d) None of these

6. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder.
   (a) 8004 (b) 13004
   (c) 18004 (d) 18014

7. Area of three fields is 165 sq m, 195 sq m and 85 sq m respectively. In each of the fields a flower bed of equal length has to be made. If flower bed in each of the fields is 3 m wide then what is the maximum length of the flower bed in each of the fields?
   (a) 7 m (b) 9 m
   (c) 5 m (d) None of these

8. Find the greatest number which will divide 2112 and 2792 leaving the remainder 4 in each case.
   (a) 78 (b) 68
   (c) 65 (d) 63

9. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are:
   (a) 66, 78 (b) 70, 82
   (c) 94, 106 (d) 84, 96

10. A merchant has three different kinds of milk: 435 litres, 493 litres and 551 litres. Find the least number of casks of equal size required to store all the milk without mixing.
   (a) 51 (b) 61
   (c) 47 (d) 45

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11. Find the greatest number which will divide 25, 73 and 97 so as to leave the same remainder in each case.
(a) 12 (b) 18
(c) 24 (d) 32

12. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are:
(a) 54, 162 (b) 108, 118
(c) 27, 189 (d) None of these

13. How often will five bells toll together in one hour if they start together and toll at intervals of 5, 6, 8, 12, 20 seconds, respectively?
(a) 29 (b) 30
(c) 31 (d) 120

14. Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51, respectively.
(a) 71 (b) 81
(c) 61 (d) 73

15. Find the side of the largest square slabs which can be paved on the floor of a room 5 metres 44 cm long and 3 metres 74 cm broad.
(a) 56 (b) 42
(c) 38 (d) 34

16. The traffic lights at three different road crossings change after every 48 sec., 72 sec. and 108 sec., respectively. If they all change simultaneously at 8 : 20 : 00 hrs; then they will again change simultaneously at:
(a) 8 : 27 : 12 hrs (b) 8 : 27 : 24 hrs
(c) 8 : 27 : 36 hrs (d) 8 : 27 : 48 hrs

17. The product of two numbers is 6760 and their H.C.F. is 13. How many such pairs can be formed?
(a) 2 (b) 3
(c) 4 (d) only one

18. Find the greatest number of four digits which when divided by 10, 15, 21 and 28 leaves 4, 9, 15 and 22 as remainders, respectively.
(a) 9654 (b) 9666
(c) 9664 (d) 9864

19. The number of prime factors in the expression $(6)^{10} \times (7)^{17} \times (11)^{27}$ is:
(a) 54 (b) 64
(c) 71 (d) 81

20. Find the greatest number which will divide 3962, 4085 and 4167 leaving the same remainder in each case.
(a) 37 (b) 39
(c) 41 (d) 43

21. A wholesale tea dealer has 408 kilograms, 468 kilograms and 516 kilograms of three different qualities of tea. He wants it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.
(a) 50 (b) 36
(c) 24 (d) 12

22. A room is 4 metres 37 m long and 3 metres 23 m broad. It is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose?
(a) 485 (b) 431
(c) 391 (d) 381

23. The least perfect square number which is divisible by 3, 4, 5, 6 and 8:
(a) 900 (b) 1200
(c) 2500 (d) 3600

24. Find the least number of five digits which when divided by 12, 16, 21, 36 and 40 leaves in each case a remainder 8.
(a) 10088 (b) 10072
(c) 10080 (d) None of these

25. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?
(a) 7 m (b) 14 m
(c) 42 m (d) 63 m

26. Three men start together to travel the same way around a circular track of 11 kilometres in circumference. Their speeds are 4, $5\frac{1}{2}$ and 8 kilometres per hour, respectively. When will they meet at the starting point?
(a) 11 hours (b) 12 hours
(c) 23 hours (d) 22 hours

27. Five bells begin to toll together and toll at intervals of 36, 45, 72, 81 and 108 seconds. After what interval of time will they keep on tolling together?
(a) 3240 seconds (b) 3080 seconds
(c) 3140 seconds (d) 3200 seconds

28. Three different containers contain different quantities of mixture of milk and water, whose measurements are 403 kg, 434 kg and 465 kg. What biggest measure must be there to measure all the different quantities exactly?
(a) 1 kg (b) 7 kg
(c) 31 kg (d) 41 kg

29. The L.C.M. and G.C.D. of two numbers are 1530 and 51, respectively. Find how many such pairs are possible?
(a) 2 (b) 3
(c) 4 (d) only one

30. Find the least number of five digits which when divided by 63, 56 and 42 leaves a remainder 1 in each case.
(a) 10082 (b) 10081
(c) 10001 (d) 10071

31. The H.C.F. and L.C.M. of two numbers are 44 and 264, respectively. If the first number is divided by 2, the quotient is 44. The other number is:
(a) 33 (b) 66
(c) 132 (d) 264

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32. The largest natural number which exactly divides the product of any four consecutive natural numbers, is
(a) 6 (b) 12
(c) 24 (d) 120

33. Find the least number of six digits which is exactly divisible by 15, 21 and 28:
(a) 100480 (b) 100270
(c) 100380 (d) 100340

34. Find the greatest number of five digits which when divided by 12, 15, 21, 25 and 28 leaves 5, 8, 14, 18 and 21 as remainders, respectively.
(a) 98696 (b) 98700
(c) 97693 (d) 98693

35. What is the smallest number which when increased by 3 is divisible by 16, 24, 30 and 32?
(a) 480 (b) 475
(c) 472 (d) 477

36. The least number of square tiles required to pave the ceiling of a room 15 m 17 cm long and 9 m, 2 cm broad, is:
(a) 656 (b) 738
(c) 814 (d) 902

37. Find the least number which when divided by 2, 3, 4, 5 and 6 leaves 1, 2, 3, 4 and 5 as remainders, respectively, but when divided by 7 leaves no remainder.
(a) 210 (b) 119
(c) 126 (d) 154

38. Find the greatest number of five digits which when divided by 4, 6, 10 and 15 leaves the same remainder 3 in each case.
(a) 99993 (b) 99063
(c) 90093 (d) 99963

39. Find the least number which is a multiple of 31 and when divided by 15, 24 and 32 leaves the remainders 2, 11 and 19, respectively.
(a) 2418 (b) 2387
(c) 2356 (d) 2325

40. Find the two largest numbers of four digits having 531 as their H.C.F.
(a) 9231, 9762 (b) 9027, 9558
(c) 9037, 9568 (d) 9127, 9658

41. Find the greatest number of five digits which becomes exactly divisible by 10, 12, 15 and 18 when 3769 is added to it.
(a) 99811 (b) 99911
(c) 98911 (d) 99011

42. Find the least number which when decreased by 11 is divisible by 14, 15, 21, 32 and 60.
(a) 4371 (b) 3271
(c) 3371 (d) 3360

43. Find the least number of five digits which when divided by 8, 12, 16 and 20 leaves remainders 1, 5, 9 and 13, respectively.
(a) 10003 (b) 10093
(c) 10073 (d) 10013

44. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.
(a) 909 (b) 119
(c) 66 (d) 99

45. Find the greatest number of four digits which is exactly divisible by 24, 28, 30 and 35.
(a) 9225 (b) 9240
(c) 9250 (d) 9260

46. Find the greatest number of four digits which must be added to 5231 so that the final number becomes exactly divisible by 12, 15, 27, 32 and 40.
(a) 7929 (b) 7829
(c) 9729 (d) 7729

47. A heap of stones can be made up into groups of 21. When made up into groups of 16, 20, 25 and 45, there are 3 stones left in each case. How many stones at least can there be in the heap?
(a) 7203 (b) 2403
(c) 3603 (d) 4803

48. Find the greatest number of five digits which when divided by 8, 9 and 10 leaves 3 as remainder in each case.
(a) 99996 (b) 99723
(c) 99983 (d) None of these

49. What is the least number of cut pieces of equal length that can be cut out of two lengths 10 metres 857 millimetres and 15 metres 87 millimetres?
(a) 174 (b) 172
(c) 164 (d) 184

EXPLANATORY ANSWERS

1. Ans. (c)

Solution H.C.F. of 27, 18 and 36

$$18\overline{)27}\,(1$$
$$\underline{18}$$
$$9\overline{)18}\,(2$$
$$\underline{18}$$
$$\times$$

$\therefore$ H.C.F. of 27 and 18 is 9

Now, H.C.F. of 9 and 36

$$9\overline{)36}\,(4$$
$$\underline{36}$$
$$\times$$

$\therefore$ H.C.F. of 9 and 36 is 9

Therefore, the required H.C.F of 27, 18 and 36 is 9.

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2. Ans. (a)

Solution L.C.M. of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25}$

$$= \frac{\text{L.C.M. of 2, 3 and 6}}{\text{H.C.F. of 5, 10 and 25}}$$

$\because$ L.C.M. of 2, 3 and 6 = 6

and, H.C.F. of 5, 10 and 25 = 5

$\therefore$ Required L.C.M. = $\frac{6}{5}$.

3. Ans. (b)

Solution

2	25,	30,	35,	40
5	25,	15,	35,	20
	5,	3,	7,	4

$\therefore$ Required L.C.M. = $2 \times 5 \times 5 \times 3 \times 7 \times 4$

= **4200.**

4. Ans. (d)

Solution H.C.F. of 852 and 1065 is 213

H.C.F. of 213 and 1491 is 213

5. Ans. (c)

Solution H.C.F. of $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{63}$

$$= \frac{\text{H.C.F. of 4,10 and 20}}{\text{L.C.M. of 9,21 and 63}}$$

$\because$ H.C.F. of 4, 10 and 20 = 2

and L.C.M. of 9, 21 and 63 = 63

$\therefore$ Required H.C.F. = $\frac{2}{63}$.

6. Ans. (c)

Solution L.C.M. of 16, 18, 20 and 25 is 3600

Required number = $3600 \times K + 4$

$= (7 \times 514 + 2)K + 4$

$= (7 \times 514)K + 2K + 4$

Now $(2K + 4)$ is divisible by 7 for $K = 5$.

$\therefore$ Required number = $5 \times 3600 + 4$

= **18004.**

7. Ans. (c)

Solution H.C.F. of 165, 195 and 85 will be maximum area of each of the flower beds.

H.C.F. of 165 and 195:

$$\begin{array}{r|l} 165 & 195 \ (1 \\ & 165 \\ \hline 30 & 165 \ (2 \\ & 150 \\ \hline 15 & 35 \ (2 \\ & 30 \\ \hline & \times \end{array}$$

$\therefore$ H.C.F. of 165 and 195 is 15.

Also, now, H.C.F. of 15 and 85 is 5.

8. Ans. (b)

Solution Subtract 4 from each of the numbers 2112 and 2792 and then take the H.C.F, i.e. H.C.F of 2108 and 2788.

9. Ans. (d)

Solution The difference of requisite numbers must be 12 and each one must be divisible by 12. So, the numbers are 84, 96.

10. Ans. (a)

Solution Since minimum number of casks are required, the size of the cask is greatest. Also the cask in three cases are of equal size. The size of the cask is the H.C.F. of 435, 493 and 551 which is 29.

Now, the number of casks required for storing the milk

$= (493 + 435 + 551) \div 29 = 51.$

11. Ans. (c)

Solution $73 - 25 = 48$

$97 - 73 = 24$

$97 - 25 = 72$

H.C.F. of 48, 24 and 72 is 24.

12. Ans. (c)

Solution Let the numbers be $27a$ and $27b$

Then, $27a + 27b = 216$ or $27(a + b) = 216$

or, $a + b = \frac{216}{27} = 8$

$\therefore$ Values of co-primes (with sum 8) are (1, 7) and (3, 5)

So, the numbers are $(27 \times 1, 27 \times 7)$,

i.e. (27, 189).

13. Ans. (c)

Solution The time after which the bells will ring together is the L.C.M. of 5, 6, 8, 12 and 20 seconds, i.e. 120 seconds. The number of times they will toll together in one hour

$= (3600 \div 120) + 1$

$= 30 + 1 =$ **31.**

14. Ans. (a)

Solution $964 - 41 = 923$

$1238 - 31 = 1207$

$1400 - 51 = 1349$

H.C.F. of 923 and 1207 is 71

H.C.F. of 71 and 1349 is 71.

15. Ans. (d)

Solution The side of the square slab is the H.C.F. of 544 and 374 cm i.e. 34.

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16. Ans. (a)

Solution Interval of change = (L.C.M. of 48, 72, 108) sec.
= 432

So, the lights will change after every 432 seconds, i.e. 7 min. 12 sec.

So, the next simultaneous change will take place at 8 : 27 : 12 hrs.

17. Ans. (a)

Solution Let the numbers be $13x$ and $13y$

$13x \cdot 13y = 6760$

$\therefore \quad x \times y = 6760 \div (13 \times 13) = 40$

Possible values of (x, y) are

(1, 40); (2, 20); (4, 10); (5, 8)

Only two acceptable values are (1, 40) and (5, 8).

18. Ans. (a)

Solution First, find the greatest number of four digits that is divisible by the L.C.M. of 10, 15, 21 and 28 and then subtract 6 from it to get the required number.

19. Ans. (b)

Solution Since 2, 3, 7, 11 are prime numbers and the given expression is $2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$, the number of prime factors in the given expression is $(10 + 10 + 17 + 27) = 64$.

20. Ans. (c)

Solution $4085 - 3962 = 123$

$4167 - 4085 = 82$

$4167 - 3962 = 205$

H.C.F. of 123, 82 and 205 is 41.

21. Ans. (d)

Solution The capacity of the box is H.C.F. of 408, 468 and 516, i.e. 12.

22. Ans. (c)

Solution Length = 437 cm

Breadth = 323 cm.

The side of the square slab is the H.C.F. of 437 and 323, i.e. 19 cm.

$\therefore$ Area of square slab = 19 cm × 19 cm = 361 cm^2

$$\text{The number of slabs} = \frac{\text{Area of the room}}{\text{Area of the slab}} = \frac{437 \times 323\ cm^2}{361\ cm^2} = \mathbf{391.}$$

23. Ans. (d)

Solution

2	3,	4,	5,	6,	8
2	3,	2,	5,	3,	4
3	3,	1,	5,	3,	2
	1,	1,	5,	1,	2

L.C.M. of 3, 4, 5, 6, 8 = 120

Required number = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
= 3600.

24. Ans. (a)

Solution Required number = the least number of 5 digits divisible by the L.C.M. of 12, 16, 21, 36, 40 + the remainder 8.

25. Ans. (a)

Solution Greatest possible length of each plank = (H.C.F. of 42, 49, 63) m = **7 m.**

26. Ans. (d)

Solution Time for one revolution by each of three men

$$= \frac{11}{4}, \frac{11}{5\frac{1}{2}}, \frac{11}{8} \text{ hours}$$

$$= \frac{11}{4}, \frac{2}{1}, \frac{11}{8} \text{ hours}$$

$\therefore$ The time when they will meet at the starting point = L.C.M. of $\frac{11}{4}, \frac{2}{1}, \frac{11}{8}$ which is $\frac{22}{1}$, i.e. 22 hours.

27. Ans. (a)

Solution The interval of time is L.C.M. of the numbers 36, 45, 72, 81 and 108.

2	36,	45,	72,	81,	108
2	18,	45,	36,	81,	54
2	9,	45,	18,	81,	27
3	9,	45,	9,	81,	27
3	3,	15,	3,	27	9
3	1,	5,	1,	9,	3
	1,	5,	1,	3,	1

L.C.M (36, 45, 72, 108) = **3240.**

28. Ans. (c)

Solution Biggest measure = H.C.F. of (403, 434, 465)
= 31 kg.

29. Ans. (c)

Solution Let the numbers be $51x$ and $51y$ where x and y are co-prime.

Now, $51x \times 51y = 51 \times 1530$

$\therefore \quad x \times y = 30$

Possible pairs are (1, 30); (2, 15); (3, 10) and (5, 6).

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30. Ans. (b)

Solution L.C.M. of 63, 56 and 42 is 504.

Least number of 5 digits divisible by 504:

$$504\overline{)10000}(19$$
$$\underline{-504}$$
$$4960$$
$$\underline{-4536}$$
$$424$$

$= 1000 + (504 - 424) = 10080$

$\therefore$ Required number $= 10080 + 1 =$ **10081.**

31. Ans. (c)

Solution First number $= 2 \times 44 = 88$

Second number $= \dfrac{44 \times 264}{88} =$ **132.**

32. Ans. (c)

Solution $1 \times 2 \times 3 \times 4 = 24$

$\therefore$ Required number $= 24.$

33. Ans. (c)

Solution L.C.M. of 15, 21 and 28 is 420.

Least number of 6 digits $= 100000$

$$420\overline{)100000}(238$$
$$\underline{-840}$$
$$1600$$
$$\underline{-1260}$$
$$3400$$
$$\underline{-3360}$$
$$40$$

Remainder $= 40.$

$\therefore$ Least number $= 100000 + (420 - 40)$

$=$ **100380.**

34. Ans. (d)

Solution Find the greatest number of five digits which is divisible by the L.C.M. of 12, 15, 21, 25 and 28 and then subtract 7 from it to get the required number.

Required number $= 98700 - 7 = 98693.$

35. Ans. (d)

Solution Required number

$=$ (L.C.M. of 16, 24, 30 and 32) $- 3$

$= 480 - 3 =$ **477.**

36. Ans. (c)

Solution Side of each tile $=$ (H.C.F. of 1517 and 902) cm

$= 41$ cm

$\therefore$ Number of tiles $= \dfrac{1517 \times 902}{41 \times 41} =$ **814.**

37. Ans. (b)

Solution L.C.M. of 2, 3, 4, 5, 6 is 60.

One of the numbers satisfying the first condition is $60 - 1 = 59$

$60 + 59 = 119$, etc.

But 119 is also divisible by **7.**

38. Ans. (d)

Solution L.C.M. of 4, 6, 10, 15 $= 60$

Greatest number of five digits which is divisible by 60 $= 99960.$

$\therefore$ Required number $= 99960 + 3 =$ **99963.**

39. Ans. (b)

Solution L.C.M. of 15, 24, 32 is 480

Required number $= 480K - 13$

$= 15 \times 31K + (15K - 13)$

$(15K - 13)$ is divisible by 31 for $K = 5$

$\therefore$ Least number $= 480 \times 5 - 13 =$ **2387.**

40. Ans. (b)

Solution The greatest number of four digits divisible by 531 is 9558, so the other number is $9558 - 531 = 9027$. Thus, the numbers are 9558 and 9027.

41. Ans. (b)

Solution L.C.M. of 10, 12, 15 and 18 $= 540$. Dividing $(99999 + 3769)$ by 540, the remainder is 88.

$\therefore$ Required number $= 99999 - 88 =$ **99911.**

42. Ans. (c)

Solution Required number

$=$ (L.C.M. of 14, 15, 21, 32, 60) $+ 11$

$= 3360 + 11 =$ **3371.**

43. Ans. (c)

Solution Least number of five digits divisible by L.C.M. of 8, 12, 16, 20 is 10080.

$\therefore$ Required number $= 10080 - 7 = 10073.$

44. Ans. (d)

Solution Required number $= \dfrac{\text{L.C.M.} \times \text{H.C.F.}}{\text{Given number}}$

$= \dfrac{693 \times 11}{77} =$ **99.**

45. Ans. (b)

Solution L.C.M. of 24, 28, 30 and 35

2	24, 28, 30, 35
2	12, 14, 15, 35
3	6, 7, 15, 35
5	2, 7, 5, 35
7	2, 7, 1, 7
	2, 1, 1, 1

$= 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

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Quotient when 9999 is divided by 840 is 11 and remainder is 759.

∴ Greatest number of four digits in this case = 9999 – 759 = **9240.**

46. Ans. (d)

Solution L.C.M. of 12, 15, 27, 32, 40 = 4320. Let us add 5231 to the greatest number of four digits and then divide by 4320 to find the remainder.

$$4320\overline{)15230}(3$$
$$\underline{12960}$$
$$2270$$

Required greatest number of four digits

= 9999 – 2270

= **7729.**

47. Ans. (a)

Solution L.C.M. of 16, 20, 25, 45 = 3600

1st number = 3600 × 1 + 3 = 3603 which is not divisible by 21.

2nd number = 3600 × 2 + 3 = 7203 which is divisible by 21.

48. Ans. (b)

Solution L.C.M. of 8, 9, 10 = 360

$$360\overline{)99999}(277$$
$$\underline{720}$$
$$2799$$
$$\underline{2520}$$
$$2799$$
$$\underline{2520}$$
$$279$$

Greatest number of 5 digits which is divisible by 360 = 99999 – 279 = 99720

∴ Required number = 99729 + 3 = **99723.**

49. Ans. (d)

Solution H.C.F. of 10857 and 15087 is 141.

The least number of cut pieces

= (10857 + 15087) ÷ 141

= **184.**

QUESTIONS FROM COMPETITIVE EXAMS

1. The L.C.M. of two numbers is 4800 and their HCF is 160. If one of the numbers is 480, then the second number is

(a) 16 (b) 16000
(c) 160 (d) 1600

[Based on MAT 2004]

2. HCF of 3240, 3600 and a third number is 36 and their LCM is $2^4 \times 3^5 \times 5^2 \times 7^2$. The third number is

(a) $2^4 \times 5^3 \times 7^2$ (b) $2^2 \times 3^5 \times 7^2$
(c) $2^3 \times 3^5 \times 7^2$ (d) $2^5 \times 5^2 \times 7^2$

[Based on MAT 2003]

3. An electronic device makes a beep after every 60 sec. Another device makes a beep after every 62 sec. They beeped together at 10 a.m. The time when they will next make a beep together at the earliest is

(a) 10.30 a.m. (b) 10.31 a.m.
(c) 10.59 a.m. (d) 11 a.m.

[Based on MAT 2005]

4. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest?

(a) $2^{1/2}$ (b) $3^{1/3}$
(c) $4^{1/4}$ (d) $6^{1/6}$
(e) $12^{1/12}$

[Based on CAT 2006]

5. An electronic device makes a beep after every 60 sec. Another device makes a beep after every 62 sec. They beeped together at 10 a.m. The next time, when they would beep together at the earliest is:

(a) 10:30 a.m. (b) 10:31 a.m.
(c) 10:59 a.m. (d) 11:00 a.m.

[Based on FMS (Delhi) 2006]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Second number $= \dfrac{\text{HCF} \times \text{LCM}}{\text{First number}}$

$= \dfrac{4800 \times 160}{480} =$ **1600.**

2. Ans. (b), (c)

3. Ans. (b)

Solution L.C.M. of 60 and 62 seconds is 1860 sec = 31 min.

∴ They will beep together at **10.31 a.m.**

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4. Ans. (a)

Solution $t_n = \dfrac{n}{n+2}$

$$t_3 = \frac{3}{3+2} = \frac{5}{3}, t_4 = \frac{4}{6}, t_5 \frac{5}{7}, \text{---} t_{53} = \frac{53}{55}$$

$$t_3 \times t_4 \times t_5 \times \text{---} \times t_{53} = \frac{3}{5} \times \frac{4}{6} \times \frac{5}{7} \times \text{---} \times \frac{53}{55}$$

$$= \frac{2}{495}$$

5. Ans. (b)

EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. Find the least multiple of 13 which when divided by 6, 8 and 12 leaves 5, 7 and 11 as remainders, respectively.
(a) 143 (b) 169
(c) 260 (d) 221

2. Find the H.C.F. of $\frac{4}{5}, \frac{5}{6}$ and $\frac{7}{15}$.
(a) $\frac{7}{90}$ (b) $\frac{1}{30}$
(c) $\frac{1}{60}$ (d) $\frac{7}{60}$

3. Three measuring rods are 64 cm, 80 cm and 96 cm in length. The least length of cloth that can be measured exact number of times using any one of the above rods is:
(a) 0.96 m (b) 19.20 m
(c) 9.60 m (d) 96.00 m

4. The L.C.M. of two numbers is 45 times their H.C.F. One number is 125; and the sum of their H.C.F. and L.C.M. is 1150. Find the other number.
(a) 275 (b) 215
(c) 230 (d) 225

5. Find the L.C.M. of $\frac{4}{5}, \frac{5}{6}$ and $\frac{7}{15}$.
(a) 140 (b) 120
(c) 70 (d) 210

6. About the number of pairs which have 16 as their H.C.F and 136 as their L.C.M., we can definitely say that
(a) Only one such pair exists
(b) Only two such pairs exist
(c) Many such pairs exist
(d) No such pair exists.

7. Find the least number which is exactly divisible by 32, 36, 45, 60 and 80.
(a) 1440 (b) 1400
(c) 1380 (d) 1360

8. Find the least square number which is exactly divisible by 10, 15, 18, 25 and 30.
(a) 8100 (b) 900
(c) 3600 (d) 6400

9. The H.C.F of two numbers of 3 digits is 310 and their L.C.M. is 1860. Find the numbers.
(a) 310, 1860 (b) 620, 310
(c) 620, 930 (d) None of these

10. Two numbers 4242 and 2903 when divided by a certain number of three digits, leave the same remainder. Find the number.
(a) 101 (b) 103
(c) 107 (d) 113

Test II

[Time: 10 min]

11. Find the least number which when divided by 12, 15, 20 and 54 leaves in each case a remainder 7.
(a) 547 (b) 533
(c) 540 (d) 587

12. A gardener was asked to plant flowers in a row containing equal number of plants. He tried to plant 6, 8, 10 and 12 in each row, but 5 plants were left in each case. When he planted 13 in a row, no plant was left. Find the number of plants with him.
(a) 245 (b) 125
(c) 485 (d) 845

13. The sum and difference of the L.C.M. and the H.C.F. of two numbers are 312 and 264, respectively. Find the numbers if their sum is 168.
(a) 96, 72 (b) 72, 96
(c) 108, 84 (d) None of these

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14. Three persons begin to walk round a circular track. The first completes revolution in $15\frac{1}{6}$ seconds, the second in $16\frac{1}{4}$ seconds and the third in $18\frac{2}{3}$ seconds. After what time will they be together at the starting point again?
(a) 1 hr 40 min. (b) 1 hr 40 sec
(c) 1.40 hr (d) 1 hr 3 min. 40 sec.

15. Find the least number which when diminished by 6 is divisible by 14, 35, 75, 96 and 210.
(a) 16800 (b) 16794
(c) 16086 (d) None of these

16. What is the least number which when divided by 8, 12, 18 and 24 leaves the remainders 4, 8, 14 and 20, respectively?
(a) 78 (b) 68
(c) 58 (d) None of these

17. The L.C.M. of 54, 90 and a third number is 1890 and their G.C.D. is 18. What is the third number?
(a) 36 (b) 180
(c) 126 (d) 108

18. H.C.F. of three numbers is 12. If they be in the ratio 1 : 2 : 3, the numbers are:
(a) 12, 24, 36 (b) 10, 20, 30
(c) 5, 10, 15 (d) 4, 8, 12

19. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. Minimum number of rows in which the above trees may be planted is:
(a) 15 (b) 17
(c) 3 (d) 20

20. The product of two numbers is 2160, their H.C.F. is 12. Find the number of such possible pairs.
(a) 2 (b) only one
(c) 3 (d) More than 3

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution L.C.M. of 6, 8 and 12 is 24

$\therefore$ Required number = $(24K - 1)$ that is divisible by 13 for the least value of K.

Now, $(24K - 1) = 13 \times 2K - (2K + 1)$

This is divisible by 13 for $K = 6$

$\therefore$ Required number $= 24 \times 6 - 1 =$ **143.**

3. Ans. (c)

Solution Required length = (L.C.M. of 64, 80, 96) cm
= 960 cm = **9.60 m.**

5. Ans. (a)

Solution Required L.C.M. $= \dfrac{\text{L.C.M. of } 4, 5, 7}{\text{H.C.F. of } 5, 6, 15}$

$= \dfrac{140}{1} =$ **140.**

7. Ans. (a)

Solution Least number is the L.C.M. of 32, 36, 45, 60 and 80:

2	32,	36,	45,	60,	80
2	16,	18,	45,	30,	40
2	8,	9,	45,	15,	20
2	4,	9,	45,	15,	10
2	2,	9,	45,	15,	5
3	1,	9,	45,	15,	5
3	1,	3,	15,	5,	5
5	1,	1,	5,	5,	5
	1,	1,	1,	1,	1

L.C.M. $= 2^5 \times 3^2 \times 5 =$ **140.**

9. Ans. (c)

Solution Let two numbers of 3 digits be $310x$ and $310y$

$\therefore$ $310x \cdot 310y = 310 \times 1860$

$\Rightarrow$ $x \times y = 6$

$\therefore$ $x = 2$ and $y = 3$

$\therefore$ Numbers are 310×2, 310×3, i.e., 620 and **930.**

11. Ans. (a)

Solution The required least number
= The L.C.M. of (12, 15, 20, 54) + 7
= 540 + 7 = **547.**

13. Ans. (a)

Solution Let x and y be the L.C.M. and the H.C.F. of the two numbers respectively, then

$x + y = 312$ and $x - y = 264$

$\Rightarrow$ $x = \dfrac{312+264}{2} = 288$ and $y = \dfrac{312-264}{2} = 24$

Since the H.C.F. is 24, let the two numbers be $24a$ and $24b$.

Then, $24a + 24b = 168$

$\therefore$ $a + b = 7$

Next, L.C.M. $\times$ H.C.F. = product of the numbers

or, $288 \times 24 = 24a \times 24b$ or $ab = 12$

$\therefore$ $a - b = \sqrt{(a+b)^2 - 4ab} = \sqrt{49-48} = 1$

Hence, $a = 4$ and $b = 3$

The numbers then are $24a$ and $24b$, i.e. **96** and **72.**

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15. Ans. (c)

Solution The required least number
= L.C.M. of (14, 35, 75, 96 and 210) + 6
= 16800 + 6 = **16806.**

17. Ans. (c)

Solution Since the H.C.F. of 54 and 90 is 18, let the third number be $18x$.

L.C.M. of 54, 90, $18x$ is $18 \times 3 \times 5 \times x$

$\therefore \quad 18 \times 3 \times 5 \times x = 1890$

$\therefore \quad x = 7$

Third number is $18 \times 7 =$ **126.**

19. Ans. (b)

Solution H.C.F. of 21, 42, 56 = 7

Number of rows of mango trees, apple trees and orange trees are $\frac{21}{7} = 3$

$\frac{42}{7} = 6$

$\frac{56}{7} = 8$

$\therefore$ Required number of rows = 3 + 6 + 8 = **17.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (d)	**6.** (d)	**8.** (b)	**10.** (b)	**12.** (d)	**14.** (b)	**16.** (b)	**18.** (a)	**20.** (a)

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Square Root and Cube Root

3

SQUARE

A number multiplied by itself is known as the *square* of the given number. For example, square of 6 is $6 \times 6 = 36$.

Square Root

Square root of a given number is that number which when multiplied by itself is equal to the given number.

For example, square root of 81 is 9 because $9^2 = 9 \times 9 = 81$.

The square root of a number is denoted by the symbol $\sqrt{}$ or $\sqrt{\ }$, called the *radical sign*.

Thus, $\sqrt{81} = 9$, $\sqrt{64} = 8$ and so on.

Note that $\sqrt{1} = 1$.

Methods of Finding the Square Root

I. Prime Factorisation Method

1. Find the prime factors of the given number.
2. Group the factors in pairs.
3. Take one number from each pair of factors and then multiply them together.

   This product is the square root of the given number.

Illustration 1 Find the square root of:

(i) 4761 (ii) 207025

Solution (i) $4761 = \underbrace{23 \times 23} \times \underbrace{3 \times 3}$

$\therefore \sqrt{4761} = 23 \times 3 = 69.$

(ii) $207025 = \underbrace{5 \times 5} \times \underbrace{7 \times 7} \times \underbrace{13 \times 13}$

$\therefore \sqrt{207025} = 5 \times 7 \times 13 = \mathbf{455.}$

***Note*:** The above method is used when the given number is a perfect square or when every prime factor of that number is repeated twice.

II. Method of Division

This method is used when the number is large and the factors cannot be easily determined.

The working rule is explained with the help of following example:

Step 1: The digits of a number, whose square root is required, are separated into periods of two beginning from the right. The last period may be either single digit or a pair.

$$\begin{array}{r|l} & 476 \\ \hline 8 & \underline{22}\ \underline{65}\ \underline{76} \\ 87 & \underline{16} \\ & 665 \\ & \underline{609} \\ 946 & 5676 \\ & \underline{5676} \\ & \times \end{array}$$

Step 2: Find a number (here, 4) whose square may be equal to or less than the first period (here, 22).

Step 3: Find out the remainder (here, 6) and bring down the next period (here, 65).

Step 4: Double the quotient (here, 4) and write to the left (here, 8).

Step 5: The divisor of this stage will be equal to the above sum (here, 8) with the quotient of this stage (here, 7) suffixed to it (here, 87).

Step 6: Repeat this process (step 4 and step 5) till all the periods get exhausted.

The quotient (here, 476) is equal to the square root of the given number (here, 226576).

Illustration 2 Find the square root of:

(i) 180625 (ii) 1498176

Solution (i)

$$\begin{array}{r|l} & 425 \\ \hline 8 & \underline{18}\ \underline{06}\ \underline{25} \\ 82 & \underline{16} \\ & 206 \\ & \underline{164} \\ 845 & 4225 \\ & \underline{4225} \\ & \times \end{array}$$

Thus, $\sqrt{180625} = \mathbf{425.}$

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(ii)

	1224			
1	1	49	81	76
	1			
22		49		
		44		
242		581		
		484		
2444		9776		
		9776		
		×		

Thus, $\sqrt{1478176} = \mathbf{1224.}$

Square Root of a Decimal

If the given number is having decimal, we separate the digits of that number into periods of two to the right and left starting from the decimal point and then proceed as in the following illustration:

Illustration 3 Find the square root of :

(i) 12.1801 (ii) 127.0129

(iii) 0.1790136 (iv) 0.000625.

Solution (i)

	3.49			
3	12.	18	01	
	16			
64	665			
	609			
689	6201			
	6201			
	×			

$\therefore \sqrt{12.1801} = \mathbf{3.49.}$

(ii)

	11.27			
1	1	27.	01	29
	1			
21		27		
		21		
222		601		
		444		
2447		15729		
		15729		
		×		

$\therefore \sqrt{127.0129} = \mathbf{11.27}$

(iii) Since the number of decimal places is odd, we make it even by affixing one 0 to the right.

	0.423				
4	0.	17	90	13	60
		16			
82		190			
		164			
843		2613			
		2529			
846		8460			

In the above solution, after bringing down the last period, we note that the remainder is not zero. So, a pair of zeros can be annexed and process can be continued to find the square root up to 4 places of decimals. The above process can be continued still further and square root up to required number of decimal places can be obtained.

Note: If a decimal has an odd number of decimal places, its square root cannot be found exactly.

(iv)

	.025			
0.	00	06	25	
	00			
02	06			
	04			
45	225			
	225			
	×			

$\therefore \sqrt{0.000625} = \mathbf{0.025.}$

Square Root of a Fraction

(a) *If the denominator is a perfect square*:

The square root is found by taking the square root of the numerator and denominator separately.

(b) *If the denominator is not a perfect square*:

The fraction is converted into decimal and then square root is obtained or the denominator is made perfect square by multiplying and dividing by a suitable number and then its square root is obtained.

Illustration 4 Find the square root of:

(i) $\frac{2704}{49}$ (ii) $\frac{44}{25}$

(iii) $\frac{354}{43}$ (iv) $\frac{461}{32}$

Solution (i) $\sqrt{\frac{2704}{49}} = \frac{\sqrt{2704}}{\sqrt{49}} = \frac{\sqrt{52\times52}}{\sqrt{7\times7}} = \frac{52}{7}$

$= 7\frac{3}{7}.$

(ii) $\sqrt{\frac{44}{25}} = \frac{\sqrt{44}}{\sqrt{25}} = \frac{\sqrt{44}}{\sqrt{5\times5}} = \frac{\sqrt{44}}{5} = \frac{6.6332}{5}$

$= \mathbf{1.3266}$ **(nearly).**

(iii) $\sqrt{\frac{354}{43}} = \sqrt{8.2325} = \mathbf{2.8692}$ **(nearly)**

(iv) $\sqrt{\frac{461}{32}} = \sqrt{\frac{461\times2}{32\times2}} = \frac{\sqrt{922}}{\sqrt{64}} = \frac{30.3644}{8}$

$= \mathbf{3.7955}$ **(nearly).**

Cube

Cube of a number is obtained by multiplying the number itself thrice.

For example, 27 is the cube of 3 as $27 = 3 \times 3 \times 3$.

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Cube Root

The *cube root* of a given number is that number which when raised to the third power produces the given number, that is the cube root of a number x is the number whose cube is x.

The cube root of x is written as $\sqrt[3]{x}$.

For example, cube root of 64 is 4 as $4 \times 4 \times 4 = 64$.

Methods to find Cube Root

I. Method of Factorisation

1. Write the given number as product of prime factors.
2. Take the product of prime numbers, choosing one out of three of each type.

   This product gives the cube root of the given number.

Illustration 5 Find the cube root of 42875.

Solution Resolving 42875 into prime factors, we get

$$42875 = \underbrace{5\times5} \times \underbrace{5\times7} \times \underbrace{7\times7}$$

$$\therefore \quad \sqrt[3]{42875} = 5 \times 7 = \mathbf{35}.$$

II. Short-cut Method to Find Cube Roots of Exact Cubes Consisting of up to 6 Digits:

Before we discuss the method to find the cube roots of exact cubes, the following two remarks are very useful and must be remembered by heart.

1. $1^3 = 1$; $2^3 = 9$; $3^3 = 27$; $4^3 = 64$; $5^3 = 125$; $6^3 = 216$; $7^3 = 343$; $8^3 = 512$; $9^3 = 729$; $10^3 = 1000$.
2. If the cube ends in 1, then its cube root ends in 1

If the cube ends in 2, then its cube root ends in 8
If the cube ends in 3, then its cube root ends in 7
If the cube ends in 4, then its cube root ends in 4
If the cube ends in 5, then its cube root ends in 5
If the cube ends in 6, then its cube root ends in 6
If the cube ends in 7, then its cube root ends in 3
If the cube ends in 8, then its cube root ends in 2
If the cube ends in 9, then it cube root ends in 9
If the cube ends in 0, then its cube root ends in 0

Clearly, from above

$1 \leftrightarrow 1$, $4 \leftrightarrow 4$, $5 \leftrightarrow 5$, $6 \leftrightarrow 6$, $9 \leftrightarrow 9$, $0 \leftrightarrow 0$
$2 \leftrightarrow 8$, $3 \leftrightarrow 7$.

The method of finding the cube root of a number up to 6 digits which is actually a cube of some number consisting of 2 digits is best illustrated with the help of following examples:

Illustration 6 Find the cube roots of the following:

(i) 2744 (ii) 9261
(iii) 19684 (iv) 54872
(v) 614125

Solution (i) Make groups of 3 digits from the right side. $\underline{2}\ \underline{744}$

2 lies between 1^3 and 2^3, so left digit is 1.
744 ends in 4, so right digit is 4.
Thus, cube root of **2744** is **14.**

(ii) $\underline{9}\ \underline{261}$

9 lies between 2^3 and 3^3, so left digit is 2.
261 ends in 1, so right digit is 1.
Thus, cube root of **9261** is **21.**

(ii) $\underline{19}\ \underline{683}$

19 lies between 2^3 and 3^3, so left digit is 2.
683 ends in 3, so right digit is 7.
Thus, cube root of **19683** is **27.**

(iv) $\underline{54}\ \underline{872}$

54 lies between 3^3 and 4^3, so left digit is 3.
872 ends in 2, so right digit is 8.
Thus, cube root of **19683** is **38.**

(iv) $\underline{614}\ \underline{125}$

614 lies between 8^3 and 9^3, so left digit is 8.
125 ends in 5, so right digit is 5.
Thus, cube root of **614125** is **85.**

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find the square root of 4356
   (a) 68 (b) 64
   (c) 66 (d) None of these
2. Find the square root of 104976
   (a) 324 (b) 424
   (c) 326 (d) None of these
3. Find the square root of 211600
   (a) 460 (b) 440
   (c) 480 (d) None of these
4. Find the value of $\sqrt{6492304}$
   (a) 2384 (b) 2484
   (c) 2548 (d) 2684

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5. Find the least number which when multiplied with 74088 will make it a perfect square.
(a) 42 (b) 44
(c) 46 (d) 48

6. $\sqrt{10} \times \sqrt{250} = ?$
(a) 46.95 (b) 43.75
(c) 50.25 (d) 50.00

7. $\sqrt{80} + 3\sqrt{245} - \sqrt{125} = ?$
(a) $20\sqrt{5}$ (b) $25\sqrt{2}$
(c) $15\sqrt{2}$ (d) None of these

8. $\frac{250}{\sqrt{?}} = 10$
(a) 25 (b) 250
(c) 625 (d) 2500

9. If $\sqrt{256} + \sqrt{x} = 2$, then x is equal to
(a) 64 (b) 128
(c) 512 (d) 1024

10. Find the smallest number by which 216 should be divided to make the result a perfect square.
(a) 4 (b) 3
(c) 6 (d) 2

11. $\frac{\sqrt{?}}{200} = 0.02$
(a) 0.4 (b) 4
(c) 16 (d) 1.6

12. $\frac{\sqrt{6727}}{\sqrt{7}} = ?$
(a) 30.79 (b) 32.29
(c) 31 (d) None of these

13. $\sqrt{.09} = ?$
(a) 0.3 (b) 0.03
(c) 0.003 (d) None of these

14. $\frac{14}{3+\sqrt{2}} = ?$
(a) 3.172 (b) 4.586
(c) 8.828 (d) None of these

15. Find the smallest number which when added to 3579 gives a perfect square.
(a) 27 (b) 24
(c) 21 (d) 18

16. If $\sqrt{\left(1+\frac{27}{169}\right)} = 1 + \frac{x}{13}$, then x equals
(a) 1 (b) 3
(c) 5 (d) 7

17. $\frac{\sqrt{4375}}{\sqrt{7}} = ?$
(a) 24.75 (b) 27.75
(c) 25 (d) 35

18. If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, then the value of $\frac{a}{b}$ is.
(a) 0.016 (b) 1.60
(c) 0.16 (d) None of these

19. $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{3}}}}}} = ?$
(a) $3^{31/64}$ (b) $3^{31/32}$
(c) $3^{1/64}$ (d) None of these

20. $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$
(a) 6 (b) 7
(c) 8 (d) 9

21. $\sqrt{176 + \sqrt{2401}} = ?$
(a) 14 (b) 15
(c) 18 (d) 24

22. $\sqrt{10} \times \sqrt{15} = ?$
(a) $5\sqrt{6}$ (b) $6\sqrt{5}$
(c) 5 (d) $\sqrt{30}$

23. $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = ?$
(a) $\frac{1}{2\sqrt{3}}$ (b) $-\frac{1}{2\sqrt{3}}$
(c) 1 (d) $\frac{5\sqrt{3}}{6}$

24. $\sqrt{248 + \sqrt{52 + \sqrt{144}}} = 1$
(a) 14 (b) 16
(c) 16.6 (d) 18.8

25. $\sqrt{0.0009} + \sqrt{0.01} = ?$
(a) 3 (b) 0.3
(c) $\frac{1}{3}$ (d) None of these

26. $\frac{1}{\sqrt{9} - \sqrt{8}} = ?$
(a) $\frac{1}{2}(3 - 2\sqrt{2})$ (b) $\frac{1}{3+2\sqrt{2}}$
(c) $3 - 2\sqrt{2}$ (d) $3 + 2\sqrt{2}$

27. If $\sqrt{\frac{x}{169}} = \frac{54}{39}$, then x is equal to:
(a) 108 (b) 324
(c) 2916 (d) 4800

28. $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}} = ?$
(a) 3 (b) 4
(c) 6 (d) greater than 6

29. $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{12} \times \frac{\sqrt{256}}{8} = ?$

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(a) 8 (b) 12
(c) 16 (d) 32

30. If $\sqrt{12} = 3.464$, value of $\sqrt{\frac{3}{4}} + 2\sqrt{\frac{4}{3}}$ is
(a) 3.17 (b) 3.464
(c) 3.1753 (d) None of these

31. If $\sqrt{15625} = 125$, then the value of $\sqrt{15625} + \sqrt{156.25} + \sqrt{1.5625}$ is
(a) 1.3875 (b) 13.875
(c) 138.75 (d) 156.25

32. If $\sqrt{0.03\times0.3\times a} = 0.03 \times 0.3 \times \sqrt{b}$, value of $\frac{a}{b}$ is
(a) 0.009 (b) 0.03
(c) 0.09 (d) None of these

33. Given that $\sqrt{4096} = 64$, the value of $\sqrt{4096} + \sqrt{40.96} + \sqrt{.004096}$ is
(a) 70.4 (b) 70.464
(c) 71.104 (d) 71.4

34. If $\sqrt{1+\sqrt{1-\frac{2176}{2401}}} = 1+\frac{x}{7}$, value of x is
(a) 3 (b) 1
(c) 5 (d) 7

35. Which of the following numbers, wherein some of the digits have been suppressed by symbols, can possibly be the perfect square of a 3 digit odd number?
(a) 65 ××× 1 (b) 9 ×× 1
(c) 10 ××× 4 (d) 9 ×××××× 5

36. $\sqrt{\frac{0.324\times0.081\times4.624}{1.5625\times0.0289\times72.9\times64}} = ?$
(a) 24 (b) 2.40
(c) 0.024 (d) None of these

37. Find the cube root of $\frac{512}{3375}$
(a) 12/15 (b) 16/25
(c) 8/15 (d) None of these

38. $\sqrt{0.01+\sqrt{0.0064}} = ?$
(a) 0.3 (b) 0.03
(c) $\sqrt{0.18}$ (d) None of these

39. Find the cube root of 15.625
(a) 3.5 (b) 2.5
(c) 4.5 (d) 5.5

40. What is the value of $\sqrt[3]{.000064}$?
(a) 0.4 (b) 0.08
(c) 0.04 (d) 0.16

41. What is the value of $\sqrt[3]{\sqrt{441}+\sqrt{16}+\sqrt{4}}$
(a) 3 (b) 5
(c) 7 (d) 9

42. The least number by which 14175 be divided to make it a perfect square is
(a) 3 (b) 5
(c) 7 (d) 15

43. $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ is equal to
(a) $4+\sqrt{15}$ (b) $4-\sqrt{15}$
(c) $\frac{1}{2}$ (d) 1

44. $\frac{\sqrt{24}+\sqrt{216}}{\sqrt{96}} = ?$
(a) $2\sqrt{6}$ (b) $6\sqrt{2}$
(c) 2 (d) $\frac{2}{\sqrt{6}}$

45. Given that $\sqrt{20} = 4.472$, find the square root of $2\frac{2}{9}$ up to two places of decimals
(a) 1.56 (b) 1.69
(c) 1.49 (d) None of these

46. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{3}{5}$ (d) $\frac{5}{3}$

47. The least number by which 10584 be multiplied to make it a perfect square is:
(a) 2 (b) 3
(c) 6 (d) 8

48. The smallest number which is a perfect square and contains 7936 as a factor is
(a) 12008 (b) 246016
(c) 61504 (d) 240616

49. $\sqrt{0.00059049} = ?$
(a) 0.243 (b) 0.0243
(c) 0.00243 (d) 0.000243

50. Given that $\sqrt{10} = 3.16$, what is the value of $\sqrt{\frac{4}{12.1}}$ to one place of decimal?
(a) 0.16 (b) 0.06
(c) 0.6 (d) 0.016

51. $\sqrt{\frac{0.256\times0.081\times4.356}{1.5625\times0.0121\times129.6\times64}} = ?$
(a) 0.0124 (b) 0.124
(c) 0.0024 (d) 0.024

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52. A general wishing to draw up his 16160 men in the form of a solid square found that he had 31 men over. The number of men in the front row is
(a) 127 (b) 123
(c) 137 (d) 129

53. The areas of two square fields are 420.25 sq. m. and 441 sq. m., respectively. The ratio of their sides is
(a) 20 : 21 (b) 40 : 41
(c) 41 : 42 (d) 40 : 42

54. A general wishing to draw up his 5180 men in the form of a solid square found that he had 4 men less. If he could get four more men and form the solid square, the number of men in the front row is
(a) 68 (b) 72
(c) 78 (d) 82

55. The largest number of three digits which is a perfect square is
(a) 900 (b) 841
(c) 961 (c) 784

56. What least number should be subtracted from the square root of $21\frac{15}{289}$ so that the result is a whole number?
(a) 15/289 (b) 7/17
(c) 10/17 (d) 5/17

57. The smallest number which when subtracted from the number 62512 makes it a perfect square is
(a) 22 (b) 32
(c) 12 (d) 2

58. The largest number of five digits which is a perfect square is
(a) 97344 (b) 98596
(c) 99856 (d) None of these

59. By what least number, 2450 be multiplied, so that the resulting number is perfect square ?
(a) 8 (b) 10
(c) 5 (d) 2

60. The smallest number by which 3600 must be multiplied to make it a perfect cube is
(a) 40 (b) 60
(c) 20 (d) 15

EXPLANATORY ANSWERS

1. Ans. (c)

Solution The prime factors of 4356 are
$2 \times 2 \times 3 \times 3 \times 11 \times 11$
$4356 = 2 \times 2 \times 3 \times 3 \times 11 \times 11$

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

$\sqrt{4356} = \sqrt{2^2 \times 3^2 \times 11^2}$
$= 2 \times 3 \times 11 = \mathbf{66}.$

2. Ans. (a)

Solution

```
          324
      ------------
   3 | 10  49  76
     |  9
     |-----------
  62 | 149
     | 124
     |-----------
 644 | 2576
     | 2576
     |-----------
     |  ×
```

∴ Square root of **104976** is **324.**

3. Ans. (a)

Solution

```
          460
      ------------
   4 | 21  16  00
     | 16
     |-----------
  86 | 516
     | 516
     |-----------
     |  ×
```

∴ Square root of **211600** is **460.**

4. Ans. (c)

Solution

```
          2548
      ------------
   2 | 6   49  23  04
     | 4
     |-----------
  45 | 249
     | 225
     |-----------
 644 | 2423
     | 2016
     |-----------
5088 | 40704
     | 40704
     |-----------
     |  ×
```

∴ $\sqrt{6492304}$ = **2548.**

5. Ans. (a)

Solution $74088 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$
$= (2 \times 2) \times (3 \times 3) \times (7 \times 7) \times (2 \times 3 \times 7)$

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Therefore, required number
$= 2 \times 3 \times 7 = 42.$

2	74088
2	37044
2	18522
3	6261
3	3087
3	1029
7	343
7	49
7	7
	1

6. Ans. (d)

Solution $\sqrt{10} \times \sqrt{250} = \sqrt{2500} = \mathbf{50}.$

7. Ans. (a)

Solution $\sqrt{80} + 3\sqrt{245} - \sqrt{125} = 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5}$
$= \mathbf{20\sqrt{5}}.$

8. Ans. (c)

Solution Let $\frac{250}{\sqrt{x}} = 10$. Then, $\sqrt{x} = \frac{250}{10} = 25$.
$\therefore \quad x = (25)^2 = \mathbf{625}.$

9. Ans. (a)

Solution $\frac{\sqrt{256}}{\sqrt{x}} = 2$ or, $\frac{16}{\sqrt{x}} = 2$
$\therefore \quad 16 = 2\sqrt{x} \Rightarrow \sqrt{x} = 8$ **or,** $\boldsymbol{x = 64}.$

10. Ans. (c)

Solution We know that
$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^2 \times 6$
Thus, $\frac{216}{6} = 2^2 \times 3^2 = 6^2$
Therefore, 216 should be divided by 6, so that the result is a perfect square.

11. Ans. (c)

Solution Let $\frac{\sqrt{x}}{200} = 0.02$. Then,
$\sqrt{x} = 200 \times 0.02$ or $\sqrt{x} = 4$
So, $\boldsymbol{x = 16}.$

12. Ans. (c)

Solution $\frac{\sqrt{6727}}{\sqrt{7}} = \frac{\sqrt{6727}}{\sqrt{7}} = \sqrt{961} = \mathbf{31}.$

13. Ans. (a)

Solution $\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = \mathbf{0.3}.$

14. Ans. (a)

Solution $\frac{14}{3+\sqrt{2}} = \frac{14(3-\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{14(3-\sqrt{2})}{9-2}$
$= 2(3 - \sqrt{2}) = 2(3 - 1.414)$
$= 2 \times 1.586 = \mathbf{3.172}.$

15. Ans. (c)

Solution The number nearest to 3579 which is a perfect square is 3600.
$\therefore$ Required number $= 60^2 - 3579 = \mathbf{21}.$

16. Ans. (a)

Solution $\sqrt{\left(1+\frac{27}{169}\right)} = 1 + \frac{x}{13}$

$\therefore \quad \sqrt{\frac{196}{169}} = 1 + \frac{x}{13}$

or, $\frac{14}{13} = 1 + \frac{x}{13}$ or, $\frac{x}{13} = \frac{14}{13} - 1$

or, $\frac{x}{13} = \frac{1}{13}$ **or,** $\boldsymbol{x = 1}.$

17. Ans. (c)

Solution $\frac{\sqrt{4375}}{\sqrt{7}} = \frac{\sqrt{4375}}{\sqrt{7}} = \sqrt{625} = \mathbf{25}.$

18. Ans. (a)

Solution $\sqrt{0.016a} = 0.016 \times \sqrt{b}$
$\Rightarrow \sqrt{\frac{a}{b}} = \frac{0.016}{\sqrt{0.016}} \Rightarrow \sqrt{\frac{a}{b}} = \sqrt{0.016} \Rightarrow \frac{a}{b} = \mathbf{0.016}.$

19. Ans. (b)

Solution $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{3.3^{1/2}}}}} = \sqrt{\sqrt[3]{\sqrt[3]{3.3^{3/4}}}} = \sqrt{\sqrt[3]{3.3^{7/8}}}$
$= \sqrt{3.3^{15/16}} = \mathbf{3^{31/32}}.$

20. Ans. (d)

Solution Let $\frac{\sqrt{1296}}{x} = \frac{x}{2.25}$

Then, $\frac{36}{x} = \frac{x}{2.25}$

or, $x^2 = 36 \times \frac{225}{100}$

$\therefore \quad x = \sqrt{\frac{36 \times 225}{100}} = \frac{6 \times 15}{10} = \mathbf{9}.$

21. Ans. (b)

Solution $\sqrt{176 + \sqrt{2401}} = \sqrt{176 + 49} = \sqrt{225} = 15.$

22. Ans. (a)

Solution $\sqrt{10} \times \sqrt{15} = \sqrt{150} = \sqrt{25 \times 6}$
$= \sqrt{25} \times \sqrt{6} = \mathbf{5\sqrt{6}}.$

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23. Ans. (a)

Solution $\frac{\sqrt{4}}{\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{4}}=\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}=\frac{4-3}{2\sqrt{3}}=\frac{1}{2\sqrt{3}}$.

24. Ans. (b)

Solution $\sqrt{248+\sqrt{52+\sqrt{144}}}=\sqrt{248+\sqrt{52+12}}$

$=\sqrt{248+\sqrt{64}}$

$=\sqrt{248+8}$

$=\sqrt{256}=\mathbf{16}.$

25. Ans. (b)

Solution Given expression $=\frac{\sqrt{0.0009}}{\sqrt{0.01}}=\sqrt{\frac{0.0009}{0.0100}}$

$=\sqrt{\frac{9}{100}}=\frac{3}{10}=\mathbf{0.3}.$

26. Ans. (d)

Solution $\frac{1}{\sqrt{9}-\sqrt{8}}=\frac{1}{\sqrt{9}-\sqrt{8}}\times\frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}}=\frac{3+2\sqrt{2}}{9-8}$

$=\mathbf{3+2\sqrt{2}}.$

27. Ans. (b)

Solution $\sqrt{\frac{x}{169}}=\frac{54}{39}\Rightarrow\frac{x}{169}=\frac{54}{39}\times\frac{54}{39}$

$\therefore\quad x=\frac{54}{39}\times\frac{54}{39}\times169=\mathbf{324}.$

28. Ans. (b)

Solution Let, given expression $= x$

Then, $\sqrt{12+x}=x\Rightarrow 12+x=x^2$

$\therefore\quad x^2-x-12=0$ or, $(x-4)(x+3)=0$

So, $\boldsymbol{x=4}$ **(neglecting** $\boldsymbol{x=-3}$**).**

29. Ans. (d)

Solution Given expression

$=\frac{112}{14}\times\frac{24}{12}\times\frac{16}{8}=\mathbf{32}.$

30. Ans. (c)

Solution $\sqrt{\frac{3}{4}}+2\sqrt{\frac{4}{3}}=\sqrt{12}\left(\frac{1}{4}+\frac{2}{3}\right)$

$=\frac{3.464\times11}{12}=\mathbf{3.1753}.$

31. Ans. (c)

Solution Given expression

$=\sqrt{15625}+\sqrt{\frac{15625}{100}}+\sqrt{\frac{15625}{10000}}$

$=125+\frac{125}{10}+\frac{125}{100}$

$=125+12.5+1.25=\mathbf{138.75}.$

32. Ans. (a)

Solution $\sqrt{0.03\times0.3\times a}=0.03\times0.3\times\sqrt{b}$

$\Rightarrow\sqrt{\frac{a}{b}}=\sqrt{0.03\times0.3}$

$\Rightarrow\frac{a}{b}=0.03\times0.3$

or, $\frac{a}{b}=\mathbf{0.009}.$

33. Ans. (b)

Solution $\sqrt{4096}+\sqrt{40.96}+\sqrt{.004096}$

$=\sqrt{4096}+\sqrt{\frac{4096}{100}}+\sqrt{\frac{4096}{1000000}}$

$=64+\frac{64}{10}+\frac{64}{1000}$

$=64+6.4+0.064=\mathbf{70.464}.$

34. Ans. (b)

Solution $\sqrt{1+\sqrt{1-\frac{2176}{2401}}}=1+\frac{x}{7}$

$\Rightarrow 1+\frac{x}{7}=\sqrt{1+\sqrt{\frac{225}{2401}}}=\sqrt{1+\frac{15}{49}}$

$=\sqrt{\frac{64}{49}}=\frac{8}{7}=1+\frac{1}{7}$

$\therefore\quad x=\mathbf{1}.$

35. Ans. (a)

Solution The square of an odd number cannot have 4 as the unit digit. The square of a 3-digit number will have at least 5 digits and at the most 6 digits.

36. Ans. (c)

Solution Given expression

$=\sqrt{\frac{324\times81\times4624}{15625\times289\times729\times64}}$

(Sum of decimal places being equal in Num. and Denom.)

$=\frac{18\times9\times68}{125\times17\times27\times8}=\frac{3}{125}=\mathbf{0.024}.$

37. Ans. (c)

Solution $\sqrt[3]{\frac{512}{3375}}=\sqrt[3]{\frac{2\times2\times2\times2\times2\times2\times2\times2\times2}{3\times3\times3\times5\times5\times5}}$

$=\frac{2\times2\times2}{3\times5}=\mathbf{\frac{8}{15}}.$

38. Ans. (a)

Solution Given expression

$=\sqrt{0.01+0.08}=\sqrt{0.09}$

$=\sqrt{\frac{9}{100}}=\frac{3}{10}=\mathbf{0.3}.$

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39. Ans. (b)

Solution $\sqrt[3]{15.625} = \sqrt[3]{\frac{15625}{1000}} = \sqrt[3]{\frac{5\times5\times5\times5\times5\times5}{2\times2\times2\times5\times5\times5}}$

$= \frac{5\times5}{2\times5} = \frac{5}{2} = \mathbf{2.5.}$

40. Ans. (c)

Solution $\sqrt[3]{0.000064} = \sqrt[3]{\frac{64}{1000000}}$

$= \sqrt[3]{\frac{4\times4\times4}{100\times100\times100}}$

$= \frac{4}{100} = \mathbf{0.04.}$

41. Ans. (a)

Solution $\sqrt[3]{\sqrt{441}+\sqrt{16}+\sqrt{4}}$

$= \sqrt[3]{21+4+2} = \sqrt[3]{27}$

$= \sqrt[3]{3\times3\times3} = \mathbf{3.}$

42. Ans. (c)

Solution $14175 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 7$

$= 5^2 \times 3^4 \times 7$

It must be multiplied by **7.**

43. Ans. (b)

Solution $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{(\sqrt{5}-\sqrt{3})^2}{5-3} = \frac{5+3-2\sqrt{15}}{2}$

$= \frac{2(4-\sqrt{15})}{2} = \mathbf{4 - \sqrt{15}.}$

44. Ans. (c)

Solution $\frac{\sqrt{24}+\sqrt{216}}{\sqrt{96}} = \frac{\sqrt{4\times6}+\sqrt{36\times6}}{\sqrt{16\times6}}$

$= \frac{2\sqrt{6}+6\sqrt{6}}{4\sqrt{6}} = \frac{8\sqrt{6}}{4\sqrt{6}} = \mathbf{2.}$

45. Ans. (c)

Solution $\sqrt{2\frac{2}{9}} = \frac{\sqrt{20}}{\sqrt{9}} = \frac{\sqrt{20}}{3}$

$= \frac{4.472}{3} = 1.491 \approx \mathbf{1.49.}$

46. Ans. (b)

Solution $a = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-(1)^2}$

$= \frac{5+1+2\sqrt{5}}{5-1}$

$= \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$

$b = \frac{\sqrt{5}-1}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{(\sqrt{5}-1)^2}{(\sqrt{5})^2-(1)^2}$

$= \frac{5+1-2\sqrt{5}}{5-1} = \frac{6+2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$

$a^2 + b^2 = \frac{(3+\sqrt{5})^2+(3-\sqrt{5})^2}{4}$

$= \frac{9+5+6\sqrt{5}+9+5-6\sqrt{5}}{5} = \frac{28}{4} = 7$

$ab = 1$

$\therefore \quad \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{7+1}{7-1} = \frac{8}{6} = \mathbf{\frac{4}{3}.}$

47. Ans. (c)

Solution $10584 = 4 \times 9 \times 2 \times 7 \times 7 \times 3$

$= 2^2 \times 3^2 \times 7^2 \times 2 \times 3$

This must be multiplied by **6.**

48. Ans. (b)

Solution $7936 = 4 \times 4 \times 4 \times 4 \times 31$

To make it a perfect square, we multiply it by 31

$\therefore$ Required smallest number

$= 7936 \times 31 = \mathbf{246016.}$

49. Ans. (b)

Solution $\sqrt{.00059049} = \sqrt{\frac{59049}{100000000}}$

$= \frac{\sqrt{59049}}{\sqrt{100000000}}$

$= \frac{243}{10000} = \mathbf{0.0243.}$

50. Ans. (c)

Solution $\sqrt{\frac{4}{12.1}} = \sqrt{\frac{4\times10}{12.1\times10}} = \frac{\sqrt{4}\times\sqrt{10}}{\sqrt{121}}$

$= \frac{2}{11} \times \sqrt{10} = \frac{2}{11} \times 3.16$

$= \frac{6.32}{11} = 0.57 \approx \mathbf{0.6.}$

51. Ans. (d)

Solution Given expression

$= \sqrt{\frac{256\times81\times4356}{15625\times121\times1296\times64}}$

$= \frac{16\times9\times66}{125\times11\times36\times8} = \mathbf{0.024.}$

52. Ans. (a)

Solution The number of men in the front row is the square root of 16160 – 31, that is **16129** which is **127.**

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53. Ans. (c)

Solution Ratio of their sides is the ratio of their square roots

$= \sqrt{420.25} : \sqrt{441}$

$= 20.5 : 21$

$= \mathbf{41 : 42.}$

54. Ans. (b)

Solution The number of men in the front row is the square root of 5180 + 4, that is 5184 which is 72.

55. Ans. (c)

Solution The largest number of three digits is 999

	31
3	999
	9
61	99
	61
	38

$\therefore$ Required number is $(31)^2 = \mathbf{961.}$

56. Ans. (c)

Solution $21\frac{15}{289} = \frac{6084}{289} = \left(\frac{78}{17}\right)^2$

$\therefore$ Square root $= \frac{78}{17} = 4\frac{10}{17}.$

$\therefore$ Least fraction to be subtracted $= \mathbf{\frac{10}{17}}.$

57. Ans. (c)

Solution

	250
2	$\overline{62}$ 5 $\overline{12}$
	4
45	225
	225
50	12

So, 12 is the smallest number which when subtracted from 62512 makes it a perfect square

$\sqrt{62500} = \mathbf{250.}$

58. Ans. (c)

Solution Largest no. of 5 digits = 99999

	316
3	9 $\overline{99}$ $\overline{99}$
	9
61	99
	61
625	3899
	3756
	143

Reqd. number $= (316)^2 = \mathbf{99856.}$

59. Ans. (d)

Solution $2450 = 5 \times 5 \times 7 \times 7 \times 2$

$\therefore$ 2450 must be multiplied by 2.

60. Ans. (b)

Solution $3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 2^2 \times 2^2 \times 3^2 \times 5^2$

$\therefore$ 3600 should be multiplied by $2 \times 2 \times 3 \times 5$ That is 60 to make it a perfect cube.

QUESTIONS FROM COMPETITIVE EXAMS

1. Taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{6} = 2.449$, then the value of $\frac{9+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{6-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ to three places of decimals is

(a) 9.231 (b) 13.716
(c) 11.723 (d) 15.892

[Based on MAT 2002]

2. The cube root of 1.061208 is

(a) 1.022 (b) 10.22
(c) 0.102 (d) 1.02

[Based on MAT 2002]

3. Which one of the following is the largest?

$2\sqrt{5}, 6\sqrt{3}, 3\sqrt{7}$ and $8\sqrt{2}$

(a) $8\sqrt{2}$ (b) $2\sqrt{5}$
(c) $6\sqrt{3}$ (d) $3\sqrt{7}$

[Based on MAT 2001]

4. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

(a) 32 (b) 34
(c) 35 (d) 36

[Based on MAT 2005]

5. $\frac{\sqrt{8750}}{\sqrt{14}}$ is equal to

(a) 24.75 (b) 27.25
(c) 25 (d) 35

[Based on MAT 2005]

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6. The value of $\sqrt[4]{(625)^3}$ is
(a) 25 (b) 125
(c) $\sqrt[3]{(1875)}$ (d) None of these
[Based on FMS (Delhi) 2006]

7. The sum of four consecutive two-digit odd number, when divide by 10, becomes a perfect square. Which of the following can possible be one of these four numbers?
(a) 21 (b) 25
(c) 41 (d) 67
(e) 73
[Based on CAT 2006]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution $\frac{9+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{6-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$=\frac{9\sqrt{5}-9\sqrt{3}+\sqrt{10}-\sqrt{6}+6\sqrt{5}+6\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$

$=\frac{15\sqrt{5}-3\sqrt{3}-2\sqrt{6}}{2}$

$=\frac{15\times 2.236-3\times 1.732-2\times 2.449}{2}$

$=\frac{33.54-5.196-4.898}{2}=$ **11.723.**

2. Ans. (d)

3. Ans. (a)

4. Ans. (d)

Solution No. of camels seen in the forest $=\frac{36}{4}=9$

No. of camels gone to the mountains $=2\sqrt{36}=12$

No. of camels seen on the bank of river = 15

∴ Total number of camels = 9 + 12 + 15 = **36.**

5. Ans. (c)

6. Ans. (b)

7. Ans. (c)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. $\frac{\sqrt{324}}{1.5}=\frac{?}{\sqrt{256}}$
(a) 192 (b) 432
(c) 288 (d) 122

2. Find the value of $\sqrt[3]{50\ \sqrt{121}\ \sqrt{9}}$
(a) 6 (b) 8
(c) 4 (d) 12

3. If $\sqrt{2^n}=64$, then the value of n is
(a) 2 (b) 4
(c) 6 (d) 12

4. Find the least number by which when 750 is divided or multiplied, then it becomes a perfect square.
(a) 25 (b) 15
(c) 30 (d) 35

5. $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}=?$
(a) $\sqrt{2}$ (b) 2
(c) 4 (d) 8

6. Each student in a class contributed as many paise as the number of students in the class. The teacher contributed Rs. 13 to make the total collection of Rs. 49. How many students were there in the class?
(a) 36 (b) 69
(c) 70 (d) None of these

7. $2+\frac{1}{\sqrt{2}}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}=?$
(a) $2-\frac{1}{\sqrt{2}}$ (b) $2+\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

8. The least perfect square number divisible by 3, 4, 5, 6, 8 is
(a) 900 (b) 1200
(c) 2500 (d) 3600

9. If $\sqrt{3}=1.732$ and $\sqrt{2}=1.414$, then the value of $\frac{1}{\sqrt{3}+\sqrt{2}}$ is
(a) 0.064 (b) 0.308
(c) 0.318 (d) 2.146

10. If $\sqrt{3}=1.732$, value of $\frac{3}{2+\sqrt{3}}$ is
(a) 0.268 (b) 11.196
(c) 0.804 (d) None of these

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Test II **[Time: 10 min]**

11. If $\sqrt{6} = 2.55$, then the value of $\sqrt{\frac{2}{3}} + 3\sqrt{\frac{3}{2}}$ is
(a) 4.48 (b) 4.49
(c) 4.50 (d) None of these

12. Find the least number by which when 2205 is divided or multiplied, then it becomes a perfect square
(a) 25 (b) 15
(c) 10 (d) 5

13. If $\sqrt{(75.24+x)} = 8.71$, then the value of x is
(a) 0.6241 (b) 6.241
(c) 62. 41 (d) None of these

14. A man after a tour, finds that he had spent every day as many Rs. 5 notes as the number of days he has been away from home, and he spent in all Rs. 9245. How long did his tour last?
(a) 95 (b) 45
(c) 43 (d) 47

15. The smallest number that should be added to 13218 to make it a perfect square is
(a) 11 (b) 3
(c) 17 (d) 7

16. By what least number, 6728 is divided so that the resulting number is the perfect square?
(a) 2 (b) 4
(c) 7 (d) None of these

17. If $\sqrt{24} = 4.899$, then the value of $\sqrt{\frac{8}{3}}$ is
(a) 0.544 (b) 2.666
(c) 1.633 (d) 1.333

18. By what smallest number must the number 21600 be multiplied in order that the resulting number may be a perfect cube?
(a) 100 (b) 10
(c) 5 (b) 20

19. The smallest number of six digits which is a perfect square is
(a) 100489 (b) 100000
(c) 101124 (d) 101761

20. Find the square root of $\dfrac{5}{6+\dfrac{7}{8\dfrac{10}{11}}}$ up to two decimal places.
(a) 0.85 (b) 0.56
(c) 0.71 (d) 0.32

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution $\dfrac{\sqrt{324}}{1.5} = \dfrac{x}{\sqrt{256}}$ or, $\dfrac{18}{1.5} = \dfrac{x}{16}$

$\therefore \quad x = \dfrac{18\times16}{1.5} = \dfrac{18\times16\times10}{5}$

$\therefore \quad x = 192.$

3. Ans. (d)

Solution $\sqrt{2^n} = 64 = 2^6 \Rightarrow 2^{n/2} = 2^6$

So, $\dfrac{n}{2} = 6$ **or,** $n = 12.$

5. Ans. (b)

Solution $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \dfrac{\sqrt{16\times2}+\sqrt{16\times3}}{\sqrt{4\times2}+\sqrt{4\times3}}$

$= \dfrac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}}$

$= \dfrac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = 2.$

7. Ans. (a)

Solution $2 + \dfrac{\sqrt{2}}{2} + \dfrac{1}{2+\sqrt{2}}\times\dfrac{2-\sqrt{2}}{2-\sqrt{2}} + \dfrac{1}{\sqrt{2}-2}\times\dfrac{\sqrt{2}+2}{\sqrt{2}+2}$

$= 2 + \dfrac{\sqrt{2}}{2} + \dfrac{1}{2}(2-\sqrt{2}) - \dfrac{1}{2}(\sqrt{2}+2)$

$= 2 + \dfrac{\sqrt{2}}{2} + 1 - \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2} - 1$

$= 2 - \sqrt{2}/2 = 2 - 1/\sqrt{2}.$

9. Ans. (c)

Solution $\dfrac{1}{\sqrt{3}+\sqrt{2}} = \dfrac{1}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \dfrac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$

$= 1.732 - 1.414 = 0.318.$

11. Ans. (d)

Solution $\sqrt{\frac{2}{3}} + 3.\sqrt{\frac{3}{2}} = \dfrac{\sqrt{2}}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}} + 3.\dfrac{\sqrt{3}}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{6}}{3} + \dfrac{3\sqrt{6}}{2} = \dfrac{2.55}{3} + \dfrac{3\times2.55}{2}$

$= \dfrac{2.55}{3} + \dfrac{7.65}{2} = \dfrac{5.10+22.98}{6}$

$= \dfrac{28.05}{6} = 4.675.$

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PA54

13. Ans. (a)

Solution $75.24 + x = 8.71 \times 8.71$

or, $x = \mathbf{0.6241.}$

15. Ans. (d)

Solution $13218 = (115)^2 - 7.$

17. Ans. (c)

Solution $\sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{24}}{3} = \frac{4.899}{3} = \mathbf{1.633.}$

19. Ans. (a)

Solution Smallest no. of six digits = 10000

		316		
3	10	$\overline{00}$	$\overline{00}$	
	9			
61	100			
	61			
626	3900			
	3756			
	144			

Reqd. no $= (317)^2 = \mathbf{100489.}$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (c)	**6.** (d)	**8.** (d)	**10.** (c)	**12.** (d)	**14.** (c)	**16.** (a)	**18.** (b)	**20.** (a)

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4 Simplification

SIMPLE ARITHMETIC OPERATIONS

It is a common need to simplify the expressions formulated according to the statements of the problems relating to practical life. To do this, it is essential to follow in sequence the mathematical operations given by the term "BODMAS".

BODMAS

Each letter of the word BODMAS stands as follows:

B for Bracket : [{(–)}]

There are four brackets, namely, – bar, ( ), { } and [ ]. They are removed, strictly in the order –, ( ), { } and [ ].

O for Of : of

D for Division : ÷

M for Multiplication : ×

A for Addition : +

S for Subtraction : –

The order of various operations in exercises involving brackets and fractions must be performed strictly according to the order of the letters of the word BODMAS.

Note: Here, $-\overline{5-8} = -(-3) = 3$.

Illustration 1 Simplify:

$$8\frac{1}{2}-\left[3\frac{1}{5}\div 4\frac{1}{2} \text{ of } 5\frac{1}{3}+\left\{11-\left(3-1\frac{\overline{1}}{4}-\frac{5}{8}\right)\right\}\right]$$

Solution Given expression

$$= \frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\left\{11-\left(3-\frac{\overline{5}}{4}-\frac{5}{8}\right)\right\}\right]$$

$$= \frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\left\{11-\left(3-\frac{5}{8}\right)\right\}\right]$$

$$= \frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\left\{11-\frac{19}{8}\right\}\right]$$

$$= \frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2} \text{ of } \frac{16}{3}+\frac{69}{8}\right]$$

$$= \frac{17}{2}-\left[\frac{16}{5}\div\frac{9}{2}\times\frac{16}{3}+\frac{69}{8}\right]$$

$$= \frac{17}{2}-\left[\frac{16}{5}\div\frac{24}{1}+\frac{69}{8}\right]$$

$$= \frac{17}{2}-\left[\frac{16}{5}\times\frac{1}{24}+\frac{69}{8}\right] = \frac{17}{2}-\left[\frac{16}{120}+\frac{69}{8}\right]$$

$$= \frac{17}{2}-\left[\frac{16+1035}{120}\right] = \frac{17}{2}-\frac{1051}{120}$$

$$= \frac{1020-1051}{120} = -\frac{\mathbf{31}}{\mathbf{120}}.$$

Illustration 2 Simplify:

$$5\frac{1}{3}-\left\{4\frac{1}{3}-\left(3\frac{1}{3}-2\frac{\overline{1}}{3}-\frac{1}{3}\right)\right\}$$

Solution Given expression

$$= \frac{16}{3}-\left\{\frac{13}{3}-\left(\frac{10}{3}-\frac{\overline{7}}{3}-\frac{1}{3}\right)\right\}$$

$$= \frac{16}{3}-\left\{\frac{13}{3}-\left(\frac{10}{3}-\frac{6}{3}\right)\right\} = \frac{16}{3}-\left\{\frac{13}{3}-\frac{4}{3}\right\}$$

$$= \frac{16}{3}-\left\{\frac{9}{3}\right\} = \frac{16}{3}-\frac{9}{3} = \frac{7}{3} = \mathbf{2\frac{1}{3}}.$$

Use of Algebraic Formulae

The following important formulae are sometimes found useful in dealing with the simplifications:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$
4. $(a+b)^2 - (a-b)^2 = 4ab$
5. $a^2 - b^2 = (a+b)(a-b)$
6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab(a+b)$
7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab(a-b)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

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10. $\dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a+b+c)$.

11. $a^4 - a^4 = (a^2+b^2)(a+b)(a-b)$.

Illustration 3 Simplify the following:

(i) $0.32 \times 0.32 + 0.64 \times 0.68 + 0.68 \times 0.68$

Solution Given expression

$= 0.32 \times 0.32 + 2 \times 0.32 \times 0.68 + 0.68 \times 0.68$

$= (0.32)^2 + 2 \times 0.32 \times 0.68 + (0.68)^2$

$= (0.32 + 0.68)^2 \quad [\because a^2 + 2ab + b^2 = (a+b)^2]$

$= 1^2 = 1.$

(ii) $2.45 \times 2.45 - 0.9 \times 2.45 + 0.45 \times 0.45$

Solution Given expression

$= 2.45 \times 2.45 - 2 \times 2.45 \times 0.45 + 0.45 \times 0.45$

$= (2.45)^2 - 2 \times 2.45 \times 0.45 + (0.45)^2$

$= (2.45 - 0.45)^2 \quad [\because a^2 - 2ab + b^2 = (a-b)^2]$

$= (2)^2 = 4.$

(iii) $\dfrac{7\times\{(146+92)^2+(146-92)^2\}}{(146)^2+(92)^2}$

Solution Given expression

$= \dfrac{7\times 2\{(146)^2+(92)^2\}}{(146)^2+(92)^2}$

$[\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)]$

$= \mathbf{14.}$

(iv) $\dfrac{(0.345+0.255)^2-(0.345-0.255)^2}{0.345\times 1.02}$

Solution Given expression

$= \dfrac{(0.345+0.255)^2-(0.345-0.255)^2}{4\times 0.345\times 0.255}$

$= \dfrac{4\times 0.345\times 0.255}{4\times 0.345\times 0.255}$ $[\because (a+b)^2 - (a-b)^2 = 4ab]$

$= \mathbf{1.}$

(v) $\dfrac{0.682\times 0.682-0.318\times 0.318}{0.682-0.318}$

Solution Given expression

$= \dfrac{(0.682)^2-(0.318)^2}{0.682-0.318}$

$= (0.682 + 0.318) \quad \left[\because \dfrac{a^2-b^2}{a-b} = a+b\right]$

$= \mathbf{1.}$

(vi) $\dfrac{(3.29)^2-(0.81)^2}{4}$

Solution Given expression

$= \dfrac{(3.29)^2-(0.81)^2}{3.29+0.81}$

$= (3.29 - 0.81) \quad \left[\because \dfrac{a^2-b^2}{a+b} = a-b\right]$

$= \mathbf{2.48.}$

(vii) $(2.35)^3 + 1.95 \times (2.35)^2 + 7.05 \times (0.65)^2 + (0.65)^3$

Solution Given expression

$= (2.35)^3 + 3 \times 0.65 \times (2.35)^2$

$+ 3 \times 2.35 \times (0.65)^2 + (0.65)^3$

$= (2.35 + 0.65)^3$

$[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3]$

$= (3)^3 = \mathbf{27.}$

(viii) $\dfrac{(4.32)^3-0.96\times(4.32)^2+12.96\times(0.32)^2-(0.32)^3}{4\times 4\times 4}$

Solution Given expression

$= \dfrac{(4.32)^3-3\times 0.32\times(4.32)^2+3\times 4.32\times(0.32)^2-(0.32)^3}{4\times 4\times 4}$

$= \dfrac{(4.32-0.32)^3}{4^3}$ $[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3]$

$= \left(\dfrac{4}{4}\right)^3 = \mathbf{1.}$

(ix) $\dfrac{885\times 885\times 885+115\times 115\times 115}{885\times 885+115\times 115-885\times 115}$

Solution Given expression

$= \dfrac{(885)^3+(115)^3}{(885)^2+(115)^2-885\times 115}$

$= (885 + 115) \quad \left[\because \dfrac{a^3+b^3}{a^2-ab+b^2} = a+b\right]$

$= \mathbf{1000.}$

(x) $\dfrac{0.62\times 0.62\times 0.62-0.41\times 0.41\times 0.41}{0.62\times 0.62+0.62\times 0.41+0.41+0.41}$

Solution Given expression

$= \dfrac{(0.62)^3-(0.41)^3}{(0.62)^2+0.62\times 0.41+(0.41)^2}$

$= (0.62 - 0.41) \quad \left[\because \dfrac{a^3-b^3}{a^2+ab+b^2} = a-b\right]$

$= \mathbf{0.21.}$

(xi) $\dfrac{(2.3)^3+(1.5)^3+(1.2)^3-3\times 2.3\times 1.5\times 1.2}{2.3\times 2.3+1.5\times 1.5+1.2\times 1.2-2.3\times 1.5-2.3\times 1.2-1.5\times 1.2}$

Solution Given expression

$= \dfrac{(2.3)^3+(1.5)^3+(1.2)^3-3\times 2.3\times 1.5\times 1.2}{(2.3)^2+(1.5)^2+(1.2)^2-2.3\times 1.5-2.3\times 1.2-1.5\times 1.2}$

$= (2.3 + 1.5 + 1.2)$

$\left[\because \dfrac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-ac-bc} = a+b+c\right]$

$= \mathbf{5.}$

Surds and Indices

a^n is called a **surd** if n is a fraction and a^n is called an **index** if n is an integer. a is called the **base**.

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SOME USEFUL FORMULAE

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = (a^n)^m = a^{mn}$
4. $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$.
5. $a^m \div b^{-n} = a^m \times b^n$
6. $(\sqrt[n]{a})^n = a$, where 'n' is a +ve integer and 'a' a +ve rational number
7. $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$, where '$n$' is a +ve integer and '$a$', '$b$' are rational numbers
8. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, where '$n$' is a +ve integer and '$a$', '$b$' are rational numbers
9. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where '$m$', '$n$' are +ve integers and '$a$' is a +ve rational number
10. $\sqrt[n]{\sqrt[m]{(a^k)^m}} = \sqrt[n]{a^k} = \sqrt[mn]{a^{km}}$, where '$m$', '$n$', '$k$' are +ve integers and '$a$' is a +ve rational number
11. $\sqrt{a} \times \sqrt{a} = a$
12. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
13. $(\sqrt{a}+\sqrt{b})^2 = a + b + 2\sqrt{ab}$
14. $(\sqrt{a}-\sqrt{b})^2 = a + b - 2\sqrt{ab}$
15. $a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c$ and $b = d$.
16. $\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})} = \frac{\sqrt{a}+\sqrt{b}}{a-b}$
17. $\frac{1}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{a}-\sqrt{b}}{a-b}$
18. If $x = n(n+1)$, then
    (a) $\sqrt{x-\sqrt{x-\sqrt{x-\ldots\infty}}} = n$ and
    (b) $\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}} = (n+1)$.

Illustration 4 Find the value of $(243)^{0.8} \div (243)^{0.4}$.

Solution $(243)^{0.8} \div (243)^{0.4} = (243)^{0.8-0.4}$ $[\because a^m \div a^n = a^{m-n}]$

$= (243)^{0.4}$

$= (3^5)^{\frac{2}{5}} = 3^2 = \mathbf{9}$.

Illustration 5 Find the value of $(27)^{2/3} \div (64)^{-4/3}$

Solution $(27)^{2/3} \div (64)^{-4/3} = (3^3)^{2/3} \times (64)^{4/3}$ $[\because a^m \div b^{-n} = a^m \times b^n]$

$= 3^2 \times (4^3)^{4/3}$

$= 9 \times (4^4) = 9 \times 256 = \mathbf{2304}$.

Illustration 6 Find the value of $(-3)^{(-2)^{(-2)^{(-4)}}}$

Solution $(-3)^{(-2)^{(-2)^{(-4)}}} = \left(-\frac{1}{3}\right)^{(2)^{(-2)^{(-4)}}} = \left(\frac{1}{9}\right)^{(-2)^{(-4)}}$

$= (9)^{(2)^{(-4)}}$

$= (81)^{-4} = \left(\frac{1}{81}\right)^4$

$= \left(\frac{1}{3^4}\right)^4 = \frac{\mathbf{1}}{\mathbf{3}}$.

Illustration 7 Find the value of x if $\sqrt[5]{2x-7} - 3 = 0$.

Solution We have,

$\sqrt[5]{2x-7} - 3 = 0 \Rightarrow \sqrt[5]{2x-7} = 3$

$\Rightarrow (\sqrt[5]{2x-7})^5 = 3^5$

$\Rightarrow 2x - 7 = 243$ $[\because (\sqrt[n]{a})^n = a]$

$\Rightarrow 2x = 250$ or $x = \mathbf{125}$.

Illustration 8 Find the value of $\sqrt[5]{64} \times \sqrt[5]{512}$.

Solution $\sqrt[5]{64} \times \sqrt[5]{512}$

$= \sqrt[5]{64 \times 512}$ $[\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}]$

$= \sqrt[5]{8^2 \times 8^3} = \sqrt[5]{8^5} = \mathbf{8}$. $[\because \sqrt[n]{a^n} = a]$

Illustration 9 Find the value of $\sqrt[3]{\sqrt[2]{729}}$.

Solution $\sqrt[3]{\sqrt[2]{729}} = \sqrt[6]{729}$ $[\because \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}]$

$= \sqrt[6]{3^6} = \mathbf{3}$. $[\because \sqrt[n]{a^n} = a]$

Illustration 10 Find the value of $\frac{\sqrt[7]{\sqrt[5]{(21^7)^5}}}{\sqrt[5]{\sqrt[3]{(7^5)^3}}}$.

Solution Given expression

$= \frac{\sqrt[7]{(21)^7}}{\sqrt[5]{(7)^5}}$ $\left[\because \sqrt[n]{\sqrt[m]{(a^p)^m}} = \sqrt[n]{a^p}\right]$

$= \frac{21}{7} = \mathbf{3}$. $[\because \sqrt[n]{a^n} = a]$

Illustration 11 Find the value of $\sqrt{5} \times \sqrt{125}$.

Solution $\sqrt{5} \times \sqrt{125} = \sqrt{625}$ $[\because \sqrt{a} \times \sqrt{b} = \sqrt{ab}]$

$= \mathbf{25}$.

Illustration 12 Simplify each of the following by rationalising the denominator.

(i) $\frac{1}{2+\sqrt{3}}$ (ii) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Solution (i) $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2-\sqrt{3}}{4-3} = \mathbf{2-\sqrt{3}}.$

(ii) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{4^2\times3}+\sqrt{3^2\times2}}$

$= \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

$= \frac{(7\sqrt{3}-5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3}+3\sqrt{2})(4\sqrt{3}-3\sqrt{2})}$

$= \frac{7\sqrt{3}\times4\sqrt{3}-7\sqrt{3}\times3\sqrt{2}-5\sqrt{2}\times4\sqrt{3}+5\sqrt{2}\times3\sqrt{2}}{(4\sqrt{3})^2-(3\sqrt{2})^2}$

$= \frac{28\sqrt{3\times3}-21\sqrt{3\times2}-20\sqrt{2\times3}+15\sqrt{2\times2}}{16\times3-9\times2}$

$= \frac{28\times3-21\times\sqrt{6}-20\sqrt{6}+15\times2}{48-18}$

$= \frac{84-(21\times20)\sqrt{6}+30}{30} = \mathbf{\frac{114-41\sqrt{6}}{30}}.$

Illustration 13 If a and b are rational numbers, find the values of a and b in the following equation:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}.$$

Solution $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2} = \frac{5+2\sqrt{6}}{1}$

$= 5 + 2\sqrt{6}$

$\therefore \quad \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6} \Rightarrow 5 + 2\sqrt{6} = a + b\sqrt{6}.$

On equating rational and irrational parts, we get

$$\boldsymbol{a = 5 \text{ and } b = 2.}$$

Illustration 14 Find the value of

$$\left(\sqrt{72+\sqrt{72+\sqrt{72+\ldots\infty}}}\right) \div \left(\sqrt{12-\sqrt{12-\sqrt{12-\ldots\infty}}}\right)$$

Solution Since $72 = 9 \times 8$,

therefore, $72 + \sqrt{72+\sqrt{72+\ldots\infty}} = 9$

Also, since $12 = 4 \times 3$

therefore, $\sqrt{12-\sqrt{12-\sqrt{12-\ldots\infty}}} = 3.$

Thus, the given expression $= \frac{9}{3} = \mathbf{3}.$

Fractions

1. Continued Fraction

Fractions of the form $7 + \cfrac{2}{5+\cfrac{3}{4+\cfrac{2}{3+\cfrac{1}{4}}}}$

are called continued fractions.

To simplify a continued fraction, we start from the bottom and work upwards.

Illustration 15 Simplify $3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{5}{6+\cfrac{2}{3}}}}$.

Solution $3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{5}{6+\cfrac{2}{3}}}} = 3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{15}{20}}}$

[Multiply the numerator and denominator of lowest term $\frac{5}{\left(6+\frac{2}{5}\right)}$ by 3 to get $\frac{15}{20}$.]

$= 3 - \cfrac{1}{4+\cfrac{7}{9-\cfrac{3}{4}}} = 3 - \cfrac{1}{4+\cfrac{28}{33}}$

[Multiply the numerator and denominator of the lowest term $\frac{7}{\left(9-\frac{3}{4}\right)}$ by 4 to get $\frac{28}{33}$]

$= 3 - \frac{33}{160}$

[Multiply the numerator and denominator of the term $\frac{1}{\left(4+\frac{28}{33}\right)}$ by 33 to get $\frac{33}{160}$]

$= \frac{480-33}{160} = \frac{447}{160} = \mathbf{2\frac{127}{160}}.$

2. Comparison of Fractions

The following points are found useful while comparing two or more fractions:

(a) If the denominators of the fractions are same, the largest is one whose numerator is the largest.

Illustration 16 Which is the largest fraction among $\frac{3}{8}$, $\frac{7}{8}$ and $\frac{5}{8}$?

Solution $\mathbf{\frac{7}{8}}.$

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(b) If the numerators of the fractions are same, the largest is one whose denominator is the smallest.

Illustration 17 Which is the largest fraction among $\frac{5}{2}, \frac{5}{7}$ and $\frac{5}{9}$?

Solution $\frac{5}{2}$.

(c) If neither the numerators nor denominators of the fractions are same then they are converted into equivalent fractions of the same denominator by taking the L.C.M. of the denominators of the given fractions. Then, the fractions are compared according to (1).

Illustration 18 Which is the largest fraction among $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$ and $\frac{5}{8}$?

Solution L.C.M. of 2, 3, 5 and 8 = 120.

Then, $\frac{1}{2} = \frac{1\times60}{2\times60} = \frac{60}{120}$

$\frac{2}{3} = \frac{2\times40}{3\times40} = \frac{80}{120}$

$\frac{4}{5} = \frac{4\times24}{5\times24} = \frac{96}{120}$

and, $\frac{5}{8} = \frac{5\times15}{8\times15} = \frac{75}{120}$

Now, the denominator of these fractions are same and the largest numerator is 96. Hence, the largest fraction is $\frac{96}{120}$, that is $\frac{4}{5}$.

(d) Two fractions can also be compared by cross multiplication method.

Illustration 19 Which is greater $\frac{6}{13}$ or $\frac{5}{7}$?

Solution **Step 1.** Cross multiply the two given fractions

$$\frac{6}{13} \times \frac{5}{7},$$

we get $6 \times 7 = 42$ and $13 \times 5 = 65$.

Step 2. Since 65 is greater than 42 and in 65, the numerator of $\frac{5}{7}$ is included, $\therefore$ $\frac{5}{7}$ is greater than $\frac{6}{13}$.

(e) If the difference of the numerator and denominator of each of the given fractions be same then the fraction of the largest numerator is the smallest.

Illustration 20 Which one of the following fractions is the largest?

$$\frac{2}{3}, \frac{3}{4}, \frac{5}{6} \text{ and } \frac{9}{10}$$

Solution Since in each of the given fractions the difference between the numerator and denominator is same and the largest numerator is 9, therefore, the largest fraction is $\frac{9}{10}$.

(f) In the given fractions, $\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \ldots, \frac{x+na}{y+nb}$, where $a < b$

(a) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}} >$ first fraction, the last value is the greatest

(b) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}} <$ first fraction, the last value is the least.

(c) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}} =$ first fraction, all values are equal.

Illustration 21 Which one the following fractions is the greatest?

$$\frac{3}{8}, \frac{4}{11}, \frac{5}{14}, \frac{6}{17}, \frac{7}{20}.$$

Solution Since, $\frac{\text{increase in numerator}}{\text{increase in denominator}} = \frac{1}{3}$ is less than the first fraction $\frac{3}{8}$, therefore, the first fraction $\frac{3}{8}$ is the greatest.

Illustration 22 Which of the following fractions is the least?

$$\frac{2}{5}, \frac{4}{11}, \frac{6}{17}, \frac{8}{23}$$

Solution Since, $\frac{\text{increase in numerator}}{\text{increase in denominator}} = \frac{2}{6} = \frac{1}{3}$ is less than the first fraction $\frac{2}{5}$, therefore, the last fraction $\frac{8}{23}$ is the least.

3. Inserting a fraction between two given fractions

To insert a fraction between two given fractions $\frac{a_1}{b_1}$ and $\frac{a_2}{b_2}$, the following steps may be useful:

Step 1 The numerators of the two given fractions are added to get the numerator of the resulting fraction, that is, $a_1 + a_2$.

Step 2 The denominators of the two given fractions are added to get the denominator of the resulting fraction, that is, $b_1 + b_2$.

Step 3. Resulting fraction = $\frac{a_1+a_2}{b_1+b_2}$.

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Illustration 23 Insert one fraction between $\frac{2}{5}$ and $\frac{4}{7}$.

Solution Using the above method,

$$\frac{2}{5}, \frac{2+4}{5+7}, \frac{4}{7} = \frac{2}{5}, \frac{6}{12}, \frac{4}{7} \text{ or, } \frac{2}{5}, \frac{1}{2}, \frac{4}{7}.$$

Illustration 24 Insert three fractions between $\frac{5}{7}$ and $\frac{9}{11}$.

Solution Using the above method,

$$\frac{5}{7}, \frac{5+9}{7+11}, \frac{9}{11} = \frac{5}{7}, \frac{14}{18}, \frac{9}{11} \text{ or } \frac{5}{7}, \frac{7}{9}, \frac{9}{11}.$$

Further,

$$\frac{5}{7}, \frac{5+7}{7+9}, \frac{7}{9}, \frac{7+9}{9+11}, \frac{9}{11} = \frac{5}{7}, \frac{12}{16}, \frac{7}{9}, \frac{16}{20}, \frac{9}{11}$$

or $$\frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}.$$

Thus, the three fractions inserted between $\frac{5}{7}$ and $\frac{9}{11}$ are $\frac{3}{4}, \frac{7}{9}$ and $\frac{4}{5}$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Simplify

$$\frac{3}{10}+\frac{3}{7} \text{ of } \left(2\frac{3}{10}+2\frac{3}{5}\right) \div \frac{1}{5} \div 1\frac{2}{5}-\frac{2}{7}$$

(a) 1 (b) 2
(c) 0 (d) 3

2. $1+1+\left\{1+1+\left(1-\frac{1}{3}\right)\right\} = ?$

(a) $\frac{7}{5}$ (b) $\frac{2}{3}$
(c) $\frac{4}{5}$ (d) None of these

3. $48+12\times\left(\frac{9}{8} \text{ of } \frac{4}{3}+\frac{3}{4} \text{ of } \frac{2}{3}\right) = ?$

(a) 9 (b) 12
(c) 15 (d) None of these

4. Simplify:

$2 + [2 + 2 + \{2 + 2 + (2 + 2 + 3)\}]$

(a) 13/15 (b) 17/15
(c) 11/15 (d) None of these

5. $7\frac{1}{2}-\left[2\frac{1}{4}+\left\{1\frac{1}{4}-?\left(1\frac{1}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right] = 3$

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{4}{3}$ (d) None of these

6. The simplification of $\frac{0.8\times0.8\times0.8-0.5\times0.5\times0.5}{0.8\times0.8+0.8\times0.5+0.5\times0.5}$ gives

(a) 0.8 (b) 0.4
(c) 0.3 (d) 0.13

7. The simplification of $\left[\frac{1}{2}+\frac{1}{2}\left\{\frac{3}{4}-\frac{1}{2}\left(\frac{7}{8}-\frac{3}{4}\right)\right\}\right]$ yields

(a) $\frac{27}{16}$ (b) $\frac{27}{32}$
(c) $\frac{27}{64}$ (d) $\frac{107}{112}$

8. Simplify: $1 - [2 - \{5 - (4 - \overline{3-2})\}]$

(a) 1 (b) 2
(c) 3 (d) 4

9. $3 + \left[(8-5)+\left\{(4-2)+\left(2+\frac{8}{13}\right)\right\}\right] = ?$

(a) $\frac{33}{71}$ (b) $\frac{55}{17}$
(c) $\frac{13}{17}$ (d) None of these

10. $\frac{69842\times69842-30158\times30158}{69842-30158} = ?$

(a) 100000 (b) 69842
(c) 39684 (d) 30158

11. Simplify $\frac{2\frac{1}{7}-2\frac{1}{2}}{2\frac{1}{4}+1\frac{1}{7}} \div \frac{1}{2+\frac{1}{2+\frac{1}{2-\frac{1}{2}}}}$

(a) $\frac{-1}{2}$ (b) $-\frac{1}{8}$
(c) $-\frac{1}{6}$ (d) $-\frac{1}{4}$

12. The value of $\frac{2.75\times2.75\times2.75-2.25\times2.25\times2.25}{2.75\times2.75+2.75\times2.25+2.25\times2.25}$ is

(a) 0.30 (b) 0.50
(c) 3.00 (d) 5.00

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13. $\dfrac{\frac{1}{2}+4+20}{\frac{1}{2}\times 4+20} = ?$

(a) $\frac{81}{88}$ (b) $2\frac{3}{11}$
(c) $\frac{161}{176}$ (d) 1

14. Evaluate $\dfrac{0.53\times 0.53-2\times 0.53\times 0.41+0.41\times 0.41}{0.53\times 0.41}$

(a) 0.16 (b) 0.8
(c) 0.12 (d) None of these

15. The value of $\dfrac{9^2\times 18^4}{3^{16}}$ is

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$
(c) $\frac{16}{81}$ (d) $\frac{32}{243}$

16. The simplification of $1+\dfrac{1}{2+\dfrac{1}{1-\frac{1}{3}}}$ yields the result

(a) $\frac{2}{7}$ (b) $\frac{7}{9}$
(c) $\frac{9}{7}$ (d) $\frac{13}{7}$

17. $108 + ?$ of $\frac{1}{3}+\frac{2}{5}\times 3\frac{3}{4} = 10\frac{1}{2}$

(a) 15 (b) 63
(c) 24 (d) 36

18. The value of

$1+\frac{1}{4\times 3}+\frac{1}{4\times 3^2}+\frac{1}{4\times 3^3}$ up to four places of decimals is:

(a) 1.1202 (b) 1.1203
(c) 1.1204 (d) None of these

19. $\frac{3}{48}$ is what part of $\frac{1}{12}$?

(a) $\frac{3}{7}$ (b) $\frac{1}{12}$
(c) $\frac{3}{4}$ (d) None of these

20. The simplification of

$1+\dfrac{1}{1+\dfrac{1}{1-\frac{2}{3}}}$ yields the result

(a) $\frac{7}{4}$ (b) $\frac{4}{5}$
(c) $\frac{5}{4}$ (d) None of these

21. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?

(a) $\frac{1}{4}$ (b) $\frac{23}{24}$
(c) $\frac{11}{12}$ (d) $\frac{17}{24}$

22. How many $\frac{1}{8}$'s are there in $37\frac{1}{2}$?

(a) 300 (b) 400
(c) 500 (d) Cannot be determined

23. In a college, $\frac{1}{5}$th of the girls and $\frac{1}{8}$th of the boys took part in a social camp. What of the total number of students in the college took part in the camp?

(a) $\frac{13}{40}$ (b) $\frac{13}{80}$
(c) $\frac{2}{13}$ (d) Data inadequate

24. $\left\{7\frac{1}{2}+\frac{1}{2}+\frac{1}{2} \text{ of } \frac{1}{4}-\frac{2}{5}\times 2\frac{1}{3}+1\frac{7}{8} \text{ of } \left(1\frac{2}{5}-1\frac{1}{3}\right)\right\} = ?$

(a) $3\frac{1}{5}$ (b) $2\frac{1}{24}$
(c) $4\frac{1}{30}$ (d) None of these

25. When simplified, the product

$\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\left(2-\frac{999}{1001}\right)$ is equal to:

(a) $\frac{991}{1001}$ (b) $\frac{1001}{13}$
(c) $\frac{1003}{3}$ (d) None of these

26. The value of $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\frac{1}{4.5.6}$ is

(a) $\frac{7}{30}$ (b) $\frac{1}{3}$
(c) $\frac{13}{30}$ (d) None of these

27. $15\frac{2}{3}\times 3\frac{1}{6}+6\frac{1}{3} = 11\frac{7}{18} + ?$

(a) $39\frac{5}{9}$ (b) $137\frac{4}{9}$
(c) $29\frac{7}{9}$ (d) None of these

28. $3+\left[(8-5)+\left\{(4-2)+\left(2+\frac{8}{13}\right)\right\}\right] = ?$

(a) $\frac{13}{17}$ (b) $\frac{68}{13}$
(c) $\frac{17}{13}$ (d) $\frac{13}{68}$

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29. If the numbers $\frac{3}{5}, \frac{2}{3}, \frac{3}{4}$ are given, then we can say that

(a) $\frac{3}{4} > \frac{3}{5} > \frac{2}{3}$ (b) $\frac{2}{3} > \frac{3}{5} > \frac{3}{4}$

(c) $\frac{3}{4} > \frac{2}{3} > \frac{3}{5}$ (d) $\frac{3}{5} > \frac{2}{3} > \frac{3}{4}$

30. $\frac{(272-32)(124+176)}{17\times15-15} = ?$

(a) 0 (b) 2.25

(c) 300 (d) None of these

31. If $\frac{a}{b} = \frac{1}{3}$, then $\frac{3a+2b}{3a-2b}$ is equal to

(a) 3 (b) −3

(c) −5 (d) −1

32. $(20 \div 5) \div 2 + (16 \div 8) \times 2 + (10 \div 5) \times (3 + 2) = ?$

(a) 9 (b) 12

(c) 15 (d) 18

33. $\frac{5}{6} + \frac{6}{7} \times ? - \frac{8}{9} \div 1\frac{3}{5} + \frac{3}{4} \times 3\frac{1}{3} = 2\frac{7}{9}$

(a) $\frac{7}{6}$ (b) $\frac{6}{7}$

(c) 1 (d) None of these

34. $4\frac{1}{2} + 3\frac{1}{6} + ? \div 2\frac{1}{3} = 13\frac{2}{5}$

(a) $3\frac{2}{5}$ (b) $1\frac{2}{5}$

(c) $4\frac{1}{5}$ (d) $4\frac{1}{6}$

35. The simplification of $\frac{0.67\times0.67\times0.67-0.001}{0.67\times0.67+0.067+0.01}$ gives

(a) 0.57 (b) 0.66

(c) 0.68 (d) 0.77

36. In a certain college the number of girls is twice the number of boys. $\frac{1}{5}$th of the girls and $\frac{1}{8}$th of the boys took part in a social camp. What part of the total number of students took part in the camp?

(a) $\frac{7}{40}$ (b) $\frac{7}{80}$

(c) $\frac{2}{12}$ (d) $\frac{1}{24}$

37. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The fraction is

(a) $\frac{8}{27}$ (b) $2\frac{2}{3}$

(c) $1\frac{1}{3}$ (d) None of these

38. Which of the following numbers is the greatest?

(a) $(0.3)^2$ (b) 1 ÷ 0.3

(c) $\frac{1}{8}$ (d) $\sqrt{0.49}$

39. What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the three fractions?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

40. A person was to multiply a fraction by $\frac{6}{7}$. Instead, he divided and got an answer which exceeds the correct answer by $\frac{1}{7}$. The correct answer was

(a) $\frac{6}{13}$ (b) $\frac{36}{91}$

(c) $\frac{7}{13}$ (d) None of these

41. $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2} = ?$

(a) 2 (b) 4

(c) 0 (d) Cannot be determined

42. The value of $\frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5}$ correct to three places of decimal is

(a) 0.713 (b) 0.715

(c) 0.717 (d) 0.718

43. $\frac{?+12}{0.2\times3.6} = 2$

(a) 17.82 (b) 17.22

(c) 17.28 (d) 17.12

44. $\sqrt{?\times7} \times 18 = 84$

(a) 3.11 (b) 3.12

(c) 3.13 (d) 3.14

45. The difference between the sum of $1\frac{3}{4}, 2\frac{1}{3}, 3\frac{5}{12}, 5\frac{1}{5}$ and $2\frac{1}{6}$ and the nearest whole number is

(a) $\frac{2}{15}$ (b) $\frac{13}{15}$

(c) $\frac{11}{60}$ (d) None of these

46. $\left(2\frac{3}{x}\right) \times \left(y\frac{1}{2}\right) = 7\frac{3}{4}$, find the values of x and y

(a) (3, 19) (b) (3, 14)

(c) (14, 3) (d) (24, 6)

47. If we multiply a fraction by itself and divide the product by the square of its reciprocal, the fraction so obtained is $3\frac{13}{81}$. The original fraction is

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(a) $\frac{16}{9}$ (b) $\frac{8}{9}$
(c) $\frac{4}{3}$ (d) $\frac{1}{3}$

48. If $x * y = (x + 2)^2(y - 2)$ then $7 * 5 = ?$
(a) 234 (b) 243
(c) 343 (d) 423

49. If m and n are whole numbers such that $m^n = 121$, then $(m - 1)^{n+1} = ?$
(a) 10 (b) 10^2
(c) 10^3 (d) 10^4

50. Between two fractions $\frac{1}{2}$ and $\frac{1}{8}$, how many fractions are there in all?
(a) four (b) zero
(c) sixteen (d) Infinite

51. A boy was asked to multiply a given number by 8/17. Instead, he divided the given number by $\frac{8}{17}$ and got the result 225 more than what he should have got if he had multiplied the number by 8/17. The given number was
(a) 8 (b) 17
(c) 64 (d) 136

52. The value of $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{9.10}$ is
(a) $\frac{11}{10}$ (b) $\frac{8}{9}$
(c) $\frac{9}{10}$ (d) $\frac{25}{1089}$

53. $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$
(a) 6 (b) 3
(c) 9 (d) 12

54. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The original fraction is
(a) $\frac{8}{27}$ (b) $2\frac{2}{3}$
(c) $1\frac{1}{3}$ (d) None of these

55. If $\frac{a}{a+b} = \frac{17}{23}$, what is $\frac{a+b}{a-b}$ equal to?
(a) $\frac{11}{23}$ (b) $\frac{17}{32}$
(c) $\frac{23}{11}$ (d) $\frac{23}{17}$

56. A tin of oil was $\frac{4}{5}$ full, when six bottles of oil were taken out and four bottles of oil were poured into it, it was $\frac{3}{4}$ full. How many bottles of oil can it contain?
(a) 10 (b) 20
(c) 30 (d) 40

57. In an examination a student was asked to find (3/14) of a certain number. By mistake he found (3/4) of it. His answer was 150 more than the correct answer. The given number is
(a) 180 (b) 240
(c) 280 (d) 290

58. The value of $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)...\left(2-\frac{999}{1001}\right)$ is
(a) $\frac{1003}{3}$ (b) $\frac{1003}{1001}$
(c) $\frac{1}{1001}$ (d) None of these

59. If $\sqrt{2^n} = 64$, then find the value of n
(a) 8 (b) 10
(c) 12 (d) 16

60. If $10^{2y} = 25$ then what is the value of 10^y?
(a) –5 (b) 5
(c) $\frac{1}{25}$ (d) $\sqrt{\frac{1}{25}}$
(e) None of these

61. $11\frac{1}{3} \times 4\frac{8}{10} \div ? = 22\frac{2}{3}$
(a) 2.4 (b) 4.2
(c) 2.6 (d) 2.8

62. Simplify

$$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$

(a) $\frac{a}{a-1}$ (b) $\frac{a-1}{2}$
(c) $\frac{2}{a-1}$ (d) $\frac{2}{1-a}$

63. If $a^2 + b^2 = 45$ and $ab = 18$, find $\frac{1}{a}+\frac{1}{b}$.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) Cannot be determined

64. If $\frac{a^2+b^2}{c^2+d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only
(a) $\frac{c+d}{cd}$ (b) $\frac{cd}{c+d}$
(c) $\frac{c-d}{c+d}$ (d) $\frac{c+d}{c-d}$

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65. $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$
(a) 1.04 (b) 1.4
(c) 1.5 (d) Cannot be determined

66. The highest score in an inning was $\frac{2}{9}$ of the total score and the next highest was $\frac{2}{9}$ of the remainder. These scores differ by 8 runs. What was the total score in the innings?
(a) 162 (b) 152
(c) 142 (d) 132

67. Simplify $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$
(a) $17\frac{1}{8}$ (b) $17\frac{3}{8}$
(c) $11\frac{7}{8}$ (d) $17\frac{7}{8}$

68. $\dfrac{\frac{64}{121} - \frac{9}{64}}{\frac{8}{11} + \frac{3}{8}} = ?$
(a) $\frac{88}{31}$ (b) $\frac{31}{88}$
(c) $\frac{41}{99}$ (d) $\frac{99}{41}$

69. When $\frac{1}{4}$ of a number is subtracted from $\frac{1}{3}$ of the same number, the remainder obtained is 12. The number is
(a) 144 (b) 72
(c) 120 (d) 63
(e) None of these

70. What is the difference between the largest and the smallest fractions
$\frac{5}{8}, \frac{21}{35}, \frac{9}{16}$ and $\frac{6}{7}$?
(a) $\frac{33}{112}$ (b) $\frac{11}{37}$
(c) $\frac{13}{41}$ (d) $\frac{9}{35}$
(e) None of these

71. If a man spends $\frac{5}{6}$th part of money and then again earns $\frac{1}{2}$ part of the remaining money, what part of his money is with him now?
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$
(e) $\frac{1}{3}$

72. Manmohan spends $\frac{1}{5}$ part of his money as pocket money and $\frac{4}{5}$ of the remainder in other affairs. If he is left with Rs 48 per month, what is monthly income?
(a) Rs 360 (b) Rs 400
(c) Rs 320 (d) Rs 300
(e) None of these

73. If the difference between $\frac{4}{5}$ part and $\frac{3}{4}$ part of a number is 4, what is the number?
(a) 60 (b) 100
(c) 80 (d) 40
(e) None of these

74. If $\frac{2}{3}$ part of a number is 96, what is the value of $\frac{3}{4}$ part of the same number?
(a) 48 (b) 192
(c) 108 (d) 72
(e) None of these

75. A man completes $\frac{2}{15}$ of his journey by aeroplane, $\frac{2}{5}$ by train and the rest by taxi. What part of his journey does he complete by taxi?
(a) $\frac{8}{15}$ (b) $\frac{7}{15}$
(c) $\frac{9}{15}$ (d) None of these

76. If $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{70}\right) = \frac{x}{70}$, then what is the value of x ?
(a) 69 (b) 35
(c) 20 (d) 15
(e) 1

77. The value of
$\dfrac{1.073 \times 1.073 - 0.927}{1.073 - 0.927} + \dfrac{(3^4)^4 \times 9^6}{(27)^7 \times (3)^9}$ is
(a) 2 (b) $\frac{1}{9}$
(c) $2\frac{1}{9}$ (d) $3\frac{1}{9}$
(e) None of these

78. The value of
$\dfrac{2^{1/2}.3^{1/3}.4^{1/4}}{10^{-1/5}.5^{3/5}} + \dfrac{3^{4/3}.5^{-7/5}}{4^{-3/5}.6}$ is
(a) 5 (b) 6
(c) 10 (d) 15
(e) None of these

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EXPLANATORY ANSWERS

1. Ans. (c)

Solution Given expression:

$$= \frac{3}{10} + \frac{3}{7} \text{ of } \left(\frac{23}{10} + \frac{13}{5}\right) + \frac{1}{5} \times \frac{5}{7} - \frac{2}{7}$$

$$= \frac{3}{10} + \frac{3}{7} \text{ of } \frac{49}{10} + \frac{1}{7} - \frac{2}{7} = \frac{3}{10} + \frac{21}{10} - \frac{1}{7}$$

$$= \frac{3}{10} \times \frac{10}{21} - \frac{1}{7} = \frac{1}{7} - \frac{1}{7} = \mathbf{0}.$$

2. Ans. (a)

Solution Given expression

$$= 1 + 1 \div \left\{1 + 1 \div \left(\frac{2}{3}\right)\right\}$$

$$= 1 + 1 \div \left\{1 + 1 \times \frac{3}{2}\right\}$$

$$= 1 + 1 \div \left\{1 + \frac{3}{2}\right\} = 1 + 1 \div \frac{5}{2}$$

$$= 1 + 1 \times \frac{2}{5} = 1 + \frac{2}{5} = \mathbf{\frac{7}{5}}.$$

3. Ans. (b)

Solution Given expression

$$= 48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} \div \frac{3}{4} \text{ of } \frac{2}{3}\right)$$

$$= \frac{48}{12} \times \left\{\left(\frac{9}{8} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{2}{3}\right)\right\}$$

$$= \frac{48}{12} \times \left(\frac{3}{2} \times 2\right) = 4 \times 3 = \mathbf{12}.$$

4. Ans. (c)

Solution Given expression

$$= 2 \div \left[2 + 2 \div \left\{2 + 2 \div \left(2 + \frac{2}{3}\right)\right\}\right]$$

$$= 2 \div \left[2 + 2 \div \left\{2 + 2 \times \frac{3}{8}\right\}\right]$$

$$= 2 \div \left[2 + 2 \div \frac{11}{4}\right] = 2 \div \left[2 + 2 \times \frac{4}{11}\right]$$

$$= 2 \div \frac{30}{11} = 2 \times \frac{11}{30} = \mathbf{\frac{11}{15}}.$$

5. Ans. (b)

Solution Let missing figure = x

$$\frac{15}{2} - \left[\frac{9}{4} \div \left\{\frac{5}{4} - x\left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right] = 3$$

$$\frac{15}{2} - \left[\frac{9}{4} \div \left\{\frac{5}{4} - x\right\}\right] = 3$$

$$\frac{15}{2} - 3 = \frac{9/4}{5/4 - x}$$

$$\frac{9}{2} = \frac{9}{5 - 4x}$$

$$5 - 4x = 2$$

$$x = \frac{3}{4}.$$

6. Ans. (c)

Solution We know that $\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$

$\therefore$ The given expression = 0.8 – 0.5 = 0.3.

7. Ans. (b)

8. Ans. (a)

Solution Given expression

$$= 1 - [2 - \{5 - (4 - 1)\}]$$

$$= 1 - [2 - \{5 - 3\}]$$

$$= 1 - [2 - 2] = 1 - 0 = \mathbf{1}.$$

9. Ans. (c)

Solution Given expression

$$= 3 \div \left[3 \div \left\{2 + \frac{34}{13}\right\}\right]$$

$$= 3 \div \left[3 \div \left\{2 \times \frac{13}{34}\right\}\right] = 3 \div \left[3 \times \frac{17}{13}\right]$$

$$= 3 \div \frac{51}{13} = 3 \times \frac{13}{51} = \mathbf{\frac{13}{17}}.$$

10. Ans. (a)

Solution Given expression

$$= \frac{(69842)^2 - (30158)^2}{69842 - 30158}$$

$$= \frac{(69842 - 30158)(69842 + 30158)}{69842 - 30158}$$

$$= \mathbf{100000}.$$

11. Ans. (d)

Solution Given expression

$$= \frac{\frac{15}{7} - \frac{5}{2}}{\frac{9}{4} + \frac{8}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{2}{3}}}$$

$$= \frac{-5}{14} \times \frac{28}{95} \div \frac{1}{2 + \frac{3}{8}}$$

$$= \frac{-2}{19} \div \frac{8}{19} = \frac{-2}{19} \times \frac{19}{8} = \mathbf{\frac{-1}{4}}.$$

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12. Ans. (b)

Solution The given expression

$= 2.75 - 2.25 = \mathbf{0.50}.$

13. Ans. (c)

Solution Given expression

$$= \frac{\frac{1}{2}\times\frac{1}{4}+20}{2+20} = \frac{161}{8}\times\frac{1}{22} = \mathbf{\frac{161}{176}}.$$

14. Ans. (c)

Solution Given expression

$$= \frac{(0.53)^2 - 2\times0.53\times0.41+(0.41)^2}{0.12}$$

$$= \frac{(0.53-0.41)^2}{0.12} = \frac{(0.12)^2}{0.12} = \mathbf{0.12}.$$

15. Ans. (c)

Solution Given expression

$$= \frac{(3^2)^2\times(3\times3\times2)^4}{3^{16}}$$

$$= \frac{3^4\times3^8\times2^4}{3^{16}} = \frac{2^4}{3^4} = \mathbf{\frac{16}{81}}.$$

16. Ans. (c)

Solution Given expression

$$= 1 + \frac{1}{2+\frac{1}{\frac{2}{3}}} = 1 + 1 + \frac{1}{2+\frac{3}{2}}$$

$$= 1 + \frac{1}{\frac{7}{2}} = 1 + \frac{2}{7} = \mathbf{\frac{9}{7}}.$$

17. Ans. (d)

Solution Let x be the missing number

$$= 108 \div x \text{ of } \frac{1}{3}+\frac{2}{5}\times3\frac{3}{4} = 10\frac{1}{2}$$

$$= 108 \div \frac{x}{3}+\frac{2}{5}\times\frac{15}{4} = \frac{21}{2}$$

$$= \frac{3\times108}{x}+\frac{3}{2} = \frac{21}{2}$$

$$= \frac{3\times108}{x} = \frac{21}{2}-\frac{3}{2}$$

$$= \frac{3\times108}{x} = 9$$

$$x = \frac{3\times108}{9}$$

$$x = \mathbf{36}.$$

18. Ans. (b)

Solution Given expression

$$= \frac{108+9+3+1}{108} = \frac{121}{108} = \mathbf{1.1203}.$$

19. Ans. (d)

Solution Let x of $\frac{1}{12} = \frac{3}{48}$

Then, $x = \frac{3}{48}\times12 = \frac{3}{4}$.

20. Ans. (c)

Solution Given expression

$$= 1 + \frac{1}{1+3} = 1 + \frac{1}{4} = \mathbf{\frac{5}{4}}.$$

21. Ans. (d)

Solution $\frac{1}{3} = 0.33$ and $\frac{7}{8} = 0.875$

$\frac{1}{4} = 0.25$ does not lie between 0.33 and 0.875

$\frac{23}{24} = 0.96$ which exceeds 0.875

$\frac{11}{12} = 0.92$ which exceeds 0.875

$\frac{17}{24} = \mathbf{0.708}$ which lies between **0.33** and **0.875.**

22. Ans. (a)

Solution Number of $\frac{1}{8}$'s $= \frac{75}{2}\div\frac{1}{8} = \frac{75}{2}\times8 = \mathbf{300}.$

23. Ans. (c)

Solution Out of the 5 girls, 1 took part in the camp.
Out of the 8 boys, 1 took part in the camp.
Out of the 13 students, 2 took part in the camp.

$\therefore \frac{2}{13}$ of total number of students took part in the camp.

24. Ans. (c)

Solution Given expression

$$= \frac{15}{2}+\frac{1}{2}\div\frac{1}{8}-\frac{2}{5}\times\frac{7}{3}\div\frac{15}{8} \text{ of } \left(\frac{7}{5}-\frac{4}{3}\right)$$

$$= \frac{15}{2}+4-\frac{2}{5}\times\frac{7}{3}\div\frac{15}{8} \text{ of } \frac{1}{15}$$

$$= \frac{15}{2}+4-\frac{2}{5}\times\frac{7}{3}\div\frac{1}{8}$$

$$= \frac{15}{2}+4-\frac{2}{5}\times\frac{7}{3}\times\frac{8}{1}$$

$$= \frac{15}{2}+4-\frac{112}{15} = \frac{23}{2}-\frac{112}{15}$$

$$= \frac{121}{30} = \mathbf{4\frac{1}{30}}.$$

25. Ans. (c)

Solution Given expression

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$$= \frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\ldots\times\frac{1003}{1001} = \frac{1003}{3}.$$

26. Ans. (a)

Solution Given expression

$$= \frac{1}{6}+\frac{1}{24}+\frac{1}{60}+\frac{1}{120}$$

$$= \frac{20+5+2+1}{120} = \frac{28}{120} = \frac{7}{30}.$$

27. Ans. (d)

Solution Let $\frac{47}{3}\times\frac{19}{6}+\frac{19}{3} = \frac{205}{18} + x$

Then, $x = \frac{893}{18}+\frac{19}{3}-\frac{205}{18}$

$$= \frac{893+114-205}{18} = \frac{802}{18} = 44\frac{5}{9}.$$

28. Ans. (a)

Solution Given expression

$$= 3 \div \left[3\div\left\{2\div\frac{34}{13}\right\}\right]$$

$$= 3 \div \left[3\div\left(2\times\frac{13}{34}\right)\right]$$

$$= 3 \div \left[3\times\frac{17}{13}\right] = 3 \times \frac{13}{51} = \frac{13}{17}.$$

29. Ans. (c)

Solution $\frac{3}{5} = 0.60$

$\frac{3}{4} = 0.75$

$\frac{2}{3} = 0.66$

$\therefore \quad \frac{3}{4} > \frac{2}{3} > \frac{3}{5}.$

30. Ans. (c)

Solution Given expression

$$= \frac{240\times300}{240} = 300.$$

31. Ans. (b)

Solution $\frac{3a+2b}{3a-2b} = \frac{3\left(\frac{a}{b}\right)+2}{3\left(\frac{a}{b}\right)-2} = \frac{3\left(\frac{1}{3}\right)+2}{3\left(\frac{1}{3}\right)-2}$

$$= \frac{3}{-1} = -3.$$

32. Ans. (a)

Solution Given expression

$$= \frac{4}{2}+2\times2+2\times\frac{3}{2} = 2 + 4 + 3 = 9.$$

33. Ans. (b)

Solution Let $\frac{5}{6}+\frac{6}{7}\times x-\frac{8}{9}\div\frac{8}{5}+\frac{3}{4}\times\frac{10}{3} = \frac{25}{9}$. Then,

$$\frac{5}{6}\times\frac{7}{6}\times x-\frac{8}{9}\times\frac{5}{8}+\frac{3}{4}\times\frac{10}{3} = \frac{25}{9}$$

or, $\frac{35}{36}x-\frac{5}{9}+\frac{10}{4} = \frac{25}{9}$ or, $\frac{35}{36}x = \frac{25}{9}+\frac{5}{9}-\frac{5}{2}$

or, $\frac{35}{36}x = \frac{30}{9}-\frac{5}{2}$ or, $\frac{35}{36}x = \frac{60-45}{18}$

or, $x = \frac{15}{18}\times\frac{36}{35}$

$\therefore \quad x = \frac{6}{7}.$

34. Ans. (a)

Solution Let $\frac{9}{2}+\frac{19}{6}+x+\frac{7}{3} = \frac{67}{5}$. Then,

$$x = \frac{67}{5}-\left(\frac{9}{2}+\frac{19}{6}+\frac{7}{3}\right)$$

$$= \frac{67}{5}-10 = \frac{17}{5} = 3\frac{2}{5}.$$

35. Ans. (a)

Solution $0.001 = (0.1)^3$; $0.067 = 0.1 \times 0.67$

The given expression

$$= \frac{a^3-b^3}{a^2+ab+b^2} = a - b$$

$$= 0.67 - 0.10 = 0.57.$$

36. Ans. (a)

Solution Let total number of students = x

Number of girl students = $\frac{2x}{3}$

Number of boy students = $\frac{x}{3}$

Number of girls who took part in camp

$$= \frac{1}{5}\left(\frac{2x}{3}\right) = \frac{2}{15}x$$

Number of boys who took part in camp

$$= \frac{1}{8}\left(\frac{x}{3}\right) = \frac{x}{24}$$

Total number of students who took part in camp

$$= \frac{2}{15}x+\frac{x}{24} = \left(\frac{16+5}{120}\right)x = \frac{7}{40}x.$$

37. Ans. (b)

Solution Let the fraction be $\frac{a}{b}$. Then,

$$\left(\frac{a}{b}\times\frac{a}{b}\right)\div\frac{b}{a} = 18\frac{26}{27} = \frac{512}{27}$$

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or, $\left(\frac{a}{b}\right)^3 = \left(\frac{8}{3}\right)^3$

$\therefore \frac{a}{b} = \frac{8}{3} = 2\frac{2}{3}$.

38. Ans. (b)

39. Ans. (d)

Solution Let $\frac{1}{4}+\frac{1}{6}-x = 3 \times \frac{1}{12}$. Then,

$\frac{1}{4}+\frac{1}{6}-x = \frac{1}{4}$ or, $x = \frac{1}{6}$.

40. Ans. (b)

Solution Let x be the fraction

$\frac{7}{6}x-\frac{6}{7}x = \frac{1}{7} \Rightarrow x = \frac{6}{13}$

Correct answer $= \frac{6}{7}x = \frac{6}{7}\times\frac{6}{13} = \frac{36}{91}$.

41. Ans. (a)

Solution $2 + \sqrt{2} + \left[\frac{\sqrt{2}-2+2+\sqrt{2}}{(\sqrt{2})^2-(2)^2}\right]$

$= 2 + \sqrt{2} + \frac{2\sqrt{2}}{2-4}$

$= 2 + \sqrt{2} - \sqrt{2} = 2.$

42. Ans. (c)

Solution $\frac{1}{2} = 0.50000$ ...(i)

$\frac{1}{2.3} = 0.16667$ ...(ii) (divide (i) by 3)

$\frac{1}{2.3.4} = 0.04167$ ...(iii) (divide (ii) by 4)

$\frac{1}{2.3.4.5} = 0.00833$ ...(iv) (divide (iii) by 5)

Adding, we have 0.71667 or 0.717 up to three places.

43. Ans. (c)

Solution Putting x in place of ?

$\frac{x+12}{0.2\times3.6} = 2$ or, $x + 12 = 2 \times 0.2 \times 3.6$

$\Rightarrow x = 2 \times 0.2 \times 3.6 \times 12$ or, $x = 17.28.$

44. Ans. (a)

Solution Substituting x for ?, we get

$\sqrt{x\times7} \times 18 = 84$

or, $\sqrt{x\times7} = \frac{84}{18}$ or, $(\sqrt{x\times7})^2 = \left(\frac{84}{18}\right)^2$

or, $x \times 7 = \frac{84\times84}{18\times18}$ or, $x = \frac{84\times84}{18\times18\times7} = 3.11.$

45. Ans. (a)

Solution Sum $= \frac{7}{4}+\frac{7}{3}+\frac{41}{12}+\frac{26}{5}+\frac{13}{6}$

$= \frac{105+140+205+312+130}{60}$

$= \frac{892}{60} = 14\frac{13}{15}$

which is nearer to 15 than 14

Difference : $15 - 14\frac{13}{15} = \frac{2}{15}$.

46. Ans. (c)

Solution Taking the quotient 2, y and 7, we get $2y = 7$, which gives the quotient as 3

$\therefore y = 3$. Substituting the value of y, we get

$2\frac{3}{x}\times3\frac{1}{2} = 7\frac{3}{4}$

Now, $\frac{7\frac{3}{4}}{3\frac{1}{2}} = 2\frac{3}{x} \Rightarrow 2\frac{3}{14} = 2\frac{3}{x}$

$\therefore x = 14, y = 3.$

47. Ans. (c)

Solution Let x be the fraction

$x \times x + \left(\frac{1}{x}\right)^2 = 3\frac{13}{81} \Rightarrow x^4 = \frac{256}{81} = \left(\frac{4}{3}\right)^4$

$\therefore x = \frac{4}{3}$.

48. Ans. (b)

Solution Substituting $x = 7$ and $y = 5$, we get

$7 * 5 = (7 + 2)^2 (5 - 2) = (9)^2 \times 3 = 243.$

49. Ans. (c)

Solution Given that $m^n = 121 \Rightarrow m^n = 11^2$

Hence, $m = 11$, $n = 2$. Substituting these values

$(m - 1)^{n+1} = (11 - 1)^{2+1} = 10^3 = 1000.$

50. Ans. (d)

51. Ans. (d)

Solution $x \times \frac{17}{8} - x \times \frac{8}{17} = 225$ or, $\frac{225}{136}x = 225$

$\therefore x = 136.$

52. Ans. (c)

Solution $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{9.10}$

$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right) + ... + \left(\frac{1}{9}-\frac{1}{10}\right)$

$= 1 - \frac{1}{10} = \frac{9}{10}$.

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53. Ans. (c)

Solution Putting x for ?

$$\sqrt{1296} \times 2.25 = x^2 \quad \text{or,} \quad 36 \times 2.25 = x^2$$

or, $x = \sqrt{36 \times 2.25}$ or, $x = 6 \times 1.5$

$\therefore$ $x = \mathbf{9}$.

54. Ans. (b)

Solution $x \times x + \frac{1}{x} = 18\frac{26}{27}$ or, $x^3 = \frac{512}{27}$

$\therefore$ $x^3 = \left(\frac{8}{3}\right)^3$ and so $x = \frac{8}{3} = \mathbf{2\frac{2}{3}}$.

55. Ans. (c)

Solution Given that $\frac{a}{a+b} = \frac{17}{23}$

i.e. if $a = 17$, then $a + b = 23$ or, $b = 6$

$a - b = 17 - 6 = 11$

$\therefore$ $\frac{a+b}{a-b} = \mathbf{\frac{23}{11}}$.

56. Ans. (d)

57. Ans. (c)

Solution $\frac{3}{4}x - \frac{3}{14}x = 150$ or, $\frac{15}{28}x = 150$

$\therefore$ $x = \frac{150 \times 28}{15} = \mathbf{280}$.

58. Ans. (a)

Solution Given product

$$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \ldots \times \frac{1003}{1001} = \mathbf{\frac{1003}{3}}.$$

59. Ans. (c)

Solution $\sqrt{2^n} = 64$ or, $(2^n)^{1/2} = 2^6$

$\Rightarrow \frac{n}{2} = 6 \Rightarrow n = \mathbf{12}$.

60. Ans. (b)

Solution $10^y = \sqrt{10^{2y}} = \sqrt{25} = \mathbf{5}$.

61. Ans. (a)

Solution Putting x for ? and solving

$$11\frac{1}{3} \times 4\frac{8}{10} + x = 22\frac{2}{3}$$

or, $11\frac{1}{3} \times 4\frac{8}{10} = 22\frac{2}{3}x$

or, $x = \frac{11\frac{1}{3} \times 4\frac{8}{10}}{22\frac{2}{3}}$ or, $x = \frac{1}{2} \times 4\frac{8}{10}$

$\therefore$ $x = \frac{1}{2} \times \frac{48}{10} = \frac{24}{10} = \mathbf{2.4}$.

62. Ans. (d)

Solution $\frac{a^{1/2} + a^{-1/2}}{1-a} + \frac{1-a^{-1/2}}{1+\sqrt{a}}$

$$= \frac{a^{1/2} + a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})} + \frac{1-a^{-1/2}}{1+a^{1/2}}$$

$$= \frac{a^{1/2} + a^{-1/2} + (1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})}$$

$$= \frac{a^{1/2} + a^{-1/2} + 1 - a^{-1/2} - a^{1/2} + 1}{1-a}$$

$$= \mathbf{\frac{2}{1-a}}.$$

63. Ans. (c)

Solution $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{\sqrt{a^2+b^2+2ab}}{ab}$

$$= \frac{\sqrt{45+2\times18}}{18} = \pm\frac{9}{18} = \pm\mathbf{\frac{1}{2}}.$$

64. Ans. (d)

Solution $\frac{a^2+b^2}{c^2+d^2} = \frac{ab}{cd}$ or, $\frac{a^2+b^2}{c^2+d^2} = \frac{2ab}{2cd}$

or, $\frac{a^2+b^2+2ab}{a^2+b^2-2ab} = \frac{c^2+d^2+2cd}{c^2+d^2-2cd}$

[by componendo and dividendo]

or, $\left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$

$\therefore$ $\frac{a+b}{a-b} = \mathbf{\frac{c+d}{c-d}}$.

65. Ans. (a)

Solution Putting x for ? and solving

$$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$$

Here, $1.06 = a$ and $0.04 = b$

$\therefore$ $(a + b)^2 - x = 4ab$

$\therefore$ $x = (a + b)^2 - 4ab = (a - b)^2 = (1.06 - 0.04)^2$

$= (1.02)^2 = \mathbf{1.0404}$.

66. Ans. (a)

Solution Let the total score be x runs, such that

$$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8 \text{ or, } \frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$$

or, $\frac{2}{9}x \times \frac{2}{9} = 8$ or, $x = \mathbf{162}$.

67. Ans. (a)

Solution $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$= 1 + (8^2)^{-1/2} + (-1 \times 32)^{4/5}$

$= 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}]$

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$$= 1 + 8^{-1} + [((-1)^2)^{2/5} \times (2^5)^{4/5}]$$

$$= 1 + \frac{1}{8} + [1 \times 16] = \mathbf{17\frac{1}{8}}.$$

68. Ans. (b)

Solution $x = \dfrac{(64)^2 - 9\times121}{121\times64} \times \dfrac{8\times11}{(8)^2 + 3\times11}$

$$x = \frac{(64)^2 - 3\times3\times11\times11}{11\times11\times8\times8} \times \frac{8\times11}{64+33}$$

or, $$x = \frac{(64+33)(64-33)}{88} \times \frac{1}{64+33}$$

or, $$x = \mathbf{\frac{31}{88}}.$$

69. Ans. (a)

Solution Let the number be 1

$\therefore$ $\frac{1}{3}$ of 1 $= \frac{1}{3}$ and, $\frac{1}{4}$ of 1 $= \frac{1}{4}$

$\therefore$ $\frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

$\therefore$ Number $12 \div \frac{1}{12} = \mathbf{144.}$

70. Ans. (a)

Solution L.C.M. of 7, 8, 16 and 35 = 560

$\therefore$ $\frac{5}{8} = \frac{5\times70}{8\times70} = \frac{350}{560}$

$\frac{21}{35} = \frac{21\times16}{35\times16} = \frac{336}{560}$

$\frac{9}{16} = \frac{9\times35}{16\times35} = \frac{315}{560}$

and, $\frac{6}{7} = \frac{6\times80}{7\times80} = \frac{480}{560}$

$\therefore$ Difference between the largest and the smallest fractions

$$= \frac{6}{7} - \frac{9}{16} = \frac{480}{560} - \frac{315}{560} = \frac{165}{560} = \mathbf{\frac{32}{112}}.$$

71. Ans. (b)

Solution Let the money with the man at first be Re 1

$\therefore$ Money spent $= \frac{5}{6}$ of 1 $=$ Rs $\frac{5}{6}$

$\therefore$ Remaining money $= 1 - \frac{5}{6} =$ Rs $\frac{1}{6}$

and money earned $= \frac{1}{2}$ of Rs $\frac{1}{6} =$ Rs $\frac{1}{12}$

$\therefore$ Total money with him now

$$= \frac{1}{6} + \frac{1}{12} = \text{Rs } \frac{3}{12} = \text{Rs } \frac{1}{4}$$

$\therefore$ $\frac{1}{4}$th part of his money is with him now.

72. Ans. (d)

Solution Let the monthly income of Manmohan be Re 1

$\therefore$ Pocket money $= \frac{1}{5}$ of Re 1 $=$ Rs $\frac{1}{5}$

and remainder $= 1 - \frac{1}{5} =$ Rs $\frac{4}{5}$

$\therefore$ Other expenses $= \frac{4}{5}$ of Rs $\frac{4}{5} =$ Rs $\frac{16}{25}$

$\therefore$ Saving $= \frac{4}{5} - \frac{16}{25} =$ Rs $\frac{4}{25}$

$\therefore$ Monthly income $= 48 \div \frac{4}{25} =$ **Rs 300.**

73. Ans. (c)

Solution Let the number be 1

$\therefore$ $\frac{4}{5}$ of 1 $= \frac{4}{5}$ and, $\frac{3}{4}$ of 1 $= \frac{3}{4}$

$\therefore$ Difference $= \frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

$\therefore$ Number $= 4 \div \frac{1}{20} =$ **80.**

74. Ans. (c)

Solution $\because$ $\frac{2}{3}$ part $= 96$

$\therefore$ $\frac{3}{4}$ part $= 96 \times \frac{3}{2} \times \frac{3}{4} =$ **108.**

75. Ans. (b)

Solution Journey completed by aeroplane and train

$$= \frac{2}{15} + \frac{2}{5} = \frac{2+6}{15} = \frac{8}{15}$$

$\therefore$ Remaining journey $= 1 - \frac{8}{15} = \frac{7}{15}$

$\therefore$ He completed $\frac{7}{15}$ part of his journey by taxi.

76. Ans. (e)

Solution $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right) \ldots \left(1-\frac{1}{70}\right) = \frac{x}{70}$

$\therefore$ $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\ldots\frac{69}{70} = \frac{x}{70}$

$\therefore$ $\frac{1}{70} = \frac{x}{70}$

$\therefore$ $x = \mathbf{1}$.

77. Ans. (c)

Solution $\dfrac{1.073\times1.073 - 0.927\times0.927}{1.073 - 0.927} + \dfrac{(3^4)^4 \times (9)^6}{(27)^7 \times (3)^9}$

$$= \frac{(1.073)^2 - (0.927)^2}{1.073 - 0.927} + \frac{(3^4)^4 \times (3^2)^6}{(3^3)^7 \times (3)^9}$$

$$= \frac{(1.073+0.927)(1.073-0.927)}{1.073-0.927} + \frac{3^{28}}{3^{30}}$$

$$= 2 + \frac{1}{3^2} = 2 + \frac{1}{9} = \mathbf{2\frac{1}{9}}.$$

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78. Ans. (c)

Solution $\dfrac{2^{1/2}\times3^{1/3}\times4^{1/4}}{10^{-1/5}\times5^{3/5}}\div\dfrac{3^{4/3}\times5^{-7/5}}{4^{-3/5}\times6}$

$=\dfrac{2^{1/2}\times3^{1/3}\times(2^2)^{1/4}\times10^{1/5}}{5^{3/5}}\div\dfrac{3^{4/3}\times4^{3/5}}{5^{7/5}\times6}$

$=\dfrac{2^{1/2}\times3^{1/3}\times2^{1/2}\times2^{1/5}\times5^{1/5}}{5^{3/5}}\times\dfrac{5^{7/5}\times2\times3}{3^{4/3}\times2^{6/5}}$

$=2^{\frac{1}{2}+\frac{1}{2}+\frac{1}{5}-\frac{6}{5}+1}\times3^{\frac{1}{3}-\frac{4}{3}+1}\times5^{\frac{1}{5}-\frac{3}{5}+\frac{7}{5}}$

$=2^1\times3^0\times5^1=2\times5=10.$

QUESTIONS FROM COMPETITIVE EXAMS

1. If $\left(a+\dfrac{1}{a}\right)^2=3$, then what is the value of $a^3+\dfrac{1}{a^3}$?

(a) $\dfrac{10\sqrt{3}}{3}$ (b) 0

(c) $3\sqrt{3}$ (d) $6\sqrt{3}$

[Based on MAT 2003]

2. If $a=\dfrac{x}{x+y}$ and $b=\dfrac{y}{x-y}$ then $\dfrac{ab}{a+b}$ is equal to

(a) $\dfrac{xy}{x^2+y^2}$ (b) $\dfrac{x^2+y^2}{xy}$

(c) $\dfrac{x}{x+y}$ (d) $\left(\dfrac{y}{x+y}\right)^2$

[Based on MAT 2003]

3. If $x=\dfrac{4ab}{a+b}$, then the value of $\dfrac{x+2a}{x-2a}+\dfrac{x+2b}{x-2b}$ is equal to

(a) 0 (b) 1

(c) 2 (d) None of these

[Based on MAT 2003]

4. Given $\dfrac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\dfrac{5}{2}$. The value of x is

(a) 1 (b) $\dfrac{331}{5}$

(c) $\dfrac{263}{20}$ (d) $\dfrac{17}{21}$

[Based on MAT 2002]

5. If the fractions $\dfrac{9}{13},\dfrac{2}{3},\dfrac{8}{11},\dfrac{5}{7}$ are arranged in ascending order, then the correct sequence is

(a) $\dfrac{9}{13},\dfrac{2}{3},\dfrac{8}{11},\dfrac{5}{7}$ (b) $\dfrac{2}{3},\dfrac{9}{13},\dfrac{5}{7},\dfrac{8}{11}$

(c) $\dfrac{2}{3},\dfrac{8}{11},\dfrac{5}{7},\dfrac{9}{13}$ (d) $\dfrac{5}{7},\dfrac{8}{11},\dfrac{2}{3},\dfrac{9}{13}$

[Based on MAT 2001]

6. 9^6+7, when divided by 8, would have a remainder of

(a) 0 (b) 6

(c) 5 (d) None of these

[Based on MAT 2001]

7. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4\times3^5\times5^2\times7^2$. The third number is

(a) $2^2\times5^3\times7^2$ (b) $2^2\times3^5\times7^2$

(c) $2^3\times3^5\times7^2$ (d) $2^5\times5^2\times7^2$

[Based on MAT 2001]

8. There are four prime numbers written in ascending order. The product of the first three is 385 and that of the last three is 1001. Find the first number.

(a) 5 (b) 7

(c) 11 (d) 17

[Based on MAT 2001]

9. The value of $\left(\dfrac{2.75\times2.75\times2.75-2.25\times2.25\times2.25}{2.75\times2.75+2.75\times2.25+2.25\times2.25}\right)$ is

(a) 0.30 (b) 0.50

(c) 3 (d) 5

[Based on MAT 2005]

10. The simplification of

$\left(\dfrac{75983\times75983-45983\times45983}{30000}\right)$

(a) 121796 (b) 121866

(c) 121956 (d) 121966

[Based on MAT 2005]

11. $\dfrac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3-\left(\frac{1}{3}\right)^{-1}}$ is equal to

(a) $-\dfrac{3}{2}$ (b) $-\dfrac{1}{2}$

(c) $\dfrac{2}{3}$ (d) $\dfrac{3}{2}$

[Based on MAT 2005]

12. What is the missing figure in the expression given below?

$\dfrac{16}{7}\times\dfrac{16}{7}-\dfrac{*}{7}\times\dfrac{9}{7}+\dfrac{9}{7}\times\dfrac{9}{7}=1$

(a) 1 (b) 7

(c) 4.57 (d) 32

[Based on MAT 2000]

13. If the numerator and the denominator of a proper fraction are increased by the same quantity, then the resulting fraction is

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(a) Always greater than the original fraction
(b) Always less than the original fraction
(c) Always equal to the original fraction
(d) None of the above **[Based on MAT 2001]**

14. If $\frac{a^3-b^3}{a^3+b^3}=\frac{13}{14}$, then find $\frac{a+b}{a-b}$

(a) $\frac{3}{2}$ (b) 1
(c) 2 (d) None of these
[Based on FMS (Delhi) 2004]

15. Arrange the following in ascending order of values:

$$\frac{87}{83},\left(\frac{87}{83}\right)^2,\frac{87}{89},\left(\frac{87}{89}\right)^2$$

(a) $\frac{87}{89},\frac{87}{89},\left(\frac{87}{89}\right)^2,\left(\frac{87}{83}\right)^2$

(b) $\frac{87}{83},\frac{87}{89},\left(\frac{87}{83}\right)^2,\left(\frac{87}{89}\right)^2$

(c) $\left(\frac{87}{89}\right)^2,\frac{87}{89},\frac{87}{83},\left(\frac{87}{83}\right)^2$

(d) $\left(\frac{87}{83}\right)^2,\frac{87}{83},\frac{87}{89},\left(\frac{87}{89}\right)^2$

[Based on FMS (Delhi) 2004]

16. Which of the following is the lowest?

(a) $\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{2^2}$ (b) $\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{6^2}$
(c) $\frac{1}{3^2}+\frac{1}{3^2}+\frac{1}{5^2}$ (d) $\frac{1}{7^2}+\frac{1}{2^2}+\frac{1}{2^2}$
[Based on FMS (Delhi) 2004]

17. What digit exists at unit's place in $(483)^{82}$?
(a) 8 (b) 9
(c) 10 (d) 11
[Based on FMS (Delhi) 2004]

18. Which of the following can't be the difference of two four-digit numbers formed by the four digits 4, 6, 7 and 9, where each digit can be repeated?
(a) 5085 (b) 5095
(c) 5795 (d) 2095
[Based on FMS (Delhi) 2004]

19. Find the remainder when $7^{21}+7^{22}+7^{23}+7^{24}$ is divided by 25.
(a) 0 (b) 2
(c) 4 (d) 6
[Based on FMS (Delhi) 2004]

20. A four-digit number is formed, using digits 1, 2, 3 and 4, without repeating any one of them. What is the sum of all such possible numbers?
(a) 66600 (b) 66660
(c) 66666 (d) 60000
[Based on FMS (Delhi) 2004]

21. How many multiples of 9 can be found which are less than 9999 and are perfect cubes?
(a) 5 (b) 6
(c) 7 (d) 8
[Based on FMS (Delhi) 2004]

22. If $\sqrt{a^b}=5b+a^2$ then (a, b) could be
(a) (3, 4) (b) (2, 12)
(c) (4, 18) (d) (6, 4)
[Based on IIT Joint Man. Ent. Test 2004]

23. Find the greatest number which when divides 51,141 and 276 leaves the same remainder.
(a) 15 (b) 30
(c) 45 (d) 60
[Based on IIT Joint Man. Ent. Test 2004]

24. What is the smallest number with which 1800 must be multiplied to make it a perfect cube?
(a) 12 (b) 5
(c) 18 (d) 15
[Based on IIT Joint Man. Ent. Test 2004]

25. A number when divided by 192 gives a remainder of 54. What remainder would be obtained on dividing the same number by 16?
(a) 2 (b) 4
(c) 6 (d) 8
[Based on IIT Joint Man. Ent. Test 2004]

26. 2.53×0.154 is the same as if
(a) 253×0.00154 (b) 25.3×1.54
(c) 253×0.154 (d) 2.53×0.154
[Based on Narsee Manjee Inst. of Man. Studies, 2003]

27. The difference between the squares of two consecutive odd integers is always divisible by
(a) 8 (b) 7
(c) 6 (d) 3
[Based on Narsee Manjee Inst. of Man. Studies, 2003]

28. The number $3^9+3^{12}+3^{15}+3^n$ is a perfect cube of an integer for natural number n equalling
(a) 12 (b) 13
(c) 14 (d) 15
[Based on IITTM, Gwalior, 2003]

29. The number of ways, in which 8064 can be resolved as the product of two factors, is
(a) 10 (b) 16
(c) 24 (d) 48
[Based on IITTM, Gwalior, 2003]

30. Find the positive integer, which when added to the numerator and denominator of $\frac{2}{3}$, will result in a fraction nearest to $\frac{13}{15}$.
(a) 6 (b) 5
(c) 4 (d) 3
[Based on IITTM, Gwalior, 2003]

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31. If $x = 5+2\sqrt{6}$, then $\left(\frac{\sqrt{x-1}}{\sqrt{x}}\right)$ is equal to

(a) $2\sqrt{3}$ (b) $\sqrt{3}$
(c) $2\sqrt{2}$ (d) None of these

[Based on IITTM, Gwalior, 2003]

32. The largest number in the sequence $1, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}$ is

(a) 1 (b) $2^{\frac{1}{2}}$
(c) $3^{\frac{1}{3}}$ (c) $4^{\frac{1}{4}}$

[Based on IITTM, Gwalior, 2003]

33. $\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what?

(a) 15 (b) 8
(c) $\frac{7}{225}$ (d) 7

[Based on REC Tiruchiraplli 2003]

34. If p is a positive fraction less than 1, then

(a) $\frac{2}{p} - p$ is a positive number
(b) p^2 is less than p
(c) $\frac{1}{p}$ is a positive integer
(d) $\frac{1}{p}$ is less than 1

[Based on REC Tiruchiraplli 2003]

35. More than half of the members of a club are ladies. If $\frac{4}{7}$ of the ladies and $\frac{7}{11}$ of the gents in the club attended the meeting, then what is the smallest number of members that the club could have?

(a) 25 (b) 18
(c) 39 (d) 22

[Based on REC Tiruchiraplli 2003]

36. The term $(r^2 + s)^{1/2}$ is approximately equal to $r+\frac{s}{2r}$. Which of the following is the closest approximation to $(85)^{1/2}$?

(a) 9.06 (b) 9.34
(c) 9.22 (d) 9.28

[Based on REC Tiruchiraplli 2003]

37. $2^8 \times 3^6$ is divisible by

(a) $2^7 \times 3^7$ (b) $2^6 \times 3^5$
(c) $2^4 \times 3^7$ (d) $2^5 \times 3^8$

[Based on REC Tiruchiraplli 2003]

38. What is the value of $\frac{\frac{7}{9}\times\frac{7}{9}}{\frac{7}{9}+\frac{7}{9}+\frac{7}{9}}$?

(a) $\frac{7}{27}$ (b) $\frac{7}{18}$
(c) $\frac{49}{81}$ (d) $\frac{7}{81}$

[Based on REC Tiruchiraplli 2003]

39. $\frac{885\times885\times885+115\times115\times115}{885\times885+115\times115-885\times115} = ?$

(a) 1000 (b) 770
(c) 885 (d) 115

[Based on FMS (Delhi) 2003]

40. If $\left(x+\frac{1}{x}\right) = 3$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ is

(a) 927 (b) 414
(c) 364 (d) 322

[Based on FMS (Delhi) 2003]

41. The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is

(a) 2 (c) 3
(d) 2.327 (d) 2.273

[Based on FMS (Delhi) 2003]

42. The value of $\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\times\left(1+\frac{1}{x+3}\right)\left(1+\frac{1}{x+4}\right)$ is

(a) $1+\frac{1}{x+5}$ (b) $\frac{1}{x+5}$
(c) $x+\frac{1}{x+5}$ (d) $\frac{x+5}{x+1}$

[Based on FMS (Delhi) 2003]

43. The expression $\frac{(x-1)(x-2)(x^2-9x+14)}{(x-7)(x^2-3x+2)}$ simplifies to

(a) $(x-1)$ (b) $(x-2)$
(c) $(x-7)$ (d) $1/(x-7)$

[Based on FMS (Delhi) 2003]

44. The value of $\frac{1-x^4}{1+x}+\frac{1+x^2}{x}\times\frac{1}{x(1-x)} = ?$

(a) $\frac{1}{x}$ (b) $1+x$
(c) $1-x^2$ (d) 1

[Based on FMS (Delhi) 2003]

45. If $(a/b)^{x-1} = (b/a)^{x-3}$, then x is equal to

(a) 1 (b) 1/2
(c) 7/2 (d) 2

[Based on FMS (Delhi) 2003]

46. The expression $\frac{1}{1+x^{(b-a)}+x^{(c-a)}}+\frac{1}{1+x^{(a-b)}+x^{(c-b)}}+\frac{1}{1+x^{(b-c)}+x^{(a-c)}}$

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(a) x^{a-b-c} (b) 1
(c) 0 (d) None of these
[Based on FMS (Delhi) 2003]

47. $\frac{1}{1+a^{n-m}}+\frac{1}{1+a^{m-n}}$ is equal to
(a) 0 (b) 1
(c) 1/2 (d) a^{m+n}
[Based on FMS (Delhi) 2003]

48. Consider the following statements:
(a) If $a^x = b$, $b^y = c$, $c^z = a$, then $xyz = 0$.
(b) If $p = a^x$, $q = a^y$, $(p^y q^x)^z = a^2$, then $xyz = 1$
(c) $x^a = y^b = z^c$, $ab + bc + ca = 0$, then $xyz = 1$.
Of these statements:
(a) (a) and (b) are correct (b) (b) and c are correct
(c) Only (a) is correct (d) (a) and c are correct
[Based on FMS (Delhi) 2003]

49. If $r = -2$, then $r^4 + 2r^3 + 3r^2 + r = ?$
(a) –8 (b) –4
(c) 0 (d) 10
(e) None of the above **[Based on IIFT 2003]**

50. Let $(a/b) - (b/a) = x : y$ and $(x - y) = (a/b) + (b/a)$, then x is equal to
(a) $(a + b)/a$ (b) $(a + b)/b$
(c) $(a - b)/a$ (d) $(a - b)/b$
(e) None of the above **[Based on IIFT 2003]**

51. If $(3)^{(x+y)} = 81$ and $81^{(x-y)} = 3$, then the value of x and y are
(a) 17/8, 9/8 (b) 17/8, 15/8
(c) 17/8, 11/8 (d) 15/8, 11/8
(e) None of the above **[Based on IIFT 2003]**

52. Given that x and y are real numbers, let $P(x, y) = x^2 - y^2$. Then $P(3, P(3,4)) = ?$
(a) –40 (b) –7
(c) 40 (d) 7
(e) None of the above **[Based on IIFT 2003]**

53. Of the two-digit numbers (those from 10 to 99, both inclusive), how many have a second digit greater than the first digit?
(a) 38 (b) 40
(c) 36 (d) 41
[Based on SCMHRD Ent. Exam. 2003]

54. How much more is 1/2 of 2/3 than 3/4 of 1/3?
(a) 1/4 (b) 1/3
(c) 1/12 (d) 7/12
[Based on SCMHRD Ent. Exam. 2003]

55. Assuming that in Harappan era, rocks, stones and pebbles were used for money. The following used to be the currency valuations : 1 rock = 7 stones, 1 stone = 7 pebbles
If a person used 6 rocks to purchase a cave that costs 5 rocks, 2 stones and 3 pebbles, then how much would the change be?
(a) 4 stones, 4 pebbles
(b) 5 stones, 4 pebbles
(c) 1 rock, 5 stones, 4 pebbles
(d) 5 stones, 5 pebbles
[Based on SCMHRD Ent. Exam. 2003]

56. If $\frac{x^4}{x^9} = 5^{-5}$, then x is equal to
(a) 5 (b) 4
(c) 9 (d) None of these
[Based on IMT Ghaziabad, 2002]

57. The sum of the two digits of a number is 13 and the difference between the number (x) and that formed by reversing the digits (y) is 27, i.e. $(x - y)$ is 27. Find the number.
(a) 72 (b) 58
(c) 27 (d) 85
[Bases on IMT Ghaziabad, 2002]

58. Find a positive number which when increased by 17 is equal to 60 times the reciprocal of the number.
(a) 20 (b) 10
(c) 3 (d) 17
[Based on IMT Ghaziabad, 2002]

59. Which of the following numbers can fill in both the blank spaces?
$$\frac{*}{108}=\frac{12}{*}$$
(a) 36 (b) 24
(c) 72 (d) 54
[Based on IMT Ghaziabad, 2002]

60. If $x = 1 - q$ and $y = 2q + 1$, then for what value of q, x is equal to y?
(a) $\frac{1}{2}$ (b) –1
(c) 0 (d) 2
[Based on IMT Ghaziabad, 2002]

61. Four of the following five parts numbered (1), (2), (3), (4) and (5) are exactly equal. The number of the part which is not equal to the remaining four parts will be your answer.
(a) $36 \times 15 + 27 \times 13$
(b) $53 \times 4 + 64 + 16 \times 7$
(c) $328 + 41 \times 21 + 9 \times 2^3$
(d) $\sqrt{1024} \times 11 - 16 \times 7$
(e) $17 \times 18 - \sqrt{121} \times 6$ **[Based on IRMA, 2002]**

62. What approximate value should come in place of question mark (?) in the following equation?
$95.975^{3.5} + 16.001^{3.5} \times 6.002^{1.5} + 35.99^2 = ?$
(a) 36 (2) 16
(c) 96 (d) 32
(e) 6 **[Based on IRMA, 2002]**

63. The rank of 2/9 in the following fraction when expressed in ascending order is –2/3, 1/7, 0, 4/9,

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2/9, 14/15, 9/11
(a) 4 (b) 5
(c) 6 (d) 9
[Based on Narsee Monjee Inst. of Man. Studies 2002]

64. Which of the following fractions is the smallest ?
(a) 13/16 (b) 15/19
(c) 17/21 (d) 7/8
[Based on Narsee Monjee Inst. of Man. Studies 2002]

65. Which of the following fractions is less than 7/8 and greater than 1/3?
(a) 1/4 (b) 23/24
(c) 11/12 (d) 17/24
[Based on Narsee Monjee Inst. of Man. Studies 2002]

66. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\left[\frac{a^2+ab+b^2}{a^2-ab+b^2}\right]$ would be
(a) 3/4 (b) 4/3
(c) 3/5 (d) 5/3
[Based on FMS (Delhi) 2002]

67. If $(x + y) : (x - y) = 4 : 1$, then $(x^2 + y^2) : (x^2 - y^2) = ?$
(a) 25 : 9 (b) 16 : 1
(c) 8 : 17 (d) 17 : 8
[Based on FMS (Delhi) 2002]

68. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is
(a) 536 (b) 544
(c) 548 (d) 504
[Based on FMS (Delhi) 2002]

69. The difference between five times a certain number and two times the same number is 36. The number is
(a) 10 (b) 11
(c) 12 (d) 15
[Based on FMS (Delhi) 2002]

70. A number consists of two digits whose sum is 7. If the digits are reversed, then the number is increased by 27. The number is
(a) 25 (b) 34
(c) 16 (d) 52
[Based on FMS (Delhi) 2002]

71. Suppose $a = 2/3\ b$, $b = 2/3\ c$, and $c = 2/3\ d$. What would be the value of b as a fraction of d?
(a) 2/3 (b) 4/3
(c) 4/9 (d) 8/27
[Based on I.P. Univ. 2002]

72. Find x and y:
$x^2 - xy = 4$ and $y^2 - xy = -3$
(a) (4, 3) or (–4, –3) (b) (4, 1) or (–4, –1)
(c) (3, 4) or (–3, –4) (d) $x = y = 0$
(e) None of these **[Based on SCMHRD 2002]**

73. If $a * b = \frac{ab}{a+b}$, find $3 * (3 * -1)$
(a) –3 (b) –1.5
(c) 2/3 (d) 3
(e) –1 **[Based on SCMHRD 2002]**

74. Solve $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these **[Based on SCMHRD 2002]**

75. The expression
$(2a - 3b)(4a + 0.5b) - a(8a - 11b)$
(a) Is always positive for all a and b
(b) Is always negative for all a and b
(c) Is zero
(d) Depends on the values of a and b
(e) None of these **[Based on SCMHRD 2002]**

76. $\frac{1}{1+\frac{1}{x}}$ is equal to which of the following?
(a) $x + 1$ (b) $1/(x + 1)$
(c) $x/(x + 1)$ (d) $(x + 1)/x$
[Based on REC Tiruchirapalli, 2002]

77. If $x = 2k - 2$ and $y = 4k^2$, then what is y in terms of x?
(a) $x + 2$ (b) $(x + 2)^2/2$
(c) $(x + 2)^2$ (d) $x^2 + 4$
[Based on REC Tiruchirapalli, 2002]

78. At the first stop on his route, a driver unloaded 2/5 of the packages in his van. After he unloaded another three packages at his next stop, 1/2 of the original number of packages in the van remained. How many packages were in the van before the first delivery?
(a) 10 (b) 25
(c) 30 (d) 36
[Based on REC Tiruchirapalli, 2002]

79. Which of the given numbers is the greatest?
(a) $6\sqrt[3]{5}$ (b) $8\sqrt[3]{2}$
(c) $2\sqrt[3]{130}$ (d) $\sqrt[3]{900}$
[Based on REC Tiruchirapalli, 2002]

80. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is
(a) 3 (b) 2
(c) 1 (d) None of these
[Based on REC Tiruchirapalli, 2002]

81. $\left[1+\frac{1}{1+\frac{1}{1+\frac{1}{3}}}\right]+1\frac{4}{7}$ is equal to
(a) $1\frac{1}{3}$ (b) $1\frac{1}{4}$
(c) $1\frac{1}{7}$ (d) None of these
[Based on FMS (Delhi) 2006]

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EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\left(a+\frac{1}{a}\right)^2 = 3 \Rightarrow a+\frac{1}{a} = \sqrt{3}$

$\therefore \quad \left(a+\frac{1}{a}\right)^3 = a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$

$\Rightarrow \quad 3\sqrt{3} = a^3+\frac{1}{a^3}+3\sqrt{3} \Rightarrow a^3+\frac{1}{a^3} = \mathbf{0}.$

2. Ans. (a)

Solution $a = \frac{x}{x+y}, \; b = \frac{y}{x-y}$

$\therefore \quad ab = \frac{xy}{x^2-y^2}, \; a+b = \frac{x^2+y^2}{x^2-y^2}$

$\therefore \quad \frac{ab}{a+b} = \frac{xy}{\mathbf{x^2+y^2}}.$

3. Ans. (a)

Solution $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b} = \frac{\frac{6ab+2a^2}{a+b}}{\frac{2ab-2a^2}{a+b}}+\frac{\frac{6b^2+2ab}{a+b}}{\frac{2ab-2b^2}{a+b}}$

$= \frac{2a(3b+a)}{2a(b-a)}+\frac{2b(3b+a)}{2b(a-b)}$

$= \frac{3b+a}{b-a}-\frac{3b+a}{b-a} = \mathbf{0}.$

4. Ans. (c)

Solution Given expression

$\Rightarrow \quad \frac{(\sqrt{x+4}+\sqrt{x-10})^2}{(x+4)-(x-10)} = \frac{25}{4}$

$\Rightarrow \quad \frac{x+4+x-10+2\sqrt{x^2-6x-40}}{14} = \frac{5}{2}$

$\Rightarrow \quad 2x+2\sqrt{x^2-6x-40} = 41$

$\Rightarrow \quad (2x-41)^2 = [-2\sqrt{x^2-6x-40}]^2$

$\Rightarrow \quad 4x^2+1681-164x = 4(x^2-6x-40)$

$= 4x^2-24x-160$

$\Rightarrow \quad 140x = 1841$

$\Rightarrow \quad x = \mathbf{\frac{263}{20}}.$

5. Ans. (b)

Solution $\frac{9}{13} = 0.692, \; \frac{2}{3} = 0.666$

$\frac{8}{11} = 0.727, \; \frac{5}{7} = \mathbf{0.714}.$

6. Ans. (a)

Solution Consider $9, 9^2, 9^3, 9^4, 9^6$. Each of these numbers, when divided by 8, will leave a remainder of 1.

$\therefore 9^6 + 7$, when divided by 8, will leave a remainder of **0.**

7. Ans. (b)

Solution The product of the numbers = H.C.F × L.C.M.

Let the third number be x.

$\therefore \; 3240 \times 3600 \times x = 36 \times 2^4 \times 3^5 \times 5^2 \times 7^2$

$\Rightarrow \quad x = \frac{36\times 2^4\times 3^5\times 5^2\times 7^2}{(2^3\times 3^4\times 5)\times(2^4\times 3^2\times 5^2)}$

$\left[\because \begin{array}{l} 3240 = 2^3\times 3^4\times 5 \\ 3600 = 2^4\times 3^2\times 5^2 \end{array}\right]$

$\Rightarrow \quad x = \frac{(2^2\times 3^2)\times 2^4\times 3^5\times 5^2\times 7^2}{2^3\times 3^4\times 5\times 2^4\times 3^2\times 5^2}$

$= \frac{2^6\times 3^7\times 5^2\times 7^2}{2^7\times 3^6\times 5^3} = \mathbf{2^2\times 3^5\times 7^2}.$

8. Ans. (a)

Solution The four prime numbers are 5, 7, 11 and 13.

9. Ans. (b)

Solution By using $\frac{a^3-b^3}{a^2+ab+b^2} = a-b$

$\frac{(2.75)^3-(2.25)^3}{(2.75)^2+2.75\times 2.25+(2.25)^2} = 2.75-2.25 = \mathbf{0.50}.$

10. Ans. (d)

11. Ans. (a)

Solution $\frac{1-\left(\frac{1}{10}\right)^{-1}}{\left(\frac{2^3}{3}\right)\left(\frac{3}{2}\right)^3+(-3)} = \frac{1-10}{\frac{2^3}{3}\cdot\frac{3^3}{2^3}-3}$

$= \frac{-9}{9-3} = \mathbf{-\frac{3}{2}}.$

12. Ans. (d)

Solution $-\frac{*}{7}\times\frac{9}{7} = -\frac{256}{49}-\frac{81}{49}+1$

$\Rightarrow \quad -*\times 9 = -256-81+49$

$\Rightarrow \quad * = \mathbf{32}.$

13. Ans. (a)

Solution $\frac{1}{2} < \frac{1+1}{2+1} \Rightarrow \frac{1}{2} < \frac{2}{3}$

$\frac{2}{3} < \frac{2+1}{3+1} \Rightarrow \frac{2}{3} < \frac{3}{4}$

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$$\frac{2}{5} < \frac{2+1}{5+1} \Rightarrow \frac{2}{5} < \frac{1}{2}.$$

14. Ans. (c)

Solution $\frac{a^3-b^3}{a^3+b^3} = \frac{13}{14} \Rightarrow 14a^3 - 14b^3 = 13a^3 + 13b^3$

$$\Rightarrow a^3 = 27b^3 \Rightarrow \frac{a}{b} = 3$$

$$\therefore \quad \frac{a+b}{a-b} = \frac{\frac{a}{b}+1}{\frac{a}{b}-1} = \frac{4}{2} = \mathbf{2.}$$

15. Ans. (c)

Solution $\left(\frac{87}{89}\right)^2 = 0.9555$

$$\frac{87}{89} = 0.9775$$

$$\frac{87}{83} = 1.0481$$

$$\left(\frac{87}{83}\right)^2 = 1.0985$$

If $x < 1 \Rightarrow x^2 < x$

If $x > 1 \Rightarrow \boldsymbol{x^2 > x.}$

16. Ans. (c)

17. Ans. (b)

18. Ans. (c)

19. Ans. (a)

Solution Given expression $= 7^{21}(1 + 7 + 7^2 + 7^3)$
$= 7^{21} \times 400,$

which is completely divisible by 25.

Hence, remainder is zero.

20. Ans. (b)

Solution Sum of the digits in the thousand's place
$= 6000 + 12000 + 18000 + 24000 = 60000$

Sum of the digits in the hundred's place
$= 600 + 1200 + 1800 + 2400 = 6000$

Sum of the digits in the ten's place
$= 60 + 120 + 180 + 240 = 600$

Sum of the digits in the unit's place
$= 6 + 12 + 18 + 24 = \mathbf{60.}$

21. Ans. (b)

Solution 27, 216, 729, 1728, 3375, 5832, 9261.

22. Ans. (b)

Solution Put $a = 2$, $b = 12$ in $\sqrt{a^b} = 5b + a^2$

$\therefore \quad \sqrt{2^{12}} = 5 \times 12 + 4 = 64$

$\Rightarrow \quad 2^6 = 64$, which is true.

23. Ans. (c)

24. Ans. (d)

Solution $1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$\therefore$ 1800 must be multiplied by $3 \times 5 = 15$ so that the resulting number becomes a perfect cube.

25. Ans. (c)

26. Ans. (a)

27. Ans. (a)

Solution $(2x + 3)^2 - (2x + 1)^2$
$= 4x^2 + 12x + 9 - (4x^2 + 4x + 1)$
$= 8x + 8 = \mathbf{8(x + 1).}$

28. Ans. (c)

Solution For $n = 14$, $3^9(1 + 3^3 + 3^6 + 3^5)$
$= 3^9(1 + 27 + 729 + 243)$
$= \mathbf{3^9 \times 10^3.}$

29. Ans. (c)

Solution 8064 is divisible by 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 24, 28, 32, 36, 42, 48, 56, 63, 64, 72, 84.

30. Ans. (b)

Solution Let $\frac{2+K}{3+K} = \frac{13}{15} \Rightarrow 2K = 9 \Rightarrow K = 4.5 \approx \mathbf{5.}$

31. Ans. (d)

Solution $x = 5+2\sqrt{6}$

$\Rightarrow \quad \sqrt{x} = \sqrt{5+2\sqrt{6}} = \sqrt{A} + \sqrt{B}$, say

$\therefore \quad x = 5+2\sqrt{6} = A + B + 2\sqrt{AB}$

$\Rightarrow \quad A + B = 5, AB = 6$

$\Rightarrow \quad A = 3, B = 2$

$\Rightarrow \quad \sqrt{x} = \sqrt{3} + \sqrt{2}$

$$\therefore \quad \frac{\sqrt{x}-1}{\sqrt{x}} = \frac{\sqrt{3}+\sqrt{2}-1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= 1 - \sqrt{3} + \sqrt{2} = \mathbf{1 - (\sqrt{3} - \sqrt{2}).}$$

32. Ans. (c)

Solution $1^{1/2} = 1$, $(2^{1/2})^{12} = 2^6 = 64$

$(3^{1/3})^{12} = 3^4 = 81$, $(4^{1/4})^{12} = 4^3 = \mathbf{64.}$

33. Ans. (d)

Solution $\frac{5}{8}$ of $24 = \frac{15}{7}$ of K, say

$$\Rightarrow K = \frac{7}{15} \times \frac{5}{8} \times 24 = \mathbf{7.}$$

34. Ans. (a), (c)

35. Ans. (a)

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36. Ans. (c)

Solution $r = 9,\ s = 4$

$$\therefore\ 85^{1/2} = r + \frac{s}{2r} = 9 + \frac{4}{18} = 9\frac{2}{9} = \mathbf{9.22.}$$

37. Ans. (b)

38. Ans. (a)

Solution $$? = \frac{\frac{49}{81}}{\frac{21}{9}} = \frac{49}{81} \times \frac{9}{21} = \frac{7}{27}.$$

39. Ans. (a)

Solution $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

Put $a = 885$, $b = 115$, we have

$$? = \frac{a^3 + b^3}{a^2 + b^2 - ab} = a + b = \mathbf{1000.}$$

40. Ans. (d)

Solution $$x + \frac{1}{x} = 3 \Rightarrow \left(x + \frac{1}{x}\right)^3 = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 18$$

$$\Rightarrow \left(x^3 + \frac{1}{x^3}\right)^2 = 324$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 2 = 324$$

$$\Rightarrow x^6 + \frac{1}{x^6} = \mathbf{322.}$$

41. Ans. (a)

Solution $$\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$$

$$= \frac{(2.3)^2 - (.3)^3}{(2.3)^2 + 2.3 \times .3 + (.3)^2}$$

$$= 2.3 - .3 = \mathbf{2}$$

$$[\because\ a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

42. Ans. (d)

Solution Given expression

$$= \frac{x+2}{x+1} \times \frac{x+3}{x+2} \times \frac{x+4}{x+3} \times \frac{x+5}{x+4}$$

$$= \frac{x+5}{x+1}.$$

43. Ans. (b)

Solution Given expression

$$= \frac{(x-1)(x-2)(x-7)(x-2)}{(x-7)(x-2)(x-1)} = \mathbf{x - 2.}$$

44. Ans. (d)

Solution

$$? = \frac{(1+x^2)(1+x)(1-x)}{(1+x)} \times \frac{x}{1+x^2} \times \frac{1}{x(1-x)} = 1.$$

45. Ans. (d)

Solution $$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3} = \left(\frac{a}{b}\right)^{-x+3}$$

$$\therefore\ x - 1 = -x + 3 \Rightarrow \mathbf{x = 2.}$$

46. Ans. (b)

Solution Given expression

$$= \frac{1}{1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}} + \frac{1}{1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}} + \frac{1}{1 + \frac{x^b}{x^c} + \frac{x^a}{x^c}} = \mathbf{1.}$$

47. Ans. (b)

Solution $$\frac{1}{1 + \frac{a^n}{a^m}} + \frac{1}{1 + \frac{a^m}{a^n}} = \frac{a^m}{a^m + a^n} + \frac{a^n}{a^m + a^n} = \mathbf{1.}$$

48. Ans. (b)

Solution (b) is correct.

$$(p^y q^x)^z = a^2 \Rightarrow (a^{xy} \cdot a^{yx})^z = a^2$$

$$\Rightarrow (a^{2xy})^z = a^2$$

$$\Rightarrow a^{2xyz} = a^2$$

$$\Rightarrow 2xyz = 2 \Rightarrow xyz = 1$$

(c) is correct.

$$x^a = y^b = z^c$$

$$\Rightarrow\ a \log x = b \log y = c \log z$$

$$\therefore\ ab + bc + ca = 0$$

$$\Rightarrow\ c\frac{\log z}{\log x} \times c\frac{\log z}{\log y} + c\frac{\log z}{\log y} \times c + c \times c\frac{\log z}{\log x} = 0.$$

$$\Rightarrow\ \log z \log z + \log x \log z + \log y \log z = 0$$

$$\Rightarrow\ \log z + \log x + \log y = 0$$

$$\Rightarrow \log(xyz) = 0 = \log 1 \Rightarrow \mathbf{xyz = 1.}$$

49. Ans. (d)

Solution $r^4 + 2r^3 + 3r^2 + r$

$$= 16 + 2 \times (-8) + 3 \times 4 - 2$$

$$= \mathbf{10.}$$

50. Ans. (e)

Solution $$\frac{x}{y} = \frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{ab}$$

$$x - y = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab}$$

$$\Rightarrow\ x = \frac{a^2 + b^2}{ab} + y = \frac{a^2 + b^2}{ab} + \frac{abx}{a^2 - b^2}$$

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$$\Rightarrow \quad x\left[1-\frac{ab}{a^2-b^2}\right]=\frac{a^2+b^2}{ab}$$

$$\Rightarrow \quad x=\frac{a^2+b^2}{ab}\times\frac{(a^2-b^2)}{a^2-b^2-ab}.$$

51. Ans. (b)

Solution $3^{x+y}=81=34,\ 81^{x-y}=3$

$\Rightarrow \quad x+y=4,\ (3^4)^{(x-y)}=3^1$

$\Rightarrow \quad x+y=4,\ 4x-4y=1$

$x=\frac{17}{8},\ y=\frac{15}{8}.$

52. Ans. (a)

Solution $P(x, y)=x^2-y^2$

$\therefore \quad P(3, 4)=3^2-4^2=9-16=-7$

$\Rightarrow \quad P(3, P(3, 4))=P(3, -7)$

$=(3)^2-(-7)^2=9-49$

$=$ **–40.**

53. Ans. (c)

Solution 12 to 19 : 8, 23 to 29 : 7,
34 to 39 : 6, 45 to 49 : 5,
56 to 59 : 4, 67 to 69 : 3,
78 to 79 : 2, 89 : 1
Total : 36.

54. Ans. (c)

Solution $\frac{1}{2}\text{ of }\frac{2}{3}-\frac{3}{4}\text{ of }\frac{1}{3}=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}.$

55. Ans. (a)

Solution Cost of Cave
= 5 rocks + 2 stones + 3 pebbles
= 35 stones + 14 pebbles + 3 pebbles
= 245 pebbles + 14 pebbles + 3 pebbles
= 262 pebbles

1 rock = 49 pebbles

$\therefore$ To use 6 rocks, it requires 294 pebbles

$\therefore$ Change required = 294 – 262 = 32 pebbles
= **4 stones and 4 pebbles.**

56. Ans. (a)

57. Ans. (d)

Solution Let the number be $10p + q$

$\therefore \quad p+q=13$ and $(10p+q)-(10q+p)=27$

$(x=10p+q,\ y=10q+p)$

i.e. $9p-9q=27$, i.e. $p-q=3$

$\therefore \quad p=8,\ q=5$

Hence the required number = **85.**

58. Ans. (c)

Solution Let the number be x

$\therefore \quad x+17=\frac{60}{x} \Rightarrow x^2+17x-60=0$

$\Rightarrow (x+20)(x-3)=0$

$\Rightarrow x=3\ (x>0).$

59. Ans. (a)

60. Ans. (c)

61. Ans. (a)

Solution (1) $=36\times15+27\times13$

$=36\times\frac{15}{27}\times13=260$

(2) $=53\times5+64\div16\times7$

$=53\times4+\frac{64}{16}\times7$

$=212+28=240$

(3) $=\frac{328}{41}\times21+9\times2^3$

$=168+9\times8$

$=168+72=240$

(4) $=\sqrt{1024}\times11-16\times7$

$=32\times11-112$

$=352-112=240$

(5) $=17\times18-\sqrt{121}\times6$

$=17\times18-11\times6$

$=306-66=$ **240.**

62. Ans. (e)

Solution Given expression $=\frac{96^{3.5}\times6^{1.5}}{16^{3.5}}\times\frac{1}{36^2}$

$=\frac{6^5}{6^4}=$ **6.**

63. Ans. (a)

Solution $-\frac{2}{3}, 0, \frac{1}{7}, \frac{2}{9}, \frac{4}{9}, \frac{9}{11}, \frac{14}{15}.$

64. Ans. (b)

Solution $\frac{13}{16}=.8125,\ \frac{15}{19}=.79,\ \frac{17}{21}=.8095,$

$\frac{7}{8}=$ **.875.**

65. Ans. (d)

Solution $\frac{7}{8}=.875,\ \frac{1}{3}=.3333\ ...,\ \frac{17}{24}=.7083.$

$\therefore \quad \frac{1}{3}<\frac{17}{24}<\frac{7}{8}.$

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66. Ans. (b)

Solution $a = \dfrac{\sqrt{5}+1}{\sqrt{5}-1} = \dfrac{(\sqrt{5}+1)^2}{4} = \dfrac{6+2\sqrt{5}}{4} = \dfrac{3+\sqrt{5}}{2}$

$b = \dfrac{\sqrt{5}-1}{\sqrt{5}+1} = \dfrac{(\sqrt{5}-1)^2}{4} = \dfrac{6-2\sqrt{5}}{4} = \dfrac{3-\sqrt{5}}{2}$

$a^2 = \left(\dfrac{3+\sqrt{5}}{2}\right)^2 = \dfrac{14+6\sqrt{5}}{4}$

$b^2 = \left(\dfrac{3-\sqrt{5}}{2}\right)^2 = \dfrac{14-6\sqrt{5}}{4}$

$ab = \left(\dfrac{\sqrt{5}+1}{\sqrt{5}-1}\right)\left(\dfrac{\sqrt{5}-1}{\sqrt{5}+1}\right) = \dfrac{4}{1} = 1.$

$\therefore \quad \dfrac{a^2+ab+b^2}{a^2-ab+b^2} = \dfrac{32}{24} = \mathbf{\dfrac{4}{3}}.$

67. Ans. (d)

Solution $\dfrac{x+y}{x-y} = \dfrac{4}{1} \Rightarrow x = \dfrac{5}{3}y$

$\therefore \quad x^2 + y^2 = \dfrac{25}{9}y^2 + y^2 = \dfrac{34y^2}{9}$

$x^2 - y^2 = \dfrac{25}{9}y^2 - y^2 = \dfrac{16y^2}{9}$

$\therefore \quad \dfrac{x^2+y^2}{x^2-y^2} = \dfrac{34}{16} = \mathbf{\dfrac{17}{8}}.$

68. Ans. (c)

Solution L.C.M of 12, 15, 20 and 54 = 540

$\therefore$ Required number = 540 + 8 = **548.**

69. Ans. (c)

Solution $5X - 2X = 36 \Rightarrow X = \mathbf{12}.$

70. Ans. (a)

Solution Let the number be $10x + y$.

$\therefore \quad x + y = 7$ and $10y + x = 10x + y + 27$

$\Rightarrow \quad y = 5, x = 2$

$\therefore$ The number = $10x + y$ = **25.**

71. Ans. (c)

Solution $b = \dfrac{2}{3}c,\ c = \dfrac{2}{3}d$

$\Rightarrow \quad b = \dfrac{2}{3} \times \dfrac{2}{3}d = \mathbf{\dfrac{4}{9}}d.$

72. Ans. (a)

Solution $x^2 - xy = 4 \Rightarrow x(x - y) = 4$ ...(i)

$y^2 - xy = -3 \Rightarrow y(y - x) = -3$ ...(ii)

(i) and (ii) $\Rightarrow \dfrac{4}{x} = \dfrac{3}{y}, \Rightarrow \dfrac{x}{y} = \mathbf{\dfrac{4}{3}}.$

73. Ans. (a)

Solution Given expression

$= 3 * \left(\dfrac{3\times(-1)}{3+(-1)}\right) = 3 * \left(\dfrac{-3}{2}\right)$

$= \dfrac{3\times\left(\dfrac{-3}{2}\right)}{3+\left(\dfrac{-3}{2}\right)} = \dfrac{\dfrac{-9}{2}}{\dfrac{3}{2}} = \mathbf{-3}.$

74. Ans. (d)

Solution $\left(\dfrac{9}{4}\right)^x \times \left(\dfrac{8}{27}\right)^{x-1} = \dfrac{2}{3}$

$\Rightarrow \quad \left(\dfrac{3}{2}\right)^{2x} \times \left(\dfrac{2}{3}\right)^{3(x-1)} = \dfrac{2}{3}$

$\Rightarrow \quad \left(\dfrac{2}{3}\right)^{-2x} \times \left(\dfrac{2}{3}\right)^{3x-3} = \left(\dfrac{2}{3}\right)^1$

$\Rightarrow \quad -2x + (3x - 3) = 1 \Rightarrow \mathbf{x = 4}.$

75. Ans. (b)

Solution Given expression = $-\dfrac{3}{2}b^2$. Since b^2 is always positive therefore, $-\dfrac{3}{2}b^2$ is always negative.

76. Ans. (c)

Solution $\dfrac{1}{1+\dfrac{1}{x}} = \dfrac{1}{\dfrac{x+1}{x}} = \mathbf{\dfrac{x}{x+1}}.$

77. Ans. (c)

Solution $x = 2k - 2 \Rightarrow y = 4k^2 = (2k)^2 = \mathbf{(x + 2)^2}.$

78. Ans. (c)

Solution Suppose there were x packages in the van before delivery.

$\therefore$ After 1st delivery, the number of packages in the van = $x - \dfrac{2}{5}x = \dfrac{3}{5}x$

After 2nd delivery, the number of packages in the van = $\dfrac{3}{5}x - 3 = \dfrac{3x-15}{5}$

$\therefore \quad \dfrac{3x-15}{5} = \dfrac{x}{2}$ (Given)

$\Rightarrow \mathbf{x = 30}.$

79. Ans. (a)

Solution $(6\times\sqrt[3]{5})^3 = 216 \times 5 = 1080$

$(8\times\sqrt[3]{2})^3 = 512 \times 2 = 1024$

$(2\times\sqrt[3]{130})^3 = 8 \times 130 = 1040$

$(\sqrt[3]{900})^3 = \mathbf{900}.$

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80. Ans. (b)

Solution $x = 2 + 2^{2/3} + 2^{1/3}$

$\Rightarrow \quad (x-2) = 2^{2/3} + 2^{1/3}$

$\Rightarrow (x-2)^3 = (2^{2/3} + 2^{1/3})^3$

$= 4 + 2 + 3 \times 2^{2/3} \times 2^{1/3} [2^{2/3} + 2^{1/3}]$

$= 6 + 3 \times 2 (x-2)$

$\Rightarrow \quad (x-2)^3 = 6 + 6x - 12 = 6x - 6$

$\Rightarrow \quad x^3 - 8 - 6x(x-2) = 6x - 6$

$\Rightarrow \quad x^3 - 6x + 6x = \mathbf{2}.$

81. Ans. (d)

Solution $\left[1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{3}}}\right]+1\dfrac{4}{7}$

$= \left[1+\dfrac{1}{1+\dfrac{3}{4}}\right]+\dfrac{11}{7} = \left[1+\dfrac{4}{7}\right]+\dfrac{11}{7}$

$= \dfrac{11}{7}+\dfrac{11}{7} = \dfrac{22}{7}$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. $\dfrac{31}{10} \times \dfrac{3}{10} + \dfrac{7}{5} \div 20 = ?$

(a) 0 (b) 1

(c) 100 (d) $\dfrac{107}{200}$

2. The value of $1 + \dfrac{1}{1.2} + \dfrac{1}{1.2.4} + \dfrac{1}{1.2.4.8}$ up to 2 places of decimal is

(a) 1.64 (b) 1.65

(c) 1.79 (d) 1.78

3. $\dfrac{\frac{1}{5}+\frac{1}{5}\text{ of }\frac{1}{5}}{\frac{1}{5}\text{ of }\frac{1}{5}+\frac{1}{5}} = ?$

(a) 1 (b) 5

(c) $\dfrac{1}{5}$ (d) 25

4. In a certain office, $\left(\dfrac{1}{3}\right)$ of the workers are women, $\left(\dfrac{1}{2}\right)$ of the women are married and $\left(\dfrac{1}{3}\right)$ of the married women have children. If $\left(\dfrac{3}{4}\right)$ of the men are married and $\dfrac{2}{3}$ of the married men have children, what part of the workers are without children?

(a) $\dfrac{5}{18}$ (c) $\dfrac{4}{9}$

(c) $\dfrac{11}{18}$ (d) $\dfrac{17}{36}$

5. The value of $48 \div 12 \times \left(\dfrac{9}{8}\text{of}\dfrac{4}{3}+\dfrac{3}{4}\text{of}\dfrac{2}{3}\right)$ is

(a) $1\dfrac{1}{3}$ (b) $5\dfrac{1}{3}$

(c) 3 (d) 12

6. In a bank there are equal number of male and female workers. $\left(\dfrac{1}{2}\right)$ of the female are married and $\left(\dfrac{1}{3}\right)$ of the married women have children, $\left(\dfrac{3}{4}\right)$ of the males are married and $\left(\dfrac{2}{3}\right)$ of the married men have children, what part of workers are without children?

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{3}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{12}$

7. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\dfrac{26}{27}$. The original fraction is

(a) $\dfrac{16}{27}$ (b) $\dfrac{8}{3}$

(c) $\dfrac{8}{27}$ (d) $\dfrac{4}{3}$

8. $\left(\dfrac{?}{18}\right) \times \left(\dfrac{?}{162}\right) = 1$

(a) 64 (b) 34

(c) 54 (d) 94

9. $\dfrac{(0.55)^2 + (0.07)^2 + (0.027)^2}{(0.055)^2 + (0.007)^2 + (.0027)^2} = ?$

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Test II **[Time: 10 min]**

(a) 10 (b) 100
(c) 50 (d) 1000

10. Ravi earns twice as much in January as in each of the other months. What part of his annual earnings he earns in that month?

(a) $\frac{2}{13}$ (c) $\frac{1}{10}$
(c) $\frac{5}{7}$ (d) $\frac{5}{7}$

11. The value of $\frac{1}{3+\frac{2}{2+\frac{1}{2}}}$ is

(a) $\frac{5}{19}$ (b) $\frac{19}{5}$
(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

12. $7\frac{1}{2}-\left[2\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(1\frac{1}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right]$ is equal to

(a) $\frac{2}{9}$ (b) 1
(c) $4\frac{1}{2}$ (d) $1\frac{77}{288}$

13. $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\text{----}\left(1-\frac{1}{1001}\right)$ = ?

(a) $\frac{1}{1001}$ (b) $\frac{1000}{1001}$
(c) $\frac{2}{1001}$ (d) None of these

14. A number divided by 68 gives the quotient 269 and the remainder zero. If the same number is divided by 67, the remainder is

(a) 3 (b) 2
(c) 1 (d) 0

15. $(4^3)^4 \div (4^2)^3 \times (4^5)^0$ = ?

(a) 6^4 (b) 4^6
(c) 2^6 (d) 6^2

16. $\sqrt{\frac{67.6}{?}} = 0.26$

(a) 10 (b) 10^2
(c) 10^3 (d) 50^2

17. In a family, the father took $\frac{1}{4}$ of the cake and he had 3 times as much as others had. The total number of family members is

(a) 3 (b) 7
(c) 10 (d) 12

18. If $\sqrt{18\times14\times x} = 84$, x is equal to?

(a) 82 (b) 28
(c) 32 (d) 42

19. The smallest fraction which should be subtracted from the sum of $1\frac{3}{4}$, $2\frac{1}{2}$, $5\frac{7}{12}$, $3\frac{1}{3}$ and $2\frac{1}{4}$ to make the result a whole number is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$
(c) $\frac{1}{2}$ (d) 7

20. The value of $1+\frac{1}{1+\frac{1}{1+\frac{1}{9}}}$ is

(a) $\frac{29}{19}$ (b) $\frac{10}{19}$
(c) $\frac{29}{10}$ (d) $\frac{10}{9}$

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution Given expression

$$= \frac{93}{100}+\frac{7}{5}\times\frac{1}{20} = \frac{93}{100}+\frac{7}{100} = \frac{100}{100} = \mathbf{1.}$$

3. Ans. (d)

Solution Given expression

$$= \frac{\frac{1}{5}+\frac{1}{25}}{\frac{1}{25}+\frac{1}{5}} = \frac{\frac{1}{5}\times25}{\frac{1}{25}\times5} = 5\times5 = \mathbf{25.}$$

5. Ans .(d)

Solution Given expression

$$= \frac{48}{12}\times\left(\frac{3}{2}\div\frac{1}{2}\right) = 4\times\left(\frac{3}{2}\times2\right) = \mathbf{12.}$$

7. Ans. (b)

Solution Let x be the fraction

$$x\times x\div\frac{1}{x} = 18\ \frac{26}{27} = \frac{512}{27}$$

$$x^3 = \left(\frac{8}{3}\right)^3 \qquad x = \mathbf{\frac{8}{3}.}$$

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9. Ans. (b)

Solution Let $0.55 = a$

$0.07 = b$ and $0.027 = c$

Then, the given expression becomes

$$\frac{a^2+b^2+c^2}{(0.1\times a)^2+(0.1\times b)^2+(0.1\times c)^2}$$

$$= \frac{a^2+b^2+c^2}{0.01[a^2+b^2+c^2} = \frac{1}{0.01} = \mathbf{100.}$$

11. Ans. (a)

Solution Given expression $= \dfrac{1}{3+\dfrac{2}{\dfrac{5}{2}}} = \dfrac{1}{3+2\times\dfrac{2}{5}}$

$$= \frac{1}{\left(\frac{19}{5}\right)} = \mathbf{\frac{5}{19}}.$$

13. Ans. (c)

Solution Given product:

$$= \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)\cdots\left(\frac{1000}{1001}\right) = \mathbf{\frac{2}{1001}}.$$

15. Ans. (b)

Solution Putting x for (?) and since all bases are equal to 4, hence, put $a = 4$

$\therefore \quad x = (a^3)^4 \div (a^2)^3 \times (a^5)^0$

or, $\quad x = a^{12} \div a^6 \times 1$ or, $x = a^{12-6}$ or, $x = a^6$

$\therefore \quad x = \mathbf{4^6}.$

17. Ans. (c)

Solution Let there be n numbers in the family

Then, according to question,

$$3\left[\frac{3}{4}+n-1\right] = \frac{1}{4}$$

$$3\left[\frac{3}{4(n-1))}\right] = \frac{1}{4}$$

$n - 1 = 9$

$n = \mathbf{10.}$

19. Ans. (a)

Solution $1\frac{3}{4}+2\frac{1}{2}+5\frac{7}{12}+3\frac{1}{3}+2\frac{1}{4}$

$$= \frac{7}{4}+\frac{5}{2}+\frac{67}{12}+\frac{10}{3}+\frac{9}{4} = \frac{185}{12}$$

Since 180 is completely divisible by 12

$\therefore \quad \dfrac{5}{12}$ should be subtracted.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (c)	**6.** (a)	**8.** (c)	**10.** (a)	**12.** (c)	**14.** (c)	**16.** (c)	**18.** (b)	**20.** (a)

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5 Percentage

INTRODUCTION

The term *per cent* means per hundred or for every hundred. It is the abbreviation of the Latin phrase *per centum*.

Scoring 60 per cent marks means out of every 100 marks the candidate scored 60 marks.

The term per cent is sometimes abbreviated as p.c. The symbol % is often used for the term per cent.

Thus, 40 per cent will be written as 40%.

A fraction whose denominator is 100 is called a *percentage* and the numerator of the fraction is called *rate per cent*, e.g. $\frac{5}{100}$ and 5 per cent means the same thing, i.e. 5 parts out of every hundred parts.

SOME BASIC FORMULAE

1. To Convert a Fraction into a Per cent:

To convert any fraction $\frac{l}{m}$ to rate per cent, multiply it by 100 and put % sign, i.e. $\frac{l}{m} \times 100\%$.

Illustration 1 What percentage is equivalent to $\frac{3}{5}$?

Solution $\frac{3}{5} \times 100 = \mathbf{60\%}$.

2. To Convert a Per cent into a Fraction:

To convert a per cent into a fraction, drop the per cent sign and divide the number by 100.

Illustration 2 What fraction is $16\frac{2}{3}\%$?

Solution $16\frac{2}{3}\% = \frac{\left(\frac{50}{3}\right)}{100} = \left(\frac{50}{3} \times \frac{1}{100}\right) = \mathbf{\frac{1}{6}}$.

3. To find a percentage of a given number:

x % of given number (N) $= \frac{x}{100} \times$ N.

Illustration 3 75% of 400 = ?

Solution 75% of $400 = \frac{75}{100} \times 400 = \mathbf{300}$.

Illustration 4 Find a number whose 4% is 72.

Solution Let the required number be x.

Then, 4% of $x = 72$

$\Rightarrow \frac{4}{100} \times x = 72 \Rightarrow x = \frac{100}{4} \times 72 = \mathbf{1800}$.

Illustration 5 What per cent of 25 kg is 3 5 kg?

Solution Let x% of 25 kg be 3 5 kg.

Then, x% of 25 kg = 3 5 kg

$\Rightarrow \frac{x}{100} \times 25 = 3\ 5 \Rightarrow x = \frac{3{\cdot}5 \times 100}{25} = 14$.

Hence, 3 5 kg is 14% of 25 kg.

SOME USEFUL SHORT-CUT METHODS

1. (a) If A is x% more than that of B, then B is less than that of A by

$$\left[\frac{x}{100+x} \times 100\right]\%.$$

(b) If A is x% less than that of B, then B is more than that of A by

$$\left[\frac{x}{100-x} \times 100\right]\%.$$

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Explanation

Given: $A = B + \frac{x}{100}B = \frac{100+x}{100}B$

$\therefore\ A - B = \frac{100+x}{100}B - B$

$= \left(\frac{100+x}{100} - 1\right)B = \frac{x}{100}B.$

So, $\frac{A-B}{A} = \frac{\frac{x}{100}B}{\frac{100+x}{100}B} = \frac{x}{100+x}$

$\Rightarrow\ A - B = \left(\frac{x}{100+x} \times 100\right)\%$ of A.

Therefore, B is less than that of A by

$$\left(\frac{x}{100+x} \times 100\right)\%.$$

Similarly, (b) can be proved.

Illustration 6 If Mohan's salary is 10% more than that of Sohan, then how much per cent is Sohan's salary less than that of Mohan?

Solution Here, $x = 10$.

$\therefore$ Required answer $= \left(\frac{x}{100+x} \times 100\right)\%$

$= \left(\frac{10}{100+10} \times 100\right)\%$

$= \mathbf{11\frac{1}{9}\%}.$

Illustration 7 If A's income is 40% less than B's income, then how much per cent is B's income more than A's income?

Solution Here, $x = 40$.

$\therefore$ Required answer $= \left(\frac{x}{100-x} \times 100\right)\%$

$= \left(\frac{40}{100-40} \times 100\right)\%$

$= \mathbf{66\frac{2}{3}\%}.$

2. If A is $x\%$ of C and B is $y\%$ of C, then

$$A = \frac{x}{y} \times 100\% \text{ of } B.$$

Explanation

Given :

$$A = \frac{x}{100}C \Rightarrow C = 100\frac{A}{x}$$

and, $B = \frac{y}{100}C \Rightarrow C = 100\frac{B}{y}$

$\therefore\ C = 100\frac{A}{x} = 100\frac{B}{y} \Rightarrow A = \frac{x}{y}B$

or, $\frac{x}{y} \times 100\%$ of B.

Illustration 8 If A is 20% of C and B is 25% of C, then what percentage is A of B?

Solution Here, $x = 20$ and $y = 25$.

$A = \frac{x}{y} \times 100\%$ of B

$= \mathbf{\frac{20}{25} \times 100\% \text{ of } B, \text{ i.e. } 80\% \text{ of } B.}$

3. (a) If two numbers are respectively $x\%$ and $y\%$ more than a third number, then the first number is $\left(\frac{100+x}{100+y} \times 100\right)\%$ of the second and the second is $\left(\frac{100+y}{100+x} \times 100\right)\%$ of the first.

(b) If two numbers are respectively $x\%$ and $y\%$ less than a third number, then the first number is $\left(\frac{100-x}{100-y} \times 100\right)\%$ of the second and the second is $\left(\frac{100-y}{100-x} \times 100\right)\%$ of the first.

Explanation

Let A, B and C be the three numbers.

Given:

$$A = C + \frac{x}{100}C = \left(\frac{100+x}{100}\right)C \Rightarrow C = A\left(\frac{100}{100+x}\right)$$

and, $B = C + \frac{y}{100}C = \left(\frac{100+y}{100}\right)C \Rightarrow C = B\left(\frac{100}{100+y}\right)$

$\therefore\ A\left(\frac{100}{100+x}\right) = B\left(\frac{100}{100+y}\right)$

$\Rightarrow\ A = \left(\frac{100+x}{100+y}\right)B$ or $\left(\frac{100+x}{100+y}\right) \times 100\%$ of B

and, $B = \left(\frac{100+y}{100+x}\right)A$ or $\left(\frac{100+y}{100+x}\right) \times 100\%$ of A.

Similarly, (b) can be proved.

Illustration 9 Two numbers are respectively 20% and 50% more than a third number. What per cent is the first of the second?

Solution Here, $x = 20$ and $y = 50$.

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$\therefore$ First number $= \left(\frac{100+x}{100+y}\right) \times 100\%$ of the second

$= \left(\frac{100+20}{100+50}\right) \times 100\%$ of the second

i.e. 80% of the second.

Illustration 10 Two numbers are respectively 32% and 20% less than a third number. What percent is the first of the second?

Solution Here, $x = 32$ and $y = 20$.

$\therefore$ First number $= \left(\frac{100-x}{100-y} \times 100\right)\%$ of the second

$= \left(\frac{100-32}{100-20} \times 100\right)\%$ of the second

i.e. 85% of the second.

4. (a) If the price of a commodity increases by $P\%$, then the reduction in consumption so as not to increase the expenditure is

$$\left(\frac{P}{100+P} \times 100\right)\%.$$

(b) If the price of a commodity decreases by $P\%$, then the increase in consumption so as not to decrease the expenditure is

$$\left(\frac{P}{100-P} \times 100\right)\%.$$

Explanation

Let the original price of the commodity be Rs. 100.

Then, the increased price $= 100 + \frac{P}{100} \times 100$

$=$ Rs. $(100 + P)$.

Therefore, to keep the price unchanged, there should be a reduction in the consumption of the commodity by Rs. P.

$\because$ Decrease in Rs. $(100 + P)$ = Rs. P

$\therefore$ Decrease in Rs. $100 = \frac{P}{100+P} \times 100$

$\therefore$ Required reduction in consumption is

$$\left(\frac{P}{100+P} \times 100\right)\%.$$

Similarly, (b) part can be proved.

Illustration 11 If the price of sugar increases by 25%, find how much per cent its consumption be reduced so as not to increase the expenditure.

Solution Reduction in consumption

$= \left(\frac{P}{100+P} \times 100\right)\%$

$= \left(\frac{25}{100+25} \times 100\right)\%$ **or 20%.**

Illustration 12 If the price of a commodity decreases by 25%, find how much per cent its consumption be increased so as not to decrease the expenditure.

Solution Increase in consumption

$= \left(\frac{P}{100-P} \times 100\right)\%$

$= \left(\frac{25}{100-25} \times 100\right)\%$ **or** $33\frac{1}{3}\%$.

5. If a number is changed (increased/decreased) successively by $x\%$ and $y\%$, then net % change is given by $\left(x + y + \frac{xy}{100}\right)\%$ which represents increase or decrease in value according as the sign is +ve or −ve.

If x or y indicates decrease in percentage, then put −ve sign before x or y, otherwise +ve sign.

Explanation

Let the given number be N.

If it is increased by $x\%$, then it becomes

$$N + x\% \text{ of } N = N + \frac{Nx}{100} = \frac{N(x+100)}{100}.$$

If it is further increased by $y\%$, then it becomes

$$\frac{N(x+100)}{100} + \frac{y}{100} \times \frac{N(x+100)}{100}$$

$$= \frac{N(x+100)(y+100)}{(100)^2}$$

$\therefore$ Net change $= \frac{N(x+100)(y+100)}{(100)^2} - N$

$= \frac{N(100x+100y+xy)}{(100)^2}$

$\therefore$ % change $= N\left(x + y + \frac{xy}{100}\right) \times \frac{1}{100} \times \frac{100}{N}$

$= \left(x + y + \frac{xy}{100}\right)\%$.

Illustration 13 If salary of a person is first increased by 15% and thereafter decreased by 12%, what is the net change in his salary?

Solution Here, $x = 15$ and $y = -12$.

$\therefore$ The net % change in the salary

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$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(15-12-\frac{15\times12}{100}\right)\% \text{ or } \mathbf{1.2\%}.$$

Since the sign is +ve, the salary of the person increases by 1.2%.

Illustration 14 The population of a town is decreased by 25% and 40% in two successive years. What per cent population is decreased after two years?

Solution Here, $x = -25$ and $y = -40$.

$\therefore$ The net % change in population

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-25-40+\frac{25\times40}{100}\right)\% \text{ or } -55\%.$$

Since the sign is –ve, there is decrease in population after two years by **55%.**

6. If two parameters A and B are multiplied to get a product and if A is changed (increased/decreased) by x% and another parameter B is changed (increased/decreased) by y%, then the net % change in the product $(A \times B)$ is given $\left(x+y+\frac{xy}{100}\right)\%$ which represents increase or decrease in value according as the sign in +ve or –ve.

   If x or y indicates decrease in percentage, then put –ve sign before x or y, otherwise +ve sign.

Illustration 15 If the side of a square is increased by 20%, its area is increased by k%. Find the value of k.

Solution Since side × side = area

$\therefore$ Net % change in area

$$= \left(x+y+\frac{xy}{100}\right)\% = \left(20+20+\frac{20\times20}{100}\right)\%$$

[Here, $x = 20$ and $y = 20$]

$= 44\%$.

Therefore, the area is increased by 44%.

Here, $\boldsymbol{k = 44}$.

Illustration 16 The radius of a circle is increased by 2%. Find the percentage increase in its area.

Solution Since $\pi \times$ radius × radius = area

$\therefore$ Net % change in area

$$= \left(x+y+\frac{xy}{100}\right)\% = \left(2+2+\frac{2\times2}{100}\right)\%$$

[Here, $x = 2$ and $y = 2$]

$$= 4\frac{1}{25}\%.$$

Therefore, the percentage increase in area is $\mathbf{4\frac{1}{25}\%}$.

Illustration 17 The tax on a commodity is diminished by 15% and its consumption increases by 10%. Find the effect on revenue.

Solution Since tax × consumption = revenue

$\therefore$ Net % change in revenue

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-15+10-\frac{15\times10}{100}\right)\%$$

[Here, $x = -15$ and $y = 10$]

$= -6.5\%$.

$\therefore$ The revenue decreases by **6.5%.**

7. If the present population of a town (or value of an item) be P and the population (or value of item) changes at r% per annum, then

   (a) Population (or value of item) after n years

   $$= P\left(1+\frac{r}{100}\right)^n$$

   (b) Population (or value of item) n years ago

   $$= \frac{P}{\left(1+\frac{r}{100}\right)^n}.$$

   where r is +ve or –ve according as the population (or value of item) increases or decreases.

Explanation

Population at the end of first year

$$= P + \frac{r}{100}P = P\left(1+\frac{r}{100}\right).$$

Now, the population at the beginning of second year

$$= P\left(1+\frac{r}{100}\right).$$

$\therefore$ Population at the end of second year

$$= P\left(1+\frac{r}{100}\right)+\frac{r}{100}P\left(1+\frac{r}{100}\right) = P\left(1+\frac{r}{100}\right)^2$$

$$\vdots \qquad \vdots \qquad \vdots$$

Population at the end of n years $= P\left(1+\frac{r}{100}\right)^n$.

Illustration 18 The population of a town increases 5% annually. If its present population is 84000, what will it be in 2 years time?

Solution Here, $P = 84000$, $r = 5$ and $n = 2$.

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∴ Population after 2 years

$$= P\left(1+\frac{r}{100}\right)^n$$

$$= 84000\left(1+\frac{5}{100}\right)^2$$

$$= 84000 \times \frac{105}{100}\times\frac{105}{100} = \mathbf{92610.}$$

Illustration 19 The population of a town increases at the rate of 5% annually. If the present population is 4410, what it was 2 years ago?

Solution Here, $P = 4410$, $r = 5$ and $n = 2$.

∴ Population of the town 2 years ago

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n}$$

$$= \frac{4410}{\left(1+\frac{5}{100}\right)^2} = \frac{4410}{\frac{105}{100}\times\frac{105}{100}} = 4000.$$

8. If a number A is increased successively by x% followed by y% and then by z%, then the final value of A will be

$$A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

In case a given value decreases by any percentage, we will use a negative sign before that.

Illustration 20 The population of a town is 144000. It increases by 5% during the first year. During the second year, it decreases by 10% and increases by 15% during the third year. What is the population after 3 years?

Solution Here, $P = 144000$, $x = 5$, $y = -10$ and $z = 15$.

∴ Population after 3 years

$$= A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

$$= 144000\left(1+\frac{5}{100}\right)\left(1-\frac{10}{100}\right)\left(1+\frac{15}{100}\right)$$

$$= \frac{144000\times105\times90\times115}{100\times100\times100}$$

$$= 156492.$$

9. In an examination, the minimum pass percentage is x%. If a student secures y marks and fails by z marks, then the maximum marks in the examination is $\frac{100(y+z)}{x}$.

Explanation

Let the maximum marks be m.

Given: x% of $m = y + z$

$$\Rightarrow \quad \frac{x}{100} \times m = y + z \text{ or } m = \frac{100(y+z)}{x}.$$

Illustration 21 In an examination, a student must get 60% marks to pass. If a student who gets 120 marks, fails by 60 marks, find the maximum marks.

Solution Here, $x = 60$, $y = 120$ and $z = 60$.

$$\therefore \text{ Maximum marks} = \frac{100(y+z)}{x} = \frac{100(120+60)}{60}$$

$$= \frac{100\times180}{60} = \mathbf{300.}$$

10. In an examination x% and y% students respectively fail in two different subjects while z% students fail in both the subjects, then the percentage of students who pass in both the subjects will be $(100 - (x + y - z))$%.

Explanation

%age of students who failed in one subject only $= (x - z)$%.

%age of students who failed in other subject only $= (y - z)$%.

%age of students who failed in both the subjects $= z$%.

∴ % age of students who passed in both the subjects

$$= [100 - [(x - z) + (y - z) + z]]\%$$

$$= (100 - (x + y - z))\%.$$

Illustration 22 In an examination, 42% students failed in Mathematics and 52% failed in Science. If 17% failed in both the subjects, find the percentage of those who passed in both the subjects.

Solution Here, $x = 42$, $y = 52$ and $z = 17$.

∴ Percentage of students passing both the subjects.

$$= (100 - (x + y - z))\%$$

$$= (100 - (42 + 52 - 17))\% \textbf{ or } \mathbf{23\%.}$$

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. What percentage is equivalent to $5\frac{1}{4}$?
(a) 2 (b) 425%
(c) 625% (d) None of these

2. 0·005 = (... ? ...)%
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$%
(c) $\frac{1}{3}$% (d) None of these

3. $6\frac{2}{3}$% expressed as a fraction in its lowest term is
(a) $\frac{2}{15}$ (b) $\frac{1}{15}$
(c) $\frac{3}{20}$ (d) None of these

4. What fraction is 0.6%?
(a) $\frac{7}{500}$ (b) $\frac{9}{500}$
(c) $\frac{3}{500}$ (d) None of these

5. ·025 in terms of rate per cent is
(a) 3.5% (b) 2.5%
(c) 1.5% (d) None of these

6. What per cent of 12 is 84?
(a) 800% (b) 600%
(c) 700% (d) None of these

7. Express $\frac{7}{8}$ as percentage.
(a) $67\frac{1}{2}$ (b) $87\frac{1}{2}$%
(c) $97\frac{1}{4}$% (c) None of these

8. Express $8\frac{1}{3}$% as a fraction.
(a) $\frac{1}{12}$ (b) $\frac{1}{16}$
(c) $\frac{1}{18}$ (d) None of these

9. $37\frac{1}{2}$% of Rs. 48 is
(a) Rs. 20 (b) Rs. 16
(c) Rs. 18 (d) None of these

10. What per cent of $\frac{2}{7}$ is $\frac{1}{35}$?
(a) 15% (b) 18%
(c) 10% (d) None of these

11. 60% of 480 = (?) × 15
(a) 12 (b) 36
(c) 24 (d) None of these

12. If 200% of a number is 90, then what is the 80% of that number?
(a) 48 (b) 36
(c) 24 (d) None of these

13. If $37\frac{1}{2}$% of a number is 45, then $87\frac{1}{2}$% of the number will be
(a) 115 (b) 135
(c) 105 (d) None of these

14. ? × 15 = 37·5% of 220.
(a) 5.5 (b) 7.5
(c) 6.5 (d) None of these

15. What per cent of 4 km is 8 metres?
(a) 0·4 (b) 0·2
(c) 0·8 (d) None of these

16. x% of y + y% of x = ?
(a) 3% of xy (b) 2% of xy
(c) 5% of xy (d) None of these

17. 0·35% of a number is equivalent to multiplying it by the number
(a) 0.0025 (b) 0.0045
(c) 0.0035 (d) None of these

18. If 8% of x = 4% of y, then 20% of x is
(a) 15% of y (b) 10% of y
(c) 20% of y (d) None of these

19. x% of y + ?% of x = x% of (x + y).
(a) $x + y$ (b) x
(c) y (d) None of these

20. A number x is 125% of y. To compute y, the number x has to be multiplied by
(a) 0.08 (b) 0.4
(c) 0.8 (d) None of these

21. 25% of 25% = ?
(a) 6.25 (b) 0.0625
(c) 0.625 (d) None of these

22. Which number is 60% less than 80?
(a) 24 (b) 36
(c) 32 (d) None of these

23. 20% of 30% of 20% of Rs. 850 is
(a) Rs. 9.50 (b) Rs. 10.20
(c) Rs. 10.50 (d) None of these

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24. The greatest of $16\frac{2}{3}\%$, $6\frac{2}{3}\%$, 0·3 is
(a) $16\frac{2}{3}\%$ (b) $6\frac{2}{3}\%$
(c) $0 \cdot 3$ (d) Cannot be compared

25. 40% of 20% + 30% of 25% + 50% of 28% is equivalent to
(a) 29.5% (b) 28.5%
(c) 30.5% (d) None of these

26. If 90% of A = 30% of B and $B = x\%$ of A, then the value of x is
(a) 800 (b) 300
(c) 700 (d) None of these

27. 1 quintal 25 kg is what per cent of 1 metric tonne?
(a) $16\frac{1}{2}\%$ (b) $8\frac{1}{2}\%$
(c) $12\frac{1}{2}\%$ (d) None of these

28. If 12% of x is equal to 6% of y, then 18% of x will be equal to how much per cent of y?
(a) 7% (b) 9%
(c) 11% (d) None of these

29. If a number is 20% more than the other, how much per cent is the second number less than the first?
(a) $12\frac{1}{3}\%$ (b) $16\frac{2}{3}\%$
(c) $16\frac{1}{3}\%$ (d) None of these

30. If A's income is 25% less than that of B, then how much per cent is B's income more than that of A?
(a) $33\frac{1}{3}\%$ (b) $66\frac{2}{3}\%$
(c) $11\frac{2}{3}\%$ (d) None of these

31. If the given two numbers are respectively 7% and 28% of a third number, then what percentage is the first of the second?
(a) 20% (b) 25%
(c) 18% (d) None of these

32. Two numbers are respectively 60% and 20% more than a third number. Second number expressed as a percentage of first is
(a) 75% (b) 90%
(c) 80% (d) None of these

33. Two numbers are less than a third number by 30% and 37% respectively. How much per cent is the second number less than the first?
(a) 15% (b) 10%
(c) 20% (d) None of these

34. Two numbers are respectively 20% and 10% more than a third number. How much per cent is the first number more than the second?
(a) $9\frac{1}{11}\%$ (b) $7\frac{1}{11}\%$
(c) $11\frac{1}{11}\%$ (d) None of these

35. The price of cooking oil has increased by 15%. The percentage of reduction that a family should effect in the use of cooking oil so as not to increase the expenditure on this account is
(a) $15\frac{2}{23}\%$ (b) $13\frac{1}{23}\%$
(c) $17\frac{1}{23}\%$ (d) None of these

36. If the price of apples goes down by 10%, find the percentage of increase that a family should effect in its consumption so as not to increase expenditure on this account
(a) $13\frac{1}{9}\%$ (b) $15\frac{1}{9}$
(c) $11\frac{1}{9}\%$ (d) None of these.

37. A number is increased by 20% and then decreased by 20%, the final value of the number
(a) does not change (b) decreases by 2%
(c) increases by 4% (d) decreases by 4%.

38. A man's wages were decreased by 50%. Again, the reduced wages were increased by 50%. He has a loss of
(a) 35% (b) 25%
(c) 20% (d) None of these.

39. The population of a town is decreased by 20% and 25% in two successive years. What per cent population is decreased after two years?
(a) 50% (b) 40%
(c) 60% None of these

40. The difference between a discount of 35% and two successive discounts of 20% and 20% on a certain bill was Rs. 22. Find the amount of the bill.
(a) Rs. 3200 (b) Rs. 2200
(c) Rs. 1800 (d) None of these

41. A shopkeeper marks the prices of his goods at 25% higher than the original price. After that, he allows a discount of 12%. What profit or loss did he get?
(a) 10% profit (b) 15% profit
(c) 10% loss (d) 15% loss

42. Two shopkeepers sell a ratio of similar brand and type at the same list price of Rs. 1000. The first allows two successive discounts of 20% and 10% and the second allows the successive discounts of 15% and 15%. Find the difference in discounts offered by the two shopkeepers.
(a) Rs. 3.50 (b) Rs. 1.50
(c) Rs. 2.50 (d) None of these

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43. The tax on a commodity is diminished by 10% and its consumption increases by 10%. Find the effects on revenue.
(a) 1% (b) 2%
(c) 3% (d) None of these

44. The radius of a sphere is increased by 10%. The surface area increases by
(a) 21% (b) 31%
(c) 41% (d) None of these

45. When the price of an article is reduced by 15%, the sales increases by 35%. The percentage change in the total amount of receipts is
(a) $14\frac{3}{4}$% decrease (b) $14\frac{3}{4}$% increase
(c) $13\frac{3}{4}$% decrease (d) None of these

46. If the side of a square is increased by 30%, its area is increased by
(a) 49% (b) 69%
(c) 79% (d) None of these

47. The length and breadth of a square are increased by 30% and 20%, respectively. The area of the rectangle so formed exceeds the area of the square by
(a) 56% (b) 46%
(c) 66% (d) None of these

48. In measuring the sides of a rectangle, one side is taken 10% in excess and the other 20% in deficit. Find the error per cent in area calculated from the measurement.
(a) 12% deficit (b) 10% deficit
(c) 12% excess (d) None of these

49. For a rectangle, the length and breadth are increased by 10% and 20%, respectively. The percentage increase in area is
(a) 24% (b) 48%
(c) 32% (d) None of these

50. Water tax is increased by 20% but its consumption is decreased by 20%. The increase or decrease in the expenditure is
(a) 4% decrease (b) 4% increase
(c) 8% decrease (d) 8% increase

51. On decreasing the price of a colour TV by 30%, its sale is increased by 20%. The effect on the revenue is
(a) 16% decrease (b) 16% increase
(c) 20% increase (d) None of these

52. The population of a city increases at the rate of 10% annually. Its present population is 90.51 lacs. The population 3 years ago was nearly
(a) 72 lacs (b) 68 lacs
(c) 80 lacs (d) None of these

53. The value of a machine depreciates at the rate of 10% every year. It was purchased 3 years ago. If its present value is Rs. 8748, its purchase price was
(a) Rs. 16000 (b) Rs. 18000
(c) Rs. 12000 (d) None of these

54. The income of a company increases 20% per annum. If its income is Rs. 2664000 in the year 1999 what was its income in the year 1997?
(a) Rs. 1750000 (b) Rs. 1650000
(c) Rs. 1850000 (d) None of these

55. The population of a town is 32000. It increases 15% annually. What will it be in 2 years?
(a) 52340 (b) 42320
(c) 62430 (d) None of these

56. The value of a machine is Rs. 6250. It decreases by 10% during the first year, 20% during the second year and 30% during the third year. What will be the value of the machine after 3 years?
(a) Rs. 2650 (b) Rs. 3050
(c) Rs. 3150 (d) None of these

57. The population of a town increases by 12% during first year and decreases by 10% during second year. If the present population is 50400, what it was 2 years ago?
(a) 40000 (b) 35000
(c) 50000 (d) None of these

58. Ramesh loses 20% of his pocket money. After spending 25% of the remainder he has Rs. 480 left. What was his pocket money?
(a) Rs. 600 (b) Rs. 800
(c) Rs. 900 (d) None of these

59. An army lost 10% its men in war, 10% of the remaining due to diseases and 10% of the rest were disabled. Thus, the strength was reduced to 729000 active men. Find the original strength.
(a) 1000000 (b) 1200000
(c) 1500000 (d) None of these

60. The daily wage is increased by 25% and a person now gets Rs. 25 per day. What was his daily wage before the increase?
(a) Rs. 25 (b) Rs. 20
(c) Rs. 30 (d) None of these

61. A student has to secure 15% marks to get through. If he gets 80 marks and fails by 70 marks, find the maximum marks set for the examination.
(a) 900 (b) 1000
(c) 1200 (d) None of these

62. In an examination, 30% and 35% students respectively failed in History and Geography while 27% students failed in both the subjects. If the number of students passing the examination is 248, find the total number of students who appeared in the examination.
(a) 425 (b) 380
(c) 400 (d) None of these

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63. Mr. Katial buys a house for Rs. 100000 and rents it. He puts 12.5% of each month's rent aside for upkeep and repairs, pays Rs. 325 per year as taxes and realizes 5.5% annually on his investment. Find the monthly rent.
(a) Rs. 634.76 (b) Rs. 554.76
(c) Rs. 654.76 (d) None of these

64. In an examination, there were 2000 candidates, out of which 900 candidates were boys and rest were girls. If 32% of the boys and 38% of the girls passed, then the total percentage of failed candidates is
(a) 35.3% (b) 64.7%
(c) 68.5% (d) 70%

65. From the salary of an officer, 10% is deducted as house rent, 15% of the rest he spends on children's education and 10% of the balance, he spends on clothes. After this expenditure he is left with Rs. 1377. His salary is
(a) Rs. 2000 (b) Rs. 2040
(c) Rs. 2100 (d) Rs. 2200

66. If the price of gold increases by 30%, find by how much the quantity of ornaments must be reduced so that the expenditure may remain the same as before?
(a) $27\frac{2}{13}\%$ (b) $23\frac{1}{13}\%$
(c) 30% (d) 19%

67. The price of sugar has fallen by 10%. How many quintals can be bought for the same money which was sufficient to buy 18 quintals at the higher price?
(a) 20 (b) 22
(c) 25 (d) 30

68. In an examination, there are 1000 boys and 800 girls, 60% of boys and 40% girls passed. The percentage of candidates that failed is
(a) 48·88 (b) 45·88
(c) 50·00 (d) 49·88

69. The price of an article is cut by 20%. To restore it to its original price, the new price must be increased by
(a) 20% (b) $22\frac{1}{2}\%$
(c) 25% (d) 40%

70. In a fraction, numerator is increased by 25% and the denominator is diminished by 10%. The new fraction obtained is $\frac{5}{9}$. The original fraction is
(a) $\frac{2}{5}$ (b) $\frac{5}{9}$
(c) $\frac{3}{5}$ (d) None of these

71. One side of a square is increased by 30%. To maintain the same area the other side will have to be decreased by
(a) $23\frac{1}{13}\%$ (b) $76\frac{12}{13}\%$
(c) 30% (d) 15%

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $5\frac{1}{4} = \frac{21}{4} = \frac{21}{4} \times 100 = \mathbf{525\%}.$

2. Ans. (b)

Solution $0 \cdot 005 = \frac{5}{1000} = \frac{5}{1000} \times 100 = \mathbf{\frac{1}{2}\%}.$

3. Ans. (b)

Solution $6\frac{2}{3}\% = \frac{\left(\frac{20}{3}\right)}{100} = \left(\frac{20}{3} \times \frac{1}{100}\right) = \mathbf{\frac{1}{15}}.$

4. Ans. (c)

Solution $0 \cdot 6\% = \frac{0 \cdot 6}{100} = \frac{6}{1000} = \mathbf{\frac{3}{500}}.$

5. Ans. (b)

Solution $.025 = \left(\frac{25}{1000} \times 100\right)\% = \mathbf{2.5\%}.$

6. Ans. (c)

Solution Let $x\%$ of $12 = 84$

$\Rightarrow \frac{x}{100} \times 12 = 84 \Rightarrow x = \frac{84 \times 100}{12} = 700.$

$\therefore$ 700% of **12** is **84.**

7. Ans. (b)

Solution $\frac{7}{8} = \left(\frac{7}{8} \times 100\right)\% = \frac{175}{2}\% = \mathbf{87\frac{1}{2}\%}.$

8. Ans. (a)

Solution $8\frac{1}{3}\% = \frac{25}{3}\% = \frac{25}{3} \times \frac{1}{100} = \mathbf{\frac{1}{12}}.$

9. Ans. (c)

Solution $37\frac{1}{2}\%$ of Rs. $48 = 48 \times \frac{75}{2 \times 100} = \mathbf{Rs.\ 18.}$

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10. Ans.

Solution Let $x\%$ of $\frac{2}{7} = \frac{1}{35} \Rightarrow x = \frac{100\times7}{2\times35} = 10.$

$\therefore$ 10% of $\frac{2}{7}$ is $\frac{1}{35}$.

11. Ans. (c)

Solution Let 75% of 480 = $x \times 15$.

Then, $\frac{75}{100} \times 480 = 15x$ or $x = \frac{75\times480}{100\times15} = \mathbf{24.}$

12. Ans. (b)

Solution Let 200% of $x = 90 \Rightarrow \frac{200}{100} \times x = 90$

$$\Rightarrow x = \frac{100\times90}{200} = 45.$$

$\therefore$ 80% of 45 $= \frac{80}{100} = 45 = \mathbf{36.}$

13. Ans. (c)

Solution Let the number be x, then

$$37\frac{1}{2}\% \text{ of } x = 45 \Rightarrow \frac{75}{2}\times\frac{1}{100} \times x = 45$$

or, $\frac{3}{8}x = 45 \Rightarrow x = \frac{45\times8}{3} = 120.$

$\therefore$ $87\frac{1}{2}\%$ of 120 $= \frac{175}{2}\times\frac{1}{100} \times 120 = \mathbf{105.}$

14. Ans. (a)

Solution Let $x \times 15 = 37.5\%$ of 220

$\Rightarrow 15x = \frac{37.5}{100} \times 220$

$\Rightarrow x = \frac{37.5\times220}{15\times100} = \mathbf{5.5.}$

15. Ans. (b)

Solution Let $x\%$ of 4 km = 8 metre

$\Rightarrow \frac{x}{100} \times 4000 = 8$ ($\because$ 1 km = 1000 metre)

$\Rightarrow x = \frac{8\times100}{4000} = \frac{1}{5} = 0.2$

$\therefore$ 0.2% of 4 km $= \mathbf{8}$ metre.

16. Ans. (b)

Solution $x\%$ of $y + y\%$ of $x = \left(\frac{x}{100}\times y\right) + \left(\frac{y}{100}\times x\right)$

$$= \frac{2}{100}xy = 2\% \text{ of } xy.$$

17. Ans. (c)

Solution $0.35\% = 0{\cdot}35 \times \frac{1}{100} = \mathbf{0.0035.}$

18. Ans. (b)

Solution We have, 8% of x = 4% of y

$$\Rightarrow \frac{8}{100} \times x = \frac{4}{100} \times y \Rightarrow x = \left(\frac{4}{100}\times\frac{100}{8}\right)y = \frac{y}{2}.$$

$\therefore$ 20% of $x = \frac{20}{100} \times x = \frac{20}{100} \times \frac{y}{2}$

$$= \frac{10}{100} \times y$$

$$= \mathbf{10\% \text{ of } y.}$$

19. Ans. (b)

Solution Let the missing figure be a.

We have, $\frac{x}{100} \times y + \frac{a}{100} \times x = \frac{x}{100} \times (x + y)$

$\Rightarrow xy + ax = x(x + y) \Rightarrow ax = x^2$. $\therefore$ $\mathbf{a = x.}$

20. Ans. (c)

Solution We have, $x = 125\%$ of $y \Rightarrow x = \frac{125}{100}y = \frac{5}{4}y$

or, $y = \frac{4}{5}x = \mathbf{0.8x.}$

21. Ans. (b)

Solution 25% of $25\% = \frac{25}{100}\times\frac{25}{100} = \frac{625}{10000} = 0.0625.$

22. Ans. (c)

Solution Required number = 80 – 60% of 80

$$= 80 - \frac{60}{100} \times 80 = \mathbf{32.}$$

23. Ans. (b)

Solution 20% of 30% of 20% of 850

$$= \frac{20}{100}\times\frac{30}{100}\times\frac{20}{100}\times850 = \frac{1020}{100} = \mathbf{Rs.\ 10.20.}$$

24. Ans. (c)

Solution $16\frac{2}{3}\% = \frac{50}{3}\times\frac{1}{100} = \frac{1}{6} = 0.167 < 0.3.$

$$6\frac{2}{3}\% = \frac{20}{3}\times\frac{1}{100} = \frac{1}{15} = \mathbf{0.067 < 0.3.}$$

$\therefore$ 0.3 is greatest.

25. Ans. (a)

Solution $40\% \text{ of } 20\% = \frac{40}{100}\times\frac{20}{100} = \frac{8}{100} = 8\%$

$$30\% \text{ of } 25\% = \frac{30}{100}\times\frac{25}{100} = \frac{75}{100} = 7.5\%$$

and, $50\% \text{ of } 28\% = \frac{50}{100}\times\frac{28}{100} = \frac{14}{100} = 14\%$

$\therefore$ (40% of 20% + 30% of 25% + 50% of 28%)

$= 8\% + 7.5\% + 14\% = \mathbf{29.5\%.}$

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26. Ans. (b)

Solution We have, $\frac{90}{100} \times A = \frac{30}{100} \times B$

$= \frac{30}{100} \times \frac{x}{100} \times A$

$\therefore \quad x = \left(100 \times \frac{100}{30} \times \frac{90}{100}\right) = \mathbf{300.}$

27. Ans. (c)

Solution Let 1 quintal 25 kg = x% of 1 metric tonne

$\Rightarrow \quad x\% = \frac{125}{1000} = \frac{1}{8} = \frac{1}{8} \times 100\% = \mathbf{12\frac{1}{2}\%.}$

28. Ans. (b)

Solution We have, 12% of x = 6% of y

$\Rightarrow$ 2% of x = 1% of y

$\Rightarrow$ (2 × 9)% of x = (1 × 9)% of y

$\Rightarrow$ 18% of x = **9% of y.**

29. Ans. (b)

Solution Here, x = 20.

$\therefore$ Required answer $= \left(\frac{x}{100+x} \times 100\right)\%$

$= \left(\frac{20}{100+20} \times 100\right)\%$

$= \mathbf{16\frac{2}{3}\%.}$

30. Ans. (a)

Solution Here, x = 25.

$\therefore$ Required answer $= \left(\frac{x}{100-x} \times 100\right)\%$

$= \left(\frac{25}{100-25} \times 100\right)\%$

$= \mathbf{33\frac{1}{3}\%.}$

31. Ans. (b)

Solution Here, l = 7 and m = 28.

$\therefore$ First number $= \frac{l}{m} \times 100\%$ of second number

$= \frac{7}{28} \times 100\%$ of second number

or 25% of second number.

32. Ans. (a)

Solution Here, x = 60 and y = 20.

$\therefore$ Second number $= \left(\frac{100+y}{100+x} \times 100\right)\%$ of the first

$= \left(\frac{100+20}{100+60} \times 100\right)\%$ of the first

i.e. 75% of the first.

33. Ans. (b)

Solution Here, x = 30 and y = 37.

$\therefore$ Second number $= \left(\frac{100-y}{100-x} \times 100\right)\%$ of the first

$= \left(\frac{100-37}{100-30} \times 100\right)\%$ of the first

i.e. 90% of the first.

34. Ans. (a)

Solution Here x = 20 and y = 10.

$\therefore$ First number $= \left(\frac{100+x}{100+y} \times 100\right)\%$ of the second

$= \left(\frac{100+20}{100+10} \times 100\right)\%$ of the second

i.e. $109\frac{1}{11}\%$ of the second.

$\therefore$ The first number is $9\frac{1}{11}\%$ more than the second.

35. Ans. (b)

Solution Reduction in consumption

$= \left(\frac{P}{100+P} \times 100\right)\%$

$= \left(\frac{15}{100+15} \times 100\right)\%$ **or** $\mathbf{13\frac{1}{23}\%.}$

36. Ans. (c)

Solution Increase in consumption

$= \left(\frac{P}{100-P} \times 100\right)\%$

$= \left(\frac{10}{100-10} \times 100\right)\%$ **or** $\mathbf{11\frac{1}{9}\%.}$

37. Ans. (d)

Solution Here, x = 20 and y = – 20.

$\therefore$ The net % change in value

$= \left(x+y+\frac{xy}{100}\right)\%$

$= \left(20-20-\frac{20\times 20}{100}\right)\%$ or –4%.

Since the sign is –ve, there is decrease in value by **4%.**

38. Ans. (b)

Solution Here, x = –50 and y = 50.

$\therefore$ The net % change in wages

$= \left(x+y+\frac{xy}{100}\right)\%$

$= \left(-50+50-\frac{50\times 50}{100}\right)\%$ or –25%.

Since the sign is –ve, he has a loss of **25%.**

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39. Ans. (b)

Solution Here, $x = -20$ and $y = -25$.

$\therefore$ The net % change in population

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-20-25+\frac{20\times 25}{100}\right) \text{ or } -40\%.$$

Since the sign is –ve, the population is decreased by 40% after two years.

40. Ans. (b)

Solution The equivalent discount of two successive discounts of 20% and 20%

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-20-20+\frac{20\times 20}{100}\right)\% \text{ or } -36\%.$$

Given : 36% –35% = Rs. 22.

$\therefore$ Amount of the bill = 22 × 100 = **Rs. 2200.**

41. Ans. (a)

Solution Here, $x = 25$ and $y = -12$.

$\therefore$ The net % change in original price

$$= \left(x+y+\frac{xy}{100}\right)$$

$$= \left(25-12-\frac{25\times 12}{100}\right)\% \text{ or } 10\%.$$

Since the sign is +ve, there is a profit of **10%.**

42. Ans. (c)

Solution The equivalent discount of two successive discounts of 20% and 10%

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-20-10+\frac{20\times 10}{100}\right)\% \text{ or } 28\%.$$

$\therefore$ Discount on the list price of radio offered by the first shopkeeper

$$= 28\% \text{ of } 1000 = \frac{28}{100} \times 1000 = \text{Rs. } 280.$$

Also, the equivalent discount of two successive discounts of 15% and 15%

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-15-15+\frac{15\times 15}{100}\right)\% \text{ or } 27\frac{3}{4}\%.$$

$\therefore$ Discount on the list price of radio offered by the second shopkeeper

$$= 27\frac{3}{4}\% \text{ of } 1000 = \frac{111}{400} \times 1000$$

$$= \text{Rs. } 277.50.$$

$\therefore$ Difference in discounts offered by the two shopkeepers = Rs. 280 – Rs. 277.50 = **Rs. 2.50.**

43. Ans. (a)

Solution Since tax × consumption = revenue

$\therefore$ Net % change in revenue

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-10+10-\frac{10\times 10}{100}\right)\%$$

[Here $x = -10$ and $y = 10$]

$$= -1\%.$$

$\therefore$ The revenue decreases by **1%.**

44. Ans. (a)

Solution Since $4\pi \times$ radius × radius = surface area

$\therefore$ Net % change in area $= \left(x+y+\frac{xy}{100}\right)\%$

$$= \left(10+10+\frac{10\times 10}{100}\right)\%$$

$$= \mathbf{21\%.}$$

45. Ans. (b)

Solution We have, receipts = price × sales.

$\therefore$ Net % change in receipts

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-15+35-\frac{15\times 35}{100}\right)\%$$

$$= \mathbf{14\frac{3}{4}\%.}$$

46. Ans. (b)

Solution Since side × side = area

$\therefore$ Net % change in area

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(30+30+\frac{30\times 30}{100}\right)\%$$

$$= 69\%.$$

$\therefore$ The area is increased by **69%.**

47. Ans. (a)

Solution Since $\text{side}_1 \times \text{side}_2$ = area

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∴ Net % change in area

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(30+20+\frac{30\times 20}{100}\right)\%$$

[Here, $x = 30$ and $y = 20$]

$= 56\%$.

∴ The area of the rectangle so formed exceeds the area of the square by **56%.**

48. Ans. (a)

Solution Since $\text{side}_1 \times \text{side}_2 = \text{area}$

∴ Error % in area $= \left(x+y+\frac{xy}{100}\right)\%$

$$= \left(10-20-\frac{10\times 20}{100}\right)\%$$

[Here, $x = 10$ and $y = -20$]

$= -12\%$, i.e. 12% deficit.

49. Ans. (c)

Solution Since $\text{side}_1 \times \text{side}_2 = \text{area}$

∴ Net % change in area

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(10+20+\frac{10\times 20}{100}\right)\%$$

$= 32\%$.

∴ The area of the rectangle increases by **32%.**

50. Ans. (a)

Solution Since tax × consumption = expenditure

∴ Net % change in expenditure

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(20-20-\frac{20\times 20}{100}\right)\%$$

[Here, $x = 20$ and $y = -20$]

$= -4\%$.

∴ Expenditure decreases by **4%.**

51. Ans. (a)

Solution Net % change in revenue

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(-30+20\frac{30\times 20}{100}\right)\%$$

[Here, $x = -30$ and $y = 20$]

$= -16\%$.

52. Ans. (b)

Solution We have, $P = 90.51$, $r = 10$ and $n = 3$.

∴ The population 3 years ago

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n} = \frac{90\cdot 51}{\left(1+\frac{10}{100}\right)^3}$$

$$= \frac{9051}{100}\times\frac{100}{110}\times\frac{100}{110}\times\frac{100}{110}$$

= 68 lacs.

53. Ans. (c)

Solution Here, $P = 8748$, $r = -10$ and $n = 3$.

∴ Purchase price of the machine

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n} = \frac{8748}{\left(1-\frac{10}{100}\right)^3}$$

$$= \frac{8748\times 100\times 100\times 100}{90\times 90\times 90} = \textbf{Rs. 12000.}$$

54. Ans. (c).

Solution Here, $P = 2664000$, $r = 20$ and $n = 2$.

∴ Company's income in the year 1997

$$= \frac{P}{\left(1+\frac{r}{100}\right)^n} = \frac{2664000}{\left(1+\frac{20}{100}\right)^2}$$

$$= \frac{2664000\times 5\times 5}{6\times 6}$$

= Rs. 1850000.

55. Ans. (b)

Solution Here, $P = 32000$, $r = 15$ and $n = 2$.

∴ Population of the town in 2 years

$$= P\left(1+\frac{r}{100}\right)^n = 32000\left(1+\frac{15}{100}\right)^2$$

$$= 32000 \times \frac{115}{100}\times\frac{115}{100} = \textbf{42320.}$$

56. Ans. (c)

Solution Here, $A = 6250$, $x = -10$, $y = -20$ and $z = -30$.

∴ Value of the machine after 3 years

$$= A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

$$= 6250\left(1-\frac{10}{100}\right)\left(1-\frac{20}{100}\right)\left(1-\frac{30}{100}\right)$$

$$= \frac{6250\times 90\times 80\times 70}{100\times 100\times 100} = \textbf{Rs. 3150.}$$

57. Ans. (c)

Solution Here, $A = 50400$, $x = 12$ and $y = -10$.

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$\therefore$ Population of the town 2 years ago

$$= \frac{A}{\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)}$$

$$= \frac{50400}{\left(1+\frac{12}{100}\right)\left(1-\frac{10}{100}\right)}$$

$$= \frac{50400\times100\times100}{112\times90}$$

$$= \mathbf{50000.}$$

58. Ans. (b)

Solution Let Rs. A be the pocket money.

Then, $A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right) = 480$ (Given)

Here, $x = -20$ and $y = -25$.

$\therefore$ $A\left(1-\frac{20}{100}\right)\left(1-\frac{25}{100}\right) = 480$

$\Rightarrow$ $A = \frac{480\times100\times100}{80\times75}$ = **Rs. 800.**

59. Ans. (a)

Solution Let A be the original strength.

Then, $A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right) = 729000$ (Given)

Here, $x = -10$, $y = -10$ and $z = -10$.

$\therefore$ $A\left(1-\frac{10}{100}\right)\left(1-\frac{10}{100}\right)\left(1-\frac{10}{100}\right) = 729000$

$\Rightarrow$ $A = \frac{729000\times100\times100\times100}{90\times90\times90}$ = **1000000 men.**

60. Ans. (b)

Solution Let the daily wage before the increase was Rs. A.

Then, $A\left(1+\frac{x}{100}\right) = 25.$

Here, $x = 25$.

$\therefore$ $A\left(1+\frac{25}{100}\right) = 25 \Rightarrow A = \frac{25\times100}{125}$ = **Rs. 20.**

61. Ans. (b)

Solution Here, $x = 15$, $y = 80$ and $z = 70$.

$\therefore$ Maximum marks $= \frac{100(y+z)}{x} = \frac{100(80+70)}{15}$

$= 1000.$

62. Ans. (c)

Solution Percentage of students passing the examination

$= (100 - (30 + 35 - 27))\%$

[Here, $x = 30$, $y = 35$ and $z = 27$]

$= 62(100 - 38)\% = 62\%.$

Let the total number of students appearing in the examination be x.

Given: 62% of $x = 248$

or, $\frac{62}{100} \times x = 248$ or $x = \frac{248\times100}{62} = 400.$

Therefore, 400 students appeared in the examination.

63. Ans. (b)

Solution Let the monthly rent be Rs. x.

We have, $\frac{5.5\%}{12}$ of $100000 = x - 12.5\%$ of $x - 325$

$\Rightarrow$ $\frac{5500}{12} = x - \frac{x}{8} - \frac{325}{12}$

$\Rightarrow$ $\frac{5500}{12} + \frac{325}{12} = \frac{7}{8} \times x$

$\Rightarrow$ $x = \frac{5825}{12}\times\frac{8}{7}$ = **Rs. 554.76 per month.**

64. Ans. (b)

Solution Boys = 900, Girls = 1100

Passed = (32% of 900) + (38% of 1100)

= 288 + 418 = 706

Failed = 2000 − 706 = 1294

Failed % = $\left(\frac{1294}{2000}\times100\right)\%$ = **64.7%**

65. Ans. (a)

Solution Suppose that his salary = Rs. 100

House rent = Rs. 10, balance = Rs. 90

Expenditure on education = Rs. $\left(\frac{15}{100}\times90\right)$

= Rs. 13.50

Balance = Rs. 76.50.

Expenditure on clothes = Rs. $\left(\frac{10}{100}\times76{\cdot}50\right)$.

= Rs. 7.65

Balance now = Rs. 68.85

If balance is Rs. 68.85, salary = Rs. 100

If balance is Rs. 1377, salary = Rs. $\frac{100}{68{\cdot}85} \times 1377$

= **Rs. 2000.**

66. Ans. (b)

Solution Reduction = $\frac{30}{100+30} \times 100\% = 23\frac{1}{13}\%$

67. Ans. (a)

Solution 90% of original price can buy = 18 quintals

$\therefore$ He can buy $\frac{18\times100}{90}$ = 20 quintals at the lower price.

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68. Ans. (a)

Solution Number of candidates who failed
$= 40\%$ of $1000 + 60\%$ of 800
$= 400 + 480$
$= 880$

$\therefore$ Fail $\% = \left(\frac{880}{1800}\times 100\right)\% = \mathbf{48.88\%}.$

69. Ans. (c)

Solution New price must be increased by

$$\left(\frac{20}{100-20}\times 100\right)\% = 25\%.$$

70. Ans. (a)

Solution Let the fraction be $\frac{x}{y}$

Then, $\frac{x+0.25x}{y-0.10y} = \frac{5}{9}$

$$\frac{x(1.25)}{y(0.9)} = \frac{5}{9}$$
$$\frac{x}{y} = \frac{5}{9}\times\frac{90}{125}$$
$$\frac{x}{y} = \frac{\mathbf{2}}{\mathbf{5}}.$$

71. Ans. (a)

Solution Let the side of the square $= x$
After increase, length of one side $= 1.3x$
Let after decrease, length of other side $= y$
Then, $(1.3x)(y) = x^2$

$$y = \frac{10x}{13}$$

Decrease in other side $= x - \frac{10x}{13} = \frac{3x}{13}$
Percentage decrease in other side

$$= \frac{\frac{3x}{13}}{x}\times 100 = \frac{300}{13} = \mathbf{23\frac{1}{13}\%}.$$

QUESTIONS FROM COMPETITIVE EXAMS

1. In a shipment of 120 machine parts, 5 percent were defective. In a shipment of 80 machine parts, 10 percent were defective. For the two shipments combined, what percent of the machine parts were defective?
(a) 6.5% (b) 7.0%
(c) 7.5% (d) 8.0%
[Based on MAT 2004]

2. The population of a city increases at the rate of 4% per annum. There is an additional annual increase of 1% in the population due to the influx of job seekers. Therefore, the % increase in the population after 2 years will be
(a) 10 (b) 10.25
(c) 10.50 (d) 10.75
[Based on MAT 2004]

3. $x\%$ of y is $y\%$ of
(a) x (b) $y/100$
(c) $x/100$ (d) $100x$
[Based on MAT 2004]

4. A recipe gives directions to mix 4 parts of substance A with 7 parts of substance B. These substances are to be taken by weight, but by mistake they were taken by volume. Find the error in the percentage of the weight of A in the mixture, if 117 cm^3 of the substance A weighs as much as 151cm^3 of the substance B.
(a) 5.05% (b) 6.00%
(c) 7.05% (d) 6.08%
(e) None of these **[Based on MAT 2003]**

5. Given that Carbon-14 decays at a constant rate in such a way that it reduces to 50% in 5568 years. Find the age of an old wood piece in which carbon is only $12\frac{1}{2}\%$ of the original.
(a) 1856 years (b) 16704 years
(c) 1392 years (d) 22727 years
[Based on MAT 2003]

6. $p\%$ of a number P is $q\%$ more than $r\%$ of the number R. If the difference between P and R is $r\%$ of R and if the sum of P and R is 210, then which of the following statements is always true?
(a) $P = 110$; $R = 100$
(b) $P = 220$; $R = 200$
(c) $P = 3300$; $R = 3000$
(d) All of the above **[Based on MAT 2002]**

7. A manufacturer offers a 20% rebate on the marked price of a product. The retailer offers another 30% rebate on the reduced price. The two reductions are equal to a single reduction of
(a) 50% (b) 44%
(c) 46% (d) 40%
[Based on MAT 2002]

8. The population of a variety of tiny bush in an experimental field increased by 10% in the first year, increased by 8% in the second year but decreased by 10% in the third year. If the present number of bushes in the experimental field is 26730, then the number of variety of bushes in the beginning was
(a) 35000 (b) 27000

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(c) 25000 (d) 36000

[Based on MAT 2002]

9. If A's salary is 25% higher than B's salary, then how much percent is B's salary lower than A's?

(a) 15% (b) 20%

(c) 25% (d) $33\frac{1}{3}\%$

[Based on MAT 2001]

10. A number is increased by 10% and then reduced by 10%. After this operation, the number

(a) Does not change (b) Decreases by 1 %

(c) Increases by 1 % (d) Increases by 0.1 %

[Based on MAT 2001]

11. A reduction of 20% in the price of sugar enables a purchaser to obtain $2\frac{1}{2}$ kg more for Rs. 160. Find the original price per kg of sugar.

(a) Rs 12 (b) Rs 15

(c) Rs 16 (d) Rs 18

[Based on MAT 2001]

12. Successive discounts of 20% and 15% are equivalent to a single discount of

(a) 35% (b) 32%

(c) 17.5% (d) 17%

[Based on MAT 2001]

13. After having spent 35% of the money on machinery, 40% on raw material and 10% on staff, a person is left with Rs. 60,000. The total amount of money spent on machinery and the raw material is

(a) Rs. 1,76,000 (b) Rs. 1,70,000

(c) Rs. 3,00,000 (d) Rs. 3,40,000

[Based on MAT 1999]

14. 5% of (25% of Rs. 1600)

(a) Rs. 5 (b) Rs. 17.50

(c) Rs. 20 (d) Rs. 25

[Based on MAT 2005]

15. x% of y is y% of

(a) x (b) $\frac{y}{100}$

(c) $\frac{x}{100}$ (d) $100x$

[Based on MAT 2000]

16. X is what percent of 4/5th of X?

(a) 75 (b) 80

(c) 120 (d) 125

[Based on MAT 2000]

17. Mr. X's salary is increased by 20%. On the increase, the tax rate is 10% higher. Find the percentage of increase in the tax liability.

(a) 20 (b) 22

(c) 23 (d) Indeterminate

[Based on MAT 2001]

18. Vishal goes to a shop to buy a radio costing Rs. 2568. The rate of sales tax is 7%. He tells the shopkeeper to reduce the price of the radio to such an extent that he has to pay Rs. 2568, inclusive of sales tax. Find the reduction needed in the price of the radio.

(a) Rs. 179.76 (b) Rs. 170

(c) Rs. 168 (d) Rs. 169

[Based on MAT 2001]

19. Sunder purchased an office bag with a price tag of Rs. 600 in a sale where 25% discount was being offered on the tag price. He was given a further discount of 10% on the amount arrived at after giving usual 25% discount. What was the final amount paid by Sunder?

(a) Rs. 210 (b) Rs. 540

(c) Rs. 405 (d) Rs. 450

[Based on MAT 2001]

20. In a society, there are 100 members. Each of them has been allotted membership number from 1 to 100. They started a business in which the nth member contributed Rs $(10 \times 2^n - 5)$. After one year, 4th member gets Rs 62 as his share. Find the total profit in the business after one year.

(a) Rs $8\,[2^{100} - 26]$ (b) Rs $4\,[2^{99} - 26]$

(c) Rs $2\,[2^{100} - 26]$ (d) None of these

[Based on FMS (Delhi) 2004]

21. In a school, 60% of the students of Class X were boys. 75% of the boys passed the Class X exam. 40% of the passed boys got first division. 80% of the total students passed the exam and 50% of the passed students got first division. Which of the following conclusions is not correct?

(a) 75% of the failed students are boys

(b) 55% of the first-divisioners are girls

(c) Number of passed girls is more than that of the boys

(d) If x students failed, $2x$ got first division.

[Based on FMS (Delhi) 2004]

22. A shopkeeper has certain number of eggs of which 5% are found to be broken. He sells 93% of the remainder and still has 266 eggs left. How many eggs did he originally have?

(a) 3800 (b) 4000

(c) 4200 (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

23. If 3/5 of a number is 23 more than 50% of the same number, then what will be 80% of the number?

(a) 92 (b) 184

(c) 180 (d) 186

[Based on IIT Joint Man. Ent. Test 2004]

24. One kg of tea and one kg of sugar together cost Rs 95. If the price of tea falls by 10% and that of sugar rises by 20%, then the price of one kg of

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each combined comes to Rs 90. The original price of tea in Rs per kg is
(a) Rs 72 (b) Rs 55
(c) Rs 60 (d) Rs 80
[Based on IIT Joint Man. Ent. Test 2004]

25. Two numbers are respectively 30% and 40% less than a third number. What is the second number as a percentage of the first?
(a) 70% (b) 75%
(c) $85\frac{5}{7}\%$ (d) $116\frac{2}{3}\%$
[Based on Narsee Monjee Inst. of Man. Studies, 2003]

26. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. The error percent in the area calculated from these measurements is
(a) 4/5 % less (b) 4/5 % excess
(c) 20 % excess (d) None of these
[Based on Narsee Monjee Inst. of Man. Studies, 2003]

27. If 1 micron = 10,000 angstroms, then 100 angstroms is what percent of 10 microns?
(a) 0.0001% (b) 0.001%
(c) 0.01% (d) 0.1%
[Based on REC Tiruchirapalli 2003]

28. A man walked diagonally across a square lot. Approx., what was the percent saved by not walking along the edges?
(a) 30 (b) 20
(c) 33 (d) 24
[Based on REC Tiruchirapalli 2003]

29. The normal dosage of a particular medicine is t tablets per day for each patient. A hospital's current supply of these tablets will last p patients for d days. If the recommended dosage increases by 20% and the number of patients decreases by one-third, then for how many days will the hospital's supply last?
(a) $\frac{5d}{4}$ (b) $\frac{4d}{5}$
(c) $\frac{4pt}{5}$ (d) None of these.
[Based on REC Tiruchirapalli 2003]

30. The price of a garment is reduced by 20 percent. During a special offer, all the garments are marked "Take an additional 30 percent off the reduced price". The two reductions are equivalent to a single reduction of
(a) 25% (b) 44%
(c) 50% (d) 60%
[Based on REC Tiruchirapalli 2003]

31. A clothing supplier stores 800 coats in a warehouse, of which 15 percent are full-length-coats. If 500 of the shorter length coats are removed from the warehouse, then what percent of the remaining coats are full-length?
(a) 5.62% (b) 34%
(c) 40% (d) 48%
[Based on REC Tiruchirapalli 2003]

32. 5% income of A is equal to 15% income of B and 10% income of B is equal to 20% income of C. If the income of C is Rs. 2000, then the total income of A, B and C (in rupees) is
(a) 6000 (b) 9000
(c) 12000 (d) 18000
[Based on FMS (Delhi) 2003]

33. The price of cooking oil has increased by 25%. The percentage reduction that a family should effect in the use of cooking oil so as not to increase the expenditure on this account is
(a) 15% (b) 20%
(c) 25% (d) 30%
[Based on FMS (Delhi) 2003]

34. If the numerator of a fraction is increased by 20% and the denominator is diminished by 10%, then the value of the fraction is 16/21. The original fraction is
(a) $\frac{3}{5}$ (b) $\frac{4}{7}$
(c) $\frac{2}{3}$ (d) $\frac{5}{7}$
[Based on FMS (Delhi) 2003]

35. At what percentage above the cost price must an article be marked so as to gain 33% after allowing the customer a discount of 5%?
(a) 48% (b) 43%
(c) 40% (d) 38%
(e) None of the above **[Based on IIFT 2003]**

36. In a certain month, Mr. Khan's call charges were Rs 124.80. If in addition to the telephone rent of Rs 380, he has to pay 5% tax on the total amount, then what is his telephone bill for that month?
(a) Rs 529.30 (b) Rs 530.24
(c) Rs 530.04 (d) Rs 530.20
(e) None of the above **[Based on IIFT 2003]**

37. In a group of 20 singers and 40 dancers, 20% of the singers are less than 25 years old and 40% of the entire group are under 25 years. What percentage of dancers are under 25 years?
(a) 20% (b) 40%
(c) 50% (d) 60%
(e) None of the above **[Based on IIFT 2003]**

38. Tom's salary is 125% of Tina's salary. Tito's salary is 80% of Tina's salary. The total of all the three salaries is Rs. 61,000. What is Tito's salary?
(a) Rs 16,000 (b) Rs 16,500
(c) Rs 15,500 (d) Rs 15,000
(e) None of the above **[Based on IIFT 2003]**

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39. Mayur weighs twice as much as Shweta. Shweta's weight is 60% of Deepika's weight. Rakesh weighs 50% of Vikas's weight. Vikas weighs 190% of Mayur's weight. Which of these people weighs the least?
(a) Mayur (b) Deepika
(c) Shweta (d) Rakesh
[Based on SCMHRD Ent. Exam. 2003]

40. What percentage of numbers from 1 to 70 have squares that end in the digit 1?
(a) 20 (b) 14
(c) 1 (d) 21
[Based on SCMHRD Ent. Exam. 2003]

41. Ajay and Vikas are sharing a flat in Delhi, with an arrangement of equally dividing the household expenses. Ajay went to Pune, where a sale was going on and bought batteries for the house, worth Rs 150 on 20% discount. But he lost them on his way back and had to buy new ones, after he reached Delhi. How much did he end up spending on the batteries?
(a) Rs 280 (b) Rs 195
(c) Rs 270 (d) Rs 75
[Based on SCMHRD Ent. Exam. 2003]

Directions (Q 42 to 45): It is given that 5% increase in X always means 3% increase in Y and 5% increase in Y always implies 2.5% increase in Z.

42. Relationship between X and Z could be
(a) $1.05X = 1.025Z$ (b) $0.5X = 0.25Z$
(c) $25X = 7.5Z$ (d) $250X = 213Z$
[Based on IMT Ghaziabad, 2002]

43. If Y is increased by 30%, then Z^2 should increase by
(a) 32.25% (b) 60%
(c) 69% (d) 90%
[Based on IMT Ghaziabad, 2002]

44. If X is increased to 23 from 20, then Y should increase from 100 to
(a) 103 (b) 106
(c) 109 (d) 112
[Based on IMT Ghaziabad, 2002]

45. If X is increased from 10 to 15, then Z must increase from 30 to
(a) 35 (b) 45
(c) 60 (d) 75
[Based on IMT Ghaziabad, 2002]

46. Salary of an officer increases every year by 20%. His salary in the year 2001 was Rs 26640. What was his salary in 1999?
(a) Rs 20000 (b) Rs 19028
(c) Rs 18500 (d) Rs 18840
(e) None of the above **[Based on IRMA, 2002]**

47. The owner of a restaurant decides to calculate the percentage of vegetarian Jain dishes sold. If 70 percent of the dishes sold are vegetarian and of this 20 percent are Jain dishes, then what percentage of the total dishes sold are vegetarian Jain dishes?
(a) 50% (b) 18%
(c) 20% (d) 14%
[Based on Narsee Monjee Inst. of Man. Studies 2002]

48. In a certain shop, which stocks four types of caps, there are 1/3 as many red caps as blue and 1/2 as many green caps as red caps. There are equal numbers of green caps and yellow caps. If there are 42 blue caps, then what percent of the total caps in the shop are blue?
(a) 70% (b) 28%
(c) 60% (d) 14%
[Based on Narsee Monjee Inst. of Man. Studies 2002]

49. A bag contains 600 pens of brand A and 1200 pens of brand B. If 12% of brand A pens and 25% of brand B pens are removed, then what is the approximate percentage of total pens removed from the bag?
(a) 37% (b) 36%
(c) 22% (d) 18%
[Based on Narsee Monjee Inst. of Man. Studies 2002]

50. A invested 125% as much money as B. C invested 80% as much money as B. The total of all the three is 61,000. How much did C invest?
(a) 25000 (b) 16000
(c) 20000 (d) 45000
[Based on Narsee Monjee Inst. of Man. Studies 2002]

51. A retailer buys a stereo for Rs 225. His overhead expenses are Rs. 15. He sells the stereo for Rs 300. What percentage return on this cost did he get on this transaction?
(a) 10 (b) 25
(c) 33 (d) 50
[Based on Narsee Monjee Inst. of Man. Studies 2002]

52. A shopkeeper offers to sell $33\frac{1}{3}\%$ more quantity of a product for a price that is 20% higher. What is the effective discount that he is offering?
(a) 10.8% (b) 16.67%
(c) 13% (d) 10%
[Based on FMS (Delhi) 2002]

53. The price of cooking oil has increased by 25%. The percentage of reduction that a family should effect in the use of cooking oil so as not to increase the expenditure on this account is
(a) 25% (b) 30%
(c) 20% (d) 15%
[Based on FMS (Delhi) 2002]

54. A flat costs Rs 10 lakh. Incidental expenses and taxes amount to Rs 36000 a year. What rent per month must the owner receive to earn at least 6 percent on his investment?

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(a) Rs 6000 (b) Rs 6500
(c) Rs 8000 (d) Rs 8500

[Based on FMS (Delhi) 2002]

55. A businessman charges 20% over cost price. He allows his customers 10% off on their bills for cash payment. His net gain percent is
(a) 10 (b) 8
(c) 12 (d) 15

[Based on FMS (Delhi) 2002]

56. X gets 25% more than Y and Y gets 20% more than Z. The share of Z out of a sum of Rs 740 is
(a) Rs 200 (b) Rs 300
(c) Rs 250 (d) Rs 350

[Based on FMS (Delhi) 2002]

57. A man's wage was reduced by 50%. The reduced wage was then increased by 50%. His loss in percentage term is
(a) 10% (b) 25%
(c) 20% (d) 50%

[Based on FMS (Delhi) 2002]

58. In a municipal election, there were two candidates. One who got 30% of the votes polled was defeated by 16000 votes. Calculate the total number of votes polled.
(a) 24000 (b) 28000
(c) 30000 (d) 40000

[Based on I.P. Univ. 2002]

59. A person usually spent Rs 48 to buy groundnuts for roasting and resale. On one occasion he could buy 1.5 kg of groundnuts less for Rs 48 as the price had gone up by 25%. What was the earlier price of groundnut per kg?
(a) Rs 8 (b) Rs 6.40
(c) Rs 7.20 (d) None of these

[Based on I.P. Univ. 2002]

60. Two-thirds of one-seventh of a number is 87.5% of 240. What is the number?
(a) 2670 (b) 2450
(c) 2205 (d) 1470

[Based on I.P. Univ. 2002]

61. X, Y and Z shared Rs 7400 so that X received 25% more than Y, and Y received 20% more than Z. What amount did Z receive?
(a) Rs 3500 (b) Rs 3000
(c) Rs 2400 (d) Rs 2000

[Based on I.P. Univ. 2002]

62. What would be the value of 20% of m as a percentage of p, if 8% of m = 4% of p?
(a) 80% (b) 16%
(c) 10% (d) None of these

[Based on I.P. Univ. 2002]

63. A trader dealing in pressure cookers reduced the price by 20% as a result of which his sale went up by 80%. What was the net effect on his sales income?
(a) 44% decrease (b) 44% increase
(c) 66% increase (d) 66% decrease

[Based on I.P. Univ. 2002]

64. If S is 150 percent of T, then T is what percent of $S + T$?
(a) $33\frac{1}{3}\%$ (b) 40%
(c) 75% (d) 80%

[Based on REC Tiruchirapalli, 2002]

65. At a school, 20% of the students are seniors. If all of the seniors attended the school play, and 60% of all the students attended the play,. then what percent of the non-seniors attended the play?
(a) 20% (b) 40%
(c) 50% (d) 100%

[Based on REC Tiruchirapalli, 2002]

66. The price of LPG increases by 20%. By what percent must a family reduce the consumption of LPG, so that the expenditure on gas is the same as before:
(a) 83.33 (b) 80
(c) 20 (d) 16.64

[Based on FMS (Delhi) 2006]

67. If a exceeds b by x%, then which one of the following equation is correct:
(a) $a - b = \frac{x}{100}$ (b) $b = a + 100x$
(c) $a = \frac{bx}{100+x}$ (d) $a = b + \frac{bx}{100}$

[Based on FMS (Delhi) 2006]

68. If the price of a DVD player is increased by 25%, then by what percentage the new price be reduce to bring it down to the original level?
(a) 15% (b) 20%
(c) 25% (d) 30%

[Based on FMS (Delhi) 2006]

69. A rainy day occurs once in every 10 days. Half of the rainy days produce rainbows. What per cent of all the days do not produce rainbow?
(a) 95% (b) 10%
(c) 50% (d) 5%

[Based on SNAP 2007]

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70. If S is 150 per cent of T, then T is what per cent of S + T?

(a) 40% (b) $33\frac{1}{3}$ (c) 75% (d) 80%

[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution 14 out of 200 machine parts were defective, i.e.. 7%.

2. Ans. (b)

Solution $100 \to 105 \to 110.25$, **i.e. 10.25%.**

3. Ans. (a)

Solution Let $x\%$ of $y = y\%$ of $z \Rightarrow xy = yz \Rightarrow z = x$.

4. Ans. (e)

Solution Required ratio $= 4V_A d_A : 7V_B d_B$

$$= \frac{4V_A d_A}{d_B} : 7V_B,$$

where d is density of the substance

Given $117d_A = 151d_B$

$$\therefore \quad \frac{d_A}{d_B} = \frac{151}{117}$$

Now with $7V_B$ of substance B, $4V_A$ of substance A is used in place of $4V_A \times \frac{151}{117}$

$$\Rightarrow \quad \% \text{ error} = \frac{34}{117} \times \frac{117}{151} \times 100 \approx \mathbf{22\%}.$$

5. Ans. (b)

6. Ans. (a)

7. Ans. (b)

Solution $100 - 20\% = 80$

$80 - 30\% = 56$

$\therefore$ Single discount = **44%.**

8. Ans. (c)

Solution Suppose the number of bushes in the experimental field = x

$\therefore$ No. of bushes after one year

$$= x + 10\% \text{ of } x = \frac{11x}{10}$$

No. of bushes after 2nd year

$$= \frac{11x}{10} + 8\% \text{ of } \frac{11x}{10} = \frac{297x}{250}$$

No. of bushes after 3rd year

$$= \frac{297x}{250} - 10\% \text{ of } \frac{297x}{250}$$

$$= \frac{2673}{2500}x$$

$$\therefore \quad \frac{2673}{2500}x = 26730 \Rightarrow x = \mathbf{25000}.$$

9. Ans. (b)

Solution $A = B + 25\%$ of B

$$\Rightarrow \quad A = B + \frac{B}{4} = \frac{5B}{4}$$

$$\Rightarrow \quad B = \frac{4}{5}A = A - \frac{1}{5}A = \mathbf{A - 20\% \text{ of } A}.$$

10. Ans. (b)

11. Ans. (c)

Solution Let the original price be Rs x per kg

$\therefore$ Reduced price = Rs $\frac{4}{5}x$ per kg

$$\Rightarrow \quad \frac{5}{4x} \times 160 = \frac{160}{x} + 2\frac{1}{2}$$

$$\Rightarrow \quad \frac{200}{x} = \frac{160}{x} + \frac{5}{2}$$

$$\Rightarrow \quad \frac{40}{x} = \frac{5}{2} \Rightarrow \mathbf{x = 16}.$$

12. Ans. (b)

Solution $(-20) + (-15) + \frac{(-20) \times (-15)}{100} = \mathbf{-32}.$

13. Ans. (c)

Solution 15% = 60000.

75% = 300000

= Total amount spent on machinery and Raw material.

14. Ans. (c)

Solution $\frac{5}{100} \times \left(\frac{25}{100} \times 1600\right) = \mathbf{20}.$

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15. Ans. (a)

Solution $x\%$ of $y = y\%$ of k, say

$$\therefore \quad \frac{xy}{100} = \frac{yk}{100} \Rightarrow \boldsymbol{k = x.}$$

16. Ans. (d)

Solution Suppose $X = k\%$ of $\frac{4}{5}X$

$$\Rightarrow X = \frac{k}{100} \times \frac{4}{5}X \Rightarrow \boldsymbol{k = 125.}$$

17. Ans. (b)

Solution Let Mr X's salary = Rs. 100

$\therefore$ Salary after increase = Rs. 120, i.e. Rs. 20 is the increase in salary on Rs. 100.

Let the tax on the original salary was 30%.

Hence the tax on the increased salary (Rs. 20) will be 33%, i.e. Rs 6.60.

$$\therefore \text{ Increase in tax liability } = \frac{6.60}{30} \times 100 = \boldsymbol{22\%.}$$

18. Ans. (c)

Solution Let reduced price of the radio be Rs. x.

$$\therefore \quad x + 7\% \text{ of } x = 2568 \Rightarrow 107x = 256800$$
$$\Rightarrow x = 2400$$

$\therefore$ Reduction needed in the price of radio = **Rs. 168.**

19. Ans. (c)

Solution $600 - 25\% = 450$

$450 - 10\% = \boldsymbol{405.}$

20. Ans. (a)

Solution nth member contributed Rs $(10 \times 2^n - 5)$

$\Rightarrow$ 1st member contributed Rs 15

2nd member contributed Rs 35

3rd member contributed Rs 75

4th member contributed Rs 155

And so on.

Since 4th member gets Rs 62 as his share in the profit, therefore we conlcude that 40% profit is earned by each member.

$\therefore$ Total profit earned

= 40% of [15 + 35 + 75 + ... + upto 100 terms]

= 2 [3 + 7 + 15 + 31 + ... + upto 100 terms]

= 2 [(4 + 8 + 16 + 32 + ... + upto 100 terms]

= 8 [(1 + 2 + 4 + 8 + ... + upto 100 terms) – 25]

$$= 8\left[\left(\frac{2^{100} - 1}{2 - 1}\right) - 25\right]$$

$= 8(2^{100} - 1 - 25)$

$= \boldsymbol{8(2^{100} - 26).}$

21. Ans. (c)

Solution Total No. of students = K, say

$$\therefore \text{ No. of boys } = \frac{3}{5}K$$

$$\text{No. of boys who passed } = 75\% \text{ of } \frac{3}{5}K = \frac{9}{20}K$$

No. of boys who got 1st Division

$$= 40\% \text{ of } \frac{9}{20}K = \frac{9}{50}K$$

$$\text{No. of students passed } = \frac{4}{5}K$$

No. of students who get Ist

$$\text{Division } = \frac{2}{5}K$$

$$\text{No. of girls passed } = \frac{4}{5}K - \frac{9}{20}K = \frac{7K}{20}$$

No. of girls who got 1st Divison

$$= \frac{2}{5}K - \frac{9}{50}K = \frac{11K}{50}$$

$$\text{No. of girls failed } = \frac{2K}{5} - \frac{7K}{20} = \boldsymbol{\frac{K}{20}.}$$

22. Ans. (b)

Solution Suppose the shopkeeper had x eggs, originally.

$$\text{No. of broken eggs } = 5\% \text{ of } x = \frac{x}{20}$$

$$\text{Balance } = x - \frac{x}{20} = \frac{19x}{20}$$

$$\therefore \quad 7\% \text{ of } \frac{19x}{20} = 266 \Rightarrow \boldsymbol{x = 4000.}$$

23. Ans. (b)

Solution Let the number be K

$$\therefore \quad \frac{3}{5}K = \frac{1}{2}K + 23$$
$$\Rightarrow \quad K = 230$$
$$\Rightarrow \quad 80\% \text{ of } K = 80\% \text{ of } 230 = \boldsymbol{184.}$$

24. Ans. (d)

Solution Let C.P. of 1 kg of tea be Rs x and 1 kg of sugar be Rs y.

$$\therefore \quad x + y = 95 \qquad \ldots(i)$$

$$\Rightarrow \quad \left(x - \frac{x}{10}\right) + \left(y + \frac{y}{5}\right) = 90$$

$$\Rightarrow \quad 3x + 4y = 300 \qquad \ldots(ii)$$

From (i) and (ii) we, get

$$\boldsymbol{x = 80, y = 15.}$$

25. Ans. (c)

Solution Let the third number be x.

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$\therefore$ 1st number = x – 30% of $x = x - \frac{3x}{10} = \frac{7x}{10}$

2nd number = x – 40% of $x = x - \frac{4x}{10} = \frac{6x}{10}$

Suppose 2nd number = K% of 1st number

$\therefore \quad \frac{6x}{10} = \frac{K}{100} \times \frac{7x}{10} \Rightarrow K = \frac{600}{7} = \mathbf{85\frac{5}{7}}.$

26. Ans. (b)

27. Ans. (d)

Solution Let 100 angstroms = x% of 10 microns

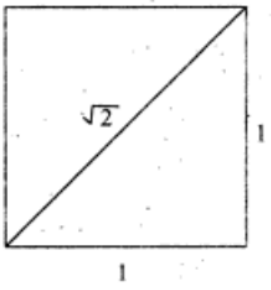
$\Rightarrow$ 100 angstronms = x% of 100000 angstroms

$\Rightarrow \quad x = \frac{100 \times 100}{100000} = \frac{1}{10} = 0.1.$

28. Ans. (a)

Solution Suppose side of the square = 1 metre

$\therefore$ Diagonal = $\sqrt{2}$ m



Distance saved by not walking along the edges

$= 2 - \sqrt{2}$

i.e. $\left(\frac{2-\sqrt{2}}{2} \times 100\right)\%$ i.e. $29.3\% \approx \mathbf{30\%}.$

29. Ans. (a)

Solution $\frac{ptd}{(t + 20\% \text{ of } t)\left(p - \frac{p}{3}\right)} = \frac{ptd}{\frac{6t}{5} \times \frac{2p}{3}} = \mathbf{\frac{5}{4}d}.$

30. Ans. (b)

31. Ans. (c)

Solution 120 coats for full length. 500 shorter length coats are removed.

$\therefore$ Percentage of full length coats out of the remaining 300 coats

$= \frac{120}{300} \times 100 = \mathbf{40}.$

32. Ans. (d)

Solution 5% of A = 15% of $B \Rightarrow 5A = 15B \Rightarrow A = 3B$

10% of B = 20% of $C \Rightarrow 10B = 20C \Rightarrow B = 2C$

If C = 2000, then B = 4000

$\therefore \quad A$ = 12000

Hence the total income of A, B and C = **18000.**

33. Ans. (b)

34. Ans. (b)

Solution Let the fraction be $\frac{p}{q}$

$\therefore \quad \frac{p + 20\% \text{ of } p}{q - 10\% \text{ of } q} = \frac{16}{21} \Rightarrow \frac{p + \frac{p}{5}}{q - \frac{q}{10}} = \frac{16}{21}$

$\Rightarrow \frac{6p}{5} \times \frac{10}{9q} = \frac{16}{21}$

$\Rightarrow \frac{p}{q} = \frac{16}{21} \times \frac{9}{12} = \frac{144}{252} = \frac{4}{7}.$

35. Ans. (c)

Solution Let the marked price be Rs x

Let the C.P. be Rs y

S.P. = x – 5% of $x = \frac{95x}{100} = \frac{19x}{20}$

y + 33% of $y = \frac{19x}{20}$

$\Rightarrow \quad 7y = 5x$

$\Rightarrow \quad x = \frac{7}{5}y = y + \frac{2}{5}y = \mathbf{y + 40\ \%\ of\ y.}$

36. Ans. (c)

37. Ans. (c)

Solution No. of singers = 20

No. of dancers = 40

$\therefore$ 4 singers are less than 25 years old.

24 out of 60 members of the group are less than 25 years old.

$\therefore$ 20 dancers are less than 25 years old, i.e. 50% of the dancers are less than 25 years old.

38. Ans. (a)

Solution Suppose Tito's salary = x,

Tom's salary = y and Tina's salary = z

$\therefore \quad y = 125\% \text{ of } z = \frac{5z}{4}$

$x = 80\% \text{ of } z = \frac{4}{5}z \Rightarrow z = \frac{5}{4}x$

$\therefore \quad y = \frac{5z}{4} = \frac{5}{4} \times \frac{5}{4}x = \frac{25}{16}x$

Also $x + y + z = 61000$

$\Rightarrow \quad x + \frac{25}{16}x + \frac{5}{4}x = 61000$

$\Rightarrow \quad \frac{61x}{16} = 61000$

$\Rightarrow \quad x = 16000$

Also $y = 25000$, $z = \mathbf{20000}.$

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39. Ans. (c)

Solution Mayur = 2 (Shweta)

Shweta = 60% of Deepika = $\frac{3}{5}$ of Deepika

Rakesh = 50% of Vikas = $\frac{1}{2}$ of Vikas

Vikas = 190% of Mayur = $\frac{19}{10}$ of Mayur

$\therefore$ $M > D$, $S < D$, $R < V$, $V > M$

$\Rightarrow$ $V > M > D > S$, $V > R$

Rakesh = $\frac{19}{20}$ of Mayur

$\Rightarrow$ Mayur = $\frac{20}{19}$ of Rakesh

Shweta = $\frac{10}{19}$ of Rakesh

$\Rightarrow$ $S < R \Rightarrow R > S$

$\therefore$ **Shweta** weighs the least.

40. Ans. (a)

Solution 1, 9, 11, 19, 21, 29, 31, 39, 41, 49, 51, 59, 61, 69:

These numbers have their squares ending in digit 1, i.e. 14 out of 70, **i.e. 20%.**

41. Ans. (b)

Solution Expenditure by Ajay on batteries

$= 80\% \text{ of } 150 + \frac{150}{2}$

$= 120 + 75 =$ **Rs. 195.**

42. Ans. (c)

Solution 5% increase in $X \Rightarrow$ 3% increase in Y

5% increase in $Y \Rightarrow$ 2.5% increase in Z

$\therefore$ 3% increase in $Y \Rightarrow$ 1.5% increase in Z

$\therefore$ 5% increase in X

$\Rightarrow$ 1.5% increase in Z i.e. $5X = 1.5Z$

$\Rightarrow$ $\mathbf{25X = 7.5Z.}$

43. Ans. (a)

Solution 30% of increase in Y

$\Rightarrow$ 15% of increase in Z i.e. 32.25% increase in Z^2.

44. Ans. (c)

Solution X is increased from 20 to 23, i.e. there is 15% increase in X.

$\Rightarrow$ There is 9% increase in Y i.e. Y will increase from **100** to **109.**

45. Ans. (a)

Solution X is increased from 10 to 15, i.e. there is 50% increase in X.

$\therefore$ There must be 15% increase in Z, i.e. Z must increase from 30 to 34.5, i.e. **35 approx.**

46. Ans. (c)

Solution Suppose salary in 1999 was Rs x

$\therefore$ $x\left(1+\frac{20}{100}\right)^2 = 26640$

$\Rightarrow$ $x = \frac{26640 \times 25}{36} =$ **Rs. 18500.**

47. Ans. (d)

48. Ans. (c)

Solution $R = \frac{1}{3}B$, $G = \frac{1}{2}R = G = Y$

Since $B = 42$, $R = 14$, $G = 7$ and $Y = 7$,

$\therefore$ Percentage of blue caps = $\frac{42}{70} \times 100 =$ **60.**

49. Ans. (c)

Solution No. of pens removed

$= 12\% \text{ of } 600 + 25\% \text{ of } 1200$

$= 72 + 300 = 372$

$\therefore$ Percentage of total pens removed

$= \frac{372}{1800} \times 100 = 20.67 =$ **22.**

50. Ans. (b)

Solution $A = 125\%$ of B, $C = 80\%$ of B

$\Rightarrow$ $A = \frac{5}{4}B$, $C = \frac{4}{5}B = \frac{4}{5} \times \frac{4}{5}A = \frac{16}{25}A$

$A + B + C = 61000 \Rightarrow A + \frac{4}{5}A + \frac{16}{25}A = 61000$

$\Rightarrow A = 25000$

$\therefore$ $B = 20000$, $C =$ **16000.**

51. Ans. (b)

52. Ans. (d)

Solution Suppose the list pirce of 6 kg is Rs 600

$\therefore$ Price of 1 kg = Rs 100

The shopkeeper offers to sell 8 kg for Rs 720, i.e. Rs 90 per kg

Hence, Discount = **10%.**

53. Ans. (c)

Solution Let the price of the cooking oil = Rs 100/unit

Let the family consumes 100 units

$\therefore$ Total expenditure on this account = Rs 10000

If the price becomes Rs 125 and the family consumes, say x units, then the total expenditure will become Rs $125x$.

In order that after increase in price, the total expenditure of the family remains unaffected, therefore

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$125x = 10000 \Rightarrow x = 80$

$\therefore$ Reduction in consumption of units

$= \mathbf{20, i.e.\ 20\%.}$

54. Ans. (c)

Solution $\frac{6\% \text{ of } 1000000}{12} = \text{Rs } 5000/\text{month}$

Incidental Expenses + Taxes = Rs 3000/month

Total Rent per month = **Rs 8000.**

55. Ans. (b)

Solution $100 + 20\% = 120$

$120 - 10\% = 108$

$\therefore$ Gain % = **8.**

56. Ans. (a)

Solution $X = Y + 25\%$ of $Y = \frac{5Y}{4}$

$Y = Z + 20\%$ of $Z = \frac{6Z}{5}$

$\therefore \quad 4X = 5Y = 6Z \Rightarrow \frac{X}{15} = \frac{Y}{12} = \frac{Z}{10}$

i.e. X, Y, Z share their profit in the ratio 15 : 12 : 10.

$\therefore$ Z's share $= \frac{10}{37} \times 740 =$ **Rs 200.**

57. Ans. (b)

Solution $100 \rightarrow 50 \rightarrow 75$, $\therefore$ Loss = **25%.**

58. Ans. (d)

Solution Let the total number of votes polled = x

$\therefore$ The winning candidate got 70% of the votes polled.

Hence, 40% of $x = 16000 \Rightarrow \boldsymbol{x = 40000.}$

59. Ans. (b)

Solution Let the earlier price of groundnut be Rs x/kg

$\therefore$ $\frac{48}{x}$ kg of groundnuts could be purchased for Rs 48.

$\Rightarrow \quad \left(\frac{48}{x} - \frac{3}{2}\right) \times \frac{5x}{4} = 48 \Rightarrow \boldsymbol{x = 6.40.}$

60. Ans. (c)

Solution $\frac{2}{3}$ of $\frac{1}{7}$ of $K = 87\frac{1}{2}\%$ of 240

$\Rightarrow \quad 2K = 21 \times \frac{175}{2} \times \frac{1}{100} \times 240$

$\Rightarrow \quad \boldsymbol{K = 2205.}$

61. Ans. (d)

Solution $X + Y + Z = 7400$ ...(i)

$X = Y + 25\%$ of $Y = \frac{5Y}{4}$ ...(ii)

$Y = Z + 20\%$ of $Z = \frac{6Z}{5}$ ...(iii)

$\therefore$ (i) gives

$\frac{5Y}{4} + \frac{6Z}{5} + Z = 7400 \Rightarrow \frac{3Z}{2} + \frac{6Z}{5} + Z = 7400$

$\Rightarrow 37Z = 74000 \Rightarrow Z = \mathbf{2000}$

$\therefore$ $Y = 2400$, $X = 3000$.

62. Ans. (c)

Solution 8% of m = 4% of $p \Rightarrow 2m = p$

$\therefore$ 20% of $m = \frac{m}{5} = \frac{p}{10} =$ **10% of p.**

63. Ans. (b)

Solution Using PDPT, we get

Net effect on sales $= -20 + 80 + \frac{(-20) \times 80}{100}$

$= 60 - 16 =$ **44% increase.**

64. Ans. (b)

Solution $S = 150\%$ of $T \Rightarrow S = \frac{150T}{100} \Rightarrow S = \frac{3}{2}T$

$\Rightarrow S + T = \frac{3}{2}T + T = \frac{5T}{2}$

$\Rightarrow T = \frac{2}{5}(S + T)$

$=$ **40% of $(S + T)$.**

65. Ans. (b)

Solution Suppose total number of students = 100

$\therefore$ No. of seniors who attended the play = 20

Total number of students who attended the play = 60

$\therefore$ No. of non-seniors who attended the play

$= 60 - 20 =$ **40 i.e. 40%.**

66. Ans. (d)

Solution Required Percentage $= \left(\frac{P}{100 + P} \times 100\right)\%$

$= \left(\frac{20}{120} \times 100\right)\% = 16.66\%$

67. Ans. (d)

68. Ans. (b)

69. Ans. (a)

Rainbow will occur once in 20 days

Rest 19 days will not producer rainbow

$\% = \frac{19}{20} \times 100 = 95\%$

70. Ans. (a)

Solution Let T = 100

S = 150

T + S = 250

$\therefore$ Required Percentage $= \frac{100}{250} \times 100 = 40\%$

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PA108

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. In a certain examination there were 2500 candidates of whom 20% were girls and the rest boys. Suppose 5% of boys and 40% of girls failed. The percentage of candidates who passed was
   (a) 78% (b) 88%
   (c) 68% (d) 70%

2. In an election between two candidates, one got 55% of the total valid votes, 20% of the votes were invalid. If the total number of votes was 7500, the number of valid votes that the other candidate got was
   (a) 2700 (b) 2900
   (c) 3000 (d) 3100

3. The population of a town increases 4% annually but is decreased by emigration annually to the extent of $\frac{1}{2}$%. What will be the increase per cent in three years?
   (a) 9.8 (b) 10
   (c) 10.5 (d) 10.8

4. The current birth rate per thousand is 32 whereas corresponding death rate is 11 per thousand. The net growth rate in terms of population increase in percent is given by
   (a) 0.021% (b) 0.0021%
   (c) 21% (d) 2.1%

5. A man loses $12\frac{1}{2}$% of his money and after spending 70% of the remainder, is left with Rs. 210. Find the amount of money he had.
   (a) Rs. 800 (b) Rs. 600
   (c) Rs. 840 (d) Rs. 900

6. The value of a machine depreciates from Rs 32768 to Rs. 21952 in three years. What is the rate percentage of depreciation?
   (a) 11% (b) 12.5%
   (c) 33% (d) 12.25%

7. A monthly return railway ticket costs 25 per cent more than a single ticket. A week's extension can be had for the former by paying 5 per cent of the monthly ticket's cost. If the money paid for the monthly ticket (with extension) is Rs 84, the price of the single ticket is
   (a) Rs. 48 (b) Rs. 80
   (c) Rs. 64 (d) Rs. 72

8. Vineet spends 40% of his salary on food articles and $\frac{1}{3}$rd of the remaining on transport. If he saves Rs. 450 per month, which is half of the balance after spending on food items and transport, what is his monthly salary?
   (a) Rs. 1125 (b) Rs. 2250
   (c) Rs. 2500 (d) Rs. 4500

9. A mixture of 50 litres of milk and water contains 10% water. The quantity of water to be added, so that new mixture contains 20% of water, is
   (a) 6.25 litres (b) 6 litres
   (c) 5.4 litres. (d) None of these

10. The price of a commodity is raised by 20%. For the expenditure not to increase, the consumption must be reduced in the ratio :
    (a) 1 : 2 (b) 1 : 3
    (c) 1 : 6 (d) 1 : 4

11. The price of tea increases from Rs. 8 per kg to Rs. 9.50 per kg. How much consumption of tea should be reduced by a housewife so that the expenditure on it may not increase?
    (a) 16% (b) $15\frac{15}{19}$%
    (c) $14\frac{4}{5}$% (d) 15%.

12. Due to increase of 15% in the price of milk, a family reduces its consumption of milk by 15%. What was the effect on expenditure of that family on account of milk?
    (a) 2.50% decrease (b) 2.25% decrease
    (c) 3% decrease (d) 3.5% decrease

13. In an examination, 20% students passed in 1st division, 25% in 2nd division and 45% passed in 3rd division. If 124 students failed in the examination, what is the total number of students who appeared in the examination?
    (a) 1216 (b) 1240
    (c) 800 (d) 1364

14. An author gets a royalty of 13% on the printed price of a book written by him. If the printed price of a copy of his book is Rs. 4.50 and 600 copies of the book are sold, how much royalty did he earn?
    (a) Rs 2349 (b) Rs 2700
    (c) Rs 312 (d) Rs 351

15. A papaya tree was planted 2 years ago. It increases at the rate of 20% every year. If at present, the

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Test II **[Time: 10 min]**

height of the tree is 540 cm, what was it when the tree was planted?
(a) 324 cm (b) 375 cm
(c) 400 cm (d) 432 cm

16. One litre of water is evaporated from 6 litres of a solution containing 5% salt. The percentage of salt in the remaining solution is:
(a) $4\frac{4}{9}\%$ (b) $5\frac{5}{7}\%$
(c) 5% (d) 6%

17. A mixture of 40 litres of milk and water contains 10% water. How much water should be added to this so that water may be 20% in the new mixture?
(a) 4 litres (b) 5 litres
(c) 6.5 litres (d) 7.5 litres

18. In an examination of n questions, a student answered 15 of the first 20 correctly. Of the remaining questions he answered one-third correctly. All the questions have the same credit. If the student scored 50% marks, how many different values of n can be there?
(a) 4 (b) 3
(c) 2 (d) 1

19. The amount of water (in ml) that should be added to reduce 9 ml lotion, containing 50% alcohol, to a lotion containing 30% alcohol, is
(a) 3 ml (b) 4 ml
(c) 5 ml (d) 6 ml

20. A person saves every year 20% of his income. if his income increases every year by 10%, then his savings increase every year by
(a) 10% (b) 5%
(c) 2% (d) 1%

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution No. of girls $= \frac{20}{100} \times 2500 = 500$

No. of boys $= 2500 - 500 = 2000$

No. of boys who failed $= \frac{5}{100} \times 2000 = 100$

No. of girls who failed $= \frac{40}{100} \times 500 = 200$

No. of candidates who passed

$= 2500 - (100 + 200)$

$= 2200$

Percentage of candidates who passed

$= \frac{2200}{2500} \times 100$

$= \mathbf{88\%}.$

3. Ans. (d)

Solution Increase in 3 years over 100

$$= 100 \times \left(1+\frac{7}{2\times100}\right)^3$$

$$= 100 \times \frac{207}{200}\times\frac{207}{200}\times\frac{207}{200}$$

$$= \frac{(200+7)^3}{80000}$$

$$= \frac{(200)^3+7^3+4200\,(200+7)}{80000}$$

$$= \frac{8869743}{80000} = 110.8718$$

Increase % $= \mathbf{10.8\%}.$

5. Ans. (a)

Solution Let the money he had be Rs. x

He lost $12\frac{1}{2}\%$, i.e. $\frac{25}{2}\times\frac{1}{100}\times x = \frac{x}{8}$

Money left $= x - \frac{x}{8} = \frac{7}{8}x$

He spends 70% of $\frac{7}{8}x$ or, $\frac{70}{100}\times\frac{7}{8} = \frac{49}{80}x$

Finally, he is left with Rs. $\frac{7}{8}x - \frac{49}{80}x$ or, $\frac{21}{80}x$

But $\frac{21}{80}\times x = 210$ (given)

$\therefore \quad x = \frac{210\times80}{21} = \mathbf{Rs.\ 800.}$

7. Ans. (c)

Solution Let the price of single ticket be Rs. x

Price of return ticket $= x + \frac{25}{100}x = \frac{5}{4}x$

Price of ticket after extension $= \frac{5}{4}x + \frac{5}{100}\times\frac{5}{4}x$

$\therefore \quad 84 = \frac{5}{4}x + \frac{x}{16} = \frac{21x}{16}$

or, $\quad x = \frac{16\times84}{21} = \mathbf{Rs.\ 64.}$

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9. Ans. (a)

Solution Quantity of water in given mixture

$$= \frac{10}{100} \times 50 = 5 \text{ litres}$$

Let quantity of water to be added = x litres

The new mixture is of $(50 + x)$ litres and contains $(5 + x)$ litres of water.

$$\therefore \quad \frac{5+x}{50+x} = \frac{20}{100}$$

or, $25 + 5x = 50 + x$

or, $4x = 25$

or, $x = \frac{25}{4} = \mathbf{6.25.}$

11. Ans. (b)

Solution In Rs. 9.50, the expenditure should be reduced by Rs. 1.50.

$\therefore$ In Re. 1, the expenditure should be reduced by Rs. $\frac{1.50}{9.50}$

$\therefore$ In Rs. 100, the expenditure should be reduced by Rs. $\frac{1.50}{9.50} \times 100 = 15\frac{15}{19}\%$

Hence, the housewife should reduce her consumption by $15\frac{15}{19}\%$.

13. Ans. (b)

Solution Pass percentage = 20% + 25% + 45% = 90%

Fail percentage = 100% – 90%% = 10%

Number of failed students = 124

Suppose the total number of students who appeared in the examination = x

$\therefore$ 10% of $x = 124$

or, $\frac{10 \times x}{100} = 124$

or, $x = \frac{124 \times 100}{10} = 1240.$

Hence, 1240 students appeared in the examination.

15. Ans. (b)

Solution $540 = x\left(1+\frac{20}{100}\right)^2$

or, $x = 540 \times \frac{5}{6} \times \frac{5}{6} = 375$ cm.

17. Ans. (b)

Solution Milk = 90% of 40 = 36 litres, water = 4 litres

$\therefore \quad \frac{4+x}{40+x} \times 100 = 20$ or, $20(40 + x) = 100(4 + x)$

So, $80x = 400$ **or,** $\boldsymbol{x = 5.}$

19. Ans. (d)

Solution 30% of new mixture = 5% of 9

$\therefore$ New mixture = 15 ml

$\therefore$ Water added = 15 ml – 9 ml = **6 ml.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (d)	**6.** (b)	**8.** (b)	**10.** (c)	**12.** (b)	**14.** (d)	**16.** (d)	**18.** (d)	**20.** (a)

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Average

6

INTRODUCTION

Whenever we are asked the marks scored by us in any examination, we usually tell the marks in percentage, taking the percentage of total marks of all subjects. This percentage is called *average percentage*. Also, in a class, if there are 100 students, instead of knowing the age of individual student, we usually talk about average age.

The *average* or *mean* or *arithmetic mean* of a number of quantities of the same kind is equal to their sum divided by the number of those quantities. For example, the average of 3, 9, 11, 15, 18, 19 and 23 is

$$\frac{3+9+11+15+18+19+23}{7} = \frac{98}{7} = 14.$$

SOME BASIC FORMULAE

1. $\text{Average} = \dfrac{\text{Sum of quantities}}{\text{Number of quantities}}$
2. $\text{Sum of quantities} = \text{Average} \times \text{Number of quantities}$
3. $\text{Number of quantities} = \dfrac{\text{Sum of quantities}}{\text{Average}}$

Illustration 1 A man purchased 5 toys at the rate of Rs 200 each, 6 toys at the rate of Rs 250 each and 9 toys at the rate of Rs 300 each. Calculate the average cost of one toy.

Solution Price of 5 toys = 200 × 5 = Rs 1000
Price of 6 toys = 250 × 6 = Rs 1500
Price of 9 toys = 300 × 9 = Rs 2700
Total number of toys = 5 + 6 + 9 = 20.

$$\text{Average price of 1 toy} = \frac{1000+1500+2700}{20}$$

$$= \frac{5200}{20} = \textbf{Rs 260.}$$

Illustration 2 The average marks obtained by 200 students in a certain examination is 45. Find the total marks.

Solution Total marks
= Average marks × Number of students
= 200 × 45 = **900.**

Illustration 3 Total temperature for the month of September is 840°C. If the average temperature of that month is 28°C, find of how many days is the month of September.

Solution Number of days in the month of September

$$= \frac{\text{Total temperature}}{\text{Average temperature}} = \frac{840}{28}$$

= 30 days.

SOME USEFUL SHORT-CUT METHODS

1. Average of two or more groups taken together
   (a) If the number of quantities in two groups be n_1 and n_2 and their average is x and y, respectively, the combined average (average of all of them put together) is

$$\frac{n_1x+n_2y}{n_1+n_2}$$

Explanation

No. of quantities in first group = n_1
Their average = x
$\therefore$ Sum = $n_1 \times x$
No. of quantities in second group = n_2
Their average = y
$\therefore$ Sum = $n_2 \times y$

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No. of quantities in the combined group $= n_1 + n_2$

Total sum (sum of quantities of first group and second group)

$= n_1x + n_2y.$

$\therefore$ Average of the two groups

$$= \frac{n_1x+n_2y}{n_1+n_2}.$$

(b) If the average of n_1 quantities is x and the average of n_2 quantities out of them is y, the average of remaining group (rest of the quantities) is

$$\frac{n_1x-n_2y}{n_1-n_2}.$$

Explanation

No. of quantities $= n_1$

Their average $= x$

$\therefore$ Sum $= n_1 x$

No. of quantities taken out $= n_2$

Their average $= y$

$\therefore$ Sum $= n_2y$

Sum of remaining quantities $= n_1x - n_2y$

No. of remaining quantities $= n_1 - n_2$

$\therefore$ Average of remaining group $= \frac{n_1x-n_2y}{n_1-n_2}.$

Illustration 4 The average weight of 24 students of section A of a class is 58 kg whereas the average weight of 26 students of section B of the same class is 60.5 kg. Find the average weight of all the 50 students of the class.

Solution Here, $n_1 = 24$, $n_2 = 26$, $x = 58$ and $y = 60.5$.

$\therefore$ Average weight of all the 50 students

$$= \frac{n_1x+n_2y}{n_1+n_2}$$

$$= \frac{24\times58+26\times60.5}{24+26}$$

$$= \frac{1392+1573}{50} = \frac{2965}{50} = \mathbf{59.3\ kg.}$$

Illustration 5 Average salary of all the 50 employees including 5 officers of a company is Rs 850. If the average salary of the officers is Rs 2500, find the average salary of the remaining staff of the company.

Solution Here, $n_1 = 50$, $n_2 = 5$, $x = 850$ and $y = 2500$.

$\therefore$ Average salary of the remaining staff

$$= \frac{n_1x-n_2y}{n_1-n_2} = \frac{50\times850-5\times2500}{50-5}$$

$$= \frac{42500-12500}{45} = \frac{30000}{45}$$

= Rs 667 (approx.)

2. If $\bar{x}$ is the average of $x_1, x_2, ..., x_n$, then

(a) The average of $x_1 + a, x_2 + a, ..., x_n + a$ is $\bar{x} + a$.

(b) The average of $x_1 - a, x_2 - a, ... x_n - a$ is $\bar{x} - a$.

(c) The average of $ax_1, ax_2, ..., ax_n$ is $a\bar{x}$, provided $a \neq 0$.

(d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, ..., \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$, provided $a \neq 0$.

Illustration 6 The average value of six numbers 7, 12, 17, 24, 26 and 28 is 19. If 8 is added to each number, what will be the new average?

Solution The new average $= \bar{x} + a$

$= 19 + 8 =$ **27.**

Illustration 7 The average of x numbers is $5x$. If $x - 2$ is subtracted from each given number, what will be the new average?

Solution The new average $= \bar{x} - a$

$= 5x - (x - 2) =$ **$4x + 2$.**

Illustration 8 The average of 8 numbers is 21. If each of the numbers is multiplied by 8, find the average of a new set of numbers.

Solution The average of a new set of numbers

$= a\bar{x} = 8 \times 21 =$ **168.**

3. The average of n quantities is equal to x. If one of the given quantities whose value is p, is replaced by a new quantity having value q, the average becomes y, then $q = p + n(y - x)$

Illustration 9 The average weight of 25 persons is increased by 2 kg when one of them whose weight is 60 kg, is replaced by a new person. What is the weight of the new person?

Solution The weight of the new person

$= p + n(y - x)$

$= 60 + 25(2) =$ **110 kg.**

4. (a) The average of n quantities is equal to x. When a quantity is removed, the average becomes y. The value of the removed quantity is $n(x-y)+y$.

(b) The average of n quantities is equal to y. When a quantity is added, the average becomes y. The value of the new quantity is $n(y-x)+y$.

Illustration 10 The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average age reduces by 1 year. What is the age of the class teacher?

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Solution The age of class teacher
$= n(x - y) + y$
$= 25(16 - 15) + 15 =$ **40 years.**

Illustration 11 The average age of 30 children in a class is 9 years. If the teacher's age be included, the average age becomes 10 years. Find the teacher's age.

Solution The teacher's age
$= n(y - x) + y$
$= 30(10 - 9) + 10 =$ **40 years.**

5. (a) The average of first n natural numbers is $\frac{n+1}{2}$.
   (b) The average of square of natural numbers till n is $\frac{(n+1)(2n+1)}{6}$.
   (c) The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$.
   (d) The average of odd numbers from 1 to n is $\frac{\text{last odd number} + 1}{2}$.
   (e) The average of even numbers from 1 to n is $\frac{\text{last even number} + 2}{2}$.

Illustration 12 Find the average of first 81 natural numbers.

Solution The required average
$= \frac{n+1}{2} = \frac{81+1}{2} =$ **41.**

Illustration 13 What is the average of squares of the natural numbers from 1 to 41?

Solution The required average
$= \frac{(n+1)(2n+1)}{6} = \frac{(41+1)(2\times41+1)}{6}$
$= \frac{42\times83}{6} = \frac{3486}{6} =$ **581.**

Illustration 14 Find the average of cubes of natural numbers from 1 to 27.

Solution The required average
$= \frac{n(n+1)^2}{4} = \frac{27\times(27+1)^2}{4}$
$= \frac{27\times28\times28}{4} = \frac{21168}{4} =$ **5292.**

Illustration 15 What is the average of odd numbers from 1 to 40?

Solution The required average
$= \frac{\text{last odd number} + 1}{2} = \frac{39+1}{2} =$ **20.**

Illustration 16 What is the average of even numbers from 1 to 81?

Solution The required average
$= \frac{\text{last even number} + 2}{2} = \frac{80+2}{2} =$ **41.**

6. (a) If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.
   (b) If n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.
   (c) The average of first n consecutive even numbers is $(n + 1)$.
   (d) The average of first n consecutive odd numbers is n.
   (e) The average of squares of first n consecutive even numbers is $\frac{2(n+1)(2n+1)}{3}$.
   (f) The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$.
   (g) The average of squares of consecutive odd numbers till n is $\frac{n(n+2)}{3}$.
   (h) If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is $2(n - 1)$.

Illustration 17 Find the average of 7 consecutive numbers 3, 4, 5, 6, 7, 8, 9.

Solution The required average = middle number = **6.**

Illustration 18 Find the average of consecutive odd numbers 21, 23, 25, 27, 29, 31, 33, 35.

Solution The required average
= average of middle two numbers
= average of 27 and 29
$= \frac{27+29}{2} =$ **28.**

Illustration 19 Find the average of first 31 consecutive even numbers.

Solution The required average $= (n + 1) = 31 + 1 =$ **32.**

Illustration 20 Find the average of first 50 consecutive odd numbers.

Solution The required average $= n =$ **50.**

Illustration 21 Find the average of squares of first 19 consecutive even numbers.

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Solution The required average $= \frac{2(n+1)(2n+1)}{3}$

$= \frac{2(19+1)(2\times19+1)}{3}$

$= \frac{2\times20\times39}{3}$

$= \frac{1560}{3} = \mathbf{520.}$

Illustration 22 Find the average of squares of consecutive even numbers from 1 to 25.

Solution The required average

$= \frac{(n+1)(n+2)}{3} = \frac{(25+1)(25+2)}{3}$

$= \frac{26\times27}{3} = \frac{702}{3} = 234.$

Illustration 23 Find the average of squares of consecutive odd numbers from 1 to 31.

Solution The required average

$= \frac{n(n+2)}{3} = \frac{31\times(31+2)}{3}$

$= \frac{31\times33}{3} = \mathbf{341.}$

Illustration 24 If the average of 6 consecutive numbers is 48, what is the difference between the smallest and the largest number?

Solution The required difference

$= 2(n-1) = 2(6-1) = \mathbf{10.}$

7. Geometric Mean or Geometric Average

Geometric mean of $x_1, x_2, \ldots, x_n$ is denoted by

$$\text{G.M.} = \sqrt[n]{x_1 \times x_2 \times \ldots \times x_n}.$$

Geometric mean is useful in calculating averages of ratios such as average population growth rate, average percentage increase and so on.

Illustration 25 The production of a company for three successive years has increased by 10%, 20% and 40%, respectively. What is the average annual increase of production?

Solution Geometric mean of x, y and $z = (x \times y \times z)^{1/3}$.

$\therefore$ Average increase $= (10 \times 20 \times 40)^{1/3}\% = \mathbf{20\%.}$

Illustration 26 The population of a city in two successive years increases at the rates of 16% and 4%, respectively. Find the average increase of two years.

Solution In case of population increase, the geometric mean is required.

$\therefore$ Geometric mean of 16% and 4% is

$= \mathbf{(16 \times 4)^{1/2}\%,\ i.e.\ 8\%.}$

8. Harmonic Mean or Harmonic Average

Harmonic means of $x_1, x_2, \ldots, x_n$ is denoted by

$$\text{H.M.} = \frac{1}{\frac{1}{n}\left(\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}\right)}$$

Harmonic mean is useful for finding out average speed of a vehicle, average production per day and so on.

Illustration 27 A man runs 1 km at 15 km per hour and another 1 km he walks at 5 km per hour. Find his average speed for the whole distance in covering 2 km.

Solution When the distance is constant and the speed varies, harmonic mean is used. Harmonic mean of x and y is $\frac{2}{\frac{1}{x}+\frac{1}{y}}$ or $\frac{2xy}{x+y}$.

$\therefore$ Average speed for the whole distance

$= \frac{2\times15\times5}{15+5} = \mathbf{7.5\ km/hr.}$

9. If a certain distance is covered at a speed of x km/hr and the same distance is covered at a speed of y km/hr, the average speed during the entire journey is

$$\left(\frac{2xy}{x+y}\right)\text{km/hr.}$$

Illustration 28 If half of the journey is travelled at a speed of 15 km/hr and the next half at a speed of 12 km/hr, find the average speed during the entire journey

Solution The average speed

$= \left(\frac{2xy}{x+y}\right) = \left(\frac{2\times15\times12}{15+12}\right)$

$= \frac{360}{27} = \mathbf{13\frac{1}{3}\ km/hr.}$

Illustration 29 A man goes to a certain place at a speed of 30 km/hr and returns to the original place at a speed of 20 km/hr, find the average speed during up and down journey.

Solution The average speed

$= \left(\frac{2xy}{x+y}\right) = \left(\frac{2\times30\times20}{30+20}\right) = \frac{1200}{50}$

$= \mathbf{24\ km/hr.}$

10. If a person or a motor car covers three equal distances at the speed of x km/hr, y km/hr and z km/hr, respectively, then for the entire journey average speed of the person or motor car is

$$\left(\frac{3xzy}{xy+yz+zx}\right)\text{km/hr.}$$

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Illustration 30 A train covers the first 160 kms at a speed of 120 km/hr, another 160 kms at 140 km/hr and the last 160 kms at 80 km/hr. Find the average speed of the train for the entire journey.

Solution Average speed $= \dfrac{3xyz}{xy+yz+zx}$

$$= \frac{3\times120\times140\times80}{120\times140+140\times80+80\times120}$$

$$= \frac{360\times140\times80}{16800+11200+9600} = \frac{4032000}{37600}$$

$$= \mathbf{107\,\frac{11}{47}\ km/hr.}$$

11. If a person covers A km at a speed of x km/hr, B km at a speed of y km/hr and C km at a speed of z km/hr, the average speed during the entire journey is

$$\left(\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) \text{km/hr.}$$

Illustration 31 A person covers 9 km at a speed of 3 km/hr, 25 km at a speed of 5 km/hr and 30 km at a speed of 10 km/hr. Find the average speed for the entire journey.

Solution The average speed $= \left(\dfrac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)$

$$= \left(\frac{9+25+30}{\frac{9}{3}+\frac{25}{5}+\frac{30}{10}}\right)$$

$$= \frac{64}{11} = \mathbf{5\frac{9}{11}\ km/hr.}$$

12. If a person covers Ath part of the distance at x km/hr, Bth part of the distance at y km/hr and the remaining Cth part at z km/hr, then the average speed during the entire journey is

$$\left(\frac{1}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) \text{km/hr.}$$

Illustration 32 A person covers the first $\frac{1}{4}$th of the distance at 8 km/hr, the next $\frac{3}{5}$th at 6 km/hr and the remaining distance at 15 km/hr. Find the average speed during the entire journey.

Solution The average speed

$$= \frac{1}{\left(\frac{A}{x}+\frac{B}{y}+\frac{C}{z}\right)}$$

$$= \left(\frac{1}{\frac{1/4}{8}+\frac{3/5}{6}+\frac{3/20}{15}}\right)$$

$$\left[\text{Here, } A=\frac{1}{4},\ B=\frac{3}{5} \text{ and } C=1-\left(\frac{1}{4}+\frac{3}{5}\right)=\frac{3}{20}\right]$$

$$= \frac{1}{\frac{1}{32}+\frac{1}{10}+\frac{1}{100}} = \frac{3200}{452} = \mathbf{7\frac{9}{113}\ km/hr.}$$

Illustration 33 A train covers 50% of the journey at 30 km/hr, 25% of the journey at 25 km/hr and the remaining at 20 km/hr. Find the average speed of the train during the entire journey.

Solution The average speed

$$= \left(\frac{100}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) = \left(\frac{100}{\frac{50}{30}+\frac{25}{25}+\frac{25}{20}}\right)$$

[Here, $A = 50$, $B = 25$ and $C = 25$]

$$= \frac{100}{47/12} = \frac{1200}{47}$$

$$= \mathbf{25\frac{25}{47}\ km/hr.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The daily earnings of a taxi driver during a week are : Rs 60, Rs 65, Rs 70, Rs 52. 50, Rs 63, Rs 73 and Rs 68. What is his average daily earning for the week?

(a) Rs 74.50 (b) Rs 54.50
(c) Rs 64.50 (d) Rs 84.50

2. The average of 10 numbers is 7. What will be the new average if each of the numbers is multiplied by 8?

(a) 45 (b) 52
(c) 56 (d) 55

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3. The average weight of 5 persons sitting in a boat is 38 kg. If the average weight of the boat and the persons sitting in the boat is 52 kg, what is the weight of the boat?
(a) 228 kg (b) 122 kg
(c) 232 kg (d) 242 kg

4. There are 35 students in a hostel. If the number of students increased by 7, the expenses of the mess were increased by Rs 42 per day while the average expenditure per head diminished by Re 1. Find the original expenditure of the mess.
(a) Rs 480 (b) Rs 440
(c) Rs 520 (d) Rs 420

5. The daily maximum temperature in Delhi for 7 consecutive days in May 1988 were 42.7°C, 44.6°C, 42.0°C, 39.1°C, 43.0°C, 42.5°C and 38.5°C. Find the average daily maximum temperature.
(a) 42.63° C (b) 45.65° C
(c) 41.77° C (d) 39.60° C

6. The average salary per head of all the workers in a workshop is Rs 850. If the average salary per head of 7 technicians is Rs 1000 and the average salary per head of the rest is Rs 780, find the total number of workers in the workshop.
(a) 26 (b) 24
(c) 28 (d) 22

7. An aeroplane travels distances 2500 km, 1200 km and 500 km at the rate of 500 km/hr, 400 km, and 250 km/hr, respectively. The average is
(a) 420 km/hr (b) 410 km/hr
(c) 405 km/hr (d) 575 km/hr

8. In a class of 20 students in an examination in Mathematics 2 students scored 100 marks each, 3 got zero each and the average of the rest was 40. What is the average of the whole class?
(a) 40 marks (b) 35 marks
(c) 32 marks (d) 45 marks

9. The average weight of 24 students of section *A* of a class is 58 kg whereas the average weight of 26 students of section *B* of the same class is 60.5 kg. Find the average weight of all the 50 students of the class.
(a) 57.4 kg (b) 59.3 kg
(c) 58.9 kg (d) 59.7 kg

10. The average age of 5 members is 21 years. If the age of the youngest member be 5 years, find the average age of the family at the birth of the youngest member.
(a) 24 years (b) 25 years
(c) 20 years (d) 28 years

11. The average of 7 numbers is 5. If the average of first six of these numbers is 4, the seventh number is
(a) 14 (b) 12
(c) 11 (d) 15

12. Three years ago the average age of a family of 5 members was 27 years. On the addition of a child to the family, the present average age of the family is still 27 years. Find the present age of the child.
(a) 16 years (b) 12 years
(c) 24 years (d) 20 years

13. The average weight of 10 students is increased by half a kg when one of the students weighing 50 kg is replaced by a new student. Find the weight of the new student.
(a) 55 kg (b) 60 kg
(c) 45 kg (d) 40 kg

14. The average monthly salary of a staff of 9 persons is Rs 2450. One member of the staff whose monthly salary is Rs 2650 is transferred. Find the average salary of the remaining 8 persons of the staff.
(a) Rs 2425 (b) Rs 2625
(c) Rs 3025 (d) Rs 2825

15. The mean marks of 10 boys in a class is 70% whereas the mean marks of 15 girls is 60%. The mean marks of all the 25 students is
(a) 64% (b) 60%
(c) 55% (d) 52%

16. The average income of *A* for 15 days is Rs 70. The average for first five days is Rs 60 and that for the last nine days is Rs 80. The income for the sixth day is
(a) Rs 80 (b) Rs 60
(c) Rs 40 (d) Rs 30

17. The average of five consecutive even numbers starting with 4, is
(a) 6 (b) 7
(c) 8 (d) 7.5

18. 3 years ago the average age of a family of 5 members was 17 years. With the birth of a new baby, the average remains the same even today. Find the age of the baby.
(a) 1 years (b) 3 years
(c) $2\frac{1}{2}$ years (d) 2 years

19. The average of 17 numbers is 10.9. If the average of first nine numbers is 10.5 and that of the last nine numbers is 11.4, the middle no. is
(a) 11.8 (b) 11.4
(c) 10.9 (d) 11.7

20. A batsman has a certain average of runs for 12 innings. In the 13th innings he scores 96 runs there-by increasing his average by 5 runs. What is his average after the 13th innings?
(a) 48 (b) 64
(c) 36 (d) 72

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21. A batsman in his 17th innings makes a score of 85, and thereby increases his average by 3. What is his average after the 17th innings? He had never been 'not out'.
(a) 47 (b) 37
(c) 39 (d) 43

22. The sum of three numbers is 98. If the ratio between first and second be 2 : 3 and that between second and third be 5 : 8, then the second number is
(a) 30 (b) 20
(c) 58 (d) 48

23. The average weight of 8 sailors in a boat is increased by 1 kg if one of them weighing 56 kg is replaced by a new sailor. The weight of the new sailor is
(a) 57 kg (b) 60 kg
(c) 64 kg (d) 62 kg

24. A number x equals 80% of the average of 5, 7, 14 and a number y. If the average of x and y is 26, the value of y is
(a) 13 (b) 26
(c) 39 (d) None of these

25. The average age of A, B, C, D five years ago was 45 years. By including x, the present average age of all the five is 49 years. The present age of x is
(a) 64 years (b) 48 years
(c) 45 years (d) 40 years

26. It rained as much on Wednesday as on all the others days of the week combined. If the average rainfall for the whole week was 3 cm, how much did it rain on Wednesday?
(a) 2.625 cm (b) 3 cm
(c) 10.5 cm (d) 15 cm

27. The average monthly expenditure of a family for the first four months is Rs 2750, for the next three months is Rs 2940 and for the last five months Rs 3130. If the family saves Rs 5330 during the whole year, find the average monthly income of the family during the year.
(a) Rs 3800 (b) Rs 3500
(c) Rs 3400 (d) Rs 4200

28. The average age of 8 men is increased by 2 years. When two of them, whose ages are 20 years and 24 years respectively, are replaced by two women. What is the average age of these women?
(a) 36 years (b) 30 years
(c) 40 years (d) 42 years

29. The average of 50 numbers is 38. If two numbers 45 and 55 are discarded, the average of the remaining set of numbers is
(a) 38.5 (b) 37.5
(c) 37.0 (d) 36.5

30. The average speed of a train running at a speed of 30 km/hr during the first 100 kilometres, at 40 km/hr during the second 100 kilometres and at 50 km/hr during the last 100 kilometres is nearly
(a) 38.5 km/hr (b) 38.3 km/hr
(c) 40.0 km/hr (d) 39.2 km/hr

31. The average of 6 observations is 12. A new seventh observation is included and the new average is decreased by 1. The seventh observation is
(a) 1 (b) 3
(c) 5 (d) 6

32. The average age of 20 boys in the class is 15.6 years. Five new boys join and the new average becomes 15.56 years. What was the average age of five new boys?
(a) 15.5 (b) 15.4
(c) 15.25 (d) 15.3
(e) None of these

33. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A, then average weight of B, C, D and E becomes 79 kg. The weight of A is
(a) 70 kg (b) 72 kg
(c) 75 kg (d) 80 kg

34. There was one mess for 30 boarders in a certain hostel. On the number of boarders being increased by 10, the expenses of the mess were increased by Rs 40 per month while the average expenditure per head diminished by Rs 2. Find the original monthly expenses.
(a) Rs 390 (b) Rs 410
(c) Rs 360 (d) Rs 480

35. Of the three numbers the first is twice the second and the second is thrice the third. If the average of the three numbers is 10, the numbers are
(a) 18, 3, 9 (b) 9, 3, 18
(c) 3, 9, 18 (d) 18, 9, 3

36. The average weight of 36 students is 50 kg. It was found later that the figure of 37 kg was misread as 73 kg. What is the correct average?
(a) 49 kg (b) 51 kg
(c) 50.5 kg (d) None of these

37. The average earning of a mechanic for the first four days of a week is Rs 18 and for the last four days is Rs 22. If he earns Rs 20 on the fourth day, his average earning for the whole week is
(a) Rs 18.95 (b) Rs 16
(c) Rs 20 (d) Rs 25.71

38. The average of marks obtained by 120 candidates was 35. If the average of marks of passed candidates was 39 and that of failed candidates was 15, the number of candidates who passed the examination is
(a) 100 (b) 110
(c) 120 (d) 150

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39. In a class, there are 20 boys whose average age is decreased by 2 months, when one boy aged 18 years in replaced by a new boy. The age of the new boy is
(a) 14 yrs 8 months (b) 15 yrs
(c) 16 yrs 4 months (d) 17 yrs 10 months

40. The average temperature from Monday to Thursday is 48° and from Tuesday to Friday is 52°. If the temperature on Monday is 42°, what was it on Friday?
(a) 52° (b) 55°
(c) 58° (d) 51°

41. A man spends on an average Rs 269.47 for the first 7 months and Rs 281.05 for the next 5 months. Find his monthly salary if he saves Rs 308.46 during the whole year.
(a) Rs 400 (b) Rs 500
(c) Rs 300 (d) Rs 600

42. The average of two numbers is 62. If 2 is added to the smaller number, the ratio between the numbers becomes 1 : 2. The smaller number is
(a) 60 (b) 30
(c) 84 (d) 40

43. In a coconut grove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year per tree be 100, find x.
(a) 4 (b) 2
(c) 8 (d) 6

44. Average temperature of first 4 days of a week is 38.6°C and that of the last 4 days is 40.3°C. If the average temperature of the week be 39.1°C, the temperature on 4th day is.
(a) 36.7°C (b) 38.6°C
(c) 39.8°C (d) 41.9°C

45. The average daily wages of A, B and C is Rs 120. If B earns Rs 40 more than C per day and A earns double of what C earns per day, the wages of A per day is
(a) Rs 80 (b) Rs 120
(c) Rs 160 (d) Rs 100

46. With an average speed of 40 km/hr a train reaches its destination on time. If it goes with an average speed of 35 km/hr, it is late by 15 minutes. The total journey is
(a) 30 km (b) 40 km
(c) 70 km (d) 80 km

47. In a competitive examination, the average marks obtained was 45. It was later discovered that there was some error in computerisation and the marks of 90 candidates had to be changed from 80 to 50 and the average came down to 40 marks. The total no. of candidates who appeared in examination is
(a) 520 (b) 550
(c) 540 (d) 525

48. Visitors to a show were charged Rs 15.00 each on the first day, Rs 7.50 on the second, Rs 2.50 on the third day and total attendance on the three days were in the ratio 2 : 5 : 13, respectively. Find the average charge per person for the whole show.
(a) Rs 7 (b) Rs 5
(c) Rs 9 (d) Rs 11

49. The mean daily profit made by a shopkeeper in a month of 30 days was Rs 350. If the mean profit for the first fifteen days was Rs 275, then the mean profit for the last fifteen days would be
(a) Rs 200 (b) Rs 275
(c) Rs 350 (d) Rs 425

50. A man whose bowling average is 12.4 takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets, taken by him, before his last match is
(a) 85 (b) 78
(c) 72 (d) 64

51. Out of three numbers, the first is twice the second and is half of the third. If the average of the three numbers is 56, the three numbers in order are
(a) 48, 96, 24 (b) 48, 24, 96
(c) 96, 24, 48 (d) 96, 48, 24

52. There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by Rs 42 per day while the average expenditure per head diminishes by Re 1. Find the original expenditure of the mess.
(a) Rs 480 (b) Rs 420
(c) Rs 520 (d) Rs 460

53. The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, what is the average of the remaining numbers?
(a) 37.5 (b) 38.5
(c) 39.5 (d) 36.5

54. In a cricket team of 11 boys, one player weighing 42 kg is injured and his place is taken by another player. If the average weight of the team is increased by 100 gm as a result of this, then what is the weight of the new player?
(a) 42.1 kg (b) 45.1 kg
(c) 44.1 kg (d) 43.1 kg

55. The average of these consecutive numbers is n. If the next two consecutive numbers are also included, the average of the five numbers will
(a) remain the same (b) increase by 0.5
(c) increase by 1 (d) increase by 1.5

56. The average salary of 20 workers in an office is Rs 1900 per month. If the manager's salary is added, the average becomes Rs 2000 per month. The manager's salary (in Rs) is
(a) 24000 (b) 25200
(d) 45600 (d) None of these

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57. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of number of boys to the number of girls in the class is
(a) 1 : 2 (b) 3 : 4
(c) 3 : 5 (d) 2 : 3

58. The average expenditure of a man for the first five months is Rs 3600 and for next seven months it is Rs 3900. If he saves Rs 8700 during the year, his average income per month is
(a) Rs 4500 (b) Rs 4200
(c) Rs 4050 (d) Rs 3750

59. Of the three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, the largest number is
(a) 24 (b) 36
(c) 72 (d) 108

60. The average age of a committee of 8 members is 40 years. A member, aged 55 years, retired and he was replaced by a member aged 39 years. The average age of the present committee is
(a) 39 years (b) 38 years
(c) 36 years (d) 35 years

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Total earning for 7 days
= Rs (60 + 65 + 70 + 52.50 + 63 + 73 + 68)
= Rs 451.50

Average daily earning = Rs $\frac{451.50}{7}$ = **Rs 64.50.**

2. Ans. (c)

Solution The average of 10 numbers = 7
Total of 10 numbers = 10 × 7 = 70
New total of 10 numbers after each of given numbers is multiplied by 8 = 70 × 8 = 560

∴ New average = $\frac{560}{10}$ = **56.**

3. Ans. (b)

Solution Average weight of 5 persons = 38 kg
∴ Total weight of these five persons
= 38 × 5 = 190 kg
Now, average weight of (the boat + 5 persons)
= 52 kg
∴ Total weight of (the boat + 5 persons)
= 52 × 6 = 312 kg
∴ Weight of the boat = 312 − 190 = **122 kg.**

4. Ans. (d)

Solution Let the original expenditure = Rs x

Original average expenditure = $\frac{x}{35}$

New average expenditure = $\frac{x+42}{42}$

$\Rightarrow \frac{x}{35} - \frac{x+42}{42} = 1 \Rightarrow x = 420$

∴ Original expenditure = **Rs 420.**

5. Ans. (c)

Solution Average daily maximum temperature
$= \frac{42.7+44.6+42.0+39.1+43.0+42.5+38.5}{7}$

$= \frac{292.4}{7}$ = **41.77°C.**

6. Ans. (d)

Solution Let the total number of workers be x
$\Rightarrow$ 850 × x = 7 × 1000 + (x − 7) × 780
$\Rightarrow$ x = **22.**

7. Ans. (a)

Solution The total time taken can be calculated as shown below:

Distance	*Speed*	*Time*
2500 km	500 km/hr	5 hrs.
1200 km	400 km/hr	3 hrs.
500 km	250 km/hr	2 hrs.
Total 4200 km		10 hrs.

Average speed = $\frac{4200}{10}$ = **420 km/hr.**

8. Ans. (a)

Solution Marks scored by 2 students = 100 × 2 = 200
Marks scored by 3 students = 3 × 0 = 0
Marks scored by 15 students = 15 × 40 = 600
∴ Marks scored by 20 students
= 200 + 0 + 600 = 800

∴ Average marks = $\frac{800}{20}$ = **40.**

9. Ans. (b)

Solution Average weight of 24 students of section A = 58 kg

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Total weight of 24 students of section $A = 58 \times 24 = 1392$ kg

Average weight of 26 students of section $B = 60.5$ kg

Total weight of 26 students of section B

$= 60.5 \times 26 = 1573$ kg

Total weight of 50 students $= (1392 + 1573)$ kg

$= 2965$ kg

Average weight of students in the class

$$= \frac{2965}{50} = \textbf{59.3 kg.}$$

10. Ans. (c)

Solution Total age of 5 members $= 21 \times 5 = 105$ years

Total age of 4 members at the birth of the younger member, that is, 5 years ago

$= 105 - (5 \times 5) = 80$ years

Before the birth of the youngest member, the family consisted of only 4 members.

Average age of 4 members 5 years ago

$$= \frac{80}{4} = \textbf{20 years.}$$

11. Ans. (c)

Solution Sum of seven numbers $= 7 \times 5 = 35$

Sum of first six numbers $= 6 \times 4 = 24$

Therefore, the seventh number $= 35 - 24 = \textbf{11.}$

12. Ans. (b)

Solution Present average age of 5 members

$= 27 + 3 = 30$ years

Sum of present age of 5 members

$= 30 \times 5 = 150$ years

Let the present age of the child be x years

Present average age of 6 members

$= \frac{150+x}{6}$ and this is equal to 27 yrs.

So, $\frac{150+x}{6} = 27$

or, $x = 27 \times 6 - 150$ or, $x = \textbf{12 yrs.}$

13. Ans. (a)

Solution Weight of the new student

$= 50 + 10 \times 1/2 = 55$ kg.

14. Ans. (a)

Solution Average salary of 9 persons = Rs 2450

Total salary of 9 persons

$=$ Rs $2450 \times 9 =$ Rs 2650

Total salary of the person who is transferred

$=$ Rs 2650

Thus, the total salary of remaining 8 persons

$=$ Rs 22050 $-$ Rs 2650 $=$ Rs 19400

The average salary of the remaining 8 persons

$$= \text{Rs } \frac{19400}{8} = \textbf{Rs 2425.}$$

15. Ans. (a)

Solution The mean marks of 10 boys = 70%

Total marks of 10 boys $= 70\% \times 10 = 700\%$

The mean marks of 15 girls = 60%

Total marks of 15 girls $= 60\% \times 15 = 900\%$

$\therefore$ Sum of the total marks of 25 students

$= 700 + 900 = 1600\%$

$\therefore$ The mean marks of all the 25 students

$$= \frac{1600}{25} = \textbf{64\%.}$$

16. Ans. (d)

Solution Income for 6th day in rupees

$= 15 \times 70 - 5 \times 60 - 9 \times 80 = \textbf{30.}$

17. Ans. (c)

Solution The five even consecutive numbers are 4, 6, 8, 10 and 12

Their average $= \frac{4+6+8+10+12}{5} = \frac{40}{5} = \textbf{8.}$

18. Ans. (d)

Solution Present age of 5 members

$= (5 \times 17 + 3 \times 5)$ years.

$= 100$ years.

Present age of 5 members and a baby

$= 17 \times 6 = 102$ yrs.

$\therefore$ Age of the baby $= (102 - 100)$ years.

$= \textbf{2 years.}$

19. Ans. (a)

Solution Sum of first nine numbers + sum of last nine numbers $= 10.5 \times 9 + 11.4 \times 9 = 21.9 \times 9 = 197.1$

Hence, the middle number

$= 197.1 - 17 \times 10.9$

$= 197.1 - 185.3 = \textbf{11.8.}$

20. Ans. (c)

Solution To improve his average by 5 runs per innings he has to contribute $12 \times 5 = 60$ runs for the previous 12 innings. Thus, the average after the 13th innings

$= 96 - 60 = \textbf{36.}$

21. Ans. (b)

Solution Average score before 17th innings

$= 85 - 3 \times 17 = 34$

$\therefore$ Average score after 17th innings

$= 34 + 3 = \textbf{37.}$

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22. Ans (a)

Solution Let the numbers be x, y and z. Then,

$$x + y + z = 98,\ \frac{x}{y} = \frac{2}{3} \text{ and } \frac{y}{z} = \frac{5}{8}$$

$$\therefore \quad x = \frac{2y}{3} \text{ and } z = \frac{8y}{5}$$

So, $\frac{2y}{3} + y + \frac{8y}{5} = 98$

or, $\frac{49y}{15} = 98$ **or, $y = 30$.**

23. Ans. (c)

Solution The sailor weighing 56 kg is replaced and the average is increased by 1 kg. Hence, the weight of the new sailor is (56 + increase in total weight) = $56 + 1 \times 8$

$= 56 + 8 =$ **64 kg.**

24. Ans. (c)

Solution Average of 5, 7, 14 and $y = \frac{5+7+14+y}{4}$

Therefore, $x = 80\%$ of $\frac{5+7+14+y}{4} = \frac{80}{100} \times \frac{26+y}{4}$

$\Rightarrow x = \frac{26+y}{5}$ ...(i). Also $\frac{x+y}{2} = 26$ ...(ii)

From (i) and (ii), we get $52 - y = \frac{26+y}{5} \Rightarrow y =$ **39.**

25. Ans. (c)

Solution Present age of x

$= [(49 \times 5) - (4 \times 45 + 4 \times 5)]$ years

= 45 years.

26. Ans. (c)

Solution Let the rainfall on Wednesday be x cm so that on the other 6 days, the total is also x.

Since average rainfall for the week = 3 cm

$\therefore \quad x + x = 3 \times 7$ **or, $x = 10.5$ cm.**

27. Ans. (c)

Solution Average monthly expenditure of 4 months

= Rs 2700

Total expenditure for 4 months = Rs 2700×4

= Rs 11000

Average monthly expenditure of 3 months

= Rs 2940

Total expenditure of 3 months = Rs 2940×5

= Rs 8820 ...(i)

Average monthly expenditure of 5 months

= Rs 3130

Total expenditure of 5 months = Rs 31020

= Rs 1560 ...(ii)

Total expenditure in the whole year

= Rs 11000 + 8820 + Rs 15650

= Rs 35470.

Saving during the whole year = Rs. 5330

Total income of the family during the year

= Rs 35470 + Rs 5330 = Rs 40800

$\therefore$ Average monthly income during the year

$= \frac{40800}{12} =$ **Rs 3400.**

28. Ans. (b)

Solution Let the average age of 8 men be x years

$\therefore$ Sum of the ages of 8 men = 84 years

Now, according to the condition of the question, average age of (6 men + 2 women) = $(x + 2)$ years

$\therefore$ Sum of the ages of (6 men + 2 women)

$= 8(x + 2) = 8x + 16$ yrs

Hence, it is clear that on replacing two men by two women, sum of their ages increased by 16 years

Therefore, sum of the ages of two women

$= (20 + 24) + 16 = 60$ yrs

$\therefore$ Average age of the women $= \frac{60}{2} =$ **30 yrs.**

29. Ans. (b)

Solution Average of the remaining set of numbers

$$= \frac{50 \times 38 - (45+55)}{50-2}$$

$$= \frac{1900-100}{48} = 37.5.$$

30. Ans. (b)

Solution Time taken to cover first 100 kilometres

$$= \frac{100}{30} = 3\frac{1}{3}\text{hrs}$$

Time taken to cover second 100 kilometres

$$= \frac{100}{40} = 2\frac{1}{2}\text{hrs}$$

Time take to cover last 100 kilometres

$$= \frac{100}{5} = 2 \text{ hrs}$$

Total time taken $= 3\frac{1}{3} + 2\frac{1}{2} + 2 = \frac{10}{3} + \frac{5}{2} + 2$

$$= \frac{47}{6}\text{hrs}$$

Total distance covered = 300 km

$\therefore$ Average speed $= \frac{300}{47/6} = \frac{300 \times 6}{47} =$ **38.3 km/hr.**

31. Ans. (c)

Solution Seventh observation

$= 7 \times 11 - 6 \times 12 =$ **5.**

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32. Ans. (b)

Solution Average age of five new boys
$= (25 \times 15.56 - 20 \times 15.6) \div 5$
$= \mathbf{15.4.}$

33. Ans. (c)

Solution Weight of $D = (80 \times 4 - 84 \times 3)$ kg $= 68$ kg
Weight of $E = (68 + 3)$ kg $= 71$ kg
$(B + C + D + E)$'s weight $= (79 \times 4)$ kg $= 316$ kg
$\therefore$ $(B + C)$'s weight $= [316 - (68 + 71)]$ kg
$= 177$ kg
Hence, A's weight $= [(84 \times 3) - 177]$ kg $= \mathbf{75\ kg.}$

34. Ans. (c)

Solution Let x rupees be the average of 30 boarders
$\therefore$ $30x + 40 = (x - 2) \times 40$ or, $x = 12$
Hence, original expenditure
$=$ Rs $12 \times 30 =$ **Rs 360.**

35. Ans. (d)

Solution Let the numbers be $2x$, x and $\frac{x}{3}$

Then, average $= \dfrac{2x + x + \frac{x}{3}}{3} = 10 \Rightarrow \dfrac{9x + x}{3 \times 3} = 10$

or, $\dfrac{10x}{9} = 10$ or, $x = 9$

Hence, the numbers are **18, 9** and **3.**

36. Ans. (a)

Solution Correct average

$$= \frac{50 \times 36 - 73 + 37}{36}$$

$$= \frac{1764}{36} = \mathbf{49\ kg.}$$

37. Ans. (c)

Solution Total earning for the week
$=$ Rs $(4 \times 18 + 4 \times 22 - 20) =$ Rs 140

$\therefore$ Average earning $=$ Rs $\dfrac{140}{7} =$ **Rs 20.**

38. Ans. (a)

Solution Let the number of candidates who passed $= x$.
Then, $39 \times x + 15 \times (120 - x) = 120 \times 35$
$\therefore$ $24x = 4200 - 1800$

or, $x = \dfrac{2400}{24}$

$\mathbf{x = 100.}$

39. Ans. (a)

Solution Total decrease $= (20 \times 2)$ months
$= 3$ years 4 months
$\therefore$ Age of the new boy $= 18$ years $- 3$ years 4 months
$=$ **14 years 8 months.**

40. Ans. (c)

Solution Temperature on Monday + Tuesday + Wednesday + Thursday $= 4 \times 48° = 192°$
Temperature on Monday $= 42°$
$\therefore$ Tuesday + Wednesday + Thursday
$= (192° - 42°) = 150°$
Temperature on Tuesday + Wednesday + Thursday + Friday $= 4 \times 52° = 208°$
$\therefore$ Friday's temperature $= 208° - 150° =$ **58°.**

41. Ans. (c)

Solution Total spending in 12 months
$=$ Rs $[269.47 \times 7 + 281.05 \times 5]$
$=$ Rs 3291.54
Total income = spending + saving
$=$ Rs 3291.54 + Rs 308.46
$=$ Rs 3600.00

$\therefore$ Monthly salary $=$ Rs $\dfrac{3600}{12} =$ **Rs 300.**

42. Ans. (d)

Solution Let the numbers be x and y, $x < y$

Then, $x + y = 124$; $\dfrac{x+2}{y} = \dfrac{1}{2} \Rightarrow y = 2x + 4.$

Solving above equations, we get
$\mathbf{y = 84,\ x = 40.}$

43. Ans. (a)

Solution $\dfrac{(x+2) \times 60 + x \times 120 + (x-2) \times 180}{x + 2 + x + x - 2} = 100$

$\therefore$ $\mathbf{x = 4.}$

44. Ans. (d)

Solution Let temperature on 4th day be x°C
$\therefore$ $4 \times 38.6 + 4 \times 40.3 - x = 7 \times 39.1$
or, $x = 41.9$
$\therefore$ Temperature on 4th day $=$ **41.9°C.**

45. Ans. (c)

Solution Let daily wages of $C = x$
Then, daily wages of $A = 2x$
and, daily wages of $B = x + 40$
Hence, average daily wages of A, B and C

$$= \frac{x + 2x + x + 40}{3} = \frac{4x + 40}{3}$$

$\therefore$ $\dfrac{4x + 40}{3} = 120$ or, $4x + 40 = 360$
$\Rightarrow$ $4x = 320$ or, $x = 80$
$\therefore$ Wages of A per day $= 2 \times 80 =$ **Rs 160.**

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46. Ans. (c)

Solution $\frac{x}{35}-\frac{x}{40}=\frac{15}{60}$ or, $\frac{5x}{35\times40}=\frac{1}{4}$

or, $x=\frac{35\times40}{5\times4}=70$

$\therefore$ Total journey = **70 km.**

47. Ans. (c)

Solution Let the number of candidates be x. Then, total marks obtained by all the candidates = $45x$

Marks reduced for 90 candidates

$= 30 \times 90 = 2700$

Total reduced marks = $45x - 2700$

Reduced average = $\frac{45x-2700}{x}$

$\therefore \quad 40 = \frac{45x-2700}{x}$ or, $40x = 45x - 2700$

$\Rightarrow \quad 5x = 2700$ **or,** x **= 540.**

48. Ans. (b)

Solution Let attendance on first, second and third day be 2, 5 and 13, respectively.

Total attendance of three days = 2 + 5 + 13 = 20

Total money collected

$= 2 \times 15 + 5 \times 7.50 + 13 \times 2.50$

$= 30 + 37.5 + 32.5 = 100$

Average charge per person = $\frac{100}{20}$ **= 5.**

49. Ans. (d)

Solution Total profit for 30 days = 30 × 350

= Rs 10500

Profit for first 15 days = 15 × 275 = Rs 4125

$\therefore$ Profit for last 15 days = Rs 10500 – 4125

= Rs 6375

Average profit for the last 15 days

= Rs $\frac{6375}{15}$ **= Rs 425.**

50. Ans. (a)

Solution Let the number of wickets taken before the last match = x

Then, $\frac{12.4x+26}{x+5}=12 \Rightarrow x$ **= 85.**

51. Ans. (b)

Solution Let the numbers be $2x$, x and $4x$

Average = $\frac{2x+x+4x}{3} \Rightarrow \frac{7x}{3}=56$

$\therefore \quad x=\frac{3\times56}{7}=24$

Hence, the numbers in order are **48, 24** and **96.**

52. Ans. (b)

Solution Suppose the average expenditure was Rs x

Then, total expenditure = $35x$

When 7 more students join the mess, total expenditure

$= 35x + 42$

Now, the average expenditure

$= \frac{35x+42}{35+7} = \frac{35x+42}{42}$

Now, we have $\frac{35x+42}{42} = x - 1$

$\therefore \quad x = 12$

Thus, the original expenditure of the mass

$= 35 \times 12 =$ **Rs 420.**

53. Ans. (a)

Solution Sum total of 50 numbers = 50 × 38 = 1900

Sum total of the remaining 48 numbers

$= 1900 - (45 + 55) = 1800$

$\therefore$ Average = $\frac{1800}{48}$ **= 37.5.**

54. Ans. (d)

Solution Average weight of 11 boys is increased by 100 gm (= 0.1 kg.)

$\therefore$ Their total increase in weight

$= 0.1 \times 11 = 1.1$ kg

Hence, the weight of new boy = 42 + 1.1 = **43.1kg**

55. Ans. (c)

Solution Let the numbers be $n - 1$, n and $n + 1$. Their average = n

Next two consecutive numbers are $n + 2$ and $n + 3$

Therefore, the average of five numbers

$= \frac{(n-1)+n+(n+1)+(n+2)+(n+3)}{5}$

$= \frac{5n+5}{5} = n + 1.$

56. Ans. (d)

Solution Total monthly salary of 21 persons

= Rs (21 × 2000) = Rs 42000

Total monthly salary of 20 persons

= Rs (20 × 1900) = Rs 38000

Monthly salary of the manager = Rs 4000

Annual salary of the manager = **Rs 48000.**

57. Ans. (d)

Solution Let the number of boys be x and the number of girls be y.

Sum of ages of boys = $16.4x$

Sum of ages of girls = $15.4y$

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Average age of all the students

$$= \frac{16.4x + 15.4y}{x + y} = 15.8$$

$\Rightarrow$ $16.4x + 15.4y = 15.8x + 15.8y$

or, $16.4x - 15.8x = 15.8y - 15.4y$

or, $0.6x = 0.4y$

or, $\frac{x}{y} = \frac{0.4}{0.6} = \frac{2}{3}$ **or $x : y = 2 : 3$.**

58. Ans. (a)

Solution Total expenditure for the first five months

$= 5 \times 3600 =$ Rs 18000

Total expenditure for the next seven months

$= 7 \times 3900 =$ Rs 27300

Savings = Rs 8700

Total income during the year

$= 18000 + 27300 + 8700 =$ Rs 54000

Average income per month $= \frac{54000}{12} =$ **Rs 4500.**

59. Ans. (c)

Solution Let the numbers be x, $2x$, $\frac{2}{3}x$

$$\text{Average} = \frac{x + 2x + \frac{2}{3}x}{3} \Rightarrow \frac{11x}{9} = 44$$

$$\therefore \quad x = \frac{44 \times 9}{11} = 36$$

So, the numbers are 36, 72 and 24

Hence, the largest one is **72.**

60. Ans. (b)

Solution New average of the committee (in years)

$$= \frac{8 \times 40 - 55 + 39}{8} = \frac{320 - 16}{8} = \frac{304}{8}$$

= 38 years.

QUESTIONS FROM COMPETITIVE EXAMS

1. The average of 11 numbers is 10.9. If the average of the first six numbers is 10.5 and that of the last six numbers is 11.4, then the middle number is

(a) 11.5 (b) 11.4
(c) 11.3 (d) 11.0

[Based on MAT 2003]

2. In a class, the average score of girls in an examination is 73 and that of boys is 71. The average score for the whole class is 71.8. The percentage of the girls in the class is

(a) 60% (b) 40%
(c) 1.8% (d) 18%

[Based on MAT 2003]

3. The average (arithmetic mean) of x and y is 40. If $z = 10$; then what is the average of x, y and z?

(a) $16\frac{2}{3}$ (b) 30
(c) 25 (d) $7\frac{1}{2}$

[Based on MAT 2002]

4. The average of 11 numbers is 10.9. If the average of the first six numbers is 10.5 and that of the last six numbers is 11.4, then the middle number is

(a) 11.5 (b) 11.4
(c) 11.3 (d) 11.0

[Based on MAT 2001]

5. A salesman averages Rs. 240 during a normal 40-hour week. During a sale, his rates are increased by 50%. What is his commission if he puts in 60 hours during the sale?

(a) 390 (b) 540
(c) 600 (d) 640

[Based on MAT 2000]

6. A car owner buys petrol at Rs. 7.50, Rs. 8.00 and Rs. 8.50 per litre for three successive years. What approximately is the average cost per litre of petrol if he spends Rs. 4000 each year.

(a) Rs. 8 (b) Rs. 9
(c) Rs. 7.98 (d) Rs. 8.50

[Based on MAT 2001]

7. In a one-day cricket match, Agarkar, Sehwag, Sachin, Dravid and Ganguly scored an average of 39 runs. Dravid scored 7 more than Ganguly. Ganguly scored 9 fewer than Agarkar. Sehwag scored as many as Dravid and Ganguly combined; and Sehwag and Sachin scored 110 runs between them. How many runs did Sachin score?

(a) 47 (b) 51
(c) 53 (d) None of the above

[Based on FMS (Delhi) 2004]

8. There is a sequence of 11 consecutive odd numbers. If the average of first 7 numbers is X, then find the average of all the 11 integers.

(a) $X + 3$ (b) $X + 4$
(c) $X + 5$ (d) $X + 7$

[Based on FMS (Delhi) 2004]

9. There are 197 boys and 591 girls in a college. If the average weight of boys is 63 kg and average weight of girls is 51 kg, then what is the average weight of the students in the college?

(a) 56 kg (b) 57 kg
(c) 54 kg (d) 60 kg

[Based on IIT Joint Man. Ent. Test 2004]

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10. A student on his birthday distributed on an average 5 chocolates per student. If on the arrival of the teacher and the headmaster to whom the student gives 10 and 15 chocolates respectively, the average chocolate distributed per head increases to 5.5, then what is the strength of the class?
(a) 28 (b) 30
(c) 32 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

11. The average temperature of the town in the first four days of a month was 58 degrees. The average for the second, third, fourth and fifth days was 60 degrees. If the temperatures of the first and fifth days were in the ratio 7 : 8, then what is the temperature on the fifth day?
(a) 240 degrees (b) 232 degrees
(c) 64 degrees (d) None of these
[Based on Narsee Manjee Inst. of Man. Studies, 2003]

12. If the mean of a, b, c is M and $ab + bc + ca = 0$, then the mean of a^2, b^2, c^2 is
(a) M^2 (b) $3M^2$
(c) $6M^2$ (d) $9M^2$
[Based on IITTM, Gwalior, 2003]

13. What is the average of n, $n + 1$, $n + 2$, $n + 3$, $n + 4$, $n + 5$?
(a) $n + 2$ (b) $n + 2\frac{1}{2}$
(c) $6n + 15$ (d) $n + 3\frac{1}{2}$
[Based on REC Tiruchirapalli 2003]

14. An aeroplane flies along the four sides of a square field at the speeds of 200, 400, 600 and 800 km/hr. The average speed of the plane around the field in km/hr is
(a) 384 (b) 400
(c) 500 (4) 284
[Based on FMS (Delhi) 2003]

15. The average of marks obtained by 120 candidates was 35. If the average of the passed candidates was 39 and that of the failed candidates was 15, then the number of those candidates, who passed the examination, was
(a) 100 (b) 110
(c) 120 (d) 150
[Based on FMS (Delhi) 2003]

16. Three years ago, the average age of A and B was 18 years. With C joining them, the average age becomes 22 years. How old is C now?
(a) 24 years (b) 27 years
(c) 28 years (d) 30 years
[Based on FMS (Delhi) 2003]

17. The average of three numbers is 135. The largest number is 180 and the difference of the other numbers is 25. The smallest number is
(a) 130 (b) 125
(c) 120 (d) 100
(e) None of the above **[Based on IIFT 2003]**

18. The ratio of the arithmetic mean of two numbers to one of the numbers is 3 : 5. What is the ratio of the smaller number to the larger one?
(a) 1 : 5 (b) 1 : 4
(c) 1 : 3 (d) 1 : 2
(e) None of the above **[Based on IIFT 2003]**

19. A person travels from X to Y at a speed of 40 kmph and returns by increasing his speed by 50%. What is his average speed for both the trips?
(a) 36 kmph (b) 45 kmph
(c) 48 kmph (d) 50 kmph
(e) None of the above **[Based on IIFT 2003]**

20. A painter is paid x rupees for painting every 10 metres of a wall and y rupees for painting every extra metre. During one week, he painted 10 metres on Monday, 13 metres on Tuesday, 12 metres on Wednesday, 11 metres on Thursday and 12 metres on Friday. What is his average daily earning in rupees for the five day week?
(a) $x + (8/5)\,y$ (b) $(5x + 9y)/5$
(c) $10x + (8/5)y$ (d) $5x + 8y$
[Based on SCMHRD Ent. Exam. 2003]

21. The average of 5 consecutive numbers is n. If the next two numbers are also included,. the average will
(a) Increase by 1 (b) Remain the same
(c) Increase by 1.4 (d) Increase by 2
[Based on MAT 2005]

22. For ten hours, a train travels at a constant speed of 20 miles per hour and during the next 15 hours, it travels 240 miles. What is the average speed of the train for the whole journey?
(a) 17.6 miles/hour (b) 20.8 miles/hour
(c) 130 miles/hour (d) 176 miles/hour
[Based on IMT Ghaziabad, 2002]

23. Average of ten positive numbers is $\overline{X}$. If each number increases by 10%, then $\overline{X}$
(a) Remains unchanged
(b) Is increased by 10%
(c) May decrease
(d) May either increase or decrease
[Based on IMT Ghaziabad, 2002]

24. On an 800 miles trip, car W travelled half the distance at 80 miles per hour and the other half at 100 miles per hour. What was the average speed of the car?
(a) 18.00 (b) 180.00
(c) 90.00 (d) $88\frac{8}{9}$
[Based on Narsee Monjee Inst. of Man. Studies 2002]

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(a) 28 (b) 26
(c) 24 (d) 22

[Based on I.P. Univ. 2002]

26. Angad was conducting an experiment in which the average of 11 observations came to be 90, while the average of first five observations was 87, and that of the last five was 84. What was the measure of the 6th observation?

(a) 165 (b) 150
(c) 145 (d) 135

[Based on I.P. Univ. 2002]

27. A student who scored 30% marks in the first paper of Physics out of 180 marks, has to get an overall score of at least 50% in two papers, the second paper carrying 150 marks. What percentage of marks should he score in the second paper to get the overall average score?

(a) 80% (b) 76%
(c) 74% (d) 70%

[Based on I.P. Univ. 2002]

28. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many papers were there in the examination?

(a) 8 (b) 9
(c) 10 (d) 11
(e) 12

[Based on SCMHRD 2002]

29. A batsman has a certain average of runs for 12 innings. In the 13th innings he scores 96 runs, there by increasing his average by 5 runs. What is his average after the 13th innings?

(a) 64 (b) 48
(c) 36 (d) 72

[Based on MAT 2008]

30. The average of nine number is M and the average of three of these is P. If the average of remaining number is N, then

(a) M = N + P (b) 2M = N + P
(c) 3M = 2N + P (d) 3M = 2P + N

[Based on SNAP 2007]

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $6 \times 10.5 + 6 \times 11.4 - 11 \times 10.9$
$= 63 + 68.4 - 119.9$
$= 131.4 - 119.9 = \mathbf{11.5}$

2. Ans. (b)

Solution Total score for the whole class $= (B + G) \times 71.8$

$\therefore \quad (B + G) \times 71.8 = 73G + 71B \Rightarrow 1.2G = 0.8B$

$\Rightarrow 3G = 2B$

$\Rightarrow \frac{B}{G} = \frac{3}{2}.$

3. Ans. (b)

4. Ans. (c)

Solution $6 \times 10.5 + 6 \times 11.4 - 11 \times 10.9$
$= 63 + 68.4 - 119.9$
$= 131.4 - 119.9 = \mathbf{11.3.}$

5. Ans. (b)

Solution Rate is increased from Rs. 6 per hour to Rs. 9 per hour.

6. Ans. (c)

Solution Let average cost of petrol per litre be Rs. x.

$$\therefore \quad x = \frac{12000}{\frac{4000}{7.50} + \frac{4000}{8} + \frac{4000}{8.50}} = \mathbf{7.98.}$$

7. Ans. (d)

Solution (Agarkar + Sehwag + Sachin + Dravid + Ganguly) make $= 39 \times 5 = \mathbf{195}$ **runs.**

With respect to scoring runs

Dravid = Ganguly + 7
Ganguly = Agarkar – 9
Sehwag = Dravid + Ganguly
Sehwag + Sachin = 110

$\Rightarrow$ Agarkar, Dravid Ganguly, Sehwag and Sachin scored 32, 30, 23, 53 and 57 runs respectively.

8. Ans. (b)

Solution Average of first 7 numbers will be the 4th number $= X$ (Given)

Average of all the 11 numbers will be the 6th number, i.e. $X + 4$.

9. Ans. (c)

Solution $\frac{197 \times 63 + 591 \times 51}{197 + 591} = \frac{197 \times 63 + 591 \times 51}{788}$

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$$= \frac{63}{4} + \frac{3}{4} \times 51 = \frac{63+153}{4}$$

$$= \frac{216}{4} = \mathbf{54\ kg.}$$

10. Ans. (a)

Solution Suppose strength of the class = x

$\therefore \quad 5x + 10 + 15 = 5.5\,(x + 2)$

$\Rightarrow \quad 0.5x = 14 \Rightarrow \mathbf{x = 28.}$

11. Ans. (c)

Solution Suppose Temperature on 1st day = $7K$

Suppose Temperature on 5th day = $8K$

$\therefore$ Temperature on $M + T + W + Th = 232$

Average Temperatue on $T + W + Th + F = 240$

$\therefore \quad 232 - 7K = 240 - 8K \Rightarrow K = 8$

$\therefore$ Temperature on 5th day = **64°.**

12. Ans. (b)

Solution $a + b + c = 3M$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2\,(ab + bc + ca)$$

$$= a^2 + b^2 + c^2$$

$$\Rightarrow \quad a^2 + b^2 + c^2 = (3M)^2 = 9M^2$$

$$\Rightarrow \quad \text{Mean of } a^2, b^2 \text{ and } c^2 = \frac{9M^2}{3} = \mathbf{3M^2.}$$

13. Ans. (b)

Solution $\dfrac{6n+15}{6} = \dfrac{2n+5}{2} = n + \dfrac{5}{2} = n + 2\dfrac{1}{2}.$

14. Ans. (a)

Solution Let each side of the square field be x km

$\therefore$ Average speed of the plane

$$= \frac{4x}{\frac{x}{800} + \frac{x}{600} + \frac{x}{400} + \frac{x}{200}}$$

$$= \frac{4 \times 2000}{3+4+6+12} = \frac{9600}{25}$$

$$= \mathbf{384\ km/hr.}$$

15. Ans. (a)

Solution Suppose the number of candidates passed = x

$\therefore \quad 39x + 15\,(120 - x) = 120 \times 15$

$\Rightarrow \quad 24x = 120 \times 35 - 120 \times 35$

$$= 120\,(35 - 15) = 120 \times 20$$

$\Rightarrow \quad \mathbf{x = 100.}$

16. Ans. (a)

Solution $(A - 3) + (B - 3) = 36 \Rightarrow A + B = 42$

Also $A + B + C = 66 \Rightarrow \mathbf{C = 24.}$

17. Ans. (d)

Solution Let the three numbers be X, Y and Z.

$$\therefore \quad \frac{X+Y+Z}{3} = 135 \Rightarrow X + Y + Z = 405$$

Let X be the largest number

$\therefore \quad X = 180 \Rightarrow Y + Z = 225$

$Y - Z = 25$

$\therefore \quad Y = 125,\ Z = \mathbf{100}$ **(smallest number).**

18. Ans. (a)

Solution Let X and Y be the two numbers

$$\therefore \quad \frac{\frac{X+Y}{2}}{X} = \frac{3}{5} \Rightarrow \frac{X+Y}{2X} = \frac{3}{5}$$

$$\Rightarrow 5X + 5Y = 6X \Rightarrow X = 5Y$$

$$\Rightarrow \frac{X}{Y} = \frac{1}{5}.$$

19. Ans. (c)

Solution Let the distance between X and Y be x km,

$\therefore$ Time taken from X to Y @ 40 km/hr = $\dfrac{x}{40}$ hours

Time taken from Y to X @ 60 km/hr = $\dfrac{x}{60}$ hours

$\therefore$ Average speed = $\dfrac{2x}{\frac{x}{40} + \frac{x}{60}}$ = **48 km/hr.**

20. Ans. (a)

Solution On Monday, the payment = Rs x

On Tuesday, the payment = Rs $(x + 3y)$

On Wednesday, the payment = Rs $(x + 2y)$

On Thursday, the payment = Rs $(x + y)$

On Friday, the payment = Rs $(x + 2y)$

$\therefore$ Average daily earnings during the five-day week

$$= \frac{5x+8y}{5} = x + \frac{8y}{5}.$$

21. Ans. (a)

Solution Let the consecutive numbers be $x, x+1, x+2, x+3, x+4$

$$\text{Average} = \frac{5x+10}{5} = x + 2$$

$$\text{Average of 7 number} = \frac{5x+10+x+5+x+6}{7}$$

$$= \frac{7x+21}{7} = x + 3$$

$\therefore$ The average increased by **1.**

22. Ans. (a)

Solution $\dfrac{20 \times 10 + 240}{25} = \dfrac{200+240}{25} = \dfrac{440}{25}$

$$= \mathbf{17.6\ miles/hour.}$$

23. Ans. (b)

24. Ans. (d)

Solution The distance of 800 miles was covered in $5 + 4 = 9$ hours.

$\therefore$ Average speed of $W = \dfrac{800}{9} = 88\dfrac{8}{9}$ miles per hour.

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25. Ans. (b)

Solution Let the average score of the 1st 18 innings be x

$\therefore \quad 18x + 98 = 19(x + 4)$

$\Rightarrow \quad x = 22$

$\therefore$ Average score after 19th innings $= x + 4 =$ **26.**

26. Ans. (d)

Solution $11 \times 90 - 5 \times 87 - 5 \times 84$

$= 990 - 435 - 420 =$ **135.**

27. Ans. (c)

Solution Out of 330 marks, the student is required to obtain 165 marks, i.e. 50%.

$\therefore$ In first paper, because of scoring 54 marks only, he has to score 111 marks out of 150 in the 2nd paper, i.e., 74%.

28. Ans. (d)

29. Ans. (c)

Solution Suppose his average after 12 inning $= x$

Then $\frac{12+96}{13} = x+5$

$x = 31$

$\therefore$ Required average $= x + 5 = 31 + 5 =$ **36.**

30. Ans. (c)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?
(a) 200 (b) 100
(c) 300 (d) 400

2. The average monthly salary of 8 workers and one supervisor in a factory was Rs 430. When the supervisor, whose salary was Rs 870 per month, retired, a new person was appointed and then the average salary of 9 persons was Rs 400 per month. The salary of the new supervisor is
(a) Rs 700 (b) Rs 600
(c) Rs 430 (d) Rs 400

3. The average of 9 results is 50. The average of first four results is 52 and average of last 4 results is 49. What is the fifth result?
(a) 48 (b) 46
(c) 52 (d) 44

4. The average height of 30 girls out of a class of 40 is 160 cm and that of the remaining girls is 156 cm. What is the average height of the whole class?
(a) 161 cm (b) 163 cm
(c) 169 cm (d) 159 cm

5. The average salary of a staff of 18 officers and 32 clerks is Rs 800. If the average salary of the officers is Rs 1200, find the average salary of the clerks
(a) Rs 565 (b) Rs 550
(c) Rs 575 (d) Rs 600

6. The average temperature of Monday, Tuesday, Wednesday and Thursday was 38°C and that of Tuesday, Wednesday, Thursday and Friday was 40°C. If the temperature on Monday was 30°C, what was the temperature on Friday?
(a) 39°C (b) 36°C
(c) 38°C (d) 42°C

7. The average age of an adult class is 40 years. 12 new students with an average age of 32 years join the class, thereby decreasing the average by 4 years. The original strength of the class was
(a) 10 (b) 11
(c) 12 (d) 15

8. The average income of a group of 50 persons is Rs 1690. It was later discovered that one person's income was wrongly taken as Rs 1340 instead of Rs 1430. The correct average income is
(a) Rs 1680 (b) Rs 1698
(c) Rs 1691.80 (d) None of these

9. The average of the first and the second of three numbers is 10 more than the average of the second and the third of these numbers. What is the difference between the first and the third of these three numbers?
(a) 30 (b) 20
(c) 40 (d) 50

10. The average age of four children in a family is 12 yrs. If the spacing between their ages is 4 yrs, the age of the youngest child is
(a) 6 yrs (b) 7 yrs
(c) 8 yrs (d) 9 yrs

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Test II **[Time: 10 min]**

11. An investor earns 3% return on $\frac{1}{4}$th of his capital, 5% on $\frac{2}{3}$rd and 11% on the remainder. What is the average rate of return he earns on his total capital?
(a) 5% (b) 10%
(c) 5.5% (d) 10.5%

12. The average of three consecutive numbers is n. If the next two consecutive numbers are also included, the average of the five numbers will
(a) remain the same (b) increase by 0.5
(c) increase by 1 (d) increase by 1.5

13. The average age of 8 persons in a committee is increased by 2 yrs when two men aged 35 yrs and 45 yrs are substituted by two women. Find the average of these two women.
(a) 48 yrs (b) 56 yrs
(c) 52 yrs (d) 44 yrs

14. The average salary of the entire staff in an office is Rs 120 per month. The average salary of officers is Rs 460 and that of non-officers is Rs 110. If the number of officers is 15, then find the number of non-officers in the office.
(a) 410 (b) 610
(c) 510 (d) 710

15. The average temperature of the first three days of a week is 27°C and that of next three days is 29°C. If the average temperature of the whole week is 28.5°C, find the temperature of the last day.
(a) 32.5°C (b) 31.5°C
(c) 33.5°C (d) 34.5°C

16. A candidate scores an aggregate of 60% marks, scoring an average of 56% in 4 of the papers and 68% in the others. How many papers were there totally?
(a) 6 (b) 7
(c) 10 (d) None of these

17. The average of five consecutive even numbers is 38. The largest of these is
(a) 40 (b) 38
(c) 44 (d) 42

18. The average score of 35 students in a class is 37. If every student is given 3 grace marks, the new average of the class is
(a) 40 (b) 34
(c) 43 (d) None of these

19. The average of 5 consecutive numbers is 43. The average of the three smallest of these numbers is
(a) 43 (b) 38
(c) 44 (d) 42

20. The average weight of 29 students is 40 kg. If the weight of their teacher be included, the average weight is increased by 300 gm. The weight of the teacher is
(a) 49 kg (b) 48.7 kg
(c) 49.3 kg (d) None of these

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution Let the number of passed candidates be x
Then, total marks = $120 \times 35 = 39x + (120 - x) \times 15$
or, $4200 = 39x + 1800 - 15x$
or, $24x = 2400$
$\therefore x = 100$
$\therefore$ Number of passed candidates = **100.**

3. Ans. (b)

Solution Average of 9 results = 50
$\therefore$ Total of 9 results = $50 \times 9 = 450$
Average of 1st four results = 52
$\therefore$ Total of 1st four results = $52 \times 4 = 208$
Average of last four results = 49
$\therefore$ Total of last four results = $49 \times 4 = 196$
Hence, total of 8 results = 208 + 196 = 404
Therefore, the fifth result is 450 – 404 = **46.**

5. Ans. (c)

Solution Average salary of 18 officers = Rs 1200
$\therefore$ Total salary of 18 officers
= Rs 1200×18
= Rs 21600 ...(i)
Average salary of 32 clerks and 18 officers
= Rs 800
$\therefore$ Total salary of 32 clerks + 18 officers
= Rs 800×50
= Rs 40000
$\therefore$ Total salary of 32 clerks
= Rs 40000 – 21600 [Using (i)]
= 18400
$\therefore$ Average salary of clerk = Rs $\frac{18400}{32}$ = **Rs 575.**

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7. Ans. (c)

Solution Let original strength = x. Then,

$40x + 12 \times 32 = (x + 12) \times 36$

or, $40x + 384 = 36x + 432$

or, $4x = 48$ **or,** $x = 12$.

9. Ans. (b)

Solution Average of the first and second numbers

$$= \frac{\text{First} + \text{second}}{2}$$

Average of the second and third numbers

$$= \frac{\text{Second} + \text{third}}{2}$$

According to the question

$$\frac{\text{First} + \text{second}}{2} - \frac{\text{second} + \text{third}}{2} = 10$$

$\therefore$ First − third = **20.**

11. Ans. (a)

Solution Remainder capital $= 1 - \left(\frac{1}{4} + \frac{2}{3}\right)$

$$= 1 - \frac{3+8}{12}$$

$$= 1 - \frac{11}{12} = \frac{1}{12}$$

Total return $= 3 \times \frac{1}{4} + 5 \times \frac{2}{3} + 11 \times \frac{1}{12}$

$$= \frac{9+40+11}{12} = \frac{60}{12} = 5$$

$\therefore$ Average percent return = **5%.**

13. Ans. (a)

Solution Total increase in ages = 2 × 8 = 16 yrs

Sum of ages of persons to be replaced

= 35 + 45 = 80 yrs

Sum of ages of two women

= 80 + 16 = 96 yrs

$\therefore$ Average age of women $= \left(\frac{96}{2}\right)$ yrs = **48 yrs.**

15. Ans. (b)

Solution Sum of the temperatures of first three days = $3 \times 27 = 81°$ C

Sum of the temperatures of the next three days

$= 3 \times 29 = 87°$ C

Sum of the temperatures of the whole week, that is 7 days

$= 7 \times 28.5 = 199.5°$ C

So, the temperature of the last day

$= 199.5 - (81 + 87) =$ **31.5° C.**

17. Ans. (d)

Solution Let the five consecutive even numbers be,

$n - 4, n - 2, n, n + 2, n + 4$

Then, average $= \frac{n-4+n-2+n+n+2+n+4}{5} = n$

$\therefore$ $38 = n$.

Hence, largest number is $n + 4$ **or 42.**

19. Ans. (d)

Solution Let the numbers be $n - 2$, $n - 1$, n, $n + 1$ and $n + 2$

Average $= \frac{(n-2)+(n-1)+n+(n+1)+(n+2)}{5}$

$= 43$ or, $n = 43$

Therefore, the numbres are 41, 42, 43, 44, and 45.

Average of the smallest three numbers

$$= \frac{41+42+43}{3} = \mathbf{42.}$$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (d)	**6.** (c)	**8.** (c)	**10.** (a)	**12.** (c)	**14.** (c)	**16.** (a)	**18.** (a)	**20.** (a)

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7 Ratio and Proportion

RATIO

A *ratio* is a comparison of two quantities by division. It is a relation that one quantity bears to another with respect to magnitude. In other words, ratio means what part one quantity is of another. The quantities may be of same kind or different kinds. For example, when we consider the ratio of the weight 45 kg of a bag of rice to the weight 29 kg of a bag of sugar we are considering the quantities of same kind but when we talk of allotting 2 cricket bats to 5 sportsmen, we are considering quantities of different kinds. Normally, we consider the ratio between quantities of the same kind.

If a and b are two numbers, the ratio of a to b is $\frac{a}{b}$ or $a \div b$ and is denoted by $a : b$. The two quantities that are being compared are called *terms*. The first is called *antecedent* and the second term is called *consequent*.

For example, the ratio 3 : 5 represents $\frac{3}{5}$ with antecedent 3 and consequent 5.

Note:

1. A ratio is a number, so to find the ratio of two quantities, they must be expressed in the same units.
2. A ratio does not change if both of its terms are multiplied or divided by the same number. Thus,

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} \text{ etc.}$$

TYPES OF RATIOS

1. **Duplicate Ratio** The ratio of the squares of two numbers is called the *duplicate ratio* of the two numbers.

   For example, $\frac{3^2}{4^2}$ or $\frac{9}{16}$ is called the duplicate ratio of $\frac{3}{4}$.

2. **Triplicate Ratio** The ratio of the cubes of two numbers is called the *triplicate ratio* of the two numbers.

   For example, $\frac{3^3}{4^3}$ or $\frac{27}{64}$ is triplicate ratio of $\frac{3}{4}$.

3. **Sub-duplicate Ratio** The ratio of the square roots of two numbers is called the *sub-duplicate ratio* of two numbers.

   For example, $\frac{3}{4}$ is the sub-duplicate ratio of $\frac{9}{16}$.

4. **Sub-triplicate Ratio** The ratio of the cube roots of two numbers is called the *sub-triplicate ratio* of two numbers.

   For example, $\frac{2}{3}$ is the sub-triplicate ratio of $\frac{8}{27}$.

5. **Inverse Ratio or Reciprocal Ratio** If the antecedent and consequent of a ratio interchange their places, the new ratio is called the *inverse ratio* of the first.

   Thus, if $a : b$ be the given ratio, then $\frac{1}{a} : \frac{1}{b}$ or $b : a$ is its inverse ratio.

   For example, $\frac{3}{5}$ is the inverse ratio of $\frac{5}{3}$.

6. **Compound Ratio** The ratio of the product of the antecedents to that of the consequents of two or more given ratios is called the *compound ratio*. Thus, if $a : b$ and $c : d$ are two given ratios, then $ac : bd$ is the compound ratio of the given ratios.

   For example, if $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{7}$ be the given ratios, then their compound ratio is $\frac{3\times4\times5}{4\times5\times7}$, that is, $\frac{3}{7}$.

PROPORTION

The equality of two ratios is called *proportion*.

If $\frac{a}{b} = \frac{c}{d}$, then a, b, c and d are said to be in proportion and we write $a : b :: c : d$. This is read as "a is to b as c is to d".

For example, since $\frac{3}{4} = \frac{6}{8}$, we write 3 : 4 :: 6 : 8 and say 3, 4, 6 and 8 are in proportion.

Each term of the ratio $\frac{a}{b}$ and $\frac{c}{d}$ is called a *proportional*. a, b, c and d are respectively the first, second, third and fourth proportionals.

Here, a, d are known as *extremes* and b, c are known as *means*.

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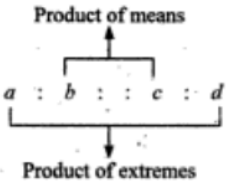
SOME BASIC FORMULAE

1. If four quantities are in proportion, then
Product of Means = Product of Extremes
For example, in the proportion $a : b : : c : d$, we have $bc = ad$.



From this relation we see that if any three of the four quantities are given, the fourth can be determined.

2. **Fourth proportional** If $a : b : : c : x$, x is called the *fourth proportional* of a, b, c.
We have, $\frac{a}{b} = \frac{c}{x}$ or, $x = \frac{b \times c}{a}$.
Thus, fourth proportional of a, b, c is $\frac{b \times c}{a}$.

Illustration 1 Find a fourth proportional to the numbers 2, 5, 4.

Solution Let x be the fourth proportional, then
$2 : 5 : : 4 : x$ or $\frac{2}{5} = \frac{4}{x}$.
$\therefore \quad x = \frac{5 \times 4}{2} = \mathbf{10.}$

3. **Third Proportional** If $a : b : : b : x$, x is called the third proportional of a, b.
We have, $\frac{a}{b} = \frac{b}{x}$ or $x = \frac{b^2}{a}$.
Thus, third proportional of a, b is $\frac{b^2}{a}$.

Illustration 2 Find a third proportional to the numbers 2.5, 1.5.

Solution Let x be the third proportional, then
$2{\cdot}5 : 1{\cdot}5 : : 1{\cdot}5 : x$ or $\frac{2{\cdot}5}{1{\cdot}5} = \frac{1{\cdot}5}{x}$.
$\therefore \quad x = \frac{1{\cdot}5 \times 1{\cdot}5}{2{\cdot}5} = \mathbf{0.9.}$

4. **Mean Proportional** If $a : x : : x : b$, x is called the *mean* or *second proportional* of a, b.
We have, $\frac{a}{x} = \frac{x}{b}$ or $x^2 = ab$ or $x = \sqrt{ab}$.
$\therefore$ Mean proportional of a and b is $\sqrt{ab}$.
We also say that a, x, b are in *continued proportion.*

Illustration 3 Find the mean proportional between 48 and 12.

Solution Let x be the mean proportional. Then,
$48 : x : : x : 12$ or, $\frac{48}{x} = \frac{x}{12}$ or, $x^2 = 576$ **or, $x = 24$.**

5. If $\frac{a}{b} = \frac{c}{d}$, then
(i) $\frac{a+b}{b} = \frac{c+d}{d}$ **(Componendo)**
(ii) $\frac{a-b}{b} = \frac{c-d}{d}$ **(Dividendo)**
(iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ **(Componendo and Dividendo)**
(iv) $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$.

Illustration 4 The sum of two numbers is c and their quotient is $\frac{p}{q}$. Find the numbers.

Solution Let the numbers be x, y.
Given : $x + y = c$ ...(1)
and, $\frac{x}{y} = \frac{p}{q}$ ...(2)
$\therefore \quad \frac{x}{x+y} = \frac{p}{p+q} \Rightarrow \frac{x}{c} = \frac{p}{p+q}$ [Using (1)]
$\Rightarrow \quad x = \frac{pc}{p+q}$.

SOME USEFUL SHORT-CUT METHODS

1. (a) If two numbers are in the ratio of $a : b$ and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b}$ and $\frac{bx}{a+b}$, respectively.
or
If in a mixture of x litres, two liquids A and B are in the ratio of $a : b$, then the quantities of liquids A and B in the mixture will be

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$\frac{ax}{a+b}$ litres and $\frac{bx}{a+b}$ litres, respectively.

(b) If three numbers are in the ratio of $a : b : c$ and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b+c}$, $\frac{bx}{a+b+c}$ and $\frac{cx}{a+b+c}$, respectively.

Explanation

Let the three numbers in the ratio $a : b : c$ be A, B and C. Then,

$$A = ka,\ B = kb,\ C = kc$$

and, $A + B + C = ka + kb + kc = x$

$\Rightarrow$ $k(a + b + c) = x \Rightarrow k = \frac{x}{a+b+c}$.

$\therefore$ $A = ka = \frac{ax}{a+b+c}$

$B = kb = \frac{bx}{a+b+c}$

$C = kc = \frac{cx}{a+b+c}$.

Illustration 5 Two numbers are in the rauo of 4 : 5 and the sum of these numbers is 27. Find the two numbers.

Solution Here, $a = 4$, $b = 5$ and $x = 27$.

$\therefore$ The first number $= \frac{ax}{a+b} = \frac{4\times27}{4+5} = 12$

and, the second number $= \frac{bx}{a+b} = \frac{5\times27}{4+5} = \mathbf{15}$.

Illustration 6 Three numbers are in the ratio of 3 : 4 : 8 and the sum of these numbers is 975. Find the three numbers.

Solution Here, $a = 3$, $b = 4$, $c = 8$ and $x = 975$.

$\therefore$ The first number $= \frac{ax}{a+b+c} = \frac{3\times975}{3+4+8} = 195$.

The second number $= \frac{bx}{a+b+c} = \frac{4\times975}{3+4+8} = 260$.

and, the third number $= \frac{cx}{a+b+c} = \frac{8\times975}{3+4+8} = \mathbf{520}$.

2. If two numbers are in the ratio of $a : b$ and difference between these numbers is x, then these numbers will be

(a) $\frac{ax}{a-b}$ and $\frac{bx}{a-b}$, respectively (where $a > b$)

(b) $\frac{ax}{b-a}$ and $\frac{bx}{b-a}$, respectively (where $a < b$).

Explanation

Let the two numbers be ak and bk.

Let $a > b$.

Given : $ak - bk = x$

$\Rightarrow$ $(a - b)k = x$ or $k = \frac{x}{a-b}$.

Therefore, the two numbers are $\frac{ax}{a-b}$ and $\frac{bx}{a-b}$.

Illustration 7 Two numbers are in the ratio of 4 : 5. If the difference between these numbers is 24, then find the numbers.

Solution Here, $a = 4$, $b = 5$ and $x = 24$.

$\therefore$ The first number $= \frac{ax}{b-a} = \frac{4\times24}{5-4} = 96$

and, the second number $= \frac{bx}{b-a} = \frac{5\times24}{5-4} = \mathbf{120}$.

3. (a) If $a : b = n_1 : d_1$ and $b : c = n_2 : d_2$, then $a : b : c = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$.

(b) If $a : b = n_1 : d_1$, $b : c = n_2 : d_2$ and $c : d = n_3 : d_3$, then $a : b : c : d = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$.

Illustration 8 If $A : B = 3 : 4$ and $B : C = 8 : 9$, find $A : B : C$.

Solution Here, $n_1 = 3$, $n_2 = 8$, $d_1 = 4$ and $d_2 = 9$.

$\therefore$ $a : b : c = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (3 \times 8) : (4 \times 8) : (4 \times 9)$

$= 24 : 32 : 36$ **or, 6 : 8 : 9.**

Illustration 9 If $A : B = 2 : 3$, $B : C = 4 : 5$ and $C : D = 6 : 7$, find $A : D$.

Solution Here, $n_1 = 2$, $n_2 = 4$, $n_3 = 6$, $d_1 = 3$, $d_2 = 5$ and $d_3 = 7$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 : n_3) : (d_1 \times d_2 \times d_3)$

$= (2 \times 4 \times 6) : (3 \times 4 \times 6) : (3 \times 5 \times 6) : (3 \times 5 \times 7)$

$= 48 : 72 : 90 : 105$ or, $16 : 24 : 30 : 35$.

Thus, $A : D = \mathbf{16 : 35}$.

4. (a) The ratio between two numbers is $a : b$. If x is added to each of these numbers, the ratio becomes $c : d$. The two numbers are given as: $\frac{ax(c-d)}{ad-bc}$ and $\frac{bx(c-d)}{ad-bc}$.

Explanation

Let two numbers be ak and bk.

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Given: $\frac{ak+x}{bk+x} = \frac{c}{d} \Rightarrow akd + dx = cbk + cx$

$\Rightarrow k(ad - bc) = x(c - d)$

$\Rightarrow k = \frac{x(c-d)}{ad-bc}$.

Therefore, the two numbers are $\frac{ax(c-d)}{ad-bc}$ and $\frac{bx(c-d)}{ad-bc}$.

(b) The ratio between two numbers is $a : b$. If x is subtracted from each of these numbers, the ratio becomes $c : d$.

The two numbers are given as:

$\frac{ax(d-c)}{ad-bc}$ and $\frac{bx(d-c)}{ad-bc}$.

Explanation

Let the two numbers be ak and bk.

Given : $\frac{ak-x}{bk-x} = \frac{c}{d} \Rightarrow akd - xd = bck - xc$

$\Rightarrow k(ad - bc) = x(d - c)$

$\Rightarrow k = \frac{x(d-c)}{ad-bc}$.

Therefore, the two numbers are $\frac{ax(d-c)}{ad-bc}$ and $\frac{bx(d-c)}{ad-bc}$.

Illustration 10 Given two numbers which are in the ratio of 3 : 4. If 8 is added to each of them, their ratio is changed to 5 : 6. Find the two numbers.

Solution We have,

$a : b = 3 : 4$, $c : d = 5 : 6$ and $x = 8$.

$\therefore$ The first number $= \frac{ax(c-d)}{ad-bc}$

$= \frac{3\times8\times(5-6)}{(3\times6-4\times5)} = 12$

and, the second number $= \frac{bx(c-d)}{ad-bc}$

$= \frac{4\times8\times(5-6)}{(3\times6-4\times5)} = 16.$

Illustration 11 The ratio of two numbers is 5 : 9. If each number is decreased by 5, the ratio becomes 5 : 11. Find the numbers.

Solution We have,

$a : b = 5 : 9$, $c : d = 5 : 11$ and $x = 5$.

$\therefore$ The first number $= \frac{ax(d-c)}{ad-bc}$

$= \frac{5\times5\times(11-5)}{(5\times11-9\times5)} = 15$

and, the second number $= \frac{bx(d-c)}{ad-bc}$

$= \frac{9\times5\times(11-5)}{(5\times11-9\times5)} = 27.$

5. (a) If the ratio of two numbers is $a : b$, then the numbers that should be added to each of the numbers in order to make this ratio $c : d$ is given by

$\frac{ad-bc}{c-d}$.

Explanation

Let the required number be x.

Given : $\frac{a+x}{b+x} = \frac{c}{d} \Rightarrow ad + xd = bc + xc$

$\Rightarrow x(d - c) = bc - ad$

or, $x = \frac{ad-bc}{c-d}$.

(b) If the ratio of two numbers is $a : b$, then the number that should be subtracted from each of the numbers in order to make this ratio $c : d$ is given by

$\frac{bc-ad}{c-d}$.

Explanation

Let the required number be x.

Given: $\frac{a-x}{b-x} = \frac{c}{d} \Rightarrow ad - xd = bc - xc$

$\Rightarrow x(c - d) = bc - ad$

or, $x = \frac{bc-ad}{c-d}$.

Illustration 12 Find the number that must be subtracted from the terms of the ratio 5 : 6 to make it equal to 2 : 3.

Solution We have $a : b = 5 : 6$ and $c : d = 2 : 3$.

$\therefore$ The required number $= \frac{bc-ad}{c-d}$

$= \frac{6\times2-5\times3}{2-3} = 3.$

Illustration 13 Find the number that must be added to the terms of the ratio 11 : 29 to make it equal to 11 : 20.

Solution We have, $a : b = 11 : 29$ and $c : d = 11 : 20$.

$\therefore$ The required number $= \frac{ad-bc}{c-d}$

$= \frac{11\times20-29\times11}{11-20} = 11.$

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6. There are four numbers a, b, c and d.

(i) The number that should be subtracted from each of these numbers so that the remaining numbers may be proportional is given by

$$\frac{ad-bc}{(a+d)-(b+c)}.$$

Explanation

Let x be subtracted from each of the numbers.

The remainders are $a-x$, $b-x$, $c-x$ and $d-x$.

Given: $\frac{a-x}{b-x} = \frac{c-x}{d-x}$

$\Rightarrow (a-x)(d-x) = (b-x)(c-x)$

$\Rightarrow ad - x(a+d) + x^2 = bc - x(b+c) + x^2$

$\Rightarrow (b+c)x - (a+d)x = bc - ad$

$\therefore \quad x = \frac{bc-ad}{(b+c)-(a+d)}$ or, $\frac{ad-bc}{(a+d)-(b+c)}$

(ii) The number that should be added to each of these numbers so that the new numbers may be proportional is given by

$$\frac{bc-ad}{(a+d)-(b+c)}.$$

Explanation

Let x be added to each of the numbers.

The new numbers are $a+x$, $b+x$, $c+x$ and $d+x$.

Given: $\frac{a+x}{b+x} = \frac{c+x}{d+x}$.

$\Rightarrow (a+x)(d+x) = (b+x)(c+x)$

$\Rightarrow ad + x(a+d) + x^2 = bc + x(b+c) + x^2$

$\Rightarrow (a+d)x - (b+c)x = bc - ad.$

$\therefore \quad x = \frac{bc-ad}{(a+d)-(b+c)}.$

Illustration 14 Find the number subtracted from each of the numbers 54, 71, 75 and 99 leaves the remainders which are proportional.

Solution We have, $a = 54$, $b = 71$, $c = 75$ and $d = 99$.

$$\text{The required number} = \frac{ad-bc}{(a+d)-(b+c)}$$

$$= \frac{54\times99 - 71\times75}{(54+99)-(71+75)} = 3.$$

7. The incomes of two persons are in the ratio of $a:b$ and their expenditures are in the ratio of $c:d$. If the saving of each person be Rs. S, then their incomes are given by

$$\text{Rs. } \frac{aS(d-c)}{ad-bc} \text{ and Rs. } \frac{bS(d-c)}{ad-bc}.$$

and, their expenditures are given by

$$\text{Rs. } \frac{cS(b-a)}{ad-bc} \text{ and Rs. } \frac{dS(b-a)}{ad-bc}.$$

Explanation

Let their incomes be Rs. ak and Rs. bk, respectively.

Since each person saves Rs. S,

$\therefore$ expenditure of first person = Rs. $(ak - S)$

and expenditure of second person = Rs. $(bk - S)$.

Given : $\frac{ak-S}{bk-S} = \frac{c}{d}$

$\Rightarrow akd - Sd = bkc - Sc$

$\Rightarrow k(ad - bc) = (d-c)S$ or, $k = \frac{(d-c)S}{ad-bc}$.

Therefore, the incomes of two persons are

$$\frac{a(d-c)S}{ad-bc} \text{ and } \frac{b(d-c)S}{ad-bc}$$

and their expenditures are

$$ak - S \text{ and } bk - S$$

that is, $\frac{a(d-c)S}{ad-bc} - S$ and $\frac{b(d-c)S}{ad-bc} - S$

or, $\frac{cS(b-a)}{ad-bc}$ and $\frac{dS(b-a)}{ad-bc}$.

Illustration 15 Annual income of A and B is in the ratio of 5 : 4 and their annual expenses bear a ratio of 4 : 3. If each of them saves Rs. 500 at the end of the year, then find their annual income.

Solution We have, $a : b = 5 : 4$, $c : d = 4 : 3$ and $S = 500$.

$$\therefore \text{ Annual income of } A = \frac{aS(d-c)}{ad-bc}$$

$$= \frac{5\times500\times(3-4)}{(5\times3-4\times4)}$$

$$= \text{Rs. } 2500.$$

$$\text{and annual income of } B = \frac{bS(d-c)}{ad-bc}$$

$$= \frac{4\times500\times(3-4)}{(5\times3-4\times4)}$$

$$= \textbf{Rs. 2000.}$$

Illustration 16 The incomes of Mohan and Sohan are in the ratio 7 : 2 and their expenditures are in the ratio 4 : 1. If each saves Rs. 1000, find their expenditures.

Solution We have, $a : b = 7 : 2$, $c : d = 4 : 1$ and $S = 1000$.

$$\therefore \text{ A's expenditure} = \frac{cS(b-a)}{ad-bc} = \frac{4\times1000\times(2-7)}{(7\times1-2\times4)}$$

$$= \text{Rs. } 20000$$

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and, B's expenditure $= \frac{dS(b-a)}{ad-bc} = \frac{1\times1000\times(2-7)}{(7\times1-2\times4)}$

$=$ **Rs. 5000.**

8. (a) If in a mixture of x litres of two liquids A and B, the ratio of liquids A and B is $a : b$, then the quantity of liquid B to be added in order to make this ratio.

$c : d$ is $\frac{x(ad-bc)}{c(a+b)}$.

Explanation

Quantity of liquid A in the mixture $= \frac{ax}{a+b}$.

Quantity of liquid B in the mixture $= \frac{bx}{a+b}$.

Let litres of liquid B to be added in order to make this ratio as $c : d$.

Then, $\frac{ax}{a+b} : \frac{bx}{a+b} + l = c : d$

or, $\frac{ax}{a+b} : \frac{bx+l(a+b)}{a+b} = c : d$

or, $\frac{ax}{bx+l(a+b)} = \frac{c}{d}$ or, $axd = bcx + cl(a+b)$

or, $l = \frac{x(ad-bc)}{(a+b)c}$.

(b) In a mixture of two liquids A and B, the ratio of liquids A and B is $a : b$. If on adding x litres of liquid B to the mixture, the ratio of A to B becomes $a : c$, then in the beginning the quantity of liquid A in the mixture was $\frac{ax}{c-b}$ litres and that of liquid B was $\frac{bx}{c-b}$ litres.

Explanation

Let the quantity of mixture be M litres.

Then, the quantity of liquid $A = \frac{aM}{a+b}$ litres

and the quantity of liquid $B = \frac{bM}{a+b}$ litres.

If x litres of liquid B is added, then

$\frac{aM}{a+b} : \frac{bM}{a+b} + x = a : c$

or, $\frac{aM}{a+b} : \frac{bM+x(a+b)}{a+b} = a : c$

or, $\frac{aM}{bM+x(a+b)} = \frac{a}{c}$ or, $cM = bM + x(a+b)$

or, $M = \frac{x(a+b)}{c-b}$.

$\therefore$ Quantity of liquid $A = \frac{ax(a+b)}{(c-b)(a+b)}$

$= \frac{ax}{c-b}$ litres

and, quantity of liquid $B = \frac{bx(a+b)}{(c-b)(a+b)}$

$= \frac{bx}{c-b}$ litres.

Illustration 17 729 ml of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio of 7 : 3.

Solution Here, $x = 729$, $a : b = 7 : 2$ and $c : d = 7 : 3$.

$\therefore$ The quantity of water to be added

$= \frac{x(ad-bc)}{c(a+b)} = \frac{729\times(7\times3-2\times7)}{7(7+2)} =$ **81 ml.**

Illustration 18 A mixture contains alcohol and water in the ratio of 6 : 1. On adding 8 litres of water, the ratio of alcohol to water becomes 6 : 5. Find the quantity of water in the mixture.

Solution We have, $a : b = 6 : 1$, $a : c = 6 : 5$ and $x = 8$.

$\therefore$ The quantity of water in the mixture

$= \frac{bx}{c-b} = \frac{1\times8}{5-1} =$ **2 litres.**

9. When two ingredients A and B of quantities q_1 and q_2 and cost price/unit c_1 and c_2 are mixed to get a mixture c having cost price/unit c_m, then

(a) $\frac{q_1}{q_2} = \frac{c_2-c_m}{c_m-c_1}$ and

(b) $c_m = \frac{c_1\times q_1 + c_2\times q_2}{q_1+q_2}$.

Illustration 19 In what ratio the two kinds of tea must be mixed together into one at Rs. 9 per kg and another at Rs. 15 per kg, so that mixture may cost Rs. 10.2 per kg?

Solution We have, $c_1 = 9$, $c_2 = 15$, $c_m = 10.2$

$\therefore \frac{q_1}{q_2} = \frac{c_2-c_m}{c_m-c_1} = \frac{15-10.2}{10.2-9} = \frac{4.8}{1.2} = \frac{4}{1}$.

Thus, the two kinds of tea are mixed in the ratio **4 : 1.**

Illustration 20 In a mixture of two types of oils O_1 and O_2, the ratio $O_1 : O_2$ is 3 : 2. If the cost of oil O_1 is Rs. 4 per litre and that of O_2 is Rs. 9 per litre, then find the cost/litre of the resulting mixture.

Solution We have, $q_1 = q_2 = 2$, $c_1 = 4$ and $c_2 = 9$.

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$\therefore$ The cost of resulting mixture

$$= \frac{c_1 \times q_1 + c_2 \times q_2}{q_1 + q_2} = \frac{4 \times 3 + 9 \times 2}{3+2} = \frac{30}{5} = \textbf{Rs. 6.}$$

10. (a) If a mixture contains two ingredients A and B in the ratio $a : b$, then

percentage of A in the mixture $= \frac{a}{a+b} \times 100\%$ and percentage of B in the mixture $= \frac{b}{a+b} \times 100\%$.

(b) If two mixtures M_1 and M_2 contain ingredients A and B in the ratios $a : b$ and $c : d$ respectively, then a third mixture M_3 obtained by mixing M_1 and M_2 in the ratio $x : y$ will contain

$$\left[\frac{\frac{a \times x}{a+b} + \frac{c \times y}{c+d}}{x+y}\right] \times 100\% \text{ ingredient } A, \text{ and}$$

$$\left[100\% - \left\{\frac{\frac{ax}{a+b} + \frac{cy}{c+d}}{x+y}\right\}\right]$$

or, $$\left[\frac{\frac{bx}{a+b} + \frac{dy}{c+d}}{x+y}\right] \times 100\% \text{ ingredient } B.$$

Illustration 21 If a mixture contains water and alcohol in the ratio 2 : 3, what is the percentage quantity of water in the mixture?

Solution Here, $a = 2$, $b = 3$.

$\therefore$ percentage quantity of water in the mixture

$$= \frac{a}{a+b} \times 100\% = \frac{2}{2+3} \times 100\%$$

$$= \frac{2}{5} \times 100\%$$

$$= \frac{200}{5} \textbf{ or } \textbf{40\%}.$$

Illustration 22 Two alloys contain silver and copper in the ratio 3 : 1 and 5 : 3. In what ratio the two alloys should be added together to get a new alloy having silver and copper in the ratio of 2 : 1?

Solution We have, $a : b = 3 : 1$, $c : d = 5 : 3$

Let the two alloys be mixed in the ratio $x : y$.

Then, percentage quantity of sliver in the new alloy

$$= \left[\frac{\frac{ax}{a+b} + \frac{cy}{c+d}}{x+y}\right] \times 100\%$$

$$= \left[\frac{\frac{3x}{4} + \frac{5y}{8}}{x+y}\right] \times 100\%$$

$$= \frac{6x+5y}{8(x+y)} \times 100\% \quad \ldots(1)$$

Since the ratio of silver and copper in the new alloys is 2 : 1.

$\therefore$ percentage quantity of sliver in the new alloy

$$= \frac{2}{2+1} \times 100\% = \frac{200}{3}\% \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{6x+5y}{8(x+y)} = \frac{2}{3} \text{ or } 18x + 15y = 16x + 16y$$

or, $2x = y$

or, $x : y = 1 : 2$.

Hence, the two alloys should be mixed in the ratio **1:2.**

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find a fourth proportional to the numbers 60, 48, 30.
   (a) 36 (b) 24
   (c) 48 (d) None of these

2. Find the value of x in the following proportion: $27 : 72 :: x : 8$
   (a) 5 (b) 7
   (c) 3 (d) None of these

3. Find a third proportional to the numbers 4, 42.
   (a) 441 (b) 541
   (c) 641 (d) None of these

4. If $18 : x = x : 8$, then x is equal to
   (a) 12 (b) 16
   (c) 18 (d) None of these

5. The third proportional to 0.8 and 0.2 is
   (a) 0.6 (b) 0.05
   (c) 0.7 (d) None of these

6. The fourth proportional to 0.2, 0.12 and 0.3 is
   (a) 0.24 (b) 0.16
   (c) 0.18 (d) None of these

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7. In a ratio 11 : 14, if the antecedent is 55, the consequent is
(a) 70 (b) 90
(c) 60 (d) None of these

8. The mean proportional between 64 and 81 is
(a) 48 (b) 68
(c) 72 (d) None of these

9. The mean proportional of 0.25 and 0.04 is
(a) 0.01 (b) 0.1
(c) $10\sqrt{10}$ (d) None of these

10. The ratio of two numbers is 3 : 4 and their sum is 420. The greater of the two numbers is
(a) 360 (b) 240
(c) 180 (d) None of these

11. The ratio of boys and girls in a school is 9 : 5. If the total number of students in the school is 1050, then the number of boys is
(a) 785 (b) 890
(c) 675 (d) None of these

12. An amount of Rs. 1200 is distributed among A, B and C in the ratio of 5 : 7 : 13. What is the difference between the shares of C and B?
(a) Rs. 288 (b) Rs. 328
(c) Rs. 296 (d) None of these

13. Amit, Sumit and Puneet share an amount of Rs. 660 in the ratio of 3 : 4 : 5. What is the share of Puneet?
(a) Rs. 375 (b) Rs. 275
(c) Rs. 575 (d) None of these

14. Three numbers A, B and C are in the ratio of 12 : 15 : 25. If sum of these numbers is 312, find the ratio between the difference of B and A and the difference of C and B.
(a) 3 : 7 (b) 10 : 3
(c) 3 : 10 (d) None of these

15. The prices of a scooter and a television set are in the ratio of 3 : 2. If a scooter costs Rs. 600 more than the television set, then the price of television set is
(a) Rs. 1800 (b) Rs. 1200
(c) Rs. 2400 (d) None of these

16. Two numbers are in the ratio of 4 : 9. If the larger number is 35 more than the smaller number, then the product of the numbers is
(a) 1764 (b) 1564
(c) 1864 (d) None of these

17. If the income of A, B and C is in the ratio of 2 : 5 : 11 and the income of B is Rs. 291 more than that of A, then the income of C is
(a) Rs. 907 (b) Rs. 1127
(c) Rs. 1067 (d) None of these

18. If $A : B = 7 : 5$ and $B : C = 9 : 11$, then $A : B : C$ is equal to
(a) 55 : 45 : 63 (b) 63 : 45 : 55
(c) 45 : 63 : 55 (d) None of these

19. If $A : B = 3/4$, $B : C = 4/5$, $C : D = 5/6$, then $A : D$ will be
(a) 2 : 3 (b) 4 : 3
(c) 1 : 2 (d) None of these

20. If $3A = 4B = 5C$, then $A : B : C$ is
(a) 16 : 20 : 18 (b) 15 : 20 : 16
(c) 20 : 15 : 12 (d) None of these

21. If $3A = 5B$ and $2B = 3C$, then $A : C$ is
(a) 5 : 2 (b) 7 : 2
(c) 3 : 2 (d) None of these

22. Ajay, Aman, Suman and Geeta rented a house and agreed to share the rent as follows:
Ajay : Aman = 8 : 15, Aman : Suman = 5 : 8 and Suman and Geeta = 4 : 5. The part of rent paid by Suman will be
(a) $\frac{24}{77}$ (b) $\frac{12}{55}$
(c) $\frac{13}{66}$ (d) None of these

23. The ratio of money with Anju and Sanju is 4 : 5 and that with Sanju and Manju is 5 : 6. If Anju has Rs. 280, then the amount of money Manju has
(a) Rs. 320 (b) Rs. 420
(c) Rs. 640 (d) None of these

24. There are three sections A, B and C in a school of class I. The ratio of students in sections A and B is 3 : 5 and that in B and C is 4 : 7. If the total number of students in the class be 201, then the number of students in section A are
(a) 24 (b) 36
(c) 48 (d) None of these

25. The sum of three numbers is 124. If the ratio between the first and second be 2 : 3 and that between the second and third be 7 : 9, then the third number is
(a) 54 (b) 64
(c) 48 (d) None of these

26. A, B, C and D share a property worth Rs. 77500. If $A : B = 3 : 2$, $B : C = 5 : 4$ and $C : D = 3 : 7$, find the share of B.
(a) Rs. 20000 (b) Rs. 15000
(c) Rs. 25000 (d) None of these

27. Two numbers are in the ratio 3 : 5. If each number is increased by 10, the ratio becomes 5 : 7. The numbers are
(a) 15, 25 (b) 30, 45
(c) 48, 60 (d) None of these

28. The ratio between two numbers is 2 : 3. If each number is increased by 4, the ratio becomes 5 : 7. The numbers are
(a) 8, 16 (b) 16, 24
(c) 24, 32 (d) None of these

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29. Two numbers are in the ratio of 5 : 6. If 5 is subtracted from each number, the ratio becomes 4 : 5. The numbers are
(a) 25, 30 (b) 30, 45
(c) 15, 20 (d) None of these

30. The ratio of present ages of Suresh and Mahesh is 7 : 5. If after 6 years their ages will be in the ratio of 4 : 3, the present age of Mahesh is
(a) 32 years (b) 36 years
(c) 30 years (d) None of these

31. The ratio of present ages of Sita and Gita is 4 : 3. If, 4 years before, the ratio of their ages was 2 : 1, the present age of Sita is
(a) 8 years (b) 10 years
(c) 12 years (d) None of these

32. Two numbers are in the ratio of 5 : 8. If 12 be added to each, they are in the ratio of 3 : 4. Find the sum of two numbers.
(a) 43 (b) 39
(c) 47 (d) None of these

33. Two numbers are in the ratio of 5 : 7. If 25 be subtracted from each, they are in the ratio of 35 : 59. Find the difference of the two numbers.
(a) 48 (b) 52
(c) 24 (d) None of these

34. When x is added to each term of 7 : 13 the ratio becomes 2 : 3. The value of x is
(a) 7 (b) 11
(c) 5 (d) None of these

35. Find the number which, when subtracted from the terms of the ratio 12 : 17 makes it equal to the ratio 2 : 3.
(a) 2 (b) 6
(c) 8 (d) None of these

36. The value of k that must be added to 7, 16, 43, 79 so that they are in proportion, is
(a) 7 (b) 5
(c) 9 (d) None of these

37. What should be subtracted from 15, 28, 20 and 38 so that the remaining numbers may be proportional?
(a) 2 (b) 4
(c) 6 (d) None of these

38. The number that must be added to each of the numbers 8, 21, 13 and 31 to make the ratio of first two numbers equal to the ratio of last two numbers is
(a) 5 (b) 7
(c) 9 (d) None of these

39. The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves Rs. 1000, A's income is
(a) Rs. 5000 (b) Rs. 6000
(c) Rs. 8000 (d) None of these

40. The annual incomes and expenditures of man and his wife are in the ratios of 5 : 3 and 3 : 1, respectively. If they decide to save equally and find a balance of Rs. 4000 at the end of year, their incomes were
(a) Rs. 5000, Rs. 3000 (b) Rs. 6000, Rs. 4000
(c) Rs. 3000, Rs. 2000 (d) None of these

41. The incomes of Gupta and Verma are in the ratio 9 : 4 and their expenditures are in the ratio 7 : 3. If each saves Rs. 2000, then Gupta's expenditure is
(a) Rs. 6000 (b) Rs. 8000
(c) Rs. 7000 (d) None of these

42. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. What amount of water must be added to make the ratio of milk and water as 1 : 2?
(a) 75 litres (b) 55 litres
(c) 60 litres (d) None of these

43. A mixture contains alcohol and water in the ratio of 12 : 5. On adding 14 litres of water, the ratio of alcohol to water becomes 4 : 3. The quantity of alcohol in the mixture is
(a) 18 litres (b) 24 litres
(c) 26 litres (d) None of these

44. If an alloy contains copper and silver in the ratio 3 : 7, then the percentage quantity of silver in the alloy is
(a) 90% (b) 70%
(c) 60% (d) None of these

45. Two alloys contain zinc and copper in the ratio of 2 : 1 and 4 : 1. In what ratio the two alloys should be added together to get as new alloy having zinc and copper in the ratio of 3 : 1?
(a) 7 : 5 (b) 5 : 7
(c) 3 : 5 (d) None of these

46. Mixture of milk and water has been kept in two separate containers. Ratio of milk to water in one of the containers is 5 : 1 and that in the other container is 7 : 2. In what ratio the mixtures of these two containers should be added together so that the quantity of milk in the new mixture may become 80%?
(a) 2 : 3 (b) 3 : 2
(c) 4 : 5 (d) None of these

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EXPLANATORY ANSWERS

1. Ans. (b)

Solution Let x be the fourth proportional, then

$$60 : 48 : 30 : x \text{ or, } \frac{60}{48} = \frac{30}{x}$$

$$\therefore \quad x = \frac{30\times 38}{60} = \mathbf{24.}$$

2. Ans. (c)

Solution We have, $27 : 72 :: x : 8$ or, $\frac{27}{72} = \frac{x}{8}$

$$\therefore \quad x = \frac{8\times 27}{72} = \mathbf{3.}$$

3. Ans. (a)

Solution Let x be the third proportional, then

$$4 : 42 :: 42 : x \text{ or, } \frac{4}{42} = \frac{42}{x}.$$

$$\therefore \quad x = \frac{42\times 42}{4} = \mathbf{441.}$$

4. Ans. (a)

Solution We have, $\frac{18}{x} = \frac{x}{8}$ or, $x^2 = 18 \times 8$

or, $\quad x = \sqrt{144} = \mathbf{12.}$

5. Ans. (b)

Solution Let x be the third proportional. Then,

$$0.8 : 0.2 :: 0.2 : x \text{ or, } \frac{0\cdot 8}{0\cdot 2} = \frac{0\cdot 2}{x}$$

$$\therefore \quad x = \frac{0\cdot 2\times 0\cdot 2}{0\cdot 8} = \mathbf{0.05.}$$

6. Ans. (c)

Solution Let x be the fourth proportional. Then,

$$0.2 : 0\cdot 2 :: 0.2\ x \text{ or. } \frac{0.2}{0.12} = \frac{0.3}{x}.$$

$$\therefore \quad x = \frac{0.3\times 0.12}{0.2} = \mathbf{0.18.}$$

7. Ans. (a)

Solution $\frac{11}{14} = \frac{11\times 5}{14\times 5} = \frac{55}{70}$.

$\therefore$ Consequent = **70.**

8. Ans. (c)

Solution Let x be the mean proportional. Then,

$$64 : x :: x : 81 \text{ or, } \frac{64}{x} = \frac{x}{81} \text{ or, } x^2 = 5184 \textbf{ or } x = \mathbf{72.}$$

9. Ans. (b)

Solution Let x be the mean proportional. Then,

$$0.25 : x : x : 0\cdot 04 \text{ or, } \frac{0.25}{x} = \frac{x}{0\cdot 04}$$

or, $\quad x^2 = 0.01$ **or, $x = 0.1$**

10. Ans. (b)

Solution Here, $a = 3$, $b = 4$ and $x = 420$.

$\therefore$ The first number $= \frac{ax}{a+b} = \frac{3\times 420}{3+4} = 180.$

and, the second number $= \frac{bx}{a+b} = \frac{4\times 420}{3+4} = \mathbf{240.}$

11. Ans. (c)

Solution Here, $a = 9$, $b = 5$ and $x = 1050$.

$\therefore$ Number of boys $= \frac{ax}{a+b} = \frac{9\times 1050}{9+5} = \mathbf{675.}$

12. Ans. (a)

Solution We have, $a = 5$, $b = 7$, $c = 13$ and $x = 1200$.

$\therefore$ Share of $B = \frac{bx}{a+b+c} = \frac{7\times 1200}{5+7+13} = 336$

and, share of $C = \frac{cx}{a+b+c} = \frac{13\times 1200}{5+7+13} = \mathbf{624.}$

13. Ans. (b)

Solution Here, $a = 3$, $b = 4$, $c = 5$ and $x = 660$.

$\therefore$ Share of Puneet $= \frac{cx}{a+b+c} = \frac{5\times 660}{3+4+5} = \mathbf{Rs.\ 275.}$

14. Ans. (c)

Solution We have, $a = 12$, $b = 15$, $c = 25$ and $x = 312$.

$$\therefore \quad A = \frac{ax}{a+b+c} = \frac{12\times 312}{12+15+25} = 72,$$

$$B = \frac{bx}{a+b+c} = \frac{15\times 312}{12+15+25} = 90$$

and, $C = \frac{cx}{a+b+c} = \frac{25\times 312}{12+15+25} = 150$

$\therefore \quad B - A = 18$ and $C - B = 60$

Thus, their ratio = 18 : 60 **or, 3 : 10.**

15. Ans. (b)

Solution Here, $a = 3$, $b = 2$ and $x = 600$.

$\therefore$ The price of a television set $= \frac{bx}{a-b} = \frac{2\times 600}{3-2}$

$= \mathbf{Rs.\ 1200.}$

16. Ans. (a)

Solution Here, $a = 4$, $b = 9$ and $x = 35$.

$\therefore$ The first number $= \frac{ax}{b-a} = \frac{4\times 35}{9-4} = 28$

and, the second number $= \frac{bx}{b-a} = \frac{9\times 35}{9-4} = 63.$

Thus, the product of the numbers $= 28 \times 63 = \mathbf{1764.}$

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17. Ans. (c)

Solution Ratio of the income of A, B and $C = 2 : 5 : 11$.

$\therefore$ Ratio of the income of A and $B = 2 : 5$.

Difference between income of A and B = Rs. 291.

$\therefore$ Income of $C = \frac{cx}{b-a} = \frac{11 \times 291}{5-2}$ **= Rs. 1067.**

[Here, $a = 2$, $b = 5$, $c = 11$ and $x = 291$]

18. Ans. (b)

Solution Here, $n_1 = 7$, $n_2 = 9$, $d_1 = 5$ and $d_2 = 11$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (7 \times 9) : (5 \times 9) : (5 \times 11)$

$= 63 : 45 : 55$.

19. Ans. (c)

Solution We have, $n_1 = 3$, $n_2 = 4$, $n_3 = 5$,

$d_1 = 4$, $d_2 = 5$ and $d_3 = 6$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$

$= (3 \times 4 \times 5) : (4 \times 4 \times 5) : (4 \times 5 \times 5) : (4 \times 5 \times 6)$

$= 60 : 80 : 100 : 120$ or, $3 : 4 : 5 : 6$.

Thus, $A : D$ **= 3 : 6 or, 1 : 2.**

20. Ans. (c)

Solution We have $A : B = 4 : 3$ and $B : C = 5 : 4$.

Here, $n_1 = 4$, $n_2 = 5$, $d_1 = 3$ and $d_2 = 4$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (4 \times 5) : (3 \times 5) : (3 \times 4)$

$= 20 : 15 : 12$.

21. Ans. (a)

Solution We have, $A : B = 5 : 3$ and $B : C = 3 : 2$.

Here, $n_1 = 5$, $n_2 = 3$, $d_1 = 3$ and $d_2 = 2$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (5 \times 3) : (3 \times 3) : (3 \times 2)$

$= 15 : 9 : 6$ or $5 : 3 : 2$.

Thus, $A : C$ **= 5 : 2.**

22. Ans. (a)

Solution We have, $A : B = 8 : 15$, $B : C = 5 : 8$ and $C : D = 4 : 5$.

Here, $n_1 = 8$, $n_2 = 5$, $n_3 = 4$, $d_1 = 15$, $d_2 = 8$ and $d_3 = 5$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$

$= (8 \times 5 \times 4) : (15 \times 5 \times 4) : (15 \times 8 \times 4) : (15 \times 8 \times 5)$

$= 160 : 300 : 480 : 600$ or, $8 : 15 : 24 : 30$.

Thus, Suman pays $= \frac{24}{8+15+24+30}$ of the rent

$= \frac{24}{77}$ of the rent.

23. Ans. (b)

Solution We have, $A : B = 4 : 5$ and $B : C = 5 : 6$.

Here, $n_1 = 4$, $n_2 = 5$, $d_1 = 5$ and $d_2 = 6$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (4 \times 5) : (5 \times 5) : (5 \times 6)$

$= 20 : 25 : 30$ or, $4 : 5 : 6$.

Thus, ratio of money with Anju, Sanju and Manju is $4 : 5 : 6$. Since Anju has Rs. 280, the amount of money Manju has $= \frac{280}{4} \times 6$ **= Rs. 420.**

24. Ans. (b)

Solution We have, $A : B = 3 : 5$ and $B : C = 4 : 7$.

Here, $n_1 = 3$, $n_2 = 4$, $d_1 = 5$ and $d_2 = 7$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (3 \times 4) : (5 \times 4) : (5 \times 7)$

$= 12 : 20 : 35$.

The total number of students = 201.

$\therefore$ The number of students in section A

$= \frac{12}{12+20+35} \times 201$ **= 36.**

25. Ans. (a)

Solution We have, $A : B = 2 : 3$ and $B : C = 7 : 9$.

Here, $n_1 = 2$, $n_2 = 7$, $d_1 = 3$ and $d_2 = 9$.

$\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$

$= (2 \times 7) : (3 \times 7) : (3 \times 9)$

$= 14 : 21 : 27$.

Since the sum of the numbers is 124, the third number is $\frac{27}{14+21+27} \times 124$ **= 54.**

26. Ans. (b)

Solution We have, $A : B = 3 : 2$, $B : C = 5 : 4$ and $C : D = 3 : 7$.

Here, $n_1 = 3$, $n_2 = 5$, $n_3 = 3$, $d_1 = 2$, $d_2 = 4$ and $d_3 = 7$.

$\therefore$ $A : B : C : D = (n_1 \times n_2 \times n_3) : (d_1 \times n_2 \times n_3) : (d_1 \times d_2 \times n_3) : (d_1 \times d_2 \times d_3)$

$= (3 \times 5 \times 3) : (2 \times 5 \times 3) : (2 \times 4 \times 3) : (2 \times 4 \times 7)$

$= 45 : 30 : 24 : 56$

$\therefore$ B's share of property worth Rs. 77500 is

$= \left(\frac{30}{45+30+24+56}\right) \times 77500$ **= Rs. 15000.**

27. Ans. (a)

Solution We have, $a : b = 3 : 5$, $c : d = 5 : 7$ and $x = 10$.

$\therefore$ The first number $= \frac{ax(c-d)}{ad-bc}$

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$$= \frac{3\times10\times(5-7)}{(3\times7-5\times5)} = 15$$

and, the second number $= \frac{bx(c-d)}{ad-bc}$

$$= \frac{5\times10\times(5-7)}{(3\times7-5\times5)} = \mathbf{235.}$$

28. Ans. (b)

Solution We have, $a : b = 2 : 3$, $c : d = 5 : 7$ and $x = 4$.

$\therefore$ The first number $= \frac{ax(c-d)}{ad-bc}$

$$= \frac{2\times4\times(5-7)}{(2\times7-3\times5)} = 16.$$

and, the second number $= \frac{bx(c-d)}{ad-bc}$

$$= \frac{3\times4\times(5-7)}{(2\times7-3\times5)} = \mathbf{24.}$$

29. Ans. (a)

Solution We have, $a : b = 5 : 6$, $c : d = 4 : 5$ and $x = 5$.

$\therefore$ The first number $= \frac{ax(d-c)}{ad-bc}$

$$= \frac{5\times5\times(5-4)}{(5\times5-6\times4)} = 25$$

and, the second number $= \frac{bx(d-c)}{ad-bc}$

$$= \frac{6\times5\times(5-4)}{(5\times5-6\times4)} = \mathbf{30.}$$

30. Ans. (c)

Solution We have, $a : b = 7 : 5$, $c : d = 4 : 3$ and $x = 6$.

$\therefore$ The present age of Mahesh $= \frac{bx(c-d)}{ad-bc}$

$$= \frac{5\times6\times(4-3)}{(7\times3-5\times4)}$$

$$= \mathbf{30\ years.}$$

31. Ans. (a)

Solution We have, $a : b = 4 : 3$, $c : d = 2 : 1$ and $x = 4$.

$\therefore$ The present age of Sita $= \frac{ax(d-c)}{ad-bc}$

$$= \frac{4\times4\times(1-2)}{(4\times1-3\times2)}$$

$$= \mathbf{8\ years.}$$

32. Ans. (b)

Solution We have, $a : b = 5 : 8$, $c : d = 3 : 4$ and $x = 12$.

$\therefore$ The first number $= \frac{ax(c-d)}{ad-bc}$

$$= \frac{5\times12\times(3-4)}{(5\times4-8\times3)} = 15$$

and, the second number $= \frac{bx(c-d)}{ad-bc}$

$$= \frac{8\times12\times(3-4)}{(5\times4-8\times3)} = \mathbf{24.}$$

33. Ans. (c)

Solution We have, $a : b = 5 : 7$, $c : d = 35 : 59$ and $x = 25$.

$\therefore$ The first number $= \frac{ax(d-c)}{ad-bc}$

$$= \frac{5\times25\times(59-35)}{(5\times59-7\times35)} = 60$$

and, the second number $= \frac{bx(d-c)}{ad-bc}$

$$= \frac{7\times25\times(59-35)}{(5\times59-7\times35)} = 84$$

$\therefore$ The difference of two numbers $= 84 - 60 = \mathbf{24.}$

34. Ans. (c)

Solution We have, $a : b = 7 : 13$ and $c : d = 2 : 3$.

$\therefore$ $x = \frac{ad-bc}{c-d} = \frac{7\times3-13\times2}{2-3} = \mathbf{5.}$

35. Ans. (a)

Solution We have, $a : b = 12 : 17$ and $c : d = 2 : 3$.

$\therefore$ The required number $= \frac{bc-ad}{c-d}$

$$= \frac{17\times2-12\times3}{2-3} = \mathbf{2.}$$

36. Ans. (b)

Solution Here, $a = 7$, $b = 16$, $c = 43$ and $d = 79$.

$\therefore$ $k = \frac{bc-ad}{(a+d)-(b+c)} = \frac{16\times43-7\times79}{(7+79)-(16+43)} = \mathbf{5.}$

37. Ans. (a)

Solution Here, $a = 15$, $b = 28$, $c = 20$ and $d = 38$.

$\therefore$ The required number $= \frac{ad-bc}{(a+d)-(b+c)}$

$$= \frac{15\times38-28\times20}{(15+38)-(28+20)}$$

$$= \mathbf{2.}$$

38. Ans. (a)

Solution Here, $a = 8$, $b = 21$, $c = 13$ and $d = 31$.

$\therefore$ The required number $= \frac{bc-ad}{(a+d)-(b+c)}$

$$= \frac{21\times13-8\times31}{(8+31)-(21+13)}$$

$$= \mathbf{5.}$$

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39. Ans. (b)

Solution We have, $a : b = 3 : 2$, $c : d = 5 : 3$ and $S = 1000$.

$$\therefore \quad A\text{'s income} = \frac{aS(d-c)}{ad-bc}$$
$$= \frac{3\times1000\times(3-5)}{(3\times3-2\times5)}$$
$$= \textbf{Rs. 6000.}$$

40. Ans. (a)

Solution We have, $a : b = 5 : 3$, $c : d = 3 : 1$ and $S = 2000$.

$$\therefore \quad \text{Income of man} = \frac{aS(d-c)}{ad-bc}$$
$$= \frac{5\times2000\times(1-3)}{(5\times1-3\times3)}$$
$$= \text{Rs. } 5000$$

and, the income of his wife

$$= \frac{bS(d-c)}{ad-bc}$$
$$= \frac{3\times2000\times(1-3)}{(5\times1-3\times3)}$$
$$= \textbf{Rs. 3000.}$$

41. Ans. (c)

Solution We have, $a : b = 9 : 4$, $c : d = 7 : 3$ and $S = 2000$.

$$\therefore \quad \text{Gupta's expenditure} = \frac{cS(b-a)}{ad-bc}$$
$$= \frac{7\times2000\times(4-9)}{(9\times3-4\times7)}$$
$$= \textbf{Rs. 70000.}$$

42. Ans. (c)

Solution Here, $x = 60$, $a : b = 2 : 1$ and $c : d = 1 : 2$.

$\therefore$ Required amount of water to be added

$$= \frac{x(ad-bc)}{c(a+b)} = \frac{60\times(2\times2-1\times1)}{1(2+1)} = \textbf{60 litres.}$$

43. Ans. (b)

Solution The two given ratios are 12 : 5 and 4 : 3.

In order to equate the antecedents of the two ratios, we write the second ratio as 12 : 9.

Now, we have, $a : b = 12 : 5$, $c : d = 12 : 9$ and $x = 14$

$\therefore$ The quantity of alcohol in the mixture

$$= \frac{ax}{c-b} = \frac{12\times14}{12-5} = \textbf{24 litres.}$$

44. Ans. (b)

Solution Here, $a = 3$ and $b = 7$.

$\therefore$ percentage quantity of silver in the alloy

$$= \left(\frac{b}{a+b}\right)\times100\% = \left(\frac{7}{3+7}\right)\times100\% = \textbf{70\%.}$$

45. Ans. (c)

Solution We have, $a : b = 2 : 1$, $c : d = 4 : 1$.

Let the two alloys be mixed in the ratio $x : y$.

Then, percentage quantity of zinc in the new alloy

$$= \left[\frac{\frac{ax}{a+b}+\frac{cy}{c+d}}{x+y}\right]\times100\%$$
$$= \left[\frac{\frac{2x}{3}+\frac{4y}{5}}{x+y}\right]\times100\%$$
$$= \frac{10x+12y}{15(x+y)}\times100\% \qquad ...(1)$$

Since the ratio of zinc and copper in the new alloy is 3 : 1

$\therefore$ percentage quantity of zinc in the new alloy

$$= \frac{3}{3+1}\times100\% = \frac{300}{4}\% \text{ or, } 57\% \qquad ...(2)$$

From (1) and (2), we get

$$\frac{10x+12y}{15(x+y)} = \frac{3}{4} \quad \text{or,} \quad 40x + 48y = 45(x + y)$$

$$\text{or,} \quad 5x = 3y \quad \text{or,} \quad x : y = 3 : 5.$$

Hence, the alloys should be mixed in the ratio **3 : 5.**

46. Ans. (a)

Solution Here, $a : b = 5 : 1$, $c : d = 7 : 2$.

Let the mixtures of the two containers be added together in the ratio of $x : y$.

$\therefore$ percentage quantity of milk in the new mixture

$$= \left[\frac{\frac{ax}{a+b}+\frac{cy}{c+d}}{x+y}\right]\times100\%$$
$$= \left[\frac{\frac{5x}{6}+\frac{7y}{9}}{x+y}\right]\times100\% = \frac{45x+42y}{(x+y)}\times100\%$$

Since the percentage quantity of milk in the new mixture is 80%

$$\therefore \quad \frac{45x+42y}{54(x+y)}\times100\% = 80\%$$
$$\Rightarrow \quad (45x + 42y)\times5 = 4\times54(x + y)$$
$$\Rightarrow \quad 225x + 210y = 216x + 216y$$
$$\Rightarrow \quad \textbf{9x = 6y or x : y = 2 : 3.}$$

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QUESTIONS FROM COMPETITIVE EXAMS

1. Let a, b, c, d and e be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$. Then which of the following pairs contain a number that is not an integer?

(a) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$

(d) $\left(\frac{a}{12}, \frac{bd}{18}\right)$ (d) $\left(\frac{a}{6}, \frac{c}{d}\right)$

[Based on CAT 2004]

2. The ratio of the rate of flow of water in pipes varies inversely as the square of the radius of the pipes. What is the ratio of the rates of flow in two pipes of diameters 2cm and 4cm?

(a) 1 : 2 (b) 2 : 1
(c) 1 : 8 (d) 4 : 1

[Based on MAT 2004]

3. Half the girls and one-third of the boys of a college reside in the hostel. What fractional part of the student body is hostel dwellers if the total number of girls in the college is 100 and is 1/4 of the total strength?

(a) 2/5 (b) 5/12
(c) 1/5 (d) 3/8

[Based on MAT 2004]

4. Given that 24 carat gold is pure gold; 18 carat gold is 3/4 gold and 20 carat gold is 5/6 gold, the ratio of the pure gold in 18 carat gold to the pure gold in 20 carat gold is

(a) 5 : 8 (b) 9 : 10
(c) 15 : 24 (d) 8 : 5

[Based on MAT 2004]

5. Ravi's salary is 150% of Amit's salary. Amit's salary is 80% of Ram's salary. What is the ratio of Ram's salary to Ravi's salary ?

(a) 1 to 2 (b) 2 to 3
(c) 5 to 6 (d) 6 to 5

[Based on MAT 2004]

6. In a class room, three-fourth of the boys are above 160 cm in height and they are 18 in number. Also out of the total strength, the boys form only two-third and the rest are girls. The total number of girls in the class is

(a) 18 (b) 24
(c) 12 (d) 20

[Based on MAT 2004]

7. A bag contains Rs. 216 in the form of one rupee, 50 paise and 25 paise coins in the ratio of 2 : 3 : 4. The number of 50 paise coins is

(a) 96 (b) 144
(c) 114 (d) 141

[Based on MAT 2003]

8. At the start of a seminar, the ratio of the number of male participants to the number of female participants was 3:1. During the tea break, 16 participants left and 6 more female participants registered. The ratio of the male to the female participants became 2:1. What was the total number of participants at the start of the seminar?

(a) 64 (b) 48
(c) 54 (d) 72

[Based on MAT 2003]

9. A person spends 1/3 of the money with him on clothes, 1 /5 of the remaining on food and 1/4 of the remaining on travel. Now, he is left with Rs. 100. How much did he have with him in the beginning?

(a) Rs 200 (b) Rs 250
(c) Rs 300 (d) Rs 450

[Based on MAT 2003]

10. 1/3rd of the contents of a container evaporated on the 1st day. 3/4th of the remaining evaporated on the second day. What part of the contents of the container is left at the end of the second day?

(a) 1/4 (b) 1/2
(c) 1/18 (d) 1/6

[Based on MAT 2003]

11. The monthly incomes of two persons are in the ratio of 4 : 5 and their monthly expenditures are in the ratio of 7 : 9. If each saves Rs. 50 a month, then what are their monthly incomes?

(a) Rs 100, Rs 125 (b) Rs 200, Rs 250
(c) Rs 300, Rs 375 (d) Rs 400, Rs 500

[Based on MAT 2002]

12. When a bus started from the first stop, the number of male passengers to the number of female passengers was 3 : 1. At the first stop, 16 passengers got down and 6 more female passengers got in. The ratio of the male to female passengers now became 2 : 1. What was the total number of passengers in the bus when it started from the first stop?

(a) 64 (b) 48
(c) 54 (d) 72

[Based on MAT 2002]

13. If the ratio of boys to girls in a class is B and the ratio of girls to boys is G, then $3(B + G)$ is

(a) Equal to 3. (b) Less than 3.
(c) More than 3 (d) Less than $\frac{1}{3}$

[Based on MAT 2001]

14. If $a : b = 2 : 5$, then the value of $(2a + 3b) : (7a + 5b)$ is

(a) $\frac{19}{39}$ (b) $\frac{99}{13}$

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(c) $\frac{31}{19}$ (d) $\frac{19}{31}$

[Based on MAT 1999]

15. If two numbers are in the ratio 6 : 13 and their least common multiple is 312, the sum of the numbers is

(a) 75 (b) 57
(c) 76 (d) 67

[Based on MAT 1999]

16. If $\frac{1}{3}A = \frac{1}{4}B = \frac{1}{5}C$. Then, $A : B : C$ is

(a) 4 : 3 : 5 (b) 5 : 4 : 3
(c) 3 : 4 : 5 (d) 20 : 15 : 12

[Based on MAT 2005]

17. If $P = \frac{x^2 - 36}{x^2 - 49}$ and $Q = \frac{x+6}{x+7}$, then the value of P/Q is

(a) $\frac{x-6}{x-7}$ (b) $\frac{x-6}{x+7}$
(c) $\frac{x-6}{x+6}$ (d) $\frac{x+6}{x-7}$

[Based on MAT 1999]

18. Two numbers are in the ratio 2 : 3. If eight is added to both the numbers, the ratio becomes 3 : 4. The numbers are

(a) 15 and 20 (b) 16 and 24
(c) 13 and 17 (d) 17 and 9

[Based on MAT 1999]

19. What is the value of $\frac{P+Q}{P-Q}$ if $\frac{P}{Q} = 7$?

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$
(c) $\frac{2}{6}$ (d) $\frac{7}{8}$

[Based on MAT 1999]

20. If two numbers are in the ratio 6 : 13 and their least common multiple is 312, then the sum of the numbers is

(a) 75 (b) 57
(c) 76 (d) 67

[Based on MAT 2000]

21. If $\frac{5a+3b}{2a-3b} = \frac{23}{5}$, then the value of $a : b$ is

(a) 2 : 1 (b) 1 : 4
(c) 1 : 2 (d) 4 : 1

[Based on MAT 2000]

22. What is the value of $\frac{P+Q}{P-Q}$ if $\frac{P}{Q} = 7$?

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$
(c) $\frac{2}{6}$ (d) $\frac{7}{8}$

[Based on MAT 2000]

23. A bag contains Rs. 216 in the form of one-rupee, 50-paise and 25-paise coins in the ratio of 2 : 3 : 4. The number of 50-paise coins is

(a) 96 (b) 144
(c) 114 (d) 141

[Based on MAT 2001]

24. Rs. 770 have been divided among A, B and C such that A receives 2/9th of what B and C together receive. Then A's share is

(a) Rs. 140 (b) Rs. 154
(c) Rs. 165 (d) Rs. 170

[Based on MAT 2001]

25. What least number must be subtracted from each of the numbers 14, 17, 34 and 42 so that the remainders are proportional?

(a) 0 (b) 1
(c) 2 (d) 7

[Based on MAT 2001]

26. If $a : b = 2 : 5$ then $(3a + 4b) : (5a + 6b)$ is equal to

(a) 13/20 (b) 26/33
(c) 16/27 (d) 18/35

[Based on MAT 2001]

27. 5 : 7 : : ? : 28

(a) 20 (b) 14
(c) 56 (d) None of these

[Based on MAT 2001]

28. At Narmada Sarovar Bachao demonstration, supporters of Ms. Patkar outnumbered the police by 9 : 1. The police arrested 135 NSB supporters averaging 5 for every 3 policemen. How many supporters of NSB were there in the demonstration?

(a) 405 (b) 665
(c) 1215 (d) None of the above

[Based on FMS (Delhi) 2004]

29. Monthly incomes of two persons are in the ratio 5 : 4 and their monthly expenditures are in the ratio of 9 : 7. If each person saves Rs 500 per month, then what are their monthly incomes?

(a) Rs 8000 and Rs 10000
(b) Rs 3750 and Rs 3000
(c) Rs 5000 and Rs 4000
(d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

30. Which of the following is the ratio between a number and the number obtained by adding one-fifth of that number to it?

(a) 6 : 5 (b) 5 : 6
(c) 5 : 4 (d) 4 : 5

[Based on Narsee Monjee Inst. of Man. Studies, 2003]

31. The intensity of illumination on a surface from a source of light varies inversely as the square of

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the distance of the surface from the source. The effect of moving a piece of paper 3 times as far from the source is to
(a) Divide the intensity by 3
(b) Multiply the intensity by 3
(c) Divide the intensity by 9
(d) Multiply the intensity by 9
[Based on REC Tiruchirapalli 2003]

32. Suppose y varies as the sum of two quantities of which one varies directly, as x and the other varies inversely as x. If $y = 6$ when $x = 4$ and $y = 3\frac{1}{3}$ when $x = 3$, then the relation between x and y is
(a) $y = x + \frac{4}{x}$ (b) $y = -2x + \frac{4}{x}$
(c) $y = 2x + \frac{8}{x}$ (d) $y = 2x - \frac{8}{x}$
[Based on FMS (Delhi) 2003]

33. The ratio of the age of a man and his wife is 4 : 3. After 4 years, this ratio will be 9 : 7. If at the time of the marriage, the ratio was 5 : 3, then how many years ago they were married ?
(a) 12 years (b) 8 years
(c) 10 years (d) 15 years
(e) None of the above **[Based on IIFT 2003]**

34. Pressure varies inversely with volume while temperature varies directly with volume. At a time, Volume = 50 m^3, Temperature = 25° K and Pressure = 1 atmosphere. If the volume is increased to 200 m^3, then the temperature will be
(a) 100° K (b) 50° K
(c) $12\frac{1}{2}$° K (d) 0° K
(e) None of these **[Based on IIFT 2003]**

35. For each 200 rupees spent by the research department, sales department spends 20 rupees. For every 400 rupees spent by the sales department, the advertising department spends 150 rupees. The triple ratio of the money spent by the research department to the money spent by the sales department to the money spent by the advertising department can be expressed as
(a) 40 : 8 : 3 (b) 80 : 8 : 3
(c) 20 : 4 :.1 (d) 2 : 1 : 5
[Based on SCMHRD Ent. Exam. 2003]

36. x^2 varies directly as y^3 and when $x = 6$, $y = 3$. Which of the following equations correctly represents the relationship between x and y?
(a) $6x^2 = 3y^3$ (b) $3y^2 = 6x^3$
(c) $3x^2 = 2y^3$ (d) $3x^2 = 4y^3$
[Based on IMT Ghaziabad, 2002]

37. It is given that $y \propto \frac{1}{x^3 - x}$. For $x = 2$, value of y is $\frac{1}{6}$. If $x = 1$, then the value of y will be
(a) 1 (b) 0
(c) –1 (d) None of these
[Based on IMT Ghaziabad, 2002]

38. The ratio of boys and the girls in a club is 3 : 2. Which of the following could be the actual number of members?
(a) 16 (b) 18
(c) 24 (d) 25
[Based on IMT Ghaziabad, 2002]

39. Rs. 510 are divided among A, B and C in such a way that A gets $\frac{2}{3}$ of what B gets, and B gets $\frac{1}{4}$ of what C gets. Find the share of each.
(a) Rs 60, Rs 90, Rs 360
(b) Rs 150, Rs 300, Rs 60
(c) Rs 120, Rs 240, Rs 150
(d) None of these
[Based on IMT Ghaziabad, 2002]

40. Some one-rupee, 50-paise and 25-paise coins make up Rs 93.75 and their numbers are in the proportion of 3 : 4 : 5. Find the number of each type of coins.
(a) 40, 70, 75 (b) 46, 58, 75
(c) 42, 56, 70 (d) 45, 60, 75
[Based on IMT Ghaziabad, 2002]

41. A man spends Rs 500 in buying 12 tables and chairs. The cost of one table is Rs 50 and that of one chair is Rs 40. What is the ratio of the numbers of the chairs and the tables purchased ?
(a) 5 : 4 (b) 3 : 2
(c) 1 : 5 (d) None of these
[Based on IMT Ghaziabad, 2002]

42. If x varies directly as $3y + 1$ and $x = 9$ when $y = 1$, then what is the value of x when $y = 5$?
(a) 11 (b) 10
(c) 20 (d) 36
[Based on IMT Ghaziabad, 2002]

Directions (Qs. 43 to 44): These questions are based on the following information:

Out of the total 390 students studying in a college of Arts and Science, boys and girls are in the ratio of 7 : 6 respectively and the number of students studying Arts and Science are in the ratio of 3 : 7 respectively. The boys and girls studying Arts are in the ratio of 4 : 5 respectively.

43. How many boys are studying Science?
(a) 52 (b) 65
(c) 115 (d) 180
(e) None of these **[Based on IRMA, 2002]**

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44. What is the ratio between the girls studying Arts & Science respectively?
(a) 13 : 23 (b) 26 : 79
(c) 8 : 13 (d) 23 : 36
(e) None of these **[Based on IRMA, 2002]**

45. There are 361 doctors and nurses in a hospital. If the ratio of the doctors to the nurses is 8 : 11, then how many nurses are there in the hospital?
(a) 152 (b) 209
(c) 57 (d) 171
[Based on Narsee Monjee Inst. of Man. Studies 2002]

46. Which of the following ratios is the largest ?
(a) 7 : 15 (b) 15 : 23
(c) 17 : 25 (d) 21 : 29
[Based on Narsee Monjee Inst. of Man. Studies 2002]

47. Rs 180 contained in a box consist of one-rupee, 50-paise and 25-paise coins in the proportion of 2 : 3 : 4. How many 50-paise coins are there in the box?
(a) 120 (b) 60
(c) 40 (d) 240
[Based on Narsee Monjee Inst. of Man. Studies 2002]

48. The profits of a firm are to be distributed in a suitable ratio. Suitable ratio is the ratio whose terms differ by 40 and the measure of which is 2/7.
(a) 280 : 2 (b) 16 : 56
(c) 80 : 7 (d) 40 : 14
[Based on Narsee Monjee Inst. of Man. Studies 2002]

49. From a number of mangoes, a man sells half the number of existing mangoes plus 1 to the first customer, then sells 1/3rd of the remaining number of mangoes plus 1 to the second customer, then 1/4th of the remaining number of mangoes plus 1 to the third customer and 1/5th of the remaining number of mangoes plus 1 to the fourth customer. He then finds that he does not have any mango left. How many mangoes did he have originally?
(a) 12 (b) 14
(c) 15 (d) 13
[Based on FMS (Delhi) 2002]

50. Two numbers are in the ratio of 1 : 2. If 7 be added to both, their ratio changes to 3 : 5. The greater number is
(a) 20 (b) 24
(c) 28 (d) 32
[Based on FMS (Delhi) 2002]

51. The prices of a room air-conditioner and an automatic washing machine are in the ratio of 3 : 2. What would be the price of the washing machine if it costs Rs 6000 less than the air-conditioner?
(a) Rs 18000 (b) Rs 10000
(c) Rs 12000 (d) Rs 6000
[Based on I.P. Univ. 2002]

52. The ratio between the number of passengers travelling by I and II class between the two railway stations is 1 : 50, whereas the ratio of I and II class fares between the same stations is 3 : 1. If on a particular day, Rs 1325 were collected from the passengers travelling between these stations, then what was the amount collected from the II class passengers?
(a) Rs 750 (b) Rs 850
(c) Rs 1000 (d) Rs 1250
[Based on I.P. Univ. 2002]

53. Which is the larger among two numbers if they are in the ratio of 6 : 13 and their least common multiple is 312?
(a) 52 (b) 26
(c) 24 (d) 12
[Based on I.P. Univ. 2002]

54. Two numbers A and B are such that the sum of 5% of A and 4% of B is 2/3rd of the sum of 6% of A and 8% of B. Find the ratio of $A : B$.
(a) 4/3 (b) 3/4
(c) 1 (d) 2/3
(e) None of these **[Based on SCMHRD 2002]**

55. A jar contains black and white marbles. If there are ten marbles in the jar, then which of the following could NOT be the ratio of black to white marbles?
(a) 9 : 1 (b) 7 : 3
(c) 1 : 10 (d) 1 : 4
[Based on REC Tiruchirapalli, 2002]

56. Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally the entire rost of the car, then the share of each of the remaining persons increased by
(a) 1/9 (b) 1/8
(c) 1/7 (d) 7/8
[Based on REC Tiruchirapalli, 2002]

57. Determine the ratio of the number of people having characteristic X to the number of people having characteristic Y in a population of 100 subjects from the following table:

Having X and Y	10
Having X but not Y	30
Having Y but not X	20
Having neither X nor Y	40

(a) 4 : 3 (b) 3 : 2
(c) 1 : 2 (d) 2 : 3
[Based on REC Tiruchirapalli, 2002]

58. The number of employees in Obelix Menhir Co. is a prime number and is less than 300. The ratio of the number of employees who are graduates and above, to that of employees who are not, can possibly be
(a) 101 : 88 (b) 87 : 100
(c) 110 : 111 (d) 85 : 98
(e) 97 : 84 **[Based on CAT 2006]**

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59. Two persons are climbing up on two moving escalators which have 120 steps. The ratio of 1st person's speed to that of 1st escalator is 2 : 3 (steps). The ratio of 2nd person's speed to that of 2nd escalator is 3 : 5 (steps). Find the total number of steps they both have taken together.

(a) 85 (b) 93
(c) 80 (d) 75

[Based on SNAP 2007]

60. Two vessels contain spirit of 0.5 and 0.75 concentrations. If two litres from the first vessel and three litres from the second vessel are mixed, then what will be the ratio of the spirit and the water in the resultant solution?

(a) 7 : 17 (b) 13 : 7
(c) 15 : 17 (d) 17 : 15

[Based on MAT 2008]

61. To win an election, a candidate needs 3/4 of the votes cast. If, after 2/3 of the votes have been counted, a candidates has 5/6 of what he needs, then what part of the remaining ratio does he still need?

(a) 1/8 (b) 1/10
(c) 1/4 (d) 3/8

[Based on MAT 2008]

62. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20kg of fresh grapes?

(a) 2kg (b) 2.4kg
(c) 2.5kg (d) None of these

[Based on CAT 2001]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution $a = 6b = 12c \Rightarrow a : b : c = 1 : \frac{1}{6} : \frac{1}{12}$

$\Rightarrow a : b : c = 12 : 2 : 1$ ...(i)

It is also given,

$2b = 9d = 12e \Rightarrow b : d : e = \frac{1}{2} : \frac{1}{9} : \frac{1}{12}$

$\Rightarrow b : d : e = 18 : 4 : 3$ ...(ii)

From (i) and (ii),

$a : b : c : d : e = 108 : 18 : 9 : 4 : 3$

$\Rightarrow a = 108K, b = 18K, c = 9K, d = 4K, e = 3K$

Now, going through the options, the pair in option (d) is not an integer:

$\frac{a}{6}$ is an integer but $\frac{c}{d}$ is not an integer.

2. Ans. (d)

Solution Radii of the two pipes are 1cm and 2cm.

Squares of the radii of the two pipes are 1cm and 4cm.

$\therefore$ Rates of flow of the two pipes are in the ratio

$1 : \frac{1}{4}$, **i.e. 4 : 1.**

3. Ans. (d)

Solution Number of girls = 100

$\therefore$ Number of boys = 300

$\therefore$ Number of hostel dwellers = 50 + 100 = 150

$\Rightarrow$ Required ratio = 150 : 400 = 3 : 8 = $\frac{3}{8}$.

4. Ans. (b)

Solution $\frac{3}{4} : \frac{5}{6}$ **= 9 : 10.**

5. Ans. (c)

Solution $R_v = 150\%$ of A

$A = 80\%$ of R_m

$\Rightarrow \frac{2R_v}{3} = \frac{4}{5}R_m \Rightarrow \frac{R_m}{R_v} = \frac{5}{6}$.

6. Ans. (c)

Solution Total number of boys in the class = 24

Total number of girls in the class = 12

Total strength of the class = **36.**

7. Ans. (b)

Solution Let number of one-rupee coins = $2K$

Number of 50-paise coins = $3K$

Number of 25-paise coins = $4K$

$\therefore 2K + 0.50 \times 3K + 0.25 \times 4K = 216$

$\Rightarrow 2K + 1.5K + K = 216$

$\Rightarrow K = 48$

$\therefore$ No. of 50-paise coins = $3K$ **= 144.**

8. Ans. (a)

9. Ans. (b)

Solution Suppose the amount in the beginning was Rs x

Money spent on clothes = Rs $\frac{1}{3}x$

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Balance = Rs $\frac{2}{3}x$

Money spent on food = $\frac{1}{5}$ of $\frac{2}{3}x$ = Rs $\frac{2}{15}x$

Balance = $\frac{2}{3}x - \frac{2}{15}x$ = Rs $\frac{8x}{15}$

Money spent on travel = $\frac{1}{4}$ of $\frac{8x}{15}$ = Rs $\frac{2x}{15}$

$= \frac{8x}{15} - \frac{2x}{15} = \frac{6x}{15}$ = Rs $\frac{2x}{5}$

$\therefore \quad \frac{2x}{5} = 100 \Rightarrow x = \mathbf{250}.$

10. Ans. (d)

Solution After first day, $\frac{2}{3}$rd of the contents remain.

After second day, $\frac{2}{3} - \frac{3}{4}\left(\frac{2}{3}\right) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ of the contents remain.

11. Ans. (d)

Solution Let the incomes of the two persons be 4I and 5I respectively.

Let the expenditure of the two persons be 7E and 9E respectively.

$\therefore \quad 4I - 7E = 50$ and $5I - 9E = 50 \Rightarrow 20I - 35E = 250$

$20I - 36E = 200 \Rightarrow E = 50$ and $I = 100$

$\therefore$ Monthly incomes of the two persons are Rs 400 and Rs 500 respectively.

12. Ans. (a)

Solution Let, initially, the number of males and females in the bus be $3x$ and x respectively.

At the first stop, suppose m males and f females left the bus.

$\therefore$ At the first stop:

No. of the Males	No. of Females
$3x - m$	$(x - f) + 6$

$\therefore \quad \frac{3x - m}{(x - f) + 6} = \frac{2}{1} \Rightarrow 3x - m = 2x - 2f + 12$

Also $m + f = 16 \Rightarrow x = 28 - 3f$

$\therefore$ For $f = 4$, $x = 16$

$\therefore$ Total number of passengers in the beginning = $4x = 64$

For no other value of f any of the other alternatives holds good.

13. Ans. (c)

Solution Let number of boys = x

Let number of girls = y

$\therefore \quad \frac{x}{y} = B$ and $\frac{y}{x} = G$

$\therefore \quad 3(B + G) = 3\left[\frac{x}{y} + \frac{y}{x}\right] = \frac{3(x^2 + y^2)}{xy} > 3.$

14. Ans. (a)

Solution $\frac{a}{b} = \frac{2}{5}$

$$\frac{2a + 3b}{7a + 5b} = \frac{2\frac{a}{b} + 3}{7\frac{a}{b} + 5} = \frac{\frac{4}{5} + 3}{\frac{14}{5} + 5} = \frac{19}{39}.$$

15. Ans. (c)

Solution Let the two numbers be $6K$ and $13K$.

L.C.M. of $6K$ and $13K = 78K$

$\therefore \quad 78K = 312 \Rightarrow K = 4$

$\therefore$ Sum of the numbers = $6K + 13K = 19K = \mathbf{76}.$

16. Ans. (c)

Solution $\frac{A}{3} = \frac{B}{4} = \frac{C}{5} = K$

$\Rightarrow \quad A = 3K, B = 4K, C = 5K$

$\therefore \quad A : B : C = 3K : 4K : 5K = \mathbf{3 : 4 : 5}.$

17. Ans. (a)

Solution $\frac{P}{Q} = \frac{x^2 - 36}{x^2 - 49} \times \frac{x + 7}{x + 6} = \frac{x - 6}{x - 7}.$

18. Ans. (b)

Solution Let x and y be the two numbers

$\therefore \quad \frac{x}{y} = \frac{2}{3}, \quad \frac{x + 8}{y + 8} = \frac{3}{4} \Rightarrow x = 16, y = 24.$

19. Ans. (a)

Solution $\frac{P + Q}{P - Q} = \frac{\frac{P}{Q} + 1}{\frac{P}{Q} - 1}$

$= \frac{7 + 1}{7 - 1} = \frac{8}{6} = \frac{4}{3}.$

20. Ans. (c)

Solution Let the numbers be $6k$ and $13k$.

L.C.M. = 312

L.C.M. of $6k$ and $13k = 6 \times 13 \times k$

$\therefore \quad 6 \times 13 \times k = 312 \Rightarrow k = 4$

$\therefore$ Numbers are 24 and 52.

$\therefore$ Sum = $24 + 52 = \mathbf{76}.$

21. Ans. (d)

Solution $\frac{5a + 3b}{2a - 3b} = \frac{23}{5} \Rightarrow 25a + 15b = 46a - 69b$

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$\Rightarrow 21a = 84b \Rightarrow a = 4b$

$\Rightarrow \frac{a}{b} = \frac{4}{1}$.

22. Ans. (a)

Solution $\frac{P+Q}{P-Q} = \frac{\frac{P}{Q}+1}{\frac{P}{Q}-1} = \frac{7+1}{7-1} = \frac{8}{6} = \frac{4}{3}$.

23. Ans. (b)

Solution Let the number of one-rupee coins, 50-paise coins and 25-paise coins respectively be $2k$, $3k$ and $4k$.

$\therefore \quad 2k + 3k \times (0.50) + 4k \times (0.25) = 216$

$\Rightarrow \quad 2k + 1.5k + k = 216$

$\Rightarrow \quad 4.5k = 216 \Rightarrow k = 48$

$\therefore$ No. of fifty-paise coins $= 3k =$ **144.**

24. Ans. (a)

Solution $A = \frac{2}{9}(B+C)$

$A + B + C = 770 \Rightarrow A + \frac{9A}{2} = 770$

$\Rightarrow 11A = 770 \times 2 \Rightarrow A =$ **140.**

25. Ans. (c)

Solution $\frac{14-x}{17-x} = \frac{34-x}{42-x}$

$\Rightarrow \quad 588 - 56x + x^2 = 578 - 51x + x^2$

$\Rightarrow \quad x =$ **2.**

26. Ans. (a)

Solution $\frac{a}{b} = \frac{2}{5}$.

Therefore, $\frac{3a+4b}{5a+6b} = \frac{3\frac{a}{b}+4}{5\frac{a}{b}+6} = \frac{3\times\frac{2}{5}+4}{5\times\frac{2}{5}+6}$

$= \frac{26}{40} = \frac{13}{20}$.

27. Ans. (a)

Solution $\frac{5}{7} = \frac{?}{28} \Rightarrow ? = \frac{28\times 5}{7} =$ **20.**

28. Ans. (d)

Solution No. of policemen $= \frac{3}{5}\times 135 = 81$

Since number of supporters: number of policmen $= 9 : 1$, therefore number of supporters must be **729.**

29. Ans. (c)

Solution Let the monthly incomes of two persons be $5I$ and $4I$ and their monthly expenditures be $9E$ and $7E$ respectively.

$\therefore \quad 5I - 9E = 500$

$4I - 7E = 500$

$\Rightarrow \quad I = 1000, E = 500$

$\therefore$ Monthly incomes of the two persons are **Rs 5000** and **Rs 4000.**

30. Ans. (b)

Solution Let the number be x

$\therefore$ Required ratio $= \frac{x}{x+\frac{1}{5}x} = \frac{x}{\frac{6x}{5}} = \frac{5}{6}$.

31. Ans. (c)

Solution $I \propto \frac{1}{R^2}$.

32. Ans. (d)

Solution Let $y \propto p + q$, where $p \propto x$ and $q \propto \frac{1}{x}$

$\therefore \quad y = K(p + q)$,

$p = Mx$, $q = \frac{N}{x} \Rightarrow y = K\left(Mx + \frac{N}{x}\right)$

$y = \frac{10}{3}$, when $x = 3 \Rightarrow K\left(3M + \frac{N}{3}\right) = \frac{10}{3}$

$\therefore \quad MK = 2$ and $NK = -8$

$\therefore \quad p = \frac{2x}{K}$, $q = \frac{-8}{Kx}$ and so $y = 2x - \frac{8}{x}$.

33. Ans. (a)

Solution Man's age $= 4k$, say

Wife's age $= 3k$, say

$\therefore \quad \frac{4k+4}{3k+4} = \frac{9}{7} \Rightarrow k = 8.$

$\therefore$ Man's age $= 32$ years

Wife's age $= 24$ years

Suppose they were married x years ago.

$\therefore \quad \frac{32-x}{24-x} = \frac{5}{3} \Rightarrow x =$ **12.**

34. Ans. (a)

Solution $\because$ Temperature $\propto$ Volume

If Temperature $= 25°\ K$,

Volume $= 50\ m^3$

If Volume $= 200 m^3$, then

Temerpature $= 100°\ K$.

35. Ans. (b)

Solution $RD : SD : AD = 80 : 8 : 3$

(i.e. 4000 : 400 : 150).

36. Ans. (d)

Solution $x^2 \propto y^3 \Rightarrow x^2 = Ky^3$

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$\therefore$ $x = 6, y = 3 \Rightarrow K = \frac{4}{3}$

$\therefore$ $\mathbf{3x^2 = 4y^3.}$

37. Ans. (d)

Solution $y \propto \frac{1}{x^3 - x} \Rightarrow y = \frac{K}{x^3 - x}$

When $x = 2, y = \frac{1}{6}$

$\therefore$ $\frac{1}{6} = \frac{K}{8-2} \Rightarrow K = 1$

$\therefore$ $y = \frac{1}{x^3 - x}$

$\therefore$ $x = 1 \Rightarrow y = \frac{1}{0} = \infty.$

38. Ans. (d)

39. Ans. (a)

Solution $A = \frac{2}{3}B, B = \frac{1}{4}C$

$A + B + C = 510$

$\therefore$ $\mathbf{A = 60, B = 90, C = 360.}$

40. Ans. (d)

Solution Let the number of one-rupee, 50-paise, 25-paise coins be $3K$, $4K$ and $5K$ respectively.

$\Rightarrow$ $3K \times 1 + 4K \times 0.50 + 5K \times 0.25 = 93.75$

$\Rightarrow$ $3K + 2K + 1.25K = 93.75$

$\Rightarrow$ $\mathbf{K = 15.}$

41. Ans. (d)

Solution $50T + 40C = 500$

$T + C = 12 \Rightarrow C = 10, T = 2$

$\therefore$ Ratio of the number of chairs and tables = **5 : 1.**

42. Ans. (d)

Solution $x \propto 3y + 1 \Rightarrow x = K(3y + 1)$

Put $x = 9, y = 1$, we get $K = \frac{9}{4}$

$\therefore$ $x = \frac{9}{4}(3y + 1)$

$\therefore$ When $y = 5$, $\mathbf{x = 36.}$

43. Ans. (e)

Solution Number of boys = 210

Number of girls = 180

Number of students studying Arts and Science are in the ratio of 3 : 7

$\therefore$ No. of student studying Arts = 117

No. of students studying Science = 273

No. of boys studying Arts = 52

No. of girls studying Arts = 65

$\therefore$ No. of boys studying Science = 210 − 52

= **158.**

44. Ans. (a)

Solution No. of girls studying Arts = 65

No. of girls studying Science = 115

$\therefore$ Required ratio = 65 : 115 = **13 : 23**

45. Ans. (b)

Solution $\frac{11}{19} \times 361 = \mathbf{209.}$

46. Ans. (d)

Solution $\frac{7}{15} = .47, \frac{15}{23} = .65, \frac{17}{25} = .68, \frac{21}{29} = \mathbf{.72.}$

47. Ans. (a)

Solution Let the number of one-rupee, 50-paise and 25-paise coins be $2k$, $3k$ and $4k$ respectively.

$\Rightarrow$ $2k + 3k \times 0.50 + 4k \times 0.25 = 180$

$\Rightarrow$ $2k + \frac{3k}{2} + k = 180 \Rightarrow k = 40$

$\therefore$ No. of 50-paise coins = $3k$ = **120.**

48. Ans. (b)

Solution $56 - 16 = 40, \frac{16}{56} = \mathbf{\frac{2}{7}.}$

49. Ans. (b)

Solution Let the number of mangoes that the man had originally = x

No. of Mangoes sold to Balance

1st customer = $\frac{x}{2} + 1 \quad \frac{x-2}{2}$

2nd customer = $\frac{x-2}{6} + 1 \quad \frac{x-5}{3}$

3rd customer = $\frac{x-5}{12} + 1 \quad \frac{x-9}{4}$

4th customer = $\frac{x-9}{20} + 1 \quad 0$

$\therefore$ $\frac{x-9}{20} + 1 = \frac{x-9}{4} \Rightarrow \mathbf{x = 14.}$

50. Ans. (c)

Solution $\frac{x}{y} = \frac{1}{2}, \frac{x+7}{y+7} = \frac{3}{5} \Rightarrow \mathbf{x = 14, y = 28.}$

51. Ans. (c)

Solution $\frac{AC}{W} = \frac{3}{2}$

$W = AC - 6000 \Rightarrow W = \frac{3W}{2} - 6000$

$\Rightarrow 2W = 3W - 12000$

$\Rightarrow \mathbf{W = 12000.}$

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52. Ans. (d)

Solution Ratio of the amount collected from the 1st class and the 2nd class passengers = 3 : 50

∴ Amount collected from the 2nd class passengers

$$= \frac{50}{53} \times 1325 = \textbf{Rs 1250.}$$

53. Ans. (a)

Solution Let the numbers be $6K$ and $13K$

∴ $78K = 312 \Rightarrow K = 4$

∴ Larger number = 52.

54. Ans. (a)

Solution 5% of A + 4% of $B = \frac{2}{3}$[6% of A + 8% of B]

$$\Rightarrow \quad 5A + 4B = \frac{12A}{3} + \frac{16B}{3}$$

$$\Rightarrow \quad 3A = 4B \Rightarrow \frac{A}{B} = \frac{4}{3}.$$

55. Ans. (c)

Solution 1 : 10 ⇒ There are atleast 11 marbles in the jar.

56. Ans. (c)

Solution When there are eight people, the share of each person is $\frac{1}{8}$ of the total cost.

When there are seven people, the share of each person is $\frac{1}{7}$ of the total cost.

∴ Increase in the share of each person

$$= \frac{1}{7} - \frac{1}{8} = \frac{1}{56}, \text{ i.e. } \frac{1}{7} \text{ of } \frac{1}{8}, \text{ i.e. } \frac{1}{7}$$

of the original share of each person.

57. Ans. (a)

Solution No. of people having characteristic X

$$= 10 + 30 = 40$$

No. of people having characteristic Y

$$= 10 + 20 = 30$$

∴ Required ratio = 40 : 30 = **4 : 3.**

58. Ans. (e)

59. Ans. (b)

60. Ans. (d)

61. Ans. (d)

Solution Suppose total votes = x

To win a candidate required $= \frac{3}{4} \times x = \frac{3}{4}x$

When $\frac{2}{3} \times x = \frac{2}{3}x$ votes were tainted a candidate has $\frac{5}{6} \times \frac{3}{4}x = \frac{5}{8}x$

So now he needed $\frac{3}{4}x - \frac{5}{8}x = \frac{6x - 5x}{8} = \frac{x}{8}$

Votes out of remaining $x - \frac{2}{3}x = \frac{x}{3}$

∴ required ratio $= \frac{x}{8} \times \frac{3}{8} = \frac{3}{8}$

62. Ans. (c)

Solution Fresh grapes contain 10% pulp.

∴ 20 kg fresh grapes contain 2 kg pulp.

Dry grapes contain 80% pulp.

∴ 2 kg pulp would contain $\frac{2}{0.8} = \frac{20}{8}$ = **2.5 kg dry grapes.**

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Rs. 675 was divided between A, B and C. If each of them had received Rs. 5 less, their shares would have been in the ratio of 1 : 2 : 3. How much did B receive?
(a) Rs. 112.50 (b) Rs. 215
(c) Rs. 220 (d) Rs. 225

2. A merchant blends two varieties of tea from two different gardens, one costing Rs. 45 per kg and other Rs. 60 per kg, in the ratio of 7 : 3, respectively. He sells the blended variety at Rs. 54.45 per kg. The profit per cent in the transaction is
(a) 5% (b) 10%
(c) $9\frac{1}{11}\%$ (d) $11\frac{1}{9}\%$

3. The ages of Amit and Puneet are in the ratio 2 : 3. After 12 years, their ages will be in the ratio 11 : 15. The age of Puneet is
(a) 32 years (b) 42 years
(c) 48 years (d) 56 years.

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4. The proportion of zinc and copper in a brass piece is 13 : 7. How much zinc will be there in 100 kg of such a piece?
(a) 20 kg (b) 35 kg
(c) 55 kg (d) 65 kg

5. 15 litres of mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed in it, the percentage of alcohol in the new mixture will be
(a) 17 (b) $16\frac{2}{3}$
(c) $18\frac{1}{2}$ (d) 15

6. Rs. 427 is to be divided among *A*, *B* and *C* in such a manner that three times the *A's* share may be equal to four times the *B's* share and seven times the *C's* share. Share of *C* will be
(a) Rs. 81 (b) Rs. 104
(c) Rs. 84 (d) Rs. 90

7. If incomes of *A*, *B* and *C* are in the ratio of 9 : 3 : 7 and income of *C* exceeds the income of *B* by Rs. 1200 then the income of *A* is
(a) Rs. 2800 (b) Rs. 3100
(c) Rs. 2700 (d) Rs. 3200

8. If Rs. 600 is divided among *A*, *B*, *C* so that Rs. 40 more than $\frac{2}{5}$ of *A's* share, Rs. 20 more than $\frac{2}{7}$ of *B's* share and Rs. 10 more than $\frac{9}{17}$ of *C*'s share may all be equal. *C's* share is
(a) Rs. 170 (b) Rs. 280
(c) Rs. 150 (d) Rs. 210

9. Two numbers are in the ratio 2 : 3 and their H.C.F. is 6. The numbers are
(a) 12 : 18 (b) 18 : 27
(c) 14 : 21 (d) None of these

10. Two numbers are in the ratio 3 : 4. If their L.C.M. is 144, the numbers are
(a) 27, 36 (b) 24, 32
(c) 36, 48 (c) None of these

Test I

[Time: 10 min]

11. Arrange 5 : 6, 7 : 8 and 13 : 16 in ascending order of magnitude.
(a) 13 : 16 < 5 : 6 , 7 : 8
(b) 5 : 6 < 7 : 8 < 13 : 16
(c) 7 : 8 > 5 : 6 > 13 : 16
(d) None of these

12. The ratio of ages of Usha and Madhuri is 4 : 3. The sum of their ages is 28 years. The ratio of their ages after 8 years will be
(a) 2 : 3 (b) 5 : 3
(c) 6 : 5 (d) 3 : 5

13. The average age of 3 girls is 20 years and their ages are in the proportion 3 : 5 : 7. The age of youngest girl is
(a) 4 years (b) 6 years 8 months
(c) 8 years 3 months (d) 12 years

14. A father's age was 5 times his son's age 5 years ago and will be 3 times son's age after 2 years, the ratio of their present ages is
(a) 5 : 2 (b) 5 : 3
(c) 10 : 3 (d) 11 : 5

15. An alloy contains copper and zinc in the ratio 7 : 3. If the alloy contains 10.5 kg zinc, then the quantity of copper in the alloy is:
(a) 17.35 kg (b) 24.5 kg
(c) 28.2 kg (d) 31.5 kg

16. Rs. 1900 is divided between *A*, *B* and *C* so that *A's* share is $1\frac{1}{2}$ times *B's* and *B's* is $1\frac{1}{2}$ times *C's*. What is *C's* share?
(a) Rs. 800 (b) Rs. 420
(c) Rs. 400 (d) Rs. 900

17. In a bag there are coins of 50-paise, 25-paise and one rupee in the proportion 5 : 6 : 2. If there are in all Rs. 42, the number of 25-paise coins is
(a) 32 (b) 42
(c) 52 (d) 60

18. Rs. 1300 be divided into three parts *A*, *B* and *C*. How much *A* is more than *C* if their ratio is $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$?
(a) Rs. 100 (b) Rs. 200
(c) Rs. 300 (d) Rs. 600

19. In an aptitude test boys and girls both appeared. If $\frac{1}{4}$th of the boys and $\frac{3}{8}$th of the girls qualified, the fractional part of the total number of candidates who qualified is
(a) $\frac{1}{3}$ (b) $\frac{3}{8}$
(c) $\frac{5}{7}$ (d) Data is inadequate

20. A and *B* are alloys of gold and copper prepared by mining metals in proportions 7 : 2 and 7 : 11, respectively if equal quantities of alloys are melted to form a third alloy *C*, the proportion of gold and copper in *C* will be:
(a) 7 : 5 (b) 3 : 5
(c) 5 : 7 (d) 11 : 7

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution Rs. $(675 - 3 \times 5)$ is divided in the proportion $1 : 2 : 3$

$$B\text{'s share} = \frac{2}{6} \times \text{Rs. } 660 = \text{Rs. } 220$$

$\therefore$ *B's* share in Rs. 675 = Rs. $(220 + 5)$ = **Rs. 225.**

3. Ans. (c)

Solution Let their ages be $2x$ and $3x$ years

$$\frac{2x+12}{3x+12} = \frac{11}{15}$$

or, $3x = 48$ or, $x = 16$

$\therefore$ Age of Puneet = $3x$ = **48 years.**

5. Ans. (b)

Solution Alcohol = $\left(\frac{20}{100} \times 15\right)$ litres = 3 litres,

$\therefore$ water = 12 litres

New mixture contains alcohol = 3 litres, water = 15 litres

$\therefore$ Percentage of alcohol in the new mixture

$$= \left(\frac{3}{18} \times 100\right)\% = \mathbf{16\frac{2}{3}\%}.$$

7. Ans. (c)

Solution Difference between proportional incomes of *C* and *B* = 7 – 3 = 4

$\therefore$ Income of $A = \frac{9}{4} \times 1200$ = **Rs. 2700.**

9. Ans. (a)

Solution Let the numbers be $2x$, $3x$. Since 2, 3 are relatively prime, H.C.F. $\{2x, 3x\} = x$

$\therefore$ $x = 6$ $\therefore$ Numbers are **12, 18.**

11. Ans. (a)

Solution The ratios are $\frac{5}{6}, \frac{7}{8}, \frac{13}{16}$

L.C.M. of 6, 8, 16 is 48

or $\frac{5\times8}{6\times8}, \frac{7\times6}{8\times6}, \frac{13\times3}{16\times3}$ or, $\frac{40}{48}, \frac{42}{48}, \frac{39}{48}$

The denominators are the same now. Clearly the ascending series bear the relation **13 : 16 < 5 : 6 < 7 : 8.**

13. Ans. (d)

Solution Their total age = (3×20) years = 60 years.

Let their ages be $3x$, $5x$ and $7x$ years. Then,

$3x + 5x + 7x = 60$ or $x = 4$

$\therefore$ Youngest girl is $3x$ = **12 years old.**

15. Ans. (b)

Solution If zinc is 3 kg, then copper is 7 kg

If zinc is 10.5 kg, then copper is $\frac{7}{3} \times 10.5$ kg

$= 24.5$ kg

or, If copper is x kg, then $7 : 3 :: x : 10.5$

$\Rightarrow$ $x = \frac{7\times10.5}{3}$

x = **24.5 kg.**

17. Ans. (b)

Solution Ratio of 50 *P*, 25 *P* and Re. 1 coins is 5 : 6 : 2. Value of one set of 5 coins of 50 *P*, 6 coins of 25 *P* and 2 coins of one rupee

= Rs. 2.50 + Rs. 1.50 + Rs. 2 = Rs. 6.

$\therefore$ Number of such sets = 42 ÷ 6 = 7

$\Rightarrow$ Number of 25 P coins = **6 × 7 = 42**

19. Ans. (d)

Solution Let x, y be the number of boys and girls, respectively who appeared in an aptitude test.

Then, total number of students who appeared in the test = $x + y$ and $\frac{1}{4}x$ boys and $\frac{3}{8}y$ girls qualified.

$\therefore$ Total number of students who qualified = $\frac{x}{4} + \frac{3}{8}y$

$\therefore$ Ratio of qualified students to the total number $= \frac{\frac{x}{4} + \frac{3}{8}y}{x+y}$ which cannot be obtained unless x and y are known.

$\therefore$ Data is inadequate.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (d)	**6.** (c)	**8.** (a)	**10.** (c)	**12.** (c)	**14.** (c)	**16.** (c)	**18.** (c)	**20.** (a)

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8 Partnership

INTRODUCTION

In *partnership,* two or more persons carry on a business and share the profits of the business at an agreed proportion. Persons who have entered into partnership with one another are individually called *partners* and collectively called a *firm* and the name under which their business is carried on is called the *firm name*. The partnership may be simple or compound.

Simple Partnership is one in which the capital of each partner is in the business for same time.

Compound Partnership is one in which the capitals of partners are invested for different periods.

Again partner can be working partner or sleeping partner.

Sleeping Partner is one who invests the capital in the business but does not actively participate in the conduct of business.

Working Partner besides investing capital, takes part in running the business. For his work, he is either paid some salary or given a certain per cent of profit, in addition.

SOME BASIC FORMULAE

1. (a) If capitals of two partners be Rs. C_1 and Rs. C_2 for the same period and the total profit be Rs. P, then shares of the partners in the profits are

$$\text{Rs.}\left(\frac{C_1 \times P}{C_1 + C_2}\right) \text{ and Rs.}\left(\frac{C_2 \times P}{C_1 + C_2}\right).$$

(b) If the capitals of three partners be Rs. C_1, Rs. C_2 and Rs. C_3 for the same period and the total profit be Rs. P, then shares of the partners in the profits are

$$\text{Rs.}\left(\frac{C_1 \times P}{C_1 + C_2 + C_3}\right), \text{Rs.}\left(\frac{C_2 \times P}{C_1 + C_2 + C_3}\right)$$

$$\text{and Rs.}\left(\frac{C_3 \times P}{C_1 + C_2 + C_3}\right).$$

Illustration 1 *A*, *B* and *C* invested Rs. 20000, Rs. 50000 and Rs. 40000, respectively in a business. The net profit for the year was Rs. 12100 which was divided in proportion to investments. Find the amount of profit each partner earned.

Solution We have, $C_1 = 20000$, $C_2 = 50000$, $C_3 = 40000$ and $P = 12100$.

Therefore, profit share of *A*

$$= \frac{C_1 \times P}{C_1 + C_2 + C_3}$$

$$= \frac{20000 \times 12100}{20000 + 50000 + 40000}$$

$$= \frac{2}{11} \times 12100 = \text{Rs. } 2200$$

Profit share of *B*

$$= \frac{C_2 \times P}{C_1 + C_2 + C_3}$$

$$= \frac{50000 \times 12100}{20000 + 50000 + 40000}$$

$$= \frac{5}{11} \times 12100 = \text{Rs. } 5500.$$

and, profit share of *C*

$$= \frac{C_3 \times P}{C_1 + C_2 + C_3}$$

$$= \frac{40000 \times 12100}{20000 + 50000 + 40000}$$

$$= \frac{4}{11} \times 12100 = \textbf{Rs. 4400.}$$

2. (a) If the capitals of two partners be Rs. C_1 and Rs. C_2 for the periods t_1 and t_2, respectively and the total profit be Rs. P, then shares of the partners in the profits are:

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Rs. $\left(\frac{C_1 \times t_1 \times P}{C_1t_1 + C_2t_2}\right)$ and Rs. $\left(\frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2}\right)$

(b) If the capitals of three partners be Rs. C_1, Rs. C_2 and Rs. C_3 for the periods t_1, t_2 and t_3, respectively and the total profit be Rs. P, then shares of the partners in the profits are

Rs. $\left(\frac{C_1 \times t_1 \times P}{C_1t_1 + C_2t_2 + C_3t_3}\right)$, Rs. $\left(\frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2 + C_3t_3}\right)$

and, Rs. $\left(\frac{C_3 \times t_3 \times P}{C_1t_1 + C_2t_2 + C_3t_3}\right)$

Illustration 2 A, B are two partners in a business. A contributes Rs. 1200 for 5 months and B Rs. 750 for 4 months. If total profit is Rs. 450, find their respective shares.

Solution We have, $C_1 = 1200$, $C_2 = 750$, $t_1 = 5$, $t_2 = 4$ and $P = 450$.

$\therefore$ Profit share of A

$$= \frac{C_1 \times t_1 \times P}{C_1t_1 + C_2t_2} = \frac{1200 \times 5 \times 450}{1200 \times 5 + 750 \times 4}$$

$$= \frac{2700000}{9000} = \text{Rs. } 300$$

and, profit share of B

$$= \frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2}$$

$$= \frac{750 \times 4 \times 450}{1200 \times 5 + 750 \times 4}$$

$$= \frac{1350000}{9000} = \textbf{Rs. 150.}$$

SOME USEFUL SHORT-CUT METHODS

1. (a) If the capitals of two partners be Rs. C_1 and Rs. C_2 for the periods t_1 and t_2 respectively, then

$$\frac{\text{Profit of } A}{\text{Profit of } B} = \frac{C_1 \times t_1}{C_2 \times t_2}.$$

(b) If the capitals of three partners be Rs. C_1, Rs. C_2 and Rs. C_3 for the periods t_1, t_2 and t_3 respectively, then profit of A : profit of B : profit of $C = C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$.

Note: If there is a loss in the business, then

Loss of A : Loss of B : Loss of C

$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$.

Illustration 3 There are three partners A, B and C in a certain business. A puts in Rs. 2000 for 5 months, B Rs. 1200 for 6 months and C Rs. 2500 for 3 months. Find the ratio of their shares in the profit.

Solution Here, $C_1 \times t_1 = 2000 \times 5 = 10000$, $C_2 \times t_2 = 1200 \times 6 = 7200$ and $C_3 \times t_3 = 2500 \times 3 = 7500$.

$\therefore$ Profit of A : Profit of B : Profit of C

$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$

$= 10000 : 7200 : 7500$ **or 100 : 72 : 75.**

2. If the capitals of three partners are invested in the ratio $C_1 : C_2 : C_3$ and their profits are in the ratio $P_1 : P_2 : P_3$, then the ratio of timing of their investments $= \frac{P_1}{C_1} : \frac{P_2}{C_2} : \frac{P_3}{C_3}$.

Illustration 4 Anu, Manu and Tanu invested capitals in the ratio 4 : 6 : 9. At the end of the business term, they received the profits in the ratio 2 : 3 : 5. Find the ratio of time for which they invested their capitals.

Solution We have, $C_1 : C_2 : C_3 = 4 : 6 : 9$

and, $P_1 : P_2 : P_3 = 2 : 3 : 5$

Therefore, the ratio of time for which Anu, Manu and Tanu invested their capitals

$$= \frac{P_1}{C_1} : \frac{P_2}{C_2} : \frac{P_3}{C_3} = \frac{2}{4} : \frac{3}{6} : \frac{5}{9} \quad \text{or,} \quad \frac{1}{2} : \frac{1}{2} : \frac{5}{9}$$

or, **9 : 9 : 10.**

3. Three partners invested their capitals in a business. If the timing of their investments is in the ratio $t_1 : t_2 : t_3$ and their profits are in the ratio $P_1 : P_2 : P_3$, then the ratio of their capitals invested is $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$.

Illustration 5 Gupta, Singhal and Kansal start a business. If the ratio of their periods of investments are 1 : 2 : 5 and their profits are in the ratio of 3 : 4 : 5, find the ratio of capitals of Gupta, Singhal and Kansal.

Solution We have, $P_1 : P_2 : P_3 = 3 : 4 : 5$ and $t_1 : t_2 : t_3 = 1 : 2 : 5$.

$\therefore$ Required ratio $= \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3}$

$$= \frac{3}{1} : \frac{4}{2} : \frac{5}{5} \quad \text{or} \quad 3 : 2 : 1.$$

Thus, Gupta, Singhal and Kansal invested their capitals in the ratio **3 : 2 : 1.**

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Nikita and Nishita enter into a partnership investing Rs. 50000 and Rs. 40000, respectively. They agree to share profits in the ratio of their capitals. Find the share of Nikita in a profit of Rs. 22500 after one year.
(a) Rs. 12500 (b) Rs. 9500
(c) Rs. 10500 (d) None of these

2. Niki, Nisha and Anu formed a partnership with investments of Rs. 75000, Rs. 60000 and Rs. 40000, respectively. After 3 years of operation, the partnership had a net profit of Rs. 26250. What was the share of Anu in the profit?
(a) Rs. 6000 (b) Rs. 5000
(c) Rs. 8000 (d) None of these

3. Mahesh, Suresh and Ganesh entered into a partnership. Mahesh invested Rs. 16000 for 9 months. Suresh invested Rs. 12000 for 6 months and Ganesh invested Rs. 8000 for 12 months. At the end of a year there was a profit of Rs. 26000. Find the share of Suresh in the profit.
(a) Rs. 8000 (b) Rs. 7500
(c) Rs. 6000 (d) None of these

4. Sita and Gita enter into a partnership. Sita contributes Rs. 5000 while Gita contributes Rs. 4000. After 1 month, Gita withdraws $\frac{1}{4}$ part of her contribution and after 3 months from the starting Sita puts Rs. 2000 more. When Gita withdraws her money at the same time Rita also joins them with Rs. 7000. If at the end of 1 year there is a profit of Rs. 1218, what will be share of Rita in the profit?
(a) Rs. 488.47 (b) Rs. 8447.37
(c) Rs. 588.47 (d) None of these

5. *A* starts business with Rs. 3500 and 5 months after *B* joins *A* as his partner. After a year the profits are divided in the ratio of 2 : 3. How much did *B* contribute?
(a) Rs. 7000 (b) Rs. 11000
(c) Rs. 9000 (d) None of these

6. Gupta and Bansal enter into a partnership with their capitals in the ratio 5 : 6. At the end of 8 months, Gupta withdraws his capital. If they receive the profits in the ratio of 5 : 9, find how long Bansal's capital was used?
(a) 10 months (b) 12 months
(c) 14 months (d) None of these

7. Arvind began a business with Rs. 550 and was joined afterwards by Brij with Rs. 330. When did Brij join if the profits at the end of the year were divided in the ratio 10 : 3?
(a) After 4 months (b) After 6 months
(c) After 4.5 months (d) None of these

8. *A* began business with Rs. 3750 and was joined afterwards by *B* with Rs. 5000. When did *B* join if the profits at the end of the year were divided equally?
(a) After 5 months (b) After 9 months
(c) After 7 months (d) None of these

9. Anju and Brijesh enter into a partnership with their capitals in the ratio 5 : 9. At the end of 8 months, Anju withdraws her capital. If they receive the profits in the ratio 4 : 9, find how long Brijesh's capital was used?
(a) 8 months (b) 10 months
(c) 12 months (d) None of these

10. *A*, *B* and *C* invested capitals in the ratio 3 : 5 : 9; the timing of their investments being in the ratio 2 : 3 : 1. In what ratio would their profits be distrubuted?
(a) 2 : 5 : 3 (b) 3 : 2 : 5
(c) 7 : 5 : 3 (d) None of these

11. Sumit, Punit and Ramit started a business by investing their capitals in the ratio 1 : 2 : 3. At the end of the business term, they received the profits in the ratio 1 : 2 : 3. Find the ratio of time for which they invested their capitals.
(a) 1 : 1 : 1 (b) 2 : 3 : 4
(c) 2 : 4 : 3 (d) None of these

12. *A*, *B* and *C* start a business. If the ratio of their periods of investments are 2 : 3 : 6 and their profits are in the ratio of 4 : 5 : 6, then the ratio of capitals of *A*, *B* and *C* is
(a) 6 : 8 : 10 (b) 12 : 10 : 6
(c) 10 : 12 : 6 (d) None of these

13. *A*, *B* and *C* rented a pasture. *A* puts in 12 oxen for 6 months, *B* 8 oxen for 7 months and *C* 6 oxen for 8 months. If the rent of the field is Rs. 396, what rent is paid by *A*?
(a) Rs. 126 (b) Rs. 108
(c) Rs. 162 (d) Rs. 168.

14. *A*, *B*, *C* and *D* enter into partnership. *A* subscribes $\frac{1}{3}$ of the capital, *B* $\frac{1}{4}$, *C* $\frac{1}{5}$ and *D* the rest. What is the share of *D* out of a profit of Rs. 6000?

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(a) Rs. 2000 (b) Rs. 1600
(c) Rs. 1200 (d) Rs. 1300

15. *A* and *B* enter into partnership for a year. *A* contributes Rs. 1500 and *B* Rs. 2000. After 4 months, they admit *C* who contributes Rs. 2250. If *B* withdraws his contribution after 9 months, at the end of the year they share profit in the ratio
(a) 2 : 1 : 3 (b) 1 : 3 : 2
(c) 1 : 1 : 2 (d) 1 : 1 : 1

16. *A* and *B* started a business with initial investments in the ratio 5 : 7. If after one year their profits were in the ratio 1 : 2 and the period for *A*'s investment was 7 months, *B* invested the money for
(a) 6 months (b) $2\frac{1}{2}$ months
(c) 10 months (d) 4 months

17. *A* and *B* jointly invest Rs. 2100 and Rs. 3100, respectively in a firm. *A* is an active partner and hence he gets 25% of the profit separately. If their business yields them total Rs. 1040 as profit, what will be the gain of each of them?
(a) Rs. 415, Rs. 625 (b) Rs. 575, Rs. 465
(c) Rs. 515, Rs. 525 (d) Rs. 560, Rs. 480

18. Two partners invested Rs. 12500 and Rs. 8500, respectively in a business and decided that 60% of the profit incurred from the business will be equally divided between them while remaining profit will be assumed as interest on their capitals. If one of the partners gets Rs. 300 more profit than the other, what is the total profit in the business?
(a) Rs. 3937.50 (b) Rs. 4940.50
(c) Rs. 3936.50 (d) Rs. 4156

19. *A*, *B*, *C* enter into a partnership with shares in the ratio $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}$. After 4 months, *A* increases his share by 50%. If the total profit at the end of one year be Rs. 21600, then *B*'s share in the profit is
(a) Rs. 2100 (b) Rs. 2400
(c) Rs. 3600 (d) Rs. 4000

20. *A* and *B* invest in a business in the ratio 3 : 2. If 5% of the total profit goes to charity and *A*'s share is Rs. 855, total profit is
(a) Rs. 1576 (b) Rs. 1537.50
(c) Rs. 1500 (d) Rs. 1425

21. *B* is a sleeping partner and *A* working. *A* puts in Rs. 5000 and *B* puts in 6000. *A* received $12\frac{1}{2}\%$ of profit for managing the business and rest is divided in propoftion to their capitals. *A*'s share of profit in a profit of Rs. 880 is
(a) Rs. 350 (b) Rs. 400
(c) Rs. 420 (d) Rs. 460

22. A starts business with a capital of Rs. 1200. *B* and *C* join with some investments after 3 and 6 months, respectively. If at the end of a year, the profit is divided in the ratio 2 : 3 : 5 respectively, what is *B*'s investment in the business?
(a) Rs. 2400 (b) Rs. 1800
(c) Rs. 3600 (d) Rs. 6000

23. *A*, *B* and *C* entered into a partnership with Rs. 3512420, Rs. 4222180 and Rs. 4065400. After 2 years *A* withdrew Rs. 11 lacs and at the same time *C* invested Rs. 8 lacs more. If at the end of 3 years the profit is Rs. 1053000, what is the share of *C* in the profit?
(a) Rs. 283117.80 (b) Rs. 379996.20
(c) Rs. 399866 (d) Rs. 299866
(e) None of these

24. *A* and *B* enter into partnership. A supplies whole of the capital amounting to Rs. 45000 with the condition that the profits are to be equally divided and that *B* pays *A* interest on half of the capital at 10% per annum, but receives, Rs. 120 per month for carrying on the concern. When *B*'s income is one-half of *A*'s income their total yearly profit is
(a) Rs. 9180 (b) Rs. 7150
(c) Rs. 3060 (d) Rs. 1440
(e) None of these

25. *A*, *B* and *C* entered into a partnership by investing Rs. 12000, Rs. 15000 and Rs. 18000, respectively. *A* is also a working partner and getting 15% of the annual profit for his work. If *B* and *C* got Rs. 8500 and Rs. 10200, respectively from the annual profit as their shares, what amount did *A* get from the annual profit?
(a) Rs. 10,500 (b) Rs. 11,500
(c) Rs. 11,300 (d) Rs. 14,000

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $C_1 = 50000$, $C_2 = 40000$ and $P = 22500$.
$\therefore C_1 + C_2 = 50000 + 40000 = 90000$.

$\therefore$ Nikita's share $= \frac{C_1 \times P}{C_1 + C_2} = \frac{50000 \times 22500}{90000}$

$= \frac{5}{9} \times 22500 =$ **Rs. 12500.**

2. Ans. (a)

Solution Ratio of the capitals
$= 75000 : 60000 : 40000 = 15 : 12 : 8$

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$\therefore$ Profit-sharing ratio = 15 : 12 : 8

Sum of the profit-sharing ratio = 15 + 12 + 8 = 35

Total profit = Rs. 26250

$\therefore$ Anu's share = $\frac{8}{35} \times 26250$ = **Rs. 6000.**

3. Ans. (c)

Solution Here, $C_1 = 16000$, $C_2 = 12000$, $C_3 = 8000$, $t_1 = 9$, $t_2 = 6$, $t_3 = 12$ and $P = 26000$.

Suresh's share in the profit

$$= \frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2 + C_3t_3}$$

$$= \frac{12000 \times 6 \times 26000}{16000 \times 9 + 12000 \times 6 + 8000 \times 12}$$

$$= \frac{1872000000}{312000} = \textbf{Rs. 6000.}$$

4. Ans. (a)

Solution Here,

$C_1 \times t_1 = 5000 \times 12 + 2000 \times 9 = 78000$,

$C_2 \times t_2 = 4000 \times 1 + 3000 \times 11 = 37000$,

$C_3 \times t_3 = 7000 \times 11 = 77000$ and $P = 1218$.

$\therefore$ Share of Rita in the profit

$$= \frac{C_3 \times t_3 \times P}{C_1t_1 + C_2t_2 + C_3t_3}$$

$$= \frac{77000 \times 1218}{78000 + 37000 + 77000} = \textbf{Rs. 488.47.}$$

5. Ans. (c)

Solution We have, $C_1 \times t_1 = 3500 \times 12 = 42000$ and $C_2 \times t_2 = x \times 7 = 7x$.

Then, $\frac{\text{Profit for } A}{\text{Profit for } B} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \quad \frac{2}{3} = \frac{42000}{7x}$ or, $x = \frac{42000 \times 3}{2 \times 7}$ = **Rs. 9000.**

6. Ans. (b)

Solution Let Bansal's capital be used for x months.

Then, we have $\frac{5 \times 8}{6 \times x} = \frac{5}{9}$

$\Rightarrow \quad x = \frac{5 \times 8 \times 9}{6 \times 5} = 12$ months.

$\therefore$ Bansal's capital was used for **12** months.

7. Ans. (b)

Solution Let Brij remain in the business for x months.

We have, $C_1 \times t_1 = 550 \times 12 = 6600$

$C_2 \times t_2 = 330 \times x = 330x$

$\therefore \quad \frac{\text{Arvind's share of profit}}{\text{Brij's share of profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{10}{3} = \frac{6600}{330x} \Rightarrow x = \frac{6600 \times 3}{330 \times 10}$ = **6 months.**

8. Ans. (b)

Solution Let B remain in the business for x months.

We have, $C_1 \times t_1 = 3750 \times 12 = 45000$

and, $C_2 \times t_2 = 5000 \times x = 5000x$

$\therefore \quad \frac{A\text{'s share in profit}}{B\text{'s share in profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{1}{1} = \frac{45000}{5000x}$ or, $x = \frac{45000}{5000}$ = **9 months**

9. Ans. (b)

Solution Let Brijesh's capital be used for x months.

Ratio of capitals of Anju and Brijesh are 5 : 9.

Let the capitals of Anju and Brijesh be Rs. $5y$ and Rs. $9y$.

We have, $C_1 \times t_1 = 5y \times 8 = 40y$

and, $C_2 \times t_2 = 9y \times x = 9yx$

$\therefore \quad \frac{\text{Anju's share of profit}}{\text{Brijesh's share of profit}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$\Rightarrow \frac{4}{9} = \frac{40y}{9yx} \Rightarrow x = \frac{40 \times 9}{4 \times 9}$ = **10 months.**

10. Ans. (a)

Solution Ratio of capitals of A, B and C are 3 : 5 : 9. Let the capitals of A, B and C be $3x$, $5x$ and $9x$, respectively.

Ratio of timing of their investments are 2 : 3 : 1. Let A, B and C invest their capitals for $2y$, $3y$ and y months, respectively.

Then, profit of A : profit of B : profit of C

$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$

$= 3x \times 2y : 5x \times 3y : 9x \times y$

$= 6 : 15 : 9$ **or, 2 : 5 : 3.**

11. Ans. (a)

Solution We have, $C_1 : C_2 : C_3 = 1 : 2 : 3$

and, $P_1 : P_2 : P_3 = 1 : 2 : 3$.

$\therefore$ Required ratio = $\frac{P_1}{C_1} : \frac{P_2}{C_2} : \frac{P_3}{C_3} = \frac{1}{1} : \frac{2}{2} : \frac{3}{3}$

or, 1 : 1 : 1.

Thus, Sumit, Punit and Ramit invested their capitals for equal period of time.

12. Ans. (b)

Solution We have, $P_1 : P_2 : P_3 = 4 : 5 : 6$ and $t_1 : t_2 : t_3 = 2 : 3 : 6$.

$\therefore$ Required ratio = $\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{4}{2} : \frac{5}{3} : \frac{6}{6}$

or, 12 : 10 : 6.

Thus, A, B and C invested their capitals in the ratio 12 : 10 : 6.

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13. Ans. (c)

Solution Ratio in which A, B, C pay the rent
$= (6 \times 12) : (8 \times 7) : (6 \times 8)$
$= 72 : 56 : 48$
$= 9 : 7 : 6$

Rent paid by $A = \frac{9}{22} \times \text{Rs. } 396 = \mathbf{Rs.\ 162.}$

14. Ans. (d)

Solution D's capital $= 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{5} = \frac{13}{60}$

Profit ratio of A, B, C, D is $\frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{13}{60}$

$\Rightarrow \quad 20 : 15 : 12 : 13$

$\therefore$ Share of $D = \frac{13}{60} \times \text{Rs. } 6000 = \mathbf{Rs.\ 1300.}$

15. Ans. (d)

Solution Obviously, C invests for $12 - 4 = 8$ months.

$\therefore$ Equivalent capitals are:

Rs. 1500×1; Rs. $2000 \times \frac{9}{12} = \text{Rs. } 1500$,

Rs. $2250 \times \frac{8}{12} = \text{Rs. } 1500$

$\therefore$ Profit is to be shared in the ratio
$1500 : 1500 : 1500 = \mathbf{1 : 1 : 1.}$

16. Ans. (c)

Solution Let investments of A and B respectively be $5x$ and $7x$ and period of B's investment be y months

Then, $\frac{(5x) \times 7}{(7x) \times y} = \frac{1}{2} \Rightarrow \mathbf{y = 10.}$

17. Ans. (b)

Solution Total profit in a business = Rs. 1040

Separate profit for A = 25% of Rs. 1040

$= \frac{1040 \times 25}{100} = \text{Rs. } 260$

Remaining profit = Rs. $(1040 - 260)$ = Rs. 780

The remaining profit will be divided in proportion to their capitals.

$\therefore$ Ratio between capitals of A and B = 2100 : 3100 = 21 : 31

Sum of proportional = 21 + 31 = 52

$\therefore$ A's profit $= \frac{21}{52} \times 780 = \text{Rs. } 315$

B's profit $= \frac{31}{52} \times 780 = \text{Rs. } 465$

Total profit of A = Rs. $(315 + 260)$ = Rs. 575

Therefore, A and B will make profit of Rs. 575 and Rs. 465, respectively.

18. Ans. (a)

Solution Suppose total profit in the business = Rs. x

$\therefore$ 60% of total profit = 60% of x = Rs. $\frac{3x}{5}$

$\therefore$ The two partners will get profit of Rs. $\frac{3x}{10}$ and Rs. $\frac{3x}{10}$, respectively.

$\therefore$ Remaining profit $= x - \frac{3x}{5} = \text{Rs. } \frac{2x}{5}$.

The remaining profit assumed as interest on the capital will be divided in the proportion of their capitals.

Ratio between their capitals = 12500 : 8500 = 25 : 17

Sum of proportionals = 25 + 17 = 42

$\therefore$ Ist partner's profit $= \frac{25}{42} \times \frac{2x}{5} = \text{Rs. } \frac{5x}{21}$

2nd partner's profit $= \frac{17}{42} \times \frac{2x}{5} = \text{Rs. } \frac{17x}{105}$

Given : $\frac{3x}{10} + \frac{5x}{21} = \frac{3x}{10} + \frac{17x}{105} + 300$

$\therefore \quad \frac{8x}{105} = 300$

or, $x = \frac{105 \times 300}{8} = \text{Rs. } 3937.50$

$\therefore$ Total profit in the business = **Rs. 3937.50.**

19. Ans. (d)

Solution Given ratio $= \frac{7}{2} : \frac{4}{3} : \frac{6}{5} = 105 : 40 : 36$

Let they initially invest Rs. 105, Rs 40 and Rs. 36, respectively.

Ratio of investments
$= [105 \times 4 + (150\% \text{ of } 105) \times 8] : (40 \times 12) : (36 \times 12)$
$= 1680 : 480 : 432 = 35 : 10 : 9$

B's share = Rs. $\left(21600 \times \frac{10}{54}\right) = \mathbf{Rs.\ 4000.}$

20. Ans. (c)

Solution Let the total profit be Rs. 100.

After paying the charity, A's share = Rs. $\left(95 \times \frac{3}{5}\right)$
= Rs. 57

If A's share is Rs. 57, total profit = Rs. 100

If A's share is Rs. 855, total profit = Rs. $\left(855 \times \frac{100}{57}\right)$
$= \mathbf{Rs.\ 1500.}$

21. Ans. (d)

Solution A's share for managing the business

$= 12\frac{1}{2}\%$ of Rs. 880 = Rs. 110

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Remaining profit = Rs. 770

Profit ratio of A and B = 5 : 6

A's share = $\frac{5}{11}$ of Rs. 770 = Rs. 350

A's total profit = Rs. 350 + Rs. 110 = **Rs. 460.**

22. Ans. (a)

Solution Profit ratio of A, B and C is

$(1200 \times 12) : (x \times 9) : (y \times 6) = 2 : 3 : 5$

$\Rightarrow \frac{1200 \times 12}{2} = \frac{9x}{3}$ $\therefore$ x = **Rs. 2400.**

23. Ans. (e)

Solution Share of C in profit = $\frac{C_3 t_3 \times P}{C_1 t_1 + C_2 t_2 + C_3 t_3}$

[Here, $C_1 \times t_1 = 3512420 \times 2 + 2412420 \times 1$

$C_2 \times t_2 = 4222180 \times 3$ $C_3 \times t_3$

$= 4065400 \times 2 + 4865400 \times 1$ and $P = 105300$]

$$= \frac{(4065400 \times 2 + 4865400 \times 1) \times 1053000}{(3512420 \times 2 + 2412420 \times 1) + 4222180 \times 3 + (4065400 \times 2 + 4865400 \times 1)}$$

= **Rs. 389886.**

24. Ans. (a)

Solution Let the total profit be Rs. x

Salary given to B = Rs. 1440

$\therefore$ Net profit = $x - 1440$

$\therefore$ Share of A and B each = $\frac{x - 1440}{2}$

Interest given by B to A = $\frac{10}{100} \times 22500$ = Rs. 2250.

$$\therefore \frac{\frac{x-1440}{2} + 1440 - 2250}{\frac{x-1440}{2} + 2250} = \frac{1}{2} \text{ or, } \frac{x - 3060}{x + 3060} = \frac{1}{2}$$

$\therefore$ x = **Rs. 9180.**

25. Ans. (c)

Solution Capital ratio of A, B, C is 12000 : 15000 : 18000 = 4 : 5 : 6

Profit ratio of B and C = 8500 : 10200 = 5 : 6

$\therefore$ A's share of profit = $\frac{4}{15} \times$ Rs. 8500 = Rs. 6800

85% of x = 6800 + 8500 + 10200 = 25500

$\therefore$ 15% of x = $\frac{25500 \times 15}{85}$ = 4500

Total profit of A = Rs. 6800 + Rs. 4500 = **Rs. 11300.**

QUESTIONS FROM COMPETITIVE EXAMS

1. A, B, C subscribe Rs 50000 for business. A subscribes 4000 more than B and B Rs 5000 more than C. Out of total profit of Rs 35000 A receives:

(a) Rs 11900 (b) Rs 8400

(c) Rs 14700 (d) Rs 13600

[Based on MAT 2005]

2. A and B enter into a partnership with Rs 50,000 and Rs 60,000 respectively. C joins them after x months contributing Rs 70,000 and B leaves x months before the end of the year. If they share the profit in the ratio of 20 : 18 : 21, then find the value of x.

(a) 9 (b) 3

(c) 6 (d) 8

[Based on IIT Joint Man. Ent. Test 2004]

3. Rs 120 are divided among X, Y and Z so that X's share is Rs 20 more than Y's share and Rs 20 less than Z's share. What is Y's share?

(a) Rs 40 (b) Rs 30

(c) Rs 25 (d) Rs 20

(e) None of the above **[Based on IIFT 2003]**

4. Surendra, Rajendra and Manindra invested some amount in a business in the ratio of 5 : 7 : 6 respectively. In the next year, they increased their investments by 26%, 20% and 15% respectively. The profit earned during the second year should be distributed in what ratio among Surendra, Rajendra and Manindra

(a) 31 : 27 : 21 (b) 21 : 28 : 23

(c) 26 : 20 : 15 (d) Cannot be determined

(e) None of these **[Based on IRMA, 2002]**

5. X and Y start a business. X invests Rs 3000 for 4 months and Y invests Rs 2000 for 6 months. How much should X be paid out of a total profit of Rs 500?

(a) Rs 200 (b) Rs 300

(c) Rs 250 (d) Rs 350

[Based on FMS (Delhi) 2002]

6. Rahul started a business with a capital of Rs 8,000. After six months, Sanjay joined him with an investment of some capital. If at the end of the year each of them gets equal amount as profit, how much did Sanjay invest in the business?

(a) Rs 17,500 (b) Rs 18,000

(c) Rs 16,000 (d) Rs 16,500

[Based on MAT 2008]

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EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let $C = x$

$B = x + 5000$, $A = x + 5000 + 4000$

$\therefore x + x + 5000 + x + 9000 = 50000$

$\therefore x = 12000$

$\therefore \quad A : B : C = 21000 : 17000 : 12000 = 21 : 17 : 12$

A's share $= 35000 \times \frac{21}{50} =$ **Rs 14700.**

2. Ans. (b)

Solution Let B leaves x months before the end of the year.

$\therefore$ B stays for $(12 - x)$ months.

C joins after x months.

$\therefore$ C also remains for $(12 - x)$ months.

$\therefore$ Profits are shared among A, B and C in the ratio.

$50000 \times 12 : 60000\,(12 - x) : 70000\,(12 - x)$

$= 20 : 18 : 21$ (Given)

$\therefore \quad \frac{60}{6(12-x)} = \frac{20}{18} \Rightarrow$ **$x = 3$.**

3. Ans. (d)

Solution $X + Y + Z = 120$

$X = Y + 20$, $X = Z - 20$

$\Rightarrow \quad (Y + 20) + Y + (X + 20) = 120$

$\Rightarrow \quad X + 2Y = 80$

$\Rightarrow \quad (Y + 20) + 2Y = 80$

$\Rightarrow \quad$ **$3Y = 60 \Rightarrow Y = 20$.**

4. Ans. (b)

Solution $5K + 26\%$ of $5K : 7K + 20\%$ of $7K : 6K + 15\%$ of $6K$

$\Rightarrow \quad \frac{630}{100}K : \frac{840}{100}K : \frac{690}{100}K$

$\Rightarrow \quad 63 : 84 : 69$

$\Rightarrow \quad$ **21 : 28 : 23.**

5. Ans. (c)

Solution The ratio in which X and Y should be paid the prfoit out of Rs 500.

$= 3000 \times 4 : 2000 \times 6 = 12 : 12 = 1 : 1$

$\therefore$ Each of X and Y should be paid **Rs 250.**

6. Ans. (c)

Solution $\frac{8000 \times 12}{x \times 6} = \frac{1}{1} \Rightarrow x = 16000$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. A began business with Rs. 6000 and was joined afterwards by B with Rs. 8000. At the end of the year B got one-third of the total profit. When did B join?

(a) After 3 months (b) After 4 months

(c) After 8 months (d) After 9 months

2. Dolly, Veena and Aruna started a shop by making investments in the ratio $\frac{5}{2} : \frac{4}{3} : \frac{7}{5}$. After 4 months Dolly increased her capital by 70%. If the total profit at the end of one year be Rs. 9600, Aruna's share in profit was

(a) Rs. 2100 (b) Rs. 4500

(c) Rs. 2000 (d) Rs. 3200

3. A and B are two business partners. They decided to distribute equally 60% of the profit and remaining 40% as the interest on their capital. If they invested Rs. 1250 and Rs. 850, respectively and A received Rs. 30 more than B, the total profit was

(a) Rs. 371.25 (b) Rs. 423.75

(c) Rs. 325.50 (d) Rs. 393.75

4. A, B and C are partners in a business and their shares are in the proportion $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$. A withdraws half his capital at the end of 4 months and after 8 months more a profit of Rs. 1694 is divided. A's share is

(a) Rs. 560 (b) Rs. 630

(d) Rs. 440 (d) Rs. 504

5. A, B and C enter partnership by making investments in the ratio 3 : 5 : 7. After a year, C invests another Rs. 337600 while A withdrew Rs. 45600. The ratio of investments then changes to 24 : 59 : 167. How much did A invest initially?

(a) Rs. 45600 (b) Rs. 96000

(c) Rs. 141600 (d) None of these

6. A and B enter into partnership, investing Rs. 12000 and Rs. 16000, respectively. After 8 months, C joins them with a capital of Rs. 15000. The share of C in a profit of Rs. 45600 after 2 years will be

(a) Rs. 21200 (b) Rs. 19200

(c) Rs. 14400 (d) Rs. 12000

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7. *A* began business with Rs. 4200 and was joined afterwards by *B* with Rs. 7200. When did *B* join if the profits at the end of the year were divided equally?
(a) After 7 months (b) After 4 months
(c) After 5 months (d) After 8 months

8. *A* and *B* began business with Rs. 3000 and Rs. 4000. After 8 months, *A* withdraws Rs. 1000 and *B* advances Rs. 1000 more. At the end of the year, their profits, amounted to Rs. 630. Find the share of *A*
(a) Rs. 240 (b) Rs. 350
(c) Rs. 340 (d) Rs. 390

9. Rahul started a business with a capital of Rs. 8000. After six months, Sanjay joined him with investment of some capital. If at the end of the year each of them gets equal amount as profit, how much did Sanjay invest in the business?
(a) Rs. 18000 (b) Rs. 17500
(c) Rs. 16000 (d) Rs. 16500

10. Rakesh and Mukesh are partners in a business. They divide between them the profit incurred in the business in such a proportion that $\frac{2}{5}$th portion of the profit gained by Rakesh is equal to $\frac{1}{3}$rd portion of the profit gained by Mukesh. If total profit in the business is Rs. 1210, find the profit earned by Mukesh
(a) Rs. 780 (b) Rs. 550
(c) Rs. 680 (d) Rs. 590

Test II

[Time: 10 min]

11. *A* started a business with Rs. 3200. After sometime *B* joined him with Rs. 4800. If at the end of one year they got equal profit, after how many months did *B* join?
(a) 6 (b) 5
(c) 3 (d) 4
(e) None of these

12. Ramesh, Mahesh and Naresh jointly hired a vedio cassette. If they use it for 6 hours, 10 hours and 12 hours, respectively, what is the share of Naresh for rent?
(a) Rs. 75 (b) Rs. 125
(c) Rs. 35 (d) Rs. 150
(e) None of these

13. Two partners invest Rs. 12500 and Rs. 8500, respectively in a business. If one partner gets Rs. 300 more than the other in the profit, what is the total profit?
(a) Rs. 1475 (b) Rs. 1575
(c) Rs. 1675 (d) Rs. 1570
(e) Rs. 157

14. *A* and *B* enter into partnership investing Rs. 1200 and Rs. 1600 respectively. After 8 months *C* joins them with a capital of Rs. 1500. If in 2 years profit earned is Rs. 4560, *A*'s share will be
(a) Rs. 1440 (b) Rs. 1200
(c) Rs. 1920 (d) Rs. 1320

15. *A* puts in Rs. 600 more in a business than *B*, but *B* has invested his capital for 4 months. If *A* gets Rs. 48 more than *B* out of the total profit of Rs. 528, money invested by *B* was
(a) Rs. 1800 (b) Rs. 600
(c) Rs. 1200 (d) None of these

16. *A*, *B* and *C* have respectively invested Rs. 20, Rs. 18 and Rs. 12 (all in thousands) jointly in a business. *A* and *B* receive respectively, 12% and 8% of the annual profits for services, the remainder being divided among *A*, *B* and *C* in proportion to their capitals. If at the end of the year *A* receives altogether Rs. 648 more than *B*, *C*'s share is
(a) Rs. 3960 (b) Rs. 3312
(c) Rs. 1728 (d) Rs. 2400

17. *A*, *B* and *C* are three partners. They altogether invested Rs. 1400 in business. At the end of the year *A* got Rs. 337.50, *B* Rs. 1125.00 and *C* Rs. 637.50 as profit. The difference between investments of *B* and *A* was
(a) Rs. 4200 (b) Rs. 3200
(c) Rs. 2200 (d) Rs. 5250

18. If 2 (*A*'s capital) = 3 (*B*'s capital) = 5 (*C*'s captial), out of a total profit of Rs. 930, *C* gets
(a) Rs. 180 (b) Rs. 450
(c) Rs. 300 (d) Rs. 350

19. *A* and *B* rent a pasture for 10 months. A puts in 80 oxen for 7 months. How many can *B* put in for the remaining 3 months if he pays half as much again as *A*?
(a) 280 (b) 250
(c) 220 (d) 180

20. *A*, *B* and *C* purchase a form for Rs. 10000 of which *A* pays Rs. 4000. They sell it so as to gain a certain sum of which *B* gets Rs. 275 and *C* Rs. 175. *A*'s share is
(a) Rs. 275 (b) Rs. 300
(c) Rs. 325 (d) Rs. 250

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution Profit ratio of A and $B = \frac{2}{3} : \frac{1}{3} = 3 : 1$

If B joins after x months, then

$(6000 \times 12) : (8000) \times (12 - x) :: 2 : 1$

$\Rightarrow 12 \times 6000 = 2 \times 8000\ (12 - x) \Rightarrow \boldsymbol{x = 9.}$

3. Ans. (d)

Solution Let Rs. y be the total profit.

Then, Rs. $(0.4\,y)$ is to be divided in the ratio 1250 : 850, i.e. 25 : 17 and Rs. $(0.6y)$ equally

$\therefore$ A gets $= \frac{(0.6\,y)}{2} + \frac{25}{42} \times 0.4\,y = 0.3y + \frac{5}{21}y$

B gets $= 0.3y + \frac{17}{42} \times 0.4y = 0.3y + \frac{3.4}{21}y$

$\therefore$ Difference between A's and B's share $= \frac{1.6}{21}y$

$$\frac{1.6}{21}y = 30 \quad \therefore \quad y = \frac{30 \times 21}{1.6} = \mathbf{393.75.}$$

5. Ans. (c)

Solution Let initial investments be $3x$, $5x$ and $7x$ rupees.

$(3x - 45600) : 5x : (7x + 337600) = 24 : 59 : 167$

$\therefore \frac{3x - 45600}{5x} = \frac{24}{59}$ or, $x = 47200$

$\therefore$ Initial investment of A = Rs. (47200×3)

= **Rs. 141600.**

7. Ans. (c)

Solution Let B join after x months so that he remains in the business for $(12 - x)$ months

$\Rightarrow 4200 \times 12 = 7200 \times (12 - x) \Rightarrow \boldsymbol{x = 5.}$

9. Ans. (c)

Solution Investment by Rahul = Rs. 8000 for 12 months

Capital of Rahul = Rs. 8000×12 = Rs. 96000 for 1 month

Suppose Sanjay invested Rs. x for 6 months

$\therefore$ Capital of Sanjay = Rs. $6x$ for 1 month

Since they get equal amounts as profit

$\therefore$ Their investment will also be same

Therefore, $6x = 96000$

or, $x = \frac{96000}{6}$ = Rs. 16000

Sanjay invested Rs. 16000 in the business.

11. Ans. (d)

Solution Capital of A for 1 month $= 3200 \times 12$

= Rs. 38400

Since A and B got equal profit

$\therefore$ Capital of B for 1 month should be Rs. 38400

$\therefore$ Duration of B to Rs. 4800 $= \frac{38400}{4800} = 8$ months

$\therefore$ B joined A after $12 - 8$, i.e. 4 months.

13. Ans. (b)

Solution Ratio in investment = 12500 : 8500 = 25 : 17

Sum of ratios = 25 + 17 = 42

Difference of ratio = 25 – 17 = 8

$\therefore$ Total profit $= \frac{42 \times 300}{8}$ = **Rs. 1575.**

15. Ans. (c)

Solution Let A's and B's investments were Rs. $(600 + x)$, and Rs. x, respectively.

Then, equivalent capitals are :

Rs. $4\,(600 + x)$, Rs. $5x$

$\therefore$ Profit of Rs. 528 is to be divided in the ratio $4(600 + x) : 5x$

$\therefore \frac{\{4(600 + x) - 5x\} \times 528}{4(600 + x) + 5x} = 48$

$\therefore (2400 - x) \times 11 = 2400 + 9x$

$\therefore 20x = 24000 \quad \therefore \quad x = 1200$

$\therefore$ B's capital = **Rs. 1200.**

17. Ans. (d)

Solution Shares are in the ratio : 337.5 : 1125 : 637.5, i.e. 0.9 : 3 : 1.7, i.e. 9 : 30 : 17

$\therefore$ The capitals of A, B and C were in the ratio 9 : 30 : 17

$\therefore$ Difference between B's and A's capitals

$$= \frac{(30 - 9)}{9 + 30 + 17} \times 14000$$

$$= \frac{21}{56} \times 14000 = \mathbf{Rs.\ 5250.}$$

19. Ans. (a)

Solution A's share : B's share (of rent) $= 1 : 1\frac{1}{2} = 2 : 3$

A puts in 80 oxen for 7 months or he can put 80×7, i.e. 560 oxen for one month.

Since B's share of rent is $\frac{3}{2}$ of A's rent

$\therefore$ B can put in $\frac{3}{2} \times 560$ oxen for one month

$\therefore \frac{1}{3} \times \left(\frac{3}{2} \times 560\right)$, i.e. 280 oxen for **3 months.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (a)	**6.** (d)	**8.** (a)	**10.** (b)	**12.** (d)	**14.** (a)	**16.** (c)	**18.** (a)	**20.** (a)

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9 Profit and Loss

INTRODUCTION

Business transactions have now-a-days become common feature of life.When a person deals in the purchase and sale of any item, he either gains or loses some amount generally. The aim of any business is to earn profit. The commonly used terms in dealing with questions involving sale and purchase are:

Cost Price The cost price of an article is the price at which an article has been purchased. It is abbreviated as C.P.

Selling Price The selling price of an article is the price at which an article has been sold. It is abbreviated as S.P.

Profit or Gain If the selling price of an article is more than the cost price, there is a gain or profit.

Thus, Profit or Gain = S.P. – C.P.

Loss If the cost price of an article is greater than the selling price, the seller suffers a loss.

Thus, Loss = C.P. – S.P.

Note that profit and loss are always calculated with respect to the cost price of the item.

Illustration 1. (i) If C.P. = Rs. 235, S.P. = Rs. 240, then profit = ?

(ii) If C.P. = Rs. 116, S.P. = Rs. 107, then loss = ?

Solution (i) Profit = S.P. – C.P. = 240 –235 = Rs. 5.

(ii) Loss = C.P. – S.P. = 116 – 107 = **Rs. 9.**

SOME BASIC FORMULAE

1. Gain on Rs. 100 is *Gain per cent*

$$\text{Gain\%} = \frac{\text{Gain} \times 100}{\text{C.P.}}$$

Loss on Rs. 100 is *Loss per cent*

$$\text{Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}}$$

Illustration 2 The cost price of a shirt is Rs. 200 and selling price is Rs. 250. Calculate the % profit.

Solution We have, C.P. = Rs. 200, S.P. = Rs. 250.

Profit = S.P. – C.P. = 250 – 200 = Rs. 50.

$$\therefore \quad \text{Profit\%} = \frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{50 \times 100}{200} = \mathbf{25\%}.$$

Illustraton 3 Anu bought a necklace for Rs. 750 and sold it for Rs. 675. Find her percentage loss.

Solution Here, C.P. = Rs. 750, S.P. = Rs. 675.

Loss = C.P. – S.P. = 750 – 675 = Rs. 75.

$$\therefore \quad \text{Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{75 \times 100}{750} = \mathbf{10\%}.$$

2. When the selling price and gain% are given:

$$\text{C.P.} = \left(\frac{100}{100 + \text{Gain\%}}\right) \times \text{S.P.}$$

3. When the cost and gain per cent are given:

$$\text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

Explanation

$$\text{Since Profit\%} = \frac{\text{Profit} \times 100}{\text{C.P.}} = \left[\frac{(\text{S.P.} - \text{C.P.}) \times 100}{\text{C.P.}}\right]$$

$$\therefore \quad \frac{\text{Profit\%}}{100} = \frac{\text{S.P.}}{\text{C.P.}} - 1$$

$$\text{or,} \quad \frac{\text{S.P.}}{\text{C.P.}} = 1 + \frac{\text{Profit\%}}{100}$$

$$\therefore \quad \text{S.P.} = \left(\frac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$$

$$\text{and,} \quad \text{C.P.} = \left(\frac{100}{100 + \text{Profit\%}}\right) \times \text{S.P.}$$

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4. When the cost and loss per cent are given:

$$\text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$$

5. When the selling price and loss per cent are given:

$$\text{C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.}$$

Explanation

$$\text{Since Loss\%} = \frac{\text{Loss} \times 100}{\text{C.P.}}$$

$$= \left[\frac{(\text{C.P.} - \text{S.P.}) \times 100}{\text{C.P.}}\right]$$

$$\therefore \quad \frac{\text{Loss\%}}{100} = 1 - \frac{\text{S.P.}}{\text{C.P.}} \text{ or } \frac{\text{S.P.}}{\text{C.P.}} = 1 - \frac{\text{Loss\%}}{100}$$

$$\therefore \quad \text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$$

$$\text{and, C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.}$$

Illustration 4 Mr. Sharma buys a cooler for Rs. 4500. For how much should he sell so that there is a gain of 8%?

Solution We have, C.P. = Rs. 4500, gain% = 8%.

$$\therefore \quad \text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$= \left(\frac{100 + 8}{100}\right) \times 4500$$

$$= \frac{108}{100} \times 4500 = \textbf{Rs. 4860.}$$

Illustration 5 By selling a fridge for Rs. 7200, Pankaj loses 10%. Find the cost price of the fridge.

Solution We have, S.P. = Rs. 7200, loss% = 10%.

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 - \text{Loss\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 - 10}\right) \times 7200$$

$$= \frac{100}{90} \times 7200 = \textbf{Rs. 8000.}$$

Illustration 6 By selling a pen for Rs. 99, Mohan gains $12\frac{1}{2}\%$. Find the cost price of the pen.

Solution Here, S.P. = Rs. 99, gain% = $12\frac{1}{2}\%$ or $\frac{25}{2}\%$.

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100 + \text{Gain\%}}\right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 + \frac{25}{2}}\right) \times 99$$

$$= \left(\frac{100 \times 2}{225}\right) \times 99 = \textbf{Rs. 88.}$$

SOME USEFUL SHORT-CUT METHODS

1. If a man buys x items for Rs. y and sells z items for Rs. w, then the gain or loss per cent made by him is

$$\left(\frac{xw}{zy} - 1\right) \times 100\%.$$

Explanation

S.P. of z items = Rs. w

S.P. of x items = Rs. $\frac{w}{z}x$

Net profit = $\frac{w}{z}x - y$.

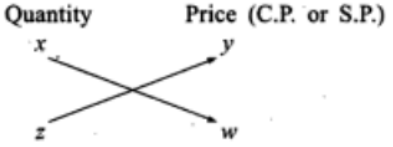
$$\therefore \quad \%\ \text{Profit} = \frac{\frac{w}{z}x - y}{y} \times 100\%$$

i.e. $\left(\frac{xw}{zy} - 1\right) \times 100\%$,

which represents loss, if the result is negative.

Note: In the case of gain per cent the result obtained bears positive sign whereas in the case of loss per cent the result obtained bears negative sign.

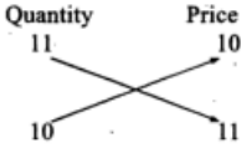
How to remember:



1. Cross-multiply the numbers connected by the arrows (xw and zy)
2. Mark the directon of the arrows for cross-multiplicaton. The arrow going down forms the numerator while the arrow going up forms the denominator $\left(\frac{xw}{zy}\right)$.

Illustration 7 If 11 oranges are bought for Rs. 10 and sold at 10 for Rs. 11, what is the gain or loss%?

Solution



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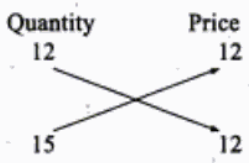
$$\% \text{ profit} = \left(\frac{xw}{zy} - 1\right) \times 100\%$$

$$= \left(\frac{11 \times 11}{10 \times 10} - 1\right) \times 100\%$$

$$= \frac{21}{100} \times 100\% = \mathbf{21\%}.$$

Illustration 8 A fruit seller buys apples at the rate of Rs. 12 per dozen and sells them at the rate of 15 for Rs. 12. Find his percentage gain or loss.

Solution



$$\% \text{ gain or loss} = \left(\frac{xw}{zy} - 1\right) \times 100\%$$

$$= \left(\frac{12 \times 12}{15 \times 12} - 1\right) \times 100\%$$

$$= -\frac{36}{180} \times 100\% = -20\%.$$

Since the sign is –ve, there is a loss of **20%.**

2. If the cost price of m articles is equal to the selling price of n articles, then

$$\% \text{ gain or loss} = \left(\frac{m-n}{n}\right) \times 100$$

[If $m > n$, it is % gain and if $m < n$, it is % loss]

Explanation

Let the C.P. of one article be Re. 1

$\therefore$ C.P. of m articles = Rs. $m \times 1$ = Rs. m

$\because$ S.P. of n articles = Rs. m

$\therefore$ S.P. of 1 article = Rs. $\frac{m}{n}$

$\therefore$ Profit on 1 article = Rs. $\left(\frac{m}{n} - 1\right)$

i.e. Rs. $\left(\frac{m-n}{n}\right)$

$\therefore$ % profit = $\frac{m-n}{n} \times \frac{100}{1}$ i.e. $\left(\frac{m-n}{n}\right) \times 100$.

Illustration 9 A shopkeeper professes to sell his goods on cost price but uses 800 gm, instead of 1kg. What is his gain %?

Solution Here, cost price of 1000 gm is equal to selling price of 800 gm,

$\therefore$ % gain = $\left(\frac{m-n}{n}\right) \times 100$

$$= \left(\frac{1000 - 800}{800}\right) \times 100$$

$$= \frac{200}{800} \times 100 = \mathbf{25\%}.$$

Illustraton 10 If the selling price of 12 articles is equal to the cost price of 18 articles, what is the profit %?

Solution Here, $m = 18$, $n = 12$

$\therefore$ Profit% = $\left(\frac{m-n}{n}\right) \times 100$

$$= \left(\frac{18 - 12}{12}\right) \times 100$$

$$= \frac{6}{12} \times 100 = \mathbf{50\%}.$$

3. If an article is sold at a price $S.P_1$., then % gain or % loss is x and if it is sold at a price $S.P_2$., then % gain or % loss is y. If the cost price of the article is C.P., then

$$\frac{S.P_1}{100 + x} = \frac{S.P_2}{100 + y} = \frac{C.P.}{100} = \frac{S.P_1 - S.P_2}{x - y},$$

where x or y is –ve, if it indicates a loss, otherwise it is +ve.

Illustration 11 By selling a radio for Rs. 1536, Suresh lost 20%. What per cent shall he gain or lose by selling it for Rs. 2000?

Solution Here, $S.P_1 = 1536$, $x = -20$

(–ve sign indicates loss)

$S.P_2$ = Rs. 2000, y = ?

Using the formula,

$$\frac{S.P_1}{100 + x} = \frac{S.P_2}{100 + y}$$

we get $\frac{1536}{100 - 20} = \frac{2000}{100 + y}$

$$\Rightarrow \quad 100 + y = \frac{2000 \times 80}{1536} = 104\frac{1}{6}$$

$$\Rightarrow \quad y = 4\frac{1}{6}\%.$$

Thus, Suresh has a gain of $4\frac{1}{6}\%$ by selling it for **Rs. 2000.**

4. If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%. If 'C' pays Rs. z for it to 'B' then the cost price for 'A' is

$$\left[\frac{100^2 z}{(100 + m)(100 + n)}\right],$$

where m or n is –ve, of it indicates a loss, otherwise it is +ve.

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Illustration 12 Mohit sells a bicycle to Rohit at a gain of 10% and Rohit again sells it to Jyoti at a profit of 5%. If Jyoti pays Rs. 462 to Rohit, what is the cost price of the bicycle for Mohit?

Solution Here, $m = 10$, $n = 5$, z = Rs. 462.

Using the formula,

$$\text{C.P.} = \left[\frac{100^2 z}{(100+m)(100+n)}\right],$$

we get, C.P. for Mohit $= \left[\frac{100^2 \times 462}{(100+10)(100+5)}\right]$

$$= \frac{462 \times 10000}{110 \times 105} = \textbf{Rs. 400.}$$

Illustration 13 'A' sells a DVD to 'B' at a gain of 17% and 'B' again sells it to 'C' at a loss of 25%. If 'C' pays Rs. 1053 to 'B', what is the cost price of the DVD to 'A'?

Solution We have, $m = 17$, $n = -25$, z = Rs. 1053.

$\therefore$ Cost price of DVD to 'A'

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \frac{100 \times 100 \times 1053}{(100+17)(100-25)}$$

$$= \frac{100 \times 100 \times 1053}{117 \times 75} = \textbf{Rs. 1200.}$$

5. If 'A' sells an article to 'B' at a gain/loss of m% and 'B' sells it to 'C' at a gain/loss of n%, then the resultant profit/loss per cent is given by

$$\left(m + n + \frac{mn}{100}\right) \quad \text{...(1)}$$

where m or n is –ve, if it indicates a loss, otherwise it is +ve.

Note: The expression given by (1) represents resultant profit or loss according as it is +ve or –ve.

Illustration 14 'A' sells a horse to 'B' at a profit of 5% and 'B' sells it to 'C' at a profit of 10%. Find the resultant profit per cent.

Solution We have, $m = 5$ and $n = 10$

$\therefore$ Resultant profit% $= \left(m + n + \frac{mn}{100}\right)$

$$= \left(5 + 10 + \frac{5 \times 10}{100}\right)$$

$$= \frac{31}{2}\% \text{ or } 15\frac{1}{2}\%.$$

Illustraton 15 Manoj sells a shirt to Yogesh at a profit of 15% and Yogesh sells it to Suresh at a loss of 10%. Find the resultant profit or loss.

Solution Here, $m = 15$, $n = -10$

$\therefore$ Resultant profit/loss% $= \left(m + n + \frac{mn}{100}\right)$

$$= \left(15 - 10 + \frac{15 \times -10}{100}\right) = \left(15 - 10 - \frac{150}{100}\right)$$

$$= 7/2\% \text{ or } 3\frac{1}{2}\%,$$

which represents profit as the sign is +ve.

6. When two different articles are sold at the same selling price, getting gain/loss of x% on the first and gain/loss of y% on the second, then the overall% gain or % loss in the transaction is given by

$$\left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

The above expression represent overall gain or loss according as its sign is +ve or –ve.

7. When two different articles are sold at the same selling price getting a gain of x% on the first and loss of x% on the second, then the overall% loss in the transaction is given by

$$\left(\frac{x}{10}\right)^2\%.$$

Note that in such questions there is always a loss.

Explanation

Let each article be sold at Rs. z.

Since gain/loss of x% is made on the first, cost price of the first article

$$= \text{Rs. } z\left(\frac{100}{100+x}\right).$$

Also, gain/loss of y% is made on the second, therefore cost price of the second article

$$= \text{Rs. } z\left(\frac{100}{100+y}\right).$$

$\therefore$ Total C.P. $= z\left(\frac{100}{100+x}\right) + z\left(\frac{100}{100+y}\right)$

$$= z\left[\frac{100(100+y)+100(100+x)}{(100+x)(100+y)}\right]$$

Total S.P. $= 2z$.

$\therefore$ Overall % gain or loss $= \frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$

$$= \frac{2z - \frac{100z[100+x+100+y]}{(100+x)(100+y)}}{\frac{100z[100+x+100+y]}{(100+x)(100+y)}} \times 100$$

$$= \frac{2(100+x)(100+y) - 100(200+x+y)}{100(200+x+y)} \times 100$$

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$$= \frac{100x + 100y + 2xy}{(100+x)+(100+y)}\%$$

$$= \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%.$$

Note: In case $y = -x$, we have

Overall% gain or loss $= -\frac{x^2}{100}\%$

Since the sign is –ve, there is always a loss.

Illustration 16 Mahesh sold two scooters, each for Rs. 24000. If he makes 20% profit on the first and 15% loss on the second, what is his gain or loss per cent in the transaction?

Solution Here, $x = 20$ and $y = -15$.

$\therefore$ Over all gain/loss%

$$= \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

$$= \left[\frac{100(20-15)+2\times 20\times -15}{(100+20)+(100-15)}\right]\%$$

$$= -\frac{100}{205}\% = -\frac{20}{41}\%,$$

which represents loss, being a –ve expression.

Illustration 17 Rajesh sold two horses for Rs. 990 each; gaining 10% on the one and losing 10% on the other. Find his total gain or loss per cent.

Solution Here, $x = 10$.

$\therefore$ Overall loss% $= \left(\frac{x}{10}\right)^2\% = \left(\frac{10}{10}\right)^2\% = \mathbf{1\%}.$

8. A merchant uses faulty measure and sells his goods at gain/loss of x%. The overall % gain/loss(g) is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{Faulty measure}}$$

Note: If the merchant sells his goods at cost price, then $x = 0$.

9. A merchant uses y% less weight/length and sells his goods at gain/loss of x%. The overall % gain/loss is given by

$$\left[\left(\frac{y+x}{100-y}\right)\times 100\right]\%.$$

Illustration 18 A dis-honest shopkeeper professes to sell cloth at the cost price but he uses faulty metre rod. His metre rod measures 95 cms only. Find his gain per cent.

Solution Here, True measure = 100 cms

False measure = 95 cms.

Since the shopkeeper sells the cloth at cost price,

$\therefore$ $x = 0$.

$\therefore$ Over all gain% is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{Faulty measure}}$$

$$\Rightarrow \frac{100+g}{100} = \frac{100}{95} \Rightarrow 100 + g = \frac{100\times 100}{95}$$

$$\Rightarrow g = \frac{10000}{95} - 100 = \mathbf{5\frac{5}{19}\%}.$$

Illustration 19 A dis-honest shopkeeper professes to sell his goods at cost price but he uses a weight of 800 g for the kg weight. Find his gain per cent.

Solution True measure = 1000 g

False measure = 800 g. Also, $x = 0$.

$\therefore$ Overall gain% is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{False measure}}$$

$$\Rightarrow \frac{100+g}{100} = \frac{1000}{800} \Rightarrow 100 + g = \frac{1000\times 100}{800}$$

$$\Rightarrow g = \frac{1000}{8} - 100 = \mathbf{25\%}.$$

Illustration 20 A shopkeeper sells the goods at 44% loss on cost price but uses 30% less weight. What is his percentage profit or loss?

Solution Here, $x = -44$ and $y = 30$.

$\therefore$ Overall gain/loss% $= \left(\frac{y+x}{100-y}\right)\times 100\%$

$$= \left(\frac{30-44}{100-30}\times 100\right)\%$$

$$= \left(\frac{-14}{70}\times 100\right)\% = -20\%,$$

which represents loss being a negative expression.

10. A person buys two items for Rs. A and sells one at a loss of l% and other at a gain of g%. If each item was sold at the same price, then

(a) The cost price of the item sold at loss

$$= \frac{A(100 + \%\text{gain})}{(100 - \%\text{loss}) + (100 + \%\text{gain})}$$

(b) The cost price of the item sold at gain

$$= \frac{A(100 - \%\text{loss})}{(100 - \%\text{loss}) + (100 + \%\text{gain})}$$

Illustration 21 Ramesh buys two books for Rs. 410 and sells one at a loss of 20% and the other at a gain of 25%. If both the books are sold at the same price, find the cost price of two books.

Solution Cost price of the book sold at a loss of 20%

$$= \frac{410(100+25)}{(100-20)+(100+25)}$$

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$$= \frac{410 \times 125}{80+125} = \text{Rs. } 250.$$

Cost price of the book sold at a profit of 25%

$$= \frac{410(100-20)}{(100-20)+(100+25)} = \frac{410 \times 80}{80+125}$$

$$= \textbf{Rs. 160.}$$

11. If two successive discounts on an article are $m\%$ and $n\%$ respectively, then a single discount equivalent to the two successive discounts will be

$$\left(m+n-\frac{mn}{100}\right)\%.$$

Explanation

Let the marked price of the article be Rs. 100.

$\therefore$ S.P. after the first discount = Rs. $(100-m)$ and

discount at n% on Rs. $(100-m)$ = Rs. $\frac{(100-m)\times n}{100}$.

$\therefore$ Single equivalent discount

$$= \left[m+\frac{(100-m)\times n}{100}\right]\%$$

$$= \left(\frac{100m+100n-mn}{100}\right)\%$$

$$= \left(m+n-\frac{mn}{100}\right)\%.$$

12. If three successive discounts on an article are $l\%$, $m\%$ and $n\%$ respectively, then a single discount equivalent to the three successive discounts will be

$$\left[l+m+n-\frac{(lm+ln+mn)}{100}+\frac{lmn}{100^2}\right]\%.$$

Explanation

Let the marked price of the article be Rs. 100

$\therefore$ S.P. after the first discount = Rs. $(100-l)$.

Second discount at $m\%$ on Rs. $(100-l)$

$$= \text{Rs. } \frac{(100-l)\times m}{100}.$$

$\therefore$ S.P. after second discount

$$= \text{Rs. } (100-l) - \frac{(100-l)m}{100}$$

$$= \text{Rs. } \frac{100(100-l)-(100-l)m}{100}$$

$$= \text{Rs. } \frac{(100-l)\cdot(100-m)}{100}$$

Third discount at $n\%$ on Rs. $\frac{(100-l)(100-m)}{100}$

$$= \text{Rs. } \frac{(100-l)(100-m)n}{100\times 100}$$

$\therefore$ S.P. after third discount

$$= \text{Rs. } \frac{(100-l)(100-m)}{100} - \frac{(100-l)(100-m)n}{100\times 100}$$

$$= \text{Rs. } \frac{(100-l)(100-m)(100-n)}{100\times 100}$$

$$= \left(l+m+n-\frac{(lm+ln+mn)}{100}+\frac{lmn}{(100)^2}\right)$$

$\therefore$ Single equivalent discount

$$= \left(l+m+n-\frac{(lm+ln+mn)}{100}+\frac{lmn}{(100)^2}\right)\%.$$

Illustration 22 Find a single discount equivalent to two successive discounts of 10% and 20%.

Solution The equivalent single discount is given by

$$\left(10+20-\frac{10\times 20}{100}\right)\% \textbf{ i.e., 28\%.}$$

Illustration 23 Find a single discount equivalent to three successive discounts of 10%, 20% and 30%.

Solution The equivalent single discount is given by

$$\left(10+20+30-\frac{(10\times 20+10\times 30+20\times 30)}{100}+\frac{10\times 20\times 30}{100^2}\right)\%$$

i.e. $\left(60-11+\frac{6}{10}\right)\% = \textbf{\frac{496}{10}\% or 49.6\%.}$

Illustration 24 Two shopkeepers sell machines at the same list price. The first allows two successive discounts of 30% and 16% and the second 20% and 26%. Which discount series is more advantageous to the purchaser?

Solution A single discount equivalent to the two successive discounts of 30% and 16% is

$$\left(30+16-\frac{30\times 16}{100}\right)\%$$

or, $\left(46-\frac{24}{5}\right)\%$ or $41\frac{1}{5}\%$

Also, a single discount equivalent to the two successive discounts of 20% and 26% is

$$\left(20+26-\frac{20\times 26}{100}\right)\%$$

or, $\left(46-\frac{26}{5}\right)\%$ or $40\frac{4}{5}\%$.

Clearly, the discount series being offered by the first shopkeeper is more advantageous to the purchaser.

13 A shopkeeper sells an item at Rs. z after giving a discount of $d\%$ on labelled price. Had he not given the discount, he would have earned a profit of $p\%$ on the cost price.

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The cost price of each item is given by

$$\text{C.P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right].$$

Illustration 25 A shopkeeper sold sarees at Rs. 266 each after giving 5% discount on labelled price. Had he not given the discount, he would have earned a profit of 12% on the cost price. What was the cost price of each saree?

Solution We have, labelled price z = Rs. 266, discount d = 5% and profit p = 12%.

Using the formula

$$\text{C.P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right]$$

we get the cost price of each saree

$$= \left[\frac{100\times 100\times 266}{(100-5)(100+12)}\right]$$

$$= \frac{100\times 100\times 266}{95\times 112} = \textbf{Rs. 250.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Mohan buys a watch for Rs. 350 and sells it for Rs. 392. Find his percentage of profit.
(a) 9% (b) 12%
(c) 14% (d) None of these

2. Ramesh purchased a bicycle for Rs. 5200 and spent Rs. 800 on its repairs. He had to sell it for Rs. 5500. Find his profit or loss per cent.
(a) $8\frac{1}{3}$% loss (b) $7\frac{1}{2}$% gain
(c) 9% (d) None of these

3. A man buys 10 articles for Rs. 8 and sells them at the rate of Rs. 1.25 per article. His gain per cent is
(a) 55% (b) $56\frac{1}{4}$%
(c) 40% (d) None of these

4. A tooth paste marked at Rs. 80 is sold for Rs. 68. The rate of discount is
(a) 12% (b) 14%
(c) 15% (d) None of these

5. Sardar Singh bought 200 dozen oranges at Rs. 10 a dozen. He spent Rs. 500 on transportation. He sold them at Re. 1 each. What was his profit or loss percent?
(a) 4% (b) 6%
(c) 5% (d) None of these

6. Mr. Verma sold his scooter for Rs. 10500 at a gain of 5%. Find the cost price of the scooter.
(a) Rs. 10300 (b) Rs. 10700
(c) Rs. 10000 (d) None of these

7. Suresh buys a camera for Rs. 1800 and sells it at 10% loss. Find its selling price.
(a) Rs. 1620 (b) Rs. 1730
(c) Rs. 1650 (d) None of these.

8. Hemant purchased 120 rims of paper at Rs. 80 per rim. He spent Rs. 280 on transportation, paid octroi at the rate of 40 paise per rim and paid Rs. 72 to the coolie. If he wants to have a gain of 8%, the selling price per rim must be:
(a) Rs. 89 (b) Rs. 90
(c) Rs. 95 (d) None of these

9. A shopkeeper loses 7 % by selling a cricket ball for Rs. 31. For how much should he sell the ball so as to gain 5 %?
(a) Rs. 50 (b) Rs. 65
(c) Rs. 35 (d) None of these

10. A shopkeeper sold some articles at Rs. 35 per article and gained 40% on it. What would be the selling price of each article to get 60% profit?
(a) Rs. 40 (b) Rs. 45
(c) Rs. 50 (d) None of these

11. A man bought apples at the rate of 6 for Rs. 20 and sold them at 4 for Rs. 16. His estimated profit % is
(a) 23% (b) 18%
(c) 20% (d) None of these

12. A fruitseller buys 10 bananas for Rs. 14 and sells them at 12 for Rs. 15. Find his percentage gain or loss.
(a) $10\frac{5}{7}$% loss (b) $10\frac{5}{9}$% gain
(c) 9% gain (d) None of these

13. If eggs are bought 12 for Rs. 10 and sold at 10 for Rs. 12. What is the gain or loss %.
(a) 40% loss (b) 44% gain
(c) 44% loss (d) None of these

14. If the cost price of 21 watches is equal to the selling price of 18 of them, what will be the gain per cent in this transaction?
(a) $6\frac{1}{2}$% (b) 7%
(c) $16\frac{2}{3}$% (d) None of these

15. A shopkeeper gains the cost of 8 metres of thread by selling 40 metres of thread. Find his gain per cent.

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(a) 19% (b) 20%
(c) 22% (d) None of these

16. If the selling price of 2/3 rd of a certain quantity of milk be equal to the cost price of whole milk, what will be the gain per cent in this transaction?
(a) 50% (b) 48%
(c) 53% (d) None of these

17. A shopkeeper sells 20 pencils for the same money as he paid for 25. What is his gain per cent?
(a) 20% (b) 25%
(c) 24% (d) None of these

18. Mohit lost 18% by selling a bicycle for Rs. 1230. What per cent shall he gain or loss by selling it for Rs. 1600?
(a) $6\frac{2}{3}$% loss (b) 4% gain
(c) $6\frac{2}{3}$% gain (d) None of these

19. A shopkeeper sells an article at a gain of 10%. Had he sold it at a loss of 20%, its selling price would have been Rs. 180 less. What is the cost price of the article?
(a) Rs. 630 (b) Rs. 600
(c) Rs. 580 (d) None of these

20. A person sells 36 oranges per rupee and suffers a loss of 4%. Find how many oranges per rupee to be sold to have a gain of 8 %?
(a) 1/32 (b) 5
(c) 1/16 (d) None of these

21. A person sells a colour TV at 10% below cost price. Had he received Rs. 1494 more, he would have made a profit of $12\frac{1}{2}$%. What was the cost price of colour TV ?
(a) Rs. 6400 (b) Rs. 7200
(c) Rs. 6640 (d) None of these

22. Vijay sold a watch at a gain of 5 %. Had he sold it for Rs. 72 more, he would have gained 13 %. Find the cost price of the watch.
(a) Rs. 900 (b) Rs. 910
(c) Rs.870 (d) None of these

23. Sita sells a calculator to Gita at a gain of 17% and Gita sells it to Anu at a loss of 25%. If Anu pays Rs. 1842.75 for it, what did Sita pay for it?
(a) Rs. 2080 (b) Rs. 2100
(c) Rs. 2110 (d) None of these

24. '*A*' buys an article and sells it to '*B*' at a profit of 10%, '*B*' sells it to '*C*' gaining 20 %. If '*C*' gives Rs. 924, what did '*A*' give?
(a) Rs. 700 (b) Rs. 724
(c) Rs. 780 (d) None of these

25. '*A*' sells an article to '*B*' at a gain of 20% and '*B*' sells it to '*C*' at a gain of 10% and '*C*' sells it to '*D*' at a gain of $12\frac{1}{2}$%. If '*D*' pays Rs. 29.70 what did it cost '*A*'?
(a) Rs. 20 (b) Rs. 24
(c) Rs. 18 (d) None of these

26. Rajesh sells taperecorder to Mihir at a loss of 10 % and Mihir sells it to Shiv at a loss of 20 %. If Shiv pays Rs. 1440 for it, at what price did Rajesh buy?
(a) Rs. 1920 (b) Rs. 2000
(c) Rs. 1800 (d) None of these

27. A man sells a scooter to his friend at 10 % loss. If the friend sells it for Rs. 54000 and gains 20 %, find the original cost price of the scooter.
(a) Rs. 50000 (b) Rs. 45000
(c) Rs. 40000 (d) None of these

28. '*A*' sells a good to '*B*' at a profit of 10 % and *B* sells it to '*C*' at a profit of 20 %. Find the resultant profit.
(a) Rs. 35% (b) 20%
(c) Rs. 32% (d) None of these

29. A manufacturer sells an article to a wholesale dealer at a profit of 20 % and the wholesale dealer sells it to a retail merchant at a loss of 5 %. Find the resultant profit or loss.
(a) 14% loss (b) 14% gain
(c) 12% gain (d) None of these

30. A man sold two watches for Rs. 3750 each; on one he gained 5 % and on the other he lost 5 %. What was his total gain or loss as a percentage ?
(a) $1\frac{1}{4}$% (b) $\frac{1}{2}$%
(c) $\frac{1}{4}$% (d) None of these

31. A men sells two houses at the rate of Rs. 1.995 lakh each. On one he gains 20 % and on the other he loses 20%. His gain or loss per cent in the whole transaction is:
(a) 5% loss (b) 4%
(c) 4% loss (d) None of these

32. A shopkeeper sold two bicycles for Rs. 1500 each. On one he gains 25 % and on the other he loses 20 %. His gain or loss per cent in the whole transaction is:
(a) $2\frac{18}{41}$% loss (b) $2\frac{18}{41}$% gain
(c) 2% gain (d) None of these

33. A man sells two articles, each for the same price of Rs. 640. He earns 20 % profit on the first and 40 % profit on the second. Find his overall per cent profit.
(a) $29\frac{1}{2}$% (b) $28\frac{1}{2}$%
(c) $29\frac{3}{13}$% (d) None of these

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34. A person sells two articles, each for the same price Rs. 1040. He incurs 20 % loss on the first and 10 % loss on the second. Find his overall per cent loss.

(a) $12\frac{5}{17}\%$ (b) $15\frac{5}{17}$

(c) $13\frac{3}{4}\%$ (d) None of these

35. A grocer sells rice at a profit of 20 % and uses a weight which is 25 % less. Find his overall percentage gain.

(a) 60% (b) 65%

(c) 58% (d) None of these

36. A shopkeeper sells the goods at 10% loss on cost price but uses 20% less weight. What is his percentage profit or loss?

(a) 2% gain (b) $2\frac{1}{2}\%$ loss

(c) $12\frac{1}{2}\%$ gain (d) None of these.

37. A cloth merchant says that due to slump in the market, he sells the cloth at 10 % loss but he uses a false metre scale and actually gains 15 %. Find the actual length of the scale.

(a) 72.4 cm (b) 71.34 cm

(c) 78.25 cm (d) None of these

38. A cloth dealer professes to sell cotton at cost price, but uses a meter having a length of 80 cm only and charges for the meter. Find his gain per cent.

(a) 25% (b) 30%

(c) 40% (d) None of these

39. Sudeep buys two *CD*s for Rs. 380 and sells one at a loss of 22 % and the other at a gain of 12 %. If both the *CD*s are sold at the same price, then the cost price of two *CD*s is

(a) Rs. 196, Rs. 225 (b) Rs. 230, Rs. 140

(c) Rs. 224, Rs. 156 (d) None of these

40. An article is listed at Rs. 65. A customer bought this article for Rs. 56.16 and got two successive discounts of which one is 10 %. Find the other discount of this discount scheme that was allowed by the shopkeeper.

(a) 4% (b) 3%

(c) 6% (d) None of these

41. A cash payment that will settle a bill for 250 chairs at Rs. 50 per chair less 20 % and 15 % with a further discount of 5 % on cash payment is

(a) Rs. 7025 (b) Rs. 8075

(c) Rs. 8500 (d) None of these

42. A person sells taperecorders at Rs. 1134 each after giving a discount of 19 % on the marked price. Had he not given the discount, he would have earned a profit of 40 % on the cost price. The cost price of each taperecorder is

(a) Rs. 1000 (b) Rs. 1200

(c) Rs. 1400 (d) None of these

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Here, C.P. = Rs. 350, S.P. = Rs. 392

Profit = S.P. – C.P. = 392 – 350 = Rs. 42.

$\therefore$ Profit % $= \frac{\text{Profit} \times 100}{\text{C.P.}} = \frac{42 \times 100}{350} = \mathbf{12\%.}$

2. Ans. (a)

Solution C.P. of the bicycle = 5200 + 800 = Rs. 6000.
S.P. = Rs. 5500.

Since S.P. < C.P.,

$\therefore$ loss = C.P. – S.P. = 6000 – 5500 = Rs. 500.

$\therefore$ Loss% $= \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{500 \times 100}{6000}$

$= \mathbf{\frac{25}{3}\%}$ **or** $\mathbf{8\frac{1}{3}\%.}$

3. Ans. (b)

Solution Cost price of 10 articles (C.P.) = Rs. 8

Selling price of 10 articles (S.P.) = 1.25 × 10 = Rs. 12.50

Profit = S.P. – C.P. = 12.50 – 8 = Rs. 4.50.

$\therefore$ Gain% $= \frac{\text{Gain} \times 100}{\text{C.P}} = \frac{4.50 \times 100}{8} = \mathbf{56\frac{1}{4}\%.}$

4. Ans. (c)

Solution Marked price (M.P.) = Rs. 80.

Selling price (S.P.) = Rs. 68.

Discount = M.P. – S.P. = 80 – 68 = Rs. 12.

$\therefore$ Rate of discount $= \frac{\text{Discount} \times 100}{\text{Marked Price}}$

$= \frac{12 \times 100}{80} = \mathbf{15\%.}$

5. Ans. (a)

Solution Cost price of 200 dozen oranges = 200 × 10 = Rs. 2000

Transportation cost = Rs. 500.

$\therefore$ Cost Price (C.P) = 2000 + 500 = Rs. 2500.

Selling Price (S.P.) = 200 × 12 × 1 = Rs. 2400

Loss = C.P. – S.P. = 2500 – 2400 = Rs. 100.

$\therefore$ Loss% $= \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{100 \times 100}{2500} = \mathbf{4\%.}$

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6. Ans. (c)

Solution We have, S.P. = Rs. 10500, gain % = 5 %

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100+\text{Gain\%}}\right)\times \text{S.P.}$$

$$= \left(\frac{100}{100+5}\right)\times 10500$$

$$= \frac{100}{105}\times 10500 = \textbf{Rs. 10000.}$$

7. Ans. (a)

Solution Here, C.P. = Rs. 1800, loss% = 10%.

$$\therefore \text{S.P.} = \left(\frac{100-\text{Loss\%}}{100}\right)\times \text{C.P.}$$

$$= \left(\frac{100-10}{100}\right)\times 1800$$

$$= \frac{90}{100}\times 1800 = \textbf{Rs. 1620.}$$

8. Ans. (b)

Solution We have,

Cost price (C.P.) $= (120\times 80 + 280 + 72 + 120\times 40)$

$= \text{Rs. } 10000.$

Gain% = 8%.

$\therefore$ Selling Price (S.P.) of 120 rims

$$= \left(\frac{100+\text{Gain\%}}{100}\right)\times \text{C.P.}$$

$$= \left(\frac{100+8}{100}\right)\times 10000$$

$$= \frac{108}{100}\times 10000 = \text{Rs. } 10800.$$

Thus, selling price per rim $= \frac{10800}{120} = \textbf{Rs. 90.}$

9. Ans. (c)

Solution In the first case, S.P. = Rs. 31 and loss% = 7 %.

$$\therefore \text{C.P.} = \left(\frac{100}{100-\text{Loss\%}}\right)\times \text{S.P.} = \left(\frac{100}{100-7}\right)\times 31$$

$$= \frac{100}{93}\times 31 = \text{Rs. } 33\frac{1}{3}.$$

In the second case, C.P. = Rs. $33\frac{1}{3}$ and gain% = 5%

$$\therefore \text{S.P.} = \left(\frac{100+\text{Gain\%}}{100}\right)\times \text{C.P.}$$

$$= \left(\frac{100+5}{100}\right)\times\frac{100}{3} = \frac{105}{100}\times\frac{100}{3} = \textbf{Rs. 35.}$$

10. Ans. (a)

Solution In the first case, we have, S.P. = Rs. 35 and gain % = 40 %

$$\therefore \quad \text{C.P.} = \left(\frac{100}{100+\text{Gain\%}}\right)\times \text{S.P.}$$

$$= \left(\frac{100}{100+40}\right)\times 35$$

$$= \frac{100}{140}\times 35 = \text{Rs. } 25.$$

In the second case, C.P. = Rs. 25 and gain% = 60 %.

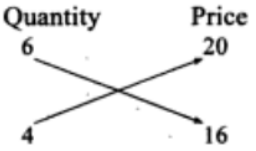
$$\therefore \quad \text{S.P.} = \left(\frac{100+\text{Gain\%}}{100}\right)\times \text{C.P.}$$

$$= \left(\frac{100+60}{100}\right)\times 25$$

$$= \frac{160}{100}\times 25 = \textbf{Rs. 40.}$$

11. Ans. (c)

Solution



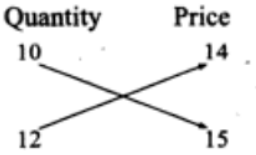
$$\%\ \text{profit} = \left(\frac{xw}{zy}-1\right)\times 100\% = \left(\frac{6\times 16}{4\times 20}\right)\times 100\%$$

$$= \frac{16}{80}\times 100\%$$

$$= \textbf{20 \%.}$$

12. Ans. (a)

Solution



$$\%\ \text{gain or loss} = \left(\frac{xw}{zy}-1\right)\times 100\%$$

$$= \left(\frac{10\times 15}{12\times 14}-1\right)\times 100\%$$

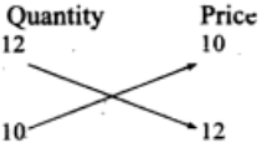
$$= \left(\frac{150}{168}-1\right)\times 100\%$$

$$= -\frac{18}{168}\times 100\% = -10\frac{5}{7}\%.$$

Since the sign is – ve, since there is a loss of $\mathbf{10\frac{5}{7}\%}$.

13. Ans. (b)

Solution



$$\%\ \text{gain or loss} = \left(\frac{xw}{zy}-1\right)\times 100\%$$

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$$= \left(\frac{12\times12}{10\times10}-1\right)\times100\% = \left(\frac{144}{100}-1\right)\times100\%$$

$$= \frac{44}{100}\times100\% = 44\%.$$

Since the sign is + ve, therefore there is a profit of **44 %.**

14. Ans. (c)

Solution Here, $m = 21$, $n = 18$.

$$\therefore \text{ gain \%} = \left(\frac{m-n}{n}\right)\times100 = \left(\frac{21-18}{18}\right)\times100$$

$$= \left(\frac{3}{18}\times100\right)\% = \mathbf{16\frac{2}{3}\%}.$$

15. Ans. (b)

Solution Here, $m = 48$, $n = 40$.

$$\therefore \text{ gain\%} = \left(\frac{m-n}{n}\right)\times100 = \left(\frac{48-40}{40}\right)\times100$$

$$= \frac{8}{40}\times100 = \mathbf{20\ \%}.$$

16. Ans. (a)

Solution Let the quantity of milk be x litres.

Then, $m = x$, $n = \frac{2}{3}x$.

$$\therefore \text{ gain\%} = \left(\frac{m-n}{n}\right)\times100 = \left(\frac{x-\frac{2}{3}x}{\frac{2}{3}x}\right)\times100$$

$$= \frac{1}{2}\times100 = \mathbf{50\%}.$$

17. Ans. (b)

Solution Here, $m = 25$, $n = 20$.

$$\therefore \text{ \% gain} = \left(\frac{m-n}{n}\right)\times100 = \left(\frac{25-20}{20}\right)\times100$$

$$= 25\%.$$

18. Ans. (c)

Solution Here, $S.P_1 = \text{Rs. } 1230$, $x = -18$,
$S.P_2 = \text{Rs. } 1600$, $y = ?$

Using the formula,

$$\frac{S.P_1}{100+x} = \frac{S.P_2}{100+y}$$

we get $\dfrac{1230}{100-18} = \dfrac{1600}{100+y}$

$$\Rightarrow 100 + y = \frac{1600\times82}{1230} = 106\frac{2}{3} \Rightarrow y = 6\frac{2}{3}\%$$

Thus, Mohit has a gain of $6\frac{2}{3}\%$ by selling it for **Rs. 1600.**

19. Ans. (b)

Solution Here, $S.P_1 - S.P_2 = \text{Rs. } 180$, $x = 10$, $y = -20$, C.P. = ?

Using the formula, $\dfrac{C.P}{100} = \dfrac{S.P_1 - S.P_2}{x-y}$

we get $C.P. = \left(\dfrac{180}{10+20}\right)\times100 = \dfrac{180}{30}\times100 = \mathbf{Rs.\ 600.}$

20. Ans. (a)

Solution We have,

$S.P_1$ = Selling price of one orange = Rs. $\frac{1}{36}$,

$x = -4$, $y = 8$.

Using the formula, $\dfrac{S.P_1}{100+x} = \dfrac{S.P_2}{100+y}$

we get $\dfrac{1/36}{100-4} = \dfrac{S.P_2}{100+8} \Rightarrow S.P_2 = \dfrac{108}{96}\times\dfrac{1}{36} = \mathbf{\dfrac{1}{32}}.$

21. Ans. (c)

Solution Here, $S.P_1 - S.P_2 = -1494$, $x = -10$, $y = \frac{25}{2}$.

Using the formula,

$$\frac{C.P.}{100} = \frac{S.P_1 - S.P_2}{x-y}$$

we get $\dfrac{C.P.}{100} = \dfrac{-1494}{-10-25/2} = \dfrac{1494}{45/2}$

$$\Rightarrow C.P. = \frac{1494\times2\times100}{45} = \mathbf{Rs.\ 6640.}$$

22. Ans. (a)

Solution Here, $S.P_1 - S.P_2 = -72$, $x = 5$, $y = 13$.

Using the formula,

$$\frac{C.P}{100} = \left(\frac{S.P_1 - S.P_2}{x-y}\right)$$

we get $\dfrac{C.P.}{100} = \left(\dfrac{-72}{5-13}\right)$

$$\Rightarrow C.P. = \frac{72}{8}\times100 = \mathbf{Rs.\ 900.}$$

23. Ans. (b)

Solution Here, $m = 17$, $n = -25$, $z = \text{Rs. } 1842.75$.

$\therefore$ Cost price of calculator to Sita

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \frac{100\times100\times1842.75}{(100+17)(100-25)}$$

$$= \frac{100\times100\times1842.75}{117\times75} = \mathbf{Rs.\ 2100.}$$

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24. Ans. (a)

Solution We have, $m = 10$, $n = 20$, z = Rs. 924.

$\therefore$ Cost price of A

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \left[\frac{100\times100\times924}{(100+10)(100+20)}\right] = \frac{100\times100\times924}{110\times120}$$

= Rs. 700.

25. Ans. (a)

Solution Here, $m = 20$, $n = 10$, $p = \frac{25}{2}$,

z = Rs. 29.70.

$\therefore$ Cost price of A

$$= \left[\frac{100^3 z}{(100+m)(100+n)(100+p)}\right]$$

$$= \left(\frac{100\times100\times100\times29.70\times2}{120\times110\times225}\right)$$

$$= \left(\frac{100\times100\times100\times29.70\times2}{120\times110\times225}\right) = \textbf{Rs. 20.}$$

26. Ans. (b)

Solution We have, $m = -10$, $n = -20$, z = Rs. 1440

$\therefore$ Cost price of taperecorder for Rajesh

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \left[\frac{100\times100\times1440}{(100-10)(100-20)}\right] =$$

= Rs. 2000.

27. Ans. (a)

Solution Here, $m = -10$, $n = 20$, z = Rs. 54000.

$\therefore$ Original cost price of the scooter

$$= \left[\frac{100^2 z}{(100+m)(100+n)}\right]$$

$$= \left[\frac{100\times100\times54000}{(100-10)(100+20)}\right]$$

$$= \left(\frac{100\times100\times54000}{90\times120}\right) = \textbf{Rs. 50000.}$$

28. Ans. (c)

Solution We have, $m = 10$, $n = 20$.

$\therefore$ Resultant profit % $= \left(m+n+\frac{mn}{100}\right)$

$$= \left(10+20+\frac{10\times20}{100}\right)$$

= 32 %.

29. Ans. (b)

Solution Here, $m = 20$, $n = -5$

$\therefore$ Resultant profit or loss % $= \left(m+n+\frac{mn}{100}\right)$

$$= \left(20-5+\frac{20\times-5}{100}\right) = 14\,\%,$$

which represents profit as the sign is + ve.

30. Ans. (c)

Solution Here, $x = 5$

$\therefore$ Over all loss % $= \left(\frac{x}{10}\right)^2\% = \left(\frac{5}{10}\right)^2\% = \textbf{\frac{1}{4}\%.}$

31. Ans. (c)

Solution Here, $x = 20$

$\therefore$ over all loss % $= \left(\frac{x}{10}\right)^2\% = \left(\frac{20}{10}\right)^2\% = \textbf{4 \%.}$

32. Ans. (a)

Solution Here $x = 25$ and $y = -20$.

$\therefore$ Overall gain/loss %

$$= \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

$$= \left[\frac{100(25-20)+2\times25\times-20}{(100+25)+(100-20)}\right]\%$$

$$= -\frac{100}{41}\% \text{ or } -2\frac{18}{41}\%,$$

which represents loss being a –ve expression.

33. Ans. (c)

Solution Here, $x = 20$ and $y = 40$.

$\therefore$ Over all gain %

$$= \left[\frac{100(x+y)+2xy}{(100+x)+(100+y)}\right]\%$$

$$= \left[\frac{100(20+40)+2\times20\times40}{(100+20)+(100+40)}\right]\%$$

$$= \frac{7600}{260}\% = \textbf{29\frac{3}{13}\%.}$$

34. Ans. (b)

Solution We have, $x = -20$ and $y = -10$.

$\therefore$ Over all gain/loss

$$= \left[\frac{100(-20-10)+2\times-20\times-10}{(100-20)+(100-10)}\right]\%$$

$$= -\frac{2600}{170}\%$$

$$= -15\frac{5}{17}\%,$$

which represents loss being a negative expression.

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35. Ans. (a)

Solution We have, $x = 20$ and $y = 25$

$$\therefore \text{ Overall gain } \% = \left[\frac{y+x}{100-y}\times 100\right]\%$$

$$= \left[\frac{25+20}{100-25}\times 100\right]\% = \mathbf{60\%.}$$

36. Ans. (c)

Solution Here, $x = -10$ and $y = 20$.

$$\therefore \text{ Overall gain/loss } \% = \left[\frac{y+x}{100-y}\times 100\right]\%$$

$$= \left[\frac{20-10}{100-20}\times 100\right]\%$$

$$= 12\frac{1}{2}\%,$$

which represents gain being a positive expression.

37. Ans. (c)

Solution Here, $x = -10$ and $g = 15$.

Let false scale length = y cm.

Using the formula, $\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{False measure}}$,

we get $\frac{100+15}{100-10} = \frac{100}{y} \Rightarrow y = \frac{100\times 90}{115} = 78.25$ cm.

$\therefore$ Actual length of the scale is 78.25 cm instead of **1 m.**

38. Ans. (a)

Solution True measure = 100 cm.

False measure = 80 cm. Also, $x = 0$.

$\therefore$ Overall gain % is given by

$$\frac{100+g}{100+x} = \frac{\text{True measure}}{\text{False measure}}$$

$$\Rightarrow \quad \frac{100+g}{100} = \frac{100}{80} \Rightarrow 100 + g = \frac{100\times 100}{80}$$

$$\Rightarrow \quad g = \frac{1000}{8} - 100 = \mathbf{25\ \%.}$$

39. Ans. (c)

Solution Cost price of the CD sold at a loss of 22 %

$$= \frac{A(100+\%\,\text{gain})}{(100-\%\,\text{loss})+(100+\%\,\text{gain})}$$

$$= \frac{380(100+12)}{(100-22)+(100+12)} = \frac{380\times 112}{78+112}$$

= Rs. 224.

Cost price of the CD sold at a gain of 12 %

$$= \frac{A(100-\%\,\text{loss})}{(100-\%\,\text{loss})+(100+\%\,\text{gain})}$$

$$= \frac{380(100-22)}{(100-22)+(100+12)} = \frac{380\times 78}{78+112}$$

= **Rs. 156.**

40. Ans. (a)

Solution List price of the article = Rs. 65.

Selling price of the article = Rs. 56.16.

$\therefore$ Discount = 65 − 56.16 = Rs. 8.84.

$\therefore$ Discount $\% = \frac{8.84\times 100}{65} = 13.6\ \%$.

The first discount of the given discount series is 10 %

Let the second discount of the series be m %

$$\therefore \text{ Single discount } \% = \left(m+n-\frac{mn}{100}\right)\%$$

or, $13.6\ \% = \left(m+10-\frac{10m}{100}\right)\%$

or, $13.6 = m + 10 - \frac{m}{10}$

or, $136 = 100 + 10m - m$ or, $9m = 36$

or, $m = 4\ \%$.

41. Ans. (b)

Solution A single discount equivalent to three given successive discounts of 20 %, 15 % and 5 % is given by

$$= \left(20+15+5-\frac{(20\times 15+5\times 15+5\times 20)}{100}+\frac{20\times 15\times 5}{100^2}\right)\%$$

i.e. $(40 - 4.75 + 0.15)\ \% = 35.4\ \%$.

Marked price of 250 chairs = 250 × 50 = Rs. 12500.

$$\therefore \text{ Cash payment } = 12500 - 12500\times\frac{35.4}{100}$$

= **Rs. 8075.**

42. Ans. (a)

Solution Using the formula

$$\text{C.P.} = \left[\frac{100^2 z}{(100-d)(100+p)}\right],$$

the cost price of each taperecorder is given by

$$\text{C.P.} = \left[\frac{100\times 100\times 1134}{(100-19)(100+40)}\right]$$

$$= \frac{100\times 100\times 1134}{81\times 140} = \mathbf{Rs.\ 1000.}$$

QUESTIONS FROM COMPETITIVE EXAMS

1. The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, then the value of x is

(a) 25 (b) 18

(c) 16 (d) 15

[Based on MAT 2004]

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2. A grocer bought 10 kg of apples for Rs. 81 out of which one kg were found rotten. If he wishes to make a profit of 10%, then he should sell it at.... per kg.
(a) Rs. 9.00 (b) Rs. 9.90
(c) Rs. 10.10 (d) Rs. 10.20
[Based on MAT 2003]

3. A man sells an article at 5% profit. If he had bought it at 5% less and sold it for Re. 1 less, he would have gained 10%. The cost price of the article is
(a) Rs 200 (b) Rs 150
(c) Rs 250 (d) Rs 240
[Based on MAT 2003]

4. A shopkeeper sold a TV set for Rs. 17,940, with a discount of 8% and gained 19.6%. If no discount is allowed, then what will be his gain per cent?
(a) 25% (b) 26.4%
(c) 24.8% (d) None of these
[Based on MAT 2003]

5. My friend collects antique stamps. She purchased two, but found that she needed to raise money urgently. So she sold them for Rs. 8000 each. On one she made 20% and on the other she lost 20%. How much did she gain or lose in the entire transaction?
(a) No Loss/Profit (b) Rs. 667 Loss
(c) Rs. 667 Profit (d) None of these
[Based on MAT 2003]

6. A trader wants 10% profit on the selling price of a product whereas his expenses amount to 15% on sales. What should his rate of mark up be on an article of Rs. 9?
(a) 20% (b) 25%
(c) 30% (d) $\frac{100}{3}\%$
[Based on MAT 2003]

7. On selling a pen at 5% loss and a book at 15% gain, Karim gains Rs. 7. If he sells the pen at 5% gain and the book at 10% gain, then he gains Rs. 13. The actual price of the book is
(a) Rs 100 (b) Rs 80
(c) Rs 10 (d) Rs 400
[Based on MAT 2003]

8. By selling 33 metres of cloth, a shopkeeper gains the price of 11 metres of cloth. His gain percent is
(a) 7% (b) 50%
(c) 20% (d) 22%
[Based on MAT 2002]

9. If 7% of the sale price of an article is equivalent to 8% of its cost price and 9% of its sale price exceeds 10% of its cost price by Re. 1, then what is the cost price of the article?
(a) Rs 400 (b) Rs 350
(c) Rs 300 (d) Rs 280
[Based on MAT 2002]

10. A cycle agent buys 30 bicycles, of which 8 are first grade and the rest are second grade, for Rs 3150, Find at what price he must sell the first grade bicycles so that if he sells the second grade bicycles at three quarters of the price, he may make a profit of 40% on his outlay?
(a) Rs 200 (b) Rs 240
(c) Rs 180 (d) Rs 210
[Based on MAT 2002]

11. The retail price of a water geyser is Rs. 1265. If the manufacturer gains 10%, the wholesale dealer gains 15% and the retailer gains 25%, then the cost of the product is
(a) Rs. 800 (b) Rs. 900
(c) Rs. 700 (d) Rs. 600
[Based on MAT 2002]

12. If the cost of 12 pencils is equal to the selling price of 10 pencils, the profit percent in the transaction is
(a) $16\frac{2}{3}\%$ (2) 18%
(c) 20% (4) 25%
[Based on MAT 2001]

13. Two motor cars were sold for Rs. 9,900 each, gaining 10% on one and losing 10% on the other. The gain or loss percent in the whole transaction is
(a) Neither loss nor gain
(b) 1% profit
(c) $\frac{100}{99}\%$ profit
(d) 1% loss **[Based on MAT 2001]**

14. There would be 10% loss if a toy is sold at Rs. 10.80 per piece. At what price should it be sold to earn a profit of 20%?
(a) Rs. 12 (b) Rs. 1296
(c) Rs. 14.40 (d) None of these
[Based on MAT 2005]

15. Ravi sells an article at a gain of $12\frac{1}{2}\%$. If he had sold it at Rs. 22.50 more, he would have gained 25%. The cost price of the article is
(a) Rs. 162 (b) Rs. 140
(c) Rs. 196 (d) Rs. 180
[Based on MAT 1999]

16. A machine is sold at a profit of 10%. Had it been sold for Rs. 40 less, there would have been a loss of 10%. What was the cost price?
(a) Rs. 175 (b) Rs. 200
(c) Rs. 225 (d) Rs. 250
[Based on MAT 2000]

17. A house costs C rupees. Later it was sold for a profit of 25%. What is the capital gains tax if it is 50% of the profit?
(a) $C/24$ (b) $C/8$

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(c) $C/4$ (d) $C/2$

[Based on MAT 2000]

18. If selling price is doubled, the profit triples. Find the profit percent.

(a) $66\frac{2}{3}$ (b) 100

(c) $105\frac{1}{3}$ (d) 120

[Based on MAT 2001]

19. A showroom owner sells a leather jacket for Rs X and claims to make a profit of 10%. He plans to have a stall in the trade fair and marks the same jacket at Rs $2X$. At the stall, he allows a discount of 20%. What will be the percentage profit that he will make at the trade fair?

(a) 80% (b) 60%

(c) 76% (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

20. If a person makes a profit of 10% on 1/4th of the quantity sold and a loss of 20% on the rest, then what is his average percent profit or loss?

(a) 12.5% profit (b) 11.25% loss

(c) 11.75% profit (d) 12.5% loss

[Based on IIT Joint Man. Ent. Test 2004]

21. Rawat sold a book at 10% loss. Had he sold it for Rs 85 more, then he would have made a profit of 7%. What was the cost price of the book?

(a) Rs 500 (b) Rs 850

(c) Rs 615 (d) Rs 585

[Based on IIT Joint Man. Ent. Test 2004]

22. If the cost price of 12 pencils is equal to sale price of 10 pencils, then find the gain or loss as a percentage of the sale price.

(a) 20% gain (b) 10% gain

(c) 20% loss (d) 15% loss

[Based on Narsee Monjee Inst. of Man. Studies, 2003]

23. What is the percentage profit made by selling an umbrella at a certain price, if by selling at 2/3 of that price, there would be a loss of 10%?

(a) 25% (b) 30%

(c) 35% (d) None of these

[Based on Narsee Monjee Inst. of Man. Studies, 2003]

24. A manufacturer of a certain item can sell all he can produce at the selling price of Rs 60 each. It costs him Rs 40 in materials and labour to produce each item and he has overhead expenses of Rs. 3000 per week in order to operate that plant. The number of units he should produce and sell in order to make a profit of at least Rs. 1000 per week is

(a) 400 (b) 300

(c) 250 (d) 200

[Based on FMS (Delhi) 2003]

25. Profit after selling an article for Rs 425 is the same as the loss after selling it for Rs 355. The cost of the article is

(a) Rs 385 (b) Rs 390

(c) Rs 395 (c) Rs 400

[Based on FMS (Delhi) 2003]

26. A tradesman by means of false balance defrauds to the extent of 8% in buying goods and also defrauds to the extent of 8% in selling. His gain percent is

(a) 16% (b) 15.48%

(c) 16.64% (d) 36%

[Based on FMS (Delhi) 2003]

27. If 7 kg of tea of price Rs 72/kg is blended wirh 33 kg of tea of Rs 87/kg and 35 kg of Rs 85/kg and the mixture is sold at 15% profit, then what is the selling price of the mixture?

(a) Rs 80.66/kg (b) Rs 102.33/kg

(c) Rs 91.22/kg (d) Rs 97.37/kg

(e) None of the above **[Based on IIFT 2003]**

28. A property dealer sells a house for Rs 6,30,000 and in the bargain makes a profit of 5%. Had he sold it for Rs 5,00,000, then what percentage of loss or gain he would have made?

(a) 15% loss (b) 15% gain

(c) $16\frac{2}{3}$% gain (d) $16\frac{2}{3}$% loss

(e) None of the above **[Based on IIFT 2003]**

29. In a game show, each participation costs Rs 150. The 1st person to play wins Rs 10, the second person wins Rs 50, the 3rd person wins Rs 100 and the 4th person wins Rs 250. This cycle is repeated with 5th person winning Rs 10. After 83 people have played, how much profit has the game show made for itself?

(a) 4090 (b) 3990

(c) 3900 (d) 8180

[Based on SCMHRD En. Exam. 2003]

30. A furniture store owner has determined that he can sell 100 chairs a month at a selling price of Rs 200 each. For each rise of Rs 4 in the selling price, he will sell 2 less chairs a month. If he sells the chairs for Rs Z each, then how much money a month will he receive from the sale of chairs ?

(a) $200 - Z/2$ (b) $50Z + Z^2/4$

(c) $150Z - Z^2/4$ (d) $200Z - Z^2/2$

[Based on SCMHRD Ent. Exam. 2003]

31. For how much must the goods bought for Rs 7560 be sold so as to gain $7\frac{1}{2}$ per cent of the cost?

(a) Rs 567 (b) Rs 10000

(c) Rs 8127 (d) None of these

[Based on IMT Ghaziabad, 2002]

32. A shopkeeper sold an article for Rs 6,750 after giving a discount of 10% on the labelled price. He would have earned a profit of 50%, had there been no discount. What was the actual percentage of profit earned?

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(a) 36 (b) 40
(c) 35 (d) None of these
[Based on IRMA, 2002]

33. A shopkeeper sells wheat at a profit of 10% and uses weights which are 20% less than the actual weight. The total gain earned by him will be
(a) 30% (b) 88%
(c) 37.5% (d) None of these
[Based on Narsee Monjee Inst. of Man. Studies 2002]

34. Mr. *X* bought a music system with 20% discount on the labelled price. Had he bought it with 25% discount, he would have saved Rs 500. At what price did he buy the music system?
(a) Rs 16000 (b) Rs 12000
(c) Rs 10000 (d) None of these
[Based on Narsee Monjee Inst. of Man. Studies 2002]

35. Sita buys a fridge at 15/16 of its original value and sells it for 10% more than its value. Then the gain % is
(a) 15.55 (b) 11.67
(c) 16.67 (d) None of these
[Based on MAT 2008]

36. A cash payment that will settle a bill for 250 chairs are Rs 50 per chair less 20% and 15% with a further discount of 5% on cash payment is
(a) Rs 8075 (b) Rs 7025
(c) Rs 8500 (d) None of these
[Based on MAT 2008]

37. A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. How much was sold at 10% gain and how much at 5% loss?
(a) 40 kg, 15 kg (b) 30 kg, 10 kg
(c) 35 kg, 40 kg (d) 40 kg, 10 kg
[Based on MAT 2008]

38. Prof. Chatterjee bought a car and got 15% of its original price ad dealer's discount. He then sold it with 20% profit on his purchase price. What percentage profit did he get on the original price?
(a) 2% (b) 12%
(c) 5% (d) 17%
[Based on FMS (Delhi) 2006]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let S.P. of x articles = Rs 100
= C.P. of 20 articles
$\therefore$ C.P. of one articles = Rs 5
Profit = 25%
$\therefore$ S.P. of one article = 6.25
$\Rightarrow$ S.P. of x articles = $6.25x$
$\therefore$ $6.25x = 100 \Rightarrow x = \mathbf{16}$.

2. Ans. (b)

3. Ans. (a)

Solution Let the C.P. of the article be Rs. x.
Profit = 5%
$\therefore$ S.P. = x + 5% of x = Rs $\frac{21x}{20}$
If C.P. would have been (x – 5% of x), i.e. Rs $\frac{19x}{20}$ and S.P. would have been Rs $\left(\frac{21x}{20}-1\right)$, then gain % = 10

$$\therefore \quad \frac{\left(\frac{21x}{20}-1\right)-\frac{19x}{20}}{\frac{19x}{20}}\times 100 = 10$$

$$\Rightarrow \quad \frac{2x-20}{19x}\times 100 = 10$$

$\Rightarrow$ $19x = 20x - 200 \Rightarrow x = \mathbf{200}$.

4. Ans. (d)

Solution S.P. = Rs 17940
Discount = 8%
$\therefore$ Marked Price = $\frac{17940\times 100}{100-8}$ = Rs 19500
Gain = 19.6%
C.P. = $\frac{17940\times 100}{100+19.6}$ = Rs 15000

If no discount is allowed on the Marked Price, then S.P. = Rs 19500
$\therefore$ Gain % = $\frac{4500}{15000}\times 100 = \mathbf{30}$.

5. Ans. (b)

Solution C.P.of one stamp = $\frac{8000\times 100}{100+20}$

$$= \frac{8000\times 100}{120} = \frac{20000}{3}$$

C.P. of another stamp = $\frac{8000\times 100}{100-20}$

$$= \frac{8000\times 100}{80} = 10000$$

$\therefore$ C.P. of both the stamps = $\frac{50000}{3}$
S.P. of both the stamps =16000

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$\therefore$ Loss $= \frac{2000}{3}$ = Rs 666.67 $\approx$ **Rs 667.**

6. Ans. (d)

Solution Let S.P. = Rs x

$\therefore$ Expenses = 15% of $x = 0.15x$

Profit = 10% of $x = 0.10x$

C.P. = Rs 9 (Given)

$\therefore$ $9 + 0.15x + 0.1x = x \Rightarrow x = 12$

$\therefore$ The rate of mark up on the article should be $\frac{100}{3}$%.

7. Ans. (b)

Solution Let actual price of the book = Rs x

Let actual price of the pen = Rs y

$\therefore$ $(x + 15\% \text{ of } x) + (y - 5\% \text{ of } y) = x + y + 7$

$\Rightarrow$ $15x - 5y = 700$ ...(i)

Also $(x + 10\% \text{ of } x) + (y + 5\% \text{ of } y) = x + y + 13$

$\Rightarrow$ $10x + 5y = 1300$ ...(ii)

Using (i) and (ii), we get $x = 80$, $y = 100$

$\therefore$ Actual price of the book = **Rs 80.**

8. Ans. (b)

Solution Suppose S.P. of 33 metres of cloth = Rs 33

$\therefore$ Gain = Rs 11 $\Rightarrow$ C.P. = Rs 22

$\Rightarrow$ **Gain = 50%.**

9. Ans. (b)

Solution Suppose C.P. = x and S.P. = y

$\Rightarrow$ 7% of y = 8% of x and 9% of y = 10% of x + 1

$\Rightarrow$ $7y = 8x$ and $9y = 10x + 100$

$\Rightarrow$ $9 \times \frac{8x}{7} = 10x + 100 \Rightarrow x = 350.$

10. Ans. (c)

Solution Suppose the price of first grade cycle = Rs x and the price of second grade cycle = Rs y.

$\therefore$ $8x + 22y = 3150$

Suppose he sells the first grade bicycles @ Rs z per bicycle

$\therefore$ $8z + 22 \times \frac{3z}{4} = 3150 + 40\% \text{ of } 3150$

i.e. $98z = 17640 \Rightarrow z = 180$

$\therefore$ S.P. of the first grade bicycle = **Rs 180**

S.P. of the second grade bicycle = Rs 135.

11. Ans. (a)

Solution Suppose the cost = Rs x

$\therefore$ S.P. of the manufacturer = $x + 10\% \text{ of } x = \frac{11x}{10}$

$\Rightarrow$ S.P. of the wholesale dealer

$= \frac{11x}{10} + 15\% \text{ of } \frac{11x}{10} = \frac{11x}{10} + \frac{33x}{200} = \frac{253}{200}x$

$\Rightarrow$ S.P. of the retailer

$= \frac{253}{200}x + 25\% \text{ of } \frac{253}{200}x$

$= \frac{253}{200}x + \frac{253}{800}x = \frac{1265}{800}x$

$\therefore$ $\frac{1265}{800}x = 1265 \Rightarrow x = 800.$

12. Ans. (c)

Solution Let C.P. of 12 pencils = Rs 12

$\therefore$ S.P. of 10 pencils = Rs 12

C.P. of 10 pencils = Rs 10

$\therefore$ Profit = **20%.**

13. Ans. (d)

Solution C.P. of the 1st Motor Car $= \frac{9900 \times 100}{100 + 10}$

= Rs 9000

C.P. of the 2nd Motor Car $= \frac{9900 \times 100}{100 - 10}$

= Rs 11000

$\therefore$ Total C.P. = Rs 20000

Total S.P. = Rs 19800

$\therefore$ Loss % $= \frac{200}{20000} \times 100 = 1.$

14. Ans. (c)

Solution $90 : 10.80 = 120 : x$

$\frac{90}{10.80} = \frac{120}{x}$

$\therefore$ $x = \frac{120 \times 10.80}{90} = 14.40.$

15. Ans. (d)

Solution $12\frac{1}{2}\%$ = Rs. 22.50

$\Rightarrow$ **C.P. = Rs. 180.**

16. Ans. (b)

Solution Let the C.P. be Rs. k

$\therefore$ S.P. = $k + 10\% \text{ of } k = \frac{11k}{10}$

$\therefore$ $k - 10\% \text{ of } k = \frac{11k}{10} - 40 \Rightarrow \frac{9k}{10} = \frac{11k}{10} - 40$

$\Rightarrow$ $\frac{2k}{10} = 40$

$\Rightarrow$ $k = 200.$

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17. Ans. (b)

Solution C.P. = Rs. C

Profit = 25% of $C = \frac{C}{4}$

S.P. = $C + \frac{C}{4} = \frac{5C}{4}$

If $\frac{C}{4}$ = 50% profit, then Capital Gains Tax = 50% of $\frac{C}{4} = \frac{C}{8}$.

18. Ans. (b)

Solution Let C.P. = Rs. x

$\therefore$ S.P. = Rs. y.

Profit = Rs. $(y - x)$

If S.P. = $2y$, then profit = $3(y - x)$

$\therefore$ $2y - x = 3(y - x) \Rightarrow y = 2k$

$\therefore$ Profit on Rs. x = Rs. x; i.e. **100%.**

19. Ans. (c)

Solution S.P. at the stall at the trade fair

$= 2x - 20\%$ of $2x = \frac{8x}{5}$

C.P. of the jacket = $\frac{x \times 100}{100 + 10} = \frac{10x}{11}$

$\therefore$ Profit = $\frac{8x}{5} - \frac{10x}{11} = \frac{38x}{55}$

$\therefore$ Profit % made at the trade fair

$$= \frac{\frac{38x}{55}}{\frac{10x}{11}} \times 100$$

$$= \frac{38}{5} \times \frac{1}{10} \times 100 = \mathbf{76.}$$

20. Ans. (d)

Solution Suppose C.P. = Rs 100

$\therefore$ Gain on Rs 25 = Rs 2.50 $\Rightarrow$ S.P. = Rs 27.50

Loss on Rs 75 = Rs 15 $\Rightarrow$ S.P. = Rs 60

$\therefore$ Total S.P. = Rs 87.50 $\Rightarrow$ **Loss = Rs 12.50.**

21. Ans. (a)

Solution Let C.P. of the book = Rs x

$\therefore$ $x - 10\%$ of $x + 85 = x + 7\%$ of x

$\Rightarrow$ 17% of $x = 85 \Rightarrow x = \mathbf{500.}$

22. Ans. (a)

Solution C.P. of 12 Pencils = Rs 12 say

$\therefore$ S.P. of 10 Pencils = Rs 12

$\Rightarrow$ S.P. of 12 pencils = $\frac{12}{10} \times 12$ = Rs 14.40

$\therefore$ Gain% = $\frac{2.40}{12} \times 100 = \mathbf{20.}$

23. Ans. (c)

Solution Let C.P of the umbrella = Rs x

Let S.P. of the umbrella = Rs y

$\therefore$ $\frac{x - \frac{2}{3}y}{x} \times 100 = 10 \Rightarrow \frac{3x - 2y}{3x} \times 100 = 10$

$$\Rightarrow \left(1 - \frac{2}{3} \times \frac{y}{x}\right) \times 100 = 10$$

$$\Rightarrow \frac{y}{x} = 1.35$$

$\therefore$ Profit % = $\frac{y - x}{x} \times 100 = 100\frac{y}{x} - 100$

$= 135 - 100 = \mathbf{35.}$

24. Ans. (d)

Solution Suppose the manufacturer should produce x items

$\therefore$ $60x - [40x + 3000] = 1000$

$\Rightarrow$ $20x = 4000 \Rightarrow \boldsymbol{x = 200.}$

25. Ans. (b)

Solution Suppose C.P. = Rs x

$\therefore$ $425 - x = x - 355 \Rightarrow 2x = 780$

$\Rightarrow \boldsymbol{x = 390.}$

26. Ans. (c)

Solution The tradesman pays for 100 kg and 108 kg of goods by means of false balance.

Actually, he sells 108 kg of goods, but due to false balance, he sells 116.64 kg of goods.

27. Ans. (d)

Solution C.P./kg of the mixture

$$= \frac{7 \times 72 + 33 \times 87 + 35 \times 85}{75}$$

$$= \frac{504 + 2871 + 2975}{75}$$

$$= \frac{6350}{75} = \text{Rs } 84.66$$

Profit = 15%

$\therefore$ S.P. per kg = Rs 84.66 + 15% of Rs 84.66

$= \mathbf{Rs\ 97.37.}$

28. Ans. (d)

Solution S.P. = Rs 630000

Profit = 5%

$\therefore$ C.P = Rs 600000

If S.P. would have been Rs 500000, then there would have been a loss of $\mathbf{16\frac{2}{3}\%}$.

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29. Ans. (a)

Solution Profit per cycle = Rs 190

Total profit $= 20 \times 190 + 140 + 100 + 50$

$= 3800 + 290 =$ **Rs 4090.**

30. Ans. (d)

Solution If S.P. of each chair is Rs $Z = 200 + 4K$, then number of chairs purchased

$$= 100 - 2K = 100 - \frac{2(Z-200)}{4}$$

$$= 100 - \frac{Z}{2} + 100 = 200 - \frac{Z}{2}$$

$\therefore$ Total money received from the sale of chairs

$$= Z\left(200 - \frac{Z}{2}\right) = \mathbf{200Z - \frac{Z^2}{2}}.$$

31. Ans. (c)

Solution $7560 + 7\frac{1}{2}\%$ of $7560 =$ **Rs 8127.**

32. Ans. (c)

Solution Marked Price = Rs x, say

$$\text{S.P.} = x - 10\% \text{ of } x = \frac{9x}{10}$$

$$\therefore \quad \frac{9x}{10} = 6750 \Rightarrow \boldsymbol{x = 7500}$$

If S.P. = Rs 7500, then the C.P. would have been Rs 5000, due to 50% profit earned by the shopkeeper .

$\therefore$ Actual percentage of profit by selling the article for Rs 6750

$$= \frac{1750}{5000} \times 100 = \mathbf{35\%}.$$

33. Ans. (c)

Solution Suppose C.P. = Rs 100/kg

S.P. = Rs 110/800 gms due to using false weight

= **Rs 137.50/kg.**

34. Ans. (None)

Solution Let the labelled price be Rs 100

$\therefore$ Discounted price = Rs 80

Had he bought the music system for Rs 75, he would have saved **Rs 5.**

35. Ans. (d)

Solution Let original value = x

$$\text{C. P} = \frac{15}{16} \times x = \frac{15}{16}x$$

$$\text{S. P} = x + \frac{10}{100} \times x = \frac{11}{10}x$$

$$\text{gain\%} = \frac{\frac{11}{10}x - \frac{15}{16}x}{\frac{15}{16}x} \times 100\% = \frac{52}{3} = 17.33\%.$$

36. Ans. (a)

Solution Rs $250 \times 50\left(\frac{80}{100}\right)\left(\frac{85}{100}\right)\left(\frac{95}{100}\right) =$ Rs 8075

37. Ans. (d)

38. Ans. (a)

Solution Let L.P = x

$$\text{discount} = \frac{15}{100}x = \frac{3}{20}x$$

$$\text{C.P} = x - \frac{3}{20}x = \frac{17}{20}x$$

$$\text{Profit} = \frac{17}{20}x \times \frac{20}{100}x = \frac{17}{100}x$$

$$\text{S.P} = \frac{17}{20}x + \frac{17}{100}x = \frac{51}{50}x$$

$$\text{Profit \%} = \left(\frac{x}{50} \times \frac{1}{x} \times 100\right)\% = 2\%$$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. A fruitseller buys oranges at 6 for Re. 1 and an equal number at 4 for Re. 1. He mixes them and sells at 20 paise per orange. Find his gain or loss per cent.

(a) 3 % profit (b) 5 % loss

(c) 4 % loss (d) 10 % profit

2. Mohan purchased 7 oranges for Rs. 6 and sold the whole lot at the rate of 3 for Rs. 4. In the process he gained Rs. 40. He purchased oranges:

(a) 63 (b) 80

(c) 84 (d) 90

3. *A* purchased an article for Rs. 500. He sold it to *B* at a profit of 10 %. *B* sold it back to *A* at a loss of 10 %. In the whole process how much did *A* gain?

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(a) Rs. 40 (b) Rs. 4
(c) Rs. 55 (d) Rs. 24
(e) None of these

4. A pressure cooker was sold for Rs. 120. $\frac{1}{5}$th part of S.P. was sales tax and $\frac{1}{3}$rd part of the remaining was profit. What is C.P. of the pressure cooker?
(a) Rs. 75 (b) Rs. 64
(c) Rs. 80 (d) Rs. 72
(e) None of these

5. By selling tea at Rs. 120 per kg, a shopkeeper makes a profit of 20 %. By how much per cent should he increase his S.P. so as to gain 30 %?
(a) $9\frac{1}{3}\%$ (b) $8\frac{2}{3}\%$
(c) $8\frac{1}{3}\%$ (d) $7\frac{1}{3}\%$

6. A man sold a watch for Rs. 600 at a loss. Had he sold it for Rs. 750, his gain would have been double the former loss. The cost price of the watch is
(a) Rs. 800 (b) Rs. 700
(c) Rs. 670 (d) Rs. 650

7. Sita buys a fridge at $\frac{15}{16}$th of its value and sells it for 10 % more than its value. Then, gain % is
(a) 11.67 (b) 15.55
(c) 16.67 (d) None of these

8. A shopkeeper earns 15 % profit on a shirt even after allowing 31 % discount on the list price. If list price is Rs. 125, then cost price of shirt is
(a) Rs. 87 (b) Rs. 80
(c) Rs. 75 (d) Rs. 69
(e) None of these

9. A man bought 70 kg rice worth Rs. 175. If he sells it at Rs. 2.75 per kg, his total gain or loss per cent is:
(a) 10% profit (b) 5% loss
(c) 8% profit (d) 4% loss

10. Tarun bought a T.V. with 20 % discount on the labelled price. Had he bought it with 25 % discount, he would have saved Rs. 500. At what price did he buy T.V.?
(a) Rs. 5000 (b) 10000
(c) Rs. 12000 (d) 16000

Test II

[Time: 10 min]

11. A trader sells his articles 20 % above C.P. and allows a discount of 10% on cash payment, his gain per cent is:
(a) 10% (b) 8%
(c) 6% (d) 5%

12. A fruit vendor purchased 88 dozens of bananas at Rs. 10 per dozen. Out of this, 12 dozens got rotten and he sold them at Rs. 6 per dozen. Remaining bananas were sold by him at Rs. 14 per dozen. What profit did the make in this transaction ?
(a) 26.5 % (b) 29.1 %
(c) 31% (d) 33.3 %

13. If an article is successively sold at a profit of 8% and 11%, difference in two selling prices is Rs. 15. What is the cost price of this article ?
(a) Rs. 490 (b) Rs. 480
(c) Rs. 510 (d) Rs. 500

14. A person bought some oranges at Rs 10 per dozen and bought the same number of oranges at Rs. 8 per dozen. He sold these oranges at Rs. 11 per dozen and gained Rs. 120. The total number of oranges bought by him was
(a) 30 dozens (b) 40 dozens
(c) 50 dozens (d) 60 dozens

15. A tradesman marks his goods 30% more than the cost price. If he allows a discount of $6\frac{1}{4}$, then his gain per cent is:
(a) $22\frac{3}{4}\%$ (b) 22%
(c) $21\frac{7}{8}\%$ (d) none of these

16. 20 percent profit is made on goods when a 10 % discount is given on the list price. When 20 % discount is given on the list price, the profit will be
(a) $6\frac{1}{2}\%$ (b) 15%
(c) $6\frac{2}{3}$ (d) 10 %

17. If a shopkeeper buys almonds at Rs. 250 per kg and sells at Rs. 10 per 50 gm, then he
(a) loses 20% (b) Loses 25 %
(c) Gains 20 % (d) gains 25 %
(e) None of these

18. A trader bought some knives at the rate of 8 knives for Rs. 34 and sold them at 12 knives for Rs. 57. How many knives must be sold by him to gain Rs. 900 ?
(a) 1400 (b) 1600
(c) 1800 (d) 200
(e) None of these

19. A shopkeeper sold an article at a profit of 17.5%. If he had bought it at 8% less and sold it at 30% profit, he would have earned Rs. 11.55 more as profit. Cost price of the article is:
(a) Rs. 550 (b) Rs. 675

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(a) Rs. 550 (b) Rs. 675
(c) Rs. 750 (d) Rs. 475

20. Ajay sold an article at a loss of 5%. Had he sold it at a gain of 5%, he would have gained Rs. 67.50 more. Find the cost price of the article:
(a) Rs. 670 (b) Rs. 575
(c) Rs. 675 (d) Rs. 560

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans (c)

Solution C.P. of 12 oranges at 6 for Re 1 = Rs. 2
and C.P. of 12 oranges at 4 for Re 1 = Rs. 3

$\therefore$ Total C.P. of 24 oranges = Rs. 5

$\therefore$ C.P. of 1 orange = Rs. $\frac{5}{24}$

and, S.P. of 1 orange = 20 paise = Rs. 1/5

$\therefore$ Loss = C.P. – S.P.

$= \text{Rs. } \frac{5}{24} - \text{Rs. } \frac{1}{5}$

$= \text{Rs. } \frac{25-24}{120} = \text{Rs. } \frac{1}{120}$

$$\text{Loss \%} = \frac{\text{Loss} \times 100}{\text{C.P.}} = \frac{\frac{1}{120} \times 100}{\frac{5}{24}} = 4\%$$

Hence, he suffered a loss of 4 % by selling oranges at the rate of 20 paise per orange.

3. Ans (c)

Solution C.P. of A = Rs. 500

$\therefore$ S.P. of $A = 500\left(\frac{100+10}{100}\right) = \text{Rs. } 550$

$\therefore$ C.P. for B = Rs. 550

$\therefore$ S.P. for $B = 550\left(\frac{100-10}{100}\right) = \text{Rs. } 495$

$\therefore$ Total profit for A = Rs. (550 – 495) = **Rs. 55.**

5. Ans (c)

Solution **In the first case:**

S.P. = Rs. 120 per kg

Profit = 20 %

$\therefore$ C.P. $= 120 \times \frac{100}{120} =$ **Rs. 100 per kg**

In the second case:

S.P. of tea with 30 % profit for shopkeeper

$= 100 \times \frac{130}{100} = \text{Rs. } 130 \text{ per kg}$

$\therefore$ S.P. of tea worth Rs. 120 per kg is increased by Rs. 10

$\therefore$ S.P. of tea worth Rs. 1 per kg is increased by Rs. $\frac{10}{120}$

$\therefore$ S.P. of tea worth Rs. 100 per kg is increased by Rs. $\frac{10 \times 100}{120} = \text{Rs. } 8\frac{1}{3}$.

$\therefore$ Percentage increase in S.P. = $\mathbf{8\frac{1}{3}\%}$.

7. Ans. (d)

Solution Let the value of the fridge be Rs. x.

Then, C.P. $= \frac{15}{16}x$

S.P. $= \frac{100+10}{100}x = \frac{11}{10}x$

Profit $= \frac{11}{10}x - \frac{15}{16}x = \frac{13}{80}x$.

Profit % $= \frac{13}{80}x \times \frac{16}{15x} \times 100 =$ **17.33 %.**

9. Ans (a)

Solution C.P. of 70 kg rice = Rs. 175

$\therefore$ C.P. of 1 kg rice = Rs. $\frac{175}{70}$ = Rs. 2.50

S.P. of 1 kg rice = Rs. 2.75

$\therefore$ Profit = Rs. 2.75 – Rs. 2.50 = Rs. 0.25

$\therefore$ Profit % $= \frac{\text{Profit}}{\text{C.P}} \times 100 = \frac{0.25}{2.50} \times 100 =$ **10 %.**

11. Ans. (b)

Solution Let C.P. Rs. 100.

Then, marked price = Rs. 120

S.P. = 90% of Rs. 120 = Rs. 108

$\therefore$ **Gain = 8 %.**

13. Ans. (d)

Solution Suppose C.P. of the article = Rs. x

$\therefore$ In the first case, C.P. = Rs. x; profit = 8 %

S.P. $= x \times \frac{108}{100} = \text{Rs. } \frac{108}{100}x$

In the second case, C.P. = Rs. x; profit = 11%

S.P. $= x \times \frac{111}{100} = \text{Rs. } \frac{111}{100}x$

According to question,

there is a difference of Rs. 15 in the two selling prices.

$\therefore \quad \frac{111x}{100} - \frac{108x}{100} = 15$

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or, $\frac{3x}{100} = 15$

or, $x = \frac{15 \times 100}{3}$ = **Rs. 500.**

15. Ans. (c)

Solution Let the C.P. be Rs. 100.

Then, marked price = Rs. 130

$$\text{S.P.} = \left(93\frac{3}{4}\% \text{ of Rs. } 130\right)$$

$$= \text{Rs.} \left(\frac{375}{4 \times 100} \times 130\right) = \text{Rs. } 121\frac{7}{8}$$

$\therefore$ Gain % = $\mathbf{21\frac{7}{8}}$.

17. Ans. (a)

Solution C.P. of 1 kg of almonds = Rs. 250

and, S.P. of 1 kg of almonds = $\frac{100 \times 1000}{50}$ = Rs. 200

$\therefore$ Loss on 1 kg almonds = Rs. 250 – Rs. 200 = Rs. 50

$\therefore$ Loss % = $\frac{50 \times 100}{250}$ = 20 %.

19. Ans. (a)

Solution Suppose C.P. of the article = Rs. 100

In the first case:

S.P. of the article = Rs. 117.50

In the second case:

C.P. of the article = Rs. 100 – 8% of Rs. 100

= Rs. 100 – Rs. 8 = Rs. 92

Profit = 30%

$\therefore$ S.P. of the article = $92 \times \frac{130}{100}$ = Rs. 119.60

In the second case, S.P. of the article exceeds the S.P. of the article in the first case by Rs. 119.60 – Rs. 117.50 = Rs. 2.10

$\therefore$ When profit is Rs. 2.10 more, C.P. of the article = Rs. 100

$\therefore$ When profit is Re. 1 more, C.P. = Rs. $\frac{100}{2.10}$

$\therefore$ When profit is Rs. 11.55 more, C.P. of the article = Rs. $\frac{100}{2.10} \times 11.55$ = **Rs. 550.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (b)	**6.** (d)	**8.** (c)	**10.** (b)	**12.** (b)	**14.** (d)	**16.** (c)	**18.** (c)	**20.** (c)

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Time and Work, Work and Wages 10

INTRODUCTION

In our daily life, we come across situations where we need to complete a particular job in a reasonable time. We have to complete the project earlier or later depending upon the needs. Accordingly, the men on duty have to be increased or decreased, i.e. the time allowed and the men engaged for a project are inversely proportional to each other, i.e. the more the number of men involved, the lesser is the time required to finish a job. We also come across situations where time and work or men and work are in direct proportion to each other.

For solving problems on time and work, we follow the following general rules:

1. If 'A' can do a piece of work in n days, then at a uniform rate of working 'A' will finish $\frac{1}{n}$th work in one day.
2. If $\frac{1}{n}$ of a work is done by 'A' in one day, then 'A' will take n days to complete the full work.
3. If 'A' does $\frac{1}{n}$th of a work in one hour then to complete the full work, 'A' will take $\frac{n}{m}$ hours.
4. If 'A' does three times faster work than 'B', then ratio of work done by A and B is 3 : 1 and ratio of time taken by A and B is 1 : 3.
5. A, B and C can do a piece of work in T_1, T_2 and T_3 days, respectively. If they have worked for D_1, D_2 and D_3 days respectively, then

   Amount of work done by $A = \frac{D_1}{T_1}$

   Amount of work done by $B = \frac{D_2}{T_2}$

   and, Amount of work done by $C = \frac{D_3}{T_3}$

   Also, the amount of work done by A, B and C together

   $$= \frac{D_1}{T_1} + \frac{D_2}{T_2} + \frac{D_3}{T_3}.$$

   which will be equal to 1, if the work is complete.

SOME USEFUL SHORT-CUT METHODS

1. If A can do a piece of work in X days and B can do the same work in Y days, then both of them working together will do the same work in $\frac{XY}{X+Y}$ days.

Explanation

A's 1 day's work $= \frac{1}{X}$

B's 1 day's work $= \frac{1}{Y}$

Then, $(A + B)$'s 1 day's work $= \frac{1}{X} + \frac{1}{Y} = \frac{X+Y}{XY}$

$\therefore$ A and B together can complete the work in

$$= \frac{XY}{X+Y} \text{ days.}$$

Illustration 1 A can finish a piece of work by working alone in 6 days and B, while working alone, can finish the same work in 12 days. If both of them work together, then in how many days, the work will be finished?

Solution Here, $X = 6$ and $Y = 12$.

$\therefore$ Working together, A and B will complete the work in $= \frac{XY}{X+Y}$ days $= \frac{6 \times 12}{6+12}$ days, i.e. **4 days.**

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2. If A, B and C, while working alone, can complete a work in X, Y and Z days respectively, then they will together complete the work in $\frac{XYZ}{XY+YZ+ZX}$ days.

Explanation

A's 1 day's work $= \frac{1}{X}$

B's 1 day's work $= \frac{1}{Y}$

C's 1 day's work $= \frac{1}{Z}$

$\therefore$ $(A + B + C)$'s 1 day's work

$$= \frac{1}{X}+\frac{1}{Y}+\frac{1}{Z} = \frac{XY+YZ+ZX}{XYZ}.$$

So, A, B and C together can complete the work in

$$= \left(\frac{XYZ}{XY+YZ+ZX}\right) \text{ days.}$$

Illustration 2 A, B and C can complete a piece of work in 10, 15 and 18 days, respectively. In how many days would all of them complete the same work working together?

Solution Here, $X = 10$, $Y = 15$ and $Z = 18$.

Therefore, the work will be completed in

$$= \frac{XYZ}{XY+YZ+ZX} \text{ days}$$

$$= \frac{10\times15\times18}{10\times15+15\times18+18\times10} \text{ days,}$$

i.e. $\frac{2700}{600}$ or, $4\frac{1}{2}$ **days.**

3. Two persons A and B, working together, can complete a piece of work in X days. If A, working alone, can complete the work in Y days, then B, working alone, will complete the work in $\frac{XY}{Y-X}$ days.

Explanation

A and B together can complete the work in X days.

$\therefore$ $(A + B)$'s 1 day's work $= \frac{1}{X}$

Similarly, A's 1 day's work $= \frac{1}{Y}$

Therefore, B's 1 day's work $= \frac{1}{X}-\frac{1}{Y} = \frac{Y-X}{XY}$.

$\therefore$ B alone can complete the work in $\left(\frac{XY}{Y-X}\right)$ days.

$\therefore$ B alone will complete the work in

$$= \frac{XY}{Y-X} \text{ days} = \frac{15\times20}{20-15}, \text{ i.e. 60 days.}$$

Illustration 3 A and B working together take 15 days to complete a piece of work. If A alone can do this work in 20 days, how long would B take to complete the same work?

Solution Here, $X = 15$ and $Y = 20$.

4. If A and B, working together, can finish a piece of work in X days, B and C in Y days, C and A in Z days, then

(a) A, B and C working together, will finish the job in $\left(\frac{2XYZ}{XY+YZ+ZX}\right)$ days.

(b) A alone will finish the job in $\left(\frac{2XYZ}{XY+YZ-ZX}\right)$ days.

(c) B alone will finish the job in $\left(\frac{2XYZ}{YZ+ZX-XY}\right)$ days.

Explanation

$(A + B)$'s 1 day's work $= \frac{1}{X}$

$(B + C)$'s 1 day's work $= \frac{1}{Y}$

$(C + A)$'s 1 day's work $= \frac{1}{Z}$.

So, $[(A + B) + (B + C) + (C + A)]$'s 1 day's work $= \frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

or, $2(A + B + C)$'s 1 day's work $= \left(\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}\right)$

or, $(A + B + C)$'s 1 day's work $= \frac{1}{2}\left(\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}\right)$

i.e. $\left(\frac{XY+YZ+ZX}{2XYZ}\right)$

$\therefore$ A, B and C working together, will complete the work in $\left(\frac{2XYZ}{XY+YZ+ZX}\right)$ days.

Also, A's 1 day's work = $(A + B + C)$'s 1 day's work $-$ $(B + C)$'s 1 day's work

$$= \frac{1}{2}\left(\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}\right)-\frac{1}{Y}$$

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$$= \frac{1}{2}\left(\frac{1}{X} - \frac{1}{Y} + \frac{1}{Z}\right)$$

$$= \frac{XY + YZ - ZX}{2XYZ}.$$

So, A alone can do the work in $\left(\frac{2XYZ}{XY + YZ - ZX}\right)$ days.

Similarly, B alone can do the work in $\left(\frac{2XYZ}{YZ + ZX - XY}\right)$ days and C alone can do the work in $\left(\frac{2XYZ}{ZX + XY - YZ}\right)$ days.

Illustration 4 A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?

Solution Here, $X = 12$, $Y = 15$ and $Z = 20$.

$\therefore$ A alone can do the work in

$$= \frac{2XYZ}{XY + YZ - ZX}$$

$$= \frac{2 \times 12 \times 15 \times 20}{12 \times 15 + 15 \times 20 - 20 \times 12} \text{ days}$$

or, $\frac{7200}{240}$, i.e. 30 days.

B alone can do the work in

$$= \frac{2XYZ}{YZ + ZX - XY} \text{ days}$$

$$= \frac{2 \times 12 \times 15 \times 20}{15 \times 20 + 20 \times 12 - 12 \times 15} \text{ days}$$

or, $\frac{7200}{360}$, i.e. 20 days.

C alone can do the work in

$$= \frac{2XYZ}{ZX + XY - YZ} \text{ days}$$

$$= \frac{2 \times 12 \times 15 \times 20}{20 \times 12 + 12 \times 15 - 15 \times 20} \text{ days}$$

or, $\frac{7200}{120}$, **i.e. 60 days.**

5. (a) If A can finish a work in X days and B is k times efficient than A, then the time taken by both A and B working together to complete the work is $\frac{X}{1+k}$.

(b) If A and B working together can finish a work in X days and B is k times efficient than A, then the time taken by

(i) A, working alone, to complete the work is $(k + 1)X$.

(ii) B, working alone, to complete the work is $\left(\frac{k+1}{k}\right)X$.

Illustration 5 Harbans Lal can do a piece of work in 24 days. If Bansi Lal works twice as fast as Harbans Lal, how long would they take to finish the work working together?

Solution Here, $X = 24$ and $k = 2$.

$\therefore$ Time taken by Harbans Lal and Bansi Lal, working together, to complete the work

$$= \left(\frac{X}{1+k}\right) \text{ days.}$$

$$= \left(\frac{24}{1+2}\right) \text{ days, i.e. 8 days.}$$

Illustration 6 A and B together can do a piece of work in 3 days. If A does thrice as much work as B in a given time, find how long A alone would take to do the work?

Solution Here, $X = 3$ and $k = 3$.

$\therefore$ Time taken by A, working alone, to complete the work

$$= \left(\frac{k+1}{k}\right)X = \left(\frac{3+1}{3}\right)3 = \textbf{4 days.}$$

6. If A working alone takes a days more than A and B working alone takes b days more than A and B together, then the number of days taken by A and B, working together, to finish a job is given by $\sqrt{ab}$.

Illustration 7 A alone would take 8 hours more to complete the job than if both A and B worked together. If B worked alone, he took $4\frac{1}{2}$ hours more to complete the job than A and B worked together. What time would they take if both A and B worked together?

Solution Here, $a = 8$ and $b = \frac{9}{2}$.

$\therefore$ Time taken by A and B, working together, to complete the job

$$= \sqrt{ab} \text{ days}$$

$$= \sqrt{8 \times \frac{9}{2}} \text{ or, 6 days.}$$

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7. If A is k times more efficient than B and is therefore able to finish a work in l days less than B, then
   (a) A and B, working together, can finish the work in $\frac{kl}{k^2-1}$ days.
   (b) A, working alone, can finish the work in $\frac{l}{k-1}$ days.
   (c) B, working alone, can finish the work in $\frac{kl}{k-1}$ days.

Illustration 8 A is thrice as good a workman as B and takes 10 days less to do a piece of work than B takes. Find the time in which B alone can complete the work.

Solution Here, $k = 3$ and $l = 10$.

$\therefore$ Time taken by B, working alone, to complete the work

$$= \frac{kl}{k-1} \text{ days}$$

$$= \frac{3\times 10}{3-1} \text{ days, i.e. 15 days.}$$

8. If A can complete $\frac{a}{b}$ part of work in X days, then $\frac{c}{d}$ part of the work will be done in $\frac{b\times c\times X}{a\times d}$ days.

Illustration 9 A can do $\frac{3}{4}$ of a work in 12 days. In how many days can he finish $\frac{1}{8}$ of the work?

Solution Here, $a = 3$, $b = 4$, $X = 12$, $c = 1$ and $d = 8$.

Therefore, number of days required to finish $\frac{1}{8}$ of the work

$$= \frac{b\times c\times X}{a\times d} = \frac{4\times 1\times 12}{3\times 8} = \textbf{2 days.}$$

9. (a) There are two groups of people with same efficiency. In one M_1 persons can do W_1 works in D_1 time and in the other M_2 persons can do W_2 works in D_2 time. The relationship between the two groups is given by
   $M_1D_1W_2 = M_2D_2W_1$

   (b) There are two groups of people with same efficiency. In one M_1 persons can do W_1 works in D_1 time working t_1 hours a day and M_2 persons can do W_2 works in D_2 time working t_2 hours a day. The relationship between the two groups is given by
   $M_1D_1t_1W_2 = M_2D_2t_2W_1$.

Illustration 10 If 10 persons can complete 2/5th of a work in 8 days, then find the number of persons required to complete the remaining work in 12 days.

Solution We have, $M_1 = 10$, $W_2 = \frac{2}{5}$, $D_1 = 8$

$M_2 = ?$, $W_2 = \frac{3}{5}$, $D_2 = 12$.

$\therefore$ $M_1D_1W_2 = M_2D_2W_1$

$\Rightarrow$ $10 \times 8 \times \frac{23}{5} = M_2 \times 12 \times \frac{2}{5}$

$\Rightarrow$ $\boldsymbol{M_2 = 10.}$

Illustration 11 If 10 persons can cut 20 trees in 3 days working 12 hours a day. Then, in how many days can 24 persons cut 32 trees working 4 hours a day?

Solution We have, $M_1 = 10$, $W_1 = 20$, $D_1 = 3$, $t_1 = 12$

$M_2 = 24$, $W_2 = 32$, $D_2 = ?$, $t_2 = 4$

$\therefore$ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

$\Rightarrow$ $10 \times 3 \times 12 \times 32 = 24 \times D_2 \times 4 \times 20$

$\Rightarrow$ $\boldsymbol{D_2 = 6}$ **days.**

10. If a men and b women can do a piece of work in n days, then c men and d women can do the work in $\left(\frac{nab}{bc+ad}\right)$ days

Illustration 12 12 men or 15 women can do a work in 14 days. In how many days, 7 men and 5 women would complete the work?

Solution Here, $a = 12$, $b = 15$, $n = 14$, $c = 7$ and $d = 5$.

Required number of days

$$= \frac{nab}{bc+ad} = \left(\frac{14\times 12\times 15}{15\times 7+12\times 5}\right) \text{ days}$$

$$= \frac{168}{11} \text{ days or, } 15\frac{3}{11} \textbf{ days.}$$

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed?

(a) $6\frac{2}{3}$ days (b) $8\frac{1}{3}$ days

(c) $7\frac{2}{3}$ days (d) None of these

2. A can do a piece of work in 30 days while B can do it in 40 days. A and B working together can do it in

(a) $15\frac{2}{7}$ days (b) $17\frac{1}{7}$ days

(c) $18\frac{3}{7}$ days (d) None of these

3. A can do 1/3 of a work in 5 days and B can do 2/5 of the work in 10 days. In how many days both A and B together can do the work?

(a) $13\frac{2}{3}$ days (b) $9\frac{3}{8}$ days

(c) $18\frac{5}{8}$ days (d) None of these

4. A, B and C can do a piece of work in 6, 12 and 24 days, respectively. They altogether will complete the work in

(a) $5\frac{2}{7}$ days (b) $4\frac{3}{7}$ days

(c) $3\frac{3}{7}$ days (d) None of these.

5. A is thrice as good as B and is therefore able to finish a piece of work in 60 days less than B. Find the time in which they can do it, working together.

(a) $22\frac{3}{4}$ days (b) $22\frac{1}{2}$ days

(c) 24 days (d) None of these

6. Ramesh takes twice as much time as Mahesh and thrice as much time as Suresh to complete a job. If working together, they can complete the job in 4 days, then the time taken by each of them separately to complete the work is

(a) 36, 24 and 16 days (b) 20, 16 and 12 days

(c) 24, 42 and 18 days (d) None of these

7. Sita takes twice as much time as Gita to complete a work and Rita does it in the same time as Sita and Gita together. If all three working together can finish the work in 6 days, then the time taken by each of them to finish the work is

(a) 18, 36, and 12 days (b) 20, 38 and 14 days

(c) 24, 42 and 18 days (d) None of these

8. 5 men can complete a work in 2 days, 4 women can complete the same work in 3 days and 5 children can do it in 3 days. 1 man, 1 woman and 1 child, working together, can complete the work in

(a) 6 days (b) 4 days

(c) 8 days (d) None of these

9. A and B can do a piece of work in 6 days and A alone can do it in 9 days. The time taken by B alone to do the work is

(a) 20 days (b) 18 days

(c) 24 days (d) None of these

10. A and B can finish a piece of work in 30 days, B and C in 40 days while C and A in 60 days. A, B, C together can do the work in

(a) $24\frac{3}{4}$ days (b) $28\frac{2}{3}$ days

(c) $26\frac{2}{3}$ days (d) None of these

11. A and B can do a piece of work in 18 days; B and C in 24 days; C and A in 36 days. A alone can do the work in

(a) 48 days (b) 56 days

(c) 40 days (d) None of these

12. Ajay and Sunil can do a piece of work in 10 days, Sunil and Sanjay in 15 days and Sanjay and Ajay in 20 days. They together work at it for 6 days and then Ajay leaves and Sunil and Sanjay go on together for 4 days more. If Sunil then leaves, how long will Sanjay take to complete the work?

(a) 12 days (b) 10 days

(c) 16 days (d) None of these

13. Anu can complete a work in 10 days. Manu is 25% more efficient than Anu and Sonu is 60% more efficient than Manu. Working together, how long would they take to finish the job?

(a) $3\frac{5}{8}$ days (b) $5\frac{6}{7}$ days

(c) $2\frac{6}{17}$ days (d) None of these

14. A and B finish a job in 12 days while A, B and C can finish it in 8 days. C alone will finish the job in

(a) 24 days (b) 36 days

(c) 28 days (d) None of these.

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15. Bansal, Gupta and Singhal together can complete a work in 4 days. If Bansal and Gupta together can complete the work in $4\frac{4}{5}$ days, Gupta and Singhal togehter can do it in 8 days, then Gupta alone can complete the work in
(a) 16 days (b) 12 days
(c) 20 days (d) None of these.

16. Nikita, Nishita and Kavita can complete a work in $2\frac{2}{3}$ days. If Nishita and Kavita can do it in 4 days and Nishita alone can do it in 6 days, then Nikita and Nishita can complete the work in
(a) $5\frac{4}{7}$ days (b) $4\frac{2}{7}$ days
(c) $3\frac{3}{7}$ days (d) None of these

17. A is twice as good a workman as B and together they finish a piece of work in 1 day. A alone can finish the work in
(a) 28 days (b) 21 days
(c) 24 days (d) None of these

18. Bindal can finish a work in 10 days. Jindal is twice as efficient as Bindal. If they work together, in how many days, the work will be finished?
(a) $3\frac{1}{3}$ days (b) $5\frac{2}{3}$ days
(c) $4\frac{1}{3}$ days (d) None of these

19. A alone would take 27 days more to complete the job than if both A and B would together. If B worked alone, he took 3 days more to complete the job than A and B worked together. What time would they take if both A and B worked together?
(a) 7 days (b) 9 days
(c) 11 days (d) None of these.

20. A is 4 times as fast as B and is therefore able to finish a work in 45 days less than B. A and B, working together, can complete the work in
(a) 12 days (b) 16 days
(c) 8 days (d) None of these.

21. If A can complete a work in 16 days, then in how many days can he complete $\frac{3}{4}$th of the work?
(a) 16 days (b) 20 days
(c) 12 days (d) None of these

22. Working 7 hours daily 24 men can complete a piece of work in 27 days. In how many days would 14 men complete the same piece of work working 9 hours daily?
(a) 36 days (b) 30 days
(c) 32 days (d) None of these

23. 10 men can cut 15 trees in 2 hours. If 2 men leave the job, how many trees will be cut in 3 hours?
(a) 20 trees (b) 18 trees
(c) 24 trees (d) None of these

24. 45 men complete a piece of work in 30 days working 12 hours a day. In how many days will 60 men complete the work working 10 hours a day?
(a) 27 days (b) 30 days
(c) 24 days (d) None of these

25. The work done by a woman in 8 hours is equal to the work done by a man in 6 hours and by a boy in 12 hours. If working 6 hours per day 9 men can complete a work in 6 days then in how many days can 12 men, 12 women and 12 boys together finish the same work working 8 hours per day?
(a) $2\frac{1}{2}$ days (b) $1\frac{1}{2}$ days
(c) $3\frac{1}{2}$ days (d) None of these

26. 4 men or 6 women can finish a piece of work in 20 days. In how many days can 6 men and 11 women finish the same work?
(a) 9 days (b) 6 days
(c) 7 days (d) None of these

27. 10 men can finish a piece of work in 10 days whereas it takes 12 women to finish it in 10 days. If 15 men and 6 women undertake to complete the work, how many days will they take to complete it?
(a) 7 days (b) 5 days
(c) 9 days (d) None of these

28. A can do a piece of work in 10 days, while B alone can do it in 15 days. They work together for 5 days and the rest of the work is done by C in 2 days. If they get Rs. 450 for the whole work, how should they divide the money?
(a) Rs. 250, Rs. 100, Rs. 100
(b) Rs. 225, Rs. 150, Rs. 75
(c) Rs. 200, Rs. 150, Rs. 100
(d) Rs. 175, Rs. 175, Rs. 100

29. Two men undertake to do a piece of work for Rs. 1400. First man alone can do this work in 7 days while the second man alone can do this work in 8 days. If they working together complete this work in 3 days with the help of a boy, how should money be divided?
(a) Rs. 600, Rs. 500, Rs. 300
(b) Rs. 600, Rs. 525, Rs. 275
(c) Rs. 600, Rs. 550, Rs. 250
(d) Rs. 500, Rs. 525, Rs. 375

30. A does half as much work as B in three-fourths of the time. If together they take 18 days to complete a work, how much time shall B take to do it?

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(a) 30 days (b) 35 days
(c) 40 days (d) None of these

31. Two men A and B working together complete a piece of work which it would have taken them respectively 30 and 40 days to complete if they worked separately. If they received a payment of Rs. 2100, B's share is
(a) Rs. 900 (b) Rs. 1200
(c) Rs. 800 (d) Rs. 1300

32. Two men undertake to do a piece of work for Rs. 600. One alone could do it in 6 days and the other in 8 days. With the assistance of a boy they finish it in 3 days. Boy's share should be
(a) Rs. 300 (b) Rs. 225
(c) Rs. 75 (d) Rs. 100

33. A can do a piece of work in 8 days. A undertook to do it for Rs. 320. With the help of B, he finishes the work in 6 days. B's share is
(a) Rs. 80 (b) Rs. 240
(c) Rs. 100 (d) Rs. 120

34. 5 men and 2 boys working together can do four times as much work per hour as a man and a boy together. The work done by a man and a boy should be in the ratio
(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 4 : 1

35. A, B and C can do a piece of work in 16, 32 and 48 days, respectively. They started working together but C left after working 4 days and B 2 days before the completion of work. Total number of days taken for completion of work was
(a) 8 days (b) $9\frac{1}{9}$ days
(c) 11 days (d) $10\frac{4}{9}$ days

36. A and B working separately can do a piece of work in 9 and 12 days, respectively. If they work for a day alternately, A beginning, in how many days the work will be completed?
(a) $10\frac{1}{2}$ days (b) $10\frac{1}{4}$ days
(c) $10\frac{2}{3}$ days (d) $10\frac{1}{2}$ days

37. A and B can do a piece of work in 45 and 40 days, respectively. They began the work together, but A leaves after some days and B finished the remaining work in 23 days. After how many days did A leave?
(a) 6 days (b) 8 days
(c) 9 days (d) 12 days

38. A and B working together can complete a piece of work in 12 days and B and C working together can complete the same work in 16 days. A worked at it for 5 days and B worked at it for 7 days. C finished the remaining work in 13 days. How many days would C alone take to complete it?
(a) 10 days (b) 24 days
(c) 32 days (d) 40 days

39. A can do a piece of work in 90 days. He starts working, but having some other engagements he drops out after 5 days. Thereafter B completes this work in 21 days. How many days would A and B take to complete this work working together?
(a) 15 days (b) 16 days
(c) 17 days (d) 11 days

40. A can do a piece of work in 30 days, B in 50 days and C in 40 days. If A is assisted by B on one day and by C on the next day alternately work will be completed in
(a) $17\frac{32}{35}$ days (b) $19\frac{2}{3}$ days
(c) $16\frac{31}{37}$ days (d) $18\frac{1}{3}$ days

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $X = 15$ and $Y = 12$

$\therefore$ Working together, 10 men and 15 women will complete the work in

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{15\times 12}{15+12}, \text{ i.e. } \frac{20}{3} \textbf{ or, } \mathbf{6\frac{2}{3}} \textbf{ days.}$$

2. Ans. (b)

Solution Here, $X = 30$ and $Y = 40$.

$\therefore$ A and B working together can do the work in

$$= \frac{XY}{X+Y} \text{ days.}$$

$$= \frac{30\times 40}{30+40} \text{ days, i.e. } \frac{120}{7} \textbf{ or, } \mathbf{17\frac{1}{7}} \textbf{ days.}$$

3. Ans. (b)

Solution A can do the complete work in $5 \times 3 = 15$ days.

B can do the complete work in $10 \times \frac{5}{2} = 25$ days. Here, $X = 15$ and $Y = 25$.

$\therefore$ A and B, working together, can complete the work in

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$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{15\times 25}{15+25} \text{ days, i.e. } \frac{75}{8} \textbf{ or, } 9\frac{3}{8} \textbf{ days.}$$

4. Ans. (c)

Solution Here, $X = 6$, $Y = 12$ and $Z = 24$.

$\therefore$ Working together, A, B and C will complete the work in

$$= \frac{XYZ}{XY+YZ+ZX} \text{ days}$$

$$= \frac{6\times 12\times 24}{6\times 12+12\times 24+24\times 6} \text{ days,}$$

i.e. $\frac{24}{7}$ days **or,** $3\frac{3}{7}$ **days.**

5. Ans. (b)

Solution Let B takes x days to do the work. Then, A takes $(x - 60)$ days to complete the work.

Since ratio of work done by A and B is 3 : 1, ratio of time taken by A and B is 1 : 3.

We have, $\frac{x-60}{x} = \frac{1}{3}$

$\Rightarrow 3(x - 60) = x$ or, $x = 90$.

$\therefore$ Time taken by B to finish the work = 90 days and time taken by A to finish the work = $\frac{90}{3}$ = 30 days.

$\therefore$ A and B, working together, will complete the work in

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{90\times 30}{90+30} \text{ days, i.e. } \frac{45}{2} \textbf{ or, } 22\frac{1}{2} \textbf{ days.}$$

6. Ans. (c)

Solution Let Ramesh takes x days to finish the work.

Then, Mahesh takes $\frac{x}{2}$ and Suresh takes $\frac{x}{3}$ days to finish the same work.

$\therefore$ Ramesh, Mahesh and Suresh, working together, will complete the work in

$$= \frac{XYZ}{XY+YZ+ZX} \text{ days}$$

$$= \frac{x+\frac{x}{2}\times\frac{x}{3}}{x\times\frac{x}{2}+\frac{x}{2}\times\frac{x}{3}+x\times\frac{x}{3}} \text{ days}$$

i.e. $\frac{x^3/6}{x^2}$ or, $\frac{x}{6}$ days.

Given : $\frac{x}{6} = 4$.

$\therefore$ $x = 24$

$\therefore$ Ramesh takes 24 days, Mahesh takes $\frac{24}{2}$ or 12 days and Suresh takes $\frac{24}{3}$ or 8 days to finish the work.

7. Ans. (a)

Solution Let Gita takes x days to complete the work. Then, Sita takes $2x$ days to complete the same work.

$\therefore$ Time taken by Rita to complete the work

$$= \frac{XY}{X+Y} \text{ days}$$

$$= \frac{x\times 2x}{x+2x} \text{ days or, } \frac{2x}{3} \text{ days.}$$

$\therefore$ Working together, Gita, Sita and Rita will finish the work in

$$= \frac{XYZ}{XY+YZ+ZX} \text{ days}$$

$$= \frac{x\times 2x\times\frac{2x}{3}}{x\times 2x+2x\times\frac{2x}{3}+\frac{2x}{3}\times x} \text{ days or, } \frac{x}{3} \text{ days.}$$

Given: $\frac{x}{3} = 6$. $\therefore$ $x = 18$ days.

$\therefore$ Gita takes 18 days, Sita takes 36 days and Rita takes 12 days to finish the work.

8. Ans. (b)

Solution 1 Man can complete the work in $5 \times 2 = 10$ days,

1 woman can complete the work in $4 \times 3 = 12$ days and 1 child can complete the work in $5 \times 3 = 15$ days.

$\therefore$ 1 man, 1 woman and 1 child, working together, can complete the work in

$$= \frac{XYZ}{XY+YZ+ZX} \text{ days}$$

$$= \frac{10\times 12\times 15}{10\times 12+12\times 15+15\times 10} = \textbf{4 days.}$$

9. Ans. (b)

Solution Here $X = 6$ and $Y = 9$.

$\therefore$ B alone will complete the work in = $\frac{XY}{Y-X}$ days

$$= \frac{6\times 9}{9-6}, \textbf{ i.e. 18 days.}$$

10. Ans. (c)

Solution Here, $X = 30$, $Y = 40$ and $Z = 60$.

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$\therefore$ A, B and C together will finish the work in

$$= \left(\frac{2XYZ}{XY + YZ + ZX}\right) \text{ days}$$

$$= \left(\frac{2 \times 30 \times 40 \times 60}{30 \times 40 + 40 \times 60 + 60 \times 30}\right) \text{ days}$$

or, $\frac{144000}{5400}$, **i.e.** $\mathbf{26\frac{2}{3}}$ **days.**

11. Ans. (a)

Solution Here, $X = 18$, $Y = 24$ and $Z = 36$.

$\therefore$ A alone can do the work in

$$= \left(\frac{2XYZ}{XY + YZ - ZX}\right) \text{ days}$$

$$= \left(\frac{2 \times 18 \times 24 \times 36}{18 \times 24 + 24 \times 36 - 36 \times 18}\right) \text{ days}$$

or, $\frac{31104}{648}$, **i.e. 48 days.**

12. Ans. (b)

Solution Ajay, Sunil and Sanjay, working together, can complete the work in

$$= \left(\frac{2XYZ}{XY + YZ + ZX}\right) \text{ days}$$

$$= \left(\frac{2 \times 10 \times 15 \times 20}{10 \times 15 + 15 \times 20 + 20 \times 10}\right) \text{ days}$$

or, $\frac{6000}{650}$, i.e. $\frac{120}{13}$ days.

$\therefore$ Work done by all of them together in 6 days

$$= 6 \times \frac{13}{120}, \text{ i.e. } \frac{13}{20}.$$

Also, work done by Sunil and Sanjay in 4 days $= \frac{4}{15}$.

$\therefore$ Remaining work $= 1 - \left(\frac{13}{20} + \frac{4}{15}\right) = \frac{1}{12}$, which is to be done by Sanjay.

Now, Ajay, Sunil and Sanjay, can complete the work in $\frac{120}{13}$ days and Ajay and Sunil can complete the work in 10 days.

$\therefore$ Sanjay alone can complete the work in

$$= \frac{\frac{120}{13} \times 10}{10 - \frac{120}{13}} \text{ days} = 120 \text{ days.}$$

$\therefore$ $\frac{1}{12}$ of the work is done by Sanjay in $\frac{120}{12}$ = **10 days.**

13. Ans. (c)

Solution Since Anu can complete the work in 10 days,

Manu can complete the work in $= 10 \times \frac{100}{125} = 8$ days.

Also, Sonu can complete the work in $= 8 \times \frac{100}{160} = 5$ days.

$\therefore$ Anu, Manu and Sonu will complete the work in

$$= \frac{XYZ}{XY + YZ + ZX} \text{ days}$$

$$= \left(\frac{10 \times 8 \times 5}{10 \times 8 + 8 \times 5 + 5 + 10}\right) \text{ days}$$

or, $\frac{400}{170}$, **i.e.** $\mathbf{2\frac{6}{17}}$ **days.**

14. Ans. (a)

Solution C alone will finish the job in

$$= \frac{XY}{Y - X} \text{ days} = \frac{8 \times 12}{12 - 8} \text{ days}$$

[Here, $X = 8$ and $Y = 12$]

= 24 days.

15. Ans. (b)

Solution Bansal, Gupta and Singhal together can finish the work in 4 days.

Bansal and Gupta together can do it in $\frac{24}{5}$ days.

Gupta and Singhal together can do it in 8 days. Therefore, Bansal alone can complete the work in

$$= \frac{XY}{Y - X} \text{ days}$$

$$= \left(\frac{8 \times 4}{8 - 4}\right) \text{ days}$$

[Here, $X = 4$ and $Y = 8$]

$= 8$ days.

Also, Singhal alone can complete the work in

$$= \left(\frac{XY}{Y - X}\right) \text{ days} = \left(\frac{\frac{24}{5} \times 4}{\frac{24}{5} - 4}\right) \text{ days}$$

$\left[\text{Here, } X = 4 \text{ and } Y = \frac{24}{5}\right]$

$= 24$ days.

$\therefore$ Bansal and Singhal can complete the work in

$$= \left(\frac{XY}{X + Y}\right) \text{ days} = \left(\frac{24 \times 8}{24 + 8}\right) \text{ days}$$

[Here, $X = 24$ and $Y = 8$]

$= 6$ days.

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∴ Gupta alone can complete the work in

$$= \left(\frac{XY}{Y-X}\right) \text{ days} = \left(\frac{4\times 6}{6-4}\right) \text{ days}$$

[Here, $X = 4$ and $Y = 6$]

= 12 days.

16. Ans. (c)

Solution Nishita and Kavita complete the work in 4 days and Nishita alone does it in 6 days.

∴ Kavita alone can do the work in

$$= \left(\frac{XY}{Y-X}\right) \text{ days} = \left(\frac{4\times 6}{6-4}\right) \text{ days}$$

[Here, $X = 4$ and $Y = 6$]

$= 12$ days.

∴ Nikita and Nishita can complete the work in

$$= \left(\frac{XY}{Y-X}\right) \text{ days} = \left(\frac{\frac{8}{3}\times 12}{12-\frac{8}{3}}\right) \text{ days}$$

$$\left[\text{Here, } X = \frac{8}{3} \text{ and } Y = 12\right]$$

$$= \frac{96}{28} = \mathbf{3\frac{3}{7}} \textbf{ days.}$$

17. Ans. (b)

Solution Here, $X = 14$ and $K = 2$

∴ A alone can finish the work in

$$= \left(\frac{K+1}{K}\right) X \text{ days} = \left(\frac{2+1}{2}\right) 14 \text{ days, } \textbf{i.e. 21 days.}$$

18. Ans. (a)

Solution Here, $X = 10$ and $K = 2$.

∴ The time taken by Bindal and Jindal, working together, to complete the work

$$= \frac{X}{1+K} = \frac{10}{1+2}, \text{ i.e. } \frac{10}{3} \textbf{ or, } \mathbf{3\frac{1}{3}} \textbf{ days.}$$

19. Ans. (b)

Solution Here, $a = 27$ and $b = 3$.

∴ Time taken by A and B, working together, to complete the job

$$= \sqrt{ab} \text{ days} = \sqrt{27\times 3} \text{ or, } 9 \text{ days.}$$

20. Ans. (a)

Solution Here, $k = 4$ and $l = 45$.

Therefore, A and B, working together, can complete the work in

$$= \frac{kl}{k^2-1} \text{ days} = \frac{4\times 45}{16-1} \text{ days, } \textbf{i.e. 12 days.}$$

21. Ans. (c)

Solution Here, $a = 1$, $b = 1$, $X = 16$, $c = 3$ and $d = 4$.

Therefore, number of days required to finish $\frac{3}{4}$ of the work

$$= \frac{b\times c\times X}{a\times d} = \frac{1\times 3\times 16}{1\times 4} = \textbf{12 days.}$$

22. Ans. (a)

Solution We have, $M_1 = 24$, $D_1 = 27$, $W_1 = 1$, $t_1 = 7$

$M_2 = 14$, $D_2 = ?$, $W_2 = 1$, $t_2 = 9$

∴ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

⇒ $24 \times 27 \times 7 \times 1 = 14 \times D_2 \times 9 \times 1$

⇒ $\boldsymbol{D_2 = 36}$ **days.**

23. Ans. (b)

Solution We have, $M_1 = 10$, $D_1 = 2$, $W_1 = 15$

$M_2 = 10 - 2 = 18$, $D_2 = 3$, $W_2 = ?$

∴ $M_1D_1W_2 = M_2D_2W_1$

⇒ $10 \times 2 \times W_2 = 8 \times 3 \times 15$

⇒ $W_2 = 18$ trees.

Thus, 18 trees will be cut in 3 hours.

24. Ans. (a)

Solution We have, $M_1 = 45$, $D_1 = 30$, $t_1 = 12$, $W_1 = 1$

$M_2 = 60$, $D_2 = ?$, $t_2 = 10$, $W_2 = 1$

∴ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

⇒ $45 \times 30 \times 12 \times 1 = 60 \times D_2 \times 10 \times 1$

⇒ $\boldsymbol{D_2 = 27}$ **days.**

25. Ans. (b)

Solution Given: 8 women = 6 Men = 12 Boys

∴ 12 Men + 12 Women + 12 Boys

= 12 Men + 9 Men + 6 Men = 27 Men

We have, $M_1 = 9$, $D_1 = 6$, $t_1 = 6$, $W_1 = 1$

$M_2 = 27$, $D_2 = ?$, $t_2 = 8$, $W_2 = 1$

∴ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

⇒ $9 \times 6 \times 6 \times 1 = 27 \times D_2 \times 8 \times 1$

⇒ $\boldsymbol{D_2 = \frac{3}{2}}$ **days or,** $\mathbf{1\frac{1}{2}}$ **days.**

26. Ans. (b)

Solution Here, $a = 4$, $b = 6$, $n = 20$, $c = 6$ and $d = 11$.

∴ Required number of days

$$= \left(\frac{nab}{bc+ad}\right) \text{ days} = \left(\frac{20\times 4\times 6}{6\times 6+4\times 11}\right) \text{ days}$$

= 6 days.

27. Ans. (b)

Solution Here, $a = 10$, $b = 12$, $n = 10$, $c = 15$ and $d = 6$.

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$\therefore$ Required number of days

$$= \left(\frac{nab}{bc+ad}\right) \text{ days}$$

$$= \left(\frac{10\times10\times12}{12\times15+10\times6}\right) \text{ days}$$

$$= \mathbf{5 \text{ days.}}$$

28. Ans. (b)

Solution $(A + B)$'s 5 day's work $= 5\left(\frac{1}{10}+\frac{1}{15}\right) = \frac{5}{6}$

Remaining work $= 1-\frac{5}{6} = \frac{1}{6}$

$\therefore$ C's 2 days' work $= \frac{1}{6}$

Now, A's 5 days' work : B's 5 days' work

$\therefore$ C's 2 days' work $= \frac{5}{10}:\frac{5}{15}:\frac{1}{6} = 3:2:1$

$\therefore$ A's share $= \text{Rs.}\left(450\times\frac{3}{6}\right) = \text{Rs. } 225$

B's share $= \text{Rs.}\left(450\times\frac{2}{6}\right) = \text{Rs. } 150$

C's share $= \text{Rs. } [450 - (225 + 150)] = \mathbf{Rs.\ 75}$

29. Ans. (b)

Solution Wages of the first man for 3 days

= work done by him in 3 days × Rs. 1400

$= \frac{3}{7} \times 1400 = \text{Rs. } 600$

Wages of second man for 3 days

= work done by him in 3 days × Rs. 1400

$= \frac{3}{8} \times 1400 = \text{Rs. } 525$

$\therefore$ Wages of the boy for 3 days

$= \text{Rs. } 1400 - \text{Rs. } (600 + 525) = \text{Rs. } 275$

$\therefore$ Their shares will be Rs. 600, Rs. 525 and Rs. 275, respectively.

30. Ans. (a)

Solution Suppose, B takes x days to do the work

$\therefore$ A takes $\left(2\times\frac{3}{4}x\right)$, i.e. $\frac{3x}{2}$ days to do it.

Now, $(A + B)$'s 1 day's work $= \frac{1}{18}$

$\therefore$ $\frac{1}{x}+\frac{2}{3x} = \frac{1}{18}$ **or,** $\mathbf{x = 30.}$

31. Ans. (a)

Solution A's 1 days' work $= \frac{1}{30}$

B's 1 days' work $= \frac{1}{40}$

$\therefore$ Share of A and B should be in the ratio

$$\frac{1}{30}:\frac{1}{40} = 4:3$$

$\therefore$ B's share $= \text{Rs.}\left(\frac{3}{7}\times2100\right) = \mathbf{Rs.\ 900.}$

32. Ans. (c)

Solution 1st man's 3 days' work $= \frac{3}{6} = \frac{1}{2}$

2nd man's 3 days' work $= \frac{3}{8}$

Boy's 3 days' work $= 1-\left(\frac{1}{2}+\frac{3}{8}\right) = \frac{1}{8}$

$\therefore$ They should get money in the ratio

$$\frac{1}{2}:\frac{3}{8}:\frac{1}{8} \quad \text{i.e. } 4:3:1$$

$\therefore$ Boy's share $= \text{Rs.}\ \frac{1}{8}\times600 = \text{Rs. } 75.$

33. Ans. (a)

Solution $(A + B)$'s 1 day's work $= \frac{1}{6}$

A's 1 days' work $= \frac{1}{8}$

$\therefore$ B's 1 days' work $= \frac{1}{6}-\frac{1}{8} = \frac{1}{24}$

$\therefore$ Money should be divided in the ratio

$$\frac{1}{8}:\frac{1}{24} = 3:1$$

$\therefore$ B gets $= \text{Rs.}\left(\frac{1}{4}\times320\right) = \mathbf{Rs.\ 80.}$

34. Ans. (b)

Solution Obviously,

$(5M + 2B) = 4(1M + 1B)$

$\therefore$ $M = 2B$

$\therefore$ Work done by a man and a boy are in the ratio 2 : 1.

35. Ans. (d)

Solution $(A + B + C)$'s 1 day's work

$$= \frac{1}{16}+\frac{1}{32}+\frac{1}{48} = \frac{11}{96}$$

Work done in first 4 days $= \frac{11}{96}\times4 = \frac{11}{24}$

For last two days, A works alone

$\therefore$ Work done in last 2 days $= \frac{1}{16}\times2 = \frac{1}{8}$

Rest of the work $\left(\text{i.e. } 1-\frac{11}{24}-\frac{1}{8}=\frac{5}{12}\right)$ is done by A and B together.

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$(A + B)$'s 1 day's work $= \frac{1}{16} + \frac{1}{32} = \frac{3}{32}$

$\therefore$ $\frac{5}{12}$ work is finished in $\frac{32}{3} \times \frac{5}{12} = \frac{40}{9}$ days

$\therefore$ Total number of days in which whole work is finished $= 4 + 2 + \frac{40}{9} = \mathbf{10\frac{4}{9}}$ **days.**

36. Ans. (b)

Solution $(A + B)$'s 2 days' work $= \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$

Evidently, the work done by A and B during 5 pairs of days

$$= 5 \times \frac{7}{36} = \frac{35}{36}$$

Remaining work $= 1 - \frac{35}{36} = \frac{1}{36}$

Now, on 11th day, it is A's turn

Now, $\frac{1}{9}$ work is done by A in 1 day

$\therefore$ $\frac{1}{36}$ work will be done by A in $9 \times \frac{1}{36} = \frac{1}{4}$ day

So, total time taken $= 10\frac{1}{4}$ days.

37. Ans. (c)

Solution B's 23 days' work $= \frac{23}{40}$

Remaining work $= 1 - \frac{23}{40} = \frac{17}{40}$

Now, $(A + B)$'s 1 day's work $= \frac{1}{45} + \frac{1}{40} = \frac{17}{360}$

$\frac{17}{360}$ work is done by A and B in 1 day

$\therefore$ $\frac{17}{40}$ work is done by A and B in $\frac{360 \times 17}{17 \times 40} = 9$ days

So, A left after 9 days.

38. Ans. (b)

Solution Suppose, C alone can do this work in x days

$\therefore$ C will do $\frac{1}{x}$ work in 1 day

Now, work done by $(B + C)$ in 1 day $= \frac{1}{16}$

$\therefore$ Work done by B in 1day $= \left(\frac{1}{16} - \frac{1}{x}\right)$

And, work done by $(A + B)$ in 1 day $= \frac{1}{12}$

$\therefore$ Work done by A in 1 day $= \frac{1}{12} - \left(\frac{1}{16} - \frac{1}{x}\right)$

$$= \frac{1}{48} + \frac{1}{x}$$

According to question,

Work done by A in 5 days + work done by B in 7 days + work done by C in 13 days = whole work

$\therefore$ $5\left(\frac{1}{48} + \frac{1}{x}\right) + 7\left(\frac{1}{16} - \frac{1}{x}\right) + \frac{13}{x} = 1$

or, $\frac{5}{48} + \frac{5}{x} + \frac{7}{16} - \frac{7}{x} + \frac{13}{x} = 1$

or, $\frac{26}{48} + \frac{11}{x} = 1$ or, $\frac{11}{x} = 1 - \frac{26}{48}$

or, $\frac{11}{x} = \frac{22}{48}$ **or,** $\boldsymbol{x = 24}$

$\therefore$ C alone would do this work in 24 days.

39. Ans. (a)

Solution Work done by A in 5 days $= \frac{5}{40} = \frac{1}{8}$

$\therefore$ Remaining work $= 1 - \frac{1}{8} = \frac{7}{8}$

$\therefore$ B completes $\frac{7}{8}$ work in 21 days

$\therefore$ B would complete 1 work in $\frac{21 \times 8}{7} = 24$ days

Here, $x = 40$, $y = 24$.

$\therefore$ Working together, A and B would complete this work in

$$= \frac{xy}{x + y} = \frac{40 \times 24}{40 + 24} = \frac{40 \times 24}{64} = \mathbf{15\ days.}$$

40. Ans. (a)

Solution $(A + B)$'s 1 day's work $= \frac{1}{30} + \frac{1}{50} = \frac{8}{150}$

$\therefore$ $(A + C)$'s 1 day's work $= \frac{1}{30} + \frac{1}{40} = \frac{7}{120}$

$\therefore$ Work done in first 2 days $= \frac{8}{150} + \frac{7}{120} = \frac{67}{600}$

Work done in $8 \times 2 = 16$ days $= \frac{67 \times 8}{600} = \frac{67}{75}$

Work left $= 1 - \frac{67}{75} = \frac{8}{75}$

On 17th day $(A + B)$ will work and they will finish $\frac{8}{150}$ work

$\therefore$ Work left $= \frac{8}{75} - \frac{8}{150} = \frac{8}{150} = \frac{4}{75}$

On 18th day $(A + C)$ will work and they will finish it in $\frac{120}{7} \times \frac{4}{75} = \frac{32}{35}$ days

$\therefore$ Whole work will be done in $\mathbf{17\frac{32}{35}}$ **days.**

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QUESTIONS FROM COMPETITIVE EXAMS

1. Running at the same constant rate, 6 identical machines can produce a total of 270 bottles per minute. At this rate, how many bottles could 10 such machines produce in 4 minutes?
(a) 648 (b) 1,800
(c) 2,700 (d) 10,800
[Based on MAT 2004]

2. Machine *A* produces bolts at a uniform rate of 120 every 40 seconds, and machine *B* produces bolts at a uniform rate of 100 every 20 seconds. If the two machines run simultaneously, then how many seconds will it take them to produce a total of 200 bolts?
(a) 22 (b) 25
(c) 28 (d) 32
[Based on MAT 2004]

3. Gunjan went to the stationers and bought things worth Rs. 25, out of which 30 paise went on sales tax on taxable purchases. If the tax rate was 6 percent, then what was the cost of the tax free items?
(a) Rs. 15 (b) Rs. 20
(c) Rs. 15.70 (d) Rs. 19.70
[Based on MAT 2003]

4. In an examination, there were 1100 boys and 900 girls. 50% of the boys and 40% of the girls passed the examination. The percentage of candidates failed is
(a) 45% (b) 45.5%
(c) 54.5% (d) 59.2%
[Based on MAT 2003]

5. A discount series of 10%, 20% and 40% is equal to a single discount of
(a) 50% (b) 56.80%
(c) 60% (d) 70.28%
[Based on MAT 2003]

6. The price of cooking oil has increased by 25%. The percentage of reduction that a family should effect in the use of cooking oil so as not to change its expenditure on cooking oil is
(a) 15% (b) 20%
(c) 25% (d) 30%
[Based on MAT 2003]

7. Two men undertake to do a piece of work for Rs. 200. One alone can do it in 6 days and the other in 8 days. With the help of a boy, they finish it in 3 days. How much is the share of the boy?
(a) Rs. 45 (b) Rs. 40
(c) Rs. 30 (d) Rs. 25
[Based on MAT 2003]

8. A worker makes a basket in $\frac{2}{3}$ of an hour. If he works for $7\frac{1}{2}$ hours, then how many baskets will he make?
(a) $10\frac{3}{4}$ (b) $11\frac{1}{4}$
(c) $12\frac{1}{2}$ (4) 13
[Based on MAT 2003]

9. If 5 men take an hour to dig a ditch, then how long should 12 men take to dig a ditch of the same type?
(a) 25 minutes (b) 30 minutes
(c) 28 minutes (d) 20 minutes
[Based on MAT 2003]

10. Rohit, Harsha and Sanjeev are three typists who, working simultaneously, can type 216 pages in four hours. In one hour, Sanjeev can type as many pages more than Harsha as Harsha can type more than Rohit. During a period of rive hours, Sanjeev can type as many pages as Rohit can during seven hours. How many pages does each of them type per hour?
(a) 16, 18, 22 (b) 14, 17, 20
(c) 15, 17, 22 (d) 15, 18, 21
[Based on MAT 2003]

11. A can do a work in 9 days. If *B* is 50% more efficient to *A*, then in how many days can *B* do the same work?
(a) 13.5 (b) 4.5
(c) 6 (d) 3
[Based on MAT 2003]

12. Two men and 7 children complete a certain piece of work in 4 days while 4 men and 4 children complete the same work in only 3 days. The number of days required by 1 man to complete the work is
(a) 60 days (b) 15 days
(c) 6 days (d) 51 days
[Based on MAT 2003]

13. Shatabadi Express has a capacity of 500 seats of which 10% are in the Executive Class and the rest being Chair cars. During one journey, the train was booked to 85% of its capacity. If Executive Class was booked to 96% of capacity, then how many Chair Car seats were empty during that journey
(a) 75 (b) 73
(c) 71 (d) None of these
[Based on MAT 2002]

14. '*A*' can do a piece of work in 25 days and *B* in 20 days. They work together for 5 days and then '*A*' goes away. In how many days will '*B*' finish the remaining work?
(a) 17 days (b) 11 days

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(c) 10 days (d) None of these
[Based on MAT 2002]

15. If Ajit can do $\frac{1}{4}$ of a work in 3 days and Sujit can do $\frac{1}{6}$ of the same work in 4 days, how much will Ajit get if both work together and are paid Rs. 180 in all?
(a) Rs 120 (b) Rs 108
(c) Rs 60 (d) Rs 36
[Based on MAT 2001]

16. *A* and *B* weave a carpet in 10 days and 15 days respectively. They begin to work together but *B* leaves after 2 days. In what time will *A* complete the remaining work?
(a) $6\frac{1}{3}$ days (b) $6\frac{2}{3}$ days
(c) 7 days (d) 8 days
[Based on MAT 2001]

17. *A* can do a work in 18 days, *B* in 9 days and *C* in 6 days. *A* and *B* start working together and after 2 days *C* joins them. What is the total number of days taken to finish the work?
(a) 4.33 (b) 4.5
(c) 4.66 (d) 5.33
[Based on MAT 2000]

18. If 18 persons can build a wall 140 m long in 42 days, the number of days that 30 persons will take to complete a similar wall 100 m long is
(a) 18 (b) 21
(c) 24 (d) 28
[Based on MAT 2005]

19. A can do piece of work in 14 days which *B* can do in 21 days. They begin together but 3 days before the completion of the work *A* leaves off. The total number of days to complete the work is
(a) $6\frac{3}{5}$ (b) $8\frac{1}{2}$
(c) $10\frac{1}{5}$ (d) $13\frac{1}{2}$
[Based on MAT 2005]

20. Rohit, Harsha and Sanjeev are three typists who, working simultaneously, can type 216 pages in four hours. In one hour, Sanjeev can type as many pages more than Harsha as Harsha can type more than Rohit. During a period of five hours, Sanjeev can type as many passges as Rohit can, during seven hours. How many pages does each of them type per hour?
(a) 16, 18, 22 (b) 14, 17, 20
(c) 15, 17, 22 (d) 15, 18, 21
[Based on MAT 2001]

21. If 7 maids with 7 mops cleaned 7 floors in 7 hours, how long would it takes 3 maids to mop 3 floors with 3 mops?
(a) 3 hours (b) 49/3 hours
(c) 7 hours (d) 7/3 hours
[Based on MAT 2001]

22. A contractor undertook to do a piece of work in 9 days. He employed certain number of labourers, but 6 of them being absent from the very first day, the rest could finish the work in 15 days. Find the number of men originally employed.
(a) 15 (b) 6
(c) 13 (d) 9
[Based on MAT 2001]

23. *A* and *B* together can do a piece of work in 6 days. *A* alone can do it in 10 days. What time will *B* require to do it alone?
(a) 20 days (b) 15 days
(c) 25 days (d) 30 days
[Based on MAT 2001]

24. If a man and a half can build a wall and a half in a day and a half, then how many walls do six men build in six days?
(a) 3 (b) 6
(c) 12 (d) None of these
[Based on FMS (Delhi) 2004]

25. A tailor had a number of shirt pieces to cut from a roll of fabric. He cut each roll of equal length in 10 pieces. He cut at the rate of 45 cuts a minute. How many rolls would he cut in 24 minutes?
(a) 32 rolls (b) 54 rolls
(c) 108 rolls (d) 120 rolls
[Based on FMS (Delhi) 2004]

26. 30 men can produce 1500 units in 24 days working 6 hours a day. In how many days, can 18 men produce 1800 units workfng 8 hours a day?
(a) 36 days (b) 45 days
(c) 18 days (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

27. A man completes $\frac{5}{8}$ of a job in 10 days. At this rate, how many more days will it take him to finish the job?
(a) 5 (b) 6
(c) 7 (d) $7\frac{1}{2}$
[Based on REC Tiruchirapalli 2003]

28. A computer can perform 30 identical tasks in six hours. At that rate, what is the minimum number of computers that should be assigned to complete 80 of the tasks within 3 hours?
(a) 6 (b) 7
(c) 12 (d) 16
[Based on REC Tiruchirapalli 2003]

29. A certain industrial loom weaves 0.128 metres of cloth every second. Approximately, how many seconds will it take for the loom to weave 25 metres of cloth?

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(a) 178 (b) 195
(c) 204 (d) 488
[Based on REC Tiruchirapalli 2003]

30. If four examiners can examine a certain number of answer books in 8 days by working 5 hours a day, then for how many hours a day, would two examiners have to work in order to examine twice the number of answer books in 20 days?
(a) 6 hours (b) $7\frac{1}{2}$ hours
(c) 8 hours (d) 9 hours
[Based on FMS (Delhi) 2003]

31. 15 men take 21 days of 8 hours each to do a piece of work. How many days of 6 hours each would 21 women take, if 3 women do as much work as 2 men?
(a) 20 (b) 18
(c) 25 (d) 30
[Based on FMS (Delhi) 2003]

32. A certain job was assigned to a group of men to do in 20 days. But 12 men did not turn up for the job and the remaining men did the job in 32 days. The original number of men in the group was
(a) 32 (b) 34
(c) 36 (d) 40
[Based on FMS (Delhi) 2003]

33. A man is paid Rs 5 per hour for regular working day and Rs 8 per hour overtime. A regular working day has 8 hours and a regular week has 5 working days. If he earns Rs 920 in 4 weeks, then what is the total number of hours that he worked assuming that he was present on all regular working days?
(a) 200 hours (b) 180 hours
(c) 175 hours (d) 160 hours
(e) None of the above **[Based on IIFT 2003]**

34. Ram finishes a work in 7 days. Rahim finishes the same job in 8 days and Robert in 6 days. They take turns to finish the work. Ram worked on the first day, Rahim on the second day and Robert on the third and then again Ram and so on. Who was working on the last day when work got finished?
(a) Ram (b) Rahim
(c) Robert (d) Rahim and Robert
(e) Can't be determined **[Based on IIFT 2003]**

35. Ritu and Somaya are working on an assignment. Ritu takes 6 hours to type 32 pages on computer, while Somaya takes 5 hours for typing 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?
(a) 8 hours (b) 7 hours 30 minutes
(c) 8 hours 15 minutes (d) 8 hours 25 minutes
[Based on SCMHRD Ent. Exam. 2003]

36. If a person makes '*J*' jobs in 'M' minutes, then how many jobs will he make in 2/3 hour?
(a) $\frac{2}{3}\frac{J}{M}$ (b) $\frac{2}{3}\frac{M}{J}$
(c) $40\frac{J}{M}$ (d) $40\frac{M}{J}$
[Based on IMT Ghaziabad, 2002]

37. A boy read $\frac{3}{8}$th of the book on one day and $\frac{4}{5}$th of the remainder on another day. If there were 30 pages unread, then how many pages did the book contain?
(a) 600 (b) 300
(c) 240 (d) None of these
[Based on IMT Ghaziabad, 2002]

38. A mill worker's basic pay for a 40-hour week is Rs 20. Overtime is paid for at 25% above the basic rate. In a certain week, he worked overtime and his total wage was Rs 25. He therefore worked for a total of
(a) 40 hours (b) 42 hours
(c) 48 hours (d) 50 hours
[Based on Narsee Monjee Inst. of Man. Studies 2002]

39. Three workers, working all days, can do a work in 10 days, but one of them having other employment can work only half time. In how many days the work can be finished?
(a) 15 days (b) 16 days
(c) 12 days (d) 12.5 days
[Based on FMS (Delhi) 2002]

40. Sam, Bob and Kirm can do a job alone in 15 days, 10 days and 30 days respectively. Sam is helped by Bob and Kim every third day. In how many days will the job be completed?
(a) 9 days (b) $8\frac{1}{3}$ days
(c) 8 days (d) $6\frac{1}{3}$ days
[Based on I.P. Univ. 2002]

41. Construction of a road was entrusted to a civil engineer. He was to finish the work in 124 days for which he employed 120 workmen. Two-thirds of the work was completed in 64 days. How many workmen can be reduced now without affecting the completion of the work on time?
(a) 56 (b) 64
(c) 80 (d) 24
[Based on I.P. Univ. 2002]

42. Two workers earned Rs 225. The first worked for 10 days and the second for 9 days. How much did each of them get daily if the first worker got Rs. 15 more for working 5 days than the second worker got for working 3 days?
(a) Rs 11.70; Rs 12.00 (b) Rs 10.80; Rs 13.00
(c) Rs 11.25; Rs 12.50 (d) Rs 12.60; Rs 11.00
(e) None of the above **[Based on SCMHRD 2002]**

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43. Eight children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, in how many days will 12 men finish the same work?

(a) 15 (b) 8
(c) 9 (d) 12

[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution 6 machines in 4 minutes can produce 1080 bottles.

10 machines in 4 minutes can produce

$$\frac{1080}{6}\times 10 = \textbf{1800 bottles.}$$

2. Ans. (b)

Solution 160 bolts/20 seconds, i.e. 8 bolts/second, i.e. 200 bolts/25 seconds.

3. Ans. (d)

Solution Since tax @ 6% = 30 paise, therefore, the cost of taxable items = Rs 5

$\Rightarrow$ Cost of non-taxable items

$= 25 - 5.30 = \textbf{Rs 19.70.}$

4. Ans. (c)

Solution No. of students passed = 550 + 360 = 910

$\therefore$ No. of students failed = 1090

$\therefore$ Percentage of students failed $= \frac{1090}{2000}\times 100 = \textbf{54.5\%.}$

5. Ans. (b)

Solution $100 - 10\% = 90$

$90 - 20\% = 90 - 18 = 72$

$72 - 40\% = 72 - 28.8 = 43.2$

$\therefore$ Single Discount $= 100 - 43.2 = \textbf{56.8\%.}$

6. Ans. (b)

7. Ans. (d)

Solution Suppose A and B are two men who can finish a certain work in 6 day and 8 days respectively. Let the boy C can finish the same work in x days.

$\therefore \quad \frac{1}{6}+\frac{1}{8}+\frac{1}{x} = \frac{1}{3}$

$\Rightarrow \quad x = 24$

$\Rightarrow$ Rs. 200 must be distributed among A, B and C in the ratio $\frac{1}{6}:\frac{1}{8}:\frac{1}{24}$, i.e. 4 : 3 : 1

$\therefore \quad C$'s share $= \frac{1}{8}\times 200$

$= \textbf{Rs 25.}$

8. Ans. (b)

Solution $\frac{7\frac{1}{2}}{\frac{2}{3}} = \frac{\frac{15}{2}}{\frac{2}{3}} = \frac{15}{2}\times\frac{3}{2} = \frac{45}{4} = 11\frac{1}{4}.$

9. Ans. (a)

10. Ans. (d)

Solution Suppose Rohit types x pages per hour.

Harsha types y pages per hour

Sanjeev types z pages per hour

$\therefore \quad z - y = y - x \Rightarrow y = \frac{x+z}{2}$ ...(i)

$\Rightarrow 5z = 7x$ ...(ii)

$4(x + y + z) = 216 \Rightarrow x + y + z = 54$ ...(iii)

Solving (i), (ii) and (iii) we get $y = \textbf{18}$, $x = \textbf{15}$, $z = \textbf{21}$.

11. Ans. (c)

Solution If A can do a work in 9 days then B can do the same work in 6 days. (B being 50% more efficient than A).

12. Ans. (b)

Solution $(2M + 7C)$'s one day's work $= \frac{1}{4}$

$\therefore$ $8M + 28C$ can finish the whole work.

Also, $(4M + 4C)$'s one day's work $= \frac{1}{3}$

$\therefore \quad (12M + 12C)$ can finish the whole work

$\therefore \quad 8M + 28C = 12M + 12C \Rightarrow M = 4C$

$\therefore \quad 4M + 4C = 5M$

Since 5 Men can complete the work in 3 days

$\Rightarrow$ 1 Man can complete the work in **15 days.**

13. Ans. (b)

Solution Total number of seats = 500

No. of seats in Executive Class = 50

No. of seats in Chair Car = 450

Total seats occupied = 85% of 500 = 425

Total seats occupied in Executive Class

= 96% of 50 = 48

Total seats occupied in the Chair Car

= 425 – 48 = 377

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$\therefore$ No. of seats vacant in the Chair Car

$= 450 - 377 = 73.$

14. Ans. (b)

Solution A's one day's work $= \frac{1}{25}$

B's one day's work $= \frac{1}{20}$

$(A + B)$'s one day's work $= \frac{1}{25} + \frac{1}{20} = \frac{9}{100}$

$(A + B)$'s five day's work $= \frac{45}{100} = \frac{9}{20}$

Remaining $\frac{11}{20}$ of the work is to be finished by B in **11 days.**

15. Ans. (a)

Solution Ajit's one day's work $= \frac{1}{12}$

Sujit's one day's work $= \frac{1}{24}$

$\therefore$ Rs 180 are divided among them in the ratio

$\frac{1}{12} : \frac{1}{24}$, i.e. 2 : 1.

$\therefore$ Ajit's share $= \frac{2}{3} \times 180 =$ **Rs 120.**

16. Ans. (b)

Solution $(A + B)$'s one day's work $= \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$

$\therefore$ $(A + B)$'s two day's work $= \frac{1}{3}$

Remaining $\frac{2}{3}$ of the work is done by A alone in $10 \times \frac{2}{3} = \frac{20}{3} = 6\frac{2}{3}$ **days.**

17. Ans. (None)

Solution $(A + B + C)$'s one day's work

$= \frac{1}{18} + \frac{1}{9} + \frac{1}{6} = \frac{1+2+3}{18} = \frac{1}{3}$

$(A + B)$'s one day's work $= \frac{1}{18} + \frac{1}{9} = \frac{1+2}{18} = \frac{1}{6}$

$(A + B)$'s 2 days' work $= \frac{1}{3}$

Remaining 2/3 of the work is finished by A, B and C together in 2 days.

$\therefore$ Total number of days taken to finish the work = 4.

18. Ans. (a)

Solution $140 \times 40 \times x = 100 \times 18 \times 42$

$\Rightarrow x = \frac{100 \times 18 \times 42}{140 \times 30} = 18$

$\therefore$ Required no. of days **= 18.**

19. Ans. (c)

Solution B's 3 days work $= \frac{3}{21} = \frac{1}{7}$

Remaining work $= 1 - \frac{1}{7} = \frac{6}{7}$

$(A + B)$'s 2 days' work $= \frac{1}{14} + \frac{1}{14} = \frac{5}{42}$

$\frac{5}{42}$ work is done by A and B in 1 day

$\therefore$ $\frac{6}{7}$ work is done by A and B in $\frac{42}{5} \times \frac{6}{7} = \frac{36}{5}$ days

$\therefore$ Total time taken $= 3 + \frac{36}{5} = 10\frac{1}{5}$ **days.**

20. Ans. (d)

Solution Let Rohit, Harsha and Sanjeev can type x, y and z pages respectively in one hour. Therefore, they together can type $4(x + y + z)$ pages in 4 hours.

$\therefore$ $4(x + y + z) = 216 \Rightarrow x + y + z = 54$ ...(1)

Also, $z - y = y - x$

i.e. $2y = x + z$ ...(2)

and $5z = 7x$ ...(3)

(1) and (2) $\Rightarrow$ $3y = 54 \Rightarrow y = 18$

$\therefore$ (2) $\Rightarrow x + z = 36$ ...(4)

(3) and (4) $\Rightarrow$ $x =$ **15,** $z =$ **21.**

21. Ans. (c)

22. Ans. (a)

Solution Let the number of men originally employed $= x$.

$\therefore$ $(x - 6)$ men could finish the work in 15 days and x men could finish the work in 9 days.

$\therefore$ $9x = 15(x - 6) \Rightarrow x =$ **15.**

23. Ans. (b)

Solution $(A + B)$'s one day's work $= \frac{1}{6}$

A's one day's work $= \frac{1}{10}$

$\therefore$ B's one day's work $= \frac{1}{6} - \frac{1}{10} =$ **$\frac{1}{15}$.**

24. Ans. (b)

Solution $\frac{3}{2}$ Men can build $\frac{3}{2}$ wall in $\frac{3}{2}$ days.

$\frac{3}{2}$ Men can build 6 walls in 6 days.

6 Men can build 24 walls in **6 days.**

25. Ans. (d)

Solution 5 rolls are cut in one minute.

26. Ans. (a)

Solution 30 men can produce 1500 units in 144 hours.

30 men can produce 1800 units in

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$\frac{144}{1500} \times 1800 = \frac{864}{5}$ hours

18 men can produce 1800 units in

$\frac{864}{5} \times \frac{30}{18}$ hours = 288 hours

= 36 days of 8 hours each.

27. Ans. (b)

Solution $\frac{5}{8}$ of the job is done in 10 days.

$\therefore$ Remaining $\frac{3}{8}$ of the job is done in

$10 \times \frac{8}{5} \times \frac{3}{8}$ **= 6 days.**

28. Ans. (a)

29. Ans. (b)

Solution $\frac{25}{0.128} = 195$ seconds.

30. Ans. (c)

Solution 4 examiners can examine certain number of Answer Books in 40 hours.

Suppose for x hours a day, 2 examiners examine the same number of Answer Books in 10 days.

$\Rightarrow$ 4 examiners examine the same number of answer books in $5x$ hours.

$\therefore$ $5x = 40 \Rightarrow x = 8$.

31. Ans. (d)

Solution 15 men can do a piece of work in 168 hours.

3 women = 2 men

$\Rightarrow$ 21 women = 14 men

Let 14 men can do the same piece of work in x days of 6 hours each i.e. 14 men can do the same piece of work in 6x hours.

$\therefore$ $6x \times 14 = 15 \times 168 \Rightarrow$ **$x = 30$.**

32. Ans. (a)

Solution Suppose the original number of men was x.

$\therefore$ x men could do the job in 20 days and $(x - 12)$ men could do the same job in 32 days.

$\therefore$ $20x = 32(x - 12) \Rightarrow 12x = 32 \times 12 \Rightarrow$ **$x = 32$.**

33. Ans. (c)

Solution No. of regular hours during 4 weeks

$= 8 \times 5 \times 4 = 160$.

Amount paid for regular working hours

$= 160 \times 5 =$ Rs 800

Balance of Rs 120 was paid as overtime @ Rs 8 per hour for **15 hours.**

34. Ans. (a)

Solution Three day's work $= \frac{1}{7} + \frac{1}{8} + \frac{1}{6} = \frac{73}{168}$

Six day's work $= \frac{73}{84}$

Seventh day work $= \frac{1}{7}$, done by Ram

Since $\frac{73}{84} + \frac{1}{7} = \frac{85}{84} > 1$, therefore Ram was working on the last day.

35. Ans. (c)

Solution In one hour, $\frac{32}{6} + \frac{40}{5} = \frac{16}{3} + 8 = \frac{40}{3}$ of the pages are typed.

$\therefore$ To type 110 pages, it will take

$\frac{3}{40} \times 110 = \frac{33}{4} =$ **$8\frac{1}{4}$ hrs.**

36. Ans. (c)

37. Ans. (c)

Solution Book read on the first day $= \frac{3}{8}$

Remaining book to be read $= \frac{5}{8}$

Book read on the second day $= \frac{4}{5} \times \frac{5}{8} = \frac{1}{2}$

Remaining book to be read $= \frac{5}{8} - \frac{1}{2} = \frac{1}{8}$

$\therefore$ $\frac{1}{8}$ of the book contains 30 pages.

$\Rightarrow$ The book contains **240 pages.**

38. Ans. (c)

39. Ans. (c)

Solution $(A + B + C)$'s one day's work

$= \frac{1}{30} + \frac{1}{30} + \frac{1}{60} = \frac{5}{60} = \frac{1}{12}$

$\therefore$ Work can be finished in **12 days.**

40. Ans. (a)

Solution In 3 days, $\frac{3}{15} + \frac{1}{10} + \frac{1}{30} = \frac{6+3+1}{30} = \frac{1}{3}$ of the work is done by Sam, Bob and Kim.

$\therefore$ The whole job will be finished by them in 9 days.

41. Ans. (a)

Solution $\frac{2}{3}$rd of the work was completed in 64 days by 120 men.

$\frac{1}{3}$rd of the work was completed in 32 days by 120 men.

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Also $\frac{1}{3}$rd of the work is to be completed in 60 days by $(120 - x)$ men, where x is the number of men reduced in order to finish the work on schedule.

$\therefore \quad (120 - x) \times 60 = 32 \times 120 \Rightarrow x = 56$

M recevied 5% of Rs 7400 extra for being the working member, i.e. Rs 370.

42. Ans. (b)

Solution Let A got Rs x per day and B got Rs y per day.

$\therefore \quad 10x + 9y = 225$ and $5x = 3y + 15$

$\Rightarrow \quad x = 10.80, y = 13.$

43. Ans. (d)

Solution Time taken by child is twice the time taken by man

$4 + 12 = 16$ men do the work in 9 days

$\therefore$ 12 men can do it in $= 9 \times \frac{16}{12} = 12$ days

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Two men undertake to do a piece of work for Rs. 400. One alone can do it in 6 days, the other in 8 days. With the help of a boy, they finish it in 3 days. The boy's share is
(a) Rs. 40 (b) Rs. 50
(c) Rs. 60 (d) Rs. 80

2. 2 men and 3 women can finish a piece of work in 10 days, while 4 men can do it in 10 days. In how many days will 3 men and 3 women finish it?
(a) 6 days (b) $5\frac{2}{3}$ days
(c) 8 days (d) $8\frac{1}{3}$ days

3. A, B and C can complete a work in 6, 8 and 12 days, respectively. A and B start the work together but B drops out after 2 days and C replaces him. How many days in whole they take to finish the job?
(a) $3\frac{2}{3}$ days (b) $2\frac{2}{3}$ days
(c) $3\frac{1}{3}$ days (d) $3\frac{2}{3}$ days

4. 8 children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, 12 men will finish the same work in
(a) 15 days (b) 12 days
(c) 14 days (d) 8 days

5. A, B and C can do a piece of work in 12, 18 and 24 days, respectively. They started work at it together. A stops the work after 4 days and B is called off 2 days before the work is done. The work was finished in
(a) 6 days (b) 10 days
(c) 8 days (d) 9 days

6. One man, 3 women and 4 boys can do a piece of work in 96 hours; 2 men and 8 boys can do it in 80 hours; 2 men and 3 women can do it in 120 hours. 5 men and 12 boys can do it in
(a) $42\frac{7}{11}$ hours (b) $43\frac{7}{11}$ hours
(c) 44 hours (d) $39\frac{1}{11}$ hours

7. 8 children and 12 men complete a certain piece of work in 9 days. If each child takes twice the time taken by a man to finish the work, in how many days will 12 men finish the same work?
(a) 8 (b) 15
(c) 9 (d) 12

8. A is twice as good a workman as B and thrice as good a workman as C. If C alone can do a piece of work in 24 days, how long would the three persons take to finish the work working together?
(a) $3\frac{3}{11}$ days (b) $4\frac{4}{7}$ days
(c) $4\frac{4}{11}$ days (d) $3\frac{4}{11}$ days

9. Two persons A and B can complete a piece of work in 8 hours and 16 hours, respectively. If they work at it alternately for an hour, A starting first, in how many hours will the work be finished?
(a) $9\frac{1}{3}$ hours (b) $10\frac{1}{2}$ hours
(c) $11\frac{1}{2}$ hours (d) $8\frac{1}{2}$ hours

10. A can do a certain job in 12 days. B is 60% more efficient than A. The number of days, it takes B to do the same piece of work is
(a) 6 (b) $6\frac{1}{4}$
(c) $7\frac{1}{2}$ (d) 8

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Test II **[Time: 10 min]**

11. 15 men can complete a work in 210 days. They started the work but at the end of 10 days 15 additional men, with double efficiency, were inducted. How many days, in whole, did they take to finish the work?

(a) $76\frac{2}{3}$ days (b) $84\frac{3}{4}$ days

(c) $72\frac{1}{2}$ days (d) 70 days

12. *A*, *B* and *C* can do a work in 6, 8 and 12 days' respectively. Doing that work together they get an amount of Rs. 1350. What is the share of *B* in that amount?

(a) Rs. 450 (b) Rs. 500

(c) Rs. 550 (d) Rs. 700

13. *A* and *B* undertake to do a work for Rs. 56. *A* can do it alone in 7 days and *B* in 8 days. If with the assistance of a boy they finish the work in 3 days, then the boy gets

(a) Rs. 11 (b) Rs. 12

(c) Rs. 15 (d) Rs. 18

14. Efficiencies of two men are in the ratio 2 : 3. If the first man takes 30 days to do a piece of work, the time required when both of them work together on a job twice as big as the first, will be

(a) 30 days (b) 18 days

(c) 24 days (d) 72 days

15. If 4 men and 6 boys can finish a piece of work in 24 days, in what time will 8 men and 9 boys finish it, supposing that 8 men can do as much work in 16 hours as 24 boys can do in 12 hours

(a) 12 days (b) $13\frac{1}{3}$ days

(c) 15 days (d) $12\frac{1}{2}$ days

16. *A*, *B* and *C* contract a work for Rs. 550. Together *A* and *B* are supposed to do $\frac{7}{11}$ of the work. How much does *C* get?

(a) Rs. 200 (b) Rs. 300

(c) Rs. 250 (d) Rs. 350

17. *A* can complete a job in 9 days, *B* in 10 days and *C* in 15 days. *B* and *C* start the work and are forced to leave after 2 days. The time taken to complete the remaining work is

(a) 6 days (b) 9 days

(c) 10 days (d) 13 days

18. 12 men can complete a work in 8 days. Three days after they started the work, 3 more men joined them. In how many days will all of them together complete the remaining work?

(a) 2 days (b) 4 days

(c) 5 days (d) 6 days

19. Sumit and Amit can complete a work in 10 days and 15 days, respectively. Amit starts the work and after 5 days Sumit also joins him. In all, the work would be completed in

(a) 7 days (b) 9 days

(c) 11 days (d) None of these

20. *A* and *B* working separately can do a piece of work in 8 and 10 hrs, respectively. If they work for an hour alternately, *A* beginning, work will be completed in

(a) $8\frac{4}{5}$ hours (b) 9 hours

(c) $6\frac{4}{5}$ hours (d) None of these

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution One man's 1 day's work $= \frac{1}{6}$

Another man's 1 day's work $= \frac{1}{8}$

Boy's 1 day's work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) = \frac{1}{24}$

Ratio of their shares $= \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1$

$\therefore$ Boy's share = Rs. $\left(400 \times \frac{1}{8}\right)$ = **Rs. 50.**

3. Ans. (a)

Solution Work done by *A* in 1 day $= \frac{1}{6}$

Work done by *B* in 1 day $= \frac{1}{8}$

Work done by *C* in 1 day $= \frac{1}{12}$

Work done by $(A + B)$ in 2 days $= \left(\frac{1}{6} + \frac{1}{8}\right) \times 2$

$= \frac{7}{24} \times 2 = \frac{7}{12}$

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$\therefore$ Remaining work $= 1 - \frac{7}{12} = \frac{5}{12}$

Work done by $(A + C)$ in 1 day $= \frac{1}{6} + \frac{1}{12} = \frac{1}{4}$

Since $(A + C)$ do $\frac{1}{4}$ work in 1 day

$\therefore$ $(A + C)$ will do $\frac{5}{12}$ work in $4 \times \frac{5}{12} = \frac{5}{3}$ days

$\therefore$ Total time required for completing the work

$= 2 + 1\frac{2}{3} = \mathbf{3\frac{2}{3}}$ **days.**

5. Ans. (c)

Solution $(A + B + C)$'s 1 day's work $= \frac{1}{12} + \frac{1}{18} + \frac{1}{24}$

$= \frac{13}{72}$

$(A + B + C)$'s 4 day's work $= \frac{13}{72} \times 4 = \frac{13}{18}$

Remaining work $= 1 - \frac{13}{18} = \frac{5}{18}$

Work is completed by C alone in last two days

C's last 2 day's work $= 2 \times \frac{1}{24} = \frac{1}{12}$

$\therefore$ Remaining work $= \frac{5}{18} - \frac{1}{12} = \frac{7}{36}$

This much work is done by B and C together

$(B + C)$'s 1 day's work $= \frac{1}{18} + \frac{1}{24} = \frac{7}{36}$

$\therefore$ B and C together finish $\frac{7}{36}$ work in

$\frac{72}{7} \times \frac{7}{36} = 2$ days

$\therefore$ Whole work is completed in $4 + 2 + 2 = \mathbf{8}$ **days.**

7. Ans. (d)

Solution 2 children $\equiv$ 1 man

$\therefore$ (8 children + 12 men) = 16 men

Now, less men, more days

12 : 16 : : 9 : x

$\therefore$ $x = \frac{16 \times 9}{12} = \mathbf{12}$ **days.**

9. Ans. (b)

Solution Work done by A in 1 hour $= \frac{1}{8}$ and

Work done by B in 1 hour $= \frac{1}{16}$

According to question:

They work at it alternately for an hour

$\therefore$ Work done by A and B in 2 hours

$= \frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

$\therefore$ Work done by A and B in 10 hours

$= \frac{3}{16} \times 5 = \frac{15}{16}$

$\therefore$ Remaining work $= 1 - \frac{15}{16} = \frac{1}{16}$

Now, comes the turn of A

Since A does 1 work in 8 hours = 8 × 60 minutes

$\therefore$ A does $\frac{1}{16}$ work in $\frac{8 \times 60}{16} = 30$ minutes

$\therefore$ Total time taken

$= 10$ hours + 30 minutes $= \mathbf{10\frac{1}{2}}$ **hours.**

11. Ans. (a)

Solution 15 men do in 210 days = 1 work

$\therefore$ 15 men will do in 10 days $= \frac{1}{210} \times 10 = \frac{1}{21}$ work

$\therefore$ Remaining work $= 1 - \frac{1}{21} = \frac{20}{21}$

According to question:

15 new men with double efficiency were inducted after 10 days.

i.e. work done by previously employed 15 men

$=$ work done by $\frac{1}{2} \times 15$ new men

$\therefore$ Work done by previously employed 30 men = work done by 15 new men

$\therefore$ 15 men do $\frac{1}{21}$ work in 10 days

$\therefore$ 45 men will do $\frac{20}{21}$ work in $\frac{10 \times 21 \times 20 \times 15}{45 \times 21}$

$= \frac{200}{3} = 66\frac{2}{3}$ days

$\therefore$ Total time spent in completing the work

$= 10$ days $+ 66\frac{2}{3}$ days $= \mathbf{76\frac{2}{3}}$ **days.**

13. Ans. (a)

Solution A's 3 days' work + B's 3 days' work + Boys' 3 days' work = 1

or, $\frac{3}{7} + \frac{3}{8}$ + Boy's 3 days' work = 1

or, Boy's 3 days' work $= 1 - \left(\frac{3}{7} + \frac{3}{8}\right) = \frac{11}{56}$

Ratio of shares $= \frac{3}{7} : \frac{3}{8} : \frac{11}{56} = 24 : 21 : 11$

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$\therefore$ Boy's share $= \frac{56}{24+21+11} \times 11 =$ **Rs. 11.**

15. Ans. (b)

Solution 8×16 man-hours $= 12 \times 24$ boy-hours

$\therefore$ 1 man hour/man day $= \frac{9}{4}$ boy-hours/boy days

The work required 96 men-days + 144 boy-days
or 96 men-days + 64 men-days = 160 men-days
8 men + 9 boys = 8 men + 4 men = 12 men

$\therefore$ They required $= \frac{160}{12}$ days $= \mathbf{13\frac{1}{3}}$ **days.**

17. Ans. (a)

Solution $(B + C)$'s 2 day's work $= 2\left(\frac{1}{10}+\frac{1}{15}\right) = \frac{1}{3}$

Remaining work $= 1-\frac{1}{3} = \frac{2}{3}$

$\frac{1}{9}$ work is done by A in 1 day

$\therefore \frac{2}{3}$ work is done by A in $\left(\frac{9\times 2}{3}\right) =$ **6 days.**

19. Ans. (b)

Solution Amit's 5 days' work $= 5\times\frac{1}{15} = \frac{1}{3}$

Remaining work $= 1-\frac{1}{3} = \frac{2}{3}$

$\left(\frac{1}{10}+\frac{1}{15}\right)$ work is done by both in 1 day

$\therefore \frac{2}{3}$ work is done by both in $6\times\frac{2}{3} = 4$ days

Hence, the work was completed in **9 days.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (b)	**6.** (b)	**8.** (c)	**10.** (c)	**12.** (a)	**14.** (c)	**16.** (a)	**18.** (b)	**20.** (a)

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Pipes and Cisterns 11

INTRODUCTION

Pipes are connected to a *tank* or *cistern* and are used to fill or empty the tank.

Inlet A pipe connected with a tank or a cistern that fills it is known as *inlet*.

Outlet A pipe connected with a tank or cistern emptying it is known as *outlet*.

Pipes and Cistern problems are similar to those on Time and Work. The only difference here is, the work done is in terms of filling or emptying a cistern and the time taken is the time taken by a pipe or leak (crack) to fill or empty a cistern, respectively.

Generally, the time taken to fill a cistern is taken as positive and the time taken to empty a cistern is taken as negative. The amount of work done, i.e. filling or emptying a cistern is generally taken as unity, unless otherwise specified.

SOME BASIC FORMULAE

1. If an inlet can completely fill the empty tank in X hours, the part of the tank filled in 1 hour $= \frac{1}{X}$.
2. If an outlet can empty the full tank in Y hours, the part of the tank emptied in 1 hour $= \frac{1}{Y}$.
3. If both inlet and outlet are open, net part of the tank filled in 1 hour $= \frac{1}{X} - \frac{1}{Y}$.

Illustration 1 A pipe can fill a tank in 5 hours. Find the part of tank filled in one hour.

Solution The part of the tank filled in 1 hour $= \mathbf{\frac{1}{5}}$.

Illustration 2 A pipe can fill a tank in 28 minutes. Find the time in which $\frac{1}{7}$ part of the tank will be filled.

Solution We have, $\frac{1}{28}$ part of the tank is filled in 1 minute.

$\therefore$ $\frac{1}{7}$ part of the tank is filled in $\frac{28}{7}$ minutes

$= \mathbf{4\ minutes.}$

Illustration 3 A pipe can empty a cistern in 40 minutes. Find the time in which $\frac{3}{4}$ part of the cistern will be emptied.

Solution We have, $\frac{1}{40}$ part of the cistern is emptied in = 1 minute.

$\therefore$ $\frac{3}{4}$ part of the cistern is emptied in

$= 40 \times \frac{3}{4} = \mathbf{30\ minutes.}$

Illustration 4 A pipe can empty a cistern in 12 hours. Find the part of the cistern emptied in 4 hours.

Solution We have, part of the cistern emptied in 1 hour $= \frac{1}{12}$,

$\therefore$ part of the cistern emptied in 4 hours $= \frac{1}{12} \times 4 = \mathbf{\frac{1}{3}}$.

Illustration 5 A tap can fill a cistern is 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, find the time (in hours) to fill the cistern.

Solution Here, $X = 8$ and $Y = 16$.

$\therefore$ Part of the cistern filled in 1 hour

$$= \frac{1}{X} - \frac{1}{Y}$$

$$= \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$$

$\therefore$ Total time taken to fill the cistern = **16 hours.**

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SOME USEFUL SHORT-CUT METHODS

1. Two pipes A and B can fill (or empty) a cistern in X and Y hours respectively, while working alone. If both the pipes are opened together, then the time taken to fill (or empty) the cistern is given by

$$\left(\frac{XY}{X+Y}\right)\text{hours.}$$

Explanation

Part of the cistern filled (or emptied) by pipe A alone in 1 hour $= \frac{1}{X}$

Part of the cistern filled (or emptied) by pipe B alone in 1 hour $= \frac{1}{Y}$.

$\therefore$ Part filled (or emptied) by $(A + B)$ in 1 hour

$$= \frac{1}{X}+\frac{1}{Y} = \frac{X+Y}{XY}.$$

Therefore, both the pipes A and B together will fill (or empty) the cistern in $\left(\frac{XY}{X+Y}\right)$ hours.

Illustration 6 Two pipes A and B can fill a cistern in 20 and 30 minutes, respectively. If both the pipes are opened simultaneously, how long will it take to fill the cistern?

Solution Here, $X = 20$ and $Y = 30$.

$\therefore$ Part of the cistern filled by $(A + B)$ in 1 minute

$$= \frac{1}{X}+\frac{1}{Y} = \frac{1}{20}+\frac{1}{30} = \frac{5}{60} = \frac{1}{12}.$$

$\therefore$ Both the pipes A and B together will fill the cistern in **12 minutes.**

2. Three pipes A, B and C can fill a cistern in X, Y and Z hours respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern is given by

$$\left(\frac{X\times Y\times Z}{XY+YZ+ZX}\right)\text{hours.}$$

Explanation

Part of the cistern filled by A alone in 1 hour $= \frac{1}{X}$

Part filled by B alone in 1 hour $= \frac{1}{Y}$

Part filled by C alone in 1 hour $= \frac{1}{Z}$

All the three pipes are opened.

$\therefore$ Part filled in 1 hour $= \frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

$$= \frac{XY+YZ+ZX}{XYZ}$$

$\therefore$ Cistern will be filled in $\frac{XYZ}{XY+YZ+ZX}$ hours.

Note: We can generate more formulae like above by replacing negative sign wherever a pipe starts emptying a cistern instead of the standard positive sign.

Illustration 7 Two pipes A and B can separately fill a cistern is 8 hours and 12 hours respectively, while a third pipe C can empty it in 6 hours. In what time will the cistern be full, if all the pipes are opened together?

Solution Here, $X = 8$, $Y = 12$ and $Z = -6$.

$\therefore$ The cistern will be full in

$$= \left(\frac{8\times12\times-6}{8\times12-12\times6-6\times8}\right)\text{hours}$$

$$= \left(\frac{576}{24}\right)\textbf{ hours or 24 hours.}$$

3. Two pipes A and B can fill a cistern in X hours and Y hours, respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in Z hours. The time taken by C to empty the full tank is given by

$$\left(\frac{XYZ}{XZ+YZ-XY}\right)\text{hours.}$$

Explanation

Part of the tank emptied by C in 1 hour

$$= \left(\frac{1}{X}+\frac{1}{Y}-\frac{1}{Z}\right)$$

$\therefore$ C can empty the full tank in $\left(\frac{XYZ}{XZ+YZ-XY}\right)$ hours.

Illustration 8 Two taps A and B can fill a cistern in 30 minutes and 60 minutes, respectively. There is third exhaust tap C at the bottom of the tank. If all taps are opened at the same time, the cistern will be full in 45 minutes. In what time can exhaust tap C empty the cistern when full?

Solution Here, $X = 30$, $Y = 60$ and $Z = 45$.

$\therefore$ Exhaust tap C can empty the cistern in

$$= \left(\frac{XYZ}{XZ+YZ-XY}\right)\text{ minutes}$$

$$= \left(\frac{30\times60\times45}{30\times45\times60\times45-30\times60}\right)\text{ minutes}$$

$$= \textbf{36 minutes.}$$

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4. A tank takes X hours to be filled by a pipe. But due to a leak, it is filled in Y hours. The amount of time in which the leak can empty the full tank

$$= \left(\frac{XY}{Y-X}\right) \text{hours.}$$

Illustration 9 A pipe can fill a tank in 12 hours. Due to leakage in the bottom, it is filled in 24 hours. If the tank is full, how much time will the leak take to empty it?

Solution Here, $X = 12$ and $Y = 24$.

$\therefore$ The time taken by the leak to empty the full tank

$$= \left(\frac{XY}{Y-X}\right) \text{hours}$$

$$= \left(\frac{12\times 24}{24-12}\right) \textbf{hours or 24 hours.}$$

5. A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. The capacity of the cistern is

$$\left(\frac{XYZ}{Z-X}\right) \text{ litres.}$$

Illustration 10 A leak in the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres per minute. When the tank is full, the inlet is opened and due to leak, the tank is empty in 8 hours. Find the capacity of the tank.

Solution Here, $X = 6$, $Y = 4 \times 60 = 240$ and $Z = 8$.

$\therefore$ The capacity of the tank is

$$= \left(\frac{XYZ}{Z-X}\right) \text{litres} = \left(\frac{6\times 240\times 8}{8-6}\right) \text{litres}$$

$$= \textbf{5760 litres.}$$

6. One fill pipe A is k times faster than the other fill pipe B.

(a) If B can fill a cistern in x hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{x}{k+1}\right)$ hours.

(b) If A can fill a cistern in y hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{k}{k+1}\right)y$ hours.

Illustration 11 One fill pipe A is 10 times faster than second fill pipe B. If B can fill a cistern in 55 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution Here, $k = 10$ and $x = 55$.

$\therefore$ Cistern will be full in

$$= \left(\frac{x}{k+1}\right) \text{ minutes}$$

$$= \left(\frac{55}{10+1}\right) \textbf{ minutes or 5 minutes.}$$

Illustration 12 One fill pipe A is 4 times faster than second fill pipe B. If A can fill a cistern in 15 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution Here, $k = 4$ and $y = 15$.

$\therefore$ Cistern will be full in $= \left(\frac{k}{k+1}\right)y$ minutes

$$= \left(\frac{4}{4+1}\right)15 \text{ minutes}$$

$$= \textbf{12 minutes.}$$

7. If one fill pipe A is k times faster and takes x minutes less time than the other fill pipe B, then

(a) the time taken to fill a cistern, if both the pipes are opened together is $\left(\frac{kx}{(k-1)^2}\right)$ minutes.

(b) A will fill the cistern in $\left(\frac{x}{k-1}\right)$ minutes.

(c) B will fill the cistern in $\left(\frac{kx}{k-1}\right)$ minutes.

Illustration 13 One fill pipe A is 5 times faster than second fill pipe B and takes 32 minutes less than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

Solution Here, $k = 5$ and $x = 32$.

$\therefore$ Cistern will be full in $= \frac{kx}{(k-1)^2}$ minutes

$$= \frac{5\times 32}{(5-1)^2} \text{ minutes}$$

$$= \textbf{10 minutes.}$$

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?
(a) 6 hours (b) 7 hours
(c) 6.30 hours (d) None of these

2. A tap can fill a tank in 25 minutes and another can empty it in 50 minutes. Find whether the tank will be filled up or emptied and in how many minutes?
(a) Tank is filled up in 50 minutes
(b) Tank is emptied in 25 minutes
(c) Tank is filled up in 25 minutes
(d) None of these

3. A water tank is $\frac{2}{5}$th full. Pipe A can fill the tank in 10 minutes and the pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely?
(a) 6 minutes to fill (b) 6 minutes to empty
(c) 8 minutes to fill (d) None of these

4. Two taps A and B can fill a tank in 10 hours and 15 hours, respectively. If both the taps are opened together the tank will be full in
(a) 8 hours (b) 6 hours
(c) 5 hours (d) None of these

5. Two pipes A and B can separately empty a cistern in 12 hours and 15 hours, respectively. In what time will the cistern be emptied, if both the pipes are opened together?
(a) 5 hours 30 minutes (b) 7 hours
(c) 6 hours 40 minutes (d) None of these

6. Two pipes can fill a tank in 10 hours and 12 hours, respectively. While a third pipe emptied the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled?
(a) 7 hours 30 minutes (b) 6 hours 40 minutes
(c) 8 hours 30 mintues (d) None of these

7. Three pipes A, B and C can fill a cistern in 10, 12 and 15 hours, respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern will be
(a) 4 hours (b) 6 hours
(c) 7 hours (d) None of these

8. Two pipes A and B can fill a cistern in 24 minutes and 30 minutes, respectively. There is also an outlet C. If all the three pipes are opened together, the cistern is full in 20 minutes. How much time will be taken by C to empty the full cistern?
(a) 30 minutes (b) 40 mintues
(c) 45 mintues (d) None of these

9. A cistern is normally filled in 8 hours but takes 2 hours longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in
(a) 35 hours (b) 45 hours
(c) 40 hours (d) None of these

10. A cistern has a leak which would empty in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern and it is now emptied in 12 hours. The cistern can hold
(a) 6840 litres (b) 7860 litres
(c) 8640 litres (d) None of these

11. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
(a) 35 hours (b) 30 hours
(c) 40 hours (d) None of these

12. One fill pipe A is 3 times faster than second fill pipe B and takes 32 minutes less than the fill pipe B. When will the cistern be full if both pipes are opened together?
(a) 28 mintues (b) 24 minutes.
(c) 30 minutes (d) Data inadequate.

13. Two pipes A and B can fill a cistern in 4 minutes and 6 minutes, respectively. If these pipes are turned on alternately for 1 minute each how long will it take for the cistern to fill?
(a) 4 min 40 sec (b) 3 min 20 sec
(c) 4 min 50 sec (d) 3 min 30 sec.

14. There are two taps to fill a tank while a third to empty it. When the third tap is closed, they can fill the tank in 10 minutes and 12 minutes, respectively. If all the three taps be opened, the tank is filled in 15 minutes. If the first two taps are closed, in what time can the third tap empty the tank when it is full?
(a) 7 min (b) 9 min and 32 sec
(c) 8 min and 34 sec (d) 6 min.

15. Two pipes A and B can separately fill a cistern in 15 minutes and 18 minutes, respectively, while a third pipe C can empty it in 6 minutes. Two pipes A and B are kept open for 6 minutes in the beginning and then the third pipe is also opened. In what

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time will the cistern be emptied?

(a) $16\frac{1}{2}$ minutes (b) 15 minutes

(c) $15\frac{1}{2}$ minutes (d) 16 minutes

16. A reservoir is provided by two pipes A and B. A can fill the reservoir 5 hours faster than B. If both together fill the reservoir in 6 hours, the reservoir will be filled by A alone in

(a) 10 hours (b) 8 hours

(c) 12 hours (d) 11 hours

17. A cistern is provided by two taps A and B. A can fill it in 20 minutes and B in 25 minutes. Both the taps are kept open for 5 minutes and then the second is turned off. The cistern will be completely filled in another

(a) 11 minutes (b) 10 minutes

(c) 15 minutes (d) 12 minutes

18. Two pipes A and B can separately fill a tank in 6 hours and 8 hours, respectively. Both the pipes are opened together, but $1\frac{1}{2}$ hours after the start, the pipe A is turned off. How much time will it take to fill the tank?

(a) 5 hours (b) 6 hours

(c) $4\frac{1}{2}$ hours (d) $5\frac{1}{2}$ hours

19. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty a full cistern?

(a) 8 minutes (b) 10 minutes

(c) 12 minutes (d) 16 minutes

20. Two taps can separately fill a cistern in 10 minutes and 15 minutes, respectively and when the waste pipe is open, they can together fill it in 18 minutes. The waste pipe can empty the full cistern in

(a) 7 min (b) 9 min

(c) 13 min (d) 23 min

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $X = 2$ and $Y = 3$.

$\therefore$ Part of the cistern filled in 1 hour

$$= \frac{1}{X} - \frac{1}{Y}$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

$\therefore$ Total time taken to fill the cistern **= 6 hours.**

2. Ans. (a)

Solution Here, $X = 25$ and $Y = 50$.

$\therefore$ Part of the tank filled or emptied in 1 minute

$$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50},$$

which is positive, therefore the tank will be filled.

$\therefore$ Total time taken to fill the tank **= 50 minutes.**

3. Ans. (b)

Solution Here, $X = 10$ and $Y = 6$.

$\therefore$ Part of the tank filled or emptied in 1 minute

$$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{10} - \frac{1}{6} = -\frac{1}{15}$$

which is negative, therefore tank will be emptied.

Thus, $\frac{2}{5}$th full of the tank will be emptied in

$$= 15 \times \frac{2}{5} = \textbf{6 minutes.}$$

4. Ans. (b)

Solution Here, $X = 10$ and $Y = 15$.

$\therefore$ The tank will be full in

$$= \left(\frac{XY}{X+Y}\right) \text{ hours}$$

$$= \left(\frac{10\times15}{10+15}\right) \textbf{ hours or 6 hours.}$$

5. Ans. (c)

Solution Here, $X = 12$ and $Y = 15$.

$\therefore$ The cistern will be empty in

$$= \left(\frac{XY}{X+Y}\right) \text{ hours}$$

$$= \left(\frac{12\times15}{12+15}\right) \text{ hours}$$

$$= \frac{20}{3} \textbf{ hours or, 6 hours 40 minutes.}$$

6. Ans. (a)

Solution Here, $X = 10$, $Y = 12$ and $Z = -20$.

$\therefore$ The tank will be full in

$$= \left(\frac{X\times Y\times -Z}{XY - YZ - ZX}\right) \text{ hours}$$

$$= \left(\frac{10\times12\times -20}{10\times12 - 12\times20 - 20\times10}\right) \text{ hours}$$

$$= \left(\frac{15}{2}\right) \textbf{ hours or, 7 hours 30 minutes.}$$

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7. Ans. (a)

Solution Here, $X = 10$, $Y = 12$ and $Z = 15$.

$\therefore$ The total time taken to fill the cistern

$$= \left(\frac{XYZ}{XY+YZ+ZX}\right) \text{ hours}$$

$$= \left(\frac{10\times12\times15}{10\times12+12\times15+10\times15}\right) \text{ hours}$$

$$= \mathbf{4 \text{ hours.}}$$

8. Ans. (b)

Solution Here, $X = 24$, $Y = 30$ and $Z = 20$.

$\therefore$ The total time taken by C to empty the full cistern

$$= \left(\frac{XYZ}{XZ+YZ-XY}\right) \text{ minutes}$$

$$= \left(\frac{24\times30\times20}{24\times20+30\times20-24\times30}\right) \text{ minutes}$$

$$= \mathbf{40 \text{ minutes.}}$$

9. Ans. (c)

Solution Here, $X = 8$ and $Y = 8 + 2 = 10$.

$\therefore$ The leak will empty the cistern in

$$= \left(\frac{XY}{Y-X}\right) \text{ hours}$$

$$= \left(\frac{8\times10}{10-8}\right) \text{ hours or, } \mathbf{40 \text{ hours.}}$$

10. Ans. (c)

Solution Here, $X = 8$, $Y = 6 \times 60 = 360$ and $Z = 12$.

$\therefore$ The capacity of the cistern is

$$= \left(\frac{XYZ}{Z-X}\right) \text{ litres}$$

$$= \left(\frac{8\times360\times12}{12-8}\right) \text{ litres}$$

$$= \mathbf{8640 \text{ litres.}}$$

11. Ans. (b)

Solution Let one pipe take x hours to fill the reservoir. Then, another pipe takes $(x - 10)$ hours.

$$\therefore \frac{1}{x}+\frac{1}{x-10} = \frac{1}{12} \Rightarrow x(x-10) = 12(x+x-10)$$

$$\Rightarrow x^2 - 34x + 120 = 0$$

or, $(x - 30)(x - 4) = 0$

$\therefore$ $x = 30$ or $x = 4$.

$\therefore$ The faster pipe takes 30 hours to fill the reservoir.

12. Ans. (b)

Solution Here, $k = 3$ and $x = 32$

$\therefore$ Cistern will be full in $= \frac{kx}{(k-1)^2}$ minutes

$$= \frac{3\times32}{(3-1)^2} \text{ minutes}$$

$$= \mathbf{24 \text{ minutes.}}$$

13. Ans. (a)

Solution As the pipes are operating alternately, thus their 2 minutes job is $\frac{1}{4}+\frac{1}{6} = \frac{5}{12}$.

In the next 2 minutes the pipes can fill another $\frac{5}{12}$ part of cistern. Therefore, in 4 minutes the two pipes which are operating alternately will fill $\frac{5}{12}+\frac{5}{12} = \frac{10}{12} = \frac{5}{6}$ part.

The part of the cistern left unfilled $= 1 - \frac{5}{6} = \frac{1}{6}$

Pipe A can fill $\frac{1}{4}$ of the cistern in 1 minute

Pipe A can fill $\frac{1}{6}$ of the cistern in $4 \times \frac{1}{6} = \frac{2}{3}$ minutes.

Total time taken to fill the cistern $4 + \frac{2}{3} = 4\frac{2}{3}$ minutes or, 4 minutes **40 seconds.**

14. Ans. (c)

Solution Part emptied by the third pipe in 1 minute

$$= \left(\frac{1}{10}+\frac{1}{12}\right)-\frac{1}{15} = \frac{7}{60}$$

So, the full tank will be emptied by third pipe in $\left(\frac{60}{7}\right)$ minute $=$ 8 minute **34 seconds**

15. Ans. (a)

Solution Pipe $(A + B)$'s 6 minutes job $= 6\left(\frac{1}{15}+\frac{1}{18}\right)$

$$= \frac{11}{15}$$

Net work done by the three pipes $(A + B + C)$ in 1 minute $= \left(\frac{1}{15}+\frac{1}{18}\right)-\frac{1}{6} = \frac{-4}{90} = \frac{-2}{45}$

Net $\frac{2}{45}$ part of the tank is emptied by the pipe C in 1 minute

Net $\frac{11}{15}$ part of the tank is emptied by the pipe C in $\frac{45}{2}\times\frac{11}{15} = \frac{33}{2}$ minutes $= \mathbf{16\frac{1}{2}}$ **minutes.**

16. Ans. (a)

Solution Let A alone can fill the reservoir in x hours.

Then, B can fill the reservoir in $(x + 5)$ hours

$$\therefore \frac{1}{x}+\frac{1}{x+5} = \frac{1}{6}$$

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$\therefore \quad 6(2x + 5) = x(x + 5)$

or, $x^2 - 7x - 30 = 0$

or, $(x - 10)(x + 3) = 0$

or, x = 10 hours.

17. Ans. (a)

Solution Part filled in 1 minute $= \frac{1}{20} + \frac{1}{25} = \frac{9}{100}$

Part filled in 5 minutes $= \frac{9}{100} \times 5 = \frac{9}{20}$

Unfilled part $= 1 - \frac{9}{20} = \frac{11}{20}$

This is to be filled by A alone and hence will be filled in $20 \times \frac{11}{20}$ **= 11 minutes.**

18. Ans. (b)

Solution Pipe $(A + B)$'s $1\frac{1}{2}$ hours job $= \frac{3}{2}\left(\frac{1}{6} + \frac{1}{8}\right) = \frac{7}{16}$

Part unfilled $= 1 - \frac{7}{16} = \frac{9}{16}$

B can fill $\frac{1}{8}$ of the tank in = 1 hour

B can fill $\frac{9}{16}$th of the tank in $= 8 \times \frac{9}{16} = \frac{9}{2}$ hours

Total time taken to fill the tank $= \left(\frac{3}{2} + \frac{9}{2}\right)$ hours

= 6 hours.

19. Ans. (b)

Solution Work done by waste pipe in 1 minute

$$= \left(\frac{1}{12} + \frac{1}{15}\right) - \frac{1}{20}$$

$$= \left(\frac{3}{20} - \frac{1}{20}\right) = \frac{1}{10}$$

$\therefore$ Waste pipe can empty the cistern in **10 minutes**

20. Ans. (b)

Solution Work done by waste pipe in 1 minute

$$= \left(\frac{1}{10} + \frac{1}{15}\right) - \frac{1}{18}$$

$$= \left(\frac{1}{6} - \frac{1}{18}\right) = \frac{1}{9}$$

$\therefore$ Waste pipe can empty the cistern in **9 minutes.**

QUESTIONS FROM COMPETITIVE EXAMS

1. A tap can fill a tank in 16 minutes and another can empty it in 8 minutes. If the tank is already 1/2 full and both the taps are opened together, will the tank be filled or emptied? How long will it take before the tank is either filled or emptied completely as the case may be?

(a) Emptied; 16 minutes
(b) Filled; 8 minutes
(c) Emptied; 8 minutes
(d) Filled; 12 minutes **[Based on MAT 2004]**

2. Three-fourth of a tank is full of water. If 5 litres are added to it, then four-fifth of the tank becomes full. What is the capacity of the tank?

(a) 75 litres (b) 80 litres
(c) 100 litres (d) 120 litres
[Based on MAT 2004]

3. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank?

(a) 50 minutes (b) 12 minutes
(c) 25 minutes (d) 15 minutes
[Based on MAT 2003]

4. A vessel is fully filled with a special liquid. Four litres of liquid is drawn out of this vessel and is replaced with water. If the ratio of the special liquid to the water becomes 1 : 2, then what is the capacity of the vessel?

(a) 8 litres (b) 10 litres
(c) 12 litres (d) 14 litres
[Based on MAT 2002]

5. A pipe can fill a cistern in 6 hours. Due to a leak in its bottom, it is filled in 7 hours. When the cistern is full, in how much time will it be emptied by the leak?

(a) 42 hours (b) 40 hours
(c) 43 hours (d) 45 hours
[Based on MAT 2002]

6. Two pipes can fill a tank in 10 hours and 12 hours respectively, while the third can empty it in 20 hours. If all the pipes are opened together, then the tank will be filled in

(a) $7\frac{1}{2}$ hours (b) 10 hours
(c) 8 hours (d) $9\frac{1}{10}$
[Based on MAT 2001]

7. A leak in the bottom of a tank can empty the full tank in 8 hours. An inlet pipe fills water at the rate of 6 litres a minute. When the tank is full, the inlet is opened and due to the leak, the tank is empty in 12 hours. How many litres does the cistern hold?

(a) 7580 (b) 7960
(c) 8290 (d) 8640
[Based on MAT 2005]

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8. Pipes A and B running together can fill a cistern in 6 minutes. If B takes 5 minutes more than A to fill the cistern, then the time in which A and B will fill the cistern separately will be respectively
(a) 15 minutes, 20 minutes
(b) 15 minutes, 10 minutes
(c) 10 minutes, 15 minutes
(d) 25 minutes, 20 minutes **[Based on MAT 2001]**

9. Two taps can fill a tank in 20 minutes and 30 minutes respectively. There is an outlet tap at exactly half level of that rectangular tank which can pump out 50 litres of water per minute. If the outlet tap is open, then it takes 24 minutes to fill an empty tank. What is the volume of the tank ?
(a) 1200 litres (b) 1500 litres
(c) 1800 litres (d) 2400 litres
[Based on IIT Joint Man. Ent. Test 2004]

10. A tap can fill a tank in 48 minutes, whereas another tap can empty it in 2 hours. If both the taps are opened at 11:40 A.M., then the tank will be filled at
(a) 12 : 40 P.M. (b) 1 : 30 P.M.
(c) 1 : 00 P.M. (d) 1 : 20 P.M.
[Based on IIT Joint Man. Ent. Test 2004]

11. A cistern can be filled by pipes A and B in 4 hours and 6 hours respectively. When full, the tank can be emptied by pipe C in 8 hours. If all the taps were turned on at the same time, then the cistern will be full in
(a) 3 hrs 18 min. (b) 3 hrs 26 min.
(c) 3 hrs 42 min. (d) 3 hrs 48 min.
[Based on FMS (Delhi) 2003]

12. Two pipes A and B can fill a cistern in 12 minutes and 15 minutes respectively but a third pipe C can empty the full tank in 6 minutes. A and B are kept open for 5 minutes in the beginning and then C is also opened. In what time will the cistern be emptied?
(a) 30 min. (b) 33 min.
(c) 37.5 min. (d) 45 min.
[Based on FMS (Delhi) 2002]

13. Two pipes P and Q would fill a cistern in 24 and 32 minutes respectively. Both pipes are kept open. When should the first pipe be turned off so that the cistern may be just filled in 16 minutes.
(a) After 10 minutes (b) After 12 minutes
(c) After 14 minutes (d) None of these
[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution In one minute, $\frac{1}{16}-\frac{1}{8}=-\frac{1}{16}$ of the tank will be filled, i.e. $\frac{1}{16}$ of the tank will be emptied.

2. Ans. (c)

Solution Let the capacity of the tank be x litres.

$$\therefore \quad \frac{3}{4}x+5=\frac{4}{5} \Rightarrow \boldsymbol{x = 100.}$$

3. Ans. (b)

Solution In one minute, $\frac{1}{20}+\frac{1}{30}=\frac{1}{12}$ of the tank can be filled.

4. Ans. (c)

Solution Suppose the volume of the vessel be x litres.

Therefore, the quantity of special liquid in the vessel $= x$ litres.

After 1st operation Quantity of special liquid $= x-4$
Water $= 4$

After 2nd operation

Quantity of special liquid

$$= (x-4)-\frac{6}{x}(x-4) \text{ i.e. } x+\frac{24}{x}-10$$

$$\text{Water} = 4-\frac{6}{x}\times 4+6 = 10-\frac{24}{x}$$

$$\therefore \quad \frac{x+\frac{24}{x}-10}{10-\frac{24}{x}}=\frac{1}{2} \Rightarrow 2x+\frac{48}{x}-20=10-\frac{24}{x}$$

$$\Rightarrow 2x+\frac{72}{x}-30=0$$
$$\Rightarrow 2x^2-30x+72=0$$
$$\Rightarrow x^2-15x+36=0$$
$$\Rightarrow \boldsymbol{x = 12.}$$

5. Ans. (a)

Solution In one hour, $\frac{1}{6}$ of the cistern can be filled.

In one hour, only $\frac{1}{7}$ of the cistern can be filled due to leak in its bottom.

$\therefore$ In one hour, $\frac{1}{6}-\frac{1}{7}=\frac{1}{42}$ of the cistern is empty.

$\therefore$ The whose cistern will be emptied in 42 hours.

6. Ans. (a)

Solution In one hour, $\frac{1}{10}+\frac{1}{12}-\frac{1}{20}=\frac{6+5-3}{60}=\frac{2}{15}$ of the tank can be filled.

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7. Ans. (d)

Solution Work done by inlet in 1 hour = $\frac{1}{8}-\frac{1}{12}=\frac{1}{24}$

Work done by inlet in 1 min = $\frac{1}{24}\times\frac{1}{60}=\frac{1}{1440}$

$\because$ Volume of $\frac{1}{1440}$ part = 6 litres

$\therefore$ Volume of whole = 1440×6 = **8640 litres.**

8. Ans. (c)

Solution Suppose pipe A fills the cistern in x minutes. Therefore pipe B will fill the cistern in $(x+5)$ minutes.

$\therefore$ In one minute, pipes A and B together can fill $\left[\frac{1}{x}+\frac{1}{x+5}\right]$ of the cistern.

$\Rightarrow \frac{1}{x}+\frac{1}{x+5}=\frac{1}{6} \Rightarrow x = 10$

$\therefore$ Pipe A and B can fill the cistern in 10 minutes and 15 minutes respectiely.

9. Ans. (a)

Solution In one minute, $\frac{1}{20}+\frac{1}{30}=\frac{1}{12}$ of the tank can be filled.

The whole tank can be filled in 12 minutes.

Total time to fill the tank = 24 minutes

$\therefore$ Outlet tap is taking 12 minutes to empty half tank at the speed of 50 litre/minute

$\therefore$ Volume of half tank = $50 \times 12 = 600$ litres

$\therefore$ Volume of full tank = **1200 litres.**

10. Ans. (c)

Solution In one minute, $\frac{1}{48}-\frac{1}{120}=\frac{1}{80}$ of tank can be filled.

$\therefore$ The whole tank can be filled in 80 minutes.

11. Ans. (b)

Solution In one hour, $\frac{1}{4}+\frac{1}{6}-\frac{1}{8}=\frac{6+4-3}{24}=\frac{7}{24}$ of the cistern can be filled.

$\therefore$ Whole cistern could be filled in $\frac{24}{7}$ hours, i.e., 3 hours and 26 minutes appox.

12. Ans. (d)

Solution In one minute, pipes A, B and C can fill $\frac{1}{12}+\frac{1}{15}-\frac{1}{6}=-\frac{1}{60}$ of the cistern, i.e. if all the three pipes are opened simultaneously, then the cistern can be emptied in 60 minutes.

In one minute, pipes A and B can fill $\frac{1}{12}+\frac{1}{15}$ $=\frac{3}{20}$ of the cistern.

In five minutes, $\frac{15}{20}=\frac{3}{4}$ of the cistern will be filled.

$\therefore$ $\frac{3}{4}$ of the cistern can be emptied in

$\frac{3}{4}\times 60$ = **45 min.**

13. Ans. (b)

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. If two pipes function simultaneously, the reservoir is filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?
(a) 18 hours (b) 19 hours
(c) 20 hours (d) 17 hours

2. Two pipes A and B can separately fill a cistern in 15 and 10 minutes, respectively and a waste pipe C can carry off 7 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 2 hours. How many litres does the cistern hold?
(a) 30 litres (b) 40 litres
(c) 35 litres (d) None of these

3. Three pipes P, Q and R together can fill a tank in 12 hours. After working at it together for 3 hours, P is closed and Q and R fill it in 15 hours. The time (in hours) taken by P alone to fill the tank is
(a) 30 (b) 32
(c) 25 (d) 28

4. Two pipes A and B can fill a tank in 36 minutes and 45 minutes, respectively. A waste pipe C can empty the tank in 30 minutes. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank is full?
(a) 60 minutes (b) 46 minutes
(c) 42 minutes (d) 48 minutes

5. If three taps are opened together, a tank is filled in 12 hours. One of the taps can fill it in 10 hours and another in 15 hours How does the third tap work?
(a) 12 hours (b) 14 hours
(c) 18 hours (d) 16 hours

6. One pipe can fill a tank three times as fast as another

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another pipe. If together the two pipes can fill the tank in 36 minutes, the slower pipe alone will be able to fill the tank in
(a) 81 minutes (b) 108 minutes
(c) 144 minutes (d) 192 minutes

7. Two pipes A and B can separately fill a cistern in $7\frac{1}{2}$ and 5 minutes, respectively and a waste pipe C can carry off 14 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 1 hour. The capacity of the cistern is
(a) 40 litres (b) 35 litres
(c) 25 litres (d) 32 litres

8. A tap can fill a tank in 16 minutes and another can empty it in 8 minutes. If the tank is already half full, and both the taps are opened together, the tank will be
(a) filled in 12 minutes (b) emptied in 12 minutes
(c) filled in 8 minutes (d) emptied in 8 minutes

9. A cistern can be filled by two pipes in 20 and 30 minutes, respectively. Both pipes being opened, when must the first pipe be turned off so that the cistern may be filled in 10 minutes more?
(a) after 10 minutes (b) after 12 minutes
(c) after 8 minutes (d) after 6 minutes

10. There is a leak in the bottom of a cistern. When the cistern is in thorough repair, it would be filled in $3\frac{1}{2}$ hours. It now takes half an hour longer. If the cistern in full, time (in hours) taken by the leak to empty it is
(a) 28 (b) 32
(c) 24 (d) 30

Test II

[Time: 10 min]

11. A cistern can be filled by one pipe in 6 hours and by another in 5 hours. There is a waste pipe also and if this is open when the second pipe is working the cistern takes $7\frac{1}{2}$ hours to fill. If all the three pipes are open, time (in hours) taken by the cistern to be full is:
(a) $4\frac{2}{3}$ hours (b) 2 hours
(c) $3\frac{1}{3}$ hours (d) $5\frac{1}{2}$ hours

12. Taps A and B can fill a bucket in 12 minutes, respectively. If both taps are opened and tap A is closed after 3 minutes, how much further time would it take for tap B to fill the bucket?
(a) 8 minutes 5 seconds
(b) 8 minutes 15 seconds
(c) 7 minutes 45 seconds
(d) 7 minutes 15 seconds

13. Two pipes P and Q would fill a cistern in 24 and 32 minutes, respectively. Both pipes being opened, find when the first pipe must be turned off so that the cistern may be just filled in 16 mimutes.
(a) after 12 minutes (b) after 10 minutes
(c) after 14 minutes (d) None of these

14. A pipe can fill a tank in 12 minutes and another pipe in 15 minutes, but a third pipe can empty it in 6 minutes. The first two pipes are kept open for 5 minutes in the beginning and then the third pipe is also opened. In what time is the cistern emptied?
(a) 40 minutes (b) 45 minutes
(c) 55 minutes (d) 50 minutes

15. A cistern is filled by three taps A, B and C. A can fill it in 12 minutes, B in 15 minutes and C can empty in 1 hour. If A, B and C be opened successively for a minute each, the cistern will be filled in
(a) $22\frac{3}{5}$ minutes (b) 21 minutes
(c) $21\frac{4}{5}$ minutes (d) 20 minutes

16. Two fill pipes A and B can fill a cistern in 12 and 16 minutes, respectively. Both fill pipes are opened together, but 4 minutes before the cistern is full, one pipe A is closed. How much time will the cistern take to fill?
(a) $8\frac{1}{7}$ minutes (b) 8 minutes
(c) $9\frac{1}{7}$ minutes (d) 9 minutes

17. Three pipes A, B and C can fill a cistern in 6 hours. After working together for 2 hours, C is closed and A and B fill the cistern in 8 hours. Then, find the time in which the cistern can be filled by pipe C.
(a) 12 hours (b) 16 hours
(c) 18 hours (d) 10 hours

18. Two pipes A and B fill a tank in 15 hours and 20 hours, respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. The time to fill the tank is
(a) 12 hours (b) $13\frac{1}{2}$ hours
(c) 16 hours (d) 18 hours

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19. A tap can fill a bath in 20 minutes and another tap can fill it in 30 minutes. Amit opens both the taps simultaneously. When the bath should have been full, he finds that the waste pipe was open. He then closes the waste pipe and in another 4 minutes the bath is full. In what time would the waste pipe empty it?
(a) 35 minutes (b) 38 minutes
(c) 36 minutes (d) 39 minutes

20. A cistern has a leak which would empty it in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold?
(a) 7580 litres (b) 7960 litres
(c) 8290 litres (d) 8640 litres

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (c)

Solution Let the faster pipe fills the tank in x hours then the slower pipe fills the tank in $x + 10$ hours. When both of them are opened, the reservoir will be filled in $\frac{x(x+10)}{x+(x+10)} = 12$

or, $x^2 - 14x - 120 = 0$

$\therefore \quad x = 20, -6$

But x connot be –ve, hence the faster pipe will fill the reservoir in 20 hours.

3. Ans. (a)

Solution Part filled by $P + Q + R$ in 1 hour $= \frac{1}{12}$

Part filled by $(P + Q + R)$ in 3 hours $= 3 \times \frac{1}{12} = \frac{1}{4}$

Unfilled part $= 1 - \frac{1}{4} = \frac{3}{4}$

This is filled by $(Q + R)$ in 15 hours

$\therefore$ Part filled by $(Q + R)$ in 1 hour $= \frac{3}{4} \times \frac{1}{15} = \frac{1}{20}$

$\therefore$ Part filled by P in 1 hour $= \frac{1}{12} - \frac{1}{20} = \frac{1}{30}$

$\therefore$ P alone can fill the tank in **30 hours.**

5. Ans. (a)

Solution We have to find the nature of the third tap—whether it is a filler or a waste pipe.

Let it be a filler pipe which fill in x hours

Then, $\frac{10 \times 15 \times x}{10 \times 15 + 10x + 15x} = 12$

or, $150x = 150 \times 12 + 25x \times 12$

or, $-150x = 1800$

$\therefore \quad x = -12$

–ve sign shows that the third pipe is a waste pipe which vacates the tank in **12 hours.**

7. Ans. (a)

Solution Let waste C can empty the full cistern in x minutes.

Then, part of cistern emptied in 1 minute

$$= \frac{1}{x} - \left(\frac{2}{15} + \frac{1}{5}\right) = \frac{1}{x} - \frac{1}{3}$$

Also, cistern is emptied in 1 hour (i.e. 60 minutes)

$\therefore \quad \frac{1}{x} - \frac{1}{3} = \frac{1}{60}$

$\therefore \quad \frac{1}{x} = \frac{1}{60} + \frac{1}{3} = \frac{21}{60}$

$\therefore \quad x = \frac{60}{21}$

$\therefore$ Quantity carried off by C in $\frac{60}{21}$ minutes

$$= 14 \times \frac{60}{21} = 40 \text{ litres.}$$

This is the capacity of tank.

9. Ans. (c)

Solution In 1 minute both pipes can fill $= \frac{1}{20} + \frac{1}{30}$ part of the cistern.

In 10 minutes second pipe can fill

$$= \frac{1}{30} \times 10 = \frac{1}{3} \text{ part.}$$

Cistern filled by both pipes $= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ Time taken by both the pipes to fill $\frac{2}{3}$ part of cistern $= \frac{12 \times 2}{3} = 8$ minutes.

Therefore, the first pipe can be turned off after **8 minutes.**

11. Ans. (c)

Solution Let a waste pipe can empty the full cistern in x hours

Then, $\frac{1}{5} - \frac{1}{x} = \frac{2}{15}$

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When all the three pipes are open, part filled in

$$1 \text{ hour} = \frac{1}{6}+\frac{1}{5}-\frac{1}{x} = \frac{1}{6}+\frac{2}{15} = \frac{9}{30} = \frac{3}{10}$$

$\therefore$ Cistern will be full in $\frac{10}{3} = 3\frac{1}{3}$ hours.

13. Ans. (a)

Solution Let the pipe P be turned off after x minutes.

Now, part filled by $(P + Q)$ in 1 minute

$$= \frac{1}{24}+\frac{1}{32} = \frac{7}{96}$$

Part filled by $(A - B)$ in x minutes $= \frac{7x}{96}$.

For the time $(16 - x)$ minutes, pipe Q alone fills the tank.

Now, part filled by Q alone in $(16 - x)$ minutes

$$= \frac{16-x}{32}.$$

Also, tank is just filled in 16 minutes.

$$\therefore \quad \frac{7x}{96}+\frac{16-x}{32} = 1$$

$$\Rightarrow \quad 7x + 3(16 - x) = 96 \quad \text{or.} \quad 4x = 48$$

$\therefore \quad x = 12.$

$\therefore$ Pipe P must be turned off after **12 minutes.**

15. Ans. (c)

Solution Part filled in first 3 minutes

$$= \frac{1}{12}+\frac{1}{15}-\frac{1}{60} = \frac{2}{15}$$

Part filled in $7 \times 3 = 21$ minutes

$$= \frac{2}{15} \times 7 = \frac{14}{15}$$

Unfilled part $= 1 - \frac{14}{15} = \frac{1}{15}$

Now, tap A is opened and will fill this remaining part in $12 \times \frac{1}{15} = \frac{4}{5}$ minutes.

$\therefore$ Tank will be filled in $21 + \frac{4}{5} = \mathbf{21\frac{4}{5}}$ **minutes.**

17. Ans. (a)

Solution $A + B + C$ can fill in 1 hour $= \frac{1}{6}$ of cistern

$A + B + C$ can fill in 2 hours $= \frac{2}{6} = \frac{1}{3}$ of cistern

Unfilled part $= \left(1-\frac{1}{3}=\frac{2}{3}\right)$ is filled by $A + B$ in 8 hours.

$\therefore$ $(A + B)$ can fill the cistern in $\frac{8\times3}{2} = 12$ hours

And, we have $(A + B + C)$ can fill the cistern in 6 hours.

$\therefore$ $C = (A + B + C) - (A + B)$ can fill the cistern in $\frac{12\times6}{12-6} =$ **12 hours.**

19. Ans. (c)

Solution Work done in 1 minute by both the taps

$$\left(\frac{1}{20}+\frac{1}{30}\right) = \frac{5}{60} = \frac{1}{12}$$

Both the taps can fill the bath in 12 minutes. Clearly, the waste pipe empties as much the bath in 12 minutes as the other two taps together can fill it in 4 minutes.

Work done by the two taps in 4 minutes

$$= \frac{1}{12} \times 4 = \frac{1}{3}$$

In 12 minutes, the waste pipe empties $= \frac{1}{3}$ part

In 1 minute, the waste pipe empties $= \frac{1}{3}\times\frac{1}{12} = \frac{1}{36}$ part.

Thus, the waste pipe can empty the bath in **36 minutes.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (b)	**6.** (c)	**8.** (d)	**10.** (a)	**12.** (b)	**14.** (b)	**16.** (c)	**18.** (a)	**20.** (d)

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12 Time and Distance

INTRODUCTION

The terms '*Time*' and '*Distance*' are related to the speed of a moving object.

Speed: We define the *speed* of an object as the distance covered by it in a unit time interval. It is obtained by dividing the distance covered by the object, by the time it takes to cover that distance.

Thus, $$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}.$$

Notes:

1. If the time taken is constant, the distance travelled is proportional to the speed, that is, more the speed: more the distance travelled in the same time.
2. If the speed is constant, the distance travelled is proportional to the time taken, that is, more the distance travelled; more the time taken at the same speed.
3. If the distance travelled is constant, the speed is inversely proportional to the time taken, that is, more the speed: less the time taken for the same distance travelled.

SOME BASIC FORMULAE

1. $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$
2. $\text{Distance} = \text{Speed} \times \text{Time}$
3. $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Units of Measurement

Generally, if the distance is measured in kilometre, we measure time in hours and speed in kilometre per hour and is written as km/hr and if the distance is measured in metre then time is taken in second and speed in metre per second and is written as m/sec.

Conversion of Units

$$\text{One kilometre/hour} = \frac{1000 \text{ metre}}{60 \times 60 \text{ seconds}} = \frac{5}{18} \text{ m/sec.}$$

$$\therefore \quad \text{One metre/second} = \frac{18}{5} \text{ km/hr.}$$

$$\text{Thus,} \quad x \text{ km/hr} = \left(x \times \frac{5}{18}\right) \text{ m/sec.}$$

$$\text{and,} \quad x \text{ m/sec.} = \left(x \times \frac{18}{5}\right) \text{ km/hr.}$$

Illustration 1 Calculate the speed of a train which covers a distance of 150 km in 3 hours.

Solution $\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{150}{3}$

= 50 km/hr.

Illustration 2 How long does a train 100 metres long running at the rate of 40 km/hr take to cross a telegraphic pole?

Solution In crossing the pole, the train must travel its own length.

$\therefore$ Distance travelled is 100 metres.

$$\text{Speed} = 40 \text{ km/hr.} = \frac{40 \times 1000}{60 \times 60} = \frac{100}{9} \text{ m/sec.}$$

$$\therefore \quad \text{Time taken to cross the pole} = \frac{100}{100/9}$$

= 9 seconds.

Illustration 3 A train running at a speed of 90 km/hr passes a pole on the platform in 20 seconds. Find the length of the train in metres.

Solution Speed of the train = 90 km/hr

$$= 90 \times \frac{5}{18} = 25 \text{ m/sec.}$$

$\therefore$ Length of the train = Speed of the train × time taken in crossing the pole

$= 25 \times 20 =$ **500 m.**

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SOME USEFUL SHORT-CUT METHODS

1. (a) If A covers a distance d_1 km at s_1 km/hr and then d_2 km at s_2 km/hr, then the average speed during the whole journey is given by

$$\text{Average speed} = \frac{s_1 s_2 (d_1 + d_2)}{s_1 d_2 + s_2 d_1} \text{km/hr.}$$

(b) If A goes from X to Y at s_1 km/hr. and comes back from Y to X at s_2 km/hr., then the average speed during the whole journey is given by

$$\text{Average speed} = \frac{2 s_1 s_2}{s_1 + s_2}$$

Explanation

(a) Time taken to travel d_1 km at s_1 km/hr is

$$t_1 = \frac{d_1}{s_1} \text{hr.}$$

Time taken to travel d_2 km at s_2 km/hr is

$$t_2 = \frac{d_2}{s_2} \text{hr.}$$

$$\text{Total time taken} = t_1 + t_2 = \left(\frac{d_1}{s_1} + \frac{d_2}{s_2}\right) \text{hr.}$$

$$= \left(\frac{s_1 d_2 + s_2 d_1}{s_1 s_2}\right) \text{hr.}$$

Total distance covered = $(d_1 + d_2)$ km. Therefore,

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{s_1 s_2 (d_1 + d_2)}{(s_1 d_2 + s_2 d_1)} \text{km/hr.} \quad ...(i)$$

(b) Let the distance from X to Y be d km.

Take $d_1 = d_2 = d$ in (i), we get

$$\text{Average speed} = \frac{2 d s_1 s_2}{d(s_1 + s_2)} = \frac{2 s_1 s_2}{s_1 + s_2}.$$

Illustration 4 A ship sails to a certain city at the speed of 15 knots/hr and sails back to the same point at the rate of 30 knots/hr. What is the average speed for the whole journey?

Solution Here, $s_1 = 15$ and $s_2 = 30$.

$$\therefore \quad \text{Average speed} = \frac{2 s_1 s_2}{s_1 + s_2} = \frac{2 \times 15 \times 30}{15 + 30}$$

$$= \mathbf{20 \text{ knots/hr.}}$$

2. A person goes certain distance (A to B) at a speed of s_1 km/hr. and returns back (B to A) at a speed of s_2 km/hr. If he takes T hours in all, the distance between A and B is

$$T\left(\frac{s_1 s_2}{s_1 + s_2}\right).$$

Explanation

Let the distance between A and B be d km.

Time taken during onward journey = $t_1 = \frac{d}{s_1}$ hrs.

Time taken during return journey = $t_2 = \frac{d}{s_2}$ hrs.

$\therefore$ Total time taken during the entire journey is

$$T = t_1 + t_2 = \frac{d}{s_1} + \frac{d}{s_2} = \frac{d(s_1 + s_2)}{s_1 s_2}$$

$$\therefore \quad d = T\left(\frac{s_1 s_2}{s_1 + s_2}\right).$$

Thus the distance between A and B is

$$= T\left(\frac{s_1 s_2}{s_1 + s_2}\right)$$

$$= \text{Total time taken} \times \frac{\text{Product of two speeds}}{\text{Sum of two speeds}}.$$

Illustration 5 A boy goes to school with the speed of 3 km an hour and returns with a speed of 2 km/hr. If he takes 5 hours in all, find the distance in km between the village and the school.

Solution Here, $s_1 = 3$, $s_2 = 2$ and $T = 5$.

$\therefore$ The distance between the village and the school

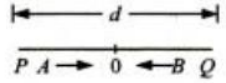
$$= T\left(\frac{s_1 s_2}{s_1 + s_2}\right) = 5\left(\frac{3 \times 2}{3 + 2}\right) = \mathbf{6 \text{ km.}}$$

3. If two persons A and B start at the same time from two points P and Q towards each other and after crossing they take T_1 and T_2 hours in reaching Q and P respectively, then

$$\frac{A\text{'s speed}}{B\text{'s speed}} = \frac{\sqrt{T_2}}{\sqrt{T_1}}.$$

Explanation

Let the total distance between P and Q be d km. Let the speed of A be s_1 km/hr and that of B be s_2 km/hr.



Since they are moving in opposite directions, their relative speed is $(s_1 + s_2)$ km/hr.

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They will meet after $\left(\frac{d}{s_1+s_2}\right)$ hr.

Distance travelled by A in $\left(\frac{d}{s_1+s_2}\right)$ hr.

$= PO = \left(\frac{ds_1}{s_1+s_2}\right)$ km.

Distance travelled by B in $\left(\frac{d}{s_1+s_2}\right)$ hr.

$= QO = \left(\frac{ds_2}{s_1+s_2}\right)$ km.

Time taken by A to travel QO

$$= \frac{\left(\frac{ds_2}{s_1+s_2}\right)}{s_1}$$

$= T_1$ (given). ...(1)

Time taken by B to travel PO

$$= \frac{\left(\frac{ds_1}{s_1+s_2}\right)}{s_2}$$

$= T_2$ (given) ...(2)

Dividing equation (2) by equation (1), we get

$$\frac{s_1/s_2}{s_2/s_1} = \frac{T_2}{T_1}$$

or, $\left(\frac{s_1}{s_2}\right)^2 = \frac{T_2}{T_1}$ or, $\frac{s_1}{s_2} = \sqrt{\frac{T_2}{T_1}}$.

$\therefore$ $\frac{A\text{'s speed}}{B\text{'s speed}} = \sqrt{\frac{T_2}{T_1}}$.

Illustration 6 Nikita starts her journey from Delhi to Bhopal and simultaneously Nishita starts from Bhopal to Delhi. After crossing each other they finish their remaining journey in $5\frac{4}{9}$ hours and 9 hours, respectively. What is Nishita's speed if Nikita's speed is 36 km/hr?

Solution $\frac{\text{Nikita's speed}}{\text{Nishita's speed}} = \sqrt{\frac{T_2}{T_1}} = \frac{\sqrt{9}}{\sqrt{5\frac{4}{9}}} = \frac{\sqrt{9}}{\sqrt{\frac{49}{9}}}$

$$= \sqrt{\frac{81}{49}} = \frac{9}{7}.$$

$\therefore$ Nishita's speed $= \frac{7}{9}$ Nikita's speed

$= \frac{7}{9} \times 36 =$ **28 km/hr.**

4. If a body travels $d_1, d_2, d_3, \ldots, d_n$ metres with different speeds $s_1, s_2, s_3, \ldots, s_n$ m/sec. in time $T_1, T_2, T_3, \ldots, T_n$ seconds respectively, then the average speed of the body throughout the journey is given by

$$V_a = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{d_1+d_2+d_3+\ldots+d_n}{T_1+T_2+T_3+\ldots+T_n}$$

[If $d_1, d_2, \ldots, d_n$ and $T_1, T_2, \ldots, T_n$ are known]

and, $V_a = \frac{s_1T_1+s_2T_2+s_3T_3+\ldots+s_nT_n}{T_1+T_2+T_3+\ldots T_n}$

[If $d_1, d_2, \ldots, d_n$ and $s_1, s_2, \ldots, s_n$ are known]

Illustration 7 A car during its journey travels 40 minutes at a speed of 30 km/hr, another 50 minutes at a speed of 60 km/hr and 1 hour at a speed of 30 km/hr. Find the average speed of the car.

Solution Here, $T_1 = \frac{40}{60}$, $T_2 = \frac{50}{60}$, $T_3 = 1$, $s_1 = 30$, $s_2 = 60$, $s_3 = 30$.

$\therefore$ Average speed of the car

$$= \frac{s_1T_1+s_2T_2+s_3T_3}{T_1+T_2+T_3} = \frac{30\times\frac{40}{60}+60\times\frac{50}{60}+30\times1}{\frac{40}{60}+\frac{50}{60}+1}$$

$=$ **40 km/hr.**

5. If the new speed is $\frac{a}{b}$ of the original speed, then the change in time taken to cover the same distance is given by

Change in time $= \left(\frac{b}{a}-1\right) \times$ original time.

Illustration 8 By walking at $\frac{4}{5}$ of his usual speed, Mohan is 6 minutes late to his office. Find his usual time to cover the distance.

Solution Here, change in time = 6 and $\frac{a}{b} = \frac{4}{5}$.

We have, change in time $= \left(\frac{b}{a}-1\right) \times$ original time

$\Rightarrow$ original time $= \frac{\text{change in time}}{\left(\frac{b}{a}-1\right)}$

$= \frac{6}{\left(\frac{5}{4}-1\right)} =$ **24 minutes.**

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6. A body covers a distance d in time T_1 with speed s_1, but when it travels with speed s_2 covers the same distance in time T_2.
The following relations hold

$$\frac{\text{Product of speed}}{d} = \frac{s_1}{T_2} = \frac{s_2}{T_1} = \frac{\text{Difference of speed}}{\text{Difference of time}}$$

Equating any two of the above, we can find the unknowns as per the given question.

Illustration 9 Two bicyclists do the same journey by travelling respectively, at the rates of 9 and 10 km an hour. Find the length of the journey when one takes 32 minutes longer than the other.

Solution Here, change in speed = 10 – 9 = 1; product of speed = 9 × 10 = 90 and difference of time = $\frac{32}{60}$.

We have, $\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}}$

$$\Rightarrow \quad d = \text{Product of speed} \times \left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$$

$$= 90 \times \frac{32}{60} = \mathbf{48\ km.}$$

7. A train travels a certain distance at a speed of s_1 km/hr without stoppages and with stoppages, it covers the same distance at a speed of s_2 km/hr. The stoppage time per hour is given by

$$\left(\frac{s_1 - s_2}{s_1}\right) \text{hr.} \quad \text{or,} \quad \left(\frac{\text{Difference of speed}}{\text{Speed without stoppages}}\right)$$

Explanation

Let the distance travelled be d km.

∴ Time taken by the train without stopping any where

$$= \frac{d}{s_1} \text{hr.}$$

Also, time taken by the train with stoppages

$$= \frac{d}{s_2} \text{hr.}$$

$$\text{Total stoppage time} = \frac{d}{s_2} - \frac{d}{s_1} = \left(\frac{s_1 - s_2}{s_1 s_2}\right) d \text{ hr.}$$

$$\therefore \quad \text{Stoppage time per hour} = \frac{\left(\frac{s_1 - s_2}{s_1 s_2}\right) d}{\frac{d}{s_2}}$$

$$= \left(\frac{s_1 - s_2}{s_1}\right) \text{hr.}$$

Illustration 10 Without stoppages, a train travels certain distance with an average speed of 80 km/hr. and with stoppages, it covers the same distance with an average speed of 60 km/hr. How many minutes per hour the train stops?

Solution Here, $s_1 = 80$ and $s_2 = 60$

∴ Stoppage time/hr.

$$= \frac{s_1 - s_2}{s_1} = \frac{80 - 60}{80} = \frac{1}{4} \text{hour} = \mathbf{15\ minutes.}$$

8. (a) If a train overtakes a pole or a man or a milestone, then the distance covered in overtaking is equal to the length of the train.
(b) If a train overtakes a bridge or a tunnel or a platform or another train, then the distance covered is equal to the sum of the two lengths.

Illustration 11 A train 600 m long crosses a pole in 9 seconds. What is the speed of the train in km/hr?

Solution Speed of the train

$$= \frac{\text{Length of the train}}{\text{time taken in crossing the pole}}$$

$$= \frac{600}{9} \text{m/s} = \frac{600}{9} \times \frac{18}{5} = \mathbf{240\ km/hr.}$$

Illustration 12 A train 130 m long passes a bridge in 21 seconds moving with a speed of 90 km/hr. Find the length of the bridge.

Solution We have, speed of the train

$$= \frac{\text{length of the train + length of the bridge}}{\text{time taken in crossing the bridge}}$$

$$\Rightarrow \quad \frac{5}{18} \times 90 = \frac{130 + \text{length of the bridge}}{21}$$

∴ Length of the bridge = 525 – 130 = **395 m.**

9. **Relative Speed**
(a) If two trains of lengths L_1 km and L_2 km, respectively are travelling in the same direction at s_1 km/hr and s_2 km/hr respectively such that $s_1 > s_2$, then $s_1 - s_2$ is called their relative speed and the time taken by the faster train to cross the slower train is given by

$$\left(\frac{L_1 + L_2}{s_1 - s_2}\right) \text{hr.}$$

(b) If two trains of length L_1 km and L_2 km, respectively are travelling in the opposite directions at s_1 km/hr and s_2 km/hr respectively, then $s_1 + s_2$ is called their relative speed and the time taken by the trains to cross each other is given by

$$\left(\frac{L_1 + L_2}{s_1 + s_2}\right) \text{hr.}$$

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Illustrations 13 A train 135 metres long is running with a speed of 49 km/hr. In what time will it pass a man who is walking at 5 km/hr in the direction opposite to that of the train?

Solution Here, $L_1 = 135$, $L_2 = 0$, $s_1 = 49$ km/hr, $s_2 = 5$ km/hr.

$\therefore \quad s_1 + s_2 = 49 + 5 = 54 \text{ km/hr.} = 54 \times \frac{5}{18} \text{ m/sec.}$

$\therefore$ The time taken $= \frac{L_1 + L_2}{s_1 + s_2} = \frac{135}{54 \times \frac{5}{18}}$

$= \frac{135 \times 18}{54 \times 5} = \mathbf{9 \text{ sec.}}$

Illustration 14 Two trains of length 110 metres and 90 metres are running on parallel lines in the same direction with a speed of 35 km/hr and 40 km/hr, respectively. In what time will they pass each other.

Solution Here, $L_1 = 110$ m, $L_2 = 90$ m, $s_1 = 35$ km/hr and $s_2 = 40$ km/hr

$\therefore \quad s_2 - s_1 = 40 - 35 = 5 \text{ km/hr} = 5 \times \frac{5}{18} \text{ m/s}$

$\therefore$ Time taken $= \frac{L_1 + L_2}{s_2 - s_1}$

$= \frac{110 + 90}{5 \times \frac{5}{18}} = \frac{200 \times 18}{5 \times 5} = \mathbf{144 \text{ sec.}}$

10. Two trains of lengths L_1 m and L_2 m run on parallel tracks. When running in the same direction, the faster train passes the slower one in T_1 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in T_2 seconds.

Then, the speed of the faster train

$$= \frac{L_1 + L_2}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right) \text{ m/s}$$

and, the speed of the slower train

$$= \frac{L_1 + L_2}{2}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \text{ m/s.}$$

Explanation

Let the speed of the faster train be s_1 m/s and that of the slower train be s_2 m/s.

Total distance covered when the two trains cross each other $= L_1 + L_2$.

When the two trains are running is the same direction, their relative speed $= (s_1 - s_2)$ m/s.

$\therefore \quad (s_1 - s_2) = \frac{L_1 + L_2}{T_1} \quad \ldots(1)$

When the two trains are running in the opposite directions, their relative speed $= (s_1 + s_2)$ ~~m/s.~~

$\therefore \quad s_1 + s_2 = \frac{L_1 + L_2}{T_2} \quad \ldots(2)$

Adding equations (1) and (2), we get

$$2s_1 = \frac{L_1 + L_2}{T_1} + \frac{L_1 + L_2}{T_2} = (L_1 + L_2)\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$$

or, $$s_1 = \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right).$$

On subtracting equation (1) from (2), we get

$$2s_2 = (L_1 + L_2)\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

or, $$s_2 = \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 T_2}\right) \text{m/s.}$$

Therefore,

speed of the faster train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right) \text{m/sec.}$$

and,

speed of the slower train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 - T_2}{T_1 T_2}\right) \text{m/sec.}$$

Note: If the two trains are of equal length that is, $L_1 = L_2 = L$ (say), then

$$s_1 = L\left(\frac{T_1 + T_2}{T_1 T_2}\right) \text{m/s and, } s_2 = L\left(\frac{T_1 - T_2}{T_1 T_2}\right) \text{m/s.}$$

Illustration 15 Two trains of lengths 200 metres and 175 metres run on parallel tracks. When running in the same direction the faster train crosses the slower one in $37\frac{1}{2}$ seconds. When running in opposite directions at speeds same as their earlier speeds, they pass each other completely in $7\frac{1}{2}$ seconds. Find the speed of each train.

Solution We have, $L_1 = 200$, $L_2 = 175$, $T_1 = \frac{75}{2}$ and $T_2 = \frac{15}{2}$.

Therefore, speed of the faster train

$$= \left(\frac{L_1 + L_2}{2}\right)\left(\frac{T_1 + T_2}{T_1 T_2}\right) = \left(\frac{200 + 175}{2}\right)\left(\frac{\frac{75}{2} + \frac{15}{2}}{\frac{75}{2} \times \frac{15}{2}}\right)$$

$$= \frac{375}{2} \times \frac{45 \times 4}{75 \times 15} = 30 \text{ m/s.}$$

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Speed of slower train

$$= \left(\frac{L_1+L_2}{2}\right)\left(\frac{T_1-T_2}{T_1T_2}\right) = \left(\frac{200+175}{2}\right)\left(\frac{\frac{75}{2}-\frac{15}{2}}{\frac{75}{2}\times\frac{15}{2}}\right)$$

$$= \frac{375}{2}\times\frac{30\times4}{75\times15} = \mathbf{20\ m/s.}$$

11. (a) A train starts from a place at s_1 km/hr and another fast train starts from the same place after T hours at s_2 km/hr in the same direction. Then, the distance from the starting place at which both the trains will meet is given by

$$\left(\frac{s_1\times s_2\times T}{s_2-s_1}\right) \text{ km.}$$

Also, the time after which the two trains will meet is given by

$$\left(\frac{s_1T}{s_2-s_1}\right) \text{ hr.}$$

(b) The distance between two stations A and B is d km. A train starts from A to B at s_1 km/hr. T hours later another train starts from B to A at s_2 km/hr. Then, the distance from A, at which both the trains will meet is given by

$$s_1\left(\frac{d+s_2T}{s_1+s_2}\right) \text{ km.}$$

Also, the time after which the two trains will meet is given by

$$\left(\frac{d+s_2T}{s_1+s_2}\right) \text{ hr.}$$

Illustration 16 A train starts from Mumbai at 10 A.M. with a speed of 25 km/hr and another train starts from there on the same day at 3 P.M. in the same direction with a speed of 35 km/hr. Find at what distance from Mumbai both the trains will meet and find also the time of their meeting.

Solution Time from 10 A.M. to 3 P.M. = 5 hr.

Distance of meeting point from Mumbai

$$= \left(\frac{s_1\times s_2\times T}{s_2-s_1}\right) \text{ km.}$$

$$= \left(\frac{25\times35\times5}{35-25}\right) \text{km} = 437\frac{1}{2} \text{km.}$$

Also, time of their meeting

$$= \left(\frac{s_1T}{s_2-s_1}\right) \text{hr.} = \left(\frac{25\times5}{35-25}\right) \text{hr.}$$

$$= \frac{125}{10} = 12\frac{1}{2} \text{hr. after 3 P.M.}$$

That is, 3.30 A.M. next day

Illustration 17 Chennai is at a distance of 560 km from Mumbai. A train starts from Mumbai to Chennai at 6 A.M. with a speed of 40 km/hr. Another train starts from Chennai to Mumbai at 7 A.M. with a speed of 60 km/hr. At what distance from Mumbai and at what time will the two trains be at the point of crossing?

Solution Time from 6 A.M. to 7 A.M. = 1 hr.

Therefore, distance of meeting point from Mumbai

$$= s_1\left(\frac{d+s_2T}{s_1+s_2}\right) \text{km.}$$

$$= 40\left(\frac{560+60\times1}{40+60}\right) = 248 \text{ km.}$$

Also, time of their meeting

$$= \left(\frac{d+s_2T}{s_1+s_2}\right) \text{hr}$$

$$= \left(\frac{560+60\times1}{40+60}\right) = \frac{31}{5} \text{ hr}$$

= 6 hr 12 min. after 6 A.M.

That is, at 12.12 noon.

12. Two trains start simultaneously from the stations A and B towards each other with speeds s_1 km/hr and s_2 km/hr, respectively. When they meet it is found that the second train had travelled d km more than the first. Then the distance between the two stations is given by

$$d\left(\frac{s_1+s_2}{s_2-s_1}\right) \text{km.}$$

Explanation

Let the distance between the two stations be x km. If the first train travels y km then the second travels $y + d$ km.

$\therefore \quad x = y + y + d = 2y + d.$

Since the time taken by both the trains is same

$$\therefore \quad \frac{y+d}{s_2} = \frac{y}{s_1}$$

$$\Rightarrow \quad s_1y + s_1d = s_2y$$

$$\Rightarrow \quad (s_2 - s_1)y = s_1d \text{ or, } y = \frac{s_1d}{s_2-s_1}.$$

$$\therefore \quad x = 2\left(\frac{s_1d}{s_2-s_1}\right)+d = \frac{d(s_1+s_2)}{(s_2-s_1)} \text{km.}$$

Illustration 18 Two trains start at the same time from Delhi and Rohtak and proceed towards each other at the rate of 75 km and 65 km per hour, respectively. When they meet, it is found that one train has travelled 10 km more than the other. Find the distance between Delhi and Rohtak.

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Solution Distance between Delhi and Rohtak

$$= d\left(\frac{s_1+s_2}{s_1-s_2}\right)\text{km}.$$

$$= 10\left(\frac{75+65}{75-65}\right)\text{km}$$

$$= \mathbf{140\ km.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Ramesh crosses a street 600 m long in 5 minutes. His speed in km/hr is
(a) 8.2 (b) 7.2
(c) 9.2 (d) None of these

2. Compare the speed of two trains, one moving at the speed of 80 km/hr and the other at 10 m/sec.
(a) 30 : 9 (b) 40 : 9
(c) 20 : 9 (d) None of these

3. Mohan covers 10.2 km in 3 hours, the distance covered by him in 5 hours is
(a) 15 km (b) 17 km
(c) 19 km (d) None of these

4. A train 100 metres long passes a bridge at the rate of 72 km/hr in 25 seconds. What is the length of the bridge?
(a) 400 m (b) 17 m
(c) 600 m (d) None of these

5. A train passes a railway bridge 150 m long in 18 seconds. If the train is running at a speed of 60 km/hr., then the length of the train in metres is
(a) 160 m (b) 150 m
(c) 180 m (d) None of these

6. Sound travels 330 metres a second. If the sound of a thunder-cloud follows the flash after 10 seconds, the thunder-cloud is at a distance of
(a) 3.7 km (b) 3.5 km
(c) 3.3 km (d) None of these

7. A train travels 92.4 km/hr. How many metres will it travel in 10 minutes?
(a) 14500 m (b) 15400 m
(c) 15200 m (d) None of these

8. The distance of the sun from the earth is one hundred and forty-three million four hundred thousand kilometres and light travels from the former to the latter in 7 minutes and fifty eight seconds. The velocity of light per second is
(a) 3×10^5 km/sec (b) 0.3×10^5 km/sec.
(c) 30×10^5 km/sec. (d) None of these

9. A train covers a distance in 50 minutes if it runs at a speed of 48 km/hr. The speed at which the train must run to reduce the time of journey to 40 minutes, will be
(a) 70 km/hr (b) 80 km/hr
(c) 60 km/hr (d) None of these

10. The wheel of an engine is $3\frac{3}{4}$ metres in circumference and makes 4 revolutions in 2 seconds. The speed of the train is
(a) 27 km/hr (b) 31 km/hr
(c) 35 km/hr (d) None of these

11. A person covers half of his journey at 30 km/hr and the remaining half at 20 km/hr. The average speed for the whole journey is
(a) 25 km/hr (b) 28 km/hr
(c) 32 km/hr (d) None of these

12. Rajesh covers a certain distance by bus at 16 km/hr and returns back at the starting point on a cycle at 9 km/hr. His average speed for the whole journey is
(a) 13.54 km/hr (b) 11.52 km/hr
(c) 15.52 km/hr (d) None of these

13. A and B are two towns. A car goes from A to B at a speed of 64 km/hr and returns to A at a slower speed. If its average speed for the whole journey is 56 km/hr, it returned with speed
(a) 52.54 km/hr (b) 47.74 km/hr
(c) 49.78 km/hr (d) None of these

14. A bicycle rider covers his onward journey from A to B at 10 km/hr and during the return journey from B to A he covers the same distance at 8 km/hr. If he finishes the onward and return journey in $4\frac{1}{2}$ hours, then the total distance covered by him during the entire journey is
(a) 30 km (b) 40 km
(c) 50 km (d) None of these

15. On a tour a man travels at the rate of 64 km an hour for the first 160 km, then travels the next 160 km at the rate of 80 km an hour. The average speed in km per hour for the first 320 km of the tour is
(a) 81.13 km/hr (b) 73.11 km/hr
(c) 71.11 km/hr (d) None of these

16. A car completes a journey in 6 hours. It covers half the distance at 50 km/hr and the rest at 70 km/hr

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The length of the journey is
(a) 165 km (b) 175 km
(c) 185 km (d) None of these

17. Rakesh sets out to cycle from Delhi to Mathura and at the same time Suresh starts from Mathura to Delhi. After passing each other they complete their journeys in 9 and 16 hours, respectively. At what speed does Suresh cycle if Rakesh cycles at 16 km per hour?
(a) 12 km/hr (b) 16 km/hr
(c) 14 km/hr (d) None of these

18. A train travels 225 km in 3·5 hours and 370 km in 5 hours. Find the average speed of train.
(a) 80 km/hr (b) 60 km/hr
(c) 70 km/hr (d) None of these

19. A man walks 6 km at a speed of $1\frac{1}{2}$ km/hr, runs 8 km at a speed of 2 km/hr and goes by bus another 32 km. Speed of the bus is 8 km/hr. If the speed of the bus is considered as the speed of the man, find the average speed of the man.
(a) $4\frac{5}{6}$ km/hr (b) $3\frac{5}{6}$ km/hr
(c) $5\frac{7}{6}$ km/hr (d) None of these

20. A car during its journey travels 30 minutes at speed of 40 km/hr, another 45 minutes at a speed of 60 km/hr, and 2 hours at a speed of 70 km/hr. The average speed of the car is
(a) 63 km/hr (b) 65 km/hr
(c) 70 km/hr (d) None of these

21. By walking at $\frac{3}{4}$ of his usual speed, a man reaches office 20 minutes later than usual. His usual time is
(a) 65 minutes (b) 60 minutes
(c) 70 minutes (d) None of these

22. Two men start together to walk a certain distance, one at 4 km/hr and another at 3 km/hr. The former arrives half an hour before the latter. Find the distance.
(a) 6 km (b) 9 km
(c) 8 km (d) None of these

23. A car starts from A for B travelling 20 km an hour. $1\frac{1}{2}$ hours later another car starts from A and travelling at the rate of 30 km an hour reaches B $2\frac{1}{2}$ hours before the first car. Find the distance from A to B.
(a) 280 km (b) 260 km
(c) 240 km (d) None of these

24. Mohan walks from Tilak Nagar to Moti Nagar and back in a certain time at the rate of $3\frac{1}{2}$ km/hr. But if he had walked from Tilak Nagar to Moti Nager at the rate of 3 km/hr and back from Moti Nagar to Tilak Nagar at the rate of 4 km/hr., he would have taken 10 minutes longer. The distance between Tilak Nagar and Moti Nagar is
(a) 28 km (b) 32 km
(c) 24 km (d) None of these

25. A train does a journey without stopping in 8 hours. If it had travelled 5 km an hour faster, it would have done the journey in 6 hours 40 min. What is its slower speed?
(a) 35 km/hr (b) 25 km/hr
(c) 40 km/hr (d) None of these

26. Without any stoppage a person travels a certain distance at an average speed of 42 km/hr and with stoppages he covers the same distance at an average speed of 28 km/hr. How many minutes per hour does he stop?
(a) 25 minutes (b) 30 minutes
(c) 20 minutes (d) None of these

27. A train is running at a uniform speed of 60 km/hr. It passes a railway platform in 15 seconds. If the length of the platform is 130 m, then the length of the train is
(a) 160 m (b) 120 m
(c) 140 m (d) None of these

28. A train passes through a telegraph post in 9 seconds moving with a speed of 54 km per hour. The length of the train is
(a) 135 metres (b) 145 metres
(c) 125 metres (d) None of these

29. A train 135 m long is running with a speed of 54 km per hour. In what time will it pass a telegraph post?
(a) 11 sec. (b) 9 sec.
(c) 7 sec. (d) None of these

30. A train 160 metres long passes a standing man in 18 seconds. The speed of the train is
(a) 35 km/hr (b) 45 km/hr
(c) 32 km/hr (d) None of these

31. A train 280 m long is moving at a speed of 60 km/hr. The time taken by the train to cross a platform 220 m long is
(a) 30 sec. (b) 40 sec.
(c) 60 sec. (d) 20 sec.

32. A train 50 m long passes a platform 100 m long in 10 seconds. The speed of the train in m/sec. is
(a) 25 sec. (b) 15 sec.
(c) 35 sec. (d) None of these

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33. A train 300 metres long is running at a speed of 90 km/hr. How many seconds will it take cross a 200 metres long train running in the same direction at a speed of 60 km/hr?
(a) 70 sec. (b) 60 sec
(c) 50 sec. (d) None of these

34. Two trains are running in opposite directions with the same speed. If the length of each train is 135 metres and they cross each other in 18 seconds, the speed of each train is
(a) 29 km/hr (b) 35 km/hr
(c) 27 km/hr (d) None of these

35. A train 150 m long is running at 95 km/hr. How much time will it take to pass a man moving in the same direction at 5 km/hr?
(a) 9 sec. (b) 6 sec.
(c) 7 sec. (d) None of these

36. A train 100 metres long takes $3\frac{3}{5}$ seconds to cross a man walking at the rate of 6 km/hr in a direction opposite to that of the train. Find the speed of the train.
(a) 76 km/hr (b) 94 km/hr
(c) 86 km/hr (d) None of these

37. Two trains are moving in the same direction at 50 km/hr and 30 km/hr. The faster train crosses a man in the slower train in 18 seconds. Find the length of the faster train.
(a) 120 m (b) 110 m
(c) 100 m (d) None of these

38. Two trains, 130 m and 110 m long, while going in the same direction, the faster train takes one minute to pass the other completely. If they are moving in opposite direction, they pass each other completely in 3 seconds. Find the speed of each train.
(a) 42 m/sec., 38 m/sec. (b) 38 m/sec, 36 m/sec.
(c) 36 m/sec., 42 m/sec. (d) None of these

39. Two trains, each of length 90 metres, run on parallel tracks. When running in the same direction, the faster train passes the slower train completely in 18 seconds, but when they are running in opposite directions approaching each other at the same speeds as before they cross each other in 9 seconds. Find the speed of each train.
(a) 9 m/sec., 15 m/sec. (b) 7 m/sec., 5 m/sec.
(c) 15 m/sec., 5 m/sec. (d) None of these

40. A train leaves the station at 5 A.M. at 60 km/hr. Another train leaves the same station at 6.30 A.M. at 75 km/hr and travels in the direction of the first train. At what time and at what distance from the station will they meet?
(a) 12.30. A.M., 450 km (b) 1.30 P.M., 375 km
(c) 11.30 A.M., 425 km (d) None of these

41. Two stations A and B are 100 km apart on a straight line. One train starts from A at 7 A.M. and travels towards B at 20 km/hr speed. Another train starts from B at 8 A.M. and travels towards A at 25 km/hr speed. At what time will they meet?
(a) 10.30 A.M. (b) 11 A.M.
(c) 10 A.M. (d) None of these

42. A train starts from station A at 9 A.M. travels at 50 km/hr towards station B, 210 km away. Another train starts from station B at 11 A.M. and travels at 60 km/hr towards station A. At what distance from A, will they meet?
(a) 150 km (b) 200 km
(c) 250 km (d) None of these

43. Two trains start at the same time from Mumbai and Pune and proceed towards each other at the rate of 60 km and 40 km per hour, respectively. When they meet, it is found that one train has travelled 20 km more than the other. Find the distance between Mumbai and Pune.
(a) 150 km (b) 100 km
(c) 120 km (d) None of these

44. A car covers four successive three km stretches at speed of 10 km/hr, 20 km/hr, 30 km/hr and 60 km/hr. respectively. Its average speed over this distance is
(a) 10 km/hr (b) 20 km/hr
(c) 30 km/hr (d) 25 km/hr

45. Two men A and B walk from P to Q at a distance of 21 km at rates 3 and 4 km an hour, respectively. B reaches Q and returns immediately and meets A at R. The distance from P to R is
(a) 14 km (b) 20 km
(c) 16 km (d) 18 km

46. A boy takes as much time in running 12 metres as a car takes in covering 36 metres. The ratio of the speeds of the boy and the car is
(a) 1 : 3 (b) 1 : 2
(c) 2 : 3 (d) 2 : 5

47. A and B are two stations. A train goes from A to B at 64 km/hr and returns to A at a slower speed. If its average speed for the whole journey is 56 km/hr, at what speed did it return?
(a) 48 km/hr (b) 49.77 km/hr
(c) 30 km/hr (d) 47.46 km/hr

48. Excluding stoppages, the speed of a bus is 54 km/hr and including stoppages, it is 45 km/hr. For how many minutes does the bus stop per hour?
(a) 9 (b) 10
(c) 12 (d) 20

49. Two boys begin to together to write out a booklet containing 817 lines. The first boy starts with the first line, writing at the rate of 200 lines an hour

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and the second boy starts with the last line, then writes line 816 and so on, backwards proceeding at the rate of 150 lines an hour. They will meet on
(a) 467th line (b) 466th line
(c) 460th line (d) 472th line

50. Ramesh sees a train passing over 1 km long bridge. The length of the train is half that of bridge. If the train clears the bridge in 2 minutes, the speed of the train is
(a) 45 km/hr (b) 43 km/hr
(c) 50 km/hr (d) None of these

51. A bullock cart has to cover a distance of 80 km in 10 hours. If it covers half of the journey in (3/5) the time, What should be its speed to cover the remaining distance in the time left?
(a) 8 km/hr (b) 20 km/hr
(c) 6.4 km/hr (d) 10 km/hr

52. Amit started cycling along the boundaries of a square field from cover point *A*. After half an hour, he reached the corner point *C*, diagonally opposite to *A*. If his speed was 8 km/hr, what is the area of the field in square km
(a) 64 (b) 8
(c) 4 (d) Cannot be determined

53. A train 100 metres long completely passes a man walking in the same direction at 6 km p.h. in 5 seconds and a car travelling in the same direction in 6 seconds. At what speed was the car travelling?
(a) 18 km p.h. (b) 48 km p.h.
(c) 24 km p.h. (d) 30 km p.h.

54. A motor cyclist goes from Mumbai to Pune, a distance of 192 kms, at an average speed of 32 km p.h. Another man starts from Mumbai by car, $2\frac{1}{2}$ hours after the first and reaches Pune half an hour earlier. What is the ratio of the speed of the motor cycle and the car?
(a) 1 : 2 (b) 1 : 3
(c) 10 : 27 (d) 5 : 4

55. Two trains are running in opposite directions with speed of 62 km p.h. and 40 km p.h, respectively. If the length of one train is 250 metres and they cross each other in 18 seconds, the length of the other train is
(a) 145 metres (b) 230 metres
(c) 260 metres (d) Cannot be determined

56. A train speeds past a pole in 15 seconds and speeds past a platform 100 metres long in 25 seconds. Its length in metres is
(a) 200 (b) 150
(c) 50 (d) Data inadequate

57. A train of length 150 metres takes 10 seconds to pass over another train 100 metres long coming from the opposite direction. If the speed of the first train be 30 km p.h., the speed of the second train is:
(a) 54 km/hr (b) 60 km/hr
(c) 72 km/hr (d) 36 km/hr

58. A person sets to cover a distance of 12 km in 45 minutes. If he covers $\frac{3}{4}$ of the distance in $\frac{2}{3}$rd time, what should be his speed to cover the remaining distance in the remaining time?
(a) 16 km/hr (b) 8 km/hr
(c) 12 km/hr (d) 14 km/hr

59. A train 110 metres in length passes a man walking at the speed of 6 km/hr, against it in 6 seconds. The speed of the train in km per hour is
(a) 60 km/hr (b) 45 km/hr
(c) 50 km/hr (d) 55 km/hr

60. If a train 110 metres in length passes a man walking at the rate of 6 km/hr against it in 6 seconds, it will pass another man walking at the same speed in the same direction in time of
(a) $9\frac{1}{3}$sec (b) $10\frac{2}{3}$sec
(c) 8 sec (d) $7\frac{1}{3}$sec

61. A man performs $\frac{2}{15}$ of the total journey by rail, $\frac{9}{20}$ by tonga and the remaining 10 km on foot. The total journey is :
(a) 15.6 km (b) 12.8 km
(c) 16.4 km (d) 24 km

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$

$= \left(\frac{600}{5\times 60}\right)$ m/sec

$= \left(\frac{600}{5\times 60}\times\frac{18}{5}\right)$ km/hr

= 7.2 km/hr.

2. Ans. (c)

Solution 80 km/hr means $\left(80\times\frac{5}{18}\right)$ m/sec.

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$\therefore$ Required comparison is $80 \times \frac{5}{18} : 10$ **or, 20 : 9.**

3. Ans. (b)

Solution Mohan's speed $= \left(\frac{10.2}{3}\right)$ km/hr $= 3.4$ km/hr

$\therefore$ Distance covered by him in 5 hours
$= (3.4 \times 5)$km $=$ **17 km.**

4. Ans. (a)

Solution Distance travelled by the train in 25 seconds at 72 km/hr.

$$= 72 \times \frac{5}{18} \times 25 = 500 \text{ m.}$$

$\therefore$ Length of the Bridge = 500 – length of train
$= 500 - 100 =$ **400 m.**

5. Ans. (b)

Solution Let the length of the train be x m.

$\therefore$ Total distance covered by the train $= (x + 150)$ m.

Speed of the train $= 60$ km/hr $= 60 \times \frac{5}{18} = \frac{50}{3}$ m/sec.

Since, Distance = Speed × time

$\therefore$ $x + 150 = \frac{50}{3} \times 18 = 300$

or, $x = 300 - 150 = 150$ m.

$\therefore$ Length of the train = **150 m.**

6. Ans. (c)

Solution Distance of thunder-cloud
= distance travelled by sound in 10 seconds
$= (330 \times 10)$ metres
= 3.3 km.

7. Ans. (b)

Solution Speed of the train = 92.4 km/hr

$$= \left(92.4 \times \frac{5}{18}\right) \text{m/sec}$$

$$= \frac{77}{3} \text{m/sec.}$$

Distance covered in 10 minutes or 10×60 (= 600) seconds

$$= \frac{77}{3} \times 600 = \textbf{15400 m.}$$

8. Ans. (a)

Solution Distance of sun from earth
$= (143 \times 10^6 + 400 \times 10^3)$ km
$= 1434 \times 10^5$ km.

Time taken by light to travel from sun to earth
$= 7 \times 60 + 58 = 478$ seconds.

$\therefore$ Velocity of light $= \frac{1434 \times 10^5}{478}$
$= \mathbf{3 \times 10^5}$ **km/sec.**

9. Ans. (c)

Solution Distance travelled = Speed × Time $= \left(48 \times \frac{50}{60}\right)$
$= 40$ km.

$\therefore$ Speed $= \frac{\text{Distance}}{\text{Time}} = \left(\frac{40 \times 60}{40}\right)$ km/hr
= 60 km/hr.

10. Ans. (a)

Solution Distance covered in 2 seconds $= \frac{15}{4} \times 4$
$= 15$ m.

$\therefore$ Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{15}{2}$ m/sec.
$= \left(\frac{15}{2} \times \frac{18}{5}\right)$km/hr **= 27 km/hr.**

11. Ans. (a)

Solution Here, $s_1 = 30$ and $s_2 = 20$.

$\therefore$ Average speed $= \frac{2s_1s_2}{s_1 + s_2}$
$= \frac{2 \times 30 \times 20}{30 + 20} =$ **25 km/hr.**

12. Ans. (b)

Solution Here, $s_1 = 16$ and $s_2 = 9$.

$\therefore$ Average speed $= \frac{2s_1s_2}{s_1 + s_2} = \frac{2 \times 16 \times 9}{16 + 9}$
= 11.52 km/hr.

13. Ans. (c)

Solution Let the speed on the return journey be x km/hr.

Then, $56 = \frac{2s_1s_1}{s_1 + s_2} = \frac{2 \times 64 \times x}{64 + x}$

$\Rightarrow$ $7(64 + x) = 16x$ or $9x = 448$

$\therefore$ $x = \frac{448}{9} =$ **49.78 km/hr.**

14. Ans. (b)

Solution Here, $s_1 = 10$ and $s_2 = 8$.

$\therefore$ Average speed $= \frac{2s_1s_2}{s_1 + s_2} = \frac{2 \times 10 \times 8}{10 + 8}$
$= \frac{80}{9}$km/hr.

Total time taken for the entire journey $= \frac{9}{2}$ hours.

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∴ Total distance covered
= Average speed × total time taken
$= \frac{80}{9} \times \frac{9}{2}$ = **40 km.**

15. Ans. (c)

Solution Here, $s_1 = 64$ and $s_2 = 80$.

∴ Average speed $= \frac{2s_1s_2}{s_1+s_2} = \frac{2\times64\times80}{64+80}$
= **71.11 km/hr.**

16. Ans. (b)

Solution Here, $s_1 = 50$, $s_2 = 70$ and $T = 6$.

∴ The length of the journey is
$= T\left(\frac{s_1s_2}{s_1+s_2}\right) = 6\left(\frac{50\times70}{50+70}\right)$ = **175 km.**

17. Ans. (a)

Solution $\frac{\text{Rakesh's speed}}{\text{Suresh's speed}} = \frac{\sqrt{T_2}}{\sqrt{T_1}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$.

∴ Suresh's speed $= \frac{3}{4}$ Rakesh's speed
$= \frac{3}{4}\times16$ = **12 km/hr.**

18. Ans. (c)

Solution Here, $x_1 = 225$, $x_2 = 370$, $T_1 = 3\cdot5$ and $T_2 = 5$.

∴ Average speed of train $= \frac{x_1+x_2}{T_1+T_2} = \frac{225+370}{3\cdot5+5}$
= **70 km/hr.**

19. Ans. (b)

Solution Here, $x_1 = 6$, $x_2 = 8$, $x_3 = 32$, $s_1 = \frac{3}{2}$, $s_2 = 2$ and $s_3 = 8$.

∴ Average speed of the man
$= \frac{x_1+x_2+x_3}{\frac{x_1}{s_1}+\frac{x_2}{s_2}+\frac{x_3}{s_3}} = \frac{6+8+32}{\frac{6}{3/2}+\frac{8}{2}+\frac{32}{8}}$
$= \frac{46}{12} = 3\frac{5}{6}$ **km/hr.**

20. Ans. (a)

Solution Here, $T_1 = \frac{30}{60}$, $T_2 = \frac{45}{60}$, $T_3 = 2$, $s_1 = 40$, $s_2 = 60$ and $s_3 = 70$.

∴ The average speed of the car
$= \frac{s_1T_1+s_2T_2+s_3T_3}{T_1+T_2+T_3}$
$= \frac{40\times\frac{30}{60}+60\times\frac{45}{60}+70\times2}{\frac{30}{60}+\frac{45}{60}+2}$
= **63 km/hr.**

21. Ans. (b)

Solution Here, change in time = 20 and $\frac{a}{b} = \frac{3}{4}$.

We have, change in time $= \left(\frac{b}{a}-1\right)$ × original time

⇒ Original time $= \frac{\text{Change in time}}{\left(\frac{b}{a}-1\right)}$
$= \frac{20}{\left(\frac{4}{3}-1\right)}$ = **60 minutes.**

22. Ans. (a)

Solution Here, difference in speed = 4 – 3 = 1
difference in time $= \frac{1}{2}$
and, product of speed = 4 × 3 = 12.

We have,
$\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}}$

⇒ d = Product of speed × $\left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$
$= 12 \times \frac{1/2}{1}$ = **6 km.**

23. Ans. (c)

Solution Here, difference in speed = 30 – 20 = 10.
difference in time $= 2\frac{1}{2} + 1\frac{1}{2} = 4$
and, product of speed = 20 × 30 = 600.

We have, $\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}}$

⇒ d = Product of speed × $\left(\frac{\text{Difference of time}}{\text{Difference of speed}}\right)$
$= 600 \times \frac{4}{10}$ = **240 km.**

24. Ans. (a)

Solution Mohan's speed in the first case $= \frac{7}{2}$ km/hr.
Mohan's average speed in the second case
$= \frac{2s_1s_2}{s_1+s_2} = \frac{2\times3\times4}{3+4} = \frac{24}{7}$ km/hr.

∴ Difference of speed $= \frac{7}{2} - \frac{24}{7} = \frac{1}{14}$.

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Product of speed $= \frac{7}{2} \times \frac{24}{7} = 12$

and, difference of time $= \frac{10}{60}$.

We have,

d = product of speed $\times \left(\frac{\text{Difference of time}}{\text{Difference of speed}} \right)$

$= 12 \times \frac{10/60}{1/14} =$ **28 km.**

25. Ans. (b)

Solution Let the slower speed = s km/hr.

Since the distance travelled is same in both the cases, therefore,

$$\frac{s_1}{T_2} = \frac{s_2}{T_1} \Rightarrow s_1 \times T_1 = s_2 \times T_2$$

$$\Rightarrow s \times 8 = (s + 5) \times \frac{20}{3}$$

$$\Rightarrow 24s = 20(s + 5)$$

$$\Rightarrow s = \textbf{25 km/hr.}$$

26. Ans. (c)

Solution Here, $s_1 = 42$ and $s_2 = 28$.

$\therefore$ Stoppage time/hr $= \frac{s_1 - s_2}{s_1} = \frac{42 - 28}{42}$

$= \frac{1}{3}$ hour = **20 minutes.**

27. Ans. (b)

Solution We have, speed of the train

$$= \frac{\text{Length of the train + length of the platform}}{\text{Total time taken in crossing the platform}}$$

$\Rightarrow 60 \times \frac{5}{18} = \frac{\text{Length of the train + 130}}{15}$

$\Rightarrow$ Length of the train $= 250 - 130 =$ **120 m.**

28. Ans. (a)

Solution Speed of the train

$$= \frac{\text{Length of the train}}{\text{Time taken in crossing the post}}$$

$\Rightarrow$ Length of the train = Speed of the train $\times$ Time taken in crossing the post

$= \left(54 \times \frac{5}{18}\right) \times 9 =$ **135 m.**

29. Ans. (b)

Solution Time taken in crossing the telegraph post

$$= \frac{\text{Length of the train}}{\text{Speed of the train}}$$

$= \frac{135}{54 \times \frac{5}{18}} = \frac{135 \times 18}{54 \times 5} =$ **9 sec.**

30. Ans. (c)

Solution Speed of the train

$$= \frac{\text{Length of the train}}{\text{Time taken in crossing the man}}$$

$= \frac{160}{18}$ or, $\frac{80}{9}$ m/s $= \left(\frac{80}{9} \times \frac{18}{5}\right)$ km/hr

$=$ **32 km/hr.**

31. Ans. (a)

Solution Time taken in crossing the platform

$$= \frac{\text{Length of the train + Length of the platform}}{\text{Speed of the train}}$$

$= \frac{280 + 220}{60 \times \frac{5}{18}} = \frac{500 \times 18}{60 \times 5} =$ **30 sec.**

32. Ans. (b)

Solution We have,

Speed of the train

$$= \frac{\text{Length of the train + Length of the platform}}{\text{Time taken in crossing the platform}}$$

$= \frac{50 + 100}{10} =$ **15 sec.**

33. Ans. (b)

Solution Here, $L_1 = 300$ m, $L_2 = 200$ m,

$s_1 = 90$ km/hr and $s_2 = 60$ km/hr

$\therefore s_1 - s_2 = 90 - 60 = 30$ km/hr $= 30 \times \frac{5}{18}$ m/s

$\therefore$ Time taken $= \frac{L_1 + L_2}{s_1 - s_2} = \frac{300 + 200}{30 \times \frac{5}{18}}$

$= \frac{500 \times 18}{30 \times 5} =$ **60 sec.**

34. Ans. (c)

Solution Let the speed of each train be x m/sec.

We have, $L_1 = L_2 = 135$ m and $s_1 = s_2 = x$ m/sec

Therefore, time taken $= \frac{L_1 + L_2}{s_1 + s_2}$

$\Rightarrow 18 = \frac{135 + 135}{x + x}$ or, $x = \frac{270}{2 \times 18}$ m/s

$= \frac{270}{2 \times 18} \times \frac{18}{5}$ km/hr $=$ **27 km/hr.**

35. Ans. (c)

Solution Here, $L_1 = 150$ m, $L_2 = 0$, $s_1 = 95$ km/hr and $s_2 = 5$ km/hr.

$\therefore s_1 - s_2 = 90$ km/hr $= 90 \times \frac{5}{18}$ m/s $= 25$ m/s.

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$\therefore$ Time taken $= \dfrac{L_1+L_2}{s_1-s_2} = \dfrac{150}{25}$ **= 6 sec.**

36. Ans. (b)

Solution Here, $L_1 = 100$ m, $L_2 = 0$ and $s_2 = 6$ km/hr.

Let $s_1 = x$ km/hr.

Then, $s_1 + s_2 = (x + 6)$ km/hr $= (x + 6)\dfrac{5}{18}$ m/sec.

$\therefore$ Time taken $= \dfrac{L_1+L_2}{s_1+s_2}$

$\Rightarrow \dfrac{18}{5} = \dfrac{100}{(x+6)\frac{5}{18}}$

$\therefore$ $x = 100 - 6$ **= 94 km/hr.**

37. Ans. (c)

Solution Relative speed = (50 – 30) km/hr = 20 km/hr

$$= \left(20\times\frac{5}{18}\right) = \left(\frac{50}{9}\right) \text{m/sec.}$$

Distance covered in 18 sec at this speed

$$= \left(18\times\frac{50}{9}\right)\text{m} = 100 \text{ m.}$$

$\therefore$ Length of faster train **= 100 m.**

38. Ans. (a)

Solution Speed of the faster train

$$= \left(\frac{L_1+L_2}{2}\right)\left(\frac{T_1+T_2}{T_1T_2}\right) = \left(\frac{130+110}{2}\right)\left(\frac{60+3}{60\times3}\right)$$

= 42 m/sec.

Speed of the slower train

$$= \left(\frac{L_1+L_2}{2}\right)\left(\frac{T_1-T_2}{T_1T_2}\right) = \left(\frac{130+110}{2}\right)\left(\frac{60-3}{60\times3}\right)$$

= 38 m/sec.

39. Ans. (c)

Solution Speed of faster train

$$= L\left(\frac{T_1+T_2}{T_1T_2}\right) = 90\left(\frac{18+9}{18\times9}\right)$$

$$= \frac{90\times27}{18\times9} = 15 \text{ m/sec.}$$

Speed of the slower train

$$= L\left(\frac{T_1-T_2}{T_1T_2}\right) = 90\left(\frac{18-9}{18\times9}\right)$$

$$= \frac{90\times9}{18\times9} = \textbf{5 m/sec.}$$

40. Ans. (a)

Solution Time from 5 A.M. to 6.30 A.M. $= 1\dfrac{1}{2}$ hours.

Therefore, distance of meeting from station

$$= \left(\frac{s_1\times s_2\times T}{s_2-s_1}\right)\text{km}$$

$$= \left(\frac{60\times75\times\frac{3}{2}}{75-60}\right)\text{km} = 450 \text{ km}$$

Also, time of their meeting

$$= \left(\frac{s_1T}{s_2-s_1}\right)\text{hr}$$

$$= \left(\frac{60\times\frac{3}{2}}{75-60}\right)\text{hr}$$

= 6 hr after 6.30 A.M.

That is, **12.30 P.M.**

41. Ans. (d)

Solution Time from 7 A.M. to 8 A.M. = 1 hour.

Therefore, time of their meeting

$$= \left(\frac{d+s_2T}{s_1+s_2}\right)\text{hr}$$

$$= \left(\frac{100+25\times1}{20+25}\right)\text{hr}$$

= 2 hrs 47 min after 7 A.M.

42. Ans. (a)

Solution Time from 9 A.M. to 11 A.M. = 2 hours.

Therefore, distance of meeting point from station A

$$= s_1\left(\frac{d+s_2T}{s_1+s_2}\right)\text{km}$$

$$= 50\left(\frac{210+60\times2}{50+60}\right)\text{km} = 150 \text{ km.}$$

43. Ans. (b)

Solution Distance between Mumbai and Pune

$$= d\left(\frac{s_1+s_2}{s_1-s_2}\right)\text{km}$$

$$= 20\left(\frac{60+40}{60-40}\right)\text{km} = \textbf{100 km.}$$

44. Ans. (b)

Solution Total time taken $= \left(\dfrac{3}{10}+\dfrac{3}{20}+\dfrac{3}{30}+\dfrac{3}{60}\right)\text{hr}$

$$= \frac{3}{5}\text{hr}$$

$\therefore$ Average speed $= \left\{\dfrac{12}{3/5}\right\}\text{km/hr} = \dfrac{12\times5}{3}\text{km/hr}$

= 20 km/hr.

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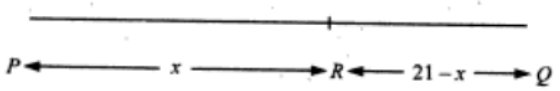
45. Ans. (d)

Solution Let the distance between P to $R = x$ km



Then, $\frac{x}{3} = \frac{21}{4} + \frac{21-x}{4}$

$\therefore \quad 7x = 2 \times 3 \times 21$

$\therefore \quad x = \mathbf{18.}$

46. Ans. (a)

Solution Let speeds of the boy and the car by x km/hr and y km/hr, respectively

Then, $\frac{12/1000}{x} = \frac{36/1000}{y}$

$\therefore \quad \frac{x}{y} = \frac{1}{3}.$

47. Ans. (b)

Solution Let the required speed be x km/hr.

Then, $\frac{2 \times 64 \times x}{64 + x} = 56$

$\Rightarrow \quad 128x = 64 \times 56 + 56x$

$\therefore \quad x = \frac{64 \times 56}{72} = \mathbf{49.77\ km/hr.}$

48. Ans. (b)

Solution Due to stoppages, it covers 9 km less per hour.

Time taken to cover 9 km $= \left(\frac{9}{54} \times 60\right)$ min

$= 10$ min.

So, the bus stops for 10 min. per hr.

49. Ans. (a)

Solution Let the boys meet after first has written x lines

Then 2nd has written $817 - (x + 1) = 816 - x$ lines

Then, $\frac{x}{200} = \frac{816-x}{150}$

$\therefore \quad x = 466.28$

$\therefore$ Two boys will meet on **467th line.**

50. Ans. (a)

Solution Distance travelled in 2 minutes

$= \left(1 + \frac{1}{2}\right)$ km i.e. $\frac{3}{2}$ kms.

Distance covered in 1 hr $= \left(\frac{3}{2} \times \frac{60}{2}\right)$ km $= 45$ km.

$\therefore$ Speed of the train = **45 km/hr.**

51. Ans. (d)

52. Ans. (c)

53. Ans. (a)

Solution Let x km p.h. be the speed of the train

$\Rightarrow \quad (x - 6) \times \frac{5}{18} \times 5 = 100$

$\Rightarrow \quad x = 78$ km p.h.

Let the speed of the car be y km p.h.

$\Rightarrow \quad (78 - y) \times \frac{5}{18} \times 6 = 100 \Rightarrow \mathbf{y = 18.}$

54. Ans. (a)

Solution Speed of the first man = 32 km/hr.

Time taken = 192 ÷ 32 = 6 hr

Second man covers 192 km in 3 hr

$\therefore$ Speed of the second man = 192 ÷ 3

= 64 km/hr

Ratio = 32 : 64 or **1 : 2**

55. Ans. (c)

Solution Let the length of another train = x m

Their relative speed = (62 + 40) km/hr

$= \left(102 \times \frac{5}{18}\right)$ m/sec $= \frac{85}{3}$ m/sec

$\frac{250+x}{\frac{85}{3}} = 18 \Rightarrow \frac{3(250+x)}{85} = 18$

$\Rightarrow 250 + x = 510$

$\Rightarrow x = 260$

$\therefore$ Length of another train = **260 m.**

56. Ans. (b)

Solution Let the length of the train be x metres and its speed by y metres/sec.

Then, $\frac{x}{y} = 15 \Rightarrow y = \frac{x}{15}$

Now, $\frac{x+100}{25} = \frac{x}{15} \Rightarrow x = 150$ m.

57. Ans. (b)

Solution Relative speed $= \left(\frac{150+100}{10}\right)$ m/sec.

$= 25$ m/sec.

$= \left(25 \times \frac{18}{5}\right)$ km/hr

$= 90$ km/hr

$\therefore$ Speed of second train = (90 – 30) km/hr

= **60 km/hr.**

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58. Ans. (c)

Solution Distance already covered $= \frac{3}{4} \times 12 = 9$ km

Time spent $= \frac{2}{3} \times 45$ min $= 30$ min

Distance left $= (12 - 9)$ km

$= 3$ km

Time left $= (45 - 30)$ min $= 15$ min

$\therefore$ Required speed $= \frac{3}{15/60}$ km/hr $=$ **12 km/hr.**

59. Ans. (a)

Solution Let the speed of the train in km/hr $= x$

Then, relative speed $= (x + 6)$ km/hr

$= (x + 6) \times \frac{5}{18}$ m/sec.

$\therefore \quad (x + 6) \times \frac{5}{18} \times 6 = 110$

$\therefore \quad x = 60$

$\therefore$ Speed of the train $=$ **60 km/hr.**

60. Ans. (d)

Solution Let the speed of the train $= x$ km/hr

Relative speed $= (x + 6)$ km/hr

$= (x + 6) \times \frac{5}{18}$ m/sec.

$\therefore \quad (x + 6) \times \frac{5}{18} \times 6 = 110$

$\therefore \quad x = 60$

$\therefore$ Speed of train $= 60$ km/hr for 2nd person,

Relative speed $= (60 - 6)$ km/hr

$= 54 \times \frac{5}{18}$ m/sec $= 15$ m/sec.

$\therefore$ Time taken to cross 2nd person

$= \frac{110}{15} = \frac{22}{3} = 7\frac{1}{3}$ **sec.**

61. Ans. (d)

Solution Let the total journey be x km. Then,

$\frac{2}{15}x + \frac{9}{20}x + 10 = x$

$\Rightarrow \quad 8x + 27x + 600 = 60x \Rightarrow x = 24$

$\therefore$ Total journey $=$ **24 km.**

QUESTIONS FROM COMPETITIVE EXAMS

1. Excluding stoppages, the speed of a train is 45 kmph and including stoppages, it is 36 kmph. For how many minutes does the train stop per hour?

(a) 10 (b) 12
(c) 15 (d) 18

[Based on MAT 2004]

2. In covering a distance of 30 km, Amit takes 2 hours more than Suresh. If Amit doubles his speed, then he would take 1 hour less than Suresh. Amit's speed is

(a) 5 kmph (b) 7.5 kmph
(3) 6 kmph (d) 6.25 kmph

[Based on MAT 2003]

3. An express train was running at an average speed of 100 kilometres per hour, stopping for 3 minutes after every 75 kilometres. How long did it take to reach its destination of 600 kilometres from the starting point?

(a) 6 hr. 21 min. (b) 6 hr. 24 min.
(c) 6 hr. 30 min. (d) 6 hr. 27 min.

[Based on MAT 2003]

4. Two cars are running along the same road. The first one, which is running at the rate of 30 kmph, starts 6 hours ahead of the second one, which is running at the rate of 50 kmph. How long will it take the second car to catch up with the first one?

(a) 6 hours (b) 9 hours
(c) 12 hours (d) 15 hours

[Based on MAT 2003]

5. A train can travel 20% faster than a car. Both start from a point A at the same time and reach point B 75 km away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. Find the speed of the car in km per hour.

(a) 50 kmph (b) 55 kmph
(c) 60 kmph (d) 65 kmph

[Based on MAT 2003]

6. A train can travel 20% faster than a car. Both start from the point A at the same time and reach point B 75 km away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is

(a) 50 kmph (b) 55 kmph
(c) 60 kmph (d) 65 kmph

[Based on MAT 2003]

7. A starts 3 min after B for a place 4.5 km distant B, on reaching his destination, immediately returns and after walking a km meets A. If A can walk 1 km in 18 minutes, then what is B's speed ?

(a) 5 kmph (b) 4 kmph
(c) 6 kmph (d) 3.5 kmph

[Based on MAT 2003]

8. Wheels of diameters 7 cm and 14 cm start rolling

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simultaneously from X and Y, which are 1980 cm apart, towards each other in opposite directions. Both of them make same number of revolutions per second. If both of them meet after 10 seconds, the speed of the smaller wheel is
(a) 22 cm/sec (b) 44 cm/sec
(c) 66 cm/sec (d) 132 cm/sec
[Based on MAT 2003]

9. The remainder when 7^{84} is divided by 342 is
(a) 0 (b) 1
(c) 49 (d) 341
[Based on MAT 2003]

10. An express train travelled at an average speed of 100 kmph, stopping for 3 minutes after every 75 km. A local train travelled at a speed of 50 kmph, stopping for 1 minute after every 25 km. If the trains began travelling at the same time, then how many kilometres did the local train travel in the time it took the express train to travel 600 km?
(a) 307.5 km (b) 900 km
(c) 1200 km (d) 1000 km
[Based on MAT 2003]

11. Two trains of equal length are running on parallel lines in the same direction at 46 km and 36 km per hr. The faster train passes the slower train in 36 sec. The length of each train is
(a) 50 m (b) 80 m
(c) 72 m (d) 82 m
[Based on MAT 2003]

12. There are 20 poles with a constant distance between each pole. A car takes 24 seconds to reach the 12th pole. How much time will it take to reach the last pole?
(a) 25.25 seconds (b) 17.45 seconds
(c) 35.75 seconds (d) 41.45 seconds
[Based on MAT 2003]

13. The jogging track in a sports complex is 726 metres in circumference. Suresh and his wife start from the same point and walk in opposite directions at 4.5 km/h and 3.75 km/h respectively. They will meet for the first time in
(a) 5.5 min. (b) 6.0 min.
(c) 5.28 min. (d) 4.9 min.
[Based on MAT 2003]

14. A car driver travels from the plains to the hill station, which are 200 km apart, at an average speed of 40 km/h. In the return trip, he covers the same distance at an average speed of 20 km/h. The average speed of the car over the entire distance of 400 km is
(a) 16.56 km/h (b) 17.89 km/h
(c) 26.67 km/h (d) 35 km/h
[Based on MAT 2002]

15. A man in a train notices that he can count 21 telephone posts in one minute. If they are known to be 50 metres apart, then at what speed is the irain travelling?
(a) 57 km/h (b) 60 km/h
(c) 63 km/h (d) 55 km/h
[Based on MAT 2002]

16. A circular running path is 726 metres in circumference. Two men start from the same point and walk in opposite directions @ 3.75 km/h and 4.5 km/h respectively. When will they meet for the first time?
(a) 5.5 minutes (b) 6.0 minutes
(c) 5.28 minutes (d) 4.9 minutes
[Based on MAT 2002]

17. A gun is fired at a distance of 3.32 km from Babar. He hears the sound 10 seconds later. Find the speed of the sound.
(a) 301 m/sec (b) 302 m/sec
(c) 332 m/sec (d) 340 m/sec
[Based on MAT 2002]

18. A train is running at 36 km/hr. If it crosses a pole in 25 seconds, find its length.
(a) 210 metres (b) 230 metres
(c) 250 metres (d) 270 metres
[Based on MAT 2002]

19. When two trains were running in the same direction at 90 km/hr and 70 km/hr respectively, then the faster train passed a man sitting in the slower train in 36 seconds. Find the length of the faster train.
(a) 100 metres (b) 150 metres
(c) 190 metres (s) 200 metres
[Based on MAT 2002]

20. The speed of a metro train is 54 km/hr excluding stoppage time and including stoppage, the speed is 45 km/hr then for how many minutes does it stop per hour?
(a) 9 (b) 10
(c) 20 (d) 11
[Based on MAT 2001]

21. Three men A, B and C go walking round a circle 1 kilometre in circumference at the rates of 10, 20 and 40 metres per minute respectively. If they all start together and walk in the same direction, when will they again be together at the same place?
(a) After 50 minutes (b) After 240 minutes
(c) After 800 minutes (d) After 100 minutes
[Based on MAT 2001]

22. X and Y start from the same point and run around a circular stadium, whose circumference is 4200 m, at the rate of 500 m and 700 m per minute respectively in the opposite directions. They will meet each other in
(a) 3.5 min. (b) 8.4 min.
(c) 6.0 min. (d) 21 min.
[Based on MAT 2001]

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23. It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to the speed of the car is
(a) 4 : 3 (b) 3 : 4
(c) 3 : 2 (d) 2 : 3
[Based on MAT 1999]

24. Points *A* and *B* are 70 km apart on a highway. One car starts from *A* and another one from *B* at the same time. If they travel in the same direction, they meet in 7 hours. But if they travel towards each other, they meet in one hour. The speeds of the two cars are
(a) 45 and 25 km/hr (b) 70 and 10 km/hr
(c) 40 and 30 km/hr (d) 60 and 40 km/hr
[Based on MAT 1999]

25. If train runs at 40 kmph, it reaches its destination late by 11 minutes but if it runs at 50 kmph it is late by 5 minutes only. The correct time for the train to complete its journey is
(a) 13 min (b) 15 min
(c) 19 min (d) 21 min
[Based on MAT 2005]

26. A train of length 150 m takes 40.5 seconds to cross a tunnel of length 300 m. The speed of the train (in km/hr) is
(a) 13.33 (b) 26.67
(c) 40 (d) 400
[Based on MAT 2005]

27. A train 100 m long passes a bridge at the rate of 72 km per hour in 25 seconds. The length of the bridge is
(a) 150 m (b) 400 m
(c) 300 m (d) 200 m
[Based on MAT 1999]

28. A train 110 m in length travels at 60 km/hr. How much time does the train take in passing a man walking at 6 km/hr against the train?
(a) 6 sec. (b) 12 sec.
(c) 16 sec. (d) 18 sec.
[Based on MAT 1999]

29. *A* travels from *B* to *C* a distance of 250 miles in 5.5 hours. He returns to *B* in 4 hours 40 minutes. His average speed is
(a) 44 (b) 46
(c) 48 (d) 50
[Based on MAT 2000]

30. Two rackets approach each other, one at 42,000 mph and the other at 18,000 mph. They start 3256 miles apart. How far are they apart (in miles) 1 minute before impact?
(a) 1628 (b) 1000
(c) 826 (d) 1200
[Based on MAT 2000]

31. A bought 4 bottles of beer and *B* bought one bottle of lager, per bottle costing twice that of the beer, *C* bought nothing but paid Rs. 50 for his share of the drink which they mixed together and shared equally. If *C*'s Rs. 50 covered his share, then what is the cost of the lager?
(a) 50 (b) 75
(c) 30 (d) 46
[Based on MAT 2000]

32. Wheels of diameters 7 cm and 14 cm start rolling simultaneously from *X* and *Y*, 1,980 cm apart, towards each other in opposite directions. Both of them make same number of revolutions per second. If both of them meet after 10 seconds, find the speed of the smaller wheel.
(a) 22 cm/sec. (b) 44 cm/sec
(c) 66 cm/sec. (d) 132 cm/sec
[Based on MAT 2001]

33. An express train runs at an average speed of 100 kmph, stopping for 3 minutes after 75 km. A local train runs at a speed of 50 kmph, stopping for 1 minute after every 25 km. If the trains began running at the same time, how many kilometers did the local train run in the time it took the express train to run 600 km?
(a) 307.5 km (b) 900 km
(c) 1200 km (d) 1000 km
[Based on MAT 2001]

34. Two trains of equal length are running on parallel lines in the same direction @ 46 km & 36 km per hr. The faster train passes the slower train in 36 sec. The length of each train is
(a) 50 mts. (b) 80 mts.
(c) 72 mts. (d) 82 mts.
[Based on MAT 2001]

35. Walking at 3/4 of his usual pace, a man reaches his office 20 minutes late. Find his usual time.
(a) 2 hrs (b) 1 hr
(c) 3 hrs (d) 4 hrs
[Based on MAT 2001]

36. Dinesh travels 760 km to his home, partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by car. He takes 12 minutes more if he travels 240 km by train and the rest by car. The speeds of the train and the car respectively are
(a) 80 km/h, 100 km/h
(b) 100 km/h, 80 km/h
(c) 120 km/h, 100 km/h
(d) 100 km/h, 120 km/h **[Based on MAT 2001]**

37. In a flight of 3000 km, an aircraft was slowed down by bad weather. Its averae speed for the trip was

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was reduced by 100 km/hour and the time increased by one hour. Find the original duration of the flight.
(a) 5 hours (b) 6 hours
(c) 4 hours (d) 10 hours
[Based on MAT 2001]

38. The diameter of a cycle wheel is 70 cm. A cyclist takes 30 hours to reach a destination at the speed of 22 km/h. How many revolutions will the wheel make during this journey?
(a) 3 million (b) 3 lakh
(c) 4 lakh (d) None of these
[Based on MAT 2001]

39. Rashmi leaves office at 6.00 p.m. and catches a 6.30 p.m. local train that arrives in her town at 7.00 p.m. Her father leaves home to pick her up at 7.00 p.m. from the station as she gets off the train. Yesterday, Rashmi left her office early and took a 6.00 pm train and arrived at 6.30 p.m. As her father was not there to pick her up, she started walking towards home. Her rather left home at the usual time, saw her daughter walking, turned around, picked her up and drove home, arriving there 10 minutes earlier than the usual. For how long did Rashmi walk before her father picked her up?
(a) 10 min (b) 15 min
(c) 20 min (d) 25 min
[Based on FMS (Delhi) 2004]

40. Rampur is 100 km from Sitapur. At 3 p.m. Bharat Express leaves Rampur for Sitapur and travels at a constant speed of 30 kmph. One hour later, Laxman Mail leaves Sitapur for Rampur and travels at a constant speed of 40 kmph. Each train makes one stop only at a station 10 km from its starting point and remains there for 15 min. Which train is nearer to Rampur when they meet?
(a) Bharat Express
(b) Laxman Mail
(c) Both are equidistant
(d) None of these **[Based on FMS (Delhi) 2004]**

41. Ram travels from P to Q at 10 km/hr and returns at 15 km/hr. Sham travels from P to Q and returns at 12.5 km/hr. If he takes 12 minutes less than Ram, then what is the distance between P and Q?
(a) 60 km (b) 45 km
(c) 36 km (d) 30 km
[Based on IIT Joint Man. Ent. Test 2004]

42. A train of length 500 m travelling at 72 km/hr approaches a man standing on the platform. How many seconds are required to completely cross him?
(a) 25 (b) 500/17
(c) 450/11 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

43. A student reached his school late by 20 minutes by travelling at the speed of 9 kmph. Had he travelled at the speed of 12 kmph, he would have reached his school 20 minutes early. Find the distance between his house and the school.
(a) 12 km (b) 6 km
(c) 3 km (d) 24 km
[Based on IIT Joint Man. Ent. Test 2004]

44. Every Sunday, Giri jogs 3 miles. For the rest of the week, each day he jogs 1 mile more than the previous day. How many miles does Giri jog in 2 weeks?
(a) 42 (b) 63
(c) 84 (d) 98
[Based on REC Tiruchirapalli 2003]

45. To get to a business meeting, John drove m miles in h hours, and arrived 1/2 hour early. At what rate should he have driven to arrive exactly on time?
(a) $\frac{m}{2h}$ (b) $\frac{2m}{2h+1}$
(c) $\frac{2m}{2h-1}$ (d) $\frac{2m-h}{2h}$
[Based on REC Tiruchirapalli 2003]

46. A is twice as fast as B and B is thrice as fast as C. The journey covered by C in 42 minutes will be covered by A in
(a) 14 min. (b) 7 min.
(c) 28 min. (d) 54 min.
[Based on FMS (Delhi) 2003]

47. Points A and B are 60 km apart. A bus starts from A and another from B at the same time. If they go in the same direction, then they meet in 6 hours and if they go in opposite directions, they meet in 2 hours. The speed of the bus with greater speed is
(a) 10 km/hr (b) 20 km/hr
(c) 30 km/hr (d) 40 km/hr
[Based on FMS (Delhi) 2003]

48. Walking at 3/4 of his usual pace, a man reaches his office 20 minutes late. Find out his usual time.
(a) 1 hr (b) 2 hrs
(c) 3 hrs (d) 4 hrs
(e) None of the above **[Based on IIFT 2003]**

49. A train travelled 75% of the way from town X to town Y by travelling for A hours at an average speed of B kmph. The train travels at an average speed of S kmph for the remaining part of the journey. Which of the following expressions represents the average speed of the entire journey?
(a) $0.75B + 0.25$ (b) $(4BS)/(3S + B)$
(c) $AB/3S$ (d) $0.75A + 0.25S$
[Based on SCMHRD Ent. Exam. 2003]

50. Shalini jogs 9 km at a speed of 6 km per hour. At what speed would she need to travel during the next 1.5 hours to have an average speed of 9 km per hour for the entire jogging session?
(a) 12 kmph (b) 10 kmph

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(c) 14 kmph (d) 9 kmph

[Based on SCMHRD Ent. Exam. 2003]

51. Plane A takes off at 4 pm and flies at a constant speed of X mph. Plane B takes off at 5.30 pm and flies the same route as that of A at a constant speed of Y mph. Assuming that Y is greater than X, how many hours after 5:30 pm will the plane B overtake plane A?

(a) $3/2\ X$ hr (b) $3/(2Y)$ hr

(c) $3/2\ (Y - X)$ hr (d) $3X/2\ (Y - X)$ hr

[Based on SCMHRD Ent. Exam. 2003]

52. The ratio between the speeds of two trains is 7 : 8. If the second train runs 400 km in 5 hours, then find the speed of the first train per hour.

(a) 250 km (b) 70 km

(c) 350 km (d) 200 km

[Based on IMT Ghaziabad, 2002]

53. A car starts running with the initial speed of 40 kmph, with its speed increasing every hour by 5 kmph. How many hours will it take to cover a distance of 385 km?

(a) $8\frac{1}{2}$ hrs (b) $9\frac{1}{2}$ hrs

(c) 9 hrs (d) 7 hrs

[Based on IRMA, 2002]

54. Car A leaves Mumbai for Daman which is 180 kilometers away. At the same time car B leaves Daman for Mumbai. If car A travels at 60 km/ hour and car B travels at 30 km/hour, where will the 2 cars meet?

(a) 120 km from Mumbai

(b) 150 km from Daman

(c) Between Mumbai and Daman

(d) None of these

[Based on Narsee Monjee Inst. of Man. Studies 2002]

55. A person travelled a distance of 20 km out of a total distance of 50 km. What percentage of the total distance he has yet to travel?

(a) 30 percent (b) 25 percent

(c) 60 percent (d) 40 percent

[Based on FMS (Delhi) 2002]

56. A passenger train running at the speed of 80km/hr leaves the railway station 6 hours after a goods train leaves and overtakes it in 4 hours. The speed of the goods train is

(a) 32 km/hr (b) 40 km/hr

(c) 50 km/hr (d) 60 km/hr

[Based on FMS (Delhi) 2002]

57. If the speed of a railway train is increased by 5 km per hour from its normal speed, then it would have taken 2 hours less for a journey of 300 km. What is its normal speed?

(a) 10 km/hr (b) 25 km/hr

(c) 20 km/hr (d) 30 km/hr

[Based on I.P. Univ. 2002]

58. An aircraft was to take off from a certain airport at 8 a.m. but it was delayed by 30 minutes. To make up for the lost time, it was to increase its speed by 250 km per hour from the normal speed to reach its distination 1500 km away, on time. What was the normal speed of the aircraft?

(a) 650 km/hr (b) 750 km/hr

(c) 850 km/hr (d) 1000 km/hr

[Based on I.P. Univ. 2002]

59. Raja was on a long distance trip. He travelled by air 2/5 of the distance which was 1200 km. Then he hired a car and travelled 1/3 of the whole trip. Thereafter, he completed the rest of the journey by train. Calculate the distance that Raja travelled by train

(a) 480 km (b) 800 km

(c) 1600 km (d) 1800 km

[Based on I.P. Univ. 2002]

60. A monkey could climb 6 metres of a round pole in one minute but slipped 3 metres in the next minute. What time will it take to reach the top of the pole which is 21 metres long?

(a) $14\frac{2}{3}$ mins (b) 14 mins

(c) 11 mins (d) 9 mins

[Based on I.P. Univ. 2002]

61. An express train left Delhi for Howrah at 3 p.m. at an average speed of 60 km/hour. At 1 p.m., a goods train also had left Delhi for Howrah on a parallel track at an average speed of 40 km/hour. How far from Delhi, is the Express train expected to overtake the goods train?

(a) 230 km (b) 240 km

(c) 280 km (d) None of these

[Based on I.P. Univ. 2002]

62. A circular playground has an area of 616 sq. metres. What time will it take for a runner to run around the circular ground at the speed of 22 km per hour?

(a) 4 hrs (b) 3 hrs

(c) 2 hrs (d) 1 hr

(e) None of these **[Based on I.P. Univ. 2002]**

63. It is 200 miles from Madras to Coimbatore. If a bus takes 2 hours to travel the first 75 miles, how long must the bus take to travel the final 125 miles in order to have an average of 50 miles per hour for the entire trip?

(a) 60 min (b) 94 min

(c) 120 min (d) 110 min

[Based on REC Tiruchirapalli, 2002]

64. Only a single rail track exists between stations A and B on a railway line. One hour after the north bound superfast train N leaves station A for station B, a south bound passenger train S reaches station A from station B. The speed of the superfast train

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is twice that of a normal express train E, while the speed of a passenger train S is half that of E. On a particular day N leaves for station B from station A, 20 minutes behind the normal schedule. In order to maintain the schedule, both N and S increased their speeds. If the superfast train doubles its speed, what should be the ratio (approximately) of the speed of passenger train to that of the superfast train so that passenger train S reaches exactly at the scheduled time at station A on that day.

(a) 1 : 3 (b) 1 : 4
(c) 1 : 5 (d) 1:6

[Based on CAT 2002]

65. On a 20 km tunnel connecting two cities A and B, there are three gutters. The distance between gutters 1 and 2 is half the distance between gutters 2 and 3. The distance from city A to its nearest gutter, gutter 1 is equal to the distance of city B from gutter 3. On a particular day, the hospital in city A receives information that an accident has happened at the third gutter. The victim can be saved only if an operation is stated within 40 minutes. An ambulance started from city A at 30 km/hr and crossed the first gutter after 5 minutes. If the driver had doubled the speed after that, what is the maximum amount of time the doctor would get to attend the patient at the hospital. Assume 1 minute is elapsed for taking the patient into and out of the ambulance.

(a) 4 minutes (b) 2.5 minutes
(c) 1.5 minutes
(d) Patient died before reaching the hospital.

[Based on CAT 2002]

66. At a bookstore, "MODERN BOOK STORE" is flashed using neon lights. The words are individually flashed at intervals of $2\frac{1}{2}, 4\frac{1}{4}, 5\frac{1}{8}$ seconds respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again, is:

(a) 49.5 seconds (b) 73.5 seconds
(c) 1744.5 seconds (d) 855 seconds

[Based on CAT 2002]

67. A train approaches a tunnel AB. Inside the tunnel a cat located at a point that is $\frac{3}{8}$ of the distance AB measured from the entrance A. When the train whistles, the cat runs. If the cat moves to the entrance of the tunnel, A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat at exactly the exit. The speed of the train is greater than the speed of the cat by what order?

(a) 3 : 1 (b) 4 : 1
(c) 5 : 1 (d) None of these

[Based on CAT 2002]

68. Six technicians working at the same rate completely work of one server in 10 hrs. If they start at 11: 00 a.m. and one additional technician per hour being added beginning at 5 : 00 p.m., at what time the server will be complete?

(a) 6 : 40p.m. (b) 7p.m.
(c) 7 : 20p.m. (d) 8 : 00p.m

[Based on CAT 2002]

69. Shyam visited Ram on vacation. In the mornings, they both would go for yoga. In the evenings, they would play tennis. To have more fun, they indulge only in one activity per day, i.e., either they went for yoga or played tennis each day. There were days when they were lazy and stayed home all day long. There were 24 mornings when they did nothing, 14 evenings when they stayed at home, and a total of 22 days when they did yoga or played tennis. For how many days Shyam stayed with Ram?

(a) 32 (b) 24
(c) 30 (d) None of these

[Based on CAT 2002]

70. A can complete a piece of work in 4 days. B takes double the time taken by A, C takes double that of B, and D takes double that of C to complete the same task. They are paired in groups of two each. One pair takes two-thirds the time needed by the second pair to complete the work. Which is the first pair?

(a) A, B (b) A, C
(c) B, C (d) A, D

[Based on CAT 2001]

71. Two men X and Y started working for a certain company at similar jobs on January 1,1950. X asked for an initial salary of Rs. 300 with an annual increment of Rs. 30. Y asked for an initial salary of Rs. 200 with a rise of Rs. 15 every six months. Assume that the arrangements remained unaltered till December 31,1959. Salary is paid on the last day of the month. What is the total amount paid to them as salary during the period?

(a) Rs.93,300 (b) Rs.93,200
(c) Rs.93,100 (d) None of these

[Based on CAT 2001]

72. At his usual rowing rate, Rahul can travel 12 miles downstream in a certain river in six hours less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for this 24 mile round trip, the downstream 12 miles would then take only one hour less than the upstream 12 miles. What is the speed of the current in miles per hour?

(a) 7/3 (b) 4/3
(c) 5/3 (d) 8/3

[Based on CAT 2001]

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73. Shyam and Vyom walk up an escalator (moving stairway). The escalator moves at a constant speed. Shyama takes three steps for every two of Vyom's steps. Shyam gets to the top of the escalator after having taken 25 steps, while Vyom (because his slower pace lets the escalator do a little more of the work) takes only 20 steps to reach the top. If the escalator were turned off, how many steps would they have to take to walk up?

(a) 40 (b) 50
(c) 60 (d) 80

[Based on CAT 2001]

74. There's a lot of work in preparing a birthday dinner. Even after the turkey is in the oven, there's still the potatoes and gravy, yams, salad, and cranberries, not to mention setting the table.

Three friends, Asit, Arnold, and Afzal, work together to get all of these chores done. The time it takes them to do the work together is six hours less than Asit would have taken working alone, one hour less than Arnold would have taken alone, and half the time Afzal would have taken working alone.

How long did it take them to do these chores working together?

(a) 20 minutes (b) 30 minutes
(c) 40 minutes (d) 80 minutes

[Based on CAT 2001]

75. Two trains, 130 m and 110 m long, are going in the same direction. The faster train takes one minute to pass the other completely. If they are moving in opposite directions, they pass each other completely in 3 seconds. Find the speed of each train.

(a) 38 m/sec, 36 m/sec
(b) 42 m/sec, 38 m/sec
(c) 36 m/sec, 42 m/sec
(d) None of these

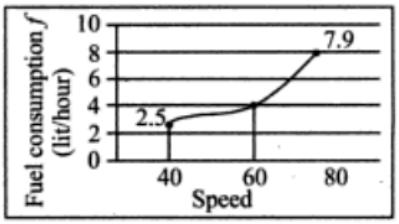
[Based on MAT 2008]

76. Two identical trains A and B running is opposite directions at same speed take 2 minutes to cross each other completely. The number of bogies of A are increased from 12 to 16. How much more time would they now require to cross each other?

(a) 40 s (b) 50 s
(c) 60 s (d) 20 s

[Based on SNAP 2007]

Directions for questions (77 to 78): The petrol consumption rate of a new model car 'Palto' depends on its speed and may be described by the graph below:



s (Km/hr)

77. Manasa makes the 200 km trip from Mumbai to Pune at a steady speed of 60 km per hour. What is the amount of petrol consumed for the journey?

(a) 12.5 litres (b) 13.33 litres
(c) 16 litres (d) 19.75 litres

[Based on CAT 2001]

78. Manasa would like to minimize the fuel consumption for the trip by driving at the appropriate speed. How should she change the speed?

(a) Increase the speed
(b) Decrease the speed
(c) Maintain the speed at 60 km/hour
(d) None of these **[Based on CAT 2001]**

Directions for questions (79 to 80): Answer the questions based on the following in formuation.

There are five machines — *A*, *B*, *C*, *D*, and *E* — situated on a straight line at distances of 10m, 20m, 30m, 40m and 50m respectively from the origin of the line. A robot is stationed at the origin of the line. The robot serves the machines with raw material whenever a machine becomes idle. All the raw materials are located at the origin. The robot is in an idle state at the origin at the beginning of a day. As soon as one or more machines become idle, they send messages to the robot-station and the robot starts and serves all the machines from which it received messages. If a message is received at the station while the robot is away from it, the robot takes notice of the message only when it returns to the station. While moving, it serves the machines in the sequence in which they are encountered, and then returns to the origin. If any messages are pending at the station when it returns, it repeats the process again. Otherwise, it remains idle at the origin till the next message(s) is (are) received.

79. Suppose on a certain day, machines *A* and *D* have sent the first two messages to the origin at the beginning of the first second, *C* has sent a message at the beginning of the 5th second, *B* at the beginning of the 6th second and *E* at the beginning of the 10th second. How much distance has the robot travelled since the beginning of the day, when it notices the message of *E*? Assume that the speed of movement of the robot is 10 m/s.

(a) 140m (b) 80m
(c) 340m (d) 360m

[Based on CAT 2000]

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80. Arun, Barun and Kiranmala start from the same place and travel in the same direction at speed of 30, 40 and 60 km per hour respectively. Barun starts two hour after Arun. If Barun and Kiranamala overtake Arun at the same instant, how many hours after Arun did Kiranamla starts?

(a) 3 (b) 3.5
(c) 4 (d) 4.5
(e) 5

[Based on CAT 2006]

81. Poorva Express destined for Howrah starts from New Delhi at the speed of 50 km/h. After one hour, Howrah Rajdhani starts from the same station for Howrah at the speed of 70 km/h. Taking basic geography into account, Rajdhani Express would catch up with Poorva Express in the following segment

(a) Tundle-Kanpur
(b) Kanpur-Mughalsarai
(c) Mughalsarai-Dhanbad
(d) Dhanbad-Howrah

[Based on FMS (Delhi) 2007]

EXPLANATORY ANSWERS

1. Ans. (b)

2. Ans. (a)

Solution Let Amit's speed = x km/hr

Let Suresh's speed = y km/hr

$$\therefore \quad \frac{30}{x} = \frac{30}{y} + 2 \qquad \text{...(i)}$$

If Amit's speed becomes $2x$ km/hr, then

$$\frac{30}{2x} = \frac{30}{y} - 1 \qquad \text{...(ii)}$$

From (i) and (ii)

$$\Rightarrow \quad \frac{30}{x} - 2 = \frac{30}{2x} + 1 \Rightarrow \mathbf{x = 5,\ y = 7.5.}$$

3. Ans. (a)

4. Ans. (b)

Solution Suppose car 1 starts at 12 noon.

$\therefore$ Car II starts at 6 p.m.

Time	Distance Covered by Car I	Distance Covered by Car II
7 p.m.	210 km	50 km
12.00 night	360 km	300 km
3 a.m.	450 km	450 km

$\therefore$ Car II will crosss Car I after **9 hours.**

5. Ans. (c)

Solution Let the speed of the car be x km/hr.

$\therefore$ Speed of the train = x + 20% of x

$$= \frac{6x}{5} \text{ km/hr}$$

$$\therefore \quad \frac{5}{6x} \times 75 + \frac{25}{2 \times 60} = \frac{75}{x} \Rightarrow \mathbf{x = 60.}$$

6. Ans. (c)

Solution Let the speed of the car be x km/hr

$\therefore$ Speed of the train = x + 20% of x

$$= \frac{6x}{5} \text{ km/hr}$$

$$\therefore \quad \frac{5}{6x} \times 75 + \frac{25}{2 \times 60} = \frac{75}{x} \Rightarrow \mathbf{x = 60.}$$

7. Ans. (a)

Solution A covers 3.5 km before meeting B in $(18 \times 3.5) + 3 = 66$ minutes

B covers a distance of 5.5 km in 66 minutes, i.e. $\frac{66}{60}$ hours, i.e. $\frac{11}{10}$ hours.

$$\therefore \quad B\text{'s speed} = \frac{11}{2} \times \frac{10}{11} = \mathbf{5 \text{ km/hr.}}$$

8. Ans. (c)

Solution Wheel of diameter 7cm starts from X.

Its circumference = $2 \times \pi \times 3.5 = 22$m

Wheel of diameter 14cm starts from Y.

Its circumference = 44m

Distance from X to Y = 1980cm

Let both the wheels make x revolutions per second.

$\therefore$ Distance covered by both the wheels in one second

$$= 22x + 44x = 66x \text{ cm}$$

Distance covered by both the wheels in 10 seconds

$$= 660x \text{ cm}$$

$\therefore$ $660x = 1980 \Rightarrow x = 3$

$\therefore$ Speed of the smaller wheel = $22x$/sec

$$= \mathbf{66 \text{ cm/sec.}}$$

9. Ans. (b)

10. Ans. (a)

Solution For express train, the time needed to travel 75 km

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$= \frac{60}{100} \times 75$

= 45 (plus 3 min. stoppage) = 48 minutes.

The time needed to travel 300 km = 192 min

The time needed to travel 600 km

= 384 min – 3 min = 381 min

For local train, time needed to travel 25 km

= 30 (plus 1 min. stoppage) = 31 minutes

The time needed to travel 300 km = 372 min

In 9 min, the local train will cover $\frac{50}{60} \times 9 = 7.5$ km

∴ In 381 min, the local train will cover **307.5 km.**

11. Ans. (a)

Solution Let the length of each train be x metres.

Relative motion of the faster train w.r.t the slower train is the difference of their speeds i.e. 10 km per hour.

∴ The two trains will be clear of each other in 36 seconds which is the time taken in moving the distance of $2x$ metres @ 10 km/hr.

i.e. 100 m/36 sec

$\Rightarrow \quad 2x = 100 \Rightarrow x = 50$

∴ Length of each train = **50 m.**

12. Ans. (d)

Solution Suppose distance between each pole is 1 metre.

∴ Total distance = 19 metres

It takes 24 seconds to cover 11 metres.

∴ To cover 19 metres, it will take $\frac{24}{11} \times 19$ = **41.45.**

13. Ans. (c)

Solution Suppose they meet after x minutes.

4500 meters are covered by Suresh in 60 mintues.

∴ In x mintues, Suresh covers $\frac{4500}{60} x$ metres.

3750 meters are covered by Suresh's wife in 60 minutes.

∴ In x minute, Suresh's wife covers $\frac{3750}{60} x$ metres

$\Rightarrow \quad \frac{4500}{60} x + \frac{3750}{60} x = 726 \Rightarrow x = 5.28.$

14. Ans. (c)

Solution 400 km in 15 hours, i.e.

Average speed = $\frac{400}{15}$ km/hr = **26.67 km/hr.**

15. Ans. (b)

16. Ans. (c)

Solution The two men walk in opposite directions @ 62.5 metres/minute and 75 metres/minute respectively.

∴ They will meet for the first time after

$\frac{726}{75 + 62.5} = \frac{726}{137.5}$ = **5.28 min.**

17. Ans. (c)

18. Ans. (c)

19. Ans. (d)

Solution 20 km/hr ⇒ 20000 m/3600 seconds

⇒ 200 metres/36 seconds

∴ Length of the faster train = **200 metres.**

20. Ans. (b)

Solution 54 km in 60 minutes

$\Rightarrow$ 45 km in $\frac{60}{54} \times 45 = 50$ minutes

∴ Stoppage time = **10 minutes.**

21. Ans. (d)

22. Ans. (a)

23. Ans. (b)

Solution Let speed of the train be x km/hr

Let speed of the car be y km/hr

$\therefore \quad \frac{120}{x} + \frac{480}{y} = 8$

$\frac{200}{x} + \frac{400}{y} = 8\frac{1}{3} = \frac{25}{3} \Rightarrow \frac{600}{x} + \frac{2400}{y} = 40$

$\frac{600}{x} + \frac{1200}{y} = 25 \Rightarrow \frac{1200}{y} = 15 \Rightarrow y = 80, x = 60.$

∴ Required ratio = 60 : 80 = **3 : 4.**

24. Ans. (c)

Solution Let x km/hr and y km/hr be the speeds of the two cars respectively.

$7x - 70 = 7y$ (If go in same direction)

$y = 70 - x$ (If go in opposite direction)

$\Rightarrow \quad x + y = 70$ and $x - y = 10$

$\Rightarrow \quad x = 40, y = 30.$

25. Ans. (c)

Solution Let the correct time to complete the journey be x minutes.

Distance covered in $(x + 11)$ minutes at 40 km/hr.

= distance covered in $(x + 5)$ minutes at 50 km/hr.

$\therefore \quad \frac{x+11}{60} \times 40 = \frac{x+5}{60} \times 50$

$\Rightarrow \quad x = 19$ **min.**

26. Ans. (c)

Solution Total distance covered = 150 + 300 = 450 m

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Time = $\frac{81}{2}$ sec.

Speed = $450 \times \frac{2}{81} \times \frac{18}{5}$ = **40 km/hr.**

27. Ans. (b)

Solution Let the length of the bridge be x meters.

$\therefore \quad \frac{100+x}{72000} = \frac{25}{3600} \quad \Rightarrow \quad x = 400.$

28. Ans. (a)

Solution The relative speed of the train w.r.t. the speed of the man

= speed of the train + speed of the man

= 60 + 6 = 66 km/hr

$\therefore$ Time taken by the train in crossing the man = Time taken in moving a distance equal to the length of the train (110 m) @ 66 km/hr.

$= \frac{3600}{66000} \times 100$ = **6 seconds.**

29. Ans. (d)

Solution A travel 500 miles in 10 hours

10 minutes, i.e. $10\frac{1}{6}$ hours

$\therefore$ Average speed = $\frac{500}{10\frac{1}{6}} = \frac{500 \times 6}{61}$

$= \frac{3000}{61} = 49.18$ = **50.**

30. Ans. (b)

Solution Because they cover 60000 miles in 60 minutes, i.e. 1000 miles in one minute, therefore they should be 1000 miles apart, 1 minute before impact.

31. Ans. (a)

Solution Let cost of beer = Rs. x/bottle

$\therefore$ Cost of Lager = Rs. $2x$/bottle

$\therefore$ A spent Rs. $4x$ and B spent Rs. $2x$. C paid Rs. 50.

$\therefore$ Cost of Lager = Rs. 50 per bottle

Cost of beer = **Rs. 12.50 per bottle.**

32. Ans. (c)

Solution Let the speed on the smaller wheel be x cm/second.

$\therefore$ Distance covered by the smaller wheel in 10 seconds = $10x$ cm.

Distance covered by the smaller wheel in one revolution

$= 2 \times \frac{22}{7} \times 3.5 = 22$ cm

$\therefore$ No. of revolutions made by the smaller wheel in 10 seconds = $\frac{10x}{22} = \frac{5x}{11}$ ...(1)

Distance covered by the bigger wheel in 10 seconds = $(1980 - 10x)$ cm.

$\therefore$ No. of revolutions made by the bigger wheel in 10 seconds = $\frac{1980-10x}{44}$ ...(2)

$\therefore \quad \frac{5x}{11} = \frac{1980-10x}{44} \Rightarrow x = 66.$

33. Ans. (a)

Solution While running for 600 km, the Express Train stopped 7 times, i.e. for 21 minute (3 minute each time) Running time = 6 hours (@ 100 km/hr for 600 km).

$\therefore$ Total time taken by the express train

= 6 hours 21 minutes

Now local train moves @ 50 km per hour, stops for one minute after every 25 km, i.e. after every half an hour.

$\therefore$ Local train's stopage time = 12 minutes

Running Time = 6 hours 9 minutes

In 6 hours 9 minutes, the local train will cover the distance of **307.5 km.**

34. Ans. (a)

Solution Let the length of each train be x meters.

Then the time in which the two trains are clear of each other is the time taken by the faster train in moving a distance $2x$ meters (sum of the lengths of the two trains) w.r.t. the relative speed of 46 – 36 = 10 km/hr.

$\therefore \quad \frac{36}{60} = \frac{60}{10} \times \frac{2x}{1000} \Rightarrow x = 50.$

35. Ans. (b)

36. Ans. (a)

Solution Let speed of the train be x km/hr and speed of the car be y km/hr.

$\therefore \quad \frac{160}{x} + \frac{600}{y} = 8$

$\frac{240}{x} + \frac{520}{y} = 8\frac{1}{5} = \frac{41}{5} \Rightarrow y = 100, x = 80.$

37. Ans. (a)

Solution Let original duration of the flight be x hours.

$\therefore$ Original Speed = $\frac{3000}{x}$ km/hr

$\therefore \quad \frac{3000}{x+1} = \frac{3000}{x} - 100 \Rightarrow x = 5.$

38. Ans. (b)

Solution Circumference of the wheel

$= 2 \times \frac{22}{7} \times 35 = 220$ cm

$\therefore$ Number of revolutions made by the wheel during the journey

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$$= \frac{2200000 \times 30}{220} = 300000 = \textbf{3 lakh.}$$

39. Ans. (d)

Solution As her father reached home 10 minutes earlier than the usual time, therefore he saw his daughter five mintues before the scheduled time and he turned back (saving 5 minutes each side). So he met her at 6.55 p.m., i.e. she walked for 25 minutes from the station towards her home before her father picked her up.

40. Ans. (c)

41. Ans. (d)

Solution Let the distance between P and Q be x km.

$$\therefore \quad \frac{x}{10} + \frac{x}{15} = \frac{x}{12.5} + \frac{x}{12.5} + \frac{12}{60} \Rightarrow \boldsymbol{x = 30.}$$

42. Ans. (a)

Solution 72 km/hr.

$\Rightarrow$ 72000 m/3600 seconds

$\Rightarrow$ 20 m/second

$\Rightarrow$ **500 m/25 seconds.**

43. Ans. (d)

Solution Let the distance of the house from the school be x km.

$$\therefore \quad \frac{x}{9} - \frac{1}{3} = \frac{x}{12} + \frac{1}{3} \Rightarrow \boldsymbol{x = 24.}$$

44. Ans. (c)

Solution In one week, Giri jogs

$3 + 4 + 5 + 6 + 7 + 8 + 9 = 42$ miles

In two weeks, Giri jogs **84 miles.**

45. Ans. (c)

Solution m miles in $h - \frac{1}{2} = \frac{2h-1}{2}$ hours

$$\Rightarrow \frac{\boldsymbol{2m}}{\boldsymbol{2h-1}} \textbf{ miles per hour.}$$

46. Ans. (b)

Solution C covers the journey in 42 min.

$\Rightarrow$ B covers the journey in 14 min

$\Rightarrow$ $\boldsymbol{A}$ **covers the journey in 7 min.**

47. Ans. (b)

Solution If buses I and II move in the same direction, then they meet after covering 120 km and 60 km respectively @ 20 km/hr and 10 km/hr respectively. If the buses move in opposite directions, then they meet after covering 40 km and 20 km respectively.

48. Ans. (a)

Solution Suppose the distance covered = $4x$ km

Suppose the usual speed = $4x$ km/hr

$\therefore$ Time taken = 1 hr

Reduced speed = $3x$ km/hr

With this speed, the distance of $4x$ km will be covered in 1 hour and **20 minutes.**

49. Ans. (b)

Solution Suppose distance between X and Y be K km

$X \to Y$:

It takes $\frac{\frac{3}{4}K}{B}$ hours to cover $\frac{3}{4}K$ km.

It takes $\frac{\frac{1}{4}K}{S}$ hours to cover $\frac{1}{4}K$ km.

$$\therefore \text{Average speed} = \frac{\frac{3}{4}K + \frac{1}{4}K}{\frac{\frac{3}{4}K}{B} + \frac{\frac{1}{4}K}{S}} = \frac{K(BS)}{\frac{3}{4}KS + \frac{1}{4}KB}$$

$$= \frac{\boldsymbol{4BS}}{\boldsymbol{3S + B}} \textbf{ km/hr.}$$

50. Ans. (a)

51. Ans. (d)

Solution Upto 5.30 p.m., the plane A has covered $\frac{3}{2}X$ miles. Suppose that the plane B overtakes plane A, T hours after 5.30 p.m. Then

$$TY = \frac{3}{2}X + TX \Rightarrow \boldsymbol{T = \frac{3X}{2(Y-X)}}.$$

52. Ans. (b)

Solution Speed of the second train = 80 km/hr

$\therefore$ Speed of the first train = 70 km/hr.

53. Ans. (d)

Solution $40 + 45 + 50 + 55 + 60 + 65 + 70 = \textbf{385.}$

54. Ans. (a)

Solution After **2 hours.**

55. Ans. (c)

56. Ans. (a)

Solution Let speed of the goods train be x km/hr

$\therefore$ $100x = 320 \Rightarrow \boldsymbol{x = 32.}$

57. Ans. (b)

Solution Let the normal speed be a km/hr

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$$\therefore \frac{300}{a} = \frac{300}{a+5} + 2 \Rightarrow 300(a+5) = 300a + 2a(a+5)$$
$$\Rightarrow 2a^2 + 10a - 1500 = 0$$
$$\Rightarrow a^2 + 5a - 750 = 0$$
$$\Rightarrow (a+30)(a-25) = 0$$
$$\Rightarrow \boldsymbol{a = 25.}$$

58. Ans. (b)

Solution Let the normal speed of the aircraft be x km/hr.

$$\therefore \frac{1500}{x} - \frac{1}{2} = \frac{1500}{250+x} \Rightarrow x^2 + 250x - 750000 = 0$$
$$\Rightarrow \boldsymbol{x = 750.}$$

59. Ans. (b)

Solution $\frac{2}{5}$ of the distance = 1200 km

$\therefore$ Whole distance = 3000 km

Distance travelled by car = 1000 km

$\therefore$ Distance travelled by train = **800 km.**

60. Ans. (c)

61. Ans. (b)

Solution At 7 p.m.

62. Ans. (None)

Solution $\pi r^2 = 616 \Rightarrow r^2 = \frac{616 \times 7}{22} = 196$

$$\Rightarrow r = 14$$

$\therefore$ $2\pi r$ = Circumference $= 2 \times \frac{22}{7} \times 14 = 88$ meters

$\therefore$ It will take 14.4 seconds for a runner to run around a circular ground @ **22 km/hr.**

63. Ans. (c)

64. Ans. (d)

Solution If speed of N = 4 units

Speed of S = 1 unit

$\Rightarrow$ Average speed $= \frac{2 \times 4 \times 1}{4+1} = 1.6$ units

Because time available is $\frac{2}{3}$, speed $= \frac{3}{2}$.

Now average speed = 2.4

Speed of N now = 8

Speed of S now = y

$$\frac{2 \times 8 \times y}{8+y} = 2.4 \Rightarrow y = 1.3$$

Required ratio = 1.3 : 8 ≈ **1 : 6.**

65. Ans. (c)

Solution AG_1 = 5 min at 30km/hr = 2.5km

G_1G_3 = 15 km

Time for AG_1 = 5 min

Time for $G_1G_3 + G_3A$ = 32.5 min

= total of 37.5 mins

1 min is taken for transferring the patient into and out of the ambulance.

Hence (40 – 37.5 – 1 =) **1.5 min is remaining.**

66. Ans. (b)

Solution Because each word is lit for a second

$$\text{LCM}\left(\frac{5}{2}+1, \frac{17}{4}+1, \frac{41}{8}+1\right) = \text{LCM}\left(\frac{7}{2}, \frac{21}{4}, \frac{49}{8}\right)$$
$$= \frac{\text{LCM}(7, 21, 49)}{\text{HCF}(2, 4, 8)}$$
$$= \frac{49 \times 3}{2} = \textbf{73.5 sec.}$$

67. Ans. (b)

Solution Let tunnel = 8km and speed of cat = 8km/hr

Time taken to reach entrance of tunnel by cat = 3 hr

Time taken to reach exit of tunnel by cat = 5 hr.

Train will cover sum (length of tunnel) = 2 hr

Therefore, ratio of speeds of train and cat = **4 : 1.**

68. Ans. (d)

Solution Total amount of work = 60 man hrs

From 11 am to 5 pm, 6 technicians = 36 man hrs

From 5 pm to 6 pm, 7 technicians = 7 man hrs

From 6 pm to 7 pm, 8 technicians = 8 man hrs

From 7 pm to 8 pm, 9 technicians = 9 man hrs

Total = **60 man hrs.**

69. Ans. (d)

70. Ans. (d)

Solution Work done in one day by A, B, C and D are $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{32}$ respectively. Using answer choices, we note that the pair of B and C does $\frac{3}{16}$ of work in one day; the pair of A and D does $\frac{1}{4} + \frac{1}{32} = \frac{9}{32}$ of the work in one day.

Hence, A and D take $\frac{32}{9}$ days.

B and C take $\frac{16}{3} = \frac{32}{6}$ days. Hence, the first pair must comprise of A and D.

71. Ans. (a)

Solution Amount of money given to X

= 12 × 300 + 12 × 330 + ... + 12 × 570

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$= 12\,[300 + 330 + \ldots + 540 + 570]$

$= 12 \times \frac{10}{2}\,[600 + 9 \times 30] = 52{,}200$

Amount of money given to Y is

$6 \times 200 + 6 \times 215 + 6 \times 230 + 6 \times 245$

$+ \ldots$ to 20 terms

$= 6 \times \frac{20}{2}\,[400 + 19 \times 15] = 6\,[400 + 285]$

$= 60 \times 685 = 41{,}100$

$\therefore$ Total amount paid = 52,200 + 41,100 = **93,300.**

72. Ans. (d)

Solution Let x be rowing rate of Rahul, and y, be the rate of flow of current in mph.

$$\frac{12}{x-y} - \frac{12}{x+y} = 6 \Rightarrow \frac{y}{x^2 - y^2} = \frac{1}{4}$$

$$\Rightarrow y = \frac{x^2 - y^2}{4} \quad \ldots(i)$$

When Rahul dobles his rowing rate, then we have

$$\frac{12}{2x-y} - \frac{12}{2x+y} = 1 \Rightarrow \frac{2y}{4x^2 - y^2} = \frac{1}{12}$$

$$\Rightarrow y = \frac{4x^2 - y^2}{4} \quad \ldots(ii)$$

Hence, from (i) and (ii), we have $2x^2 = 5y^2$

Putting $x^2 = \frac{5}{2}y^2$ in (i), we get

$$y = \frac{\frac{3}{2}y^2}{4} \Rightarrow y = \frac{8}{3}.$$

73. Ans. (b)

Solution If Shyam takes 1 min for every 3 steps, then he takes $\frac{1}{3}$ min for every step.

For 25 steps, he takes $\frac{25}{3}$ min, i.e., 8.33 min.

So, Vyom takes $\frac{1}{2}$ min for every step.

For 20 steps, he takes $\frac{20}{2}$ min, i.e. 10 min.

Difference between their times = 1.66 min.

Escalator takes 5 steps in 1.66 min.

Speed of escalator is 1 step for 0.33 min. i.e. 3 steps per minute.

If escalator is moving, then Shyam takes 25 steps and escalator also takes 25 steps.

Hence, total number of steps = **50.**

74. Ans. (c)

Solution Let the time taken by Asit, Arnold and Afzal to do the work alone be x, y and z hours respectively.

$\therefore$ Time taken to do the work together is

$$x - 6 = y - 1 = \frac{z}{2}$$

In one hour, they can do $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ of the work.

$\therefore$ Total work can be completed by them in

$\frac{xyz}{xy + yz + zx}$ hours, i.e. $\frac{xyz}{xy + yz + zx} = x - 6$

Now, put $y = x - 5$ and $z = 2(x - 6)$ in the above equation and solve for x. We get $x = \frac{20}{3}$ hours.

Hence, time taken by them to complete the work when working together = $\frac{20}{3} - 6 = \frac{2}{3}$ hour = **40 minutes.**

75. Ans. (a)

Solution Suppose the speeds of the trains are x m/sec and y m/sec respectively.

Then $\frac{110+130}{x-y} = 60 \quad \ldots(1)$

$\frac{110+130}{x+y} = 3 \quad \ldots(2)$

By equations (1) and (2) we get

$x = 42$ m/s

and $y = 38$ m/s

76. Ans. (b)

Solution Total bogies initially is 12 + 12 = 24

Additional bogies = 16 − 12 = 4

24 bogies takes = 2 minutes

1 bogy takes = $\frac{2}{24}$

4 bogies takes $= \frac{2}{21} \times 4 = \frac{1}{3}$ minutes

$= \frac{1}{3} \times 60$ sec = **20 sec.**

77. Ans. (b)

Solution 60km/hr is travelled in $4L$ petrol (from the graph)

$\therefore$ 1 L is required for 15 km.

i.e., for 15 km, 1 L petrol is required.

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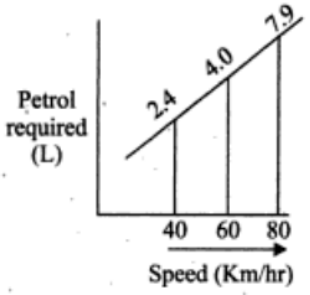

For 200 km, $\frac{200}{5}$ = 13.33 L is required.

78. Ans. (b)

Solution $\frac{40}{2.4}$ = 16.66 km; $\frac{60}{4}$ = 15 km

$\frac{80}{7.9}$ = 10 km aprroximately

∴ Answer is 16.66 km, decrease the speed.

79. Ans. (a)

Solution The robot begins to give material to machine *A* and then to *D*, it thus covers 40 m in that time span and takes 4s. Also then it returns to the origin, and takes 4s, while covering 40 m again. When it arrives at the origin, the messages of *B* and *C* are already there, thus it moves to give the material to them, which takes it in total 6s, and covers 30 + 30 = 60 m in total. Hence the distance travelled by the robot will be 40m + 40m + 60 m = **140m.**

80. Ans. (a)

81. Ans. (a)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Walking with $\frac{6}{5}$ of my usual speed, I reach my office 10 minutes too early. Find my usual time?

(a) 50 min (b) 60 min

(c) 45 min (d) 40 min

2. Two trains start from a place moving with 45 km/hr and 75 km/hr, respectively in the opposite direction. After how many hours they will be 1280 km away.

(a) $12\frac{2}{3}$ (b) $10\frac{2}{3}$

(c) $14\frac{1}{3}$ (d) $10\frac{1}{2}$

3. Two scooterists do the same journey by driving at the rates of 50 km/hr and 45 km/hr, respectively. Find the length of the journey when one takes 32 min longer than the other.

(a) 168 km (b) 240 km

(c) 210 km (d) 480 km

4. A man walking at 3 km/hr crosses a square field diagonally in 2 min. The area of the field is

(a) 2.5 km (b) 3 km

(c) 5 km (d) 6 km

5. A car travels a distance of 840 km at a uniform speed. If the speed of the car is 10 km/hr more, it takes two hours less to cover the same distance. The original speed of the car was

(a) 45 km/hr (b) 50 km/hr

(c) 60 km/hr (d) 75 km/hr

6. A certain passenger train travels at the rate of 42 km/hr and a goods train, half as long as the travelling train, travels at 33 km/hr. If, while travelling in the same direction, it takes 50 seconds for the passenger train to clear the goods train, while travelling in opposite direction they will cross each other completely in

(a) 6 sec. (b) 8 sec.

(c) 9 sec. (d) 12 sec.

7. Two persons are walking in the same direction at rates 3 km/hr and 6 km/hr. A train comes running from behind and passes them in 9 and 10 seconds. The speed of the train is

(a) 33 km/hr (b) 40 km/hr

(c) 22 km/hr (d) 35 km/hr

8. The poles on the rail road are 50 metres apart, how many will be passed by a train in one and a half hour if the speed of the train be 72 km/hr?

(a) 2160 (b) 2060

(c) 2100 (d) 2050

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9. A train 120 m long takes 6 seconds to pass a man, who is running in the opposite direction at the speed of 12 km/hr. Find the speed of the train.
(a) 60 km/hr (b) 65 km/hr
(c) 55 km/hr (d) 50 km/hr

10. Two trains 121 metres and 99 metres in length respectively, are running in opposite direction, one at the rate of 40 km/hr and the other at the rate of 32 km/hr. From the moment trains meet, they will be completely clear of each other in
(a) 20 sec. (b) 16 sec.
(c) 11 sec. (d) 9 sec.

Test II

[Time: 10 min]

11. If a train runs at 40 km/hr, it reaches its destination late by 11 min. but if it runs at 50 km/hr, it is late by 5 min. only. The correct time for the train to cover its journey is
(a) 13 min (b) 15 min
(c) 21 min (d) 19 min

12. A train travelling at 54 km/hr completely crosses another train of half its length and travelling in opposite direction at 72 km/hr in 12 seconds. If it also passes a railway platform in one minute, the length of the platform is
(a) 620 metres (b) 840 metres
(c) 920 metres (d) 750 metres

13. Two trains start at the same time from Aligarh and Delhi and proceed towards each other at 16 km/hr and 21 km/hr, respectively. When they meet, it is found that one train has travelled 60 km more than the other. The distance between the two stations is
(a) 445 km (b) 444 km
(c) 440 km (d) 450 km

14. A man travels 600 km in 15 hours partly by rail and partly by car. If he had gone all the way by car he would have arrived at his destination 3 hours earlier, find the speed of the car.
(a) $33\frac{1}{3}$ km/hr (b) 40 km/hr
(c) 50 km/hr (d) 45 km/hr

15. Two trains 108 m and 112 m in length are running towards each other on parallel lines at a speed of 45 km/hr and 54 km/hr, respectively. How long will it take then to cross each other after they meet?
(a) 5 seconds (b) 7 seconds
(c) 8 seconds (d) 6 seconds

16. Two buses start from a certain town at 9.00 a.m. and move in opposite direction. One goes towards east at an average speed of 44 km/hr and the other goes towards west at an average speed of 40 km/hr. After what time will they be 210 km apart from each other?
(a) 2 hours (b) $2\frac{1}{4}$ hours
(c) $2\frac{1}{2}$ hours (d) 3 hours

17. Two trains A and B running on parallel lines enter a tunnel 2 km long at the opposite ends at the same time. They meet in $1\frac{1}{2}$ minutes and A has then travelled in the tunnel 400 metres more than B. In $4\frac{1}{2}$ seconds they cross each other and 1 minute later A has completely passed through the tunnel. The length (in metres) of B is
(a) 40 (b) 60
(c) 50 (d) 55

18. Sound travels at 330 metres a second. How many kilometres is a thunder-cloud distance when the sound follows the flash after 10 seconds?
(a) 3.3 m (b) 3.8 m
(c) 4.2 m (d) None of these

19. Two places P and Q are 162 km apart. A train leaves P for Q and at the same time another train leaves Q for P. The two trains meet at the end of 6 hours. If the train travelling from P to Q travels 8 km an hour faster than the other, find the speed of the two trains.
(a) 16 km/hr (b) 17 km/hr
(c) $17\frac{1}{2}$ km/hr (d) 18 km/hr

20. A man takes 8 hours to walk to a certain place and ride back. However, he could have gained 2 hours, if he had covered both ways riding. How long would he have taken to walk both ways?
(a) 10 hrs (b) 8 hrs
(c) 6 hrs (d) None of these

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution $\frac{6}{5}$ of usual speed $\Rightarrow \frac{5}{6}$ of usual time

$\therefore \frac{5}{6}t = t - 10$. Thus, $t =$ **60 min.**

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3. Ans. (b)

Solution Let x km be the length of the journey so that

$$\frac{x}{45}-\frac{x}{50}=\frac{32}{60} \quad \textbf{or,} \quad \boldsymbol{x = 240\ km.}$$

5. Ans. (c)

Solution Let the original speed be x km/hr. Then,

$$\frac{840}{x}-\frac{840}{x+10}=2$$

$\Rightarrow$ $840(x + 10) - 840x = 2x(x + 10)$

$\therefore$ $x^2 + 10x - 4200 = 0$ or, $(x + 70)(x - 60) = 0$

$\therefore$ $\boldsymbol{x = 60\ km/hr.}$

7. Ans. (a)

Solution Let speed of the train be x km/hr and its length be y metres

Then, $y = (x-3)\times\frac{5}{18}\times 9$ and, $y = (x-6)\times\frac{5}{18}\times 10$

Subtracting,

$0 = 9(x - 3) - 10(x - 6)$

$\therefore$ $x = 60 - 27 = 33$

$\therefore$ Speed of train $=$ **33 km/hr.**

9. Ans. (a)

Solution Length of the train $=\frac{120}{1000}=\frac{3}{25}$km

Let the speed of the train be x km/hr. As both train and the man are running in the opposite directions, the relative speed of the train with respect to the man $= (x + 12)$ km/hr.

Time taken by the train to pass the man

$$=\frac{\text{Distance covered}}{\text{Relative speed}}=\frac{3}{25(x+12)}\ \text{hours}$$

or, $\frac{3\times60\times60}{25(x+12)}=6$ or, $3 \times 12 \times 12 = 6(x + 12)$

or, $x+12=\frac{3\times12\times12}{6}=72$ $\quad\therefore\ \boldsymbol{x = 60\ km/hr.}$

11. Ans. (d)

Solution Let the required time $= x$ min.

Then, distance covered in $(x + 11)$ min, at 40 km/hr = distance covered in $(x + 5)$ min at 50 km/hr

$$\therefore\ 40\times\frac{x+11}{60}=50\times\frac{x+5}{60}\Rightarrow \boldsymbol{x = 19.}$$

13. Ans. (b)

Solution Suppose, they meet after x hours.

Then, $21x - 16x = 60$ or, $x = 12$

$\therefore$ Required distance $= (16 \times 12 + 21 \times 12)$ km

$=$ **444 km.**

15. Ans. (c)

Solution As the trains are running in the opposite direction their relative speed $= (45 + 54)$ km/hr

$= 99$ km/hr

Relative speed in metre per second

$$=\frac{99\times1000}{60\times60}=\frac{55}{2}\text{m/sec}$$

Sum of the lengths of both the trains

$= 108 + 112 = 220$ m.

Time taken by the trains to cross each other

$$=\frac{\text{Distance covered}}{\text{Relative speed of the trains}}$$

$$=220\div\frac{55}{2}=\frac{220\times2}{55}=\textbf{8 seconds.}$$

17. Ans. (a)

Solution Let lengths (in metres) and speeds (in km/hr.) of trains A and B respectively be x, y and U, V. Obviously, trains are moving in opposite directions.

Trains enter the tunnel at the same time and meet in $1\frac{1}{2}$ minutes, that is, 90 sec.

$\therefore$ Sum of distances covered by trains in 90 sec. = length of the tunnel

$\therefore$ $\frac{5}{18}U\times90+\frac{5}{18}V\times90=2000$

$\therefore$ $U + V = 80$ ...(i)

Also, A has moved 400 m more than B

$\therefore$ $\frac{5}{18}U\times90-\frac{5}{18}V\times90=400$

$\therefore$ $U - V = 16$ ...(ii)

Adding and subtracting (i) and (ii)

$U = 48$

$V = 32$

Also, after meeting trains cross each other in $4\frac{1}{2}$ sec.

$\therefore$ $x+y=(U+V)\times\frac{5}{18}\times\frac{9}{2}=80\times\frac{5}{4}$

$= 100$ ...(iii)

A completely passes the tunnel in

$$\left(90+4\frac{1}{2}+60\right)\text{ sec.}=154\frac{1}{2}\text{ sec.}$$

$\therefore$ $x + 2000 =$ distance covered by A in $154\frac{1}{2}$ sec.

$$=48\times\frac{5}{18}\times\frac{309}{2}=2060$$

$\therefore$ $x = 60$

(iii) $\Rightarrow$ $y = 40$

$\therefore$ Lengths of A and B are 60 m and 40 m.

19. Ans. (c)

Solution The relative speed of the two trains

$= 162 + 6$ km/hr $= 27$ km/hr

The speed of the slower train started from Q

$$=\frac{27-8}{2}=\frac{19}{2}=9\frac{1}{2}\text{km/hr}$$

The speed of the faster train $=9\frac{1}{2}+8=\boldsymbol{17\frac{1}{2}}$**km/hr.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (c)	**6.** (a)	**8.** (a)	**10.** (c)	**12.** (a)	**14.** (c)	**16.** (c)	**18.** (a)	**20.** (a)

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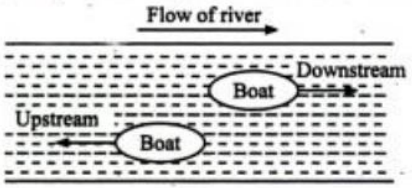
Boats and Streams 13

Some Important Terms

1. **Still Water** If the speed of the water in the river is zero, it is *still water*.
2. **Stream** If the water of the river is moving, it is called a *stream*.
3. **Upstream** If a boat (or a swimmer) moves against the stream, i.e. in the direction opposite to that of the stream, it is called *upstream*.
4. **Downstream** If a boat (or a swimmer) moves with the stream, i.e. along the direction of the stream, it is called *downstream*.

Note: When the speed of a boat or a swimmer is given, it usually means speed in the still water.



SOME BASIC FORMULAE

1. If the speed of a boat (or a swimmer) be x km/hr and the speed of the stream or the current be y km/hr, then
   (a) speed of the boat (or swimmer) downstream $= (x + y)$ km/hr.
   (b) speed of the boat (or swimmer) upstream $= (x - y)$ km/hr.

Illustration 1 The speed of a boat in still water is 20 km/hr. If the speed of the stream be 4 km/hr, find its downstream and upstream speeds.

Solution Speed of the boat $(x) = 20$ km/hr

Speed of the stream $(y) = 4$ km/hr

$\therefore$ Downstream speed $= x + y = (20 + 4) = 24$ km/hr

and, upstream speed $= x - y = (20 - 4) =$ **16 km/hr.**

2. (a) Speed of the boat (or swimmer) in still water

$$= \frac{1}{2}(\text{Downstream Speed} + \text{Upstream Speed})$$

(b) Speed of the stream

$$= \frac{1}{2}(\text{Downstream Speed} - \text{Upstream Speed})$$

Illustration 2 A boat is rowed down a river 40 km in 5 hr and up a river 21 km in 7 hr. Find the speed of the boat and the river.

Solution Speed of the boat downstream $= \frac{40}{5} = 8$ km/hr.

Speed of the boat upstream $= \frac{21}{7} = 3$ km/hr.

$\therefore$ Speed of the boat

$$= \frac{1}{2}(\text{Downstream Speed} + \text{Upstream Speed})$$

$$= \frac{1}{2}(8 + 3) = \frac{11}{2} = \text{or } 5 \quad 5 \text{ km/hr.}$$

and, speed of the river

$$= \frac{1}{2}(\text{Downstream Speed} - \text{Upstream Speed})$$

$$= \frac{1}{2}(8 - 3) = \frac{5}{2} \textbf{ or 2.5 km/hr.}$$

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SOME USEFUL SHORT-CUT METHODS

1. If a man capable of rowing at the speed of x km/hr in still water, rows the same distance up and down a stream which flows at a rate of y km/hr, then his average speed throughout the journey is

$$= \frac{\text{Uptream} \times \text{Downstream}}{\text{Man's rate in still water}}$$

$$= \frac{(x-y)(x+y)}{x}\text{km/hr.}$$

Illustration 3 A man rows at a speed of 8 km/hr in still water to a certain distance upstream and back to the starting point in a river which flows at 4 km/hr. Find his average speed for total journey.

Solution Average Speed

$$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}}$$

$$= \frac{(8-4)(8+4)}{8} = \mathbf{6\ km/hr.}$$

2. A man can row a boat in still water at x km/hr. In a stream flowing at y km/hr, if it takes t hours more in upstream than to go downstream for the same distance, then the distance is given by

$$\frac{(x^2-y^2)t}{2y}\text{km}$$

Illustration 4 A man can row 7 km/hr in still water. If the river is running at 3 km/hr, it takes 6 hours more in upstream than to go downstream for the same distance. How far is the place?

Solution The required distance

$$= \frac{(x^2-y^2)t}{2y}$$

$$= \frac{(49-9)6}{2\times 3} = \mathbf{40\ km.}$$

3. A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream be y km/hr, then the speed of the man in still water is given by

$$y\left(\frac{t_2+t_1}{t_2-t_1}\right)\text{km/hr.}$$

Explanation

Let the speed of the man in still water be x km/hr.
Then, downstream speed = $(x + y)$ km/hr
and upstream speed = $(x - y)$ km/hr.

Since the distance covered downstream and upstream are equal, we have

$$(x+y)t_1 = (x-y)t_2$$

or, $$xt_1 + yt_1 = xt_2 - yt_2$$

or, $$x(t_2 - t_1) = y(t_2 + t_1)$$

$$\therefore \quad x = y\left(\frac{t_2+t_1}{t_2-t_1}\right)\text{km/hr.}$$

Illustration 5 A motorboat covers a certain distance downstream in 6 hours but takes 8 hours to return upstream to the starting point. If the speed of the stream be 6 km/hr, find the speed of the motor boat in still water.

Solution Speed of the motorboat in still water

$$= y\left(\frac{t_2+t_1}{t_2-t_1}\right)\text{km/hr}$$

$$= 6\left(\frac{8+6}{8-6}\right) = \mathbf{42\ km/hr.}$$

4. A man can row a boat in still water at x km/hr. In a stream flowing at y km/hr if it takes him t hours to row to a place and come back, then the distance between the two places is

$$\frac{t(x^2-y^2)}{2x}\text{km.}$$

Explanation

Downstream speed = $(x + y)$km/hr

Upstream speed = $(x - y)$km/hr.

Let the distance between the two places be d km. We have,

Total time = Sum of time taken downstream and upstream

$$\Rightarrow \quad t = \frac{d}{x+y} + \frac{d}{x-y}$$

$$= d\left[\frac{(x-y)+(x+y)}{(x-y)(x+y)}\right]$$

$$= d\left[\frac{2x}{x^2-y^2}\right],$$

$$\therefore \quad d = \frac{t(x^2-y^2)}{2x}\text{km.}$$

Illustration 6 A man can row 6 km/hr in the still water. If the river is running at 2 km/hr, it takes him 3 hours to row to a place and back. How far is the place?

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Solution The required distance

$$= \frac{t(x^2 - y^2)}{2x} \text{km}$$

$$= \frac{3(36-4)}{2\times 6} = \mathbf{8\ km.}$$

5. A boat (or a swimmer) takes n times as long to row upstream as to row downstream the river. If the speed of boat (or swimmer) be x km/hr and the speed of stream be y km/hr, then

$$x = y\left(\frac{n+1}{n-1}\right).$$

Illustration 7 A man can row at the rate of 4 km/hr. in still water. If the time taken to row a certain distance upstream is 3 times as much as to row the same distance downstream, find the speed of the current.

Solution We have,

$$\text{Speed of the man} = \left(\frac{n+1}{n-1}\right) \text{speed of the current}$$

$$\Rightarrow \quad 4 = \left(\frac{3+1}{3-1}\right) \text{speed of the current.}$$

$$\therefore \quad \text{Speed of the current} = \mathbf{2\ km/hr.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. A boat goes 13 km upstream in 39 minutes. The speed of stream is 3 km/hr. The speed of boat in still water is:
(a) 23 km/hr (b) 27 km/hr
(c) 25 km/hr (d) None of these

2. The speed of a boat in still water is 8 km/hr. If it speed downstream be 15 km/hr, then speed of th stream is:
(a) 7.5 km/hr (b) 7 km/hr
(c) 9 km/hr (d) None of these

3. Speed of a man is 10 km/hr in still water. If the rate of current is 3 km/hr, then the effective speed of the man upstream is:
(a) 7 km/hr (b) 8.5 km/hr
(c) 9 km/hr (d) None of these

4. A man can row with the stream at 7 km/hr and against the stream at 3 km/hr. His speed in still water is:
(a) 6.5 km/hr (b) 7 km/hr
(c) 5 km/hr (d) None of these

5. A swimmer covers a distance of 28 km against the current and 40 km in the direction of the current. If in each case he takes 4 hours, then the speed of the current is
(a) 3.5 km/hr (b) 1.5 km/hr
(c) 2.5 km/hr (d) None of these

6. A boat moves downstream at the rate of one km in 10 minutes and upstream at the rate of 4 km an hour. What is the velocity of the current
(a) 5 km/hr (b) 3 km/hr
(c) 1 km/hr (d) None of these

7. If a man's rate with the current is 12 km/hr and the rate of the current is $1\frac{1}{2}$ km/hr, then his rate against the current is
(a) 13 km/hr (b) 7 km/hr
(c) 9 km/hr (d) None of these

8. A boatman can row 2 km against the stream in 20 minutes and return in 18 minutes. Find the rate of current.
(a) 1/3 km/hr (b) 2/3 km/hr
(c) 5/3 km/hr (d) None of these

9. A boatman can row 48 km downstream in 4 hr. If the speed of the current is 5 km/hr, then find in what time will he be able to cover 8 km upstream?
(a) 6 hr (b) 4 hr
(c) 8 hr (d) None of these

10. A man can row at a speed of 10 km/hr in still water to a certain upstream point and back to the starting point in a river which flows at 4 km/hr. Find his average speed for total journey.
(a) $9\frac{2}{5}$ km/hr (b) $8\frac{2}{5}$ km/hr
(c) $11\frac{2}{5}$ km/hr (d) None of these

11. A man can row 6 km/hr in still water. If the river is running at 2 km/hr, it takes 3 hours more in upstream than to go downstream for the same distance. How far is the place?
(a) 24 km (b) 28 km
(c) 36 km (d) None of these

12. A boat covers a certain distance downstream in 2 hours but takes 4 hours to return upstream to the starting point. If the speed of the stream be 3 km/hr, find the speed of the boat in still water.
(a) 11 km/hr (b) 13 km/hr
(c) 9 km/hr (d) None of these

13. In a river flowing at 2 km/hr, a boat travels 32 km

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upstream and then returns downstream to the starting point. If its speed in still water be 6 km/hr, find the total journey time.
(a) 16 hours (b) 12 hours
(c) 14 hours (d) None of these

14. A boat travels upstream from B to A and downstream from A to B in 3 hrs. If the speed of the boat in still water is 9 km/hr and the speed of the current is 3 km/hr, the distance between A and B is:
(a) 8 km (b) 16 km
(c) 12 km (d) None of these

15. A boat travels 2 km upstream in a stream flowing at 3 km/hr and then returns downstream to the starting point in 30 minutes. The speed of the boat in still water is:
(a) 17 km/hr (b) 9 km/hr
(c) 13 km/hr (d) None of these

16. A man swimming in a stream which flows $1\frac{1}{2}$ km/hr finds that in a given time he can swim twice as far with the stream as he can against it. At what rate does he swim?
(a) $4\frac{1}{2}$ km/hr (b) $5\frac{1}{2}$ km/hr
(c) $7\frac{1}{2}$ km/hr (d) None of these

17. A boat goes 40 km upstream in 8 hours and 36 km downstream in 6 hours. The speed of the boat in still water in km/hr is
(a) 5.5 (b) 6.6
(c) 4.5 (d) None of these

18. A man rows upstream 12 km and downstream 28 km taking 5 hours each time. The velocity of water current is
(a) $2\frac{1}{5}$ km/hr (b) $2\frac{1}{2}$ km/hr
(c) 3 km/hr (d) $1\frac{3}{5}$ km/hr

19. Twice the speed downstream is equal to the thrice the speed upstream, the ratio of speed in still water to the speed of the current is:
(a) 1 : 5 (b) 5 : 1
(c) 1 : 3 (d) 2 : 3

20. A man can swim 3 km/hr in still water. If the velocity of the stream be 2 km/hr, the time taken by him to swim to a place 10 km upstream and back, is:
(a) $8\frac{1}{3}$ hr (b) $9\frac{1}{5}$ hr
(c) 10 hr (d) 12 hr

21. A boat covers 24 km upstream and 36 km downstream in 6 hr, while it covers 36 km upstream and 24 km downstream in $6\frac{1}{2}$ hrs. The velocity of the current is:
(a) 1.5 km/hr (b) 1 km/hr
(c) 2 km/hr (d) 2.5 km/hr

22. A boatman goes 2 km against the current of the stream in 1 hr and goes 1 km along the current in 10 min. How long will he take to go 5 km in stationary water?
(a) 1 hr (b) 1 hr 15 min
(c) $1\frac{1}{2}$ hr (d) 40 min

23. P, Q, R are three towns on a river which flows uniformly. Q is equidistant from P and R. A man rows from P to Q and back in 10 hr. He can row from P to R in 4 hr. The ratio of speed of man in still water to the speed of the current is:
(a) 5 : 3 (b) 3 : 5
(c) 2 : 5 (d) 1 : 2

24. In a stream running at 2 km/hr, a motor boat goes 10 km upstream and back again to the starting point in 55 min. Find the speed of the motorboat in still water.
(a) 20 km/hr (b) 21 km/hr
(c) 22 km/hr (d) 24 km/hr

25. A man can row 30 km upstream and 44 km downstream in 10 hr. Also, he can row 40 km upstream and 55 km downstream in 13 hours. Find the rate of the current and the speed of the man in still water
(a) 3 km/hr, 8 km/hr (b) 3·5 km/hr, 7·5 km/hr
(c) 4 km/hr, 7 km/hr (d) 4·5 km/hr, 6·5 km/hr

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Speed of the boat upstream
$$= \frac{13 \times 60}{39} = 20 \text{ km/hr}$$
Speed of the stream = 3 km/hr
Let the speed of the boat in still water = x km/hr.
We have, $x - 3 = 20$
$\therefore \quad x = 20 + 3 =$ **23 km/hr.**

2. Ans. (b)

Solution Speed of the boat downstream = 15 km/hr.
Speed of the boat in still water = 8 km/hr.
Let the speed of the stream = y km/hr.

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We have, $15 = 8 + y$

$\therefore \quad y = 15 - 8 = \mathbf{7\ km/hr.}$

3. Ans. (a)

Solution Speed of man in still water = 10 km/hr

Speed of current = 3 km/hr

$\therefore$ Speed of man upstream = $10 - 3 = \mathbf{7\ km/hr.}$

4. Ans. (c)

Solution Speed of the man upstream = 7 km/hr.

Speed of the man downstream = 3 km/hr.

$\therefore$ Speed of the man in still water

$= \frac{1}{2}$ (Downstream Speed + Upstream Speed)

$= \frac{1}{2}(7 + 3) = \mathbf{5\ km/hr.}$

5. Ans. (b)

Solution Speed of the swimmer upstream

$= \frac{28}{4} = 7$ km/hr.

Speed of the swimmer downstream

$= \frac{40}{4} = 10$ km/hr.

$\therefore$ Speed of the stream

$= \frac{1}{2}$ (Downstream Speed – Upstream Speed)

$= \frac{1}{2}(10 - 7) = \frac{3}{2} = \mathbf{1.5\ km/hr.}$

6. Ans. (c)

Solution Speed of the boat downstream

$= \frac{60}{10} = 6$ km/hr.

Speed of the boat upstream = 4 km/hr

$\therefore$ Velocity of the current

$= \frac{1}{2}$(Downstream speed – Upstream Speed)

$= \frac{1}{2}(6 - 4) = \mathbf{1\ km/hr.}$

7. Ans. (c)

Solution Speed of the man downstream = 12 km/hr.

Speed of the stream $= \frac{3}{2}$ km/hr.

Let the speed of the man upstream = x km/hr.

We have,

Speed of the stream

$= \frac{1}{2}$(Downstream Speed – Upstream Speed)

$\Rightarrow \quad \frac{3}{2} = \frac{1}{2}(12 - x)$.

$\therefore \quad x = 12 - 3 = \mathbf{9\ km/hr.}$

8. Ans. (a)

Solution Speed of the boatman upstream

$= \frac{2}{20} \times 60 = 6$ km/hr.

Speed of the boatman downstream

$= \frac{2}{18} \times 60 = \frac{20}{3}$ km/hr.

$\therefore$ Rate of current

$= \frac{1}{2}$(Downstream Speed – Upstream Speed)

$= \frac{1}{2}\left(\frac{20}{3} - 6\right) = \mathbf{\frac{1}{3}\ km/hr.}$

9. Ans. (b)

Solution Speed of the boatman downstream

$= \frac{48}{4} = 12$ km/hr.

Speed of the current = 5 km/hr.

Let the boatman takes t hours to cover 8 km upstream.

Then, speed of the current

$= \frac{1}{2}$(Downstream Speed – Upstream Speed)

$\Rightarrow \quad 5 = \frac{1}{2}\left(12 - \frac{8}{t}\right)$

$\therefore \quad \mathbf{t = 4\ hours.}$

10. Ans. (b)

Solution Average Speed

$= \frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}}$

$= \frac{(10-4)(10+4)}{10} = 8\frac{2}{5}$ km/hr.

11. Ans. (a)

Solution The required distance

$= \frac{(x^2 - y^2)t}{2y} = \frac{(36-4)3}{2 \times 2} = \mathbf{24\ km.}$

12. Ans. (c)

Solution Speed of the boat in still water

$= y\left(\frac{t_2 + t_1}{t_2 - t_1}\right)$ km/hr

$= 3\left(\frac{4+2}{4-2}\right) = \mathbf{9\ km/hr.}$

13. Ans. (b)

Solution Let the total journey time be t hr.

Then, we have $d = \frac{t(x^2 - y^2)}{2x}$

$\Rightarrow \quad 32 = \frac{t(36-4)}{2 \times 6}$

$\therefore \quad \mathbf{t = 12\ hr.}$

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14. Ans. (c)

Solution The distance between A and B is

$$= \frac{t(x^2-y^2)}{2x}\text{km}$$

$$= \frac{3(81-9)}{2\times 9} = \textbf{12 km.}$$

15. Ans. (b)

Solution Let the speed of the boat be x km/hr.

We have,

$$d = \frac{t(x^2-y^2)}{2x}$$

$$\Rightarrow \quad 2 = \frac{1/2(x^2-9)}{2x}, \text{ i.e. } 2 = \frac{x^2-9}{4x}$$

or, $x^2 - 8x - 9 = 0$

or, $(x - 9)(x + 1) = 0$

or, $x = -1$ or 9.

Since the speed cannot be negative, we neglect -1.

Therefore, speed of the boat in still water = **9 km/hr.**

16. Ans. (a)

Solution Speed of man

$$= \left(\frac{n+1}{n-1}\right) \text{ speed of stream}$$

$$= \left(\frac{2+1}{2-1}\right)\times\frac{3}{2} = \frac{9}{2} \textbf{ or } 4\frac{1}{2}\textbf{ km/hr.}$$

17. Ans. (a)

Solution Boat's upstream $= \frac{40}{8} = 5$ km/hr

Boat's downstream speed $= \frac{36}{6} = 6$ km/hr.

$\therefore$ Speed of boat in still water $= \frac{5+6}{2} = \textbf{5.5 km/hr.}$

18. Ans. (d)

Solution Let man's rowing speed in still water = x km/hr and speed of current = y km/hr

Speed upstream = $(x - y)$ km/hr

and, speed downstream = $(x + y)$ km/hr

$\therefore \quad 5(x - y) = 12$ and $5(x + y) = 28$

subtracting $\quad 10y = 16$

$\therefore \quad y = \frac{8}{5} = 1\frac{3}{5}$ **km/hr.**

19. Ans. (b)

Solution Let speed in still water = x km/hr

Speed of current = y km/hr

Speed downstream = $(x + y)$ km/hr

Speed upstream = $(x - y)$ km/hr

$\therefore \quad 2(x + y) = 3(x - y)$

$\therefore \quad x = 5y$

or, $\frac{x}{y} = \frac{5}{1}$ **or 5 : 1.**

20. Ans. (d)

Solution Speed upstream = (3 – 2) km/hr = 1 km/hr

Speed downstream = (3 + 2) km/hr = 5 km/hr

Total time taken $= \left(\frac{10}{1}+\frac{10}{5}\right)$hr

$= \textbf{12 hr.}$

21. Ans. (c)

Solution Let the speed upstream be x km/hr and the speed downstream by y km/hr. Then,

$$\frac{24}{x}+\frac{36}{y} = 6 \Rightarrow 24u + 36v = 6$$

where $\frac{1}{x} = u$ and $\frac{1}{y} = v$

And, $\frac{36}{x}+\frac{24}{y} = \frac{13}{2} \Rightarrow 36u + 24v = \frac{13}{2}$

Adding these equations, we get

$$60(u + v) = \frac{25}{2} \text{ or } u + v = \frac{5}{24}$$

Subtracting, we get $12(u - v) = \frac{1}{2}$ or $u - v = \frac{1}{24}$

Solving, $u + v = \frac{5}{24}$ and $u - v = \frac{1}{24}$, we get

$u = \frac{1}{8}$ and $v = \frac{1}{12}$

$\therefore \quad x = 8$ km/hr and $y = 12$ km/hr

$\therefore \quad$ Velocity of current $= \frac{1}{2}(12 - 8) = \textbf{2 km/hr.}$

22. Ans. (b)

Solution Upstream speed = 2 km/hr

Downstream speed = 6 km/hr

$\therefore \quad$ Speed in still water $= \frac{2+6}{2} = 4$ km/hr

$\therefore \quad$ Time required to go 5 km in still water

$= \frac{5}{4}$ **hr = 1 hr 15 min.**

23. Ans. (a)

Solution Let the speed of man in still water = x km/hr

Speed of the current = y km/hr

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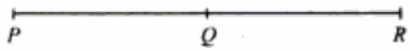
Speed downstream = $(x + y)$ km/hr

Speed upstream = $(x - y)$ km/hr

Let river be flowing from P to R and $PQ = QR = a$

Then, $PR = 2a$



$\therefore \quad \frac{a}{x+y} + \frac{a}{x-y} = 10.$ ...(i)

and, $\frac{2a}{x+y} = 4$

$\therefore \quad \frac{a}{x+y} = 2$ ...(ii)

$\therefore \quad$ (i) $\Rightarrow \frac{a}{x-y} = 8$ ...(iii)

Dividing (ii) and (iii), we get

$$\frac{x-y}{x+y} = \frac{1}{4}$$

$\therefore \quad 4x - 4y = x + y$

or, $3x = 5y$

or, $\frac{x}{y} = \frac{5}{3}$ **or 5 : 3.**

24. Ans. (c)

Solution Let the speed of the motorboat in still water be x km/hr

$$\frac{10}{x+2} + \frac{10}{x-2} = \frac{55}{60}$$

or, $240x = 11x^2 - 44$

or, $11x^2 - 240x - 44 = 0$

$\therefore \quad (x - 22)(11x + 2) = 0$

So, $x = 22$ km/hr (neglecting the –ve value)

$\therefore$ Speed of the motorboat in still water

= 22 km/hr.

25. Ans. (a)

Solution Let, upstream speed = x km/hr and downstream speed = y km/hr

Then, $\frac{30}{x} + \frac{44}{y} = 10$, and $\frac{40}{x} + \frac{55}{y} = 13$

or, $30u + 44v = 10$, and $40u + 55v = 13$

where $u = \frac{1}{x}$ and $v = \frac{1}{y}$

Solving, we get $u = \frac{1}{5}$ and $v = \frac{1}{11}$

$\therefore \quad x = 5$ and $y = 11$

$\therefore$ Rate in still water $= \frac{5+11}{2} = 8$ km/hr

Rate of current $= \frac{11-5}{2} =$ **3 km/hr.**

QUESTIONS FROM COMPETITIVE EXAMS

1. A man can row three-quarters of kilometer against the stream in $11\frac{1}{4}$ minutes and returns in $7\frac{1}{2}$ minutes. The speed of the man in still water is

(a) 2 km/hr (b) 3 km/hr

(c) 4 km/hr (d) 5 km/hr

[Based on MAT 2005]

2. A man swimming in a stream which flows $1\frac{1}{2}$ km/h finds that in a given time he can swim twice as far with the stream as he can against it. At what rate does he swim?

(a) $5\frac{1}{2}$ km/h (b) $4\frac{1}{2}$ km/h

(c) $7\frac{1}{2}$ km/h (d) None of these

[Based on MAT 2008]

3. A boat goes 24 km upstream and 28 km downstream in 6 hours. If it goes 30 km upstream and 21 km downstream in 6 hours and 30 minutes, find the speed of the stream.

(a) 10 km/h (b) 5 km/h

(c) 4 km/h (d) 6 km/h

[Based on MAT 2001]

4. A person can row with the stream at 8 km per hour and against the stream at 6 km an hour. The speed of the current is

(a) 1 km/hr (b) 2 km/hr

(c) 4 km/hr (d) 5 km/hr

[Based on FMS (Delhi) 2002]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Speed upstream $= \frac{3}{4} \times \frac{4}{45} \times 60 = 4$ km/hr.

Speed downstream $= \frac{3}{4} \times \frac{2}{15} = 6$ km/hr.

Speed in still water $= \frac{1}{2}(6+4) =$ **5 km/hr.**

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2. Ans. (b)

Solution $x+\frac{3}{2}=2\left(x-\frac{3}{2}\right)$

$\therefore x=4\frac{1}{2}$ km/h

3. Ans. (c)

Solution Let the speed of the stream be x km per hour.

Let speed of the boat in still water be y km/hr.

$\therefore$ Boat will travel downstream @ $(y + x)$ km/hr and upstream @ $(y - x)$ km/hr.

$$\therefore \quad \frac{24}{y-x}+\frac{28}{y+x}=6$$

and $$\frac{30}{y-x}+\frac{21}{y+x}=6\frac{1}{2}=\frac{13}{2}$$

$\Rightarrow$ $y + x = 14$, $y - x = 6 \Rightarrow x = 4$, $y = 10$.

4. Ans. (a)

Solution Let the speed of the current be x km/hr and speed of the person in still water be y km/hr.

$\therefore$ $y + x = 8$

$y - x = 6$

$\Rightarrow$ $y = 7$, $x = 1$

$\therefore$ Speed of the current = 1 km/hr.

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. A man can row three quarters of a km against the stream in $11\frac{1}{4}$ min and return in $7\frac{1}{2}$ min. The speed of the man in still water is:
(a) 2 km/hr (b) 3 km/hr
(c) 4 km/hr (d) 5 km/hr

2. The speed of a boat in still water is 12 km/hr and rate of flow of stream is 4 km/hr. If it travels upstream for 3 hr 15 min, find the distance in kms travelled by the boat during this journey.
(a) 24 km (b) 25 km
(c) 26 km (d) 27 km

3. A man can row $7\frac{1}{2}$ km an hr in still water and a river is running at $1\frac{1}{2}$ km an hr. How far is the place if it takes him 50 min to row to a place and back?
(a) 2 km (b) 2.5 km
(c) 3 km (d) 3.5 km

4. A boat's crew rowed down a stream from A to B and up again in $7\frac{1}{2}$ hr. If the stream flows at 3 km an hr and the crew can row in still water at 5 km an hr, the distance from A to B is
(a) 10 km (b) 9 km
(c) 15 km (d) 12 km

5. A man who can swim 48 m/min in still water swims 200 m against the current and 200 m with the current. If the difference between those two times is 10 min, find the speed of the current in m per min.
(a) 30 m/min (b) 31 m/min
(c) 29 m/min (d) 32 m/min

6. A boat travels upstream from Q to P and downstream from P to Q in 3 hr. If the distance between P to Q is 4 km and the speed of the current is 1 km/hr, then what is the speed of the boat in still water?
(a) 4.5 km/hr (b) 5.2 km/hr
(c) 2.5 km/hr (d) 3 km/hr
(e) None of these

7. A swimmer can swim a certain distance in the direction of current in 5 hr and return the same distance in 7 hr. If the stream flows at the rate of 1 km/hr, find the speed of the swimmer in still water.
(a) 4 km/hr (b) 5 km/hr
(c) 6 km/hr (d) 7 km/hr

8. A man rowed against a stream flowing $1\frac{1}{2}$ km/hr to a certain point and then turned back, stopping 2 km. short of the place where he originally started. If the whole time occupied in rowing be 2 hr 10 min and his uniform speed in still water be $4\frac{1}{2}$ km/hr, the man went up the stream a distance of:
(a) 7 km (b) 3.5 km
(c) 4 km (d) 5 km

9. A man rows to a place 48 km distant and back in 14 hr. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. The rate of the stream is
(a) 0.5 km/hr (b) 1 km/hr
(c) 3.5 km/hr (d) 1.8 km/hr

10. On a river, B is intermediate to and equidistant from A and C. If a boat can go from A to B and back in 5 hr 15 min and from A to C in 7 hr, it would go from C to A in
(a) 3 hr 30 min (b) 3 hr 45 min
(c) 2 hr 20 min (d) 2 hr 55 min

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution Speed upstream $= \left(\frac{3}{4}\times\frac{4}{45}\times 60\right)$km/hr
$= 4$ km/hr

Speed downstream $= \left(\frac{3}{4}\times\frac{2}{15}\times 60\right)$km/hr $= 6$ km/hr

Speed in still water $= \frac{1}{2}(4 + 6)$ km/hr $=$ **5 km/hr.**

3. Ans. (c)

Solution Downstream speed $= \left(7\frac{1}{2}+1\frac{1}{2}\right) = 9$ km/hr

Upstream speed $= \left(7\frac{1}{2}-1\frac{1}{2}\right) = 6$ km/hr

Let x be the distance, then time taken to a place and back is equal to $\left(\frac{x}{9}+\frac{x}{6}\right)$hr which is given to be 50 min or $\frac{50}{60}$ hr.

$\therefore \quad \frac{x}{9}+\frac{x}{6} = \frac{5}{6}$ or $x =$ **3 km.**

5. Ans. (d)

Solution Let the speed of the stream be x km/hr.

$\therefore \quad \frac{200}{48-x} - \frac{200}{48+x} = 10$

$\Rightarrow \quad 200(48 + x) - 200(48 - x) = 10(2304 - x^2)$

$\Rightarrow \quad x^2 + 40x - 2304 = 0$

$\therefore \quad x = 32$ or $x = -72$ (not acceptable)

$\therefore \quad$ Speed of stream $=$ **32 m/min.**

7. Ans. (c)

Solution Let the speed of the swimmer in still water be x km/hr.

$\therefore \quad$ Distance covered in going $= 5(x + 1)$ km

and distance covered in returning $= 7(x - 1)$ km

$\therefore \quad 5(x + 1) = 7(x - 1)$

or, $\quad 7x - 7 = 5x + 5 \quad$ or, $\quad 2x = 12$

$\therefore \quad x =$ **6.**

9. Ans. (b)

Solution Suppose he moves 4 km downstream in x hr

Then, speed downstream $= \left(\frac{4}{x}\right)$ km/hr

Speed upstream $= \left(\frac{3}{x}\right)$ km/hr

$\therefore \quad \frac{48}{4/x}+\frac{48}{3/x} = 14 \Rightarrow 12x + 16x = 14$

or, $\quad x = \frac{1}{2}$

$\therefore \quad$ Speed downstream $= 8$ km/hr

Speed upstream $= 6$ km/hr

$\therefore \quad$ Rate of stream $= \frac{1}{2}(8 - 6)$ km/hr
$=$ **1 km/hr.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (d)	**6.** (d)	**8.** (d)	**10.** (a)

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Races and Games of Skill

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INTRODUCTION

Race A contest of speed in running, driving, riding sailing or rowing over a specified distance is called *race*.

Race Course The ground or path on which contests are arranged is called a *race course*.

Starting Point The point from where a race begins is called the *starting point*.

Winning Post (or Goal) The point where the race finishes is called the *winning post* or *finishing point* or *goal*.

Dead-heat Race If all the persons contesting a race reach the finishing point exactly at the same time, then the race is called a *dead-heat race*.

Winner The person who first reaches the finishing point is the *winner*.

Suppose, A and B are two contestants in a race. We give below certain statements and their corresponding mathematical meanings, which are frequently used:

Statements	Mathematical Interpretation
1. A beats B by t seconds	A finishes the race t seconds before B finishes.
2. A gives B a start of t seconds	A starts t seconds after B starts from the same point.
3. A gives B a start of x metres	While A starts at the starting point, B starts x metres ahead from the starting point at the same time.
4. Game of 100	A game in which the participant scoring 100 points first is the winner.
5. In a game of 100, "A can give B 20 points"	While A scores 100 points, B scores only (100 – 20) or 80 points.

SOME USEFUL SHORT-CUT METHODS

1. If A is n times as fast as B and A gives B a start of x metres, then the length of the race course, so that both A and B reach the winning post at the same time, must be

$$x\left(\frac{n}{n-1}\right) \text{ metres.}$$

Illustration 1 A is $1\frac{2}{3}$ times as fast as B. If A gives B a start of 60 m, how long should the race course be so that both of them reach at the same time?

Solution Here, $n = \frac{5}{3}$ and $x = 60$.

$\therefore$ Length of race course $= x\left(\frac{n}{n-1}\right)$

$$= 60\left(\frac{5/3}{5/3-1}\right) = 60\left(\frac{5}{5-3}\right) = \mathbf{150\ m.}$$

2. If A can run xm race in t_1 seconds and B in t_2 seconds, where $t_1 < t_2$, then A beats B by a distance

$$\frac{x}{t_2} \times (t_2 - t_1)\,m.$$

Illustration 2 A can run 100 m in 27 sec and B in 30 sec. By what distance A beats B?

Solution A beasts B by a distance

$$= \frac{x}{t_2} \times (t_2 - t_1)$$

$$= \frac{100}{30}(30-27) = \frac{300}{30} = \mathbf{10\ m.}$$

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. In a km race A beats B by 25 m or 5 sec, find the time taken by A to complete the race.
(a) 3 min 15 sec (b) 4 min. 20 sec
(c) 2 min 30 sec (d) 5 min. 10 sec

2. In a race of 300 m A beats B by 15 m or 5 sec. A's time over the course is
(a) 100 sec (b) 95 sec
(c) 105 sec (d) 90 sec

3. A can run 500 m in 30 sec and B in 35 sec. How many m start can A give to B in a km race so that the race may end in a dead-heat?
(a) $139\frac{5}{7}$ (b) $138\frac{5}{7}$
(c) $142\frac{6}{7}$ (d) $140\frac{5}{7}$

4. A runs $1\frac{3}{8}$ times as fast as B. If A gives B a start of 120 m and they reach the goal at the same time, the goal is at a distance of
(a) 360 m (b) 440 m
(c) 460 m (d) 380 m

5. In a game of 100 points A can give B 20 points and C 28 points. Number of points B can give C in a game of 100 points is
(a) 10 (b) 90
(c) 15 (d) 85

6. A's speed is $1\frac{1}{2}$ times of B's. In a race A gives B a start of 300 m. How long should the race course be so that both reach the winning post simultaneously?
(a) 700 m (b) 900 m
(c) 800 m (d) 850 m

7. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 50 m. By how many m will A beat C in a race of 400 m?
(a) 364 m (b) 254 m
(c) 324 m (d) 354 m

8. In a 100 m race, A runs at 5 km/hr. A gives B a start of 8 m and still beats him by 8 sec. Find the speed of B.
(a) 6.14 km/hr (b) 4.14 km/hr
(c) 3.14 km/hr (d) 2.14 km/hr

9. In a game, A can give B ... points, A can give C 32 points and B can give C 15 points. How many points make the game?
(a) 100 (b) 200
(c) 300 (d) 400

10. At a game of billiards A can give B 6 points in 50 and he can give C 13 in 65. In a game of 55, number of points B can give C is
(a) 3 (b) 4
(c) 5 (d) 8

11. In a km race A can beat B by 80 m and B can beat C by 60 m. In the same race A can beat C by
(a) 135.2 m (b) 130.5 m
(c) 142 m (d) 132.5 m

12. In a game of 90 points A can give B 15 points and C 30 points. How many points can B give C in a game of 100 points?
(a) 140 (b) 20
(c) 50 (d) 30

13. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 50 m. By how many m will A beat C in a race of 400 m?
(a) 78 m (b) 56 m
(c) 76 m (d) 86 m

14. In a game A can give B 25 points in 75 and C 18 points in 90. How many points can C give B in a game of 120?
(a) 20 (b) 30
(c) 40 (d) 50

15. A and B run a 5 km race on a round course of 400 m. If their speeds be in the ratio 5 : 4, how often does the winner pass the other?
(a) $4\frac{1}{2}$ times (b) $3\frac{1}{2}$ times
(c) $2\frac{3}{4}$ times (d) $2\frac{1}{2}$ times

16. In a 500 m race, the ratio of speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by
(a) 60 m (b) 40 m
(c) 20 m (d) 10 m

17. In a km race A beats B by 100 m and C by 200 m, by how much can B beat C in a race of 1350 m?
(a) 150 m (b) 120 m
(c) 1200 m (d) 210 m

18. Two boys A and B run at $4\frac{1}{2}$ and 6 km an hour, respectively, A having 190 m start. The course being 1 km, B wins by a distance of
(a) 60 m (b) 65 m
(c) 45 m (d) 75 m

19. A and B run a km race. If A gives B a start of

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50 m, A wins by 14 sec and if A gives B a start of 22 sec km, B wins by 20 m. The time taken by A to run a km is:

(a) 100 sec (b) 120 sec
(c) 105 sec (d) 125 sec

20. A and B take part in a 100 m race. A runs at 5 km per hr. A gives B a start of 8 m and still beat him by 8 sec. Speed of B is

(a) 5.15 km/hr (b) 4.14 km/hr
(c) 4.25 km/hr (d) 4.4 km/hr

EXPLANATORY ANSWERS

1. Ans. (a)

Solution B runs 25 m in 5 sec

$\therefore$ B's time to cover one km $= \frac{5}{25} \times 100$
$= 200$ sec

A's time to cover one km $= 200 - 5$
$= 195$ sec
$=$ **3 min 15 sec.**

2. Ans. (b)

Solution 15 m is covered by B in 5 sec

$\therefore$ 300 m is covered by B in $\frac{5}{15} \times 300 = 100$ sec

$\therefore$ A takes $100 - 5 =$ **95 sec.**

3. Ans. (c)

Solution Time taken by A to run 1 km
$= 30 \times 2 = 60$ sec.

Time taken by B to run 1 km $= 35 \times 2 = 70$ sec.

$\therefore$ A can give B a start of $(70 - 60) = 10$ sec.

In 35 sec B runs 500 m

$\therefore$ In 10 sec B runs $= \frac{500}{35} \times 10 = \frac{1000}{7}$
$= 142\frac{6}{7}$ m

So, A can give B a start of $142\frac{6}{7}$ metres in a km race.

4. Ans. (b)

Solution The speed of A and B are in the ratio 11 : 8.

Let speeds be $11s$ and $8s$ (in m/sec)

Let race be of x m

Then time taken by A to run xm is same as that of B to run $(x - 120)$ m.

$\therefore$ $\frac{x}{11s} = \frac{x-120}{8s}$

$\therefore$ $3x = 11 \times 120$

$\therefore$ $x =$ **440.**

5. Ans. (a)

Solution A scores 100 while B scores $100 - 20 = 80$ and C scores $100 - 28 = 72$.

$\therefore$ While B scores 80, C scores 72

$\therefore$ While B scores 100, C scores $\frac{72}{80} \times 100 = 90$

$\therefore$ B can give C $100 - 90 =$ **10 points.**

6. Ans. (b)

Solution A's speed : B's speed
$= 1\frac{1}{2} : 1 = \frac{3}{2} : 1 = 3 : 2$

It means that in a race of 3 m, A gains $(3 - 2)$
$= 1$ m over B

1 m is gained by A in a race of 3 m

$\therefore$ 300 m is gained by A in a race of
$= \frac{3}{1} \times 300 =$ **900 m.**

7. Ans. (c)

Solution Clearly, if A runs 600 metres, B runs = 540 m

$\therefore$ If A runs 400 m, B runs $= \left(\frac{540}{600} \times 400\right)$
$= 360$ metres

Again, when B runs 500 m, C runs = 450 m

$\therefore$ When B runs 360 m, C runs
$= \left(\frac{450}{500} \times 360\right)$ m $=$ **324 m.**

8. Ans. (b)

Solution Time taken by A to cover 100 m
$= 100 + \left(5 \times \frac{5}{18}\right) = 72$ sec.

$\therefore$ B covers $(100 - 8)$ or, 92 m in $(72 + 8)$ or, 80 sec.

$\therefore$ Speed of $B = \frac{92}{80} \times \frac{16}{5} =$ **4.14 km/hr.**

9. Ans. (a)

Solution Suppose, x points make the game.

Clearly, when A scores x points, B scores $(x - 20)$ points and C scores $(x - 32)$ points.

Now, when B scores x points, C scores $(x - 15)$ points.

When B scores $(x - 20)$ points,

C scores $\left[\frac{(x-15)}{x} \times (x-20)\right]$ points

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$\therefore \quad \frac{(x-15)(x-20)}{x} = x - 20 \quad \text{or,} \quad x = 100$

Hence, **100 points make the game.**

10. Ans. (c)

Solution In a game of 50

While A scores 50, B scores $50 - 6 = 44$ and in a game of 65,

While A scores 65, C scores $65 - 13 = 52$.

$\therefore$ While A scores 50, C scores $\frac{52}{65} \times 50 = 40$

$\therefore$ While B scores 44, C's score = 40

While B scores 55, C's score $= \frac{40}{44} \times 55 = 50$

$\therefore$ In a game of 55,

B can give C $55 - 50 =$ **5 points.**

11. Ans. (a)

Solution While A runs 1000 m, B runs $1000 - 80 = 920$ m and while B runs 1000 m, C runs $1000 - 60 = 940$ m.

$\therefore$ While B runs 920 m; C runs

$$\frac{940}{1000} \times 920 = \frac{4324}{5} \text{ m}$$

$\therefore$ While A runs 1000 m, C runs $\frac{4324}{5}$ m

$\therefore$ A can beat C by $1000 - \frac{4324}{5} = \frac{676}{5} = \mathbf{135\frac{1}{5}}$ **m.**

12. Ans. (b)

Solution $A : B : C = 90 : 75 : 60$

$$B : C = \frac{75}{60} = \frac{70 \times \frac{100}{75}}{60 \times \frac{100}{75}} = \frac{100}{80}$$

Hence, in a game of 100 points, B can give $C(100 - 80) =$ **20 points.**

13. Ans. (c)

Solution If A runs 600 m, B runs $600 - 60$ or, 540 m

If A runs 400 m, B runs $= \frac{540 \times 400}{600} = 360$ m

Now, when B runs 500 m, C runs $500 - 50 = 450$ m

$\therefore$ When B runs 360 m, C runs

$$= \frac{450 \times 360}{500} = 324 \text{ m}$$

$\therefore$ A beats C by $400 - 324 =$ **76 m.**

14. Ans. (a)

Solution When A scores 75 points, B scores 50 points

When A scores 90 points, C scores $(90 - 18) = 72$ points

$\therefore$ When A scores 75 points, C scores

$$= \frac{72}{90} \times 75 = 60 \text{ points}$$

$A : B : C = 75 : 50 : 60$

$$C : B = \frac{60}{50} = \frac{120}{100}$$

So, in a game of 120, C can give B

$(120 - 100) =$ **20 points.**

15. Ans. (d)

Solution When A makes 5 rounds, B makes 4 rounds.

In order to pass each other, the difference in number of rounds made by each must be one. Here, A passes B each time, when A makes 5 rounds.

Distance covered by A in 5 rounds

$$= \frac{5 \times 400}{100} = 2 \text{ km}$$

In covering 2 km A passes B 1 time

$\therefore$ In covering 5 km, A passes B

$$= \frac{5}{2} = \mathbf{2\frac{1}{2}} \textbf{ times.}$$

16. Ans. (c)

Solution To reach the winning post A will have to cover a distance of $(500 - 140)$,

i.e. 360 m

While A covers 3 m, B covers 4 m

While A covers 360 m, B covers $\frac{4}{3} \times 360 = 480$ m

So, A reaches the winning post while B remains 20 m behind.

$\therefore$ A wins by **20 m.**

17. Ans. (a)

Solution While A runs 1000 m, B runs $1000 - 100 = 900$ m and C runs $1000 - 200 = 800$ m.

$\therefore$ While B runs 900 m, C runs 800 m.

$\therefore$ While B runs 1350 m; C runs $\frac{800}{900} \times 1350$

$= 1200$ m

$\therefore$ B can beat C by $1350 - 1200 =$ **150 m.**

18. Ans. (a)

Solution Speeds (in m/sec) of A and B are

$\frac{9}{2} \times \frac{5}{18} = \frac{5}{4}$ and $6 \times \frac{5}{18} = \frac{5}{3}$, respectively

A has a start of 190 m. So A has to run $1000 - 190 = 810$ m while B 1000 m.

Time taken by B to cover 1000 m $= \frac{3}{5} \times 1000$

$= 600$ sec.

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In this time A covers $\frac{5}{4} \times 600 = 750$ m.

So B reaches the winning post while A remains $810 - 750 = 60$ m behind.

$\therefore$ B wins by **60 m.**

19. Ans. (a)

Solution Let times (in sec) taken by A and B to run a km, be x and y, respectively.

When B gets a start of 50 m, B runs $1000 - 50 = 950$ m while A runs 1000 m.

$\therefore$ $\frac{950}{100}y - x = 14$,

i.e. $0.95y - x = 14$ ...(i)

and, When B gets a start of 22 sec, A runs for $(y - 22)$ sec, while B runs for y sec.

$\therefore$ $1000 - \frac{1000}{x}(y - 22) = 20$

i.e. $50y - 49x = 1100.$ ...(ii)

Multiplying (i) by 49 and subtract from (ii)

$3.45y = 414$

$\therefore$ $y = 120$ sec.

$\therefore$ (i) $\Rightarrow x = 0.95 \times 120 - 14 =$ **100 sec.**

20. Ans. (b)

Solution A's speed $= \left(5 \times \frac{5}{18}\right)$ m/sec $= \frac{25}{18}$ m/sec

Time taken by A to cover 100 m

$= \left(100 \times \frac{18}{25}\right)$ sec $= 72$ sec.

$\therefore$ B covers 92 m in $72 + 8 = 80$ sec.

B's speed $= \left(\frac{92}{80} \times \frac{18}{5}\right)$ km/hr $=$ **4.14 km/hr.**

QUESTIONS FROM COMPETITIVE EXAMS

1. In a 800 m race around a stadium having the circumference of 200 m, the top runner meets the last runner on the 5th minute of the race. If the top runner runs at twice the speed of the last runner, what is the time taken by the top runner to finish the race?

(a) 20 min. (b) 15 min

(c) 10 min (d) 5 min

[Based on MAT 2003]

2. A racecourse is 400 metres long. A and B run a race and A wins by 5 metres. B and C run over the same course and B wins by 4 metres. C and D run over it and D wins by 16 metres. If A and D run over it, then who would win and by how much?

(a) A, by 8.4 metres (b) D, by 8.4 metres

(c) D, by 7.2 metres (d) A, by 7.2 metres

[Based on MAT 2002]

3. In a race, Guninder was not the first. Joginder came in after Harinder. Inderjeet was not ahead of Maninder. Guninder was not in front of Joginder. Inderjeet was not fourth or fifth. Maninder was not the first. Who finished first and second in the race?

(a) Harinder followed by Maninder

(b) Harinder followed by Joginder

(c) Harinder followed by Guninder

(d) Cannot be determined from the clues.

[Based on FMS (Delhi) 2004]

4. In a kilometre race, A beats B by 100 metres and B beats C by 150 metres. In the same race, by how many metres does A beat C ?

(a) 250 metres (b) 240 metres

(c) 225 metres (d) 235 metres

[Based on IIT Joint Man. Ent. Test 2004]

EXPLANATORY ANSWERS

1. Ans. (c)

2. Ans. (c)

3. Ans. (c)

Solution When A covers 400 m, B covers 395 metres

When B covers 400 m, C covers 396 metres

When B covers 395 m, C covers

$\frac{396}{400} \times 395 = 391.05$ meters

$\therefore$ When A covers 400 m, B covers 391.05 metres

When D covers 400 m, C covers 384 metres

When C covers 391.05 m, D covers 407.3 metres

$\therefore$ When A covers 400 m, D covers 407.3 metres

When D covers 400 m, A covers 392.8 metres

$\therefore$ D wins by 7.2 metres, when D and A run the race.

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4. Ans. (d)

Solution By the time A covers 1000 m, B has covered 900 m.

$\therefore$ By the time B covers 900 m,

By the time B covers 1000 m, C has covered 850 m, C will cover

$$\frac{850}{1000} \times 900 = 765 \text{ meters}$$

$\therefore$ A beats C by 235 meters.

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. A can run a km in 3 min 10 sec and B in 3 min 20 sec. A can beat B by a distance of
(a) 50 m (b) 60 m
(c) 45 m (d) 65 m

2. A takes $7\frac{1}{2}$ hr and B 6 hr to walk round a circular track. If they start together in the same direction, they will next meet after time of
(a) 15 hr (b) 25 hr
(c) 22 hr (d) 30 hr

3. A and B start from the same point and travel in the same direction round a circular track 3 km in circumference. If their speeds be 200 and 150 m a min, they will be together again at the starting point after time of
(a) 60 min (b) 45 min
(c) 38 min (d) 42 min

4. A can give B 25 points, A can give C 40 points and B can give C 20 points. Number of points in the game is
(a) 80 (b) 75
(c) 100 (d) 90

5. A takes $7\frac{1}{2}$ hrs and B 6 hr to walk round a circular track. If they start together in opposite directions, they will meet after
(a) $3\frac{1}{3}$hr (b) $2\frac{2}{3}$hr
(c) $\frac{1}{2}$hr (d) $1\frac{1}{2}$hr

6. A, B and C run round a course 97 m in circumference. A in direction opposite to that of B and C. Their rates per minute are : A 30 m, B 26 m and C 18 m. They will be together for the first time after starting in time of
(a) $12\frac{1}{8}$ min (b) $\frac{97}{48}$ min
(c) $\frac{97}{56}$ min (d) $12\frac{1}{4}$ min

7. Three boys agree to start together and run, until all come together again, round a circular court 15 m in circumference. One runs at the rate of 6, the sec 7 and 3rd 8 kms per hr. The race will end after
(a) 50 sec (b) 42 sec
(c) 36 sec (d) 54 sec

8. In running a 2 km race on a course $\frac{1}{4}$ of a km round, A overlaps B at the middle of 6th round. Running at the same rate A will win by distance of
(a) $\frac{1}{11}$km (b) $\frac{9}{8}$km
(c) $\frac{18}{11}$km (d) $\frac{4}{11}$km

9. A can give B 20 points, A can give C 32 points and B can give C 15 points. Number of points that make the game are
(a) 100 (b) 60
(c) 85 (d) 90

10. In a km race A, B and C are the three participants. A can give B a start of 50 m and C a start of 69 m. How many m start can B give to C?
(a) 40 m (b) 25 m
(c) 20 m (d) 30 m

Test II **[Time: 10 min]**

11. A can give B 50 m start and C 80 m start in a km. race. In the same race B can give C a start of
(a) 30 m (b) $31\frac{11}{19}$m
(c) $32\frac{11}{19}$m (d) $29\frac{1}{19}$m

12. A can give B 10 m and C 20 m in a 100 m race. In the same race B can give C:

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(a) $12\frac{2}{9}$ m (b) $13\frac{1}{9}$ m

(c) $11\frac{1}{9}$ m (d) 10 m

13. In a race of 1000 m A can beat B by 100 m. In a race of 400 m B beats C by 40 m. By how many metres will A beat C in a race of 500 m?

(a) 85 m (b) 95 m

(c) 75 m (d) 105 m

14. Two men A and B run a 4 km race on a course $\frac{1}{4}$ km round. If their rates be 5 : 4, how often does the winner pass the other?

(a) 2 (b) 4

(c) 5 (d) 3

15. In a game of billiards, A can give B 12 points in 60 and A can give C 10 in 90. How many can C give B in a game of 70?

(a) 63 points (b) 73 points

(c) 53 points (d) 83 points

16. A can give B 15 points, A can give C 22 points and B can give C 10 points. Number of points that make the game is

(a) 60 (b) 40

(c) 45 (d) 50

17. In a km race, A gives B a start of 100 m and still wins by 20 sec. But if A gives B a start of 25 sec, B wins by 50 m. Find the time taken by each to run a km.

(a) $\frac{500}{29}$ sec, $\frac{1200}{29}$ sec (b) $\frac{500}{29}$ sec, $\frac{700}{29}$ sec

(c) $\frac{700}{29}$ sec, $\frac{400}{29}$ sec (d) None of these

18. Two men A and B walk round a circle 1200 m in circumference. They both start at the same time from the same point and walk in the same direction if they walk at rates 150 and 80 m per min, they will be together again after time of

(a) $16\frac{2}{3}$ min (b) 15 min

(c) $19\frac{1}{7}$ min (d) $17\frac{1}{7}$ min

19. In a race A runs at 10 km/hr and B at 13 km/hr. A has a start of 100 m and also A sets off 3 min before B. How soon will B overtake A?

(a) 16 min (b) 12 min

(c) 13 min (d) 14 min

20. In a km race A can give B 50 m's start and B can give C 40 m's start. In a 2 km race A can give C a start of

(a) 195 m (b) 185 m

(c) 176 m (d) 188 m

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution Obviously A can beat B by a time of 3 min 20 sec, – 3 min 10 sec = 10 sec.

Also, in 3 min 20 sec = 200 sec B can run 1000 m

$\therefore$ In 10 sec B can run $\frac{1000}{200} \times 10$ m = 50 m

$\therefore$ A can beat B by **50 m.**

3. Ans. (a)

Solution A makes one complete round in $\frac{3000}{200}$ = 15 m

B makes one complete round in $\frac{3000}{150}$ = 20 m

$\therefore$ Required time = L.C.M. (15, 20) = **60 min.**

5. Ans. (a)

Solution Let length of circular track = x km

Let A, B meet after t hr

Then, distance covered by A in t hr + distance covered by B in t hr = x

$\therefore$ $\frac{x}{15/2}t + \frac{x}{6}t = x$

$\therefore$ $t\left(\frac{2}{15} + \frac{1}{6}\right) = 1$

$\therefore$ $t = \frac{30}{9} = \frac{10}{3} = 3\frac{1}{3}$ **hr.**

7. Ans. (d)

Solution Let boys be numbered I, II and III and their speeds be 8, 7, 6 km per hr. Speeds in m/sec are:

$8 \times \frac{5}{18} = \frac{20}{9}$, $7 \times \frac{5}{18} = \frac{35}{18}$ and $6 \times \frac{5}{18} = \frac{5}{3}$, respectively

I and II will meet in $\frac{15}{\frac{20}{9} - \frac{35}{18}}$ = 54 sec.

and, I and III will meet in $\frac{15}{\frac{20}{9} - \frac{5}{3}}$ = 27 sec.

$\therefore$ I, II and III will be together after time of L.C.M. (54, 27) = 54 sec.

$\therefore$ Race will end in **54 sec.**

9. Ans. (a)

Solution Let x points make the game.

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When A scores x, B scores $(x - 20)$ and C scores $(x - 32)$ ...(i)

and, when B scores x, C scores $(x - 15)$

From (i) when B scores x, C scores $\frac{x-32}{x-20} \times x$

$\therefore \quad x - 15 = \frac{x(x-32)}{x-20}$

$\therefore \quad (x - 20)(x - 15) = x(x - 32)$

$\therefore \quad 3x = 300$

$\therefore \quad \mathbf{x = 100.}$

11. Ans. (b)

Solution While A runs 1000 m; B runs $1000 - 50 = 950$ m and C runs $1000 - 80 = 920$ m

$\therefore$ While B runs 950 m, C runs 920 m

$\therefore$ While B runs 1000 m,

$$C \text{ runs } \frac{920}{950} \times 1000 \text{ m} = \frac{18400}{19} \text{ m}$$

$\therefore$ B can give C a start of $1000 - \frac{18400}{19}$

$$= \frac{600}{19} = \mathbf{31\frac{11}{19}\ m.}$$

13. Ans. (b)

Solution When A runs 1000 m, B runs $(1000 - 100) = 900$ m

$\therefore$ When A runs 500 m, B runs $900 \times \frac{500}{1000} = 450$ m

When B runs 400 m, C runs $(400 - 40) = 360$ m

$\therefore$ When B runs 450 m, C runs $= \frac{360}{400} \times 450 = 405$ m

So, A beats C by $(500 - 405) = \mathbf{95\ m.}$

15. Ans. (a)

Solution If A scores 60 points, then B scores 48 points

Also, if A scores 90 points, then C scores 80 points

$\therefore$ If A scores 60 points, then C scores $\left(\frac{80}{90} \times 60\right)$

$$= \frac{160}{3} \text{ points}$$

Now, when C scores $\frac{160}{3}$ points, B scores 48 points

$\therefore$ When C scores 70 points, B scores $\frac{48 \times 3}{160} \times 70$

$$= \mathbf{63\ points.}$$

17. Ans. (a)

Solution Let the time taken by A and B to run 1 km be x and y sec, respectively.

When A gives B a start of 100 m, A runs 1000 m while B runs 900 m

Time taken by A to run 1000 m $= x$ sec

Time taken by B to run 900 m $= \frac{900}{1000}y = \frac{9}{10}y$ sec.

Then, $\frac{9}{10}y - x = 20$ sec.

or, $\quad 9y - 10x = 200$ ...(i)

When A gives B a start of 25 sec; B runs for y sec while A runs for $(y - 25)$ sec.

In x sec A covers 1000 m

$\therefore$ In $(y - 25)$ sec, A will cover $\frac{1000}{x}(y - 25)$ m

So, $1000 - \frac{1000(y-25)}{x} = 50$

or, $\quad 950x - 1000y + 25000 = 0$

or, $\quad 20y - 19x = 500$ ...(ii)

Solving (i) and (ii), we get

$$\mathbf{x = \frac{500}{29}\ sec\ and\ y = \frac{1200}{29}\ sec.}$$

19. Ans. (b)

Solution Distance covered by A in 3 min

$$= 10 \times \frac{3}{60} = \frac{1}{2}\text{km} = 500 \text{ m}$$

Hence, A has a start of total $(100 \text{ m} + 500 \text{ m}) = 600$ m

B's speed $-$ A's speed $= (13 - 10)$ km/hr

$$= 3 \text{ km/hr} = \frac{3000}{60}\text{m/min} = 50 \text{ m/min.}$$

B gains 50 m. over A in 1 min

$\therefore$ B will cover 600 m lead in $\frac{600}{50} = \mathbf{12\ min.}$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (d)	**4.** (c)	**6.** (a)	**8.** (d)	**10.** (c)	**12.** (c)	**14.** (d)	**16.** (d)	**18.** (d)	**20.** (c)

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Alligation or Mixture 15

INTRODUCTION

Alligation literally means "*linking*". It is a rule to find:

(a) the ratio in which two or more ingredients at their respective prices should be mixed to give a mixture at a given price.

(b) The mean or average price of a mixture when the prices of two or more ingredients which may be mixed together and the proportion in which they are mixed are given.

Here, cost price of a unit quantity of mixture is called the *mean price*.

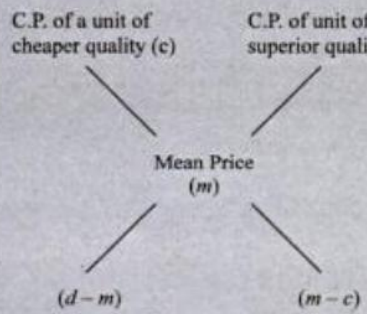
Alligation Rule

Suppose, Rs. d per unit be the price of first ingredient (superior quality) mixed with another ingredient (cheaper quality) of price Rs. c per unit to form a mixture whose mean price is Rs. m per unit, then the two ingredients must be mixed in the ratio:

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of superior}} = \frac{\text{C. P. superior} - \text{Mean price}}{\text{Mean price} - \text{C. P. of cheaper}}$$

i.e. the two ingredients are to be mixed in the inverse ratio of the differences of their prices and the mean price.

The above rule may be represented schematically as under:



$$\frac{\text{Quantity of cheaper quality}}{\text{Quantity of superior quality}} = \frac{d-m}{m-c}$$

Explanation

Suppose x kg of cheaper quality is mixed with y kg of superior quality.

Price of cheaper ingredient = Rs. cx

Price of superior ingredient = Rs. dy

$\therefore$ Price of mixture = Rs. $(cx + dy)$

and quantity of mixture = $(x + y)$ kg.

$\therefore$ Price of mixture/kg = Rs. $\left(\frac{cx+dy}{x+y}\right)$

$$\therefore \frac{cx + dy}{x + y} = m \Rightarrow cx + dy = mx + my$$

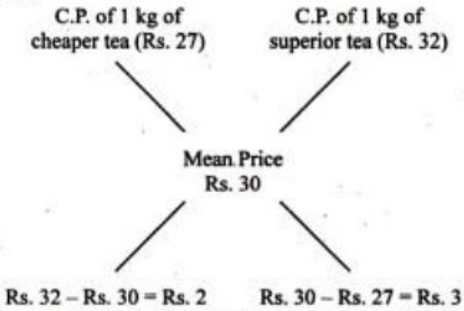
$$\Rightarrow dy - my = mx - cx$$

$$\Rightarrow y(d - m) = x(m - c)$$

$$\Rightarrow \frac{x}{y} = \frac{d-m}{m-c}.$$

Illustration 1 In what ratio two varieties of tea one costing Rs. 27 per kg and the other costing Rs. 32 per kg should be blended to produce a blended variety of tea worth Rs. 30 per kg. How much should be the quantity of second variety of tea, if the first variety is 60 kg.

Solution



The required ratio of the two varieties of tea is 2 : 3, i.e.

$$\frac{\text{Quantity of cheaper tea}}{\text{Quantity of superior tea}} = \frac{2}{3}$$

$\therefore$ Quantity of superior tea = $\frac{60\times3}{2}$ = 90 kg

Thus, the second variety of tea is 90 kg.

Illustration 2 Sugar at Rs. 15 per kg is mixed with sugar at Rs. 20 per kg in the ratio 2 : 3. Find the price per kg of the mixture.

Solution Let the mean price of the mixture be Rs. x

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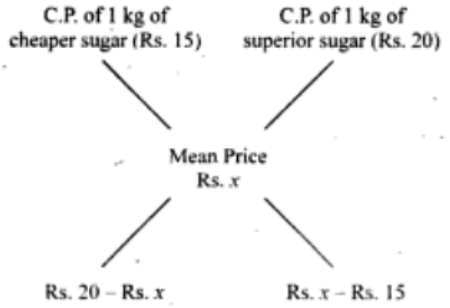

$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{20-x}{x-15}$$

$$\therefore \quad \frac{20-x}{x-15} = \frac{2}{3} \Rightarrow 60 - 3x = 2x - 30$$

$$\Rightarrow 5x = 90 \text{ or } x = 18.$$

Thus, the price per kg of the mixture is Rs. 18.

SOME USEFUL METHODS

1. A vessel, full of wine, contains 'a' litres of it of which 'b' litres are withdrawn. The vessel is then filled with water. Next, 'b' litres of the mixture are withdrawn and again the vessel is filled with water. This process is repeated n times. Then,

$$\frac{\text{Wine left in the vessel after } n\text{th operation}}{\text{Original quantity of wine in the vessel}} = \left(\frac{a-b}{a}\right)^n.$$

Explanation

Amount of wine after 1st operation

$$= a - b = \left(1-\frac{b}{a}\right) \times a$$

Ratio of wine and water after 1st operation is $(a - b) : b$.

$\therefore$ In 'b' litres of mixture withdrawn in 2nd operation,

amount of wine withdrawn

$$= \frac{a-b}{(a-b)+b} \times b = (a-b)\frac{b}{a}$$

$\therefore$ Amount of wine left after 2nd operation

$$= (a-b) - (a-b) \times \frac{b}{a} = \left(1-\frac{b}{a}\right)^2 a$$

In general, quantity of wine left after nth operation

$$= \left(1-\frac{b}{a}\right)^n a$$

$$\therefore \quad \frac{\text{Wine left after } n\text{th operation}}{\text{Original quantity of wine}} = \left(1-\frac{b}{a}\right)^n.$$

Illustration 3 A vessel contains 125 litres of wine. 25 litres of wine was taken out of the vessel and replaced by water. Then, 25 litres of mixture was withdrawn and again replaced by water. The operation was repeated for third time. How much wine is now left in the vessel?

Solution Amount of wine left in the vessel

$$= \left(1-\frac{25}{125}\right)^3 \times 125 = \frac{100\times100\times100\times125}{125\times125\times125} = 64 \text{ litres.}$$

2. There are n vessels of equal size filled with mixtures of liquids A and B in the ratio $a_1 : b_1 : a_2; b_2 \ldots, a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{\text{Quantity of liquid } A}{\text{Quantity of liquid } B} = \frac{\left(\frac{a_1}{a_1+b_1} + \frac{a_2}{a_2+b_2} + \ldots + \frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1} + \frac{b_2}{a_2+b_2} + \ldots + \frac{b_n}{a_n+b_n}\right)}$$

Explanation

Let the capacity of each vessel be c litres.

Amount of liquid A in different vessels

$$= \frac{a_1c}{a_1+b_1}, \frac{a_2c}{a_2+b_2}, \frac{a_3c}{a_3+b_3}, \ldots, \frac{a_nc}{a_n+b_n}$$

Amount of liquid B in different vessels

$$= \frac{b_1c}{a_1+b_1}, \frac{b_2c}{a_2+b_2}, \frac{b_3c}{a_3+b_3}, \ldots \frac{b_nc}{a_n+b_n}$$

So, in the resulting mixture, amount of liquid A

$$= \left(\frac{a_1}{a_1+b_1} + \frac{a_2}{a_2+b_2} + \ldots + \frac{a_n}{a_n+b_n}\right) \times c$$

Amount of liquid B

$$= \left(\frac{b_1}{a_1+b_1} + \frac{b_2}{a_2+b_2} + \ldots + \frac{b_n}{a_n+b_n}\right) \times c$$

$$\frac{\text{Quantity of liquid } A}{\text{Quantity of liquid } B} = \frac{\left(\frac{a_1}{a_1+b_1} + \frac{a_2}{a_2+b_2} + \ldots + \frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1} + \frac{b_2}{a_2+b_2} + \ldots + \frac{b_n}{a_n+b_n}\right)}.$$

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Illustration 4 Three equal glasses are filled with mixture of milk and water. The proportion of milk and water in each glass is as follows: In the first glass as 3 : 1, in the second glass as 5 : 3 and in the third as 9 : 7. The contents of the three glasses are emptied into a single vessel. What is the proportion of milk and water in it?

Solution

$$\frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\frac{3}{3+1}+\frac{5}{5+3}+\frac{9}{9+7}}{\frac{1}{3+1}+\frac{3}{5+3}+\frac{7}{9+7}} = \frac{31/16}{17/16}$$

$$= 31 : 17.$$

3. There are n vessels of sizes $c_1, c_2, \ldots, c_n$ filled with mixtures of liquids A and B in the ratio $a_1 : b_1, a_2 : b_2, \ldots, a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{\text{Quantity of Liquid } A}{\text{Quantity of liquid } B} = \frac{\frac{a_1c_1}{a_1+b_1}+\frac{a_2c_2}{a_2+b_2}+\ldots+\frac{a_nc_n}{a_n+b_n}}{\frac{b_1c_1}{a_1+b_1}+\frac{b_2c_2}{a_2+b_2}+\ldots+\frac{b_nc_n}{a_n+b_n}}$$

Illustration 5 Three glasses of sizes 3 litres, 4 litres and 5 litres contain mixture of milk and water in the ratio 2 : 3, 3 : 7 and 4 : 11, respectively. The contents of all the three glasses are poured into a single vessel. Find the ratio of milk to water in the resulting mixture.

Solution

$$\frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\left(\frac{2\times3}{2+3}+\frac{3\times4}{3+7}+\frac{4\times5}{4+11}\right)}{\left(\frac{3\times3}{2+3}+\frac{7\times4}{3+7}+\frac{11\times5}{4+11}\right)}$$

$$= \frac{\frac{6}{5}+\frac{12}{10}+\frac{20}{15}}{\frac{9}{5}+\frac{28}{10}+\frac{55}{15}} = 56 : 124 \text{ or, } 14 : 31.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. How many kg of tea worth Rs. 25 per kg must be blended with 30 kg of tea worth Rs. 30 per kg so that by selling the blended variety at Rs. 30 per kg there should be a gain of 10%?
   (a) 36 kg (b) 40 kg
   (c) 32 kg (c) 42 kg

2. How much water be added to 14 litres of milk worth Rs. 5.40 a litre so that the value of the mixture may be Rs. 4. 20 a litre?
   (a) 7 litres (b) 6 litres
   (c) 5 litres (d) 4 litres

3. In what ratio two varieties of tea, one costing Rs. 25 per kg and the other costing Rs. 30 per kg should be blended to produce blended variety of tea worth Rs. 28 per kg?
   (a) 3 : 4 (b) 4 : 3
   (c) 2 : 3 (d) 3 : 5

4. In an examination out of 80 students 85% of the girls and 70% of the boys passed. How many boys appeared in the examination if total pass percentage was 75%?
   (a) 370 (b) 340
   (c) 320 (d) 360

5. In what proportion must tea worth 75 paise per packet be mixed with tea worth Rs. 5.50 per packet so that the mixture may cost Rs. 4.50 per packet?
   (a) 3 : 11 (b) 4 : 15
   (c) 15 : 11 (d) 4 : 5

6. How many kg of sugar costing Rs. 5.50 per kg must be mixed with 60 kg of sugar costing Rs. 4.80 per kg so that the mixture is worth Rs. 5.25 per kg?
   (a) 90 kg (b) 95 kg
   (c) 108 kg (e) 106 kg

7. How many kg of sugar costing Rs. 5.75 per kg should be mixed with 75 kg of cheaper sugar costing Rs. 4.50 per kg so that the mixture is worth Rs. 5.50 per kg?
   (a) 250 kg (b) 300 kg
   (c) 350 kg (d) 325 kg

8. In what ratio must water be added to spirit to gain 10% by selling it at the cost price?
   (a) 1 : 11 (b) 1: 5
   (c) 1 : 10 (d) 1 : 9

9. 300 gm of salt solution has 40% salt in it. How much salt should be added to make it 50% in the solution?
   (a) 40 gm (b) 60 gm
   (c) 70 gm (d) 80 gm

10. A man buys two cows for Rs. 1350 and sells one so as to lose 6% and the other so as to gain 7.5% and on the whole he neither gains nor loses. What does each cow cost?

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(a) 36, 39 (b) 42, 45
(c) 39, 36 (d) 45, 48

11. There are 65 students in a class, 39 rupees are distributed among them so that each boy gets 80 P and girl gets 30 P. Find the number of boys and girls in that class.
(a) 43, 40 (b) 36, 33
(c) 39, 26 (d) 45,42

12. A trader has 50 kg of sugar, a part of which he sells at 10 % profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss ?
(a) 40 kg, 10 kg (b) 10 kg, 35 kg
(c) 25 kg, 15 kg (d) 30 kg, 20 kg

13. A person has Rs. 5000. He invests a part of it at 3% per annum and the remainder at 8% per annum simple interest. His total income in 3 years is Rs. 750. Find the sum invested at different rates of interest.
(a) Rs. 2000, Rs. 1000 (b) Rs. 3000, Rs. 1000
(c) Rs. 1000, Rs. 4000 (d) Rs. 3000, Rs. 2000

14. Some amount out of Rs. 7000 was lent at 6% p.a. and the remaining at 4% p.a. If the total simple interest from both the fractions in 5 years was Rs. 1600, the sum lent at 6% p.a. was
(a) Rs. 3000 (b) Rs. 4000
(c) Rs. 5000 (d) Rs. 2000

15. 729 ml of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7 : 3 ?
(a) 600 ml (b) 710 ml
(c) 520 ml (d) None of these

16. In what proportion water must be added to spirit to gain 20 % by selling it at the cost price ?
(a) 1 : 5 (b) 2 : 5
(c) 3 : 5 (d) 4 : 5

17. The average monthly salary of employees, consisting of officers and workers of an organisation is Rs. 3000. The average salary of an officer is 10000 while that of a worker is Rs. 2000 per month. If there are total 400 employees in the organisation, find the number of officers and workers separately.
(a) 50, 275 (b) 350, 450
(c) 50, 350 (d) 325, 350

18. A person covers a distance 100 kms in 10 hr, partly by walking at 7 km per hr and rest by running at 12 km per hr. Find the distance covered in each part.
(a) 28 km, 72 km (b) 32 km, 82 km
(c) 24 km, 68 km (d) 26 km, 70 km

19. The average weekly salary per head of all employees (supervisors and labourers) is Rs. 100. The average weekly salary per head of all the supervisors is Rs. 600 while the average weekly salary per head of all the labourers is Rs. 75. Find the number of supervisors in the factory if there are 840 labourers in it.
(a) 46 (b) 42
(c) 44 (d) 48

20. A person has a chemical of Rs. 25 per litre. In what ratio should water be mixed in that chemical so that after selling the mixture at Rs 20/litre he may get a profit of 25% ?
(a) 13 : 16 (b) 16 : 19
(c) 12 : 15 (d) 19 : 22

21. A person travels 285 km in 6 hr in two stages. In the first part of the journey, he travels by bus at the speed of 40 km per hr. In the second part of the journey, he travels by train at the speed of 55 km per hr. How much distance did he travel by train?
(a) 205 km (b) 145 km
(c) 165 km. (d) 185 km

22. A trader has 50 kg of pulses, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit?
(a) 30 kg (b) 25 kg
(c) 20 kg (d) 40 kg

23. A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss?
(a) 30 kg, 10 kg (b) 40 kg, 15 kg
(c) 35 kg, 40 kg (d) 40 kg, 10 kg

24. Mira's expenditure and savings are in the ratio 3 : 2. Her income increases by 10%. Her expenditure also increases by 12%. By how many % does her saving increase?
(a) 7% (b) 10%
(c) 9% (d) 13%

EXPLANATORY ANSWERS

1. Ans. (a)

Solution S.P. = Rs. 30 per kg
Gain = 10 %

$$\text{C.P.} = \text{Rs. } \frac{30\times100}{110} = \text{Rs. } \frac{300}{11}$$

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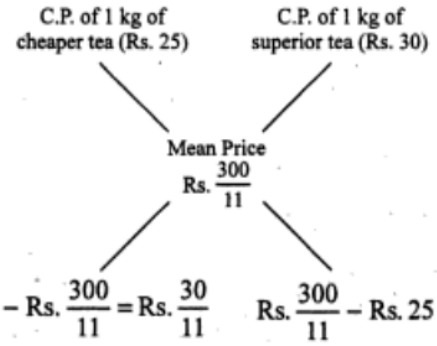


The required ratio of the two varieties of tea

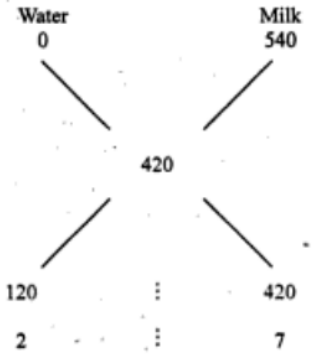
$$= \frac{30}{11} : \frac{25}{11} = 6 : 5$$

$$\frac{\text{Quantity of cheaper tea}}{\text{Quantity of superior tea}} = \frac{6}{5}$$

$$\text{Quantity of cheaper tea} = \frac{6}{5} \times 30 = 36 \text{ kg.}$$

2. Ans. (d)

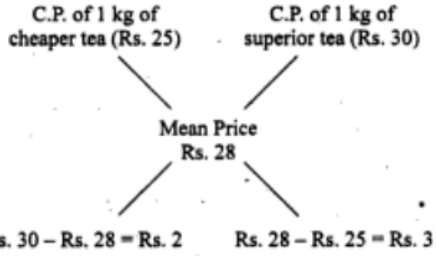
Solution



Water is free, its cost can be taken as 0 paisa per litre. By method of alligation the ratio of water and milk is 2 : 7, i.e. with 7 litres of milk, 2 litres of water is added, with 14 litres, water added is 4 litres.

3. Ans. (c)

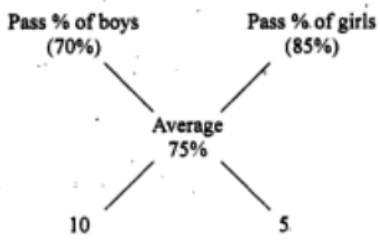
Solution



∴ The required ratio of two varieties of tea is 2 : 3.

4. Ans. (c)

Solution



By Alligation rule:

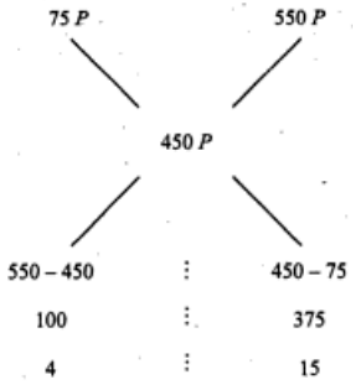
$$\frac{\text{Number of boys}}{\text{Number of girls}} = \frac{10}{5} = \frac{2}{1}$$

Total number of students = 480

$$\text{Number of boys} = \frac{480 \times 2}{3} = 320.$$

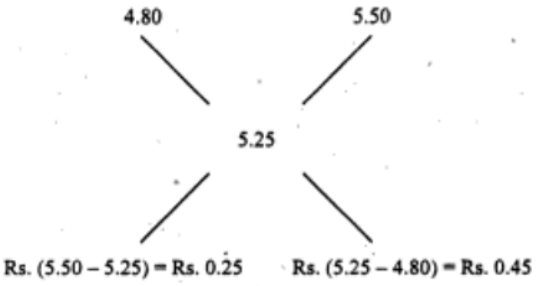
5. Ans. (b)

Solution By method of alligation the required ratio is 4 : 15



6. Ans. (c)

Solution



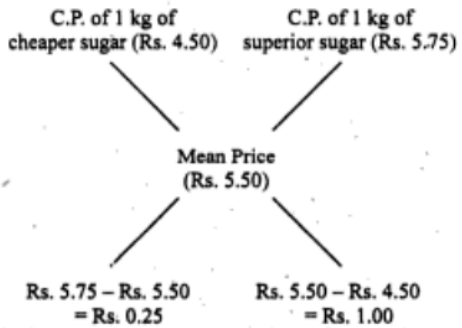
Now, $\frac{.25}{.45} = \frac{5}{9}$

$\therefore \frac{5}{9} = \frac{60}{\text{superior sugar}}$

$\therefore$ Quantity of superior sugar $= \frac{60 \times 9}{5} = 108$ kg.

7. Ans. (b)

Solution



Quantity of cheaper sugar : Quantity of superior sugar = 0.25 : 1

or, $\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of superior sugar}} = \frac{0.25}{1} = \frac{1}{4}$

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$\Rightarrow \dfrac{75}{\text{Quantity of dearer sugar}} = \dfrac{1}{4}$

$\therefore$ Quantity of dearer sugar $= 75 \times 4 = 300$ kg.

8. Ans. (c)

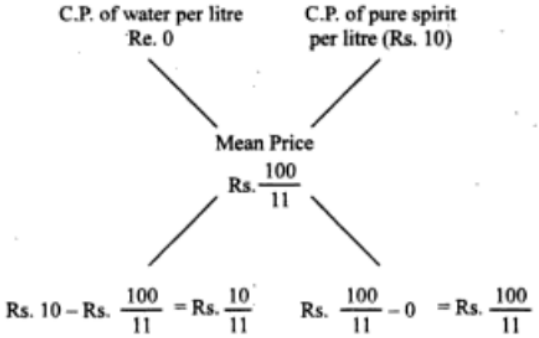
Solution Let the C.P. of spirit = Rs. 10 per litre

S.P. of the mixture = Rs. 10 per litre

Profit = 10 %

$\therefore$ C.P. of the mixture $= \dfrac{10 \times 100}{110}$

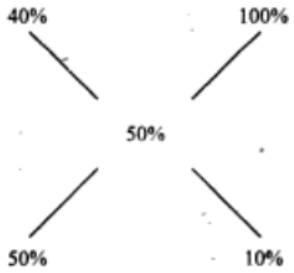
$= \text{Rs. } \dfrac{100}{11}$ per litre



$\therefore \dfrac{\text{Quantity of water}}{\text{Quantity of spirit}} = \dfrac{10/11}{100/11} = 1 : 10.$

9. Ans. (b)

Solution The existing solution has 40% salt. More salt is to be mixed to make 100% salt solution. So, by alligation method:

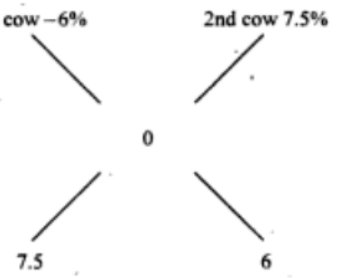


$\therefore$ The two mixtures should be added in the ratio 5 : 1

$\therefore$ Required salt $= \dfrac{300}{5} \times 1 = 60$ gm.

10. Ans. (b)

Solution



Thus, we see that the ratio of the cost of two cows is 5 : 4

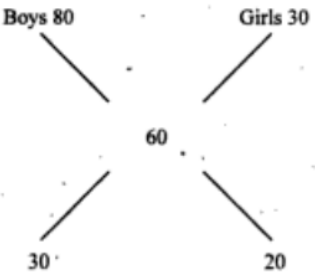
$\therefore$ Cost of 1st cow $= \dfrac{1350}{5+4} \times 5 = \text{Rs. } 750$

and, cost of 2nd cow $= \dfrac{1350}{5+4} \times 4 = \text{Rs. } 600.$

11. Ans. (c)

Solution Here, alligation is applicable for "money per boy or girl".

Mean value of money per student $= \dfrac{3900}{65} = 60\ P.$



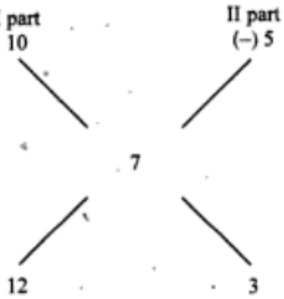
$\therefore$ Boys : Girl = 3 : 2

$\therefore$ Number of boys $= \dfrac{65}{3+2} \times 3 = 39$

and, number of girls = 65 – 39 = 26.

12. Ans. (a)

Solution



$\therefore$ Ratio of quantities sold at 10% profit and 5% loss = 12 : 3 = 4 : 1

$\therefore$ The quantity sold at 10% profit $= \dfrac{50}{4+1} \times 4$

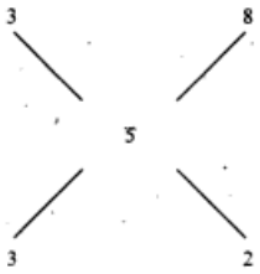
$= 40$ kg

and, the quantity sold at 5% loss = 50 – 40 = 10 kg.

13. Ans. (d)

Solution Average rate of interest

$= \dfrac{100 \times 750}{5000 \times 3} = 5\%$ per annum



Investment at 3% per annum

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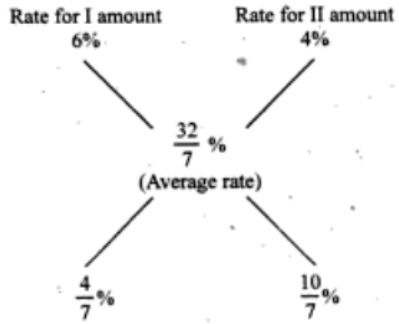
$$= \frac{3}{3+2} \times 5000 = \text{Rs. } 3000$$

Investment at 8% per annum

$$= \frac{2}{3+2} \times 5000 = \text{Rs. } 2000.$$

14. Ans. (d)

Solution Overall rate of interest $= \frac{1600 \times 100}{5 \times 7000} = \frac{32}{7}\%$



$\therefore$ Ratio of two amounts = 2 : 5

$\therefore$ Amount lent at 6% $= \frac{2}{7} \times 7000 = \text{Rs. } 2000.$

15. Ans. (d)

Solution Milk $= \left(729 \times \frac{7}{9}\right) = 567$ ml

Water = 729 – 567 = 162 ml

Now, $\frac{567}{162 + x} = \frac{7}{3} \Rightarrow x = 81$ ml.

16. Ans. (a)

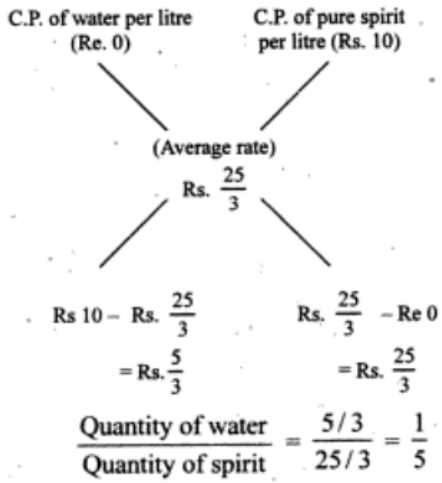
Solution Let the C. P. of the spirit = Rs. 10 per litre

S.P. of the mixture = Rs. 10 per litre

Profit = 20%

$\therefore$ C. P. of the mixture = Rs. $\frac{10 \times 100}{120}$

$= \text{Rs. } \frac{25}{3}$ per litre

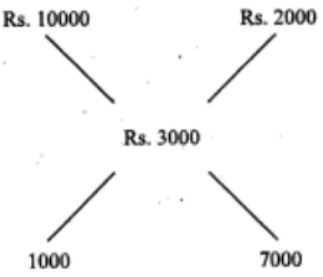


$$\frac{\text{Quantity of water}}{\text{Quantity of spirit}} = \frac{5/3}{25/3} = \frac{1}{5}$$

Ratio of water and spirit = 1 : 5.

17. Ans. (c)

Solution



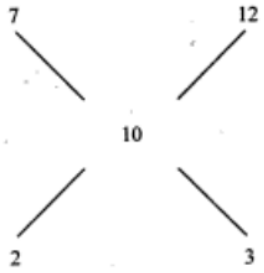
$$\frac{\text{Number of officers}}{\text{Number of workers}} = \frac{1000}{7000} = \frac{1}{7}$$

$$\text{Number of officers} = \frac{1}{1+7} \times 400 = 50$$

Numbers of worker = 400 – 50 = 350.

18. Ans. (a)

Solution Average speed $= \frac{100}{10} = 10$ kms/hr



Ratio of time taken at 7 kms per hr to 12 km per hr = 2 : 3

Time taken at 7 km per hr

$$= \frac{2}{2+3} \times 10 = 4 \text{ hr.}$$

Distance covered at 7 km per hr

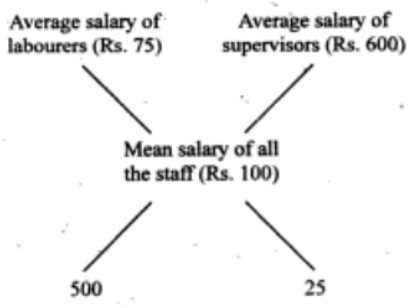
$$= 7 \times 4 = 28 \text{ km.}$$

Distance covered at 12 km per hr

$$= 100 - 28 = 72 \text{ km.}$$

19. Ans. (b)

Solution



The required ratio is 500 : 25 or 20 : 1

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$$\frac{\text{Number of labourers}}{\text{Number of supervisors}} = \frac{20}{1}$$

$$\Rightarrow \frac{840}{\text{Number of supervisors}} = \frac{20}{1}$$

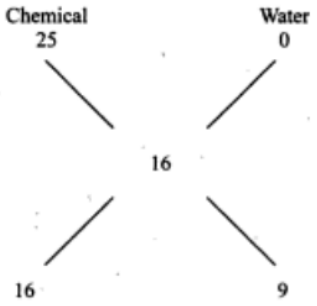
$$\therefore \text{Number of supervisors} = \frac{840}{20} = 42.$$

20. Ans. (b)

Solution In this question the alligation method is applicable on prices, so we should get the average price of mixture.

S.P. of mixture = Rs. 20/litre, profit = 25%

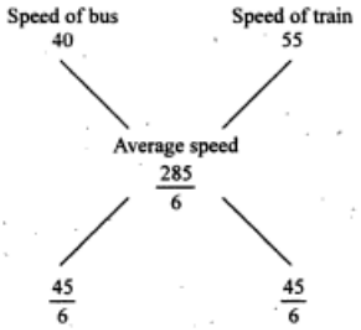
$$\therefore \text{Average price} = 20 \times \frac{100}{125} = \text{Rs. } 16/\text{litre}$$



$\therefore$ Chemical : water = 16 : 9.

21. Ans. (c)

Solution In this question, the alligation method is applicable for the speed.



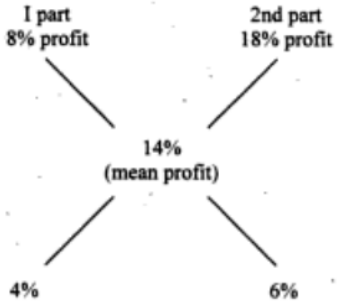
$\therefore$ Time spent in bus : time spent in train

$$= \frac{45}{6} : \frac{45}{6} = 1 : 1.$$

$\therefore$ Distance travelled by train = $55 \times 3 = 165$ km.

22. Ans. (a)

Solution



$\therefore$ Ratio of quantities sold at 8% profit and 18% profit

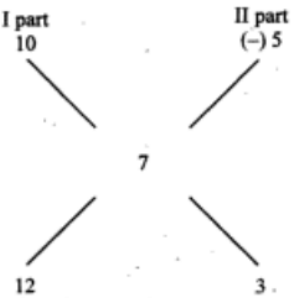
$= 4 : 6 = 2 : 3$

Therefore, the quantity sold at 18% profit

$$= \frac{50}{2 + 3} \times 3 = 30 \text{ kg.}$$

23. Ans. (d)

Solution



$\therefore$ Ratio of quantities sold at 10 % profit and 5% loss $= 12 : 3 = 4 : 1$

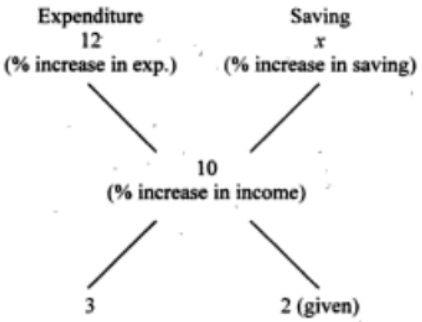
$\therefore$ The quantity sold at 10% profit

$$= \frac{50}{4 + 1} \times 4 = 40 \text{ kg}$$

and, the quantity sold at 5% loss = 50 – 40 = 10 kg.

24. Ans. (a)

Solution



We get two values of x, 7 and 13. But to get a viable answer, we must keep in mind that the central value (10) must lie between x and 12. Thus, the value of x should be 7 and not 13.

$\therefore$ Required % increase = 7%.

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QUESTIONS FROM COMPETITIVE EXAMS

1. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, then the price of the third variety per kg will be
(a) Rs 169.50 (d) Rs 170
(c) Rs 175.50 (d) Rs 180
[Based on MAT 2001]

2. In a mixture of 45 litres, the ratio of milk and water is 4 : 1. How much water must be added to make the mixture ratio 3 : 2?
(a) 72 litres (b) 24 litres
(c) 15 litres (d) 1.5 litres
[Based on MAT 2003]

3. In a mixture of 45 litres, the ratio of milk and water is 3 : 2. How much water must be added to make the ratio 9 : 11?
(a) 10 litres (b) 15 litres
(c) 17 litres (d) 20 litres
[Based on MAT 2002]

4. How many kg of pure salt must be added to a 30kg of 2% solution of salt and water to increase it to a 10% solution?
(a) 3kg (b) 15kg
(c) $2\frac{2}{3}$kg (d) $14\frac{1}{3}$ kg
[Based on MAT 2004]

5. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. If this ratio is to be 1 : 2, then the quantity of water (in litres) to be further added is
(a) 20 (b) 30
(c) 40 (d) 60
[Based on MAT 2003]

6. A dairy man pays Rs. 6.4 per litre of milk. He adds water and sells the mixture at Rs. 8 per litre, thereby making 37.5% profit. Find the proportion of the water to that of the milk received by the customers.
(a) 1 : 15 (b) 1 : 10
(c) 1 : 20 (d) 1 : 12
[Based on MAT 2003]

7. A dairyman pays Rs. 64 per litre of miik. He adds water and sells the mixture at Rs. 8 per litre, thereby making 37.5% profit. The proportion of water to milk received by the customers is
(a) 1 : 15 (b) 1 : 10
(c) 1 : 20 (d) 1 : 12
[Based on MAT 2003]

8. Several litres of acid were drawn off a 54-litre vessel full of acid and an equal amount of water added. Again the same volume of the mixture was drawn off and replaced by water. As a result, the vessel contained 24 litres of pure acid. How much of the acid was drawn off initially.
(a) 12 litres (b) 16 litres
(c) 18 litres (d) 24 litres
[Based on MAT 2001]

9. If 50% of the 2 : 3 solution of milk and water is replaced with water, then the concentration of the solution is reduced by
(a) 25% (b) 33.33%
(c) 50% (d) 75%
[Based on MAT 2001]

10. A bottle contains 3/4 of milk and the rest water. How much of the mixture must be taken away and replaced by equal quantity of water so that mixture has half milk and half water?
(a) 25% (b) 45%
(c) 33 + 1/3% (d) 50%
[Based on IIT Joint Man. Ent. Test 2004]

11. Two vessels contain spirit of 0.5 and 0.75 concentrations. If two litres from the first vessel and three litres from the second vessel are mixed, then what will be the ratio of the spirit and the water in the resultant solution?
(a) 13 : 7 (b) 7 : 17
(c) 15 : 17 (d) 17 : 15
[Based on IIT Joint Man. Ent. Test 2004]

12. How many litres of pure acid are there in 8 litres of a 20% solution?
(a) 1.6 (b) 1.5
(c) 1.4 (d) 2.4
[Based on REC Tiruchirapalli 2003]

13. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. If the ratio of milk and water is to be 1 : 2, then the amount of water (in litres) to be further added must be
(a) 20 (b) 30
(c) 40 (d) 60
[Based on FMS (Delhi) 2003]

14. Milk sold by a milkman contains 5% water. What quantity of pure milk should be added to 20 litres so that water content comes down to 2%?
(a) 16 litres (b) 20 litres
(c) 25 litres (d) 10 litres
(e) None of these **[Based on IRMA, 2002]**

15. There are 3 containers A, B and C which contain water, milk and acid respectively in equal quantities. 10% of the content of A is taken out and poured into B. Then, the same amount from B is transferred to C, from which again the same amount is transferred to A. What is the proportion of milk in container A at the end of the process?
(a) 9/10 (b) 1/11

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(c) 1/121 (d) 2/13

[Based on FMS (Delhi) 2002]

16. Milk and water are in the ratio of 3 : 2 in a mixture of 80 litres. How much water should be added so that the ratio of the milk and water becomes 2 : 3?

(a) 25 litres (b) 40 litres

(c) 35 litres (d) 20 litres

[Based on FMS (Delhi) 2002]

17. Ten litres of water were added to ten litres of a 30% strong solution of sulphuric acid. Find the strength of the resulting solution.

(a) 10% (b) 12%

(c) 12.5% (d) 15%

(e) 16.66% **[Based on SCMHRD 2002]**

18. A mixture of milk and water has been kept in two separate containers. Ratio of milk to water in one of the containers is 5 : 1 and that in the other container is 7 : 2. In what ratio should the mixtures of these two containers be added together so that the quantity of milk in the new mixture may become 80%.

(a) 3 : 2 (b) 2 : 3

(c) 4 : 5 (d) None of these

[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Suppose the quantities of Tea worth Rs 126 per kg, Rs 135 per kg and Rs x per kg purchased are y, y and $2y$ kg respectively.

$$\therefore \quad \frac{126y + 135y + 2xy}{4y} = 153$$

$$\Rightarrow \quad 261 + 2x = 612$$

$$\Rightarrow \quad x = \frac{351}{2} = 175.50$$

$\therefore$ Tea of the third variety is purchased @ Rs **175.50 per kg.**

2. Ans. (c)

Solution Milk = 36 litres

Water = 9 litres

To make the ratio of Milk : Water as 3 : 2, **15 litres of water must be added.**

3. Ans. (b)

Solution Let $M = 3K$, $W = 2K$

$$\therefore \quad 3K + 2K = 45 \Rightarrow K = 9$$

$\Rightarrow$ Milk = 27 litres and Water = 18 litres

Now suppose x litres of water is added to the mixture such that

$$\frac{27}{18 + x} = \frac{9}{11} \Rightarrow 162 + 9x = 297$$

$$\Rightarrow 9x = 135 \Rightarrow \boldsymbol{x = 15.}$$

4. Ans. (a)

5. Ans. (d)

Solution Milk = 40 litres

Water = 20 litres

To make the ratio $M : W = 1 : 2$, 60 litres of water shoule be added.

6. Ans. (b)

Solution Suppose the quantity of milk purchased = x litres

Suppose quantity of water mixed = y litres

$\therefore$ Required ratio of the water and the milk in the mixture = $y : x$

C.P. of x litres of milk = Rs 6.4 x

S.P. of x litres of milk = Rs 8 $(x + y)$

Profit % = 37.5

$$\therefore \text{C.P.} = \frac{\text{S.P.} \times 100}{100 + \text{Gain\%}} \Rightarrow 6.4x = \frac{(8x + 8y) \times 100}{100 + 37.5}$$

$$\Rightarrow 880x = 800x + 800y$$

$$\Rightarrow 80x = 800y \Rightarrow x = 10y$$

$$\Rightarrow \frac{x}{10} = \frac{y}{1}$$

$\therefore$ Required ratio = **1 : 10.**

7. Ans. (b)

Solution Suppose the quantity of milk purchased = x litres

Suppose quantity of water mixed = y litres

$\therefore$ Required ratio of the water and the milk in the mixture = $y : x$

C.P. of x litres of milk = Rs 64.4x

S.P of x litres of milk = 8 $(x + y)$

$$\therefore \text{C.P.} = \frac{\text{S.P.} \times 100}{100 + \text{Gain\%}} \Rightarrow 6.4x = \frac{(8x + 8y) \times 100}{100 + 37.5}$$

$$\Rightarrow 880x = 800x + 800y$$

$$\Rightarrow \frac{x}{10} = \frac{y}{1}$$

$\Rightarrow$ required ratio = 1 : 10

8. Ans. (c)

Solution Suppose x litres of acid was drawn off initially.

$\therefore$ Remaining acid in the 54-litre vessel full of acid.

= $(54 - x)$ litres and water added = x litres.

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Now out of the 54 litres of mixture of acid and water, x litres of mixture is drawn off.

$\Rightarrow$ Quantity of acid drawn off $= \left(\frac{54-x}{54} \times x\right)$ litres

and quantity of water drawn off $= \frac{x^2}{54}$ litres.

Now the vessel contains $\left[54 - x - \left(\frac{54-x}{54}\right)x\right]$ litres of acid.

$\therefore \quad 54 - x - \left(\frac{54-x}{54}\right)x = 24$

$\Rightarrow \quad x^2 - 108x + 1620 = 0$

$\Rightarrow \quad x = 90, 18.$

Since $90 > 54$, therefore $x = 90$ is ruled out. Hence $x = 18$.

9. Ans. (c)

Solution

Milk	:	Water
20		30
−10		− 15
		+25
10		40

Threfore, the concentration of the solution is reduced from 20 to 10, i.e. reduced by 50%. The dilution of the solution increased from 30 to 40, i.e. increased by $33\frac{1}{3}$ %.

10. Ans. (c)

11. Ans. (a)

Solution From vessel A (0.5 concentration), 1 litre of spirit and 1 litre of water is taken and from vessel B (0.75 concentration), 2.25 litres of spirit and 0.75 litre of water is taken.

$\therefore$ Ratio of spirit to water in the resulting mixture

$$= \frac{1+2.25}{1+0.75} = \frac{3.25}{1.75} = \frac{13}{7}.$$

12. Ans. (a)

Solution 20% of 8 litres, i.e. $\frac{20}{100} \times 8 = 1.6$ litres

13. Ans. (d)

Solution Milk = 40 litres, Water = 20 litres in the mixture of 60 litres.

14. Ans. (e)

Solution Total milk = 20 Litres

Water	Milk	Total
1 Litre	19 Litres	20
1	$19 + x$	$20 + x$

Now $1 = 2\%$ of $(20 + x)$, where x is the quantity of pure milk added to the mixture.

$\Rightarrow \quad x = 30$

where x = Quantity of pure milk added to the mixture.

15. Ans. (c)

Solution Suppose there is 100 cc of liquid in each container initially.

I Step: Container B has 10 cc water in 110 cc of mixture.

II Step: 10 cc of this mixture contains

$10 \times \frac{100}{100} = \frac{100}{11}$ cc of milk and $\frac{10}{11}$ cc of water, which is transferred to container C.

Container C has 100 cc of acid, $\frac{100}{11}$ cc of milk and $\frac{10}{11}$ cc of water.

Total quantity of mixture in container C = 110 cc.

III Step: 10 cc of this mixture is transferred to A which contains.

$$\frac{100}{1210} \times 10 \text{cc of milk} = \frac{100}{121} \text{ cc of milk}$$

Thus, in 100 cc of mixture, there is $\frac{100}{121}$ cc of milk of the $\frac{1}{121}$ th proportion of the mixture.

16. Ans. (b)

Solution Quantity of Milk in the mixture = 48 litres.

Quantity of water in the mixture = 32 litres

Let x litres of water be added in the mixture so that

$$\frac{48}{32+x} = \frac{2}{3} \Rightarrow 2x + 64 = 144 \Rightarrow x = 40.$$

17. Ans. (d)

18. Ans. (a)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. 15 litres of a mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed in it the percentage of alcohol in the mixture will be

(a) 17 (b) $16\frac{2}{3}$

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(c) $18\frac{2}{3}$ (d) 15

2. In what proportion must water be mixed with spirit to gain $16\frac{2}{3}\%$ by selling it at cost price?
(a) 1 : 6 (b) 1 : 9
(c) 1 : 7 (d) 1 : 9

3. Two vessels A and B contain milk and water in the ratio of 5 : 2 and 4 : 1, respectively. Find the ratio in which these two qualities are to be mixed to get a new mixture of milk and water in the ratio 3 : 1.
(a) 9 : 6 (b) 7 : 5
(c) 8 : 5 (d) 6 : 5

4. 300 gm of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% in the solution?
(a) 50 gm (b) 70 gm
(c) 60 gm (d) 80 gm

5. A person buys two watches for Rs. 1000. He sells one at a loss of 5% and the other at 20% gain and on the whole he gains Rs. 50. Find the cost price of each watch.
(a) Rs. 300 (b) Rs. 450
(c) Rs. 350 (d) Rs. 400

6. A man borrows a total sum of Rs. 10000 from two sources. To one he pays 10% and to the other 5% per annum simple interest. If the total interest paid by him every year is Rs. 700, how much did he borrow at each rate of interest?
(a) Rs. 4000, Rs. 6000 (b) Rs. 3000, Rs. 5000
(c) Rs. 3500, Rs. 5500 (d) Rs. 5000, Rs. 7000

7. One type of liquid contains 25% of milk, the other contains 30% of milk. A can is filled with 6 parts of the first liquid and 4 parts of the second liquid. Find the percentage of milk in the mixture.
(a) 29% (b) 31%
(c) 27% (d) 33%

8. A man lends Rs. 4000 for 3 years. A part he lends at the rate of 10% per annum at simple interest and the rest at the rate of 12% per annum at simple interest. If he receives Rs. 1350 in all as interest on both the sums, how much money does he lend at 12%?
(a) Rs. 2000 (b) Rs. 2250
(c) Rs. 2500 (d) Rs. 2750

9. A mixture of 45 litres of spirit and water, contains 20% of water in it. How much water must be added to it to make the water 25% in the new mixture?
(a) 5 litres (b) 4 litres
(c) 3 litres (d) 6 litres

10. How many kg of salt at 42 P per kg must a man mix with 25 kg of salt at 24 P per kg, so that he may, on selling the mixture at 40 kg, gain 25% on the outlay?
(a) 5 (b) 20
(c) 25 (d) 30

Test II

[Time: 9 min]

11. A mixture of a certain quantity of milk with 16 litres of water is worth 90 P per litre. If pure milk be worth 108 P per litre how much milk is there in the mixture?
(a) 80 litres (b) 60 litres
(c) 70 litres (d) 90 litres

12. Vimal mixes 80 kg of sugar worth Rs. 6.75 per kg with 120 kg of sugar worth Rs. 8 per kg. At what rate shall he sell the mixture to gain 20%?
(a) Rs. 7.50 (b) Rs. 9
(c) Rs. 8.20 (d) Rs. 8.85

13. How many kg of tea worth Rs. 29 per kg must be blended with 25 kg of tea worth Rs. 32 per kg so that by selling the blended variety at Rs. 33 per kg there should be 10% gain?
(a) 45 kg (b) 33 kg
(c) 25 kg (d) 50 kg

14. The expenditure and savings of an employee are in the ratio 3 : 1. His income increases by 16 % but at the same time his expenditure also increases by 20%. Find increase or decrease in his savings.
(a) 5.5% (b) 6%
(c) 4% (d) 4.5%

15. A vessel of 80 litres is filled with milk and water. 70% of milk and 30% of water is taken out of the vessel. It is found that the vessel is vacated by 55%. Find the initial quantity of milk and water.
(a) 50 litres, 30 litres (b) 60 litres, 40 litres
(c) 40 litres, 20 litres (d) 70 litres, 50 litres

16. In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, these are 580. How many pigeons are there?
(a) 120 (b) 130
(c) 110 (d) 140

17. A person bought two tables for Rs. 2200. He sells one at 5% loss and the other at 6% profit and thus on the whole he neither gains nor loses. Find the cost price of each table.
(a) Rs. 1000, Rs. 1200 (b) Rs. 2000, Rs. 1200
(c) Rs. 1500, Rs. 1000 (d) Rs. 2500, Rs. 1200.

18. The average daily wages of staff, consisting of supervisors and labourers of a company is Rs. 50. The average wages of supervisors is Rs. 150 while that of labourers is Rs. 40 per day. If the number of supervisors is 15, find the number of labourers in the company.

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(a) 350 (b) 150
(c) 200 (d) 250

19. A jar full of whisky contains 40% of alcohol. A part of this whisky is replaced by another containing 19% alcohol and the percentage of alcohol was found to be 26. The quantity of whisky replaced is

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{5}$

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution Initially, the mixture contains 3 litres of alcohol and 12 litres of water.

Afterwards, the mixture contains 3 litres of alcohol and 15 litres of water

$\therefore$ Percentage of alcohol $= \left(\frac{3}{18}\times 100\right)\% = \mathbf{16\frac{13}{7}\%}$.

3. Ans. (b)

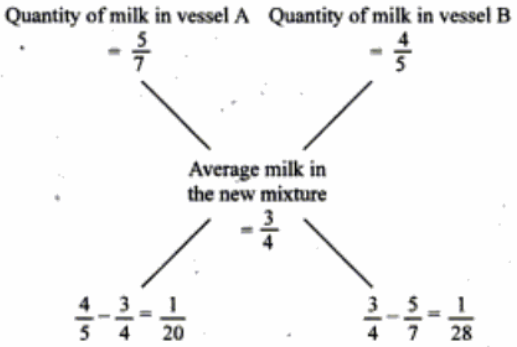
Solution Ratio of milk and water in vessel $A = 5 : 2$

$\therefore$ The quantity of milk in the vessel $A = \frac{5}{7}$ part of the whole

Ratio of milk and water in vessel $B = \frac{4}{5}$ part of the whole

Ratio of milk and water in the new mixture = 3 : 1

$\therefore$ The quantity of milk in new mixture $= \frac{3}{4}$ part of the whole



Quantity of cheaper milk : Quantity of superior milk (Mixture in vessel A): (mixture in vessel B)

$= \frac{1}{20} : \frac{1}{28}$ **or 7 : 5.**

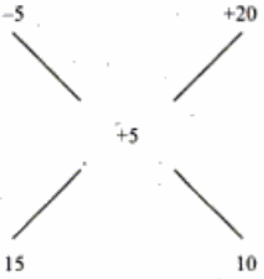
5. Ans. (d)

Solution Overall % profit $= \frac{50}{1000} \times 100 = 5\%$

15 : 10 or 3 : 2

C.P. of watch sold at 5% loss

$= \frac{3}{3+2} \times 1000 =$ Rs. 600

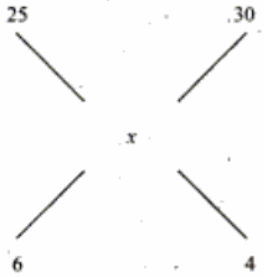


C. P. of watch sold at 20% gain

$= 1000 - 600 =$ **Rs. 400.**

7. Ans. (c)

Solution This question can be solved by the method of alligation



$\frac{30-x}{x-25} = \frac{6}{4} = \frac{3}{2}$

or, $60 - 2x = 3x - 75$, or, $5x = 60 + 75$

$\therefore$ $\mathbf{x = 27\%}$.

9. Ans. (c)

Solution Quantity of water in mixture $= 45 \times \frac{20}{100} = 9$ litres

Quantity of spirit in mixture = 36 litres

Ratio of spirit to water in the new mixture = 1 : 3

$\frac{\text{Quantity of water}}{\text{Quantity of spirit}} = \frac{1}{3}$, $\frac{\text{Quantity of water}}{36} = \frac{1}{3}$

Quantity of water $= \frac{36}{3} = 12$ litres

$\therefore$ Water to be added $= 12 - 9 =$ **3 litres.**

11. Ans. (a)

Solution The mean value is 90 P and the price of water is OP

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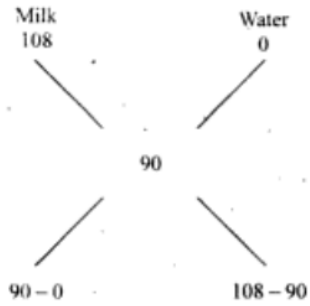


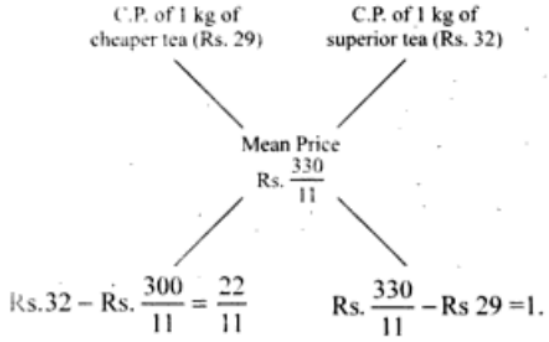
By the Alligation rule, milk and water are in the ratio of 5 : 1

∴ Quantity of milk in the mixture

$= 5 \times 16 =$ **80 litres.**

13. Ans. (d)

Solution $C.P. = \frac{33 \times 100}{110} = Rs.\ \frac{330}{11}$



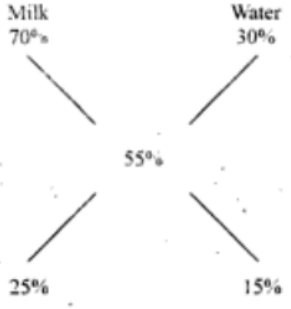
$$\frac{\text{Quantity of cheaper tea}}{\text{Quantityof superior tea}} = \frac{2}{1}$$

$$\Rightarrow \frac{\text{Quantity of cheaper tea}}{25} = \frac{2}{1}$$

Quantity of cheaper tea = **50 kg.**

15. Ans. (a)

Solution Here, the % values of milk and water that is taken from the vessel should be taken into consideration.



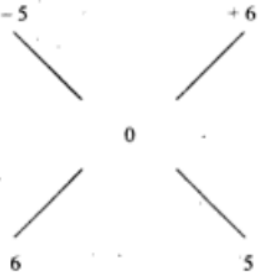
Ratio of milk to water = 5 : 3

∴ Quantity of milk $= \frac{80}{5+3} \times 5 = 50$ litres

and, Quantity of water $= \frac{80}{5+3} \times 3 =$ **30 litres.**

17. Ans. (a)

Solution



C.P. of the table sold at 5% loss

$= \frac{6}{6+5} \times 2200 = Rs.\ 1200$

C.P. of the table sold at 6% profit

$= \frac{5}{6+5} \times 2200 =$ **Rs. 1000.**

19. Ans. (c)

Solution Using the method of alligation,

Required ratio = 7 : 14 = 1 : 2

∴ Required quantity = $\frac{2}{3}$.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (c)	**6.** (a)	**8.** (c)	**10.** (b)	**12.** (b)	**14.** (c)	**16.** (c)	**18.** (c)

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16 Problems on Ages

INTRODUCTION

Problems based on ages are generally asked in most of the competitive examinations. To solve these problems, the knowledge of linear equations is essential. In such problems, there may be three situations:

(i) Age some years ago
(ii) Present age (iii) Age some years hence

Two of these situations are given and it is required to find the third. The relation between the age of two persons may also be given. Simple linear equations are framed and their solutions are obtained. Sometimes, short cut methods given below are also helpful in solving such problems.

SOME USEFUL SHORT-CUT METHODS

1. If the age of A, t years ago, was n times the age of B and at present A's age is n_2 times that of B, then

$$A\text{'s present age} = \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2\, t \text{ years}$$

and, $$B\text{'s present age} = \left(\frac{n_1 - 1}{n_1 - n_2}\right) t \text{ years}$$

Explanation

Let the present age of B be x years.

Then, the present age of $A = n_2 x$ years

Given, t years ago,

$n_1 (x - t) = n_2 x - t$ or, $(n_1 - n_2)\, x = (n_1 - 1)\, t$

or, $$x = \left(\frac{n_1 - 1}{n_1 - n_2}\right) t.$$

Therefore, B's present age $= \left(\frac{n_1 - 1}{n_1 - n_2}\right) t$ years

and, A's present age $= \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2\, t$ years .

Illustration 1 The age of father is 4 times the age of his son. If 5 years ago father's age was 7 times the age of his son at that time, what is father's present age?

Solution The father's present age

$$= \left(\frac{n_1 - 1}{n_1 - n_2}\right) n_2 t \quad [\text{Here, } n_1 = 7,\ n_2 = 4 \text{ and } t = 5]$$

$$= \left(\frac{7 - 1}{7 - 4}\right) 4 \times 5 = \frac{6 \times 4 \times 5}{3} = \mathbf{40\ years.}$$

2. The present age of A is n_1 times the present age of B. If t years hence, the age of A would be n_2 times that of B, then

$$A\text{'s present age} = \left(\frac{n_2 - 1}{n_1 - n_2}\right) n_2\, t \text{ years}$$

and B's present age $= \left(\frac{n_2 - 1}{n_1 - n_2}\right) t$ years

Explanation

Let the present age of B be x years.

Then, the present age of $A = n_1 x$

Given, t years hence,

$$(n_1 x + t) = n_2 (x + t)$$

or, $$(n_1 - n_2) x = (n_2 - 1) t$$

or, $$x = \left(\frac{n_2 - 1}{n_1 - n_2}\right) t$$

Therefore, B's present age $= \left(\frac{n_2 - 1}{n_1 - n_2}\right) n_1 t$ years

and, A's present age $= \left(\frac{n_2 - 1}{n_1 - n_2}\right) n_1 t$ years.

Illustration 2 The age of Mr. Gupta is four times the age of his son. After ten years, the age of Mr. Gupta will be only twice the age of his son. Find the present age of Mr. Gupta's son.

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Solution The present age of Mr. Gupta's son

$$= \left(\frac{n_2 - 1}{n_1 - n_2}\right) t$$

$$= \left(\frac{2-1}{4-2}\right) 10$$

[Here, $n_1 = 4$, $n_2 = 2$ and $t = 10$]

$$= \mathbf{5 \text{ years.}}$$

3. The age of A, t_1 years ago, was n_1 times the age of B. If t_2 years hence A's age would be n_2 times that of B, then,

$$A\text{'s present age} = \frac{n_1(t_1 + t_2)(n_2 - 1)}{n_1 - n_2} + t_1 \text{ years}$$

and, B's present age $= \dfrac{t_2(n_2 - 1) + t_1(n_1 - 1)}{n_1 - n_2}$ years.

Explanation

Let A's present age $= x$ years and B's present age $= y$ years.

Given : $x - t_1 = n_1 (y - t_1)$ and $x + t_2 = n_2 (y + t_2)$

i.e. $x - n_1 y = (1 - n_1) t_1$ ...(1)

and $x - n_2 y = (-1 + n_2) t_2$ ...(2)

Solving (1) and (2), we get

$$x = \frac{n_1(t_1 + t_2)(n_2 - 1)}{n_1 - n_2} + t_1$$

and, $$y = \frac{t_2(n_2 - 1) + t_1(n_1 - 1)}{n_1 - n_2}.$$

Illustration 3 10 years ago Anu's mother was 4 times older than her daughter. After 10 years, the mother will be twice older than the daughter. Find the present age of Anu.

Solution Present age of Anu

$$= \frac{t_2(n_2 - 1) + t_1(n_1 - 1)}{n_1 - n_2}$$

[Here, $n_1 = 4$, $n_2 = 2$, $t_1 = 10$ and $t_2 = 10$]

$$= \frac{10(2-1) + 10(4-1)}{4-2} = \frac{10 + 30}{2} = \mathbf{20 \text{ years.}}$$

4. The sum of present ages of A and B is S years. If, t years ago, the age of A was n times the age of B, then

$$\text{Present age of } A = \frac{Sn - t(n-1)}{n+1} \text{ years,}$$

$$\text{and Present age of } B = \frac{S + t(n-1)}{n+1} \text{ years.}$$

Explanation

Let the present ages of A and B be x and y years respectively.

Given: $x + y = S$ ...(1)

and, $x - t = n (y - t)$

or, $x - ny = (1 - n) t$ ...(2)

Solving (1) and (2), we get

$$x = \frac{Sn - t(n-1)}{n+1}$$

and, $$y = \frac{S + t(n-1)}{n+1}.$$

Illustration 4 The sum of the ages of A and B is 42 years. 3 years back, the age of A was 5 times the age of B. Find the difference between the present ages of A and B.

Solution Here, $S = 42$, $n = 5$ and $t = 3$

$\therefore$ Present age of A

$$= \frac{Sn - t(n-1)}{n+1} = \frac{42 \times 5 - 3(5-1)}{5+1}$$

$$= \frac{198}{6} = 33 \text{ years}$$

and, present age of B

$$= \frac{5 + t(n+1)}{n+1} = \frac{42 + 3(5-1)}{5+1}$$

$$= \frac{54}{6} = \mathbf{9 \text{ years.}}$$

$\therefore$ Difference between the present ages of A and B $= 33 - 9 = 24$ years.

Note: If, instead of sum (S), difference (D) of their ages is given, replace S by D and in the denominator $(n + 1)$ by $(n - 1)$ in the above formula.

5. The sum of present ages of A and B is S years. If, t years hence, the age of A would be n times the age of B, then

$$\text{present age of } A = \frac{Sn + t(n-1)}{n+1} \text{ years}$$

$$\text{and, present age of } B = \frac{S - t(n-1)}{n+1} \text{ years.}$$

Explanation

Let the present ages of A and B be x and y years, respectively

Given: $x + y = S$ ...(1)

and, $x + t = n (y + t)$

or, $x - ny = t (n - 1)$ ...(2)

Solving (1) and (2), we get

$$x = \frac{Sn + t(n-1)}{n+1}$$

and, $$y = \frac{S - t(n-1)}{n+1}.$$

Illustration 5 The sum of the ages of a son and father is 56 years. After four years, the age of the father will be three times that of the son. Find their respective ages.

Solution The age of father

$$= \frac{Sn + t(n-1)}{n+1} = \frac{56 \times 3 + 4(3-1)}{3+1}$$

[Here, $S = 56$, $t = 4$ and $n = 3$]

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$$= \frac{176}{4} = 44 \text{ years.}$$

The age of son $= \dfrac{S - t(n-1)}{n+1}$

$$= \frac{56 - 4(3-1)}{3+1}$$

$$= \frac{48}{4} = \mathbf{12 \text{ years.}}$$

6. If the ratio of the present ages of A and B is $a : b$ and t years hence, it will be $c : d$, then

A's present age $= \dfrac{at(c-d)}{ad-bc}$

and, B's present age $= \dfrac{bt(c-d)}{ad-bc}$.

Illustration 6 The ratio of the age of father and son at present is 6 : 1. After 5 years, the ratio will become 7 : 2. Find the present age of the son.

Solution The present age of the son $= \dfrac{bt(c-d)}{ad-bc}$

[Here, $a = 6$, $b = 1$, $c = 7$, $d = 2$ and $t = 5$]

$$= \frac{1 \times 5(7-2)}{6 \times 2 - 1 \times 7} = \mathbf{5 \text{ years.}}$$

Note: If, with the ratio of present ages, the ratio of ages t years ago is given, then replace t by $(-t)$ in the above formula.

Illustration 7 6 years ago Mahesh was twice as old as Suresh. If the ratio of their present ages is 9 : 5 respectively, what is the difference between their present ages?

Solution Present age of Mahesh

$$= \frac{-at(c-d)}{ad-bc}$$

$$= \frac{-9 \times 6(2-1)}{1 \times 9 - 5 \times 2}$$

[Here, $a = 9$, $b = 5$, $c = 2$, $d = 1$ and $t = 6$]

$= 54$ years

Present age of Suresh

$$= \frac{-bt(c-d)}{ad-bc}$$

$$= \frac{-5 \times 6(2-1)}{1 \times 9 - 5 \times 2} = 30 \text{ years.}$$

$\therefore$ Difference of their ages $= 54 - 30 = \mathbf{24 \text{ years.}}$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Ten years ago, Mohan was thrice as old as Ram was but 10 years hence, he will be only twice as old. Find Mohan's present age.
(a) 60 years (b) 80 years
(c) 70 years (d) 76 years

2. The ages of Ram and Shyam differ by 16 years. Six years ago, Mohan's age was thrice as that of Ram's, find their present ages.
(a) 14 years, 30 years (b) 12 years, 28 years
(c) 16 years, 34 years (d) 18 years, 38 years

3. 15 years hence, Rohit will be just four times as old as he was 15 years ago. How old is Rohit at present?
(a) 20 (b) 25
(c) 30 (d) 35

4. A man's age is 125% of what it was 10 years ago, but $83\frac{1}{3}\%$ of what it will be after ten 10 years. What is his present age?
(a) 45 years (b) 50 years
(c) 55 years (d) 60 years

5. If twice the son's age in years be added to the father's age, the sum is 70 and if twice the father's age is added to the son's age, the sum is 95. Father's age is
(a) 40 years (b) 35 years
(c) 42 years (d) 45 years

6. Three years ago, the average age of a family of 5 members was 17. A baby having been born the average age of the family is the same today? What is the age of the child?
(a) 3 years (b) 5 years
(c) 2 years (d) 1 year

7. The ratio of A's and B's ages is 4 : 5. If the difference between the present age of A and the age of B 5 years hence is 3, then what is the total of present ages of A and B?
(b) 68 years (b) 72 years
(c) 76 years (d) 64 years

8. The ages of A and B are in the ratio of 6 : 5 and sum of their ages is 44 years. The ratio of their ages after 8 years will be
(a) 4 : 5 (b) 3 : 4
(c) 3 : 7 (d) 8 : 7

9. One year ago the ratio between Samir and Ashok's age was 4 : 3. One year hence the ratio of their ages will be 5 : 4. What is the sum of their present ages in years?

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(a) 12 years (b) 15 years
(c) 16 years (d) Cannot be determined

10. Ratio of Ashok's age to Pradeep's age is 4 : 3. Ashok will be 26 years old after 6 years. How old is Pradeep now?
(a) 18 years (b) 21 years
(c) 15 years (d) 24 years

11. Jayesh is as much younger to Anil as he is older to Prashant. If the sum of the ages of Anil and Prashant is 48 years, what is the age of Jayesh?
(a) 20 years (b) 24 years
(c) 30 years (d) Cannot be determined

12. Five years ago Mr. Sohanlal was thrice as old as his son and ten years hence he will be twice as old as his son. Mr. Sohanlal's present age (in years) is
(a) 35 (b) 45
(c) 50 (d) 55

13. Three times the present age of a father is equal to eight times the present age of his son. Eight years hence the father will be twice as old as his son at that time. What are their present ages?
(a) 35, 15 (b) 32, 12
(c) 40, 15 (d) 27, 8

14. The sum of ages of a father and son is 45 years. Five years ago, the product of their ages was four times the father's age at that time. The present age of the father is
(a) 39 years (b) 36 years
(c) 25 years (d) None of these

15. One year ago a father was four times as old as his son. In 6 years time his age exceeds twice his son's age by 9 years. Ratio of their ages is
(a) 13 : 4 (b) 12 : 5
(c) 11 : 3 (d) 9 : 2

16. The ages of A, B and C together total 185 years. B is twice as old as A and C is 17 years older than A. Then, the respective ages of A, B and C are
(a) 40, 86 and 59 years (b) 42, 84 and 59 years
(c) 40, 80 and 65 years (d) None of these

17. A father's age is three times the sum of the ages of his two children, but 20 years hence his age will be equal to the sum of their ages. Then the father's age is
(a) 30 years (b) 40 years
(c) 35 years (d) 45 years

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let Mohan's present age be x years and Ram's present age be y years.

Then, according to the first condition,

$x - 10 = 3(y - 10)$

or, $x - 3y = -20$ ...(1)

Now, Mohan's age after 10 years $= (x + 10)$ years

Ram's age after 10 years $= (y + 10)$

$\therefore \quad (x + 10) = 2(y + 10)$

or, $x - 2y = 10$ ...(2)

Solving (1) and (2), we get

$x = 70$ and $y = 30$

$\therefore$ Mohan's age = 70 years and Ram's age **= 30 years.**

2. Ans. (a)

Solution Let Ram's age = x years

So, Mohan's age = $(x + 16)$ years

Also, $3(x - 6) = x + 16 - 6$ or, $x = 14$

$\therefore$ Ram's age = 14 years

and, Mohan's age = $14 + 16$ **= 30 years.**

3. Ans. (b)

Solution Let the present age of Rohit be x years

Then, given: $x + 15 = 4(x - 15) \Rightarrow x = 25$.

4. Ans. (b)

Solution Let the present age be x years.

Then, 125% of $(x - 10) = x$

and, $83\frac{1}{3}\%$ of $(x + 10) = x$

$\therefore$ 125 % of $(x - 10) = 83\frac{1}{3}\%$ of $(x + 10)$

or, $\frac{5}{4}(x - 10) = \frac{5}{6}(x + 10)$

or, $\frac{5}{4}x - \frac{5}{6}x = \frac{50}{6} + \frac{50}{4}$

or, $\frac{5x}{12} = \frac{250}{12}$ or, x **= 50 years.**

5. Ans. (a)

Solution Let son's age (in years) = x and father's age (in years) = y

Given: $2x + y = 70$ and, $x + 2y = 95$

Solving for y, we get y **= 40.**

6. Ans. (c)

Solution Present age of 5 members

$= 5 \times 17 + 3 \times 5 = 100$ years

Also, present ages of 5 members + Age of the baby

$= 6 \times 17 = 102$ years

$\therefore$ Age of the baby = $102 - 100$ **= 2 years.**

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7. Ans. (b)

Solution Given: $\frac{A}{B} = \frac{4}{5}$ or, $B = \frac{5}{4}A$

and, $B - (A + 5) = 3$ or, $B = A + 8$

$\therefore \quad \frac{5}{4}A = A + 8$

or, $A\left(\frac{5}{4} - 1\right) = 8$

$\therefore \quad A = 32$ years

and, $B = \frac{5}{4} \times 32 = 40$ years

$\therefore \quad A + B = 40 + 32 = \mathbf{72\ years.}$

8. Ans. (d)

Solution Let present ages (in years) of A and B respectively, be $6x$ and $5x$

Given: $6x + 5x = 44 \Rightarrow x = 4$

Ratio of ages after 8 years will be

$6x + 8 : 5x + 8$

or, $32 : 28$ **or, 8: 7.**

9. Ans. (c)

Solution Let one year ago

Samir's age be $4x$ years

and, Ashok's age be $3x$ years

Present age of Samir = $(4x + 1)$ years

Present age of Ashok = $(3x + 1)$ years

One year hence

Samir's age = $(4x + 2)$ years

Ashok's age = $(3x + 2)$ years

According to question,

$\frac{4x+2}{3x+2} = \frac{5}{4} \Rightarrow 16 + 8 = 15x + 10$

or, $x = 2.$

$\therefore$ Sum of their present ages = $4x + 1 + 3x + 1$

$= 7x + 2$

$= 7 \times 2 + 2 = \mathbf{16\ years.}$

10. Ans. (c)

Solution Let the present ages of Ashok and Pradeep be $4x$ and $3x$

So that $4x + 6 = 26 \Rightarrow x = 5$

$\therefore$ Present age of Pradeep is $3x = 3 \times 5$, i.e. 15 years

11. Ans. (b)

12. Ans. (c)

Solution Let Mr. Sohanlal's age (in years) = x

and his son's age = y

Then, $x - 5 = 3(y - 5)$ i.e. $x - 3y + 10 = 0$

and, $x + 10 = 2(y + 10)$ i.e. $x - 2y - 10 = 0$

Solving the two equations, we get

$x = 50, y = \mathbf{20.}$

13. Ans. (b)

14. Ans. (b)

Solution Let father's present age = x years

Then, son's present age = $(45 - x)$ years

Given: $(x - 5)(45 - x - 5) = 4(x - 5)$

or, $x^2 - 41x + 180 = 0$ or, $(x - 36)(x - 5) = 0$

$\therefore \quad x = \mathbf{36\ years.}$

15. Ans. (c)

Solution Let the present ages of father and son be x and y years, respectively

Then, $(x - 1) = 4(y - 1)$

or, $4y - x = 3$ ...(1)

and, $(x + 6) - 2(y + 6) = 9$

or, $-2y + x = 15$ ...(2)

Solving (1) and (2), we get, $x = 33, y = 9$

$\therefore$ Ratio of their ages $= \mathbf{33 : 9 = 11 : 3.}$

16. Ans. (b)

Solution Let A's age be x years

B's age be $2x$ years

C's age = $(x + 17)$ years

According to the question,

$x + 2x + (x + 17) = 185$

$\therefore \quad 4x = 185 - 17 = 168 \quad \therefore \quad x = 42$

$\therefore$ A's age = 42 years

B's age = 84 years

C's age = $42 + 17 = \mathbf{59\ years.}$

17. Ans. (a)

Solution Let the present age of father be x years and the present age of son be y years.

$\therefore \quad x = 3y$ ...(1)

Also, $(x + 20) = (y + 20 + 20)$ ...(2)

Solving (1) and (2), we get

$x = \mathbf{30\ years.}$

QUESTIONS FROM COMPETITIVE EXAMS

1. A's age is 1/6th of B's age. B's age will be twice of C's age after 10 years. If C's eighth birthday was celebrated two years ago, then the present age of A must be

(a) 5 years (b) 10 years

(c) 15 years (d) 20 years

[Based on MAT 2002]

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2. Sachin was twice as old as Ajay 10 years back. How old is Ajay today if Sachin will be 40 years old 10 years hence?
(a) 20 years (b) 10 years
(c) 30 years (d) None of these
[Based on MAT 2005]

3. A demographic survey of 100 families in which two parents were present revealed that the average age A, of the oldest child, is 20 years less than 1/2 the sum of the ages of the two parents. If F represents the age of one parent and M, the age of the other parent, then which of the following equivalent to A?
(a) $\frac{F+M-20}{2}$ (b) $\frac{F+M}{2}+20$
(c) $\frac{F+M}{2}-20$ (d) $F+M-10$
[Based on MAT 2001]

4. If 10 years are subtracted from the present age of Ram and the remainder divided by 14, then you would get the present age of his grandson Shyam. If Shyam is 9 years younger to Sunder whose age is 14, then what is the present age of Ram?
(a) 80 years (b) 70 years
(c) 60 years (d) None of these
[Based on MAT 2001]

5. Two groups of student, whose average ages are 20 years and 30 years, combine to form a third group whose average age is 23 years. What is the ratio of the number of students in the first group to the number of students in the second group?
(a) 5 : 2 (b) 2 : 5
(c) 7 : 3 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

6. A year ago, a father was four times his son's age. In six years, his age will be 9 more than twice his son's age. What is the present age of the son?
(a) 10 years (b) 9 years
(c) 20 years (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

7. In 1932, I was as old as the number formed by the last two digits of my birth year. When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him also. The year of birth of my grandfather is
(a) 1866 (b) 1886
(c) 1906 (d) 1916
[Based on IITTM, Gwalior, 2003]

8. The average age of three boys is 16 years. If their ages are in the ratio 4 : 5 : 7, then the age of the youngest boy is
(a) 8 years (b) 9 years
(c) 12 years (d) 16 years
[Based on FMS (Delhi) 2003]

9. "I am eight times as old as you were when I was as old as you are", said a man to his son. Find out their present ages if the sum of their ages is 75 years.
(a) 47 yrs and 28 yrs (b) 42 yrs and 33 yrs
(c) 48 yrs and 27 yrs (d) 46 yrs and 26 yrs
(e) None of the above **[Based on IIFT 2003]**

10. My brother was 3 years of age when my sister was born, while my mother was 26 years of age when I was born. If my sister was 4 years of age when my brother was born, then what was the age of my father and mother respectively when my brother was born?
(a) 35 yrs, 33 yrs (b) 35 yrs, 29 yrs
(c) 32 yrs, 23 yrs (d) 32 yrs, 29 yrs
(e) None of these **[Based on IIFT 2003]**

11. Namrata's father is now four times her age. In five years, he will be three times her age. In how many years, will he be twice her age?
(a) 5 (b) 20
(c) 25 (d) 15
[Based on SCMHRD Ent. Exam. 2003]

12. A father is twice as old as his son. 20 years back, he was twelve times as old as the son. What are their present ages?
(a) 24, 12 (b) 44, 22
(c) 48, 24 (d) None of these
[Based on IMT Ghaziabad, 2002]

13. The present ages of three persons are in the proportion of 4 : 7 : 9. Eight years ago, the sum of their ages was 56. Find their present ages.
(a) 20, 35, 45 (b) 8, 20, 28
(c) 16, 28, 36 (d) None of these
[Based on IMT Ghaziabad, 2002]

14. If 6 years is subtracted from the present age of Randheer and the remainder is divided by 18, then the present age of his grandson Anup is obtained. If Anup is 2 years younger to Mahesh, whose age is 5 years, then what is the age of Randheer?
(a) 84 years (b) 96 years
(c) 48 years (d) 60 years
[Based on MAT 2008]

15. The ratio of Ashok's age to Pardeep's is 4 : 3. Ashok will be 26 year old after 6 years. How old is Pardeep now?
(a) 18 years (b) 21 years
(c) 15 years (d) 24 years
[Based on MAT 2008]

16. Ten years ago, the ages of the members of a joint family of eight people added up to 231 years. Three years later, one member died at the age of 60 years and a child was born during the same year. After another three years, one more member died, again at 60, and a child was born during the same year. The current average age of this eight-member joint family is nearest to
(a) 21 years (b) 25 years
(c) 24 years (d) 23 years
(e) 22 years **[Based on CAT 2007]**

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EXPLANATORY ANSWERS

1. Ans. (a)

Solution $A = \frac{1}{6}B$, $B + 10 = 2(C + 10)$, $C = 10$

$\therefore$ $B = 30$, $A = 5$.

2. Ans. (a)

Solution Sachin's age today = 30 years

Sachin's age 10 years back = 20 years

Ajay's age 10 years back = 10 years

Ajay's age today = **20 years.**

3. Ans. (c)

Solution $A = \frac{F + M}{2} - 20$.

4. Ans. (a)

Solution Let present age of Ram be x years.

$\therefore$ $\frac{x - 10}{14} = 14 - 9 \Rightarrow x = 80$.

5. Ans. (c)

Solution Let the number of students in the two groups be x and y respectively.

$\therefore$ $20x + 30y = 23(x + y) \Rightarrow 3x = 7y \Rightarrow \frac{x}{y} = \frac{7}{3}$.

6. Ans. (b)

Solution $(F - 1) = 4(S - 1)$ ...(i)

where F and S are the Father's and the Son's ages respectively at present.

$\therefore$ $(F + 6) = 2(S + 6) + 9$ ...(ii)

From (i) and (ii), $S = 9$.

7. Ans. (a)

8. Ans. (c)

Solution Let the ages of the three boys be $4K$, $5K$, $7K$ respectively.

$\therefore$ $16K = 48 \Rightarrow K = 3$

$\therefore$ Age of the youngest boy = $4K = 12$.

9. Ans. (c)

Solution When I was 27 years old, which you are at present, you were 6 years old.

And, so now, I am 48 years old, i.e. 8 times of 6 years.

10. Ans. (e)

11. Ans. (b)

Solution Let Namrata's age = x years

Let Namrata's father's age = y years

$\therefore$ $y = 4x$ and $y + 5 = 3(x + 5)$

$\therefore$ $x = 10$, $y = 10$

Let $y + K = 2(x + K)$

(i.e. After K years, the father will be double of her age)

$\Rightarrow$ $40 + K = 2(10 + K) \Rightarrow K = 20$

$\therefore$ After 20 years, Namrata's father will be double of her age.

12. Ans. (b)

Solution $F = 2S$, $F - 20 = 12(S - 20)$

$\Rightarrow$ $2S - 20 = 12S - 240 \Rightarrow S = 22$

$\therefore$ $F = 44$.

13. Ans. (c)

Solution Let the present ages of three persons be $4K$, $7K$ and $9K$ respectively.

$\therefore$ $(4K - 8) + (7K - 8) + (9K - 8) = 56$

$\Rightarrow$ $20K = 80 \Rightarrow K = 4$.

14. Ans. (D)

Solution Anup's age = 5 – 2 = 3 years

$\therefore$ If Randheer's age is x years

$\frac{x - 6}{18} = 3$

$\therefore$ $x = 60$ years.

15. Ans. (c)

16. Ans. (a)

Solution 10 years age, total age of 8 members = 231

after there years, sum of the ages = $231 + 8 \times 3 - 60$

$= 195$

There more years later sum of ages

$= 198 + 8 \times 3 - 60 = 159$

Current average age = $\frac{191}{8} = 24$ years.

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EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. Today Mohan is 20 years younger than his father. Ten years ago he was one-half as old as his father. How old will his father be 10 years hence?
   (a) 40 years (b) 60 years
   (c) 50 years (d) Data inadequate
   (e) None of these

2. The difference between the ages of two persons is 10 years. 15 years ago, the elder one was twice as old as the younger one. The present age of the elder person is
   (a) 25 years (b) 35 years
   (c) 45 years (d) 55 years

3. The ratio of Laxmi's age to the age of her mother is 3 : 11. The difference of their ages is 24 years. The ratio of their ages after 3 years will be
   (a) 1 : 3 (b) 2 : 3
   (c) 3 : 5 (d) None of these

4. The age of a man is three times the sum of the ages of his three children. After three years his age will be twice the sum of their ages. Find the present age of the man.
   (a) 40 years (b) 25 years
   (c) 35 years (d) 45 years

5. If 6 years are subtracted from the present age of Randheer and the remainder is divided by 18, then the present age of his grandson Anup is obtained. If Anup is 2 years younger to Mahesh whose age is 5 years, then what is the age of Randheer?
   (a) 96 years (b) 84 years
   (c) 48 years (d) 60 years

6. The difference of present ages of A and B is 12 years. If 6 years back their ages were in the ratio 3 : 2, how old are A and B?
   (a) 42 years, 30 years (b) 36 years, 24 years
   (c) 48 years, 36 years (d) 44 years, 32 years

7. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is
   (a) 29 years (b) 39 years
   (c) 19 years (d) 49 years

8. Ages of A and B are in the ratio 4 : 3. A will be 26 years old after 6 years. B's present age is:
   (a) 15 years (b) 20 years
   (c) 12 years (d) 18 years

9. A mother tells her daughter " I was half your present age when you were born". If the mother is 18 years older than her daughter, what is the present age of the mother?
   (a) 54 (b) 63
   (c) 36 (d) 72

10. Ten years ago A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages?
    (a) 8 years (b) 20 years
    (c) 35 years (d) 45 years

Test II

[Time: 9 min]

11. The sum of the ages of A and B is 60 years. After 10 years A will be twice as old as B. Find their present ages.
    (a) 40 years, 8 years (b) 50 years, 10 years
    (c) 60 years, 20 years (d) 70 years, 30 years

12. If Rahul as $\frac{1}{3}$rd as old as Vineet 5 years back and Rahul is 17 years old now, how old is Vineet now?
    (a) 41 years (b) 36 years
    (c) 9 years (d) 51 years
    (e) None of these

13. After five years the age of a father will be twice the age of his son, whereas five years ago, he was seven times as old as his son was. What is father's present age?
    (a) 35 years (b) 40 years
    (c) 45 years (d) 50 years

14. 20 years ago my age was $\frac{1}{3}$ of what it is now. What is my present age?
    (a) 30 years (b) 35 years
    (c) 40 years (d) 45 years

15. The ratio of father's age to son's age is 3 : 1. The product of their ages is 243. The ratio of their ages 3 years hence will be
    (a) 3 : 1 (b) 5 : 1
    (c) 7 : 3 (d) 5 : 2

16. Vinay was twice as old as Dev 10 years back. How old is Dev today if Vinay will be 40 years old 10 years hence?
    (a) 20 years (b) 10 years
    (c) 30 years (d) 15 years

17. Neena was married 6 years ago. Today, her age is $1\frac{1}{4}$ times of her age at the time of marriage. The age

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of her son is $\frac{1}{10}$ times of her present age. What is the age of her son?
(a) 3 years (b) 2 years
(c) 4 years (d) 4 years
(e) None of these

18. If $1\frac{1}{2}$ years age are added to $\frac{3}{7}$ age of Sohan, he will be half his age. What is his actual age?
(a) 20 years (b) 21 years
(c) 25 years (d) 9 years

19. Rakesh is as much younger to Ajay as he is older to Anil. If the sum of the ages of Ajay and Anil is 48 years, what is the age of Rakesh?
(a) 36 years (b) 24 years
(c) 30 years (d) 28 years

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution Let Mohan's father's present age = x years
Mohan's present age = $(x - 20)$ years

Given: $(x - 20) - 10 = \frac{1}{2}(x - 10)$

$\Rightarrow \quad 2(x - 30) = x - 10$

$\therefore \quad x = 50.$

$\therefore$ 10 years hence father's age will be $50 + 10 =$ **60 years.**

3. Ans. (a)

Solution Given: $11x - 3x = 24 \Rightarrow x = 3$

$\therefore$ Ratio of their ages after 3 years

$= 3x + 3 : 11x + 3 = 12 : 36 =$ **1 : 3.**

5. Ans. (d)

Solution Anup's age = $(5 - 2)$ years = 3 years
Let Randheer's age be x years

Given: $\frac{x-6}{18} = 3$

or, $\quad x = 54 + 6 =$ **60 years.**

7. Ans. (b)

Solution Let the present age of B and A be x years and $(x + 9)$ years.

Given: $(x + 9 + 10) = 2(x - 10)$ or $x =$ **39.**

9. Ans. (a)

Solution Let the present age of the mother = 54 years
$\therefore$ Daughter's age = 36 years
When mother was 36 years of age, daughter's age = 18 years

Now, $18 = \frac{1}{2} \times$ **36.**

11. Ans. (b)

Solution Here $A + B = 60$ ... (1)
Also, $(A + 10) = 3(B + 10)$

or, $\quad A - 3B = 20$ ... (2)

Solving (1) and (2), we get
$A = 50$ and $B = 10$
$\therefore$ A's Age = 50 years
B's age = **10 years.**

13. Ans. (b)

Solution Let son's age 5 years hence = x years
Then, father's age 5 years hence = $(3x)$ years
Given: $7(x - 10) = 3x - 10$ or $x = 15$
$\therefore$ Father's present age = $3x - 5 =$ **40 years.**

15. Ans. (d)

Solution Let ages (in years) of father and son be $3x$ and x.
Then, $(3x)(x) = 243$

$\therefore \quad x^2 = 81$ or, $x = 9$

$\therefore$ Present age (in years) of father and son are 27 and 9.

After 3 years, Ratio of ages will be
$27 + 3 : 9 + 3$

i.e. **30 : 12 i.e. 5 : 2.**

17. Ans. (a)

Solution Let Neena's present age = x years
Neena's age at the time of marriage = $(x - 6)$ years

$\therefore \quad x = \frac{5}{4}(x - 6)$

$\therefore \quad x = 30$

Age of Neena's son = $\frac{1}{10} \times 30 =$ **3 years**

19. Ans. (b)

Solution Let there be three numbers a, b, and c. If b is as much less than a as it is more than c. than $b = \frac{a+c}{2}$

$\because \quad a - b = b - c \Rightarrow 2b = a + c$

or, $\quad b = \frac{a+c}{2}$

Applying this rule, the age of Rakesh is $\frac{48}{2}$, **i.e. 24 years.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (d)	**6.** (a)	**8.** (c)	**10.** (c)	**12.** (a)	**14.** (a)	**16.** (a)	**18.** (b)

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17 Simple Interest

INTRODUCTION

When a person A borrows some money from another person B, then A has to pay certain amount to B for the use of this money. This amount paid by A is called *Interest*. The total amount of money borrowed by A from B is called the *Principal*. The money paid back to B, which comprises of the principal and the interest is called the *Amount*.

In other words,

Amount = Principal + Interest

The interest is usually charged according to a specified term, which is expressed as some per cent of the principal and is called the *rate of interest* for the fixed period of time. This fixed period may be a year, six months, three months or a month and correspondingly the rate of interest is charged annually, semi-annually, quarterly or monthly. For example, the rate of interest is 5% per annum means the interest payable on Rs. 100 for one year is Rs. 5.

Interest can be of two types :

1. Simple Interest 2. Compound Interest

SIMPLE INTEREST

When the interest is payable on the principal only, it is called *simple interest*. For example, simple interest on Rs. 100 at 5% per annum will be Rs. 5 each year, that is, at the end of one year, total amount will be Rs. 105. At the end of second year, it will be Rs. 110 and so on.

Thus, simple interest is the interest computed on the principal for the entire period it is borrowed.

In this chapter, we shall limit ourselves to simple interest. Compound interest will be discussed in the next chapter.

SOME BASIC FORMULAE

If P stands for Principal, R the rate per cent per annum, T the number of years, I the simple interest and A the amount, then

$$1.\ \text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

$$\text{or,}\quad I = \frac{P \times R \times T}{100}$$

Illustration 1 Find the simple interest on Rs. 5200 for 2 years at 6% per annum.

Solution Here, P = Rs. 5200, T = 2 years and R = 6%

$$\therefore\ \text{Simple interest} = \frac{P \times R \times T}{100} = \frac{5200 \times 6 \times 2}{100}$$

= Rs. 624.

$$2.\ \text{Principal} = \frac{100 \times \text{Simple Interest}}{\text{Rate} \times \text{Time}}$$

$$\text{or,}\quad P = \frac{100 \times I}{R \times T}$$

Illustration 2 A man earns Rs. 450 as an interest in 2 years on a certain sum invested with a company at the rate of 12 per cent per annum. Find the sum invested by the man in the company.

Solution We have, I = Rs. 450, T = 2 years, R = 12% per annum

$$\therefore\ P = \frac{I \times 100}{R \times T} = \frac{450 \times 100}{12 \times 2} = \text{Rs. } 1875.$$

Thus, the money invested by the man was **Rs. 1875.**

$$3.\ \text{Rate} = \frac{100 \times \text{Simple Interest}}{\text{Principal} \times \text{Time}}$$

$$\text{or,}\quad R = \frac{100 \times I}{P \times T}$$

Illustration 3 At what rate per annum will a sum of Rs. 5000 amount to Rs. 6000 in 4 years?

Solution Here, P = Rs. 5000, A = Rs. 6000, T = 4 years

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So, $I = A - P$ = Rs. (6000 − 5000) = Rs. 1000

$$\therefore \quad R = \frac{100 \times I}{P \times T} = \frac{100 \times 1000}{5000 \times 4} = \textbf{5\%.}$$

4. $$\text{Time} = \frac{100 \times \text{Simple Interest}}{\text{Rate} \times \text{Principal}}$$

or, $$T = \frac{100 \times I}{R \times P}$$

Illustration 4 In what time will Rs. 1200 earn an interest of Rs. 240 at 5% per annum?

Solution Here, P = Rs. 1200, I = Rs. 240, R = 5%

$$\therefore \quad T = \frac{100 \times I}{P \times R} = \frac{100 \times 240}{1200 \times 5} = \textbf{4 years.}$$

5. $$\text{Amount} = \text{Principal} + \text{Simple Interest}$$
$$= \text{Principal} + \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$
$$= \text{Principal}\left(1 + \frac{\text{Rate} \times \text{Time}}{100}\right)$$

or, $$A = P\left(1 + \frac{R \times T}{100}\right)$$

Illustration 5 Mahesh borrowed Rs. 3000 from his friend Suresh at 15 per cent per annum for 3 years. Find the interest and money returned by Mahesh to Suresh.

Solution Here, P = Rs. 3000, R = 15% per annum, T = 3 years

$$\therefore \quad I = \frac{P \times R \times T}{100} = \frac{3000 \times 15 \times 3}{100} = \text{Rs. } 1350.$$

$$\therefore \quad A = P + I = \text{Rs. } 3000 + \text{Rs. } 1350 = \text{Rs. } 4350.$$

Thus, Mahesh paid Rs. 1350 as interest to Suresh and the amount returned by Mahesh to Suresh **= Rs. 4350.**

SOME USEFUL SHORT-CUT METHODS

1. If a certain sum in T years at R% per annum amounts to Rs. A, then the sum will be

$$P = \frac{100 \times A}{100 + R \times T}$$

Explanation

Let the principal be Rs. x

$\therefore$ Simple interest = Rs. $(A - x)$

$$\therefore \quad A - x = \frac{x \times R \times T}{100}$$

$$\Rightarrow \quad 100A - 100x = xRT$$

$$\Rightarrow \quad (100 + RT)x = 100A$$

$$\therefore \quad x = \frac{100 \times A}{100 + R \times T}.$$

Illustration 6 What principal will amount to Rs 570 at 4% per annum in $3\frac{1}{2}$ years?

Solution We have, A = Rs. 570, R = 4% per annum, $T = \frac{7}{2}$ years.

$$\therefore \quad P = \frac{100 \times A}{100 + R \times T} = \frac{100 \times 570}{100 + 4 \times 7/2}$$

$$= \frac{100 \times 570}{114} = \text{Rs. } 500$$

Thus, Rs. 500 will amount to Rs. 570 at 4% per annum in $\mathbf{3\frac{1}{2}}$ **years.**

2. The annual payment that will discharge a debt of Rs. A due in T years at R% per annum is

$$\text{Annual payment} = \text{Rs.} \left(\frac{100A}{100T + \frac{RT(T-1)}{2}}\right)$$

Explanation

Let the annual payment be Rs. x.

Since the first instalment is paid at the end of first year,

$\therefore$ Amount of first instalment at the end of t years

$$= x + \frac{(T-1) \times R \times x}{100}$$

Similarly, amount of second instalment at the end of t years

$$= x + \frac{(T-2) \times R \times x}{100}, \text{ and so on.}$$

Thus, total amount of T instalments

$$A = \left[x + \frac{(T-1) \times R \times x}{100}\right] + \left[x + \frac{(T-2) \times R \times x}{100}\right] + \ldots + x$$

$$= Tx + \frac{Rx}{100}[(T-1) + (T-2) + \ldots + 1]$$

$$= Tx + \frac{Rx}{100}\left[\frac{(T-1) \times T}{2}\right]$$

or, $$100\,Tx + Rx\left[\frac{T(T-1)}{2}\right] = 100A$$

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$$\text{or, } x\left[100T+\frac{RT(T-1)}{2}\right]=100A$$

$$\therefore\ x=\frac{100A}{100T+\frac{RT(T-1)}{2}}.$$

Illustration 7 Find the annual instalment that will discharge a debt of Rs. 12900 due in 4 years at 5% per annum simple interest.

Solution Here, A = Rs. 12900, T = 4 years, R = 5% per annum

$$\therefore \text{Annual instalment} = \frac{100\times A}{100\times T+\frac{RT(T-1)}{2}}$$

$$=\frac{100\times 12900}{(100\times 4)+\frac{5(4-1)\times 4}{2}}$$

$$=\frac{100\times 12900}{400+30}=\frac{100\times 12900}{430}$$

$$= \textbf{Rs. 3000.}$$

3. If a certain sum is invested in n types of investments in such a manner that equal amount is obtained on each investment where interest rates are R_1, R_2, R_3, ..., R_n respectively and time periods are T_1, T_2, T_3, ..., T_n respectively, then the ratio in which the amounts are invested is

$$\frac{1}{100+R_1T_1}:\frac{1}{100+R_2T_2}:\frac{1}{100+R_3T_3}:\ldots\frac{1}{100+R_nT_n}.$$

Explanation

Let $P_1, P_2, \ldots, P_n$ be invested in n types of investments whose interest rates are $R_1, R_2, \ldots, R_n$ and time periods are $T_1, T_2, \ldots, T_n$.

Then, $P_1=\frac{100\times A}{100+R_1T_1}$

$P_2=\frac{100\times A}{100+R_2T_2}$

$\vdots \qquad \vdots$

$P_n=\frac{100\times A}{100+R_nT_n}$.

$\therefore\ P_1 : P_2 : \ldots : P_n$

$$=\frac{100\times A}{100+R_1T_1}:\frac{100\times A}{100+R_2T_2}:\ldots\frac{100\times A}{100+R_nT_n}$$

$$=\frac{1}{100+R_1T_1}:\frac{1}{100+R_2T_2}:\ldots\frac{1}{100+R_nT_n}$$

[$\because$ the amount A is same for all]

Illustration 8 A sum of Rs. 1586 is divided among three such parts that amount obtained on these three parts of money after 2, 3 and 4 years, respectively at the rate of 5% per annum remains equal. Find such three parts of the sum.

Solution Since the amount accured from each of the three parts of Rs. 1586 at the rate of 5% p.a. in 2, 3 and 4 years, respectively remains equal, such three parts of Rs. 1586 will be in the ratio of

$$\frac{1}{100+R_1T_1}:\frac{1}{100+R_2T_2}:\frac{1}{100+R_3T_3}$$

Hence, the ratio

$$=\frac{1}{100+5\times 2}:\frac{1}{100+5\times 3}:\frac{1}{100+5\times 4}$$

$$=\frac{1}{110}:\frac{1}{115}:\frac{1}{120}$$

$$=\frac{1\times 30360}{110}:\frac{1\times 30360}{115}:\frac{1\times 30360}{120}$$

($\because$ L.C.M. of 110, 115 and 120 is 30360)

$\therefore$ ratio = 276 : 264 : 253

Sum of proportionals = 276 + 264 + 253 = 793

$\therefore$ 1st part $=\frac{276}{793}\times 1586$ = Rs. 552,

2nd part $=\frac{264}{793}\times 1586$ = Rs. 528

and, 3rd part $=\frac{253}{793}\times 1586$ = **Rs. 506.**

4. If a certain sum of money becomes n times itself in T years at simple interest, then the rate of interest per annum is

$$R=\frac{100(n-1)}{T}\%$$

Explanation

Let Rs. P become Rs. nP in t years

$\therefore$ Simple interest I is given by

$$I = nP - P = (n-1)P$$

$\therefore$ Rate of interest R is given by

$$R=\frac{100\times I}{P\times T}=\frac{100\times(n-1)P}{P\times T}=\frac{100(n-1)}{T}.$$

Illustration 9 A certain sum of money trebles itself in 5 years simple interest. Find the rate per cent per annum.

Solution Here, n = 3, T = 5 years

$$\therefore\ R=\frac{100(n-1)}{T}\%=\frac{100(3-1)}{5}\%=\textbf{40\%}.$$

5. If a certain sum of money becomes n times itself at R% per annum simple interest in T years, then

$$T=\left(\frac{n-1}{R}\right)\times 100 \text{ years.}$$

Illustration 10 In what time a sum of money will double itself at a rate of simple interest of 8% p.a.?

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Solution Required time $(T) = \dfrac{(n-1)\times 100}{R}$ years

$= \dfrac{(2-1)\times 100}{8}$ years

$= \mathbf{12\frac{1}{2}}$ **years.**

6. If a certain sum of money becomes n times itself in T years at a simple interest, then the time T' in which it will become m times itself is given by

$$T' = \left(\frac{m-1}{n-1}\right)\times T \text{ years.}$$

Explanation

Let the principal be Rs. P.

Let it become m times in T' years.

Then, the amount in T years = Rs. nP

and the amount in T' years = Rs. mP.

$\therefore \quad nP - P = \dfrac{P\times R\times T}{100}$

or, $\quad (n-1)P = \dfrac{P\times R\times T}{100}$ ...(1)

and, $\quad (m-1)P = \dfrac{P\times R\times T'}{100}$ ...(2)

$\therefore \quad \dfrac{(m-1)P}{(n-1)P} = \dfrac{P\times R\times T'}{100}\times\dfrac{100}{P\times R\times T}$

or, $\quad \dfrac{m-1}{n-1} = \dfrac{T'}{T}$

$\therefore \quad T' = \left(\dfrac{m-1}{n-1}\right) T$ years.

Illustration 11 A sum of money put out on simple interest doubles itself in $12\frac{1}{2}$ years. In how many years would it treble itself?

Solution Here, $n = 2$, $m = 3$, $T = \dfrac{25}{2}$ years.

$\therefore$ Required time $(T') = \left(\dfrac{m-1}{n-1}\right)\times T$ years

$= \left(\dfrac{3-1}{2-1}\right)\times\dfrac{25}{2}$ years

$= \mathbf{25}$ **years.**

7. Effect of change of P, R and T on simple interest is given by the following formula :

Change in Simple Interest

$= \dfrac{\text{Product of fixed parameter}}{100}$

$\times$ [difference of product of variable parameters]

For example, if rate (R) changes from R_1 to R_2 and P, T are fixed, then

Change in SI $= \dfrac{PT}{100}\times(R_1 - R_2)$

Similarly, if principal (P) changes from P_1 to P_2 and R, T are fixed, then change in SI $= \dfrac{RT}{100}\times(P_1 - P_2)$

Also, if rate (R) changes from R_1 to R_2 and time (T) changes from T_1 to T_2 but principal (P) is fixed, then change in SI $= \dfrac{P}{100}\times(R_1T_1 - R_2T_2)$.

Illustration 12 If simple interest on Rs. 600 increases by Rs. 30, when the rate % increases by 4% per annum, find the time.

Solution Here, $P = 600$, change in SI $= 30$, $R_1 - R_2 = 4$, $T = ?$

Using, change in SI $= \dfrac{PT}{100}\times(R_1 - R_2)$

we have, $30 = \dfrac{600T}{100}\times 4 \Rightarrow T = \dfrac{5}{4}$, i.e. $\mathbf{1\frac{1}{4}}$ **years.**

Illustration 13 If the simple interest on Rs. 1400 be more than the interest on Rs. 1000 by Rs. 60 in 5 years, find the rate per cent per annum.

Solution Here, change in SI $= 60$, $P_1 - P_2 = 400$, $T = 5$, $R = ?$

Using change in SI $= \dfrac{RT}{100}\times(P_1 - P_2)$

We have, $60 = \dfrac{5R}{100}\times 400 \Rightarrow \boldsymbol{R = 3\%}$.

Illustration 14 If the simple interest on a certain sum at 4% per annum for 4 years is Rs. 80 more than the interest on the same sum for 3 years at 5% per annum, find the sum.

Solution Here, change in SI $= 80$, $R_1 = 4$, $R_2 = 5$, $T_1 = 4$, $T_2 = 3$, $P = ?$

Using change in SI $= \dfrac{P}{100}\times(R_1T_1 - R_2T_2)$

We have, $80 = \dfrac{P}{100}\times(4\times 4 - 5\times 3)$

$\Rightarrow \quad \boldsymbol{P}$ **= Rs. 8000.**

8. If a debt of Rs. Z is paid in 'n' number of instalments and if the value of each instalment is Rs. a, then the borrowed (debt) amount is given by

$$Z = na + \frac{Ra}{100\times b}\times\frac{n(n-1)}{2}$$

where, R = rate of interest per annum

b = no. of instalments/year

$b = 1$, when each instalment is paid yearly

$b = 2$, when each instalment is paid half-yearly

$b = 4$ when each instalment is paid quarterly

$b = 12$, when each instalment is paid monthly.

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Illustration 15 A sum of Rs. 2 is lent to be paid back in 3 equal monthly instalments of Re. 1 each. Find the rate per cent.

Solution Here, Z = Rs. 2, a = Re. 1, $n = 3$, $b = 12$, $R = ?$

Using the formula

$$Z = na + \frac{Ra}{100 \times b} \times \frac{n(n-1)}{2},$$

we have, $2 = 3 \times 1 + \frac{R \times 1}{100 \times 12} \times \frac{3 \times 2}{2} \Rightarrow R = 400\%$

$\therefore$ The rate % p.a. is **400%.**

9. If a certain sum of money P lent out at SI amounts to A_1 in T_1 years and to A_2 in T_2 years, then

$$P = \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$$

and, $$R = \frac{A_1 - A_2}{A_1T_2 - A_2T_1} \times 100\%$$

Illustration 16 If a certain sum of money at simple interest amounts to Rs. 5184 in 2 years and to Rs. 5832 in 3 years, what is the sum and the rate of interest?

Solution Principal $= \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$

$\left[\begin{matrix}\text{Here, } A_1 = 5184, A_2 = 5832 \\ T_1 = 2, T_2 = 3\end{matrix}\right]$

$$= \frac{5184 \times 3 - 5832 \times 2}{3 - 2} = \text{Rs. } 3888$$

and, Rate $= \frac{(A_2 - A_1) \times 100}{T_1A_2 - T_2A_1} = \frac{(5832 - 5184) \times 100}{2 \times 5832 - 3 \times 5184}$

$$= \frac{64800}{3888} = \mathbf{16\frac{2}{3}\%.}$$

10. If a certain sum of money P lent out for a certain time T amounts to A_1 at R_1% per annum and to A_2 at R_2% per annum, then

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$$

and, $$T = \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100 \text{ years.}$$

Illustration 17 A certain sum is invested for certain time. It amounts to Rs. 450 at 7% per annum. But when invested at 5% per annum, it amounts to Rs. 350. Find the sum and time.

Solution Here, $A_1 = 450$, $R_1 = 7$, $A_2 = 350$, $R_2 = 5$.

Using the formula,

$$P = \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$$

We get, $P = \frac{350 \times 7 - 450 \times 5}{7 - 5} = \text{Rs. } 100$

Also, using the formula,

$$T = \left(\frac{A_1 - A_2}{A_2R_1 - A_1R_2}\right) \times 100$$

we get, $T = \left(\frac{450 - 350}{350 \times 7 - 450 \times 5}\right) \times 100 = \mathbf{5 \text{ years.}}$

11. If an amount P_1 lent at simple interest rate of R_1% per annum and another amount P_2 at simple interest rate of R_2% per annum, then the rate of interest for the whole sum is

$$R = \left(\frac{P_1R_1 + P_2R_2}{P_1 + P_2}\right).$$

Illustration 18 Mohan deposits Rs. 5000 in NSC at 2% per annum and Rs. 2000 in mutual funds at 4% per annum. Find the rate of interest for the whole sum.

Solution Here, $P_1 = 5000$, $R_1 = 2$, $P_2 = 2000$, $R_2 = 4$.

Using the formula

$$R = \left(\frac{P_1R_1 + P_2R_2}{P_1 + P_2}\right)$$

We get, $R = \frac{5000 \times 2 + 2000 \times 4}{5000 + 2000} = \mathbf{2\frac{4}{7}\%.}$

12. If a certain sum of money is lent out in n parts in such a manner that equal sum of money is obtained as simple interest on each part where interest rates are $R_1, R_2, \ldots, R_n$ respectively and time periods are $T_1, T_2, \ldots, T_n$ respectively, then the ratio in which the sum will be divided in n parts is given by

$$\frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \ldots \frac{1}{R_nT_n}.$$

Explanation

Let the n equal parts be $P_1, P_2, \ldots, P_n$ and let I be the equal interest earned on each part.

Then, $$P_1 = \frac{I \times 100}{R_1T_1}$$

$$P_2 = \frac{I \times 100}{R_2T_2}$$

$$\vdots \qquad \vdots$$

$$P_n = \frac{I \times 100}{R_nT_n}.$$

$$\therefore P_1 : P_2 : \ldots : P_n = \frac{I \times 100}{R_1T_1} : \frac{I \times 100}{R_2T_2} : \ldots \frac{I \times 100}{R_nT_n}$$

$$= \frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \ldots \frac{1}{R_nT_n}.$$

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Illustration 19 If a sum of Rs. 1600 is divided into two such parts that the simple interest on the first part for $2\frac{1}{2}$ years at the rate of 4% p.a. equals the simple interest on the second part for 5 years at the rate of 3% p.a., then find two such divisions of the sum.

Solution Ratio of one part to other part of Rs. 1600

$$= \frac{1}{R_1T_1} : \frac{1}{R_2T_2}$$

$\therefore$ 1st part : 2nd part $= \frac{1}{4\times 5/2} : \frac{1}{3\times 5}$

[Here, R_1 = 4% p.a., $T_1 = \frac{5}{2}$ years, R_2 = 3% p.a., T_2 = 5 years]

or, 1st part : 2nd part $= \frac{1}{10} : \frac{1}{15} = 3 : 2$

Sum of proportionals = 3 + 2 = 5

$\therefore$ 1st part $= \frac{3}{5}\times 1600$ = Rs. 96

and, 2nd part $= \frac{2}{5}\times 1600$ = **Rs. 640.**

13. When there is a change in principal (P), Rate (R) and Time (T), then the value of simple interest I also changes and is given by

$$\frac{I_1}{I_2} = \frac{P_1\times R_1\times T_1}{P_2\times R_2\times T_2}$$

$$\Rightarrow \quad \frac{A_1 - P_1}{A_2 - P_2} = \frac{P_1\times R_1\times T_1}{P_2\times R_2\times T_2}$$

as $I_1 = A_1 - P_1$ and $I_2 = A_2 - P_2$.

Illustration 20 If Rs. 85 amounts to Rs. 95 in 3 years, what Rs. 102 will amount to in 5 years at the same rate per cent?

Solution Here, P_1 = Rs. 85, A_1 = Rs. 95, T_1 = 3 years, P_2 = Rs. 102, T_2 = 5 years, $R_1 = R_2 = R$ (say).

Then, using the formula

$$\frac{A_1 - P_1}{A_2 - P_2} = \frac{P_1\times R_1\times T_1}{P_2\times R_2\times T_2}$$

We have, $\frac{95-85}{A_2 - 102} = \frac{85\times R\times 3}{102\times R\times 5}$

$\Rightarrow \quad A_2 - 102 = 20$

$\Rightarrow \quad A_2 = 122$

$\therefore$ The amount is **Rs. 122.**

14. Out of a certain sum P, $\frac{1}{a}$ part is invested at R_1%, $\frac{1}{b}$ part at R_2% and the remainder $\left(1-\frac{1}{a}-\frac{1}{b}\right)$ say $\frac{1}{c}$ part at R_3%. If the annual income from all these investments is Rs. A, then the original sum is given by

$$P = \left(\frac{A\times 100}{\frac{R_1}{a}+\frac{R_2}{b}+\frac{R_3}{c}}\right).$$

Illustration 21 Out of a certain sum, $\frac{1}{3}$rd is invested at 3%, $\frac{1}{6}$th at 6% and the rest at 8%. If the annual income is Rs. 300, then the original sum is

Solution Here, $\frac{1}{a} = \frac{1}{3}$, $\frac{1}{b} = \frac{1}{6}$,

$$\frac{1}{c} = 1 - \left(\frac{1}{3}+\frac{1}{6}\right) = \frac{1}{2},$$

R_1 = 3%, R_2 = 6%, R_3 = 8%, A = Rs. 300.

$\therefore$ The original sum $= \frac{A\times 100}{\frac{R_1}{a}+\frac{R_2}{b}+\frac{R_3}{c}}$

$$= \frac{300\times 100}{\frac{3}{3}+\frac{6}{6}+\frac{8}{2}} = \frac{30000}{6}$$

= **Rs. 5000.**

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The simple interest on Rs. 500 at 6% per annum from May 3rd to July 15th in the same year is
   (a) Rs. 9 (b) Rs. 6
   (c) Rs. 4 (d) None of these

2. Mr. Irani borrowed a sum of Rs. 10000 from a finance company for 6 years at 8% per annum. The amount returned by Mr. Irani to the finance company is

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(a) Rs. 14800 (b) Rs. 12600
(c) Rs. 13300 (d) None of these

3. The principal that will yield Rs. 60 as simple interest at 6% per annum in 5 years is
(a) Rs. 175 (b) Rs. 350
(c) Rs. 200 (d) None of these

4. The sum of money that will produce Rs. 1770 interest in $7\frac{1}{2}$ years at 8% simple interest per annum is
(a) Rs.2950 (b) Rs. 3120
(c) Rs. 2800 (d) None of these

5. If the simple interest on a certain sum of money after $6\frac{1}{4}$ years is 3/8 of the principal, then the rate of interest per annum is
(a) 5% (b) 6%
(c) 4% (d) None of these

6. Rakesh borrowed Rs. 5000 from Ganesh at simple interest. If Ganesh got Rs. 500 more than his capital after 5 years, then the rate of interest per annum is
(a) 2% (b) 3%
(c) 4% (d) None of these

7. The rate per cent per annum at which Rs. 1200 amount to Rs. 1440 in 4 years, is
(a) 5% (b) 4%
(c) 6% (d) None of these

8. If simple interest on a certain sum of money is Rs. 256 and the rate of interest per annum equals the number of years, then the rate of interest is
(a) 13% (b) 14%
(c) 16% (d) None of these

9. If the simple interest on a certain sum of money for 2 years is one-fifth of the sum, then the rate of interest per annum is
(a) 9% (b) 10%
(c) 8% (d) None of these

10. If the simple interest on a certain sum of money is 4/25th of the sum and the rate per cent equals the number of years, then the rate of interest per annum is
(a) 2% (b) 3%
(c) 4% (d) None of these

11. If a certain sum of money borrowed at 5% per annum simple interest amounts to Rs. 1020 in 4 years, then the sum of money borrowed is
(a) Rs. 850 (b) Rs. 925
(c) Rs. 750 (d) None of these

12. In what time will Rs. 1200 amount to Rs. 1344 at 6% per annum?
(a) $2\frac{1}{2}$ years (b) 3 years
(c) 2 years (d) None of these

13. In what time will Rs. 8100 produce the same income at 3% as Rs. 225 in 4 years at 3%.
(a) $\frac{1}{7}$ years (b) $\frac{1}{9}$ years
(c) $\frac{1}{6}$ years (d) None of these

14. If Rs. 1000 be invested at interest rate of 5% and the interest be added to the principal every 10 years, then the number of years in which it will amount to Rs. 2000 is
(a) $16\frac{2}{3}$ years (b) $16\frac{1}{4}$ years
(c) 16 years (d) None of these

15. If Rs. 500 amounts to Rs. 725 at 9% simple interest in some time, what will Rs. 600 amount to at 11% in the same time?
(a) Rs. 870 (b) Rs. 930
(c) Rs. 910 (d) None of these

16. Sumit lends Rs. 10000 for 2 years at 20% per annum simple interest. After 1 year he gets Rs. 6000. How much will he get next year?
(a) Rs. 5900 (b) Rs. 6400
(c) Rs. 7200 (d) None of these

17. What principal will amount to Rs. 15000 at 10% per annum in 5 years?
(a) Rs. 10000 (b) Rs. 8700
(c) Rs. 10500 (d) None of these

18. The annual payment that will discharge a debt of Rs. 47250 due 3 years hence at the rate of 5% simple interest is
(a) Rs. 8000 (b) Rs. 10000
(c) Rs. 15000 (d) None of these

19. The annual instalment that will discharge a debt of Rs. 4200 due in 5 years at 10% simple interest is
(a) Rs. 700 (b) Rs. 750
(c) Rs. 800 (d) None of these

20. If the amount obtained by Mahesh by investing Rs. 1500 for $2\frac{1}{2}$ years at the rate of 8% p.a. is equal to the amount obtained by Suresh by investing a certain sum for 2 years at 5% p.a. simple interest, then the sum invested by Suresh is
(a) Rs. $1636\frac{4}{11}$ (b) Rs. 1636
(c) Rs. $1636\frac{1}{2}$ (d) None of these

21. A man invests Rs. 3965 in the names of his three daughters Neeta, Sita and Gita in such a way that they get the same amount after 2, 3 and 4 years, respectively. If the rate of interest is 5% p.a., then the amount invested for Neeta, Sita and Gita is
(a) Rs. 1380, Rs. 1320, Rs. 1265 (b) Rs. 1330, Rs. 1360, Rs. 1380

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(c) Rs. 1265, Rs. 1320, Rs. 1340 (d) None of these

22. A sum of money at simple interest becomes four times in 24 years. The rate per cent of interest per annum is

(a) $13\frac{3}{4}\%$ (b) $12\frac{1}{2}\%$

(c) $11\frac{3}{4}\%$ (d) None of these

23. In how many years will a sum of money treble itself at 10% per annum simple interest?

(a) 15 years (b) 19 years

(c) 20 years (d) None of these

24. A sum of money doubles itself in 8 years. In how many years will it treble?

(a) 16 years (b) 15 years

(c) 14 years (d) None of these

25. A sum was put at simple interest at a certain rate for 4 years. Had it been put at 2% higher rate, it would have fetched Rs. 56 more. Find the sum.

(a) Rs. 680 (b) Rs. 700

(c) Rs. 720 (d) None of these

26. If the interest on Rs. 800 be more than the interest on Rs. 400 by Rs. 40 in 2 years, then the rate of interest per annum is

(a) 5% (b) $5\frac{1}{2}\%$

(c) 6% (d) None of these

27. If the difference between the simple interest on a certain sum for 4 years at $2\frac{1}{2}\%$ per annum and the simple interest on the same sum for the same period at 3% per annum is Rs. 60, then the sum is

(a) Rs. 3000 (b) Rs. 2900

(c) Rs. 3100 (d) None of these

28. If a certain sum of money at simple interest amounts to Rs. 2800 in 2 years and to Rs. 3250 in 5 years, then the rate of interest per annum is

(a) 4% (b) 6%

(c) 5% (d) None of these

29. If a certain sum of money amounts to Rs. 1760 in two years and Rs. 2000 in 5 years at simple interest, then the sum is

(a) Rs. 1960 (b) Rs. 1590

(c) Rs. 1600 (d) None of these

30. A certain sum is invested for certain time. It amounts to Rs. 450 at 7% per annum. But when invested at 5% per annum, it amounts to Rs. 350. Find the sum.

(a) Rs. 60 (b) Rs. 100

(c) Rs. 120 (d) None of these

31. A certain sum is invested for T years. It amounts to Rs. 400 at 10% per annum. But when invested at 4% per annum, it amounts to Rs. 200. Find the time (T).

(a) 41 years (b) 39 years

(c) 50 years (d) None of these

32. If a sum of Rs. 9 is lent to be paid back in 10 equal monthly instalments of Re. 1 each, then the rate of interest is

(a) $266\frac{2}{3}\%$ (b) $265\frac{3}{4}\%$

(c) 266% (d) None of these

33. A sum of Rs. 7700 is to be divided among three brothers Vikas, Vijay and Viraj in such a way that simple interest on each part at 5% per annum after 1, 2 and 3 years, respectively remains equal. The share of Vikas is more than that of Viraj by

(a) Rs. 2800 (b) Rs. 2500

(c) Rs. 3000 (d) None of these

34. If simple interest on a certain sum of money for 4 years at 5% p.a. is same as the simple interest on Rs. 560 for 10 years at the rate of 4% p.a., then the sum of money is

(a) Rs. 1190 (b) Rs. 1120

(c) Rs. 1210 (d) None of these

35. Mr. Mani invested an amount of Rs. 12000 at the simple interest rate of 10% per annum and another amount at the simple interest rate of 20% per annum. The total interest earned at the end of one year on the total amount invested became 14% per annum. Find the total amount invested.

(a) Rs. 20000 (b) Rs. 20800

(c) Rs. 21000 (d) None of these

36. Mr. Gupta deposits Rs. 3000 in a bank at 10% per annum and Rs. 5000 in another bank at 8% per annum. The rate of interest for the whole sum is

(a) $8\frac{1}{2}\%$ (b) $8\frac{3}{4}\%$

(c) 8% (d) None of these

37. A person invested $\frac{2}{3}$rd of his capital at 3%, $\frac{1}{6}$th at 6% and the remainder at 12%. If his annual income is Rs. 25, then the capital is

(a) Rs. 490 (b) Rs. 510

(c) Rs. 500 (d) None of these

38. The simple interest on a sum of money will be Rs. 600 after 10 years. If the principal is trebled after 5 years, what will be the total interest at the end of the tenth year?

(a) Rs. 1200 (b) Rs. 1190

(c) Rs. 1210 (d) None of these

39. Rs. 1500 is invested at a rate of 10% simple interest and interest is added to the principal after every 5 years. In how many years will it amount to Rs. 2500.

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(a) $6\frac{1}{9}$ years (b) $6\frac{1}{4}$ years

(c) 7 years (d) None of these

40. Sumit lent some money to Mohit at 5% per annum simple interest. Mohit lent the entire amount to Birju on the same day at $8\frac{1}{2}$% per annum. In this transaction after a year Mohit earned a profit of Rs. 350. Find the sum of money lent by Sumit to Mohit.

(a) Rs. 9000 (b) Rs. 10000

(c) Rs. 10200 (d) None of these

41. Brinda borrowed Rs. 1000 to build a hut. She pays 5% simple interest. She lets the hut to Ramu and receives the rent of Rs. $12\frac{1}{2}$ per month from Ramu. In how many years Brinda ought to clear off the debt?

(a) 10 years (b) $10\frac{1}{4}$ years

(c) $10\frac{1}{2}$ years (d) None of these

42. The rate of interest on a sum of money is 4% per annum for the first 2 years, 6% per annum for the next 4 years and 8% per annum for the period beyond 6 years. If the simple interest accrued by the sum for a total period of 9 years is Rs. 1120, then the sum is

(a) Rs. 2400 (b) Rs. 2200

(c) Rs. 2000 (d) None of these.

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Time from May 3rd to July 15th
= 28 days of May + 30 days of June and 15 days of July
= 73 days = $\frac{73}{365}$ years, i.e. $\frac{1}{5}$ years.

$\therefore\ I = \frac{P \times R \times T}{100} = \frac{500 \times 6 \times \frac{1}{5}}{100}$ = **Rs. 6.**

2. Ans. (a)

Solution We have, P = Rs. 10000, R = 8% per annum, T = 6 years.

$\therefore\ I = \frac{P \times R \times T}{100} = \frac{10000 \times 8 \times 6}{100}$ = Rs. 4800

$\therefore\ A = P + I$ = 10000 + 4800 = Rs. 14800

Thus, Mr. Irani returned Rs. 14800 to the finance company.

3. Ans. (c)

Solution Here, I = Rs. 60, R = 6% per annum, T = 5 years.

$\therefore$ Principal $(P) = \frac{100 \times I}{R \times T} = \frac{100 \times 60}{6 \times 5}$ = **Rs. 200.**

4. Ans. (a)

Solution Here, I = Rs. 1770, R = 8% per annum, $T = \frac{15}{2}$ years.

$\therefore$ Principal $(P) = \frac{100 \times I}{R \times T} = \frac{100 \times 1770}{8 \times \frac{15}{2}}$

= **Rs. 2950.**

5. Ans. (b)

Solution Let the sum of money be Rs. x.

Then, simple interest = $\frac{3}{8}x$

Also, time = $6\frac{1}{4}$ years, i.e. $\frac{25}{4}$ years

$\therefore$ Rate (R) $= \frac{100 \times I}{P \times T} = \frac{100 \times \frac{3x}{8}}{x \times \frac{25}{4}}$

$= \frac{100 \times 3}{2 \times 25}$ = **6%.**

6. Ans. (a)

Solution Here, P = Rs. 5000, I = Rs. 500, T = 5 years.

Therefore, using the formula

$R = \frac{100 \times I}{P \times T}$

We have, rate of interest $(R) = \frac{100 \times 500}{5000 \times 5}$ = **2% p.a.**

7. Ans. (a)

Solution We have, P = Rs. 1200, T = 4 years,
I = 1440 −1200 = Rs. 240.

$\therefore$ Rate $(R) = \frac{100 \times I}{P \times T} = \frac{100 \times 240}{1200 \times 4}$ = **5% per annum.**

8. Ans. (c)

Solution Here, I = Rs. 256.

Let the principal be Rs. 100.

Let the rate of interest per annum be x%.

Then, time $(T) = x$ years.

Therefore, using the formula

$R = \frac{100 \times I}{P \times T}$

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We have, $x = \frac{100 \times 256}{100 \times x} \Rightarrow x^2 = 256$ or $x = 16\%$.

$\therefore$ Rate of interest per annum is **16%.**

9. Ans. (b)

Solution We have, $T = 2$ years.

Let the principal be Rs. x

Then, simple interest (I) = Rs. $x/5$.

$$\text{Rate of interest } (R) = \frac{100 \times I}{P \times T} = \frac{100 \times \frac{x}{5}}{x \times 2}$$

$$= \frac{100}{5 \times 2} = \textbf{10\% p.a.}$$

10. Ans. (c)

Solution Let the principal be Rs. x, then the simple interest (I) = $\frac{4}{25}x$.

Let the rate of interest p.a. be r% then time (T) = r years.

$$\therefore \quad R = \frac{100 \times I}{P \times T} \Rightarrow r = \frac{100 \times \frac{4}{25}x}{x \times r}$$

$$\Rightarrow r^2 = \frac{400}{25} \text{ or } r = \frac{20}{5} = \textbf{4\%.}$$

11. Ans. (a)

Solution We have, A = Rs. 1020, T = 4 years, R = 5% p.a.

Let the principal be Rs. x.

Then, Interest (I) = $A - P = 1020 - x$.

Therefore, by using formula,

$$P = \frac{100 \times I}{R \times T}$$

We have, $x = \frac{100 \times (1020 - x)}{5 \times 4}$

$\Rightarrow \quad x = 5100 - 5x$ or, $6x = 5100$

or, $x = \frac{5100}{6}$ = Rs. 850.

$\therefore$ The sum of money borrowed = **Rs. 850.**

12. Ans. (c)

Solution Here, P = Rs. 1200, A = Rs. 1344, R = 6% p.a.

$\therefore$ Interest (I) = 1344 –1200 = 144

$\therefore$ Time (T) = $\frac{100 \times I}{P \times R} = \frac{100 \times 144}{1200 \times 6}$ = **2 years.**

13. Ans. (b)

Solution Income on Rs. 225 in 4 years at 3%

$$= \frac{P_1 \times R \times T_1}{100} = \frac{225 \times 3 \times 4}{100} = \text{Rs. } 27.$$

Now, interest of Rs. 27 is earned on Rs. 8100 at 3% simple interest.

$\therefore$ Time (T_2) = $\frac{100 \times I}{P_2 \times R} = \frac{100 \times 27}{8100 \times 3} = \frac{1}{9}$ **year.**

14. Ans. (a)

Solution The interest earned in 10 years on Rs. 1000 at 5% per annum

$$= \frac{1000 \times 5 \times 10}{100} = \text{Rs. } 500.$$

The principal now becomes = Rs. 1000 + Rs. 500 = Rs. 1500. We now find the time in which Rs. 1500 becomes Rs. 2000 at 5% p.a.

P = Rs. 1500, A = Rs. 2000,

$I = A - P$ = 2000 – 1500 = Rs. 500, R = 5% p.a.

$\therefore$ Time (T) = $\frac{100 \times I}{R \times P} = \frac{100 \times 500}{5 \times 1500} = 6\frac{2}{3}$ years.

$\therefore$ Total time = $\left(10 + 6\frac{2}{3}\right)$ years = $\mathbf{16\frac{2}{3}}$ **years.**

15. Ans. (b)

Solution Interest on Rs. 500 is = 725 – 500 = Rs. 225.

$$\text{Time} = \frac{225 \times 100}{500 \times 9} = 5 \text{ years.}$$

$$\therefore \text{ Required amount } (A) = P\left(1 + \frac{R \times T}{100}\right)$$

$$= 600\left(1 + \frac{11 \times 5}{100}\right)$$

$$= \textbf{Rs. 930.}$$

16. Ans. (c)

Solution Amount after 1 year $= P\left(1 + \frac{R \times T}{100}\right)$

$$= 10000\left(1 + \frac{20 \times 1}{100}\right)$$

$$= \text{Rs. } 12000.$$

After paying Rs. 6000, the remaining sum = Rs. 6000.

$\therefore$ Amount obtained in the next year

$$= P\left(1 + \frac{R \times T}{100}\right)$$

$$= 6000\left(1 + \frac{20 \times 1}{100}\right) = \textbf{Rs. 7200.}$$

17. Ans. (a)

Solution We have, A = Rs. 15000, R = 10% p.a., T = 5 years.

$$\therefore \quad P = \frac{100 \times A}{100 + R \times T} = \frac{100 \times 15000}{100 + 10 \times 5} = \textbf{Rs. 10000.}$$

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18. Ans. (c)

Solution We have, A = Rs. 47250, T = 3 years, R = 5% p.a.

$$\therefore \quad \text{Annual payment} = \frac{100 \times A}{100 \times T + \frac{RT(T-1)}{2}}$$

$$= \frac{100 \times 47250}{100 \times 3 + \frac{5 \times 3 \times 2}{2}}$$

$$= \textbf{Rs. 15000.}$$

19. Ans. (a)

Solution Here, A = Rs. 4200, T = 5 years. R = 10% p.a.

$$\therefore \text{ Annual instalment} = \frac{100 \times A}{100 \times T + \frac{RT(T-1)}{2}}$$

$$= \frac{100 \times 4200}{100 \times 5 + \frac{10 \times 5 \times 4}{2}}$$

$$= \textbf{Rs. 700.}$$

20. Ans. (a)

Solution Let the sum of money invested by Suresh be Rs. x.

Since the amount obtained in both the cases is equal, the ratio in which the sums are invested is

$$\frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2}$$

where $R_1 = 8\%$, $T_1 = \frac{5}{2}$ years, $R_2 = 5\%$, $T_2 = 2$ years.

That is, $\frac{1}{100 + 8 \times \frac{5}{2}} : \frac{1}{100 + 5 \times 2}$ or, $\frac{1}{120} : \frac{1}{110}$

Given : $1500 : x :: \frac{1}{120} : \frac{1}{110}$

$$\Rightarrow \quad \frac{1500 \times 1}{110} = \frac{1}{120} \times x$$

or, $$x = \frac{1500 \times 1 \times 120}{110} = 1636\frac{4}{11}.$$

$\therefore$ The sum invested by Suresh is **Rs. $1636\frac{4}{11}$.**

21. Ans. (a)

Solution We have, T_1 = 2 years, T_2 = 3 years, T_3 = 4 years.

$$R_1 = R_2 = R_3 = 5\% \text{ p.a.}$$

$\therefore$ The ratio in which the amount is invested

$$= \frac{1}{100 + R_1T_1} : \frac{1}{100 + R_2T_2} : \frac{1}{100 + R_3T_3}$$

i.e. $\frac{1}{100 + 2 \times 5} : \frac{1}{100 + 3 \times 5} : \frac{1}{100 + 4 \times 5}$

i.e. $\frac{1}{110} : \frac{1}{115} : \frac{1}{120}$ or, 276 : 264 : 253.

Their sum = 276 + 264 + 253 = 793

$\therefore$ The amount invested for

$$\text{Neeta} = \frac{3965}{793} \times 276 = \text{Rs. } 1380$$

$$\text{Sita} = \frac{3965}{793} \times 264 = \text{Rs. } 1320$$

$$\text{Gita} = \frac{3965}{793} \times 253 = \textbf{Rs. 1265.}$$

22. Ans. (b)

Solution We have, n = 4 and T = 24 years.

$$\therefore \text{ Rate of interest} = \frac{100(n-1)}{T} = \frac{100(4-1)}{24}$$

$$= \textbf{12}\frac{\textbf{1}}{\textbf{2}}\textbf{\%.}$$

23. Ans. (c)

Solution Here, n = 3, R = 10% p.a.

$$\therefore \text{ Required time} = \left(\frac{n-1}{R}\right) \times 100 = \left(\frac{3-1}{10}\right) \times 100$$

$$= \textbf{20 years.}$$

24. Ans. (a)

Solution We have, n = 2, T = 8 years, m = 3.

$$\therefore \text{ Required Time } (T') = \left(\frac{m-1}{n-1}\right) \times T$$

$$= \left(\frac{3-1}{2-1}\right) \times 8 = \textbf{16 years.}$$

25. Ans. (b)

Solution Here, change in SI = Rs. 56, $R_1 - R_2 = 2$, T = 4 years, P = ?

Therefore, using the formula

$$\text{Change in } SI = \frac{PT}{100} \times (R_1 - R_2)$$

We get $56 = \frac{P \times 4}{100} \times 2 \Rightarrow P$ = Rs. 700.

$\therefore$ The sum is **Rs. 700.**

26. Ans. (a)

Solution Here, change in SI = Rs. 40,

$P_1 - P_2 = 800 - 400$ = Rs. 400, T = 2 years.

Using, change in $SI = \frac{RT}{100} \times (P_1 - P_2)$

We have, $40 = \frac{R \times 2}{100} \times 400 \Rightarrow \boldsymbol{R = 5\%}$.

27. Ans. (a)

Solution Here, change in SI = Rs. 60,

$R_1 - R_2 = 3 - \frac{5}{2} = \frac{1}{2}$, T = 4 years, P = ?

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Using, change in $SI = \frac{PT}{100} \times (R_1 - R_2)$

We have, $60 = \frac{P \times 4}{100} \times \frac{1}{2} \Rightarrow P =$ **Rs. 3000.**

28. Ans. (b)

Solution We have, A_1 = Rs. 2800, A_2 = Rs. 3250, T_1 = 2 years, T_2 = 5 years

$\therefore$ Rate of interest per annum (R)

$$= \frac{A_1 - A_2}{A_1T_2 - A_2T_1} \times 100\%$$

$$= \frac{2800 - 3250}{2800 \times 5 - 3250 \times 2} \times 100\%$$

$$= \frac{450 \times 100}{14000 - 6500}\% = \frac{45000}{7500}\% = \mathbf{6\%.}$$

29. Ans. (c)

Solution We have, A_1 = Rs. 1760, A_2 = 2000, T_1 = 2 years, T_2 = 5 years.

$\therefore$ Principal $(P) = \frac{A_1T_2 - A_2T_1}{T_2 - T_1}$

$$= \frac{1760 \times 5 - 2000 \times 2}{5 - 2} = \frac{4800}{3}$$

$$= \textbf{Rs. 1600.}$$

30. Ans. (b)

Solution We have, A_1 = Rs. 450, A_2 = Rs. 350, R_1 = 7%, R_2 = 5%.

$\therefore$ Principal $(P) = \frac{A_2R_1 - A_1R_2}{R_1 - R_2}$

$$= \frac{350 \times 7 - 450 \times 5}{7 - 5} = \frac{200}{2}$$

$$= \textbf{Rs. 100.}$$

31. Ans. (c)

Solution We have, A_1 = Rs. 400, A_2 = Rs. 200, R_1 = 10%, R_2 = 4%.

$\therefore$ Time $(T) = \frac{A_1 - A_2}{A_2R_1 - A_1R_2} \times 100$

$$= \frac{400 - 200}{200 \times 10 - 400 \times 4} \times 100 = \frac{20000}{400}$$

$$= \textbf{50 years.}$$

32. Ans. (a)

Solution Here, z = Rs. 9, a = Rs. 1, b = 12, n = 10, R = ?

Using the formula

$$z = na. + \frac{Ra}{100 \times b} \times \frac{n(n-1)}{2},$$

We get $9 = 10 \times 1 + \frac{R \times 1}{100 \times 12} \times \frac{10 \times 9}{2}$

$$\Rightarrow \frac{90R}{2400} = 1 \Rightarrow R = \frac{2400}{9} = \mathbf{266\frac{2}{3}\%.}$$

33. Ans. (a)

Solution Here, $T_1 = 1$, $T_2 = 2$, $T_3 = 3$,
$R_1 = R_2 = R_3 = 5\%$.

The shares of Vikas, Vijay and Viraj will be in the ratio

$$\frac{1}{R_1T_1} : \frac{1}{R_2T_2} : \frac{1}{R_3T_3} = \frac{1}{1 \times 5} : \frac{1}{2 \times 5} : \frac{1}{3 \times 5}$$

$$= \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2.$$

Sum of proportionals = 6 + 3 + 2 = 11.

$\therefore$ Share of Vikas = $\frac{6}{11} \times 7700$ = Rs. 4200

Share of Vijay = $\frac{3}{11} \times 7700$ = Rs. 2100

Share of Viraj = $\frac{2}{11} \times 7700$ = Rs. 1400

Therefore, Vikas's share is 4200 – 1400 = Rs. 2800 more than that of Viraj.

34. Ans. (b)

Solution Let the required sum of money be Rs. x. Here R_1 = 5%, T_1 = 4 years, R_2 = 4%, T_2 = 10 years.

Given: $x : 560 = \frac{1}{R_1T_1} : \frac{1}{R_2T_2} = \frac{1}{5 \times 4} : \frac{1}{4 \times 10}$

$$\Rightarrow \frac{x}{560} = \frac{2}{1} \quad \text{or,} \quad \mathbf{x = Rs.\ 1120.}$$

35. Ans. (a)

Solution Here, P_1 = Rs. 12000, R_1 = 10%, P_2 = ?, R_2 = 20%, R = 14%.

Therefore, using the formula

$$R = \frac{P_1R_1 + P_2R_2}{P_1 + P_2}$$

We get, $14 = \frac{12000 \times 10 + P_2 \times 20}{12000 + P_2}$

or, P_2 = Rs. 8000.

$\therefore$ Total amount invested = Rs. (12000 + 8000)

$$= \textbf{Rs. 20000.}$$

36. Ans. (b)

Solution We have, P_1 = Rs. 3000, R_1 = 10%, P_2 = Rs. 5000, R_2 = 8%.

$\therefore$ Required rate of interest

$$= \frac{P_1R_1 + P_2R_2}{P_1 + P_2}$$

$$= \frac{3000 \times 10 + 5000 \times 8}{3000 + 5000} = \frac{70}{8} = \mathbf{8\frac{3}{4}\%.}$$

37. Ans. (c)

Solution We have, $\frac{1}{a} = \frac{2}{3}$, $\frac{1}{b} = \frac{1}{6}$,

$$\frac{1}{c} = 1 - \left(\frac{2}{3} + \frac{1}{6}\right) = \frac{1}{6},$$

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$R_1 = 3\%$, $R_2 = 6\%$, $R_3 = 12\%$, A = Rs. 25.

$$\therefore \text{ The capital } = \frac{A \times 100}{\frac{R_1}{a} + \frac{R_2}{b} + \frac{R_3}{c}}$$

$$= \frac{25 \times 100}{3 \times \frac{2}{3} + \frac{6}{6} + \frac{12}{6}} = \frac{2500}{5}$$

$$= \textbf{Rs. 500.}$$

38. Ans. (a)

Solution Interest for 5 years on the sum = Rs. 300.

When the principal is trebled, the interest is also trebled.

∴ Interest for another 5 years on this increased sum = Rs. (300 × 3) = Rs. 900.

∴ Total interest = Rs. 300 + Rs. 900 = Rs. 1200.

39. Ans. (a)

Solution The simple interest on Rs. 1500 invested at a rate of 10% p.a. for 5 years is

$$= \frac{1500 \times 10 \times 5}{100} = \text{Rs. } 750.$$

Now, principal after 5 years = Rs. 1500 + 750

= Rs. 2250.

Also, final amount = Rs. 2500.

∴ Simple interest = Rs. 2500 – 2250 = Rs. 250.

$$\therefore \text{ Time } (T) = \frac{250 \times 100}{2250 \times 10} = \frac{10}{9} \text{ years}$$

Hence, total time $= 5 + \frac{10}{9} = \frac{55}{9}$ **or** $6\frac{1}{9}$ **years.**

40. Ans. (c)

Solution Let the sum of money lent by Sumit to Mohit be Rs. x.

Then, simple interest paid by Mohit after 1 year

$$= \frac{x \times 5 \times 1}{100} = \text{Rs. } \frac{5x}{100}.$$

Also, the simple interest received by Mohit from Birju after 1 year

$$= \frac{x \times \frac{17}{2} \times 1}{100} = \text{Rs. } \frac{17x}{200}.$$

Given: $\frac{5x}{100} + 350 = \frac{17x}{200}$

$$\Rightarrow \quad \frac{5x + 35000}{100} = \frac{17x}{200}$$

$$\Rightarrow \quad 1700x - 1000x = 7000000$$

$$\text{or,} \quad 700x = 7000000$$

$$\text{or,} \quad x = \frac{7000000}{700} = \text{Rs. } 10000.$$

Thus, the sum of money lent by Sumit to Mohit is **Rs. 10000.**

41. Ans. (a)

Solution Simple interest paid by Brinda on Rs. 1000 for 1 year $= \frac{1000 \times 5 \times 1}{100} = \text{Rs. } 50.$

Rent received by Brinda from Ramu in 1 year

$$= 12\frac{1}{2} \times 12 = \text{Rs. } 150.$$

∴ Net savings = Rs. 100.

Thus, Brinda will clear the debt of Rs. 1000 in **10 years.**

42. Ans. (c)

Solution Let the sum be Rs. x.

Given: $\frac{x \times 4 \times 2}{100} + \frac{x \times 6 \times 4}{100} + \frac{x \times 8 \times 3}{100} = 1120$

$$\Rightarrow \quad 56x = 112000 \quad \text{or,} \quad x = \frac{112000}{56} = \textbf{Rs. 2000.}$$

QUESTIONS FROM COMPETITIVE EXAMS

1. How much interest will Rs. 10,000 earn in 9 months at an annual rate of 6 per cent?

(a) 450 (b) 475

(c) 500 (d) 600

[Based on MAT 2003]

2. Ravi gave Rs 1200 on loan. Some amount he gave at 4% per annum simple interest and remaining at 5% per annum simple interest. After two years, he got Rs. 110 as interest. Then the amounts given at 4% and 5% per annum simple interest are respectively,

(a) Rs 500, Rs 700 (b) Rs 400, Rs 800

(c) Rs 800, Rs 300 (d) Rs 1100, Rs 1100

[Based on MAT 2003]

3. Two equal sums of money were invested, one at 4% and the other at $4\frac{1}{2}$%. At the end of 7 years, the simple interest received from the latter exceeded that received from the former by Rs 31.50. Each sum was

(a) Rs 1,000 (b) Rs 500

(c) Rs 750 (d) Rs 900

[Based on MAT 2001]

4. I derive an annual income of Rs. 688.25 from Rs. 10,000 invested partly at 8% p.a. and partly at 5% p.a. simple interest. How much of my money is invested at 5%?

(a) Rs. 3,725 (b) Rs. 4225

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(c) Rs 4,800 (d) Rs 5,000

[Based on MAT 2000]

5. A certain sum amounts to Rs 2,300 in 3 years and Rs 2,500 in 5 years at simple interest. Find the sum and the rate of interest.

(a) Rs 1200, 6% (b) Rs 1800, 5%

(c) Rs 2000, 5% (d) Rs 1500, 6%

[Based on IIT Joint Man. Ent. Test 2004]

6. Mr. Anand deposited a total amount of Rs 65000 in three different schemes *A*, *B* and *C* with rates of interest 12 p.c.p.a., 16 p.c.p.a. and 18 p.c.p.a. respectively and earned a total interest of Rs 10,180 in one year. If the amount invested in Scheme *A* was 72% of the amount invested in Scheme *C*, then what was the amount invested in Scheme *B*?

(a) Rs 25000 (b) Rs 22000

(c) Rs 18000 (d) Cannot be determined.

(e) None of these **[Based on Based on IRMA, 2002]**

7. What annual rate of interest was paid, if Rs 10000 earned Rs 1000 as interest in 2 years?

(a) 2% (b) 5%

(c) 10% (d) 20%

[Based on Narsee Monjee Inst. of Man. Studies 2002]

8. In 4 years, the simple interest on a certain sum of money is 7/25 of the principal. The annual rate of interest is

(a) 4% (b) 4.5%

(c) 7% (d) 9%

[Based on SNAP, 2007]

9. A man earns 6% SI on his deposits in bank A while he earns 8% simple interest on his deposits in bank B. If the total interest he earns is Rs 1800 in three years on an investment of Rs 9000, what is the amount invested at 6%?

(a) 3000 (b) 6000

(c) 4000 (d) 4500

[Based on I.P. Univ. 2002]

10. Rs 2000 amount of Rs 2600 in 5 years at simple interest. If the interest rate were increased by 3%, it would amount to how much?

(a) Rs 2900 (b) Rs 3200

(c) Rs 3600 (d) None of these

[Based on FMS (Delhi) 2007]

11. A sum of Rs 4000 is lent out in two parts, one at 8% simple interest and other at 10% simple interest. If the annual interest is Rs 352, the sum lent at 8% is

(a) Rs 1600 (b) Rs 2400

(c) Rs 1800 (d) Rs 2800

[Based on MAT 2005]

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $10000 \times \frac{9}{12} \times \frac{6}{100}$

$= \text{Rs } 450.$

2. Ans. (a)

Solution Let the amount given at 4% per annum be Rs x

$\therefore$ Amount given at 5% per annum = Rs $(1200 - x)$

$\Rightarrow \frac{x \times 4 \times 2}{100} + \frac{(1200 - x) \times 5 \times 2}{100}$

$= 110$

$\Rightarrow$ $\boldsymbol{x = 500.}$

3. Ans. (d)

Solution Let each sum be Rs x

$\therefore$ $\frac{x \times 4\frac{1}{2} \times 7}{100} - \frac{x \times 4 \times 7}{100} = 31.50$

$\Rightarrow \frac{7x}{100} \times \frac{1}{2} = \frac{63}{2}$

$\Rightarrow$ $\boldsymbol{x = 900.}$

4. Ans. (a)

Solution Let money invested at 5% be Rs k

$\therefore$ $\frac{k \times 1 \times 5}{100} + \frac{(10000 - k) \times 1 \times 8}{100}$

$= 688.25$

$\Rightarrow 5k - 8k + 80000 = 68825$

$\Rightarrow 3k = 11175$

$\Rightarrow$ $\boldsymbol{k = 3725.}$

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5. Ans. (c)

Solution Let P be the sum and R be the rate %

$$\therefore \quad P + \frac{P \times 3 \times R}{100} = 2300 \qquad ...(i)$$

and $$P + \frac{P \times 5 \times R}{100} = 2500 \qquad ...(ii)$$

(1) – (2) gives $\frac{2PR}{100} = 2000 \Rightarrow PR = 10000$, which is satisfied only by alternative (c).

6. Ans. (b)

Solution Suppose Amount invested in scheme A = Rs x

Amount invested in scheme B = Rs y

Amount invested in scheme C = Rs z

$$\therefore \quad x + y + z = 65000 \qquad ...(i)$$

$$x = 72\% \text{ of } z = \frac{18}{25}z \qquad ...(ii)$$

$$12x + 16y + 18z = 1018000$$

i.e., $$6x + 8y + 9z = 509000 \qquad ...(iii)$$

Using (ii) in (i) and (iii), we get

$$\frac{43}{25}z + y = 65000 \qquad ...(iv)$$

$$\frac{108}{25}z + 8y + 9z = 509000$$

$$\Rightarrow \quad \frac{333}{25}z + 8y = 509000 \qquad ...(v)$$

From (iv), we have

$$\frac{344}{5z} + 8y = 520000 \qquad ...(vi)$$

$\therefore$ (vi) – (v)

$$\Rightarrow \quad \frac{11z}{25} = 11000$$

$$\Rightarrow \quad 11z = 275000$$

$$\Rightarrow z = 25000$$

$$\therefore \quad x = 18000$$

$$y = \textbf{22000.}$$

7. Ans. (b)

8. Ans. (c)

Solution (c) Interest $= \frac{P \times R \times T}{100}$

$$\frac{7}{25}x = \frac{x \times r4 \times r}{100}$$

$$r = \frac{7x}{25x} \times \frac{100}{400} = 7\%$$

9. Ans. (b)

10. Ans. (a)

Solution $I = \frac{P \times R \times T}{100}$

$$600 = \frac{2000 \times R \times 5}{100}$$

$$R = 6\%$$

New $R = 6 + 3 = 9\%$

$$I = \frac{200 \times 9 \times 5}{100}$$

$$= 900$$

Amount = $P = I = 2000 + 900$

= **Rs 2900.**

11. Ans. (b)

Solution Total interest on Rs 4000 in 1 year = Rs 352

Average rate $= \frac{352 \times 100}{4000 \times 1} = 8.8\%$

1.2 : 0.8 = 3.2

First part = **Rs 2400.**

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Rs. 1000 is invested at 5% simple interest. If the interest is added to the principal every 10 years, the amount will become Rs. 2000 after

(a) 15 years (b) 18 years

(c) 20 years (d) $16\frac{2}{3}$ years

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2. Divide the sum of Rs. 1175 in three such parts that simple interest accured from them after 3, 4 and 5 years, respectively at the rate of 4% per annum remains equal.

(a) Rs. 490, Rs. 375, Rs. 300

(b) Rs. 500, Rs. 375, Rs. 300

(c) Rs. 500, Rs. 350, Rs. 300

(d) None of these

3. In what time will Rs. 4000 lent at 3% per annum on S.I. earn as much interest as Rs. 5000 will earn in 5 years at 4% per annum on S.I.?

(a) $8\frac{1}{3}$ years (b) 9 years

(c) $7\frac{1}{2}$ years (d) $7\frac{1}{3}$ years

4. A man lends Rs. 10000 in four parts. If he gets 8% on Rs. 2000; $7\frac{1}{2}$% on Rs. 4000 and $8\frac{1}{2}$% on Rs. 1400, what per cent must he get for the remainder, if the average interest is 8.13%?

(a) 8% (b) 10%

(c) 9% (d) None of these

5. A man invests an amount of Rs. 15860 in the names of his three sons *A*, *B* and *C* in such a way that they get the same interest after 2, 3 and 4 years, respectively. If the rate of simple interest is 5%, the ratio of amount invested among *A*, *B* and *C* will be

(a) 10 : 15 : 20 (b) 110 : 115 : 120

(c) $\frac{1}{10} : \frac{1}{15} : \frac{1}{20}$ (d) $\frac{1}{110} : \frac{1}{115} : \frac{1}{120}$

6. What is the least number of years for which simple interest must be reckoned at 3% on Rs. 212.50, so that the interest may be an exact number of rupees.

(a) 6 years (b) 8 years

(c) 10 years (d) 7 years

7. A person invested Rs. 500 at a certain rate of interest and Rs. 700 at 1% higher and the total interest from these investments in 3 years was Rs. 165. Find the rate per cent.

(a) 6% (b) 5%

(c) $4\frac{1}{2}$% (d) 4%

8. Amitabh invested an amount of Rs. 29000 in two parts under two different schemes *A* and *B* and earned a total interest of Rs. 3840. Schemes *A* and *B* offered 15% and 12% interest, respectively. What was the amount invested in scheme '*A*'?

(a) Rs. 17000 (b) Rs. 12000

(c) Rs. 14000 (d) data inadequate

(e) None of these

9. Out of a sum of Rs. 625, a part was lent at 5% and the other at 10% simple interest. If the interest on the first part after 2 years is equal to the interest on the second part after 4 years, then the second sum (in Rs.) is

(a) 125 (b) 200

(c) 250 (d) 300

10. Find in what time a given sum of money will quadruple itself at simple interest at the rate of one paise per rupee per month?

(a) 25 years (b) 27 years

(c) 23 years (d) 28 years

Test II

[Time: 10 min]

11. The simple interest of Rs. 800 is Rs. 216. If the rate per cent of the interest per annum is $\frac{1}{3}$ of the number of years, the rate per cent per annum is

(a) 2% (b) 4%

(c) 6% (d) 3%

12. The simple interest on Rs. 1820 from Oct. 9, 2003 to Dec. 21, 2003 at $7\frac{1}{2}$% will be

(a) Rs. 22.50 (b) Rs. 27.30

(c) Rs. 28.80 (d) Rs. 29

13. A sum of Rs. 1550 was lent partly at 5% and partly at 8% simple interest. The total interest received after 3 years is Rs. 300. The ratio of money lent at 5% to that at 8% is

(a) 5 : 8 (b) 8 : 5

(c) 31 : 6 (d) 16 : 15

14. Rs. 800 becomes Rs. 956 in 3 years at certain simple rate of interest. If the rate of interest is increased by 4%, what amount will Rs. 800 become in 3 years?

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(a) Rs. 1020.80 (b) Rs. 1025
(c) Rs. 1052 (d) data inadequate

15. Mr. Rao invested a total amount of Rs. 16500 for two years in two Schemes *A* and *B* with rate of simple interest 10% p.a., and 12% p.a., respectively. If the total amount of interest earned was Rs. 3,620, what was the amount invested in scheme *B*?
(a) Rs. 8000 (b) Rs. 8600
(c) Rs. 8150 (d) data inadequate
(e) None of these

16. Rs. 4000 was divided into two parts in such a way that when first part was invested at 3% and the second at 5% the whole annual interest from both the investments was Rs. 144, how much was put at 3%?
(a) Rs. 2500 (b) Rs. 2700
(c) Rs. 2800 (d) Rs. 3000

17. The simple interest on a sum of money is 1/6 of the principal and the number of months is equal to twice the rate per cent. Find the time and the rate
(a) $3\frac{1}{3}$ years; 10% (b) $1\frac{2}{3}$ years; 10%
(c) 4 years; 8% (d) $5\frac{1}{3}$ years; $10\frac{2}{3}$%

18. Ram borrowed a sum of Rs. 12000 at 15% per annum from a money lender on 13th January, 2003 and returned the amount on 8th June, 2003 to clear his debt. What was the amount paid by Ram to the money-lender to clear his debt?
(a) Rs. 12520 (b) Rs. 12820
(c) Rs. 12720 (d) None of these

19. Pankaj deposited a certain sum of money in a post office at 12% per annum for 4 years and deposited an equal amount in fixed deposit in a bank for 5 years at 15% per annum. If the difference in the interest from two sources is Rs. 1350, find the sum deposited in each case.
(a) Rs. 4500 (b) Rs. 5000
(c) Rs. 5500 (d) None of these

20. What equal half yearly payment will discharge the debt of Rs. 4360 due in 2 years at 12% per annum.
(a) Rs. 1000 (b) Rs. 1020
(c) Rs. 990 (d) None of these

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution Since the interest is added after 10 years to the principal, the principal after 10 years will be
P + Interest for 10 years

$$\text{or, amount after 10 years} = 1000 + \frac{1000\times10\times5}{100}$$
$$= 1000 + 500$$
$$= \text{Rs. } 1500$$

Suppose the amount becomes Rs. 2000, after n more years, then the interest on Rs. 1500 for n years is Rs. 2000 – Rs. 1500 = Rs. 500

We have $500 = \frac{1500\times5\times n}{100}$

$$\therefore \quad n = \frac{500\times100}{1500\times5} = \frac{20}{3} = 6\frac{2}{3} \text{ years}$$

$$\therefore \text{ Total time} = 10 + 6\frac{2}{3} = \mathbf{16\frac{2}{3}\ years.}$$

3. Ans. (a)

Solution Interest in the later case

$$= \text{Rs. } \frac{5000\times5\times4}{100} = \text{Rs. } 1000$$

$\therefore$ Time in the former case

$$= \frac{1000\times100}{4000\times3} \text{ years} = \mathbf{8\frac{1}{3}\ years.}$$

5. Ans. (c)

Solution Let the amount invested be x, y, z, respectively.

Then,

$$\frac{x\times2\times5}{100} = \frac{y\times3\times5}{100} = \frac{z\times4\times5}{100}.$$

$$\therefore \quad 10x = 15y = 20z = k$$

$$\text{So, } x = \frac{k}{10},\ y = \frac{k}{15},\ z = \frac{k}{20}$$

$$\therefore \quad x : y : z = \mathbf{\frac{1}{10} : \frac{1}{15} : \frac{1}{20}}.$$

7. Ans. (d)

Solution Let x be the rate per cent. Interest for 1 year on both the investments is Rs. (165 + 3), i.e. Rs. 55

$$\therefore \quad \frac{x\times500\times1}{100} + \frac{(x+1)\times700\times1}{100} = 55$$

$$\therefore \quad 5x + 7x + 7 = 55$$

$$\text{or, } \quad 12x = 48 \quad \text{or} \quad \mathbf{x = 4.}$$

9. Ans. (a)

Solution Let first part = x. Then, second part = $(625 - x)$

We have,

$$\frac{x\times5\times2}{100} = \frac{(625-x)\times10\times4}{100} \Rightarrow x = 500$$

$\therefore$ Second part = Rs. (625 – 500) = **Rs. 125.**

11. Ans (d)

Solution Suppose, the number of years = x

$$\therefore \quad \text{Rate per cent} = \frac{x}{3}\%$$

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$\therefore$ Simple interest $= \dfrac{P \times R \times T}{100}$

or, $216 = \dfrac{800 \times x \times x}{3 \times 100}$ or, $x^2 = \dfrac{216 \times 3}{8}$

or, $x = 9$

$\therefore$ Rate per cent per annum $= \dfrac{x}{3} = \dfrac{9}{3} =$ **3%.**

13. Ans (d)

Solution Let the sum lent at the rate of interest 5% per annum is Rs. x and at the rate of interest 8% per annum is Rs. $(1550 - x)$.

According to question,

$$\frac{x \times 5 \times 3}{100} + \frac{(1550 - x) \times 8 \times 3}{100} = 300$$

or, $\dfrac{15x}{100} + \dfrac{37200 - 24x}{100} = 300$

or, $15x + 37200 - 24x = 30000$

or, $9x = 7200 \quad \therefore \quad x =$ Rs. 800

and, $1550 - x = 1550 - 800 =$ Rs. 750

$\therefore$ Ratio of money lent at 5% to that at 8% $= 900 : 750$ **or, 16 : 15.**

15. Ans (a)

Solution Let the amount invested in scheme B be Rs. x

$\therefore$ Amount invested in scheme A = Rs. $(16500 - x)$

According to question,

$$\frac{(16500 - x) \times 10 \times 2}{100} + \frac{x \times 12 \times 2}{100} = 3620$$

or, $\dfrac{16500 - x}{5} + \dfrac{6x}{25} = 3620$

or, $5 \times 16500 - 5x + 6x = 3620 \times 25$

or, $x = 90500 - 82500$

$x = 8000$

$\therefore$ Required amount = **Rs. 8000.**

17. Ans (b)

Solution Let x be the principal so that $\dfrac{x}{6}$ is the interest. Let the rate % be r so that number of months is $2r$ or, $\dfrac{2r}{12}$ i.e. $\dfrac{r}{6}$ years

$\therefore \quad \dfrac{x}{6} = x \times \dfrac{r}{100} \times \dfrac{r}{6}$, i.e. $r = 10$

Time $= 2 \times 10$ months or, $\dfrac{20}{12}$, i.e. $\dfrac{5}{3}$ **or,** $\mathbf{1\frac{2}{3}}$ **years.**

19. Ans (b)

Solution Let the sum deposited in each case be Rs. 100.

Interest in the case of post office

$P =$ Rs. 100, $R = 12\%$ p.a., $T = 4$ years

$$I = \text{Rs.}\left(\frac{100 \times 12 \times 4}{100}\right) = \text{Rs. } 48$$

Interest in the case of bank

$P =$ Rs. 100, $R = 15\%$ p.a., $R = 5$ years

$$\therefore \quad I = \text{Rs.}\left(\frac{100 \times 15 \times 5}{100}\right) = \text{Rs. } 75$$

Difference in interest from two sources = Rs. 75 – Rs. 48 = Rs. 27.

If the difference in interest is Rs. 27, then the sum deposited = Rs. 100

If the difference in interest is Re. 1, then the sum deposited = Rs. $\left(\dfrac{100}{27}\right)$.

If the difference in interest is Rs. 1350, then the sum deposited = Rs. $\dfrac{100}{27} \times 1350$ **= Rs. 5000.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (c)	**6.** (b)	**8.** (b)	**10.** (a)	**12.** (b)	**14.** (c)	**16.** (c)	**18.** (c)	**20.** (a)

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18 Compound Interest

INTRODUCTION

In the previous chapter we discussed simple interest. A second method of paying interest is the *compound interest* method, where the interest for each period is added to the principal before interest is calculated for the next period. With this method the principal grows as the interest is added to it. This method is used in investments such as savings account and bonds. An understanding of compound interest is important not only for people planning careers with financial institutions, but also for anyone planning to invest money.

SOME BASIC FORMULAE

1. (a) The amount A due after t years, when a principal P is given on compound interest at the rate $R\%$ per annum is given by

$$A = P\left(1+\frac{R}{100}\right)^t.$$

(b) Compound interest (CI) $= A - P$

$$= P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

(c) Rate of interest $(R) = \left[\left(\frac{A}{P}\right)^{1/t} - 1\right]\%$ p.a.

Note: Simple interest and compound interest for 1 year at a given rate of interest p.a. are always equal.

Illustration 1 Mohan invested an amount of Rs. 15000 at compound interest rate 5% p.a. for a period of 2 years. What amount will he receive at the end of 2 years?

Solution Here, $P = 15000$, $R = 5$ and $t = 2$.

$$\therefore \text{ Amount} = P\left(1+\frac{R}{100}\right)^t$$

$$= 15000\left(1+\frac{5}{100}\right)^2 = 15000\left(1+\frac{1}{20}\right)^2$$

$$= \frac{15000\times 21\times 21}{20\times 20} = \textbf{Rs. 16537.50.}$$

Illustration 2 Find compound interest on Rs. 5000 for 2 years at 4% p.a.

Solution Here, $P = 5000$, $R = 4$ and $t = 2$.

$$\therefore \text{ CI} = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

$$= 5000\left[\left(1+\frac{4}{100}\right)^2 - 1\right]$$

$$= 5000\left[\left(\frac{26}{25}\right)^2 - 1\right] = 5000((1.04)^2 - 1)$$

$$= 5000(1.0816 - 1) = \text{Rs. } 408.$$

$\therefore$ The compound interest is **Rs. 408.**

Illustration 3 Rashi invested Rs. 16000 for two years at compound interest and received an amount of Rs. 17640 on maturity. What is the rate of interest?

Solution Here, $P = 16000$, $t = 2$ and $A = 17640$.

$$\therefore R = 100\left[\left(\frac{A}{P}\right)^{1/t} - 1\right]\% \text{ p.a.}$$

$$= 100\left[\left(\frac{17640}{16000}\right)^{1/2} - 1\right]\% \text{ p.a.}$$

$$= 100\left[\left(\frac{441}{400}\right)^{1/2} - 1\right]\% \text{ p.a.}$$

$$= 100\left[\left(\frac{21}{20}\right)^{2\times\frac{1}{2}} - 1\right]\% \text{ p.a.}$$

$$= 100\times\frac{1}{20} = \textbf{5\% p.a.}$$

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2. If the interest is compounded half-yearly, then

(a) Amount $(A) = P\left(1+\frac{R}{100\times 2}\right)^{2t}$.

(b) Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times 2}\right)^{2t}-1\right]$$

(c) Rate $(R) = 2\times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times 2}-1\right]$ % p.a.

Illustration 4 Find the amount of Rs. 8000 in $1\frac{1}{2}$ years at 5% per annum compound interest payable half-yearly.

Solution Here, $P = 8000$, $R = 5$ and $t = \frac{3}{2}$.

$$\therefore \text{ Amount} = P\left(1+\frac{R}{100\times 2}\right)^{2t}$$

$$= 8000\left(1+\frac{5}{100\times 2}\right)^{2\times\frac{3}{2}} = 8000\left(\frac{41}{40}\right)^{3}$$

$$= \frac{8000\times 41\times 41\times 41}{40\times 40\times 40} = \textbf{Rs. 8615.13.}$$

3. If the interest is compounded quarterly, then

(a) Amount $(A) = P\left(1+\frac{P}{100\times 4}\right)^{4t}$

(b) Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times 4}\right)^{4t}-1\right]$$

(c) Rate $(R) = 4\times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times 4}-1\right]$ % p.a.

In general, if the interest is compounded n times a year, then

(a) Amount $(A) = P\left(1+\frac{R}{100\times n}\right)^{n\times t}$

(b) Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times n}\right)^{n\times t}-1\right]$$

(c) Rate of interest (R)

$$= n\times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times n}-1\right] \text{ \% p.a.}$$

Illustration 5 Find the compound interest on Rs. 1000 at 40% per annum compounded quarterly for 1 year.

Solution Here, $P = 1000$, $R = 40$ and $t = 1$.

$\therefore$ Compound interest (CI)

$$= P\left[\left(1+\frac{R}{100\times 4}\right)^{4\times t}-1\right]$$

$$= 1000\left[\left(1+\frac{40}{100\times 4}\right)^{4\times 1}-1\right]$$

$$= 1000\left[\left(\frac{11}{10}\right)^{4}-1\right]$$

$$= 1000\left[\frac{14641-10000}{10000}\right]$$

$$= \textbf{Rs. 464.10.}$$

Illustration 6 Find the compound interest on Rs. 4000 at 24% per annum for 3 months, compounded monthly.

Solution Here, $P = 4000$, $R = 24$ and $t = \frac{3}{12}$.

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 12}\right)^{12\times t}-1\right]$$

$$= 4000\left[\left(1+\frac{24}{100\times 12}\right)^{12\times\frac{3}{12}}-1\right]$$

$$= 4000\left[\left(\frac{51}{50}\right)^{3}-1\right] = \frac{4000\times 7651}{50\times 50\times 50}$$

$$= \textbf{Rs. 244.83.}$$

SOME USEFUL SHORT-CUT METHODS

1. When the rates of interest are different for different years, say R_1, R_2, R_3 per cent for first, second and third year respectively, then

$$\text{Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

Explanation

Let the given sum of money be Rs. P. Amount after first year $= P\left(1+\frac{R_1}{100}\right)$.

This amount will be the principal for the second year.

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$\therefore$ Amount after second year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right).$$

This amount will be the principal for the third year.

$\therefore$ Amount after third year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

Illustration 7 Anu invests Rs. 5000 in a bond which gives interest at 4% per annum during the first year, 5% during the second year and 10% during the third year. How much does she get at the end of the third year.

Solution Here, $P = 5000$, $R_1 = 4$, $R_2 = 5$ and $R_3 = 10$.

$\therefore$ Amount at the end of third year

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= 5000\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)$$

$$= 5000 \times \frac{26}{25}\times\frac{21}{20}\times\frac{11}{10} = \textbf{Rs. 6006.}$$

2. When the time is given in the form of fraction, say $3\frac{3}{4}$ years, then

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{3}{4}R}{100}\right).$$

Illustration 8 What will be the compound interest on Rs. 15625 for $2\frac{1}{2}$ years at 4% per annum?

Solution $$\text{CI} = 15625\left[\left(1+\frac{4}{100}\right)^2\left(1+\frac{4\times\frac{1}{2}}{100}\right)-1\right]$$

$$= 15625\left[\frac{26}{25}\times\frac{26}{25}\times\frac{51}{50}-1\right]$$

$$= \frac{15625\times 3226}{31250} = \textbf{Rs. 1613.}$$

3. (a) The difference between the compound interest and the simple interest on a certain sum of money for 2 years at $R\%$ per annum is given by

$$\text{CI} - SI = P\left(\frac{R}{100}\right)^2 \quad \text{[in terms of } P \text{ and } R\text{]}$$

$$\text{and, CI} - SI = \frac{R\times SI}{2\times 100} \quad \text{[in terms of } SI \text{ and } R\text{]}$$

Explanation

Let Rs. P be given sum of money. Simple interest on Rs. P for 2 years at $R\%$ per annum

$$= \frac{P\times R\times 2}{100}$$

and compound interest on Rs. P for 2 years at $R\%$ per annum

$$= P\left[\left(1+\frac{R}{100}\right)^2 - 1\right].$$

$$\therefore\ CI - SI = P\left[\left(1+\frac{R}{100}\right)^2 - 1\right] - \frac{P\times R\times 2}{100}$$

$$= P\left[1+\frac{R^2}{10000}+\frac{2R}{100}-1-\frac{2R}{100}\right]$$

$$= P\left(\frac{R}{100}\right)^2.$$

$$\text{Also, } CI - SI = P\left(\frac{R}{100}\right)^2 = \frac{R}{100\times 2}\times\left(\frac{P\times R\times 2}{100}\right)$$

$$= \frac{R\times SI}{2\times 100}.$$

(b) The difference between the compound interest and the simple interest on a certain sum of money for 2 years at $R\%$ per annum is given by

$$CI - SI = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right] \quad \text{[in terms of } P \text{ and } R\text{]}$$

$$\text{and, } CI - SI = \frac{SI}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right] \quad \text{[in terms of } SI \text{ and } R\text{]}$$

Explanation

Let Rs. P be the given sum of money. Simple interest on Rs. P for 3 years at $R\%$ per annum

$$= \frac{P\times R\times 3}{100}$$

and compound interest on Rs. P for 3 years at $R\%$ per annum

$$= P\left[\left(1+\frac{R}{100}\right)^3 - 1\right]$$

$$\therefore\ CI - SI = P\left[\left(1+\frac{R}{100}\right)^3 - 1\right] - \frac{P\times R\times 3}{100}$$

$$= P\left[1+\frac{R^3}{100000}+\frac{3R^2}{10000}+\frac{3R}{100}-1-\frac{3R}{100}\right]$$

$$= P\left[\frac{R^3}{1000000}+\frac{3R^2}{1000}\right]$$

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$$= P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right].$$

$$= \frac{P\times R\times 3}{100}\times\frac{1}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right]$$

$$= \frac{SI}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right].$$

Illustration 9 What will be the difference between simple and compound interest on a sum of Rs. 4500 put for 2 years at 5% per annum?

Solution Here, $P = 4500$ and $R = 5$.

$$\therefore\quad CI - SI = P\left(\frac{R}{100}\right)^2 = 4500\left(\frac{5}{100}\right)^2$$

$$= \frac{4500}{20\times 20} = \textbf{Rs. 11·25.}$$

Illustration 10 If the difference between the compound interest and simple interest on a certain sum of money for 3 years at 5% per annum is Rs. 61, find the sum.

Solution Here, CI $-$ $SI = 61$ and $R = 5$.

$$\therefore\quad \text{CI} - SI = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$$

$$\Rightarrow\qquad 61 = P\left[\left(\frac{5}{100}\right)^3 + 3\left(\frac{5}{100}\right)^2\right]$$

$$= P\left[\left(\frac{1}{20}\right)^3 + 3\left(\frac{1}{20}\right)^2\right]$$

$$= P\left[\frac{1+3\times 20}{20\times 20\times 20}\right] = P\left(\frac{61}{20\times 20\times 20}\right)$$

$$\Rightarrow\qquad P = \textbf{Rs. 8000.}$$

4. If a certain sum becomes n times in t years at compound interest, then the same sum becomes n^m times in mt years.

Explanation

Let Rs. P be the given sum of money. We have,

$$nP = P\left(1+\frac{R}{100}\right)^t \Rightarrow n = \left(1+\frac{R}{100}\right)^t \qquad ...(1)$$

Let the sum become n^m times in T years.

Then, $$n^m = \left(1+\frac{R}{100}\right)^T$$

or, $$n = \left(1+\frac{R}{100}\right)^{T/m} \qquad ...(2)$$

On comparing equations (1) and (2), we get

$T/m = t$ or $T = mt$ years.

Therefore, the sum becomes n^m times in mt years.

Illustration 11 A sum of money placed at compound interest doubles itself in 3 years. In how many years will it amount to four times itself?

Solution Here, $n = 2$, $t = 3$ and $m = 2$

$\therefore$ The given sum of money will become four times itself in mt, i.e. $2 \times 3 = \textbf{6 years.}$

5. If a certain sum becomes n times in t years, then the rate of compound interest is given by

$$R = 100[(n)^{1/t} - 1].$$

Illustration 12 At what rate per cent compound interest does a sum of money become four-fold in 2 years?

Solution The required rate per cent is

$$R = 100[(n)^{1/t} - 1] = 100[(4)^{1/2} - 1]$$

$$= 100(2 - 1) = \textbf{100\%.}$$

[Here $n = 4$ and $t = 2$]

6. If a certain sum of money at compound interest amounts to Rs. x in A years and to Rs. y in B years, then the rate of interest per annum is

$$R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%.$$

Explanation

Let the principal be Rs. P and the rate of interest be R% p.a.

$$\text{Given : } x = P\left(1+\frac{R}{100}\right)^A \text{ and } y = P\left(1+\frac{R}{100}\right)^B.$$

$$\therefore\quad \frac{y}{x} = \frac{\left(1+\frac{R}{100}\right)^B}{\left(1+\frac{R}{100}\right)^A} = \left(1+\frac{R}{100}\right)^{B-A}.$$

$$\therefore\quad \left(\frac{y}{x}\right)^{1/B-A} = 1 + \frac{R}{100}$$

or, $$\frac{R}{100} = \left(\frac{y}{x}\right)^{1/B-A} - 1$$

or, $$R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100.$$

Illustration 13 A sum of money at compound interest amounts to Rs. 4050 in one year and to Rs. 4723.92 in 3 years. Find the rate of interest per annum.

Solution Here, $x = 4050$, $y = 4723.92$, $A = 1$ and $B = 3$.

$$\therefore\quad R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$$

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$$= \left[\left(\frac{4723 \cdot 92}{4050}\right)^{1/2} - 1\right] \times 100\%$$

$$= \left(\frac{27}{25} - 1\right) \times 100\% = \mathbf{8\%}.$$

7. If a loan of Rs. P at R% compound interest per annum is to be repaid in n equal yearly instalments, then the value of each instalment is given by

$$\text{Rs. } \frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2 + \ldots \left(\frac{100}{100+R}\right)^n}.$$

Explanation

Let each instalment be of Rs. X.

$\therefore$ Principal for the amount of Rs. X due at end of first year at R%

$$= \frac{100X}{100+R}.$$

Principal for the amount of Rs. X due at the end of second year at R%

$$= \left(\frac{100}{100+R}\right)^2 X$$

$\vdots \qquad \vdots \qquad \vdots$

Principal for the amount of Rs. X due at the end of nth year at R%

$$= \left(\frac{100}{100+R}\right)^n X.$$

$$\therefore \frac{100X}{100+R} + \left(\frac{100}{100+R}\right)^2 X + \ldots \left(\frac{100}{100+R}\right)^n X = P.$$

$$\text{or, } X = \text{Rs. } \frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2 + \ldots \left(\frac{100}{100+R}\right)^n}.$$

Illustration 14 If a sum of Rs. 13040 is to be paid back in two equal annual instalments at $3\frac{3}{4}$% per annum, what is the amount of each instalment?

Solution Each instalment $= \dfrac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2}$

$$= \frac{13040}{\left(\frac{100}{100+\frac{15}{4}}\right) + \left(\frac{100}{100+\frac{15}{4}}\right)^2}$$

$$\left[\text{Here } P = 13040 \text{ and } R = \frac{15}{4}\right]$$

$$= \frac{13040}{\frac{400}{415} + \left(\frac{400}{415}\right)^2} = \frac{13040}{\frac{400}{415}\left(1 + \frac{400}{415}\right)}$$

$$= 13040 \times \frac{415}{400} \times \frac{815}{400} = \mathbf{Rs.\ 6889.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Nikita invested Rs. 8000 for 3 years at 5% CI in a post office. If the interest is compounded once in a year, what sum will she get after 3 years?
   (a) Rs. 9261 (b) Rs. 8265
   (c) Rs. 9365 (d) None of these

2. The compound interest on Rs. 2000 at 5% per annum, compounded yearly, for 2 years is
   (a) Rs. 315 (b) Rs. 425
   (c) Rs. 205 (d) None of these

3. At what rate per cent per annum will Rs. 1000 amount to Rs. 1331 in 3 years? The interest is compounded yearly.
   (a) 10% p.a. (b) 12% p.a.
   (c) 13% p.a. (d) None of these

4. Find the present worth of Rs. 9261 due 3 years hence at 5% per annum compounded yearly.
   (a) Rs. 7000 (b) Rs. 8000
   (c) Rs. 9000 (d) None of these

5. The compound interest on Rs. 10000 at 20% per annum at the end of 1 year 6 months if the interest is calculated half-yearly will be
   (a) Rs. 5320 (b) Rs. 3310
   (c) Rs. 4340 (d) None of these

6. A sum put out at 4% compound interest payable half-yearly amounts to Rs. 6632.55 in $1\frac{1}{2}$ years. The sum is
   (a) Rs. 6530 (b) Rs. 6250
   (c) Rs. 6470 (d) None of these

7. The compound interest on Rs. 12000 for 9 months at 20% per annum, interest being compounded quarterly, is
   (a) Rs. 1891.50 (b) Rs. 1901.50
   (c) Rs. 1791.50 (d) None of these

8. The difference of compound interest on Rs. 800 for 1 year at 20% per annum when compounded half yearly and quarterly is

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(a) Rs. 4.40 (b) Nil
(c) Rs. 6.40 (d) None of these

9. The difference between the simple interest and the compound interest on Rs. 60 for 1 year at 10% per annum, reckoned half-yearly is
(a) Re. 1 (b) Rs. $1\frac{1}{2}$
(c) Rs. 2 (d) None of these

10. Rs. 800 at 5% per annum compound interest amount to Rs. 882 in
(a) 6 years (b) 2 years
(c) 4 years (d) None of these

11. What will be the compound interest on a sum of Rs. 1875 after 2 years if the rate of interest for the first year is 4% and that for the second year is 8%?
(a) Rs. 231 (b) Rs. 341
(c) Rs. 241 (d) None of these

12. What will be the amount if a sum of Rs. 5000 is placed at compound interest for 3 years while rate of interest for the first, second and third years is 2, 3 and 4 per cent, respectively?
(a) Rs. 5643·12 (b) Rs. 5463·12
(c) Rs. 6413·12 (d) None of these

13. What sum will amount to Rs. 15916.59 in 3 years at compound interest, the interest for first, second and third year being 3, 2 and 1 per cent, respectively?
(a) Rs. 18000 (b) Rs. 12000
(c) Rs. 15000 (d) None of these

14. The compound interest on Rs. 800 in $2\frac{1}{2}$ years at 5% is
(a) Rs. 105.05 (b) Rs. 104.05
(c) Rs. 106.05 (d) None of these

15. On what sum will the compound interest for $2\frac{1}{2}$ years at 10% amount to Rs. 6352.50?
(a) Rs. 7000 (b) Rs. 8000
(c) Rs. 5000 (d) None of these

16. The compound interest on a sum of money for 3 years at 5% is Rs. 1324·05. What is the simple interest?
(a) Rs. 1460 (b) Rs. 1365
(c) Rs. 1260 (d) None of these

17. The simple interest on a certain sum at 4% per annum for 2 years is Rs. 80. The compound interest on the same sum for the same period is
(a) Rs. 91.60 (b) Rs. 81.60
(c) Rs. 71.60 (d) None of these

18. If the compound interest on a certain sum for 2 years is Rs. 60·60 and the simple interest is Rs. 60, then the rate of interest per annum is
(a) 2% (b) 3%
(c) 4% (d) None of these

19. If the compound interest on a certain sum for 2 years is Rs. 105 and simple interest is Rs. 100, then the sum is
(a) Rs. 300 (b) Rs. 500
(c) Rs. 400 (d) None of these

20. The difference between simple interest and compound interest on Rs. 1250 for 2 years at 4% p.a. is
(a) Rs. 3 (b) Rs. 4
(c) Rs. 2 (d) None of these

21. On a certain sum of money, the simple interest for 2 years is Rs. 200 at the rate of 7% per annum. Find the difference in *CI* and *SI*.
(a) Rs. 7 (b) Rs. 9
(c) Rs. 11 (d) None of these

22. The difference between the compound interest and simple interest on a certain sum at 5% for 2 years is Rs. 1.50. The sum is
(a) Rs. 700 (b) Rs. 600
(c) Rs. 500 (d) None of these

23. The difference between the compound interest and simple interest on a certain sum at 3% per annum for 3 years is Rs. 27.27. The sum is
(a) Rs. 12000 (b) Rs. 15000
(c) Rs. 10000 (d) None of these

24. The difference between the compound interest and the simple interest on Rs. 8000 for 3 years at 5% per annum is
(a) Rs. 61 (b) Rs. 63
(c) Rs. 65 (d) None of these

25. If a sum of money at compound interest amounts to thrice itself in 3 years, then in how many years will it be 9 times itself?
(a) 9 years (b) 6 years
(c) 7 years (d) None of these

26. At what rate per cent compound interest does a sum of money become 16 times in 4 years?
(a) 75% (b) 100%
(c) 50% (d) None of these

27. A certain sum of money at compound interest grows up to Rs. 12960 in 2 years and up to Rs. 13176 in 3 years. Find the rate per cent per annum.
(a) $1\frac{1}{3}\%$ (b) $2\frac{1}{3}$
(c) $1\frac{2}{3}\%$ (d) None of these

28. What sum of money at compound interest will amount to Rs. 650 at the end of the first year and Rs. 676 at the end of the second year?
(a) Rs. 825 (b) Rs. 925
(c) Rs. 625 (d) None of these

29. A sum of Rs. 1260 is borrowed from a money lender at 10% p.a. compounded annually. If the

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amount is to be paid back in two equal annual instalments, find the annual instalment.
(a) Rs. 726 (b) Rs. 626
(c) Rs. 526 (d) None of these

30. A tree increases annually by $\frac{1}{8}$th of its height. By how much will it increase after 2 years, if it stands today 64 cm high?
(a) 72 cm (b) 74 cm
(c) 75 cm (d) 81 cm

31. The least number of completed years in which a sum of money put out at 20% C.I. will be more than doubled is
(a) 3 (b) 4
(c) 5 (d) 6

32. A man borrows Rs. 4000 from a bank at $7\frac{1}{2}\%$ compound interest. At the end of every year he pays Rs. 1500 as part repayment of loan and interest. How much does he still owe to the bank after three such instalments?
(a) Rs. 123.25 (b) Rs. 125
(c) Rs. 400 (d) Rs. 469.18

33. If in a certain number of years Rs. 3000 amount to Rs. 4320 at compound interest, in half that time Rs. 3000 will amount to:
(a) Rs. 3400 (b) Rs. 3600
(c) Rs. 3800 (d) Rs. 3520

34. Rs. 3757 is to be divided between A and B such that A's share at the end of 7 years may be equal to B's share at the end of 9 years. If rate per cent be 10% p.a. compound interest, B's share is
(a) Rs. 1700 (b) Rs. 1500
(c) Rs. 2057 (d) Rs. 1400

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Here, $P = 8000$, $t = 3$ and $R = 5$.

$$\therefore \text{Amount} = P\left(1+\frac{R}{100}\right)^t = 8000\left(1+\frac{5}{100}\right)^3$$
$$= 8000\left(\frac{21}{20}\right)^3 = \frac{8000\times21\times21\times21}{20\times20}$$
$$= \text{Rs. } 9261.$$

$\therefore$ Nikita will get Rs. 9261 after **3 years.**

2. Ans. (c)

Solution Here, $P = 2000$, $R = 5$ and $t = 2$.

$$\therefore CI = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$
$$= 2000\left[\left(1+\frac{5}{100}\right)^2 - 1\right] = 2000\left[\left(\frac{21}{20}\right)^2 - 1\right]$$
$$= 2000\left(\frac{441}{400} - 1\right) = 2000 \times \frac{41}{400} = \textbf{Rs. 205.}$$

3. Ans. (a)

Solution Here, $P = 1000$, $A = 1331$ and $t = 3$.

$$\therefore P = 100\left[\left(\frac{A}{P}\right)^{1/t} - 1\right]\% \text{ p.a.}$$
$$= 100\left[\left(\frac{1331}{1000}\right)^{1/3} - 1\right]\% \text{ p.a.}$$
$$= 100\left[\left(\frac{11}{10}\right)^{3\times\frac{1}{3}} - 1\right] = 100 \times \frac{1}{10} = \textbf{10\% p.a.}$$

4. Ans. (b)

Solution Here, $A = 9261$, $t = 3$ and $R = 5$.

$$\therefore P = \frac{A}{\left(1+\frac{R}{100}\right)^t} = \frac{9261}{\left(1+\frac{5}{100}\right)^3}$$
$$= \frac{9261\times20\times20\times20}{21\times21\times21} = \textbf{Rs. 8000.}$$

5. Ans. (b)

Solution Here, $P = 10000$, $R = 20$ and $t = \frac{3}{2}$.

$$\therefore CI = P\left[\left(1+\frac{R}{100\times2}\right)^{2t} - 1\right]$$
$$= 10000\left[\left(1+\frac{20}{100\times2}\right)^{2\times\frac{3}{2}} - 1\right]$$
$$= 10000\left[\left(\frac{11}{10}\right)^3 - 1\right]$$
$$= 10000\left(\frac{11\times11\times11}{10\times10\times10} - 1\right)$$
$$= \frac{10000\times331}{1000} = \textbf{Rs. 3310.}$$

6. Ans (b)

Solution Let Rs. x be the sum. Then,

$$6632.55 = x\left(1+\frac{4}{2\times100}\right)^{2\times\frac{3}{2}} = x\left(\frac{51}{50}\right)^3$$
$$\therefore x = \frac{6632\cdot55\times50\times50\times50}{51\times51\times51}$$

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7. Ans. (a)

Solution Here, $P = 12000$, $R = 20$ and $t = \frac{9}{12}$

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 4}\right)^{4\times t} - 1\right]$$

$$= 12000\left[\left(1+\frac{20}{100\times 4}\right)^{4\times \frac{9}{12}} - 1\right]$$

$$= 12000\left[\left(1+\frac{1}{20}\right)^{3} - 1\right] = \frac{12000\times 1261}{20\times 20\times 20}$$

$$= \textbf{Rs. 1891·50.}$$

8. Ans. (a)

Solution When compounded half-yearly:

Here, $P = 800$, $R = 20$ and $t = 1$.

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 2}\right)^{2\times t} - 1\right]$$

$$= 800\left[\left(1+\frac{20}{100\times 2}\right)^{2\times 1} - 1\right]$$

$$= 800\left[\left(\frac{11}{10}\right)^{2} - 1\right] = \frac{800\times 21}{10\times 10} = \text{Rs. } 168.$$

When compounded quarterly:

Here, $P = 8000$, $R = 20$ and $t = 1$.

$$\therefore \quad CI = P\left[\left(1+\frac{R}{100\times 4}\right)^{4\times t} - 1\right]$$

$$= 800\left[\left(1+\frac{20}{100\times 4}\right)^{4\times 1} - 1\right]$$

$$= 800\left[\left(\frac{21}{20}\right)^{4} - 1\right] = \frac{800\times 34481}{20\times 20\times 20\times 20}$$

$$= \text{Rs. } 172.40.$$

$\therefore$ Difference = Rs. $(172.40 - 168)$ = **Rs. 4.40.**

9. Ans. (b)

Solution $SI = \frac{600\times 10\times 1}{100} = \text{Rs. } 60.$

$$CI = 600\left[\left(1+\frac{10}{100\times 2}\right)^{2\times 1} - 1\right]$$

$$= 600\left[\left(\frac{21}{20}\right)^{2} - 1\right] = \frac{600\times 41}{20\times 20} = \text{Rs. } 61·50.$$

$\therefore$ Difference = Rs. $(61.50 - 60)$ = **Rs. 1.50.**

10. Ans. (b)

Solution Let the time be t years. Then,

$$882 = 800\left(1+\frac{5}{100}\right)^{t} \Rightarrow \frac{882}{800} = \left(\frac{21}{20}\right)^{t}$$

$$\Rightarrow \left(\frac{21}{20}\right)^{2} = \left(\frac{21}{20}\right)^{t}$$

$$\Rightarrow \textbf{t = 2 years.}$$

11. Ans. (a)

Solution Here, $P = 1875$, $R_1 = 4$ and $R_2 = 8$.

$$\therefore \quad CI = P\left[\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) - 1\right]$$

$$= 1875\left[\left(1+\frac{4}{100}\right)\left(1+\frac{8}{100}\right) - 1\right]$$

$$= 1875\left[\frac{26}{25}\times\frac{27}{25} - 1\right]$$

$$= \frac{1875\times 77}{625} = \textbf{Rs. 231.}$$

12. Ans.

Solution Here, $P = 5000$, $R_1 = 2$, $R_2 = 3$ and $R_3 = 4$.

$\therefore$ Amount after 3 years

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$$

$$= 5000\left(1+\frac{2}{100}\right)\left(1+\frac{3}{100}\right)\left(1+\frac{4}{100}\right)$$

$$= 5000 \times \frac{51}{50}\times\frac{103}{100}\times\frac{26}{25}$$

$$= \textbf{Rs. 5463·12.}$$

13. Ans. (c)

Solution Let Rs. P be the required sum.

Then, $15916.59 = P\left(1+\frac{3}{100}\right)\left(1+\frac{2}{100}\right)\left(1+\frac{1}{100}\right)$

$$= P\left(\frac{103}{100}\times\frac{102}{100}\times\frac{101}{100}\right).$$

$$\therefore \quad P = \frac{1591659\times 100\times 100}{103\times 102\times 101}$$

$$= \textbf{Rs. 15000.}$$

14. Ans. (b)

Solution $CI = 800\left[\left(1+\frac{5}{100}\right)^{2}\left(1+\frac{\frac{1}{2}\times 5}{100}\right) - 1\right]$

$$= 800\left[\frac{21\times 21\times 41}{20\times 20\times 40} - 1\right]$$

$$= \frac{800\times 2081}{16000}$$

$$= \textbf{Rs. 104.05.}$$

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15. Ans. (c)

Solution We have, $6352{\cdot}50 = P\left(1+\frac{10}{100}\right)^2\left(1+\frac{\frac{1}{2}\times 10}{100}\right)$

$= P\left(\frac{11}{10}\right)^2\left(\frac{21}{20}\right)$

$\Rightarrow \quad P = \frac{6352{\cdot}50\times 10\times 10\times 20}{11\times 11\times 21}$ = **Rs. 5000.**

16. Ans. (c)

Solution We have, $CI - SI = \frac{SI}{3}\left[\left(\frac{R}{100}\right)^2 + 3\left(\frac{R}{100}\right)\right]$

$\Rightarrow \quad 1324{\cdot}05 - SI = \frac{SI}{3}\left[\left(\frac{5}{100}\right)^2 + 3\left(\frac{5}{100}\right)\right]$

$= \frac{SI}{3}\left[\frac{1+60}{20\times 20}\right] = \frac{61SI}{1200}$

$\Rightarrow \quad \left(1+\frac{61}{1200}\right)SI = 1324{\cdot}05$

$\Rightarrow \quad SI = \frac{1324{\cdot}05\times 1200}{1261}$ = **Rs. 1260.**

17. Ans. (b)

Solution We have, $CI - SI = \frac{R\times SI}{200}$

$\Rightarrow CI = SI + \frac{R\times SI}{200} = SI\left(1+\frac{R}{200}\right) = 80\left(1+\frac{4}{200}\right)$

[Here, $R = 4$ and $SI = 80$]

$= \frac{80\times 51}{50}$ = **Rs. 81.60.**

18. Ans. (a)

Solution We have,

$CI - SI = \frac{R\times SI}{200} \Rightarrow 60.60 - 60 = \frac{R\times 60}{200}$

$\Rightarrow R = \frac{0{\cdot}60\times 200}{60}$ = **2%.**

19. Ans. (b)

Solution We have,

$CI - SI = \frac{R\times SI}{200} \Rightarrow 105 - 100 = \frac{R\times 100}{200}$

$\Rightarrow R = 10.$

Also, $CI - SI = P\left(\frac{R}{100}\right)^2 \Rightarrow 105 - 100 = P\left(\frac{10}{100}\right)^2$

$\Rightarrow P$ = **Rs. 500.**

20. Ans. (c)

Solution We have,

$CI - SI = P\left(\frac{R}{100}\right)^2 = 1250\left(\frac{4}{100}\right)^2$

[Here, $P = 1250$ and $R = 4$]

$= \frac{1250}{15\times 25}$ = **Rs. 2.**

21. Ans. (a)

Solution We have,

$CI - SI = \frac{R\times SI}{200} = \frac{7\times 200}{200}$

[Here, $R = 7$ and $SI = 200$]

= **Rs. 7.**

22. Ans. (b)

Solution We have, $CI - SI = P\left(\frac{R}{100}\right)^2$

$\Rightarrow \quad \frac{3}{2} = P\left(\frac{5}{100}\right)^2$ $\left[\text{Here, CI} - \text{SI} = \frac{3}{2} \text{ and } R = 5\right]$

$\Rightarrow \quad P = \frac{3\times 20\times 20}{2}$ = **Rs. 600.**

23. Ans. (c)

Solution We have, $CI - SI = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$

$\Rightarrow 27{\cdot}27 = P\left[\left(\frac{3}{100}\right)^3 + 3\left(\frac{3}{100}\right)^2\right]$

[Here, $CI - SI = 27{\cdot}27$ and $R = 3$]

$= P\left[\frac{27+2700}{100\times 100\times 100}\right]$

$\Rightarrow P = \frac{27{\cdot}27\times 100\times 100\times 100}{2727}$ = **Rs. 10000.**

24. Ans. (a)

Solution We have,

$CI - SI = P\left[\left(\frac{R}{100}\right)^3 + 3\left(\frac{R}{100}\right)^2\right]$

$= 8000\left[\left(\frac{5}{100}\right)^3 + 3\left(\frac{5}{100}\right)^2\right]$

[Here, $P = 8000$ and $R = 5$]

$= 8000\left[\frac{125+7500}{100\times 100\times 100}\right]$

= **Rs. 61.**

25. Ans. (b)

Solution Here, $n = 3$, $t = 3$ and $m = 2$.

$\therefore$ The given sum will become 9 times itself in mt, i.e. 2×3 = **6 years.**

26. Ans. (b)

Solution The required rate per cent is

$R = 100[(n)^{1/t} - 1] = 100[(16)^{1/4} - 1]$

[Here, $n = 16$ and $t = 4$]

$= 100(2 - 1)$ = **100%.**

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27. Ans. (a)

Solution Here, $x = 12960$, $y = 13176$, $A = 2$ and $B = 3$.

$$\therefore \quad R = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$$

$$= \left(\frac{13176}{12960} - 1\right) \times 100\%$$

$$= \left(\frac{216}{12960} \times 100\right)\%$$

$$= \frac{4}{3}\% \text{ or } 1\frac{1}{3}\%.$$

28. Ans. (c)

Solution Here, $x = 650$, $y = 676$, $A = 1$ and $B = 2$.

$$\therefore \text{ Rate of interest } (R) = \left[\left(\frac{y}{x}\right)^{1/B-A} - 1\right] \times 100\%$$

$$= \left[\frac{676}{650} - 1\right] \times 100\%$$

$$= \frac{26}{650} \times 100\% = 4\%.$$

$$\therefore \quad 650 = P\left(1 + \frac{4}{100}\right) \Rightarrow P = \frac{650 \times 25}{26} = \textbf{Rs. 625.}$$

29. Ans. (a)

Solution Here, $P = 1260$ and $R = 10$.

$\therefore$ Annual instalment

$$= \frac{P}{\left(\frac{100}{100+R}\right) + \left(\frac{100}{100+R}\right)^2} = \frac{1260}{\frac{100}{110} + \left(\frac{100}{110}\right)^2}$$

$$= \frac{1260}{\frac{100}{110}\left(1 + \frac{100}{110}\right)} = 1260 \times \frac{110}{100} \times \frac{110}{210}$$

$$= \textbf{Rs. 726.}$$

30. Ans. (d)

Solution Increase % $= \left(\frac{1}{8} \times 100\right)\% = 12.5\%$

$$\text{Height after 2 years} = 64 \times \left(1 + \frac{25}{2 \times 100}\right)^2$$

$$= 64 \times \frac{9}{8} \times \frac{9}{8}$$

$$= \textbf{81 cm.}$$

31. Ans. (b)

Solution $x\left(1 + \frac{20}{100}\right)^n > 2x$ or $\left(\frac{6}{5}\right)^n > 2$

Now, $\left(\frac{6}{5} \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5}\right) > 2$

$\therefore$ $\boldsymbol{n = 4}$ **years.**

32. Ans. (a)

Solution Balance = Rs. $\left[\left\{4000 \times \left(1 + \frac{15}{2 \times 100}\right)^3\right\}\right.$

$$\left. - \left\{1500 \times \left(1 + \frac{15}{2 \times 100}\right)^2 + 1500 \times \left(1 + \frac{15}{2 \times 100}\right) + 1500\right\}\right]$$

$$= \textbf{Rs. 123·25}$$

33. Ans. (b)

Solution Let $r\%$ be the rate and n years be the time.

Then, $4320 = 3000\left(1 + \frac{r}{100}\right)^n$

$$\therefore \quad \left(1 + \frac{r}{100}\right)^n = \frac{4320}{3000} = 1.44$$

$$\therefore \quad \left(1 + \frac{r}{100}\right)^{n/2} = \sqrt{1.44} = 1.2$$

$\therefore$ In $\frac{n}{2}$ years, Rs. 3000 will amount to

$$3000\left(1 + \frac{r}{100}\right)^{n/2} = 3000 \times 1.2$$

$$= \textbf{Rs. 3600.}$$

34. Ans. (a)

Solution Let A's share = Rs. x

B's share = Rs. $(3757 - x)$

$$x\left(1 + \frac{10}{100}\right)^7 = (3757 - x)\left(1 + \frac{10}{100}\right)^9$$

$$x = (3757 - x)\left(\frac{11}{10}\right)^2$$

$$\therefore \quad x\left(1 + \frac{121}{100}\right) = \frac{3757 \times 121}{100}$$

$$\therefore \quad x = \frac{375 \times 121}{221} = \text{Rs. } 2057$$

$\therefore$ B's share = Rs. $(3757 - 2057)$ = **Rs. 1700.**

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. A scooter costs Rs. 25,000 when it is brand new. At the end of each year, its value is only 80% of what it was at the beginning of the year. What is the value of the scooter at the end of 3 years?

(a) Rs 10,000 (b) Rs 12,500
(c) Rs 12,800 (d) Rs 12,000

[Based on MAT 2004]

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2. What.will Rs. 1000 be worth after three years if it earns interest at the rate of 5% compounded annually?
(a) 1057 (b) 1257
(c) 1157 (d) 1300
[Based on MAT 2003]

3. Sanjay borrowed a certain sum from Anil at a certain rate of simple interest for 2 years. He lent this sum to Ram at the same rate of interest compounded annually for the same period. At the end of two years, he received Rs. 4200 as compound interest but paid Rs. 4000 only as simple interest. Find the rate of interest.
(a) 15% (b) 20%
(c) 35% (d) 10%
[Based on MAT 2002]

4. The difference between the simple interest and the compound interest compounded annually at the rate of 12% per annum on Rs. 5000 for two years will be
(a) Rs 17.50 (b) Rs 36
(c) Rs 45 (d) Rs 72
[Based on MAT 2001]

5. If the difference between the simple and the compound interests on some principal amount at 20% for 3 years is Rs. 48, then the principal amount must be
(a) Rs. 650 (b) Rs. 600
(c) Rs. 375 (d) Rs. 400
[Based on MAT 2001]

6. A bank offers 10% interest rate compounded annually. A person deposits Rs. 10,000 every year in his account. If he does not withdraw any amount, then how much balance will his account show after four years?
(a) Rs 51051 (b) Rs 45095
(c) Rs 36410 (d) Rs 51000
[Based on FMS (Delhi) 2004]

7. Two customers borrowed the same amount of money, one at compound interest and the other at simple interest. If after 2 years, the interest payable by one was Rs 220 and by the other Rs 200, then, what was the principal money lent to each one of them?
(a) Rs 450 (b) Rs 500
(c) Rs 550 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

8. What is the present value (in lakh of Rs) of an income of Rs 2 lakh to be received after 2 years if the rate of interest is 5%?
(a) 1.81 (b) 1.67
(c) 2.2 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

9. A sum is invested for 3 years compounded at 5%, 10% and 20% respectively. In three years, if the sum amounts to Rs. 16,632, then find the sum.
(a) Rs 11000 (b) Rs 12000
(c) Rs 13000 (d) Rs 14000
[Based on IIT Joint Man. Ent. Test 2004]

10. The value of a fixed asset depreciates at the rate of 10% on the value at the beginning of each year. If the value of the asset, two years ago, was Rs 12,000 more than the value of the asset one year ago, then find the present value of the asset, given that the asset was bought two years ago.
(a) Rs 14520 (b) Rs 96,000
(c) Rs 97,200 (d) Rs 17,520
[Based on IIT Joint Man. Ent. Test 2004]

11. What will be the difference in the compound interest on Rs 50,000 at 12% for one year, when the interest is paid yearly and half-yearly?
(a) Rs 500 (b) Rs 600
(c) Rs 180 (d) Rs 360
[Based on IIT Joint Man. Ent. Test 2004]

12. Akash borrows Rs 65,000 at 10% p.a. simple interest for 3 years and lends it at 10% p.a, compound interest for 3 years. Find his gain after three years.
(a) Rs 2015 (b) Rs 1330
(c) Rs 1300 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

13. A father divides his property between his two sons *A* and *B*. A invests the amount at compound interest of 8% p.a. *B* invests the amount at 10% p,a. simple interest. At the end of 2 years, the interest received by *B* is Rs. 1336 more than the interest received by *A*. Find the share of *A* in the father's property of Rs 25,000.
(a) Rs 12,000 (b) Rs 13,000
(c) Rs 12,500 (d) Rs 10,000
[Based on IIT Joint Man. Ent. Test 2004]

14. What effective rate of interest per annum does a person get who is paid at the rate of 10% p.a. interest payable half yearly?
(a) 11% (b) 10.75%
(c) 10.5% (d) 10.25%
[Based on IIT Joint Man. Ent. Test 2004]

15. Rohit earns an interest of Rs 1656 for the third year and Rs 1440 for the second year on the same sum. Find the rate of interest if it is lent at compound interest.
(a) 18% (b) 12%
(c) 15% (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

16. The population of a town increases annually by 25%. If the present population is one crore, then what is the difference between the population 3 years ago and 2 years ago?
(a) 25,00,000 (b) 12,80,000
(c) 15,60,000 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

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in the interest on the amount between the 3rd and 2nd years is Rs 1,100. What amount has Anuj deposited ?

(a) Rs 10000 (b) Rs 11000

(c) Data inadequate (d) None of these

[Based on Narsee Manjee Inst. of Man. Studies, 2003]

18. What sum invested for 2 years at 12% compounded annually will grow to Rs 4390.40?

(a) Rs 4000 (b) Rs 3500

(c) Rs 3800 (d) Rs 3875

[Based on REC Tiruchirapalli 2003]

19. Divide Rs 1301 between *A* and *B*, so that the amount of *A* after 7 years is equal to the amount of *B* after 9 years, the interest being compounded at 4% per annum.

(a) Rs 676 and Rs 625 (b) Rs 650 and Rs 651

(c) Rs 670 and Rs 631 (d) Rs 660 and Rs 630

(e) None of the above **[Based on IIFT 2003]**

20. A sum of money placed at compound interest doubles itself in 5 years. If would amount eight times in

(a) 10 years (b) 12 years

(c) 15 years (d) 20 years

[Based on FMS 2006]

21. Two friends A and B jointly lent out Rs 81,600 at 4% per annum compound interest. After 2 years A gets the same amount as B gets after 3 years. The investment made by B was

(a) Rs 40,000 (b) Rs 30,000

(c) Rs 45,000 (d) Rs 38,000

[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution $25000\left(1-\frac{20}{100}\right)^3 = 25000\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}$

$= \mathbf{12800.}$

2. Ans. (c)

Solution $1000\left(1+\frac{5}{100}\right)^3 = 1000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$

$= \frac{9261}{8} = \text{Rs } 1157.62$

$\approx \mathbf{Rs\ 1157.}$

3. Ans. (d)

Solution Suppose the sum borrowed = Rs x

Rate of interest = $R\%$

Time = 2 years

$\therefore \quad 4000 = \frac{x\times R\times 2}{100} \Rightarrow Rx = 200000 \quad ...(i)$

Now

$x\left(1+\frac{R}{100}\right)^2 = x + 4200 \Rightarrow \frac{xR^2}{10000} + \frac{2RX}{100} = 4200$

$\Rightarrow 20R + 4000 = 4200$

$\Rightarrow \boldsymbol{R = 10.}$

4. Ans. (d)

Solution $\text{C.I.} - \text{S.I.} = \left[5000+\left(1+\frac{12}{100}\right)^2 - 5000\right] - \frac{5000\times 12\times 2}{100}$

$= 5000\left(\frac{28}{25}\times\frac{28}{25}-1\right) - 1200$

$= \mathbf{72.}$

5. Ans. (c)

Solution Let the principal be Rs. 100.

$\therefore \quad \text{S.I.} = \frac{100\times 20\times 3}{100} = \text{Rs. } 60$

$\text{C.I.} = 100\left(1+\frac{20}{100}\right)^3 - 100$

$= 100\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5} - 10C$

$= 100\left[\frac{216-125}{125}\right] = \frac{4}{5}\times 91 = \frac{364}{5}$

$\therefore \quad \text{C.I.} - \text{S.I.} = \frac{364}{5} - 60 = \frac{64}{5}$

If difference is Rs. $\frac{64}{5}$, principal = Rs. 100

If difference is Rs. 48, principal

$= \frac{100\times 5}{64}\times 48 = \mathbf{Rs.\ 375.}$

6. Ans. (a)

Solution Rs 10000 after 4 years become

$= 10000\left[1+\frac{10}{100}\right]^4$

$= 10000\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10} = 14641$

Rs 10000 after 3 years become

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$$= 10000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = 13310$$

Rs 10000 after 2 years become

$$= 10000 \times \frac{11}{10} \times \frac{11}{10} = 12100$$

Rs 10000 after 1 year become Rs 11000

$\therefore$ Total amount after 4 years = **Rs 51051.**

7. Ans. (b)

Solution S.I. for two years = Rs 200

S.I. for one years = Rs 100

C.I. for two years = Rs 220

$\Rightarrow$ Rs 20 is the interest on Rs 100 for one year.

If interest is Rs 20, then amount = Rs 100.

If interest is Rs 100, then amount

$$= \frac{100}{20} \times 100 = \textbf{Rs 500.}$$

8. Ans. (a)

Solution P = Present value of Rs 2 lakh

$$\Rightarrow \quad P\left(1+\frac{5}{100}\right)^2 = 2 \text{ Lakh}$$

$$\Rightarrow \quad P = \frac{2 \times 20 \times 20}{21 \times 21} = \textbf{1.81.}$$

9. Ans. (b)

Solution Let P be the sum.

$\therefore$ Amount after one year = P + 5% of $P = \frac{21P}{20}$

Amount after two years $= \frac{21P}{20} + 10\%$ of $\frac{21P}{20}$

$$= \frac{21P}{20} + \frac{21P}{200} = \frac{231P}{200}$$

Amount after three years

$$= \frac{231P}{200} + 20\% \text{ of } \frac{231P}{200}$$

$$= \frac{231P}{200} + \frac{231P}{1000} = \frac{1386P}{1000}$$

$\therefore \quad \frac{1386P}{1000} = 16632 \Rightarrow \boldsymbol{P = 12000.}$

10. Ans. (c)

Solution Suppose value of the asset two years ago = Rs x

$\therefore$ Value of the asset one year ago

$$= x - 10\% \text{ of } x = \frac{9x}{10}$$

$\therefore \quad x - \frac{9x}{10} = 12000 \Rightarrow x = 120000$

$\therefore$ Value of the asset at present

$$= \frac{9x}{10} - 10\% \text{ of } \frac{9x}{10}$$

$$= \frac{81x}{100} = 81 \times 1200 = \textbf{Rs 97200.}$$

11. Ans. (c)

Solution C.I. on Rs 50000 @ 12% for one year, when the interest is paid half yearly

$$= 50000\left(1+\frac{6}{100}\right)^2 - 50000$$

$$= 50000 \times \frac{53}{50} \times \frac{53}{50} - 50000$$

$$= \text{Rs } 6180$$

C.I. when the interest is paid yearly

$$= \frac{50000 \times 12 \times 1}{100} = \text{Rs } 6000$$

$\therefore$ Required difference = C.I. – S.I.

$$= 6180 - 6000 = \textbf{Rs 180.}$$

12. Ans. (a)

Solution S.I. on Rs 65000 @ 10% for 3 years

$$= \frac{65000 \times 10 \times 3}{100} = \text{Rs } 19500$$

C.I. on Rs 65000 @ 10% for 3 years

$$= 65000\left[1+\frac{10}{100}\right]^3 - 65000$$

$$= 65000\left[\frac{11 \times 11 \times 11 - 10 \times 10 \times 10}{1000}\right]$$

$$= \text{Rs } 21515$$

$\therefore$ Required gain = 21515 – 19500 = **Rs 2015.**

13. Ans. (d)

Solution Suppose A get Rs x

$\therefore$ B gets Rs $(25000 - x)$

Interest received by A @ 8% p.a. C.I.

$$= x\left[1+\frac{8}{100}\right]^2 - x$$

$$= \left(\frac{27}{25}\right)^2 x - x = \frac{104x}{625}$$

Interest received by B @ 10% of p.a. S.I.

$$= \frac{(25000-x) \times 10 \times 2}{100}$$

$$= \frac{25000-x}{5}$$

Given: $\frac{25000-x}{5} = \frac{104x}{625} + 1336 \Rightarrow \boldsymbol{x = 10000.}$

14. Ans. (d)

15. Ans. (c)

Solution Interest on Rs 1440 = Rs 216 for the third year

$$\Rightarrow \quad \text{Rate } \% = \frac{216 \times 100}{1440 \times 1} = \textbf{15.}$$

16. Ans. (b)

Solution Let the population be x, three years ago

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$\therefore$ Population two years ago = x + 25% of $x = \frac{5x}{4}$

Population one year ago = $\frac{5x}{4}$ + 5% of $\frac{5x}{4}$

$= \frac{5x}{4} + \frac{5x}{16} = \frac{25x}{16}$

$\therefore$ Present population = $\frac{25x}{16}$ + 25% of $\frac{25x}{16}$

$= \frac{125x}{64} = 10000000$ (Given) $\Rightarrow x = 5120000$

$\therefore$ Required difference = $\frac{5x}{4} - x = \frac{x}{4}$ = **1280000.**

17. Ans. (b)

Solution S.I. for one year = Rs 1100

$\therefore$ Amount at the end of 2nd year = $\frac{1100 \times 100}{10 \times 1}$

= **Rs 11000.**

18. Ans. (b)

Solution $P\left(1+\frac{R}{100}\right)^T$ = Amount

$\Rightarrow P\left(1+\frac{12}{100}\right)^2 = 4390.40$

$\Rightarrow P\left[\frac{112}{100}\right]^2 = 4390.40$

$\Rightarrow P = \frac{43904000}{112 \times 112}$ = **3500.**

19. Ans. (a)

Solution Let Amount of $A = P$

$\therefore$ Amount of $B = 1301 - P$

$\Rightarrow P\left(1+\frac{4}{100}\right)^7 = (1301 - P)\left(1+\frac{4}{100}\right)^9$

$\Rightarrow P = (1301 - P)\left(\frac{26}{25} \times \frac{26}{25}\right)$

$\Rightarrow 625P = 1301 \times 676 - 676P \Rightarrow P = 676$

$\therefore$ A gets Rs 676 and B gets **Rs 625.**

20. Ans. (c)

Solution $x\left(1+\frac{4}{100}\right)^3 = (81600 - x)\left(1-\frac{4}{100}\right)^2$

$x\left(\frac{104}{100}\right) = (81600 - x)$; $x = 40000$

21. Ans. (a)

Solution $2x = x\left(1+\frac{R}{100}\right)^5$; $2 = \left(1+\frac{R}{100}\right)^5$

$2^{\frac{1}{5}} = 1+\frac{R}{100}$ ...(1)

$8x = x\left(1+\frac{R}{100}\right)^n$; $8 = \left(1+\frac{R}{100}\right)^n$

$2^{\frac{3}{n}} = 1+\frac{R}{100}$ ...(2)

From equations (1) and (2) we get $2^{\frac{1}{5}} = 2^{\frac{3}{n}}$

$\frac{1}{5} = \frac{3}{n} \Rightarrow n = 15$ years

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. If certain sum lent at $3\frac{3}{4}$% p.a. for 16 months produces as much simple interest as Rs. 750 lent at 7% p.a. in 5 years, the sum is
(a) Rs. 525 (b) Rs. 5250
(c) Rs. 4250 (d) Rs. 5000

2. A lent Rs. 600 to B for 2 years and Rs. 150 to C for 4 years at the same rate of interest and received altogether from both Rs. 90 as simple interest. The rate of interest was
(a) 5% (b) $4\frac{1}{2}$%
(c) 3% (d) $2\frac{1}{2}$%

3. If Rs 5600 amounts to Rs. 6678 in $3\frac{1}{2}$ years, at the same rate p.c. Rs. 9400 in $5\frac{1}{4}$ years will amount to
(a) Rs. 12114.25 (b) Rs. 11214.25
(c) Rs. 10914.50 (d) None of these

4. A sum of money placed at C.I. doubles itself in 5 years. It will amount to eight times itself in
(a) 15 years (b) 20 years
(c) 12 years (d) 10 years

5. What is the principal amount which earns Rs. 132 at 10% per annum?
(a) Rs. 1000 (b) Rs. 1200
(c) Rs. 1320 (d) None of these

6. What will be the amount if a sum of Rs 5000 is placed at compound interest for 3 years while rate of interest for the first, second and third year is 2, 3 and 4%, respectively?
(a) Rs. 5231.20 (b) Rs. 5000.00
(c) Rs. 5463.12 (d) Rs. 5713.20

7. Divide the sum of Rs. 3903 between A and B in such

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such a manner that if both of them put their parts of money at 4% per annum compound interest, amount received by A after 7 years remains equal to the amount received by B after 9 years.
(a) Rs. 1875 (b) Rs. 1587
(c) Rs. 1785 (d) Rs. 1758

8. The amount obtained from an investment of Rs. 3025 at 10% per annum compound interest for 3 years is equal to the amount obtained from the investment of a certain sum of money at 10% per annum compound interest for 5 years. Find the sum of investment in the second case.
(a) Rs 2500 (b) Rs 2415
(c) Rs 2480 (d) Rs 2550

9. Two friends A and B jointly lent out Rs 81600 at 4% per annum compound interest. After 2 years A gets the same amount as B gets after 3 years. The investment made by B was
(a) Rs. 30000 (b) Rs. 40000
(c) Rs. 45000 (d) Rs. 38000

10. A borrowed from a bank Rs 8000 for 2 years at 5% per annum compound interest and B borrowed from a money lender a certain sum of money for 3 years at 4% per annum compound interest. If after expiry of their respective loan period, each of them pay an equal amount to clear off their debts, find the sum of money borrowed by B from the money lender.
(a) Rs. 7840.25 (b) Rs. 7820.75
(c) Rs. 8840.25 (d) Rs. 8200

Test II

[Time: 10 min]

11. Sanjay put equal amounts of money one at 10% p.a. compound interest payable half-yearly and other at r% p.a. compound interest payable yearly. If he gets equal amounts after 3 years, the value of r is
(a) $10\frac{1}{4}$ (b) $9\frac{1}{2}\%$
(c) 10% (d) None of these

12. A certain sum put at compound interest amounts to Rs 699.66 in 2 years. If the rate p.c. for the first year be 4% and $3\frac{1}{2}\%$ for the second year, the sum is
(a) Rs. 650 (b) Rs. 625
(c) Rs. 599.25 (d) Rs. 602.27

13. If a certain sum of money amounts to Rs. 800 for 2 years and Rs 880 for 3 years then how much will it amount to in 4 years?
(a) Rs 920 (b) Rs 968
(c) Rs 898 (d) Rs 1000
(e) None of these

14. If the simple interest on a certain sum of money at 5% per annum for 2 years is Rs 160, what will be the compound interest on the same sum for the same time and the same rate of interest?
(a) Rs. 146 (b) Rs. 168
(c) Rs. 165 (d) Rs. 164
(e) Rs. 170

15. On what sum of money will be the difference between simple interest and compound interest for 2 years at 4% per annum be equal to Rs. 50?
(a) Rs 31250 (b) Rs 20400
(c) Rs 100000 (b) Rs 25000
(e) None of these

16. If a certain sum of money invested at compound interest amounts to Rs. 1452 in 2 years and Rs 1597.20 in 3 years, what is the rate of interest?
(a) 11% (b) 12%
(c) 10% (d) 9%
(e) None of these

17. A money lender lends Rs. 2000 for 6 months at 20% per annum whereas the interest is compounded quarterly. After the given period he will get the amount of
(a) Rs. 2205 (b) Rs. 2200
(c) Rs. 2160 (d) Rs. 2040
(e) None of these

18. The population of a village decreases at the rate of 20% per annum. If its population 2 years ago was 10,000, what is its present population?
(a) 6000 (b) 9000
(c) 6400 (d) 7600
(e) 19000

19. If the first year's interest on a certain sum of money placed at 5% per annum compound interest is Rs. 1200, what will be the interest for the third year?
(a) Rs. 1220 (b) Rs. 1323
(c) Rs. 1423 (d) Rs. 1330

20. For obtaining the difference between compound and simple interest on a certain sum of money invested for 3 years at 5% p.a., we must multiply the sum of money by
(a) 0.007625 (b) 0.007645
(c) 0.008625 (d) 0.007525

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution S.I. on Rs. 750 $= \frac{750\times7\times5}{100}$ = Rs. 262.50

This is also the S.I. of required sum.

$\therefore$ Required sum $= \frac{262.5\times100}{\frac{15}{4}\times\frac{16}{12}} = \frac{26250\times48}{15\times16}$

= Rs 5250.

3. Ans. (a)

Solution Interest in former case = Rs (6678 – 5600)
= Rs 1078

$\therefore$ Rate $= \frac{1078\times100}{5600\times7/2} = \frac{11}{2}\%$

$\therefore$ Interest on Rs. 9400 $= 9400 \times \frac{11}{200}\times\frac{21}{4}$
= Rs. 2714.25

$\therefore$ Required amount = Rs. (9400 + 2714.25)
= Rs. 12114.25.

5. Ans. (b)

Solution Let x be the principal at the end of first year

Then, $\frac{x\times10\times1}{100} = 132 \Rightarrow x = 1320$

Let y be the original principal

Then, $y + \frac{y+10\times1}{100} = 1320 \Rightarrow y =$ **Rs 1200.**

7. Ans. (a)

Solution Suppose A's part = Rs. x

$\therefore$ B's part = Rs. $(3903 - x)$

For A Amount $= x\left(1+\frac{4}{100}\right)^7 = x\left(\frac{26}{25}\right)^7$

For B Amount $= (3903 - x)\left(1+\frac{4}{100}\right)^9$

$= (3903 - x)\left(\frac{26}{25}\right)^9$

According to question, the two amounts are equal

i.e. $x\left(\frac{26}{25}\right)^7 = (3903 - x)\left(\frac{26}{25}\right)^9$

or, $\frac{x}{(3903-x)} = \left(\frac{26}{25}\right)^2 = \frac{676}{625}$

or, $625x = 2638428 - 676x$

or, $625x + 676x = 2638428$

or, $1301x = 26388428$

or, $x = \frac{2638428}{1301}$ = Rs 2028

$\therefore$ A's part = Rs 2028

Hence, B's part = 3903 – 2028 **= Rs 1875.**

9. Ans. (b)

Solution 2 years after the amount received by A is equal to the amount received by B after 3 years while rate of compound interest in both cases is same, that is, 4% p.a.

Hence, their sum of money will be in the ratio of

$$\frac{1}{\left(1+\frac{4}{100}\right)^2} : \frac{1}{\left(1+\frac{4}{100}\right)^3}$$

$\therefore$ Ratio of A's part to B's part

$= \frac{1}{\left(\frac{26}{25}\right)^2} : \frac{1}{\left(\frac{26}{25}\right)^3} = \frac{26}{25} : 1 = 26 : 25$

Sum of proportionals = 26 + 25 = 51

$\therefore$ A's part in total investment $= \frac{26}{51}\times81600$
= Rs. 41600

while B's part $= \frac{25}{51} \times 81600$ **= Rs 40000.**

11. Ans. (a)

Solution Let Rs x be the sum in each investment

Then, $x\left(1+\frac{\frac{1}{2}\times10}{100}\right)^{2\times3} = x\left(1+\frac{r}{100}\right)^3$

$\therefore$ $\left(1+\frac{1}{20}\right)^6 = \left(1+\frac{r}{100}\right)^3$

$\therefore$ $1 + \frac{r}{100} = \left(\frac{21}{20}\right)^2 = \frac{441}{400}$

$\therefore$ $r = \frac{41}{4} = 10\frac{1}{4}\%$.

12. Ans. (a)

Solution Let Rs x be the sum

$x\left(1+\frac{4}{100}\right)\left(1+\frac{7}{200}\right) = 699{\cdot}66$

$\therefore$ $x = \frac{699{\cdot}66\times25\times200}{26\times207}$ = Rs. 650.

13. Ans. (b)

Solution $800 = P\left(1+\frac{r}{100}\right)^2$ and, $880 = P\left(1+\frac{r}{100}\right)^3$

$\therefore$ Amount after 4 years

$= P\left(1+\frac{r}{100}\right)^4 = P\left(1+\frac{r}{100}\right)^2\left(1+\frac{r}{100}\right)$

$= 880 \times \frac{880}{800}$ **= Rs. 968.**

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15. Ans. (a)

Solution $\because$ Difference $= P\left(\frac{R}{100}\right)^2$

$\therefore \quad 50 = P\left(\frac{4}{100}\right)^2$

$\therefore \quad P = \frac{50\times100\times100}{4\times4} =$ **Rs. 31250.**

17. Ans. (a)

Solution Amount $= 2000\left(1+\frac{5}{100}\right)^2 = 2000\times\frac{21}{20}\times\frac{21}{20}$

= Rs. 2205.

19. Ans. (b)

Solution Since for the first year simple interest is equal to the compound interest

$$P = \frac{\text{S.I}\times100}{R\times T} = \frac{1200\times100}{5\times1} = \text{Rs. } 24000$$

Principal for second year = Rs. 24000 + Rs. 1200
= Rs. 25200.

$\therefore$ Interest for the second year $= \frac{25200\times5\times1}{100}$
= Rs 1260

$\therefore$ Interest for the third year $= \frac{26460\times5\times1}{100}$
= Rs 1323.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (a)	**6.** (c)	**8.** (a)	**10.** (a)	**12.** (a)	**14.** (b)	**16.** (c)	**18.** (c)	**20.** (a)

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19 Logarithms

INTRODUCTION

Logarithm, in mathematics, is the 'exponent' or 'power' to which a stated number called the *base*, is raised to yield a specific number. For example, in the expression $10^2 = 100$, the logarithm of 100 to the **base** 10 is 2. This is written as $\log_{10} 100 = 2$. Logarithms were originally invented to help simplify the arithmetical processes of multiplication, division, expansion to a power and extraction of a 'root', but they are nowadays used for a variety of purposes in pure and applied mathematics.

Logarithm

If for a positive real number ($a \neq 1$), $a^m = b$, then the index m is called the logarithm of b to the base a. We write this as

$$\log_a b = m$$

'log' being the abbreviation of the word 'logarithm'. Thus,

$$a^m = b \Leftrightarrow \log_a b = m$$

where, $a^m = b$ is called the *exponential form* and $\log_a b = m$ is called the *logarithmic form*.

Illustration 1 Refer to the following Table

Exponential form	logarithmic form
$3^5 = 243$	$\log_3 243 = 5$
$2^4 = 16$	$\log_2 16 = 4$
$3^0 = 1$	$\log_3 1 = 0$
$8^{1/3} = 2$	$\log_8 2 = \frac{1}{3}$

LAWS OF LOGARITHMS

1. **Product formula**
   The logarithm of the product of two numbers is equal to the sum of their logarithms.
   i.e. $\log_a (mn) = \log_a m + \log_a n$.
   Generalisation: In general, we have
   $\log_a (mnpq...) = \log_a m + \log_a n + \log_a p + \log_a q + ...$
2. **Quotient formula**
   The logarithm of the quotient of two numbers is equal to the difference of their logarithms.
   i.e. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$,
   where, a, m, n are positive and $a \neq 1$.
3. **Power formula**
   The logarithm of a number raised to a power is equal to the power multiplied by logarithm of the number.
   i.e. $\log_a (m^n) = n \log_a m$,
   where, a, m are positive and $a \neq 1$.
3. **Base changing formula**
   $\log_n m = \frac{\log_a m}{\log_a n}$. So, $\log_n m = \frac{\log m}{\log n}$.
   where, m, n, a are positived and $n \neq 1$, $a \neq 1$.
4. **Reciprocal relation**
   $\log_b a \times \log_a b = 1$,
   where, a, b are positive and not equal to 1.
5. $\log_b a = \frac{1}{\log_a b}$
6. ${}_a\log_a x = x$, where, a and x are positive, $a \neq 1$.
7. If $a > 1$ and $x > 1$, then $\log_a x > 0$.
8. If $0 < a < 1$ and $0 < x < 1$, then $\log_a x > 0$.
9. If $0 < a < 1$ and $x > 1$, then $\log_a x > 0$.
10. If $a > 1$ and $0 < x < 1$, then $\log_a x < 0$.

SOME USEFUL FORMULAE

1. Logarithm of 1 to any base is equal to zero.
   i.e. $\log_a 1 = 0$, where $a > 0$, $a \neq 1$.

2. Logarithm of any number to the same base is 1.
   i.e. $\log_a a = 1$, where $a > 0$, $a \neq 1$.

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Common Logarithms

There are two bases of logarithms that are extensively used these days. One is base e (e = 2. 71828 approx.) and the other is base 10. The logarithms to base e are called natural logarithms. The logarithms to base 10 are called the common logarithms.

$\log_{10}10 = 1$, since $10^1 = 10$.
$\log_{10}100 = 2$, since $10^2 = 100$.
$\log_{10}10000 = 4$, since $10^4 = 10000$.
$\log_{10}0.01 = -2$, since $10^{-2} = 0.01$.
$\log_{10}0.001 = -3$, since $10^{-3} = 0.001$
and, $\log_{10}1 = 0$, since $10^0 = 1$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find $\log_{3/2} 3.375$.
(a) 2 (b) 3
(c) 5/2 (d) 17/2

2. If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, find $yz\ (2 - x)$.
(a) 1 (b) –1
(c) 2 (d) –2

3. $\dfrac{\log x}{l+m-2m} = \dfrac{\log y}{m+n-2l} = \dfrac{\log z}{n+l-2m}$, find $x^2y^2z^2$.
(a) 2 (b) –1
(c) 4 (d) 1

4. If $\log \dfrac{x+y}{5} = \dfrac{1}{2}(\log x + \log y)$, then $\dfrac{x}{y} + \dfrac{y}{x} =$
(a) 20 (b) 23
(c) 22 (d) 21

5. If $\log (x + y) = \log\left(\dfrac{3x-3y}{2}\right)$, then $\log x - \log y =$
(a) log 2 (b) log 3
(c) log 5 (d) log 6

6. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, then $x =$
(a) 8 (b) 4
(c) 2 (d) 16

7. $7 \log \dfrac{16}{15} + 5 \log \dfrac{25}{24} + 3 \log \dfrac{81}{80} =$
(a) log 2 (b) log 3
(c) log 5 (d) None of these

8. If $0 < a \leq x$, the minimum value of $\log_a x + \log_x a$ is
(a) 1 (b) 2
(c) 3 (d) 5

9. If $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$, then $xyz = x^a \cdot y^b \cdot z^c = x^{b+c} \cdot y^{c+a} \cdot z^{a+b} =$
(a) 1 (b) 0
(c) 2 (d) None of these

10. $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} =$
(a) 0 (b) 2
(c) 1 (d) None of these

11. If $\log_{10}[98 + \sqrt{x^2 - 12x + 36}] = 2$, then $x =$
(a) 4 (b) 8
(c) 12 (d) 4,8

12. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then
(a) $xyz = x + y + z + 2$
(b) $xyz = x + y + z + 1$
(c) $x + y + z = 1$
(d) $xyz = 1$.

13. If $a^x = b^y = c^z = d^w$, then $\log_a (bcd) =$
(a) $\dfrac{1}{x}\left(\dfrac{1}{y} + \dfrac{1}{z} + \dfrac{1}{w}\right)$ (b) $x\left(\dfrac{1}{y} + \dfrac{1}{z} + \dfrac{1}{w}\right)$
(c) $\dfrac{y+z+w}{x}$ (d) None of these

14. If $\log_{10} 2 = 0.3010$, then $\log_{10} (1/2) =$
(a) –0.3010 (b) 0.6990
(c) $\bar{1}.6990$ (d) $\bar{1}.3010$

15. If $\log_2 (3^{2x-2} + 7) = 2 + \log_2 (3^{x-1} + 1)$, then $x =$
(a) 0 (b) 1
(c) 2 (d) 1 or 2

16. If $\log_a b = \log_b c = \log_c a$, then
(a) $a > b \geq c$ (b) $a < b < c$
(c) $a = b = c$ (b) $a < b \leq c$.

17. If $\dfrac{1}{\log_x 10} = \dfrac{2}{\log_a 10} - 2$, then $x =$
(a) $a/2$ (b) $a/100$
(c) $a^2/10$ (d) $a^2/100$

18. If $a^2 + b^2 = c^2$, then $\dfrac{1}{\log_{c+a} b} + \dfrac{1}{\log_{c-a} b} =$
(a) 1 (b) 2
(c) –1 (d) –2

19. If $\log_{10} 87.5 = 1.9421$, then the number of digits in $(875)^{10}$ is

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(a) 30 (b) 29
(c) 20 (d) 19

20. If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, then the number of zeros between the decimal point and the first significant figure in $(0.0432)^{10}$ is
(a) 10 (b) 13
(c) 14 (d) 15

21. If $(4.2)^x = (0.42)^y = 100$, then $\frac{1}{x} - \frac{1}{y} =$
(a) 1 (b) 2
(c) 1/2 (d) –1

22. $\frac{\log_9 11}{\log_5 13} - \frac{\log_3 11}{\log_{\sqrt{5}} 13} =$
(a) 1 (b) –1
(c) 0 (d) None of these

23. If $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5}$, then yz in terms of x is
(a) x (b) x^2
(c) x^3 (d) x^4

24. If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$, then $x =$
(a) 1/2 (b) 3/2
(c) 5/2 (d) 1

25. $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5} =$
(a) 5 (b) 2
(c) 5/2 (d) 3/2

26. $\log_{10} \tan 40° \cdot \log_{10} 41° \ldots \log_{10} \tan 50° =$
(a) 1 (b) 0
(c) –1 (d) None of these

27. If $\log_8 p = 2.5$, $\log_2 q = 5$, then p in terms of q is
(a) $q\sqrt{q}$ (b) $2q$
(c) q (d) $q/2$

28. If $y = \frac{1}{a^{1-\log_a x}}$, $z = \frac{1}{a^{1-\log_a y}}$ and $x = a^k$, then $k =$
(a) $\frac{1}{a^{1-\log_a z}}$ (b) $\frac{1}{1-\log_a z}$
(c) $\frac{1}{1+\log_z a}$ (d) $\frac{1}{1-\log_z a}$

29. If $\log_e 2 \cdot \log_b 625 = \log_{10}{}^{16} \cdot \log_e 10$, then $b =$
(a) 4 (b) 5
(c) 1 (d) e

30. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$
(a) log 2 (b) 1
(c) 0 (d) None of these

31. $2^{\log_3^7} - 7^{\log_3^2}$
(a) $\log_2 7$ (b) log 7
(c) log 2 (d) 0

32. If $\log_{30} 3 = a$, $\log_{30} 5 = b$, then $\log_{30} 8 =$
(a) $3(1 - a - b)$ (b) $a - b + 1$
(c) $1 - a - b$ (d) $1(a - b + 1)$

33. If $0 < a < 1$, $0 < x < 1$ and $x < a$, then $\log_a x$
(a) < 1 (b) > 1
(c) < 0 (d) ≤ 1

34. $\log_5 2$ is
(a) an integer
(b) a rational number
(c) an irrational number
(d) a prime number

35. $\log_5\left(1+\frac{1}{5}\right)+\log_5\left(1+\frac{1}{6}\right)+\log_5\left(1+\frac{1}{7}\right) + \ldots + \log_5\left(1+\frac{1}{624}\right)$
(a) 5 (b) 4
(c) 3 (d) 2

36. If $\log_{10} 2986 = 3.4751$, then $\log_{10} 0.02986 =$
(a) $\bar{1}.2986$ (b) $\bar{2}.4751$
(c) 0.34751 (d) None of these

37. If $\log(2a - 3b) = \log a - \log b$, then $a =$
(a) $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$
(c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$

38. If $\log(x - y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, then $\frac{x}{y}+\frac{y}{x} =$
(a) 25 (b) 26
(c) 27 (d) 28

39. If $\log x : 3 = \log y : 4 = \log z : 5$, then $zx =$
(a) $2y$ (b) y^2
(c) $8y$ (d) $4y$

40. If $3 + \log_5 x = 2\log_{25} y$, then $x =$
(a) $y/125$ (b) $y/25$
(c) $y^2/625$ (d) $3 - y^2/25$

41. If $\frac{\log_2 a}{3} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4}$ and $a^{1/2} \cdot b^{1/3} \cdot c^{1/4} = 24$, then
(a) $a = 24$ (b) $b = 81$
(c) $c = 64$ (d) $c = 256$

42. If $\frac{\log_2 x}{3} = \frac{\log_2 y}{4} = \frac{\log_2 z}{5k}$ and $\frac{z}{x^3 y^4} = 1$, then $k =$
(a) 3 (b) 4
(c) 5 (d) –5.

43. $\frac{3+\log_{10} 343}{2+\frac{1}{2}\log\left(\frac{49}{4}\right)+\frac{1}{3}\log\left(\frac{1}{125}\right)} =$

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(a) 3 (b) 3/2
(c) 2 (d) 1

44. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then $x^{a-b} \cdot y^{b-c} \cdot z^{c-a} =$
(a) 0 (b) −1
(c) 1 (d) 2

45. If $3^{x-2} = 5$ and $\log_{10} 2 = 0.20103$, $\log_{10} 3 = 0.4771$, then $x =$
(a) $1\frac{22187}{47710}$ (b) $2\frac{22187}{47710}$
(c) $3\frac{22187}{47710}$ (d) None of these

46. If $\log_2 = 0.30103$ and $\log_3 = 0.4771$, then number of digits in $(648)^5$ is
(a) 12 (b) 13
(c) 14 (d) 15

47. If $\log x = \frac{\log y}{2} = \frac{\log z}{5}$, then $x^4 \cdot y^3 \cdot z^{-2} =$
(a) 2 (b) 10
(c) 1 (d) 0

48. $\frac{\log\sqrt{27}+\log\sqrt{1000}+\log 8}{\log 120}$
(a) 1/2 (b) 1
(c) 3/2 (d) 2

49. For $x > 0$, if $y = \frac{{}_{10}\log_{10}^{x}}{x^2}$ and $x = y^a$, then $a =$
(a) 1 (b) −1
(c) 0 (d) 2

50. If $x = 100_{4/3}(1/2)$, $y = \log_{1/2}(1/3)$, then
(a) $x > y$ (b) $x < y$
(c) $x = y$ (d) $x \geq y$

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\log_{3/2} 3.375 = x \Rightarrow \left(\frac{3}{2}\right)^x = 3.375$

$\Rightarrow (1.5)^x = (1.5)^3 \Rightarrow \boldsymbol{x = 3.}$

2. Ans. (a)

Solution $yz(2-x) = 2yz - xyz = 2\log_{4a} 2a - \log_{4a} a$

$= \log_{4a}\left(\frac{4a^2}{a}\right) = \boldsymbol{1.}$

3. Ans. (d)

Solution Each is equal to k

$\Rightarrow \log x = k(l + m - 2n)$,

$\log y = k(m + n - 2l)$, $\log z = k(n + l - 2m)$.

$\Rightarrow \log xyz = k(0) \Rightarrow xyz = e^0 = 1 \Rightarrow \boldsymbol{x^2y^2z^2 = 1.}$

4. Ans. (b)

Solution $\log\left(\frac{x+y}{5}\right) = \frac{1}{2}[\log x + \log y]$

$\Rightarrow x + y = 5\sqrt{xy} \Rightarrow x^2 + y^2 = 23xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = \boldsymbol{23.}$

5. Ans. (c)

Solution $x + y = \frac{3x-3y}{2} \Rightarrow x = 5y \Rightarrow \frac{x}{y} = 5$

$\Rightarrow \log x - \log y = \boldsymbol{\log 5.}$

6. Ans. (a)

Solution $\log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4}$

$\Rightarrow \log_2 x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4} \Rightarrow \log_2 x = 3 \Rightarrow \boldsymbol{x = 8.}$

7. Ans. (a)

Solution $7\log\left(\frac{2^4}{5\times 3}\right) + 5\log\left(\frac{5^2}{2^3\times 3}\right) + 3\log\left(\frac{3^4}{2^4\times 5}\right)$

$= 28\log 2 - 7\log 5 - 7\log 3 + 10\log 5 - 15\log 2$

$-5\log 3 + 12\log 3 - 12\log 2 - 3\log 5 = \boldsymbol{\log 2.}$

8. Ans. (b)

Solution $0 < a \leq x$; Min. value of $\log_a x + \log_x a$ is 2 when we put $\boldsymbol{x = a.}$

9. Ans. (a)

Solution $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k$ (say)

$\Rightarrow \log x = k(b-c)$, $\log y = k(c-a)$, $\log z = k(a-b)$

$\Rightarrow \log x + \log y + \log z = 0 \Rightarrow xyz = 1.$

Also, $a\log x + b\log y + c\log z = 0 \Rightarrow x^a \cdot y^b \cdot z^c = 1.$

Again $(b+c)\log x + (c+a)\log y + (a+b)\log z = 0.$

$\Rightarrow x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1.$

$\therefore \quad xyz = x^a \cdot y^b \cdot z^c = \boldsymbol{x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1.}$

10. Ans. (c)

Solution $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} = k$ (say)

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$\Rightarrow$ $(\log y - \log z)\log x + (\log z - \log x)\log y + (\log x - \log y)\log z = \log k = 0$

$\Rightarrow k = 1.$

11. Ans. (d)

Solution $98 + \sqrt{x^2 - 12x + 36} = 100$

$\Rightarrow \sqrt{x^2 - 12x + 36} = 2$

$\Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow x = 8, 4.$

12. Ans. (a)

Solution $x = \log_a bc \Rightarrow a^x = bc \Rightarrow a^{x+1} = abc$

$\Rightarrow a = (abc)^{1/x+1}.$

Similarly, $b = (abc)^{1/y+1}$ and $c = (abc)^{1/z+1}$

$\therefore \quad abc = (abc)^{\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}}$

$\Rightarrow \quad 1 = \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$

$\Rightarrow \quad (x+1)(y+1)(z+1) = (y+1)(z+1) + (x+1)(z+1) + (x+1)(y+1)$

$\Rightarrow \quad xyz = x + y + z + 2.$

13. Ans. (b)

Solution $b^y = a^x \Rightarrow b = a^{\frac{x}{y}}, \; c = a^{\frac{x}{z}}, \; d = a^{\frac{x}{w}}$

$$\log_a(bcd) = \log_a\left(a^{\frac{x}{y}}.a^{\frac{x}{z}}.a^{\frac{x}{w}}\right) = \frac{x}{y}+\frac{x}{z}+\frac{x}{w}$$

$$= x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right).$$

14. Ans. (c)

Solution $\log_{10}\left(\frac{1}{2}\right) = -\log_{10} 2 = -0.3010$

$= 1 - 0.3010 - 1 = \bar{1}.6990.$

15. Ans. (d)

Solution $\log_2(3^{2x-2} + 7) = \log_2 4 + \log_2(3^{x-1} + 1)$ $[\because 2 = 2\log_2 2 = \log_2 2^2]$

$\Rightarrow 3^{2x-2} + 7 = 4(3^{x-1} + 1)$

$\Rightarrow t^2 + 7 = 4(t + 1)$, where, $3^{x-1} = t$

$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$

When $t = 1 \Rightarrow 3^{x-1} = 1 \Rightarrow x = 1$

When $t = 3 \Rightarrow 3^{x-1} = 3^1 \Rightarrow x = 2.$

16. Ans. (c)

Solution $\log_a b = \log_b c = \log_c a = k$ (say)

$\Rightarrow b = a^k, \; c = b^k, \; a = c^k$

$\Rightarrow c = (a^k)^k = a^{k^2} = (c^{k^2})^k = c^{k^3}$

$\Rightarrow k^3 = 1 \Rightarrow k = 1. \quad \therefore \quad a = b = c.$

17. Ans. (d)

Solution $\log_{10} x = 2\log_{10} a - 2$

$\Rightarrow \log_{10} x = 2(\log_{10} a - 1)$

$\Rightarrow \log_{10} x = 2\log_{10}\left(\frac{a}{10}\right) \Rightarrow x = \frac{a^2}{100}.$

18. Ans. (b)

Solution $\log_b(c + a) + \log_b(c - a)$

$= \log_b(c^2 - a^2) = \log_b b^2 = 2.$

19. Ans. (a)

Solution $x = (875)^{10} = (87.5 \times 10)^{10}$

$\therefore \quad \log_{10} x = 10(\log_{10} 87.5 + 1)$

$= 10(1.9421 + 1)$

$= 10(2.9421) = 29.421.$

$\therefore \quad x = \text{Antilog } (29.421).$

$\therefore$ No. of digits in $x = 30.$

20. Ans. (b)

Solution $x = (0.0432)^{10} = \left(\frac{432}{10000}\right)^{10} = \left(\frac{3^3.2^4}{10^4}\right)^{10}$

$\therefore \quad \log_{10} x = 10(3\log_{10} 3 + 4\log_{10} 2 - 4)$

$= 10(1.4313 + 1.2040 - 4)$

$= 10(-1.3647) = -13.647$

$= \bar{1}4.353$

$\therefore \quad x = \text{Antilog } (\bar{1}4.053)$

$\therefore$ No. of zeros between the decimal and the first significant figure $= 13.$

21. Ans. (c)

Solution $(4.2)^x = 100 \Rightarrow (42)^x = 10^{2+x}$

$\Rightarrow 42 = 10^{\frac{2}{x}+1}$ ...(1)

$\left(\frac{42}{100}\right)^y = 100 \Rightarrow (42)^y = 10^{2+2y}$

$\Rightarrow 42 = 10^{\frac{2}{y}+2}$ ...(2)

From (1) and (2), $\frac{2}{x} - \frac{2}{y} = 1 \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{2}.$

22. Ans. (c)

Solution $\frac{\log_9 11}{\log_5 13} - \frac{\log_3 11}{\log_{\sqrt{5}} 13} = \frac{\log_3 11}{2.\log_5 13} - \frac{\log_3 11}{2.\log_5 13}$

$= 0.$

23. Ans. (d)

Solution $\frac{\log x}{2} = \frac{\log y}{3} = \frac{\log z}{5} = k$ (say)

$\Rightarrow \quad \log x = 2k, \; \log y = 3k, \; \log z = 5k$

$\Rightarrow \quad \log yz = 3k + 5k = 8k; \; \log x^4 = 8k$

$\therefore \quad \log yz = \log x^4 \Rightarrow yz = x^4.$

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24. Ans. (b)

Solution $4^x + \frac{4^x}{2} = \frac{3^x}{\sqrt{3}} + 3^x.\sqrt{3}$

$\Rightarrow 4^x \cdot \frac{3}{2} = 3^x \cdot \frac{4}{\sqrt{3}} \Rightarrow \left(\frac{4}{3}\right)^x = \frac{8}{3\sqrt{3}}$

$\Rightarrow \left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{3/2} \Rightarrow x = \mathbf{\frac{3}{2}}.$

25. Ans. (c)

Solution $\frac{\log 7^{5/2} + \log 5^{5/2} - \log 2^{5/2}}{\log 17.5}$

$= \frac{5(\log 7 + \log 5 - \log 2)}{2\log\left(\frac{35}{2}\right)} = \mathbf{\frac{5}{2}}.$

26. Ans. (b)

Solution $\log_{10}\tan 40° \cdot \log_{10}\tan 41° \ \ldots \ \log_{10}\tan 50°$

$= 0$, since $\log_{10}\tan 45 = \mathbf{0}.$

27. Ans. (a)

Solution $\log_8 p = \frac{5}{2} \Rightarrow p = (8)^{5/2} = 2^{\frac{15}{2}} = (2^5)^{3/2}$

$\log_2 q = 5 \Rightarrow q = 2^5.$

$\therefore \quad \mathbf{p = q^{3/2}}.$

28. Ans. (b)

Solution $\log_a y = \frac{1}{1-\log_a x}, \ \log_a z = \frac{1}{1-\log_a y}$

$\therefore \quad \log_a z = \frac{1}{1-\left(\frac{1}{1-\log_a x}\right)} = \frac{1-\log_a x}{-\log_a x}$

$\Rightarrow \quad -\log_a z = -1 + \frac{1}{\log_a x}$

$\Rightarrow \quad \frac{1}{\log_a x} = 1 - \log_a z$

$\therefore \quad \log_a x = \frac{1}{1-\log_a z} \Rightarrow x = \frac{1}{a^{1-\log_a^z}} = a^k$ (given)

$\therefore \quad \mathbf{k = \frac{1}{1-\log_a z}}.$

29. Ans. (b)

Solution $\log_e 2 \cdot 4\log_b 5 = 4 \cdot \log_{10} 2 \cdot \log_e 10 = 4\log_e 2$

$\Rightarrow \log_b 5 = 1 \Rightarrow b = 5.$

30. Ans. (c)

Solution $5^{\sqrt{\log_5 7}} - (7^{\log_7 5})^{\frac{1}{\sqrt{\log_7 5}}}$

$= 5^{\sqrt{\log_5 7}} - \frac{1}{5^{\sqrt{\log_7 5}}}$

$\Rightarrow 5^{\sqrt{\log_5 7}} - 5^{\sqrt{\log_5 7}} = \mathbf{0}.$

31. Ans. (d)

Solution $2^{\log_3 7} - 7^{\log_3 2} = 2^{\log_2 7 \,.\, \log_3 2} - 7^{\log_3 2}$

$= 7^{\log_3 2} - 7^{\log_3 2} = \mathbf{0}.$

32. Ans. (a)

Solution $a + b = \log_{30} 15 = \log_{30}\left(\frac{30}{2}\right) = 1 - \log_{30} 2$

$\Rightarrow \quad \log_{30} 2 = 1 - a - b.$

$\therefore \quad \mathbf{\log_{30} 8 = 3(1 - a - b).}$

33. Ans. (b)

Solution $0 < a < 1, \ 0 < x < 1$ and $x < a$

$\Rightarrow \quad \log_a x > \log_a a \Rightarrow \mathbf{\log_a x > 1.}$

34. Ans. (c)

Solution $\log_5 2 = \frac{p}{q} \Rightarrow 2 = 5^{p/q} = 2^q = 5^p$

$\Rightarrow$ even no. = odd no.,

which is a contradiction.

$\therefore \quad \log_5 2$ is an irrational number.

35. Ans. (b)

Solution $\log_5\frac{6}{5} + \log_5\frac{7}{6} + \log_5\frac{8}{7} + \ \ldots \ + \log_5\frac{625}{624}$

$= \log_5\left(\frac{6}{5}\cdot\frac{7}{6}\cdot\frac{8}{7}\ldots\frac{625}{624}\right) = \log_5\left(\frac{625}{5}\right) = \mathbf{4}.$

36. Ans. (b)

Solution $\log_{10}(0.02986) = \log_{10}\left(\frac{2986}{100000}\right)$

$= 3.4751 - 5 = -1.5249$

$= \mathbf{\bar{2}.4751}.$

37. Ans. (a)

Solution $2a - 3b = \frac{a}{b} \Rightarrow 2ab - 3b^2 = a$

$\Rightarrow 3b^2 = a(2b - 1)$

$\Rightarrow \mathbf{a = \frac{3b^2}{2b-1}}.$

38. Ans. (c)

Solution $(x - y)^2 = 25xy \Rightarrow x^2 + y^2 = 27xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = \mathbf{27}.$

39. Ans. (b)

Solution $\frac{\log x}{3} = \frac{\log y}{4} = \frac{\log z}{5} = k$

$\Rightarrow \log x = 3k; \ \log y = 4k; \ \log z = 5k.$

$\Rightarrow \log(zx) = \log z + \log x = 8k = 2\log y$

$\therefore \quad \mathbf{zx = y^2}.$

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40. Ans. (a)

Solution $3 + \log_5 x = \log_5 y \Rightarrow \log_5(125x) = \log_5 y$

$\Rightarrow x = \dfrac{y}{125}$.

41. Ans. (d)

Solution $\dfrac{\log_2 a}{3} = \dfrac{\log_3 b}{3} = \dfrac{\log_4 c}{4} = k$

$\Rightarrow \quad a = 2^{2k},\ b = 3^{3k},\ c = 4^{4k}$ and

$a^{1/2} \cdot b^{1/3} \cdot c^{1/4} = 2^k \cdot 3^k \cdot 4^k = 24$

$\Rightarrow \quad 24^k = 24^1 \Rightarrow k = 1.$

$\therefore \quad a = 4,\ b = 27,\ c = \mathbf{256}.$

42. Ans. (c)

Solution $\dfrac{z}{x^3 y^4} = 1 \Rightarrow \log_2 z - 3\log_2 x - 4\log_2 y = 0$

$\Rightarrow \log_2 z - \dfrac{3.3}{5k} \cdot \log_2 z - 4 \cdot \dfrac{4}{5k} \cdot \log_2 z = 0$

$\Rightarrow 1 - \dfrac{9}{5k} - \dfrac{16}{5k} = 0$

$\Rightarrow 5k - 25 = 0 \Rightarrow \boldsymbol{k = 5}.$

43. Ans. (a)

Solution $\dfrac{3(1+\log_{10} 7)}{2 + \log\frac{7}{2} + \log\frac{1}{5}} = \dfrac{3(1+\log_{10} 7)}{2 + \log\left(\frac{7}{10}\right)}$

$= \dfrac{3(1+\log_{10} 7)}{1+\log_{10} 7} = \mathbf{3}.$

44. Ans. (c)

Solution Each ratio $= k \Rightarrow \log x = k(a^2 + ab + b^2)$

$\Rightarrow \quad (a - b)\log x = k(a^3 - b^3)$

$\Rightarrow \quad \log x^{a-b} = k(a^3 - b^3) \Rightarrow x^{a-b} = e^{k(a^3 - b^3)}$

Similarly, $y^{b-c} = e^{k(b^3 - c^3)},\ z^{c-a} = e^{k(c^3 - a^3)}$.

$\therefore \quad x^{a-b} \cdot y^{b-c} \cdot z^{c-a} = e^0 = \mathbf{1}.$

45. Ans. (c)

Solution $3^{x-2} = 5 \Rightarrow 3^x = 45 = \left(\dfrac{90}{2}\right)$

$\Rightarrow x\log_{10} 3 = \log_{10} 90 - \log_{10} 2$

$= 2\log_{10} 3 + 1 - \log_{10} 2$

$\Rightarrow x(0.4771) = 1.65317$

$\Rightarrow x = \dfrac{165317}{47710} = 3\dfrac{\mathbf{22187}}{\mathbf{47710}}.$

46. Ans. (d)

Solution $\log (648)^5 = 5\log(81 \times 8) = 20\log 3 + 15\log 2$

$= 20(0.4771) + 15(0.30103)$

$= 14.05745.$

$\therefore$ Number of digits in $(648)^5$ is **15.**

47. Ans. (c)

Solution $\dfrac{\log x}{1} = \dfrac{\log y}{2} = \dfrac{\log z}{5} = k$

$\Rightarrow \quad \log x = k,\ \log y = 2k,\ \log z = 5k.$

$\therefore \quad \log(x^4 . y^3 . z^{-2}) = 4\log x + 3\log y - 2\log z = 0$

$\Rightarrow \quad x^4 . y^3 . z^{-2} = \mathbf{1}.$

48. Ans. (c)

Solution $\dfrac{\log\sqrt{27} + \log\sqrt{1000} + \log 8}{\log 120}$

$= \dfrac{\frac{3}{2}(\log 3 + \log 10 + \log 4)}{\log 3 + \log 10 + \log 4} = \dfrac{3}{2}.$

49. Ans. (b)

Solution $y = \dfrac{10^{\log_{10} x}}{x^2} = \dfrac{1}{x} = \dfrac{1}{y^a} = y^{-a} \Rightarrow \boldsymbol{a = -1}.$

50. Ans. (b)

Solution $x = \log_{4/3}(1/2) = -\log_{4/3} 2 < 0$

and, $y = \log_{1/2}(1/3) = \log_2 3 > 0 \Rightarrow \boldsymbol{y > x}.$

QUESTIONS FROM COMPETITIVE EXAMS

1. If $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$, then a possible value of x is given by:

(a) 10 (b) 1/100

(c) 1/1000 (d) None of these

[Based on CAT 2004]

2. What is the sum of 'n' terms in the series:

$$\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3}\right) + \ldots$$

(a) $\log\left[\dfrac{n^{(n-1)}}{m^{(n+1)}}\right]^{n/2}$ (b) $\log\left[\dfrac{n^m}{n^n}\right]^{n/2}$

(c) $\log\left[\dfrac{m^{(1-n)}}{n^{(1+n)}}\right]^{n/2}$ (d) $\log\left[\dfrac{m^{(n+1)}}{n^{(n-1)}}\right]^{n/2}$

[Based on CAT 2004]

3. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s respectively. Then,

(a) $s > t > c$ (b) $c > t > s$

(c) $c > s > t$ (d) $s > c > t$

[Based on CAT 2004]

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4. If $\log_y x = (a.\log_z y) = (b.\log_x Z) = ab$, then which of the following pairs of values for (a, b) is no possible?

(a) $(-2, 1/2)$ (b) $(1, 1)$
(c) $(0.4, 2.5)$ (d) $(2.1, 3.2)$
(e) $(2, 2)$

[Based on CAT, 2006]

5. If $\log_3[(\log_2 x)] = 1$, then x is equal to

(a) 512 (b) 128
(c) 12 (d) 0

[Based on FMS (Delhi) 2006]

6. If $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$, then

(a) $a + b = 1$ (b) $a - b$
(c) $a^2 - b = 1$ (d) $a = b$

[Based on FMS (Delhi) 2007]

7. The number of real solutions of the equation $\log(-x) = 2\log(x + 1)$ is

(a) One (b) Two
(c) Three (d) Four

[Based on MAT, 2005]

8. If $\log_5(x^2 + x) - \log_5(x+1) = 2$, then the value of x is

(a) 5 (b) 32
(c) 25 (d) 10

[Based on FMS (Delhi) 2007]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$

$\Rightarrow \log_{10} x - \frac{1}{2}\log_{10} x = \frac{2}{\log_{10} x}$

$\Rightarrow \frac{\log_{10} x}{2} = \frac{2}{\log_{10} x}$

$\Rightarrow (\log_{10} x)^2 = 4$

$\Rightarrow \log_{10} x = \pm 2$

$\Rightarrow x = 10^{-2}, 10^2$

$= \frac{1}{100}, 100.$

2. Ans. (d)

Solution $\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3}\right) + \ldots$nth term

$= \log\left[\frac{m \cdot m^2 \cdot m^2 \ldots m^n}{n \cdot n^2 \cdot n^3 \ldots n^{n-1}}\right] = \log\left[\frac{m^{(1+2+3+\ldots+n)}}{n^{(1+2+3+\ldots+n-1)}}\right]$

$= \log\left[\frac{m^{\frac{n(n+1)}{2}}}{n^{\frac{(n-1)n}{2}}}\right] = \log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{n/2}.$

3. Ans. (c)

Solution If the length of the circumference of a circle equals the perimeter of a regular polygon then

Area of circle > Area of regular polygon

Also, if two regular polygons have the same perimeter, then the regular polygon having larger no. of sides will have area greater than that of the regular polygon having less number of sides.

$\therefore \quad c > s > t.$

4. Ans. (e)

5. Ans. (a)

Solution $\frac{1}{2}\log 25 - 2\log 3 + \log 18$

$= \frac{1}{2}\log 5^2 - 2\log 3 + \log(2 \times 3 \times 3)$

$= \log 5 - 2\log 3 + \log 2 + \log 3^2$

$= \log 5 + \log 2$

$= \log(5 \times 2)$

$= \log 10 = 1.$

6. Ans. (a)

Solution $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$

$\log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log(a+b)$

$\log(1) = \log(a+b)$

$a + b = 1$

7. Ans. (a)

Solution $-x = (x + 1)^2$

$\Rightarrow x^2 + 3x + 1 = 0$

$\therefore \quad x = \frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}$

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But only $= \frac{-3+\sqrt{5}}{2}$ satisfies the other condition of $x + 1 > 0$.

8. Ans. (a)

Solution

$$\frac{\log(x^2+x)}{\log 5} - \frac{\log(x+1)}{\log 5} = 2$$

$$\log(x^2+x) - \log(x+1) = 2\log 5$$

$$\frac{\log(x^2+x)}{\log(x+1)} = 2\log 5$$

$$\frac{\log[x(x+1)]}{\log(x+1)} = 2\log 5$$

$$\frac{\log x \cdot \log(x+1)}{\log(x+1)} = \log 5^2$$

$= 36.35$ minutes.

EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. If $10^x = 1.73$ and $\log_{10} 1730 = 3.2380$, then $x =$
(a) 1.2380 (b) 0.2380
(c) 2.380 (d) 2.2380

2. The value of $3^{-\frac{1}{2}\log_3 9}$ is
(a) -1 (b) $-\frac{1}{3}$
(c) $\frac{1}{3}$ (d) $-\frac{3}{2}$

3. If $\log_4 x + \log_2 x = 6$, then the value of x is
(a) 2 (b) 4
(c) 8 (d) 16

4. The value of $\left(\frac{1}{\log_{(q/p)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right)$ is
(a) 3 (b) 2
(c) 1 (d) 0

5. $\log_5 5 \quad \log_4 9 \quad \log_3 2 =$
(a) 2 (b) 1
(c) 5 (d) 3/2

6. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then the value of $a^a b^b c^a$ is
(a) abc (b) $\frac{1}{abc}$
(c) 1 (d) $\log(abc)$

7. Given $\log_2 = 0.30103$, the number of digits in 5^{20} is
(a) 14 (b) 16
(c) 18 (d) 25

8. The value of x satisfying $\log_{32} x = 0.8$ is
(a) 25.6 (b) 16
(c) 10 (d) 12.8

9. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{100}(.72)$ is
(a) $\bar{1}.9286$ (b) $\bar{1}.8573$
(c) 1.8572 (d) .9286

10. If $\log(x + y) = \log x + \log y$ and $x = 1.15683$, then the value of y is
(a) $\bar{7}.736$ (b) 7.377
(c) 3.456 (d) 1.234

Test II

[Time: 10 min]

11. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is
(a) 1.9030 (b) 1.6020
(c) 3.9030 (d) None of these

12. $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right)$ is
(a) 1 (b) 0
(c) 2 (d) abc

13. $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1} =$
(a) 1 (b) 2
(c) 3 (d) $\frac{3}{2}$

14. If $\log 2 = 0.3010$ and $5^x = 400$, then $x =$
(a) 2.40 (b) 3.72
(c) 4.36 (d) 1

15. $\log_{10} 10 + \log_{10} 100 + \log_{10} 1000 + \log_{10} 10000 + \log_{10} 100000$ is
(a) 15 (b) $\log_{10} 11111$
(c) $\log_{10} 1111$ (d) $14\log_{10} 100$

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16. The value of $\log_2(\log_2 \log_2 \log_2(65536))$ is
(a) 0 (b) 1
(c) 2 (d) 4

17. If $\log_2(\log_3(\log_2 x)) = 1$, then $x =$
(a) 512 (b) 128
(c) 12 (d) 0

18. If a, b, c are three consecutive integers, then $\log(ac + 1)$ has the value
(a) $(\log b)^2$ (b) $\log b$
(c) $2 \log b$ (d) $\log 2b$

19. If $\log\frac{m}{n} + \log\frac{n}{m} = \log(m + n)$, then
(a) $m + n = 1$ (b) $\frac{m}{n} = 1$
(c) $m - n = 1$ (d) $m^2 - n^2 = 1$

20. If $\log_{10} x = 1.9665$, then $\log_{10}(1000x)$ is
(a) 19.675 (b) 4.9675
(c) 1.9675×3 (d) 1967.5

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution $10^x = 1.73 \Rightarrow x = \log_{10}(1.73) = \mathbf{0.2380.}$

3. Ans. (d)

Solution $\log_4 x + \log_2 x = 6$

$\Rightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 6 \Rightarrow \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 6$

$\Rightarrow 3 \log x = 12 \log 2 \Rightarrow \log x^3 = \log 2^{12}$

$\Rightarrow x^3 = 2^{12} \Rightarrow \mathbf{x = 2^4 = 16.}$

5. Ans. (b)

Solution Given expression $= \frac{\log 5}{\log 5} \times \frac{\log 9}{\log 4} \times \frac{\log 2}{\log 3}$

$= \frac{\log 3^2}{\log 2^2} \times \frac{\log 2}{\log 3}$

$= \frac{2\log 3}{2\log 2} \times \frac{\log 2}{\log 3} = \mathbf{1.}$

7. Ans. (a)

Solution $\log 5^{20} = 20 \log 5 = 20\left[\log\left(\frac{10}{2}\right)\right]$

$= 20(\log 10 - \log 2)$

$= 20 \times (1 - 0.3010) = 13.98.$

$\therefore$ No. of digits in 5^{20} is **14.**

9. Ans. (a)

Solution $\log_{100}(0.72) = \log_{100}\left(\frac{72}{100}\right)$

$= \log_{100} 72 - \log_{100} 100$

$= \frac{\log_{10} 72}{\log_{10} 100} - 1$

$= \frac{\log_{10}(2^3 \times 3^2)}{2} - 1$

$= \frac{3\log_{10} 2 + 2\log_{10} 3}{2} - 1$

$= \frac{3 \times 0.301 + 2 \times 0.4771}{2} - 1$

$= .9286 - 1 = \mathbf{\bar{1}.9286.}$

11. Ans. (a)

Solution $\log_{10} 80 = \log_{10}(10 \times 2^3) = \log_{10} 10 + \log_{10} 2^3$

$= 1 + 3 \log_{10} 2$

$= (1 + 3 \times 0.3010) = \mathbf{1.9030.}$

13. Ans. (a)

Solution Given expression

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} (abc) = \mathbf{1.}$

15. Ans. (a)

Solution Given expression

$= \log_{10} 10 + \log_{10} 10^2 + \log_{10} 10^3 + \log_{10} 10^4 + \log_{10} 10^5$

$= (1 + 2 + 3 + 4 + 5) = \mathbf{15.}$

17. Ans. (a)

Solution $\log_2[\log_3(\log_2 x)] = 1 = \log_2 2$

$\Rightarrow \log_3(\log_2 x) = 2$

$\Rightarrow \log_2 x = 3^2 = 9$

$\Rightarrow x = 2^9 = \mathbf{512.}$

19. Ans. (a)

Solution $\log(m + n) = \log\frac{m}{n} + \log\frac{n}{m} = \log\left(\frac{m}{n} \times \frac{n}{m}\right)$

$= \log 1$

$\therefore \quad m + n = \mathbf{1.}$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (d)	**6.** (c)	**8.** (b)	**10.** (b)	**12.** (b)	**14.** (b)	**16.** (b)	**18.** (c)	**20.** (b)

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Stocks, Shares and Debentures 20

INTRODUCTION

To start a big business or an industry, a large sum of money is required. It may not be possible for one or two persons to arrange for the requisite finance and expertise required for the project. So a number of individuals join hands to form a company called a **Joint Stock Company**. It is a registered body under the Companies Act. The persons who join together to form the company are called its **Promoters.** The total amount of money required by the company is called the **Capital.**

The promoters of the company issue a circular giving the details of the project, its benefits and drawbacks and invite the public to come forward and subscribe towards the capital of the company. The company divides the required capital into small units of equal amount. Each unit is called a **share.** Each person, who purchases one or more shares of the company is called a **shareholder** of the **company**. The company issues a **share certificate** to each of its shareholders stating the number of shares alloted to the person and the value of each share. The value of a share, as stated on the share certificate is called the **nominal value (or face value, or par value)** of the share.

When a company earns a profit during a financial year, a part of it is used in paying for working expenses, taxes, interest on loans and keeping some part of it as reserve fund for future expansion of the project, the remaining profit is distributed among the shareholders. The distributed profit is called the **dividend.**

Dividends are declared annually, semi-annually, quarterly as per regulations of the company. The dividend on a share is expressed as certain percentage of its face value which is printed on the share certificate. Sometimes it is also expressed as a specified amount per share. For example, we may say that dividend on a share is 12% of its face value or the dividend is Rs. 2 per share.

Illustration 1 Find the annual dividend paid in each of the following cases:

Sl. No.	*Par value of a share*	*Number of Common Shares*	*Rate of dividend declared on a Common Share*
(i)	Rs. 10	500	10% per annum
(ii)	Rs. 10	800	5% semi-annually
(iii)	Rs. 100	1500	5% quarterly
(iv)	Rs. 10	2500	2% per month

Solution

(i) Annual dividend on one share

$$= 10\% \text{ of Rs. } 10 = \text{Rs.}\left(\frac{10}{100}\times 10\right) = \text{Re. } 1$$

Annual dividend on 500 shares

$$= \text{Rs. } (500 \times 1) = \textbf{Rs. 500.}$$

(ii) Annual dividend on one share

$$= \text{Rs. } (2 \times 5)\% \text{ of Rs. } 10$$

$$= \text{Rs.}\left(\frac{10}{100}\times 10\right) = \text{Re.1}$$

$\therefore$ Annual dividend on 800 shares

$$= \text{Rs. } (800 \times 1) = \textbf{Rs. 800.}$$

(iii) Annual dividend on one share

$$= (4 \times 5)\% \text{ of Rs. } 100$$

$$= \text{Rs.}\left(\frac{20}{100}\times 100\right) = \text{Rs. } 20$$

$\therefore$ Annual dividend on 1500 shares

$$= \text{Rs. } (1500 \times 20) = \textbf{Rs. 30000.}$$

(iv) Annual dividend on one share

$$= (12 \times 2)\% \text{ of Rs. } 10$$

$$= \text{Rs.}\left(\frac{24}{100}\times 10\right) = \text{Rs. } 2.40$$

$\therefore$ Annual dividend on 2500 shares

$$= \text{Rs. } (2500 \times 2.40) = \textbf{Rs. 6000.}$$

TYPES OF SHARES

The shares are generally of two types:

(i) Preferred shares and (ii) Common (ordinary) shares

(i) **Preferred shares** These shares get preference in terms of payment of dividend and return of capital over ordinary shares. The rate of dividend for these shares is decided when they are issued and dividend to preferred shareholders is paid before any dividend is paid to common shareholders.

(ii) **Ordinary shares** Ordinary shareholders are paid dividend only when profits are left after preferred shareholders have been paid dividend at specified rate. The rate of dividend on these shares is also not fixed and depends upon the amount of available profit.

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FACE VALUE AND MARKET VALUE OF A SHARE

The price at which the shares are initially issued by the company to its shareholders is called the *face value* of a share (This is also called *nominal* or *par value* of a share). In fact, this is that value of a share which is mentioned in the share certificate issued by the company to its shareholders.

As other things, shares are also sold in (or purchased from) the market. The value of a share quoted in the market is called the *market value* of the share. The market value of a share keeps on changing according to its demand and supply changes.

If the market value of a share is equal to the par value of the share, the share is said to be *at par*. If the market value of a share is more than its face (or par) value, the share is said to be at *premium*. On other hand, if the market value of a share is less than its face value, the share is said to be *at discount* (or *below par*).

For example, if the market value of a Rs. 100 share is Rs. 130, it is said to be at 30% premium.

If the market value of a Rs. 100 share is Rs. 90, it is said to be at 10% discount. If Rs. 100 share is quoted at 45 premium then its market value is

$$\text{Rs. } (100 + 45) = \text{Rs. } 145.$$

Every company declares dividend on the face value of its shares irrespective of the market value of the share.

Note: The statement "32%, Rs. 100 shares at Rs. 125" means:

(i) Face value of each share is Rs. 100
(ii) Dividend on each share is 32% of the face value
(iii) Market value of each share is Rs. 125
(iv) An investment of Rs. 125 earns an annual income of Rs. 32.

Illustration 2 Find the cost of purchasing 150 shares of a company, each of par value Rs. 10, quoted at Rs. 16 each in the market, from the original shareholder. Also, find the gain to the new shareholder if he sells each share at a premium of Rs. 10.

Solution Market value of share = Rs. 16

$\therefore$ Market value of 150 shares

$$= \text{Rs. } (150 \times 16) = \text{Rs. } 2400$$

Thus, the new shareholder spent Rs. 2400 for buying 150 shares. The new shareholder sold the shares at a premium of Rs. 10.

$\therefore$ Now, market value of a share

$$= \text{Rs. } (10 + 10) = \text{Rs. } 20$$

The selling price of 150 shares at the new market value = Rs. (150×20) = Rs. 3000

$\therefore$ Gain of the new shareholder in the transaction

$$= \text{Rs. } (3000 - 2400) = \textbf{Rs. 600.}$$

Illustration 3 Raja buys 200 shares, each of par value Rs. 10 of a company which pays annual dividend of 15% at such a price that he gets 12% on his investment. Find the market value of a share.

Solution Par value of 200 shares

$$= \text{Rs. } (200 \times 10) = \text{Rs. } 2000.$$

Dividend received by Raja

$$= \text{Rs. } \left(\frac{2000 \times 15}{100}\right) = \text{Rs. } 300.$$

Let the market value of 200 shares be Rs. x. We have to find x such that 12% of x = 300,

i.e. $\frac{12}{100} \times x = 300 \quad \therefore \quad x = \frac{100 \times 300}{12} = 2500$

i.e. Market value of 200 shares = Rs. 2500

Hence, the market value of one share = **Rs. 12.50.**

STOCKS AND BROKERAGE

Stock

In the previous section, we have learnt about shares, which can be sold and purchased by the public. The nominal value or face value of shares is fixed (usually Rs. 10 or Rs. 100) but their market value varies.

Sometimes, joint stock companies or the government also raises loans from the market by issuing *bonds* or *promisory notes*. They promise to pay a fixed amount (called *redemption value*) on a future date and interest payments at fixed periods until that time. The money paid to company or government for buying such bonds is called *stock*.

The stocks are usually known by their rates of dividend. Thus, by 9% stock we mean that the dividend on a Rs. 100 stock is Rs. 9.

If the market value of Rs. 100 stock, which yields a dividend of Rs. 5, is Rs. 115, the stock is called 5% stock at 115. Similarly, 10% stock at 120 means that a stock of face value Rs. 100 gives a dividend of Rs. 10 and is available in the market of 120.

Note: There can be stocks in units different from Rs. 100, say Rs. 500, Rs. 1000, etc., but the phrase "8% stock at 90" can be used only in case of that stock whose face value is Rs. 100. Dividend on a stock is fixed (declared at the time of issue) whereas for a share it varies with time. Usually, the *date of maturity* of the stock is fixed. In case the holder of the stock requires money before the due date he can sell his stock to some other person, whereby his claim of interest is transferred to that person.

Brokerage

The sale and purchase of stock is generally executed through a stockbroker, who charges some money, called *Brokerage* from both the seller and purchaser. *The brokerage is charged either as some fixed amount on each unit of stock or as some percentage of the market value of unit of stock.*

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Thus, the brokerage of Rs. x means that x rupees are to be added or subtracted from the market value of the stock. Similarly, brokerage 2% means that the brokerage equal to 2% of the market value of a unit of stock and be added to (or subtracted from) the market value of a unit of stock.

Notes:

(i) The brokerage is added to the market value when the stock is purchased.

(ii) The brokerage is subtracted from the market value when the stock is sold.

CALCULATION OF INCOME ON A STOCK

When the face value of the total stock is given, the income can be calculated on the assumption that the face value of each unit of stock is Rs. 100. On the contrary, if the market value of the total investment is given, the income can be calculated on the basis of the market value of a unit of stock.

Illustration 4 Find the income from Rs. 2875 of 4% stock.

Solution By 4% stock, we mean a stock of Rs. 100 will fetch a dividend of Rs. 4 p.a.

Hence, the income from Rs. 2875 of 4% stock

$$= \frac{2875\times 4}{100} = \textbf{Rs. 115.}$$

Illustration 5 Find the income on 10% stock of Rs. 25000 purchased at Rs. 120.

Solution Face value of stock = Rs. 25000

Income on Rs. 100 stock = Rs. 10

Income on Re. 1 stock = Rs. $\left(\frac{10}{100}\right)$

Income on Rs. 25000 stock = Rs. $\left(\frac{25000\times 10}{100}\right)$

$$= \textbf{Rs. 2500.}$$

COMPUTATION OF INVESTMENT OR MARKET VALUE OF A STOCK

If the face value of a stock is given, the market value of the stock can be found on the basis of market value of each unit of stock.

Illustration 6 Find the investment required to purchase Rs. 75000 of 10% stock at 95.

Solution Market value of Rs. 100 stock = Rs. 95

$\therefore$ Market value of Rs. 75000 stock

$$= \text{Rs.}\left(\frac{95}{100}\times 75000\right) = \text{Rs. } 71250$$

$\therefore$ An investment of Rs. 71250 is required to purchase Rs. 75000 of 10% stock at **95.**

Illustration 7 Find the investment required to get an income of Rs. 4200 from $10\frac{1}{2}$% stock at 80 (Brokerage: 2%).

Solution Brokerage = 2% of Rs. 80 = Rs. $\left(\frac{2}{100}\times 80\right)$

$$= \text{Rs. } 1.60$$

$\therefore$ Investment needed to buy Rs. 100 stock

$$= \text{Rs. } 81.60 \text{ on which the income is Rs. } 10\frac{1}{2}$$

For income of Rs. $10\frac{1}{2}$, the investment = Rs. 81.60

For income of Rs. 4200, the investment

$$= \text{Rs.}\left(81.60\times\frac{2}{21}\times 4200\right) = \textbf{Rs. 32640.}$$

COMPUTATION OF GAIN OR LOSS IN THE SALE AND PURCHASE OF A STOCK

When the market is favourable to stock holders, i.e. they are likely to get better proceeds for their stock, they sell the stock and may reinvest the money so obtained in another stock which may give them more income.

Illustration 8 Ram bought Rs. 12000 of 8% stock at 92 and sold it when the price rose to 98. Find his total gain and gain per cent.

Solution Investment made by Ram in buying Rs. 12000 of 8% stock at 92

$$= \text{Rs.}\left[12000\times\frac{92}{100}\right] = \text{Rs. } 11040.$$

When the price rose to Rs. 98, Ram sold the stock, thus money realised from selling the stock

$$= \text{Rs.}\left[12000\times\frac{98}{100}\right] = \text{Rs. } 11760.$$

$\therefore$ Gain realised in the transaction

$$= \text{Rs. } (11760 - 11040) = \text{Rs. } 720$$

$\therefore$ Gain per cent = $\frac{(720\times 100)}{11040} = \mathbf{6\frac{12}{23}\%}$.

CHANGE IN INCOME ON SALE OR REINVESTMENT

A person having one type of stock may sell it to buy another which gives higher income. In such problems, the income in two cases is calculated and change is found out.

Illustration 9 Ram invests Rs. 46500 in 6% stock at 93 and sells the stock when its price rose to Rs. 95. He invests the sale proceeds in 9% stock at 95. Find the change in Ram's income.

Solution Income from first stock = Rs. $\left(\frac{6}{93}\times 46500\right)$

$$= \text{Rs. } 3000$$

We have to find the amount realized on selling this stock.

Amount realised on selling Rs. 93 stock = Rs. 95

$\therefore$ Amount realised on selling Rs. 46500 stock

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$$= \text{Rs. } \frac{95}{93} \times 4650 = \text{Rs. } 47500$$

This amount is invested in 9% stock at 95

∴ Income from 2nd stock

$$= \text{Rs. } \left(\frac{9}{95} \times 47500\right) = \text{Rs. } 4500$$

Hence, increase in income

$$= \text{Rs. } (4500 - 3000) = \textbf{Rs. 1500.}$$

DEBENTURES

Sometimes a running joint stock company may require more capital for its further expansion. The company borrows the required sum of money from the general public for a fixed period of time and at a fixed rate of interest by dividing the amount required into small parts. These small parts are called *debentures*.

The debenture holders are creditors of the company and do not have any right on the profits declared by the company. However, interest at fixed rate and fixed time is payable to debenture holders, irrespective of the fact whether the company is running in profits or losses.

Like shares, debentures can also be sold in or purchased from the market. The terms used in case of shares, are also used with the same meaning in case of debentures. Thus we use the terms '*debentures at premium*', '*debentures at discount*' etc. Furthermore, the rules for calculating the brokerage on debenture are also the same as those in case of shares.

DIFFERENCE BETWEEN SHARES AND DEBENTURES

Shares	*Debentures*
(a) Share money forms a part of the capital of the company.	(a) Debentures are a mere debt.
(b) Shareholders have right on the profit declared by the company.	(b) Debentureholders are creditors of the company and do not have any right on the profit declared by the company.
(c) Shareholders may receive different dividend according as profit is more or less.	(c) Debenture holders receive interest at a fixed rate.

Illustration 10 Find the income percent of a buyer on 8% debentures of face value Rs. 120 and available in the market at Rs. 180.

Solution The market value of a debenture is Rs. 180

∴ Income on Rs. 180 is Rs. 8

∴ Income on Rs. 120 is Rs. $\left(\frac{8}{180} \times 120\right) = \text{Rs. } 5\frac{1}{3}$

∴ Per cent income on the debenture is $\mathbf{5\frac{1}{3}\%}$.

Illustration 11 Ram has 500 shares of par value Rs. 10 each of a company and 500 debentures of par value Rs. 100 each. The company pays a dividend of 8% on the shares and pays an interest of 10% on its debentures. Find the total annual income of Ram and the rate of return on her investments.

Solution Annual dividend on 500 shares

$$= \text{Rs. } \frac{(500 \times 10 \times 8)}{100}$$

$$= \text{Rs. } 400.$$

Annual interest on 500 debentures

$$= \text{Rs. } \frac{(500 \times 100 \times 10)}{100}$$

$$= \text{Rs. } 5000.$$

∴ Total annual income of Ram

$$= \text{Rs. } (5000 + 400)$$

$$= \text{Rs. } 5400.$$

Total investment of Ram

$$= \text{Rs. } (500 \times 10 + 500 \times 100)$$

$$= \text{Rs. } 55000.$$

Rate of return on Ram's investment

$$= \left(\frac{5400}{55000} \times 100\right)\%$$

$$= \frac{108}{11}\% \quad \textbf{or} \quad \mathbf{9\frac{9}{11}\%}.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. A company declared an annual dividend of 10%. Find the annual dividend of Ram owning 1500 shares of the company of par value Rs. 10 each.

(a) Rs. 1400 (b) Rs. 1500
(c) Rs. 1700 (d) Rs. 1600

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2. A company declared an annual dividend of 10%. Find the annual dividend received by Anu owning 4000 shares of the company having a par value of Rs. 100 each.
(a) Rs. 45000 (b) Rs. 40000
(c) Rs. 50000 (d) Rs. 60000

3. Jatin invested Rs. 27260 in buying Rs. 100 shares of a company at Rs. 116 each. If the company paid 16% dividend at the end of the year, find his income from the dividend.
(a) Rs. 3560 (b) Rs. 2760
(c) Rs. 3760 (d) Rs. 3660

4. A company issued 50000 shares of par value Rs. 10 each. If the total dividend declared by the company is Rs. 62500, find the rate of dividend paid by the company.
(a) $8\frac{1}{2}\%$ (b) $12\frac{1}{2}\%$
(c) 12% (d) $13\frac{3}{4}\%$

5. A company declared a semi-annual dividend of $7\frac{1}{2}\%$. Find the annual dividend of Chetan, owning 1250 shares of the company having a par value of Rs. 10 each.
(a) Rs. 1875 (b) Rs. 1757
(c) Rs. 1680 (d) Rs. 1575

6. A medicine company issued 125000 shares of par value Rs. 20 each. If the total dividend declared by the company is Rs. 375000, find the rate of dividend paid by the company.
(a) 15 % (b) 13%
(c) 10% (d) 14%

7. Seema had 50 preferred shares and 400 common shares of par value Rs. 100 each. If the dividend declared on preferred shares is 10% per annum and a semi-annual dividend of 7.5% is on common shares find the annual dividend received by Seema.
(a) Rs. 7500 (b) Rs. 6500
(c) Rs. 8500 (d) Rs. 5500

8. Find the annual dividend received by Sunil for his 200 preferred shares and 1000 common shares, both of par value Rs. 100 each if the dividend declared on a preferred share is 10% per annum and an annual dividend of $12\frac{1}{2}$ % on the common shares.
(a) Rs. 4500 (b) Rs. 550
(c) Rs. 4000 (d) Rs. 3500

9. A company issued 50000 shares of par value Rs. 100 each. If the total dividend declared by the company is Rs. 125000, out of which Rs. 50000 have been kept in reserve fund and the remaining is distributed as dividend, find the rate of dividend paid by the company.
(a) $2\frac{3}{4}\%$ (b) $1\frac{1}{2}\%$
(c) $1\frac{1}{4}\%$ (d) 2%

10. Find the annual dividend received by Nishita from 1200 preferred shares and 3000 common shares both of par value Rs. 50 each if the dividend paid on preferred shares is 10% and semi-annual dividend of $3\frac{1}{2}$ % is declared on common shares.
(a) Rs. 18500 (b) Rs. 16500
(c) Rs. 14500 (d) Rs. 14500

11. 12500 shares, of par value Rs. 20 each, are purchased from Ram by Mohan at a price of Rs. 25 each. Find the amount required to purchase the shares. If Mohan further sells the shares at a premium of Rs. 11 each, find his gain in the transaction.
(a) Rs. 75000 (b) Rs. 85000
(c) Rs. 70000 (d) Rs. 65000

12. Mac buys 200 shares of par value Rs. 10 each, of a company, which pays an annual dividend of 8% at such a price that he gets 10% on his investment. Find the market value of share.
(a) Rs. 8 (b) Rs. 10
(c) Rs. 6 (d) Rs. 12

13. Shyam purchased 12000 shares of a company, of par value Rs. 10 each, paying an annual dividend of 15% at such a price, that she gets 10% on her investment. Find the market value of a share.
(a) Rs. 25 (b) Rs 15
(c) Rs. 20 (d) Rs. 14

14. The capital of a company is made up of 50000 preferred shares with dividend of 20% and 20000 common shares, the par value of each type of share being Rs. 10. The company had a total profit of Rs. 180000 out of which Rs. 30000 were kept in reserve fund and the remaining distributed to shareholders. Find the dividend percent to the common shareholders.
(a) 24% (b) 20%
(c) 25% (d) 30%

15. A company has issued 10000 preferred shares and 50000 common shares both of par value Rs. 100 each. The dividend on a preferred share and a common share is 12% and 17.6%, respectively. The company had a total profit of 15 lakh rupees out of which some amount was kept in reserve fund and the remaining distributed as dividend. Find the amount kept in reserve fund.
(a) Rs. 5 lakh (b) Rs. 6 lakh
(c) Rs. 6.5 lakh (d) Rs. 5.5 lakh

16. A man sells 5000 common shares of a Company X (each of par value Rs. 10), which pays a dividend of 20%, at Rs. 30 per share. He invests the sale

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proceeds in ordinary shares of Company *Y* (each of par value Rs. 25) that pays a dividend of 15%. If the market value of a share of Company *Y* is Rs. 40, find the number of shares of Company *Y* purchased by the man
(a) 3850 (b) 3750
(c) 3700 (d) 3800

17. The shares of a company of par value Rs. 10 each, are available at 20% premium. Find the amount paid by the buyer who wants to buy 2500 shares. What would be the gain of the buyer if he sells those shares at the rate of Rs. 20 per share?
(a) Rs. 25000 (b) Rs. 30000
(c) Rs. 20000 (d) Rs. 22000

18. Find the income on 12% stock of Rs. 60000 purchased at Rs. 110.
(a) Rs. 7200 (b) Rs. 7500
(c) Rs. 7400 (d) Rs. 8200

19. Find the income on $7\frac{1}{2}\%$ stock of Rs. 20000 purchased at Rs. 120.
(a) Rs. 1550 (b) Rs. 1450
(c) Rs. 1500 (d) Rs. 1600

20. Find the income by investing Rs. 81000 in 9% stock at 135.
(a) Rs. 5500 (b) Rs. 6400
(c) Rs. 5400 (d) Rs. 6000

21. Find the income obtained by investing Rs. 90000 in $7\frac{1}{2}\%$ stock at $112\frac{1}{2}$.
(a) Rs. 6000 (b) Rs. 6500
(c) Rs. 7500 (d) Rs. 7000

22. A person buys $9\frac{1}{2}\%$ stock of Rs. 72000 at 144. Find his annual income.
(a) Rs. 6640 (b) Rs. 6840
(c) Rs. 6900 (d) Rs. 7240

23. Mr. Lal invested Rs. 92000 in $9\frac{1}{2}\%$ stock at 91 (Brokerage: Re. 1). Find the annual income of Mr. Lal from this investment.
(a) Rs. 9000 (b) Rs. 9500
(c) Rs. 10500 (d) Rs. 8000

24. Raja invested Rs. 99000 in $7\frac{1}{2}\%$ stocks at $81\frac{1}{2}$ (Brokerage: Re. 1). Find Ram's annual income from his investment.
(a) Rs. 9500 (b) Rs. 10000
(c) Rs. 10500 (d) Rs. 9000

25. Ram invested Rs. 88008 in $9\frac{1}{2}\%$ stock at 112 (Brokerage: Rs. 2) Find annual income of Ram from this investment.
(a) Rs. 6334 (b) Rs. 6874
(c) Rs. 7334 (d) Rs. 6534

26. Find the investment required to purchase Rs. 125000 of 8% stock at 92.
(a) Rs. 115000 (b) Rs. 120000
(c) Rs. 105000 (d) Rs. 125000

27. What investment will be required to purchase Rs. 90000 of 8% stock at 110?
(a) Rs. 88000 (b) Rs. 99000
(c) Rs. 88500 (d) Rs. 9950

28. Find the investment required to get an income of Rs. 1938 from $9\frac{1}{2}\%$ stock at 90 (Brokerage 1%).
(a) Rs. 19642.60 (b) Rs. 17543.00
(c) Rs. 18543.60 (d) Rs. 18600.60

29. A man bought Rs. 20000 of 5% stock at 90 and sold it when its price rose to Rs. $93\frac{3}{4}$. Find his gain per cent.
(a) $5\frac{1}{6}\%$ (b) $4\frac{1}{6}\%$
(c) $5\frac{5}{6}\%$ (d) $4\frac{5}{6}\%$

30. Meena bought Rs. 36000 of $7\frac{1}{2}\%$ stock at 92 and sold it when its price rose to Rs. $93\frac{3}{4}$. Find her gain per cent.
(a) 1.9% (b) 2.9%
(c) 2.3% (d) 1.4%

31. A man invests Rs. 27600 in 4% stock at 92. He sold Rs. 20000 stock when the price rose to Rs. 96, and sold the remaining stock when the market value fell to Rs. 90. How much does he gain or loss in the transaction?
(a) Gain = Rs. 600 (b) Loss = Rs. 600
(c) Loss = Rs. 650 (d) Gain = Rs. 650

32. A person invests Rs. 28500 in 5% stock at 95. He sold Rs. 15000 stock when the price rose to Rs. 98 and sold the remaining stock when the market value of the stock fell to Rs. 90. How much does he gain or loss in the transaction?
(a) Gain = Rs. 300 (b) Loss = Rs 300
(c) Gain = Rs. 400 (d) Loss = Rs. 400

33. Sushma invested Rs. 245000 in 7% stock at 98 and sold the stock when its price rose to Rs. 100. She invested the sale proceeds in 9% stock at 125. Find the change in income of Sushma.
(a) Rs. 600 (b) Rs. 400
(c) Rs. 500 (d) Rs. 650

34. Anu invested Rs. 32400 in 8% stock at 90. She sold out Rs. 18000 stock when the price rose to Rs. 95 and the remaining stock at Rs. 98. She invested the total sale proceeds in 10% stock at $96\frac{1}{2}$. Find the change in income of Anu.

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(a) Rs. 750 (b) Rs. 720
(c) Rs. 760 (d) Rs. 740

35. A man invested Rs. 50490 in 5% stock at 99 and sold it when the price rose to Rs. 102. He invested the sale proceeds in 8% stock at 96. Find the change in man's income (Brokerage: Rs. 3)
(c) Rs. 1485 (b) Rs. 1585
(c) Rs. 1385 (d) Rs. 1685

36. A man invested Rs. 260000 in 5% stock at 104. He sold the stock when the price rose to Rs. 125 and invested the sale proceeds in 6% stock. By doing this his income increased by Rs. 2500. At what price did he purchase the second stock?
(a) Rs. 225 (b) Rs. 175
(c) Rs. 125 (d) Rs. 150

37. Find the income per cent of a buyer on 5% debentures of face value Rs. 95 and available in the market for Rs. 125.
(a) 4.8% (b) 5.8%
(c) 3.8% (d) 2.8%

38. Find the income per cent on 10% debentures of par value Rs. 120 available in the market for Rs. 150.
(a) 9% (b) 8%
(c) 7% (d) 6%

39. Brij has 800 shares of par value Rs. 50 each and 600 debentures of par value Rs. 100 each of the company. The company pays an annual dividend of 6% on the shares and interest of 12% on the debentures. Find the total annual income of Brij and rate of return on his investment.
(a) Rs. 9600, 9.6% (b) Rs. 8000, 8%
(c) Rs. 10600, 10.6% (d) Rs. 9000, 8.6%

40. A man bought 20 shares of Rs. 50 at 5 discount, the rate of discount being $4\frac{3}{4}\%$. The rate of interest obtained is
(a) $4\frac{3}{4}\%$ (b) $3\frac{1}{4}\%$
(c) 5.28% (d) 4.95%

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Annual dividend on one share
= 10% of Rs. 10
$= \text{Rs.}\left(10\times\frac{10}{100}\right) = \text{Re. } 1$

Annual dividend of Ram owning 1500 shares
$= (1500\times 1) = \text{Rs. } 1500.$

Alternatively, we could have found the total par value of 1500 shares first and then find dividend at 10% of it as shown below:

Total par value of 1500 shares
$= \text{Rs. } (1500\times 10) = \text{Rs. } 15000$

$\therefore$ Total annual dividend of Ram
$= \left(15000\times\frac{10}{100}\right) = \mathbf{Rs.\ 1500.}$

2. Ans. (b)

Solution Annual dividend on one share = 10% of Rs. 100
$= \text{Rs.}\left(\frac{10}{100}\times 100\right) = \text{Rs. } 10$

$\therefore$ Annual dividend on 4000 shares
$= \text{Rs. } (4000\times 10) = \mathbf{Rs.\ 40000.}$

3. Ans. (c)

Solution Number of shares purchased by Jatin
$= \frac{27260}{116} = 235.$

Face value of 235 shares
$= \text{Rs. } (235\times 100) = \text{Rs. } 23500.$

Annual income from 235 shares
= 16% of Rs. 23500
$= \text{Rs.}\left(\frac{16}{100}\times 23500\right) = \mathbf{Rs.\ 3760.}$

4. Ans.(b)

Solution Number of shares = 50000
Par value of a share = Rs. 10
$\therefore$ Total par value of 50000 shares = Rs. 500000
Total dividend = Rs. 62500
$\therefore$ Rate of dividend paid by the company
$= \left(\frac{62500}{500000}\times 100\right)\% = \mathbf{12\frac{1}{2}\%.}$

5. Ans. (a)

Solution Annual dividend on one share $= \left(2\times 7\frac{1}{2}\right)\%$
i.e. 15% of Rs. 10
$= \left(\frac{15}{100}\times 10\right) = \text{Rs. } 1.50$

$\therefore$ Annual dividend on 1250 shares
$= \text{Rs. } (1250\times 1.50) = \mathbf{Rs.\ 1875.}$

6. Ans. (a)

Solution Number of shares = 125000
Par value of a share = Rs. 20

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∴ Total par value of 125000 shares
= Rs. (1250000 × 20) = Rs. 2500000
Total dividend = Rs. 375000
∴ Rate of dividend paid by the company
$= \left(\frac{375000}{2500000}\times 100\right)\% = \mathbf{15\%}.$

7. Ans. (b)

Solution Dividend on 50 preferred shares
$= \text{Rs.}\left(50\times 100\times\frac{10}{100}\right) = \text{Rs. } 500$
Dividend on 400 common shares
$= \text{Rs.}\left(400\times\frac{100}{100}\times\frac{15}{2}\times 2\right) = \text{Rs. } 6000.$
∴ Total dividend received by Seema
= Rs. (500 + 6000) = **Rs. 6500.**

8. Ans. (a)

Solution Dividend on 200 preferred shares
= 10% of Rs. (200 × 100)
$= \text{Rs.}\left(\frac{10}{100}\times 20000\right) = \text{Rs. } 2000$
Dividend on 1000 common shares
$= 12\frac{1}{2}\%$ of Rs. (1000 × 100)
$= \text{Rs.}\left(\frac{25/2}{100}\times 100000\right)$
$= \text{Rs.}\left(\frac{25}{2}\times 1000\right) = \text{Rs. } 12500$
∴ Total dividend received
= Rs. (2000 + 12500) = **Rs. 4500.**

9. Ans. (b)

Solution The total dividend declared = Rs. 125000
Amount kept in reserve fund = Rs. 50000
Net amount paid as dividend to the shareholders
= Rs. (125000 – 50000) = Rs. 75000
Number of shares of par value Rs.100 each = 50000
Total par value of 50000 shares
= Rs. (50000 × 100) = Rs. 5000000
Rate of dividend paid by the company
$= \left(\frac{75000}{5000000}\times 100\right)\% = \frac{3}{2}\% = \mathbf{1\frac{1}{2}\%}.$

10. Ans. (b)

Solution Dividend on 1200 preferred shares
= 10% of Rs. (1200 × 50)
$= \text{Rs.}\left(\frac{10}{100}\times 1200\times 50\right) = \text{Rs. } 6000$
Dividend on 3000 common shares
$= \left(3\frac{1}{2}\times 2\right)\%$ of Rs. (3000 × 50)
$= \text{Rs.}\left(\frac{7}{100}\times 3000\times 50\right) = \text{Rs. } 10500$
∴ Total dividend received by Nishita
= Rs. (6000 + 10500) = **Rs. 16500.**

11. Ans. (a)

Solution Market value of a share = Rs. 25
∴ Market value of 12500 shares
= Rs. (25 × 12500) = Rs. 312500
Thus, the amount required to purchase 12500 shares = Rs. 312500
Then, Mohan sells these shares at a premium of Rs. 11 each.
∴ New market rate per shares
= Rs. (20 + 11) = Rs. 31
∴ Selling price of these shares
= Rs. (31 × 12500) = Rs. 387500
∴ Gain = S.P. – C.P. = Rs. (387500 – 312500)
= **Rs. 75000.**

12. Ans. (a)

Solution Par value of 200 shares = Rs. (200 × 10)
= Rs. 2000
Dividend received by Mac = $\text{Rs.}\left(\frac{8}{100}\times 2000\right)$
= Rs. 160
Let the market value of 200 shares be Rs. x.
We have to find x such that 10% of x = 160
$\Rightarrow \frac{10}{100}\times x = 160 \Rightarrow x = 160\times 10 = 1600$
i.e. Market value of 200 shares = Rs. 1600.
Hence, the market value of one share
$= \text{Rs.}\left(\frac{1600}{200}\right) = \mathbf{Rs.\ 8.}$

13. Ans. (b)

Solution Par value of 12000 shares = Rs. (12000 × 10)
= Rs. 120000
Dividend received by Shyam = $\text{Rs.}\left(\frac{15}{100}\times 120000\right)$
= Rs. 18000
Let the market value of 12000 shares be Rs. x.
We have to find x such that 10% of x = 18000
$\Rightarrow \frac{10}{100}\times x = 18000 \Rightarrow x = 18000\times 10 = 180000$
i.e. Market value of 12000 shares = Rs. 180000.

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Hence, the market value of one share

$$= \text{Rs.}\left(\frac{180000}{12000}\right) = \textbf{Rs. 15.}$$

14. Ans. (c)

Solution The total profit of the company = Rs. 180000

Amount kept in reserve fund = Rs. 30000

∴ Net amount paid as dividend to shareholders

= Rs. (180000 – 30000) = Rs. 150000

Dividend paid by the company on 50000 preferred shares

$$= \text{Rs.}\left(50000\times\frac{10\times20}{10}\right) = \text{Rs. } 100000$$

∴ Dividend to be paid to common shareholders

= Rs. (150000 – 100000) = Rs. 50000

Thus, dividend paid on a common share

$$= \text{Rs.}\left(\frac{50000}{20000}\right) = \text{Rs. } 2.50$$

Hence, dividend percent paid on a common share

$$= \text{Rs.}\left(\frac{2.50}{10}\times100\right)\% = \textbf{25\%.}$$

15. Ans. (a)

Solution = 12% of Rs. (10000 × 100)

$$= \text{Rs.}\left(\frac{12}{100}\times10000\times100\right) = \text{Rs. } 120000$$

Dividend on 50000 common shares

= 17.6% of Rs. (50000 × 100)

$$= \text{Rs.}\left(\frac{17.6}{100}\times50000\times100\right) = \text{Rs. } 880000$$

∴ Total dividend paid = Rs. (120000 + 880000)

= Rs. 1000000 = Rs. 10 lakh

∴ Amount kept in reserve fund

= Rs. 15 lakh – 10 lakh = **Rs. 5 lakh.**

16. Ans. (b)

Solution Income of the man from 5000 ordinary shares of Company X, which pays a dividend of 20%

$$= \text{Rs.}\left(\frac{5000\times10\times20}{100}\right) = \text{Rs. } 10000.$$

Selling price of a share of Company X = Rs. 30

∴ Selling price of 5000 shares of Company X

= Rs. (5000 × 30)

= Rs. 150000.

Now, the market value of a share of Company Y is given to be Rs. 40.

∴ Number of shares of Company Y purchased by the man from Rs. 150000

$$= \left(\frac{150000}{40}\right) = \textbf{Rs. 3750.}$$

17. Ans. (c)

Solution Par value of a share = Rs. 10

$$\text{Market value of a share} = \text{Rs.}\left(10\times\frac{120}{100}\right) = \text{Rs. } 12$$

The amount to be paid by the buyer to purchase 2500 shares = Rs. (2500 × 12) = Rs. 30000

Gain of the shareholder on selling one share

= Rs. (20 – 12) = Rs. 8

∴ Gain from selling 2500 shares

= Rs. (2500 × 8) = **Rs. 20000.**

18. Ans. (a)

Solution Face value of the stock = Rs. 60000

Income on Rs. 100 stock = Rs. 12

$$\text{Income on Re. 1 stock} = \text{Rs.}\left(\frac{12}{100}\right)$$

$$\text{Income on Rs. 60000} = \text{Rs.}\left(\frac{12}{100}\times60000\right) = \textbf{Rs. 7200.}$$

19. Ans. (c)

Solution Face value of the stock = Rs. 20000

$$\text{Income on Rs. 100 stock} = \text{Rs. } 7\frac{1}{2}$$

$$\text{Income on Re. 1 stock} = \text{Rs.}\left(\frac{15/2}{100}\right) = \text{Rs.}\left(\frac{15}{200}\right)$$

$$\text{Income on Rs. 20000 stock} = \text{Rs.}\left(\frac{15}{200}\times20000\right)$$

$$= \textbf{Rs. 1500.}$$

20. Ans. (c)

Solution Here, the market value of the stock

= Rs. 81000

By investing Rs. 135, stock of par value Rs. 100 is available

∴ Income on Rs. 135 is Rs. 9

$$\therefore \text{ Income on Rs. 81000 is Rs.}\left(\frac{9}{135}\times81000\right)$$

$$= \textbf{Rs. 5400.}$$

21. Ans. (a)

Solution Here, market value of the stock = Rs. 90000

By investing Rs. $112\frac{1}{2}$, stock of par value Rs. 100 is available.

∴ Income on Rs. $112\frac{1}{2}$ is $7\frac{1}{2}\%$

$$\therefore \text{ Income on Rs. 90000 is Rs.}\left(\frac{15}{2}\times\frac{2}{225}\times90000\right)$$

$$= \textbf{Rs. 6000.}$$

22. Ans. (b)

Solution Face value of the stock = Rs. 72000

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$\therefore$ Income on stock = Rs. $\left(\frac{72000}{100}\times\frac{19}{2}\right)$ = **Rs. 6840.**

23. Ans. (b)

Solution Market value of Rs. 100 stock
= Rs. (91 + 1) = Rs. 92

Income on Rs. 92 = Rs. $9\frac{1}{2}$

$\therefore$ Income on Rs. 92000 = Rs. $\left(\frac{19}{2}\times\frac{19}{92}\times 92000\right)$
= **Rs. 9500.**

24. Ans. (d)

Solution Market value of Rs. 100 stock
= Rs. $\left(81\frac{1}{2}+1\right)$ = Rs. $82\frac{1}{2}$

Inceome on Rs. $82\frac{1}{2}$ = Rs. $7\frac{1}{2}$

$\therefore$ Income on Rs. 99000 = Rs. $\left(\frac{15}{2}\times\frac{2}{165}\times 99000\right)$
= **Rs. 9000.**

25. Ans. (c)

Solution Market value of Rs. 100 stock
= Rs. (112 + 2) = Rs. 114

Income on Rs. 114 = Rs. $9\frac{1}{2}$

$\therefore$ Income on Rs. 88008 = Rs. $\frac{19}{2}\times\frac{1}{114}\times 88008$
= **Rs. 7334.**

26. Ans. (a)

Solution Market value of Rs. 100 stock = Rs. 92

$\therefore$ Market value of Rs. 125000 stock

= Rs. $\left(\frac{92}{100}\times 125000\right)$ = Rs. 115000.

$\therefore$ An investment of Rs. 115000 is required to purchase Rs. 125000 of 8% stock at **92.**

27. Ans. (b)

Solution Market value of Rs. 100 stock = Rs. 110

$\therefore$ Market value of Rs. 90000 stock

= Rs. $\left(\frac{110}{100}\times 90000\right)$ = Rs. 99000

$\therefore$ An investment of Rs. 99000 is required to purchase Rs. 90000 of 8% stock at **110.**

28. Ans. (c)

Solution Brokerage = 1% Rs. 90 = Re. 0.90

$\therefore$ Investment needed to buy Rs. 100 stock
= Rs. 90.90 on which the income is Rs. 9%

For income of Rs. $9\frac{1}{2}$, the investment
= Rs. 90.90

For income of Rs. 1938, the investment

= Rs. $\left(\frac{90.90\times 2}{19}\times 1938\right)$ = **Rs. 18543.60.**

29. Ans. (b)

Solution Investment made by the man in buying Rs. 20000 of 5% stock at 90 = Rs. $\left(\frac{90}{100}\times 20000\right)$
= Rs. 18000.

When the price rose to Rs. $93\frac{3}{4}$, the man sold the stock. Thus, money realised from selling the stock

= Rs. $\left(\frac{375}{4}\times\frac{1}{100}\times 20000\right)$ = Rs. 18750

$\therefore$ Gain in the transaction
= Rs. (18750 – 18000) = Rs. 750

$\therefore$ Gain per cent = $\left(\frac{750}{18000}\times 100\right)\%$ = $\mathbf{4\frac{1}{6}\%}$.

30. Ans. (a)

Solution Investment made by Meena in buying Rs. 36000 of $7\frac{1}{2}\%$ stock at 92 = Rs. $\left(\frac{92}{100}\times 36000\right)$
= Rs. 33120.

When the price rose to Rs. $93\frac{3}{4}$, Meena sold the stock. Thus, money realised from selling the stock

= Rs. $\left(\frac{375}{4}\times\frac{1}{100}\times 36000\right)$ = Rs. 33750

$\therefore$ Gain in the transaction
= Rs. (33750 – 33120) = Rs. 630

$\therefore$ Gain per cent = $\left(\frac{630}{33120}\times 100\right)\%$ = **1.9 (approx).**

31. Ans. (a)

Solution Stock purchased by investing Rs. 27600 in 4% stock at 92

= Rs. $\left(\frac{27600\times 100}{92}\right)$ = Rs. 30000.

Money realised by selling Rs. 20000 stock at market value of Rs. 96

= Rs. $\left(\frac{20000\times 96}{100}\right)$ = Rs. 19200.

Remaining stock = Rs. (30000 – 20000) = Rs. 10000.

Money realised by selling Rs. 10000 stock at Rs. 90

= Rs. $\left(10000\times\frac{90}{100}\right)$ = Rs. 9000.

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∴ Total money realised by selling the whole stock
= Rs. (19200 + 9000) = Rs. 28200.

Money invested = Rs. 27600

∴ Gain = Rs. (28200 – 27600) = **Rs. 600.**

32. Ans. (b)

Solution Stock purchased by investing Rs. 28500 in 5% stock at 95

$$= \text{Rs.}\left(\frac{100}{95}\times 28500\right) = \text{Rs. } 30000$$

Money realised by selling Rs. 15000 stock market value of Rs. 98.

$$= \text{Rs.}\left(\frac{98}{100}\times 15000\right) = \text{Rs. } 14700$$

Remaining stock = Rs. (30000 – 15000) = Rs. 15000

Money realised by selling Rs. 15000 stock at Rs. 90

$$= \text{Rs.}\left(\frac{90}{100}\times 15000\right) = \text{Rs. } 13500$$

∴ Total money realised
= Rs. (14700 + 13500) = Rs. 28200

Money invested = Rs. 28500

∴ Loss = Rs. (28500 – 28200) = **Rs. 300.**

33. Ans. (c)

Solution Income from first stock = $\text{Rs.}\left(\frac{7}{98}\times 245000\right)$
= Rs. 17500

We have to find the amount realised on selling this stock.

Amount realised on selling Rs. 98 stock = Rs. 100

∴ Amount realised on selling Rs. 245000 stock

$$= \text{Rs.}\left(\frac{100}{98}\times 245000\right) = \text{Rs. } 250000$$

This amount is invested in 9% stock at 125

∴ Income from second stock

$$= \text{Rs.}\left(\frac{9}{125}\times 250000\right) = \text{Rs. } 18000$$

Hence, increase in income
= Rs. (18000 – 17500) = **Rs. 500.**

34. Ans. (b)

Solution Income from first stock

$$= \text{Rs.}\left(\frac{8}{90}\times 32400\right) = \text{Rs. } 2880$$

Amount of stock purchased by Anu

$$= \text{Rs.}\left(\frac{100}{90}\times 32400\right) = \text{Rs. } 36000$$

Amount received by selling Rs. 18000 stock at 95

$$= \text{Rs.}\left(\frac{95}{100}\times 18000\right) = \text{Rs. } 17100$$

Amount received by selling the remaining Rs. 18000 stock at 98

$$= \text{Rs.}\left(\frac{98}{100}\times 18000\right) = \text{Rs. } 17640$$

∴ Total amount received
= Rs. (17100 + 17640) = Rs. 34740

The amount of Rs. 34740 is invested in 10% stock at $96\frac{1}{2}$

∴ Income from this stock

$$= \text{Rs.}\left(10\times\frac{2}{193}\times 34740\right) = \text{Rs. } 3600$$

Hence, change in income
= Rs. (3600 – 2880) = **Rs. 720.**

35. Ans. (a)

Solution Purchase price of first stock
= Rs. (99 + 3) = Rs. 102

∴ Income on first stock

$$= \text{Rs.}\left(\frac{5}{102}\times 50490\right) = \text{Rs. } 2475$$

Sale price of stock = Rs. (102 – 3) = Rs. 99

∴ Amount received by selling the first stock

$$= \text{Rs.}\left(\frac{99}{102}\times 50490\right) = \text{Rs. } 49005$$

Purchase price of the second stock
= Rs. (96 + 3) = Rs. 99

∴ Income on second stock

$$= \text{Rs.}\left(\frac{8}{99}\times 49005\right) = \text{Rs. } 3960$$

Hence, change in income
= Rs. (3960 – 2475) = **Rs. 1485.**

36. Ans. (c)

Solution Income on first stock = $\text{Rs.}\left(\frac{5}{104}\times 260000\right)$
= Rs. 12500

Money realised by selling the stock when price rose to Rs. 125

$$= \text{Rs.}\left(\frac{125}{104}\times 260000\right) = \text{Rs. } 312500$$

Income on second stock is Rs. 2500 more on the first stock

∴ Income on second stock
= Rs. (12500 + 2500) = Rs. 15000

Let Rs. x be the market value of the second stock

$$\therefore \quad \frac{312500\times 6}{x} = 15000 \Rightarrow x = \frac{312500\times 6}{15000} = 125$$

i.e. The man purchased the stock at **Rs. 125.**

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37. Ans. (c)

Solution The market value of a debenture = Rs. 125

∴ Income on Rs. 125 is Rs. 5

∴ Income on Rs. 95 is Rs. $\left(\frac{5}{125}\times 95\right)$ = Rs. $\frac{19}{5}$

∴ Per cent income on the debentures is **3.8%.**

38. Ans. (b)

Solution The market value of a debenture = Rs. 150

∴ Income on Rs. 150 is Rs. 10

∴ Income on Rs. 120 = Rs. $\left(\frac{10}{150}\times 120\right)$ = Rs. 8

∴ Per cent income on the debentures = 8%.

39. Ans. (a)

Solution Annual dividend on 800 shares

= Rs. $\left(\frac{800\times 50\times 6}{100}\right)$ = Rs. 2400

Annual interest on 600 debentures

= Rs. $\left(\frac{600\times 100\times 12}{100}\right)$ = Rs. 7200

∴ Total annual income of Brij

= Rs. (2400 + 7200) = Rs. 9600

Total investment of Brij

= Rs. (800 × 50 + 600 × 100)

= Rs. (40000 + 60000)

= Rs. 100000

∴ Rate of return

= $\left(\frac{9600}{100000}\times 100\right)$% = **9.6%.**

40. Ans. (c)

Solution Face value = Rs (50 × 20) = Rs. 1000

Dividend = Rs. $\left(\frac{1000\times 19}{4\times 100}\right)$ = Rs. $\left(\frac{95}{2}\right)$

Investment = Rs. (45 × 20) = Rs. 900

Rate = Rs. $\left(\frac{95\times 100}{2\times 900}\right)$ = **5.28%.**

QUESTIONS FROM COMPETITIVE EXAMS

1. A man who owned 25% of the equity capital of a certain company sold 1/3 of his holding last year and 5/12 of the remaining this year. What part of the business does he now own ?

(a) 1/5 (b) 5/144
(c) 7/72 (d) 65/72

[Based on MAT 2004]

2. Arun has 800 shares of par value of Rs 50 each, and 600 debentures of par value Rs 100 each of a company. The company pays an annual dividend of 6% the shares and interest of 12% on the debentures. The rate of return of his investment is

(a) 8% (b) 9.6%
(c) 10.6% (d) 8.6%

[Based on MAT 2008]

3. A and B enter into a partnership with Rs 50,000 and Rs 60,000 respectively. C joins them after x months, contributing Rs 70,000, and B leaves x months before the end of the year. If they share the profit in the ratio of 20 : 18 : 21, then the value of x is

(a) 3 (b) 9
(c) 6 (d) 8

[Based on MAT 2008]

4. An amount of Rs 100,000 is invested in two types of shares. The first yields an interest of 9% per annum and the second yields 11% per annum. If the total interest at the end of one years is 9.75%, then the amount invested in each share respectively is:

(a) Rs 72,500, Rs 27,500
(b) Rs 62,500, Rs 37,500
(c) Rs 52,500, Rs 47,500
(d) Rs 82,500, Rs 17,500

[Based on FMS (Delhi) 2007]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let the total capital of the company = x

∴ The capital that the man held last year

= $\frac{1}{4}x - \frac{1}{3}$ of $\frac{1}{4}x = \frac{x}{4} - \frac{x}{12} = \frac{x}{6}$

The capital that the man holds this year

= $\frac{x}{6} - \frac{5}{12}$ of $\frac{x}{6} = \frac{x}{6} - \frac{5x}{72} = \frac{7x}{72}$.

2. Ans. (b)

Solution Total investment = 800× 50 + 600 × 100

= 40000 + 6 0000 = 100000

Annual return = 6% of 40000 + 12% of 60000

$= \frac{6\times 40000}{100} + \frac{12\times 60000}{100}$

= 2400 + 7200 = 9600

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Rate of return $= \frac{9600}{100000} \times 100\% = 9.6\%$

3. Ans. (a)

4. Ans. (b)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

Suppose Arun drives $2x$ miles and Balu drives x miles to the meeting.

$\therefore$ Arun drives $2x$ miles in 2 hours and Balu drives x miles in $\frac{3}{4}$ hour.

$\Rightarrow$ Required ratio $= 2 : \frac{3}{4} = \mathbf{8 : 3.}$

1. A man invests in a $4\frac{1}{2}\%$ stock at 96. The interest obtained by him is
(a) 4% (b) 4.5%
(c) 4.69% (d) $\frac{1}{2}\%$

2. Find the purchase cost of 80 shares of Rs. 10 each at $\frac{3}{8}$ discount, brokerage being $\frac{1}{8}$ per share.
(a) Rs. 780 (b) Rs. 680
(c) Rs. 980 (d) Rs. 880

3. Determine the increase in income obtained by transferring Rs. 4000 of $3\frac{1}{4}\%$ stock at 87 to $6\frac{1}{4}\%$ stock at 145.
(a) Rs. 10 increase (b) Rs. 20 increase
(c) Rs. 15 increase (d) None of these

4. A man invests Rs. 2730 in 9% at 90 and sells out part of the stock when they have risen to $94\frac{1}{2}$ and the remainder when they have fallen to 86. If he loses Rs. 10 by the transaction, the amount of stock he sold at first (brokerage : 1%) is
(a) Rs. 1500 (b) Rs. 1600
(c) Rs. 2000 (d) Rs. 2200

5. Find what a purchaser would have to pay for 300 shares of Rs. 10 each quoted at Rs. 25. What would be the gain to the shareholder, if he had purchased the share at par?
(a) Rs. 5000, Rs. 4500 (b) Rs. 5500, Rs. 4500
(c) Rs. 7500, Rs. 4500 (d) Rs. 4000, Rs. 7500

6. A man invests a certain sum in $9\frac{1}{2}\%$ government paper at 104. The price falling to Rs. 101, he sells out and loses Rs 600 by the transaction, exclusive of brokerage. The sum invested was
(a) Rs. 31200 (b) Rs. 10400
(c) Rs. 20800 (d) Rs. 15600

7. Which one of the following is the best investment?
(a) 13% stock at 104 (b) 17% stock at 140
(c) 15% stock at 125 (d) 9% stock at 70

8. A man invests half of his money in $13\frac{1}{2}\%$ stock at 120 and the other half in $10\frac{1}{2}\%$ at 90. Had he invested his money in order to buy equal amounts of each stock, he would have got Rs. 4.5 less income. His total investment was
(a) Rs 14200 (b) Rs. 17240
(b) Rs. 15120 (d) Rs. 13010

9. Divide Rs. 5300 into two parts such that if one part is invested in $10\frac{1}{2}\%$ at 98 and the other in 12% at par, the resulting income are equal (neglect brokerage).
(a) Rs. 2800, Rs. 2500 (b) Rs. 3200, Rs. 2100
(c) Rs. 2500, Rs. 2800 (d) Rs. 2100, Rs. 3200

Test II **[Time: 10 min]**

10. A man invests some money partly in 3% stock at 96 and partly in 4% stock at 120. To get equal dividends from both, he must invest the money in the ratio
(a) 16 : 15 (b) 3 : 4
(c) 4 : 5 (d) 3 : 5

11. Find the cash realised by selling Rs. 2400, $5\frac{1}{2}\%$ stock at 5 premium (brokerage $\frac{1}{4}\%$).
(a) Rs. 2614 (b) Rs. 2400
(c) Rs. 2514 (d) Rs. 2714

12. Which one of the following is the best investment?
(a) $9\frac{1}{2}\%$ stock at 120 (b) 11% stock at 125
(c) 7% stock at 84 (d) 20% stock at 230

13. A man buys Rs. 20 shares paying 9% dividend. The

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How is my income effected?
(a) Rs. 35 increase (b) Rs. 45 increase
(c) Rs. 30 increase (d) None of these

15. By investing in a company paying a dividend of 30% one obtains a return of 12%. Company's Rs. 10 share is quoted at
(a) Rs. 35 (b) Rs. 20
(c) Rs. 25 (d) Rs. 45

16. I transfer Rs. 4000 from 3% stock at 92 to a 4% stock and thus gain Rs. 5 in income. Find the price of the 4% stock.
(a) Rs. 117.76 (b) Rs. 118.26
(c) Rs. 119.76 (d) Rs. 107.76

17. Having Rs. 8370 to invest a man puts part of it in 9% stock at 96 and the remainder in 12% stock at 120. If his dividend from each investment is same, amount invested in second stock was
(a) Rs. 4140 (b) Rs. 4320
(c) Rs. 4050 (d) Rs. 4230

18. I invested Rs. 10000 partly in 4% at 120 and partly in 5% at 130 and my total income was Rs. 360. How much did I invest at 5% stock?
(a) Rs. 5200 (b) Rs 5000
(c) Rs. 4800 (d) Rs. 4700

19. The sale proceeds of Rs. 4500, $6\frac{1}{2}$% stock at 95 is transferred to 16% stock and thereby income increases by Rs 11.50. The price of 16% stock is
(a) Rs. 110 (b) Rs. 150
(c) Rs. 175 (d) Rs. 225

20. By investing in $3\frac{3}{4}$% stock at 96, one earns Rs. 100. The investment made is
(a) Rs. 36000 (b) Rs. 3600
(c) Rs. 2560 (d) Rs. 4800

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (c)

Solution On Rs. 96, he gets Rs. $\frac{9}{2}$

On Rs. 100, he gets = Rs. $\left(\frac{9\times100}{2\times96}\right)$ = **4.69%.**

3. Ans. (b)

Solution Income from 1st stock

$$= \text{Rs. } \frac{3(1/4)}{100} \times 4000 = \text{Rs. } 130$$

Sale proceeds = Rs. $\frac{87}{100} \times 4000$ = Rs. 3480

Income from 2nd stock

$$= \text{Rs. } \frac{6\frac{1}{4}}{145} \times 3480 = \text{Rs. } 150$$

Gain = Rs. 150 – Rs. 130 = **Rs. 20.**

5. Ans. (c)

Solution Face value of 1 share = Rs. 10

Market value of 1 share = Rs. 25

Amount paid by the purchaser to shareholder

$$= 300 \times 25 = \text{Rs. } 7500$$

According to the question,

If shareholder had purchased the share at par then the purchase cost of shareholder

$$= 300 \times 10 = \text{Rs. } 3000$$

∴ Gain by the shareholder

$$= \text{Rs. } 7500 - \text{Rs. } 3000 = \textbf{Rs. 4500.}$$

7. Ans. (d)

Solution To earn Re. 1 from the company, the investment required is

(a) Rs. 104 ÷ 13 = Rs. 8.00
(b) Rs. 140 ÷ 17 = Rs. 8.24
(c) Rs. 125 ÷ 15 = Rs. 8.33
(d) Rs. 70 ÷ 9 = Rs. 7.78

Minimum investment is in **(d).**

9. Ans. (a)

Solution In 1st stock for Re. 1, investment

$$= \text{Rs. } \frac{98\times2}{21} = \text{Rs. } \frac{28}{3}$$

and in 2nd stock for income of Re. 1, investment

$$= \text{Rs. } \frac{100}{12} = \text{Rs. } \frac{25}{3}$$

∴ Total sum of Rs. 5300 should be divided in the ratio $\frac{28}{3} : \frac{25}{3}$, i.e. 28 : 25

∴ 1st part = Rs. $\frac{28}{53} \times 5300$ = Rs. 2800

and, 2nd part = Rs. 5300 – Rs. 2800 = **Rs. 2500.**

11. Ans. (c)

Solution By selling Rs. 100 stock, cash realised

$$= \text{Rs. } \left(105 - \frac{1}{4}\right) = \text{Rs. } \left(\frac{419}{4}\right)$$

By selling Rs. 2400 stock, cash realised

$$= \text{Rs. } \left(\frac{419\times2400}{4\times100}\right) = \textbf{Rs. 2514.}$$

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13. Ans. (b)

Solution Dividend on Rs. 20 = Rs. $\left(\frac{9}{100}\times 20\right)$ = Rs. = $\frac{9}{5}$

Rs. 12 is an income on Rs. 100.

∴ Rs. $\frac{9}{5}$ is an income on Rs. $\left(\frac{100}{12}\times\frac{9}{5}\right)$ **= Rs. 15.**

15. Ans. (c)

Solution Let investment = Rs y
and price per share = Rs. x

No. of shares = $\frac{y}{x}$

Paid up value of these shares = Rs. $\frac{10y}{x}$

∴ Income = Rs. $\frac{30}{100}\times\frac{10y}{x}$ = Rs. $\frac{3y}{x}$

Also, return at 12% rate = Rs. $\frac{12}{100}y$

∴ $\frac{3y}{x}=\frac{12y}{100}$

∴ x **= 25.**

17. Ans. (c)

Solution In 1st stock:

If income is Rs. 9, investment = Rs. 96

∴ If income is Re. 1, investment

= Rs. $\frac{96}{9}$ = Rs. $\frac{32}{3}$

In 2nd stock:

If income is Rs. 12, investment = Rs. 120

∴ If income is Re 1, investment = Rs. $\frac{120}{12}$ = Rs. 10

∴ Total amount of Rs. 8370 should be invested in the ratio $\frac{32}{3}$: 10 i.e., 16 : 15

∴ Investment in 2nd stock

= Rs. $\frac{15}{31}\times 8370$ **= Rs. 4050.**

19. Ans. (d)

Solution Sale proceed of first stock = Rs. $\frac{95}{100}\times 4500$
= Rs. 4275

and, Income = Rs. $\frac{13}{200}\times 4500$ = Rs. $\frac{585}{2}$

∴ Income from 16% stock = Rs. $\left(\frac{585}{2}+\frac{23}{2}\right)$
= Rs. 304

This income is from Rs. 4275

∴ Price of 16% stock = Rs. $\frac{4275\times 16}{304}$ **= Rs. 225.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (b)	**6.** (c)	**8.** (c)	**10.** (a)	**12.** (b)	**14.** (b)	**16.** (a)	**18.** (a)	**20.** (c)

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Discount (True and Banker's) 21

INTRODUCTION

Suppose, a man buys a pen at a credit of one year for Rs. 105 at 5% simple interest. If the money is to be paid immediately, he shall give Rs. 100. Rs. 100 is the *present value* or *present worth* of Rs. 105 due 1 year hence. The sum due (Rs. 105) is called the *amount* and the reduction made in consideration of making the immediate payment is called *true discount*.

Present Value

The *present value* or *present worth* of a sum of money due at the end of a given time is that sum which with its interest for the given time at the given rate will amount to the sum due.

True Discount (T.D)

The *true discount* is the difference between the sum due at the end of a given time and its present worth.

Thus, T.D. = Amount (A) – present worth (P.W.)

In the above case, T.D. = Rs. (105 – 100) = Rs. 5.

Note:

1. Clearly, T.D. is the Interest on P.W. and
$$A = \text{P.W.} + \text{T.D.}$$
2. Interest is reckoned on P.W. and T.D. is reckoned on amount.

Banker's Discount

Suppose, a businessman A purchases goods worth Rs. 10000 from businessman B at a credit of say 3 months. Thus, B prepares a bill, called the *bill of exchange*. On receipt of the goods, A gives an agreement and signs the bill accepting that the money can be withdrawn from his account after 3 months of the date of the bill. Accordingly, A orders his bank to pay Rs. 10000 to B after 3 months. Besides, 3 days grace period is also added to this date (named as *nominally due date*) of expiry of 3 months to arrive at a date called *legally due date*. Thus, if 4 April 2004 is the nominally due date then 7 April 2004 will be legally due date. The amount of Rs. 10000 is called the *face value*.

Now, suppose B needs the money of this bill earlier than 7 April say 7 March. In such a case, B can approach the banker or broker to pay him money against the bill. Obviously, in such a situation, the money paid by the banker will be less than the face value of the bill. Now suppose, the bill is presented to the banker on 7 March 2004, then the banker will deduct the interest on the face value for the period 7 March 2004 to 7 April 2004 and this interest is called the *Banker's Discount* (B.D.) or *Commercial Discount*.

Thus, Banker's Discount is the simple interest on the face value for the period from the date on which the bill was discounted and the legally due date. The amount mentioned in the bill is called the *face value* of the bill. It may be noted that *banker's discount* is greater than *true discount*, because while the true discount is the interest on the present worth, banker's discount is the interest on sum due.

The difference between the present worth and cash value of a bill is called the *banker's gain* (B.G.) Thus, the interest on the bill value (or the face value) is called the banker's discount (B.D) and the difference between the banker's discount and true discount (T.D.) is called banker's gain (B.G.)

We have the following results:

Banker's gain = Banker's discount – True discount
= Interest on sum due – Interest on present worth
= Interest on (sum due – present worth)
= Interest on True Discount.

SOME BASIC FORMULAE

1. If rate = $R\%$ p.a. and time = T years, then

(i) $$\text{P.W.} = \frac{100 \times A}{100 + R \times T} = \frac{100 \times \text{T.D.}}{R \times T}$$

Illustration 1 Find the present worth of Rs. 8700 due in 3 years at 15% per annum at simple interest.

Also, find the true discount.

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Solution $\text{P.W.} = \frac{100\times A}{100+R\times T} = \frac{100\times 8700}{100+15\times 3}$

$= \frac{100\times 8700}{145} = \text{Rs. } 6000$

$\therefore$ T.D. = Amount − P.W.

= 8700 − 6000 = **Rs. 2700.**

(ii) $\text{T.D.} = \frac{\text{P.W.}\times R\times T}{100} = \frac{A\times R\times T}{100+R\times T}$.

Illustration 2 Find the true discount and the sum due 15 months hence whose present value at 8% is Rs. 1000.

Solution $\text{T.D.} = \frac{\text{P.W.}\times R\times T}{100} = \frac{1000\times 8\times 15}{100\times 12}$

= Rs. 100.

Sum due = P.W. + T.D.

= 1000 + 100 = **Rs. 1100.**

Illustration 3 Find the true discount reckoning 3% p.a. simple interest on Rs. 1802 due in 2 year's time.

Solution $\text{T.D.} = \frac{A\times R\times T}{100+R\times T} = \frac{1802\times 3\times 2}{100+3\times 2}$ = **Rs. 102.**

(iii) $\text{Sum (A)} = \frac{\text{S.I.}\times\text{T.D.}}{\text{S.I.}-\text{T.D.}}$

Explanation

$$\frac{\text{S.I.}\times\text{T.D.}}{\text{S.I.}-\text{T.D.}} = \frac{\left(A\times R\times\frac{T}{100}\right)\times\text{T.D.}}{\left(\text{T.D.}\times R\times\frac{T}{100}\right)} = A$$

$$\therefore\ A = \frac{\text{S.I.}\times\text{T.D.}}{\text{S.I.}-\text{T.D.}}.$$

Illustration 4 The true discount on a certain sum of money due 2 years hence is Rs. 1800 and the simple interest on the same sum is Rs. 2232. Find the sum.

Solution $\text{Sum (A)} = \frac{\text{S.I.}\times\text{T.D.}}{\text{S.I.}-\text{T.D.}} = \frac{2232\times 1800}{2232-1800}$

$= \frac{2232\times 1800}{432}$ = **Rs. 9300.**

(iv) S.I. − T.D. = S.I. on T.D.

Explanation

$$\text{S.I.} - \text{T.D.} = \frac{A\times R\times T}{100} - \frac{\text{P.W.}\times R\times T}{100}$$

$$= (A - \text{P.W.})\times\frac{R\times T}{100}$$

$$= \frac{\text{T.D.}\times R\times T}{100} = \text{S.I. on T.D.}$$

Illustration 5 The discount on a certain sum due 4 years hence is Rs. 100 but the interest on the same sum for the same period is Rs. 125. Find the sum and the rate per cent.

Solution We have,

S.I. on T.D. = S.I. − T.D.

= 125 − 100 = Rs. 25.

$\therefore$ Rate $(R) = \frac{25\times 100}{100\times 4} = \frac{25}{4} = 6\frac{1}{4}\%$

and, Sum $(A) = \frac{\text{S.I.}\times\text{T.D.}}{\text{S.I.}-\text{T.D.}} = \frac{125\times 100}{125-100}$

= **Rs. 500.**

(v) When the money is invested on compound interest,

$$\text{P.W.} = \frac{A}{\left(1+\frac{R}{100}\right)^T}$$

Illustration 6 Find the present worth of a bill of Rs. 3380 due 2 years hence at 4% compound interest. Also, calculate T.D.

Solution $\text{P.W.} = \frac{A}{\left(1+\frac{R}{100}\right)^T} = \frac{3380}{\left(1+\frac{4}{100}\right)^2}$

$= \frac{3380\times 25\times 25}{26\times 26} = \text{Rs. } 3125.$

T.D. = A − P.W.

= 3380 − 3125 = **Rs. 255.**

2. B.D. = S.I. on the bill for unexpired time
3. B.G. = B.D. − T.D.
4. B.D. = T.D. + Interest on T.D.
5. Sum Due $= \frac{\text{B.D.}\times\text{T.D.}}{\text{B.D.}-\text{T.D.}}$
6. T.D. $= \sqrt{\text{P.W.}\times\text{B.G.}}$
7. T.D. $= \frac{\text{B.G.}\times 100}{\text{Rate}\times\text{Time}}$
8. T.D. $= \frac{\text{Amount}\times\text{Rate}\times\text{Time}}{100+(\text{Rate}\times\text{Time})}$
9. B.D. $= \frac{\text{Amount}\times\text{Rate}\times\text{Time}}{100}$
10. Sum Due $= \frac{\text{B.D.}\times 100}{R\times T}$
11. Money paid by the banker = Amount − B.D.

Illustration 7 A bill is drawn for Rs. 5050 on the 12 June 2004 at 5 months credit. It is discounted on 3 September at 5% per annum. Find:

(i) Banker's Discount,

(ii) Money received by the holder of the bill, and

(iii) Banker's Gain.

Solution Amount = Rs. 5050.

Date of drawing = 12 June, 2004 (for 5 months)

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Date of maturing = 15 Nov, 2004 (including 3 days grace)
Date of discounting = 3 September, 2004.
No. of days from 3 Sept. to 15 Nov. =

Sept.	Oct.	Nov.	
27	+ 31	+ 15	= 73 days = $\frac{1}{5}$ year.

$\therefore$ (i) Banker's discount = $\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}$

$= \frac{5050 \times 1 \times 5}{100 \times 5}$ = Rs. 50.50.

(ii) Amount received by the holder of the bill
= Amount − B.D.
= 5050 − 50.50 = Rs. 4999.50.

(iii) True discount on Rs. 5050

$= \frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}$

$= \frac{5050 + \frac{1}{5} \times 5}{100 + \frac{1}{5} \times 5}$ = Rs. 50

Banker's gain = B.D. − T.D. = **Rs. 0.50.**

Illustration 8 The banker's discount and the true discount on a certain sum of money due 4 months hence are Rs. 48 and Rs. 45, respectively. Find the sum and the rate of interest.

Solution Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{48 \times 45}{48 - 45} = \frac{48 \times 45}{3}$

= Rs. 720.

Now, the banker's discount is simple interest on the sum due for 4 months.

Rate of interest = $\frac{100 \times 48 \times 3}{720 \times 1}$ = **20% p.a.**

Illustration 9 Find the face value of 3 months bill when the banker's discount at 3% per annum is Rs. 18.

Solution B.D. = Rs. 18, Rate (R) = 3%,

Time (T) = $\frac{1}{4}$ years.

$\therefore$ Face value = $\frac{\text{B.D.} \times 100}{R \times T} = \frac{18 \times 100 \times 4}{3 \times 1}$

= **Rs. 2400.**

Illustration 10 The present worth of a bill due sometimes hence is Rs. 1500. Find the banker's discount on the bill, if the true discount is Rs. 75.

Solution T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

$\Rightarrow 75 = \sqrt{1500 \times \text{B.G.}}$

$\Rightarrow 75 \times 75 = 1500 \times \text{B.G.}$

$\Rightarrow$ B.G. = $\frac{75 \times 75}{1500}$ = Rs. $\frac{15}{4}$ or, Rs. 3.75.

B.D. = T.D. + B.G. = 75 + 3.75 = **Rs. 78.75.**

Illustration 11 The banker's gain on a bill due 1 year hence at 12% per annum is Rs. 6. Find the true discount.

Solution T.D. = $\frac{\text{B.G.} \times 100}{R \times T} = \frac{6 \times 100}{12 \times 1}$ = **Rs. 50.**

Illustration 12 If the true discount on a certain sum due 6 months hence at 6% is Rs. 36, what is the banker's discount on the same sum for the same time and at the same rate?

Solution B.D. = T.D. + Interest on T.D.

$= \text{T.D.} + \frac{\text{T.D.} \times R \times T}{100} = 36 + \frac{36 \times 6 \times 6}{100 \times 12}$

= 36 + 1.08 = **Rs. 37.08.**

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The true discount on a bill for Rs. 1260 due 6 months hence at 10% per annum is
   (a) Rs. 60 (b) Rs. 160
   (c) Rs. 80 (d) Rs. 260

2. If the discount on a certain sum in 2 years at a certain rate is Rs. 150 and the interest in 3 years is Rs. 240. Find the sum and the rate %
   (a) Rs. 2400, $3\frac{1}{3}$% (b) Rs. 2400, $4\frac{1}{3}$%
   (c) Rs. 2200, $5\frac{1}{3}$% (d) None of these

3. If the true discount on Rs. 161 due 2 years 6 months hence is Rs. 21, find the rate of interest.
   (a) $2\frac{1}{2}$% (b) $4\frac{1}{2}$%
   (c) 5% (d) 6%

4. The present worth of Rs. 920 due at the end of 3 years at 5% simple interest per annum is
   (a) Rs. 780 (b) Rs. 850
   (c) Rs. 800 (d) Rs. 810

5. If the simple interest on a certain sum due some years hence at 6% is Rs. 180 and the discount at 5% on the same sum is Rs. 140. Find the sum and the time.
   (a) Rs 2100 and $1\frac{3}{7}$ years

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(b) Rs 2200 and $2\frac{3}{7}$ years

(c) Rs 2000 and $2\frac{3}{7}$ years

(d) None of these

6. The banker's gain on a certain sum due 9 months hence at 4% is Rs. 2.25. The sum is
(a) Rs. 2575 (b) Rs. 2500
(c) Rs. 2250 (d) Rs. 3250

7. At a given rate per cent, the simple interest and the true discount on a certain sum for a given time are Rs. 24 and Rs. 22, respectively. The sum is
(a) Rs. 264 (b) Rs. 220
(c) Rs. 288 (d) Rs. 295

8. The present worth of a bill of Rs. 1764 due 2 years hence at 5% compound interest is
(a) Rs. 1650 (b) Rs. 1700
(c) Rs. 1600 (d) Rs. 1714

9. If Rs. 21 in the true discount on Rs. 371 for a certain time, what is the true discount on the same sum for double that time, the rate being the same in both cases?
(a) Rs. 39.00 (b) Rs. 35.75
(c) Rs. 40.00 (d) Rs. 39.75

10. The T.D. on a certain sum of money due 2 years hence is Rs. 1800 and the simple interest on the same sum is Rs. 2232. Find the sum.
(a) Rs. 9300 (b) Rs. 9350
(c) Rs. 9450 (d) Rs. 9400

11. The present worth of Rs. 220.50 due in 2 years reckoning compound interest at 5% is
(a) Rs. 200 (b) Rs. 197.5
(c) Rs. 202 (d) Rs. 192.25

12. The T.D. on Rs. 936 due after a certain time at 8% is Rs. 36. Money is due after
(a) 6 months (b) 3 months
(c) 1 years (d) 9 months

13. A man bought a motor-cycle for Rs. 32500 and sold it for Rs. 35000, allowing the buyer 6 months credit. If the money be worth 4% per annum, the gain per cent is
(a) $8\frac{1}{7}\%$ (b) $7\frac{9}{13}\%$
(c) $7\frac{5}{13}\%$ (d) $8\frac{2}{3}\%$

14. Find the present worth of a bill of Rs. 3720 which is due 2 years hence at 12% compound interest, being compounded annually.
(a) Rs. 3100 (b) Rs. 3150
(c) Rs. 3125 (d) Rs. 3225

15. The holder of a bill for Rs. 17850 nominally due on 21 May, 1991 received Rs. 357 less than the amount of the bill by having it discounted at 5%. When was it doscounted?
(a) 29 Dec. 1990 (b) 30 Dec. 1989
(c) 19 Dec. 1990 (d) None of these

16. The true discount on a certain bill due nine months hence at 4% simple interest is Rs. 150. Find the amount of the bill.
(a) Rs. 5150 (b) Rs. 5250
(c) Rs. 4750 (d) Rs. 5650

17. A banker discounts a 4 months bill at 3% discount. If the proceeds be invested so that nothing is lost, the rate p.c. should be
(a) 3% (b) 4%
(c) $3\frac{1}{33}\%$ (d) None of these

18. The difference between the simple interest and the true discount on a certain sum of money for 2 years at 15% per annum at simple interest is Rs. 45. Find the sum.
(a) Rs. 700 (b) Rs. 650
(c) Rs. 675 (d) Rs. 625

19. The present worth of a sum of money due 146 days hence at 5% is Rs. 400. The sum due is
(a) Rs. 410 (b) Rs. 408
(c) Rs. 415 (d) Rs. 450

20. The present worth of a bill due sometime hence is Rs. 1500. Find the banker's discount on the bill, if the true discount is Rs. 75.
(a) Rs. 78.75 (b) Rs. 77.75
(c) Rs. 82.75 (d) Rs. 76.75

21. If the simple interest on Rs. 2000 at 5% p.a. is equal to the true discount on Rs. 2500 for the same time and at the same rate, the time is
(a) $4\frac{1}{2}$ years (b) 5 years
(c) $7\frac{1}{2}$ years (d) $2\frac{1}{2}$ years

22. Rs. 21 is the true discount on Rs. 371 for a certain time at certain rate p.c. If the rate p.c. is kept same, true discount on the same sum for double that time will be
(a) Rs. 44.38 (b) Rs. 39.75
(c) Rs. 33.25 (d) None of these

23. The true discount on a bill of Rs. 5450 due in 9 months is Rs. 450. Find the rate of interest.
(a) 12% (b) 12.5%
(c) 11.5% (d) 13.1%

24. If Rs. 10 be allowed as true discount on a bill of Rs. 110 due at the end of a certain time, then the discount allowed on the same sum due at the end of double the time is
(a) Rs. 20 (b) Rs. 21.81
(c) Rs. 22 (d) Rs. 18.33

25. A bill which being due at the end of 4 years is now

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worth Rs. 575 but if it is due $2\frac{1}{2}$ years hence it would now be worth Rs. 620. The sum of the bill is
(a) Rs. 695 (b) Rs. 725
(c) Rs. 713 (d) None of these

26. Find the present worth (PW) and the true discount reckoning 6% per annum simple interest of Rs. 176 due in 20 months time.
(a) Rs. 160, Rs. 16 (b) Rs. 130, Rs. 46
(c) Rs. 150, Rs. 26 (d) None of these

27. What rate per cent does a man get for his money when in discounting a bill due 10 months hence, he deducts 4% of the amount of the bill?
(a) 5% (b) 6%
(c) 8% (d) 4%

28. The discount on Rs. 5229 due in 1 year 9 months reckoning compound interest at 5% is
(a) Rs. 429.00 (b) Rs. 415.00
(c) Rs. 393.25 (d) None of these

29. A bill is discounted at 5% per annum. If banker's discount be allowed, at what rate perc ent must the proceeds be invested, so that nothing may be lost?
(a) 5% (b) $4\frac{19}{20}\%$
(c) $5\frac{5}{19}\%$ (d) 10%

30. If the compound interest on a certain sum of money for 2 years at 4% is Rs. 45.90, the true discount on the same sum of money due 2 years hence at 4% simple interest is
(a) Rs. 39.69 (b) Rs. 41.67
(c) Rs. 45.00 (d) Rs. 38.45

31. The true discount on a bill of Rs. 2550 due after 3 months is Rs. 50. Find the banker's discount.
(a) Rs. 53 (b) Rs. 51
(c) Rs. 55 (d) Rs. 57

32. A owes *B* Rs. 1350 due 3 months hence and *B* owes *A* Rs. 1078 due 5 months hence. If they agree to settle their account right now at rate 5% p.a., *A* should pay *B*
(a) Rs. $277\frac{1}{3}$ (b) Rs. 288.25
(c) Rs. 302 (d) None of these

33. Rs. 20 is the true discount on Rs. 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same
(a) Rs. 10 (b) Rs. 10.40
(c) Rs. 15.20 (d) Rs. 13

34. What is the rate of interest when the P.W. of Rs. 1245 due 15 months hence is Rs. 1200?
(a) 3% (b) 4%
(c) $4\frac{1}{2}\%$ (d) 5%

35. *A* has to pay Rs. 22 to *B* after 1 year. *B* asks *A* to pay Rs. 110 in cash and defer the payment of Rs. 110 for 2 years. *A* agrees to it. Counting the rate of interest at 10% per annum in this new mode of payment,
(a) there is no gain or loss to anyone
(b) *A* gains Rs. 7.34
(c) *A* loses Rs. 7.34
(d) *A* gains Rs. 11

36. The B.G. on a sum due 3 years hence at 10% is Rs. 180. The B.D. is
(a) Rs. 680 (b) Rs. 780
(c) Rs. 580 (d) Rs. 480

37. If the discount on Rs. 249 at 5% S.I. be Rs. 9, when is the sum due?
(a) 6 months (b) 4 months
(c) 9 months (d) 7 months

38. The banker's gain on a certain sum due 2 years hence at 5% per annum is Rs.8. The present worth is
(a) Rs. 800 (b) Rs. 1600
(c) Rs. 1200 (d) Rs. 880

39. The B.G. on a certain sum due 5 years hence is $\frac{3}{23}$ of B.D. the rate % is
(a) 6% (b) 5%
(c) 4% (d) 3%

EXPLANATORY ANSWERS

1. Ans. (a)

Solution T.D. $= \text{Rs}\ \dfrac{1260\times\frac{1}{2}\times10}{100+\frac{1}{2}\times10} =$ **Rs. 60.**

2. Ans. (a)

Solution Interest for 2 years $= \dfrac{240\times2}{3} =$ Rs. 160
Discount for 2 years = Rs. 150

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Sum due $= \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{160 \times 150}{160 - 150} = \text{Rs. } 2400$

Rate of interest $= \frac{240 \times 100}{2400 \times 3} = \mathbf{3\frac{1}{3}\%}$.

3. Ans. (d)

Solution Rs. 21 is the interest on Rs. (161 – 21)

or, Rs. 140 for 2 years 6 months

$\therefore$ Rate % $= \frac{21 \times 100}{140 \times 5/2} = \mathbf{6\%}$.

4. Ans. (c)

Solution Present worth $= \frac{A \times 100}{100 + R \times T} = \text{Rs. } \frac{920 \times 100}{100 + 3 \times 5}$

$= \text{Rs. } \frac{920 \times 100}{115} = \mathbf{Rs.\ 800.}$

5. Ans. (a)

Solution B.D. or Simple Interest at 5%

$= \frac{180 \times 5}{6} = \text{Rs. } 150$

Discount at 5% = Rs. 140

Sum $= \frac{\text{B.D} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{150 \times 140}{150 - 140} = \text{Rs. } 2100.$

Rate per cent $= \frac{180 \times 100}{2100 \times 6} = \mathbf{1\frac{3}{7}\ years.}$

6. Ans. (a)

Solution B.G. is the interest on T.D.

$\therefore$ T.D. $= \frac{2.25 \times 100}{\frac{3}{4} \times 4} = \text{Rs. } 75$

B.D. = Rs. 75 + Rs. 2.25 = Rs. 77.25

$\therefore$ Sum due $= \frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}} = \text{Rs. } \frac{77.25 \times 75}{2.25}$

$= \mathbf{Rs.\ 2575.}$

7. Ans. (a)

Solution Sum $= \frac{\text{T.D.} \times \text{S.I.}}{\text{S.I.} - \text{T.D.}} = \frac{24 \times 22}{24 - 22} = \mathbf{Rs.\ 264.}$

8. Ans. (c)

Solution P.W. $= \text{Rs. } 1764 \div \left(1 + \frac{5}{100}\right)^2$

$= \text{Rs. } 1764 \times \frac{400}{441} = \mathbf{Rs.\ 1600.}$

9. Ans. (d)

Solution Rs. 21 is the interest on Rs. (371 – 21) = Rs. 350

$\therefore \quad \frac{350 \times \text{no. of years} \times \text{rate}}{100} = 21$

$\Rightarrow$ No. of years $\times$ rate $= \frac{2100}{350} = 6$

$\therefore$ Twice no. of years $\times$ rate = 12

Now, on Rs. 112, the T.D. is Rs. 12

$\therefore$ on Rs. 371, the T.D. $= \text{Rs. } \frac{12}{112} \times 371$

$= \mathbf{Rs.\ 39.75.}$

10. Ans. (a)

Solution Sum $= \frac{\text{Simple interest on the sum} \times (\text{T.D.})}{\text{Simple interest on the sum} - (\text{T.D.})}$

$= \text{Rs. } \left(\frac{2232 \times 1800}{2232 - 1800}\right)$

$= \text{Rs. } \left(\frac{2232 \times 1800}{432}\right)$

$= \mathbf{Rs.\ 9300.}$

11. Ans. (a)

Solution P.W. $= \frac{220.5}{\left(1 + \frac{5}{100}\right)^2} = \frac{220.5 \times 20 \times 20}{21 \times 21}$

$= \mathbf{Rs.\ 200.}$

12. Ans. (a)

Solution P.W. = Rs. (936 – 36) = Rs. 900

Rs. 36 is S.I. on Rs. 900

$\therefore$ Time $= \frac{36 \times 100}{900 \times 8} = \frac{1}{2}$ year

$= \mathbf{6\ months.}$

13. Ans. (b)

Solution S.P. of motor = cycle = Rs. 35000

Gain = Rs. 35000 – Rs. 32500

= Rs. 2500

$\therefore$ Gain % $= \frac{2500}{32500} \times 100\ \%$

$= \frac{100}{13}\% = \mathbf{7\frac{9}{13}\%}$.

14. Ans. (c)

Solution P.W. $= \frac{A}{\left(1 + \frac{R}{100}\right)^T} = \text{Rs. } \frac{3720}{\left[1 + \frac{12}{100}\right]^2}$

$= \text{Rs. } 3720 \times \frac{25}{28} \times \frac{25}{28} = \mathbf{Rs.\ 3125.}$

15. Ans. (a)

Solution Clearly, S.I. on Rs. 17850 at 5% is Rs. 357

$\therefore$ Time $= \left(\frac{100 \times 357}{17850 \times 5}\right) = \frac{2}{5}$ years = 146 days.

So, the bill is 146 days prior to 24 May, the legally due date.

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May	April	March	Feb.	Jan.	Dec.
24	+ 30	+ 31	+ 28	+ 31	+ 2 = 146 days

So, the bill was discounted on **29 Dec. 1990.**

16. Ans. (a)

Solution $\text{P.W.} = \text{T.D.} \times \frac{100}{R \times T} = \text{Rs. } \frac{150 \times 100}{\frac{9}{12} \times 4}$

$= \text{Rs. } 5{,}000$

$\therefore$ Amount of the bill = Rs. 5000 + Rs. 150

= **Rs. 5150.**

17. Ans. (c)

Solution 4 months $= \frac{1}{3}$ year

$\therefore$ Banker deducts Rs. $3 \times \frac{1}{3}$ = Re. 1 from a bill of Rs. 100

So, the banker pays Rs. (100 – 1) = Rs. 99

So, the bill-holder loses Re. 1

So, for investment Re. 1 should be interest on Rs. 99 for 4 months.

$\therefore$ Rate $= \frac{1 \times 100}{99 \times \frac{4}{12}} = \frac{100}{33} = 1\frac{1}{33}\%$.

18. Ans. (b)

Solution Let the sum be Rs. 100

Time = 2 years

Rate = 15% per annum

$\text{S.I.} = \text{Rs. } \left(\frac{100 \times 15 \times 2}{100}\right) = \text{Rs. } 30$

$\text{Time Discount} = \text{Rs. } \left[\frac{100 \times R \times T}{100 + (R \times T)}\right]$

$= \text{Rs. } \left[\frac{100 \times 15 \times 2}{100 + (15 \times 2)}\right]$

$= \text{Rs. } \left[\frac{100 \times 15 \times 2}{130}\right] = \text{Rs. } \frac{300}{13}$.

The difference between S.I. and T.D. is

$$\text{Rs. } \left(30 - \frac{300}{13}\right) = \text{Rs. } \frac{90}{13}$$

If the difference in S.I. and T.D. is Rs. $\frac{90}{13}$, the sum = Rs. 100.

If the difference in S.I. and T.D. is Re. 1, the sum = Rs. $100 \times \frac{13}{10}$.

If the difference in S.I. and T.D. is Rs. 45, the sum = Rs. $100 \times \frac{13}{90} \times 45$ = **Rs. 650.**

19. Ans. (b)

Solution $\text{T.D.} = \text{Rs. } 400 \times \frac{146}{365} \times \frac{5}{100} = \text{Rs. } 8$

S.D. = Rs. 400 + Rs. 8 = **Rs. 408.**

20. Ans. (a)

Solution $\text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$

$75 = \sqrt{1500 \times \text{B.G.}}$

Squaring both the sides

$75 \times 75 = 1500 \times \text{B.G.}$

$\text{B.G.} = \text{Rs. } \frac{75 \times 75}{1500} = \text{Rs. } \frac{15}{4} = \text{Rs. } 3.75$

B.D. = T.D. + B.G.

= 75 + Rs. 3.75 = **Rs. 78.75.**

21. Ans. (b)

Solution T.D. = Rs. 500

Rate = 5%

P.W. = Rs. 2000

$\therefore$ Time $= \frac{500 \times 100}{2000 \times 5}$ = **5 years.**

22. Ans. (b)

Solution P.W. of Rs. 371 = Rs. (371 – 21) = Rs. 350

Also, T.D. = Simple Interest on P.W.

$\therefore$ Simple Interest on Rs. 350 for a certain period at certain rate p.c. = Rs. 21

$\therefore$ Simple Interest on Rs. 350 for double the period at same rate p.c. = Rs. 42

$\therefore$ Rs. 42 is T.D. on Rs. (350 + 42) = Rs. 392 for double the period.

$\therefore$ T.D. on Rs. 371 for double the period and same rate p.c. $= \frac{42}{392} \times 371$ = **Rs. 39.75.**

23. Ans. (a)

Solution Amount = Rs. 5450

P.W. = Amount – T.D.

= Rs. 5450 – Rs. 450

= Rs. 5000

In other words, simple interest on Rs. 5000 for 9 months is Rs. 450

$\therefore$ Rate $= \frac{\text{S.I.} \times 100}{\text{P} \times \text{T}} = \frac{450 \times 100 \times 4}{5000 \times 3}$

= **12% per annum.**

24. Ans. (d)

Solution S.I. on Rs. (110 – 10) for a given time = Rs. 10

S.I. on Rs. 100 for double the time = Rs. 20

Sum = Rs. (100 + 20) = Rs. 120

T.D. on Rs. 110 = Rs. $\left(\frac{20}{120} \times 110\right)$ = **Rs. 18.33.**

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25. Ans. (c)

Solution Let rate p.c. be r %.

Let Rs. x be the amount of the bill.

Then, $575 = \frac{x \times 100}{100+4r}$

i.e. $57500 + 2300r = 100x$

$\therefore \quad x = 575 + 23r \quad ...(i)$

and, $620 = \frac{100x}{100+\frac{5}{2}r}$

$\therefore \quad 62000 + 1550r = 100x$

$\therefore \quad 6200 + 155r = 10x$

$= 5750 + 230r \quad$ [Using (i)]

$\therefore \quad 75r = 450$

$\therefore \quad r = 6$

$\therefore$ (i) $\Rightarrow x = 575 + 138 =$ **Rs. 713.**

26. Ans. (a)

Solution Present Worth $= \frac{100 \times 76}{100+6\times\frac{20}{12}} =$ Rs. 160

True discount = Amount − Present Worth

= Rs. 176 − Rs. 160

= **Rs. 16.**

27. Ans. (a)

Solution Let the amount of the bill be Rs. 100

Money deducted = Rs. 4

Money received by holder of the bill

= Rs. (100 − 4) = Rs. 96

S.I. on Rs. 96 for 10 months = Rs. 4

Rate $= \frac{100\times4\times6}{96\times5} =$ **5%.**

28. Ans. (a)

Solution P.W. $= \frac{5229}{\left(1+\frac{5}{100}\right)\left(1+\frac{3}{4}\cdot\frac{5}{100}\right)}$

$= 5229 \times \frac{20}{21}\times\frac{80}{83} =$ Rs. 4800

$\therefore$ T.D. = Rs. (5229 − 4800) = **Rs. 429.**

29. Ans. (c)

Solution Let the sum be Rs. 100. Then, B.D. = Rs. 5

Proceeds = Rs. (100 − 5) = Rs. 95

$\therefore$ Rs. 5 must be the interest on Rs. 95 for 1 year.

So, rate $= \left(\frac{100\times5}{95\times1}\right) = 5\frac{5}{19}$ **%.**

30. Ans. (b)

Solution Let the sum be Rs. x. Then,

$45.90 = x\left\{\left(1+\frac{4}{100}\right)^2 - 1\right\}$

$= \left\{\left(\frac{26}{25}\right)^2 - 1\right\} = \left\{\frac{676-625}{625}\right\}$

$\therefore \quad x = \frac{625\times45.9}{51} =$ Rs. 562.5

$\therefore$ T.D. on Rs. 562.5

$= \frac{562.5\times4\times2}{100+4\times2} = \frac{4500}{108} =$ **Rs. 41.67.**

31. Ans. (b)

Solution T.D. = Rs. 50

P.W. = Rs. 2550 − Rs. 50 = Rs. 2500

Rate of Interest $= \frac{50\times100\times4}{2500\times1} = 8\%$ per annum

B.D. = Rs. $\left[\frac{2550}{100}\times8\times\frac{1}{4}\right] =$ **Rs. 51.**

32. Ans. (a)

Solution P.W. of Rs. 1350

$= \frac{1350\times100}{100+\frac{3}{12}\times5} = \frac{1350\times400}{405}$

$=$ Rs. $\frac{4000}{3}$

P.W. of Rs. 1078 $= \frac{1078\times100}{100+\frac{5}{12}\times5} = \frac{1078\times1200}{1225}$

= Rs. 1056

$\therefore$ A should pay B = Rs. $\left(\frac{4000}{3} - 1056\right)$

= Rs. $\frac{832}{3} =$ **Rs. $277\frac{1}{3}$.**

33. Ans. (b)

Solution S.I. on Rs. 240 for a given time = Rs. 20

S.I. on Rs. 240 for half the time = Rs. 10

$\therefore$ Rs. 10 is T.D. on Rs. 250

So, T.D. on Rs. 260 = Rs. $\left(\frac{10}{250}\times260\right) =$ **Rs. 10.40.**

34. Ans. (a)

Solution T.D. = Rs. 1245 − Rs. 1200 = Rs. 45

P.W. = Rs. 1200

$\therefore$ Rate % $= \frac{45\times100}{1200\times15/12} =$ **3%.**

35. Ans. (b)

Solution A has to pay the P.W. of Rs. 220 due 1 year hence, which is

= Rs. $\left[\frac{100\times220}{100+(10\times1)}\right] =$ Rs. 200

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A actually pays = Rs. [110 + P.W. of Rs. 110 due 2 years hence].

$$= \text{Rs.} \left[110 + \frac{100 \times 110}{100 + (8 \times 2)}\right]$$

$$= \text{Rs. } 192.66$$

$\therefore$ A gains = Rs. [200 – 192.66]

= Rs. 7.34.

36. Ans. (b)

Solution $\text{T.D.} = \dfrac{\text{B.G.} \times 100}{\text{R} \times \text{T}}$

$$= \text{Rs.} \frac{180 \times 100}{10 \times 3} = \text{Rs. } 600$$

$\therefore$ B.D. = Rs. (600 + 180) **= Rs. 780.**

37. Ans. (c)

Solution P.W. = Rs. 249 – Rs. 9 = Rs. 240

T.D. = Rs. 9

Rate = 5%

$\therefore$ $\text{Time} = \dfrac{\text{T.D.} \times 100}{\text{P.W.} \times \text{rate}} = \dfrac{9 \times 100}{240 \times 5} = \dfrac{3}{4}$ year

= 9 months.

38. Ans. (a)

Solution $\text{T.D.} = \dfrac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}$

$$= \text{Rs.} \left(\frac{8 \times 100}{5 \times 2}\right)$$

= Rs. 800.

39. Ans. (d)

Solution Let B.D. = Re. 1

$\because$ $\text{B.G.} = \text{Rs.} \dfrac{3}{23}$

$\therefore$ $\text{T.D.} = \text{Rs.} \left[1 - \dfrac{3}{23}\right] = \text{Rs.} \dfrac{20}{23}$

$\therefore$ $\text{Sum} = \dfrac{\text{B.D} \times \text{T.D.}}{\text{B.D} - \text{T.D.}} = \dfrac{1 \times \frac{20}{23}}{1 - \frac{20}{23}} = \text{Rs.} \dfrac{20}{3}$

$\because$ S.I. on Rs. $\dfrac{20}{3}$ for 5 years is Re. 1

$\therefore$ $\text{Rate} = \dfrac{100 \times 3}{20 \times 5}$ **= 3%.**

QUESTIONS FROM COMPETITIVE EXAMS

1. A retailer allows a trade discount of 20% and a cash discount of 6.25% on the market price of the products and gets a net profit of 20% on the cost. By how much above the cost, should the products be labeled for sale?

(a) 40% (b) 50%

(c) 60% (d) 70%

[Based on MAT 2003]

EXPLANATORY ANSWERS

1. Ans. (c)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. If the discount on Rs. 1050 for one year was Rs. 50, the discount on the same sum for half the year would be

(a) Rs. 25.61 (b) Rs. 27.50

(c) Rs. 22.50 (d) None of these

2. I want to sell my scooter. There are two offers, one at cash payment of Rs. 8100 and another at credit of Rs. 8250 to be paid after 6 months. If money being worth $6\frac{1}{4}$% per annum simple interest, which is the better offer?

(a) Rs. 8100 in cash

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(b) Rs. 8250 due 6 months hence
(c) Both are equally good.
(d) Cannot be said

3. The simple interest and the true discount on a certain sum for a given time and at a given rate are Rs. 25 and Rs. 20, respectively. The sum is
(a) Rs. 500 (b) Rs. 200
(c) Rs. 250 (d) Rs. 100

4. A sum of Rs. 2550 is borrowed to be paid back in two years by equal annual payments, 4 per cent compound interest being allowed. What is the annual payment?
(a) Rs. 1352 (b) Rs. 1362
(c) Rs. 1382 (d) Rs. 1772

5. The banker's discount on a sum of money for $1\frac{1}{2}$ years is Rs. 60 and the true discount on the same sum for 2 years is Rs. 75. The rate per cent is
(a) 5% (b) 6%
(c) $6\frac{2}{3}\%$ (d) $3\frac{1}{3}$

6. The banker's gain on a sum due 2 years hence at 6% is Rs. 84. The banker's discount is
(a) Rs. 616 (b) Rs. 716
(c) Rs. 784 (d) Rs. 684

7. Find the present worth of a bill of Rs. 2420 due 2 years hence at 10% compound interest. Also, find the true discount.
(a) Rs. 2000, Rs. 420 (b) Rs. 2200, Rs. 520
(c) Rs. 2100, Rs. 460 (d) None of these

8. The difference between the simple interest and true discount on a certain sum of money for $2\frac{1}{2}$ years at 6% per annum is Rs. 90. The sum is
(a) Rs. 4500 (b) Rs. 4600
(c) Rs. 4300 (d) Rs. 4450

9. A bill was drawn on May 14 at two months after due date and was discounted on July 2 at $8\frac{1}{9}\%$. If the banker's gain be Re. 1, the bill was drawn for sum
(a) Rs. 9030 (b) Rs. 90300
(c) Rs. 11200 (d) Rs. 10020

10. The discount on a sum due 4 years hence is $\frac{5}{6}$ of the interest on the same for the same time. Find the rate of interest.
(a) 4% (b) 5%
(c) 6% (d) 8%

Test II

[Time: 10 min]

11. The true discount on Rs. 1247 due 1 year 6 months hence at 5% per annum is
(a) Rs. 47 (b) Rs. 87
(c) Rs. 127 (d) Rs. 157

12. A shopkeeper gives two prices of his articles, one in terms of cash and the other on credit for 4 months at 3% per annum. If the credit price is Rs. 202, find the cash price.
(a) Rs. 190 (b) Rs. 192
(c) Rs. 195 (d) Rs. 200

13. The banker's discount on a bill due 1 year and 6 months hence is Rs. 50 and the true discount on the same sum and the same rate per cent is Rs. 45. The rate per cent is
(a) 6% (b) $6\frac{2}{3}\%$
(c) $6\frac{1}{2}\%$ (d) $8\frac{44}{59}\%$

14. A bill is discounted at 5% per annum. If banker's discount be allowed, at what rate per cent must the proceeds be invested, so that nothing may be lost?
(a) 5% (b) $4\frac{19}{21}\%$
(c) $5\frac{5}{19}\%$ (d) 10%

15. The banker's discount on Rs. 1600 at 6% is the same as the true discount on Rs. 1624 for the same time and at the same rate. Then, the time is
(a) 3 months (b) 4 months
(c) 6 months (d) 8 months

16. What is the sum due 219 days hence whose present worth at 5% per annum is Rs. 500?
(a) Rs. 515 (b) Rs. 520
(c) Rs. 525 (d) Rs. 530

17. A bill for Rs. 17850 drawn on 21 May 1999, at 7 months was discounted by a banker on the 29 Dec. 1991. If the money being worth 5%, the difference between the banker's discount and the true discount was
(a) Rs. 12 (b) Rs. 20
(c) Rs. 7 (d) Rs. 15

18. The present worth of Rs. 1404 due in two equal half-yearly instalments at 8% per annum simple interest is
(a) Rs. 1325 (b) Rs. 1300
(c) Rs. 1350 (d) Rs. 1500

19. If the banker's discount calculated for 1 year is 21 times his gain, the rate p.c. is
(a) 5% (b) 6%
(c) 4% (d) $3\frac{1}{2}\%$

20. The banker's gain on a bill due 1 year hence at 5% is Re 1. The true discount is
(a) Rs. 15 (b) Rs. 20
(c) Rs. 25 (d) Rs. 5

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution P.W of Rs. 1050 = Rs. (1050 – 50) = Rs. 1000

$\therefore$ 50 = S.I. on Rs. 1000 for 1 year

$\therefore$ Rate $= \dfrac{50\times100}{1000\times1} = 5\%$

$\therefore$ T.D. on Rs. 1050 for half the year

$$= \frac{1050\times5\times\frac{1}{2}}{100+5\times\frac{1}{2}} = \frac{5250}{205} = \textbf{Rs. 25.61.}$$

3. Ans. (d)

Solution Sum $= \dfrac{(S.I.)\times(T.D.)}{(S.I)-(T.D)} = \text{Rs.}\left(\dfrac{25\times20}{25-20}\right)$

$= \textbf{Rs. 100.}$

5. Ans. (d)

Solution B.D. for (3/2) years = Rs. 60

B.D. for 2 years = Rs. $\left(\dfrac{60\times2}{3}\times2\right)$ = Rs. 80

Now, B.D. = Rs. 80, T.D. = Rs. 75 and Time = 2 years

$\therefore$ Sum = Rs. $\left(\dfrac{80\times75}{5}\right)$ = Rs. 1200

$\therefore$ Rs. 80 is S.I. on Rs. 1200 for 2 years

So, rate $= \left(\dfrac{100\times80}{1200\times2}\right)\% = \mathbf{3\frac{1}{3}\%}$.

7. Ans. (a)

Solution Here, sum is put on compound interest, hence we have

$$\text{P.W.} = \frac{A}{\left(1+\frac{R}{100}\right)^n} = \frac{2420}{\left(1+\frac{10}{100}\right)^2} = \text{Rs. } 2000$$

T.D. = P.W. – P

$\therefore$ T.D. = 2420 – 2000 = **Rs. 420.**

9. Ans. (b)

Solution The bill legally falls due on date after 2 months + 3 days from 14 May,

i.e. on 17 July.

$\therefore$ No. of unexpired days = No. of days from 2 July to 17 July = 15

Let bill be drawn for Rs. x

B.D. = S.I. on face value of bill

$$= \frac{x}{100}\times\frac{15}{365}\times\frac{73}{9} = \frac{x}{300}$$

and, T.D. on Rs. $x = \dfrac{x\times\frac{15}{365}\times\frac{73}{9}}{100+\frac{15}{365}\times\frac{73}{9}} = \dfrac{x}{301}$

$\therefore$ B.G. = B.D. – T.D. $= \dfrac{x}{300}-\dfrac{x}{301}$

Given: $\dfrac{x}{90300} = 1$ $\therefore$ x = **Rs. 90300.**

11. Ans. (b)

Solution T.D. = Rs. $\dfrac{1247\times\frac{3}{2}\times5}{100+\frac{3}{2}\times5}$

$= \text{Rs. } 1247\times\dfrac{3}{2}\times5\times\dfrac{2}{215} = \textbf{Rs. 87.}$

13. Ans. (b)

Solution Sum $= \dfrac{\text{B.D.}\times\text{T.D.}}{\text{B.D.}-\text{T.D.}} = \text{Rs.}\left(\dfrac{50\times45}{5}\right) = \text{Rs. } 450$

Now, Rs. 50 is S.I. on Rs. 450 for (5/3) years

$\therefore$ Rate $= \left(\dfrac{100\times50}{450\times5/3}\right)\% = \mathbf{6\frac{2}{3}\%}$.

15. Ans. (a)

Solution S.I. on Rs. 1600 = T.D. on Rs. 1624

$\therefore$ Rs. 1600 is the P.W. of Rs. 1624

$\Rightarrow$ Rs. 24 is the S.I. on Rs. 1600 at 6%

$\therefore$ Time $= \dfrac{100\times24}{1600\times6}$ yrs $= \dfrac{1}{4}$ yr = **3 months.**

17. Ans. (c)

Solution Nominally due date was 21 May 1999

$\therefore$ Legally due date was 24 May 1999

To find unexpired no. of days are

Dec.	Jan.	Feb.	March	April	May
2	31	28	31	30	24

$\therefore$ Total no. of unexpired days = 146

$\therefore$ Unexpired time $= \dfrac{146}{365} = \dfrac{2}{3}$ year

$\therefore$ B.D. = S.I. on face value of bill

$$= 17850\times5\times\frac{2}{5}\times\frac{1}{100} = \text{Rs. } 357.$$

$$\text{T.D.} = \frac{A\times R\times T}{100+R\times T} = \frac{17850\times5\times\frac{2}{5}}{100+5\times\frac{2}{5}} = \frac{35700}{102}$$

$= \text{Rs. } 350$

$\therefore$ B.D. – T.D = Rs. (357 – 350) = **Rs. 7.**

19. Ans. (a)

Solution B.D. = 21 B.G.

Now, B.D. – T.D. = B.G.

$\therefore$ 21 B.G. – T.D. = B.G. $\therefore$ T.D. = 20 B.G.

Also, B.G. is simple interest on T.D.

$\therefore$ Rate $= \dfrac{\text{B.G.}\times100}{\text{T.D.}\times1} = \mathbf{5\%}$.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (a)	**6.** (c)	**8.** (b)	**10.** (b)	**12.** (d)	**14.** (c)	**16.** (a)	**18.** (a)	**20.** (b)

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Binary Number System

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INTRODUCTION

A number system is nothing more than a code. For each distinct quantity there is an assigned symbol. The most familiar number system is the decimal system which uses 10 digits, that is, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The main advantage of this system is its simplicity and long use. Most of the ancient societies used this system. Even in our everyday life we use this system and is sometimes being taken as the natural way to count. Since this system uses 10 digits it is called a system to base 10.

A *binary number system* is a code that uses only two basic symbols, that is, 0 and 1. This system is very useful in computers. Since, in this system, only two symbols are there, it can be used in electronic industry using 'on' and 'off" positions of a switch denoted by the two digits 0 and 1.

Decimal Number System

Decimal number system used 10 digits, 0 through 9, that is, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Binary Number System

Binary means two. The binary number system uses only two digits, 0 and 1.

Base or Radix

The *base* or *radix* of a number system is equal to the number of digits or symbols used in that number system. For example, decimal system uses 10 digits, so that base of decimal system (that is, decimal numbers) is 10. Binary numbers have base 2.

A subscript attached to a number indicates the base of the number. For example, 100_2 means binary 100. 100_{10} stands for decimal 100.

Weights

In any number to a given base, each digit, depending on its position in the number has a weight in powers of the base.

Illustration 1 In the number $(5342)_x$.

The weight of 2 is x^0

The weight of 4 is x^1

The weight of 3 is x^2

The weight of 5 is x^3.

The sum of all the digits multiplied by their respective weights is equal to the decimal equivalent of that number and gives the total amount represented by that number.

$$(5342)_x = (5x^3 + 3x^2 + 4x + 2x^0)_{10}$$

Illustration 2

5	7	0	3	4	Number to the base 10, that is, decimal number weights
10^4	10^3	10^2	10^1	10^0	

$\therefore 5 \times 10^4 + 7 \times 10^3 + 0 \times 10^2 + 3 \times 10 + 4 \times 10^0$

= Value represented or decimal equivalent

Illustration 3

1	1	0	0	1	Number to the base 2
2^4	2^3	2^2	2^1	2^0	that is, binary number weights

$\therefore 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 16 + 8 + 1 = 25$

= Decimal equivalent or value represented by 11001_2.

Decimal to Binary Conversion

Step 1: Divide the number by 2.

Step 2: Divide Quotient of Step 1 by 2

Continue the process till we get quotient = 0 and remainder as 1.

Then, the remainders from down upwards written from left to right give the binary number.

Illustration 4 Convert decimal 23 to binary.

Solution

		Remainders
2	23	
2	11	1
2	5	1
2	2	1
	1	0
	0	1

Reading the remainders upwards and writing from left to right we get the binary equivalent of decimal 23 as 10111.

That is, Binary 10111 is equivalent to decimal 23 or we can write $10111_2 = 23_{10}$.

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Binary to Decimal Conversion

Following steps are involved to convert a binary number to its decimal equivalent

Step 1: Write the binary number.

Step 2: Write the weights $2^0, 2^1, 2^2, 2^3, \ldots$ under the binary digits starting from extreme right.

Step 3: Cross out any weight under a zero, that is, weights under zeros in the binary number should be deleted.

Step 4: Add the remaining weights.

Illustration 5 Convert binary 1101 to its decimal equivalent.

Solution

1	1	0	1	Binary number
2^3	2^2	2^1	2^0	weights

The weight 2^1 is under 0 so it can be deleted. Sum of the remaining weights

$= 2^3 + 2^2 + 2^0 = 8 + 4 = 1 = 13.$

$\therefore$ Decimal equivalent of binary 1101 = 13

that is, $1101_2 = \mathbf{13_{10}}$.

Binary Addition

In binary number system there are only 2 digits, that is, 0 and 1. In decimal system we carry 1 for every 10 whereas in binary system we carry 1 for every 2. Hence, rules of addition are as under:

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 10$

Illustration 6 Add 1010 to 10100

Solution

$$\begin{array}{r} 10100 \\ +1010 \\ \hline 11110 \end{array}$$

Binary Subtraction

1. $0 - 0 = 0$
2. $1 - 0 = 1$
3. $1 - 1 = 0$
4. $10 - 1 = 1$
5. $0 - 1 = -1$

[Complement of a binary number is the exact reverse of the given number]

Complement of 0 = 1

Complement of 1 = 0

For subtraction of binary number the following method known as one's complement method is used.

Subtraction of a lower number from a higher number. To determine which binary number is lower and which is higher, it is advisable to find their decimal equivalents.

Step I: Make the number of digits equal in both the numbers.

Step II: Take the complement of the second number, that is, take the complement of the number to be subtracted.

Step III: Add the complement obtained in Step II to the first number. The carry over obtained from this addition indicates that the answer shall be positive.

Step IV: This carry over is taken out and added to the first digit on the right, that is, extreme right digit.

Step IV: The digits so obtained is the final answer.

Illustration 7 Subtract 11 from 101.

Solution Now, $101_2 = 4 + 1 = 5_{10}$, $11_2 = 2 + 1 = 3_{10}$. Clearly, 11 is smaller than 101. Making the number of digits equal, we write 11 as 011.

Complement of 011 = 100.

Adding 100 to 101, we get

$$\begin{array}{r} 101 \\ 100 \\ \hline (1)001 \end{array}$$ [Carry over is 1]

Taking out the carry over and adding to extreme right digit, we get

$$\begin{array}{r} 001 \\ 1 \\ \hline 010 \end{array}$$

$\therefore$ The answer is 010 **or 10.**

Subtraction of a higher number from a lower number.

Step I: Take the complement of the second number.

Step II: Add the complement obtained in Step I to the first number. In this case there is no carry over indicating that the answer is negative.

Step III: Recomplement the digits obtained after adding the complement of the second number to the first number.

Step IV: Put a negative sign before the result obtained in Step IV.

Illustration 8 Subtract 1110 from 1001.

Solution Now, $1110_2 = 8 + 4 + 2 = 14_{10}$;

$1001 = 8 + 2 = 10_{10}$.

Clearly, $1110_2 > 1001_2$.

Complement of 1110 = 0001.

Adding 0001 to 1001, we get

$$\begin{array}{r} 1001 \\ 0001 \\ \hline 1010 \end{array}$$ [There is no carry over]

Complement of 1010 = 0101.

$\therefore$ The answer is –0101 **or** $\mathbf{-101}$.

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Binary Multiplication

Rules: $1 \times 1 = 1$, $1 \times 0 = 0$.

Illustration 9 Multiply 1111 by 11.

Solution

$$\begin{array}{r} 1111 \\ \times\quad 11 \\ \hline 1111 \\ 1111 \\ \hline 101101 \end{array}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find the binary equivalent of decimal 117.
(a) 1010101 (b) 1110101
(c) 1111101 (d) None of these

2. Find the binary equivalent of decimal 52.
(a) 110100 (b) 111100
(c) Remainder (d) None of these

3. Find the decimal equivalent of binary 1110101.
(a) 110_{10} (b) 111_{10}
(c) 117_{10} (d) None of these

4. Find the binary equivalent of decimal 235.
(a) 1010111_2 (b) 1010111_2
(c) 11101011_2 (d) None of these

5. Find the binary equivalent of decimal 701.
(a) 1010111101_2 (b) 1011101101_2
(c) 1110111101_2 (d) None of these

6. Find the decimal equivalent of binary 101001.
(a) 31 (b) 41
(c) 51 (d) None of these

7. Find the decimal equivalent of binary 10000010011.
(a) 1043 (b) 1023
(c) 1033 (d) None of these

8. Find the decimal equivalent of binary 111011.
(a) 69 (b) 49
(c) 59 (d) None of these

9. Add 1001 to 0101
(a) 1111 (b) 1110
(c) 1010 (d) None of these

10. Add 11010 to 11100
(a) 110110 (b) 111110
(c) 110111 (d) None of these

11. $11111_2 + 10001_2 + 1011_2 =$
(a) 110111 (b) 111001
(c) 111011 (d) None of these

12. $11001_2 + 11011_2 + 11111_2 =$
(a) 1010011 (b) 111011
(c) 1110011 (d) None of these

13. $11_2 + 111_2 + 1111_2 + 11111_2 =$
(a) 101010 (b) 111000
(c) 101100 (d) None of these

14. $111_2 + 101_2 =$
(a) 1111 (b) 10111
(c) 1100 (d) None of these

15. $1000_2 + 1101_2 + 1111_2 =$
(a) 100100 (b) 111100
(c) 101010 (d) None of these

16. $111_2 + 101_2 + 011_2 =$
(a) 1011 (b) 1111
(c) 1101 (d) None of these

17. $111000_2 - 11001_2 =$
(a) 11111 (b) 10111
(c) 11011 (d) None of these

18. $10001_2 - 1111_2 =$
(a) 101 (b) 11
(c) 10 (d) None of these

19. $111101_2 - 10111_2 =$
(a) 111110 (b) 100110
(c) 101110 (d) None of these

20. $11111_2 - 10001_2 =$
(a) 1010 (b) 1111
(c) 1110 (d) None of these

21. $100001_2 - 11110_2 =$
(a) 11 (b) 111
(c) 10 (d) None of these

22. Multiply 1111 by 11
(a) 110101 (b) 101101
(c) 110100 (d) None of these

23. Multiply 101 by 11
(a) 1111 (b) 1011
(c) 1110 (d) None of these

24. Multiply 101101 by 1101
(a) 1111001001 (b) 1001101001
(c) 1001001001 (d) None of these

25. Multiply 11001 by 101
(a) 1111101 (b) 1110101
(c) 1011101 (d) None of these

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EXPLANATORY ANSWERS

1. Ans. (b)

Solution

		Remainder
2	117	
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
	1	1
	0	1

∴ The binary equivalent of decimal **117** is **1110101.**

2. Ans. (a)

Solution

		Remainder
2	52	
2	26	0
2	13	0
2	6	1
2	3	0
	1	1
	0	1

∴ The binary equivalent of decimal **52** is **110100.**

3. Ans. (c)

Solution

1	1	1	0	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Delete the weights 2^3 and 2^1.

Adding the remaining weights, we get

$2^6 + 2^5 + 2^4 + 2^2 + 2^0 = 64 + 32 + 16 + 4 + 1$

$= 117$

i.e., $1110101_2 = \mathbf{117_{10}}$.

4. Ans. (c)

Solution

		Remainder
2	235	
2	117	1
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
	1	1
	0	1

∴ $235_{10} = \mathbf{11101011_2}$

5. Ans. (a)

Solution

		Remainder
2	701	
2	350	1
2	175	0
2	87	1
2	43	1
2	21	1
2	10	1
2	5	0
2	2	1
	1	0
	0	1

∴ $(701)_{10} = \mathbf{1010111101_2}$.

6. Ans. (b)

Solution

1	0	1	0	0	1
2^5	2^4	2^3	2^2	2^1	2^0

Decimal equivalent

$= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 + 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 2^5 + 2^3 + 1 \times 2^0 = 32 = 32 + 8 + 1 = \mathbf{41.}$

7. Ans. (a)

Solution

1	0	0	0	0	0	1	0	0	1	1
2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Decimal equivalent

$= 1 \times 2^{10} + 0 \times 2^9 + 0 \times 2^8 + 0 \times 2^7$

$+ 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3$

$+ 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 2^{10} + 2^4 + 2^1 + 2^0 = \mathbf{1043.}$

8. Ans. (c)

Solution

1	1	1	0	1	1
2^5	2^4	2^3	2^2	2^1	2^0

Decimal equivalent

$= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 2^5 + 2^4 + 2^3 + 2^1 + 2^0 = \mathbf{59.}$

9. Ans. (b)

Solution

$$\begin{array}{r} 0101 \\ +1001 \\ \hline 1110 \end{array}$$

10. Ans. (a)

Solution

$$\begin{array}{r} 11100 \\ +\ 11010 \\ \hline 110110 \end{array}$$

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11. Ans. (c)

Solution
$$\begin{array}{r} 11111 \\ 10001 \\ \underline{\quad 1011} \\ 111011 \end{array}$$

12. Ans. (a)

Solution
$$\begin{array}{r} 11001 \\ 11011 \\ \underline{\quad 11111} \\ 1010011 \end{array}$$

Column 1 : $1+1+1=3; \frac{3}{2}$ = Quotient 1, Remainder 1

Column 2: $0+1+1+1$ (carry from first column)
$= 3; \frac{3}{2}$ = Quotient 1 and Remainder 1

Column 3: $0+0+1+1$ (carry from second column)
$= 2; \frac{2}{2}$ = Quotient 1 and Remainder 0

Column 4: $1+1+1$ (carry from column 3)
$= 4; \frac{4}{2}$ = Quotient 2 and Remainder 0

Column 5: $1+1+1+2$ (carry from column 4)
$= 5, 5_{10} = \mathbf{101_2}$.

Note: Quotient in any column is carry for next column.

13. Ans. (b)

Solution
$$\begin{array}{r} 11 \\ 111 \\ 1111 \\ \underline{11111} \\ 111000 \end{array}$$

14. Ans. (c)

Solution
$$\begin{array}{r} 111 \\ \underline{101} \\ 1100 \end{array}$$

15. Ans. (a)

Solution
$$\begin{array}{r} 1000 \\ 1101 \\ \underline{\quad 1111} \\ 100100 \end{array}$$

16. Ans. (b)

Solution
$$\begin{array}{r} 111 \\ 101 \\ \underline{\quad 011} \\ 1111 \end{array}$$

17. Ans. (a)

Solution $111000_2 = 32 + 16 + 8 = 56$
$11001_2 = 16 + 8 + 1 = 25$

Since $11001_2 < 111000_2$, so we are to subtract a lower number from a higher number.

Making the digits equal in the number to be subtracted, we get

011001.

Complement of 011001 = 100110.

Adding 100110 to 111 000, we get

$$\begin{array}{r} 111000 \\ \underline{\quad 100110} \\ [1]011110 \end{array}$$

[1 in the □ is the 1 carried over]

Adding 1 to the extreme right digit in 011 110, we get

$$\begin{array}{r} 011110 \\ \underline{\quad\quad 1} \\ 11111 \end{array}$$

$\therefore\ 111000_2 - 11001_2 = \mathbf{11111}$.

18. Ans. (c)

Solution $10001_2 = 2^4 + 1 = 17$,
$1111_2 = 2^3 + 2^2 + 2^1 + 1 = 15$.

Since $1111_2 < 10001_2$, we are to subtract a lower number from a higher number.

Making the digits equal in the number to be subtracted, we get

01111.

Complement of 01111 is 10000.

Adding 10000 to 10001, we get

$$\begin{array}{r} 10001 \\ \underline{\quad 10000} \\ [1]00001 \end{array}$$

Adding 1 to 1 in 00001, we get

$$\begin{array}{r} 00001 \\ \underline{\quad\quad 1} \\ 00010 \end{array}$$

$\therefore\ 10001_2 - 1111_2 = \mathbf{10}$.

19. Ans. (b)

Solution Complement of $010111_2 = 101000$

Now,
$$\begin{array}{r} 111101 \\ \underline{+101000} \\ [1]100101 \end{array}$$

Adding 1 to the extreme right digit in 100101, we get

$$\begin{array}{r} 100101 \\ \underline{+\quad\quad 1} \\ 100110 \end{array}$$

$\therefore\ 111101_2 - 10111_2 = \mathbf{100110}$.

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20. Ans. (c)

Solution Complement of $10001_2 = 01110$.

Now,
$$\begin{array}{r} 11111 \\ +\,01110 \\ \hline [1]01101 \end{array}$$

$$\begin{array}{r} 01101 \\ +\quad 1 \\ \hline 01110 \end{array}$$

$\therefore\ 11\,111_2 - 10001_2 =$ **1110.**

21. Ans. (a)

Solution Complement of $011110 = 100001$

Now,
$$\begin{array}{r} 100001 \\ +\,100001 \\ \hline [1]000010 \end{array}$$

$$\begin{array}{r} 000010 \\ +\quad 1 \\ \hline 000011 \end{array}$$

$\therefore\ 100001_2 - 11110_2 =$ **11.**

22. Ans. (b)

Solution
$$\begin{array}{r} 1111 \\ 11 \\ \hline 11\ 11 \\ 1\ 11\ 1 \\ \hline 10\ 1101 \end{array}$$

23. Ans. (a)

Solution
$$\begin{array}{r} 101 \\ 11 \\ \hline 101 \\ 101 \\ \hline 1111 \end{array}$$

24. Ans. (c)

Solution
$$\begin{array}{r} 101101 \\ 1101 \\ \hline 101101 \\ 000000 \\ 101101 \\ 101101 \\ \hline 1001001001 \end{array}$$

25. Ans. (a)

Solution
$$\begin{array}{r} 11001 \\ 101 \\ \hline 11001 \\ 00000 \\ 11001 \\ \hline 1111101 \end{array}$$

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Find the binary equivalent of decimal 1025.
(a) 1000000011 (b) 10000000001
(c) 1000100011 (d) None of these

2. Find the binary equivalent of decimal 381.
(a) 111111011 (b) 101101101
(c) 101111101 (d) None of these

3. Find the binary equivalent of decimal 610.
(a) 1110100010 (b) 1001100010
(c) 1011100010 (d) None of these

4. Find the decimal equivalent of binary 110011.
(a) 41 (b) 51
(c) 51 (d) None of these

5. Find the decimal equivalent of binary 1001111.
(a) 79 (b) 69
(c) 89 (d) None of these

6. $111101_2 + 11101_2 + 100011_2 =$
(a) 1011101 (b) 1111101
(c) 1001101 (d) None of these

7. $110110_2 + 101010_2 =$
(a) 11000011 (b) 1011000
(c) 1100000 (d) None of these

8. $1010_2 - 101_2 =$
(a) 101_2 (b) 111_2
(c) 100_2 (d) None of these

9. $1101_2 - 1001_2 =$
(a) 111_2 (b) 101_2
(c) 100_2 (d) None of these

10. Multiply 10111 by 1011
(a) 11100101 (b) 11111101
(c) 11101010 (d) None of these

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution

		Remainder
2	1025	
2	512	1
2	256	0
2	128	0
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	0
	0	1

$\therefore \quad 1025_{10} = \mathbf{10000000001_2}.$

3. Ans. (b)

Solution

		Remainder
2	610	
2	305	0
2	152	1
2	76	0
2	38	0
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$\therefore \quad 610_{10} = \mathbf{1001100010_2}.$

5. Ans. (a)

Solution

$$\begin{array}{cccccccc} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$\therefore \quad 1001111 = 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 64 + 8 + 4 + 2 + 1 = \mathbf{79}.$

7. Ans. (c)

Solution

$$\begin{array}{r} 110110 \\ 101010 \\ \hline 1100000 \end{array}$$

9. Ans. (c)

Solution Complement of 1001 = 0110.

$$\begin{array}{r} 1101 \\ +0110 \\ \hline [1]0011 \end{array}$$

Adding 1 to the first digit on the extreme right, we get

$$\begin{array}{r} 0011 \\ +\ 1 \\ \hline 0100 \end{array}$$

$\therefore \quad 1101_2 - 1001_2 = \mathbf{100_2}.$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (c)	**6.** (b)	**8.** (a)	**10.** (b)

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23 Series

INTRODUCTION

Now-a-days questions on series are asked in almost every competitive examination. These questions may involve numbers only, letters ($A, B,...$) only, or a combination of both.

SERIES

A series is a sequence of numbers. These numbers are called *terms* of the sequence. All the terms of the sequence are arranged according to a certain predefined rule. After carefully studying the given series and finding the specific pattern in which the terms are changing, it is possible to find out the next term of the series.

NUMBER SERIES

1. **Arithmetic Series** An arithmetic series is one in which the difference between any two consecutive terms is always the same and is called the common difference, that is, each successive number is obtained by adding (or subtracting) a fixed number to the previous number.

Illustration 1 Consider the series: 1, 3, 5, 7, 9, ...

Here, 2nd term – Ist term = 3rd term – 2nd term

= 4th term – 3rd term = ... = 2.

Hence, 1, 3, 5, 7, ... is an arithmetic series.

2. **Geometric Series** A geometric series is one in which the ratio of any two consecutive terms is always the same and is called the common ratio, that is, each successive number is obtained by multiplying (or dividing) a fixed number by the previous number.

Illustration 2 The series given below:

(a) 2, 4, 8, 16, 32, ... (b) 3, –6, 12, –24, 48,...

(c) $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{100}, \cdots$

(d) $\frac{1}{5}, \frac{1}{30}, \frac{1}{180}, \frac{1}{1080}, \cdots$

(e) $x, x^2, x^3, x^4, \ldots$ (where x is any fixed real number), are all geometric series. The ratio of any term in (a) to the preceding term is 2. The corresponding ratios in (b), (c), (d) and (e) are $-2, \frac{1}{3}, \frac{1}{6}$ and x, respectively.

3. **Series of Squares, Cubes and so on.** Simple powers of natural numbers (squares, cubes, etc.) or their combinations are sometimes used to form some series.

Illustrations 3

(a) 4, 9, 16, 25, 36, ...

Each term in this series is a perfect square. The square roots of the terms are 2, 3, 4, 5, 6, ... Clearly, the square roots of the terms of the given series are forming an arithmetic series with common difference 1. So, the next term of the series will be $(6 + 1)^2$, that is, 49.

(b) 1, 27, 125, 343, ...

Each term in this series is a perfect cube. The cube roots of its terms are 1, 3, 5, 7, ... clearly, the cube roots of the terms of the given series are forming an arithmetic series with common difference 2.

So, the next term of the series will be 9^3, that is, 729.

(c) $\frac{1}{8}, \frac{4}{27}, \frac{9}{64}, \frac{16}{125}, \cdots$

In the above series, the numerators are squares of natural number (n), while the denominators are cubes of $(n + 1)$.

So, the next term of the series will be $\frac{25}{216}$.

4. **Arithmetic Series of Second Order** We know that in an arithmetic series, the difference of any two consecutive terms is always the same. This is arithmetic series of first order.

A series in which the difference between successive terms themselves form an arithmetic series is called an arithmetic series of second order.

Illustration 4 Consider the series 1, 3, 7, 13, ...

The difference between successive terms of the above series are 2, 4, 6, ... which form an arithmetic series with common difference 2.

So, the next term of the series will be (13 + 8), that is, 21.

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5. **Arithmetic Series of Third Order** A series in which the difference between successive terms themselves form an arithmetic series of second order, is called an arithmetic series of third order.

Illustration 5 Consider the series : 2, 9, 17, 28, ...

The difference of successive terms of the above series are 7, 8, 11, 16, ...

The difference of successive terms of the above series are 1, 3, 5, ... which forms an arithmetic series with common difference 2.

So, the next term of the series will be (28 + 16), i.e., 44.

In this manner, we can construct arithmetic series of higher order.

6. **Arithmetico-Geometric Series** In this series each successive term is obtained by first adding a fixed number to the previous term and then multiplying it by another fixed number.

Illustration 6 The series: 1, 9, 33, 105, ... is an arithmetico-geometric series as each successive term is obtained by first adding 2 to the previous term and multiplying it by 3.

So, the next term of the series will be (105 + 2) × 3, that is, 321.

It is important to note that the differences of successive numbers in the above series are 8, 24, 72, ... which are forming a geometric series.

7. **Geometrico-Arithmetic Series** In this series each successive term is obtained by first multiplying (or dividing) the previous term by a fixed number and then adding (or subtracting) another fixed number.

Illustration 7 The series : 2, 5, 17, 65, ... is a geometrico-arithmetic series as each successive term is obtained by first multiplying the previous term by 4 and then subtracting 3 from it.

So, the next term of the series will be (65 × 4) –3, that is, 257.

Again, note that the differences of successive numbers in the above series are 3, 12, 48, ... which are forming a geometric series.

8. **Double Series** It consists of two series combined into a single series. The alternating terms of this series form an independent series.

Illustration 8 Consider the series:

1, 2, 4, 6, 7, 18, 10, 54, ...

Terms at odd places of the series: 1, 4, 7, 10, ... is an arithmetic series.

Terms at even places of the series: 2, 6, 18, 54, ... is a geometric series.

So, the next term of the series will be (10 + 3), that is, 13.

Finding the Wrong Term in a Series

In such questions, a number series is given of which all others except one are similar in some respect. The one term of the sequence does not follow the same pattern as is followed by the others. This one is the wrong term in the series. To find the wrong term in a given series we must study the given series carefully and find the pattern/rule in which the terms are changing. After that, we should find which of the terms is not changing according that pattern/rule. Thus, the wrong term is found.

Illustration 9 Find the wrong term in the given series: 5, 10, 17, 24, 37, 50, 65.

Solution The terms of the series are in the following order:

$2^2 + 1, 3^2 + 1, 4^2 + 1, 5^2 + 1, 6^2 + 1, 7^2 + 1, 8^2 + 1$

Clearly, fourth term of the series, that is, 24 should be replaced by 26 so that all the terms of the series follow a particular pattern. Thus, 24 is the wrong term in the given series.

Finding the Missing Term of the Series

In such questions, a number series is given in which a blank space or question mark is provided in place of any one term of the series. The term at the blank space follow the same pattern as followed by other terms. We are required to find the missing term to replace the blank space or question mark.

Illustration 10 Find the missing term in the given series: 49, 56, 64, 72, ?, 90, 100

Solution The terms of the series are in the following order

$7^2, 7^2 + 7, 8^2, 8^2 + 8, 9^2, 9^2 + 9, 10^2$

Clearly, fifth term in place of question mark will be 9^2, that is, 81.

SOME SPECIAL SERIES

1. **Series of Date or Time**

(a) Consider the series,

3 – 2 – 2004, 13 – 2 – 2004, 23 – 2 – 2004, 5 – 3 – 2004,

Here, each successive date differs by 10 days. Since 2004 is a leap year, 5 – 3 – 2004 should be replaced by 4 – 3 – 2004.

(b) Consider the series,

3.35, 5.00, 6.25, 7.40, 9.15, 10.40

Here, each successive time differs by 1 hr 25 min. Therefore, 7.40 should be replaced by 7.50.

2. **Numbers followed by their L.C.M. or H.C.F.**

(a) Consider the series,

1, 2, 3, 6, 4, 5, 6, 60, 5, 6, 7, ...?

1st part	2nd part	3rd part
1, 2, 3, 6	4, 5, 6, 60	5, 6, 7 ?

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Here, in each part fourth number is L.C.M. of first three numbers. Thus, the number in place of question mark will be 210 (L.C.M. of 5, 6, 7).

(b) Consider the series,

8, 4, 4, 7, 8, 1, 3, 9, 3, 2, 1, ?

1st part	2nd part	3rd part	4th part
8, 4, 4	7, 8, 1	3, 9, 3	2, 1 ?

Here, in each part third number is H.C.F. of first two numbers. Thus , the number in place of question mark will be 1 (H.C.F. of 2, 1,).

3. Numbers Followed by their Product

Consider the series,

1, 3, 3, 9, 27, 243, ?

Here, $1 \times 3 = 3$

$3 \times 3 = 9$

$3 \times 9 = 27$

$9 \times 27 = 243$

$27 \times 243 = 6561$

Thus, the number in place of question mark will be 27×243, that is, 6561.

4. By Use of Digit Sum

Consider the series,

11, 13, 17, 25, 32, ?

Here, $13 = 11 + (1 + 1)$

$17 = 13 + (1 + 3)$

$25 = 17 + (1 + 7)$

$32 = 25 + (2 + 5)$

That is, next number = previous number + digit sum of pervious number.

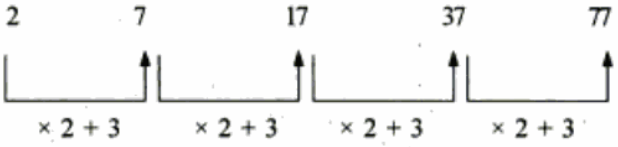
Thus, the number in place of question mark will be $32 + (3 + 2) = 37$.

Alpha – Numeric Series

Such series involve the use of both the letters of the alphabet as well as the numbers. It is a two-line series. One line is a number series while the other line is an alphabet series. The terms of both the series follow the same pattern/rule. One of these two series is completely known. We have to find the required number of the incomplete series.

Illustration 11 2, 7, 17, 37, 77,

3, *a*, *b*, *c*, *d*,



$\therefore$ $a = 3 \times 2 + 3 = 9$

$b = 9 \times 2 + 3 = 21$

$c = 21 \times 2 + 3 = 45$

$d = 45 \times 2 + 3 = 93$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Insert the missing number

5, 8, 12, 17, 23, –, 38

(a) 29 (b) 30

(c) 32 (d) 25

2. Insert the missing number

4, 9, 20, 43, 90, —

(a) 185 (b) 172

(c) 179 (d) 165

3. Insert the missing number

1, 1, 4, 8, 9, 27, 16,

(a) 25 (b) 36

(c) 125 (d) 64

4. Fill in the missing number

2, 6, 3, 4, 20, 5, 6, ?, 7

(a) 25 (b) 42

(c) 24 (d) 18

5. Fill in the missing number

1, 5, 11, 19, 29, ?

(a) 47 (b) 41

(c) 39 (d) 55

6. Fill in the missing number

3, 6, 21, 28, 55, 66, ?, 120

(a) 106 (b) 108

(c) 105 (d) 102

7. Fill in the missing number

5, 13, 25, 41, ?, 85, 113, 145

(a) 42 (b) 64

(c) 63 (d) 61

8. Fill in the missing number

4, 5, 9, 18, 34, ?

(a) 42 (b) 59

(c) 38 (d) None of these

9. Fill in the missing number

1799, 899, 449, ?

(a) 333 (b) 114

(c) 111 (d) 224

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10. Fill in the missing number
2, 1, 2, 4, 4, 5, 6, 8, 8, 10, 11, ?
(a) 12 (b) 8
(c) 10 (d) 9

11. Fill in the missing number
5, 11, 19, 29, ?
(a) 31 (b) 52
(c) 41 (d) 51

12. Fill in the missing number
0, 3, 12, 30, ?, 105, 168
(a) 61 (b) 62
(c) 60 (d) 63

13. Fill in the missing number
15, 20, 30, ?
(a) 45 (b) 40
(c) 48 (d) 50

14. Fill in the missing number
11, 10, ?, 100, 1001, 1000, 1001
(a) 110 (b) 111
(c) 101 (d) None of these

15. Fill in the missing number
99, 95, 86, 70, ?
(a) 45 (b) 62
(c) 65 (d) 55

16. Fill in the missing number
5, 18, 10, 12, 15, ?
(a) 4 (b) 8
(c) 6 (d) 10

17. Fill in the missing number
12, 8, 14, 6, 16, ?
(a) 18 (b) 4
(c) 32 (d) 10

18. Fill in the missing number
13, 21, 29, 34, 43, 92, 12, ?
(a) 84 (b) 31
(c) 92 (d) 12

19. Fill in the missing number
3, 15, 35, ... , 99, 143
(a) 68 (b) 58
(c) 63 (d) 45

20. Fill in the missing number
4, 7, 11, 18, 29, 47, ?, 123, 199
(a) 71 (b) 82
(c) 86 (d) 76

In the following number series a wrong number is given. Find out the wrong number.

21. 455, 445, 465, 435, 485, 415, 475
(a) 475 (b) 465
(c) 435 (d) 455
(e) 445

22. 3, 10, 24, 54, 108, 220, 444
(a) 108 (b) 10
(c) 24 (d) 54
(e) 220

23. 8, 18, 40, 86, 178, 370, 752
(a) 86 (b) 178
(c) 40 (d) 370
(e) 752

24. 1, 2, 6, 21, 84, 445, 2676
(a) 6 (b) 21
(c) 2676 (d) 84
(e) 445

25. 1, 16, 9, 64, 25, 216, 49
(a) 64 (b) 216
(c) 16 (d) 49
(e) 9

26. 864, 420, 200, 96, 40, 16, 6
(a) 864 (b) 200
(c) 96 (d) 16
(e) 40

27. 9, 13, 21, 37, 69, 132, 261
(a) 9 (b) 13
(c) 261 (d) 261
(e) 132

28. 2, 5, 18, 19, 24, 29, 34
(a) 18 (b) 2
(c) 19 (d) 29
(e) 34

29. 1, 5, 11, 19, 29, 55
(a) 29 (b) 55
(c) 11 (d) 5

30. 2, 4, 4, 16, 8, 256, 64
(a) 8 (b) 16
(c) 64 (d) 256
(e) 4

Directions (31–40): In each of the questions below a number series has been given followed by five alternatives. One term of the given number series is wrong. Find out that worng term and spot out a number from the alternatives which will replace the wrong term of the series.

31. 2, 9, 28, 65, 126, 216, 344
(a) 38 (b) 217
(c) 356 (d) 66

32. 58, 57, 54, 50, 42, 33, 22
(a) 48 (b) 49
(c) 52 (d) 30
(e) 18

33. 0, 9, 64, 169, 576, 1225
(a) 225 (b) 360
(c) 444 (d) 556
(e) 630

34. 1, 3, 7, 19, 42, 89, 184
(a) 8 (b) 9
(c) 24 (d) 30
(e) 182

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35. 169, 121, 80, 49, 25, 9, 1
(a) 100 (b) 81
(c) 36 (d) 16
(e) 4

36. 7, 9, 17, 42, 91, 172, 293
(a) 16 (b) 25
(c) 36 (d) 8
(e) 49

37. 8, 14, 26, 48, 98, 194, 386
(a) 60 (b) 50
(c) 72 (d) 96
(e) 108

38. 95, 86, 73, 62, 47, 30, 11
(a) 90 (b) 75
(c) 64 (d) 35
(e) 15

39. 7, 14, 56, 168, 336, 1344, 2688, 8064
(a) 3032 (b) 5032
(c) 4032 (d) 2680
(e) 332

40. 11, 15, 17, 19, 23, 25
(a) 1 (b) 18
(c) 21 (d) 10
(e) 13

Direction (41–49): In each of the following questions a number series is given. After the series a number is given followed by (A), (B), (C), (D) and (E). Complete the series starting with the number given following the sequence of the given series. Then, answer the question given below each:

41. 1 9 65 393
2 (A) (B) (C) (D) (E)
Which of the following numbers will come in place of (C)?
(a) 490 (b) 729
(c) 854 (d) 734
(e) None of these

42. 616, 496, 397, 317, 254,
838 (A), (B), (C), (D), (E),
Which of the following numbers will come in place of (E)?
(a) 428 (b) 608
(c) 426 (d) 529
(e) 712

43. 434, 353, 417, 368, 404, 379,
108 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (E)?
(a) 27 (b) 91
(c) 42 (d) 53
(e) 78

44. 4, 16, 48, 120, 272,
124, (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (C)?
(a) 4424 (b) 256
(c) 528 (d) 1080
(e) 2192

45. 1, 9, 65, 393,
2, (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (C)?
(a) 490 (b) 729
(c) 854 (d) 734
(e) None of these

46. 848, 420, 206, 99, 45.5,
664, (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (D)?
(a) 32 (b) 34
(c) 160 (d) 328
(e) 13

47. 8, 8, 12, 24,
36 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (E)?
(a) 108 (b) 36
(c) 810 (d) 54
(e) None these

48. 6, 14, 35, 111, 449,
3 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (B)?
(a) 93 (b) 377
(c) 1892 (c) 11
(e) 29

49. 8, 49, 288, 1435, 5736,
5 (A), (B), (C), (D), (E)
Which of the following numbers will come in place of (E)?
(a) 162 (b) 805
(c) 9645 (d) 3216
(e) 28

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EXPLANATORY ANSWERS

1. Ans. (b)

Solution $8-5=3, 12-8=4, 17-12=5, 23-17=6$

$\therefore \quad ?-23=7$

i.e. $\quad ?=23+7=30$

With this, $38-30=\mathbf{8}$.

2. Ans. (a)

Solution The pattern is

$9=2\times4+1$

$20=2\times9+2$

$43=2\times20+3$

$90=2\times43+4$

$\therefore$? should be $2\times90+5=\mathbf{185}$.

3. Ans. (d)

Solution The first alternate series is 1, 4, 9, 16

i.e. $1^2, 2^2, 3^2, 4^2$ and the second one is

1, 8, 27, ?

i.e. $1^3, 2^3, 3^3, 4^3$.

4. Ans. (b)

Solution $2\times3=6$

$4\times5=20$

$6\times7=\mathbf{42}$.

5. Ans. (b)

Solution $5-1=4, 11-5=6, 19-11=8$

$29-19=10$

$\therefore \quad ?-29=12$

$\therefore \quad ?=\mathbf{41}$.

6. Ans. (c)

Solution Difference between successive terms are

3, 15, 7, 27, 11, ? – 66, 120 – ?

Here, odd places terms form a series

3, 7, 11, 120 – ?

which is an A.P. with common difference 4 and even places terms form a series

15, 27, ? – 66

$\therefore \quad 120-?=11+4=15$

$\therefore \quad ?=120-15=\mathbf{105}$.

7. Ans. (d)

Solution $13=5+4\times2, 25=13+4\times3, 41=25+4\times4$

$?=41+4\times5, 85=?+4\times6$

$113=85+4\times7, 145=113+4\times8$

$\therefore \quad ?=41\times4\times5=61$. With this choice

$85=?+4\times6$

$=61+24$, which follows the pattern.

8. Ans. (b)

Solution $5=4+1^2, 9=5+2^2, 18=9+3^2,$

$34=18+4^2,$

$\therefore \quad ?=34+5^2=\mathbf{59}$.

9. Ans. (d)

Solution $1799-899=900$

$899-449=450\left(=\frac{1}{2}\times900\right)$

$\therefore \quad 449-?=\frac{1}{2}\times450=225$

$\therefore \quad ?=449-225=\mathbf{224}$.

10. Ans. (c)

Solution 1st, 4th, 7th, 10th, and 13th terms are:

2, 4, 6, 8, ?

which is an A.P. with common difference 2

$\therefore \quad ?=8+2=\mathbf{10}$.

11. Ans. (c)

Solution $11-5=6, 19-11=8, 29-19=10$

$\therefore \quad ?-29=12$

$\therefore \quad ?=12+29=\mathbf{41}$.

12. Ans. (c)

Solution 0, 3, 12, 30, ?, 105, 168

3, 9, 18, ? – 30, 105 – ?, 63

6, 9, ? – 48, 135 – 2?, ? – 42

If we take ? – 48 = 12, then ? = 60, with this choice $135-2?=135-120=15$

and, $\quad ?-42=18$

$\therefore$ 3rd row becomes

6, 9, 12, 15, 18,

which is an A.P. of common difference 3

$\therefore$? should be **60**.

13. Ans. (a)

Solution $20=15+5\times1;$

$30=20+5\times2$

$\therefore$? should be $30+5\times3=\mathbf{45}$.

14. Ans. (c)

Solution 1st, 3rd, 5th, 7th, terms are I : 11, ?, 1001 10001 and 2nd, 4th, 6th, terms are

II: 10, 100, 1000

In I:

(i) at first place there is no zero between 1's

(ii) at 3rd place there are two zeros between 1's

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(iii) at 4th place there are three zeros between 1's

∴ According to this pattern there should be one zero between 1's at 2nd place

∴ ? be **101.**

15. Ans. (a)

Solution

$99 - 95 = 4 = 2^2$, $95 - 86 = 9 = 3^2$

$86 - 70 = 16 = 4^2$

∴ $70 - ? = 5^2$

∴ $70 - 25 = ?$

∴ ? **= 45.**

16. Ans. (c)

Solution Numbers at even places form series

I : 18, 12, ?

and numbers at odd places form series

II : 5, 10, 15

Keeping the pattern in I

? should be $12 - 6 = \mathbf{6}$.

17. Ans. (b)

Solution In the first alternate series, namely, 12, 14, 16 each term is increased by 2 and in the second, namely, 8, 6 each term is decreased by 2, missing figure is of 2nd series and hence should by $6 - 2 = \mathbf{4}$.

18. Ans. (b)

Solution Making pairs taking first number from right and first from left, 2nd number from right and 2nd number from left and so on.

(13, ?); (21, 12); (29, 92); (34, 43)

In each pair numbers have their digits reversed keeping this pattern, ? should be **31.**

19. Ans. (c)

Solution

$3 = 2^2 - 1$; $15 = 4^2 - 1$; $35 = 6^2 - 1$

$99 = 10^2 - 1$; $143 = 12^2 - 1$

∴ Missing figure should be $8^2 - 1 = \mathbf{63}$.

20. Ans. (d)

Solution The pattern is

$4 + 7 = 11$

$18 + 29 = 47$

$? + 123 = 199$

∴ $? = 199 - 123 = \mathbf{76}$.

21. Ans. (a)

Solution Series formed by numbers at odd and even places respectively are:

455, 465, 485, 475 ...(I)

and, 445, 435, 415 ...(II)

The difference between successive terms of I are:

10, 20, –10

and of II are

–10, – 20

–10 in (I) is abnormal. It should be 30

∴ 475 is wrong and should be replaced by $485 + 30 = \mathbf{515}$.

22. Ans. (d)

Solution

$10 = 2 \times 3 + 4$

$24 = 2 \times 10 + 4$

$54 = 2 \times 24 + 6$

$108 = 2 \times 54 + 0$

$220 = 2 \times 108 + 4$

$444 = 2 \times 220 + 4$

Pattern is disturbed at 3rd and 4th stages.

∴ 54 is wrong and should be replaced by $2 \times 24 + 4 = 52$. With this choice at 4th stage, $108 = 2 \times 52 + 4$ which follows pattern.

23. Ans. (b)

Solution

$18 = 8 \times 2 + 2 \times 1$; $40 = 18 \times 2 + 2 \times 2$

$86 = 40 \times 2 + 2 \times 3$; $178 \neq 86 \times 2 + 2 \times 4$

$370 \neq 178 \times 2 + 2 \times 5$; $752 = 370 \times 2 + 2 \times 6$

∴ 178 is wrong and should be replaced by

$2 \times 86 + 2 \times 4 = 180$

With this choice 5th place

$2 \times 180 + 2 \times 5 = 370$

which is according to the pattern.

24. Ans. (d)

Solution

$2676 = 6 \times 445 + 6$; $445 \neq 5 \times 84 + 5$

$84 \neq 4 \times 21 + 4$; $21 = 3 \times 6 + 3$

$6 = 2 \times 2 + 2$; $2 = 1 \times 1 + 1$

Obviously, 84 is wrong and should be replaced by $4 \times 21 + 4 = 88$

With this, $445 = \mathbf{5 \times 88 + 5}$.

25. Ans. (b)

Solution Numbers at even places are

$16 = 4^2$; $64 = 8^2$; $216 \neq 12^2$

and numbers at odd places are

$1 = 1^2$; $9 = 3^2$; $25 = 5^2$; $49 = 7^2$

∴ 216 is wrong.

26. Ans. (c)

Solution

$864 = 2 \times 420 + 4 \times 6$; $420 = 2 \times 200 + 4 \times 5$

$200 \neq 2 \times 96 + 4 \times 4$; $96 \neq 2 \times 40 + 4 \times 3$

$40 = 2 \times 16 + 4 \times 2$; $16 = 2 \times 6 + 4 \times 1$

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∴ 96 is wrong and should be replaced by $2 \times 40 + 4 \times 3 = 92$. With this choice at 3rd stage

$200 = 2 \times 92 + 4 \times 4$.

27. Ans. (e)

Solution $13 - 9 = 4$; $21 - 13 = 8$; $37 - 21 = 16$

$69 - 37 = 32$; $132 - 69 = 63$; $261 - 132 = 129$

Pattern is disturbed at last two stages.

∴ 132 is wrong and should be replaced by $69 + 64 = 133$. With this choice at last stage

$261 - 133 = 128$.

28. Ans (b)

Solution Series formed by numbers at odd places and even places respectively are

I: 2, 18, 24, 34

II: 5, 19, 29

Successive terms in I and II have difference

16, 6, 10

and 14, 10, respectively.

Abnormality is at 16. It should be replaced by 2

∴ 2 is wrong and should be replaced by **16.**

29. Ans. (b)

Solution

$5 - 1 = 4$; $11 - 5 = 6$; $19 - 11 = 8$

$29 - 19 = 10$; $55 - 29 = 26$

Pattern gets disturbed at last stage

∴ 55 is wrong

It should be $29 + 12 = 41$.

30. Ans. (c)

Solution Nos. at odd and even places form respective series

I: 2, 4, 8, 64

i.e. 2^1, 2^2, 2^3, 2^6

and, II: 4, 16, 256

i.e. 2^2, 2^4, 2^8

Obviously, $2^6 = 64$ is wrong and should be replaced by $2^4 = 16$.

31. Ans. (b)

Solution If 216 is replaced by 217 the terms of the series will get arranged in the order of

$1 \times (1)^2 + 1$, $2 \times (2)^2 + 1$, $3 \times (3)^2 + 1$, $4 \times (4)^2 + 1$ .... and so on.

Therefore, alternative (b) is the correct answer.

32. Ans. (b)

Solution A careful scrutiny of the series reveals that if 50 is replaced by 49, then difference between successive terms will in the order of 1, 3, 5, 7, 9, 11. Therefore, alternative (b) is the correct answer.

33. Ans. (a)

Solution If 169 is replaced by 225 the terms will get arranged in a particular series, that is, $(1^2 - 1)^2$, $(2^2 - 1)^2$, $(3^2 - 1)^2$, $(4^2 - 1)^2$, $(5^2 - 1)^2$ and $(6^2 - 1)^2$.

Therefore, alternative (a) is the correct answer.

34. Ans. (a)

Solution If 7 is replaced by 8 these terms of the series will get arranged in order of

$1 \times 2 + 1$, $3 \times 2 + 2$, $8 \times 2 + 3$,

$19 \times 2 + 4$, $42 \times 2 + 5$, $89 \times 2 + 6$

Therefore, alternative (a) is the correct answer.

35. Ans. (b)

Solution A careful scrutiny of the series reveals that if 80 is replaced by 81 then the series will be arranged in the order of

$13^2, 11^2, 9^2, 7^2, 5^2, 3^2, 1^2$. Therefore, (b) is the correct alternative.

36. Ans. (d)

Solution It is obvious from the given series that if 9 is replaced by 8 then difference between successive terms will be in the order of 1^2, 3^2, 5^2, 7^2, 9^2 and 11^2

Therefore, alternative (d) is the correct answer.

37. Ans. (b)

Solution If 48 is replaced by 50 each term of the series is obtained by subtracting 2 from twice its previous term. Therefore, alternative (b) is the correct answer.

38. Ans. (b)

Solution If 73 is replaced by 75, difference between successive terms of the series will be in the order of 9, 11, 13, 15, 17 and 19.

Therefore, alternative (b) is the correct alternative.

39. Ans. (c)

Solution A careful scrutiny of the given series reveals that second term is two times the first, third term is four times the second and fourth term is three times the third. The same pattern is being followed by the remaining terms of the series. Therefore, 2688 should be replaced by 4032. Hence, alternative (c) is the correct alternative.

40. Ans. (e)

Solution If 15 is replaced by 13 the difference between successive terms will be in the order of 2, 4, 2, 4, ... and so on. Therefore, alternative (e) is the correct alternative.

41. Ans. (b)

Solution The pattern followed by the numbers of the given series is:

$9 = 8 \times 1 + 1$; $65 = 7 \times 9 + 2$

$393 = 6 \times 65 + 3$

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$\therefore$ (A) $= 8 \times 2 + 1 = 17$
(B) $= 7 \times 17 + 2 = 121$
(C) $= 6 \times 121 + 3 = 729$
(D) $= 5 \times 729 + 4 = 3649$
(E) $= 4 \times 3649 + 5 =$ **14601.**

42. Ans. (a)

Solution The pattern is
$616 - 496 = 120 = 12 \times 10$
$469 - 397 = 99 = 11 \times 9$
$397 - 317 = 80 = 10 \times 8$
$317 - 254 = 63 = 9 \times 7$
$\therefore$ (A) $= 838 - 120 = 718$
(B) $= 718 - 99 = 619$
(C) $= 619 - 80 = 539$
(D) $= 539 - 63 = 476$
(E) $= 476 - 48\ (= 8 \times 6) =$ **428.**

43. Ans. (d)

Solution
$434 - 353 = 9^2$
$353 - 417 = 8^2$
$417 - 368 = 7^2$
$368 - 404 = 6^2$
$404 - 379 = 5^2$
$\therefore$ $108 - (A) = 9^2 \Rightarrow (A) = 108 - 81 = 27$
$27 - (B) = -8^2 \Rightarrow (B) = 27 + 64 = 91$
$91 - (C) = 7^2 \Rightarrow (C) = 91 - 49 = 42$
$42 - (D) = -6^2 \Rightarrow (D) = 42 + 36 = 78$
$78 - (E) = 5^2 \Rightarrow (E) = 78 - 25 =$ **53.**

44. Ans. (d)

Solution The rule followed is:
$272 = 2 \times 120 + 8 \times 4$
$120 = 2 \times 48 + 8 \times 3$
$48 = 2 \times 16 + 8 \times 2$
$16 = 2 \times 4 + 8 \times 1$
$\therefore$ (A) $= 2 \times 124 + 8 \times 1 = 256$
(B) $= 2 \times 256 + 8 \times 2 = 528$
(C) $= 2 \times 528 + 8 \times 3 = 1080$
(D) $= 2 \times 1080 + 8 \times 4 = 2192$
(E) $= 2 \times 2192 + 8 \times 5 =$ **4424.**

45. Ans. (b)

Solution The pattern followed by the numbers of given series is:
$9 = 8 \times 1 + 1$; $65 = 7 \times 9 + 2$; $393 = 6 \times 65 + 3$
$\therefore$ (A) $= 8 \times 2 + 1 = 17$
(B) $= 7 \times 17 + 2 = 121$
(C) $= 6 \times 121 + 3 = 729$
(D) $= 5 \times 729 + 4 = 3649$
(E) $= 4 \times 3649 + 5 =$ **14601.**

46. Ans. (b)

Solution
$848 = 2 \times 420 + 8$ $\therefore$ (A) $= \dfrac{664-8}{2} = 328$
$420 = 2 \times 206 + 8$ (B) $= \dfrac{328-8}{2} = 160$
$206 = 2 \times 99 + 8$ (C) $= \dfrac{160-8}{2} = 76$
$99 = 2 \times 45.5 + 8$ (D) $= \dfrac{76-8}{2} = 34$
(E) $= \dfrac{34-8}{2} =$ **13.**

47. Ans. (c)

Solution The rule is
$8 = \dfrac{8}{2} \times 2$ $\therefore$ (A) $= \dfrac{36}{2} \times 2 = 36$
$12 = \dfrac{8}{2} \times 3$ (B) $= \dfrac{36}{2} \times 3 = 54$
$24 = \dfrac{12}{2} \times 4$ (C) $= \dfrac{54}{2} \times 4 = 108$
(D) $= \dfrac{108}{2} \times 5 = 270$
(E) $= \dfrac{270}{2} \times 6 =$ **810.**

48. Ans. (e)

Solution
$449 = 4 \times 111 + 5$ $\therefore$ (A) $= 1 \times 3 + 8 = 11$
$111 = 3 \times 35 + 6$ (B) $= 2 \times 11 + 7 = 29$
$35 = 2 \times 14 + 7$ (C) $= 3 \times 29 + 6 = 93$
$14 = 1 \times 6 + 8$ (C) $= 3 \times 29 + 6 = 93$
(E) $= 5 \times 377 + 4 =$ **1889.**

49. Ans. (c)

Solution The pattern is
$5736 = 4 \times 1435 - 4$
$1435 = 5 \times 288 - 5$
$288 = 6 \times 49 - 6$
$49 = 7 \times 8 - 7$
$\therefore$ (A) $= 7 \times 5 - 7 = 28$
(B) $= 6 \times 28 - 6 = 162$
(C) $= 5 \times 162 - 5 = 805$
(D) $= 4 \times 805 - 4 = 3216$
(E) $= 3 \times 3216 - 3 =$ **9645.**

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QUESTIONS FROM COMPETITIVE EXAMS

Directions (Qs. 1 to 2): In each of these questions, one term in the given number series is wrong. Find out the wrong term.

1. 8, 14, 26, 48, 98, 194, 386
(a) 14 (b) 48
(c) 98 (d) 194
[Based on MAT 2004]

2. 11, 5, 20, 12, 40, 26, 74, 54
(a) 5 (b) 20
(c) 40 (d) 26
[Based on MAT 2004]

Directions (Qs 3 to 5): In each of the following number series, two terms have been put within brackets. Mark your answer as

3. 4, 6, 10, (12), 16, (14), 22.
(a) If both the bracketed terms are right
(b) If the first bracketed term is right and the second is wrong
(c) If the first bracketed term is wrong and the second is right
(d) If both the dracketed terms are wrong.
[Based on MAT 2004]

4. 3, 10, 29, (66), (127), 218. **[Based on MAT 2004]**

5. (2), 5, (12), 25, 41, 61. **[Based on MAT 2004]**

Directions (Qs. 6 to 7): What number will replace the question mark (?) in these questions:

6.

3	8	10	2	?	1
6	56	90	2	20	0

(a) 0 (b) 3
(c) 5 (d) 7
[Based on MAT 2004]

7.

1	2	3	2	10	12
2	5	12	10	16	13
1	2	1	?	10	24

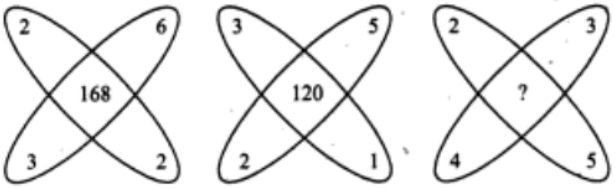
(a) 5 (b) 11
(c) 13 (d) 8
[Based on MAT 2004]

8. The missing number in the series 8, 24, 12, 36, 18, 54, — is
(a) 27 (b) 108
(c) 68 (d) 72
[Based on MAT 2002]

9. What is the next number in the series given below?
2, 5, 9, 14, 20
(a) 25 (b) 26
(c) 27 (d) 28
[Based on MAT 2000]

10. Complete the series:
10, 18, 34, ??, 130, 258
(a) 32 (b) 60
(c) 68 (d) 66
[Based on MAT 2000]

11.



(a) 240 (b) 195
(c) 84 (d) None of these.
[Based on MAT 2001]

12. 6, 15, 36, 75, ?
(a) 231 (b) 138
(c) 214 (d) None of these.
[Based on MAT 2001]

13. 15, 45, ?, 405
(a) 90 (b) 75
(c) 135 (d) 51
[Based on MAT 2001]

14. Find the next number:
2743, 2198, 1729, 1332,?
(a) 1015 (b) 1001
(c) 999 (d) 317
[Based on FMS (Delhi) 2004]

15. Find the next number:
–1, 3, –15, 105, –945, 10395,?
(a) 145535 (b) 135135
(c) –145535 (d) –135135
[Based on FMS (Delhi) 2004]

16. The first four numbers below form a series. Insert the missing number at the end of the series from the given options:
28, 33, 31, 36,...
(a) 32 (b) 34
(c) 38 (d) 40
[Based on Narsee Manjee Inst. of Man. Studeies, 2003]

17. Find the next number:
999, 730, 511, 344, 215, ?
(a) 103 (b) 104
(c) 125 (d) 126
[Based on FMS (Delhi) 2003]

18. Find the next number:
1, 3, 15, 105, 945,?
(a) 9995 (b) 10395
(c) 956 (d) 19395
[Based on FMS (Delhi) 2003]

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Directions (Qs. 19 to 23): In each of these questions, a number series is given. After the series, a number is given followed by (a), (b), (c), (d) and (e). You have to complete the series starting with the number given following the sequence of the given series to answer these questions:

19. 3 10 26 83 336 1683
7 (a) (b) (c) (d) (e)
What will come in place of (b)?
(a) 32 (b) 30
(c) 34 (d) 36
(e) None of these **[Based on IRMA, 2002]**

20. 4 3 4 7 1.5 23.5
6 (a) (b) (c) (d) (e)
What will come in place of (c) ?
(a) 8.5 (b) 9.5
(c) 8 (d) 9
(e) None of these **[Based on IRMA, 2002]**

21. 3 4 16 75 364 1945
1 (a) (b) (c) (d) (e)
What will come in place of (c)?
(a) 72 (2) 63
(c) 66 (d) 69
(e) None of these **[Based on IRMA, 2002]**

22. ABMS ciou ?
(a) BHNT bhnt (b) ABCD ascd
(c) BHNT djpv (d) bhnt DJPV **[Based on MAT, 2008]**

23. 48, 24, 72, 36, 108, ?
(a) 115 (b) 110
(c) 121 (d) 54 **[Based on MAT, 2008]**

24. K-11, M-13, P-16, T-20, ?
(a) V-22 (b) U-21
(c) Y-25 (d) W-25 **[Based on MAT, 2008]**

EXPLANATORY ANSWERS

1. Ans. (b)

Solution The sequence in the given series is +6, +12, +24, +48, +96, +192. Hence the number 48 must be replaced by **50.**

2. Ans. (c)

Solution Series 1 : 11 20 38 74.
The sequence in this series is × 2 – 2.
Series II : 5 12 26 24
The sequence in this series is × 2 + 2. Hence the number 40 must be replaced by **38.**

3. Ans. (b)

Solution The sequence in the given series is +2, +4, +2, +4, +2, +4. Therefore, 14 must be replaced by **18.**

4. Ans. (b)

Solution The terms of the given series are $1^3 + 2$, $2^3 + 2$, $3^3 + 2$, $4^3 + 2$, $5^3 + 2$, $6^3 + 2$.

5. Ans. (d)

Solution Here 2 must be replaced by 1 and 12 must be replaced by 3.
Then the sequence in the series will be +4, +8, +12, +16, +20.

6. Ans. (c)

Solution Sum of the numbers in each column is a perfect square.

7. Ans. (c)

Solution Sum of the numbers in each column is a perfect square.

8. Ans. (a)

Solution The sequence in the given series is × 2 + 2.

9. Ans. (c)

Solution The sequence in the given seies is + 3, + 4, + 5, + 6, + 7.

10. Ans. (d)

Solution The sequence in the given series is + 8, + 16, + 32, + 64, + 128.

11. Ans. (b)

Solution $(6 + 2 + 3 + 2)^2 - 1 = 169 - 1 = 168$
$(5 + 1 + 2 + 3)^2 - 1 = 121 - 1 = 120$
$(3 + 5 + 4 + 2)^2 - 1 = 196 - 1 = \mathbf{195}$

12. Ans. (b)

Solution The sequence in the given series is + 9, + 21, + 39, + 63. Therefore, the next number should be 138.

13. Ans. (c)

Solution The squence in the given series is × 3.

14. Ans. (c)

Solution The sequence in the given series is –545, –469, –397, –333, because in the series 545, 469, 397, 333 the sequence is –76, –72, –64.

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15. Ans. (d)

Solution The sequence in the given series is
$\times (-3), \times (-5) \times (-7), \times (-9), \times (-11), \times (-13)$.

16. Ans. (a)

Solution The sequence in the given series is +5, –2, +5, –2.

17. Ans. (d)

Solution The terms of the given seriers are
$10^3 - 1, 9^3 + 1, 8^3 - 1, 7^3 + 1, 6^3 - 1, 5^3 + 1$.

18. Ans. (b)

Solution The sequence in the given series is **×3, ×5, ×7, ×9, ×11.**

19. Ans. (c)

Solution The sequence in the given series is $\times 1 + 7$, $\times 2 + 6$, $\times 3 + 5$, $\times 4 + 4$, $\times 5 + 3$.

$\therefore$ The new series formed must be **7, 14, 34, 107,...**

20. Ans. (a)

Solution The sequence in the given series is $\times \frac{1}{2} + 1$, $\times 1 + 1$, $\times 1.5 + 1$, $\times 2 + 1$, $\times 2.5 + 1$

The new series formed must be **6, 4, 5, 8.5, 18, 46.**

21. Ans. (b)

Solution The sequence in the given series is $\times 1 + 1^3$, $\times 2 + 2^3$, $\times 3 + 3^3$, $\times 4 + 4^3$, $\times 5 + 5^3$.

$\therefore$ The new series must be **1, 2, 12, 63, 316,...**

22. Ans. (d)

Solution 48, 24, 72, 36, 108, ?
$\div 2, \times 3$ (series)
$48, 48 \div 2 = 24, 24 \times 3 = 72, 72 \div 3 = 36, 36 \times 3 = 108$
$108 \div 2 = 54$

23. Ans. (c)

Solution K – 11, M – 13, P – 16, T – 20, ?
K (<) M (NO) P (Q R S) T (U V W X) y
11 (12) 13 (14, 15) 16 (17, 18, 19)
20 (21, 22, 23, 24, 25)
y – 25

24. Ans. (a)

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

Solution $134 = 1 \times 3 \times 4 = 12$
$345 = 3 \times 4 \times 5 = 60$
$234 = 2 \times 3 \times 4 =$ **24.**

1. Complete the series
2, 5, 9, 19, 37, ...
(a) 76 (b) 74
(c) 75 (d) None of these

2. Find out the wrong number in the series
5, 15, 30, 135, 405, 1215, 3645
(a) 3645 (b) 1215
(c) 405 (d) 30
(e) 15

3. Find out the wrong number in the series
190, 166, 145, 128, 112, 100, 91
(a) 100 (b) 166
(c) 145 (d) 128
(e) 112

4. Find out the wrong number in the series
125, 106, 88, 76, 65, 58, 53
(a) 125 (b) 106
(c) 88 (d) 76
(e) 65

5. Insert the missing number
5, 10, 13, 26, 29, 58, 61, (?)
(a) 122 (b) 64
(c) 125 (d) 128

6. Insert the missing number
8, 24, 12, 36, 18, 54, (?)
(a) 27 (b) 108
(c) 68 (d) 72

7. Insert the missing number
7, 26, 124, 215, 342, (?)
(a) 481 (b) 511
(c) 391 (d) 421

8.

–4	–2	1	8	31
–1	*a*	*b*	*c*	*d*

Find the value of *b*.
(a) 19 (b) 17
(c) 22 (d) 21

9.

27	44	71	108	155	
34	*a*	*b*	*c*	*d*	*e*

What value should replace (e)?
(a) 319 (b) 217
(c) 219 (e) 419

10. 108 52 24 10 3

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Test I **[Time: 10 min]**

11. Find out the wrong number in the series
3, 7, 15, 27, 63, 127, 255
(a) 7 (c) 15
(d) 27 (d) 63
(e) 127

12. Find out the wrong number in the series
19, 26, 33, 46, 59, 74, 91
(a) 26 (b) 33
(c) 46 (d) 59
(e) 74

13. Find out the wrong number in the series
445, 221, 109, 46, 25, 11, 4
(a) 221 (b) 109
(c) 46 (d) 25
(e) 11

14. Find out the wrong number in the series
3, 7, 15, 39, 63, 127, 255, 511
(a) 7 (b) 15
(c) 39 (d) 63
(e) 127

15. Insert the missing number
13, 31, 63, 127, 255, (?)
(a) 513 (b) 511
(c) 517 (d) 523

16. Insert the missing number
2, 4, 12, 48, 240, ?)
(a) 960 (b) 1440
(c) 1080 (d) 1920

17. Insert the missing number
10, 5, 13, 10, 16, 20 19, (?)
(a) 22 (b) 40
(c) 38 (d) 23

18.

1296	864	576	384	256	
1080	*a*	*b*	*c*	*d*	*e*

Find the value of *c*.
(a) 380 (b) 340
(c) 320 (d) 360

19.

7	13	78	83	415	
3	*a*	*b*	*c*	*d*	*e*

Find the value of *b*
(a) 54 (b) 56
(c) 58 (d) 52

20.

3240	540	108	27	9	
3720	*a*	*b*	*c*	*d*	*e*

Find the value of *d*
(a) 11.33 (b) 10.33
(c) 12.33 (d) 9.33

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (c)

Solution Second number is one more than twice the first, third number is one less than twice the second, fourth number is one more than twice the third, fifth number is one less than twice the fourth. Therefore, the sixth number is one more than twice the fifth.

So, the missing number is **75.**

3. Ans. (d)

Solution Go on subtracting 24, 21, 18, 15, 12, 9 from the numbers to get the next numbers. Clearly, 128 is incorrect.

5. Ans. (a)

Solution Numbers are alternately multiplied by 2 and increased by 3.

So, the missing number = 61 × 2 = **122.**

7. Ans. (b)

Solution Numbers are $(2^3 - 1)$, $(3^3 - 1)$, $(4^3 - 1)$, $(5^3 - 1)$, $(6^3 - 1)$, $(7^3 - 1)$ and so on.

So, the next number is $8^3 - 1 = 512 - 1 =$ **511.**

9. Ans. (c)

Solution The difference of two successive terms of the series are 17, 27, 37, 47

$\therefore$ $a = 34 + 17 = 51$
$b = 51 + 27 = 78$
$c = 78 + 37 = 115$
$d = 115 + 47 = 162$
and, $e = 162 + 57 =$ **219.**

11. Ans. (c)

Solution Go on multiplying the number by 2 and adding 1 to it to get the next number. So, 27 is incorrect.

13. Ans. (c)

Solution Go on subtracting 3 and dividing the result by 2 to obtain the next number. Clearly, 46 is incorrect.

15. Ans. (b)

Solution Each number is double the preceding one plus 1. So, the next number is (255 × 2) + 1 = **511.**

17. Ans. (b)

Solution There are two series (10, 13, 16, 19) and (5, 10, 20, 40) one increasing by 3 and another multiplied by **2.**

19. Ans. (a)

Solution The first series is + 6, × 6, + 5, × 5
$\therefore$ $a = 3 + 6 = 9$ and $b = 9 \times 6 =$ **54.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (d)	**4.** (c)	**6.** (a)	**8.** (a)	**10.** (b)	**12.** (b)	**14.** (c)	**16.** (b)	**18.** (c)	**20.** (b)

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Clocks and Calendar 24

CLOCKS

The circumference of a dial of a clock (or watch) is divided into 60 equal parts called **minute spaces.** The clock has two hands—the hour hand and the minute hand. The hour hand (or short hand) indicates time in hours and the minute hand (or long hand) indicates time in minutes. In an hour, the hour hand covers 5 minute spaces while the minute hand covers 60 minute spaces. Thus, in one hour or 60 minutes, the minute hand gains 55 minute spaces over the hour hand.

SOME BASIC FACTS

1. In every hour:
   (a) Both the hands coincide once
   (b) The hands are straight (point in opposite directions) once. In this position, the hands are 30 minute spaces apart.
   (c) The hands are twice at right angles. In this position, the hands are 15 minute spaces apart.
2. The minute hand moves through 6° in each minute whereas the hour hand moves through $\frac{1^\circ}{2}$ in each minute. Thus, in one minute, the minute hand gains $5\frac{1^\circ}{2}$ than the hour hand.
3. (a) When the hands are coincident, the angle between them is 0°.
   (b) When the hands point in opposite directions, the angle between them is 180°.
   (c) The hands are in the same straight line, when they are coincident or opposite to each other. So, the angle between the two hands is 0° or 180°.
3. The minute hand moves 12 times as fast as the hour hand.
4. If a clock indicates 6.10, when the correct time is 6, it is said to be 10 minutes **too fast**. And if it indicates 5.50, when the correct time is 6, it is said to be 10 minutes **too slow.**

SOME USEFUL SHORT-CUT METHODS

1. The two hands of the clock will be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Explanation

At H O'clock the minute hand is $5H$ minute spaces behind the hour hand.

$\because$ the minute hand gains 55 minute spaces in 60 minute,

$\therefore$ the minute hand will gain $5H$ minute spaces in $\frac{60}{55} \times 5H = \frac{60H}{11}$ minutes. Thus, the two hands of clock will be together between H and $(H+1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Illustration 1 At what time between 5 and 6 O'clock are the hands of a clock together?

Solution Here, $H = 5$.

$$\therefore \quad \frac{60H}{11} = \frac{60}{11} \times 5 = \frac{300}{11} = 27\frac{3}{11}.$$

$\therefore$ Hands of a clock are together at $22\frac{3}{11}$ minutes past 5 O'clock.

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2. The two hands of the clock will be at right angles between H and $(H + 1)$ O'clock at $(5H \pm 15)\frac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the minute hand will be $5H$ minute spaces behind the hour hand. When the two hands are at right angle, they are 15 minute spaces apart. So there are two cases:

Case I When the minute hand is 15 minute spaces behind the hour hand, to be in this position, the minute hand will have to gain $(5H - 15)$ minute spaces over the hour hand.

Case II When the minute hand is 15 minute spaces ahead of the hour hand, to be in this position, the minute hand will have to gain $(5H + 15)$ minute spaces over the hour hand. Combining the two cases, the minute hand will have to gain $(5H \mp 15)$ minute spaces over the hour hand.

Now, 55 minute spaces are gained in 60 minutes.

$\therefore$ $(5H \mp 15)$ minute spaces are gained in

$$\frac{60}{55}(5H \mp 15) = \frac{12}{11}(5H \mp 15) \text{ minutes.}$$

So, they are at right angle at $(5H \mp 15)\frac{12}{11}$ minutes past H O'clock.

Illustration 2 At what time between 5 and 6 O'clock will the hands of a clock be at right angle?

Solution Here, $H = 5$

$$\therefore \quad (5H \mp 15)\frac{12}{11} = (5 \times 5 \mp 15)\frac{12}{11} = 10\frac{10}{11} \text{ and } 43\frac{7}{11}$$

$\therefore$ Hands of a clock are at right angle at $10\frac{10}{11}$ minutes past 5 and $43\frac{7}{11}$ minutes past 5.

3. The two hands of the clock will be in the same straight line but not together between H and $(H + 1)$ O'clock at

$(5H - 30)\frac{12}{11}$ minutes past H, when $H > 6$

and, $(5H + 30)\frac{12}{11}$ minutes past H, when $H < 6$.

Illustration 3 Find at what time between 2 and 3 O'clock will the hands of a clock be in the same straight line but not together.

Solution Here, $H = 2 < 6$.

$$\therefore \quad (5H + 30)\frac{12}{11} = (5 \times 2 + 30)\frac{12}{11}$$

$$= \frac{480}{11}, \text{ i.e., } 43\frac{7}{11}.$$

So, the hands will be in the same straight line but not together at $43\frac{7}{11}$ minutes past 2 O'clock.

4. Between H and $(H + 1)$ O'clock, the two hands of a clock are M minutes apart at $(5H \mp M)\frac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the two hands are $5H$ minute spaces apart.

Case I Minute hand is M minute spaces behind the hour hand. In this case, the minute hand has to gain $(5H - M)$ minute spaces over the hour hand.

Case II Minute hand is M minute spaces ahead of the hour hand. In this case, the minute hand has to gain $(5H + M)$ minute spaces over the hour hand.

Combining the two cases, the minute hand has to gain $(5H \pm M)$ minute spaces over the hour hand.

Now, 55 minute spaces are gained in 60 minutes.

$\therefore$ $(5H \pm M)$ minute spaces are gained in

$$\frac{60}{55}(5H \pm M) = \frac{12}{11}(5H \pm M) \text{ minutes.}$$

$\therefore$ The hands will be M minutes apart at

$$\frac{12}{11}(5H \pm M) \text{ minutes past } H \text{ O'clock.}$$

Illustration 4 Find the time between 4 and 5 O'clock when the two hands of a clock are 4 minutes apart.

Solution Here, $H = 4$ and $M = 4$.

$$\therefore \quad \frac{12}{11}(5H \pm M) = \frac{12}{11}(5 \times 4 \pm 4)$$

$$= 26\frac{2}{11} \text{ and } 17\frac{5}{11}.$$

$\therefore$ The hands will be 4 minutes apart at $26\frac{2}{11}$ minutes past 4 and $17\frac{5}{11}$ minutes past 4 O'clock.

5. Angle between hands of a clock

(a) When the minute hand is behind the hour hand, the angle between the two hands at M minutes past H O'clock $= 30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees.

(b) When the minute hand is ahead of the hourhand, the angle between the two hands, at M minutes past H O'clock $= 30\left(\frac{M}{5} - H\right) - \frac{M}{2}$ degree.

Illustration 5 Find the angle between the two hands of a clock at 15 minutes past 4 O'clock.

Solution Here, $H = 4$ and $M = 15$.

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$\therefore$ The required angle

$$= 30\left(H - \frac{M}{5}\right) + \frac{M}{2} \text{ degrees}$$

$$= 30\left(4 - \frac{15}{5}\right) + \frac{15}{2}$$

$$= \frac{75}{2}, \text{ i.e. } 37.5°.$$

6. The minute hand of a clock overtakes the hour hand at intervals of M minutes of correct time. The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60\times24}{M}\right) \text{ minutes.}$$

Illustration 6 The minute hand of a clock overtakes the hour hand at intervals of 65 minutes. How much a day does the clock gain or lose?

Solution Here, $M = 65$

$\therefore$ The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60\times24}{M}\right)$$

$$= \left(\frac{720}{11} - 65\right)\left(\frac{60\times24}{65}\right)$$

$$= \frac{5}{11} \times \frac{12\times24}{13} = \frac{1440}{143}$$

$$= 10\frac{10}{143} \text{ minutes.}$$

Since the sign is +ve, the clock gains by $10\frac{10}{143}$ minutes.

CALENDAR

In this section we shall mainly deal with finding the day of the week on a particular given date. The process of finding it depends upon the number of odd days, which are quite different from the odd numbers. So, we should be familiar with **odd days.**

Odd Days

The days more than the complete number of weeks in a given period are called odd days.

Ordinary Year

An ordinary year has 365 days.

Leap Year

That year (except century) which is divisible by 4 is called a leap year, whereas century is a leap year by itself when it is divisible by 400.

For example, 1964, 1968, 1972, 1984, and so on, are all leap years whereas 1986, 1990, 1994, 1998, and so on, are not leap years.

Further, the centuries 1200, 1600, 2000 and so on, are all leap years as they are divisible by 400 whereas 900, 1300, 1500 and so on, are not leap years.

SOME BASIC FACTS

1. An ordinary year has 365 days, i.e. 52 weeks and 1 odd day.
2. A leap year has 366 days, i.e. 52 weeks and 2 odd days.
3. A century has 76 ordinary years and 24 leap years.
   $\therefore$ 100 years = 76 ordinary years + 24 leap years
   = 76 odd days + 24 × 2 odd days
   = 124 odd days = 17 weeks + 5 days
   $\therefore$ 100 years contain 5 odd days.
4. 200 years contain 10 and therefore 3 odd days.
5. 300 years contain 15 and therefore 1 odd day.
6. 400 years contain (20 + 1) and therefore 1 odd day.
7. February in an ordinary year has no odd day, but in a leap year has one odd day.
8. Last day of a century cannot be either Tuesday, Thursday or Saturday.
9. The first day of a century must either be Monday, Tuesday, Thursday or Saturday.

Explanation

No. of odd days in first century = 5

$\therefore$ Last day of first century is Friday.

No. of odd days in two centuries = 3

$\therefore$ Wednesday is the last day.

No. of odd days in three centuries = 1

$\therefore$ Monday is the last day.

No. of odd days in four centuries = 0

$\therefore$ Last day is Sunday.

Since the order is continually kept in successive cyles, the last day of a century cannot be Tuesday, Thursday or Saturday. So, the last day of a century should be either Sunday, Monday, Wednesday or Friday. Therefore, the first day of a century must be either Monday, Tuesday, Thursday or Saturday.

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SOME USEFUL SHORT-CUT METHODS

1. Working rule to find the day of the week on a particular date when reference day is given:

Step I Find the net number of odd days for the period between the reference date and the given date (Exclude the reference day but count the given date for counting the number of net odd days).

Step II The day of the week on the particular date is equal to the number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

Illustration 7 11 January 1997 was a Sunday. What day of the week was on 7 January 2000?

Solution Total number of days between 11 January 1997 and 7 January 2000

= (365 – 11) in 1997 + (365 days in 1998)
+ (365 days in 1999) + (7 days in 2000)

= (50 weeks + 4 odd days) + (52 weeks + 1 odd day)
+ (52 weeks + 1 odd day) + (7 odd days)

= 13 days = 1 week + 6 odd days.

Hence, 7 Jan. 2000 would be 6 days ahead of Sunday, i.e. it was on Saturday.

2. Working Rule to find the day of the week on a particular date when no reference day is given

Step I Count the net number of odd days on the given date

Step II Write:

Sunday for 0 odd day
Monday for 1 odd day
Tuesday for 2 odd days
⋮ ⋮ ⋮
Saturday for 6 odd days.

Illustration 8 What day of the week was on 5 June 1999?

Solution 5 June 1999 means 1998 years + first five months up to May of 1999 + 5 days of June.

1600 years have 0 odd day.
300 years have 1 odd day.
98 years have 24 leap years + 74 ordinary years
= (24 × 2) + (74 × 1) days
= 122 days = 17 weeks + 3 odd days
Thus, 1998 years have 4 odd days.
1 January 1999 to 31 May 1999 has
= (3 + 0 + 3 + 2 + 3 + 5) 2 odd days
= 16 days = 2 weeks + odd days
Total number of odd days on 5 June 1999
= (4 + 2) odd days = 6 odd days.

Hence, 5 June 1999 was Saturday.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. At what time between 3 and 4 O'clock are the hands of a clock together?
   (a) $15\frac{7}{11}$ minutes past 4 (b) $16\frac{4}{11}$ minutes past 3.
   (c) $16\frac{2}{11}$ minutes past 2 (d) None of these

2. At what time between 7 and 8 O'clock will the hands of a clock be at right angle?
   (a) $19\frac{5}{11}$ minutes past 2 (b) $21\frac{9}{11}$ minutes past 7
   (c) 18 minutes past 4 (d) None of these

3. Find at what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together?
   (a) $11\frac{9}{11}$ minutes past 5
   (b) $9\frac{7}{11}$ minutes past 5
   (c) $10\frac{10}{11}$ minutes past 8
   (d) None of these

4. At what time between 5 and 6 O'clock are the hands of a clock 3 minutes apart?
   (a) 24 minutes past 5 (b) 22 minutes past 3
   (c) 26 minutes past 4 (d) None of these

5. Find the angle between the two hands of a clock at 30 minutes past 4 O'clock.
   (a) 40° (b) 30°
   (c) 45° (d) None of these

6. How much does a watch gain or lose per day, if its hands coincide every 64 minutes?

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(a) $32\frac{8}{11}$ minutes gain (b) $34\frac{2}{11}$ minutes gain

(c) $32\frac{8}{11}$ minutes loss (d) None of these

7. How often between 11 O'clock and 12 O'clock are the hands of the clock an integral number of minutes apart?

(a) 55 times (b) 56 times

(c) 58 times (d) 60 times

8. Number of times the hands of a clock are in a straight line everyday is

(a) 44 (b) 24

(c) 42 (d) 22

9. My watch gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 O'clock, the true time is

(a) $59\frac{7}{12}$ min. past 3 (b) $12\frac{3}{11}$ min. past 3

(c) 4 p.m. (d) $7\frac{5}{12}$ min. past 4

10. My watch gains 5 min. every hour. How many degrees the second hand moves in every minute?

(a) 375° (b) 380°

(c) 390° (d) 365°

11. At what time between 4 : 30 and 5 will the hands of a clock be in a straight line?

(a) 50 mins. past 4 (b) 42 mins. past 4

(c) $54\frac{6}{11}$ mins. past 4 (d) 46 mins. past 4

12. Two clocks are set right at 10 a.m. One gains 20 seconds and the other loses 40 seconds in 24 hours. What will be the true time when the first clock indicates 4 p.m. on the following day?

(a) $3 : 59\frac{2521}{4321}$ p.m. (b) $3 : 31\frac{1}{471}$ p.m.

(c) $3 : 59\frac{7}{12}$ p.m. (d) $3 : 57\frac{2521}{4321}$ p.m.

13. A clock strikes 4 taking 9 seconds. In order to stride 12 at the same rate, the time taken is

(a) 27 seconds (b) 36 seconds

(c) 30 seconds (d) 33 seconds

14. How often are the hands of a clock at right angle everyday?

(a) 38 times (b) 44 times

(c) 40 times (d) 48 times

15. A clock is set right at 5 a.m. The clock loses 16 min. in 24 hours. What will be the true time when the clock indicates 10 p.m. on the 4th day?

(a) 9 a.m. (b) 11 p.m.

(c) 11 a.m. (d) 9 p.m.

16. My watch was 3 minutes slow at 5 p.m. Tuesday and it was 5 minutes fast at 11 p.m. Wednesday. When did it give correct time?

(a) Wednesday 4 : 15 a.m.

(b) Wednesday 7 : 30 a.m.

(c) Tuesday 3 : 45 p.m.

(d) None of these

17. How many times do the hands of a clock point towards each other in a day?

(a) 24 (b) 20

(c) 12 (d) 22

18. A man who went out between 3 and 4 and returned between 8 and 9, found that the hands of the watch had exactly changed places. He returned at

(a) 14 mins. past 8 (b) $21\frac{1}{13}$ mins. past 8

(c) $19\frac{2}{13}$ mins. past 8 (d) $18\frac{6}{13}$ mins. past 8

19. A clock gains 10 minutes in every 24 hours. It is set right on Monday at 8 a.m. What will be the correct time on the following Wednesday, when the watch indicates 6 p.m.?

(a) 5.36 p.m. (b) 5.40 p.m.

(c) 4.36 p.m. (d) None of these

20. If the hands of a clock coincide every 65 minutes (true time), in 24 hrs the clock will gain

(a) $10\frac{10}{143}$ mins. (b) $9\frac{12}{143}$ mins.

(c) $11\frac{12}{143}$ mins. (d) $12\frac{10}{143}$ mins.

21. The watch which gains uniformly is 2 mins. slow at noon on Sunday and is 4 min. 48 secs. fast at 2 p.m. on the following Sunday. The watch was correct at

(a) 2 p.m. on Tuesday

(b) 12 noon on Monday

(c) 1 : 30 p.m. on Tuesday

(d) 12 : 45 p.m. on Monday

22. A watch which gains uniformly is 6 minutes slow at 4 p.m. on a Sunday and $10\frac{2}{3}$ minutes fast on the following Sunday at 8 p.m. During this period (Day and Time) when was the watch correct?

(a) 2.36 a.m. (b) 1.36 a.m.

(c) 2.36 p.m. (d) 1.36 p.m.

23. If a clock takes 22 seconds to strike 12, how much time will it take to strike 6?

(a) 10 seconds (b) 12 seconds

(c) 14 seconds (d) None of these

24. Mahatma Gandhi was born on 2 October 1869. The day of the week was

(a) Sunday (b) Monday

(c) Saturday (d) Friday

25. 5 March 1999 was on Friday. What day of the week was on 5 March 2000?

(a) Monday (b) Sunday

(c) Friday (d) Tuesday

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26. On what dates of August, 1988 did Fridays fall ?
(a) 5 (b) 4
(c) 14 (d) 17

27. India got independence on 15 August 1947. What was the day of the week?
(a) Monday (b) Friday
(c) Thursday (d) Sunday

28. 7 January 1992 was Tuesday. Find the day of the week on the same date after 5 years, i.e. on 7 January 1997?
(a) Tuesday (b) Wednesday
(c) Saturday (d) Friday

29. Number of times 29th day of the month occurs in 400 consecutive years is
(a) 4497 (b) 4800
(c) 4400 (d) None of these

30. The first Republic Day of India was celebrated on 26 January 1950. What was the day of the week on that date?
(a) Monday (b) Wednesday
(c) Saturday (d) Thursday

31. In an ordinary year? 'March' ? begin on the same day of the week
(a) Feb.; Nov. (b) Jan.; Nov.
(c) Feb.; Oct. (d) Jan.; Sept.

32. If 2 March 1994 was on Wednesday, 25 Jan. 1994 was on
(a) Wednesday (b) Thursday
(c) Tuesday (d) Monday

33. Calendar for 2000 will serve for also:
(a) 2003 (b) 2006
(c) 2007 (d) 2005

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Here, $H = 3$.

$$\therefore \quad \frac{60H}{11} = \frac{60}{11} \times 3 = \frac{180}{11} = 16\frac{4}{11}$$

So, the hands of a clock will coincide at $16\frac{4}{11}$ minutes past 3.

2. Ans. (b)

Solution Here, $H = 7$

$$\therefore \quad (5H \mp 15)\frac{12}{11} = (5 \times 7 \mp 15)\frac{12}{11}$$
$$= 21\frac{9}{11} \text{ and } 54\frac{6}{11}.$$

$\therefore$ The hands of a clock are at right angle at $21\frac{9}{11}$ minutes past 7 and $54\frac{6}{11}$ minutes past 7 O'clock.

3. Ans. (c)

Solution Here, $H = 8 > 6$.

$$\therefore \quad (5H - 30)\frac{12}{11} = (5 \times 8 - 30)\frac{12}{11} = \frac{120}{11} = 10\frac{10}{11}.$$

So, the hands will be in the same straight line but not together at $10\frac{10}{11}$ minutes past 8 O'clock.

4. Ans. (a)

Solution Here, $H = 5$ and $M = 3$.

$$\therefore \quad \frac{12}{11}(5H \pm M) = \frac{12}{11}(5 \times 5 \pm 3) = 31\frac{5}{11} \text{ and } 24.$$

$\therefore$ The hands will be 3 minutes apart at $31\frac{5}{11}$ minutes past 5 and 24 minutes past 5 O'clock.

5. Ans. (c)

Solution Here, $H = 4$ and $M = 30$.

$$\therefore \text{ The required angle} = 30\left(\frac{M}{5} - H\right) - \frac{M}{2} \text{ degrees}$$
$$= 30\left(\frac{30}{5} - 4\right) - \frac{30}{2}$$
$$= 60 - 15 = \mathbf{45°}.$$

6. Ans. (a)

Solution Here, $M = 64$

$\therefore$ The clock gains or loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right)$$
$$= \left(\frac{720}{11} - 64\right)\left(\frac{60 \times 24}{64}\right)$$
$$= \left(65\frac{5}{11} - 64\right) \times \left(\frac{60 \times 24}{64}\right)$$
$$= \frac{16}{11} \times \frac{60 \times 24}{64} = \frac{360}{11} = 32\frac{8}{11} \text{ minutes.}$$

Since the sign is +ve, the clock gains by $32\frac{8}{11}$ minutes.

7. Ans. (b)

Solution At 11 O'clock, the hour hand is 4 spaces apart from the minute hand. Since there are 60 spaces in one hour, so (60 – 4) times, i.e. 56 times the hands of the clock are an integral number of minutes apart.

8. Ans. (a)

Solution We know that, any relative position of the hands of a clock is repeated 11 times in every 12 hrs.

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∴ In every 12 hrs, hands coincide 11 times and are opposite to each other 11 times.

∴ In every 12 hrs, hands are in a straight line 11 + 11 = 22 times.

∴ In every 24 hrs hands are in a straight line 44 times.

9. Ans. (c)

Solution From 7 a.m. to 4 : 15, the time is 9 hours 15 min, i.e. 555 min.

Now, $\frac{37}{12}$ min of this watch = 3 minutes of correct watch.

⇒ 555 min. of this watch = $\left(\frac{3\times12}{37}\times555\right)$ on a correct watch = 540 min. or 9 hours of correct watch.

∴ correct time after 7 a.m. is 4 p.m.

10. Ans. (c)

Solution Since minute hand gains 5 minutes in every 60 minutes

⇒ second hand gains 5 seconds in every 60 seconds

∴ In every 60 seconds true time, it moves 65 seconds or 65 × 6° = 390°.

11. Ans. (c)

Solution At 4 O'clock hands are 20 min. spaces apart. At time between 4 : 30 and 5 the hands will be in straight line when they point in opposite directions and there is a space of 30 mins. between them. So, to be in this position min. hand has to gain 30 + 20 = 50 mins. spaces. Min. hand gains 50 min. in

$$\frac{60}{55}\times50=\frac{600}{11}=54\frac{6}{11}\text{ min.}$$

∴ Required time = $54\frac{6}{11}$ mins. past 4.

12. Ans. (a)

Solution From 10 a.m. to 4 p.m. on the following day = 30 hours

Now, 24 hrs 20 seconds of the first clock = 24 hrs of the current clock.

∴ 1 hour of the first clock = $\frac{24\times180}{4321}$ hours

∴ 30 hours of the first clock = $\frac{24\times180\times30}{4321}$ hours

Now, $\frac{24\times180\times30}{4321}$ hours = 29 hours $59\frac{2521}{4321}$ min.

∴ When the first clock indicates 4 p.m. on the following day the true time will be 3 hrs $59\frac{2521}{4321}$ mins.

13. Ans. (d)

Solution There are 3 intervals when the clock strikes 4

Time taken in 3 intervals = 9 seconds

∴ Time taken for 1 interval = 3 seconds

In order to strike 12, there are 11 intervals, for which the time taken is 11 × 3 seconds = 33 seconds.

14. Ans. (b)

Solution In every hour there are two positions in which hands are at right angle. Each of these positions is repeated 11 times in every 12 hrs.

∴ In every 12 hrs, hands are at right angles 11 + 11 = 22 times and in a day hands are at right angles 22 + 22 = 44 times.

15. Ans. (b)

Solution From 5 a.m. on first day to 10 p.m. on 4th day is 89 hours. When a clock loses 16 min. then 23 hours 44 min. of this clock are the same as 24 hours of correct clock, i.e. $\frac{356}{15}$ hours of this clock = 24 hours of correct clock

∴ 89 hours of this clock

$$=\left(\frac{24\times15}{356}\times89\right)\text{ hours of correct clock}$$

= 90 hours of correct clock

∴ The correct time is 11 p.m.

16. Ans. (a)

Solution Time from 5 p.m. Tuesday to 11 p.m. Wednesday = 30 hours

Clock gains 8 minutes in 30 hours

∴ It gains 3 minutes in $\frac{30}{8}\times3$ hours

= 11 hrs. 15 min.

∴ Correct time is 11 hrs. 15 min. after 5 p.m.

= 4 : 15 a.m. on Wednesday.

17. Ans. (b)

Solution The hands of a clock point towards each other 11 times in every 12 hours. (because between 5 and 7, at 6 O'clock only they point towards each other)

So, in a day the hands point towards each other 22 times.

18. Ans. (d)

19. Ans. (a)

Solution Total number of hours from Monday at 8 a.m. to the following Wednesday at 6 p.m.

24 × 2 + 10 = 58 hours

24 hours 10 minutes of this clock are the same as 24 hours of a correct clock.

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$\frac{145}{6}$ hours of the incorrect clock = 24 hours of correct clock.

58 hours of the incorrect clock = $\frac{24\times6}{145}\times58$ hours of correct clock = $57\frac{3}{5}$ hours of correct clock.

Thus, the correct time on the following Wednesday will be 5.36 p.m.

20. Ans. (a)

Solution The min. hand gains 60 mins in $\frac{60}{55}\times60$

$$=\frac{720}{11}=65\frac{5}{11}\text{ mins.}$$

∴ The hands of a correct clock coincide every $65\frac{5}{11}$ mins. But the hands of the clock in question coincide every 65 mins.

The clock in question gains $\frac{5}{11}$ mins. in 65 min.

∴ In 24 hrs = 24 × 60 mins. the clock gains

$$\frac{5}{11}\times\frac{1}{65}\times24\times60=\frac{1440}{143}\text{ mins.}=10\frac{10}{143}\text{ mins.}$$

21. Ans. (a)

Solution From Sunday noon to the following Sunday at 2 p.m., total time = 7 days + 2 hrs

= (7 × 24 + 2) hrs = 170 hrs.

In this period watch gains 2 + 4 mins. 48 seconds

$$=6\frac{48}{60}=6\frac{4}{5}\text{ mins.}$$

∴ Watch gains $6\frac{4}{5}$ mins in 170 hrs.

∴ Watch gains 2 mins. in $\frac{170}{34}\times5\times2=50$ hrs

i.e. 2 days and 2 hrs.

∴ Watch will be correct at 2 p.m. on Tuesday.

22. Ans. (b)

Solution Total time in hours from Sunday at 4 p.m. to the following Sunday at 8 a.m.

$$=6\times24+16=160\text{ hours}$$

Thus, the watch gains $6+10\frac{2}{3}$

$$=16\frac{2}{3}\text{ minutes in 160 hours}$$

Now, $\frac{50}{3}$ minutes are gained in 160 hours.

∴ 6 minutes are gained in $160\times\frac{3}{50}\times6$

$$=\frac{288}{5}\text{ hours}=57\frac{3}{5}\text{ hours.}$$

or, the watch was correct on Wednesday at 1.36 a.m.

23. Ans. (a)

Solution In order to hear 12 strikes, there are 11 intervals (12 – 1) and time of each interval is uniform.

Hence, time to hear each strike is $\frac{22}{11}=2$ seconds

Now, to hear six strikes, there are 6 – 1, i.e. 5 × 2 = 10 seconds.

Hence, it will take 10 seconds for a clock to strike 6.

24. Ans. (c)

Solution 2 October 1869 means

1868 complete years + 9 months + 2 days

1600 years give 0 odd days

200 years give 3 odd days

No. of leap years in 68 years = largest integer less than $\frac{68}{4}=17$

∴ 68 years contain 17 leap years and 51 non-leap years

∴ 68 years have 2 × 17 + 51 = 85, i.e. 1 odd day

Also count no. of days from 1 Jan. 1869 to 2 Oct. 1869

Jan.	Feb.	March	April	May	June
31	+28	+31	+30	+31	+30
July	Aug.	Sept.	Oct.		
31	+31	+30	+2	= 275	

= 39 weeks + 2 days

∴ This gives 2 odd days

∴ Total no. of odd days = 0 + 3 + 1 + 2 = 6

∴ Day on 2 Oct. 1869 was Saturday.

25. Ans. (b)

Solution Year 2000 was a leap year

Number of days remaining in 1999

= 365 – [31 days Jan. + 28 days Feb. + 5 days March]

= 301 days = 43 weeks, i.e. 0 odd day

Number of days passed in 2000:

January 31 days have 3 odd days

February 29 days (being leap year) have 1 odd day

March 5 days have 5 odd days

∴ Total number of odd days = 0 + 3 + 1 + 5 + 9 days, i.e. 2 odd days

Therefore, 5 March 2000 would be two days beyond Friday, i.e. on Sunday.

26. Ans. (c)

Solution 1 August 1988 means

1987 years + 7 months

Number of odd days in 1987 years:

1600 years have 0 odd days

300 years have 1 odd day

87 years have 21 leap years and 66 ordinary years

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So, there are 21 × 2 + 66 × 1 = 108 days, i.e. 15 weeks and 3 odd days

Number of days between 1 Jan. 1988 to 1 August 1989.

Jan. Feb. Mar. April May June July Aug.

31 +29 +31 +30 +31 +30 +31 +1 = 241 days

i.e. 30 weeks and 4 odd days

Total number of odd days = 0 + 1 + 3 + 4 = 8 odd days or 1 odd day

Thus, Friday falls on 5th, 12th, 19th and 26th in August 1988.

27. Ans. (b)

Solution 15 August 1947 = (1600 + 300 + 46) years + 1 Jan. to 15 Aug. of 1947

= (1600 + 300 + 46) years + 365 – 16 Aug to 31 Dec. 1947

= (1600 + 300 + 46) years + (365 – 138) days

No. of odd days = 0 + 1 + 1 (from 11 leap years and 35 ordinary years) + 3 = 5 odd days

∴ The day was Friday.

28. Ans. (a)

Solution During the interval we have two leap years as 1992 and 1996 and it contains February of both these years

∴ The interval has (5 + 2) = 7 odd days or 0 odd day.

Hence, 7 January 1997 was also Tuesday.

29. Ans. (a)

Solution 400 consecutive years contain 97 leap years.

∴ In 400 consecutive years Feb. has 29 days 97 times and the remaining 11 months have 29th day

400 × 11 = 4400 times

∴ 29th day of the month occurs 4400 + 97 = 4497 times.

30. Ans. (d)

Solution Total number of odd days = 1600 years have 0 odd day + 300 years have 1 odd day + 49 years (12 leap + 37 ordinary) have 5 odd days + 26 days of Jan. have 5 odd days = 0 + 1 + 5 + 5 = 4 odd days

So, the day was Thursday.

31. Ans. (a)

Solution In an ordinary year, Feb. has no odd day

∴ Feb. and March begin on same day of week.

Also we know that, November and March begin on same day of the week.

32. Ans. (c)

Solution No. of days from 25 Jan. 1994 to 2 march 1994 is

Jan. Feb. March

6 +28 +2 = 36

∴ No. of odd days = 1

∴ Day on 25 Jan. 1994 is one day before the day on 2 March 1994.

But 2 March 1994 was on Wednesday.

∴ 25 Jan. 1994 was on Tuesday.

33. Ans. (b)

Solution Starting with 2000, count for number of odd days in successive years till the sum is divisible by 7.

2000 2001 2002 2003 2004

2 +1 +1 +1 +2 = 7

∴ No. of odd days up to 2004 = 0

∴ Calendar for 2000 will serve for 2005 also.

QUESTIONS FROM COMPETITIVE EXAMS

1. A watch is 1 minute slow at 1 p.m. on Tuesday and 2 minutes fast at 1 p.m. on Thursday. When did it show the correct time?

(a) 1.00 a.m. on Wednesday
(b) 5.00 a.m. on Wednesday
(c) 1.00 p.m. on Wednesday
(d) 5.00 p.m. on Wednesday **[Based on MAT 2004]**

2. The minute and the hour hand of a watch meet every 65 minutes. How much does the watch lose or gain time?

(a) 25 seconds (b) 27 seconds
(c) 27.16 seconds (d) 30 seconds
[Based on MAT 2003]

3. If 21st July, 1999 is a Wednesday, what would have been the day of the week on 21st July, 1947?

(a) Monday (b) Sunday
(c) Thursday (d) Saturday
[Based on MAT 1999]

4. Imagme that your watch was correct at noon, but then it began to lose 30 minutes each hour. It now shows 4 pm, but it stopped 5 hours ago. What is the correct time now ?

(a) 9.30 pm (b) 11 pm
(c) 1 am (d) 1.30 am
[Based on FMS (Delhi) 2004]

5. Assume that the three of the following four dates in the year 2004 are right. Which one would be wrong?

(a) Sunday, 17th of January
(b) Sunday, 21st of February
(c) Sunday, 19th of March
(d) Sunday, 10th of April ?
[Based on FMS (Delhi) 2004]

6. A mechanical grandfather clock is at present showing 7 hr 40 min 6 sec. Assuming that it loses 4 seconds

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(d) Sunday, 10th of April ?

[Based on FMS (Delhi) 2004]

6. A mechanical grandfather clock is at present showing 7 hr 40 min 6 sec. Assuming that it loses 4 seconds in every hour, what time will it show after exactly $6\frac{1}{2}$ hrs.

(a) 2 hr 9 min 40 sec (b) 2 hr 10 min 6 sec

(c) 14 hr 9 min 34 sec (d) 14 hr 10 min 32 sec

[Based on FMS (Delhi) 2004]

7. A watch which gains uniformly, is 5 minutes slow at 8 o'clock in the morning on Sunday, and is 5 minutes 48 seconds fast at 8 pm on the following Sunday. When was it correct ?

(a) 7 : 20 pm on Tuesday

(b) 9 : 20 pm on Wednesday

(c) 7 : 20 pm on Wednesday

(d) 9 : 20 pm on Tuesday

[Based on FMS (Delhi) 2004]

Directions (Qs. 8 and 9) : Read the following information carefully to answer these questions :

A person had left his home at the age of about 14 years. He remembers that the day was Monday. Since then, he has been fasting on every Tuesday. Today he is celebrating his 60th birth anniversary in a five-star hotel with his friends. As today is Tuesday, he is not taking anything except wine. At the end of the party, he discloses that it is his 2400th Tuesday of fasting.

8. Today is 9th October 2001. On which date had he left his home?

(a) 10th October 1955 (b) 9th October 1955

(c) 8th October 1955 (d) None of these

[Based on FMS (Delhi) 2004]

9. He was born on

(a) Wednesday (b) Tuesday

(c) Monday (d) Thursday

[Based on FMS (Delhi) 2004]

10. 29th February 2000 was a Tuesday. In this century, how many times 29th February will fall on a Tuesday?

(a) 1 (b) 2

(c) 3 (d) 4

[Based on IITTM, Gwalior, 2003]

11. Between 5 and 6, a lady looked at her watch and mistaking the hour hand for the minute hand, she thought that the time was 57 minutes earlier than the correct time. The correct time was

(a) 12 minutes past 5 (b) 24 minutes past 5

(c) 36 minutes past 5 (d) 48 minutes past 5

[Based on IITTM, Gwalior, 2003]

12. From 6 : 15 p.m. to 7 : 45 p.m., on the same day, the minute hand of a standard clock describes an arc of

(a) 30° (b) 90°

(c) 180° (d) 540°

(d) None of these **[Based on IIFT 2003]**

13. In a 366-day year, how many days occur 53 times?

(a) 2 (b) 1

(c) 0 (d) 3

[Based on SCMHRD Ent. Exam. 2003]

14. A clock strikes four taking 9 seconds. In order to strike twelve at the same rate, the time taken is

(a) 36 sec (b) 27 sec

(c) 30 sec (d) 33 sec

[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Watch gains 3 minutes in 48 hours. Therefore, after 16 hours, it will show the correct time.

2. Ans. (a)

Solution In 60 minutes, the hour hand moves through an angle of 30°.

∴ In 65 minutes, the hour hand will move through an angle of $\frac{30}{60}\times 65 = 32.5°$

In 60 minutes, the minute hand moves through an angle of 360°.

∴ In 65 minutes, the minute hand moves through an angle of $\frac{360}{60}\times 65 = 390° = 360° + 30°$.

⇒ Difference of the angles made by the hour hand and the minute hand = 2.5 (In 65 mintues)

Now an angle of 30° is made by the minute hand in 5 minutes.

∴ An angle of 2.5° is made by the minute hand in $\frac{5}{30}\times 2.5$ minutes, i.e. $\frac{2.5}{6}$ min, i.e. 25 seconds.

3. Ans. (a)

4. Ans. (c)

5. Ans. (c)

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6. Ans. (a)

7. Ans. (c)

Solution From 8 A.M on Sunday to 8 P.M on the following Sunday, the watch will be 348 seconds fast.

∴ There will be difference of 648 seconds in the duration of 180 hours.

∴ Difference of 300 seconds (5 minutes), so that the watch shows the correct time will be there after $\frac{180}{648} \times 300$ hours, i.e. $\frac{250}{3}$ hours, i.e. $83\frac{1}{3}$ hours, i.e. at 7.20 pm on Wednesday.

8. Ans. (a)

9. Ans. (d)

10. Ans. (c)

Solution In 2028, 2056 and 2084, i.e. after every 28 years.

11. Ans. (b)

12. Ans. (c)

13. Ans. (a)

14. Ans. (d)

Solution When a clock strikes 4 times it takes 3 intervals. ∴ Each interval takes $9 \div 3 = 3$ seconds when a clock strikes 12 times. It takes 11 intervals it means it will take $11 \times 3 = 33$ seconds.

EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. Two clocks begin to strike 12 together. One strikes its strokes in 33 seconds and the other in 22 seconds. What is the interval of time between 6th stroke of the first and 8th stroke of the second?
(a) 2 seconds (b) 1 second
(c) 3 seconds (d) None of these

2. At what time between 3 and 4 the minute hand is 4 minutes behind the hour hand?
(a) 12 min. past 3 (b) 8 min. past 3
(c) 15 min. past 3 (d) None of these.

3. The year next to 1988 having the same calendar as that of 1988 is
(a) 1990 (b) 1992
(c) 1993 (d) 1995

4. Two watches one of which gains 1 min. 54.6 sec. and the other loses 1 min. 55.8 secs. daily were set correctly at 10 a.m. on 1 January 2000. When did the watches next indicate the same time?
(a) July 3; midnight (b) July 6; 10 p.m.
(c) June 29; 9 p.m. (d) July 2; 8 p.m.

5. A watch which is set correctly at noon indicates 10 mins. after 9 the same evening when the true time is 10. The true time when the watch indicates 11 on the same evening is
(a) 11 : 30 (b) 12 : 15
(c) 12 : 00 (i.e. midnight)
(d) 11 : 45

6. The year next to 1991 having the same calendar as that of 1990 is
(a) 1998 (b) 2001
(c) 2002 (d) 2003

7. A clock is set right at 6 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 11 a.m. on the following day?
(a) 28 hrs. 48 min. (b) 29 hrs. 48 min.
(c) 27 hrs. 46 min. (d) None of these

8. Find what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together.
(a) $8\frac{10}{11}$ min. past 4 (b) $10\frac{10}{11}$ min. past 8
(c) $12\frac{10}{11}$ min. past 6 (d) $9\frac{10}{11}$ min. past 6

9. On 2 July 1985, it was Wednesday. The day of the week on 2 July 1984 was
(a) Wednesday (b) Tuesday
(c) Monday (d) Thursday

10. The number of odd days in a leap year is
(a) 0 (b) 1
(c) 3 (d) 3

Test II

[Time: 10 min]

11. When the time in the clock is 12 : 20, then the angle between the hands of the clock is x°. Find x.
(a) 120° (b) 130°
(c) 110° (d) 145°

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12. 15 August 1992 was Saturday. What day of the week was on 15 August 1991?
(a) Friday (b) Thursday
(c) Tuesday (d) Saturday

13. The time between 3 and 4 when the min. hand is 7 minute spaces ahead of the hour hand is
(a) $8\frac{8}{11}$ mins. past 3 (b) $23\frac{1}{11}$ mins. past 3
(c) 24 mins. past 3 (d) $9\frac{2}{11}$ mins. past 3

14. Between 7 O'clock and 8 O'clock, what will be the time when the hands are 60° apart or 10 minutes apart?
(a) $21\frac{9}{4}$ min. past 7 and $54\frac{6}{11}$ min. past 7
(b) $23\frac{9}{11}$ min. past 6 and $53\frac{7}{11}$ min. past 9
(c) $25\frac{9}{11}$ min. past 7 and $55\frac{6}{11}$ min. past 7
(d) None of the above.

15. The number of odd days in 400 years is
(a) 0 (b) 1
(c) 2 (d) 5

16. Smt. Indira Gandhi died on 31 October 1984. The day of the week was
(a) Monday (b) Tuesday
(c) Wednesday (c) Friday

17. The time between 5 and 6 at which the hands of a clock are together is
(a) 25 mins. past 5 (b) $25\frac{1}{11}$ mins. past 5
(c) $27\frac{3}{11}$ mins. past 5 (d) $28\frac{2}{11}$ mins. past 5

18. My watch which gains uniformly, is 2 mins. slow at noon on Sunday, and is 4 min. 48 seconds fast at 2 p.m. on the following Sunday. When was it correct?
(a) 2 p.m. Tuesday (b) 2 a.m. Tuesday
(c) 3 p.m. Tuesday (d) 4 a.m. Tuesday

19. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of correct time. How much in a day does the clock gain or lose?
(a) $12\frac{11}{143}$ minutes (b) $10\frac{11}{143}$ minutes
(c) $13\frac{11}{143}$ minutes (d) $9\frac{11}{143}$ minutes

20. The number of times the hands of a clock coincide in 12 hrs. is
(a) 11 (b) 12
(c) 13 (d) 10

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution Since the first clock strikes (12 – 1) or 11 strokes in 33 seconds, the interval between its two successive strokes is $\frac{33}{11}$ or 3 seconds.

Similarly, the interval between two successive strokes of the second clock is $\frac{22}{11}$ or 2 seconds. Now, 6 strokes have 5 intervals which takes 5 × 3 or 15 seconds and 8 strikes of the second clock have 7 intervals which takes 7 × 2 or 14 seconds

∴ Required interval = 15 – 14 = 1 second.

3. Ans. (c)

Solution Starting with 1988, we go on counting the number of odd days till the sum is divisible by 7

Years → 1988 1989 1990 1991 1992
odd days → 2 1 1 1 2 = 7, i.e. odd days

∴ Calendar for 1993 is the same as that of 1988.

5. Ans. (c)

Solution No. of minutes from noon to 10 mins. past 9 is 9 × 60 + 10 = 550.

No. of minutes from noon to 10 the same evening
= 10 × 60 = 600

No. of minutes from noon to 11 on the same evening
= 11 × 60 = 660

∴ Watch loses 600 – 550 = 50 mins. in 550 mins.

∴ In 660 mins, watch loses $\frac{50}{550} \times 660$
= 60 mins = 1 hr.

∴ Required time is 11 + 1 = 12, i.e. midnight.

7. Ans. (a)

Solution Time from 6 a.m. on a day to 11 a.m. on the following day is 29 hours.

Now, 24 hrs. 10 min. of this clock are the same as 24 hours of correct clock, i.e. $\frac{145}{6}$ hours of this clock = 24 hrs of correct time of clock.

∴ 29 hours of this clock = $\left(\frac{6\times24}{145}\times29\right)$ hours of correct clock.

∴ The correct time is 28 hrs 48 min. after 8 a.m. or 48 min. past 12.

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9. Ans. (c)

Solution The year 1984 being a leap year, it has 2 odd days

So, the day on 2 July 1985 is two days beyond the day on 2 July 1984.

But, 2 July 1985 was Wednesday

$\therefore$ 2 July 1984 was Monday.

11. Ans. (c)

Solution Imagine the position of the hands at 12 : 20.

Angle between 12th and 4th position is 120°.

Angle made by the hour hand in 20 minutes = 10°

Required angle = 120° – 10° = 110°.

13. Ans. (c)

Solution Obviously, the min. hand has to gain 15 + 7 = 22 minutes and this it does in $\frac{60}{55} \times 22 = 24$ mins.

$\therefore$ Required time = 24 mins. past 3.

15. Ans. (a)

Solution 100 years contain 76 ordinary years + 24 leap years

= [76 × 52 weeks + 76 days] + [24 × 52 weeks + 2 × 24 days]

= 5200 weeks + 124 days

= 5217 weeks + 5 days

$\therefore$ 400 years contain 4 × 5217 weeks + 20 days + 1 day

($\because$ every 4th century is a leap year)

= 20868 weeks + 21 days

= 20871 weeks + 0 day

$\therefore$ No. of odd days in 400 years = 0.

17. Ans (c)

Solution At 5 O'clock hands are 25 mins. apart.

They will be together when min. hand has gained 25 mins. which it does in

$$\frac{60}{55} \times 25 = \frac{300}{11} = 27\frac{3}{11} \text{ mins.}$$

$\therefore$ Required time is $27\frac{3}{11}$ mins past 5.

19. Ans (b)

Solution In a correct clock, the minute hand gains 55 minute spaces over the hour hand in 60 minutes. To be together again, the minute hand must gain 60 minutes over the hour hand.

Now, 55 min. are gained in 60 min.

$\therefore$ 60 min. are gained in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min

But, they are together after 65 minutes.

Gain in 65 minutes $= \left(65\frac{5}{11} - 65\right) = \frac{5}{11}$ min.

Gain in 24 hrs. $= \left(\frac{5}{11} \times \frac{60 \times 24}{65}\right)$ min. $= 10\frac{11}{143}$ minutes.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (b)	**6.** (c)	**8.** (b)	**10.** (d)	**12.** (b)	**14.** (a)	**16.** (c)	**18.** (a)	**20.** (a)

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PART II

Algebra

CHAPTER 25 Polynomials
CHAPTER 26 H.C.F. and L.C.M. of Polynomials
CHAPTER 27 Linear Equations
CHAPTER 28 Quadratic Equations
CHAPTER 29 Progressions
CHAPTER 30 Set Theory
CHAPTER 31 Permutations and Combinations
CHAPTER 32 Probability

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Polynomials 25

POLYNOMIAL

A function $p(x)$ of the form

$$p(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$

where, $a_0, a_1, a_2, \ldots, a_n$ are real numbers, $a_n \neq 0$ and n is a non-negative integer is called a *polynomial* in x over reals.

The real number $a_0, a_1, \ldots, a_n$ are called the *coefficients of the polynomial.*

If $a_0, a_1, a_2, \ldots, a_n$ are all integers, we call it a *polynomial over integers.*

If they are rational numbers, we call it a *polynomial over rationals.*

Illustration 1

(a) $4x^2 + 7x - 8$ is a polynomial over integers.

(b) $\frac{7}{4}x^3 + \frac{2}{3}x^2 - \frac{8}{7}x + 5$ is a polynomial over rationals.

(c) $4x^2 - \sqrt{3}x + \sqrt{5}$ is a polynomial over reals.

Monomial

A polynomial having only one term is called a monomial. For example, 7, $2x$, $8x^3$ are monomials.

Binomial

A polynomial having two terms is called a binomial. For example, $2x + 3$, $7x^2 - 4x$, $x^2 + 8$ are binomials.

Trinomial

A polynomial having three terms is called a trinomial. For example, $7x^2 - 3x + 8$ is a trinomial.

Degree of a Polynomial

The exponent in the term with the highest power is called the degree of the polynomial.

For example, in the polynomial $8x^6 - 4x^5 + 7x^3 - 8x^2 + 3$, the term with the highest power is x^6. Hence, the degree of the polynomial is 6.

A polynomial of degree 1 is called a *linear polynomial.*

It is of the form $ax + b$, $a \neq 0$.

A polynomial of degree 2 is called a *quadratic polynomial.*

It is of the form $ax^2 + bx + c$, $a \neq 0$.

Division of a Polynomial by a Polynomial

Let $p(x)$ and $f(x)$ be two polynomials and $f(x) \neq 0$. Then, if we can find polynomials $q(x)$ and $r(x)$, such that

$$p(x) = f(x).\ q(x) + r(x),$$

where, degree $r(x)$ < degree $f(x)$, then we say that $p(x)$ divided by $f(x)$, gives $q(x)$ as *quotient* and $r(x)$ as *remainder.*

If the remainder $r(x)$ is zero, we say that *divisor* $f(x)$ is a factor of $p(x)$ and we have

$$p(x) = f(x).\ q(x).$$

Illustration 2 Divide $f(x) = 5x^3 - 70x^2 + 153x - 342$ by $g(x) = x^2 - 10x + 16$. Find the quotient and the remainder.

Solution

$$\begin{array}{r|l} & 5x - 20 \\ \hline x^2 - 10x + 16 & 5x^3 - 70x^2 + 153x - 342 \\ & 5x^3 - 50x^2 + 80x \\ & \quad - \quad + \quad - \\ \hline & -20x^2 + 73x - 342 \\ & -20x^2 + 200x - 320 \\ & \quad + \quad - \quad + \\ \hline & -127x - 22 \end{array}$$

$\therefore$ Quotient $= 5x - 20$ and

Remainder $= -127x - 22$.

Illustration 3 Determine if $(x - 1)$ is a factor of

$$p(x) = x^3 - 3x^2 + 4x + 2.$$

$$\begin{array}{r|l} & x^2 - 2x + 2 \\ \hline x - 1 & x^3 - 3x^2 + 4x + 2 \\ & x^3 - x^2 \\ & \quad - \quad + \\ \hline & -2x^2 + 4x \\ & -2x^2 + 2x \\ & \quad - \quad + \\ \hline & 2x + 2 \\ & 2x - 2 \\ & \quad - \quad + \\ \hline & 4 \end{array}$$

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Since the remainder is not zero, $(x - 1)$ is not a factor of $p(x)$.

SOME BASIC THEOREMS

Factor Theorem

Let $p(x)$ be a polynomial of degree $n > 0$. If $p(a) = 0$ for a real number a, then $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

Illustration 4 Use factor theorem to determine if $(x - 1)$ is a factor of $x^8 - x^7 + x^6 - x^5 + x^4 - x + 1$.

Solution Let $p(x) = x^8 - x^7 + x^6 - x^5 + x^4 - x + 1$.

Then, $p(1) = (1)^8 - (1)^7 + (1)^6 - (1)^5 + (1)^4 - 1 + 1 = 1 \neq 0$.

Hence, $(x - 1)$ is not a factor of $p(x)$.

Remainder Theorem

Let $p(x)$ be any polynomial of degree ≥ 1 and a any number.

If $p(x)$ is divided by $x - a$, the remainder is $p(a)$.

Illustration 5 Let $p(x) = x^5 + 5x^4 - 3x + 7$ be divided by $(x - 1)$. Find the remainder.

Solution Remainder $= p(1) = (1)^5 + 5(1)^4 - 3(1) + 7 = 10$.

Some Useful Results and Formulae

1. $(A + B)^2 = A^2 + B^2 + 2AB$
2. $(A - B)^2 = A^2 + B^2 - 2AB = (A + B)^2 - 4AB$
3. $(A + B)(A - B) = A^2 - B^2$
4. $(A + B)^2 + (A - B)^2 = 2(A^2 + B^2)$
5. $(A + B)^2 - (A - B)^2 = 4AB$
6. $(A + B)^3 = A^3 + B^3 + 3AB(A + B)$
7. $(A - B)^3 = A^3 - B^3 - 3AB(A - B)$
8. $A^2 + B^2 = (A + B)^2 - 2AB$
9. $A^3 + B^3 = (A + B)(A^2 + B^2 - AB)$
10. $A^3 - B^3 = (A - B)(A^2 + B^2 + AB)$
11. $(A + B + C)^2 = A^2 + B^2 + C^2 + 2(AB + BC + CA)$
12. $(A^3 + B^3 + C^3 - 3ABC) = (A + B + C)(A^2 + B^2 + C^2 - AB - CA - BC)$
13. $A + B + C = 0 \Rightarrow A^3 + B^3 + C^3 = 3ABC$.
14. $A^n - B^n$ is divisible by $(A - B)$ for all values of n.
15. $A^n - B^n$ is divisible by $(A + B)$ only for even values of n.
16. $A^n + B^n$ is never divisible by $(A - B)$.
17. $A^n + B^n$ is divisible by $(A + B)$ only when n is odd.

A Useful Short Cut Method

When a polynomial $f(x)$ is divided by $x - a$ and $x - b$, the respective remainders are A and B. Then, if the same polynomial is divided by $(x - a)(x - b)$, the remainder will be

$$\frac{A - B}{a - b}x + \frac{Ba - Ab}{a - b}.$$

Illustration 6 When a polynomial $f(x)$ is divided by $(x - 1)$ and $(x - 2)$, the respective remainders are 15 and 9. What is the remainder when it is divided by $(x - 1)(x - 2)$?

Solution Remainder $= \frac{A - B}{a - b}x + \frac{Ba - Ab}{a - b}$

$= \frac{15 - 9}{1 - 2}x + \frac{9(1) - 15(2)}{1 - 2}$

$= (-x + 21)$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. If $(x - 2)$ is a factor of the polynomial $x^3 - 2ax^2 + ax - 1$, find the value of a.
   (a) 5/6 (b) 7/6
   (c) 11/6 (d) None of these.
2. If $x + a$ is a factor of the polynomial $x^3 + ax^2 - 2x + a + 4$, find the value of a.
   (a) –4/3 (b) +2/3
   (c) +4/3 (d) None of these.
3. Find the value of k if $f(x) = x^3 - kx^2 + 11x - 6$ and $(x - 1)$ is a factor of $f(x)$.
   (a) 6 (b) 4
   (c) 8 (d) None of these.
4. If $5x^2 - 4x - 1$ is divided by $x - 1$, the remainder is
   (a) 0 (b) 2
   (c) 1 (d) None of these.
5. Find the values of m and n in the polynomial $2x^3 + mx^2 + nx - 14$ such that $(x - 1)$ and $(x + 2)$ are its factors.
   (a) $m = 4, n = 5$ (b) $m = 9, n = 3$
   (c) $m = 6, n = 7$ (d) None of these.

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6. What value should a possess so that $x + 1$ may be a factor of the polynomial.
$f(x) = 2x^3 - ax^2 - (2a - 3)\,x + 2$?
(a) 2 (b) –2
(c) 3 (d) None of these.

7. Divide the polynomial $4y^3 - 3y^2 + 2y - 4$ by $y + 2$ and find the quotient and remainder.
(a) $4y^2 - 11y + 24, -52$
(b) $6y^2 - 13y + 36, -64$
(c) $4y^2 + 13y - 24, +52$
(d) None of these.

8. Resolve into factors: $16\,(x - y)^2 - 9\,(x + y)^2$.
(a) $(x - 5y)\,(5x - y)$ (b) $(x + 7y)\,(7x + y)$
(c) $(x - 7y)\,(7x - y)$ (d) None of these.

9. Resolve into factors: $4x^2 + 12xy + 9y^2 - 8x - 12y$.
(a) $(3x + 2y)\,(4x + 2y - 3)$
(b) $(2x + 3y)\,(2x + 3y - 4)$
(c) $(2x - 3y)\,(2x + 3y + 4)$
(d) None of these.

10. Resolve into factors:
$16x^2 - 72xy + 81y^2 - 12x + 27y$.
(a) $(6x - 7y)\,(6x - 7y - 5)$
(b) $(4x - 9y)\,(4x - 9y - 3)$
(c) $(4x + 9y)\,(4x + 9y + 3)$
(d) None of these.

11. Resolve into factors: $(a + b)^2 - 14c\,(a + b) + 49c^2$.
(a) $(a - b - 9c)^3$ (b) $(a + b - 7c)^2$
(c) $(a + b + 9c)^2$ (d) None of these.

12. Resolve into factors: $81x^2y^2 + 108xyz + 36z^2$.
(a) $(6xy + 9z)^2$ (b) $(9xy - 7z)^2$
(c) $(9xy + 6z)^2$ (d) None of these.

13. Factorise: $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)\,(b + c - a)$.
(a) $4a^2$ (b) $6a^2$
(c) $8a^2$ (d) None of these.

14. Resolve into factors:
$9\,(3x + 5y)^2 - 12\,(3x + 5y)\,(2x + 3y) + 4\,(2x + 3y)^2$.
(a) $(7x + 9y)^2$ (b) $(5x + 9y)^2$
(c) $(5x - 9y)^2$ (d) None of these.

15. Factorise: $(2x + 3y)^2 + 2\,(2x + 3y)\,(2x - 3y) + (2x - 3y)^2$.
(a) $16x^2$ (b) $18x^2$
(c) $12x^2$ (d) None of these.

16. Factorise: $45a^3b + 5ab^3 - 30a^2b^2$.
(a) $5ab\,(5a - b)^2$ (b) $7ab\,(5a - b)^2$
(c) $5ab\,(3a - b)^2$ (d) None of these.

17. Find the factors of $(a - b)^3 + (b - c)^3 + (c - a)^3$.
(a) $3(a + b)\,(b + c)\,(c + a)$
(b) $5(a - b)\,(b - c)\,(c - a)$
(c) $3(a - b)\,(b - c)\,(c - a)$
(d) None of these.

18. Factorise $a^2 + \frac{1}{a^2} + 3 - 2a - \frac{2}{a}$.
(a) $\left(a+\frac{1}{a}-1\right)\left(a-\frac{1}{a}+1\right)$
(b) $\left(a+\frac{1}{a}-1\right)\left(a+\frac{1}{a}+1\right)$
(c) $\left(a+\frac{1}{a}+1\right)\left(a+\frac{1}{a}+1\right)$
(d) $\left(a+\frac{1}{a}-1\right)\left(a+\frac{1}{a}-1\right)$.

19. If $x + \frac{1}{x} = 2$, find the value of $x^4 + \frac{1}{x^4}$.
(a) 2 (b) 4
(c) 6 (d) 8

20. If $\frac{x}{y}+\frac{y}{x} = 6$, find the value of $\frac{x^3}{y^3}+\frac{y^3}{x^3}$.
(a) 176 (b) 198
(c) 184 (d) None of these.

21. If $x + y + 2 = 0$, what will be the value of $\frac{x^2+y^2+z^2}{x^2-yz}$?
(a) 4 (b) 6
(c) 2 (d) 8

22. If $\left(x^3+\frac{1}{x^3}\right) = 52$, the value of $x + \frac{1}{x}$ is:
(a) 4 (b) 3
(c) 6 (d) 13

23. If $x = 3$ and $y = 4$, find the value of
$256x^4 + 160x^2y^2 + 25y^4$.
(a) 114967 (b) 50176
(c) 103976 (d) 914976

24. If $x + \frac{1}{x} = 2$, then $x^3 + \frac{1}{x^3}$ is equal to
(a) 64 (b) 14
(c) 8 (d) 2

25. If $\sqrt{x} + \frac{1}{\sqrt{x}} = 5$, what will be the value of $x^2 + \frac{1}{x^2}$.
(a) 927 (b) 727
(c) 527 (d) 627

26. If $x + \frac{1}{x} = 3$, the value of $x^6 + \frac{1}{x^6}$ is
(a) 927 (b) 414
(c) 364 (d) 322

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27. Factors of $a^2 + \frac{1}{4} + a$ will be

(a) $\left(a+\frac{1}{2}\right)\left(a-\frac{1}{2}\right)$ (b) $\left(a+\frac{1}{2}\right)^2$

(c) $\left(a+\frac{1}{2}\right)^3$ (d) $\left(a+\frac{1}{2}\right)\cdot a$

28. If $a + b + c = 0$, the value of $\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)$ is:

(a) 1 (b) 0

(c) –1 (d) 3

29. If $x + y + z = 9$ and $xy + yz + zx = 23$, the value of $x^3 + y^3 + z^3 - 3xyz$ is

(a) 108 (b) 207

(c) 669 (d) 729

30. If $x = \sqrt{3}$, the value of $x^4 + 2 + \frac{1}{x^4}$ will be

(a) $\frac{9}{100}$ (b) $\frac{81}{100}$

(c) $\frac{101}{9}$ (d) $\frac{100}{9}$

31. If $x + \frac{1}{y} = 1$ and $y + \frac{1}{z} = 1$, find the value of $z + \frac{1}{x}$.

(a) 2 (b) 1

(c) 0 (d) 3

32. Resolve into factors:

$(a + b)^2 - 2(a^2 - b^2) + (a - b)^2$.

(a) $6b^2$ (b) $2b^2$

(c) $4b^2$ (d) None of these.

33. When $(x^3 - 2x^2 + px - q)$ is divided by $x^2 - 2x - 3$ the remainder is $(x - 6)$. The values of p and q are:

(a) $p = -2, q = -6$ (b) $p = 2, q = -6$

(c) $p = -2, q = 6$ (d) $p = 2, q = 6$

34. Let $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$, where, $a_0, a_1, a_2, \ldots, a_n$ are constants. If $f(x)$ is divided by $ax - b$, the remainder is

(a) $f\left(\frac{b}{a}\right)$ (b) $f\left(\frac{-b}{a}\right)$

(c) $f\left(\frac{a}{b}\right)$ (d) $f\left(\frac{-a}{b}\right)$

35. If $(x^{3/2} - xy^{1/2} + x^{1/2}y - y^{3/2})$ is divided by $(x^{1/2} - y^{1/2})$, the quotient is

(a) $x + y$ (b) $x - y$

(c) $x^{1/2} + y^{1/2}$ (d) $x^2 - y^2$

36. When $4x^3 - ax^2 + bx - 4$ is divided by $x - 2$ and $x + 1$, the respective remainders are 20 and –13. Find the values of a and b.

(a) $a = 3, b = 2$ (b) $a = 5, b = 4$

(c) $a = 7, b = 6$ (d) $a = 9, b = 8$

37. When a polynomial $f(x)$ is divided by $x - 3$ and $x + 6$, the respective remainders are 7 and 22. What is the remainder when $f(x)$ is divided by $(x - 3)(x + 6)$?

(a) $\frac{-5}{3}x + 12$ (b) $\frac{-7}{3}x + 14$

(c) $\frac{-5}{3}x + 16$ (d) $\frac{-7}{3}x + 12$

38. If $(x - 1)$ is a factor of $Ax^3 + Bx^2 - 36x + 22$ and $2^B = 64^A$, find A and B.

(a) $A = 4, B = 16$ (b) $A = 6, B = 24$

(c) $A = 2, B = 12$ (c) $A = 8, B = 16$

EXPLANATORY ANSWERS

1. Ans. b)

Solution Let $p(x) = x^3 - 2ax^2 + ax - 1$

Since $x - 2$ is a factor of $p(x)$, we must have $p(2) = 0$

$\therefore (2)^3 - 2a(2)^2 + 2a - 1 = 0$

$\Rightarrow 8 - 8a + 2a - 1 = 0$

$\Rightarrow -6a = -7 \Rightarrow a = \frac{7}{6}$.

2. Ans. (a)

Solution Let $p(x) = x^3 + ax^2 - 2x + a + 4$

Since $x + a$, i.e. $x - (-a)$ is a factor of $p(x)$, we must have $p(-a) = 0$

$\Rightarrow (-a)^3 + a(-a)^2 - 2(-a) + a + 4 = 0$

$\Rightarrow -a^3 + a^3 + 2a + a + 4 = 0$

$\Rightarrow 3a + 4 = 0 \Rightarrow a = -\frac{4}{3}$.

3. Ans. (a)

Solution $\because (x - 1)$ is a factor of $f(x)$.

$\therefore$ By factor theorem, $f(1) = 0$

$\Rightarrow (1)^3 - k(1)^2 + 11(1) - 6 = 0$

$\Rightarrow 1 - k + 11 - 6 = 0$

$\Rightarrow -k + 6 = 0 \Rightarrow k = 6$.

4. Ans. (a)

Solution $f(x) = 5x^2 - 4x - 1$

$\therefore f(1) = 5(1)^2 - 4(1) - 1 = 0$.

5. Ans. (b)

Solution Let $f(x) = 2x^3 + mx^2 + nx - 14$.

Since $x - 1$ is a factor of $f(x)$.

$\therefore f(1) = 0$ [By factor theorem]

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$\Rightarrow 2(1)^3 + m(1)^2 + n(1) - 14 = 0$

$\Rightarrow 2 + m + n - 14 = 0 \Rightarrow m + n = 12$ ...(1)

Since $x + 2$, i.e. $x - (-2)$ is a factor of $f(x)$.

$\therefore f(-2) = 0$ [By factor theorem]

$\Rightarrow 2(-2)^3 + m(-2)^2 + n(-2) - 14 = 0$

$\Rightarrow -16 + 4m - 2n - 24 = 0 \Rightarrow 4m - 2n - 30 = 0$

$\Rightarrow 2m - n = 15$ ...(2)

Adding (1) and (2), we get $3m = 27 \Rightarrow m = 9$

Put $m = 9$ in (1), we get $9 + n = 12 \Rightarrow n = \mathbf{3}$.

6. Ans. (c)

Solution $f(x) = 2x^3 - ax^2 - (2a - 3)x + 2$?

If $x + 1$ i.e. $x - (-1)$ is a factor of $f(x)$, then $f(-1) = 0$ [By factor theorem]

$\Rightarrow 2(-1)^3 - a(-1)^2 - (2a - 3)(-1) + 2 = 0$

$\Rightarrow -2 - a + 2a - 3 + 2 = 0$

$\Rightarrow a - 3 = 0 \Rightarrow a = \mathbf{3}$.

7. Ans. (a)

Solution

$$
\begin{array}{r|l}
 & 4y^2 - 11y + 24 \\
\hline
y + 2 & 4y^3 - 3y^2 + 2y - 4 \\
 & 4y^3 + 8y^2 \\
 & - \quad - \\
\hline
 & -11y^2 + 2y - 4 \\
 & -11y^2 - 22y \\
 & + \quad + \\
\hline
 & 24y - 4 \\
 & 24y + 48 \\
 & - \quad - \\
\hline
 & -52
\end{array}
$$

$\therefore$ Quotient $= 4y^2 - 11y + 24$

Remainder $= \mathbf{-52}$.

8. Ans. (c)

Solution $16(x - y)^2 - 9(x + y)^2$

$= [4(x - y)]^2 - [3(x + y)]^2$

$= [4(x - y) - 3(x + y)][4(x - y) + 3(x + y)]$

$= (4x - 4y - 3x - 3y)(4x - 4y + 3x + 3y)$

$= \mathbf{(x - 7y)(7x - y)}$.

9. Ans. (b)

Solution $4x^2 + 12xy + 9y^2 - 8x - 12y$

$= [(2x)^2 + 2(2x)(3y) + (3y)^2] - 4(2x + 3y)$

$= (2x + 3y)^2 - 4(2x + 3y)$

$= \mathbf{(2x + 3y)(2x + 3y - 4)}$.

10. Ans. (b)

Solution $16x^2 - 72xy + 81y^2 - 12x + 27y$

$= (4x)^2 - 2(4x)(9y) + (9y)^2 - 3(4x - 9y)$

$= (4x - 9y)^2 - 3(4x - 9y)$

$= \mathbf{(4x - 9y)(4x - 9y - 3)}$.

11. Ans. (b)

Solution $(a + b)^2 - 14c(a + b) + 49c^2$

$= (a + b)^2 - 2(a + b).(7c) + (7c)^2$

$= \mathbf{(a + b - 7c)^2}$.

12. Ans. (c)

Solution $81x^2y^2 + 108xyz + 36z^2$

$= (9xy)^2 + 2(9xy)(6z) + (6z)^2$

$= \mathbf{(9xy + 6z)^2}$.

13. Ans. (a)

Solution $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c)(b + c - a)$

$= (a - b + c)^2 + 2(a - b + c)(b + c - a) + (b - c + a)^2$ [rearranging]

$= [(a - b + c) + (b - c + a)]^2 = (2a)^2 = \mathbf{4a^2}$.

14. Ans. (b)

Solution $9(3x + 5y)^2 - 12(3x + 5y)(2x + 3y) + 4(2x + 3y)^2$

$= [3(3x + 5y)]^2 - 2[3(3x + 5y)][2(2x + 3y)] + [2(2x + 3y)]^2$

$= [3(3x + 5y) - 2(2x + 3y)]^2$

$= (9x + 15y - 4x - 6y)^2 = \mathbf{(5x + 9y)^2}$.

15. Ans. (a)

Solution $(2x + 3y)^2 + 2(2x + 3y)(2x - 3y) + (2x - 3y)^2$

$= [2x + 3y) + (2x - 3y)]^2 = (4x)^2 = \mathbf{16x^2}$.

16. Ans. (c)

Solution $45a^3b + 5ab^3 - 30a^2b^2$

$= 5ab[9a^2 + b^2 - 6ab]$

$= 5ab[9a^2 - 6ab + b^2]$

$= 5ab[(3a)^2 - 2(3a)(b) + (b)^2]$

$= \mathbf{5ab[3a - b]^2}$.

17. Ans. (c)

Solution Suppose, $a - b = x$, $b - c = y$, $c - a = z$

$\therefore (a - b) + (b - c) + (c - a) = x + y + z$

$\Rightarrow 0 = x + y + z$

$\therefore x + y = -z$ ...(i)

$\therefore (x + y)^3 = (-z)^3$

or, $x^3 + y^3 + 3xy(x + y) = -z^3$

or, $x^3 + z^3 + z^3 + 3xy(-z) = -z^3$

[On substituting $x + y = -z$ from eq. (i)]

or, $x^3 + y^3 - 3xyz = -z^3$

or, $x^3 + y^3 + z^3 = 3xyz$

$\therefore (a - b)^3 + (b - c)^3 + (c - a)^3$

$= \mathbf{3(a - b)(b - c)(c -)}$

18. Ans. (d)

Solution $a^2 + \frac{1}{a^2} + 3 - 2a - \frac{2}{a}$

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$$= \left(a^2 + \frac{1}{a^2} + 2\right) - 2a - \frac{2}{a} + 1$$

$$= \left(a + \frac{1}{a}\right)^2 - 2\left(a + \frac{1}{a}\right) + 1$$

$$= x^2 - 2x + 1 \left[\text{suppose } a + \frac{1}{a} = x\right]$$

$$= (x - 1)^2$$

$$= \left(a + \frac{1}{a} - 1\right)^2.$$

19. Ans. (a)

Solution $x + \frac{1}{x} = 2 \Rightarrow \left(x + \frac{1}{x}\right)^2 = (2)^2$

$\therefore \quad x^2 + \frac{1}{x^2} + 2x.\ \frac{1}{x} = 4 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 2$

$\therefore \left(x^2 + \frac{1}{x^2}\right)^2 = (2)^2 \Rightarrow x^4 + \frac{1}{x^4} + 2x^2.\ \frac{1}{x^2} = 4$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 4$

$\therefore \quad x^4 + \frac{1}{x^4} = \mathbf{2}.$

20. Ans. (b)

Solution $\frac{x}{y} + \frac{y}{x} = 6 \Rightarrow \left(\frac{x}{y} + \frac{y}{x}\right)^3 = (6)^3$

$\therefore \quad \frac{x^3}{y^3} + \frac{y^3}{x^3} + 3\left(\frac{x}{y} + \frac{y}{x}\right) = 216$

$\therefore \quad \frac{x^3}{y^3} + \frac{y^3}{x^3} + 3 \times 6 = 216$

$\therefore \quad \frac{x^3}{y^3} + \frac{y^3}{x^3} = 216 - 18 = \mathbf{198}.$

21. Ans. (c)

Solution $\because x + y + z = 0 \Rightarrow (x + y + z)^2 = 0$

$\therefore \quad x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$

$\therefore \quad x^2 + y^2 + z^2 = -2(xy + yz + zx)$

$= -2[x(y + z) + yz]$

$= -2(x \times -x + yz)$

$(\because x + y + z = 0)$

$= 2(x^2 - yz)$

$\therefore \quad \frac{x^2 + y^2 + z^2}{x^2 - yz} = \mathbf{2}.$

22. Ans. (a)

Solution $\left(x + \frac{1}{x}\right)^3 = \left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$

$\therefore \quad \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} = 52$

$\Rightarrow \quad y^3 - 3y = 52$ where, $y = x + \frac{1}{x}$

i.e. $y^3 - 3y - 52 = 0$

Clearly $y = 4$, satisfies $y^3 - 3y - 52 = 0$

$\therefore \quad x + \frac{1}{x} = \mathbf{4}.$

23. Ans. (b)

Solution $256x^4 + 160x^2y^2 + 25y^4$

$= (16x^2)^2 + 2.16x^2 \times 5y^2 + (5y^2)^2$

$= (16x^2 + 5y^2)^2$

On substituting $x = 3$ and $y = 4$

$(16x^2 + 5y^2)^2 = (16 \times 3^2 + 5 \times 4^2)^2$

$= (16 \times 9 + 5 \times 16)^2$

$= (144 + 80)^2 = (224)^2$

$= \mathbf{50176}.$

24. Ans. (d)

Solution $x + \frac{1}{x} = 2 \Rightarrow \left(x + \frac{1}{x}\right)^3 = 23$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 2 = 8$

$\Rightarrow x^3 + \frac{1}{x^3} = \mathbf{2}.$

25. Ans. (c)

Solution $\sqrt{x} + \frac{1}{\sqrt{x}} = 5 \Rightarrow \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = (5)^2$

$\therefore \quad x + 2.\sqrt{x}.\frac{1}{\sqrt{x}} + \frac{1}{x} = 25$

$\therefore \quad 2 + x + \frac{1}{x} = 25 \Rightarrow x + \frac{1}{x} = 23$

$\therefore \quad \left(x + \frac{1}{x}\right)^2 = (23)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 529$

$\Rightarrow x^2 + \frac{1}{x^2} = \mathbf{527}.$

26. Ans. (d)

Solution $\left(x + \frac{1}{x}\right)^2 = 3^2 \Rightarrow x^2 + \frac{1}{x^2} = 7$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^3 = 7^3$

$\therefore \quad x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = 343$

$\Rightarrow \quad x^6 + \frac{1}{x^6} + 3 \times 7 = 343$

$\therefore \quad x^6 + \frac{1}{x^6} = 343 - 21 = \mathbf{322}.$

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27. Ans. (b)

Solution $a^2 + \frac{1}{4} + a = a^2 + \left(\frac{1}{2}\right)^2 + 2.a\left(\frac{1}{2}\right)$

$= \left(a+\frac{1}{2}\right)^2$.

28. Ans. (d)

Solution $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$\therefore \quad \frac{a^3}{abc}+\frac{b^3}{abc}+\frac{c^3}{abc} = 3$ or, $\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab} = \mathbf{3}.$

29. Ans. (a)

Solution $x^3 + y^3 + z^3 - 3xyz$

$= (x+ y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$

$= 9[(9)^2 - 3(23)] = 9[81 - 69]$

$= 9 \times 12 = \mathbf{108}.$

30. Ans. (d)

Solution $x^4 + 2 + \frac{1}{x^4} = (x^2)^2 + 2.x^2.\frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2$

$= \left(x^2+\frac{1}{x^2}\right)^2$

$\therefore$ On substituting $x = \sqrt{3}$

$= \left[(\sqrt{3})^2+\frac{1}{(\sqrt{3})^2}\right]^2$

$= \left(3+\frac{1}{3}\right)^2 = \left(\frac{10}{3}\right)^2$

$= \frac{\mathbf{100}}{\mathbf{9}}$.

31. Ans. (b)

Solution $x + \frac{1}{y} = 1 \Rightarrow x = 1 - \frac{1}{y} = \frac{y-1}{y}$

$\Rightarrow \frac{1}{x} = \frac{y}{y-1}$

and, $y + \frac{1}{z} = 1 \Rightarrow \frac{1}{z} = 1-y \Rightarrow z = \frac{1}{1-y}$

$\therefore \quad z + \frac{1}{x} = \frac{1}{1-y}+\frac{y}{y-1} = \frac{1}{1-y}-\frac{y}{1-y}$

$= \frac{1-y}{1-y} = \mathbf{1}.$

32. Ans. (c)

Solution $(a + b)^2 - 2(a^2 - b^2) + (a - b)^2$

$= (a + b)^2 - 2(a + b)(a - b) + (a - b)^2$

$= \{(a + b) - (a - b)\}^2 = (2b)^2 = \mathbf{4b^2}.$

33. Ans. (c)

Solution On actual division, remainder is $(p + 3)x - q$

$\therefore (p + 3)x - q = x - 6 \Rightarrow p + 3 = 1$ and $q = 6$

$\Rightarrow p = -2, q = \mathbf{6}.$

34. Ans. (a)

Solution $ax - b = 0 \Rightarrow x = \frac{b}{a}$

So, remainder $= f\left(\frac{b}{a}\right)$.

35. Ans. (a)

Solution $x^{3/2} - xy^{1/2} + x^{1/2}y - y^{3/2}$

$= x(x^{1/2} - y^{1/2}) + y(x^{1/2} - y^{1/2})$

$= (x + y)(x^{1/2} - y^{1/2})$

$\therefore \quad \frac{x^{3/2}-xy^{1/2}+x^{1/2}y-y^{3/2}}{x^{1/2}-y^{1/2}} = \mathbf{(x + y)}.$

36. Ans. (a)

Solution Let $f(x) = 4x^3 - ax^2 + bx - 4$. When the expression $f(x)$ is divided by $x - 2$, the remainder is

$f(2) = 4(2)^3 - a(2)^2 + b(2) - 4 = 20$ (given)

$\Rightarrow \quad 2b - 4a + 28 = 20 \Rightarrow 2a - b = 4$ ...(i)

Similarly, when the expression $f(x)$ is divided by $x - (-1)$, the remainder is

$f(-1) = 4 \times (-1)^3 - a(-1) + b(-1) - 4 = -13$ (given)

$\Rightarrow \quad -4 - a - b - 4 = -13$

$\Rightarrow \quad a + b = 5$ ...(ii)

Solving (i) and (ii), we get

$\mathbf{a = 3, b = 2.}$

37. Ans. (a)

Solution The function $f(x)$ is not known

Here, $a = 3$, $b = -6$

$A = 7$ $B = 22$

Required remainder

$= \frac{A-B}{a-b}x + \frac{Ba-Ab}{a-b}$

$= \frac{7-22}{3-(-6)}x + \frac{22\times3-7\times(-6)}{3-(-6)}$

$= -\frac{5}{3}x + \mathbf{12}.$

38. Ans. (c)

Solution Since $x - 1$ is a factor of $Ax^3 + Bx^2 - 36x + 22$

$\therefore A(1)^3 + B(1)^2 - 36(1) + 22 = 0 \Rightarrow A + B = 14$

and, $2^B = (2^6)^A \Rightarrow B = 6A$

$\therefore \quad A = 2, B = \mathbf{12}.$

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QUESTIONS FROM COMPETITIVE EXAMS

1. If $(a + b + 2c + 3d)(a - b - 2c + 3d) = (a - b + 2c - 3d) \times (a + b - 2c - 3d)$, then $2bc$ is equal to

(a) $3\ ad$ (b) $\frac{3}{2}$

(c) a^2d^2 (d) $\frac{3a}{2d}$

[Based on MAT 2003]

2. What is the value of the following expression?

$(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 - x)$

(a) $1 + x^{16}$ (b) $1 - x^{16}$

(c) $x^{16} - 1$ (d) $x^8 + 1$

[Based on MAT 2000]

3. The factors of $(x^4 + 7x^2y^2 + y^4)$ are

(a) $(x^2 + y^2 - 3xy)(x^2 + y^2 + 3xy)$

(b) $(x^2 - y^2 - 3xy)(x^2 - y^2 + 3xy)$

(c) $(x^2 - 3xy + y^2)(x^2 - 3xy - y^2)$

(d) None of these **[Based on FMS (Delhi) 2003]**

4. $(x^n - a^n)$ is divisible by $(x - a)$

(a) For all values of n

(b) Only for even values of n

(c) Only for odd values of n

(d) Only for prime values of n

[Based on FMS (Delhi) 2003]

5. Amar got a job of Rs 5 lakh per annum from the campus of a reputed management institute. He pays 30% of his salary as taxes and 30% of the remainder as rent of his apartment. What percent of his income is at his disposal?

(a) 50% (b) 51%

(c) 49% (d) 40%

[Based on SCMHRD Ent. Exam. 2003]

6. If $x^2 - 6x + a$ is divisible by $x - 2$, then a is equal to

(a) 8 (b) 6

(c) 0 (d) None of these

[Based on IMT Ghaziabad, 2002]

7. Which of the following expressions are exactly equal in value?

(a) $(3x - y)^2 - (5x^2 - 2xy)$

(b) $(2x - y)^2$

(c) $(2x + y)^2 - 2xy$

(d) $(2x + 3y)^2 - 8y(2x + y)$

(a) (a) and (b) only

(b) (a), (b) and (c) only

(c) (b) and (d) only

(d) (a), (b) and (d) only

(e) None of the above **[Based on IRMA, 2002]**

EXPLANATORY ANSWERS

1. Ans. (a)

Solution Given expression

$\Rightarrow (a + b)(a - b) - (a + b)(2c - 3d) + (2c + 3d)(a - b) - (2c + 3d)(2c - 3d)$

$= (a - b)(a + b) - (a - b)(2c + 3d) + (2c - 3d)(a + b) - (2c - 3d)(2c + 3d)$

$\Rightarrow (a + b)(2c - 3d) = (a - b)(2c + 3d)$

$\Rightarrow 2ac - 3ad + 2bc - 3bd = 2ac + 3ad - 2bc - 3bd$

$\Rightarrow 4bc = 6ad \Rightarrow 2bc = 3ad.$

2. Ans. (b)

Solution Given expression

$= (1 - x)(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$

$= (1 - x^2)(1 + x^2)(1 + x^4)(1 + x^8)$

$= (1 - x^4)(1 + x^4)(1 + x^8)$

$= (1 - x^8)(1 + x^8) = 1 - x^{16}.$

3. Ans. (a)

4. Ans. (a)

5. Ans. (d)

Solution $100 - 30\% = 70$

$70 - 30\% = 40.$

6. Ans. (a)

7. Ans. (d)

Solution (a) $= (3x - y)^2 - (5x^2 - 2xy)$

$= 9x^2 + y^2 - 6xy - 5x^2 + 2xy$

$= 4x^2 + y^2 - 4xy$

$= (2x - y)^2$

(b) $= (2x - y)^2$

(c) $= (2x + y)^2 - 2xy$

$= 4x^2 + y^2 + 2xy$

(d) $= (2x + 3y)^2 - 8y(2x + y)$

$= 4x^2 + 9y^2 + 12xy - 16xy - 8y^2$

$= 4x^2 + y^2 - 4xy = (2x - y)^2.$

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EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. If $x - 2$ is a factor of the polynomial $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$, find the value of a.
(a) $\frac{3}{2}$ (b) $\frac{2}{3}$
(c) $\frac{5}{3}$ (d) None of these.

2. Find h and k so that the polynomial $x^3 - hx^2 - 13x + k$ has $(x - 1)$ and $(x + 3)$ as factors.
(a) $h = 3, k = 15$ (b) $h = 5, k = 18$
(c) $h = 5, k = 17$ (d) None of these.

3. Divide the polynomial $z^4 - 81$ *by* $z - 3$ and find the quotient and the remainder.
(a) $z^3 + 3z^2 + 9z + 27, 0$
(b) $z^3 - 3z^2 + 9z - 27, 0$
(c) $z^3 + 3z^2 - 9z + 27, 0$
(d) None of these.

4. Resolve into factors: $25(3x + 1)^2 - 9y^2$.
(a)
(b)
(c) $(15x - 3y + 5)(15x + 3y + 5)$
(d) None of these.

5. Resolve into factors: $a^2 - 10b + 25b^2 + 2a - 10b$.
(a) $(a + 5b)(a + 5b + 2)$
(b) $(a - 5b)(a + 5b - 2)$
(c) $(a - 5b)(a - 5b + 2)$
(d) None of these.

6. Factorise: $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.
(a) $(a - b - c)^2$ (b) $(a + b + c)^2$
(c) $(a + b - c)^2$ (d) None of these.

7. If the expression $x^4 + ax^2 + bx + 5$ is exactly divisible by $x^2 + 3x + 2$, the values of a and b are:
(a) $\frac{-3}{2}, \frac{9}{2}$ (b) $\frac{9}{2}, \frac{3}{2}$
(c) $\frac{4}{3}, \frac{3}{2}$ (d) $\frac{-9}{2}, \frac{3}{2}$

8. If $x = 2^{1/3} - 2^{-1/3}$, what will be the value of $(2x^3 + 6x)$?
(a) 5 (b) 7
(c) 3 (d) 2

9. If $a = x + y$, $b = x - y$ and $c = 2x - 1$, the value of $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$ will be
(a) $(2y + 2x - 1)^2$ (b) $(3y + 2x - 1)^2$
(c) $(2y + 3x - 1)^2$ (d) $(4y + 2x - 1)^2$

10. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 77$, what will be the value of $ab + bc + ca$?
(a) 22 (b) 74
(c) 20 (d) 18

Test II

[Time: 10 min]

11. If $x + a$ is a factor of the polynomial $x^4 - a^2x^2 + 3x - a$, find the value of a.
(a) 0 (b) 1
(c) –2 (d) None of these.

12. If $x + 1$ and $x - 2$ are factors of the polynomial $x^3 + ax^2 + 2x + b$, find the values of a and b.
(a) $a = -5, b = 8$ (b) $a = -3, b = 4$
(c) $a = -7, b = 12$ (d) None of these.

13. Find the value of a if $x - 2$ is a factor of $x^3 - 3x + 5a$.
(a) $-\frac{2}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) None of these.

14. Resolve into factors: $4x^2 - 25y^2 + 30y - 9$.
(a) $(2x - 5y - 3)(2x + 5y - 3)$
(b) $(5x - 2y + 3)(2x + 5y - 3)$
(c) $(2x - 5y + 3)(2x + 5y - 3)$
(d) None of these.

15. If $x + y + z = 0$ the value of $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy}$ will be:
(a) –7 (b) 3
(c) 10 (d) 4

16. Let $\frac{a}{b} - \frac{b}{a} = x : y$. If $x - y = \frac{a}{b} + \frac{b}{a}$, then $x =$
(a) $\frac{a+b}{a}$ (b) $\frac{a+b}{b}$
(c) $\frac{a-b}{a}$ (d) None of these.

17. If $x^2 = y + z$, $y^2 = z + x$ and $z^2 = x + y$, the value of $\left(\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}\right)$ will be
(a) 4 (b) 3
(c) 1 (d) 2

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18. If $a + b + c = 10$ and $ab + bc + ac = 31$, the value of $a^2 + b^2 + c^2$ will be equal to

(a) 40 (b) 38

(c) 42 (d) 45

19. $\dfrac{(a-b)^2}{(b-c)(c-a)} + \dfrac{(b-c)^2}{(a-b)(c-a)} + \dfrac{(c-a)^2}{(a-b)(b-c)} =$

(a) 2 (b) 0

(c) 3 (d) 1

20. If $x = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$, $y = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$, find the value of $x^2 + y^2$.

(a) 16 (b) 18

(c) 14 (d) 12

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution Let $p(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Since $x - 2$ is a factor of $p(x)$, we must have $p(2) = 0$

$\therefore \quad (2)^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4 = 0$

$\Rightarrow \quad 32 - 48 - 8a + 12a + 4a + 4 = 0$

$\Rightarrow \quad 8a = 12 \Rightarrow a = \dfrac{12}{8} \Rightarrow a = \mathbf{\dfrac{3}{2}}.$

3. Ans. (c)

Solution

$$
\begin{array}{r|l}
 & z^3 + 3z^2 + 9z + 27 \\
\hline
z - 3 & z^4 - 81 \\
 & z^4 - 3z^3 \\
 & - \quad + \\
\hline
 & 3z^3 - 81 \\
 & 3z^3 - 9z^2 \\
 & - \quad + \\
\hline
 & 9z^2 - 81 \\
 & 9z^2 - 27z \\
 & - \quad + \\
\hline
 & 27z - 81 \\
 & 27z - 81 \\
\hline
 & 0
\end{array}
$$

$\therefore$ Quotient $= z^3 + 3z^2 + 9z + 27$

Remainder $= \mathbf{0}$.

5. Ans. (c)

Solution $(a^2 - 10ab + 25b^2 + 2a - 10b)$

$= (a)^2 - 2(a)(5b) + (5b)^2 + 2(a - 5b)$

$= (a - 5b)^2 + (5b)^2 + 2(a - 5b)$

$= \mathbf{(a - 5b)(a - 5b + 2).}$

7. Ans. (d)

Solution $x^2 + 3x + 2 = x^2 + 2x + x + 2$

$= x(x + 2) + 1(x + 2)$

$= (x + 2)(x + 1)$

Expression $x^4 + ax^2 + bx + 5$ is exactly divisible by $x^2 + 3x + 2$

$\therefore$ $(x + 2)$ and $(x + 1)$ are also the factors of $x^4 + ax^2 + bx + 5$. Therefore, on replacing x in the given expression either by -1 or by -2, the result will be zero.

When $x = -1$

$(-1)^4 + a(-1)^2 + b(-1) + 5 = 0$

$\Rightarrow \quad 1 + a - b + 5 = 0$

$\Rightarrow \quad a - b = -6 \qquad \ldots(i)$

when $x = -2$

$(-2)^4 + a(-2)^2 + b(-2) + 5 = 0$

$\Rightarrow \quad 16 + 4a - 2b + 5 = 0$

$\Rightarrow \quad 4a - 2b = -21 \qquad \ldots(ii)$

From equations (i) and (ii),

$(4a - 2b) - 2(a - b) = -21 - (2 \times -6)$

$\Rightarrow \quad 2a = -21 + 12 = -9$

$\therefore \quad a = \mathbf{-9/2.}$

On substituting $a = -\dfrac{9}{2}$ in equation (i), we get

$$b = 6 - \frac{9}{2} = \frac{3}{2}$$

$$\therefore \quad a = \mathbf{\frac{-9}{2}} \text{ and } \mathbf{b = \frac{3}{2}}.$$

9. Ans. (a)

Solution $\because (a + c - b)^2 = a^2 + c^2 + b^2 - 2ab + 2ac - 2bc$

$\therefore \quad a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$

$= (x + y + 2x - 1 - x + y)^2$

$= \mathbf{(2y + 2x - 1)^2.}$

11. Ans. (a)

Solution Let $p(x) = x^4 - a^2x^2 + 3x - a.$

Since $x + a$, i.e. $x - (-a)$ is a factor of $p(x)$, we must have $p(-a) = 0$

$\Rightarrow \quad (-a)^4 - a^2(-a)^2 + 3(-a) - a = 0$

$\Rightarrow \quad a^4 - a^4 - 3a - a = 0 \Rightarrow -4a = 0 \Rightarrow a = \mathbf{0.}$

13. Ans. (b)

Solution Let $f(x) = x^2 - 3x + 5a$

If $x - 2$ is a factor of $f(x)$, then $f(2) = 0$ [By factor theorem]

$\Rightarrow (2)^2 - 3(2) + 5a = 0 \Rightarrow 4 - 6 + 5a = 0$

$\Rightarrow -2 + 5a = 0 \Rightarrow 5a = 2$

$\Rightarrow a = \mathbf{\dfrac{2}{5}}.$

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15. Ans. (b)

Solution $x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz$

$$\Rightarrow \frac{x^3}{xyz}+\frac{y^3}{xyz}+\frac{z^3}{xyz} = 3$$

$$\therefore \quad \frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy} = \mathbf{3.}$$

17. Ans. (c)

Solution $x^2 = y + z \Rightarrow x + x^2 = x + y + z$

$$\Rightarrow x\,(1 + x) = x + y + z$$

$$y^2 = z + x \Rightarrow y + y^2 = x + y + z$$

$$\Rightarrow y\,(1 + y) = x + y + z$$

and, $z^2 = x + y \Rightarrow z + z^2 = x + y + z$

$$\Rightarrow z\,(1 + z) = x + y + z$$

$$\therefore \quad x\,(1 + x) = y\,(1 + y) = z\,(1 + z)$$

$$= x + y + z = k$$

$$\therefore \quad x = \frac{k}{1+x}\,,\ y = \frac{k}{1+y}\,,\ z = \frac{k}{1+z}$$

or, $\frac{x}{k} = \frac{1}{1+x}, \frac{y}{k} = \frac{1}{1+y}, \frac{z}{k} = \frac{1}{1+z}$

$$\therefore \quad \frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z} = \frac{x}{k}+\frac{y}{k}+\frac{z}{k} = \frac{x+y+z}{k}$$

$$= \frac{k}{k} = \mathbf{1.}$$

19. Ans. (c)

Solution The given expression

$$= \frac{(a-b)^3}{(a-b)(b-c)(c-a)}+\frac{(b-c)^3}{(a-b)(b-c)(c-a)}$$

$$+ \frac{(c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$$

$\because \quad (a - b) + (b - c) + (c - a) = 0$

$\therefore \quad (a - b)^3 + (b - c)^3 + (c - a)^3$

$= 3\,(a - b)\,(b - c)\,(c - a)$

$$\therefore \quad \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = \mathbf{3.}$$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (c)	**6.** (b)	**8.** (c)	**10.** (b)	**12.** (a)	**14.** (c)	**16.** (d)	**18.** (b)	**20.** (c)

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H. C. F. and L. C. M. of Polynomials

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INTRODUCTION

We have already learnt in the second chapter how to find the greatest common divisor (G.C.D.) or highest common factor (H.C.F.) and least common multiple (L.C.M.) of two integers. In this chapter we shall study how to find the G.C.D. and L.C.M. of polynomials which have integral coefficients.

Divisor

A polynomial $d(x)$ is said to be a divisor of polynomial $p(x)$ if $d(x)$ is a factor of $p(x)$, i.e. $p(x)$ can be written as

$$p(x) = d(x)\ q(x),$$

where, $q(x)$ is a polynomial.

For example, $(x - 2)$ is a divisor of the polynomial $(x - 2)^3\ (x + 3)$.

Common Divisor

A polynomial $d(x)$ is said to be a common divisor of the polynomials $p(x)$ and $q(x)$, if $d(x)$ is a factor of each of $p(x)$ and $q(x)$.

For example, $(x + 4)$ is a common divisor of the polynomials $(x + 4)^3\ (x - 2)\ (x + 3)$ and $(x + 4)\ (x - 2)^3\ (x + 5)$.

G.C.D. (H.C.F) of Two Polynomials

The G.C.D. of two polynomials $p(x)$ and $q(x)$ is the common divisor which has highest degree among all common divisors and which has the highest degree term coefficient as positive.

Illustration 1 Find the G.C.D. of $(3x - 2)\ (4x + 3)$; $(3x - 2)^2\ (2x + 5)$.

Solution Here, we find that $(3x - 2)$ is a polynomial which is a common divisor and has highest degree among all common divisors. Further, the coefficient of the highest degree term $(3x)$ is 3 which is positive. Hence, $(3x - 2)$ is the G.C.D. of the given polynomial.

G.C.D. by Factorisation Method

Step 1. Resolve the given polynomials $p(x)$ and $q(x)$ in the complete factored form.

Step 2. Find the G.C.D. of the numerical factors of $p(x)$ and $q(x)$.

Step 3. Find the factors of highest degree common to the two polynomials $p(x)$ and $q(x)$.

Step 4. The product of all such common factors and the G.C.D. of the numerical factors is the G.C.D. of the two given polynomials $p(x)$ and $q(x)$.

Illustration 2 Find the G.C.D. of $4 + 9x - 9x^2$ and $9x^2 - 24x + 16$.

Solution We have the factorisation

$$\begin{aligned} p(x) &= 4 + 9x - 9x^2 = -(9x^2 - 9x - 4) \\ &= -(9x^2 - 12x + 3x - 4) \\ &= -(3x\ (3x - 4) + 1\ (3x - 4) \\ &= -(3x + 1)\ (3x - 4) \end{aligned}$$

$$q(x) = 9x^2 - 24x + 16 = (3x - 4)^2.$$

$\because$ G.C.D. of numerical factors = 1

and the highest degree common divisor = $(3x - 4)$,

$\therefore$ required G.C.D. = $(3x - 4)$.

Illustration 3 Find the G.C.D. of $8\ (x^4 + x^3 + x^2)$ and $20\ (x^3 - 1)$.

Solution Here, $p(x) = 8\ (x^4 + x^3 + x^2) = 2^3 . x^2 . (x^2 + x + 1)$.

$q(x) = 20\ (x^3 - 1) = 2^2 . 5 . (x - 1) . (x^2 + x + 1)$.

$\because$ G.C.D. of numerical factors = 2^2

and the highest degree common divisor = $x^2 + x + 1$,

$\therefore$ required G.C.D. = $2^2\ (x^2 + x + 1)$

$= 4\ (x^2 + x + 1)$.

L.C.M. of Two Polynomials

We know that if a and b are two natural numbers, the product of a and b is equal to the product of their G.C.D. and L.C.M., i.e.

$a \times b$ = (G.C.D. or H.C.F. of a and b). (L.C.M. of a and b)

or, L.C.M. of a and $b = \dfrac{a \times b}{\text{G.C.D. of } a \text{ and } b}$

Similarly, if $p(x)$ and $q(x)$ are two polynomials, then,

$$\text{L.C.M. of } p(x) \text{ and } q(x) = \frac{p(x) \times q(x)}{\text{G.C.D. of } p(x) \text{ and } q(x)}$$

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Thus, L.C.M. of two polynomials

$$= \frac{\text{Product of two polynomials}}{\text{G.C.D. of the two polynomials}}$$

Note: L.C.M. of two or more given polynomials is a polynomial of smallest degree which is divided by each one of the given polynomials.

L.C.M. by Factorisation Method

Step 1. Resolve the given polynomials $p(x)$ and $q(x)$ in the complete factored form.

Step 2. The required L.C.M. is the product of each factor of $p(x)$ and $q(x)$ and if a factor is common, we take that factor which has the highest degree in $p(x)$ or $q(x)$.

Illustration 4 Find the L.C.M. of the polynomials

$(x + 2)^2 (x - 1) (x + 4)^2$

and, $(x + 4)^3 (x + 2) (x + 7)$

Solution We have, $p(x) = (x + 2)^2 (x - 1) (x + 4)^2$

$q(x) = (x + 4)^3 (x + 2) (x + 7)$

Take the highest powers of factors common to both $p(x)$ and $q(x)$ and remaining terms for L.C.M.

$\therefore$ L.C.M. $= (x + 4)^3 (x + 2)^2 (x - 1) (x + 7)$

Illustration 5 Find the L.C.M. of the polynomials $(2x^2 - 3x - 2)$ and $(x^3 - 4x^2 + 4x)$.

Solution We have, $p(x) = 2x^2 - 3x - 2$

$= (x - 2) (2x + 1)$

$q(x) = x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x - 2)^2$.

$\therefore$ H.C.F. $= (x - 2)$

Hence, L.C.M. $= \frac{p(x) \cdot q(x)}{\text{H.C.F.}} = \frac{x(x-2)^3 .(2x+1)}{(x-2)}$

$= x(x - 2)^2 (2x + 1)$.

OR

Taking the highest powers of factors common to both $p(x)$ and $q(x)$ and remaining terms for L.C.M., we have

L.C.M. $= x(x - 2)^2 (2x + 1)$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find the G.C.D. of $3 + 13x - 30x^2$; $25x^2 - 30x + 9$.
(a) $7x - 4$ (b) $5x - 3$
(c) $6x - 5$ (d) None of these.

2. Find the L.C.M. of the polynomials
$(x + 3)^2 (x - 2) (x + 1)^2$; $(x + 1)^3 (x + 3) (x + 4)$.
(a) $(x + 3) (x + 1)^2 (x + 4)$
(b) $(x + 3)^2 (x + 1) (x - 2)$
(c) $(x + 3)^2 (x + 1)^3 (x - 2) (x + 4)$
(d) None of these.

3. Find the L.C.M. of the polynomials
$2x^2 - 3x - 2$; $x^3 - 4x^2 + 4x$.
(a) $x(x - 2)^2 (2x + 1)$ (b) $x (x - 2) (2x + 1)^2$
(c) $x (x - 2) (2x + 1)$ (d) None of these.

4. Find the G.C.D. of $8(x^3 - x^2 + x)$; $28 (x^3 + 1)$
(a) $6(x^2 + x - 1)$ (b) $4(x^2 - x + 1)$
(c) $8(x^2 + 2x - 1)$ (d) None of these.

5. Find the G.C.D. of $4x^4 + y^4$, $2x^3 - xy^2 - y^3$ and $2x^2 + 2xy + y^2$.
(a) $2x^2 + 2xy + y^2$ (b) $2x^3 + 4xy + y^2$
(c) $3x^2 + 2xy + y^2$ (d) None of these.

6. Find the G.C.D. of $(x + 4)^2 (x - 3)^2$ and $(x - 1) (x + 4) (x - 3)^2$.
(a) $(x + 3) (x + 9)^2$ (b) $(x + 4) (x - 3)^3$
(c) $(x + 4) (x - 3)^2$ (d) None of these.

7. Find the L.C.M. of the polynomials
$16 - 4x^2$; $x^2 + x - 6$.
(a) $-4(x^2 - 4) (x + 3)$ (b) $6(x^2 - 4) (x + 4)$
(c) $8(x^2 - 6) (x + 3)$ (d) None of these.

8. Find the G.C.D. of $x^2 - 4$ and $x^3 - 5x + 6$.
(a) $x - 3$ (b) $x - 2$
(c) $x + 4$ (d) None of these.

9. The H.C.F. (Highest Common Factor) of two polynomials is $(y - 7)$ and their L.C.M. is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other.
(a) $y^2 - 12y + 35$ (b) $y^2 - 8y + 35$
(c) $y^2 - 14y + 45$ (d) None of these.

10. If $x - 4$ is the G.C.D. of $x^2 - x - 12$ and $x^2 - mx - 8$, find the value of m.
(a) 4 (b) 6
(c) 2 (d) None of these.

11. Find the G.C.D. of the polynomials
$(x - 2)^2 (x + 3) (x - 4)$; $(x - 2) (x + 2) (x - 5)$.
(a) $(x - 4)$ (b) $(x - 6)$
(c) $(x - 2)$ (d) None of these.

12. For what value of a, the G.C.D. of $x^2 - 2x - 24$ and $x^2 - ax - 6$ is $(x - 6)$?

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(a) 7 (b) 5
(c) 9 (d) None of these.

13. The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $36x^3(x + a)(x^3 - a^3)$ and $x^2(x - a)$, respectively. If $p(x) = 4x^2(x^2 - a^2)$, find $q(x)$.
(a) $12x^3(x^3 - a^3)$ (b) $6x^3(x^3 - a^3)$
(c) $9x^3(x^3 - a^3)$ (d) None of these.

14. If $(x - a)$ is the G.C.D. of $x^2 - x - 6$ and $x^2 + 3x - 18$, find the value of a.
(a) 3 (b) 6
(c) 9 (d) None of these.

15. The G.C.D. and L.C.M. of two polynomials $p(x)$ and $q(x)$ are $x(x + a)$ and $12x^2(x + a)(x^2 - a^2)$, respectively. If $p(x) = 4x(x + a)^2$, find $q(x)$.
(a) $3x^2(x^2 - a^2)$ (b) $5x^2(x^3 - a^3)$
(c) $4x^2(x^2 - a^2)$ (d) None of these.

16. Find the G.C.D. of $8(x^4 - 16)$ and $12(x^3 - 8)$.
(a) $6(x - 2)$ (b) $4(x - 2)$
(c) $8(x - 2)$ (d) None of these.

17. Find the L.C.M. of the polynomials $(x + 3)(-6x^2 + 5x + 4)$; $(2x^2 + 7x + 3)(x + 3)$.
(a) $-(x + 3)^2(3x - 4)(2x + 1)$
(b) $(x + 3)^2(3x - 4)(2x + 1)$
(c) $(x + 3)^2(3x + 4)(2x + 1)$
(d) None of these.

18. Find the G.C.D. of the polynomials $36x^2 - 49$ and $6x^2 - 25x + 21$.
(a) $8x - 9$ (b) $9x - 5$
(c) $6x - 7$ (d) None of these.

19. Find the L.C.M. of the polynomials
$30x^2 + 13x - 3$; $25x^2 - 30x + 9$.
(a) $-(5x - 3)^2(5x + 3)(6x - 1)$
(b) $(5x - 3)^2(5x + 3)(6x - 1)$
(c) $(5x + 3)^2(6x - 1)$
(d) None of these.

20. Find the G.C.D. of the polynomials $6x^2 + 11x$ and $2x^2 + x - 3$.
(a) $4x + 5$ (b) $2x - 3$
(c) $2x + 3$ (d) None of these.

21. The H.C.F. of two expressions p and q is 1. Their L.C.M. is
(a) $(p + q)$ (b) $(p - q)$
(c) $p\,q$ (d) $\frac{1}{pq}$

22. The H.C.F. of $(2x^2 - 4x)$, $(3x^4 - 12x^2)$ and $(2x^5 - 2x^4 - 4x^3)$ is
(a) $2x(x + 2)$ (b) $2x(2 - x)$
(c) $2x(x - 2)$ (d) $x(x - 2)$

23. The product of two non-zero expressions is $(x + y + z)p^3$. If their H.C.F. is p^2, their L.C.M. is
(a) $(x + y)p$ (b) $(y + 2)p$
(c) $(z + x)p$ (d) $(x + y + z)p$

24. If $(x - 1)$ is the H.C.F. of $x^2 - 1$ and $px^2 - q(x + 1)$, then
(a) $p = 2q$ (b) $q = 2p$
(c) $3p = 2q$ (d) $2p = 3q$

25. The L.C.M. of $(x^2 - y^2)$, $(x^3 - y^3)$, $(x^3 - x^2y - xy^2 + y^3)$ is
(a) $(x + y)(x - y)(x^2 + y^2 + xy)$
(b) $(x + y)(x - y)^2(x^2 + y^2 + xy)$
(c) $(x + y)(x - y)^2(x^2 + y^2 - xy)$
(d) $(x + y)^2(x - y)^2$

EXPLANATORY ANSWERS

1 Ans. (b)

Solution Here,

$$p(x) = 3 + 13x - 30x^2 = 3 + 18x - 5x - 30x^2$$
$$= 3(1 + 6x) - 5x(1 + 6x)$$
$$= (3 - 5x)(1 + 6x)$$
$$= -(5x - 3)(1 + 6x)$$
$$q(x) = 25x^2 - 30x + 9 = (5x - 3)^2$$

$\therefore$ G.C.D. of numerical factors = 1 and highest degree of common divisor

$$= \mathbf{(5x - 3) = G.C.D.}$$

2. Ans. (c)

Solution $p(x) = (x + 3)^2(x - 2)(x + 1)^2$
$q(x) = (x + 3)^3(x + 3)(x + 4)$
$\therefore$ L.C.M. $= \mathbf{(x + 3)^2(x + 1)^3(x - 2)(x + 4).}$

3. Ans. (a)

Solution We have,

$$p(x) = 2x^2 - 3x - 2 = 2x^2 - 4x + x - 2$$
$$= 2x(x - 2) + 1(x - 2) = (2x + 1)(x - 2)$$
$$q(x) = x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x - 2)^2$$

$\therefore$ L.C.M. $= \mathbf{x(x - 2)^2(2x + 1).}$

4. Ans. (b)

Solution We have the factorisation

$$p(x) = 8(x^3 - x^2 + x) = 2^3 . x . (x^2 - x + 1)$$
$$q(x) = 28(x^3 + 1) = 2^2 . 7 . (x + 1)(x^2 - x + 1)$$

$\therefore$ G.C.D. of numerical factors $= 2^2$

and, the highest degree common divisor $= x^2 - x + 1$.

Therefore, required G.C.D. $= 2^2(x^2 - x + 1)$
$= 4(x^2 - x + 1).$

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5. Ans. (a)

Solution Ist Expression $= (2x^2 + y^2)^2 - (2xy)^2$

$= (2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)$

2nd Expression $= (2x^3 - 2y^3) - y^2(x - y)$

$= 2(x - y)(x^2 + xy + y^2) - y^2(x - y)$

$= (x - y)(2x^2 + 2xy + 2y^2 - y^2)$

$= (x - y)(2x^2 + 2xy + y^2)$

Hence, G.C.D. $= \mathbf{2x^2 + 2xy + y^2}$.

6. Ans. (c)

Solution Let $p(x) = (x + 4)^2 (x - 3)^2$

and $q(x) = (x - 1)(x + 4)(x - 3)^2$

The highest degree common divisor is

$(x + 4)(x - 3)^2$

$\therefore$ The G.C.D. of given polynomial is

$\mathbf{(x + 4)(x - 3)^2}$.

7. Ans. (a)

Solution We have,

$p(x) = 16 - 4x^2 = 4(4 - x^2)$

$= 4(2 - x)(2 + x) = -4(x - 2)(x + 2)$

$q(x) = x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= x(x + 3) - 2(x + 3) = (x + 3)(x - 2)$.

$\therefore$ L.C.M. $= -4(x - 2)(x + 2)(x + 3)$

$= \mathbf{-4(x^2 - 4)(x + 3)}$.

8. Ans. (b)

Solution Let $p(x) = x^2 - 4 = x^2 - 2^2$

$= (x + 2)(x - 2)$

and, $q(x) = x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$= x(x - 2) - 3(x - 2) = (x - 2)(x - 3)$,

The highest degree common divisor is $x - 2$.

$\therefore$ The G.C.D. of $p(x)$ and $q(x)$ is $\mathbf{x - 2}$.

9. Ans. (a)

Solution H.C.F. $= (y - 7)$

L.C.M. $= y^3 - 10y^2 + 11y + 70$

$p(x) = y^2 - 5y - 14$

$q(x) = ?$

L.C.M. of two polynomials

$$= \frac{\text{Ist Polynomial} \times \text{IInd Polynomial}}{\text{H.C.F. of two polynomials}}$$

$$\therefore \text{L.C.M.} = \frac{p(x)\,q(x)}{\text{H.C.F.}}$$

$$y^3 - 10y^2 + 11y + 70 = \frac{(y^2 - 5y - 14) \times q(x)}{(y - 7)}$$

$$\therefore q(x) = \frac{(y - 7)(y^3 - 10y^2 + 11y + 70)}{(y^2 - 5y - 14)}$$

$= (y - 7)(y - 5)$

$= \mathbf{y^2 - 12y + 35}$.

10. Ans. (c)

Solution H.C.F. $= (x - 4)$

$p(x) = x^2 - x - 12 = (x - 4)(x + 3)$

$q(x) = x^2 - mx - 8$

As $(x - 4)$ is common in $p(x)$ and $q(x)$. Hence, $x - 4$ should be a factor of $x^2 - mx - 8$.

Thus, putting $(x - 4) = 0$ in $q(x)$, we get (Remainder theorem)

$q(x) = x^2 - mx - 8$

$q(4) = 4^2 - m \times 4 - 8 = 0 \Rightarrow 16 - 4m - 8 = 0$

$\Rightarrow m = \mathbf{2}$.

11. Ans. (c)

Solution Let $p(x) = (x - 2)^2 (x + 3)(x - 4)$

and, $q(x) = (x - 2)(x + 2)(x - 5)$

the highest degree common divisor of the given polynomials is $x - 2$.

$\therefore$ The G.C.D. is $x - 2$.

12. Ans. (b)

Solution Here, $p(x) = x^2 - 2x - 24$

and, $q(x) = x^2 - ax - 6$

Since $(x - 6)$ is the G.C.D. of $p(x)$ and $q(x)$, $(x - 6)$ is a factor of $p(x)$ and $q(x)$ both

$\Rightarrow p(6) = q(6)$

$\Rightarrow 36 - 2 \times 6 - 24 = 36 - a \times 6 - 6$

$\Rightarrow a = \mathbf{5}$.

13. Ans. (c)

Solution We know that,

$p(x) \times q(x) = \text{L.C.M.} \times \text{H.C.F.}$

$4x^2 (x^2 - a^2) \times q(x) = 36x^3 (x + a)(x^3 - a^3) x^2 (x - a)$

$$\Rightarrow q(x) = \frac{36x^5 (x^2 - a^2)(x^3 - a^3)}{4x^2 (x^2 - a^2)} = \mathbf{9x^3 (x^3 - a^3)}.$$

14. Ans. (a)

Solution Let $p(x) = x^2 - x - 6$

and, $q(x) = x^2 + 3x - 18$

Since $(x - a)$ is the G.C.D. of $p(x)$ and $q(x)$, $(x - a)$ is a divisor of $p(x)$ and $q(x)$ or $(x - a)$ is a factor of $p(x)$ and $q(x)$ both.

$\Rightarrow p(a) = 0$ and $q(a) = 0 \Rightarrow p(a) = q(a)$

$\Rightarrow a^2 - a - 6 = a^2 + 3a - 18$

$\Rightarrow 4a = 12 \Rightarrow a = \mathbf{3}$.

15. Ans. (a)

Solution $$q(x) = \frac{\text{L.C.M.} \times \text{H.C.F.}}{p(x)}$$

$$= \frac{12x^2 (x + a)(x^2 - a^2) x(x + a)}{4x(x + a)^2}$$

$= \mathbf{3x^2 (x^2 - a^2)}$.

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16. Ans. (b)

Solution $p(x) = 8(x^4 - 16)$
$= 4 \times 2 (x^2 + 4)(x + 2)(x - 2)$
$q(x) = 12(x^3 - 8)$
$= 4 \times 3 (x - 2)(x^2 + 2x + 4)$
Hence, G.C.D. = **$4(x - 2)$.**

17. Ans. (a)

Solution $p(x) = (x + 3)(-6x^2 + 5x + 4)$
$= (x + 3)(-6x^2 + 8x - 3x + 4)$
$= -(x + 3)(3x - 4)(2x + 1)$
$q(x) = (2x^2 + 7x + 3)(x + 3)$
$= (2x + 1)(x + 3)(x + 3)$
$\therefore$ L.C.M. = **$-(x + 3)^2 (3x - 4)(2x + 1)$.**

18. Ans. (c)

Solution $p(x) = 36x^2 - 49$
$= (6x)^2 - (7)^2 = (6x + 7)(6x - 7)$
$q(x) = 6x^2 - 25x + 21$
$= 6x^2 - 18x - 7x + 21$
$= 6x(x - 3) - 7(x - 3)$
$= (6x - 7)(x - 3)$;
$\therefore$ G.C.D. = **$(6x - 7)$.**

19. Ans. (b)

Solution $30x^2 + 13x - 3 = 30x^2 + 18x - 5x - 3$
$= 6x(5x + 3) - 1(5x + 3)$
$= (5x + 3)(6x - 1)$
$q(x) = 25x^2 - 30x + 9$
$= 25x^2 - 15x - 15x + 9$
$= 5x(5x - 3) - 3(5x - 3) = (5x - 3)^2$
L.C.M. = **$(5x - 3)^2 (5x + 3)(6x - 1)$.**

20. Ans. (c)

Solution $p(x) = 6x^2 + 11x + 3$
$= 6x^2 + 9x + 2x + 3$
$= 3x(2x + 3) + 1(2x + 3)$
$= (2x + 3)(3x + 1)$
$q(x) = 2x^2 + x - 3 = 2x^2 + 3x - 2x - 3$
$= (2x + 3)(x - 1)$
$\therefore$ G.C.D. = **$(2x + 3)$.**

21. Ans. (c)

Solution L.C.M. $= \dfrac{\text{Product of expressions}}{\text{H.C.F.}}$
$= \dfrac{pq}{1} =$ **pq.**

22. Ans. (d)

Solution $2x^2 - 4x = 2x(x - 2)$
$3x^4 - 12x^2 = 3x^2 (x^2 - 4) = 3x^2 (x - 2)(x + 2)$
$2x^5 - 2x^4 - 4x^3 = 2x^3 (x^2 - x - 2)$
$= 2x^3 (x - 2)(x + 1)$
$\therefore$ H.C.F. = **$x(x - 2)$.**

23. Ans. (d)

Solution L.C.M. $= \dfrac{\text{Product}}{\text{H.C.F.}} = \dfrac{(x+y+z)p^3}{p^2}$
$=$ **$(x + y + z)p$.**

24. Ans. (a)

Solution Since $(x - 1)$ is the H.C.F., it will divide each one of the given expressions. So, $x = -1$ will make each one zero
$\therefore$ $p \times 1^2 - q(1 + 1) = 0$ or **$p = 2q$.**

25. Ans. (b)

Solution $x^2 - y^2 = (x - y)(x + y)$,
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$,
$x^3 - x^2y - xy^2 + y^3 = x^2 (x - y) - y^2 (x - y)$
$= (x - y)(x^2 - y^2)$
$= (x - y)^2 (x + y)$
$\therefore$ L.C.M. = **$(x - y)^2 (x + y)(x^2 + y^2 + xy)$.**

■ EXERCISES FOR SELF-PRACTICE ■

Test I **[Time: 10 min]**

1. Find the G.C.D. of $22x(x + 1)^2$; $36x^2 (2x^2 + 3x + 1)$.
(a) $2x(x + 1)$ (b) $3x(x + 1)$
(c) $x(x + 1)$ (d) None of these.

2. Find the G.C.D. of $16 - 4x^2$; $x^2 + x - 6$.
(a) $x - 2$ (b) $x - 3$
(c) $x - 4$ (d) None of these.

3. For what value of a, the G.C.D. of $x^2 - 2x - 24$ and $x^2 - ax - 6$ is $(x - 6)$?
(a) 7 (b) 5
(c) 9 (d) None of these.

4. Find the G.C.D. of the polynomials
$x^3 + x^2 + x + 1$ and $x^4 - 1$.
(a) $(x - 1)(x^3 + 1)$ (b) $(x + 1)(x^2 + 1)$
(b) $(x + 1)(x^3 + 1)$ (d) None of these.

5. Determine the H.C.F. of $50\ a^2b^2c^3$, $80\ ab^3c^2$ and $120a^2b^3c$.

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(a) $20a^2b^2c^3$ (b) $10ab^2c$
(c) $80ab^2c$ (d) $10ab^2c^2$

6. The H.C.F. and L.C.M. of two algebraic expressions are $(a-5)$ and $2a^3+3a^2-44a-105$, respectively. If one of the expressions is $2a^2-3a-35$, the other will be
(a) $a^2-2a-15$ (b) $a^2-3a-10$
(c) $a^2-4a+15$ (d) $2a^2-5a+10$

7. What will be the H.C.F. of $(a-b)(a-2b)$, $(a-2b)(a-3b)$, and $(a-3b)(a-4b)$?
(a) $(a-2b)$ (b) $a-3b$
(c) $a-4b$ (d) None of these

8. If $(x-1)$ is the H.C.F. of ax^2+bx+c and bx^2+ax+c, the value of c is
(a) -1 (b) 1
(c) 0 (d) $a-b$

9. If $x+k$ is the H.C.F. of x^2+ax+b and x^2+cx+d, the value of k is
(a) $\frac{b+d}{a+c}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{a-b}{c-d}$ (d) $\frac{b-d}{a-c}$

10. Find the L.C.M. of the polynomials
$(x+3)^3(x-2)(x+1)^2$; $(x+1)^3(x+3)(x+4)$.
(a) $(x+3)^3(x+1)^3(x-2)(x+4)$
(b) $(x+3)^3(x-2)^3(x+1)$
(c) $(x+3)^2(x+1)^3(x+4)$
(d) None of these.

Test II

[Time: 10 min]

11. Find the G.C.D. of
$(2x-7)(3x+4)$; $(2x-7)^2(x+3)$.
(a) $(2x-5)$ (b) $(2x-9)$
(c) $(2x-7)$ (d) None of these.

12. Find the L.C:M. of the polynomials
$8(x^3-x^2+x)$; $28(x^3+1)$.
(a) $48(x^3+1)$ (b) $56x(x^3+1)$.
(b) $64(x^3+1)$ (d) None of these.

13. Find the G.C.D. of the polynomials $(x^2-9)(x-3)$ and x^2+6x+9.
(a) $(x+3)$ (b) $(x+5)$
(c) $(x-3)$ (d) None of these.

14. If $(x-a)$ is the G.C.D. of x^2-x-6 and $x^2+3x-18$, find the value of a.
(a) 3 (b) 2
(c) 1 (d) None of these.

15. The L.C.M. and H.C.F. of two polynomials $P(x)$ and $Q(x)$ are $36x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$, respectively. If $P(x)=4x^2(x^2-a^2)$, find $Q(x)$.
(a) $9x^3(x^3-a^3)$. (b) $6x^3(x^3-a^3)$
(c) $9x^3(x^3+a^3)$ (d) None of these.

16. The HCF of a^3+b^3 and $a^4+a^2b^2+b^4$ will be
(a) a^2+ab+b^2 (b) a^2-ab+b^2
(c) a^2+ab-b^2 (d) $a^4+a^2b^2+b^4$

17. The H.C.F. of x^3-y^3, $x^4+x^2y^2+y^4$ and $x^3y^2+x^2y^3+xy^4$ will be
(a) $x^4+x^2y^2+y^4$ (b) x^2+xy+y^2
(c) x^2-xy+y^2 (d) $x(x+y)$

18. The product of two non-zero expressions is $(x+y+z)p^3$. If their H.C.F. is p^2, then their L.C.M. is
(a) $(x+y)p$ (b) $(y+z)p$
(c) $(z+x)p$ (d) $(x+y+z)p$

19. If $(x-4)$ is the H.C.F. of x^2-x-12 and (x^2-mx-8), the value of m is
(a) 0 (b) 1
(c) 2 (d) 6

20. The L.C.M. and H.C.F. of two polynomials $P(x)$ and $Q(x)$ are $27x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$, respectively. If $P(x)=3x^2(x^2-a^2)$, find $Q(x)$.
(a) $4x^3(x^3-a^3)$ (b) $6x^3(x^3-a^3)$
(c) $9x^3(x^3-a^3)$ (d) None of these.

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution Here, $p(x)=22x(x+1)^2$
$q(x)=36x^2(2x^2+3x+1)=36x^2(2x^2+2x+x+1)$
$=36x^2[2x(x+1)+1(x+1)]$
$=36x^2(2x+1)(x+1)$
$\therefore$ G.C.D. of numerical factors = 2
and, the highest degree common divisor $=x(x+1)$
Hence, required G.C.D. = $\mathbf{2x(x+1)}$.

3. Ans. (b)

Solution Obviously, $(x-6)$ divides both $x^2-2x-24$ and x^2-ax-6.
So, $x=6$, must make each polynomial zero.
$\therefore$ $(6)^2-2\times6-24=(6)^2-6a-6$
or, $6a=30$ or $\mathbf{a=5}$.

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5. Ans. (b)

Solution $50a^2b^2c^3 = 5^2 \times 2 \times a^2 \times b^2 \times c^3$

$80ab^3c^2 = 5 \times 2^4 \times a \times b^3 \times c^3$

$120a^2b^3c = 3 \times 5 \times 2^3 \times a^2 \times b^3 \times c$

$\therefore$ Required H.C.F. $= 5 \times 2 \times a \times b^2 \times c$

$= \mathbf{10ab^2c}$.

7. Ans. (d)

Solution Since no factor is common to three given expressions, 1 will be a common factor and hence H.C.F. $= \mathbf{1}$.

9. Ans. (d)

Solution Since $x + k$ is the H.C.F. it will divide each one of the given expressions. So, $x = -k$ will make each one zero.

$\therefore \quad k^2 - ak + b = 0, \; k^2 - ck + d = 0.$

So, $k^2 - a\,k + b = k^2 - ck + d$

or, $k = \dfrac{b-d}{a-c}$.

11. Ans. (c)

Solution Here, we find that $(2x - 7)$ is a polynomial which is a common divisor and has highest degree among all common divisors. Further, the coefficient of the highest degree term $(2x)$ is 2 which is positive. Hence, $\mathbf{(2x - 7)}$ is the G.C.D. of the given polynomial.

13. Ans. (d)

Solution Let $p(x) = (x^2 - 9)(x - 3) = (x^2 - 3^2)(x - 3)$

$= (x + 3)(x - 3)^2$

and, $q(x) = x^2 + 6x + 9 = x^2 + 2x \times 3 + 3^2 = (x + 3)^2$

The highest degree common divisor of the given polynomials is $(x + 3)$.

$\therefore$ The G.C.D. is $\mathbf{(x + 3)}$.

15. Ans. (a)

Solution L.C.M. $= 36x^3 (x + a)(x^3 - a^3)$

$= 36x^3 (x + a)(x - a)(x^2 + a^2 + ax)$

H.C.F. $= x^2 (x - a)$

$P(x) = 4x^2 (x^2 - a^2) = 4x^2 (x - a)(x + a)$

$\therefore Q(x) = \dfrac{(\text{L.C.M.})(\text{H.C.F.})}{P(x)}$

$= \dfrac{36x^3(x+a)(x-a)(x^2+a^2+ax)x^2(x-a)}{4x^2(x-a)(x+a)}$

$= 9x^3 (x^2 + a^2 + ax)(x - a) = \mathbf{9x^3 (x^3 - a^3)}$.

17. Ans. (b)

Solution $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + xy + y^2)(x^2 - xy + y^2)$

and, $x^3y^2 + x^2y^3 + xy^4 = xy^2 (x^2 + xy + y^2)$

$\therefore$ Required H.C.F. $= \mathbf{x^2 + xy + y^2}$.

19. Ans. (c)

Solution Clearly, $x = 4$ will make each one of the given expressions zero. So, $16 - 4m - 8 = 0$ or, $\mathbf{m = 2}$.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (b)	**6.** (a)	**8.** (c)	**10.** (a)	**12.** (b)	**14.** (a)	**16.** (b)	**18.** (d)	**20.** (c)

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27 Linear Equations

LINEAR EQUATION IN ONE VARIABLE

A linear equation in one variable is an equation of the type $ax + b = 0$ or $ax = c$, where a, b, c are constants (real numbers), $a \neq 0$ and x is an unknown variable.

The solution of the linear equation $ax + b = 0$ is $x = -\frac{b}{a}$. We also say that $-\frac{b}{a}$ is the root of the linear equation $ax + b = 0$.

For example, the equation $2x + 3 = 0$ is a linear equation in one unknown variable x. Its solution or root is $-\frac{3}{2}$.

Linear Equation in Two Variables

A linear equation in two variables is an equation of the type $ax + by + c = 0$ or $ax + by = d$, where a, b, c and d are constants, $a \neq 0$, $b \neq 0$.

For example, $3x + 4y + 7 = 0$ and $2x - 3y = 5$ are linear equations in two variables x and y.

Methods of Solving two Simultaneous Linear Equations

1. Method of Substitution

Step 1. Find the value of one variable, say y, in terms of the other, i.e. x from either equation.

Step 2. Substitute the value of y so obtained in the other equation. Thus, we get an equation in only one variable x.

Step 3. Solve this equation for x.

Step 4. Substitute the value of x, thus obtained, in step 1 and find the value of y.

Illustration 1 Solve: $2x + 3y = 7$, $3x - y = 5$.

Solution The given equations are

$$x + y = 7 \quad ...(1)$$

and, $$3x - 2y = 11 \quad ...(2)$$

From (1), we get $y = 7 - x$.

Substituting $y = 7 - x$ in (2), we get

$$3x - 2(7 - x) = 11 \Rightarrow 3x - 14 + 2x = 11$$
$$\Rightarrow 5x = 25 \Rightarrow x = 5.$$

Substituting this value of x in (1), we get

$$5 + y = 7 \Rightarrow y = 7 - 5 \text{ or } y = 2.$$

Hence, $x = 5$, $y = 2$ is the required solution.

2. Method of Elimination

Step 1. Multiply both the equations by such numbers so as to make the coefficients of one of the two unknowns numerically the same.

Step 2. Add or subtract the two questions to get an equation containing only one unknown. Solve this equation to get the value of the unknown.

Step 3. Sustitute the value of the unknown in either of the two original equations. By solving that, the value of the other unknown is obtained.

Illustration 2 Solve: $-6x + 5y = 2$, $-5x + 6y = 9$.

Solution The given equations are

$$-6x + 5y = 2 \quad ...(1)$$
$$-5x + 6y = 9 \quad ...(2)$$

Multiply equation (1) by 6,

$$-36x + 30y = 12 \quad ...(3)$$

Multiply equation (2) by 5,

$$-25x + 30y = 45 \quad ...(4)$$

Subtract (4) from (3), we get

$$-11x = -33 \text{ or } x = 3.$$

Substitute $x = 3$ in (1), we get

$$-18 + 5y = 2 \text{ or } y = 4.$$

Hence, $x = 3$ and $y = 4$ is the required solution.

3. Short-cut Method

Let the two equations be

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2.$$

The solution is written as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}.$$

i.e. $$x = -\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \; y = -\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}.$$

Illustration 3 Solve: $3x + 2y = -25$, $-2x - y = 10$.

Solution The two equations are

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$3x + 2y = -25$

$-2x - y = 10.$

The solution is given by

$$\frac{x}{2\times10-(-1)\times(-25)} = \frac{y}{(-25)\times(-2)-3\times10} = \frac{-1}{3\times(-1)-(-2)\times2}$$

or, $\frac{x}{-5} = \frac{y}{20} = \frac{-1}{1}$ or $x = 5, y = -20$.

Consistent and Inconsistent Equations

When a system of equations has a solution, the system is called consistent. When a system of equations has no solution, the system is called inconsistent.

Test for Consistency

If we are given two linear equations

$a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$. Then,

(a) If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system will have exactly one solution and will be consistent.

Note: The graphs of such equations will have intersecting lines.

(b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the system is consistent and has infinitely many solutions.

Note: The graphs of such equations will have coincident lines.

(c) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system has no solution and is inconsistent.

Note: The graphs of such equations will have parallel lines.

Illustration 4 For what values of k will the system of equations $kx + 2y = 5$ and $3x + y = 1$ have a unique solution?

Solution If the given system of equations has a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{2}{1} \Rightarrow k \neq 6.$$

Hence, for $k \neq 6$, the given system of equations will have a unique solution.

Illustration 5 For what value of k, the system of equations $3x + 4y = 6$ and $6x + 8y = k$ represent, coincident lines?

Solution If the given system of equations represents coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{6} = \frac{4}{8} = \frac{6}{k}$$

$$\Rightarrow k = \frac{6\times8}{4} = 12.$$

Illustration 6 For what value of k the equations $9x + 4y = 9$ and $7x + ky = 5$ have no solution?

Solution The given system of equations will have no solution

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{9}{7} = \frac{4}{k} \neq \frac{9}{5}$

$$\Rightarrow 9k = 28 \text{ or, } k = \frac{28}{9}.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Solve: $x + y = 3$, $2x + 3y = 7$.

(a) $x = 2, y = 1$ (b) $x = 1, y = 2$

(c) $x = 3, y = 2$ (d) None of these.

2. Solve: $x + y = 7$, $3x - 2y = 11$

(a) $x = 7, y = 3$ (b) $x = 5, y = 2$

(c) $x = 5, y = 3$ (d) None of these.

3. Solve: $7x + 11y = 1$, $8x + 13y = 2$

(a) $x = -5, y = 3$ (b) $x = -7, y = 5$

(c) $x = -3, y = 2$ (d) None of these.

4. Solve: $8u - 3v = 5uv$

$6u - 5v = -2uv$

(a) $u = \frac{13}{23}, v = \frac{22}{33}$ (b) $u = \frac{11}{5}, v = \frac{13}{22}$

(c) $u = \frac{11}{23}, v = \frac{22}{31}$ (d) None of these.

5. Solve: $2(3u - v) = 5uv$, $2(u + 3v) = 5uv$

(a) $u = 2, v = 1$ (b) $u = 3, v = 2$

(c) $u = 4, v = 3$ (d) None of these.

6. Solve: $ax + by = a - b$, $bx - ay = a + b$

(a) $x = 2, y = -1$ (b) $x = -2, y = 1$

(c) $x = 1, y = -1$ (d) None of these.

7. Solve for x and y: $\frac{2x}{a} + \frac{y}{b} = 2$, $\frac{x}{a} - \frac{y}{b} = 4$.

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(a) $x = 2a, y = -2b$ (b) $x = 3a, y = -3b$
(c) $x = 3a, y = -2b$ (d) None of these.

8. Given: $4x + \frac{6}{y} = 15$ and $6x - \frac{8}{y} = 14$. Find 'p' if, $y = px - 2$.
(a) 5/3 (b) 7/3
(c) 4/3 (d) None of these.

9. Given: $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ and $\frac{3}{x} + \frac{2}{y} = 0$. Find '$a$' for which $y = ax - 4$.
(a) 1 (b) 0
(c) 2 (d) None of these.

10. If $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$ and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$, where $x \neq 0, y \neq 0$, the values of x and y are, respectively
(a) 0 and 1 (b) 1 and 2
(c) 2 and 3 (c) 1 and 3

11. The number of solutions of the equations
$x + \frac{1}{y} = 2$ and $2xy - 3y = -2$ is
(a) 0 (b) 1
(c) 2 (d) None of these.

12. The equations $ax + b = 0$ and $cx + d = 0$ are consistent, if
(a) $ad = bc$ (b) $ad + bc = 0$
(c) $ab - cd = 0$ (d) $ab + cd = 0$

13. The solution to the system of equations $|x + y| = 1$ and $x - y = 0$ is given by:
(a) $x = y = \frac{1}{2}$ (b) $x = y = -\frac{1}{2}$
(c) $x = 1, y = 0$
(d) $x = y = \frac{1}{2}$ or $x = y = \frac{-1}{2}$

14. In the system of equations $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$, $\frac{1}{y} + \frac{1}{z} = \frac{7}{12}$ and $\frac{1}{z} + \frac{1}{x} = \frac{3}{4}$, values of x, y and z will be
(a) 4, 3 and 2 (b) 3, 2 and 4
(c) 2, 3 and 4 (d) 3, 4 and 2

15. If $2x + y = 35$ and $3x + 4y = 65$, find the value of $\frac{x}{y}$.
(a) 2 (b) 1
(c) 3 (d) None of these.

16. I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son?
(a) 45 years, 15 years (b) 40 years, 10 years
(c) 60 years, 25 years (d) 50 years, 20 years

17. A man has some hens and cows. If the number of heads be 48 and number of feet equals 140, the number of hens will be
(a) 26 (b) 24
(c) 23 (d) 22

18. Rs. 49 were divided among 150 children. Each girl got 50 paise and a boy, 25 paise. How many boys were there?
(a) 100 (b) 102
(c) 104 (d) 105

19. Find the condition for the following system of linear equations to have a unique solution:
$ax + by = c$, $lx + my = n$.
(a) $an \neq cl$ (b) $am \neq bl$
(c) $bm \neq al$ (d) None of these.

20. Find the value of k for which the system:
$kx + 2y = 5$, $3x + y = 1$
has a unique solution.
(a) $k = 6$ (b) $k \neq 9$
(c) $k \neq 6$ (d) None of these.

21. Find the value of c for which the system
$cx + 3y = c - 3$, $12x + cy = c$
has infinitely many solutions.
(a) 6 (b) 8
(c) 4 (d) None of these.

22. Find the value of k for which the system of equations $2x + 2y = 5$, $3x + ky = 7$ has no solution.
(a) 5 (b) 7
(c) 3 (d) 9

23. Find the value of k for which the system of equations $2x + ky = 1$, $3x - y = 7$ has a unique solution.
(a) $k = -\frac{2}{3}$ (b) $k \neq \frac{2}{3}$
(c) $k \neq -\frac{2}{3}$ (d) $k = \frac{2}{3}$

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $x + y = 3$ ...(1)
$2x + 3y = 7$ ...(2)
$y = 3 - x$ [From Eq. (1)];
$\therefore 2x + 3(3 - x) = 7 \Rightarrow 2x + 9 - 3x = 7$
$\Rightarrow -x = -2$, i.e., $x = 2$
and, $y = 3 - 2 = 1$.
$\therefore x = 2$ and $y = 1$.

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2. Ans. (b)

Solution $x + y = 7$ ...(1)

$3x - 2y = 11$ ...(2)

$y = 7 - x$ [From Eq. (1)]

$\therefore \quad 3x - 2(7 - x) = 11 \Rightarrow 3x - 14 + 2x = 11$

$\Rightarrow 5x = 25$, i.e., $x = 5$

$\therefore$ **$x = 5$ and $y = 2$.**

3. Ans. (c)

Solution $7x + 11y = 1$ ...(1)

$8x + 13y = 2$ ...(2)

Multiplying Eq. (1) and Eq. (2) by 8 and 7 respectively, we have

$56x + 88y = 8$ and $56x + 91y = 14$

Subtracting the second equation from the first, we get $-3y = -6$, i.e. $y = 2$

From Eq. (1), $x = \dfrac{1-11(y)}{7} = \dfrac{1-11(2)}{7} = -3$

$\therefore$ **$x = -3$ and $y = 2$.**

4. Ans. (c)

Solution $8u - 3v = 5uv$,

$6u - 5v = -2uv$

Dividing both equations by uv, we get

$$\frac{8}{v} - \frac{3}{u} = 5 \text{ and } \frac{6}{v} - \frac{5}{u} = -2.$$

Putting $\dfrac{1}{v} = x$ and $\dfrac{1}{u} = y$, we get

$8x - 3y = 5$ ...(1)

and, $6x - 5y = -2$ ...(2)

Multiplying Eq. (1) and Eq. (2) by 6 and 8, respectively, we have $48x - 18y = 30$ and $48x - 40y = -16$

Subtracting the second equation from the first, we get $22y = 46$, i.e. $y = \dfrac{23}{11}$

From Eq. (1), $x = \left(\dfrac{5+3y}{8}\right) = \dfrac{\left(5+3\left(\frac{23}{11}\right)\right)}{8} = \dfrac{31}{22}$

$\therefore$ **$u = \dfrac{1}{y} = \dfrac{11}{23}$ and $v = \dfrac{1}{x} = \dfrac{22}{31}$.**

5. Ans. (a)

Solution $2(3u - v) = 5uv \Rightarrow \dfrac{6}{v} - \dfrac{2}{u} = 5$

$2(u + 3v) = 5uv \Rightarrow \dfrac{2}{v} + \dfrac{6}{u} = 5$

Putting $1/v = x$ and $1/u = y$, we get

$6x - 2y = 5$ ...(1)

$2x + 6y = 5$ ...(2)

Multiplying Eq. (1), and Eq. (2) by 3 and 1 respectively, we have

$18x - 6y = 15$ and $2x + 6y = 5$

Adding these two equations, we get

$20x = 20$, i.e., $x = 1$

From Eq. (1), $y = \left(\dfrac{6x-5}{2}\right) = \left(\dfrac{6\times1-5}{2}\right) = \dfrac{1}{2}$

$\therefore$ **$u = \dfrac{1}{y} = 2$ and $v = \dfrac{1}{x} = 1$.**

6. Ans. (c)

Solution $ax + by = a - b$ and $bx - ay = a + b$

Here, $a_1/a_2 = a/b$ and $b_1/b_2 = -b/a \Rightarrow \dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

therefore, a unique solution exists.

Writing the coefficients in the following array

$$\begin{matrix} b & a-b \\ -a & a+b \end{matrix} \qquad \begin{matrix} a & b \\ b & -a \end{matrix}$$

we get, $\dfrac{x}{b(a+b)+a(a-b)} = \dfrac{y}{b(a-b)-a(a+b)}$

$$= \frac{-1}{-a^2-b^2}$$

$$\Rightarrow \frac{x}{b^2+a^2} = \frac{y}{-b^2-a^2} = \frac{1}{a^2+b^2}$$

$\therefore$ **$x = 1$ and $y = -1$.**

7. Ans. (a)

Solution The given equations are

$$2\frac{x}{a} + \frac{y}{b} = 2 \quad ...(1)$$

$$\frac{x}{a} - \frac{y}{b} = 4 \quad ...(2)$$

Adding (1) and (2), we get

$$3\frac{x}{a} = 6 \Rightarrow 3x = 6a \Rightarrow x = \frac{6a}{3} = 2a$$

Putting $x = 2a$ in (1), we get

$$2.\frac{2a}{a} + \frac{y}{b} = 2 \Rightarrow 4 + \frac{y}{b} = 2$$

$$\Rightarrow \frac{y}{b} = 2 - 4 = -2$$

$$\Rightarrow y = -2b$$

Hence, the required solution is **$x = 2a$, $y = -2b$.**

8. Ans. (c)

Solution We have, $4x + \dfrac{6}{y} = 15$ ...(1)

$$6x - \frac{8}{y} = 14 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by 2, we get

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$$12x + \frac{18}{y} = 45 \quad ...(3)$$

$$12x - \frac{16}{y} = 28 \quad ...(4)$$

Subtracting equation (4) from equation (3), we get

$$\frac{34}{y} = 17 \Rightarrow y = \frac{34}{17} = 2$$

Putting $y = 2$ in equation (1), we get

$$4x + \frac{6}{2} = 15 \Rightarrow 4x + 3 = 15 \Rightarrow 4x = 15 - 3$$

$$\Rightarrow 4x = 12 \Rightarrow x = \frac{12}{4} = 3$$

Hence, the solution is $x = 3$, $y = 2$. Now,

$$y = px - 2 \Rightarrow 2 = p(3) - 2$$

$$\Rightarrow 3p = 4 \Rightarrow p = \frac{4}{3}.$$

9. Ans. (b)

Solution We have, $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ ...(1)

$$\frac{3}{x} + \frac{2}{y} = 0 \quad ...(2)$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = u$, we get

$$2u + \frac{2}{3}v = \frac{1}{6} \quad ...(3)$$

$$3u + 2v = 0 \quad ...(4)$$

Multiplying equation (4) by $\frac{1}{3}$, we get

$$u + \frac{2}{3}v = 0 \quad ...(5)$$

Subtracting equation (5) from equation (3), we get

$$u = \frac{1}{6}$$

Putting $u = \frac{1}{6}$ in (4), we get

$$3\left(\frac{1}{6}\right) + 2v = 0 \Rightarrow \frac{1}{2} + 2v = 0$$

$$\Rightarrow 2v = -\frac{1}{2} \Rightarrow v = -\frac{1}{4}$$

Now, $u = \frac{1}{6} \Rightarrow \frac{1}{x} = \frac{1}{6} \Rightarrow x = 6$

and, $v = -\frac{1}{4} \Rightarrow \frac{1}{y} = -\frac{1}{4} \Rightarrow y = -4$

Hence, the solution is $x = 6$, $y = -4$. Again,

$$y = ax - 4 \Rightarrow -4 = a(6) - 4$$

$$\Rightarrow 6a = -4 + 4 \Rightarrow 6a = 0$$

$$\Rightarrow a = \frac{0}{6} = \mathbf{0}.$$

10. Ans. (d)

Solution Multiplying each equation throughout by xy, we get

$$3x + 2y = 9 \quad \text{and,} \quad 9x + 4y = 21$$

On solving these equations, we get

$$\boldsymbol{x = 1, y = 3.}$$

11. Ans. (d)

Solution First equation gives $\frac{1}{y} = 2 - x$ or $y = \frac{1}{2-x}$

second equation is $y(2x - 3) = -2$

or, $\frac{2x-3}{2-x} = -2 \quad \therefore \quad 2x - 3 = -2(2 - x)$

or, $2x - 3 = -4 + 2x$

This gives $1 = 0$

This is impossible. So, these is no solution at all.

12. Ans. (a)

Solution The equations are consistent if

$$\frac{a}{c} = \frac{b}{d} \quad \text{i.e. if} \quad \boldsymbol{ad = bc.}$$

13. Ans. (d)

Solution $|x + y| = 1 \Leftrightarrow x + y = 1$

or $-(x + y) = 1$ i.e., $x + y = -1$

Solving $x + y = 1$, $x - y = 0$, we get

$$x = \frac{1}{2} \text{ and } y = \frac{1}{2}$$

Solving $x + y = -1$ and $x - y = 0$, we get

$$x = -\frac{1}{2} \text{ and } y = \frac{-1}{2}$$

$$\therefore \quad x = y = \pm \frac{\mathbf{1}}{\mathbf{2}}.$$

14. Ans. (b)

15. Ans. (c)

Solution We have, $2x + y = 35$ ...(1)

$$3x + 4y = 65 \quad ...(2)$$

From equation (1), $y = 35 - 2x$

Putting this value of y in (2), we get

$$3x + 4(35 - 2x) = 65 \Rightarrow 3x + 140 - 8x = 65$$

$$\Rightarrow -5x = 65 - 140 = -75$$

$$\Rightarrow x = \frac{-75}{-5} = 15$$

Substituting this value of x in $y = 35 - 2x$, we get

$$y = 35 - 2(15) = 35 - 30 = 5$$

$$\therefore \quad \frac{x}{y} = \frac{15}{5} = \mathbf{3}.$$

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16. Ans. (a)

Solution Suppose, that I am x years old and my son is y years old. Then, according to the first condition of the problem,

$$x = 3y \quad ...(1)$$

Five years later, my age = $(x + 5)$ years and my son's age = $(y + 5)$ years.

Then, according to the second condition of the problem,

$$x + 5 = 2\frac{1}{2}(y + 5) \quad ...(2)$$

Putting $x = 3y$ from (1) in (2), we get

$$3y + 5 = 2\frac{1}{2}(y + 5) \Rightarrow 3y + 5 = \frac{5}{2}(y + 5)$$
$$\Rightarrow 6y + 10 = 5(y + 5)$$
[Multiplying both sides by 2]
$$\Rightarrow 6y + 10 = 5y + 25$$
$$\Rightarrow 6y - 5y = 25 - 10$$
$$\Rightarrow y = 15$$

Putting $y = 15$ in (1), we get $x = 3(15) = 45$

Hence, I am **45** years old and my son is **15** years old, at present.

17. Ans. (a)

Solution Let there be x hens and y cows

Then, $x + y = 48$ ...(i)

and, $2x + 4y = 140$ ...(ii)

Solving (i) and (ii), we get $x =$ **26.**

18. Ans. (c)

Solution Let the number of boys be x and the number of girls be y

Then, $x + y = 150$

$$\frac{25}{100}x + \frac{50}{100}y = 49$$

Solving $x + y = 150$ and $x + 2y = 196$, we get $x =$ **104.**

19. Ans. (b)

Solution We have, $ax + by = c$ ...(1)

$lx + my = n$ ...(2)

Here, $a_1 = a, b_1 = b, c_1 = c$

$a_2 = l, b_2 = m, c_2 = n$

If the given system of linear equations has a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{a}{l} \neq \frac{b}{m} \Rightarrow \boldsymbol{am \neq bl}$$

This is the required condition.

20. Ans. (c)

Solution We have, $kx + 2y = 5$ ...(1)

$3x + y = 1$ ...(2)

Here, $a_1 = k, b_1 = 2, c_1 = 5$

$a_2 = 3, b_2 = 1, c_2 = 1.$

The given system of equations will have a unique solution

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. if $\frac{k}{3} \neq \frac{2}{1}$

i.e. if $\frac{k}{3} \neq 2$ i.e. if $\boldsymbol{k \neq 6.}$

21. Ans. (a)

Solution For the equation, $cx + 3y = c - 3$ and $12x + cy = c$ to have an infinite number of solutions we must have,

$$\frac{c}{12} = \frac{3}{c} = \frac{(c-3)}{c} \Rightarrow \frac{c}{12} = \frac{3}{c} \text{ and } \frac{c}{12} = \frac{(c-3)}{c}$$
$$\Rightarrow c^2 = 36 \text{ and } c^2 = 12c - 36$$
$$\Rightarrow c = \pm 6 \text{ and } (c - 6)^2 = 0$$
$$\Rightarrow c = \pm 6 \text{ and } c = 6$$
$$\Rightarrow c = \mathbf{6.}$$

22. Ans. (c)

Solution For no solution to exist we need

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{2}{3} = \frac{2}{k}, \text{ i.e. } k = \mathbf{3.}$$

23. Ans. (c)

Solution For a unique solution to exist we require

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{3} \neq \frac{k}{-1}, \text{ i.e. } k \neq -\frac{\mathbf{2}}{\mathbf{3}}.$$

QUESTIONS FROM COMPETITIVE EXAMS

1. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p + q + r \neq 0$?

$$x + 2y - 3z = p$$
$$2x + 5y - 11z = q$$
$$x - 2y + 7z = r$$

(a) $5p - 2q - r = 0$ (b) $5p + 2q + r = 0$
(c) $5q + 2q - r = 0$ (c) $5q - 2q + r = 0$

[Based on CAT 2003]

2. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs 20 on a stamdard bag and Rs 30 on a deluxe bag. Every bag must be processed on machine A and

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on machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours/bag)	
	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximises the profit?

(a) Standard 75 bags, Deluxe 80 bags
(b) Standard 100 bags, Deluxe 60 bags
(c) Standard 50 bags, Deluxe 100 bags
(d) Standard 60 bags, Deluxe 90 bags

[Based on CAT 2003]

3. If x and y are integers then the equation

$$5x + 19y = 64 \text{ has}$$

(a) no solution for $x < 300$ and $y < 0$
(b) no solution for $x > 250$ and $y > -100$
(c) a solution for $250 < x < 300$
(d) a solution for $-59 < y < -56$

[Based on CAT 2004]

4. Village A has a population of 6800, which is decreasing at the rate of 120 per year. Village B has a population of 4200, which is increasing at the rate of 80 per year. In how many years will the population of the two villages be equal?

(a) 9 (b) 11
(c) 13 (d) 16

[Based on MAT 2004]

5. If the system of equations $2x + 3y - 5 = 0$, $4x + ky - 10 = 0$ has an infinite number of solutions, then

(a) $k = 3/2$ (b) $k \neq 3/2$
(c) $k \neq 6$ (d) $k = 6$

[Based on MAT 2003]

6. 3 chairs and 2 tables cost Rs 700, while 5 chairs and 3 tables cost Rs 1100. What is the cost of 2 chairs and 2 tables ?

(a) Rs 300 (b) Rs 350
(c) Rs 450 (d) Rs 600

[Based on MAT 2003]

7. Ram and Mohan are friends. Each has some money. If Ram gives Rs 30 to Mohan, then Mohan will have twice the money left with Ram. But, If Mohan gives Rs. 10 to Ram, then Ram will have thrice as much as is left with Mohan. How much money does each have?

(a) Rs 62, Rs 34 (b) Rs 6, Rs 2
(c) Rs 170, Rs 124 (d) Rs 43, Rs 26

[Based on MAT 2003]

8. In an examination, a student attempted 15 questions correctly and secured 40 marks. If there were two types of questions (2 marks and 4 marks questions), how many questions of 2 marks did he attempt correctly?

(a) 5 (b) 10
(c) 20 (d) 40

[Based on MAT 2003]

9. In a zoo, there are rabbits and pigeons. If their heads are counted, these are 90 while their legs are 224. Find the number of pigeons in the zoo.

(a) 70 (b) 68
(c) 72 (d) 22

[Based on MAT 2001]

10. Shyam had 85 currency notes in all, some of which were of Rs. 100 denomination and the remaining of Rs. 50 denomination. The total amount of all these currency notes was Rs. 5000. How much amount in rupees did he have in the denomination of Rs. 50?

(a) 3500 (b) 70
(c) 15 (d) 1500

11. A florist was asked to make a bouquet worth exactly Rs 1000 with 100 sticks of roses of three colours—Pink, Yellow and Red. While Pink roses cost Re 0.50 a stick, Red roses cost Rs 10.00 per stick and Yellow roses cost Rs 50.00 per stick. How many Red roses did the florist use in the bouquet ?

(a) 1 (b) 5
(c) 80
(d) Several combinations are possible

[Based on FMS (Delhi) 2004]

12. A lady went to the market with a few one-rupee notes and a few 20-paise coins. When she returned, she had as many one-rupee notes as she originally had 20-paise coins and vice versa. She actually came back with about one-third of what she had gone with. How much did she spend in the market?

(a) Rs 14.40 (b) Rs 14.70
(c) Rs 15.50 (d) Rs 17.40

[Based on FMS (Delhi) 2004]

13. A student was asked to divide a number by 17/8. Instead, he actually multiplied it by 17/8 and hence got 225 more than the expected answer. What was the expected answer?

(a) 126 (b) 136
(c) 64 (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

14. A tells B "If you give me Rs 400, then I shall have 25% more than what you have". B tells A, "If you give me Rs 200, then I shall have three-and-a-half times as much as you have." How much does B have?

(a) Rs 1200 (b) Rs 900
(c) Rs 800 (d) Rs 1600

[Based on IIT Joint Man. Ent. Test 2004]

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If $-59 < y < -56$ i.e. $y = -58, -57$, then x cannot be an integer.

Hence, (d) is also

Therefore (c) is true.

4. Ans. (c)

Solution Let the population of the two villages A and B will be equal after x years.

$\therefore \quad 6800 - 120x = 4200 + 80x \quad \Rightarrow 200x = 2600$

$\Rightarrow x = \mathbf{13}.$

5. Ans. (d)

6. Ans. (d)

Solution $3C + 2T = 700$

$5C + 3T = 1100 \Rightarrow 9C + 6T = 2100$

$10C + 6T = 2200 \Rightarrow C = 100, T = 200$

$\therefore \quad 2C + 2T = 200 + 400 = \mathbf{600}.$

7. Ans. (a)

Solution Suppose Mohan has Rs y and Ram has Rs x

$\therefore \quad 2(x - 30) = y + 30$

$x + 10 = 3(y - 10) \Rightarrow 2x - y = 90$ and $x - 3y = -40$

$\Rightarrow 6x - 3y = 270$ and $x - 3y = -40$

$\Rightarrow 5x = 310 \Rightarrow \mathbf{x = 62, y = 34.}$

8. Ans. (b)

Solution Suppose number of 2-mark questions $= x$

Suppose number of 4-mark questions $= y$

$x + y = 15$

$2x + y = 40 \Rightarrow x = \mathbf{10}, y = 5.$

9. Ans. (b)

Solution $R + P = 90$

$4R + 2P = 224$

$\therefore \quad R = 22, P = \mathbf{68}.$

10. Ans. (a)

Solution Let number of 100-rupee notes be x and number of 50-rupee notes be y.

$\therefore \quad x + y = 85$

$100x + 50y = 5000 \Rightarrow x = 15, y = 70$

$\therefore$ The amount that we had in the denomination of Rs. $50 = 50y = 50 \times 70 = \mathbf{Rs.\ 3500}$.

11. Ans. (a)

Solution

$$P + R + Y = 100 \quad \text{...(i)}$$

$$0.50P + 10R + 50Y = 1000 \quad \text{...(ii)}$$

From (i) $R = 100 - (P + Y)$

$\Rightarrow \quad 0.50P + 10(100 - P - Y) + 50Y = 1000$

or $\quad 0.50P + 1000 - 10P - 10Y + 50Y = 1000$

or $\quad 9.5P = 40Y \quad$ or $\quad \dfrac{P}{Y} = \dfrac{40}{9.5} = \dfrac{80}{19}$

$\therefore$ The florist should use

80 sticks of Pink	= Rs 40
19 Sticks of Yellow	= Rs 950
1 stick of Red	= Rs 10
100 sticks	= **Rs 1000**

12. Ans. (a)

Solution Originally, the lady had, say, x One-rupee notes and y 20-paise coins.

On returning, she had y one-rupee notes and x 20-paise coins.

The balance was Rs $\dfrac{1}{3}\left(x + \dfrac{y}{5}\right)$

$\therefore \quad \left(x + \dfrac{y}{5}\right) - \left(y + \dfrac{x}{5}\right) = \dfrac{1}{3}\left(x + \dfrac{y}{5}\right)$ ...(i)

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{13}{7}$

If we take $x = 13$ and $y = 7$, then originally she had Rs 14.40 with her.

$\Rightarrow$ On returning, she had Rs 9.60

$\therefore$ The lady spent Rs 4.80 or a multiple of Rs 4.80, i.e. Rs 14.40, which is alternative **(a).**

13. Ans. (b)

Solution $x \times \dfrac{17}{8} - \dfrac{x}{17/8} = 225 \quad \Rightarrow \dfrac{17x}{8} - \dfrac{8x}{17}$

$\Rightarrow 225x = 136 \times 225 \quad \Rightarrow x = \mathbf{136}.$

14. Ans. (a)

Solution Let A had Rs x and B had Rs y in the beginning.

If B gives, Rs 400 to A, then

$$x + 400 = \frac{5}{4}(y - 400) \Rightarrow 4x - 5y = -3600 \quad \text{...(i)}$$

If A gives Rs 200 to B, then

$$y + 200 = \frac{7}{2}(x - 200) \Rightarrow 7x - 2y = 1800 \quad \text{...(ii)}$$

Solving (i) and (ii), we get

$x = 600, y = \mathbf{1200}.$

15. Ans. (d)

Solution $\dfrac{120}{x}$ of the planned distance.

16. Ans. (d)

Solution Suppose she had x apples in the beginning.

Sold to the first customer $= \dfrac{x}{2} + \dfrac{1}{2} = \dfrac{x+1}{2}$

Sold to the second customer $= \dfrac{x+1}{4} + \dfrac{1}{2}$

$= \dfrac{2x+6}{8} = \dfrac{x+3}{4}$

Sold to the third customer $= \dfrac{x+3}{8} + \dfrac{1}{2}$

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$$= \frac{2x+6+8}{16} = \frac{x+7}{8}$$

Sold to the fourth customer $= \frac{x+7}{16} + \frac{1}{2}$

$$= \frac{x+15}{16}$$

$$\therefore \quad x - \left[\frac{x+1}{2} + \frac{x+3}{4} + \frac{x+7}{8} + \frac{x+15}{16}\right] = 15$$

$$\Rightarrow \quad x - \left[\frac{15x+49}{16}\right] = 15$$

$$\Rightarrow \quad x = 240 + 49 = \mathbf{289.}$$

17. Ans. (d)

Solution The two lines coincide.

18. Ans. (a)

Solution Let the number of students in halls P and Q be x and y respectively.

Hall P	Hall Q	
x	y	
$x - 10$	$y + 10$	
$\therefore \quad x - 10 = y + 10 \Rightarrow x - y = 20$		...(i)
Hall P	Hall Q	

$$x + 20 \qquad y - 20$$

$$\therefore \quad x + 20 = 2(y - 20) \Rightarrow x - 2y = -60 \qquad \text{...(ii)}$$

Solving (i) and (ii), we get

$$\mathbf{x = 100, y = 80.}$$

19. Ans. (d)

20. Ans. (b)

21. Ans. (c)

Solution Let the original amount be Rs x and y paise and into changed amount = Rs y and x paise

$$3(100x + y) = 100y + x - 50$$

$$y = \frac{299x+50}{97}$$

By putting the value of x from options we get (2) option is correct.

22. Ans. (c)

Solution Suppose there were x packages originally.

$$\frac{x}{2} - \frac{2x}{5} = 3 \Rightarrow \frac{5x-4x}{10} = 3 \Rightarrow \frac{x}{10} = 3$$

$$x = 30$$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Solve: $4x - 3y = 1$, $x + 2y = 3$.
(a) $x = 1, y = 1$ (b) $x = 1, y = 2$
(c) $x = 2, y = 1$ (d) None of these.

2. Solve: $\frac{11}{u} - \frac{7}{v} = 1$, $\frac{9}{u} - \frac{4}{v} = 6$.
(a) $u = \frac{1}{3}, v = \frac{1}{2}$ (b) $u = \frac{1}{2}, v = \frac{1}{3}$
(c) $u = \frac{2}{3}, v = \frac{1}{3}$ (d) None of these.

3. Solve: $\sqrt{2}x - \sqrt{3}y = 0$, $\sqrt{5}x - \sqrt{2}y = 0$.
(a) $x = 0, y = 0$ (b) $x = 0, y = 1$
(c) $x = 1, y = 2$ (d) None of these.

4. Solve: $x + y = a + b$, $ax - by = a^2 - b^2$.
(a) $x = b, y = a$ (b) $x = a + b, y + b$
(c) $x = a, y = b$ (d) None of these.

5. Find the value of k for which the system $kx + 2y = 5$, $3x + y = 1$ has no solution.
(a) 4 (b) 6
(c) 8 (d) None of these.

6. The numerator of a fraction is 4 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 1, the denominator is eight times the numerator. Find the fraction.
(a) $\frac{5}{7}$ (b) $\frac{3}{5}$
(c) $\frac{3}{7}$ (d) None of these.

7. A number consists of two digits whose sum is 10. If 72 be subtracted from the number, the digits are reversed. Find the number.
(a) 91 (b) 93
(c) 95 (d) None of these.

8. Find the value of k for which the system of equations $kx + 2y = 5$, $3x + y = 1$ has unique solution.
(a) $k \neq 6$ (b) $k \neq 7$
(c) $k \neq 8$ (d) None of these.

9. The system of equations $3x + y - 1 = 0$ and $6x + 2y - 2 = 0$
(a) has $x = 1$ and $y = 2$ as solutions
(b) has $x = -1$ and $y = -2$ as solutions
(c) does not have a solution
(d) has infinitely many solutions.

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10. The total cost of 6 books and 4 pencils is Rs. 34 and that of 5 books and 5 pencils is Rs. 30. The cost of each book and pencil (in Rs.) respectively is

(a) 1 and 5 (b) 5 and 1
(c) 6 and 1 (d) 1 and 6

Test II

[Time: 10 min]

11. Solve: $5y - 6x = 2$, $6y - 5x = 9$
(a) $x = 4, y = 3$ (b) $x = 3, y = 4$
(c) $x = 5, y = 4$ (d) None of these.

12. Solve: $3(2u + v) = 7uv$, $3(u + 3v) = 11uv$
(a) $u = 1, v = \frac{3}{2}$ (b) $u = \frac{3}{2}, v = 1$
(c) $u = 2, v = \frac{2}{3}$ (d) None of these.

13. Solve: $ax + by = \frac{a+b}{2}$, $3x + 5y = 4$.
(a) $x = \frac{1}{2}, y = \frac{1}{3}$ (b) $x = 1, y = 2$
(c) $x = \frac{1}{2}, y = \frac{1}{2}$ (d) None of these.

14. Solve: $\frac{x}{a} + \frac{y}{b} = 2$, $ax - by = a^2 - b^2$.
(a) $x = a, y = b$ (b) $a = -a, y = b$
(c) $x = a, y = -b$ (d) None of these.

15. For what value of k, the following system of equations
$4x + 5y = 3$, $kx + 15y = 9$.
has infinitely many solutions?
(a) 16 (b) 12
(c) 14 (d) None of these.

16. A lady has only 10 paise and 25 paise coins in her purse. If in all, she has 60 coins totalling Rs. 8.25, how many of each type does she have?
(a) 45, 15 (b) 60, 10
(c) 50, 20 (d) None of these.

17. There are two examination rooms A and B. If 10 candidates are sent from A to B, the number of students in each room is the same. If 20 candidates are sent from B to A, the number of students in A is double the number of students in B. Find the number of students in each room.
(a) 100, 80 (b) 80, 100
(c) 120, 60 (d) None of these.

18. For what value of k will the equations $2x + 32y + 3 = 0$ and $3x + 48y + k = 0$ represent coincident lines?
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\frac{9}{2}$ (d) 1

19. The difference between two numbers is five and the difference between their squares is 65. The larger number is
(a) 9 (b) 10
(c) 11 (d) 12

20. Find the value of k for which the system of equations $4x + 3y = 5$, $kx + y = 9$ has no solution.
(a) 5/3 (b) 4/3
(c) 7/3 (d) None of these.

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution $4x - 3y = 1$ ...(1)
$x + 2y = 3$ ...(2)
$x = 3 - 2y$ [From Eq. (2)]
$\Rightarrow 4(3 - 2y) - 3y = 1 \Rightarrow 12 - 8y - 3y = 1$
$\Rightarrow -11y = -11$, i.e., $y = 1$ and, $x = 3 - 2 \times 1 = 1$;
$\therefore$ **$x = 1$ and $y = 1$.**

3. Ans. (a)

Solution $\sqrt{2}\,x - \sqrt{3}\,y = 0$ ...(1)
$\sqrt{5}\,x - \sqrt{2}\,y = 0$ ...(2)

Multiplying Eq. (1) and Eq. (2) by $\sqrt{2}$ and $\sqrt{3}$, respectively, we get

$2x - \sqrt{6}y = 0$ and $\sqrt{15}\,x - \sqrt{6}\,y = 0$

Subtracting the second equation from the first, we get $(2 - \sqrt{15})\,x = 0$, i.e., $x = 0$

From Eq. (1), $y = \sqrt{\frac{3}{2}}\,x = \sqrt{\frac{3}{2}} \times 0 = 0$

$\therefore$ **$x = 0$ and $y = 0$.**

5. Ans. (b)

Solution The given system of equations will have no solution

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

i.e. if $\frac{k}{3} = \frac{2}{1} \neq \frac{5}{1}$ i.e. if $k = 6$.

Hence, the given system of equations has a unique solution if $k \neq 6$ and no solution if **$k = 6$.**

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7. Ans. (a)

Solution Let in the number, unit's digit = x

ten's digit = y

$\therefore \quad x + y = 10$ ...(1) [given]

number = $10y + x$

When the digits are reversed, unit's digit = y

ten's digit = x

$\therefore$ New number = $10x + y$

According to the given condition of the problem,

$10x + y = (10y + x) - 72$

$10x - x + y - 10y = -72$

or $9x - 9y = -72$

$x - y = -8$ ...(2)

[Dividing both sides by 9]

Adding (1) and (2), we get

$2x = 2$ or $x = \frac{2}{2} = 1$

Putting $x = 1$ in (1), we get

$1 + y = 10$ or $y = 10 - 1 = 9$

Hence, the number is **91.**

9. Ans. (d)

Solution The given equations are $3x + y = 1$ and $3x + y = 1$.

Thus, there in only one equation in two variables

So, the given equations have an **infinite number of solutions.**

11. Ans. (b)

Solution $-6x + 5y = 2$ ...(1)

$-5x + 6y = 9$ ...(2)

Multiplying Eq. (1) and Eq. (2) by 5 and 6 respectively, we have

$-30x + 25y = 10$ and $-30x + 36y = 54$

Subtracting the second equation from the first, we get

$-11y = -44$, i.e., $y = 4$

From Eq. (1), $x = \frac{5y-2}{6} = \frac{5\times4-2}{6} = 3$

$\therefore \quad$ **$x = 3$ and $y = 4$.**

13. Ans. (c)

Solution $ax + by = \frac{a+b}{2}$ and $3x + 5y = 4$

Here $a_1/a_2 = a/3$ and $b_1/b_2 = b/5$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore, a unique solution exists.

Writing coefficients in the following array

b	$\frac{1}{2}(a+b)$	a	b
5	4	3	5

we get, $\frac{x}{4b-\frac{5}{2}(a+b)} = \frac{y}{\frac{3}{2}(a+b)-4a} = \frac{-1}{5a-3b}$

$\Rightarrow \frac{x}{\frac{1}{2}(3b-5a)} = \frac{y}{\frac{1}{2}(3b-5a)} = \frac{1}{3b-5a}$

$\Rightarrow x = \frac{\frac{1}{2}(3b-5a)}{(3b-5a)} = \frac{1}{2}$ and $y = \frac{\frac{1}{2}(3b-5a)}{(3b-5a)} = \frac{1}{2}$

$\therefore$ **$x = \frac{1}{2}$ and $y = \frac{1}{2}$.**

15. Ans. (b)

Solution We have $4x + 5y = 3$ ...(1)

$kx + +15y = 9$ ...(2)

Here, $a_1 = 4$, $b_1 = 5$, $c_1 = 3$

$a_2 = k$, $b_2 = 15$, $c_2 = 9$

If the given system of equations has infinitely many solutions, then

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{4}{k} = \frac{5}{15} = \frac{3}{9}$

$\Rightarrow k = \frac{4\times15}{5} = 12$

Hence, for $k = 12$, the given system of equations **has infinitely many solutions.**

17. Ans. (a)

Solution Let the number of students in rooms A and B be, respectively, x and y.

According to first condition of the problem,

$x - 10 = y + 10 \Rightarrow x - y = 10 + 10$

$\Rightarrow x - y = 20$ ...(1)

According to the second condition of the problem,

$x + 20 = 2(y - 20) \Rightarrow x + 20 = 2y - 40$

$\Rightarrow x - 2y = -40 - 20$

$\Rightarrow x - 2y = -60$ ...(2)

Subtracting (2) from (1), we get

$y = 80$

Putting $y = 80$ in (1), we get

$x - 80 = 20 \Rightarrow x = 80 + 20 \Rightarrow x = 100$

Hence, the number of students in room A = **100** and, the number of students in room B = **80.**

19. Ans. (a)

Solution Let the numbers be x and y

$\therefore \quad x - y = 5$ ...(i)

and, $x^2 - y^2 = 65$

$\therefore \quad \frac{x^2-y^2}{x-y} = \frac{65}{5} \Rightarrow x + y = 13$ ...(ii)

Solving (i) and (ii), we get

$x = 9$ and $y = 9$.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (c)	**6.** (c)	**8.** (a)	**10.** (b)	**12.** (a)	**14.** (a)	**16.** (a)	**18.** (c)	**20.** (b)

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Quadratic Equations 28

INTRODUCTION

An equation of degree two is called a *quadratic equation*. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$ and x is a real variable. Some examples of quadratic equations are $x^2+4x+3=0$, $3x^2-4x+5=0$ and $3x^2+2x-3=0$.

Roots of a Quadratic Equation

A *root* of the equation $f(x) = 0$ is that value of x which makes $f(x) = 0$. In other words, $x = a$ is said to be a root of $f(x) = 0$, where $f(a)$ is the value of the polynomial $f(x)$ at $x = a$ and is obtained by replacing x by a in $f(x)$.

For example, -1 is a root of the quadratic equation $x^2 + 6x + 5 = 0$, because $(-1)^2 + 6(-1) + 5 = 0$.

Solution of a Quadratic Equation

If there is a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the roots of this equation are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Illustration 1 Solve the following quadratic equations:

(i) $6x^2 + x - 2 = 0$ (ii) $2x^2 + x - 1 = 0$.

Solution (i) Using formula:

$$\text{The roots are } x = \frac{-1\pm\sqrt{(1)^2-4(6)(-2)}}{2\times 6} = \frac{-1\pm\sqrt{49}}{12} = \frac{6}{12}, \frac{-8}{12}.$$

i.e., $\frac{1}{2}, \frac{-2}{3}$.

Using factorization:

$$6x^2 + x - 2 = 0 \Leftrightarrow 6x^2 + 4x - 3x - 2 = 0$$
$$\Leftrightarrow 2x(3x + 2) - 1(3x + 2) = 0$$
$$\Leftrightarrow (2x + 1)(3x + 2) = 0$$
$$\Leftrightarrow x = \frac{1}{2} \text{ or } x = -\frac{2}{3}.$$

(ii) Using formula:

$$\text{The roots are } x = \frac{-1\pm\sqrt{(1)^2-4(2)(-1)}}{2\times 2} = \frac{-1\pm\sqrt{9}}{4} = \frac{-1\pm 3}{4}$$
$$= \frac{2}{4}, \frac{-4}{4} \text{ i.e., } \frac{1}{2}, -1.$$

Using factorization:

$$2x^2 + x - 1 = 0 \Leftrightarrow 2x^2 + 2x - x - 1 = 0$$
$$\Leftrightarrow 2x(x + 1) - 1(x + 1) = 0$$
$$\Leftrightarrow (2x - 1)(x - 1) = 0.$$
$$\Leftrightarrow x = \frac{1}{2} \text{ or } x = -1.$$

Nature of Roots

A quadratic equation has exactly two roots, may be real or imaginary or coincident.

If $ax^2 + bx + c$, $a \neq 0$, then $D = b^2 - 4ac$ is called *discriminant*.

1. If $D > 0$, then there are two distinct and real roots given by
$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$
2. If $D = 0$, then there is a repeated real root given by
$$\alpha = -\frac{b}{2a} \text{ i.e., roots are real and equal}$$
3. If $D < 0$, then there are no real roots.

Note: The roots are rational if $D > 0$ and D is a perfect square whereas the roots are irrational if $D > 0$ but D is not a perfect square.

Illustration 2 Find the nature of the roots of the equations:

(i) $2x^2 + x - 1 = 0$ (ii) $x^2 + x + 1 = 0$

(iii) $x^2 + 5x + 5 = 0$ (iv) $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$

Solution (i) $D = (1)^2 - 4 \times 2 \times (-1) = 9 > 0$.

Also, D is a perfect square.

So, the roots are real, distinct and rational.

(ii) $D = (1)^2 - 4 \times 1 \times 1 = -3 < 0$

So, the roots are imaginary.

(iii) $D = (5)^2 - 4 \times 1 \times 5 = 5 > 0$.

Also, D is not a perfect square.

So, the roots are real, distinct and irrational.

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(iv) $D = (-2)^2 - 4 \times \frac{4}{3} \times \frac{3}{4} = 0.$

So, the roots are real and equal.

Illustration 3 For what value of k will the quadratic equation $kx^2 - 2\sqrt{5}x + 4 = 0$ have real and equal roots.

Solution $D = (-2\sqrt{5})^2 - 4 \times k \times 4 = 20 - 16k.$

The given equation will have real and equal roots if $D = 0$.

i.e., $20 - 16k = 0$ or $k = \frac{20}{16} = \frac{5}{4}$.

Note: 1. If $p + \sqrt{q}$ is a root of a quadratic equation, then its other root is $p - \sqrt{q}$.

Illustration 4 If $2 + \sqrt{3}$ is one root of a quadratic equation, find the other root.

Solution The other root is $2 - \sqrt{3}$.

2. $ax^2 + bx + c$ can be expressed as a product of two linear factors only when $D \geq 0$.

Illustration 5 For what value of k, the quadratic polynomial $kx^2 + 4x + 1$ can be factorised into two real linear factors.

Solution $D = (4)^2 - 4 \times k \times 1 = 16 - 4k.$

The given quadratic polynomial can be factorised into real linear factors if $D \geq 0$.

i.e., $16 - 4k \geq 0$ or $-4k \geq -6$ or $k \leq 4$.

Relation Between Roots and Coefficients

Let α, β be the roots of the equation,

$$ax^2 + bx + c = 0$$

Then, Sum of the roots

$$= \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and Product of the roots

$$= \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Illustration 6 Find the sum and the product of the roots of the quadratic equation $2x^2 + 5\sqrt{3}x + 6 = 0$.

Solution Here, $a = 2$, $b = 5\sqrt{3}$, $c = 6$.

$\therefore$ Sum of the roots $= -\frac{b}{a} = -\frac{5\sqrt{3}}{2}$.

Product of the roots $= \frac{c}{a} = \frac{6}{2} = 3$.

Formation of a Quadratic Equation with Given Roots

If α, β are the roots of a quadratic equation the equation, can he written as

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e., $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$.

Illustration 7 Find the quadratic equation whose roots are 5 and –6.

Solution Sum of roots $= 5 + (-6) = -1$,

Product of roots $= 5 \times (-6) = -30$.

$\therefore$ The required quadratic equation is

$x^2 - (-1)x + (-30) = 0$ i.e., $x^2 + x - 30 = 0$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. In the following determine the set of values of P for which the given quadratic equation has real roots.

$$Px^2 + 4x + 1 = 0$$

(a) $P \leq 4$ (b) $P \leq 4$

(c) $P \leq 4$ (d) $P \leq 4$

2. If one root of the quadratic equation $2x^2 + Px + 4 = 0$ is 2, find the second root and value of P.

(a) 1, –6 (b) 1, 6

(c) –1, 6 (d) –1, –6

3. One root of the quadratic equation $x^2 - 5x + 6 = 0$ is 3. Find the other root.

(a) 2 (b) –2

(c) 1 (d) –1

4. The roots of the equation

$$\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0 \text{ are}$$

(a) $-\sqrt{7}, \frac{-13\sqrt{7}}{7}$ (b) $\sqrt{7}, \frac{-13\sqrt{7}}{7}$

(c) $-\sqrt{7}, \frac{13\sqrt{7}}{7}$ (d) None of these

5. The roots of the equation

$$3a^2x^2 - abx - 2b^2 = 0 \text{ are}$$

(a) $\frac{b}{a}, \frac{-2b}{3a}$ (b) $\frac{b}{a}, \frac{2b}{3a}$

(c) $\frac{-b}{a}, \frac{-2b}{3a}$ (d) None of these

6. The roots of the equation

$$a^2x^2 - 3abx + 2b^2 = 0 \text{ are}$$

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(a) $\frac{2b}{a}, \frac{-b}{a}$ (b) $\frac{2b}{a}, \frac{b}{a}$

(c) $\frac{-2b}{a}, \frac{b}{a}$ (d) None of these

7. Construct a quadratic equation whose roots are $\sqrt{2}$ and $2\sqrt{2}$

(a) $x^2 - 3\sqrt{2}x - 4 = 0$ (b) $x^2 - 3\sqrt{2}x + 4 = 0$

(c) $x^2 + 3\sqrt{2}x - 4 = 0$ (d) $x^2 + 3\sqrt{2}x + 4 = 0$

8. The roots of the equation

$ax^2 + (4a^2 - 3b)x - 12ab = 0$ are

(a) $4a, \frac{3b}{a}$ (b) $-4a, \frac{3b}{a}$

(c) $-4a, \frac{3b}{a}$ (d) $-4a, \frac{-3b}{a}$

9. Construct a quadratic equation whose roots have the sum = 6 and product = –16.

(a) $x^2 - 6x - 16 = 0$ (b) $x^2 + 6x - 16 = 0$

(c) $x^2 - \sqrt{3}x - 6 = 0$ (d) None of these

10. In the following, find the value (s) of P so that the given equation has equal roots

$3x^2 - 5x + P = 0$

(a) –25/12 (b) 25/6

(c) 25/12 (d) –25/6

11. If α and β are the roots of the equation $ax^2 + bx + c = 0$, find the value of $\alpha^2 + \beta^2$

(a) $\frac{b^2 - 2ac}{2a^2}$ (b) $\frac{b^2 + 2ac}{a^2}$

(c) $\frac{b^2 + 2ac}{2a^2}$ (d) $\frac{b^2 - 2ac}{a^2}$

12. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, the value of $\alpha^3 + \beta^3$ is

(a) $\frac{b(b^2 - 3ac)}{a^3}$ (b) $\frac{b(3ac - b^2)}{a^3}$

(c) $\frac{b(3ac + b^2)}{a^3}$ (d) None of these

13. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

(a) $\frac{b^2 - 2ac}{ac}$ (b) $\frac{b^2 - 2ac}{2ac}$

(c) $\frac{b^2 - ac}{2ac}$ (d) $\frac{b^2 + 2ac}{ac}$

14. The quadratic equation with rational coefficients, whose one root is $\sqrt{5}$, is

(a) $x^2 + 5 = 0$ (b) $x^2 - 10 = 0$

(c) $x^2 - 5 = 0$ (d) None of these

15. The equation $x^2 - px + q = 0, p, q \in R$ has no real root if

(a) $p^2 \leq 4q$ (b) $p^2 < 4q$

(c) $p^2 > 4q$ (d) None of these

16. Determine p so that the equation $x^2 + 5px + 16 = 0$ has no real root.

(a) $\frac{-4}{5} < p < \frac{4}{5}$ (b) $\frac{-8}{5} < p < \frac{8}{5}$

(c) $p < -\frac{4}{5}$ or $p > \frac{4}{5}$ (d) None of these

17. For what value of k the quadratic polynomial $3z^2 + 5z + k$ can be factored into the product of real linear factors?

(a) $k \leq \frac{25}{6}$ (b) $k \leq \frac{25}{12}$

(c) $k \geq \frac{25}{12}$ (d) $k \geq \frac{25}{6}$

18. $x = 3$ is a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ if k has value

(a) 13 (b) –13

(c) 11 (d) –11

19. One root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$. Find the other root.

(a) 3 (b) 1/3

(c) –3 (d) None of these

20. The expression $x^4 + 7x^2 + 16$ can be factored as:

(a) $(x^2 + x + 1)(x^2 + x + 16)$

(b) $(x^2 + x + 1)(x^2 - x + 16)$

(c) $(x^2 + x + 4)(x^2 - x + 4)$

(d) $(x^2 + x - 4)(x^2 - x - 4)$

21. The common root of the equations

$x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$ is

(a) –2 (b) 3

(c) 5 (d) 2

22. The roots of the equation

$x^2 + px + q = 0$ are equal if

(a) $p^2 = 2q$ (b) $p^2 = 4q$

(c) $p^2 = -4q$ (d) $p^2 = -2q$

23. An equation equivalent to the quadratic equation $x^2 - 6x + 5 = 0$ is:

(a) $6x^2 - 5x + 1 = 0$ (b) $x^2 - 5x + 6 = 0$

(c) $5x^2 - 6x + 1 = 0$ (d) $|x - 3| = 2$

24. Divide 16 into 2 parts such that twice the square of the larger part exceeds the square of the smaller part by 164

(a) 10, 6 (b) 8, 8

(c) 12,4 (d) None of these

25. With respect to the roots of $x^2 - x - 2 - 0$, we can say that:

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8. Ans. (c)

Solution The given quadratic equation is

$ax^2 + (4a^2 - 3b)x - 12ab = 0$

$\Rightarrow ax^2 + 4ax - 3bx - 12ab = 0$

$\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$

$\Rightarrow (ax - 3b)(x + 4a) = 0$

$\Rightarrow ax - 3b = 0$ or $x + 4a = 0$

$\Rightarrow x = \frac{3b}{a}$ or $x = -4a$

Thus, the two roots of the given quadratic equation are $\boldsymbol{-4a}$ **and** $\boldsymbol{\frac{3b}{a}}$.

9. Ans. (a)

Solution The required quadratic equation is x^2 – (sum of the roots)x + (product of the roots) = 0

$\Rightarrow x^2 - 6x - 16 = \mathbf{0}$.

10. Ans. (c)

Solution The given quadratic equation is

$3x^2 - 5x + P = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 3,\ b = -5,\ c = P$

If the given quadratic equation has equal roots then its discriminant = 0

$\Rightarrow b^2 - 4ac = 0 \Rightarrow (-5)^2 - 4(3)(P) = 0$

$\Rightarrow 25 - 12P = 0 \Rightarrow \boldsymbol{P = \frac{25}{12}}$

11. Ans. (d)

Solution Since α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$,

$\therefore \quad \alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \boldsymbol{\frac{b^2 - 2ac}{a^2}}$.

12. Ans. (b)

Solution Since α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(-\frac{b}{a}\right)^3 - 3\frac{c}{a}\left(\frac{-b}{a}\right)$

$= \frac{-b^3}{a^3} + \frac{3bc}{a^2} = \frac{-b^3 + 3abc}{a^3}$

$= \boldsymbol{\frac{b(3ac - b^2)}{a^3}}$.

13. Ans. (a)

Solution Since α and β are the roots of the equation $ax^2 + bx + c = 0$

$\therefore \quad \alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\frac{c}{a}} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}$

$= \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c}{a}} = \boldsymbol{\frac{b^2 - 2ac}{ac}}$.

14. Ans. (c)

Solution One root is $\sqrt{5}$

So the other root is $-\sqrt{5}$

$\therefore$ Sum of the roots = 0

and product of the roots = $(\sqrt{5})(-\sqrt{5}) = -5$

$\therefore$ Required equation is

x^2 – (sum of the roots)x + (product of the roots) = 0

$\Rightarrow x^2 - 5 = \mathbf{0}$.

15. Ans. (b)

Solution The equation $x^2 - px + q = 0$; $p, q \in R$ has no real root if

$B^2 < 4AC$

$\Rightarrow (-p)^2 < 4.1.q \quad [\because A = 1, B = -p, C = q]$

$\Rightarrow \boldsymbol{p^2 < 4q}$.

16. Ans. (b)

Solution The given quadratic equation is

$x^2 + 5px + 16 = 0$ ...(i)

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1,\ b = 5p,\ c = 16$

If equation (i) has no real roots, then discriminent < 0

$\Rightarrow b^2 - 4ac < 0 \Rightarrow (5p)^2 - 4(1)(16) < 0$

$\Rightarrow 25p^2 - 64 < 0 \Rightarrow 25p^2 < 64$

$\Rightarrow p^2 < \frac{64}{25} \Rightarrow p^2 - \frac{64}{25} < 0$

$\Rightarrow \left(p - \frac{8}{5}\right)\left(p + \frac{8}{5}\right) < 0$

$\Rightarrow$ either $p - \frac{8}{5} > 0$ and $p + \frac{8}{5} < 0$

i.e., $p > \frac{8}{5}$ and $p < -\frac{8}{5}$,

which is not possible

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Now, we are to form an equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$. So the required equation is

x^2 – (sum of roots)x + (Product of roots) = 0

$$x^2 - \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x + \left(\frac{1}{\alpha}\cdot\frac{1}{\beta}\right) = 0$$

$$x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \left(\frac{1}{\alpha\beta}\right) = 0$$

$$x^2 - \frac{5}{6}x + \frac{1}{6} = 0$$

$$6x^2 - 5x + 1 = \mathbf{0}.$$

29. Ans. (c)

Solution Given equation is: $y + \frac{1}{y} = \frac{10}{3}$,

where $y = \frac{x+4}{x-4}$

$\therefore \quad 3y^2 - 10y + 3 = 0 \Rightarrow y = 3, \frac{1}{3}$

$\therefore \quad y = \frac{x+4}{x-4} = 3$ and $y = \frac{x+4}{x-4} = \frac{1}{3}$

$\therefore \quad x = \pm 8.$

30. Ans. (c)

Solution For reciprocal roots, product of roots must be 1

$\therefore \quad \frac{c}{a} = 1$ i.e. $c = a$

31. Ans. (a)

Solution Sum of the roots = 6

One root = $3 - \sqrt{5}$

$\therefore$ the other root = $6 - (3 - \sqrt{5}) = 3 + \sqrt{5}$

$\therefore$ Product of roots = $(3 - \sqrt{5})(3 + \sqrt{5})$

$= 9 - 5 = 4$

Hence the required equation is

x^2 – (sum of the roots)x + (product of roots) = 0

$\Rightarrow \quad x^2 - 6x + 4 = \mathbf{0}.$

32. Ans. (a)

Solution $\alpha + \beta = 6$ and $3\alpha + 2\beta = 20$

$\Rightarrow \quad \alpha = 4, \beta = 2$

Product of the roots = k

So, $k = \alpha\beta = 4 \times 2 = \mathbf{8}.$

33. Ans. (a)

Solution Let the two consecutive odd positive integers be $2x + 1$ and $2x + 3$ where x is a whole number. Now

$(2x + 1)^2 + (2x + 3)^2 = 290$

$\Rightarrow 4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 290$

$\Rightarrow 8x^2 + 16x - 280 = 0$

$\Rightarrow x^2 + 2x - 35 = 0$

$\Rightarrow (x + 7)(x - 5) = 0 \Rightarrow x = 7, -5$

But $x = -7$ is not possible, since –7 is not a whole number.

$\therefore \quad x = \mathbf{5}.$

34. Ans. (d)

Solution Let the roots be α and $-\alpha$. Then, sum of roots = 0

Also, roots being not equal, discriminant $\neq 0$

$\therefore \quad \frac{q}{p} = 0$ and $q^2 - 4pr \neq 0$

$\Leftrightarrow \quad q = 0$ and $pr \neq 0.$

35. Ans. (a)

Solution Comparing $x^2 - 2(1 + 3x)x + 7(3 + 2k) = 0$ with $ax^2 + bx + c = 0$, we get

$a = 1, b = -2(1 + 3k), c = 7(3 + 2k)$

For equal roots $D = b^2 - 4ac = 0$

$\therefore \quad 4(1 + 3k)^2 - 4 \times 1 \times 7(3 + 2k) = 0$

$\Rightarrow \quad 4(1 + k)^2 + 6k) - 84 - 56k = 0$

$\Rightarrow \quad 36k^2 - 32k - 80 = 0$

$\Rightarrow \quad 9k - 8k - 20 = 0$

$$k = \frac{8 \pm \sqrt{64 - 4(9)(-20)}}{2 \times 9}$$

$$= \frac{8 \pm \sqrt{784}}{18} = \frac{8 \pm 28}{18}$$

$$= \frac{36}{18}, \frac{-20}{18} = \mathbf{2}, \mathbf{\frac{-10}{9}}.$$

36. Ans. (d)

Solution Let α be a common root of the given equations.

Then, $\alpha^2 + 2\alpha - 3 = 0$ and $\alpha^2 + 3\alpha - k = 0$

$\therefore \quad \frac{\alpha^2}{-2k+9} = \frac{\alpha}{-3+k} = \frac{1}{3-2}$

So, $\alpha^2 = \frac{9-2k}{1}$ and $\alpha = \frac{k-3}{1}$

So, $(9 - 2k) = (k - 3)^2$ or $k^2 - 4k = 0$

or $\quad k(k - 4) = 0$, so $k = \mathbf{4}.$

37. Ans. (d)

Solution Given equation is : $2^{2x} - 3 \cdot 2^x \times 2^2 + 32 = 0$

or $\quad 2^{2x} - 12 \times 2^x + 32 = 0$

$\Rightarrow y^2 - 12y + 32 = 0$, where $2^x = y$

$\Rightarrow (y - 8)(y - 4) = 0 \Rightarrow y = 8, y = 4$

$\therefore \quad 2^x = 8 \quad$ or $\quad 2^x = 4$

$\Rightarrow \quad 2^x = 2^3 \quad$ or $\quad 2^x = 2^2$

$\Rightarrow \quad x = 3 \quad$ or $\quad x = \mathbf{2}.$

38. Ans. (a)

Solution Let the roots be 3α and 2α

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Then, $3\alpha + 2\alpha = \frac{m}{12} \Rightarrow \alpha = \frac{m}{60}$

$\therefore \quad \frac{5}{72} = \left(\frac{m}{60}\right)^2 \Leftrightarrow \frac{5}{72} = \frac{m^2}{3600}$

$\Leftrightarrow m^2 = \frac{3600 \times 5}{72} = 250$

$\therefore \quad m = \sqrt{250} = \mathbf{5\sqrt{10}}.$

39. Ans. (c)

Solution $\because$ α, β are the roots of the equation $2x^2 - 3x + 1 = 0$

$\therefore \quad \alpha + \beta = \frac{3}{2}$ ...(1) and $\alpha\beta = \frac{1}{2}$ ...(2)

We are to form a quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

S = sum of the roots

$$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{3}{2}\right)^2 - 2\left(\frac{1}{2}\right)}{\frac{1}{2}}$$ [using (1) and (2)]

$$= \frac{\frac{9}{4} - 1}{\frac{1}{2}} = \frac{5}{4} \times \frac{2}{1} = \frac{5}{2}$$

P = Product of the roots

$$= \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$$

Hence the required quadatic equation is $x^2 -$ (sum of the roots)x + (Product of the roots) = 0

$\Rightarrow \quad x^2 - \frac{5}{2}x + 1 = 0$

$\Rightarrow \quad 2x^2 - 5x + 2 = \mathbf{0}.$

40. Ans. (a)

Solution The given quadratic equation is

$3x^2 - 20x + 17 = 0$ ...(1)

Compare with $ax^2 + bx + c = 0$, we get

$a = 3, b = -20, c = 17$

The roots of (1) are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{20 \pm \sqrt{400 - 4(3)(17)}}{2 \times 3}$$

$$= \frac{20 \pm \sqrt{196}}{6} = \frac{20+14}{6}, \frac{20-14}{6}$$

$$= \frac{34}{6}, \frac{6}{6} = \frac{17}{3}, 1$$

Hence the roots of (1) are $\frac{17}{3}$ and 1. So we have to form an equation whose roots are $\frac{3}{17}$ and 1

Sum of the roots $= \frac{3}{17} + 1 = \frac{20}{17}$

Product of the roots $= \frac{3}{17} \times 1 = \frac{3}{17}$

Hence, the required equation is

$x^2 - \frac{20}{17}x + \frac{3}{17} = 0 \Rightarrow \mathbf{17x^2 - 20x + 3 = 0.}$

41. Ans. (a)

Solution $\because$ α, β are the roots of the equation

$x^2 - 3\lambda x + \lambda^2 = 0$

$\therefore \quad \alpha + \beta = 3\lambda$ ...(1) and $\alpha\beta = \lambda^2$ ...(2)

Now $\alpha^2 + \beta^2 = \frac{7}{4}$ (given)

$\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = \frac{7}{4}$

$\Rightarrow \quad (3\lambda)^2 - 2\lambda^2 = \frac{7}{4} \Rightarrow 7\lambda^2 = \frac{7}{4}$

$\Rightarrow \quad \lambda^2 = \frac{1}{4} \Rightarrow \boldsymbol{\lambda = \pm \frac{1}{2}}.$

42. Ans. (b)

Solution $\because$ α, β are the roots of the equation

$ax^2 + bx + b = 0$

$\therefore \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{b}{a}$

Now $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{b}{a}}$

$$= \frac{-\frac{b}{a}}{\sqrt{\frac{b}{a}}} + \sqrt{\frac{b}{a}}$$

$$= -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}} = \mathbf{0}.$$

43. Ans. (c)

Solution Comparing $x^2 - x + 1$ with $ax^2 + bx + c$ we have

$a = 1, b = -1, c = 1$

Here $D = b^2 - 4ac = (-1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

Since $D < 0$, so the given expression has no proper linear factor.

44. Ans. (c)

Solution Let the breadth of the rectangular plot be x m. Then the length of rectangular plot = $(x + 8)$m

$\therefore$ Area = Length $\times$ Breadth = $x(x + 8)$ m^2

But the area of the plot is given to be 308 m^2

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(a) 6 (b) 100
(c) 63 (d) 36
[Based on MAT 2003]

7. If the roots of the equation
$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$
for $a \neq 0$ are real and equal, then the value of
$a^3 + b^3 + c^3$ is
(a) abc (b) $3abc$
(c) zero (d) None of these
[Based on MAT 2003]

8. If $2x^2 - 7xy + 3y^2 = 0$, then the value of $x : y$ is
(a) 3 : 2 (b) 2 : 3
(c) 3 : 1 and 1 : 2 (d) 5 : 6
[Based on MAT 2003]

9. Which of the following equations has real roots?
(a) $3x^2 + 4x + 5 = 0$
(b) $x^2 + x + 4 = 0$
(c) $(x - 1)(2x - 5) = 0$
(d) $2x^2 - 3x + 4 = 0$ **[Based on MAT 1999]**

10. Which of the following is a quadratic equation?
(a) $x^{1/2} + 2x + 3 = 0$
(b) $(x - 1)(x + 4) = x^2 + 1$
(c) $x^4 - 3x + 5 = 0$
(d) $(2x + 1)(3x - 4) = 2x^2 + 3$
[Based on MAT 1999]

11. Which of the following equations has real roots?
(a) $3x^2 + 4x + 5 = 0$
(b) $x^2 + x + 4 = 0$
(c) $(x - 1)(2x - 5) = 0$
(d) $2x^2 - 3x + 4 = 0$ **[Based on MAT 2000]**

12. If a and b are the roots of the equation $x^2 - 6x + 6 = 0$, then the value of $a^2 + b^2$ is
(a) 36 (b) 24
(c) 12 (d) 6
[Based on MAT 2001]

13. The roots of $2x^2 - 6x + 3 = 0$ are
(a) Real, unequal and rational
(b) Real, unequal and irrational
(c) Real and equal (d) Imaginary
[Based on FMS (Delhi) 2003]

14. If the roots of $ax^2 + bx + c = 0$ be equal, then the value of c is
(a) $-b/2a$ (b) $b/2a$
(c) $-b^2/4a$ (d) $b^2/4a$
[Based on FMS (Delhi) 2003]

15. A and B solved a quadratic equation. In solving it, A made a mistake in the constant term and obtained the roots as 5, –3, while B made a mistake in the coefficient of x and obtained the roots as 1, –3. The correct roots of the equation are
(a) 1, 3 (b) –1, 3
(c) –1, –3 (d) 1, –1
[Based on FMS (Delhi) 2003]

16. The value of x in the equation
$$\sqrt{\frac{x}{1-x}} + \sqrt{1-x} = 2\frac{1}{2} \text{ is}$$
(a) 5/13 (b) 7/13
(c) 9/13 (d) None of these
[Based on FMS (Delhi) 2003]

17. For what values of k, the equation
$x^2 + 2(k - 4)x + 2k = 0$ has equal roots?
(a) 8, 2 (b) 6, 4
(c) 12, 2 (d) 10, 4
(e) None of the above **[Based on IIFT 2003]**

18. Which of the following expressions cannot be equal to zero, when $X^2 - 2X = 3$?
(a) $X^2 - 7X + 6$ (b) $X^2 - 9$
(c) $X^2 - 4X + 3$ (d) $X^2 - 6X + 9$
[Based on SCMHRD Ent. Exam. 2003]

Directions (Qs. 19 to 23): In each of these questions, two equations I and II are given. You have to solve both the equations and give answer (a), if $p < q$ (b), if $p \leq q$; (c), if $p = q$, (d), if $p \geq q$ and (e), if $p > q$.

19. I. $p^2 - 18p + 77 = 0$
II. $3q^2 - 25q + 28 = 0$ **[Based on IRMA, 2002]**

20. I. $6q^2 + q - 1 - 0$
II. $6p^2 - 7p + 2 = 0$ **[Based on IRMA, 2002]**

21. I. $7p^2 + 6p - 1 = 0$
II. $32q^2 - 20q + 3 = 0$ **[Based on IRMA, 2002]**

22. I. $4p^2 = 9$
II. $2q^2 - 9q + 10 = 0$ **[Based on IRMA, 2002]**

23. I. $2p^2 - 12p + 16 = 0$
II. $q^2 - 9q + 20 = 0$ **[Based on IRMA, 2002]**

24. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) is
(a) (2, 3) (b) (–2, 3)
(c) (4, 7) (d) (–4, 7)
[Based on FMS (Delhi) 2002]

25. The number of quadratic equations which are unchanged by squaring their roots is
(a) 2 (b) 4
(c) 5 (d) 6
[Based on FMS (Delhi) 2002]

26. If α and β are the two roots of the equation $2x^2 - 7x - 3 = 0$, then find the value of $(\alpha + 2)(\beta + 2)$.
(a) 9 (b) – 9.5
(c) 9.5 (d) 6
(e) 4.5 **[Based on SCMHRD 2002]**

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6. Ans. (d)

Solution Suppose total number of cows = x

$\frac{1}{4}x$ of the cows are in forest,

$2\sqrt{x}$ have gone to mountains and 15 are on the banks of a river.

$$\therefore 2\sqrt{x}+\frac{1}{4}x+15 = x \Rightarrow 2\sqrt{x}-\frac{3}{4}x+15 = 0$$
$$\Rightarrow 8\sqrt{x}-3x+60 = 0$$
$$\Rightarrow 3(\sqrt{x})^2-8\sqrt{x}-60 = 0.$$
$$\Rightarrow 3(\sqrt{x})^2-18\sqrt{x}+10\sqrt{x}-60 = 0$$
$$\Rightarrow 3\sqrt{x}(\sqrt{x}-6)+10(\sqrt{x}-6) = 0$$
$$\Rightarrow \sqrt{x} = 6,\ \sqrt{x} = -\frac{10}{3}$$
$$\Rightarrow \mathbf{x = 36.}$$

7. Ans. (b)

Solution Discriminant

$$= [2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

for the roots to be equal

$$\therefore\ a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc = 0$$
$$\Rightarrow\ a^3 - 2abc + c^3 + b^3 - abc = 0$$
$$\Rightarrow\ a^3 + b^3 + c^3 = 3abc.$$

8. Ans. (c)

Solution $2\frac{x^2}{y^2}-7\frac{x}{y}+3 = 0$

$$\Rightarrow\ \frac{x}{y} = \frac{7\pm\sqrt{49-24}}{2\times 2} = \frac{7\pm 5}{4} = 3, \frac{1}{2}.$$

9. Ans. (c)

Solution The equation

$ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

(3) $\Rightarrow (x - 1)(2x - 5) = 0$

$\Rightarrow b^2 - 4ac = 49 - 40 = 9 > 0.$

10. Ans. (d)

11. Ans. (c)

Solution If the equation $ax^2 + bx + c = 0$ has real roots, then $b^2 - 4ac \geq 0$

(a) $\Rightarrow b^2 - 4ac = 16 - 60 < 0,$

(b) $\Rightarrow b^2 - 4ac = 1 - 16 < 0$

(c) $\Rightarrow b^2 - 4ac = 49 - 40 = 9 > 0,$

(d) $\Rightarrow b^2 - 4ac = \mathbf{9 - 32 < 0.}$

12. Ans. (b)

Solution $a + b = 6,\ ab = 6$

$\therefore\ a^2 + b^2 = (a + b)^2 - 2ab = 36 - 12 = \mathbf{24.}$

13. Ans. (b)

Solution $\Delta = b^2 - 4ac = 36 - 24 = 12 > 0$

$\therefore$ Roots are real and distinct but irrational.

14. Ans. (d)

Solution $b^2 - 4ac = 0 \Rightarrow \boldsymbol{c = b^2/4a.}$

15. Ans. (b)

Solution A's quadratic equation was $x^2 - 2x - 15 = 0$

B's quadratic eqatuion was $x^2 + 2x - 3 = 0$

Since A made a mistake in the constant term and B made a mistake in coefficient of x, therefore the correct equation was $x^2 - 2x - 3 = 0$

$\therefore$ Roots of the correct equation are **3, –1.**

16. Ans. (d)

17. Ans. (a)

Solution For any quadractic equation, $ax^2 + bx + c = 0$, to have equal roots,

$$b^2 - 4ac = 0 \Rightarrow [2(k - 4)]^2 - 4 \times 2k = 0$$
$$\Rightarrow (k - 4)^2 - 2k = 0$$
$$\Rightarrow k + 16 - 8k - 2k = 0$$
$$\Rightarrow k^2 - 10k + 16 = 0$$
$$\Rightarrow \mathbf{k = 8, 2.}$$

18. Ans. (a)

Solution $X^2 - 2X - 3 = (X - 3)(X + 1)$

(a) $= X^2 - 7X + 6 = (X - 6)(X - 1)$

(b) $= X^2 - 9 = (X + 3)(X - 3)$

(c) $= X^2 - 4X + 3 = (X - 3)(X - 1)$

(d) $= X^2 - 6X + 9 = \mathbf{(X - 3)^2.}$

19. Ans. (d)

Solution $p^2 - 18p + 77 = 0 \Rightarrow p = 11,\ p = 7$

$3q^2 - 25q + 28 = 0 \Rightarrow 3q^2 - 21q - 4q + 28 = 0$

$$\Rightarrow 3q(q - 7) - 4(q - 7) = 0$$
$$\Rightarrow (3q - 4)(q - 7) = 0$$
$$\Rightarrow q = \frac{4}{3},\ 7.$$

$\therefore\quad p \geq q$

20. Ans. (b)

Solution $6q^2 + q - 1 - 0 \Rightarrow 6q^2 + 3q - 2q - 1 = 0$

$$\Rightarrow 3q(2q + 1) - (2q + 1) = 0$$
$$\Rightarrow (3q - 1)(2q + 1) = 0$$
$$\Rightarrow q = \frac{1}{3}, \frac{-1}{2}$$

$6p^2 - 7p + 2 = 0 \Rightarrow 6p^2 - 4p - 3p + 2 = 0$

$$\Rightarrow 2p(3p - 2) - (3p - 2) = 0$$

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EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. Find the set of values of p for which the quadratic equation $2x^2 + 3x + p = 0$ has real roots.
(a) $p \geq 9/8$ (b) $p \leq 9/8$
(c) $p < 9/8$ (d) None of these

2. If one of the roots of the quadratic equation $2x^2 + px + 4 = 0$ is 2, find the other root. Also find the value of p.
(a) 1, 6 (b) –1, 6
(c) 1, –6 (d) –1, –6

3. In the following, find the value (s) of p so that the given equation has equal roots.
$$2px^2 - 8x + p = 0$$
(a) $\pm\sqrt{2}$ (b) $\pm 3\sqrt{2}$
(c) $\pm 2\sqrt{2}$ (d) None of these

4. Find the value of k for which the equation $x^2 - 4x + k = 0$ has two equal roots
(a) $k = -4$ (b) $k = 2$
(c) $k = 4$ (d) $k = -2$

5. Determine k so that the equation $x^2 - 4x + k = 0$ has two distinct roots.
(a) $k^2 > 4$ (b) $k < 4$
(c) $k < 2$ (d) $k > 2$

6. The sum of the number and its square is 35/4. Find the numbers
(a) $\frac{5}{2}, \frac{7}{2}$ (b) $\frac{5}{2}, \frac{-7}{2}$
(c) $\frac{-5}{2}, \frac{-7}{2}$ (d) None of these

7. Which of the following is true for real values of k for which the polynomial $2x^2 + 6x + k$ can be factorized into a product of real linear factors?
(a) $k < \frac{9}{2}$ (b) $k \leq \frac{9}{2}$
(c) $k > \frac{9}{2}$ (d) $k \leq \frac{9}{2}$

8. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then find the value of $\frac{\alpha+\beta}{\alpha^{-1}+\beta^{-1}}$
(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$
(c) $\frac{b}{a}$ (d) $\frac{-b}{a}$

9. Find the quadratic equation whose roots are $3 + \sqrt{2}$ and $3 - \sqrt{2}$
(a) $x^2 - 6x + 7 = 0$ (b) $x^2 - 6x - 7 = 0$
(c) $x^2 + 6x + 7 = 0$ (d) None of these

10. The value of x in the equation
$$\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6} \text{ is:}$$
(a) $\frac{5}{13}$ (b) $\frac{7}{13}$
(c) $\frac{9}{13}$ (d) None of these

Test II

[Time: 10 min]

11. Construct a quadratic equation whose roots are
$\frac{2+\sqrt{5}}{2}, \frac{2-\sqrt{5}}{2}$
(a) $4x^2 + 8x + 1 = 0$ (b) $4x^2 - 8x - 3 = 0$
(c) $4x^2 - 8x + 1 = 0$ (d) $4x^2 - 8x - 1 = 0$

12. Construct a quadratic equation whose roots have the sum $= \sqrt{3}$ and product $= -6$.
(a) $x^2 + \sqrt{3}x - 6 = 0$ (b) $x^2 - \sqrt{3}x - 6 = 0$
(c) $x^2 - \sqrt{3}x + 6 = 0$ (d) None of these

13. If α, β are the roots of the equation $x^2 - 3x + 2 = 0$, construct a quadratic equation whose roots are $-\alpha$ and $-\beta$.
(a) $x^2 + 3x + 2 = 0$ (b) $x^2 - 3x + 2 = 0$
(c) $x^2 + 3x - 2 = 0$ (d) None of these

14. If α, β be the roots of the equation $x^2 - 4x + 8 = 0$, then the equation whose roots are $\alpha + \beta$ and $\alpha\beta$ is
(a) $x^2 - 8x + 4 = 0$ (b) $x^2 + 4x + 4 = 0$
(c) $x^2 - 12x + 32 = 0$ (d) $x^2 + 12x - 32 = 0$

15. Determine the set of values of k for which the quadratic polynomial $2x^2 + 6x + k$ can be factorized into a product of real linear factors.
(a) $k \geq 9/2$ (b) $k < 9/2$
(c) $k \leq 9/2$ (d) $k > 9/2$

16. The area of a right triangle is 63 sq. cm. If the base of the triangle exceeds that of the altitude by 5 cm, find the altitude of the triangle.
(a) 7 cm (b) 9 cm
(c) 11 cm (d) 10 cm

17. If the equations $2x^2 - 7x + 3 = 0$ and $4x^2 + ax - 3 = 0$ have common root, then the values of a are:

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$\Rightarrow \quad 36 - 8k \geq 0 \Rightarrow 36 \geq 8k$

$\Rightarrow \quad k \leq \frac{36}{8} \Rightarrow \mathbf{k \leq 9/2.}$

17. Ans. (a)

Solution Let α be the common root of the given equations

Then, $2\alpha^2 - 7\alpha + 3 = 0$ and $4\alpha^2 + a\alpha - 3 = 0$

$\therefore \quad \frac{\alpha^2}{21-3a} = \frac{\alpha}{12+6} = \frac{1}{2a+28}$

$\therefore \quad \alpha^2 = \frac{21-3a}{2a+28}$ and $\alpha = \frac{18}{2a+28} = \frac{9}{a+14}$

$\therefore \quad \frac{21-3a}{2a+28} = \left(\frac{9}{a+14}\right)^2 = \frac{81}{(a+14)^2}$

$\Leftrightarrow \quad \frac{21-3a}{2(a+14)} = \frac{81}{(a+14)^2}$

$\Leftrightarrow \quad \frac{21-3a}{2} = \frac{81}{a+14}$

$\Leftrightarrow \quad -3a^2 - 21a + 294 = 162$

$\Leftrightarrow \quad 3a^2 + 21a - 132 = 0$

$\Leftrightarrow \quad (a + 11)(3a - 12) = 0$

$\Leftrightarrow \quad (a + 11)(3a - 12) = 0$

$\therefore \quad a = -11$ or **4.**

19. Ans. (c)

Solution $\because$ α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$

$\therefore \quad \alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$

$= \frac{\left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\frac{c}{a}} = \frac{-\frac{b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}$

$= \frac{\mathbf{-b^3 + 3abc}}{\mathbf{a^2 c}}.$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (c)	**6.** (b)	**8.** (b)	**10.** (c)	**12.** (b)	**14.** (c)	**16.** (b)	**18.** (c)	**20.** (b)

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29 Progressions

INTRODUCTION

In this chapter, we shall be concerned with the study of sequences, i.e. special types of functions whose domain is the set N of natural numbers. We shall study particular types of sequences called *arithmetic* sequences, *geometric* sequences and *harmonic* sequences and also their corresponding series.

Premiums on life insurance, fixed deposits in a bank, loan instalments payments, disintegration or decay of radioactive materials and the like are some of the examples where the concept of sequence and series is used.

SEQUENCE

A *sequence* is a function whose domain is the set N of natural numbers and range, a subset of real numbers or complex numbers.

A sequence whose range is a subset of real numbers is called a *real sequence*. Since we shall be dealing with real sequences only, we shall use the term 'sequence' to denote a 'real sequence'.

Notation

The different terms of a sequence are usually denoted by $a_1, a_2, a_3, \ldots$ or by $t_1, t_2, t_3, \ldots$ The subscript (always a natural number) denotes the position of the term in the sequence. The number occurring at the nth place of a sequence, i.e. t_n is called the *general term* of the sequence.

Note : A sequence is said to be *finite* or *infinite* (according as finite or infinite number of terms it has.)

PROGRESSIONS

If the terms of a sequence follow certain pattern, then the sequence is called a *progression*.

Illustration 1 Consider the following sequences:

(i) 3, 5, 7, 9, ..., 21. (ii) 8, 5, 2, – 1, – 4, ...

(iii) 2, 6, 18, 54, ..., 1458.

(iv) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ (v) 1, 4, 9, 16, ...

We observe that each term (except the first) in (i) is formed by adding 2 to the preceding term; each term in (ii) is formed by subtracting 3 from the preceding term; each term in (iii) is formed by multiplying the preceding term by 3; each term in (iv) is formed by dividing the preceding term by 2; each term in (v) is formed by squaring the next natural number. Thus, each of (i) to (v) is a progression. Moreover, (i) and (iii) are finite sequences, whereas (ii), (iv) and (v) are infinite sequences.

However, to define a sequence we need not always have an explicit formula for the nth term. For example, for the infinite sequence 2, 3, 5, 7, 11, 13, 17, ... of all positive prime numbers, we may not be able to give an explicit formula for the nth term.

SERIES

By adding or subtracting the terms of a sequence, we obtain a *series*. A series is finite or infinite according as the number of terms in the corresponding sequence is finite or infinite.

Illustration 2 The following:

(i) 3 + 5 + 7 + 9 + ... + 21.

(ii) 8 + 5 + 2 + (– 1) + ...

(iii) 2 + 6 + 18 + 54 + ... + 1458.

(iv) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

(v) 1 + 4 + 9 + 16 + ...

are the series corresponding to the above sequences, (i) to (v).

ARITHMETIC PROGRESSION (A.P.)

A sequence whose terms increase or decrease by a fixed number is called an *arithmetic progression*. The fixed number is called the *common difference* of the A.P.

In an A.P., we usually denote the first term by a, the common difference by d and the nth term by t_n. Clearly, $d = t_n - t_{n-1}$. Thus, an A.P. can be written as

$$a, a + d, a + 2d, \ldots, a + (n - 1)\, d, \ldots$$

Illustration 3 Consider the series: 1, 3, 5, 7, 9, ...

Here, 2nd term – 1st term = 3rd term – 2nd term = 4th term – 3rd term = ... = 2.

Hence 1, 3, 5, 7, ... are in A.P. whose first term is 1 and common difference is 2.

Illustration 4 The series: 5, 3, 1, –1, –3, –5, –7, ... is in A.P. whose first term is 5 and common difference is –2.

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Notes:

- A sequence $t_1, t_2, t_3, t_4, \ldots$ will be in A.P. if $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots$, i.e. $t_n - t_{n-1}$ = constant, for $n \geq 2$.
- Three numbers a, b, c are in A.P. if and only if $b - a = c - b$, i.e. if and only if $a + c = 2b$.
- Any three numbers in an A.P. can be taken as $a - d, a, a + d$. Any four numbers in an A.P. can be taken as $a - 3d, a - d, a + d$ and $a + 3d$.

  Similarly, five numbers in an A.P. can be taken as $a - 2d, a - d, a, a + d$ and $a + 2d$.

GENERAL TERM OF AN A.P.

Let a be the first term and d be the common difference of an A.P. Then, the A.P. is $a, a + d, a + 2d, a + 3d, \ldots$. We also observe that,

t_1, the first term, is $a = a + (1 - 1)d$;

t_2, the second term, is $a + d = a + (2 - 1)d$;

t_3, the third term, is $a + 2d = a + (3 - 1)d$;

t_4, the fourth term, is $a + 3d = a + (4 - 1)d$;

t_n, the nth term, is $a + (n - 1)d$.

Thus, the formula, $t_n = a + (n - 1)d$ gives the general term of an A.P.

Notes:

- If an A.P. has n terms, then the nth term is called the *last term* of A.P. and it is denoted by l. Therefore, $l = a + (n - 1)d$.
- If a is the first term and d the common difference of an A.P. having m terms, then nth term from the end is $(m - n + 1)$th term from the beginning.

  $\therefore$ nth term from the end $= a + (m - n)d$.

Illustration 5 A sequence $< t_n >$ is given by the formula $t_n = 10 - 3n$. Prove that it is an A.P.

Solution We have,

$$t_n = 10 - 3n \Rightarrow t_{n+1} = 10 - 3(n + 1) = 7 - 3n.$$

$\therefore$ $t_{n+1} - t_n = (7 - 3n) - (10 - 3n) = -3$,

which is independent of n and hence a constant. Therefore, the given sequence $< t_n >$ is an A.P.

Illustration 6 Find the nth term and 19th term of the sequence $5, 2, -1, -4, \ldots$

Solution Clearly, the given sequence is an A.P. with $a = 5$ and $d = -3$.

$\therefore$ $t_n = a + (n - 1)d = 5 + (n - 1)(-3) = -3n + 8$.

For the 19th term, putting $n = 19$, we get $t_{19} = -3.19 + 8 = -49$.

Sum of n terms of an A.P.

The sum of n terms of an A.P. with first term 'a' and common difference 'd' is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Note:

- If S_n is the sum of n terms of an A.P. whose first term is 'a' and last term is l, then

$$S_n = \frac{n}{2}(a + l).$$

- If common difference d, number of terms n and the last term l, are given then

$$S_n = \frac{n}{2}[2l - (n - 1)d]$$

- $t_n = S_n - S_{n-1}$.

Illustration 7 Find the sum of the series $.5 + .51 + .52 + \ldots$ to 100 terms.

Solution The given series is an A.P. with first term, $a = .5$ and common difference, $d = .51 - .5 = .01$.

$\therefore$ Sum of 100 terms

$$= \frac{100}{2}[2 \times .5 + (100 - 1) \times .01]$$

$$= 50(1 + 99 \times .01) = 50(1 + .99)$$

$$= 50 \times 1.99 = \mathbf{99.5}.$$

Illustration 8 Find the sum of 20 terms of an A.P., whose first term is 3 and the last term is 57.

Solution We have, $a = 3$, $l = 57$, $n = 20$.

$\therefore$ $S_n = \frac{n}{2}(a + l)$, $\therefore$ $S_{20} = \frac{20}{2}(3 + 57) = 600$.

Hence, the sum of 20 terms is **600**.

GEOMETRIC PROGRESSION

A sequence (finite or infinite) of non-zero numbers in which every term, except the first one, bears a constant ratio with its preceding term, is called a *geometric* progression, abbreviated as G.P.

Illustration 9 The sequences given below:

(i) $2, 4, 8, 16, 32, \ldots$

(ii) $3, -6, 12, -24, 48, \ldots$

(iii) $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \frac{1}{324}, \ldots$

(iv) $\frac{1}{5}, \frac{1}{30}, \frac{1}{180}, \frac{1}{1080}, \frac{1}{6480}, \ldots$

(v) $x, x^2, x^3, x^4, x^5, \ldots$ (where x is any fixed real number),

are all geometric progressions. The ratio of any term in (i) to the preceding term is 2. The corresponding ratios in (ii), (iii), (iv) and (v) are $-2, \frac{1}{3}, \frac{1}{6}$, and x respectively. The ratio of any term of a G.P. to the preceding term is called the *common ratio* of the G.P. Thus, in the above examples, the common ratios are $2, -2, \frac{1}{3}, \frac{1}{6}$ and x respectively.

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Note: In a G.P., any term may be obtained by multiplying the preceding term by the common ratio of the G.P. Therefore, if any one term and the common ratio of a G.P. be known, any term can be written out, i.e. the G.P. is then completely known.

In particular, if the first term and the common ratio are known, the G.P. is completely known. The first term and the common ratio of a G.P. are generally denoted by a and r, respectively.

GENERAL TERM OF A G.P.

Let a be the first term and $r\ (\neq 0)$ be the common ratio of a G.P. Let $t_1, t_2, t_3, \ldots, t_n$ denote Ist, 2nd, 3rd, ... , nth terms, respectively. Then, we have

$$t_2 = t_1 r,\ t_3 = t_2 r,\ t_4 = t_3 r,\ \ldots,\ t_n = t_{n-1} r.$$

On multiplying these, we get

$$t_2 t_3 t_4 \ldots t_n = t_1 t_2 t_3 \ldots t_{n-1} r^{n-1} \Rightarrow t_n = t_1 r^{n-1};\ \text{but } t_1 = a.$$

$\therefore$ General term $= t_n = ar^{n-1}$.

Thus, if a is the first term and r the common ratio of a G.P. then the G.P. is $a, ar, ar^2, \ldots, ar^{n-1}$ or $a, ar, ar^2, \ldots$ according as it is fnite or infinite.

Cor. If the last term of a G.P. consisting of n terms is denoted by l, then $l = ar^{n-1}$.

Notes:

- If a is the first term and r the common ratio of a finite G.P. consisting of m terms, then the nth term from the end is given by ar^{m-n}.
- The nth term from the end of a G.P. with the last term l and common ratio r is l/r^{n-1}.
- Three numbers in G.P. can be taken as $a/r, a, ar$; four numbers in G.P. can be taken as $a/r^3, a/r, ar, ar^3$; five numbers in G.P. can be taken as $a/r^2, a/r, a, ar, ar^2$, and so on...
- Three numbers a, b, c are in G.P. if and only if $b/a = c/b$, i.e. if and only if $b^2 = ac$.

Illustration 10 Find the nth term and 12th term of the sequence $-6, 18, -54, \ldots$

Solution The given sequence is a G.P. with $a = -6$ and $r = -3$.

$\therefore\ t_n = ar^{n-1} = (-6)(-3)^{n-1} = (-1)^n \cdot 6 \cdot 3^{n-1}$

For the 12th term, putting $n = 12$, we get

$$t_{12} = (-1)^{12} \cdot 6 \cdot 3^{11} = \mathbf{2 \cdot 3^{12}}.$$

Sum of n terms of a G.P.

The sum of first n terms of a G.P. with first term a and common ratio r is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Notes:

(i) When $r = 1$

$S_n = a + a + \ldots$ up to n terms $= na$

(ii) It l is the last term of the G.P., then

$$S_n = \frac{lr - a}{r - a},\ r \neq 1$$

Sum of an infinite G.P. when $|r| < 1$

The sum of an infinite G.P. with first term a and common ratio r is $S_\infty = \dfrac{a}{1-r}$; when $|r| < 1$, i.e. $-1 < r < 1$.

Illustration 11 Find the sum of 8 terms and n terms of the sequence $9, -3, 1, -1/3, \ldots$

Solution The given sequence is a G.P. with $a = 9$ and $r = -1/3$.

We know that

$$S_8 = 9\frac{1-(-1/3)^8}{1-(-1/3)} = 9\frac{1-1/3^8}{4/3} = \frac{27}{4}\left(1 - \frac{1}{3^8}\right)$$

$$= \frac{27}{4}\frac{3^8 - 1}{3^8} = \frac{1}{4}\frac{6561 - 1}{3^5} = \frac{6560}{4 \times 243} = \frac{1640}{243}.$$

Also, $S_n = 9\dfrac{1-(-1/3)^n}{1-(-1/3)} = 9\dfrac{1-(-1)^n/3^n}{4/3}$

$$= \frac{27}{4}\frac{3^n - (-1)^n}{3^n} = \frac{3^n - (-1)^n}{4 \cdot 3^{n-3}}.$$

Illustration 12 Find the sum of the infinite sequence $7, -1, \dfrac{1}{7}, -\dfrac{1}{49}, \ldots$

Solution The given sequence is a G.P. with $a = 7$ and $r = -\dfrac{1}{7}$, so $|r| = \left|-\dfrac{1}{7}\right| = < 1$.

$$\therefore\ S = \frac{7}{1-(-1/7)} = \frac{7}{8/7} = \frac{\mathbf{49}}{\mathbf{8}}. \quad \left(\because S = \frac{a}{1-r}\right)$$

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2, a_3, \ldots$ is said to be a *harmonic* progression (abbreviated as H.P.) if the sequence $\dfrac{1}{a_1}, \dfrac{1}{a_2}, \dfrac{1}{a_3}, \ldots$ is an A.P.

Illustration 13 The sequence $1, \dfrac{1}{4}, \dfrac{1}{7}, \dfrac{1}{10}, \ldots$ is a H.P. The sequence obtained by taking reciprocals of its corresponding terms, i.e. 1, 4, 7, 10, ... is an A.P.

A general H.P. is $\dfrac{1}{a}, \dfrac{1}{a+d}, \dfrac{1}{a+2d}, \ldots$

nth Term of an H.P.

nth term of H.P.

$$= \frac{1}{n\text{th term of the corresponding A.P.}}$$

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Notes:

- Three numbers a, b, c are in H.P. if and only if $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., i.e.

$$\therefore \quad \frac{1}{a} + \frac{1}{c} = 2 \cdot \frac{1}{b} \text{ or } b = \frac{2ac}{a+c}.$$

- No term of H.P. can be zero.
- There is no general formula for finding the sum to n terms of H.P.
- Reciprocals of terms of H.P. are in A.P. and then properties of A.P. can be used.

Illustration 14 Find the 100th of the squence

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$$

Solution The sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ is an H.P.

Corresponding A.P. is 1, 3, 5, 7, ...

Now, for the corresponding A.P., first term $a = 1$, $d = 2$.

$\therefore$ 100th term of the corresponding A.P.

$$= a + (100 - 1)\, d$$
$$= 1 + (100 - 1)\, 2 = 199$$

Hence, the 100th term of the given sequence $= \frac{1}{199}$.

Some Special Sequences

1. The sum of first n natural numbers

$$\Sigma n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

2. The sum of squares of first n natural numbers

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

3. The sum of cubes of first n natural numbers

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Note: If nth term of a sequences is

$$T_n = an^3 + bn^2 + cn + d$$

then the sum of n terms is given by

$$S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d,$$

which can be evaluated using the above results.

Illustration 15 Find $2^2 + 4^2 + 6^2 + \ldots + (2n)^2$.

Solution nth term of the given series is $(2n)^2$. Then, $T_n = 4n^2$.

$$\therefore \quad S_n = 4\,\Sigma n^2 = \frac{4n(n+1)(2n+1)}{6}$$

$$\therefore \quad S_n = \frac{\mathbf{2n(n+1)(2n+1)}}{\mathbf{3}}.$$

Illustration 16 Sum the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$ to n terms.

Solution Here, $T_n = (1^2 + 2^2 + 3^2 + \ldots n^2)$

$$= \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{n(2n^2 + 3n + 1)}{6}$$
$$= \frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n.$$

$$\therefore \quad S_n = \Sigma\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right).$$

$$= \frac{1}{3}\Sigma n^3 + \frac{1}{2}\Sigma n^2 + \frac{1}{6}\Sigma n$$
$$= \frac{1}{3}\frac{n^2(n+1)^2}{4} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\frac{n(n+1)}{2}$$
$$= \frac{n(n+1)}{12}[n(n+1) + 2n + 1 + 1]$$
$$= \frac{n(n+1)}{12}(n^2 + 3n + 2)$$
$$= \frac{n(n+1)}{12}(n+1)(n+2)$$
$$= \mathbf{\frac{n}{12}(n+1)^2(n+2).}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Determine 25th term of an A.P. whose 9th term is -6 and common difference is $\frac{5}{4}$

(a) 16 (b) 18
(c) 12 (d) 14

2. Which term of the A.P. 5, 13, 21, ... is 181?

(a) 21st (b) 22nd
(c) 23rd (d) 24th

3. Find the nth term of the series

$$\frac{1}{n} + \frac{n+1}{n} + \frac{2n+1}{n} + \ldots$$

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26. Find the sum of all odd numbers of four digits which are divisible by 9.
(a) 2754000 (b) 2754004
(c) 2754008 (d) 2754000

27. Which term of the geometric sequence
$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?
(a) 9 (b) 7
(c) 11 (d) 13

28. Find the 10th term of the geometric series $5 + 25 + 125 + \ldots$,
(a) 5^{10} (b) 5^9
(c) 5^{11} (d) 5^8

29. Write down the 20th term of the G.P. 1, –1, 1, –1, ...
(a) 1 (b) –1
(c) +1 (d) None of these

30. Write down the 5th term of the series $\frac{1}{4} - \frac{1}{2} + 1 \ldots$
(a) 6 (b) 8
(c) 4 (d) 10

31. The 5th term of a G.P. is 2, find the product of first 9 terms.
(a) 508 (b) 512
(c) 504 (d) 516

32. What term of progression
18, –12, 8, ... is $\frac{512}{729}$?
(a) 15 (b) 18
(c) 9 (d) 12

33. The 3rd term of a G.P. is the square of the first term. If the second term is 8, determine the 6th term.
(a) 136 (b) 132
(c) 128 (d) 124

34. If 4th and 8th terms of a G.P. are 24 and 384, respectively, then find out first term and common ratio.
(a) 2, 3 (b) 5, 3
(c) 3, 2 (d) None of these

35. The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the common ratio of the G.P.
(a) 2, –3 (b) 3, –3
(c) 1, –3 (d) 5, –3

36. For what value of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?
(a) 1, –2 (b) 1, –3
(c) 1, –5 (d) 1, –1

37. A person has two parents (father and mother), four grandparents, eight great grandparents and so on. Find the number of ancestors the person has up to the 10th generation.
(a) 1028 (b) 1024
(c) 1030 (d) 1026

38. In a G.P, the first term is 7, the last term 448 and the sum 889. Find the common ratio.
(a) 4 (b) 6
(c) 8 (d) 2

39. The sum of first three terms of a G.P. is to the sum of first six terms is 125 : 152. Find the common ratio of G.P.
(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

40. Evaluate $\sum_{j=1}^{11} (2+3^j)$
(a) $22 + \frac{3}{2}(3^{11}-1)$ (b) $11 + \frac{3}{2}(3^{11}-1)$
(c) $22 + \frac{3}{2}(3^{10}-1)$ (d) None of these

41. The sum of the first two terms of a G.P. is 36 and the product of the first and the third terms is 9 times the second term; find the sum of the first 8 terms.
(a) $\frac{3480}{81}$ (b) $\frac{3280}{81}$
(c) $\frac{3680}{81}$ (d) $\frac{3880}{81}$

42. The common ratio of a G.P. is $-\frac{4}{5}$ and the sum to infinity is $\frac{80}{9}$. Find the first term.
(a) 14 (b) 16
(c) 14 (d) 10

43. Sum the series to infinity
$\frac{3}{4} - \frac{5}{4^2} + \frac{3}{4^3} - \frac{5}{4^4} + \frac{3}{4^5} - \frac{5}{45} + \ldots$
(a) $\frac{8}{15}$ (b) $\frac{7}{17}$
(c) $\frac{7}{15}$ (d) $\frac{8}{17}$

44. The product $(32)\,(32)^{1/6}\,(32)^{1/36} \ldots \infty$ is equal to
(a) 16 (b) 64
(c) 32 (d) 0

45. Find the 9th term of the H.P. 6, 4, 3, ...
(a) 7/5 (b) 6/5
(c) 5/6 (d) None of these

46. Find the nth term of the H.P. whose first two terms are 6 and 3, respectively.
(a) $6/n$ (b) $7/n$
(c) $5/n$ (d) $8/n$

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$\therefore \quad 7(a + 6d) = 11(a + 10d)$

$\Rightarrow \quad 7a + 42d = 11a + 110d \Rightarrow -4a = 68\ d$

$\therefore \quad a = -17\ d$ ...(i)

Now, $a_{18} = a + 17d = -17d + 17d$ [Using (i)]

$= \mathbf{0}.$

9. Ans. (a)

Solution Let a be the first term and d the common difference of A.P.

$\because \quad a_4 = 3a, \quad \therefore \quad a + 3d = 3a$

$\Rightarrow \quad 2a = 3d, \quad \therefore \quad a = \frac{3}{2}d$ ...(i)

Also, $a_7 = 2a_3 + 1$

$\Rightarrow \quad a + 6d = 2\ (a + 2d) + 1$

$\Rightarrow \quad a + 6d = 2a + 4d + 1$

$\therefore \quad a = 2d - 1$ ...(ii)

From (i) and (ii), $\frac{3}{2}\ d = 2d - 1 \Rightarrow 2d - \frac{3}{2}\ d = 1$

$\therefore \quad \frac{d}{2} = 1 \Rightarrow d = 2 \quad \therefore \quad a = \frac{3}{2}\ d = \frac{3}{2} \times 2 = \mathbf{3}.$

10. Ans. (a)

Solution Let a be the first term and d the common difference of A.P.

$\because \quad a_9 = 99$

$\therefore \quad a + 8d = 99$ ...(i)

Also, $a_{99} = 9 \ \therefore \ a + 98d = 9$ ...(ii)

Subtracting (ii) from (i)

$-90\ d = 90$

$\therefore \quad d = -1$ ...(iii)

Substituting this value of d in (i)

$a + 8(-1) = 99 \Rightarrow a = 99 + 8 = 107$ ...(iv)

$\therefore \quad a_{108} = a + (108 - 1)\ d = 107 + 107\ (-1) = \mathbf{0}.$

11. Ans (c)

Solution Let A be the first term and D, the common difference of A.P.

$a_p = a, \quad \therefore \quad A + (p - 1)\ D = a$ ...(i)

$a_q = b, \quad \therefore \quad A + (q - 1)\ D = b$ ...(ii)

$a_r = c, \quad \therefore \quad A + (r - 1)\ D = c$ ...(iii)

$\therefore \ a(q - r) + b(r - p) + c(p - q)$

$= [A + (p - 1)\ D]\ (q - r) + [A + (q - 1)\ D]\ (r - p)$

$+ [A + (r - 1)\ D]\ (p - q)$

$= (q - r + r - p + p - q)\ A + [(p - 1)\ (q - r)$

$+ (q - 1)\ (r - p) + (r - 1)\ (p - q)]D$

$= OA + OD = \boldsymbol{O}.$

12. Ans. (d)

Solution The distances through which the body falls in first, second, third, fourth, ... seconds form an A.P.

$16 + 48 + 80 + 112 + ...$

Here, $a = 16, \ d = 32$

Distance through which it falls in 11th second

= 11 the term of the A.P.

$= a + 10d = 16 + 10\ (32)$

$= 16 + 320 = \mathbf{336\ metres.}$

13. Ans. (a)

Solution Distance covered during the first second 36 m

Distance covered during the 2nd second = 32m

Distance covered during the 3rd second = 28m

The distance covered form an A.P.

$= 36 + 32 + 28$ ..... in which

$a = 36, \ d = -4$

$\therefore$ Distances covered in 8th second

= 8th term of the A.P.

$= a + 7d = 36 + 7(-4)$

$= 36 - 28 = \mathbf{8\ metres.}$

14. Ans. (b)

Solution Let a and d be the first term and the common difference, respectively.

$\because \quad t_n = a + (n - 1)d,$

$\therefore \quad t_2 = a + (2 - 1)\ d = 2 \Rightarrow a + d = 2$ ...(i)

and, $t_7 = a + (7 - 1)\ d = 22 \Rightarrow a + 6d = 22$ ...(ii)

Subtracting (i) from (ii), we get

$5d = 20 \Rightarrow d = 4$

$\therefore \quad a + 4 = 2 \Rightarrow a = 2 - 4 = -2$ [Using (i)]

Now, $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

$\therefore \quad S_{35} = \frac{35}{2}\ [-4 + (35 - 1)4]$

$= \frac{35}{2} \times 132, = 35 \times 66 = \mathbf{2310}.$

15. Ans. (c)

Solution Let a be the first term and d the common difference of an A.P., then

$a_5 = a + 4d = 30$ ...(i)

$a_{12} = a + 11d = 65$ ...(ii)

Subtracting (i) from (ii), we get

$7d = 35 \Rightarrow d = 5$

$\therefore$ From (i)

$a + 4\ (5) = 30 \Rightarrow a = 30 - 20 = 10$

Now, $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

$\therefore \quad S_{20} = \frac{20}{2}[2 \times 10 + (20 - 1)5]$

$= 10\ [20 + 95] = \mathbf{1150}.$

16. Ans. (a)

Solution Let a be the first term and d the common difference of the A.P.

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Then, nth term $= a + (n-1)d$

$\therefore \quad a_{12} = a + 11d = -13 \quad ...(i)$

Again, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_4 = 2(2a + 3d)$

But $\quad S_4 = 24$

$\therefore \quad 2(2a + 3d) = 24$

$\Rightarrow \quad 2a + 3d = 12 \quad ...(ii)$

Multiplying (i) by 2, we get

$2a + 22d = -26 \quad ...(iii)$

Subtracting (ii) from (iii)

$19d = -38 \Rightarrow d \quad -2$

Substituting $d = -2$ in (i), we get

$a + 11 \times (-2) = -13 \Rightarrow a = -13 + 22 = 9$

$\therefore \quad S_{10} = \frac{10}{2}[2 \times 9 + (10-1) \times -2]$

$= 5\,(18 - 18) = \mathbf{0}.$

17. Ans. (a)

Solution If n be the number of terms, then

$a_n = a + (n-1)d,$

where a is the first term and d the common difference.

$\therefore \quad 39 = 3 + (n-1)d$

or, $\quad (n-1)d = 36 \quad ...(i)$

Also, $S_n = \frac{n}{2}[a_1 + a_n]$

$\therefore \quad 525 = \frac{n}{2}[3 + 39] \Rightarrow 1050 = n\,(42)$

or, $\quad n = \frac{1050}{42} = 25$

Putting $n = 25$ in (i), we get

$(25-1)\,d = 36 \Rightarrow d = 36 \div 24 = \frac{3}{2} = \mathbf{1\frac{1}{2}}.$

18. Ans. (b)

Solution Here, $a = 100$

Let d be the common difference

Now, $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$= 5(a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12})$

$\Rightarrow \quad 6\left(\frac{a_1 + a_6}{2}\right) = 5 \times 6\left(\frac{a_7 + a_{12}}{2}\right)$

$\Rightarrow \quad a_1 + a_6 = 5\,(a_7 + a_{12})$

$\Rightarrow \quad a + a + 5d = 5[a + 6d + a + 11d]$

$\Rightarrow \quad 2a + 5d = 10a + 85d$

$\Rightarrow \quad 80d = -8a \text{ or, } d = \frac{-a}{10}$

$\therefore \quad d = \frac{-100}{10} = \mathbf{-10}.$

19. Ans. (c)

Solution Here, $a = -14$

Let d be the common difference

$a_5 = 2 \Rightarrow a + 4d = 2 \Rightarrow -14 + 4d = 2$

$\therefore \quad d = 4$

Let 40 be the sum of n terms of this A.P.

$\because \quad S_n = \frac{n}{2}[2a + (n-1)\,d]$

$\therefore \quad 40 = \frac{n}{2}[2(-14) + (n-1)4] \Rightarrow 80 = n\,(4n - 32)$

or, $\quad 4n^2 - 32n - 80 = 0 \Rightarrow n^2 - 8n - 20 = 0$

$\Rightarrow (n+2)\,(n-10) = 0$

$\therefore \quad n = 10 \text{ or, } -2.$ But $n \neq -2.$

Hence, the required number of terms are **10.**

20. Ans. (a)

Solution Here, $a = 51,\ d = -1,\ a_n = 21$

Now, $a_n = a_1 + (n-1)\,d$

or, $\quad 21 = 51 - n + 1 \quad \therefore \quad n = 52 - 21 = 31$

Now, $S_n = \frac{n}{2}(a_1 + a_n) = \frac{31}{2}(51 + 21)$

$= \frac{31}{2} \times 72 = 31 \times 36 = \mathbf{1116}.$

21. Ans (b)

Solution Here, $S_p = 3p^2 + 4p$

Putting $p = n$, we have

$S_n = 3n^2 + 4n$

Changing n to $(n-1)$, we get

$S_{n-1} = 3(n-1)^2 + 4\,(n-1)$

$= 3(n^2 - 2n + 1) + 4n - 4$

$= 3n^2 - 2n - 1$

$\therefore \quad a_n = S_n - S_{n-1}$

$= 3n^2 + 4n - 3n^2 + 2n + 1 = \mathbf{6n + 1}.$

22. Ans. (b)

Solution Here, $a = 1,\ d = 3,$

Let 715 be the sum of n terms of this A.P.

That is, $S_n = 715$

$\therefore \quad \frac{n}{2}[2a + (n-1)\,d] = 715$

Putting values, of a and d

$\frac{n}{2}[2 \times 1 + (n-1) \times 3] = 715$

$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 715$

$\Rightarrow \quad 3n^2 - n - 1430 = 0$

$\therefore \quad n = \frac{1 \pm \sqrt{1 - 4(3)(-1430)}}{2 \times 3}$

$= \frac{1 \pm \sqrt{17161}}{6} = \frac{1 \pm 131}{6}$

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$\therefore \quad n = \frac{1+131}{6} = 22, \; n = \frac{1-131}{6} = \frac{-65}{3}$

But $n \neq \frac{-65}{3} \; \therefore \; \mathbf{n = 22.}$

23. Ans. (c)

Solution Even natural numbers which are divisible by 5 are 10, 20, 30, 40,....

They form an A.P. with $a = 10, d = 10$

$$S_{100} = \frac{100}{2}[2 \times 10 + (100 - 1) \times 10]$$
$$= 50(20 + 990) = 50\ (1010) = \mathbf{50500.}$$

24. Ans. (d)

Solution The first integer, after 50 which is divisible by 7 is 56 and the last integer before 500 which is divisible by 7 is 497.

$\therefore$ The sequence of integers between 50 and 500 which are divisible by 7 is

$$56, 63, 70, ..., 497$$

It is an A.P. with

$a = 56, d = 7$

$a_n = 497 = a + (n - 1)\, d$

$\therefore \quad 497 = 56 + (n - 1) \times 7$

$\therefore \quad 7n = 497 + 7 - 56$

or, $\quad 7n = 448$

or, $\quad n = 488 \div 7 = 64$

$$\text{Required sum} = \frac{n}{2}(a_1 + a_n) = \frac{64}{2}(56 + 497)$$
$$= 32 \times (553) = \mathbf{17696.}$$

25. Ans (c)

Solution The least and the greatest number of three digits divisible by 7 are 105 and 994, respectively.

$\therefore$ It is required to find the sum of

$$105 + 112 + 119 + ... + 994$$

Here, $a = 105, d = 7, a_n = 994$

Then, $n = ?, S_n = ?$

Now, $a_n = a + (n - 1)\, d$

$\Rightarrow 994 = 105 + (n - 1) \times 7$

$\Rightarrow 994 - 105 = 7\,(n - 1)$

$\Rightarrow 889 = 7\,(n - 1)$ or, $(n - 1) = 127 \Rightarrow n = 128$

$$\text{Also, Sum} = \frac{n}{2}[2a + (n - 1)\, d]$$
$$= \frac{128}{2}[2 \times 105 + (128 - 1) \times 7]$$
$$= 64[210 + 889]$$
$$= 64 \times 1099 = \mathbf{70336.}$$

26. Ans. (a)

Solution The odd numbers of four digits which are divisible by 9 are 1017, 1035, ..., 9999

These are in A.P. with common difference 18.

Hence, nth term $= a_n = a + (n - 1)\, d$

$a = 1017, d = 18, l = 9999$

$\therefore \quad 9999 = 1017 + (n - 1) \times 18$

$\Rightarrow \quad 18n = 9999 - 999 = 9000$

$\therefore \quad n = 9000 \div 18 = 500$

$$\therefore \quad S_n = \frac{n}{2}(a_1 + a_n) = \frac{500}{2}(1017 + 9999)$$
$$= 250 \times 11016 = \mathbf{2754000.}$$

27. Ans. (a)

Solution Let nth term of the given sequence be $\frac{1}{19683}$.

Then,

$$a_n = ar^{n-1} \Rightarrow \frac{1}{19683} = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1}$$
$$\Rightarrow \left(\frac{1}{3}\right)^8 = \left(\frac{1}{3}\right)^{n-1}$$
$$\Rightarrow n - 1 = 8 \Rightarrow \mathbf{n = 9.}$$

28. Ans. (a)

Solution The given geometric series is

$$5 + 25 + 125 + ...$$

$a =$ lst term $= 5$, $r =$ common ratio $= 5$

$n = 10$

$\therefore$ 10th term $= a_{10} = ar^{10-1}$

$= (5)\ (5)^9 = \mathbf{5^{10}}.$

29. Ans. (b)

Solution Here, $a = 1, \quad r = -1$

Since $a_n = ar^{n-1}$

$\therefore \quad a_{20} = ar^{19} = 1 \times (-1)^{19} = \mathbf{-1.}$

30. Ans. (c)

Solution Here, $a = \frac{1}{4}, r = -2. \; \because \; a_n = ar^{n-1}$

$\therefore \quad a_5 = \frac{1}{4} \times (-2)^4 = \frac{1}{4} \times 16 = \mathbf{4.}$

31. Ans. (b)

Solution Let a be the first term and r the common ratio

$\therefore \quad a_5 = 2 \Rightarrow ar^4 = 2 \qquad ....(i)$

Now, product of first 9 terms

$$= a \times ar \times ar^2 \times ... \times ar^8$$
$$= a^9\, r^{1+2+...+8} = a^9\, r^{36}$$
$$= (ar^4)^9 = 2^9 = \mathbf{512.}$$

32. Ans. (c)

Solution Here $a = 18, r = \frac{-2}{3}$

Let $\frac{512}{729}$ be the nth term so that $a_n = \frac{512}{729}$

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50. Ans. (a)

Solution If r is the common ratio of G.P., then

$l = a\,r^{n-1}$ ...(i)

The first n terms of the G.P. are

$a, ar, ar^2, ar^3, ..., ar^{n-1}$

$P = a \times ar \times ar^2 \times ar^3 \times ... \times ar^{n-1}$

$= a^n \times r^{1+2+3+....+(n-1)}$

$= a^n \times r^{(n-1)\times\frac{n}{2}} = (a^2)^{n/2} \times (r^{n-1})^{n/2}$

$= (a^2\, r^{n-1})^{n/2} = (a.\, ar^{n-1})^{n/2} = \mathbf{(al)^{n/2}}.$

QUESTIONS FROM COMPETITIVE EXAMS

1. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?
(a) 1st (b) 9th
(c) 12th (d) None of these
[Based on CAT 2003]

2. If $\log_3 2, \log_3 (2^x - 5), \log_3 (2^x - 7/2)$ are in arithmetic progression, then the value of x is equal to
(a) 5 (b) 4
(c) 2 (d) 3
[Based on CAT 2003]

3. The are 8436 steel balls, each with a radius of 1 centimetre, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is
(a) 34 (b) 38
(c) 36 (d) 32
[Based on CAT 2003]

4. Let T be the set of integers {3, 11, 19, 27, ... 451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is
(a) 32 (b) 28
(c) 29 (d) 30
[Based on CAT 2003]

5. In a certain examination paper, there are n questions. For $j = 1, 2, ... n$, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is
(a) 12 (b) 11
(c) 10 (d) 9
[Based on CAT 2003]

6. If three positive real numbers x, y, z satisfy $y - x = z - y$ and $xyz = 4$, then what is the minimum possible value of y?
(a) $2^{1/3}$ (b) $2^{2/3}$
(c) $2^{1/4}$ (d) $2^{3/4}$
[Based on CAT 2004]

7. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ..., 9 such that the first digit of the code is non zero. The code, handwritten on a slip, can however potentially create confusion when read upside down — for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?
(a) 80 (b) 78
(c) 71 (d) 69
[Based on CAT 2004]

8. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?
(a) 72 (b) 90
(c) 96 (d) 144
[Based on CAT 2004]

9. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?
(a) 666 (b) 676
(c) 683 (d) 777
[Based on CAT 2004]

10. The infinite sum $1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + ...$ equals
(a) $\frac{27}{14}$ (b) $\frac{21}{13}$
(c) $\frac{49}{27}$ (d) $\frac{256}{147}$
[Based on CAT 2004]

11. If the sum of the 6th and the 15th elements of an arithmetic progression is equal to the sum of the 7th, 10th and 12th elements of the same progression, then which element of the series should necessarily be equal to zero?
(a) 10th (b) 8th
(c) 1st (d) None of these
[Based on MAT 2003]

12. The sum of the 6th and 15th elements of an arithmetic progression is equal to the sum of 7th, 10th and 12th elements of the same progression. Which element of the series should necessarily be equal to zero?
(a) 10th (b) 8th

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(c) 1st (d) None of these

[Based on MAT 2003]

13. What is the eighth term of the sequence 1, 4, 9, 16, 25, ... ?

(a) 8 (b) 64

(c) 128 (d) 200

[Based on MAT 2003]

14. If p, q, r, s are in harmonic progression and $p > s$, then

(a) $\frac{1}{ps} < \frac{1}{qr}$ (b) $q + r = p + s$

(c) $\frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$ (d) None of these

[Based on MAT 2003]

15. Mohan ate half a pizza on Monday. He ate half of what was left on Tuesday and so on. He followed this pattern for one week. How much of the pizza would he have eaten during the week?

(a) 99.22% (b) 95%

(c) 98.22% (d) 100%

[Based on MAT 2003]

16. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is

(a) $\log_a(\log_b a)$

(b) $\log_a(\log_e a) + \log_a(\log_e b)$

(c) $-\log_a(\log_a b)$

(d) $\log_a(\log_e b) - \log_a(\log_e a)$

[Based on MAT 2002]

17. A person pays Rs 975 in monthly instalments, each monthly instalment being less than the former by Rs 5. The amount of the first instalment is Rs 100. In what tune, will the entire amount be paid?

(a) 12 months (b) 26 months

(c) 15 months (d) 18 months

[Based on MAT 2002]

18. Let S_n denote the sum of the first 'n' terms of an A.P. $S_{2n} = 3S_n$. Then, the ratio S_{3n}/S_n is equal to

(a) 4 (b) 6

(c) 8 (d) 10

[Based on MAT 2002]

19. Three numbers are in G.P. Their sum is 28 and their product is 512. The numbers are

(a) 6, 9 and 13 (b) 4, 8 and 16

(c) 2, 8 and 18 (d) 2, 6 and 18

[Based on MAT 1999]

20. The sum of the series:

$1^2 + 2^2 + 3^2 + 4^2 + ... + 15^2$ is

(a) 1080 (b) 1240

(c) 1460 (d) 1620

[Based on MAT 1999]

21. If the nth term of an A.P. is $4n + 1$, then the common difference is

(a) 3 (b) 4

(c) 5 (d) 6

[Based on MAT 1999]

22. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$, then a, b, c form a/an

(a) Arithmetic progression

(b) Geometric progression

(c) Harmonic progression

(d) None of these. **[Based on MAT 1999]**

23. In a G.P., the first term is 5 and the common ratio is 2. The eighth term is

(a) 640 (b) 1280

(c) 256 (d) 160

[Based on MAT 2000]

24. If the arithmetic mean of two numbers is 5 and geometric mean is 4, then the numbers are

(a) 4, 6 (b) 4, 7

(c) 3, 8 (d) 2, 8

[Based on MAT 2000]

25. If p, q, r and s are in Harmonic Progression and $p > s$, then

(a) $\frac{1}{ps} < \frac{1}{qr}$ (b) $q + r = p + s$

(c) $\frac{1}{q} + \frac{1}{p} = \frac{1}{r} + \frac{1}{s}$ (d) None of these.

[Based on MAT 2001]

26. The value of

$(1^3 + 2^3 + 3^3 + ... 15^3) - (1 + 2 + 3 + ... 15)$ is

(a) 14280 (b) 14400

(c) 12280 (d) 13280

27. $f(a, b)$ is a series of which the first three terms are $(a + b)^2$, $(a^2 + b^2)$ and $(a - b)^2$. We add the first n terms of the series $f(a, b)$ and call it $S(a, b)$. If $a = 7$, $b = 3$, then find $S(7, 3)$ for $n = 20$.

(a) –5980 (b) 6000

(c) 6960 (d) None of these

[Based on FMS (Delhi) 2004]

28. 30 trees are planted in a straight line at intervals of 5 metres. To water them, the gardener needs to bring water for each tree, separately from a well, which is 10 metres from the first tree in line with the trees. How far will he have to walk in order to water all the trees beginning with the first tree ? Assume that he starts from the well.

(a) 4785 metres (b) 4795 metres

(c) 4800 metres (d) None of these

[Based on FMS (Delhi) 2004]

29. $f(x) + 2x$; where x is an integer. If we arrange the value of $f(x)$, for $x = 25, 24, 23....$ (continuously decreasing value of x), we get an Arithmetic Progression (A.P.) whose first term is 50. Find the maximum value of the sum of all the terms of the A.P.

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2. Ans. (d)

Solution $\log_3 2, \log_3(2^x - 5), \log_3(2^x - 7/2)$ are in arithmetic progression,

$\therefore \quad 2\log_3(2^x - 5) = \log_3 2 + \log_3(2^x - 7/2)$

$\Rightarrow \quad (2^x - 5)^2 = 2 \cdot (x^2 - 7/2)$

Putting $2x = t$

$\Rightarrow \quad (t - 5)^2 = 2 \cdot (t - 7/2)$

$\Rightarrow \quad t^2 - 10t + 25 = 2t - 7$

$\Rightarrow \quad t^2 - 12t + 32 = 0 \Rightarrow t = 4, 8$

Now $2^x = 4$

$\Rightarrow x = 2$ which is not possible because $2^x - 5$ is negative.

If $2^x = 8 \Rightarrow x = 3$

$\therefore \quad \boldsymbol{x = 3.}$

3. Ans. (c)

Solution Let there be total n layers of balls.

1st layer $\rightarrow$ 1 ball

2nd layer $\rightarrow$ 3 balls

3rd layer $\rightarrow$ 6 balls

4th layer $\rightarrow$ 10 balls

... ...

nth layer $\rightarrow \dfrac{n(n+1)}{2}$ balls.

According to the question,

$$\Sigma \frac{n(n+1)}{2} = 8436$$

$$\Rightarrow \quad \Sigma \frac{n^2}{2} + \frac{n}{2} = 8436$$

$$\Rightarrow \quad \Sigma \frac{n^2}{2} + \Sigma \frac{n}{2} = 8436$$

$$\Rightarrow \quad \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \times \frac{n(n+1)}{2} = 8436$$

$$\Rightarrow \quad \frac{1}{2}\left\{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right\} = 8436$$

Now, go through the options:

$n = 36$ satisfies

$\therefore$ Number of layers = 36.

4. Ans. (d)

Solution $T = \{3, 11, 19, 27, ..., 451, 459, 467\}$

Terms in set T are in AP.

We have to find the number of elements in S (subset of T) such that no two elements add up to 470. As we know, in a finite AP the sum of the terms equidistant from beginning and end is always the same. Sum of first and last term = 3 + 467 = 470.

$$\text{Number of terms} = \frac{467 - 3}{8} + 1 = 59$$

So, there are 29 pairs which give 470 as sum.

3, 11, 19, 27, ..., 227, 235, 243, ?? 443, 451, 459, 467

So, the number of element such that sum of no two elements is 470 = 59 – 29 = 30.

5. Ans. (a)

Solution No. of students who answered one or more questions wrongly = 2^{n-j}

The value of j lies between 1 and n (including 1 and n)

$\therefore$ Total number of wrong answers

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + ... + 2^1 + 2^0$$

According to the question,

$$2^0 + 2^1 + ... + 2^{n-3} + 2^{n-2} - 2^{n-1} = 4095$$

$$\Rightarrow \quad \frac{1(2^n - 1)}{2 - 1} = 4095 \Rightarrow 2^n - 1 = 4095$$

$$\Rightarrow \quad 2^n = 4095 + 1 = 40996 \Rightarrow n = 12.$$

6. Ans. (b)

Solution Since $y - x = z - y$

$\therefore \quad x, y$ and z are in AP.

Let x, y and z are $(a - d)$, (a) and $(a + d)$

Again, $xyz = 4$

$$\Rightarrow \quad (a - d)\, a\, (a + d) = 4 \Rightarrow a(a^2 - d^2) = 4$$

$$\Rightarrow \quad a^2 - d^2 = \frac{4}{a} \Rightarrow d^2 = a^2 - \frac{4}{a}$$

For minimum possible value of y, i.e. a, d should be equal to zero. That is,

$$a^2 - \frac{4}{a} = 0 \Rightarrow a^2 = \frac{4}{a} \Rightarrow a^3 = 2^2$$

$$\Rightarrow a = (2^2)^{1/3} = \boldsymbol{2^{2/3}}.$$

7. Ans. (c)

Solution The digits which create confusion are 0, 1, 6, 8, 9.

The total no. of two-digit codes having distinct digits and first digit non-zero.

1st place	2nd place
9 options (1 to 9)	9 options (one digit is already used out of 1 to 9 and 0 is included)

$= 9 \times 9 = 81$ such codes

Total no. of two digit codes which can create confusion

1st digit	2nd digit
4 options (1, 6, 8 or 9)	4 options (one digit is already used out of 1, 6, 8 and 9 and 0 is also included)

$= 4 \times 4 = 16$

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But these 16 two-digit codes include 69 and 96, which create no confusion. Apart from these, 10, 60, 80 and 90 are such two digit codes as create no confusion because these codes are no expected. Hence total number of two-digit codes which create no confusion

$= 81 - 16 + 6 = \mathbf{71.}$

8. Ans. (b)

Solution Total number of single direct lines $= {}^{12}C_2$
$= 66$

Now, if the towns belong to the same zone then every two towns are connected with three direct lines.

$\therefore$ No. of additional lines in one zone
$= {}^3C_2 \times 2 = 6$

Total no. of additional lines in four zones
$= 6 \times 4 = 24$

$\therefore$ Required number of direct telephone lines
$= 66 + 24 = \mathbf{90.}$

9. Ans. (b)

Solution The two-digit number is of the form $7n + 3$

First two-digit number will be for $n = 1$

i.e. $7 \times 1 + 3 = 10$

Last two-digit number will be for $n = 13$

i.e. $7 \times 13 + 3 = 94$

No. of teems $= 13$

Sum of all 13 terms $= \frac{13}{2}(10 + 94)$
$= 13 \times 52 = \mathbf{676.}$

10. Ans. (c)

Solution We have to find the sum of the series

$$1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \ldots$$

Putting $\frac{1}{7} = x$ we get

$1 + 2^2x + 3^2x^2 + 4^2x^3 + 5^2x^4 + \ldots$

Let $S = 1 + 4x + 9x^2 + 16x^3 + 25x^4 + \ldots$

$Sx = x + 4x^2 + 9x^3 + 16x^4 + \ldots$

$S - Sx = 1 + 3x + 5x^2 + 7x^3 + 9x^4 + \ldots$

$x(S - Sx) = x + 3x^2 + 5x^3 + 7x^4 + \ldots$

$(S - Sx) - \{x(S - Sx)\}$
$= 1 + 2x + 2x^2 + 2x^3 + \ldots + \text{to } \infty$

$\Rightarrow (1 - x)^2 S = 1 + \frac{2x}{1 - x}$; Since $|x| < 1$

$\Rightarrow S = \frac{1 + x}{(1 - x)^3}$

We may use it as direct formula for solving this type of problem.

Sustituting $x = \frac{1}{7}$ we get

$$S = \frac{1 + \frac{1}{7}}{\left(1 - \frac{1}{7}\right)^3} = \frac{8 \times 243}{7 \times 216} = \frac{\mathbf{49}}{\mathbf{27}}.$$

11. Ans. (b)

Solution Let a be the first term and d be the common ratio of an A.P.

$\therefore (a + 5d) + (a + 14d)$
$= (a + 6d) + (a + 9d) + (a + 11d)$

$\Rightarrow a + 7d = 0$

$\Rightarrow$ 8th term $= \mathbf{0.}$

12. Ans. (b)

Solution Let a be the first term of the series in A.P. Let d be its common difference.

$\therefore (a + 5d) + (a + 14d)$
$= (a + 6d) + (a + 9d) + (a + 11d)$

$\Rightarrow 2a + 19d = 3a + 26d \Rightarrow a = -7d$

$\Rightarrow$ 8th term $= a + 7d \Rightarrow -7d + 7d = \mathbf{0.}$

13. Ans. (b)

Solution $8^2 = 64$.

14. Ans. (d)

Solution p, q, r, s are in harmonical progression

$\Rightarrow \frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ and $\frac{1}{s}$ are in A.P.

$\Rightarrow \frac{1}{q} - \frac{1}{p} = \frac{1}{s} - \frac{1}{r}$

$\Rightarrow \frac{1}{q} + \frac{1}{r} = \frac{1}{p} + \frac{1}{s}$.

15. Ans. (a)

Solution Mohan ate $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$ of the pizza during the week. That is,

$$\frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^7\right]}{1 - \frac{1}{2}} = 1 - \frac{1}{128} = \frac{127}{128} = 99.22\%$$

$\left[\text{Here } a = \frac{1}{2}, r = \frac{1}{2} \text{ for the given G.P. of 7 terms}\right]$

16. Ans. (a)

Solution Given statement

$\Rightarrow (a^{x/2})^2 = (\log_b x) \times (\log_x a) \Rightarrow a^x = \log_b a$

$\Rightarrow x \log a = \log_a [\log_b a] \Rightarrow \mathbf{x = \log_a [\log_b a].}$

17. Ans. (c)

Solution Let n be the number of months in which all the instalments can be paid

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First instalment = Rs 100

Last instalment = Rs 5

Common Difference = –5

⇒ Sum of the series with n terms whose first term is 100 or common difference is (–5) = 975

i.e. $\frac{n}{2}[2a + (n-1)d] = 975$

i.e. $\frac{n}{2}[2 \times 100 + (n-1)(-5)] = 975$

i.e. $n^2 - 41n + 390 = 0$

i.e. $n = 26$ or $n = 15$

For $n = 15$, total amount paid

$$= \frac{15}{2}[2 \times 100 + (15-1)(-5)]$$

$$= \frac{15}{2}[200 - 70] = \mathbf{975.}$$

18. Ans. (b)

Solution $S_n = \frac{n}{2}[a+(n-1)d]$

[where a is the first term and d is the common difference]

$$S_{2n} = \frac{2n}{2}[a+(2n-1)d]$$

$$S_{3n} = \frac{3n}{2}[a+(3n-1)d]$$

Given, $S_{2n} = 3S_n$

$$\Rightarrow \quad n[a + 2nd - d] = 3\left[\frac{n}{2}(a+nd-d)\right]$$

$$\Rightarrow \quad d = \frac{a}{1+n}$$

$$\therefore \quad \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[a+3nd-d]}{\frac{n}{2}[a+nd-d]}$$

$$= \frac{3\left[a + \frac{3na}{1+n} - \frac{a}{1+n}\right]}{a + \frac{na}{1+n} - \frac{a}{1+n}} = \mathbf{6.}$$

19. Ans. (b)

Solution Let the three numbers be a, ar, ar^2, where r is the common ratio.

$\therefore \quad a + ar + ar^2 = 28$ and $a^3r^3 = 512$

$\therefore \quad ar = 8 \Rightarrow a + ar^2 = 20$

$$\Rightarrow 8r^2 - 20r + 8 = 0$$

$$\Rightarrow r = 2, \quad r = \frac{1}{2}.$$

If $r = 2$, $a = 4$. Therefore, the three numbers are **4, 8, 16.**

20. Ans. (b)

Solution The sum of the squares of the first n natural numbers is

$$\frac{n(n+1)(2n+1)}{6}$$

Put $n = 15$, we have, $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 15^2$

$$= \frac{15(15+1)(30+1)}{6} = \mathbf{1240.}$$

21. Ans. (b)

Solution nth term $= a + (n-1)d = 4n + 1$

where a = first term and d = common difference

$\therefore \quad (a-d) + nd = 1 + 4n \Rightarrow a - d = 1, \; \mathbf{d = 4}$

$$\Rightarrow a = 5.$$

22. Ans. (c)

Solution $\frac{1}{b-a} - \frac{1}{c} = \frac{1}{a} - \frac{1}{b-c}$

$$\Rightarrow \quad b = \frac{2ac}{a+c} \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

$\Rightarrow$ a, b, c are in **H.P.**

23. Ans. (a)

Solution nth term of a G.P. $= ar^{n-1}$

where a = first term and r is the common ratio.

$\therefore$ 8th term $= 5 \times (2)^7 = 5 \times 128 = \mathbf{640.}$

24. Ans. (d)

Solution Let x, y be the numbers

$\therefore \quad \frac{x+y}{2} = 5$ and $\sqrt{xy} = 4 \Rightarrow xy = 16$

$\therefore \quad x + y = 10, \; xy = 16$

$\Rightarrow \quad (x-y)^2 = (x+y)^2 - 4xy = 100 - 64 = 36$

$\Rightarrow \quad x - y = 6 \qquad \therefore \quad \mathbf{x = 8, \; y = 2.}$

25. Ans. (a)

26. Ans. (a)

Solution Given expression

$$= \left[\frac{15 \times 16}{2}\right]^2 - \frac{15 \times 16}{2} = (120)^2 - 120$$

$$= 120 \times 119 = \mathbf{14280.}$$

27. Ans. (d)

Solution $(a+b)^2$, (a^2+b^2), $(a-b)^2$, ...

This is a series in A.P. with common difference $(-2ab)$.

Given $n = 20$

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$$\therefore\ S(a, b) = \frac{20}{2}[2(a+b)^2 + (20-1)(-2ab)]$$
$$= 10\,[2a^2 + 2b^2 + 4ab - 38ab]$$
$$= 20\,[a^2 + b^2 - 17ab]$$
$$\therefore\ S(7, 3) = 20\,[49 + 9 - 357]$$
$$= 20 \times (-299) = \mathbf{-5980.}$$

28. Ans. (b)

Solution To find the sum of the series:

$$10 + 10 + 15 + 15 + (20 + 20) + \ldots + (150 + 150) + 155$$
$$= 2\,(10 + 15 + \ldots + 150) + 155$$
$$= 2\left[\frac{29}{2}(2\times 10 + (29-1)\times 5)\right] + 155$$
$$= 29\,(20 + 140) + 155$$
$$= 29 \times 160 + 155$$
$$= 4640 + 155 = \mathbf{4795.}$$

29. Ans. (c)

Solution Max. sum = S

$$= 50 + 48 + 46 + \ldots + 6 + 4 + 2$$
$$= \frac{25}{2}\,[2 \times 50 + (25 - 1)\,(-2)]$$
$$= \frac{25}{2}\,[100 - 48] = 25 \times 26 = \mathbf{650.}$$

30. Ans. (c)

Solution The roots of the given equation are –2, –1, –2/3.

These roots are neither in A.P. nor in G.P. These roots are in H.P. because $-\frac{1}{2}, -1, -\frac{3}{2}$ are in A.P. with common difference $-\frac{1}{2}$.

$\therefore$ –2, –1, $-\frac{2}{3}$ are in H.P.

31. Ans. (a)

Solution Sum of the cubes of first n natural numbers

$$= \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

$$\therefore\ \text{Mean} = \frac{n(n+1)^2}{4}.$$

32. Ans. (b)

Solution 102, 108, 114, ..., 996

There are 150 such numbers divisible by **6.**

33. Ans. (a)

Solution S_n = Sum of n terms of an A.P.

$$= \frac{n}{2}[2a + (n-1)d]$$

where a = first term, d = common difference

$$\therefore\ \frac{3+5+7+\ldots+n\,\text{terms}}{5+8+11+\ldots+10\,\text{terms}} = 7$$

$$\Rightarrow\ \frac{\frac{n}{2}[2\times 3 + (n-1)\times 2]}{\frac{10}{2}[2\times 5 + (10-1)\times 3]} = 7$$

$$\Rightarrow\ \frac{n(2n+4)}{370} = 7$$
$$\Rightarrow\ 2n^2 + 4n - 2590 = 0$$
$$\Rightarrow\ n^2 + 2n - 1295 = 0$$
$$\Rightarrow\ n^2 + 37n - 35n - 1295 = 0$$
$$\Rightarrow\ n(n + 37) - 35(n + 37) = 0$$
$$\Rightarrow\ (n - 35)(n + 37) = 0$$
$$\Rightarrow\ \boldsymbol{n = 35.}$$

34. Ans. (c)

Solution Sum of the first n natural numbers = $\frac{n(n+1)}{2}$

Sum of the squares of the first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\therefore\ \frac{n(n+1)}{2} = \frac{1}{5}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\Rightarrow\ 2n + 1 = 15 \Rightarrow \boldsymbol{n = 7.}$$

35. Ans. (a)

Solution x can take the values 1, 2, ... 98.

For x = 1, y can take the values 1, 2, ..., 98

For x = 2, y can take the values 1, 2, ..., 97

For x = 98, y can take the value 1.

Since z is dependent on x and y, therefore the required number of solutions

$$= 98 + 97 + 96 + \ldots + 1$$
$$= \frac{98(98+1)}{2} = \mathbf{4851.}$$

36. Ans. (c)

Solution $\frac{3^n - 1}{3-1} > 2000 \Rightarrow 3^n > 4001 \Rightarrow \boldsymbol{n = 8.}$

37. Ans. (None)

Solution $S_n = \frac{n}{2}[2a + (n-1)d]$

where a = 4, d = ?, n = 12

$$\therefore\ S_{12} = 256 = \frac{12}{2}[2\times 4 + (12-1)\times d]$$

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution Let a be the first term and d the common difference of an A.P.

$$a_n = a + (n-1)\,d$$

$\therefore \quad a_6 = a + 5d = 12$ ....(i)

and, $a_8 = a + 7d = 22$ ...(ii)

Solving (i) and (ii) for a and d, we get

$$d = 5$$

Putting $d = 5$ in (i), we get

$a + 25 = 12$ or $a = -13$

$\therefore \quad a_2 = a + d = -13 + 5 = \mathbf{-8}.$

3. Ans. (a)

Solution Let a be the first term and d be the common difference

$\therefore \quad ma_m = na_n$ (given)

$\Rightarrow \quad m\,[a + (m-1)\,d] = n\,[a + (n-1)\,d]$

$\Rightarrow \quad ma + (m^2 - m)\,d = n\,a + (n^2 - n)\,d$

$\Rightarrow \quad m\,a + (m^2 - m - n^2 + n)\,d - na = 0$

$\Rightarrow \quad (m-n)\,a + [(m-n)(m+n) - (m-n)]\,d = 0$

$\Rightarrow \quad (m-n)\,[a + d\,(m+n-1)] = 0$

But $m \neq n$

$\therefore \quad a + (m+n-1)\,d = 0 \Rightarrow \mathbf{a_{m+n} = 0.}$

5. Ans. (a)

Solution Here, $a = -4$, a_n = last term = 146

$\therefore \quad a + (n-1) = 146 \Rightarrow -4 + (n-1)\,d = 146$

$\therefore \quad (n-1)\,d = 150$ ...(i)

We have, $S_n = \dfrac{n}{2}\,[2a + (n-1)\,d]$

$\Rightarrow \quad 7171 = \dfrac{n}{2}\,[2 \times -4 + 150]$

$\Rightarrow \quad 7171 = \dfrac{n}{2} \times 142 \Rightarrow 71n = 7171$

$\therefore \quad n = 101$

Now, from (i), $(101 - 1)\,d = 150$

$\therefore \quad d = 150 \div 100 = \mathbf{1.5}.$

7. Ans. (c)

Solution Let 'a' be the first term and 'd' the common difference of an A.P.

Middle term, i.e. 6th term $= a + 5d = 30$

Now, $S_{11} = \dfrac{11}{2}\,[2a + (11-1)\,d]$

$= \dfrac{11}{2}[2a + 10\,d] = 11(a + 5\,d)$

$= 11 \times 30 = \mathbf{330}.$

9. Ans. (a)

Solution Sum of all the natural numbers in the first 1000 integers which are not divisible by 2, i.e. sum of odd integers

$= 1 + 3 + 5 + 7 + ... + 999$

$= \dfrac{n}{2}\,(a_1 + a_n)$

$= \dfrac{500}{2}(1 + 999) = 250 \times 1000 = 250000$

Out of this sum, sum of the numbers which are divisible by 5

$= 5 + 15 + 25 + ... + 995$

$= \dfrac{n}{2}(a_1 + a_n) = \dfrac{100}{2}(5 + 995)$

$= 50\,(1000) = 50000$

$\therefore$ Required sum

$= 250000 - 50000 = \mathbf{200000}.$

11. Ans. (c)

Solution Let 'a' be the first term and r be the common ratio of the G.P. Then, the first three terms of the G.P. are $a,\ ar,\ ar^2$

$\therefore \quad a \times ar \times ar^2 = \dfrac{27}{8} \Rightarrow a^3r^3 = \dfrac{27}{8}$

or, $(ar)^3 = \left(\dfrac{3}{2}\right)^2 \quad \Rightarrow ar = \dfrac{3}{2}$

Hence, the middle term is $ar = \mathbf{\dfrac{3}{2}}.$

13. Ans. (c)

Solution Let 'a' be the first term and r the common ratio of the G.P.

Then, $a_n = ar^{n-1}$

$\Rightarrow \quad a_5 = ar^4 = 32$ ...(i)

and, $a_8 = ar^7 = 256$ ...(ii)

Dividing (ii) by (i), $r^3 = \dfrac{256}{32} = 8 = 2^3 \ \therefore \ r = 2$

From, (i), $a(2)^4 = 32 \Rightarrow 16a = 32 \ \therefore \ a = 2$

Hence, the 4th term of the G.P.

$= ar^3 = 2(2)^3 = 2 \times 8 = \mathbf{16}.$

15. Ans. (d)

Solution Let a be the first term and r, the common ratio of G.P.

Given: $a_1 + a_3 = 36 \Rightarrow a + ar = 36$

$\Rightarrow a\,(1 + r) = 36$ ...(i)

Also, $a_1 \times a_3 = 9a_2 \Rightarrow a.\ ar^2 = 9.ar$

$\Rightarrow ar = 9$ .....(ii)

Subtracting (ii) from (i), $a = 27$

From (ii), $27\,r = 9, \ \therefore \ r = 9 \div 27 = \dfrac{1}{3}$

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$$\therefore \quad S_8 = \frac{a(1-r^8)}{1-8} = \frac{27\left[1-\left(\frac{1}{3}\right)^8\right]}{1-\frac{1}{3}}$$

$$= \frac{3}{2} \times 27\left(1-\frac{1}{6561}\right) = \frac{81}{2} \times \frac{6560}{6561} = \mathbf{\frac{3280}{81}}.$$

17. Ans. (c)

Solution Let a be the first term and r be the common ratio of the G.P.

$$a_2 = ar = 2 \qquad ...(i)$$

$$S_\infty = \frac{a}{1-r} = 8 \qquad ...(ii)$$

From (i), we get $r = \frac{2}{a}$

Putting $r = \frac{2}{a}$ in (ii), we get

$$\frac{a}{1-\frac{2}{a}} = 8 \quad \text{or,} \quad \frac{a^2}{a-2} = 8$$

$\Rightarrow a^2 = 8(a-2) = 8a - 16$

$\Rightarrow a^2 - 8a + 16 = 0$ or, $(a-4)^2 = 0$

$\therefore \quad a - 4 = 0 \Rightarrow \mathbf{a = 4}.$

19. Ans. (c)

Solution In H.P. $t_3 = \frac{5}{16}$, $t_6 = \frac{1}{5}$ (given)

$\therefore$ In the corresponding A.P.

$$a_3 = \frac{16}{5} \text{ and } a_6 = 5$$

If a be the first term and d the common difference of the corresponding A.P.,

then, $a_3 = \frac{16}{5} \Rightarrow a + 2d = \frac{16}{5}$ ...(i)

and, $a_6 = 5 \Rightarrow a + 5d = 5$ ...(iii)

Subtracting (i) from (ii)

$$3d = 5 - \frac{16}{5} = \frac{9}{5} \therefore d = \frac{3}{5}$$

Substituting this value of d in (i)

$$a + \frac{6}{5} = \frac{16}{5} \Rightarrow a = \frac{16}{5} - \frac{6}{5} = \frac{10}{5} = 2$$

Hence, a_{10} of H.P. $= \frac{1}{a_{10} \text{ of A.P.}} = \frac{1}{a+9d}$

$$= \frac{1}{2+9\left(\frac{3}{5}\right)} = \frac{5}{10+27} = \mathbf{\frac{5}{37}}.$$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (b)	**6.** (b)	**8.** (a)	**10.** (b)	**12.** (a)	**14.** (b)	**16.** (a)	**18.** (c)	**20.** (d)

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Set Theory

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INTRODUCTION

The concept of set is fundamental in all branches of mathematics. Sets are the most basic tools of mathematics which are extensively used in developing the foundations of relations and functions, logic theory, sequences and series, geometry, probability theory, etc. In fact, these days most of the concepts and results in mathematics are expressed in the set theoretic language.

The modern theory of sets was developed by the German mathematician Georg Cantor (1845–1918*A.D.*) In this chapter, we shall study some basic definitions and operations involving sets. We shall also discuss the applications of sets.

SET

We observe that in nature, varieties of objects occur in *groups*. These groups are given different names such as, a *collection* of books, a *bunch* of keys, a *herd* of cattle, an *aggregate* of points, etc., depending on the characteristic of objects they represent. In literal sense all these works have the same meaning. (i.e. a group or a collection). In mathematical language, we call this collection of objects, a *set*. From the above examples, it can be seen that each collection has a well-defined property (characteristic) of its own.

Thus, a *set is a well-defined collection of objects*. When we say well-defined, we mean that the objects follow a given rule or rules. With the help of this rule, we will be able to say whether any given object belongs to this set or not. For example, if we say that we have a collection of short students in a class, this collection is not a set as 'short students', is not well defined. However, if we say that we have a collection of students whose height is less than 5 feet, then it represents a set.

It is not necessary that a set may consist of same type of objects, For example, a book, a cup and a plate lying on a table may also form a set, their common property being that they form a collection of objects lying on the table.

Illustration 1 Some other examples of sets are:

(i) The set of numbers 1, 3, 5, 7, 9, 14
(ii) The set of vowels in the alphabets of English
(iii) The set of rivers in India
(iv) The set of all planets
(v) The set of points on a circle
(vi) The set of mathematics books in your library
(vii) The set of even positive integers (i.e. 2, 4, 6, 8, ...)
(viii) The set of multiples of 4 (i.e. 4, 8, 12, ...)
(ix) The set of factors of 12. (i.e. 1, 2, 3, 4, 6, 12)
(x) The set of integers less than zero (i.e. –1, –2, –3, ...)

Notations

Sets are usually denoted by capital letters A, B, C, etc., and their elements by small letters a, b, c, etc.

Let A be any set of objects and let 'a' be a member of A, then we write $a \in A$ and read it as 'a belongs to A' or 'a is an element of A' or 'a is a member of A'. If a is not an object of A, then we write $a \notin A$ and read it as 'a does not belong to A' or 'a is not an element of A'.

REPRESENTATION OF SETS

There are two ways of expressing a set. These are:

1. Tabular form or roster form,
2. Set-builder form or rule method.

Tabular Form or Roster Form

In this method, we list all the members of the set separating them by means of commas and enclosing them in curly brackets { }.

Illustration 2 Let A be the set consisting of the numbers 1, 3, 4 and 5, then we write $A = \{1, 3, 4, 5\}$.

Note:

- The order of writing the elements of a set is immaterial. For example, {1, 3, 5}, {3, 1, 5}, {5, 3, 1} all denote the same set.
- An element of a set is not written more than once. Thus, the set {1, 5, 1, 3, 4, 1, 4, 5} must be written as {1, 3, 4, 5}.

Set-builder Form or Rule Method

In this method, instead of listing all elements of a set, we write the set by some special property or properties satisfied by all its elements and write it as,

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$A = \{x : P(x)\}$ or, $A = \{x \mid x$ has the property $P(x)\}$ and read it as "A is the set of all elements x such that x has the property P". The symbol ':' or '|' stands for 'such that'.

Illustration 3 Let A be the set consisting of the elements 2, 3, 4, 5, 6, 7, 8, 9, 10. Then, the set A can be written as $A = \{x : 2 \le x \le 10$ and $x \in N\}$.

FINITE AND INFINITE SETS

Finite Set

A set having no element or a definite number of elements is called a *finite set*. Thus, in a finite set, either there is nothing to be counted or the number of elements can be counted, one by one, with the counting process coming to an end.

Illustration 4 Each of the following sets is a finite set:

(i) A = the set of prime numbers less than 10 $= \{2, 3, 5, 7\}$;

(ii) B = the set of vowels in English alphabets $= \{a, e, i, o, u\}$;

(iii) $C = \{x \mid x$ is divisor of $50\}$.

Cardinal Number of a Finite Set

The number of distinct elements in a finite set S is called the *cardinal number* of S and is denoted by $n(S)$.

Illustration 5 If $A = \{2, 4, 6, 8\}$ then $n(A) = 4$.

Infinite Set

A set having unlimited number of elements is called an *infinite set*. Thus, in an infinite set, if the elements are counted one by one, the counting process never comes to an end.

Illustration 6 Each of the following sets is an infinite set:

(i) the set of all natural numbers = {1, 2, 3, 4, ...);

(ii) the set of all prime numbers = {2, 3, 5, 7, ...);

(iii) the set of all points on a given line;

(iv) the set of all lines in a given plane;

(v) $\{x \mid x \in R$ and $0 < x < 1\}$.

EMPTY SET (OR NULL SET)

The set which contains no element is called the *empty set* or the *null set* or *void set*.

The symbol for the empty set or the null set is ϕ. Thus, $\phi = \{\}$, since there is no element in the empty set.

The empty set is a finite set.

Since any object x which is not equal to itself does not exist, the set $A = \{x : x \ne x\}$ is the empty set ϕ.

A set which is not empty, i.e. which has at least one element is called a *non-empty* set or a *non-void* set.

Illustration 7

(i) The set of natural numbers less than 1 is an empty set;

(ii) the set of odd numbers divisible by 2 is a null set;

(iii) $\{x \mid x \in Z$ and $x^2 = 2\} = \phi$, because there is no integer whose square is 2;

(iv) $\{x \mid x \in R$ and $x^2 = -1\} = \phi$, because the square of a real number is never negative;

(v) $\{x \mid x \in N, 4 < x < 5\}$ is the empty set;

(vi) $\{x \mid x \in Z, -1 < x < 0\}$ is the null set.

The empty set should not be confused with the set {0}. It is the set containing one element, namely 0.

SINGLETON

A set containing only one element is called a *singleton*.

Illustration 8

(i) The set {0} is a singleton since it has only one element 0;

(ii) The set of even prime numbers is the set {2} which is a singleton;

(iii) $\{x \mid x$ is an integer and $-1 < x < 1\} = \{0\}$ is a singleton.

EQUAL SETS

Two sets A and B are said to be *equal* if they have the same elements and we write $A = B$. Thus $A = B$ iff every element of A is an element of B and every element of B is an element of A.

In symbols, $A = B$ iff $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$. To indicate that two sets A and B are not equal, we shall write $A \ne B$.

Illustration 9

(i) If $A = \{2, 3, 4\}$ and $B = \{x \mid 1 < x < 5, x \in N\}$ then $A = B$.

(ii) If A = the set of letters in the word 'WOLF' and B = the set of letters in the word 'FOLLOW' then $A = B$ as each = {W, O, L, F}, remembering that in a set the repetition of elements is meaningless and order of elements is immaterial.

EQUIVALENT SETS

Two finite sets A and B are said to be *equivalent* if they have the same number of elements, i.e. if we can find a one-to-one correspondence between the elements of the two sets.

The symbol ' $\sim$ ' is used to denote equivalence. Thus $A \sim B$ is read as "A is equivalent to B". Two finite sets A and B are equivalent if $n(A) = n(B)$, i.e. if they have the same cardinal number.

Equivalent sets have the same number of elements, not necessarily the same elements. The elements in two equivalent sets may or may not be the same. Thus equal sets are always equivalent but equivalent sets may or may not be equal.

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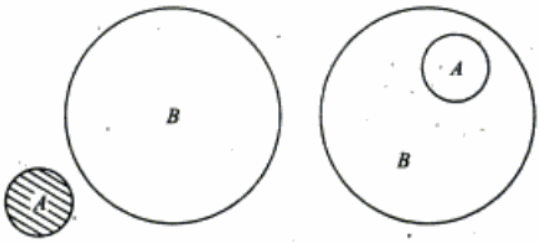

Applications of Sets

1. If a set S has only a finite number of elements, we denote by $n(S)$ the number of elements of S.

Illustration 20 If $U = \{1, 2, 3, 4, 5\}$, then $n(U) = 5$.

2. For any two sets A and B, with finite number of elements, we have the following formula:
$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$
3. If A and B are disjoint sets, then
$$n(A \cup B) = n(A) + n(B).$$

Illustration 21. X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Solution $n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$, $n(X \cap Y) = ?$

Now, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$.

Then, $38 = 17 + 23 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 17 + 23 - 38 = \mathbf{2}$.

ORDERED PAIR

Let A and B be two non-empty sets. If $a \in A$ and $b \in B$, an element of the form (a, b) is called an *ordered pair*, where 'a' is regarded as 'the first element' and 'b' as the second element. It is evident from the definition that

(i) $(a, b) \neq (b, a)$

(ii) $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

Equality of two ordered pairs. Two ordered pairs (a, b) and (c, d) are said to be equal if and only if $a = c$ and $b = d$. The ordered pairs (2, 4) and (2, 4) are equal while the ordered pairs (2, 4) and (4, 2) are different. The distinction between the set {2, 4} and the ordered pair (2, 4) must be noted carefully. We have {2, 4} = {4, 2} but (2, 4) ≠ (4, 2).

CARTESIAN PRODUCT OF SETS

Let A and B be two non-empty sets. The cartesian product of A and B is denoted by $A \times B$ (read as '*A cross B*') and is defined as the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Symbolically,
$$A \times B = \{(a, b): a \in A \text{ and } b \in B\}$$

Illustration 22 Suppose, $A = \{2, 4, 6\}$ and $B = \{x, y\}$ Then,

$A \times B = \{(2, x\}, (4, x), (6, x), (2, y), (4, y), (6, y)\}$

$B \times A = \{(x, 2), (x, 4), (x, 6), (y, 2), (y, 4), (y, 6)\}$

Thus, we note that if $A \neq B$, then $A \times B \neq B \times A$.

Illustration 23 Let $A = \{1, 2, 3\}$ and $B = \phi$. Then, $A \times B = \phi$, as there will be no ordered pair belonging to $A \times B$. Thus, we note that $A \times B = \phi$ if A or B or both of A and B are empty sets.

Illustration 24 Let $n(A)$ represents the number of elements in set A. In Illustration 22, we can see that $n(A) = 3$, $n(B) = 2$ and $n(A \times B) = 6$. Thus, we note that $n(A \times B = n(A) \times n(B)$.

In other works, if a set A has m elements and a set B has n elements, then $A \times B$ has mn elements. Further, it may be noted that $n(A \times B) = n(B \times A)$. This implies that $A \times B$ and $B \times A$ are equivalent sets.

Illustration 25 If there are three sets A, B, C and $a \in A$, $b \in B$, $c \in C$, we form an ordered triplet (a, b, c). The set of all ordered triplets (a, b, c) is called the *cartesian product* of the sets A, B, C. That is,
$$A \times B \times C = \{(a, b, c): a \in A, b \in B, c \in C\}.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Which of the following sets is non-empty?
   (a) A = set of odd natural numbers divisible by 2
   (b) $B = \{x : x + 5 = 0, x \in N\}$
   (c) C = set of even prime numbers
   (d) $D = \{x : 1 < x < 2, x \in N\}$,
2. Which of the following sets is finite?
   (a) $I = \{x; x \in Z \text{ and } x^2 - 2x - 3 = 0\}$
   (b) B = The set of natural numbers which are divisible by 2
   (c) C = The set of lines passing through a point.
   (d) $D = \{x : x \in Z \text{ and } x > -5\}$
3. Which of the following pairs of sets are not equal?
   (a) $A = \{1, 3, 3, 1\}$, $B = \{1, 4\}$
   (b) $A = \{x : x + 2 = 2\}$, $B = \{0\}$
   (c) $A = \{1, 3, 4, 4\}$, $B = \{3, 1, 4\}$
   (d) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$, $B = \left\{\frac{1}{n} : n \in N\right\}$.

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4. Which of the following sets is empty?
(a) $A = \{x : x \in N \text{ and } x \leq 1\}$
(b) $B = \{x : 3x + 1 = 0, x \in N\}$
(c) $C = \{x : x \text{ is an integer and } -1 < x < 1\}$
(d) D = set of months of the year beginning with F.

5. Which of the following sets is infinite?
(a) $\{x : x \text{ is a prime number, } x \text{ is even}\}$
(b) Set of all rivers in India
(c) Set of all concentric circles

6. Which of the following sets is finite?
(a) The set of the months of a year
(b) $\{1, 2, 3, ...\}$
(c) $\{1, 2, 3, ... 90, 100\}$
(d) The set of lines which are parallel to x-axis
(e) The set of numbers which are multiples of 5.

7. Which of the following pairs of sets are not equivalent?
(a) $A = \{2, 4, 6, 8\}$, $B = \{u, v, w, x\}$
(b) $A = \{a, b, c\}$, $B = \{a, \beta, \gamma, \delta, \nu\}$
(c) $A = \{\}$, $B = \phi$
(d) $A = \{x : x = 2n, n \in N\}$, $B = \{x : x = 2n + 1, n \in N\}$.

8. Find the cardinal number of the following set:
$\{x : x \text{ is a letter of the word 'ASSASSINATION'}\}$
(a) 4 (b) 6
(c) 8 (d) 2

9. Find the cardinal number of the following set $\{x : x \text{ is a natural number} \leq 30 \text{ and is divisible by } 7 \text{ or } 11\}$
(a) 4 (b) 8
(c) 6 (d) 2

10. Find the cardinal number of the following set $\{x : x = 2n, n \in N, 4 \leq x < 11\}$
(a) 8 (b) 6
(c) 12 (d) 4

11. Which of the following sets is finite?
(a) $\{x : x \in N \text{ and } x \text{ is a prime number}\}$
(b) $\{x : x \text{ is a quadrilateral on a plane}\}$
(c) $\{x : x \in N \text{ and } x^2 - 25 \leq 0\}$
(d) $\{x: x \in N \text{ and } x \text{ is a multiple of } 3\}$.

12. For which of the following cases A and B are equivalent?
(a) $A = \{a, b, c, ..., z\}$, $B = \{1, 2, 3, ..., 24\}$
(b) $A = \left\{\frac{1}{3}, \frac{1}{2}, \frac{3}{5}\right\}$, $B = \left\{x : x = \frac{n}{n+2}, n \in N\right\}$
(c) $A = \{2, 4, 6\}$, $B = \{(2, 4), (4, 6), (2, 6)\}$
(d) $A = \left\{x : x = \frac{n^3 - 1}{n^3 + 1}, n \in W, n \leq 3\right\}$, $B = \left\{0, \frac{7}{9}, \frac{13}{14}\right\}$.

13. In which of the following cases, $A = B$?
(a) $A = \{12, 14, 16\}$, $B = \{16, 18, 20\}$
(b) $A = \phi$, $b = \{\}$
(c) $A = \{x : x \in W \text{ and } x < 1\}$, $B = \phi$
(d) $A = \{x : x \text{ is a day of the week beginning with } S\}$, $B = \{\text{Sunday}\}$.

14. In a class, 50 students play cricket, 20 students play football and 10 play both cricket and football. How many play at least one of these two games?
(a) 60 (b) 45
(c) 55 (d) 65

15. Write down the power set of the set $\{0\}$.
(a) ϕ (b) $\{0\}$
(c) $\{\phi]$ (d) $\{\phi, \{0\}\}$

16. Find the power set of $A = \{\{a, b\}, c\}$.
(a) $\{\phi, \{a, b\}, \{c\}\}$ (b) $\{A \{a, b\}, \{c\}\}$
(c) $\{\phi, A, \{a, b\}, \{c\}\}$ (d) None of these

17. Which of the following pairs of sets are comparable?
(a) $A = \{1, 3, 5\}$ and $B = \{3, 2, 5, 6\}$
(b) $A = \{x : x \in N \text{ and } x \leq 10\}$ and $B = \{1, 2, 3, ..., 10, 11\}$
(c) $A = \{1, 2, 3, \{4, 5\}\}$ and $B = \{1, 2, 3, 4, 5\}$.
(d) None of these.

18. Let $A = \{\phi, \{\phi\}, 1, \{1, \phi\}, 7\}$. Which of the following is false?
(a) $\phi \in A$ (b) $\{\phi\} \in A$
(b) $\{1\} \in A$ (d) $\{7, \phi\} \subset A$

19. Let = $\{1, 2, \{3, 4\}, 5\}$. Which of the following statements is true?
(a) $\{3, 4\} \subset A$ (b) $\{3, 4\} \in A$
(c) $1 \subset A$ (d) $\{1, 2, 5\} \in A$

20. Let $A = \{1, 3, 5\}$ and $B = \{x : x \text{ is an odd natural number} < 6\}$. Which of the following is false?
(a) $A \subset B$ (b) $B \subset A$
(c) $A = B$ (d) None of these

21. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are true?
(a) $\{3, 4\} \subset A$; (b) $\{3, 4\} \in A$;
(c) $\{\{3, 4\}\} \not\subset A$; (d) $\{1, 3, 5\} \subset A$.

22. Write down the power set of $A = \{8, 9\}$.
(a) $\{\phi, \{8\}, \{9\}, \{8, 9\}\}$
(b) $\{\phi, \{8\}, \{9\}\}$
(c) $\{\phi, \{8\}, \{9\}, \{8, 9\}\}$
(d) None of these.

23. Write down the power set of $C = \{1, \{2\}\}$.
(a) $\{\phi, \{1\}, \{\{2\}\}\}$
(b) $\{\phi, \{1\}, \{\{2\}\}, \{1, \{2\}\}\}$
(c) $\{\{1\}, \{\{2\}\}, \{1\{2\}\}$
(d) None of these

24. If $A = \left\{x : x = \frac{n-1}{n+1}, n \in W \text{ and } n \leq 10\right\}$; point out the correct statement from the following:

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(a) 600 (b) 800
(c) 900 (d) 850

43. Let $A = \{1, 2\}$, $B = \{2, 3\}$. Evaluate $A \times B$.
(a) $\{(2, 1), (3, 1), (2, 3)\}$
(b) $\{(1, 2), (1, 3),, (2, 3)\}$
(c) $\{(1, 2),(1, 3), (2, 2), (2, 3)\}$
(d) None of these.

44. If $A = \{a, b\}$, $B = \{2, 3, 5, 6, 7\}$ and $C = \{5, 6, 7, 8, 9,\}$, find $A \times (B \cap C)$.
(a) A (b) ϕ
(c) $\{(5, a), (6, a), (7, a), (5, b), (6, b), (7, b)\}$
(d) $\{(a, 5), (a, 6), (a, 7), (b, 5), (b, 6), (b, 7)\}$

45. If $A = \{a, d\}$, $B = \{b, c, e\}$ and $C = \{b, c, f\}$ then $A \times (B \cup C) =$
(a) ϕ (b) $(A \times B) \cap (A \times C)$
(c) $(A \times B) \cup (A \times C)$ (d) None of these

46. If $A = \{a, d\}$, $B = \{b, c, e\}$ and $C = \{b, c, f\}$ then $A \times (B \cap C) =$
(a) ϕ (b) $(A \times B) \cap (A \times C)$
(c) $(A \times B) \cup (A \times C)$ (d) None of thse

47. If $A = \{a, d\}$, $B = \{b, c, e\}$ and $C = \{b, c, f\}$ then $A \times (B - C) =$
(a) $(A \times B) - (A \times C)$ (b) $(A \times B)$
(c) $A \times C$ (d) None of these

48. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$ and $D = \{2, 4, 5\}$, then $(A \times B) \cap (C \times D) =$
(a) $(A \cap D) \times (B \cap C)$ (b) $(A \cap C) \times (B \cap D)$
(c) ϕ (d) None of these

49. The catresian product $A \times A$ has 9 elements and two of them are $(-1, 0)$ and $(0, 1)$. Find the set A?
(a) $A = \{0, 1\}$ (b) $A = \{-1, 0, 1\}$
(c) $A = \{0, -1\}$ (d) $A = \{1, -1\}$

50. If A and B have n elements in common, how many elements do $A \times B$ and $B \times A$ have in common?
(a) n (b) n^3
(c) n^2 (d) None of these.

EXPLANATORY ANSWERS

1. Ans. (c)

Solution (a) As no odd natural number is divisible by 2, the set A is empty.

(b) Since no natural number satisfies the equation $x + 5 = 0$, $\therefore B = \phi$.

(c) Since 2 is an even prime number, i.e. $C = \{2\}$, C is not an empty set.

(d) Since there is no natural number between 1 and 2, D is an empty set.

2. Ans. (a)

Solution (a) $A = \{x : x \in Z \text{ and } x^2 - 2x - 3 = 0\}$
$= \{3, -1\}$. $\therefore A$ is a finite set.

(b) B = The set of natural numbers divisible by 2 $= \{2, 4, 6, 8, 10, ...\}$. $\therefore B$ is an infinite set.

(c) Since infinite number of lines pass through a point, C is an infinite set.

(d) $D = \{-4, -3, -2, ...\}$. Clearly D is an infinite set.

3. Ans. (a)

Solution (a) $A = \{1, 3\}$; $B = \{1, 4\}$. A and B have different elements, $\therefore A \neq B$.

(b) $A = \{x : x + 2 = 2\} = \{0\}$; $B = \{0\}$. A and B have same elements, $\therefore A = B$.

(c) $A = \{1, 3, 4, 4\} = \{1, 3, 4\}$; $B = \{3, 1, 4\}$. A and B have same elements, $\therefore A = B$.

(d) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, ...\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, ...\right\}$;

$B = \left\{\frac{1}{n} : n \in N\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, ...\right\}$.

A and B have same elements, $\therefore A = B$.

4. Ans. (b)
5. Ans. (c)
6. Ans. (a)
7. Ans. (b)
8. Ans. (b)
9. Ans. (c)
10. Ans. (d)
11. Ans. (c)
12. Ans. (c)
13. Ans. (b)
14. Ans. (a)

Solution Given: $n(C) = 50$, $n(F) = 20$, $n(C \cup F) = 10$..

Number of students playing at least one of these two games $= n(C \cup F) = n(C) + n(F) - n(C \cap F)$
$= 50 + 20 - 10 = 60$.

15. Ans. (d)

Solution Let $A = \{0\}$. The possible subsets of this set A are ϕ and $\{0\}$, so the power set of the given set A is $P(A) = \{\phi, \{0\}\}$.

16. Ans. (c)

Solution Let $A = \{\{a, b\}, c\}$. To determine $P(A)$: Since A contains two elements $\{a, b\}$, c, $P(A)$ will contain $2^2 = 4$ elements.

The elements of $P(A)$ are ϕ, A, $\{a, b\}$, $\{c\}$.

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17. Ans. (b)

Solution (a) $1 \in A$ but $1 \notin B$ and $6 \in B$ but $6 \neq A$.

$\therefore$ A and B are not comparable.

(b) $A = \{x : x \in N \text{ and } x \leq 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ Clearly, $A \subset B \Rightarrow A$ and B are comparable.

(c) $\{4, 5\} \in A$ but $\{4, 5\} \notin B$ and $4 \in B$ but $4 \notin A$.

$\therefore$ A and B are not comparable.

18. Ans. (c)

19. Ans. (b)

20. Ans. (d)

21. Ans. (b)

22. Ans. (a)

23. Ans. (b)

24. Ans. (a)

25. Ans. (d)

26. Ans. (a)

27. Ans. (d)

Solution $A = \{2, 4, 6, ...\}$

$B = [5, 10, 15, ...\}$

$C = \{10, 20, 30, ...\}$

$\therefore$ $(A \cap B) = \{2, 4, 6, ...\} \cap \{5, 10, 15, ...\}$

$= \{10, 20, 30, ...\} = C$

$\therefore$ $(A \cap B) \cap C = C \cap C = \mathbf{C}$.

28. Ans. (c)

Solution $B \cup C = \{5, 10, 15, ...\} \cup \{10, 20, 30, ...\}$

$= \{5, 10, 15, ...\}$

$\therefore$ $A \cap (B \cup C\} = \{2, 4, 6, ...\} \cap \{5, 10, 15, ...\}$

$= \{10, 20; 20 ...\} = \mathbf{C}$.

29. Ans. (a)

Solution Here $A \cap U = \{2, 4, 7\}$; $B \cup C = \{3, 5, 7, 8, 9, 10, 11\}$. Then $(A \cap U) \cap (B \cup C) = \{2, 4, 7\} \cap \{3, 5, 7, 8, 9, 10, 11\} = \mathbf{\{7\}}$.

30. Ans. (c)

Solution $U \cup A = \{a, b, c, d, e, f\} \cup \{a, b, c\}$

$= \{a, b, c, d, e, f\} = U$

$(U \cup A)' = \boldsymbol{\phi}$.

31. Ans. (d)

Solution $A \cup B = \{a, b, c\} \cup \{c, d, e, f\}$.

$= \{a, b, c, d, e, f\}$

$\therefore$ $(A \cup B) \cup C = \{a, b, c, d, e, f\} \cup \{c, d, e\}$

$= (a, b, c, d, e, f\}$

$= \boldsymbol{U}$.

32. Ans. (a)

Solution $A \cap B = \{a, b, c\} \cap \{c, d, e, f\} = \{c\}$

$A \cap C = \{a, b, c\} \cap \{c, d, e\} = \{c\}$

$\therefore$ $(A \cap B) \cup (A \cap C) = \mathbf{\{c\}}$.

33. Ans. (c)

Solution (a) $\{x : x$ is a natural number and $4 \leq x \leq 6\}$ $= \{4, 5, 6\}$. Now, $\{1, 2, 3, 4\}$ and $\{4, 5, 6\}$ have one element 4 common. Therefore, the given two sets are not disjoint.

(b) The sets $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ have one element e as common. Then, the given two sets are not disjoint.

(c) The sets $\{x : x$ is an even integer$\}$ and $(x : x$ is an odd integer$\}$ have no element as common and therefore they are disjoint sets.

34. Ans. (c)

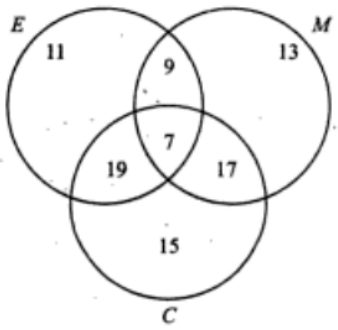
Solution $A - B$ is a set of members which belong to A but do not belong to B

$\therefore$ $A - B = \{3, 5\}$

$\therefore$ $(A - B)' = \mathbf{\{2, 4, 6, 7, 8, 9, 10, 11\}}$.

35. Ans. (a)

Solution



No. of students who passed in one or more subjects

$= 11 + 9 + 13 + 17 + 15 + 19 + 7 = 91$.

No. of students who failed in all the subjects

$= 100 - 91 = 9$.

36. Ans. (a)

Solution We are given $n(X \cup Y) = 18$, $n(X) = 8$, $n(Y) = 15$. Using the formula

$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$,

we get $n(X \cap Y) = 8 + 15 - 18 = \mathbf{5}$.

37. Ans. (b)

Solution We are given $n(A) = 40$, $n(A \cap B) = 60$ and $n(A \cap B) = 10$. Putting these values in the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we get

$60 = 40 + n(B) - 10 \Rightarrow n(B) = \mathbf{30}$.

38. Ans. (c)

Solution $n(S) = 21, n(T) = 32, n(S \cap T) = 11, n(S \cup T) = ?$

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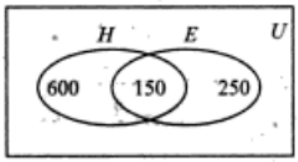
Using $n(S \cup T) = n(S) + n(T) - n(S \cap T)$
$= 21 + 32 - 11 = 42$

Hence, $S \cup T$ has **42** elements.

39. Ans. (a)

Solution Here, $n(H \cup E) = 1000$, $n(H) = 750$, $n(E) = 400$

Using $n(H \cup E) = n(H) + n(E) - n(H \cap E)$



we get $1000 = 750 + 400 - n(H \cap E)$

$\Rightarrow \quad n(H \cap E) = 1150 - 1000 = 150.$

Number of people who can speak Hindi only
$= n(H \cap E') = n(H) - n(H \cap E)$
$= 750 - 150 = \mathbf{600}.$

40. Ans. (c)

Solution Here, $n(M \cup B) = 50$, $n(M) = 35$,
$n(B) = 37$, $n(M \cap B) = ?$

Using $n(M \cup B) = n(M) + n(B) - n(M \cap B)$

we get $50 = 35 + 37 - n(M \cap B)$

$\Rightarrow \quad n(M \cap B) = 35 + 37 - 50 = 72 - 50 = 22$

$\therefore$ 22 students have opted for both Mathematics and Biology.

Again number of students who have opted for only Mathematics $= n(M) - n(M \cap B) = 35 - 22 = \mathbf{13}.$

41. Ans. (a)

Solution Let A = set of people who like coffee and B = set of people who like tea.

Then, $A \cup B$ = set of people who like at least one of the two drinks.

and $A \cap B$ = set of people who like both the drinks.

Here, $n(A) = 37$, $n(B) = 52$, $n(A \cup B) = 70$.

Using the result

$$n((A \cup B) = n(A) + n(B) - n(A \cap B)$$

we have $70 = 37 + 52 - n(A \cap B)$

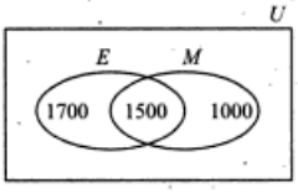
$\Rightarrow n(A \cap B) = 89 - 70 = 19$

$\therefore$ **19** people like both coffee and tea.

42. Ans. (b)

Solution Let E be the set of people who are egg-eaters and M be the set of people who are meat-eaters. We have, $n(E) = 3200$, $n(M) = 2500$, $n(E \cap M) = 1500$.

Using $n(E \cup M) = n(E) + n(M) - n(E \cap M)$
$= 3200 + 2500 - 1500$
$= 5700 - 1500 = 4200.$



$\therefore$ Number of pure vegetarians
$= n(U) - n(E \cup M)$
$= 5000 - 4200 = \mathbf{800}.$

43. Ans. (c)

Solution $A \times B = \{1, 2\} \times \{2, 3\}$
$= \mathbf{\{(1, 2), (1, 3), (2, 2), (2, 3)\}}.$

44. Ans. (d)

Solution We have,

$(B \cap C) = \{2, 3, 5, 6, 7\} \cap \{5, 6, 7, 8, 9\}$
$= \{5, 6, 7\}$

$\therefore \quad A \times (B \cap C) = \{a, b\} \times (5, 6, 7\}$
$= \mathbf{\{(a, 5), (a, 6), (a, 7), (b, 5), (b, 6), (b, 7)\}}.$

45. Ans. (c)

Solution $(B \cup C) = \{b, c, e\} \cup \{b, c, f\} = \{b, c, e, f\}$

$\therefore \quad A \times (B \cup C) = \{a, d\} \times \{b, c, e, f\}$
$= \{(a, b), (a, c), (a, e), (a, f),$
$(d, b), (d, c), (d, e), (d, f)\}$ ...(i)

Also, $(A \times B) = \{(a, b), (a, c), (a, e), (d, b), (d, c), (d, e)\}$

and, $(A \times C) = \{(a, b), (a, c), (a, f), (d, b), (d, c), (d, f)$ ...(ii)

$\therefore \quad (A \times B) \cup (A \times C) = \{(a, b), (a, c), (a, e),$
$(a, f), (d, b), (d, c), (d, e), (d, f)\}$

From (1) and (2), we have

$A \times (B \cup C) = \mathbf{(A \times B) \cup (A \times C)}.$

46. Ans. (b)

Solution $(B \cap C) = \{b, c, e\} \cap \{b, c, f\} = \{b, c\}$

$\therefore \quad A \times (B \cap C) = \{a, d\} \times \{b, c\}$
$= \{(a, b), (a, c), (d, b), (d, c)\}$

Also, $(A \times B) \cap (A \times C)$
$= \{(a, b), (a, c), (a, b), (d, c)\}$

$\therefore \quad A \times (B \cap C) = \mathbf{(A \times B) \cap (A \times C)}.$

47. Ans. (a)

Solution $(B - C) = \{b, c, e\} - \{b, c, f\} = \{e\}$

$\therefore \quad A \times (B - C) = \{(a, e), (d, e)\}$ ...(i)

Also, $(A \times B) - (A \times C) = \{(a, e), (d, e)\}$ ...(ii)

Hence, from (1) and (2), we have

$A \times (B - C) = \mathbf{(A \times B) - (A \times C)}.$

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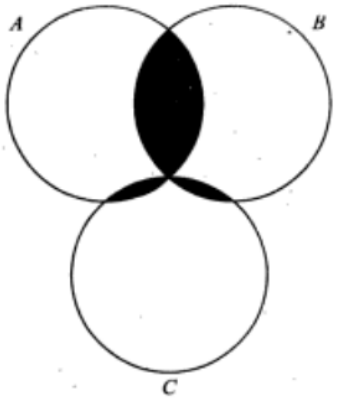
watched news on TV, 40% read in newspaper, 25% read newspaper and watched TV. What percentage of people neither watched TV nor read newspaper?
(a) 0% (b) 5%
(c) 10% (d) 20%
[Based on MAT 2000]

12. In a class, 20 opted for Physics, 17 for Maths, 5 for both and 10 for other subjects. The class contains how many students?
(a) 35 (b) 42
(c) 52 (d) 60
[Based on MAT 2000]

13. In a community of 175 pesons, 40 read the *Times*, 50 read the *Samachar* and 100 do not read any. How many persons read both the papers?
(a) 10 (b) 15
(c) 20 (d) 25
[Based on MAT 2000]

14. The shaded region in the given Venn diagram represents



(a) $A \cup (B \cup C)$ (b) $A \cup (B \cap C)$
(c) $A \cap (B \cup C)$ (d) None of these
[Based on MAT 2001]

15. One Hundred Twenty-five (125) aliens descended on a set of film as Extra Terrestrial Beings. 40 had two noses, 30 had three legs, 20 had four ears, 10 had two noses and three legs, 12 had three legs and four ears, 5 had two noses and four ears and 3 had all the three unusual features. How many were there without any of these unusual features?
(a) 5 (b) 35
(c) 80 (d) None of these
[Based on FMS (Delhi) 2004]

16. Out of 450 students in a school, 193 students read Science Today, 200 students read Junior Statesman, while 80 students read neither. How many students read both the magazines?
(a) 137 (b) 80
(c) 57 (d) 23
[Based on IITTM, Gwalior, 2003]

17. In a group of 15, 7 have studied, German, 8 have studied French, and 3 have not studied either. How many of these have studied both German and French?
(a) 0 (b) 3
(c) 4 (d) 5
[Based on REC Tiruchirapalli 2003]

18. In a class consisting of 100 students, 20 know English and 20 do not know Hindi and 10 know neither English nor Hindi. The number of students knowing both Hindi and English is
(a) 5 (b) 10
(c) 15 (d) 20
[Based on FMS (Delhi) 2003]

19. If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 7\}$, then find $A - B$ and $A \cap B$.
(a) $(3, 5\}$ and $\{2, 4, 6\}$
(b) $\{2, 4, 6\}$ and $\{1, 5\}$
(c) $\{2,4,6, 7\}$ and $\{1, 3,5,6\}$
(d) $\{2, 4\}$ and $\{1, 5\}$
(e) None of the above **[Based on IIFT 2003]**

20. In a survey of brand preference for toothpastes, 82% of the population (number of people covered for the survey) liked at least one of the brands : I, II and III. 40% of those asked liked brand I, 25% liked brand II and 35% liked brand III. If 5% of those asked, showed liking for all the three brands, then what percentage of those asked liked more than one of the three brands?
(a) 13 (b) 10
(c) 8 (d) 5
[Based on SCMHRD Ent. Exam. 2003]

21. A survey of a hundred people was conducted to find out whether they had read the recent issues of Golmal, a monthly magazine. A summary of the information regarding the readership in 3 months is given below:
September: 28; July: 48; July and August: 10;
None of the three months: 24
What is the number of surveyed people who have read exactly two consecutive issues (out of the three)?
(a) 7 (b) 9
(c) 12 (d) 14
(e) 17 **[Based on CAT, 2006]**

22. Consider the set S $\{1, 2, 3, \ldots\ldots 1000\}$. How many arithmetic progression can be formed from the element of S that start with 1 and end with 1000 and have at least 3 elements?
(a) 3 (b) 4
(c) 6 (d) 7
(e) 8 **[Based on CAT, 2006]**

23. If 40% of the Delhites read Times of India, 50% read The Hindustan times and 10% read both the paper, what per cent of e people read neither newspaper?

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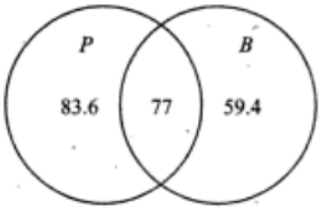
7. Ans. (None)

Solution

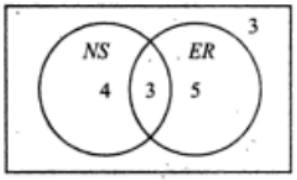


$\therefore$ 77 students out of 220 opted Psychology and Business both.

8. Ans. (c)

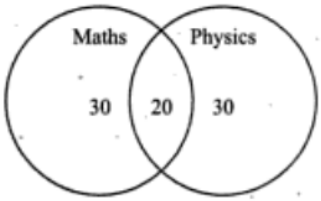
9. Ans. (c)

Solution



10. Ans. (b)

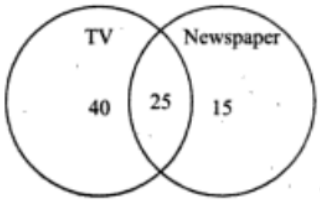
Solution $\therefore$ Number of students passed in both the subjects.



$= 100 - (30 + 20 + 30) = 20.$

11. Ans. (d)

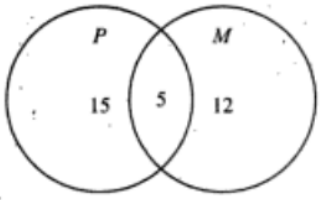
Solution 80% of the people either watched TV or read newspaper.



$\therefore$ 20% neither watched TV nor read newspaper.

12. Ans. (b)

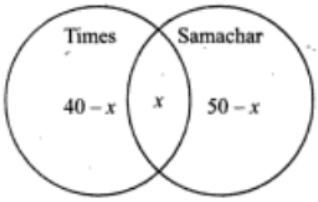
Solution



$\therefore$ Total number of students in the class
$= 15 + 5 + 12 + 10 = 42.$

13. Ans. (b)

Solution

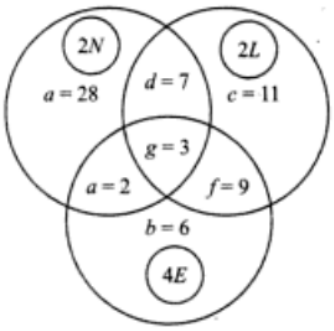


$\therefore \quad (40 - x) + x + (50 - x) = 75$
$\Rightarrow \quad x = 15.$

14. Ans. (c)

15. Ans. (d)

Solution Let x be the number of aliens who were without any of the given three unusual features.



$\therefore$ From the figure, we have

$a + b + c + d + e + f + g + x = 125 \quad ...(i)$

Also,

$$\left.\begin{aligned} a + d + g + e &= 40 \\ c + d + f + g &= 30 \\ b + e + f + g &= 20 \\ d + g &= 20 \\ g + f &= 12 \\ g + e &= 5 \end{aligned}\right\} \text{(Given)}$$

$g = 3$

$\Rightarrow \quad g = 3, d = 7, e = 2, f = 9, b = 6, c = 11, a = 28$

$\therefore \quad$ (i) $\Rightarrow x = 59.$

16. Ans. (d)

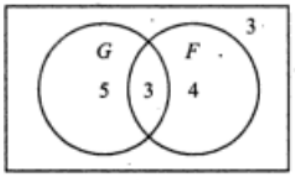
Solution $n(SR \cup JS) = n(ST) + n(JS) - n(ST \cap JS)$

$\Rightarrow \quad (450 - 80) = 193 + 200 - n(ST \cap JS)$

$\Rightarrow \quad n(ST \cap JS) = 393 - 370 = 23.$

17. Ans. (b)

Solution



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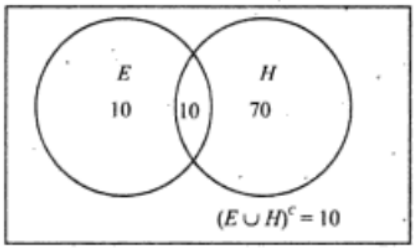
18. Ans. (b)

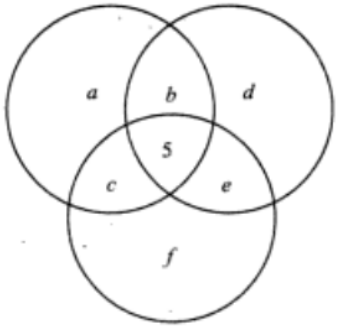


19. Ans. (e)

Solution $A - B = \{2, 4, 6, 8\}$

$A \cap B = \{1, 3, 5, 7\}$.

20. Ans. (b)



Solution Given

$a + b + c + d + e + f = 77$ ...(i)

$a + b + c = 35$ ...(ii)

$d + b + e = 20$ ...(iii)

$f + c + e = 30$ ...(iv)

Adding (ii) and (iv), we get,

$a + b + c + (d + e + f) + b + c + e = 82$ ...(v)

$\Rightarrow \quad b + c + e = 5$

$\Rightarrow \quad b + c + e + 5 = 10.$

21. Ans. (b)

22. Ans. (d)

23. Ans. (c)

24. Ans. (e)

Solution $S = \{2, 3, 4, \text{——} 2n + 1\}$; $S = 2n$

$$A = \frac{3+5+\text{----}+(2n+1)}{n}$$

$$B = \frac{2+4+\text{----}+2n}{n}$$

$$A - B = \frac{1+1+1\text{----}+(n \text{ times})}{n}\frac{n}{n} = 1$$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. The total no. of elements in the power set of a set A containing n elements is
(a) n^2 (b) 2^n
(c) $2^n - 1$ (d) None of these

2. The number of all possible subsets of a set containing n elements is
(a) n (b) $2n$
(c) 2^n (d) $n!$

3. The number of all possible subsets of the set $\{1, \{2, 3\}\}$ is
(a) 2 (b) 4
(c) 6 (d) 8

4. The set $\{x : (x - 2)(x - 3) > 0\}$ is equal to
(a) $\{x : 2 < x < 3\}$
(b) $\{x : x > 3\} \cup \{x : x < 2\}$
(c) $\{x : x > 3\} \cup \{x : x < 2\}$
(d) None of these

5. The number of elements of the set $\{x : x \in N, x^2 = 1\}$, where N is the set of all natural numbers, is
(a) 2 (b) 1
(c) 0 (d) 3

6. If $A \cap B = A$, then
(a) $A \subset B$ (b) $B \subset A$
(c) $A \not\subset B$ and $B \not\subset A$ (d) None of these

7. If $A \cup B = \phi$, then
(a) $A = \phi$ or $B = \phi$ (b) $A = \phi$ and $B = \phi$
(c) $A = \phi$ and $B \neq \phi$ (d) $A \neq \phi$ and $B = \phi$

8. If $A \subseteq B$, which of the following is correct?
(a) $A \cup B = B \Rightarrow A = B$
(b) $A \cap B = B \Rightarrow A = B$
(c) $A - B = A \Rightarrow A = B$
(d) $B - A = B \Rightarrow A = B$.

9. If $U = \{x \in N : 1 \leq x \leq 10\}$, $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 6, 10\}$ then $(A - B)'$ is
(a) $\{6, 10\}$ (b) $\{1, 4\}$
(c) $\{2, 3\}$ (d) $\{2, 3, 5, 6, 7, 8, 9, 10\}$

10. If $A \subseteq B$, then $A - B$ is equal to
(a) A (b) B
(c) ϕ (d) $B - A$.

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution Power set of A is the set of all possible subsets of A. Number of all subsets of A containing n elements is 2^n.

3. Ans. (b)

Solution The given set contains two elements namely 1 and $\{2, 3\}$.

So, the number of subsets of the given set is 2^2, i.e., 4.

5. Ans. (b)

Solution Clearly, 1 is the only natural number whose square is 1. So, the number of elements in the given set is 1.

7. Ans. (b)

Solution $A \cup B = \phi \Rightarrow A = \phi$ and $B = \phi$.

9. Ans. (d)

Solution $A - B = \{1, 4\}$. So, $(A - B)' = \{2, 3, 5, 6, 7, 8, 9, 10\}$.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (c)	**4.** (c)	**6.** (a)	**8.** (b)	**10.** (c)	**12.** (b)	**14.** (c)	**16.** (c)	**18.** (c)	**20.** (a)

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Permutations and Combinations 31

INTRODUCTION

We often come across questions such as the following:

1. In how many ways can four bottles be arranged in a row?
2. In how many ways can five students be seated at a round table?
3. In how many ways can a group of five people be selected out of a gathering of ten people?
4. In how many ways can 5 maps be selected out of 8 and displayed in a row?

Answers to these questions and many other important and more difficult ones can often be given without actually writing down all the different possibilities. In the present chapter we shall study some basic principles of the art of counting without counting which will enable us to answer such questions in an elegant manner.

FACTORIAL NOTATION

The continued product of first n natural numbers is called n factorial or factorial n and is denoted by $\lfloor n$ or $n!$

Thus, $\lfloor n$ or $n! = 1\ \ 2\ \ 3\ \ 4 \ldots\ldots (n-1)n$

$= n\ \ (n-1)\ \ (n-2) \ldots\ldots 3\ \ 2\ \ 1$ (in reverse order)

Notes:

1. When n is a negative integer or a fraction, $n!$ is not defined. Thus, $n!$ is defined only for positive integers.
2. According to the above definition, 0! makes no sense. However, we define 0! = 1.
3. $n! = n\,(n-1)!$
4. $(2n)! = 2^n\ \ n!\,[1\ \ 3\ \ 5\ \ 7 \ldots\ldots (2n-1)]$.

Illustration 1 Evaluate

(i) $\dfrac{30!}{28!}$ (ii) $\dfrac{9!}{5!3!}$

(iii) $\dfrac{12!-10!}{9!}$ (iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}$.

Solution (i) $\dfrac{30!}{28!} = \dfrac{30\times 29\times 28!}{28!} = 30 \times 29 = \mathbf{870.}$

(ii) $\dfrac{9!}{5!3!} = \dfrac{9\times 8\times 7\times 6\times 5!}{5!\times 3\times 2} = \mathbf{504.}$

(iii) $\dfrac{12!-10!}{9!} = \dfrac{12\times 11\times 10!-10!}{9!}$

$= \dfrac{10!}{9!}[132-1]$

$= 10 \times 131$

$= \mathbf{1310.}$

(iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!} = \dfrac{4\times 5}{3!\times 4\times 5}+\dfrac{5}{4!\times 5}+\dfrac{1}{5!}$

$= \dfrac{20}{5!}+\dfrac{5}{5!}+\dfrac{1}{5!}$

$= \dfrac{26}{5!} = \mathbf{\dfrac{13}{60}}.$

Illustration 2 Convert into factorials:

(i) 4.5.6.7.8.9.10.11. (ii) 2.4.6.8.10.

Solution (i) $4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11$

$= \dfrac{1.2.3.4.5.6.7.8.9.10.11}{1.2.3.}$

$= \dfrac{11!}{3!}.$

(ii) $2\ \ 4\ \ 6\ \ 8\ \ 10$

$= (2\ \ 1)\ \ (2\ \ 2)\ \ (2\ \ 3)\ \ (2\ \ 4)\ \ (2.5)$

$= (2\ \ 2\ \ 2\ \ 2\ \ 2)\ \ (1\ \ 2\ \ 3\ \ 4\ \ 5)$

$= \mathbf{2^5\ \ 5!.}$

Fundamental Principle of Counting

Multiplication Principle If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Illustration 3 How many numbers of two digits can be formed out of the digits 1, 2, 3, 4, no digit being repeated?

Solution The first digit can be any one of the four digits 1, 2, 3, 4, that is the first digit can be chosen in four ways. Having chosen the first digit, we are left with three digits

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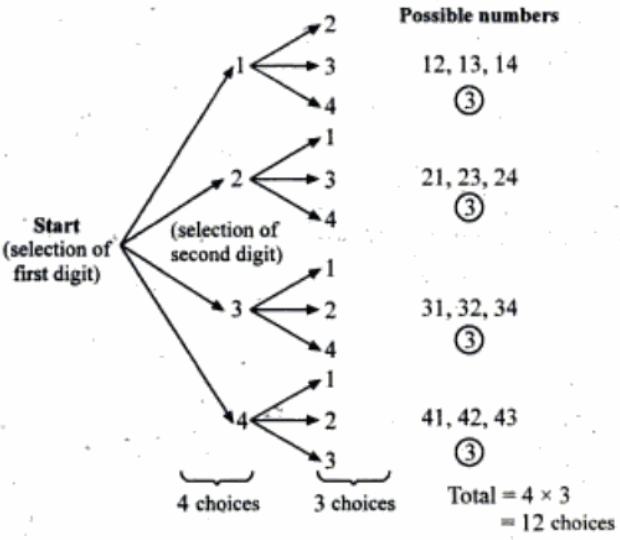
Since the first digit can be chosen in four ways and for each choice of the first digit there are three ways of choosing the second digit, therefore, there are 4×3



ways, that is, 12 ways of choosing both the digits. Thus, 12 numbers can be formed.

Illustration 4 Anu wishes to buy a birthday card for her brother Manu and send it by post. Five different types of cards are available at the card-shop, and four different types of postage stamps are available at the post-office. In how many ways can she choose the card and the stamp?

Solution She can choose the card in five ways. For each choice of the card she has four choices for the stamp. Therefore, there are 5×4 ways, that is, 20 ways of choosing the card and the stamp.

Illustration 5 Mohan wishes to go from Agra to Chennai by train and return from Chennai to Delhi by air. There are six different trains from Agra to Chennai and five different flights from Chennai to Agra. In how many ways can he perform the journey?

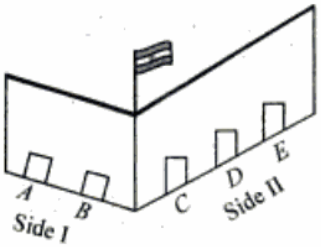
Solution Since he can choose any one of the six trains for going to Chennai, and for each such choice he has five choices for returning to Agra, he can perform the journey in 6×5 ways, that is, 30 ways.

Addition Principle

If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways, then either of the two operations can be performed in $(m + n)$ ways.

Note: The above two principles can be extended for any finite number of operations.

Illustration 6 Suppose there are 5 gates to a stadium, 2 on one side and 3 on the other. Sohan has to go out of the stadium. He can go out from any one of the 5 gates. Thus, the number of ways in which he can go out is 5. Hence, the work of going out through the gates on one side will be done in 2 ways and the work of going out through the gates on other side will be done in 3 ways. The work of going out will be done when Sohan goes out from side I or side II. Thus the work of going out can be done in $(2 + 3) = 5$ ways.



Note: Addition theorem of counting is also true for more than two opeartions.

Permutation

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a *permutation.*

Note: Permutation of things means arrangement of things. The word arrangement is used if order of things is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Notation

Let r and n be positive integers such that $1 \le r \le n$. Then, the number of permutations of n different things, taken r at a time, is denoted by the symbol ${}^nP_r$ or $P(n, r)$.

SOME BASIC RESULTS

1. ${}^nP_r$ or $P(n, r) = \dfrac{n!}{(n-r)!} = n\,(n-1)\,(n-2) \ldots [n-(r+1)],\ 0 \le r \le n.$

Illustration 7 Evaluate the following:

(i) $P(6, 4)$, (ii) $P(15, 3)$, (iii) $P(30, 2)$.

Solution (i) We have

$$P(6, 4) = \frac{6!}{(6-4)!} = \frac{6.5.4.3.2.1}{2.1} = \mathbf{360.}$$

(ii) We have

$$P(15, 3) = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{(15 \cdot 14 \cdot 13)\,(12!)}{12!} = \mathbf{2730.}$$

(iii) $P(30, 2) = \dfrac{30!}{(30-2)!} = \dfrac{30!}{28!} = \dfrac{(30.29)\,(28!)}{28!}$

$= \mathbf{870.}$

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2. The number of permutations of n things, taken all at a time, out of which p are alike and are of one type, q are alike and are of second type and rest are all different $= \frac{n!}{p!q!}$.

Illustration 8 There are 5 red, 4 white and 3 blue marbles in a bag. They are drawn one by one and arranged in a row. Assuming that all the 12 marbles ae drawn, determine the number of different arrangements.

Solution Here, $n = 12$, $p_1 = 5$, $p_2 = 4$ and $p_3 = 3$

$\therefore$ The required number of different arrangements

$$= \frac{n!}{p_1!p_2!p_3!} = \frac{12!}{5!4!3!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{2}$$

$= 990 \times 4 \times 7 =$ **27720.**

3. The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

Illustration 9 In how many ways can 5 apples be distributed among 4 boys, there being no restriction to the number of apples each boy may get?

Solution The required number of ways $= 4^5$.

4. **Permutations under Restrictions**

(a) Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is $r \cdot {}^{n-1}P_{r-1}$.

(b) Number of permutations of n different things, taken r at a time, when s particular things are to be always included in each arrangement, is $s!\,(r-(s-1)) \cdot {}^{n-s}P_{r-s}$.

(c) Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement, is ${}^{n-1}P_r$.

(d) Number of permutations of n different things, taken all at a time, when m specified things always come together, is $m! \times (n-m+1)!$

(e) Number of permutations of n different things, taken all at a time, when m specified things never come together, is $n! - m! \times (n-m+1)!$.

5. **Circular Permutations**

(a) Number of circular arrangements (permutations) of n different things $= (n-1)!$.

Illustration 10 In how many ways can eight people be seated at a round table?

Solution Required number of ways $= (8-1)!$

$= 7! =$ **5040.**

(b) Number of circular arrangements (permutations) of n different things when clockwise and anticlockwise arrangements are not different, that is, when observation can be made from both sides $= \frac{1}{2}(n-1)!$.

Illustration 11 Find the number of ways in which n different beads can be arranged to form a necklace.

Solution Required number of arrangements

$$= \frac{1}{2}(5-1)! = \frac{1}{2} \times 4! = \mathbf{12.}$$

Combination

Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a *combination*.

Note: Combination of things means selection of things. Obviously, in selection of things order of things has no importance. Thus, with the change of order of things selection of things does not change.

Notations The number of combinations of n different things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$.

Thus, ${}^nC_r = \frac{n!}{r!(n-r)!}$ $\quad (0 \le r \le n)$

$$= \frac{{}^nP_r}{r!}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots 3 \cdot 2 \cdot 1}$$

If $r > n$, then ${}^nC_r = 0$.

Illustration 12 Evaluate:

(i) ${}^{11}C_3$ (ii) ${}^{10}C_8$ (iii) ${}^{100}C_{98}$

Solution (i) ${}^{11}C_3 = \frac{11!}{3!(11-3)!} = \frac{11!}{3!8!}$

$$= \frac{11 \times 10 \times 9 \times 8!}{3 \times 2 \times 8!} = \mathbf{165.}$$

(ii) ${}^{10}C_8 = \frac{10!}{8!2!} = \frac{10 \times 9 \times 8!}{8! \times 2} = \mathbf{45.}$

(iii) ${}^{100}C_{98} = \frac{100!}{98!2!} = \frac{100 \times 99 \times 98!}{98! \times 2} = \mathbf{4950.}$

Key Points to Remember

1. ${}^nC_r = {}^nC_{n-r}$
2. ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$
3. If ${}^nC_x = {}^nC_y$ then either $x = y$ or $y = n - x$ that is, $x + y = n$.
4. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
5. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$.

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6. If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
7. If n is odd then the greatest value of ${}^nC_r$ is
$${}^nC_{\frac{n+1}{2}} \text{ or } {}^nC_{\frac{n-1}{2}}.$$
8. $${}^nC_r = \frac{r \text{ decreasing numbers starting with } n}{r \text{ increasing numbers starting with } 1} = \frac{n(n-1)(n-2)....(n-r+1)}{1\cdot2\cdot3.....r}.$$
9. $${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n.$$
10. $${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}.$$
11. Number of combinations of n different things taken r at a time
   (a) when p particular things are always included $= {}^{n-p}C_{r-p}$
   (b) when p particular things are never included $= {}^{n-p}C_r$
   (c) when p particular things are not together in any selection $= {}^nC_r - {}^{n-p}C_{r-p}$.

Illustration 13 In how many ways can 5 members forming a committee out of 10 be selected so that
(i) two particular members must be included.
(ii) two particular members must not be included.

Solution (i) When two particular members are included, then, we have to select $5-2=3$ members out of $10-2=8$.
∴ The required no. of ways
$$= C(8, 3) = \frac{8!}{3!\,5!} = \frac{8\times7\times6}{6} = \mathbf{56.}$$

(ii) When 2 particular members are not included, then, we have to select 5 members out of $10 - 2 = 8$.
∴ The required no. of ways
$$= C(8, 5) = \frac{8!}{5!\,3!} = \frac{8\times7\times6}{6} = \mathbf{56.}$$

12. (a) Number of selections of r consecutive things out of n things in a row $= n - r + 1$.
   (b) Number of selections of r consecutive things out of n things along a circle
$$\begin{cases} n, \text{ when } r < n \\ 1, \text{ when } r = n \end{cases}.$$
13. (a) Number of selections of zero or more things out of n different things.
$${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$$
   (b) Number of combinations of n different things selecting at least one of them is
$${}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n - 1.$$
   (c) Number of selections of r things ($r \le n$) out of n identical things is 1.
   (d) Number of selections of zero or more things out of n identical things $= n + 1$.
   (e) Number of selections of one or more things out of n identical things $= n$.
   (f) If out of $(p + q + r + t)$ things, p are alike of one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of selections is $(p + 1)(q + 1)(r + 1)2^t - 1$.
   (g) The number of ways of selecting some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and rest are alike of third kind is $[(p + 1)(q + 1)(r + 1)] - 1$.
14. (a) Number of ways of dividing $m + n$ different things in two groups containing m and n things, respectively ($m \ne n$):
$${}^{m+n}C_m = \frac{(m+n)!}{m!\,n!}.$$
   (b) Number of ways of dividing $m + n + p$ different things in three groups containing m, n and p things, respectively ($m \ne n \ne p$):
$$\frac{(m+n+p)!}{m!\,n!\,p!}.$$

SOME USEFUL SHORT-CUT METHODS

1. The number of triangles which can be formed by joining the angular points of a polygon of n sides as vertices are $\frac{n(n-1)(n-2)}{6}$.

Illustration 14 Find the number of triangles formed by joining the vertices of an octagon.

Solution The required number of triangles
$$= \frac{n(n-1)(n-2)}{6} = \frac{8(8-1)(8-2)}{6} = \frac{8\times7\times6}{6} = \mathbf{56.}$$

2. The number of diagonals which can be formed by joining the vertices of a polygon of n sides are $\frac{n(n-3)}{2}$.

Illustration 15 How many diagonals are there in a decagon?

Solution The required number of diagonals

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$$= \frac{n(n-3)}{2} = \frac{10(10-3)}{2}$$
$$= \frac{10\times 7}{2} = \mathbf{35.}$$

3. If there are 'm' horizontal lines and 'n' vertical lines then the number of different rectangles formed are given by $({}^{m}C_2 \times {}^{n}C_2)$.

Illustration 16 In a chess board there are 9 vertical and 9 horizontal lines. Find the number of rectangles formed in the chess board.

Solution The require number of rectangles.
$$= {}^{9}C_2 \times {}^{9}C_2 = 36 \times 36 = \mathbf{1296.}$$

4. These are 'n' points in a plane out of which 'm' points are collinear. The number of triangles formed by the points as vertices are given by ${}^{n}C_3 - {}^{m}C_3$.

Illustration 17 There are 14 points in a plane out of which 4 are collinear. Fine the number of triangles formed by the points as vertices.

Solution The required number of triangles
$$= {}^{14}C_3 - {}^{4}C_3 = 364 - 4 = \mathbf{360.}$$

5. There are 'n' points in a plane out of which 'm' points are collinear. The number of straight lines formed by joining them are given by $({}^{n}C_2 - {}^{m}C_2 + 1)$.

Illustration 18 There are 10 points in a plane out of which 5 are collinear. Find the number of straight lines formed by joining them.

Solution The required number of straight lines
$$= {}^{n}C_2 - {}^{m}C_2 + 1$$
$$= {}^{10}C_2 - {}^{5}C_2 + 1 = 45 - 10 + 1 = \mathbf{36.}$$

6. If there are 'n' points in a plane and no three points are collinear, then the number of triangles formed with 'n' points are given by $\frac{n(n-1)(n-2)}{6}$.

Illustration 19 Find the number of triangles that can be formed with 14 points in a plane of which no three points are collinear.

Solution The required number of triangles
$$= \frac{n(n-1)(n-2)}{6} = \frac{14\times 13\times 12}{6} = \mathbf{364.}$$

7. The number of quadrilaterals that can be formed by joining the vertices of a polygon of n sides are given by $\frac{n(n-1)(n-2)(n-3)}{24}$, where $n > 3$.

Illustration 20 Find the number of quadrilaterals that can be formed by joining the vertices of a septagon.

Solution The required number of quadrilaterals
$$= \frac{n(n-1)(n-2)(n-3)}{24}$$
$$= \frac{7(7-1)(7-2)(7-3)}{24}$$
$$= \frac{7\times 6\times 5\times 4}{24} = \mathbf{35.}$$

8. There are n points in a plane and no points are collinear, then the number of straight lines that can be drawn using these 'n' points are given by $\frac{n(n-1)}{2}$.

Illustration 21 How many straight lines can be drawn with 18 points on a plane of which no points are collinear?

Solution The required number of straight lines
$$= \frac{n(n-1)}{2} = \frac{18(18-1)}{2} = \frac{18\times 17}{2} = \mathbf{153.}$$

9. In a party every person shakes hands with every other person. If there was a total of H handshakes in the party, then the number of persons 'n' who were present in the party can be calculated from the equation:
$$\frac{n(n-2)}{2} = H.$$

Illustration 22 In a party every person shakes hands with every other person. If there was a total of 105 handshakes in the party, find the number of persons who were present in the party.

Solution Let 'n' be the number of persons present in the party.

We have, the equation
$$\frac{n(n-1)}{2} = H \Rightarrow \frac{n(n-1)}{2} = 105$$
$$\Rightarrow n(n-1) = 15 \times (15-1) \Rightarrow \boldsymbol{n = 15.}$$

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17. How many 3-digit numbers each less than 600 can be formed from the digits 1, 2, 3, 4, 5 and 9, if repetition of digits is allowed?
(a) 180 (b) 160
(c) 165 (d) 185

18. How many words (with or without meaning) of three distinct English alphabets are there?
(a) 15600 (b) 14650
(c) 12800 (d) 13700

19. How many numbers are there between 100 and 1000 in which all the digits are distinct?
(a) 548 (b) 648
(c) 748 (d) 756

20. How many integers between 1000 and 10000 have no digits other than 4, 5 or 6?
(a) 91 (b) 51
(c) 81 (d) 71

21. A number lock on a suitcase has 3 wheels each labelled with 10 digits from 0 to 9. If opening of the lock is a particular sequence of three digits with no repeats, how many such sequences will be possible?
(a) 720 (b) 760
(c) 680 (d) 780

22. A customer forgets a four-digit code for an Automatic Teller Machine (A.T.M.) in a bank. However, he remembers that this code consists of digits 3, 5, 6 and 9. Find the largest possible number of trials necessary to obtain the correct code.
(a) 12 (b) 24
(c) 48 (d) 36

23. If $(n + 2)! = 2550\,(n!)$, find n
(a) 38 (b) 35
(c) 49 (d) 43

24. If $(n + 1)! = 6[(n - 1)!]$, find n
(a) 6 (b) 4
(c) 8 (d) 2

25. If $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ratio 2 : 1, find the value of n.
(a) 0 (b) 1
(c) 2 (d) 3

26. Find n if ${}^{n}P_{4} = 18.\ {}^{n-1}P_{2}$.
(a) 4 (b) 8
(c) 6 (d) 12

27. If $P(56, r + 6) : P(54, r + 3) = 30800 : 1$, find r.
(a) 51 (b) 41
(c) 31 (d) 43

28. In how many ways can 10 people line up at a ticket window of a cinema hall?
(a) 3628800 (b) 3482800
(c) 344800 (d) 3328800

29. How many words, with or without meaning, can be formed using all letters of the word EQUATION, using each letter exactly once?
(a) 38320 (b) 39320
(c) 40320 (d) 38400

30. Ten students are participating in a race. In how many ways can the first three prizes be won?
(a) 920 (b) 680
(c) 820 (d) 720

31. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?
(a) 2880 (b) 2480
(c) 3680 (d) 3280

32. Four books, one each in Chemistry, Physics, Biology and Mathematics are to be arranged in a shelf. In how many ways can this be done?
(a) 12 (b) 36
(c) 24 (d) 48

33. There are three different rings to be worn in four fingers with at most one in each finger. In how many ways can this be done?
(a) 36 (b) 28
(c) 24 (d) 32

34. In an examination hall, there are four rows of chairs. Each row has 8 chairs one behind the other. There are two classes sitting for the examination with 16 students in each class. It is desired that in each row all students belong to the same class and that no two adjacent rows are allotted to the same class. In how many ways can these 32 students be seated?
(a) $2 \times 16! \times 16!$ (b) $2 \times 15! \times 15!$
(c) $2 \times 16! \times 15!$ (d) None of these

35. How many numbers lying between 1000 and 10000. can be formed by using the digits 1, 3, 5, 6, 7, 8, 9, no digits being repeated?
(a) 940 (b) 640
(c) 840 (d) 740

36. How many different numbers of six digits can be formed with the numbers 3, 1, 7, 0, 9, 5?
(a) 500 (b) 400
(c) 400 (d) 600

37. How many three-digit numbers are there, with no digits repeated?
(a) 648 (b) 548
(c) 848 (d) 748

38. If there are six periods in each working day of a school, in how many ways can one arrange 5 subjects such that each subject is allowed at least one period?
(a) 3500 (b) 3600
(c) 3550 (d) 3650

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39. Four alphabets *E*, *K*, *S* and V, one in each, were purchased from a plastic warehouse. How many ordered pairs of alphabets, to be used as initials, can be formed from them?
(a) 18 (b) 12
(c) 14 (d) 16

40. There are 8 students appearing in an examination of which 3 have to appear in a Mathematics paper and the remaining 5 in different subjects. In how many ways can they be made to sit in a row if the candidates in Mathematics cannot sit next to each other?
(a) 14400 (b) 16400
(c) 15400 (d) 17400

41. Find how many words can be formed out of the letters of the word 'ORIENTAL' so that vowels always occupy the odd places.
(a) 576 (b) 578
(c) 676 (d) None of these

42. The number of different six-digit numbers that are divisible by 10, which can be formed using the digits 1, 2, 7, 0, 9, 5?
(a) 100 (b) 120
(c) 140 (d) 160

43. In how many ways can the letters of the word 'UNIVERSAL' be arranged? In how many of these will *E*, *R*, *S* always occur together?
(a) 32240 (b) 30240
(c) 30240 (d) 31240

44. The principal wants to arrange 5 students on the platform such that the boy SUNIL occupies the second position and such that the girl GITA is always adjacent to the girl NITA. How many such arrangements are possible?
(a) 12 (b) 8
(c) 14 (d) 16

45. In how many different ways, the letters of the work ALGEBRA can be arranged in a row if
(i) The two As are together?
(ii) The two As are not together?
(a) (i) 720, (ii) 1800 (b) (i) 620, (ii) 1600
(c) (i) 780, (ii) 1860 (d) (i) 720, (ii) 1600

46. In how many ways can 6 apples be distributed among 3 boys, there being no restriction to the number of apples each boy may get?
(a) 729 (b) 739
(c) 759 (d) 749

47. In how many different ways can the letters of the word 'KURUKSHETRA' be arranged?
(a) 4497600 (b) 4979600
(c) 4989600 (d) 4789600

48. In how many different ways can the letters of the word 'ALLAHABAD' be permuted?
(a) 7560 (b) 7840
(c) 7460 (d) 7650

49. How many three-digit numbers can be formed by using the digits 1, 3, 6 and 8, when the digits may be repeated any no. of times?
(a) 48 (b) 64
(c) 80 (d) 32

50. How many three-digit numbers can be formed by using the digits 0, 2, 3, 6, 8 when the digits may be repeated any number of times?
(a) 110 (b) 120
(c) 100 (d) None of these

51. How many different words can be formed with the letters of the word 'BHARAT'?
In how many of these *B* and *H* are never together?
(a) 240, 180 (b) 360, 240
(c) 320, 200 (d) 380, 260

52. How many arrangements can be made of the letters of the word 'ARRANGEMENT'?
(a) 2492800 (b) 249300
(c) 2494800 (d) 2491800

53. If the different permutations of the word EXAMINATION are listed as in a dictionary, how many items are there in this list before the first word starting with *E*?
(a) 906200 (b) 907200
(c) 908200 (d) 905200

54. How many 5-digit even numbers can be formed using the digits 1, 2, 5, 5, 4?
(a) 16 (b) 36
(c) 24 (d) 48

55. How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?
(a) 360 (b) 240
(c) 480 (d) None of these

56. Find the number of arrangements of the letters of the word 'ALGEBRA' without altering the relative position of the vowels and the consonants.
(a) 80 (b) 48
(c) 64 (d) 72

57. In how many ways can the letters of the word BALLOON be arranged so that two Ls do not come together?
(a) 900 (b) 1200
(c) 800 (d) 600

58. How many different signals can be transmitted by arranging 3 red, 2 yellow and 2 green flags on a pole? [Assume that all the 7 flags are used to transmit a signal.]
(a) 220 (b) 240
(c) 200 (d) 210

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59. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1, so that odd digits always occupy the odd places?
(a) 36 (b) 24
(c) 18 (d) 12

60. There are 5 gentlemen and four ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?
(a) 3280 (b) 2880
(c) 2080 (d) 2480

61. Three boys and three girls are to be seated around a table in a circle. Among the boys, X does not want any girl neighbour and the girl Y does not want any boy neighbour. How many such arrangements are possible?
(a) 6 (b) 4
(c) 8 (d) 2

62. How many different necklaces can be formed with 6 white and 5 red beads?
(a) 18 (b) 24
(c) 21 (d) 27

63. The Chief Ministers of 11 States of India meet to discuss the language problem. In how many ways can they seat themselves at a round table so that the Punjab and Madras Chief Ministers sit together?
(a) 725760 (b) 625760
(c) 925760 (d) 825760

64. If $C(n, 7) = C(n, 5)$, find n
(a) 15 (b) 12
(c) 18 (d) 2

65. If $C(n, 8) = C(n, 6)$, find $C(n, 2)$
(a) 91 (b) 81
(c) 61 (d) 71

66. If the ratio $C(2n, 3) : C(n, 3)$ is equal to 11 : 1, find n.
(a) 6 (b) 9
(c) 12 (d) 18

67. If ${}^{2n}C_r = {}^{2n}C_{r+2}$; find r.
(a) $n - 1$ (b) $n - 2$
(c) $n - 4$ (d) $n - 3$

68. If ${}^{18}C_r = {}^{18}C_{r+2}$; find ${}^{r}C_5$.
(a) 56 (b) 63
(c) 49 (d) 42

69. If $12\ {}^{n}C_2 = {}^{2n}C_3$: find n.
(a) 7 (b) 5
(c) 9 (d) 3

70. Find $\sum_{r=1}^{5} C(5, r)$
(a) 41 (b) 31
(c) 51 (d) 61

71. In how many ways can 5 sportsmen be selected from a group of 10?
(a) 272 (b) 282
(c) 252 (d) 242

72. In how many ways can a cricket team of 11 players be selected out of 16 players, if two particular player are always to be included?
(a) 2006 (b) 2004
(c) 2008 (d) 2002

73. In how many ways can a cricket team of 11 players be selected out of 16 players if one particular players is to be excluded?
(a) 1565 (b) 1365
(c) 1165 (d) 1265

74. In how many ways can a cricket team of 11 players be selected out of 16 players if two particular players are to be included and one particular player is to be rejected?
(a) 715 (b) 615
(c) 915 (d) 515

75. A question paper has two parts, part A and part B, each containing 10 questions. If the student has to choose 8 from part A and 5 from part B, in how many ways can he choose the question?
(a) 11240 (b) 12240
(c) 13240 (d) 11340

76. In how many ways can a football team of 11 players be selected from 15 players? In how many cases a particular player be included?
(a) 1101 (b) 1011
(c) 1001 (d) 1111

77. How many words, each of 3 vowels and 2 consonants, can be formed from the letters of the word 'INVOLUTE'?
(a) 2280 (b) 2480
(c) 2880 (d) 2680

78. How many lines can be drawn through 21 points on a circle?
(a) 310 (b) 210
(c) 410 (d) 570

79. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls, if each selection consists of 3 balls of each colour.
(a) 3000 (b) 1000
(c) 2000 (d) 4000

80. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 courses are compulsory for every student?
(a) 45 ways (b) 35 ways
(c) 55 ways (d) 65 ways

81. In an examination, Yamini has to select 4 questions from each part. There are 6, 7 and 8 questions in Part I, Part II and Part III, respectively. What is the number of possible combinations in which she can choose the questions?
(a) 39650 (b) 37650
(c) 36750 (d) 38750

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$\therefore$ The total three-digit numbers
$= 9 \times 9 \times 8 =$ **648.**

20. Ans. (c)

Solution Any number between 1000 and 10000 is of 4 digits. The unit's place can be filled up by 4 or 5 or 6, that is, in 3 ways.

Similarly, the ten's place can be filled up by 4 or 5 or 6, that is, in 3 ways. The hundred's place can be filled up by 4 or 5 or 6, that is in 3 ways and the thousand's place can be filled up by 4 or 5 or 6, that is, in 3 ways.

Hence, the required number of numbers
$= 3 \times 3 \times 3 \times 3 =$ **81.**

21. Ans. (a)

Solution On first wheel there can be 10 digits. On the second wheel there will be one of the 9 digits and on the third wheel there will be 8 digits. Therefore, the number of numbers is $10 \times 9 \times 8 =$ **720.**

22. Ans. (b)

Solution At the first place he can try any of the four digits hence in first trial he tries four digits. In the second place he will try three remaining digits. Similarly, he will try two and one digit at the third and fourth places.

Thus, the number of trials is $= 4 \times 3 \times 2 \times 1 =$ **24.**

23. Ans. (c)

Solution $(n + 2)! = 2550\ (n!)$

$\Rightarrow (n + 2)\ (n + 1)\ (n!) = 2550(n!)$

$\Rightarrow (n + 2)\ (n + 1) = 2550$

$\Rightarrow n^2 + 3n + 2 - 2550 = 0$

$\Rightarrow n^2 + 3n - 2548 = 0$

$\Rightarrow n^2 + 52n - 49n - 2548 = 0$

$\Rightarrow n(n + 52) - 49\ (n + 52) = 0$

$\Rightarrow (n - 49)\ (n + 52) = 0$

$\Rightarrow n = 49$ or, $n = -52$

$\Rightarrow n = 49$ as $n = -52$ is rejected being $n \varepsilon N$

$\therefore$ $n = 49.$

24. Ans. (d)

Solution $(n + 1)\ ! = 6[(n - 1)!]$

$\Rightarrow (n + 1).\ n.\ [(n - 1)!] = 6[(n - 1)!]$

$\Rightarrow n^2 + n = 6 \Rightarrow n^2 + n - 6 = 0$

$\Rightarrow (n - 2)\ (n + 3) = 0$

$\therefore$ Either $n - 2 = 0$ or $n + 3 = 0$

$\Rightarrow n = 2$ or $n = -3$

n being natural number, so $n \neq -3$, $\therefore$ $n = 2.$

25. Ans. (c)

Solution $\dfrac{n!}{2!(n-2)!} : \dfrac{n!}{4!(n-4)!} = 2 : 1$

$\Rightarrow \dfrac{n!}{2!(n-2)!} \times \dfrac{4!(n-4)!}{n!} = \dfrac{2}{1} \Rightarrow \dfrac{4!(n-4)!}{2!(n-2)!} = 2$

$\Rightarrow \dfrac{4 \times 3 \times 2!}{2!} \times \dfrac{(n-4)!}{(n-2)(n-3)(n-4)!} = 2$

$\Rightarrow 12 = 2(n - 2)(n - 3)$

$\Rightarrow 6 = n^2 - 5n + 6 \Rightarrow n^2 - 5n = 0$

$\Rightarrow n(n - 5) = 0 \Rightarrow n = 0$ **or 5.**

26. Ans. (c)

Solution ${}^nP_4 = 18 \cdot {}^{n-1}P_2$

$\Rightarrow \dfrac{n!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-1-2)!}$

$\Rightarrow \dfrac{n!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-3)!}$

$\Rightarrow \dfrac{n(n-1)!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-3)(n-4)!}$

$\therefore\ n = \dfrac{18}{n-3}$

i.e. $n^2 - 3n - 18 = 0 \Rightarrow (n - 6)(n + 3) = 0$

$\Rightarrow n = 6, -3$

But n cannot be negative

$\therefore$ $n = 6.$

27. Ans. (b)

Solution $P(56, r + 6) : P(54, r + 3) = 30800 : 1$

$\Rightarrow \dfrac{56!}{(50-r)!} : \dfrac{54!}{(51-r)!} = 30800 : 1$

$\Rightarrow \dfrac{54!.55.56}{(50-r)!} : \dfrac{54!}{(50-r)!.(51-r)} = 30800 : 1$

$\Rightarrow \dfrac{3080}{1} : \dfrac{1}{51-r} = 30800 : 1$

$\Rightarrow 3080\ (51 - r) = \dfrac{30800}{1}$

$\Rightarrow 51 - r = 10 \Rightarrow r = 51 - 10$

$\therefore$ $r = 41.$

28. Ans. (a)

Solution The required number of ways = number of permutations of 10 people taking all 10 at a time.

$= P(10, 10) = 10!$

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$=$ **3628800.**

29. Ans. (c)

Solution The word EQUATION has exactly 8 letters which are all different.

$\therefore$ Number of words that can be formed = number of permutations of 8 letters taken all at a time

$= P\ (8, 8) = 8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$=$ **40320.**

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30. Ans. (d)

Solution Out of 10 students, the first three prizes can be won in

$$^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!}$$
$$= 10 \times 9 \times 8$$
$$= \mathbf{720 \text{ ways.}}$$

31. Ans. (a)

Solution Total number of candidates = 5 + 4 = 9. In the row of 9 positions, the even places are 2nd, 4th, 6th and 8th. Now, number of even places = 4. Number of women to occupy the even places = 4.

∴ Even places can be filled = $P(4, 4)$ ways

Number of men = 5

∴ The remaining 5 places can be filled by 5 men

$= P(5, 5)$ ways

∴ By the fundamental principle of counting:

∴ The required number of seating arrangements

$= P(4, 4) \times P(5, 5) = 4! \times 5!$

$= 24 \times 120 = \mathbf{2880.}$

32. Ans. (c)

Solution 4 different books can be arranged among themselves, in a shelf, in $P(4, 4)$

$= 4 \times 3 \times 2 \times 1 = \mathbf{24 \text{ ways.}}$

33. Ans. (c)

Solution Wearing 3 different rings in four fingers with at most one in each finger is equivalent to arranging 3 different objects in 4 places.

This can be done in $P(4, 3) = 4 \times 3 \times 2 = \mathbf{24 \text{ ways.}}$

34. Ans. (a)

Solution There are four rows of chairs (say I, II, III, IV) consisting of 8 chairs each. It is desired that in each row, all students belong to the same class and no two adjacent rows are allotted to same class.

Therefore, one class can be seated in either I and III or in II and IV, that is in 2 ways.

Now, 16 students of this class can be arranged in 16 chairs in $^{16}P_{16} = 16!$ ways.

16 students of other class can be arranged in remaining 16 chairs in $^{16}P_{16} = 16!$ ways

∴ Total number of ways $= \mathbf{2 \times 16! \times 16!}$.

35. Ans. (c)

Solution A number lying between 1000 and 10000 has four places which can be filled up out of 7 digits in $^7P_4 = 7 \times 6 \times 5 \times 4 = \mathbf{840 \text{ ways.}}$

36. Ans. (d)

Solution The numbers that can be formed, by taking all six digits together

$= {}^6P_6 = 6!$

But we have to neglect the numbers which begin with zero. Now, the numbers in which zero comes in the 1st place = 5!

Hence, the required number = 6! – 5!

= 720 – 120

$= \mathbf{600.}$

37. Ans. (a)

Solution The required number of three-digit numbers = The permutations of the 10 objects 0, 1, 2, 3, 4, 5, 6,7, 8, 9, take 3 at a time, with the condition that 0 is not in the hundred's place.

$= P(10, 3) - P(9, 2)$

$= \frac{10!}{7!} - \frac{9!}{7!}$

$= \frac{10 \times 9 \times 8 \times 7!}{7!} - \frac{9 \times 8 \times 7!}{7!}$

$= 10 \times 9 \times 8 - 9 \times 8$

$= 720 - 72 = \mathbf{648.}$

38. Ans. (b)

Solution Six periods can be arranged for 5 subjects in $P(6, 5)$ ways

$= 6 \times 5 \times 4 \times 3 \times 2$

$= 720$

One period is left, which can be arranged for any of the five subjects.

∴ One left period can be arranged in 5 ways.

∴ The required number of arrangements

$= 720 \times 5 = \mathbf{3600.}$

39. Ans. (b)

Solution The required number of ordered pairs of alphabets, to be used as initials, can be formed = $P(4, 2)$

$= \frac{4!}{2!} = 4 \times 3 = \mathbf{12.}$

40. Ans. (a)

Solution Total number of candidates = 8.

5 different subjects candidates can be seated in $P(5, 5) = 5!$ ways.

In between 5 candidates there are 6 places for 8 Mathematics candidates.

∴ The Mathematics candidates can be seated in $P(6, 3)$ ways

∴ By fundamental principle of counting:

The required number of ways = $5! \times P(6, 3)$

$= 120 \times \frac{6!}{3!}$

$= 120 \times 6 \times 5 \times 4 = \mathbf{14400.}$

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41. Ans. (a)

Solution The vowels in the word 'ORIENTAL' are: *O*, *I*, *E* and *A*

Total number of letters in the word 'ORIENTAL' = 8.

No. of vowels = 4

O	*R*	*I*	*E*	*N*	*T*	*A*	*L*
1	2	3	4	5	6	7	8
×		×		×		×	

∵ Vowels occupy odd places, that is 1, 3, 5 and 7. No. of odd places = 4

∴ 4 vowels can be arranged in 4 '*X*' marked places = $P(4, 4)$ ways = 4! ways

No. of consonants = 4.

∴ 4 consonants can be arranged in 4 places

$= P(4, 4)$ ways = 4! ways

∴ The required no. of words

$= 4! \times 4! = 24 \times 24 =$ **576.**

42. Ans. (b)

Solution The numbers are divisible by 10 if 0 is in the unit's place.

× × × × × 0

∴ The required numbers which are divisible by 10 = $P(5, 5) = 5! =$ **120.**

43. Ans. (b)

Solution (i) The word 'UNIVERSAL' has 9 letters which are all different. These 9 letters can be arranged amongst themselves in 9! ways.

(ii) Three letters *E*, *R*, *S* can be taken to form one block. Thus, 7 letters *U*, *N*, *I*, *V*, *ERS*, *A*, *L* can be arranged in 7! ways.

Also, 3 letters *E*, *R*, *S* can be arranged amongst themselves in 3! ways.

∴ The total number of arrangements = 7! × 3!

= 30240.

44. Ans. (a)

Solution 5 students are to be arranged on a platform. One boy SUNIL is fixed at second position.

∴ We have to arrange only 4 students. But GITA is always adjacent to NITA. Considering the two girls (NITA and GITA) as one, the three students can be arranged in 3! ways. NITA and GITA themselves can be arranged in 2! ways.

∴ The required number of arrangements

$= 2! \times 3! = (2 \times 1) \times (3 \times 2 \times 1) =$ **12.**

45. Ans. (a)

Solution ALGEBRA has seven letters where 2 – *A*, 1 – *L*, 1 – *G*, 1 – *E*, 1 – *B* and 1 – *R*.

(i) Since two A's are always together, we take both the A's as one letter.

If *P* is the number of arrangements, then

$P = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

(ii) Total number of permutations

$$q = \frac{7!}{2!} = 7 \times \frac{7\times6\times5\times4\times3\times2\times1}{2\times1} = 2520$$

In these permutations, in some permutations, two A's are together while in the rest they are not together.

Hence, the number of permutations in which two A's are not together is

$q - p = 2520 - 720 =$ **1800.**

46. Ans. (a)

Solution Each apple can be given to any one of the 3 boys and this can be done in 3 ways.

∴ The required number of ways = $3^6 =$ **729.**

47. Ans. (c)

Solution Number of letters in the word 'KURUKSHETRA' is 11 of which 2 are K's, 2 are U's, 2 are R's and remaining are different.

∴ Required number of permutations = $\frac{11!}{2!2!2!}$

= 4989600.

48. Ans. (a)

Solution The word ALLAHABAD has 9 letters in all. The letter '*A*' occurs 4 times, the letter '*L*' occurs 2 times and the remaining three letters *H*, *B*, *D* each occur once.

∴ The require number of permutations

$$= \frac{9!}{4!2!1!1!} = \frac{9\times8\times7\times6\times5\times4!}{4!\times2}$$

$= 9 \times 8 \times 7 \times 3 \times 5 =$ **7560.**

49. Ans. (b)

Solution There are three places to be filled up to form a three-digit number. Since any digit may be repeated any number of times, each one of three places can be filled up by any of the given 4 digits in 4 ways.

Hence, the number of words that can be formed

$= 4^3 =$ **64.**

50. Ans. (c)

Solution There are in all 5 digits. Now, 0 cannot be placed at the hundredth place as in that case the number will not be 3-digited. Thus, hundredth place can be filled up by any of remaining 4 digits in 4 ways.

Since the digits may be repeated any number of times, each of the remaining two places can be filled up by any of the 5 digits in 5 ways each. Thus, the total number of such arrangements = $5^2 = 25$.

Hence, the total number of words that can be formed

$= 4 \times 25 =$ **100.**

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51. Ans. (b)

Solution Out of letters in the word 'BHARAT' two letters, that is, A's are alike.

$\therefore$ Number of permutations $= \frac{6!}{2!} = 360.$

Number of words in which B and H are never together

= Total number of words – number of words in which B and H are together

$= 360 - \frac{5!}{2!}.\ 2 = 360 - 120 = \mathbf{240.}$

52. Ans. (d)

Solution The given word consists of 11 letters out of which A occurs 2 times, R occurs 2 times, N occurs 2 times and E occurs 2 times and remaining three are different.

$\therefore$ Number of arrangements

$$= \frac{11!}{2!2!2!2!} = \mathbf{2491800.}$$

53. Ans. (b)

Solution Starting with A and arranging the other ten letters $A, E, I, I, M, N, N, O, T, X$ (not all distinct, I occurs twice, N occurs twice), there are

$$\frac{10!}{2!2!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times2}$$

= 907200 words.

$\therefore$ The number of items in the list before the first word starting with E is **907200.**

54. Ans. (c)

Solution The 5-digit even numbers can be formed out of 1, 2, 5, 5, 4 by using either 2 or 4 in the unit's place. This can be done in 2 ways.

Corresponding to each such arrangement, the remaining 4 places can be filled up by any of the remaining four digits in $\frac{4!}{2!} = 12$ ways. [$\because$ 50 occurs twice]

Hence, the total number of words = $2 \times 12 = \mathbf{24.}$

55. Ans. (a)

Solution A number greater than a million has 7 places, and thus all the 7 given digits are to be used. But 2 is repeated twice and 3 is repeated thrice.

$\therefore$ Total number of ways of arranging these 7 digits amongst themselves

$$= \frac{7!}{2!.3!} = 420$$

But numbers beginning with zero are no more seven digited numbers, hence we have to reject those numbers which begin with zero, and such numbers are

$$= \frac{6!}{2!.3!} = 60.$$

Hence, the required number of arrangements,

$= 420 - 60 = \mathbf{360.}$

56. Ans. (d)

Solution There are 7 letters, of which two are A's and the rest are all different. The vowels A, A, E occupy 1st, 4th and 7th places. The number of ways in which they can be arranged in these places is $\frac{3!}{2!} = 3.$

The consonants L, G, B, R are all different. The number of ways in which they can be arranged in the remaining places in 4!. Since each way of arranging the vowels can be *associated* with each way of arranging the consonants, we find that the total number of arrangements = $3 \times 4!, = 3 \times 24 = \mathbf{72.}$

57. Ans. (a)

Solution There are in all seven letters in the word BALLOON in which L occurs 2 times and O occurs 2 times.

$\therefore$ The number of arrangements of the seven letters of the word $= \frac{7!}{2!\times2!} = 1260.$

If two L's always come together, taking them as one letter, we have to arrange 6 letters in which O occurs 2 times.

$\therefore$ The no. of arrangements in which the two L's come together

$$= \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360.$$

Hence, the required no. of ways in which the two L's do not come together

$= 1260 - 360 = \mathbf{900.}$

58. Ans. (d)

Solution Here, $n = 3 + 2 + 2 = 7$

$p_1 = 3,\ p_2 = 2$ and $p_3 = 2$

$\therefore$ The required number of different signals

$$= \frac{n!}{p_1!\,p_2!\,p_3!} = \frac{7!}{3!2!2!} = \frac{7.6.5.4}{2.2}$$

$= 7.6.5 = \mathbf{210.}$

59. Ans. (c)

Solution The given digits are: 1, 2, 3, 4, 3, 2, 1. Out of these 1, 3, 3, 1 are odd digits.

The odd digits occupy the odd places, that is, they occupy the 1st, 3rd, 5th and 7th place.

$\therefore$ 4 odd places can be filled with 4 odd digits

$$= \frac{4!}{2!2!} = \frac{4.3}{2} = 6 \text{ ways.}$$

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$\Rightarrow \frac{(2n)!}{3!(2n-3)!} : \frac{n!}{3!(n-3)!} = 11 : 1$

$\Rightarrow (2n)(2n-1)(2n-2) : n(n-1)(n-2) = 11 : 1$

$\Rightarrow 2 \times 2(2n-1)(n-1) : (n-1)(n-2) = 11 : 1$

$\Rightarrow 4(2n-1) : n-2 = 11 : 1$

$\Rightarrow \frac{8n-4}{n-2} = \frac{11}{1}$

$\Rightarrow 11n - 22 = 8n - 4$

$\Rightarrow 11n - 8n = 22 - 4$

$\Rightarrow 3n = 18 \quad \therefore \quad \boldsymbol{n = 6.}$

67. Ans. (a)

Solution ${}^{2n}C_r = {}^{2n}C_{2n-r}$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

But ${}^{2n}C_r = {}^{2n}C_{r+2}$ [Given]

$\therefore \quad {}^{2n}C_{2n-r} = {}^{2n}C_{r+2}$

$\therefore \quad 2n - r = r + 2 \Rightarrow \boldsymbol{r = n - 1.}$

68. Ans. (a)

Solution ${}^{18}C_r = {}^{18}C_{18-r}$ $\quad [\because {}^nC_r = {}^nC_{n-r}]$

But ${}^{18}C_r = {}^{18}C_{r+r}$ [Given]

$\therefore \quad {}^{18}C_{18-r} = {}^{18}C_{r+2}$ or, $18 - r = r + 2 \Rightarrow r = 8.$

$\therefore \quad {}^rC_5 = {}^8C_5 = \frac{8!}{5!\,3!} = \frac{8\times7\times6\times5!}{3\times2\times1\times5!} = \boldsymbol{56.}$

69. Ans. (b)

Solution Here, $12 \cdot {}^nC_2 = {}^{2n}C_3$

$\therefore \quad 12 \cdot \frac{n!}{2!(n-2)!} = \frac{(2n)!}{3!(2n-3)!} \quad \left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$

or, $12 \cdot \frac{n(n-1)(n-2)!}{2!.(n-2)!} = \frac{(2n)(2n-1)(2n-2)(2n-3)!}{6(2n-3)!}$

or, $\quad 6 \cdot n(n-1) = \frac{2n(2n-1)(2n-2)}{6}$

or, $\quad 18 \cdot (n-1) = (2n-1)(2n-2)$

or, $\quad 9 \cdot (n-1) = (2n-1)(n-1)$

i.e. $\quad 9 = 2n - 1 \Rightarrow \boldsymbol{n = 5.}$

70. Ans. (b)

Solution $\sum_{r=1}^{5} C(5, r) = C(5, 1) + C(5, 2) + C(5, 3) + C(5, 4) + C(5, 5)$

$= \frac{5!}{1!\,4!} + \frac{5!}{2!\,3!} + \frac{5!}{3!\,2!} + \frac{5!}{4!\,1!} + \frac{5!}{5!\,0!}$

$= \frac{5}{1} + \frac{5.4}{2} + \frac{5.4}{2} + \frac{5}{1} + 1$

$= 5 + 10 + 10 + 5 + 1 = \boldsymbol{31.}$

71. Ans. (c)

Solution The require number of ways $= C(10, 5)$

$= \frac{10!}{5!\,5!} = \frac{10.9.8.7.6}{5.4.3.2} = 3.2.42 = \boldsymbol{252.}$

72. Ans. (d)

Solution 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \frac{16!}{11!\,5!} = 4368$ ways.

When two particular players are always to be included, then 9 more players are to be selected out of remaining 14 players, which can be done in

${}^{14}C_9$ ways $= \frac{14!}{9!\,5!} = \boldsymbol{2002}$ **ways.**

73. Ans. (b)

Solution 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \frac{16!}{11!\,5!} = 4368$ ways.

If one particular player is to be excluded, then selection is to be made of 11 players out of 15 players and this can be done in ${}^{15}C_{11}$ ways

$= \frac{15!}{11!\,4!} = \boldsymbol{1365}$ **ways.**

74. Ans. (a)

Solution 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \frac{16!}{11!\,5!} = 4368$ ways

If two particular players are to be included and one particular player is to be rejected, then we have to select 9 more out of 13 in ${}^{13}C_9$ ways

$= \frac{13!}{9!\,4!} = \boldsymbol{715}$ **ways.**

75. Ans. (d)

Solution The required number of ways

$= C(10, 8) \,.\, C(10, 5)$

$= \frac{10!}{8!\,2!} \times \frac{10!}{5!\,5!} = \frac{10\times9}{2} \times \frac{10\times9\times8\times7\times6}{5\times4\times3\times2}$

$= 5 \times 9 \times 3 \times 2 \times 7 \times 6 = \boldsymbol{11340.}$

76. Ans. (c)

Solution Here, we want to find the number of ways of selecting 11 players out of 15.

$\therefore$ The required no. of ways

$= C(15, 11) = \frac{15!}{11!\,4!} = \frac{15\times14\times13\times12}{4\times3\times2\times1}$

$= 15 \times 7 \times 13 = 1365.$

A particular player to be included

So, we have to select 10 players out of 14.

$\therefore$ The required no. of ways

$= C(14, 10) = \frac{14!}{10!\,4!} = \frac{14.13.12.11}{4.3.2.1}$

$= 7.13.11 = \boldsymbol{1001.}$

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77. Ans. (c)

Solution No. of letters in the word = 8

No. of vowels in the word = 4

(I, O, U, E)

No. of consonants in the word = 4

(N, V, L, T)

Out of 4 vowels, we have to select 3.

Out of 4 consonants, we have to select 2.

Also, we have to arrange 3 vowels and 2 consonants.

$\therefore$ The required no. of words

$$= C(4, 3) \cdot C(4, 2) \cdot 5! = 4 \times \frac{4\times3}{2} = 120 = \mathbf{2880.}$$

78. Ans. (b)

Solution We get a line by joining two points, If p is the number of lines from 21 points, then,

$$p = C(21, 2) = \frac{21!}{2!(21-2)!} = \frac{21\times20(19!)}{2\times1(19!)}$$

$$= 21 \times 10 = \mathbf{210\ lines.}$$

79. Ans. (c)

Solution If p is the required number of ways, then

$$p = C(6, 3) \times C(5, 3) \times C(5, 3)$$

$$= \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!}$$

$$= \frac{6\times5\times4\times3!}{3\times2\times1\times3!} \times \frac{5\times4\times3!}{2\times1\times3!} \times \frac{5\times4\times3!}{2\times1\times3!}$$

$$= 5 \times 4 \times 5 \times 2 \times 5 \times 2 = \mathbf{2000.}$$

80. Ans. (b)

Solution Out of available nine courses, two are compulsory. Hence, the student is free to select 3 courses out of 7 remaining courses: If p is the number of ways of selecting 3 courses out of 7 courses, then

$$p = C(7, 3) = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$$

$$= 7 \times 5 = \mathbf{35\ ways.}$$

81. Ans. (c)

Solution If p is the number of ways of selection, then

$$p = C(6, 4) \times C(7, 4) \times (8, 4) = \frac{6!}{4!\,2!} \times \frac{7!}{4!\,3!} \times \frac{8!}{4!\,4!}$$

$$= \frac{6\times5\times4!}{2\times1\times4!} \times \frac{7\times6\times5\times4!}{3\times2\times1\times4!} \times \frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}$$

$$= 15 \times 35 \times 70 = \mathbf{36750\ ways.}$$

82. Ans. (c)

Solution If A and B both are not selected, then the number of permutations = $C(6, 6) = 1$.

If A and B both are selected, then we are to select 4 persons out of six persons.

The number of ways in which this can be done

$$= C(6, 4) = \frac{6!}{4!(6-2)!} = \frac{6\times5\times4!}{4!\times2\times1}$$

$$= 15$$

Hence, total number of permutations = 1 + 15 = **16.**

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. There are 6 boxes numbered 1, 2, ... 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

(a) 5 (b) 21

(c) 33 (d) 60

[Based on CAT 2003]

2. A graph may be defined as a set of points connnected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any other point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition

(a) $11 \le e \le 66$ (b) $10 \le e \le 66$

(c) $11 \le e \le 65$ (d) $0 \le e \le 11$

[Based on CAT 2003]

Directions (Q. 3 to 4): Answer the questions on the basis of the information given below.

A string of three English letters is formed as per the following rules:

(a) The first letter is any vowel.

(b) The second letter is m, n or p.

(c) If the second letter is m then the third letter is any vowel which is different from the first letter.

(d) If the second letter is n then the third letter is e or u.

(e) If the second letter is p then the third letter is the same as the first letter,

3. How may strings of letters can possibly be formed using the above rules?

(a) 40 (b) 45

(c) 30 (d) 35

[Based on CAT 2004]

4. How many strings of letters can possibly by formed using the above rules such that the third letter of

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the string is e?
(a) 8 (b) 9
(c) 10 (d) 11
[Based on CAT 2004]

5. Two series of a question booklet for an aptitude test are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical series side by side and that the students sitting one behind the other should have the same series?
(a) $2 \times {}^{12}C_6 \times (6!)^2$ (b) $6! \times 6!$
(c) $7! \times 7!$ (d) None of these
[Based on MAT 2004]

6. The letters of the word PROMISE are arranged so that no two of the vowels should come together. Find the total number of arrangements.
(a) 49 (b) 1440
(c) 7 (d) 1898
[Based on MAT 2003]

7. What is the remainder left after dividing $1! + 2! + 3! + ... + 100!$ by 7?
(a) 0 (b) 5
(c) 21 (d) 14
[Based on CAT 2003]

8. In an examination paper there are two groups, each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any group. In how many ways can 5 questions be selected?
(a) 24 (b) 48
(c) 96 (4) None of these
[Based on MAT 2002]

9. A box contains 10 balls out of which 3 are red and the rest are blue. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 red balls are included in the sample and no sample has all the 6 balls of the same colour?
(a) 105 (b) 168
(c) 189 (d) 120
[Based on MAT 2002]

10. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicket keepers. The number of ways in which a team can be formed having exactly 4 bowlers and 2 wicket keepers is
(a) 20790 (b) 6930
(c) 10790 (d) 360
[Based on MAT 2002]

11. Three boys and three girls are to be seated around a table in a circle. Among them the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour. How many such arrangements are possible?
(a) 5 (b) 6
(c) 4 (d) 2
[Based on MAT 2002]

12. In a hockey championship, three were 153 matches played. Every two teams played one match with each other. The number of teams participating in the championship is
(a) 18 (b) 19
(c) 17 (d) 16
[Based on MAT 2002]

13. Seven points lie on a circle. How many chords can be drawn by joining these points.
(a) 22 (b) 21
(c) 23 (d) 24
[Based on MAT 2005]

14. Ten different letters of an alphabet are given. Words with 5 letters are formed from these given lettes. Then the number of words which have at least one letter repeated is
(a) 69760 (b) 30240
(c) 99748 (d) None of these
[Based on MAT 2001]

15. Eight chairs are numbered from 1 to 8. Two women and three men wish to occupy one chair each. First, the women chose the chairs from amongst the chairs marked 1 to 4. Then the men selected the chairs from amongst the remaining, marked 5 to 8. The number of possible arrangements is
(a) ${}^6C_3 \times {}^4C_4$ (b) ${}^4P_2 \times {}^4P_3$
(c) ${}^4C_3 \times {}^4P_3$ (d) ${}^4C_2 \times {}^4C_3$
[Based on MAT 2001]

16. The value of ${}^{10}C_4 + {}^{10}C_5$ is
(a) 462 (b) 466
(c) 469 (d) 465
[Based on MAT 2005]

17. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is
(a) 216 (b) 240
(c) 3125 (d) 600
[Based on MAT 2005]

18. There are 6 equally spaced points A, B, C, D, E and F marked on a circle with radius R. How many convex pentagons of distinctly different areas can be drawn using these points as vertices?
(a) ${}^6P_5$ (b) 1
(c) 55 (d) None of these.
[Based on MAT 2001]

19. Five persons A, B, C, D and E occupy seats in a row such that A and B sit next to each other. In how many possible ways can these five people sit?
(a) 24 (b) 48
(c) 72 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

20. The number of zeros at the end of 601 is
(a) 12 (b) 14

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(a) 24 (b) 48
(c) 72 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

20. The. number of zeros at the end of 601 is
(a) 12 (b) 14
(c) 16 (d) 18
[Based on IITTM, Gwalior, 2003]

21. A department had 8 male and female employees each. A project team involving 3 male and 3 female members needs to be chosen from the department employees. How many different project teams can be chosen?
(a) 112896 (b) 3136
(c) 720 (d) 112
[Based on SCMHRD Ent. Exam. 2003]

22. From a group of 7 men and 6 women 5 people are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?
(a) 756 (b) 735
(c) 564 (d) 645
(e) None of these **[Based on IRMA, 2002]**

23. In how many different ways can the letters of the word TRAINER be arranged so that the vowels always come together?
(a) 1440 (b) 120
(c) 720 (d) 360
(e) None of these **[Based on IRMA, 2002]**

24. A polygon has 44 diagonals, then the number of its sides are
(a) 11 (b) 9
(c) 7 (d) 5
[Based on FMS (Delhi) 2002]

25. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is
(a) 19670 (b) 39758
(c) 69760 (d) 99748
[Based on FMS (Delhi) 2002]

26. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways in which this can be done is
(a) 211 (b) 216
(c) 221 (d) 311
[Based on FMS (Delhi) 2002]

27. How many parallelograms will be formed if 7 parallel horizontal lines intersect 6 parallel vertical lines ?
(a) 42 (b) 294
(c) 315 (d) None of these
[Based on I.P. Univ. 2002]

28. Which of the following is equal to $\frac{1\cdot3\cdot5\ldots(2n-1)}{2\cdot4\cdot6\ldots(2n)}$?
(a) $(2n)! \div (2^n(n!))^2$ (b) $(2n)! \div n!$
(c) $(2n-1) \div (n-1)!$ (d) 2^n
(e) None of these **[Based on SCMHRD 2002]**

29. How many four-digit numbers, each divisible by 4 can be formed using the digits 1, 2, 3, 4 and 5, repetitions of digits being allowed in any number?
(a) 100 (b) 150
(c) 125 (d) 75
(e) 76 **[Based on SCMHRD 2002]**

30. A dean must select three students to serve on a committee. If she is considering five students, then from how many different possible threesomes must she choose?
(a) 2 (b) 3
(c) 10 (d) 15
[Based on REC Tiruchirapalli, 2002]

31. Out of 10 red and 8 white balls, 5 red and 4 white balls can be drawn in how many ways?
(a) ${}^8C_5 \times {}^{10}C_4$ (b) ${}^{18}C_9$
(c) ${}^{10}C_5 \times {}^8C_4$ (d) None of these
[Based on REC Tiruchirapalli, 2002]

32. Hows many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?
(a) 7920 (b) 330
(c) 14640 (d) 419430
[Based on CAT 2002]

33. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?
(a) 990 (b) 2730
(c) 12870 (d) 1560000
[Based on CAT 2002]

34. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?
(a) 56 (b) 896
(c) 60 (d) 768
[Based on CAT 2002]

35. Suppose you have a currency, named Miso, in three denominations: 1 Miso, 10 Misos, and 50 Misos. In how many ways can you pay a bill of 107 Misos?
(a) 18 (b) 15
(c) 19 (d) 17
(e) 16? **[Based on CAT 2007]**

36. The figure below shows the network connecting cities *A*, *B*, *C*, *D*, *E* and *F*. The arrows indicate permissible direction of travel. What is the number of distinct paths from *A* to *F*?

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$\therefore {}^nC_2 - n = 44,\ n > 0 \Rightarrow \dfrac{n(n-1)}{2} - n = 44$

$\Rightarrow n^2 - 3n - 88 = 0$

$\Rightarrow n^2 - 11n + 8n - 88 = 0$

$\Rightarrow n(n-11) + 8(n-11) = 0$

$\Rightarrow (n-11)(n+8) = 0$

$\Rightarrow \boldsymbol{n = 11.}$

25. Ans. (c)

Solution The total number of words that can be formed with five letters out of the ten given letters = 10^5 = 100000.

The total number of words that can be formed with five distinct letters

$= 10 \times 9 \times 8 \times 7 \times 6 = 30240$

$\therefore$ The total number of words in which atleast one letter is repeated.

$= 100000 - 30240 = \mathbf{69760.}$

26. Ans. (b)

Solution All permutations formed with 1, 2, 3, 4, 5 (sum = 15) will be divisible by 3.

There are 5! = 120 such permutations. Such numbers can also be formed using 0 and 1, 2, 4, 5. There are $4 \times 4!$ such numbers, i.e. 96. (Factor of 4 for four positions of 0 and 4! for different permutations of these four numbers)

$\therefore$ Total of such numbers = 120 + 96 = **216.**

27. Ans. (c)

Solution ${}^7C_2 \times {}^6C_2 = \mathbf{315.}$

28. Ans. (a)

Solution $\dfrac{[1\cdot3\cdot5\ldots(2n-1)][2\cdot4\cdot6\ldots2n]}{[2\cdot4\cdot6\ldots2n]^2}$

$= \dfrac{(2n)!}{[2^n(1,2,3,\ldots n)]^2}$

$= \dfrac{\boldsymbol{(2n)!}}{\boldsymbol{[2^n \cdot n!]^2}}.$

29. Ans. (c)

Solution The four-digit number formed from the digits 1, 2, 3, 4, 5 will be divisible by 4 if the last two digis are 12 or 24 or 32 or 44 or 52. The first two can be chosen in $5_{P_2} = 5 \times 4 = 20$ ways and also as 11, 22, 33, 44, 55, i.e. 25 ways in all. Hence these 125 four-digit numbers are divisible by **4.**

30. Ans. (c)

Solution ${}^5C_3 = {}^5C_2 = \dfrac{5\times4}{2\times1} = \mathbf{10.}$

31. Ans. (c)

32. Ans. (a)

Solution $11 \times 10 \times 9 \times 8 = \mathbf{7920.}$

33. Ans. (c)

Solution Total number of passwords using all alphabets total number of passwords using no symmetric alphabets

$= (26 \times 25 \times 24) - (15 \times 14 \times 13) = \mathbf{12870.}$

34. Ans. (d)

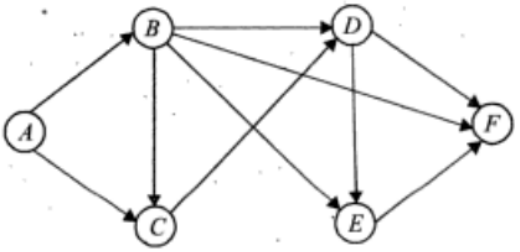
Solution A block square can be chosen in 32 ways. Once a black square is there, you cannot choose the 8 white squares in its row or column, so the number of white squares available = 24.

No. of ways = $32 \times 24 = \mathbf{768.}$

35. Ans. (a)

36. Ans. (b)

Solution The maximum routes from *A* to *F* are listed below.



(1) *ABDF* (2) *ACEF* (3) *ABF*

(4) *ABEF* (5) *ACDF* (6) *BCDEF*

(7) *ACDEF* (8) *ABDEF* (9) *ABCDF*

(10) *ABCEF*.

37. Ans. (a)

Solution Vowels = E, A, A, I, I, O = 6

Consonant = X, M, N, N, T = 5

Number of arrangements $= \dfrac{6!}{2!2!} \times \dfrac{5!}{2!} = 10800$

38. Ans. (a)

Solution Total number of ways

$= 2 \times 3 \times 4 \times 3 \times 2 \times 1$

= 144 ways

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EXERCISES FOR SELF-PRACTICE

Test I

[Time: 10 min]

1. In a monthly test, the teacher decides that there will be three questions one from each of Exercises 7, 8 and 9 of the text book. If there are 12 questions in Exercise 7, 18 in Exercise 8 and 9 in Exercise 9, in how many ways can the three questions be selected?
(a) 1894 (b) 2094
(c) 1944 (d) 2194

2. It has been decided that the flag of a newly formed forum will be in the form of three blocks, ☐☐☐ each coloured differently. If there are five different colours on the whole to choose from, how many such designs are possible?
(a) 60 (b) 50
(a) 80 (b) 70

3. A lady wants to select one cotton saree and one chiffon saree from a textile shop. If there are 10 cotton varieties and 12 chiffon varieties, in how many ways can she choose the two sarees?
(a) 124 (b) 120
(c) 132 (d) 128

4. A three-digit number is formed using the digits 0, 1, 2,..., 9. Find the number of odd numbers out of these three digit numbers.
(a) 380 (b) 650
(c) 450 (d) 550

5. How many numbers are there between 100 and 1000 in which all the digits are distinct?
(a) 548 (b) 648
(c) 748 (d) 848

6. How many numbers are there between 1000 and 10000 such that 9 is in the units place?
(a) 900 (b) 700
(c) 800 (d) 600

7. If $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$, find x.
(a) 111 (b) 121
(c) 101 (d) 131

8. Find the value of n if $\frac{(2n)!}{3!(2n-3)!} : \frac{n!}{2!(n-2)!} = 44 : 3$.
(a) 6 (b) 8
(c) 12 (d) 10

9. Find the value (s) of n such that $30\ P(n, 6) = P(n+2, 7);\ n \geq 6$.
(a) 21 (b) 25
(c) 23 (d) 19

10. How many signals can be given with 5 flags of different colours?
(a) 425 (b) 325
(c) 275 (d) 225

Test II

[Time: 10 min]

11. Find the number of different 8 letter words formed from the letters of the word EQUATION, if each word is to start with a vowel.
(a) 25200 (b) 24200
(c) 26200 (d) 27200

12. How many 6-digited telephone numbers can be constructed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. If each number starts with 35 and no digits appears more than once?
(a) 178 (b) 1880
(c) 1680 (d) 1480

13. How many different words can be formed of the letters of the word 'MALENKOV' so that no two vowels are together?
(a) 15400 (b) 14400
(c) 16400 (d) 17400

14. In how many ways can the letters of the word MODESTY be arranged so that D and E are always together?
(a) 1640 (b) 1740
(c) 1440 (d) 1840

15. Ten guests are to be seated in a row. Three of them are to be seated together. Of the remaining 7, two do not wish to sit side by side. Find the number of possible arrangements.
(a) 191400 (b) 181440
(c) 171400 (d) 161400

16. In how many ways can the letters of the word "STRANGE" be arranged so that the vowels may appear in the odd places.
(a) 1640 (b) 1440
(c) 1740 (d) 1840

17. In how many ways can 6 different rings be worn on the four fingers of hand?
(a) 4^8 (b) 4^7
(c) 4^6 (d) 4^4

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15. Ans. (b)

Solution Number of permutations of 10 guests in which three guests are always together = $8! \times 3!$.

Now, if the two guests who do not want to sit side by side, are considered to sit side by side then the number of premutations = $7! \times 3! \times 2!$

$\therefore$ Number of permutations (arrangements) of 10 guests so that three particular guests are seated together and two particular guests do not sit side by side

$$= 8! \times 3! - 7! \times 3! \times 2! = 7! \times 3! (8 - 2)$$
$$= 5040 \times 6 \times 6 = \mathbf{181440.}$$

17. Ans. (c)

Solution Each ring can be worn on any of the four fingers and this can be done in 4 ways.

$\therefore$ The required number of ways = $\mathbf{4^6}$.

19. Ans. (b)

Solution When two persons shake hands, it is counted as one handshake, not two. Therefore, the total number of handshakes is the same as the number of ways of selecting 2 persons from among 12 persons.

Hence, the total no. of handshakes

$$= C(12, 2) = \frac{12!}{2!10!}$$
$$= \frac{12.11.10!}{2.10!} = 6 \times 11 = \mathbf{66.}$$

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (c)	**6.** (a)	**8.** (a)	**10.** (b)	**12.** (c)	**14.** (c)	**16.** (b)	**18.** (a)	**20.** (a)

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32 Probability

INTRODUCTION

The word *probability* or *chance* is very frequently used in day-to-day life. For example, we generally say, 'He may come today' or 'probably it may rain tomorrow' or 'most probably he will get through the examination'. All these phrases involve an element of uncertainty and probability is a concept which measures this uncertainties. The probability when defined in simplest way is the chance of occurring of a certain event when expressed quantitatively, i.e. probability is a quantitative measure of the certainty.

The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of predictions for future.

The following remarks may be important for learning this chapter on probability.

1. **Die:** A die is a small cube used in games of chance. On its six faces dots are marked as

   ⚀ ⚁ ⚂ ⚃ ⚄ ⚅

   Plural of die is dice. The outcome of throwing (or tossing) a die is the number of dots on its uppermost face. An ace on a die means one dot.

2. **Cards:** A pack (or deck) of playing cards has 52 cards, divided into four suits:

   (i) Spades gqde (♠) (ii) Clubs fpM+h (♣)

   (iii) Hearts iku (♥) (iv) Diamonds b±V (♦)

   Each suit has 13 cards, nine cards numbered 2 to 10, an Ace (bDdk), a King (ckn'kkg), Queen (csxe) and a Jack or Knave (xqyke). Spades and Clubs are black-faced cards while Hearts and Diamonds are red-faced cards. The Aces, Kings, Queens and Jacks are called *face cards* and other cards are called *number cards*. The Kings, Queens and Jacks are called *court cards*.

3. The number of combinations of n objects taken r at a time ($r \le n$) is denoted by $C(n, r)$ or ${}^nC_r$ and is defined as

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots \text{to } r \text{ factors}}{1\cdot 2\cdot 3\ldots r.} $$

Illustration 1 $ {}^5C_3 = \frac{5\cdot 4\cdot 3}{1\cdot 2\cdot 3} = 10$, ${}^nC_0 = 1$ and ${}^nC_n = 1$.

If $r > \frac{n}{2}$, then it is better to simplify ${}^nC_r$ as ${}^nC_{n-r}$.

Illustration 2 $ {}^{52}C_{50} = {}^{52}C_{52-50} = {}^{52}C_2 = \frac{52.51}{2.1} = 26.51 = 1326.$

When, $r > n$, ${}^nC_r = 0$.

Some Important Terms and Concepts

Random Experiment or Trial: The performance of an experiment is called a *trial*. An experiment is characterised by the property that its observations under a given set of circumstances do not always lead to the same observed outcome but rather to the different outcomes. If in an experiment all the possible outcomes are known in advance and none of the outcomes can be predicted with certainity, then such an experiment is called a *random experiment*.

For example, tossing a coin or throwing a die are random experiments.

Event: The possible outcomes of a trial are called *events*. Events are generally denoted by capital letters A, B, C and so on.

Illustration 3 (i) When a coin is tossed the outcome of getting a head or a tail is an event.

(ii) When a die is thrown the outcome of getting 1 or 2 or 3 or 4 or 5 or 6 is an event.

Sample Space: The set of all possible outcomes of an experiment is called a *sample space*. We generally denote it by S.

Illustration 4 (i) When a coin is tossed, S = {H, T} where H = head, T = tail.

(ii) When a die is thrown, S = {1, 2, 3, 4, 5, 6}.

(iii) When two coins are tossed simultaneously,

S = {HH, HT, TH, TT}

Equally Likely Events: Events are said to be *equally likely* if there is no reason to expect any one in preference to other. Thus, equally likely events mean outcome is as likely to occur as any other outcome.

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Illustration 5 In throwing a die, all the six faces (1, 2, 3, 4, 5, 6) are equally likely to occur.

Simple and Compound Events

In the case of *simple events* we consider the probability of happening or non-happening of single events.

Illustration 6 We might be interested in finding out the probability of drawing an ace from a pack of cards.

In the case of *compound events* we consider the joint occurrence of two or more events.

Illustration 7 If from a bag, containing 8 red and 5 green balls, two successive draws of 2 balls are made, we shall be finding out the probability of getting 2 red balls in the first draw and 2 green balls in the second draw. We are thus dealing with a compound event.

Exhaustive Events: It is the total number of all possible outcomes of any trial.

Illustration 8 (i) When a coin is tossed, either head or tail may turn up and therefore, there are two exhaustive cases.

(ii) There are six exhaustive cases or events in throwing a die.

(iii) If two dice are thrown simultaneously, the possible outcomes are

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Thus, in this case, there are $36(=6^2)$ ordered pairs. Hence, the number of exhaustive cases in the simultaneous throw of two dice is 36.

(iv) Three dice are thrown, the number of exhaustive cases is 6^3, i.e. 216.

Algebra of Events

If A and B are two events associated with sample space S, then

(i) $A \cup B$ is the event that either A or B or both occur.

(ii) $A \cap B$ is the event that A and B both occur simultaneously.

(iii) $\overline{A}$ is the event that A does not occur.

(iv) $\overline{A} \cap \overline{B}$ is an event of non-occurrence of both A and B, i.e. none of the events A and B occurs.

Illustration 9 In a single throw of a die, let A be the event of getting an even number and B be the event of getting a number greater than 2. Then,

$$A = \{1, 3, 5\}, B = \{3, 4, 5, 6\}$$

$$\therefore \quad A \cup B = \{1, 3, 4, 5, 6\}$$

$A \cup B$ is the event of getting an odd number or a number greater than 2.

$$A \cap B = \{3, 5\}.$$

$A \cap B$ is the event of getting an odd number greater than 2.

$\overline{A} = \{2, 4, 6\}$ [Those elements of S which are not in A.]

$\overline{A}$ is the event of not getting an odd number, i.e. getting an even number.

$$\overline{B} = \{1, 2\}.$$

$\overline{B}$ is the event of not getting a number greater that 2, i.e. getting a number less than or equal to 2.

$$\overline{A} \cap \overline{B} = \{2\}.$$

$\overline{A} \cap \overline{B}$ is the event of neither getting an odd number nor a number greater than 2.

Mutually Exclusive Events

In an experiment, if the occurrence of an event precludes or rules out the happening of all the other events in the same experiment.

Illustration 10 (i) When a coin is tossed either head or tail will appear. Head and tail cannot appear simultaneously. Therefore, occurrence of a head or a tail are two mutually exclusive events.

(ii) In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive since if any one of these faces comes, the possibility of others in the same trial, is ruled out.

Note: A and B are mutually exclusive events $\Leftrightarrow A \cap B = \phi$, i.e. A and B are disjoint sets.

Illustration 11 (i) If the random experiment is '*a* die is thrown' and A, B are the events, A : the number is less than 3; B : the number is more than 4, then $A = \{1, 2\}$, $B = \{5, 6\}$.

$A \cap B = \phi$, thus A and B are mutually exclusive events.

(ii) If the random experiment is '*a* card is drawn from a well-shuffled pack of cards' and A, B are the events A: the card is Black; B: the card is an ace.

Since a black card can be an ace, $A \cap B \neq \phi$, thus A and B are not mutually exclusive events.

Mutually Exclusive and Exhaustive Events

Events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$, i.e. $\bigcup_{i=1}^{n} E_i = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$.

For example, in a single throw of a die, let A be the event of getting an even number and B be event of getting odd numbers, then

$$A = \{2, 4, 6\}, B = \{1, 3, 5\}$$

$$A \cap B = \phi, A \cup B = \{1, 2, 3, 4, 5, 6\} = S$$

$\therefore$ A and B are mutually exclusive and exhaustive events.

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Illustration 12 Two dice are thrown and the sum of the numbers which come up on the dice noted. Let us consider the following events:

A : "the sum is even"
B : "the sum is a multiple of 3"
C : "the sum is less than 4"
D : "the sum is greater than 11"

Which pairs of these events are mutually exclusive?

Solution There are $6 \times 6 = 36$ elements in the sample space (Refer to Example 2).

A is the event "the sum is even". It means we have to consider those ordered pairs (x, y) in which $x + y$ is even. Thus,

$A = [(1, 1), (2, 2) (1, 3), (1, 5), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6) (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)]$.

Similarly,

$B = [(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)]$

$C = [(1, 1), (2, 1), (1, 2)]$ $D = [(6, 6)]$.

We find that $A \cap B = [(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)] \neq \phi$

Thus, A and B are not mutually exclusive.

Similarly, $A \cap C \neq \phi$, $A \cap D \neq \phi$, $B \cap C \neq \phi$, $B \cap D \neq \phi$, $C \cap D = \phi$. Thus, C and D are mutually exclusive.

PROBABILITY OF AN EVENT

The probability of an event is defined in the following two ways:

(i) Mathematical (or *a priori*) definition
(ii) Statistical (or empirical) definition.

Mathematical Definition of Probability: Probability of an event A, denoted as $P(A)$, is defined as

$$P(A) = \frac{\text{Number of cases favourable to } A}{\text{Number of possible outcomes}}$$

Thus, if an event A can happen in m ways and fails (does not happen) in n ways and each of $m + n$ ways is equally likely to occur then the probability of happening of the event A (also called success of A) is given by

$$P(A) = \frac{m}{m+n}$$

and that the probability of non-occurrence of the A (also called its failure) is given by

$$P(\text{not } A) \text{ or } P(\overline{A}) = \frac{n}{m+n}$$

If the probability of the happening of a certain event is denoted by p and that of not happening by q, then

$$p + q = \frac{m}{m+n} + \frac{n}{m+n} = 1.$$

Here, p, q are non-negative and cannot exceed unity, i.e. $0 \leq p \leq 1$ and $0 \leq q \leq 1$.

When $p = 1$, then the event is certain to occur.

When $p = 0$, then the event is impossible. For example, the probability of throwing eight with a single die is zero.

Probability as defined above is sometimes called **Priori Probability,** i.e. it is determined before hand, that is, before the actual trials are made.

Illustration 13 A coin is tossed once. What are all possible outcomes? What is the probability of the coin coming up "tails"?

Solution The coin can come up either "heads" (H) or "tails" (T). Thus, the set S of all possible outcomes is $S = \{H, T\}$

$$\therefore \qquad P(T) = \frac{1}{2}$$

Illustration 14 What is the probability of getting an even number in a single throw of a die?

Solution Clearly, a die can fall with any of its faces upper most. The number on each of the faces is, therefore, a possible outcome. Thus, there are total 6 outcomes. Since there are 3 even numbers on the die, namely, 2, 4 and 6,

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}.$$

Illustration 15 What is the probability of drawing a 'king' from a well-shuffled deck of 52 cards?

Solution Well-shuffled ensures equally-likely outcomes. There are 4 kings in a deck. Thus,

$$P(\text{a king}) = \frac{4}{52} = \frac{1}{13}.$$

Odds of an Event

Suppose, there are m outcomes favourable to a certain event and n outcomes unfavourable to the event in a sample space, then

odds in favour of the event

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}} = \frac{m}{n}$$

and odds against the event

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcome}} = \frac{n}{m}.$$

If odds in favour of an event A are $a : b$, then the probability of happening of event $A = P(A) = \frac{a}{a+b}$ and probability of not happening of event $A = P(\overline{A}) = \frac{b}{a+b}$.

If odds against happening of an event A are $a : b$, then probability of happening of event $A = P(A) = \frac{b}{a+b}$

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and probability of not happening of event

$$A = P(\overline{A}) = \frac{a}{a+b}.$$

Illustration 16 What are the odds in favour of getting a '3' in a throw of a die? What are the odds against getting a '3'?

Solution There is only one outcome favourable to the event "getting" a 3, the other five outcomes, namely, 1, 2, 4, 5, 6 are unfavourable. Thus,

Odds in favour of getting a '3'

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$= \frac{1}{5} \text{ or 1 to 5.}$$

Odd against getting a '3'

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

$$= \frac{5}{1} \text{ or 5 to 1.}$$

Illustration 17 If the odds in favour of an event are 4 to 5, find the probability that it will occur.

Solution The odds in favour of the event are $\frac{4}{5}$. Thus,

$$\frac{P(A)}{1-P(A)} = \frac{4}{5}, \text{ i.e. } 4[1-P(A)] = 5P(A),$$

i.e. $$P(A) = \frac{4}{9}.$$

The probability that it will occur $= \frac{4}{9}$.

FUNDAMENTAL THEOREMS ON PROBABILITY

Theorem 1 In a random experiment, if S is the sample space and E is an event, then

(i) $P(E) \geq 0$ (ii) $P(\phi) = 0$ (iii) $P(S) = 1$.

Remarks: It follows from above results that

(i) probability of occurrence of an event is always non-negative;

(ii) probability of occurrence of an impossible event is 0;

(iii) probability of occurrence of a sure event is 1.

Theorem 2 If E and F are mutually exclusive events, then

(i) $P(E \cap F) = 0$ and

(ii) $P(E \cup F) = P(E) + P(F)$.

***Notes*:** **1.** For mutually exclusive events E and F, we have

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F).$$

2. If $E_1, E_2, ..., E_k$ are mutually exclusive events, then

$$P(E_1 \cup E_1 \cup, ..., \cup E_k) = P(E_1) + P(E_2) + ... + P(E_k).$$

Theorem 3 If E and F are two mutually exclusive and exhaustive events, then $P(E) + P(F) = 1$.

Theorem 4 Let E be any event and $\overline{E}$ be its complementary event, then $P(\overline{E}) = 1 - P(E)$.

Theorem 5 For any two events E and F,

$$P(E - F) = P(E) - P(E \cap F).$$

Theorem 6 (Addition Theorem). For any two events E and F,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

***Notes*:** **1.** We may express the above results as

$$P(E \text{ or } F) = P(E) + (F) - P(E \text{ and } F)$$

2. If E and F are mutually exclusive, then

$P(E \cap F) = 0$ and so $P(E \cup F) = P(E) + P(F)$.

Theorem 7 If E_1 and E_2 be two events such that $E_1 \subseteq E_2$, then prove that $P(E_1) \leq P(E_2)$.

Theorem 8 If E is an event associated with a random experiment, then $0 \leq P(E) \leq 1$.

Theorem 9. For any three events E, F, G

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

Illustration 18 A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting

(i) a jack or a queen or a king,

(ii) a two of heart or diamond.

Solution (i) In a pack of 52 cards, we have:

4 jacks, 4 queens and 4 kings.

Now, clearly a jack and a queen and a king are mutually exclusive events.

$$\text{Also, } P(\text{a jack}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}.$$

$$P(\text{a queen}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{a king}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}.$$

$\therefore$ By the addition theorem of Probability,

$P(\text{a jack or a queen or a king}) = P(\text{a jack}) + P(\text{a queen}) + P(\text{a king})$

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}.$$

(ii) $P(\text{two of heart or two of diamond})$

$$= P(\text{two of heart}) + P(\text{two of diamond})$$

$$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}.$$

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Illustration 19 Find the probability of getting a sum of 7 or 11 in a simultaneous throw of two dice.

Solution When two dice are thrown we have observed that there are 36 possible outcomes. Now, we can have a sum of 7 as

$$1 + 6 = 7,\ 2 + 5 = 7,\ 3 + 4 = 7,\ 4 + 3 = 7,\ 5 + 2 = 7,\ 6 + 1 = 7$$

Thus, the six favourable cases are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$$\therefore\quad P(\text{a sum of 7}) = \frac{6}{36} = \frac{1}{6}.$$

Again, the favourable cases of getting a sum of 11 are (5, 6), (6, 5)

$$\therefore\quad P(\text{a sum of 11}) = \frac{2}{36} = \frac{1}{18}.$$

Since the events of getting 'a sum of 7' or 'a sum of 11' are mutually exclusive:

$$\therefore\quad P(\text{a sum of 7 or 11}) = P(\text{a sum of 7}) + P(\text{a sum of 11})$$
$$= \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}.$$

Illustration 20 From a well-shuffled pack of 52 cards, a card is drawn at random, find the probability that it is either a heart or a queen.

Solution A : Getting a heart card B : Getting a queen card

$$P(A) = \frac{13}{52},\ P(B) = \frac{4}{52},\ P(A \cap B) = \frac{1}{52}$$

$$\text{Required probability} = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

INDEPENDENT EVENTS

Two event A and B are said to be independent if the occurrence (or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Illustration 21 In the simultaneous throw of two coins, 'getting a head' on first coin and 'getting a tail on the second coin are independent events.

Illustration 22 When a card is drawn from a pack of well-shuffled cards and replaced before the second card is drawn, the result of second draw is independent of first draw. We now state, without proof, the theorem which gives the probabilities of simultaneous occurrence of the independent events.

Theorem 10 If A and B are two independent events, then

$$P(A \text{ and } B) = P(A).\ P(B)$$

Illustration 23 Two dice are thrown. Find dthe probability of getting an odd number on the one die and a multiple of three on the other.

Solution Since the events of 'getting an odd number' on one die and the event of getting a multiple of three on the other are independent events,

$$P(A \text{ and } B) = P(A) \times P(B) \qquad \text{...(i)}$$

Now, $P(A) = P(\text{an odd number}) = \frac{3}{6} = \frac{1}{2}$ [There are three odd numbers 1, 3, 5] and $P(B) = P(\text{a multiple of 3})$

$= \frac{2}{6} = \frac{1}{3}$ [Multiples of 3 are 3 and 6]

$$\therefore\quad \text{From (i), required probability} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

Illustration 24 Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5. Find the probability that

(i) only one of them will be selected,

(ii) none of them be selected.

Solution Let A : Arun is s selected B : Tarun is selected.

Then, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$.

Clearly, 'A' and 'not B' are independent also 'not A' and 'not B' are independent, 'B' and 'not A' are independent.

(i) P (only one of them will be selected)

$$= P\ (A \text{ and not } B \text{ or } B \text{ and not } A)$$
$$= P\ (A)\ P\ (\text{not } B) + P\ (B)\ P\ (\text{not } A)$$
$$= \frac{1}{3}\left(1 - \frac{1}{5}\right) + \frac{1}{5}\left(1 - \frac{1}{3}\right)$$
$$= \frac{1}{3} \times \frac{4}{5} \times \frac{1}{5} \times \frac{2}{3} = \frac{4}{15} + \frac{2}{12}$$
$$= \frac{6}{15} = \frac{2}{5}.$$

(ii) P (onle one of them be selected)

$$= P\ (\text{not } A \text{ and not } B)$$
$$= P(\text{not } A) \times P(\text{not } B)$$
$$= \left(1 - \frac{1}{3}\right) \times \left(1 - \frac{1}{5}\right)$$
$$= \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. In a simultaneous toss of two coins, find the probability of 2 tails.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

2. In a simultaneous toss of two coins, find the probability of exactly 1 tail.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) None of these.

3. In a simultaneous toss of two coins, find the probability of no tail.

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) None of these.

4. Three coins are tossed. Find the probability of heads.

(a) $\frac{1}{6}$ (b)1/8

(c) $\frac{1}{4}$ (d) None of these.

5. Three coins are tossed. Find the probability of exactly 2 heads.

(b) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) None of these.

6. Three coins are tossed. Find the probability of at least 2 heads.

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$

(c) $\frac{1}{8}$ (d) None of these:

7. Three coins are tossed. Find the probability of atmost 2 heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{7}{8}$ (d) None of these.

8. Three coins are tossed. Find the probability of no heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$

(c) $\frac{1}{2}$ (d) None of these.

9. Three coins are tossed. Find the probability of at least one head and one tail.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) None of these.

10. A coin is tossed three times. Find the chance that head and tail show alternately.

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these.

11. Four coins are tossed once. Find the probability of 4 tails.

(a) $\frac{1}{16}$ (b) $\frac{5}{16}$

(c) $\frac{9}{16}$ (d) None of these.

12. Four coins are tossed once. Find the operability of exactly 3 tails.

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$

(c) $\frac{5}{16}$ (d) None of these.

13. Four coins ore tossed once. Find the probability of exactly 2 tails.

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{3}{8}$ (d) $\frac{5}{16}$

14. Four coins are tossed once. Find the probability of at least 1 tail.

(a) $\frac{1}{16}$ (b) $\frac{15}{16}$

(c) $\frac{3}{16}$ (d) None of these.

15. In a single throw of two dice, find the probability of getting a total of 3 or 5.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{16}$ (d) $\frac{5}{6}$

16. In a single throw of two dice, find the probability of getting a total of 12.

(a) $\frac{1}{36}$ (b) $\frac{1}{9}$

(c) $\frac{1}{18}$ (d) $\frac{35}{36}$

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17. In a single throw of two dice, find the probability of getting a total of 11.

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{35}{36}$

18. In a single throw of two dice, find the probability of getting a total of 8.

(a) $\frac{5}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{31}{36}$

19. In a single throw of two dice, the probability of getting a total of 7.

(a) $\frac{5}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

20. In a single throw of two dice, what is the probability of a doublet?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{18}$

21. In a single throw of two dice, what is the probability of a multiple of 2 on one and a multiple of 3 on the other?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$

(c) $\frac{11}{36}$ (d) $\frac{1}{9}$

22. (Q. Nos. 22–34) In a single throw of two dice, find the probability of getting. An odd number on one and a multiple of three on the other.

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$

(c) $\frac{11}{36}$ (d) $\frac{1}{9}$

23. Doublet of even numbers.

(a) $\frac{1}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{1}{9}$

24. A sum less than 6.

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

25. A sum more than 7.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{5}{12}$

26. A sum greater than 10.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{5}{12}$

27. A sum at least 10.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

28. An odd number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

29. An even number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

30. Six as the product.

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

31. A multiple of 3 as the sum.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{9}$ (d) $\frac{5}{36}$

32. The product a perfect square (square of a natural number)

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

33. At least one of the two numbers as 4.

(a) $\frac{1}{4}$ (b) $\frac{5}{18}$

(c) $\frac{11}{36}$ (d) $\frac{1}{3}$

34. Sum as a prime number

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$

(c) $\frac{7}{12}$ (d) $\frac{3}{4}$

35. In a single throw of three dice, find the probability of getting a total of 17 or 18.

(a) $\frac{1}{54}$ (b) $\frac{1}{27}$

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85. Only the woman will be alive.
(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

86. At least one of them will be alive.
(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

87. A man speaks truth in 80% of the cases and another in 90% of the cases. While stating the same fact, what is the probability that they contradict?
(a) $\frac{37}{50}$ (b) $\frac{13}{50}$
(c) $\frac{16}{50}$ (d) None of these.

88. There are 3 urns A, B and C, A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and a black ball?
(a) $\frac{17}{42}$ (b) $\frac{25}{42}$
(c) $\frac{19}{42}$ (d) $\frac{23}{42}$

89. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second third and fourth shots are 0.4, 0.3, 0.2 and 0.1, respectively. What is the probability that the gun hits the plane?
(a) 0.4379 (b) 0.6872
(c) 0.6976 (d) None of these.

90. A can solve 90% of the problems given in a book and B solve 70%. What is the probability that at least one of them will solve a problem selected at random from the book?
(a) $\frac{3}{100}$ (b) $\frac{97}{100}$
(c) $\frac{83}{100}$ (d) $\frac{17}{100}$

91. A and B throw a coin alternately till one of them gets a head and wins the game. If A starts the game, find the probability of winning of A.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) 1 (d) None of these.

92. Two persons A and B throw a die alternately till one of them gets a 'six' and wins the game. Find the probability of winning of B.
(a) $\frac{5}{11}$ (b) $\frac{6}{11}$
(c) $\frac{4}{11}$ (d) $\frac{3}{11}$

93. The letters of the word 'SOCIETY' are placed at row. What is probability that the three vowels come together?
(a) $\frac{4}{7}$ (b) $\frac{3}{7}$
(c) $\frac{2}{7}$ (d) $\frac{1}{7}$

94. Find the probability that in a random arrangement of the letters of the word DAUGHTER, the letter D occupies the first place.
(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There is only one favourable case TT.
$\therefore$ P (2 tails) = $\frac{1}{4}$.

2. Ans. (a)

Solution Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There are two favourable cases HT, TH.
$\therefore$ P (exactly 1 tail) = $\frac{2}{4} = \frac{1}{2}$.

3. Ans. (c)

Solution Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4.
There is only one favourable case HH.
$\therefore$ P (no tails) = $\frac{1}{4}$.

4. Ans. (b)

Solution Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
There is only one favourable case HHH.
$\therefore$ P (all heads) = $\frac{1}{8}$.

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$n(A) = 12, n(S) = 36$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{12}{36} = \dfrac{1}{3}$.

32. Ans. (b)

Solution A : Getting the product of two numbers to be a perfect square

$A = [(1, 1), (1, 4), (2, 2), (4, 1), (3, 3), (4, 4), (5, 5), (6, 6)]$

$n(A) = 8, n(S) = 36$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{8}{36} = \dfrac{2}{9}$.

33. Ans. (c)

Solution A : Getting at least one of the two number as 4.

$A = [(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)]$

$n(A) = 11, n(S) = 36$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{11}{36}$.

34. Ans. (a)

Solution A : Getting sum as a prime number

$A = [(1, 1), (1, 2), (2, 1), (2, 3), (3, 2), (4, 1), (1, 4), (4, 3), (3, 4), (6, 1), (1, 6), (2, 5), (5, 2), (6, 5), (5, 6)]$

$n(A) = 11, n(S) = 36$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{15}{36} = \dfrac{5}{12}$.

35. Ans (a)

Solution Number of exhaustive cases in a single throw of three dice

$= 6 \times 6 \times 6 = 216$

Cases favourable to a total of 17 are (5, 6, 6), (6, 5, 6), (6, 6, 5)

Number of cases favourable to a total of 17 or 18 is 4.

$\therefore$ P(a total of 17 or 18) $= \dfrac{4}{216} = \dfrac{1}{54}$.

36. Ans. (c)

Solution Number of exhaustive cases in a single throw of three dice $= 6 \times 6 \times 6 = 216$.

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1).

$\therefore$ P(a total of 5) $= \dfrac{6}{216} = \dfrac{1}{36}$.

37. Ans. (a)

Solution A total of at most 5 means a total 3, 4 or 5.

Cases favourable to a total of 3 are (1, 1, 1)

Cases favourable to a total of 4 are (1, 1, 2), (1, 2, 1), (2, 1, 1)

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 11).

Number of cases favourable to a total of 3 or 4 or 5 is 10.

$\therefore$ P (a total of at most 5) $= \dfrac{10}{216} = \dfrac{5}{108}$.

38. Ans. (c)

Solution A total of at least 5 means not a total of 3 or 4.

Number of cases favourable to a total of 3 or 4 is 4.

P(a total of 3 or 4) $= \dfrac{4}{216} = \dfrac{1}{54}$

$\therefore$ P(a total of at least 5) $= P$(not a total of 3 or 4)

$= 1 - P$(a total of 3 or 4)

$= 1 - \dfrac{1}{54} = \dfrac{53}{54}$.

39. Ans. (b)

Solution We know that a leap year has 366 days and thus a leap year has 52 weeks and 2 days over.

The two over (successive days have the following likely cases:

(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Firday and Saturday
(vii) Saturday and Sunday.

$\therefore$ Number of exhaustive cases 'n' = 7.

Out of these, the favourable cases are (i) and (vii)

$\therefore$ Number of favourable cases 'm' = 2

$\therefore$ Probability of having 53 Sundays $= \dfrac{2}{7}$.

40. Ans. (a)

Solution A : Getting a number which is a square

$A = (1, 4, 9, 16, 25, 36, 49, 64, 81, 100)$

$n(A) = 10, n(S) = 100$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{10}{100} = \dfrac{1}{10}$.

41. Ans. (c)

Solution There are 7 letters in the word 'SOCIETY' which can be arranged in 71 ways. Considering the three vowels in the word 'SOCIETY' as one letter, we can

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arrange 5 letters in a row in 5! ways. Also, three vowels can themselves be arranged in 3! ways.

$\therefore$ The total number of arrangements in which three vowels come together are $5! \times 3!$

Hence, the required probability $= \dfrac{5! \times 3!}{7!}$

$= \dfrac{3 \times 2 \times 1}{7 \times 6} = \mathbf{\dfrac{1}{7}}$.

42. Ans. (a)

Solution Out of the letters in the word 'UNIVERSITY' two letters '*I*' are alike.

$\therefore$ Number of permutations $= \dfrac{10!}{2}$ ...(i)

Number of words in which two '*I*' are never together

= Total number of words – Number of words in which two '*I*' are together

$= \dfrac{10!}{2} - 9! = \dfrac{10! - 2.9!}{2} = \dfrac{9![10-2]}{2} = \dfrac{9!.8}{2} = 9!.4$

$\therefore$ Required probability $= \dfrac{9.4}{10!/2} = \dfrac{9!.8}{10!}$

$= \dfrac{8}{10} = \mathbf{\dfrac{4}{5}}$.

43. Ans. (a)

Solution Number of ways in which 6 letters of the word PENCIL can be arranged is $P(6, 6) = 6!$.

If N is next to E, they can be considered as one and the 5 letters can be arranged in $P(5, 5) = 5!$ ways.

$\therefore$ The required probability $= \dfrac{5!}{6!} = \mathbf{\dfrac{1}{6}}$.

44. Ans. (a)

Solution Let A be the event of "getting the sum 4".

Then, $A = [(1, 32), (3, 1), (2, 2)]$.

Therefore, 3 favourable outcomes and $(36 - 3) = 33$ outcomes are unfavourable.

$\therefore$ Odds in favour of sum of $4 = \dfrac{3}{33} = \mathbf{\dfrac{1}{11}}$.

45. Ans. (b)

Solution Let A be the event of "getting a sum of 5".

Then, $A = [(1, 4), (4, 1), (2, 3), (3, 2)]$.

There are 4 favourable outcomes and $(36 - 4) = 32$ outcomes are unfavourable.

$\therefore$ Odds in favour of sum $5 = \dfrac{4}{32} = \mathbf{\dfrac{1}{8}}$.

46. Ans. (c)

Solution Let A be the event "getting the sum 6".

Then, $A = [(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)]$

There are 5 favourable outcomes and $(36 - 5) = 31$ outcomes are unfavourable.

$\therefore$ Odds against getting the sum $6 = \mathbf{\dfrac{31}{5}}$.

47. Ans. (b)

Solution A : Getting a queen B : Getting an ace

$P(A) = \dfrac{4}{52}, P(B) = \dfrac{4}{52}, P(A \cap B) = \dfrac{0}{52} = 0$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \dfrac{4}{52} + \dfrac{4}{52} - 0 = \dfrac{8}{52} = \mathbf{\dfrac{2}{13}}$.

48. Ans. (c)

Solution A : Roll number is multiple of 5 B : Roll number is multiple of 7

$A = (5, 10, 15, 20, 25)$ $B = (7, 14, 21)$

$P(A) = \dfrac{5}{25}, P(B) = \dfrac{3}{25}, P(A \cap B) = \dfrac{0}{25} = 0$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \dfrac{5}{25} + \dfrac{3}{25} - 0 = \mathbf{\dfrac{8}{25}}$.

49. Ans. (a)

Solution A : Integer chosen is divisible by 6B : Integer chosen is divisible by 8

$n(A) = 33, n(B) = 25, n(A \cap B) = 8, n(S) = 200$

$P(A) = \dfrac{33}{200}, P(B) = \dfrac{25}{200}$

$P(A \cap B) = P(A) + A(B) - P(A \cap B)$

$= \dfrac{33}{200} + \dfrac{33}{200} - \dfrac{8}{200} = \dfrac{50}{200} = \mathbf{\dfrac{1}{4}}$.

50. Ans. (c)

Solution A : Sum of two numbers is divisible by 3

B : Sum of two numbers is divisible by 4

$A = [(1, 2), (2, 1), (3, 3), (5, 1), (1, 5), (2, 4), (4, 2), (5, 4), (4, 5), (6, 3), (3, 6), 6, 6)]$

$B = [(2, 2), (3, 1), (1, 3), (5, 3), (3, 5), (4, 4), (6, 2), (2, 6), (6, 6)]$

$P(A) = \dfrac{12}{36}, P(B) = \dfrac{9}{36}, P(A \cap B) = \dfrac{1}{36}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \dfrac{12}{36} + \dfrac{9}{36} - \dfrac{1}{36} = \dfrac{20}{36} = \mathbf{\dfrac{5}{9}}$.

51. Ans. (a)

Solution A : Getting total of 11 B : Getting odd number one each die

$A = [(6, 5), (5, 6)]$

$B = [1, 1). (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)]$

$P(A) = \dfrac{2}{36}, P(B), P(B) = \dfrac{9}{36}, P(A \cap B) = \dfrac{9}{36} = 0$

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$\therefore$ Required probability $= P(A) + P(B) - P(A \cap B)$

$$= \frac{2}{36} + \frac{9}{36} - 0 = \frac{\mathbf{11}}{\mathbf{36}}.$$

52. Ans. (b)

Solution A : Number divisible by 3

B : Number is a perfect square

$A = (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36)$

$B = (1, 4, 9, 16, 25, 36)$

$$P(A) = \frac{12}{36}, P(B) = \frac{6}{36}, P(A \cap B) = \frac{2}{36}$$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B).$

$$= \frac{12}{36} + \frac{6}{36} - \frac{2}{36} = \frac{16}{36} = \frac{\mathbf{4}}{\mathbf{9}}.$$

53. Ans. (c)

Solution A : Getting rusted item B : Getting bolt

$$P(A) = \frac{100}{200}, P(B) = \frac{50}{200}, P(A \cap B) = \frac{125}{200}$$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200}$$

$$= \frac{125}{200} = \frac{\mathbf{5}}{\mathbf{8}}.$$

54. Ans. (a)

Solution A : Getting, a doublet, B : Getting a total of 10

$A = [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]$

$B = [(6, 4), (4, 6), (5, 5)]$

$$P(A) = \frac{6}{36}, P(B) = \frac{3}{36}, P(A \cap B) = \frac{1}{36}$$

$\therefore$ Required probability

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left(\frac{6}{36} + \frac{3}{36} - \frac{1}{36}\right)$$

$$= 1 - \frac{8}{36} = \frac{28}{36} = \frac{\mathbf{7}}{\mathbf{9}}.$$

55. Ans. (b)

Solution A : Getting a total of 9, B : Getting a total of 11

$A = [(5, 4), (4, 5), (6, 3), (3, 6)]$ $B = [(6, 5), (5, 6)]$

$$P(A) = \frac{4}{36}, P(B) = \frac{2}{36}, P(A \cap B) = \frac{0}{36}$$

$\therefore$ Required probability $= 1 - P(A \cup B)$

$= [P(A) + P(B) - P(A \cap B)]$

$$= 1 - \left(\frac{4}{36} + \frac{2}{36} - 0\right)$$

$$= 1 - \frac{1}{6} = \frac{\mathbf{5}}{\mathbf{6}}.$$

56. Ans. (d)

Solution A: Getting spade card B: Getting ace card C: Getting red card

$$P(A) = \frac{13}{52}, P(B) = \frac{4}{52}, P(C) = \frac{26}{52}, P(A \cap B) = \frac{1}{52},$$

$$P(B \cap C) = \frac{2}{52}, P(C \cap A) = \frac{0}{52} = 0,$$

$$P(A \cap B \cap C) = \frac{0}{52} = 0$$

$\therefore$ Required probability

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{13}{52} + \frac{4}{52} + \frac{26}{52} - \frac{1}{52} - \frac{2}{52} - 0 = \frac{40}{52} = \frac{\mathbf{10}}{\mathbf{13}}.$$

57. Ans. (a)

Solution A: Number is divisible by 2 B : Number is divisible buy 3 C : Number is divisible by 5

$$P(A) = \frac{100}{200}, P(B) = \frac{66}{200}, P(C) = \frac{40}{200},$$

$$P(B \cap C) \frac{33}{200} = P(B \cap C) = \frac{13}{200},$$

$$P(C \cap A) = \frac{20}{200}, P(A \cap B \cap C) = \frac{6}{200}$$

$\therefore$ Required probability

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{100}{200} + \frac{66}{200} + \frac{40}{200} - \frac{33}{200} - \frac{13}{200} - \frac{20}{200} + \frac{6}{200}$$

$$= \frac{146}{200} = \frac{\mathbf{73}}{\mathbf{200}}.$$

58. Ans. (b)

Solution We know $P(A) = 1 - P(\overline{A})$

$= 1 - 0.65 = 0.35$

and, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.65 = 0.35 + p - 0$

[$\because$ A and B are mutually exclusive events]

$\Rightarrow p = 0.65 - 0.35 = \mathbf{0.30}.$

59. Ans. (a)

Solution Required probability $= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [0.5 + 0.3 - 0]$

[$\because$ A and B are mutually exclusive events]

$= 1 - 0.8 = \mathbf{0.2}.$

60. Ans. (c)

Solution P (at least B grade) $= P$ (B grade) $+ P$ (A grade)

$= 0.38 + 0.30 = \mathbf{0.68}.$

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Probability of drawing a black ball from the second bag $= \frac{5}{8}$.

$\therefore$ Probability that both balls are black

$$= \frac{1}{3} \times \frac{5}{8} = \mathbf{\frac{5}{24}}.$$

69. Ans. (c)

Solution The event 'one is white and one is black' is the same as the event 'either the first is white and the second is black or the first is black and the second is white',

$\therefore$ The probability that one is white and one is black

$$= \frac{2}{3} \times \frac{5}{8} \times \frac{1}{3} \times \frac{3}{8} = \mathbf{\frac{12}{24}}.$$

70. Ans. (a)

Solution Success: Getting odd number $p = \frac{13}{25}$

$$\Rightarrow q = 1 - p = 1 - \frac{13}{25} = \frac{12}{25}$$

$$P(\text{two successes}) = pp = \frac{13}{25} \times \frac{13}{25} = \mathbf{\frac{169}{625}}.$$

71. Ans. (b)

Solution $P(\text{exactly one success}) = pq + qp$

$$= \frac{13}{25} \times \frac{12}{25} \times \frac{12}{25} \times \frac{13}{25} = \frac{156+156}{625} = \frac{312}{625}.$$

72. Ans. (c)

Solution $P(\text{at one success}) = 1\ P(\text{no success})$

$$= 1 - qq = 1 - \left(\frac{12}{25}\right)\left(\frac{12}{25}\right) = 1 - \frac{144}{625}$$

$$= \frac{625-144}{625} = \mathbf{\frac{481}{625}}.$$

73. Ans. (d)

Solution $P(\text{no success}) = qq = \frac{12}{25}\left(\frac{12}{25}\right) = \frac{144}{625}$.

74. Ans. (a)

Solution Success: getting 5 or 6 $p = \frac{2}{6} = \frac{1}{3}$

$$\Rightarrow q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(3 \text{ successes}) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \mathbf{\frac{1}{27}}.$$

75. Ans. (b)

Solution $P(\text{exactly 2 successes}) = ppq + pqp + qpp$

$$= \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{2+2+2}{27} = \frac{6}{27} = \mathbf{\frac{2}{9}}.$$

76. Ans. (c)

Solution $P(\text{at most 2 successes}) = P(\text{no success}) + P(1 \text{ success}) + P(2 \text{ successes})$

$$= 1 - P(3 \text{ successes}) = 1 - pp$$

$$= 1 - \frac{1}{3}\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = 1 - \frac{1}{27} = \mathbf{\frac{26}{27}}.$$

77. Ans. (d)

Solution $P(\text{at least 2 successes}) = + P(3 \text{ successes})$

$$= ppq + pqp + qpp + ppp$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{2+2+2+1}{27} = \mathbf{\frac{7}{27}}.$$

78. Ans. (c)

Solution A : First card is diamond card B : Second card is king card

$$P(A) = \frac{13}{52} = \frac{1}{4},\ P(B) = \frac{4}{52} = \frac{1}{13}$$

$\therefore$ Required probability $= P(A)\ P(B) = \frac{1}{4} \times \frac{1}{13} = \mathbf{\frac{1}{52}}$.

79. Ans. (a)

Solution A : Husband selected; B : Wife selected

$$P(A) = \frac{1}{7} \Rightarrow (\overline{A}) = 1 - P(A) = 1 - \frac{1}{7} = \frac{6}{7}$$

$$P(B) = \frac{6}{7} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{1}{5} = \frac{4}{5}$$

$P(\text{only one of them will be selected})$

$$= P(A)\ P(\overline{B}) + P(B)\ P(\overline{A})$$

$$= \frac{1}{7}\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{6}{7}\right) = \frac{4+6}{35} = \frac{10}{35} = \mathbf{\frac{2}{7}}.$$

80. Ans. (b)

Solution $P(\text{both of them will be selected})$

$$= P(A) \times P(B) = \frac{1}{7} \times \frac{1}{5} = \mathbf{\frac{1}{35}}.$$

81. Ans. (c)

Solution $P(\text{none of them will be selected})$

$$= P(\overline{A})\ P(\overline{B}) = \frac{6}{7} \times \frac{4}{5} = \mathbf{\frac{24}{35}}.$$

82. Ans. (d)

Solution $P(\text{at least one of them will be selected})$

$$= 1 - P(\overline{A}) \times P(\overline{B})$$

$$= 1 - \frac{6}{7} \times \frac{4}{5} = 1 - \frac{24}{35} = \mathbf{\frac{11}{35}}.$$

83. Ans. (3)

Solution A : Husband will be alive 25 years hence

B : Wife will be alive 25 years hence

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$P(A) = 0.3 \Rightarrow P(\overline{A}) = 1 - P(A) = 1.03 = 0.7$

$P(B) = 0.4 \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - 04 = 0.6$

Required probability $= P(A)\,P(B) = (0.3)\,(0.4) = \mathbf{0.12.}$

84. Ans. (b)

Solution Required probability $= P(A)\,P(\overline{B})$
$= (0.3)\,(0.6) = \mathbf{0.18.}$

85. Ans. (c)

Solution Required probability $= P(B)\,P(A)$
$= (0.4)\,(0.7) = \mathbf{0.28.}$

86. Ans. (d)

Solution Required probability $= 1 - P(\overline{A})\,P(\overline{B})$
$= 1 - (0.7)\,(0.6)$
$= 1 - 0.42 = \mathbf{0.58.}$

87. Ans. (b)

Solution Let the two men be A and B. A : A speaks truth; B : B speaks truth

$$P(A) = \frac{80}{100} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \frac{80}{100} = \frac{20}{100}$$

$$P(B) = \frac{90}{100} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{90}{100} = \frac{10}{100}$$

$\therefore$ Required probability $= P(A)\,P(\overline{B}) + P(B)\,P(\overline{A})$

$$= \frac{80}{100} \times \frac{10}{100} + \frac{90}{100} \times \frac{20}{100}$$

$$= \frac{8+18}{100} = \frac{26}{100} = \mathbf{\frac{13}{50}}.$$

88. Ans. (a)

Solution A : Getting a red ball from urn A

B : Getting a black ball from urn A

C : Getting a red ball from urn B

D : Getting a black ball fron urn C

E : Getting a red ball from urn C

F : Getting a black ball from urn C

$$P(A) = \frac{4}{7},\ P(B) = \frac{3}{7},\ P(C) = \frac{5}{9},$$

$$P(D) = \frac{4}{9},\ P(E) = \frac{4}{8}.\ P(F) = \frac{4}{8}$$

Required probability $= P(A)\,P(C)\,P(F) + P(A)\,P(D)$
$P(E) + P(B)\,P(C)\,P(E)$

$$= \frac{4}{7} \times \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{4}{8} + \frac{3}{7} \times \frac{5}{9} \times \frac{4}{8}$$

$$= \frac{80+64+60}{504} = \frac{204}{504} = \mathbf{\frac{17}{42}}.$$

89. Ans. (c)

Solution A : Plane is hit by the first shot

B : Plane is hit by the second shot

C : Plane is hit by the third shot

D : Plane is hit by the fourth shot

$P(A) = 0.4 \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - 0.4 = 0.6$

$P(B) = 0.3 \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - 0.3 = 0.7$

$P(C) = 0.2 \Rightarrow P(\overline{C}) = 1 - P(C) = 1 - 0.2 = 0.8$

$P(D) = 0.1 \Rightarrow P(\overline{D}) = 1 - P(D) = 1 - 0.1 = 0.9$

$\therefore$ Required probability

$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})\,P(\overline{D})$

$= 1 - (0.6)\,(0.7)\,(0.8)\,(0.9)$

$= 1 - 0.3024 = \mathbf{0.6976.}$

90. Ans. (b)

Solution A : A solves the problem; B : B solves the problem.

$$P(A) = \frac{90}{100} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \frac{90}{100} = \frac{10}{100}$$

$$P(B) = \frac{70}{100} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{70}{100} = \frac{30}{100}$$

Required probability $= 1 - P(\overline{A})\,P(\overline{B})$

$$= 1 - \frac{10}{100} \times \frac{30}{100} = 1 - \frac{3}{100}$$

$$= \mathbf{\frac{97}{100}}.$$

91. Ans. (b)

Solution Success : Getting head $\therefore\ p = \frac{1}{2}$

$$\Rightarrow q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}.$$

Let A start the game. A can win the game in 1st, IIIrd, Vth, ... throws. Then

$P(A \text{ winning}) = p + qqp + qqqqp + \ldots$

$= p[1 + q^2 + q^4 + \ldots]$

$$= p\frac{1}{1-q^2} = \frac{1}{2}\frac{1}{1-(1/2)^2}$$

$$= \frac{4}{2(3)} = \mathbf{\frac{2}{3}}.$$

92. Ans. (a)

Solution Success : Getting 6

$$p = \frac{1}{6} \Rightarrow q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}.$$

A can win the game in 1st, IIIrd, Vth ... throws

$P(A \text{ winning}) = p + qqp + qqqqp + \ldots$

$$= p[1 + q^2 + q^4 + \ldots] = p\frac{1}{1-q^2}$$

$$= \frac{1}{6}\frac{1}{1-(5/6)^2} = \frac{1}{6}\frac{36}{36-25} = \frac{6}{11}$$

$\therefore\ P(B \text{ winning}) = 1 - \frac{6}{11} = \mathbf{\frac{5}{11}}.$

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93. Ans (d)

Solution A : Three vowels come together

$n(A) = 5!3!, \quad n(S) = 7!$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5!\,3!}{7!}$

$= \dfrac{3 \times 2}{7 \times 6} = \dfrac{1}{7}.$

94. Ans. (a)

Solution A : D occupies the first place

$n(A) = 7!, \; n(S) = 8!$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{7!}{8!} = \dfrac{1}{8}.$

QUESTIONS FROM COMPETITIVE EXAMS

1. The probability of raining on day 1 is 0.2 and on day 2 is 0.3. What is the probability of raining on both the days?

(a) 0.2 (b) 0.1
(c) 0.06 (d) 0.25

[Based on MAT 2003]

2. A bag contains 5 red balls and 8 blue balls. It also contains 4 green and 7 black balls. If a ball is drawn at random, then find the probability that it is not green.

(a) 5/6 (b) 1/4
(c) 1/6 (d) 7/4

[Based on MAT 2003]

3. A bag contains 2 red, 3 green and 2 blue balls. 2 balls are to be drawn randomly. What is the probability that the balls drawn contain no blue ball?

(a) $\dfrac{5}{7}$ (b) $\dfrac{10}{21}$
(c) $\dfrac{2}{7}$ (d) $\dfrac{11}{21}$

[Based on MAT 2003]

4. If the probability that A will live 15 years is $\dfrac{7}{8}$ and that B will live 15 years is $\dfrac{9}{10}$, then what is the probability that both will live after 15 years?

(a) $\dfrac{1}{20}$ (b) $\dfrac{63}{80}$
(c) $\dfrac{1}{5}$ (d) None of these

[Based on MAT 2003]

5. Suppose six coins are flipped. Then the probability of getting at least one tail is

(a) $\dfrac{71}{72}$ (b) $\dfrac{53}{54}$
(c) $\dfrac{63}{64}$ (d) $\dfrac{1}{12}$

[Based on MAT 2002]

6. The probability that a student is not a swimmer is 1/5. Then the probability that out of the five students, four are swimmers, is

(a) ${}^5C_4\left(\dfrac{4}{5}\right)^2\left(\dfrac{1}{5}\right)$ (b) $\left(\dfrac{4}{5}\right)^4\left(\dfrac{1}{5}\right)$
(c) ${}^5C_1\left(\dfrac{1}{5}\right)\left(\dfrac{4}{5}\right)^4$ (d) None of these

[Based on MAT 2002]

7. A set A is containing n elements. A subset P of A is chosen at random. The set is reconstructed by replacing the elements of P. A subset of A is again chosen at random. The probability that P and Q have no common elements is

(a) 5^n (b) $\left(\dfrac{3}{4}\right)^n$
(c) $\left(\dfrac{3}{5}\right)^n$ (d) 2^n

[Based on MAT 2005]

8. If events A and B are independent and $P(A) = 0.15$, $P(A \cup B) = 0.45$, then $P(B) =$

(a) $\dfrac{6}{13}$ (b) $\dfrac{6}{17}$
(c) $\dfrac{6}{19}$ (d) $\dfrac{6}{23}$

[Based on MAT 2005]

9. One hundered identical coins each with probability p of showing up heads are tossed. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads on 51 coins; then the value of p is

(a) $\dfrac{1}{2}$ (b) $\dfrac{49}{101}$
(c) $\dfrac{50}{101}$ (d) $\dfrac{51}{101}$

[Based on MAT 1999]

10. The probability that a marksman will hit a target is given as 1/5. Then, his probability of atleast one hit in 10 shots is

(a) $\dfrac{1}{6^{10}}$ (b) $1 - \left(\dfrac{4}{5}\right)^{10}$
(c) $1 - \dfrac{1}{5^{10}}$ (d) $1 - \dfrac{1}{5^{19}}$

[Based on MAT 2005]

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11. Two dice are tossed. The probability that the total score is a prime number is

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$

(c) $\frac{1}{2}$ (d) $\frac{7}{9}$

[Based on MAT 2000]

12. If the probability that A will live 15 years is $\frac{7}{8}$ and that B will live 15 years is $\frac{9}{10}$, then what is the probability that both will live after 15 years?

(a) $\frac{1}{20}$ (b) $\frac{63}{80}$

(c) $\frac{1}{5}$ (d) None of these

[Based on MAT 2001]

13. Four different objects 1, 2, 3, 4 are distributed at random in four places marked 1, 2, 3, 4. What is the pobability that none of the objects occupy the place corresponding to its number?

(a) 17/24 (b) 3/8

(c) 1/2 (d) 5/8

[Based on MAT 2001]

14. Three students try to solve a problem independently with a probability of solving it as 1/3, 2/5, 5/12 respectively. What is the probability that the problem is solved?

(a) 1/18 (b) 12/30

(c) 23/30 (d) 1/2

[Based on IIT Joint Man. Ent. Test 2004]

15. If the probability of rain on any given day in Pune city is 50%, then what is the probability that it rains on exactly 3 days in a 5-day period?

(a) 8/125 (b) 5/16

(c) 8/25 (d) 2/25

[Based on SCMHRD Ent. Exam. 2003]

16. The probability that an event A happens In one trial of an experiment is 0.4. Three independent trials of the experiment are formed. The probability that the event A happens at least once is

(a) 0.934 (b) 0.784

(c) 0.548 (d) 0.343

[Based on FMS (Delhi) 2002]

17. Thirty days are in September, April, June and November. Some months are of thirty one days. A month is chosen at random. Then its probability of having exactly three days less than maximum of 31 is

(a) 15/16 (b) 1

(c) 3/48 (d) None of these

[Based on SNAP 2007]

18. A Charted Accountant applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7, and being rejected at Y is 0.5 and the probability of his application being rejected is 0.6. What is the probability that he will be selected in one of the firms?

(a) 0.8 (b) 0.2

(c) 0.4 (d) 0.7

[Based on MAT 2008]

19. A property tax increase was proposed by a municipal corporation and it was observed that 40% of the property owners fovoured it while 80% of the non-owners of property favoured it. If 70% of the voters are property owners, what is the probability that a voter selected at random would be the one favouring the increase?

(a) .52 (b) .80

(c) .40 (d) Data is inadequate

[Based on FMS 2007]

20. A consignment of 20 picture tubes contains 5 defectives. Two tubes are selected one after the other at random. The probability that both are defectzive assuming that the first tube is not replaced befor drawing the second, would be:

(a) $\frac{1}{16}$ (b) $\frac{1}{19}$

(c) $\frac{1}{4}$ (d) None of the above

[Based on FMS 2007]

21. A special lottery is to be held to select a student who will live in the only deluxe room in a hostel. There are 100 year-III, 150 year-II, and 200 year-I students who applied. Each year-III's name is place in the lottery 3 times; each year-II's name, 2 times; and each year-I's name, 1 time. What is the probability that a year-III's name will be chosen?

(a) 1/8 (b) 2/9

(c) 2/7 (d) 3/8

[Based on SNAP 2007]

EXPLANATORY ANSWERS

1. Ans. (d)

Solution $0.2 \times 0.3 = \mathbf{0.06}$.

2. Ans. (a)

Solution The probability that the ball drawn is of green colour

$$= \frac{4}{24} = \frac{1}{6}$$

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Probability that the ball drawn is not of green colour

$$= 1 - \frac{1}{6} = \frac{5}{6}.$$

3. Ans. (a)

4. Ans. (b)

Solution $P(A) \times P(B) = \frac{7}{8} \times \frac{9}{10} = \frac{63}{80}.$

5. Ans. (c)

Solution Total number of events that would occur by flipping six coins $= 2^6 = 64$.

Probability that no tail occurs $= \frac{1}{64}$

$\therefore$ Probability of occuring at least one tail $= 1 - \frac{1}{64} = \frac{63}{64}.$

6. Ans. (b)

7. Ans. (a)

8. Ans. (b)

Solution $P(A \cap B) = P(A) \times P(B) = 0.15 \times P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.45 = 0.15 + P(B) - (0.15) \times P(B)$

$= 0.15 + P(B)(1 - 0.15) = 0.15 + 0.85\,P(B)$

$\therefore \quad 0.85\,P(B) = 0.45 - 0.15 = 0.30$

$\therefore \quad P(B) = \frac{0.30}{0.85} = \frac{30}{85} = \frac{6}{17}.$

9. Ans. (d)

Solution Let x be the number of coins showing heads.

$\therefore$ Prob. when $x = 50$ = Prob. when $x = 51$

$\Rightarrow {}^{100}C_{50} \times p^{50} \times (1-p)^{50} = {}^{100}C_{51} \times p^{51} \times (1-p)^{49}$

$\Rightarrow p = \frac{51}{101}.$

10. Ans. (b)

Solution $P(A) = \frac{1}{5}, \; P(\overline{A}) = 1 - \frac{1}{5} = \frac{4}{5}$

The probability that he will not hit the target in 10 shots is $\left(\frac{4}{5}\right)^{10}$

So, probability that atleast once target will be hit

$$= 1 - \left(\frac{4}{5}\right)^{10}.$$

11. Ans. (b)

Solution The event "Total score is a prime number when two dice are tossed" occurs in the following 15 ways: (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5).

$\therefore$ Required probability $= \frac{15}{36} = \frac{5}{12}.$

12. Ans. (b)

Solution $\frac{7}{8} \times \frac{9}{10} = \frac{63}{80}.$

13. Ans. (c)

Solution Let the four places be

$$\boxed{1} \; \boxed{2} \; \boxed{3} \; \boxed{4}$$

Now object i cannot occupy the place i. ...(A)

Suppose Object 2 occupies the place $\boxed{1}$. Then other placements can be done in 6 ways as follows:

(1)	2	1	3	4
(2)	2	1	4	3
(3)	2	3	1	4
(4)	2	3	4	1
(5)	2	4	1	3
(6)	2	4	3	1

Here out of six ways, only three are permissible, because (1), (3) and (6) are not permissible because of the non-fulfilment of condition (A). Hence, required probability is $\frac{3}{6} = \frac{1}{2}$. Similarly you can allow objects 3 and 4 to occupy place 1 and in each case you can find that the probability is $\frac{1}{2}$.

14. Ans. (c)

Solution $1 - \left(\frac{2}{3} \times \frac{3}{5} \times \frac{7}{12}\right) = \frac{23}{30}.$

15. Ans. (b)

Solution Prob. that it rains on the 1st day $= \frac{1}{2}$

Prob. that it rains on the 2nd day $= \frac{1}{2}$

Prob. that it rains on the 3rd day $= \frac{1}{2}$

Prob. that it rains on the 4th day $= \frac{1}{2}$

Prob. that it rains on the 5th day $= \frac{1}{2}$

Prob. that it rains on the any day in a 5-day period $= \frac{1}{32}$

Prob. that it rains on exactly 3 days in a 5-day period

$$= {}^5C_3 \times \frac{1}{32} = {}^5C_2 \times \frac{1}{32} = \frac{10}{32} = \frac{5}{16}.$$

16. Ans. (b)

Solution Required Probability

$= {}^3C_1\,(0.4)\,(0.6)^2 + {}^3C_2\,(0.4)^2\,(0.6) + {}^3C_3\,(0.4)^3$

$= 3\,(0.144) + 3\,(0.096) + 1\,(0.064) = \mathbf{0.784.}$

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17. Ans. (d)

Solution February is the only month having 28 or 29 days. Probability of choosing February $= \frac{1}{12}$

18. Ans. (b)

Solution Probability of his application being rejected = 0.6

Probability of his application being not rejected = 1 – 0.6

∴ Probably of his selection must be less than 0.4. It should be 0.2

19. Ans. (a)

20. Ans. (b)

21. Ans. (d)

Solution (d) Total names in the lottery
$= 3 \times 100 + 2 \times 150 + 200 = 800$
Number of year –II names $= 2 \times 15 = 300$

Probability $= \frac{300}{800} = \frac{3}{8}$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. What is the probability that a number selected from the numbers 1, 2... 25 is a prime number? You may assume that each of the 25 numbers is equally likely to be selected.
(a) $\frac{4}{25}$ (b) $\frac{16}{25}$
(c) $\frac{9}{25}$ (d) None of these.

2. The letters of the word 'ARTICLE' are arranged at random. Find the probability that the vowels may occupy the even places.
(a) $\frac{1}{35}$ (b) $\frac{7}{35}$
(c) $\frac{6}{35}$ (d) $\frac{8}{35}$

3. Find the probability of getting an even number on the first die or a total of 8 in a single throw of two dice.
(a) $\frac{5}{9}$ (b) $\frac{4}{9}$
(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

4. Two unbiased dice are thrown. Find the probability that the sum of the numbers obtained on the two dice is neither a multiple of 2, nor a multiple of 3.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{5}{6}$ (d) None of these.

5. A card is drawn from a pack of playing cards. Find the probability of getting a king or a heart or a red card.
(a) $\frac{6}{13}$ (b) $\frac{7}{13}$
(c) $\frac{4}{13}$ (d) $\frac{9}{13}$

6. The probabilities of happening of two events A and B are 0.25 and 0.50, respectively. If the probability of happening of A and B together is 0.14, find the probability that neither A nor B happens.
(a) 0.39 (b) 0.61
(c) 0.36 (d) 0.64

7. Five letters are placed at random in five addressed envelopes. Find the probability that all the letters are not dispatched in the right envelopes.
(a) $\frac{1}{120}$ (b) $\frac{7}{120}$
(c) $\frac{119}{120}$ (d) None of these.

8. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability of drawing two aces.
(a) $\frac{1}{169}$ (b) $\frac{19}{169}$
(c) $\frac{1}{13}$ (d) $\frac{168}{169}$

9. An unbiased dice is tossed twice. Find the probability of getting a 4, 5 or 6 on the first toss and a 1, 2, 3 or 4 on the second toss.
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

10. The probability of hitting a target by three marksmen is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. Find the probability that one and only one of them will hit the target when they fire simultaneously.
(a) $\frac{9}{24}$ (b) $\frac{7}{24}$
(c) $\frac{11}{24}$ (d) $\frac{9}{24}$

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Test II **[Time: 10 min]**

11. A party of n men is to be seated round a circular table. Find the probability that two particular men sit together.

(a) $\frac{1}{n-1}$ (b) $\frac{2}{n-1}$

(c) $\frac{3}{n-1}$ (d) $\frac{n}{n-1}$

12. In a single throw of two coins, find the probability of getting both heads or both tails.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) None of these.

13. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that it is neither an ace nor a king.

(a) $\frac{3}{13}$ (b) $\frac{4}{13}$

(c) $\frac{2}{13}$ (d) $\frac{11}{13}$

14. A candidate is selected for interview for three posts. For the first post there are 5 candidates, for the second there are 8 and for the third there are 7. What are the chances for his getting at least one post?

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

15. A student takes his examination in four subjects α, β, γ, δ. He estimates his chance of passing in α as 4/5, in β as 3/4, in γ as 5/6, and in δ as 2/3. To qualify he must pass in α and at least two other subjects. What is the probability that he qualifies?

(a) $\frac{61}{90}$ (b) $\frac{29}{90}$

(c) $\frac{57}{90}$ (d) None of these.

16. A class consists of 80 students, 25 of them are girls and 55 boys, 10 of them are rich and the remaining poor, 20 of them are fair complexioned. What is the probability of selecting a fair complexioned rich girl?

(a) $\frac{507}{512}$ (b) $\frac{5}{512}$

(c) $\frac{503}{512}$ (d) None of these.

17. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

(a) 38% (b) 40%

(c) 42% (d) 47%

18. A bag contains 4 white and 3 black balls. Four balls are successively drawn out with replacement. Find the probability that they are alternately of different colours.

(a) $\frac{173}{2401}$ (b) $\frac{248}{2401}$

(c) $\frac{346}{2401}$ (d) $\frac{288}{2401}$

19. The odds that A speaks the truth are 3:2 and the odds that B speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point?

(a) 66% (b) 47%

(c) 47.5% (d) None of these.

20. In two successive throws of a pair of dice, determine the probability of getting a total of 8, each time.

(a) $\frac{17}{1296}$ (b) $\frac{25}{1296}$

(c) $\frac{48}{1296}$ (d) None of these.

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (c)

Solution A : Getting prime number

$A = (2, 3, 5, 7, 11, 13, 17, 19, 23)$

$n(A) = 9, n(S) = 25$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{9}{25}$.

3. Ans. (a)

Solution A : Getting even number on the first die B : Getting a total of 8

$A = [(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)]$

$B = [(4, 4), (5, 3), (3, 5), (6, 2), (2, 6)]$

$P(A) = \frac{18}{36}, P(B) = \frac{5}{36}, P(A \cap B) = \frac{3}{36}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$

$= \frac{20}{36} = \frac{5}{9}$.

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5. Ans. (b)

Solution A : Getting a king card B : Getting a heart card C : Getting a red card

$$P(A) = \frac{4}{52}, P(B) = \frac{13}{52}, P(C) = \frac{26}{52},$$

$$P(A \cap B) = \frac{1}{52} P(B \cap C) = \frac{13}{52},$$

$$P(C \cap A) = \frac{2}{52}, P(A \cap B \cap C) = \frac{1}{52}$$

$\therefore$ Required probability

$= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \mathbf{\frac{7}{13}}.$$

7. Ans. (c)

Solution Total number of ways of placing 5 letters in 5 envelopes = 5! = 120

All the letters can be dispatched in the right envelopes in only one way. Therefore, the probability that all the letters are placed in the right envelopes = $\frac{1}{120}$.

Hence, the probability that all the letters are not placed in the right envelopes = $1 - \frac{1}{120} = \mathbf{\frac{119}{120}}$.

9. Ans. (b)

Solution A : Getting 4, 5 or 6 on the first toss B : Getting 1, 2, 3 or 4 on the second toss

$$P(A) = \frac{3}{6}, P(B) = \frac{4}{6}$$

$\therefore$ Required probability $= P(A) \times P(B) = \frac{3}{6} \times \frac{4}{6} = \frac{12}{36} = \mathbf{\frac{1}{3}}$.

11. Ans. (b)

Solution Total number of ways in which n men can sit round a circular table = $(n-1)!$

Now, if two particular men (say A and B) sit together, then the number of ways in which $(n-1)$ men sit round the table = $(n-2)!$. But the two men can interchange their seats in two ways.

$\therefore$ Number of ways that two particular men sit together = $2(n-2)!$

$\therefore$ Probability that two particular men sit together

$$= \frac{2(n-2)!}{(n-1)!} = \mathbf{\frac{2}{n-1}}.$$

13. Ans. (d)

Solution A : Getting an ace B : Getting a king

$$P(A) = \frac{4}{52}, P(B) = \frac{4}{52}, P(A \cap B) = \frac{0}{52} = 0$$

$\therefore$ Required probability

$= 1 - P(A \cup B) = 1 - [P(A) + P(B) - (P(A \cap B)]$

$$= 1 - \left(\frac{4}{52} + \frac{4}{52} - 0\right) = 1 - \frac{8}{52} = 1 - \frac{2}{13} = \mathbf{\frac{11}{13}}.$$

15. Ans. (a)

Solution A : He passes in α; B : He passes in β; C : He passes in γ; D : He passes in δ

$$P(A) = \frac{4}{5} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$$

$$P(B) = \frac{3}{4} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(C) = \frac{5}{6} \Rightarrow P(\overline{C}) = 1 - P(C) = 1 - \frac{5}{6} = \frac{1}{6}$$

$$P(D) = \frac{2}{3} \Rightarrow P(\overline{D}) = 1 - P(D) = 1 - \frac{2}{3} = \frac{1}{3}$$

$\therefore$ Required probability

$= P(A)\,P(B)\,P(C)\,P(\overline{D}) + P(A)\,P(C)\,P(D)\,P(\overline{B}) + P(A)\,P(B)\,P(D)\,P(\overline{C}) + P(A)\,P(B)\,P(C)\,P(D)$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{1}{3} + \frac{4}{5} \times \frac{5}{6} \times \frac{2}{3} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{6} + \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{2}{3}$$

$$= \frac{60 + 40 + 24 + 120}{360} = \frac{244}{360} = \mathbf{\frac{61}{90}}.$$

17. Ans. (c)

Solution A : A speaks truth; B : B speaks truth

$$P(A) = \frac{60}{100} = \frac{6}{10} \Rightarrow P(\overline{A}) = 1 - P(A) = 1 - \frac{6}{10} = \frac{4}{10}$$

$$P(B) = \frac{90}{100} = \frac{9}{10} \Rightarrow P(\overline{B}) = 1 - P(B) = 1 - \frac{9}{10} = \frac{1}{10}$$

$\therefore$ Required probability

$$= P(A)P(\overline{B}) + P(B)P(\overline{A})$$

$$= \frac{6}{10}\left(\frac{1}{10}\right) + \frac{9}{10}\left(\frac{4}{10}\right) = \frac{6+36}{100} = \frac{42}{100}$$

Hence, they are expected to contradict each other in $\frac{42}{100} \times 100 =$ **42% cases.**

19. Ans. (c)

Solution There are two mutually exclusive ways in which A and B can contradict each other on an identical point.

(i) A speaks the truth and B tells a lie.

(ii) A tells a lie and B speaks the truth.

We define the events :

E_1 : A speaks the truth; E_2 : B speaks the truth.

$\therefore$ The required probability

$$= P(E_1) \times P(\overline{E}_2) + P(\overline{E}_1) \times P(E_2)$$

$$= \frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{5}{8} = \frac{9}{40} + \frac{10}{40} = \frac{19}{40} = 0.475$$

Hence, A and B contradict each other on an identical point in 47.5% of the cases.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (a)	**6.** (a)	**8.** (a)	**10.** (c)	**12.** (c)	**14.** (b)	**16.** (b)	**18.** (d)	**20.** (b)

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PART III

Mensuration

CHAPTER 33 Mensuration I: Area and Perimeter

CHAPTER 34 Mensuration II: Volume and Surface Area

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Mensuration I: Area and Perimeter 33

INTRODUCTION

In this chapter, we shall be dealing with plane figures of various shapes finding their sides, perimeters and areas.

Area

The area of any figure is the amount of surface enclosed within its bounding lines. Area is always expressed in square units.

Units of Measuring Area

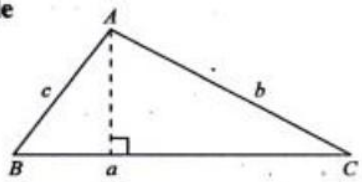
100 sq millimetres = 1 sq centimetre
100 sq centimetres = 1 sq decimetre
100 sq decimetres = 1 sq metre
100 sq metres = 1 sq decametre or arc
10,000 sq metres = 1 hectare
1,000,000 sq metres = 100 hectares
= 1 sq kilometre

Perimeter

The perimeter of a geometrical figure is the total length of the sides enclosing the figure.

SOME BASIC FORMULAE

1. Triangle



A *triangle* is a closed figure bounded by three sides. *ABC* is a triangle.

The sides *AB*, *BC* and *AC* are respectively denoted by *c*, *a* and *b*.

Area of a Triangle (*A*)

(a) $A = \frac{1}{2}$ (base × height) $= \frac{1}{2}ah$

(b) $A = \sqrt{s(s-a)(s-b)(s-c)}$,

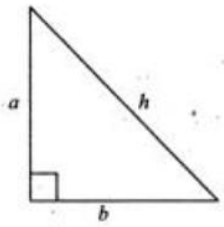
where $s = \frac{1}{2}(a + b + c)$ or semi-perimeter of the triangle

This formula is known as *Hero's formula*.

Perimeter $(P) = a + b + c = 2s$.

2. Right Angled Triangle

A triangle having one of its angles equal to 90° is called a *right-angled triangle*. The side opposite to the right angle is called the *hypotenuse*.



In a right angled triangle,

(Hypotenuse)^2 = sum of squares of sides

i.e. $h^2 = a^2 + b^2$.

Area $(A) = \frac{1}{2}$(product of the sides containing the right angle)

i.e. $A = \frac{1}{2}ab$.

Illustration 1 What is area of a triangle having sides 3 m, 4 m and 5 m?

Solution Let $a = 3$ m, $b = 4$ m, $c = 5$ m.

Then, $s = \frac{a+b+c}{2} = \frac{3+4+5}{2} = 6$ m.

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$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)}$

$= \sqrt{6\times3\times2\times1} = \sqrt{36} =$ **6 sq m**

Illustration 2 Find the area of a triangle whose base is 4.6 m and height is 67 cm.

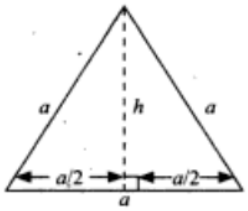
Solution Area of triangle $= \frac{1}{2}$(base × height)

$= \frac{1}{2}(4.6 \times 100 \times 67)$

= 15410 sq cm.

3. Equilateral Triangle

A triangle whose all sides are equal is called an equilateral triangle.



Area (A) of an equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}a^2$

Permeter (P) of an equilateral triangle

$= 3 \times (\text{side}) = 3a$

Altitude (h) of an equilateral triangle

$= \frac{\sqrt{3}}{2} \times (\text{side}) = \frac{\sqrt{3}}{2}a.$

In an equilateral triangle

$\angle A = \angle B = \angle c = 60°.$

Area (A) of an equilateral triangle

$= \frac{(\text{altitude})^2}{\sqrt{3}} = \frac{h^2}{\sqrt{3}}.$

Illustration 3 Find the area of an equilateral triangle each of whose sides measures 6 cm.

Solution Area of an equilateral triangle

$= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times 36 =$ **$9\sqrt{3}$ sq cm.**

Illustration 4 Length of the side of an equilateral triangle is $\frac{4}{\sqrt{3}}$cm. Find its height.

Solution Height of the equilateral triangle

$= \frac{\sqrt{3}}{2} \times (\text{side})$

$= \frac{\sqrt{3}}{2} \times \frac{4}{\sqrt{3}} =$ **2 cm.**

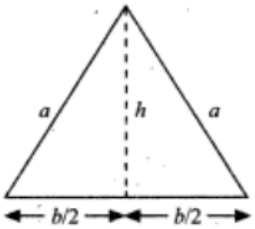
Illustration 5 Height of an equilateral triangle is $4\sqrt{3}$ cm. Find its area.

Solution Area of equilateral triangle

$= \frac{(\text{altitude})^2}{\sqrt{3}} = \frac{4\sqrt{3}\times4\sqrt{3}}{\sqrt{3}} =$ **$16\sqrt{3}$ sq cm.**

4. Isosceles Triangle

A triangle whose two sides are equal is an *isosceles triangle*.



Area (A) of an isosceles triangle

$= \frac{b}{4}\sqrt{4a^2-b^2}$

Perimeter (P) of an isosceles triangle

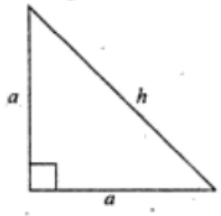
$= (2a + b)$

Height (h) of an isosceles triangle

$= \frac{1}{2}\sqrt{4a^2-b^2}.$

5. Isosceles Right-angled Triangle

An *isosceles right-angled triangle* has two sides equal with equal sides making 90° to each other.



Hypotenuse (h) $= \sqrt{2}a$

Area (A) $= \frac{1}{2}a^2$

Perimeter (P) $= 2a + \sqrt{2}a = \sqrt{2}a(\sqrt{2}+1)$

$= h(\sqrt{2}+1).$

If the perimeter of an isosceles triangle is P and the base is b, then the length of the equal sides is $\left(\frac{P-b}{2}\right)$.

If the perimeter of an isosceles triangle is P and the length of equal sides is a, then base is $(P - 2a)$.

Illustration 6 An isosceles right-angled triangle has two equal sides of length 6 m each. Find its area.

Solution Area $= \frac{1}{2}$ (equal side)$^2 = \frac{1}{2}(6)^2 =$ **18 sq m.**

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Illustration 7 The perimeter of an isosceles triangle is 80 cm. If the length of the equal sides is given by 15 cm, find the length of the base.

Solution Length of the base $= P - 2a$
$= 80 - 2(15) =$ **50 cm.**

Illustration 8 The perimeter of an isosceles triangle is 42 cm. If the base is 16 cm, find the length of equal sides.

Solution The length of equal sides $= \frac{P-b}{2}$
$= \frac{42-16}{2} = \frac{26}{2} =$ **13 cm.**

Illustration 9 If the base of an isosceles triangle is 10 cm and the length of equal sides is 13 cm, find its area.

Solution Area of the isosceles triangle
$= \frac{b}{4}\sqrt{4a^2-b^2}$
$= \frac{10}{4}\sqrt{4\times(13)^2-(10)^2}$
$= \frac{10}{4}\sqrt{676-100} = \frac{10}{4}\times 24 =$ **60 sq cm.**

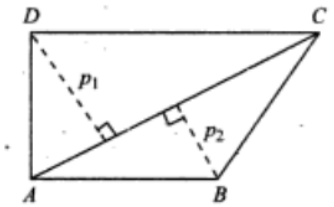
6. Quadrilateral

A closed figure bounded by four sides is called a *quadrilateral*.

It has four angles included in it.

The sum of these four angles is 360°.

i.e. $\angle A + \angle B + \angle C + \angle D = 360°$.



Area (A) of a quadrilateral
$= \frac{1}{2} \times$ one diagonal $\times$ (sum of perpendiculars to it from opposite vertices)
$= \frac{1}{2}d(p_1+p_2)$

Note: If the lengths of four sides and one of its diagonals are known, then
$A =$ Area of $\Delta ADC +$ Area of ΔABC.

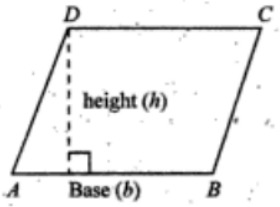
The special cases of quadrilateral are parallelogram, rectangle, square, rhombus, trapezium, etc., which are discussed below separately.

7. Parallelogram

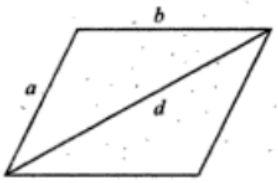
A quadrilateral in which opposite sides are equal and parallel is called a parallelogram.

The diagonals of a parallelogram bisect each other.

Area (A) of a parallelogram
= base $\times$ altitude corresponding to the base
$= b \times h$



Area (A) of a parallelogram
$= 2\sqrt{s(s-a)(s-b)(s-d)}$
where a and b are adjacent sides, d is the length of the diagonal connecting the ends of the two sides and $s = \frac{a+b+d}{2}$.



In a parallelogram, the sum of the squares of the diagonals = 2 (the sum of the squares of the two adjacent sides),

i.e.
$d_1^2 + d_2^2 = 2(a^2 + b^2)$.

Perimeter (P) of a parallelogram
$= 2(a + b)$,
where a and b are adjacent sides of the parallelogram.

Illustration 10 One side of a parallelogram is 15 cm and the corresponding altitude is 5 cm. Find the area of the parallelogram.

Solution Area of parallelogram
= base $\times$ corresponding altitude
$= 15 \times 5 =$ **75 sq cm.**

Illustration 11 In a parallelogram, the lengths of adjacent sides are 11 cm and 13 cm, respectively. If the length of one diagonal is 20 cm, find the length of the other diagonal.

Solution We have,
$d_1^2 + d_2^2 = 2(a^2 + b^2)$
$\Rightarrow (20)^2 + d_2^2 = 2(11^2 + 13^2)$
$\Rightarrow d_2^2 = 2(121 + 169) - 400 = 180.$
$\therefore d_2 = \sqrt{180} =$ **13.4 m (approx.)**

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Illustration 12 Find the area of a quadrilateral of whose diagonal is 38 cm long and the lengths of perpendiculars from the other two vertices are 31 cm and 19 cm, respectively.

Solution Area of the quadrilateral

$$= \frac{1}{2} d(p_1 + p_2)$$
$$= \frac{1}{2} \times 38 \times (31 + 19)$$
$$= 19 \times 50 = \mathbf{950 \text{ sq cm.}}$$

Illustration 13 Find the area of a parallelogram whose two adjacent sides are 130 m and 140 m and one of the diagonals is 150 m long.

Solution Here, $a = 130$, $b = 140$ and $d = 150$.

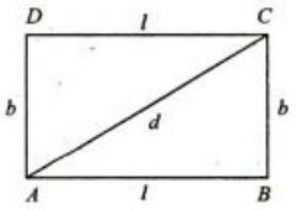
$$\therefore \quad s = \frac{a+b+d}{2} = \frac{130+140+150}{2} = \frac{420}{2} = 210.$$

$\therefore$ Area of the parallelogram

$$= 2\sqrt{s(s-a)(s-b)(s-a)}$$
$$= 2\sqrt{210(210-130)(210-140)(210-150)}$$
$$= 2\sqrt{210\times80\times70\times60} = \mathbf{2 \times 8400 \text{ sq m.}}$$

8. Rectangle

A *rectangle* is a quadrilateral with opposite sides equal and all the four angles equal to 90°.



The diagonals of a rectangle bisect each other and are equal.

(a) Area (A) of rectangle = length × breadth
$= l \times b$

OR

Area of rectangle = $(l\times\sqrt{d^2-l^2})$,
if one side (l) and diagonal (d) are given.

OR

Area of rectangle = $\left(\frac{p^2}{8}-\frac{d^2}{2}\right)$,
if perimeter (P) and diagonal (d) are given.

(b) Perimeter (P) of rectangle
= 2(length + breadth)
= $2(l + b)$.

OR

Perimeter of rectangle = $2(l+\sqrt{d^2-l^2})$,
if one side (l) and diagonal (d) are given.

(c) Diagonal of a rectangle
$= \sqrt{(\text{length})^2 + (\text{breadth})^2}$
$= \sqrt{l^2+b^2}$

(d) If area (A) and perimeter (P) of a rectangle are given, then

length of the rectangle = $\left(\sqrt{\frac{P^2}{16}-A}+\frac{P}{4}\right)$

and,

breadth of the rectangle = $\left(\frac{P}{4}-\sqrt{\frac{P^2}{16}-A}\right)$

Illustration 14 Find the diagonal of a rectangle whose sides are 8 cm and 6 cm.

Solution Diagonal of the rectangle

$$= \sqrt{l^2+b^2}$$
$$= \sqrt{8^2+6^2} = \sqrt{64+36} = \mathbf{10 \text{ cm.}}$$

Illustration 15 Find the perimeter of a rectangle of length 12 m and breadth 6 m.

Solution Perimeter of the rectangle

$$= 2(l + b)$$
$$= 2(12 + 6) = \mathbf{36 \text{ m.}}$$

Illustration 16 Calculate the area of a rectangular field whose length is 12.5 cm and breadth is 8 cm.

Solution Area of the rectangular field

$$= l \times b$$
$$= 12.5 \times 8 = \mathbf{100 \text{ sq cm.}}$$

Illustration 17 Calculate the area of a rectangular field whose one side is 16 cm and the diagonal is 20 cm.

Solution Area of the rectangular field

$$= (l\times\sqrt{d^2-l^2})$$
$$= (16\times\sqrt{20^2-16^2})$$
$$= 16 \times 12 = \mathbf{192 \text{ sq cm.}}$$

Illustration 18 A rectangular carpet has an area of 120 sq m and perimeter of 46 m. Find the length of its diagonal.

Solution We have,

Area of rectangle = $\left(\frac{P^2}{8}-\frac{d^2}{2}\right)$

$$\Rightarrow 120 = \frac{46^2}{8}-\frac{d^2}{2}$$
$$\Rightarrow 46^2 - 4d^2 = 120 \times 8$$
$$\Rightarrow 4d^2 = 2116 - 960 = 1156$$
$$\therefore \quad d = \sqrt{289} = \mathbf{17 \text{ m.}}$$

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where, $l = b - a$ if $b > a$
$= a - b$ if $a > b$

and, $s = \frac{c+d+l}{2}$

(b) Height (h) of the trapezium

$$= \frac{2}{l}\sqrt{s(s-l)(s-c)(s-d)}$$
$$= \left(\frac{2A}{a+b}\right)$$

Illustration 29 Find the area of a trapezium having parallel sides 65 m and 44 m and their separation being 20 m.

Solution Area of the trapezium

$$= \frac{1}{2} \times (a + b) \times h$$
$$= \frac{1}{2} \times (65 + 44) \times 20$$
$$= \mathbf{1100\ sq\ cm.}$$

Illustration 30 The parallel sides of a trapezium are 24 m and 52 m. If its other two sides are 26 m and 30 m, what is the area of the trapezium?

Solution Area of trapezium

$$= \frac{a+b}{l}\sqrt{s(s-l)(s-c)(s-d)}.$$

Here, $a = 24$, $b = 52$, $c = 26$, $d = 30$, $l = b - a = 28$,

$$s = \frac{c+d+l}{2} = \frac{26+30+28}{2} = 42.$$

$\therefore$ Area of trapezium

$$= \frac{24+52}{28}\sqrt{42(42-28)(42-26)(42-30)}$$
$$= \frac{76}{28} \times \sqrt{42 \times 14 \times 16 \times 12}$$
$$= \frac{76 \times 336}{28} = \mathbf{912\ sq\ m.}$$

Illustration 31 The two parallel sides of a trapezium of area 180 sq cm measure 28 cm and 12 cm. What is the height of the trapezium?

Solution Height of the trapezium

$$= \left(\frac{2A}{a+b}\right) = \left(\frac{2 \times 180}{28+12}\right) = \frac{360}{40} = \mathbf{9\ cm.}$$

12. Walls of a Room

Area of four walls of a room
= 2(length + breadth) × height

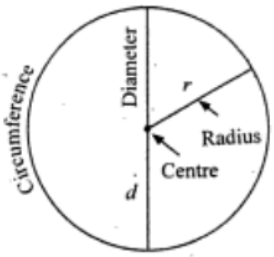
Illustration 32 Find the cost of painting the walls of a room of 6 m long, 5 m broad and 4 m high at Rs. 7.50 per sq m.

Solution Area of four walls of room
= 2(length + breadth) × height
= 2(6 + 5) × 4 = 88 sq m.

$\therefore$ Cost of painting = 88 × 7.50 = **Rs. 660.**

12. Circle

A *circle* is the path travelled by a point which moves in such a way that its distance from a fixed point remains constant.



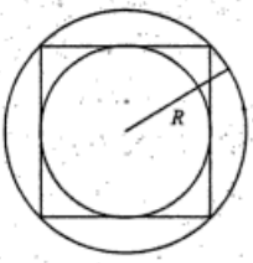
The fixed point is known as *centre* and the fixed distance is called the *radius*.

(a) Circumference or perimeter of circle
$= 2\pi r = \pi d$,
where r is radius and d is diameter of circle

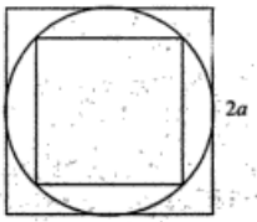
(b) Area of circle
$= \pi r^2$, r is radius
$= \frac{\pi d^2}{4}$, d is diameter
$= \frac{c^2}{4\pi}$, c is circumference
$= \frac{1}{2}$ × circumference × radius

(c) Radius of circle $= \sqrt{\frac{\text{Area}}{\pi}}$
$= \frac{\text{Perimeter or circumference}}{2\pi}$

(d) Ratio of the areas of the two circles is
$= \frac{\text{Area of circle circumscribing the square}}{\text{Area of circle inscribed in the square}} = \frac{2}{1}$.



(e) Ratio of the area of the two squares is
$= \frac{\text{Area of square circumscribing the circle}}{\text{Area of square inscribed in the circle}} = \frac{2}{1}$.



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For a cyclic quadrilateral

- Area $= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$, where $s = \dfrac{a+b+c+d}{2}$

- $\angle A + \angle B + \angle C + \angle D = 2\pi$
- $\angle A + \angle C = \angle B + \angle D = \pi$.

SOME USEFUL SHORT-CUT METHODS

1. If the length and the breadth of a rectangle are increased by $x\%$ and $y\%$ respectively, then the area of rectangle will increase by $\left(x+y+\dfrac{xy}{100}\right)\%$.

Explanation

Area of the original rectangle is

$A = l \times b$.

Area of the new rectangle is

$$A' = l\left(\frac{100+x}{100}\right)\times b\left(\frac{100+y}{100}\right)$$

$$= lb\left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right)$$

$$= A\left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right)$$

$$\therefore \quad \frac{A'}{A} - 1 = \left(\frac{100+x}{100}\right)\left(\frac{100+y}{100}\right) - 1$$

or, $$\frac{A'-A}{A} = \frac{(100+x)(100+y)-(100)^2}{(100)^2}$$

or, $$\left(\frac{A'-A}{A}\right)\times 100$$

$$= \frac{(100)^2 + 100(x+y) + xy - (100)^2}{(100)}$$

$$\therefore \quad \%\text{ increase in area} = \frac{100(x+y)+xy}{100}\%$$

or, $$\left(x+y+\frac{xy}{100}\right)\%.$$

Note: If any of x or y decreases, we put negative sign.

Illustration 1 The length and breadth of a rectangle are increased by 20% and 5%, respectively. Find the percentage increase in its area.

Solution % increase in the area of rectangle

$$= \left(x+y+\frac{xy}{100}\right)\%$$

$$= \left(20+5+\frac{20\times 5}{100}\right)\% = \mathbf{20\%}.$$

2. If the length of a rectangle is increased by $x\%$, then its breadth will have to be decreased by $\left(\dfrac{100x}{100+x}\right)\%$ in order to maintain the same area of rectangle.

Explanation

% increase in area of rectangle

$$= \left(x+y+\frac{xy}{100}\right)\%$$

or, $$0 = \left(x+y+\frac{xy}{100}\right)$$

or, $$-x = y\left(1+\frac{x}{100}\right) \quad \text{or} \quad y = -\left(\frac{100x}{100+x}\right)$$

–ve sign indicates decrease.

Therefore, breadth must be decreased by $\left(\dfrac{100x}{100+x}\right)\%$ in order to maintain the same area.

Illustration 2 The length of a rectangle is increased by 25%. By what per cent should its breadth be decreased so as to maintain the same area?

Solution The breadth must be decreased by

$$= \left(\frac{100x}{100+x}\right)\% = \left(\frac{100\times 25}{100+25}\right)\%, \textbf{ i.e. } \mathbf{20\%}.$$

3. If each of the defining dimensions or sides of any two-dimensional figure (triangle, rectangle, square, circle, quadrilateral, pentagon, hexagon, etc.) is changed by $x\%$, its area changes by $x\left(2+\dfrac{x}{100}\right)\%$.

Illustration 3 If the radius of a circle is decreased by 10%, what is the percentage decrease in its area?

Solution Here, $x = -10$ (–ve sign indicates decrease)

$$\therefore \quad \%\text{ change in area} = x\left(2+\frac{x}{100}\right)\%$$

$$= -10\left(2-\frac{10}{100}\right)\%$$

$$= (-10)\left(\frac{19}{10}\right)\% = -19\%.$$

$\therefore$ Area of the circle decreases by **19%.**

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4. If all the sides of a quadrilateral are increased (or decreased) by $x\%$, its diagonals also increase (or decrease) by $x\%$.

Illustration 4 The length and the two diagonals of a rectangle are decreased by 5% each. What is the percentage decrease in its breadth?

Solution Since the length and the two diagonals decrease by 5% each, the breadth also must decrease by **5%.**

5. If each of the defining dimensions or sides of any two-dimensional figures are increased (or decreased) by $x\%$, its perimeter also increases (or decreases) by $x\%$.

Illustration 5 If all the sides and diagonals of a square are increased by 8% each, then find the percentage increase in its perimeter.

Solution The perimeter also increases by **8%.**

6. If the ratio of the areas of two squares be $a : b$, then the ratio of their sides, ratio of their perimeters and the ratio of their diagonals, each will be in the ratio $\sqrt{a} : \sqrt{b}$.

Illustration 6 Ratio of the areas of two squares is 16 : 9. Find the ratio of their diagonals.

Solution The ratio of their diagonals

$$= \sqrt{a} : \sqrt{b}$$

$$= \sqrt{16} : \sqrt{9}, \text{ i.e. } 4 : 3.$$

7. If the diagonal of a square increases by x times, then the area of the square becomes x^2 times.

Illustration 7 The diagonal of a square is doubled. How many times will the area of the new square become?

Solution The area of the new square will become x^2 times, i.e. $(2)^2 = 4$ **times.**

8. Standard Properties of Diagonals of Quadrilaterals

Quadrilateral	*Meet at right angles*	*Bisect each other*	*Equal to each other*	*Bisect angle at vertex*
Square	√	√	√	√
Rectangle	×	√	√	×
Parallelogram	×	√	×	×
Rhombus	√	√	×	√
Trapezium	×	×	may or may not be	×

9. Carpeting the Floor of a Room

If the length and breadth of a room are l and b, respectively, and a carpet of width w is used to cover the floor, then the required length of the carpet

$$= \frac{l \times b}{w}.$$

Illustration 8 How many metres of a carpet 12 cm wide will be required to cover the floor of a room which is 600 cm long and 420 cm broad? Also, calculate the amount required in carpeting the floor if the cost of carpet is Rs. 15 per metre.

Solution Length of the carpet

$$= \frac{l \times b}{w}$$

$$= \frac{600 \times 420}{12}$$

$$= 21000 \text{ cm, i.e. } 210 \text{ m.}$$

The amount required for carpeting the floor

$$= 15 \times 210 = \textbf{Rs. 3150.}$$

10. Number of Square Tiles Required for Flooring

If the length and breadth of a room are l and b respectively, then the least number of square tiles required to cover the floor

$$= \frac{l \times b}{\text{H.C.F. } (l, b)}$$

Also, the size of the largest tile so that the tiles exactly fit

$$= \text{H.C.F. } (l, b).$$

Illustration 9 A hall of length 24 cm and breadth 20 m is to be paved with equal square tiles. What will be the size of the largest tile so that the tiles exactly fit and also find the number of tiles required.

Solution Size of the largest possible square tile

$$= \text{H.C.F. } (l, b)$$

$$= \text{H.C.F. } (24, 20) = 4 \text{ m.}$$

Number of tiles required

$$= \frac{l \times b}{\text{H.C.F. } (l, b)}$$

$$= \frac{24 \times 20}{4} = \textbf{120 tiles.}$$

11. Path around a Rectangular Space

(a) A rectangular garden l m long and b m broad is sorrounded by a path w m wide. The area of the path is given by

$$= 2w(l + b + 2w) \text{ sq m.}$$

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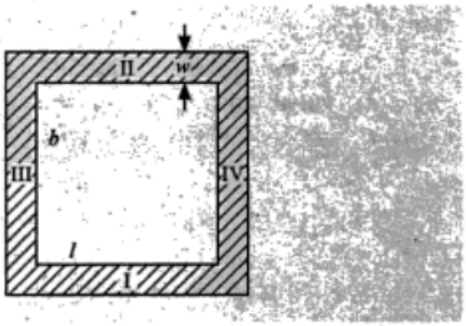


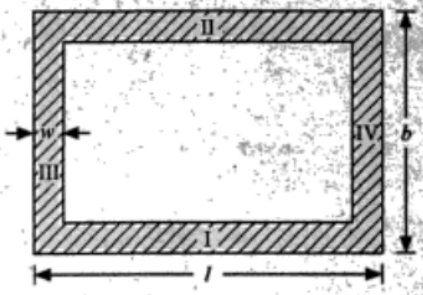
Explanation

Area of part I = Area of part II
$= (l + 2w)w$ sq m

Area of part III = Area of part IV
$= bw$ sq m.

$\therefore$ Total area of the path $= 2[(l + 2w)w + bw]$
$= 2w(l + b + 2w)$ sq m.

(b) A rectangular garden l m long and b m broad is sorrounded by a path w m wide constructed inside it along its boundary. The area of the path is given by
$= 2w(l + b - 2w)$ sq m.



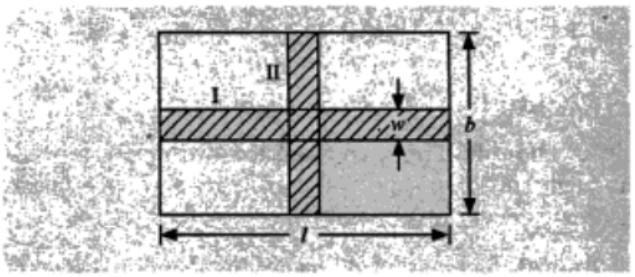
Explanation

Area of part I = Area of part II
$= lw$ sq m

Area of part III = Area of part IV
$= (b - 2w)w$ sq m

$\therefore$ Total area of the part
$= 2[lw + (b - 2w)w]$
$= 2w(l + b - 2w)$ sq m.

(c) A rectangular park is l m long and b m broad. Two paths w m wide each are perpendicular to each other inside the park. The area of the paths
$= w(l + b - w)$ sq m.
Also, area of the park minus the paths
$= (l - w)(b - w)$ sq m.



Explanation

Total area of the path
= Area of part I + Area of path II – Area of common central part
$= lw + bw - w^2$
$= w(l + b - w)$ sq m.

$\therefore$ Area of the park minus the parts
$= [lb - w(l + b - w)]$
$= lb - lw - w(b - w)$
$= l(b - w) - w(b - w)$
$= (l - w)(b - w)$ sq m.

Notes:

1. Clearly, from the figure, the area of the paths does not change on shifting their location as long as they are perpendicular to each other.
2. For a square park, take $l = b$ in all the results derived above.

Illustration 10 A rectangular park 18 m × 12 m, is sorrounded by a path 4 m wide. Find the area of the path.

Solution The area of the path
$= 2w(l + b + 2w)$
$= 2 \times 4(18 + 12 + 2 \times 4) =$ **304 sq m.**

Illustration 11 A park is square in shape with side 18 m. Find the area of the pavement 3 m wide to be laid all around it on its inside.

Solution Area of the pavement
$= 2w(l + b - 2w)$
$= 2 \times 3(18 + 18 - 2 \times 3)$ (Here, $l = b = 18$)
= 180 sq m.

Illustration 12 A playground measures 27 m × 13 m. From the centre of each side a path 2 m wide goes across to the centre of the opposite side. Calculate the area of the path and the cost of constructing it at Rs. 4 per sq m.

Solution Area of the path
$= w(l + b - w) = 2(27 + 13 - 2)$
$= 76$ sq m.

$\therefore$ Cost $= 4 \times 76 =$ **Rs. 304.**

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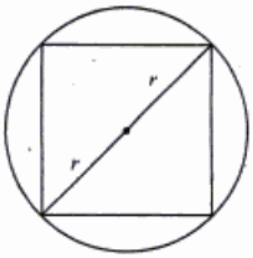


Also, side of the square = $\sqrt{\text{Area}} = \sqrt{2r^2} = \sqrt{2}\,r$.

Illustration 19 Find the side of the square inscribed in a circle whose circumference is 308 cm.

Solution Circumference of the circle $(2\pi r) = 308$

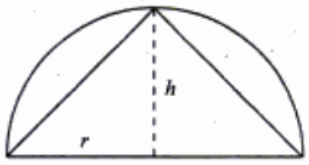
$$\Rightarrow r = \frac{308}{2\pi} = \frac{308\times 7}{2\times 22} = 49 \text{ cm.}$$

$\therefore$ Side of the inscribed square = $\sqrt{2}\,r = \mathbf{49\sqrt{2}\ cm.}$

17. The area of largest triangle inscribed in a semi-circle of radius r is r^2.

Explanation

Clearly, from the figure, the largest triangle inscribed in a semi-circle is an isosceles triangle with diameter as its base and radius as its height.



Area of the triangle = $\frac{1}{2}$ × base × height

$$= \frac{1}{2} \times 2r \times r = r^2.$$

18. The number of revolutions made by a circular wheel of radius r in travelling distance d is given by

$$= \left(\frac{d}{2\pi r}\right).$$

Explanation

Circumference of the wheel = $2\pi r$

In travelling a distance $2\pi r$, the wheel makes 1 revolution.

$\therefore$ In travelling a distance d, the wheel makes $\left(\frac{d}{2\pi r}\right)$ revolutions.

Illustration 20 The diameter of a wheel is 2 cm. If it rolls forward covering 10 revolutions, find the distance travelled by it.

Solution Radius of the wheel = 1 cm.

The distance travelled by the wheel in 10 revolutions

$$= 10 \times 2\pi r$$

$$= 10 \times 2 \times \frac{22}{7} \times 1 = \mathbf{62.8\ cm.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The area of a triangle whose sides are 15 m, 16 m and 17 m is
(a) $24\sqrt{4}$ sq m (b) $24\sqrt{3}$ sq m
(c) $24\sqrt{21}$ sq m (d) None of these

2. The area of right-angled triangle with base 6 m and hypotenuse 6.5 m is
(a) 7.5 sq m (b) 9.5 sq m
(c) 8.5 sq m (d) None of these

3. The length of each side of a triangle is 12 cm. The height of the triangle is
(a) $3\sqrt{2}$ m (b) $6\sqrt{3}$ m
(c) $6\sqrt{2}$ (d) None of these

4. The area of a triangular lawn is 1600 sq m. If one side is 64 m long and the other two sides are equal in length, the length of each equal side is
(a) 60.37 m (b) 59.36 m
(c) 60.36 cm (d) None of these

5. Three sides of a triangular field are 20 m, 21 m and 29 m long, respectively. The area of the field is
(a) 215 sq m (b) 230 sq m
(c) 210 sq m (d) None of these

6. The hypotenuse and the semi-perimeter of right triangle are 20 cm and 24 cm, respectively. The other two sides of the triangle are
(a) 16 cm, 12 cm (b) 12 cm, 16 cm
(c) 20 cm, 16 cmn (d) None of these

7. The sides of a triangle are in the ratio 3 : 4 : 5. If its perimeter is 36 cm, area of the triangle is
(a) 57 sq m (b) 54 sq m
(c) 56.5 sq m (d) None of these

8. For a triangle whose sides are 50 m, 78 m and 112 m, respectively, the length of perpendicular from the opposite angle on the side 112 m is
(a) 45 m (b) 35 m
(c) 30 m (d) None of these

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9. A ladder is resting with one end in contact with the top of a wall of height 12 m and the other end on the ground is at a distance 5 m from the wall. The length of the ladder is
(a) 13 m (b) 17 m
(c) 16 m (d) None of these

10. A ladder is placed so as to reach a window 63 cm high. The ladder is then turned over to the opposite side of the street and is found to reach a point 56 cm high. If the ladder is 65 cm long, the width of the street is
(a) 59 cm (b) 39 cm
(c) 49 cm (d) None of these

11. If the area of a triangle with base x is equal to the area of a square with side x, then the altitude of the triangle is
(a) $\frac{x}{2}$ (b) x
(c) $2x$ (d) $3x$

12. If the area of a triangle is 150 sq m and base : height is 3 : 4, find its height.
(a) 25 m (b) 35 m
(c) 20 m (d) None of these

13. The base of a triangular field is three times its height. If the cost of cultivating the field at Rs. 36.72 per hectare is Rs. 495.72, find its base and height.
(a) 950 m, 350 m (b) 800 m, 500 m
(c) 900 m, 300 m (d) None of these

14. If the sides of a triangle are doubled, its area
(a) remains same (b) is doubled
(c) becomes 4 times (d) Can't say.

15. Two sides of a triangular field are 85 m and 154 m, respectively, and its perimeter is 324 m. The cost of ploughing the field at the rate of Rs. 10 per sq m is
(a) Rs. 27720 (b) Rs. 37620
(c) Rs. 26750 (d) None of these

16. The area of an equilateral triangle each of whose sides measures $2\sqrt{3}$ cm is
(a) $5\sqrt{3}$ sq m (b) $4\sqrt{3}$ sq m
(c) $3\sqrt{3}$ sq cm (d) None of these

17. If the height of an equilateral triangle is $2\sqrt{3}$ cm, then the length of its side is
(a) 4 cm (b) 6 cm
(c) 5 cm (d) None of these

18. If the perimeter of an equilateral triangle is 12 m, find its area.
(a) $3\sqrt{4}$ m (b) $4\sqrt{3}$ m
(c) $5\sqrt{3}$ m (d) None of these

19. The height of an equilateral triangle whose perimeter is 24 cm, is
(a) $4\sqrt{3}$ cm (b) $3\sqrt{4}$ cm
(c) $5\sqrt{3}$ cm (d) None of these

20. The perimeter of a right angled triangle is 90 cm and its hypotenuse is 39 cm. Find its other sides
(a) 30 cm, 10 cm (b) 36 cm, 15 cm
(c) 48 cm, 20 cm (d) None of these

21. The perimeter of an isosceles triangle is 306 m and each of the equal sides is $\frac{5}{8}$ of the base. Find the area.
(a) 3648 cm (b) 3468 sq m
(c) 3846 cm (d) None of these

22. Find the area of an isosceles right-angled triangle whose hypotenuse is 8 cm.
(a) 32 sq m (b) 24 sq cm
(c) 16 sq cm (d) None of these

23. The perimeter of an isosceles triangle is equal to 14 cm; the lateral side is to the base in the ratio 5 to 4. The area of the triangle is
(a) $3\sqrt{21}$ sq cm (b) $2\sqrt{21}$ sq cm
(c) $4\sqrt{21}$ sq cm (d) None of these

24. If all the sides of a triangle are increased by 200%, the area of the triangle will increase by
(a) 400% (b) 600%
(c) 800% (d) None of these

25. A plot of land is in the shape of a right-angled isosceles triangle. The length of hypotenuse is $50\sqrt{2}$ m. The cost of fencing is Rs. 3 per metre. The cost of fencing the plot will be
(a) less than Rs. 300 (b) less than Rs. 500
(c) more than Rs. 500 (d) None of these

26. If the area of an equilateral triangle is equal to the area of an isosceles triangle whose base and equal sides area 16 cm and 10 cm respectively, then the side of the equilateral triangle is
(a) 10.5 cm (b) 9.5 cm
(c) 12.5 cm (d) None of these

27. If the perimeter of a right-angled isosceles triangle is $4\sqrt{2}+4$ m, then the hypotenuse is
(a) 8 m (b) 6 m
(c) 4 m (d) None of these

28. The two adjacent sides of a parallelogram are 60 m and 40 m and one of the diagonals is 80 m long. The area of the parallelogram is
(a) $600\sqrt{15}$ sq m (b) $800\sqrt{25}$ sq m
(c) $700\sqrt{15}$ sq m (d) None of these

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29. One side of the parallelogram is 14 cm. Its distance from the opposite side is 16 cm. The area of the parallelogram is
(a) 234 sq cm (b) 324 sq cm
(c) 224 sq cm (d) None of these

30. A field is in the shape of a parallelogram. Its adjacent sides and one diagonal are 65 m, 119 m and 156 m, respectively. Find the cost of gravelling it at the rate of Rs. 10 per sq m.
(a) Rs. 81400 (b) Rs. 71400
(c) Rs. 91400 (d) None of these

31. One side of a parallelogram is 10 m and the corresponding altitude is 7 m. The area of the parallelogram is
(a) 70 sq m (b) 60 sq m
(c) 80 sq m (d) None of these

32. The adjacent sides of a parallelogram are 8 m and 5 m. The distance between the longer sides is 4 m. The distance between the shorter sides is
(a) 4.6 m (b) 6.4 m
(c) 8.6 m (d) None of these

33. The area of a quadrilateral is 420 sq m and the perpendiculars drawn to one diagonal from the opposite vertices are 18 m and 12 m. Then, the length of the diagonal is
(a) 32 m (b) 24 m
(c) 28 m (d) None of these

34. The area of a parallelogram is 72 sq cm and its altitude is twice the corresponding base. The length of the base is
(a) 6 cm (b) 8 cm
(c) 4 cm (d) None of these

35. The area of a parallelogram is 240 sq cm and its height is 12 cm. The base of the parallelogram is
(a) 24 cm (b) 20 cm
(c) 28 cm (d) None of these

36. In a quadrilateral $ABCD$, the sides, AB, BC, CD, DA measure 20 m, 13 m, 17 m and 10 m, respectively and the diagonal AC is 21 m. The area of the quadrilateral is
(a) 210 sq m (b) 220 sq m
(c) 240 sq m (d) None of these

37. If the two diagonals of a parallelogram are 72 cm and 30 cm respectively, find its perimeter.
(a) 156 cm (b) 164 cm
(c) 172 cm (d) None of these

38. If the base of a parallelogram is $(x + 4)$, altitude to the base is $(x - 3)$ and the area is $(x^2 - 4)$, then the actual area is equal to
(a) 64 sq units (b) 48 sq units
(c) 60 sq units (d) None of these

39. In a parallelogram, the lengths of adjacent sides are 12 cm and 14 cm, respectively. If the length of one diagonal is 16 cm, find the length of the other diagonal.
(a) 24.8 cm (b) 20.6 cm
(c) 22.4 cm (d) None of these

40. Find the perimeter of a circular plot which occupies an area of 154 sq m.
(a) 54 m (b) 44 m
(c) 22 m (d) 11 m

41. The perimeter of a circle is equal to that of a square. Compare their areas.
(a) 14 : 11 (b) 25 : 12
(c) 24 : 7 (d) 22 : 7

42. The length of a rectangle is thrice its breadth and its perimeter is 96 m. The area of the rectangle is
(a) 288 sq m (b) 442 sq m
(c) 438 sq m (d) 432 sq m

43. A cow is tied by a rope at the corner of a rectangular field. If the length of the rope is 14 m, the area of the field which the cow could graze is
(a) 77 sq m (b) 308 sq m
(c) 23 sq m (d) 154 sq m

44. The wheel of a scooter has diameter 70 cm. How many revolutions per minute must the wheel make so that the speed of the scooter is kept 66 km per hour?
(a) 400 (b) 600
(c) 500 (d) 800

45. Two small circular parks of diameters 16 m, 12 m are to be replaced by a bigger circular park. What would be the radius of this new park, if the new park occupies same space as the two small parks?
(a) 10 (b) 15
(c) 20 (d) 25

46. A rectangular park is 65 m long and 50 m wide. Two cross paths each 2 m wide are to be constructed parallel to the sides. If these paths pass through the centre of the rectangle and cost of construction is 17.25 per sq m, find the total cost involved in the construction.
(a) Rs. 2265.59 (b) Rs. 1772.45
(c) Rs. 3898.50 (d) Rs. 8452.32

47. The area of a trapezium is 2500 sq m. One of its parallel sides is 75 m. If the distance between the two parallel sides is 40 m, find the other parallel side.
(a) 20 m (b) 30 m
(c) 40 m (d) 50 m

48. The length of a rectangle exceeds its breadth by 3 cm. If the numerical values of the area and the

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(a) 15 m (b) 16 m
(c) 17 m (d) 20 m

69. The side of a square is 22 m. What is the radius of the circle whose circumference is equal to the perimeter of the square?
(a) 28 m (b) 3.5 m
(c) 14 m (d) 7 m

70. A piece of wire 132 cm long is bent successively in the shapes of an equilateral triangle, a square, a regular hexagon, and a circle. Then, which has the largest surface area?
(a) equilateral triangle (b) square
(c) circle (d) regular hexagon

71. If the radius of one circle is twelve times the radius of another, how many times does the area of the greater contain the area of the smaller?
(a) 12 (b) 72
(c) 144 (d) 96

72. If the circumference of a circle is equal to the perimeter of a square, what is the ratio of the area of the circle to the area of the square?
(a) 22 : 7 (b) 14 : 11
(c) 11 : 7 (d) 4 : 1

73. The length of a rectangular plot of land is three times as much as its breadth. A playground measuring 1200 sq. ft. occupies one-fourth of the total area of the plot. What is the length of the plot in feet?
(a) 40 (b) 360
(c) 120 (d) Data inadequate.

74. There are two squares s_1 and s_2. The ratio of their areas are 4 : 25. If the side of s_1 is 6 cm, what is the side of s_2?
(a) 20 cm (b) 15 cm
(c) 5 cm (d) 12 cm

75. The radius of the wheel of a vehicle is 70 cm. The wheel makes 10 revolutions in 5 seconds. The speed of the vehicle is
(a) 29.46 km/hr (b) 31.68 km/hr
(c) 36.25 km/hr (d) 32.72 km/hr

76. A rectangular carpet has an area of 60 m^2. Its diagonal and longer side together equal 5 times the shorter side. The length of the carpet is
(a) 5 m (b) 13 m
(c) 14.5 m (d) 12 m

77. A playground has the shape of a rectangle with two semi-circles on its smaller sides as diameters, added outside. If the sides of the rectangle are 36 m and 24.5 m, then the area of the playground is $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) 2259.529 sq m (b) 1353.625 sq m
(c) 1139.523 sq m (d) None of these

78. A man runs a circle of 50 m radius at a speed of 12 km per hour. Find the time taken by him for going around it ten times
(a) 10 min (b) 12.5 min
(c) 15.7 min (d) None of these

79. A room 5 m × 8 m is to be carpeted leaving a margin of 10 cm from each wall. If the cost of the carpet is Rs. 18 per m^2, the cost of carpeting the room will be
(a) Rs. 702.60 (b) Rs. 691.80
(c) Rs. 682.46 (d) Rs. 673.92

80. The area of a big rectangle is equal to the area of a small equal rectangle. If the length of the big rectangle is equal to the length of the small rectangle and the width of big rectangle is 2 m, what is the width of a small rectangle?
(a) $\frac{1}{3}$m (b) 1 m
(c) 2 m (d) Cannot be determined
(e) None of these

81. If the radius of a circle is reduced by 40%, its circumference is reduced by
(a) 60% (b) 40%
(c) 35% (d) 45%

82. A figure consists of a square of side 'a' m with semi-circles drawn on the outside of the square. The area (in m^2) of the figure so formed will be
(a) a^2 (b) $a^2 + 2\pi a^2$
(c) $4\pi a^2$ (d) $a^2 + \frac{\pi a^2}{2}$

83. The area of a square field is 6050 m^2. How much time it will take to reach from one of its corner to the opposite corner at the rate of 10 m in every 30 seconds?
(a) $5\frac{1}{2}$ min (b) 11 min
(c) 22 min (d) 110 min

84. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is
(a) $3r$ (b) $6r$
(c) $9r$ (d) $12r$

85. Two poles 15 m and 30 m high stand upright in a playground. If their feet be 36 m apart, find the distance between their tops.
(a) 36 m (b) 39 m
(c) 15 m (d) None of these

86. The length of a rectangular hall is $\frac{4}{3}$ of its width. If the area of the hall is 300 m^2, what is the difference between the length and breadth?
(a) 15 m (b) 20 m
(c) 3 m (d) 5 m
(e) 6 m

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87. Each side of an equilateral triangle is increased by 1.5%. The percentage increase in its area is
(a) 1.5% (b) 3%
(c) 4.5% (d) 5.7%

88. A rope, by which a horse is tied, is increased from 12 to 23 m. How much additional ground will it be able to graze?
(a) 1315 m^2 (b) 765 m^2
(c) 1210 m^2 (d) 1012 m^2

89. If a diagonal of a square is doubled, how does the area of the square change?
(a) Becomes four-fold (b) Becomes three-fold
(c) Becomes two-fold (d) None of these

90. If the sides of a rectangle are increased by 20%, the percentage increase in its perimeter is
(a) 80 (b) 40
(c) 20 (d) None of these

91. A circle road runs around a circular garden. If the difference between the circumference of the outer circle and the inner circle is 44 m, find the width of the road.
(a) 4 m (b) 7 m
(c) 3.5 m (d) 7.5 m

92. The length of a rectangular field is double its width. Inside the field there is a square-shaped pond 8 m long. If the area of the pond is $\frac{1}{8}$ of the area of the field, what is the length of the field?
(a) 32 m (b) 64 m
(c) 16 m (d) 20 m
(e) None of these

93. A circular disc of area 0.49π sq. m rolls down a length of 1.76 km. The number of revolutions it makes is
(a) 300 (b) 400
(c) 600 (d) 4000

94. If area of a triangle whose base is 6 cm is equal to the area of a circle of radius 6 cm, find the height of this triangle
(a) 10 cm (b) 22 cm
(c) 12 cm (d) 18 cm

95. The length of a ladder exactly equals the height of a wall. If the ladder is placed on a 2-feet tall stool placed 10 feet away from the wall, its tip just touches the top of the wall. The height of the wall in feet is
(a) 15 (b) 26
(c) 28 (d) 32

96. If the diagonal of a square is doubled to make another square, the area of the new square will
(a) become four-fold (b) become two-fold
(c) become six-fold (d) become eight-fold

97. The area of a circle is 154 cm^2. The length of an arc of the circle which subtends an angle of 45° at the centre is
(a) 11 cm (b) 5.5 cm
(c) 7 cm (d) None of these

98. A lawn is in the form of a triangle having its base and height in the ratio 2 : 3. The area of the lawn is $\frac{1}{12}$ hectare. Find the base and height of the lawn.
(a) 55 m, 34 m (b) 50 m, $33\frac{1}{3}$ m
(c) 50 m, 35 m (d) Data inadequate

99. A rectangular farm has to be fenced on one long side, one short side and the diagonal. If the cost of fencing is Rs. 10 per m, the area of farm is 1200 m^2 and the short side is 30 m long, how much would the job cost?
(a) Rs. 700 (b) Rs. 1200
(c) Rs. 1400 (d) Rs. 1500

100. The diameter of a circle is 105 cm less than the circumference. What is the diameter of circle?
(a) 44 cm (b) 46 cm
(c) 48 cm (d) 49 cm

101. A garden is 24 m long and 14 m wide. There is a path 1 m wide outside the garden along its sides. If the path is to be constructed with square marble tiles 20 cm × 20 cm, find the number of tiles required to cover the path
(a) 1800 (b) 200
(c) 2000 (d) 2150

102. If the length of the diagonal of a rhombus is 80% of the length of the other diagonal, the area of the rhombus is how many times the square of the length of the longer diagonal?
(a) $\frac{4}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{4}$ (d) $\frac{1}{4}$

103. $ABCD$ is a trapezium in which $AB \parallel CD$ and $AB = 2CD$. If its diagonals intersect each other at O, the ratio of area of triangle AOB and COD is
(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

104. The ratio of the corresponding sides of two similar triangles is 3 : 4. The ratio of their areas is
(a) 4 : 3 (b) 3 : 4
(c) 9 : 16 (d) $\sqrt{3}$: 2

105. The area of the circle inscribed in an equilateral triangle of side 24 cm is
(a) 24π cm^2 (b) 36π cm^2
(c) 48π cm^2 (d) 18π cm^2

106. The radius of wheel is 1.4 decimetre. How many times does it revolve during a journey of 0.66 km?
(a) 375 (b) 750
(c) 1500 (d) 3000

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EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let $a = 15$ m, $b = 16$ m, $c = 17$ m.

Then, $s = \frac{a+b+c}{2} = \frac{15+16+17}{2} = 24$ m.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-15)(24-16)(24-17)}$

$= \sqrt{24\times9\times8\times7}$

$= \sqrt{12096} = \mathbf{24\sqrt{21}}$ **sq m.**

2. Ans. (a)

Solution Height of the triangle $= \sqrt{(6.5)^2-6^2}$

$= \sqrt{42.25-36}$

$= \sqrt{6.25} = 2.5$ m

$\therefore$ Area $= \frac{1}{2}$(base $\times$ height)

$= \frac{1}{2} \times 6 \times 2.5 =$ **7.5 sq m.**

3. Ans. (b)

Solution Each side = 12 cm

Then, $s = \frac{12+12+12}{2} = 18.$

Area $= \sqrt{18(18-12)(18-12)(18-12)}$

$= \sqrt{18\times6\times6\times6} = \frac{1}{2} \times 12 \times$ height

or, height $= \frac{36\sqrt{3}}{6} = \mathbf{6\sqrt{3}}$ **cm.**

4. Ans. (b)

Solution Let the length of equal sides be x.

Then, $s = \frac{x+x+64}{2} = x + 32.$

Area = 1600 sq m.

$= \sqrt{(x+32)(x+32-x)(x+32-x)(x+32-64)}$

$= \sqrt{(x+32)\times32\times32\times(x-32)}$

or, $1600 = 32\sqrt{x^2-32^2}$

$\Rightarrow \sqrt{x^2-32^2} = 50$

or, $x^2 = 32^2 + 50^2 = 1024 + 2500 = 3524.$

$\therefore$ $x =$ **59.36 m.**

5. Ans. (c)

Solution Here, $s = \frac{20+21+29}{2} = \frac{70}{2} = 35$ m.

$\therefore$ Area $= \sqrt{35(35-20)(25-21)(35-29)}$

$= \sqrt{35\times15\times14\times6}$

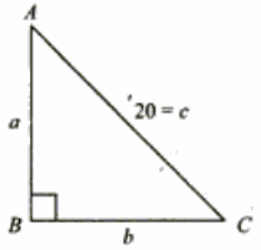
$= \sqrt{5^2\times7^2\times3^2\times2^2}$

$= 5 \times 7 \times 3 \times 2 =$ **210 sq m.**

6. Ans. (a)

Solution We have, $a + b + 20 = 2 \times 24$

$\therefore$ $a + b = 28$ ...(1)



By Pythagores Theorem, $a^2 + b^2 = 400$

$\because$ $(a + b)^2 = 28^2$

$\Rightarrow$ $ab = \frac{28^2-(a^2+b^2)}{2} = \frac{28^2-400}{2} = 192$

$\therefore$ $a - b = \sqrt{(a+b)^2-4ab}$

$= \sqrt{784-4\times192} = 4$ ...(2)

Solving (1) and (2), we get

$a = 16$ cm and $b =$ **12 cm.**

7. Ans. (b)

Solution The sides of the triangle are

$a = \frac{3}{12} \times 36 = 9$ cm, $b = \frac{4}{12} \times 36 = 12$ cm

$c = \frac{5}{12} \times 36 = 15$ cm

$s = \frac{a+b+c}{2} = \frac{36}{2} = 18$ cm

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{18\times9\times6\times3}$

$= \sqrt{9^2\times2^2\times3^2} = 9 \times 2 \times 3$

$=$ **54 sq cm.**

8. Ans. (c)

Solution We have, $2s = 50 + 78 + 112 = 240$

$\therefore$ $s = 120.$

$\therefore$ Area $= \sqrt{120(120-50)(120-78)(120-112)}$

$= \sqrt{120\times70\times42\times8}$

$= \sqrt{2^3\times5\times3\times7\times5\times2\times2\times7\times3\times2^3}$

$= 2^4 \times 5 \times 3 \times 7 = 1680$ sq m

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∴ Height of the equilateral triangle

$$= \frac{\sqrt{3}}{2} \times (\text{side}) = \frac{\sqrt{3}}{2} \times 8 = \mathbf{4\sqrt{3}\ cm.}$$

20. Ans. (b)

Solution Let x and $(51 - x)$ be the other two sides of the triangle

Then, $x^2 + (51 - x)^2 = 39^2$

$\Rightarrow \quad x^2 + 2601 - 102x + x^2 = 1521$

$\Rightarrow \quad x = \frac{51 \pm \sqrt{441}}{2} = \frac{51 \pm 21}{2} = 36,\ 15.$

∴ The other two sides are 36 cm and **15 cm.**

21. Ans. (b)

Solution Given: $a = \frac{5}{8}b.$

Now, perimeter of isosceles triangle $= 2a + b$

$\Rightarrow \quad 306 = 2 \times \frac{5}{8}b + b$ or, $b = 136.$

$\therefore \quad a = \frac{5}{8}b = \frac{5}{8} \times 136 = 85.$

∴ Area of isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{136}{4}\sqrt{4 \times (85)^2 - (136)^2}$$

$$= 34 \times 102 = \mathbf{3468\ sq\ m.}$$

22. Ans. (c)

Solution We have, hypotenuse $= \sqrt{2}\,a = 8 \Rightarrow a = 8/\sqrt{2}.$

∴ Area of isosceles triangle $= \frac{1}{2}a^2$

$$= \frac{1}{2} \times \frac{64}{2} = \mathbf{16\ sq\ cm.}$$

23. Ans. (b)

Solution Let the lateral side $= 5x$ and the base $= 4x.$

Then, $5x + 5x + 4x = 14 \Rightarrow x = 1.$

∴ The sides of the triangle are 5 cm, 5 cm and 4 cm.

∴ Area of the isosceles triangle

$$= \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{4}{4}\sqrt{4(5)^2 - (4)^2} = \sqrt{84} = \mathbf{2\sqrt{21}\ sq\ cm.}$$

24. Ans. (c)

Solution Area $= \sqrt{s(s-a)(s-b)(s-c)} = A$

where $s = \frac{a+b+c}{2}$ and a, b, c are sides of the triangle.

When the sides are increased by 200%, the sides become $3a$, $3b$ and $3c$.

$$s_1 = \frac{3a+3b+3c}{2} = 3\frac{(a+b+c)}{2} = 3s.$$

$$A_1 = \sqrt{s_1(s_1-3a)(s_1-3b)(s_1-3c)}$$

$$= \sqrt{3s.3(s-a).3(s-b).3(s-c)}$$

$$= 9\sqrt{s(s-a)(s-b)(s-c)} = 9A.$$

∴ Increase in area $= 9A - A = 8A$ or, **800%.**

25. Ans. (c)

Solution Let the length of each of equal sides of the triangle be xm.

Then, $x^2 + x^2 = (50\sqrt{2})^2 = 5000.$

$\Rightarrow \quad 2x^2 = 5000 \Rightarrow x = 50.$

∴ Perimeter of the triangle $= 50 + 50 + 50\sqrt{2}$

$= 100 + 50 \times 1.4146 = 170.73$ m.

∴ Cost of fencing = Rs. (170.73×3) = **Rs. 512.19.**

26. Ans. (a)

Solution Area of isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$

$$= \frac{16}{4}\sqrt{4 \times (10)^2 - (16)^2} = 4 \times 12 = 48 \text{ sq cm}$$

Given: Area of equilateral triangle = 48.

$\Rightarrow \quad \frac{\sqrt{3}}{4}(\text{side})^2 = 48$

$\Rightarrow$ Side of equilateral triangle = **10.5 cm.**

27. Ans. (c)

Solution Perimeter of a right-angled isosceles triangle

$= (\sqrt{2}+1) \times$ hypotenuse

$\Rightarrow \quad 4\sqrt{2} + 4 = (\sqrt{2} + 1) \times$ hypotenuse

$\Rightarrow$ hypotenuse = **4 m.**

28. Ans. (a)

Solution Here, $a = 60$, $b = 40$ and $d = 80$

$\therefore \quad s = \frac{a+b+d}{2} = \frac{60+40+80}{2} = 90.$

∴ Area of the parallelogram

$$= 2\sqrt{s(s-a)(s-b)(s-d)}$$

$$= 2\sqrt{90(90-60)(90-40)(90-80)}$$

$$= 2\sqrt{90 \times 30 \times 50 \times 10}$$

$$= \mathbf{600\sqrt{15}\ sq\ m.}$$

29. Ans. (c)

Solution Area of the parallelogram

$= 16 \times 14 =$ **224 sq cm.**

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30. Ans. (b)

Solution We have, $s = \frac{a+b+d}{2}$

$= \frac{65+119+156}{2} = 170.$

$\therefore$ Area of parallelogram

$= 2\sqrt{s(s-a)(s-b)(s-d)}$

$= 2\sqrt{170(170-65)(170-119)(170-156)}$

$= 2\sqrt{170\times51\times105\times14}$

$= 2 \times 3570 = 7140$ sq m.

$\therefore$ Cost of gravelling $= 7140 \times 10 =$ **Rs. 71400.**

31. Ans. (a)

Solution The area of the parallelogram

$= 10 \times 7 =$ **70 sq m.**

32. Ans. (b)

Solution Area of the parallelogram $= 8 \times 4 = 32$ sq m.

Distance between the shorter sides $= \frac{32}{5} =$ **6.4 m.**

33. Ans. (c)

Solution Area of quadrilateral $= \frac{1}{2}d(p_1+p_2)$

$\Rightarrow \quad 420 = \frac{1}{2}\times d\times(18+12)$

$\Rightarrow \quad d = \frac{420\times2}{30} =$ **28 m.**

34. Ans. (a)

Solution Let the base be x and altitude be $2x$.

Then, $x \times 2x = 72 \Rightarrow x^2 = 36 \Rightarrow$ **$x = 6$.**

35. Ans. (b)

Solution Let the base be x.

Area of the parallelogram = base $\times$ altitude

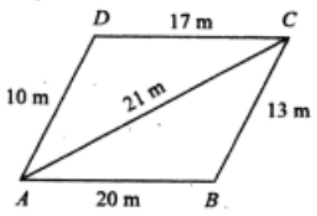
$\Rightarrow \quad 240 = x \times 12$

$\therefore \quad x = \frac{240}{12} =$ **20 cm.**

36. Ans. (a)

Solution Area of quadrilateral $ABCD$

= Area of ΔADC + Area of ΔABC, where,



Area of ΔABC

$= \sqrt{s(s-AB)(s-BC)(s-d)}$

$\left(s = \frac{AB+BC+d}{2} = \frac{20+13+21}{2} = 27\right)$

$= \sqrt{27(27-20)(27-13)(27-21)}$

$= \sqrt{27\times7\times14\times6} = 126$ sq m.

Area of ΔADC

$= \sqrt{s(s-AD)(s-DC)(s-d)}$

$\left(s = \frac{AD+DC+d}{2} = \frac{10+17+21}{2} = 24\right)$

$= \sqrt{24(24-10)(24-17)(24-21)}$

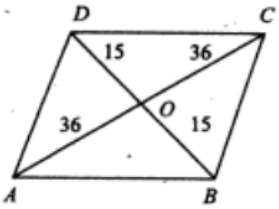
$= \sqrt{24\times14\times7\times3} = 84$ sq m.

Here, the area of quadrilateral $ABCD$

$= 126 + 84 =$ **210 sq m.**

37. Ans. (a)

Solution Since $AC = 72$ and $BD = 30$



$\therefore \quad OB = OD = \frac{BD}{2} = \frac{30}{2} = 15$

and, $\quad OA = OC = \frac{AC}{2} = \frac{72}{2} = 36$

$\therefore \quad AB = BC = CD = DA = \sqrt{36^2+15^2}$

$= \sqrt{1296+225}$

$= \sqrt{1521} = 39.$

$\therefore$ Perimeter of the parallelogram $= 4 \times 39$

$=$ **156 cm.**

38. Ans. (c)

Solution Area of the parallelogram

= base $\times$ altitude

$\Rightarrow \quad (x^2 - 4) = (x + 4) \times (x - 3)$

$\Rightarrow \quad x^2 - 4 = x^2 + 4x - 3x - 12$

$\Rightarrow \quad x = 8.$

$\therefore$ Area of the parallelogram

$= x^2 - 4 = (8)^2 - 4 =$ **60 sq units.**

39. Ans. (b)

Solution We have,

$d_1^2 + d_2^2 = 2(a^2 + b^2)$

$\Rightarrow \quad (16)^2 + d_2^2 = 2(12^2 + 14^2)$

$\Rightarrow \quad d_2^2 = 2(144 + 196) - 256 = 424.$

$\therefore \quad d_2^2 = \sqrt{424} =$ **20.6 cm.**

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40. Ans. (b)

Solution $\pi r^2 = 154\ m^2$

$\Rightarrow \quad r^2 = \frac{7}{22} \times 154\ m^2 = 49\ m^2$

$\therefore \quad r = 7\ m$

Perimeter $= 2 \times \frac{22}{7} \times 7\ m =$ **44 m.**

41. Ans. (a)

Solution If x be the side of the square and r be the radius of the circle, then

$4x = 2\pi r$

or, $x = \frac{\pi r}{2}$

Now, $\pi r^2 :: x^2 = \pi r^2 :: \frac{\pi^2 r^2}{4}$ or, $4 : \pi$

$= 4 :: \frac{22}{7}$ **or, 14 : 11.**

42. Ans. (d)

Solution Let the width of the rectangle be x m

Then length $= 3x$ m

Perimeter $= 2(x + 3x) = 96$

$\Rightarrow \quad 8x = 96$ or, $x = 12$

Area $= 12 \times 36 =$ **432 sq m.**

43. Ans. (d)

Solution As the cow is tied at the corner of a rectangular field, it will graze the area of the field enclosed between two sides of the rectangle.

$= \frac{1}{4}(\pi \times 14 \times 14)$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$=$ **154 m^2.**

44. Ans. (c)

Solution The distance covered by the wheel in one m

$= \frac{66 \times 1000 \times 100}{60} = 110000$ cm

The distance covered by the wheel in one revolution

= The circumference of the wheel

$= 2\pi r = 2 \times \frac{22}{7} \times \frac{70}{2} = 220$ cm

$\therefore$ Number of revolutions of the wheel

$= \frac{110000}{220} =$ **500.**

45. Ans. (a)

Solution $\pi(8)^2 + \pi(6)^2 = \pi R^2$

$\therefore \quad R^2 = 64 + 36$

$R = 10$

46. Ans. (c)

Solution Area of two paths $= 2 \times (65 + 50 - 2) = 226\ m^2$

Cost of construction $= 226 \times 17.25$

$=$ **Rs. 3898.50.**

47. Ans. (d)

Solution $\frac{1}{2}(75+x) \times 40 = 2500$

$\Rightarrow \quad 75 + x = 125$

$\Rightarrow \quad x = 50$

$\therefore$ The other parallel side $=$ **50 m.**

48. Ans. (c)

Solution Let the breadth be x, then length $= x + 3$

Given: $2(x + x + 3) = x(x + 3)$

$\Rightarrow \quad 4x + 6 = x^2 + 3x$

$\Rightarrow \quad 4x + 6 = x^2 + 3x$

$\Rightarrow \quad x^2 - x - 6 = 0$

$\therefore \quad x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2}$

$\therefore \quad x = 3, -2$

$\therefore$ Breadth $=$ **3 cm.**

49. Ans. (c)

Solution Let x, y be the sides of squares

$\frac{x^2}{y^2} = \frac{9}{1} \Rightarrow \frac{x}{y} = \frac{3}{1}$

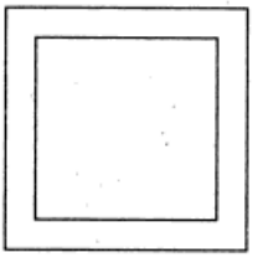
$\therefore$ The ratio of perimeters is $4x : 4y$

i.e. $x : y = 3 : 1.$

50. Ans. (d)

Solution Let x (in metres) be the width of the path

Side of outer square $= 30 + 2x$



$\therefore$ Area of path $= (30 + 2x)^2 - 30^2$

$\therefore \quad (30 + 2x)^2 - 30^2 = 256$

$\Rightarrow \quad 4x^2 + 12x - 256 = 0$

$\Rightarrow \quad x^2 + 30x - 64 = 0$

$\Rightarrow \quad (x - 2)(x + 32) = 0$

$\therefore \quad x =$ **2.** $(\because x \neq 0)$

51. Ans. (c)

Solution Additional grassy ground grazed

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64. Ans. (a)

Solution Area of rectangle = $l \times b$

Let the new width be b_1

Then, $l \times b = \frac{4}{3} l \times b_1$

$\therefore \quad b_1 = \frac{3}{4} b = 0.75b$

Thus, there should be a reduction of **25% in the width.**

65. Ans. (d)

Solution Area of rectangle = $6.4 \times 2.5 = 16\ m^2$

According to question:

Area of square = Area of the rectangle

$\therefore$ Area of square = $16\ m^2$

$\therefore$ Side of the square = **4 m.**

66. Ans. (d)

Solution $\pi \times 5 \times 5 - \pi \times 4 \times 4$

$= 25\pi - 16\pi = \mathbf{9\pi\ cm^2.}$

67. Ans. (b)

Solution The length of the room = 4 m

Since it can be partitioned into two equal square rooms, the two equal parts can only be of 2 m each.

68. Ans. (c)

Solution Let length = a m and breadth = b m

Then, $2(a + b) = 46$

or, $\quad a + b = 23$ and $ab = 120$

$\therefore$ Diagonal $= \sqrt{a^2 + b^2} = \sqrt{(a+b)^2 - 2ab}$

$= \sqrt{(23)^2 - 2 \times 120} = \sqrt{289} = \mathbf{17\ m.}$

69. Ans. (c)

Solution Perimeter of the square = 4×22 m = 88 m

$2\pi r = 88$ m

$\Rightarrow \quad r = 88 \times \frac{7}{22 \times 2}\text{m} = \mathbf{14\ m.}$

70. Ans. (c)

Solution Area of an equilateral $\Delta = \frac{\sqrt{3}}{4} a^2$

$[3a = 132 \ \therefore \ a = 44]$

$= \frac{\sqrt{3}}{4} \times 44 \times 44$

$= 838.312$ sq m.

Area of square $= a^2 = 33 \times 33$

$= 1089$ sq m. $\left[a = \frac{132}{4} = 33\right]$

Area of regular hexagon

$= \frac{3\sqrt{3}a^2}{2} = \frac{3\sqrt{3} \times 22 \times 22}{2}$

$\left[a = \frac{132}{6} = 22\right]$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 21 \times 21 \left[r = \frac{132}{2\pi} = 21\right]$

$\therefore$ Circle has largest surface area.

71. Ans. (c)

Solution Let r be the radius of smaller circle

Radius of larger circle = $12r$

$$\frac{\text{Area of larger circle}}{\text{Area of smaller circle}} = \frac{\pi(12r)^2}{\pi r^2} = \frac{144}{1}$$

$\therefore$ Area of large circle contains the area of smaller circle **144 times.**

72. Ans. (b)

Solution If a be the side of the square and r be the radius of the circle, then

$2\pi r = 4a$

or, $\quad r = \frac{4a}{2\pi} = \frac{2a}{\pi}$

$\therefore$ Area of the circle $= \pi r^2$ and,

area of the square $= a^2$

$\therefore$ Area of the circle : Area of the square

$= \pi r^2 : a^2$

$= \frac{\pi\left(\frac{2a}{\pi}\right)^2}{a^2} = \frac{\pi \frac{4a^2}{\pi^2}}{a^2}$

$= \frac{4}{\pi} = \frac{4 \times 7}{22} = \mathbf{4 : 11.}$

73. Ans. (c)

Solution Let the length and breadth of the plot be $3x$ and x feet, respectively.

Total area of the plot = $4 \times 1200 = 4800$ sq ft.

$\therefore \quad x \times 3x = 4800 \Rightarrow x = 40$ ft

$\therefore$ Length = $3 \times 40 = \mathbf{120\ ft.}$

74. Ans. (b)

Solution Ratio of the areas = 4 : 25

Ratio of the sides = 2 : 5

If the side of s_1 is 2 cm, then side of s_2 is 5 cm.

If the side of s_1 is 6 cm, the side of s_2 is $\frac{5}{2} \times 6 = \mathbf{15\ cm.}$

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75. Ans. (b)

Solution Circumference $= \left(2 \times \frac{22}{7} \times 70\right)$ cm $= 440$ cm

Distance travelled in 10 revolutions

$= 4400$ cm $= 44$ m

$\therefore$ Speed $= \frac{\text{Distance}}{\text{Time}} = \left(\frac{44}{5}\right)$ m/sec

$= \left(\frac{44}{5} \times \frac{18}{5}\right)$ km/hr $=$ **31.68 km/hr.**

76. Ans. (d)

Solution $lb = 60\ m^2$

$d^2 = l^2 + b^2$

$d + l = 5b \Rightarrow d = 5b - l$

$\therefore \quad d^2 = 25b^2 + l^2 - 10bl$

$\Rightarrow \quad l^2 + b^2 = 25b^2 + l^2 - 10bl$

$\Rightarrow \quad l^2 + b^2 = 25b^2 + l^2 - 10 \times 60$

$\Rightarrow \quad 24b^2 = 600 \quad$ or $\quad b = 5$

$\therefore \quad l = 60 \div 5 =$ **12 m.**

77. Ans. (b)

78. Ans. (c)

Solution Distance $= 2 \times 3.14 \times 50 \times 10$

$= 3140$ m

12 km/hr $= \frac{10}{3}$ m/sec

Time $= \frac{3140}{10/3} = 942$ sec

$= \frac{942}{60}$ min

$=$ **15.7 min.**

79. Ans. (d)

Solution Length of the area to be carpeted $= 8 - 0.2$

$= 7.8$ m

Width $= 5 - 0.2 = 4.8$ m

$\therefore$ Area to be carpeted $= 7.8 \times 4.8\ m^2$

Total cost $= 18 \times 7.8 \times 4.8 =$ **Rs. 673.92.**

80. Ans. (a)

Solution Let the length of the big rectangle be x m

$\therefore$ Area of the big rectangle $= x \times 2 = 2x\ m^2$

$\therefore$ Area of the small rectangle $= \frac{1}{6} \times 2x = \frac{x}{3}\ m^2$

$\therefore$ Breadth of the small rectangle $= \frac{x}{3} \div x =$ **$\frac{1}{3}$ m.**

81. Ans. (b)

Solution Let original radius $= r$

Reduced radius $= r - 0.4r = 0.6r$

$\therefore$ Percentage reduction in circumference

$= \frac{2\pi r - 2\pi(0.6)r}{2\pi r} \times 100 =$ **40%.**

82. Ans. (d)

Solution Required area = (Area of a square of side a).

$+ 4\left(\text{Area of semi-circle of radius } \frac{a}{2}\right)$

$= a^2 + 4 \times \frac{1}{2}\pi \times \left(\frac{1}{2}a\right)^2$

$= a^2 + \frac{\pi a^2}{2}.$

83. Ans. (a)

Solution $(\text{Diagonal})^2 = 2 \times 6050\ m^2 = 12100\ m^2$

Diagonal $= 110$ m

Time taken $= \frac{110}{10} \times \frac{1}{2}$ min $=$ **$5\frac{1}{2}$ min.**

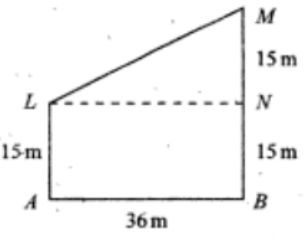
84. Ans. (b)

Solution Length of each side of hexagon $= r$

$\therefore$ Its perimeter $=$ **$6r$.**

85. Ans. (b)

Solution Let AL, BM be poles with tops at L and M, respectively.



$LA = 15$ m, $MB = 30$ m

$AB = 36$ m.

Let LN be parallel to the ground

$LN = AB = 36$ m

and, $MN = 30 - 15 = 15$ m

$\therefore \quad LM = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} =$ **39 m.**

86. Ans. (d)

Solution Let the breadth be x m

$\therefore$ Length $= \frac{4x}{3}$ m

$\therefore \quad \frac{4x}{3} \times x = 300$

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or, $x^2 = 300 \times \frac{3}{4} = 225$

$x = \sqrt{225} = 15$ m

and, length $= \frac{4}{3} \times 15 = 20$ m

length − breadth = 20 − 15 m = **5 m.**

87. Ans. (a)

Solution Let the original length of each side = a

Then, area $= \frac{\sqrt{3}}{4}a^2 = A$

New area $= \frac{\sqrt{3}}{4}\left[\left(\frac{101.5}{100}a^2\right)\right]$

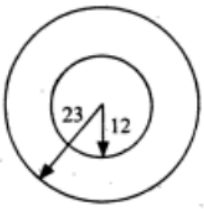
$= \frac{\sqrt{3}}{4}\left(\frac{20.3}{20}\right)a^2 = \frac{20.3}{20}A$

Increase in area $= \left(\frac{0.3}{20}A \times \frac{1}{A} \times 100\right)\%$

= 1.5%.

88. Ans. (c)

Solution Shaded area of the two (in between) concentric circle of radius R and r is $\pi(R + r)(R - r)$



∴ Additional grassy area

$= \frac{22}{7} \times (23 + 12)(23 - 12)\ m^2$

$= \frac{22}{7} \times 35 \times 11\ m^2$

$= 1210\ m^2$.

89. Ans. (a)

Solution Let originally, diagonal = x

∴ Original area $= \frac{1}{2}x^2$

After increase, diagonal = $2x$

New area $= \frac{1}{2}(2x)^2$

∴ $\frac{\text{original area}}{\text{New area}} = \frac{1}{4}$

∴ New area = **4 (original area).**

90. Ans. (c)

Solution If length of the rectangle is L and its width is B, then its perimeter = $2(L + B)$

Increased length = $1.2L$,

Increased width = $1.2B$

Increase perimeter = $2(1.2L + 1.2B)$

$= 2 \times 1.2(L + B)$

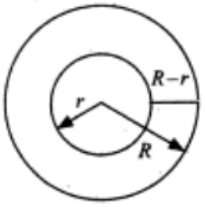
$= 2.4(L + B)$

Increase in perimeter = $0.4(L + B)$

Percentage increase $= \frac{0.4(L+B)}{2(L+B)} \times 100 =$ **20%.**

91. Ans. (b)

Solution $2\pi R - 2\pi r = 44$ m



∴ $2\pi(R - r) = 44$ m

or, $R - r = 44 \div 2\pi$

$= 44 \div \left(2 \times \frac{22}{7}\right) =$ **7 m.**

92. Ans. (a)

Solution Area of the square-shaped pond = 8×8

$= 64\ m^2$

∴ Area of field = $8 \times 64 = 512\ m^2$

If the length of the field be x m.

Then, the breadth of the field $= \frac{x}{2}$ m

∴ $x \times \frac{x}{2} = 512$

or, $x^2 = 2 \times 512.$

or, $x^2 = 1024$

or, $x = \sqrt{1024} =$ **32 m.**

93. Ans. (b)

Solution $\pi r^2 = 0.49\pi \Rightarrow r = 0.7$ m

Number of revolutions $= \frac{1.76 \times 1000}{2 \times \frac{22}{7} \times 0.7} =$ **400.**

94. Ans. (c)

Solution Radius of the circle = 6 cm

Area of the circle = 36 sq cm.

According to question:

Area of the triangle of base 6 cm

= Area of the circle of radius 6 cm

∴ $\frac{1}{2} \times$ base $\times$ height = 36

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$\Rightarrow \quad \frac{1}{2} \times 6 \times \text{height} = 36$

$\Rightarrow$ Height of the triangle $= \frac{36 \times 2}{6} =$ **12 cm.**

95. Ans. (b)

Solution Let the height of the wall be h metre when the ladder is placed at distance 10 m away from the wall on a stool of 2 m height, it will form a right triangle with sides 10 m, $(h - 2)$ m and taper side of length h m.

Hence, we have $h^2 = 10 + (h - 2)^2$

or, $h^2 - (h - 2)^2 = 100$

$\Rightarrow (h + h - 2) \times (h - h + 2) = 100$

or, $(2h - 2) \times 2 = 100$ or $4h - 4 = 100$

$\Rightarrow 4h - 4 = 100$

$\Rightarrow 4h = 104$ or, $\boldsymbol{h = 26}$ **m**

96. Ans. (a)

Solution Suppose the side of the square = x cm

Diagonal of the square = $\sqrt{2}x$ cm

Area of the square = x^2 cm, i.e. $\frac{(\text{Diagonal})^2}{2}$

According to question:

Diagonal of the new square = $2 \times \sqrt{2}x$

$= 2\sqrt{2}x$ cm

$\therefore$ Area of the new square

$= \frac{(\text{Diagonal of the new square})^2}{2}$

$= \frac{(2\sqrt{2}x)^2}{2} =$ **$4x^2$ sq cm.**

97. Ans. (b)

Solution Area of the circle = 154 cm^2

Let its radius = r, then $154 = \frac{22}{7}r^2$

$\Rightarrow \quad r^2 = \frac{154 \times 7}{22} = 49$ or, r = 7 cm

Circumference = $2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm

Length of arc = $\frac{44 \times 45}{360} =$ **5.5 cm.**

98. Ans. (b)

Solution Let the base be $2x$ m and height $3x$ m.

Then, $\frac{1}{2}(2x \times 3x) = \frac{1}{12} \times 10000$

[$\because$ 1 hectare = 10000 m^2]

or, $x = \sqrt{\frac{10000 \times 2}{6 \times 12}} = \frac{100}{6} = \frac{50}{3}$

$\therefore$ Base $= 2 \times \frac{50}{3} = 33\frac{1}{3}$ m

Height $= \frac{3 \times 50}{3} =$ **50 m.**

99. Ans. (b)

Solution Length of the longer side $= \frac{1200}{30} = 40$ m

and the length of the diagonal $= \sqrt{30^2 + 40^2} = 50$ m

$\therefore$ Length of the fence = 30 + 40 + 50 = 120 m

$\therefore$ The job cost = 120 × 10 = **Rs. 1200.**

100. Ans. (d)

Solution $\pi d - d = 105 \Rightarrow (\pi - 1)d = 105$

$\Rightarrow \left(\frac{22}{7} - 1\right)d = 105$

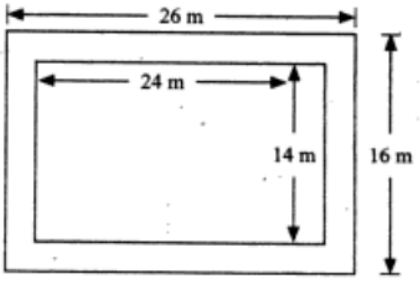
$\therefore \quad d = 105 \times \frac{7}{15} =$ **49 cm.**

101. Ans. (c)

Solution Area of garden = 24 × 14 = 336 m^2

Area of the (garden + path) = 26 × 16

= 416 m^2



$\therefore$ Area of the path = 416 − 336 = 80 m^2

Area of 1 tile = 20 × 20 = 400 cm^2 = 0.04 m^2

$\therefore$ Number of tiles required $= \frac{80}{0.04} =$ **2000.**

102. Ans. (b)

Solution Let one diagonal = x cm

Then, another diagonal

$= \left(\frac{80}{100}x\right)$ cm $= \left(\frac{4}{5}x\right)$ cm

Area of rhombus $= \frac{1}{2}x \times \frac{4}{5}x = \frac{2}{5}x^2$

$=$ **$\frac{2}{5}$ × (square of longer diagonal).**

103. Ans. (d)

Solution $\Delta OAB = \frac{1}{2} \times AB \times OE$

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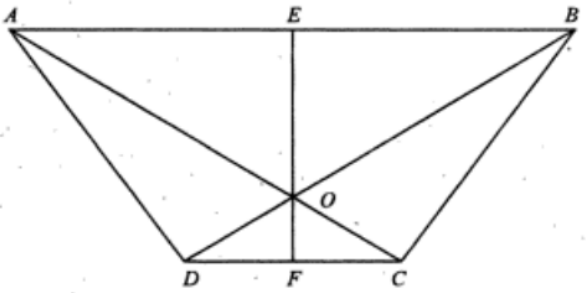
$$= \frac{1}{2} \times 2CD \times OE$$
$$= CD \times OE$$



$$\Delta OCD = \frac{1}{2} \times CD \times OF$$

$$\therefore \quad \frac{\Delta AOB}{\Delta COD} = \frac{CD \times OE}{\frac{1}{2} \times CD \times OF} = \frac{CD \times 2 \times OF}{\frac{1}{2} \times CD \times OF} = \frac{4}{1}$$

= 4 : 1.

104. Ans. (c)

Solution Ratio of areas of similar triangles
= Ratio of the squares of corresponding sides

$$= \frac{(3x)^2}{4x^2} = \frac{9x^2}{16x^2} = \frac{9}{16} = \mathbf{9 : 16.}$$

105. Ans. (c)

Solution $\frac{1}{2} \times 24 \times h = \frac{\sqrt{3}}{4} \times 24 \times 24$

or, $h = 12\sqrt{3}$

$\therefore \quad 3r = 12\sqrt{3} \Rightarrow r = 4\sqrt{3}$ cm

Area of the circle $= \pi \times (4\sqrt{3})^2$ cm^2 $= \mathbf{48\pi}$ **cm^2.**

106. Ans. (b)

Solution $r = 0.14$

Number of revolutions

$$= \left(\frac{0.66 \times 1000}{2} \times \frac{7}{22} \times \frac{1}{0.14}\right) = \mathbf{750.}$$

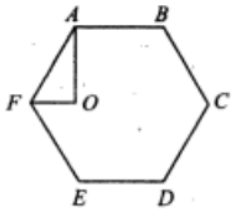
QUESTIONS FROM COMPETITIVE EXAMS

1. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be

(a) 20 (b) 0
(c) 21 (d) 22

[Based on CAT 2003]

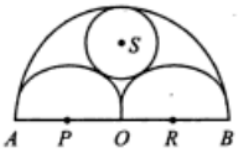
2. In the figure below, $ABCDEF$ is a regular hexagon and $\angle AOF = 90°$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon $ABCDEF$?



(a) $\frac{1}{12}$ (b) $\frac{1}{6}$
(c) $\frac{1}{24}$ (d) $\frac{1}{18}$

[Based on CAT 2003]

3. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi-circles with centres at P and R respectively, and S is the centre of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circle and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semi-circles with diameter AB that cannot be grazed by the horses is nearest to



(a) 20 (b) 28
(c) 36 (d) 40

[Based on CAT 2003]

4. A vertical tower OP stands at the centre O of a square $ABCD$. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60°$ then the relationship between h and b can be expressed as

(a) $2b^2 = h^2$ (b) $2h^2 = b^2$
(c) $3b^2 = 2h^2$ (d) $3h^2 = 2b^2$

[Based on CAT 2003]

5. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?

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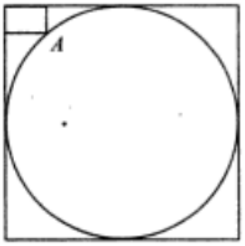


(a) 10cm (b) 40cm
(c) 50cm (d) None of these

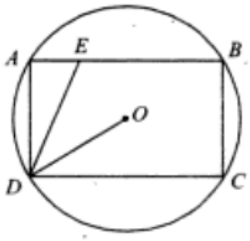
[Based on CAT 2003]

6. Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from a square cloth piece of side a in the following steps: the original square of side a is divided into n smaller squares, not necessarily of the same size; then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side a and the process ends three. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth generated in the former to that in the latter is

(a) 1 : 1 (b) $\sqrt{2}:1$
(c) $\frac{n(4-\pi)}{4n-\pi}$ (d) $\frac{4n-\pi}{n(4-\pi)}$

[Based on CAT 2004]

7. In the figure below (not drawn to scale), rectangle $ABCD$ is inscribed in the circle with centre at O. The length of side AB is greater than that of side BC. The ratio of the area of the circle to the area of the rectangle $ABCD$ is $\pi:\sqrt{3}$. The line segment DE intersects AB at E such that $\angle ODC = \angle ADE$. What is the ratio AE : AD?



(a) $1:\sqrt{3}$ (b) $1:\sqrt{2}$
(c) $1:2\sqrt{3}$ (d) 1 : 2

[Based on CAT 2004]

8. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If A_1, A_2, A_3, ... be the areas and P_1, P_2, P_3, ... be the perimeters of S_1, S_2, S_3, ..., respectively, then the ratio

$$\frac{P_1+P_2+P_3+\ldots}{A_1+A_2+A_3+\ldots}$$ equals.

(a) $\frac{2(1+\sqrt{2})}{a}$ (b) $\frac{2(2-\sqrt{2})}{a}$
(c) $\frac{2(2+\sqrt{2})}{a}$ (d) $\frac{2(1+2\sqrt{2})}{a}$

[Based on CAT 2004]

9. Let $ABCDEF$ be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon $ABCDEF$?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

[Based on CAT 2004]

10. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was a 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

(a) 16.665 (b) 16.565
(c) 15.465 (d) 14.365

[Based on CAT 2004]

11. If the length and width of a rectangular garden plot were each increased by 20 percent, then what would be the percent increase in the area of the plot?

(a) 20% (b) 24%
(c) 36% (d) 44%

[Based on MAT 2004]

12. The sides of a triangle are in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If the perimeter is 52cm, then the length of the smallest side is

(a) 9cm (b) 10cm
(c) 11cm (d) 12cm

[Based on MAT 2004]

13. A, B and C start running at the same time and from the same point around a circular track of 70 metres radius. A and B run clockwise and C counter clockwise. If A meets C every 88 seconds and B meets C every 110 seconds, then A meets B every -seconds.

(a) 22 (b) 198
(c) 440 (d) None of these

[Based on MAT 2004]

14. A pond 100m in diameter is surrounded by a circular grass walk 2m wide. How many square meters of grass is there on the walk?

(a) 98π (b) 100π
(c) 204π (d) 202π

[Based on MAT 2004]

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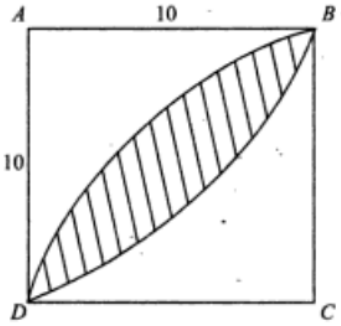
32. A person wishes to make a 100 sq m rectangular garden. Since he has only 30 m barbed wire for fencing, he fences only three sides letting the house wall act as the fourth side. The width of the garden is

(a) 10 m (b) 5 m
(c) 50 m (d) 100 m

[Based on MAT 2002]

33. In fhe figure, $ABCD$ is a square with side 10. BFD is an arc of a circle with centre C. BGD is an arc of a-circle with centre A. What is the area of the shaded region?



(a) $100 - 50\pi$ (b) $100 - 257\pi$
(c) $50\pi - 100$ (d) $25\pi - 100$

[Based on MAT 2002]

34. Let A be the area of a square inscribed in a circle of radius 'r' and let B be the area of a hexagon inscribed in the same circle. Then B/A equals

(a) $2\sqrt{3}$ (b) $\frac{3\sqrt{3}}{4}$
(c) $\frac{2\sqrt{3}}{4}$ (d) None of these

[Based on MAT 2002]

35. A steel wire bent in the form of a square of area 121 cm^2. If the same wire is bent in the form of a circle, then the area of the circle is

(a) 130 cm^2 (b) 136 cm^2
(c) 154 cm^2 (d) None of these

36. The dimensions of a rectangular box are in the ratio 1 : 2 : 4 and the difference between the costs of covering it with the cloth and sheet at the rate of Rs. 20 and Rs. 20.5 per square metre respectively is Rs. 126. Find the dimensions of the box

(a) 3 m, 6 m, 12 m (b) 6 m, 12 m, 24 m
(c) 1 m, 2 m, 4 m (d) None of these

[Based on MAT 2002]

37. The number of revolutions made by a wheel of diameter 56 cm in covering a distance of 1.1 km is $\left(\text{Use } \pi = \frac{22}{7}\right)$

(a) 31.25 (b) 56.25
(c) 625 (d) 62.5

[Based on MAT 2001]

38. Semi-circular lawns are attached to the edges of a rectangular field measuring 42 m × 35 m. The area of the total field is

(a) 3818.5 m^2 (b) 8318 m^2
(c) 5813 m^2 (d) 1358 m^2

[Based on MAT 1999]

39. A wire is in the form of a circle of radius 35 cm. If it is bent into the shape of a rhombus, what is the side of the rhombus?

(a) 32 cm (b) 70 cm
(c) 55 cm (d) 17 cm

[Based on MAT 1999]

40. The cross-section of a canal is in the form of a trapezium. If the canal top is 10 m wide and the bottom is 6 m wide, and the area of cross-section is 72 m^2, then the depth of the canal is

(a) 10 m (b) 7 m
(c) 6 m (d) 9 m

[Based on MAT 1999]

41. Two isosceles triangles have equal vertical angles and their areas are in the ratio of 9 : 16. The ratio of their corresponding heights is

(a) 3 : 4 (b) 4 : 3
(c) 2 : 1 (d) 1 : 2

[Based on MAT 1999]

42. A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.

(a) 44 cm (b) 45 cm
(c) 46 cm (d) 48 cm

[Based on MAT 2005]

43. A circular grassy plot of land, 42 m in diameter, has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at Rs. 4 per square metre.

(a) Rs. 2002 (b) Rs. 2003
(c) Rs. 2004 (d) Rs. 2000

[Based on MAT 2005]

44. If the circumference and the area of a circle are numerically equal, then what is the numerical value of the diameter?

(a) 1 (b) 2
(c) 4 (d) π

[Based on MAT 2000]

45. A wire is in the form of a circle of radius 35 cm. If it is bent into the shape of a rhombus, what is the side of the rhombus?

(a) 32 cm (b) 70 cm
(c) 55 cm (d) 17 cm

[Based on MAT 2000]

46. In the figure below, if the perimeter of ΔABC is p, then the perimeter of the regular hexagon is

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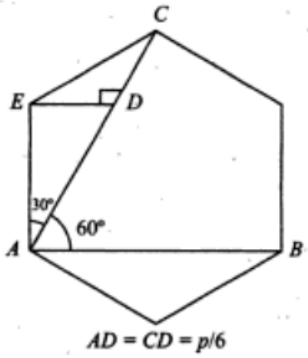


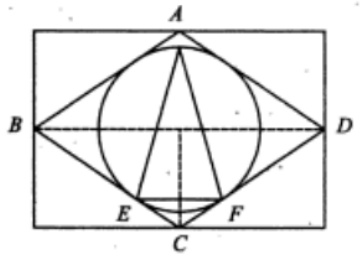
$AD = CD = p/6$

(a) $\frac{3p}{\sqrt{2}}$ (b) $\frac{\sqrt{2}p}{3}$

(c) $\frac{\sqrt{3}p}{2}$ (d) $\frac{2p}{\sqrt{3}}$

[Based on MAT 2001]

47. What is the area of the inner equilateral triangle if the side of the outermost square is 'a'? ($ABCD$ is a square)



(a) $\frac{3\sqrt{3}a^2}{32}$ (b) $\frac{3\sqrt{3}a^2}{64}$

(c) $\frac{5\sqrt{3}a^2}{32}$ (d) $\frac{\sqrt{3}a^2}{16}$

[Based on MAT 2001]

48. The sides of a rectangular field are in the ratio 3 : 4 with its area as 7500 sq. m. The cost of fencing the field @ 25-paise per meter is

(a) Rs. 87.50 (b) Rs. 86.50

(c) Rs. 67.50 (d) Rs. 55.50

[Based on MAT 2001]

49. A square and an equilateral triangle have the same perimeter. If the diagonal of the square is $12\sqrt{2}$ cm, then the area of the triangle is

(a) $24\sqrt{3}$ cm^2 (b) $24\sqrt{2}$ cm^2

(c) $64\sqrt{3}$ cm^2 (d) $32\sqrt{3}$ cm^2

[Based on MAT 2001]

50. The sides of a triangle are 6 cm, 11 cm and 15 cm. The radius of its incircle is

(a) $\frac{5\sqrt{2}}{4}$ cm (b) $3\sqrt{2}$ cm

(c) $6\sqrt{2}$ cm (d) $\frac{4\sqrt{2}}{5}$ cm

[Based on MAT 2001]

51. A horse is tethered to one corner of a rectangular grossy field 40 m by 24 m with a rope 14 m long. Over how much area of the field can it graze?

(a) 154 m^2 (b) 308 m^2

(c) 150 m^2 (d) None of these

[Based on MAT 2001]

52. Find the area in which vegetable is cultivated.

(a) 100 πm^2 (b) 90 πm^2

(c) 120 πm^2 (d) None of these

[Based on FMS (Delhi) 2004]

53. If the sides 50 m and 130 m of the triangular field meet at an angle of 72°, then find the area in which wheat is cultivated.

(a) 120 πm^2 (b) 150 πm^2

(c) 180 πm^2 (d) None of these

[Based on FMS (Delhi) 2004]

54. Four horses are tied on the four corners of a square field of 14 m length so that each horse can just touch the other two horses. They were able to graze in the area accessible to them for 11 days. For how many days is the ungrazed area sufficient for them?

(a) 3 days (b) 4 days

(c) 5 days (d) 2 days

[Based on FMS (Delhi) 2004]

55. The ratio of the area of a square inscribed in a semi-circle to that of the area of a square inscribed in the circle of the same radius is

(a) 2 : 1 (b) 2 : 3

(c) 2 : 5 (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

56. To make a marriage tent, poles are planted along the perimeter of a square field at a distance of 5 metres from each other and the total number of poles used is 20. What is the area (in sq. metres) of the square field?

(a) 500 (b) 400

(c) 900 (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

57. The circumference of a circular ground is 88 metres. A strip of land, 3 metres wide, inside and along circumference of the ground is to be levelled. What is the budgeted expenditure if the levelling costs Rs 7 per square metre?

(a) Rs 1050 (b) Rs 1125

(c) Rs 1325 (d) Rs 1650

[Based on IIT Joint Man. Ent. Test 2004]

58. If the diagonal of a square is decreased by 15%, then the area of the square is decreased by

(a) 22.5% (b) 72.25%

(c) 27.75% (d) None of these

[Based on IIT Joint Man. Ent. Test 2004]

59. The ratio of the circumference of two circles is 2 : 3. What is the ratio of their areas?

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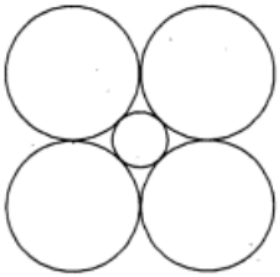
(a) 4 : 9 (b) 2 : 3
(c) 9 : 4 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

60. Student *A* gets 40% of the maximum marks in an examination and gets 10 marks more than the pass marks. Student *B* gets 50% of the maximum marks and gets 50 marks more than the pass marks. Find the pass marks.
(a) 150 (b) 250
(c) 400 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

61. By what percentage will the area or a square change if its side increases by 10%?
(a) 10% increase (b) 21% increase
(c) 10% decrease (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

62. The wheel of a cycle covers 660 metres by making 500 revolutions. What is the diameter of the wheel (in cm)?
(a) 42 (b) 21
(c) 30 (d) 60
[Based on IIT Joint Man. Ent. Test 2004]

63. In the figure, when the outer circles all have radii '*r*', then the radius of the inner circle will be



(a) $\sqrt{2}r$ (b) $(\sqrt{2}-1)r$
(c) $\frac{1}{\sqrt{2}r}$ (d) $\frac{2}{(\sqrt{2}+1)r}$
[Based on IITTM, Gwalior, 2003]

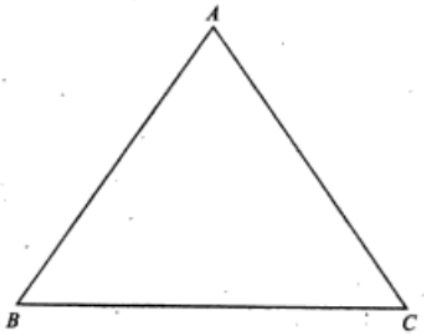
64. If the longer side of a rectangle is doubled and the other is reduced to half, then the area of the new rectangle goes up by
(a) 50% (b) 100%
(c) 0% (d) 150%
[Based on IITTM, Gwalior, 2003]

65. The front wheels of a wagon are 2π feet in circumference and the back wheels are 3π feet in circumference. When the front wheels have made 10 more revolutions than the back wheels, how many feet has the wagon travelled?
(a) 30π (b) 90π
(c) 60π (d) 150π
[Based on REC Tiruchirapalli 2003]

66. What is the radius of the largest circular disc that can be cut from a strip of metal 15″ × 21″?
(a) 7″ (b) 7.5″
(c) 15″ (d) 15.5″
[Based on REC Tiruchirapalli 2003]

67. Walter drew a square. He then erased it and drew a second -square whose sides were 3 times the sides of the first square. By what percent was the area of the square increased?
(a) 300% (b) 900%
(c) 800% (d) 600%
[Based on REC Tiruchirapalli 2003]

68. In the above figure, what is the minimum distance, in metres, that a person would have to walk to go from the point *A* to a point on the side *BC*?



(a) 80 (b) 60
(c) 100 (d) 140
Note : The diagram is not drawn to scale
[Based on REC Tiruchirapalli 2003]

69. A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?
(a) 46 (b) 81
(c) 126 (d) 252
[Based on REC Tiruchirapalli 2003]

70. A wheel makes 1000 revolutions in covering a distance of 88 km. The diameter of the wheel is
(a) 24 m (b) 40 m
(c) 28 m (d) 14 m
[Based on FMS (Delhi) 2003]

71. The area of a rectangle is given by $A = 8x^2 - 2x - 15$. If the length was larger than the breadth, then it is
(a) $(3x + 5)$ (b) $(5x + 3)$
(c) $(4x + 5)$ (d) $(2x - 3)$
[Based on FMS (Delhi) 2003]

72. The diagonal of the floor of a rectangular closet is $7\frac{1}{2}$ feet. The shorter side of the closet is $4\frac{1}{2}$ feet. What is the area of the closet in square feet?
(a) 37 (b) 27

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109. The length, breadth and height of a room are in the ratio, 3 : 2 : 1. If the breadth and height are halved and the length is doubled, then the total area of the four walls of the room will

(a) remain the same
(b) decrease by 13.64%
(c) decrease by 15%
(d) decrease by 18.75%
(e) decrease by 30%

[Based on CAT 2006]

110. Two small circular parks of diameter 16 m and 12 m are to be replaced by a bigger circular park. What would be the radius of this new park, if the new park has to occupy the same space as the two small parks?

(a) 15 m (b) 10 m
(c) 20 m (d) 25 m

[Based on MAT 2008]

EXPLANATORY ANSWERS

1. Ans. (c)

Solution Each side of the given polygon is parallel to either x or y axis.

In this part particular case, sum of interior (either concave or convex) angles must be a multiple of 180°.

Let the number of concave corners be K.

$\therefore$ $(90 \times 25 + k \times 270)$, the sum of interior angles must be a multiple of 180°.

From the options, only $k = 21$ satisfies.

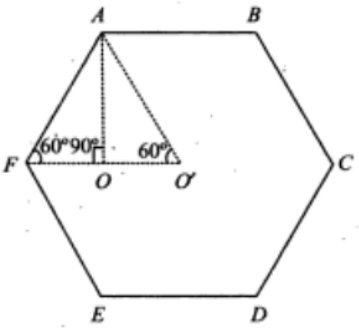
$\therefore$ $k = 21$.

In general (Quicker Method): If the number of convex polygon corners in such a polygon is n, then number of concave corners is $(n - 4)$.

$\therefore$ number of concave corners $= 25 - 4 =$ **21.**

2. Ans. (a)

Solution Let O' be the centre of the Regular Hexagon.



Since OF is parallel to ED,

$$\therefore \quad \angle AFO = \frac{1}{2}\angle AFE = \frac{1}{2} \times 120^\circ$$

(each interior angle of Hexagon is of 120°)

Joining vertices A and F to the centre of Regular Hexagon O'.

$$\angle AO'F = \frac{360^\circ}{6} = 60^\circ$$

$$\text{Area of Triangle } AFO = \frac{1}{2} \text{ Area of } \Delta AFO'$$

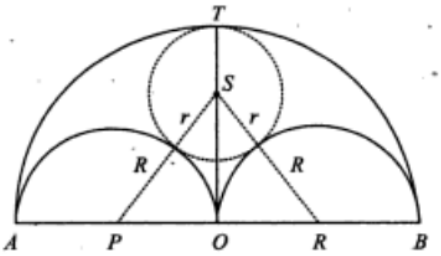
and, $$\text{Area of } \Delta AFO' = \frac{1}{6} \text{ Area of Regular Hexagon.}$$

$$\therefore \text{ Area of } \Delta AFO = \frac{1}{2} \times \frac{1}{6} \text{ Area of Regular Hexagon.}$$

$$= \frac{1}{12} \textbf{ area of Regular Hexagon.}$$

3. Ans. (b)

Solution Lines PS and RS pass through points of contacts of the circles.



In ΔPSR, triangle is isosceles.

$$PS = RS = (R + r)$$

Line TO is perpendicular bisector of AB.

$$SO = TO - TS = 2R - r$$

$$\therefore PS^2 = PO^2 + SO^2 \Rightarrow (R + r)^2 = R^2 + (2R - r)^2$$

$$\Rightarrow R^2 + r^2 + 2Rr$$

$$= R^2 + 4R + r^2 - 4Rr$$

$$\Rightarrow 4R^2 - 6Rr = 0$$

$$\Rightarrow 2R(2R - 3r) = 0$$

$$\Rightarrow R = \frac{3r}{2}$$

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$$\text{Total grazed area} = \frac{\pi R^2}{2} + \frac{\pi R^2}{2} + \pi\left(\frac{2R}{3}\right)^2$$

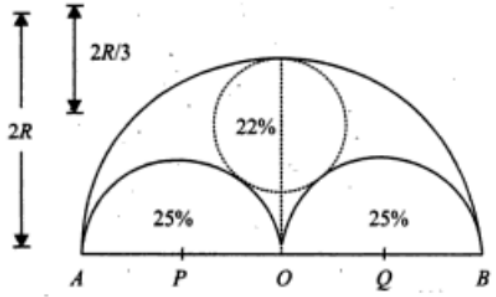
$$= \pi R^2 + \frac{4}{9}\pi R^2$$

$$\text{Ungrazed area} = \frac{\pi(2R)^2}{2} - \pi R^2 - \frac{4}{9}\pi R^2$$

$$= \frac{5\pi R^2}{9}$$

$$\% \text{ of ungrazed area} = \frac{5/9\,\pi R^2}{2\pi R^2} \times 100 = \frac{500}{18} \approx \mathbf{28\%}.$$

Alternative Method: Roughly taking the radius of smallest circle having radius r as 1/3 rd of radius of biggest circle (i.e. radius of semicircular field)



$$\text{Ratio} = \text{radius} = 2R : R : R : \frac{2R}{3}$$

$$\text{Ratio of area} = 4:1:1:2\left(\frac{4}{9}\right) = 4:1:1:\frac{8}{9}$$

$$= 100 : 25 : 25 : 22.$$

$$\% \text{ urgrazed area} = 100\% - (25\% + 25\% + 22\%)$$

$$= \mathbf{28\%}.$$

4. Ans. (b)

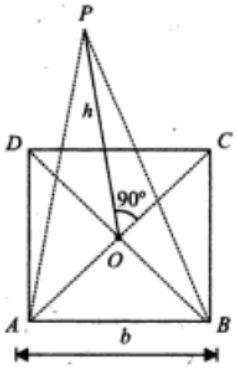
Solution It is a three-dimensional figure.

$$OA = \frac{AC}{2} = \frac{\sqrt{2}b}{2} = \frac{b}{\sqrt{2}}$$

$$\angle APB = \angle PAB = \angle PBA = 60^\circ$$

Triangle APB is equilateral

$\therefore$ $AP = b$



And, by Pythagoras theorem

$$(OA)^2 + (OP)^2 = (AP)^2 \Rightarrow \left(\frac{b}{\sqrt{2}}\right)^2 + (h)^2 = b^2$$

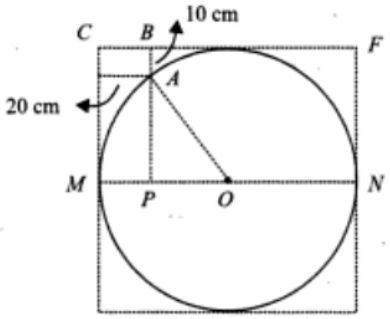
$$\Rightarrow \frac{b^2}{2} + h^2 = b^2$$

$$\Rightarrow \frac{b^2}{2} = h^2 \Rightarrow \mathbf{b^2 = 2h^2}.$$

5. Ans. (c)

Solution Rectangle at the corner measures 10cm × 20cm

We have to find the radius of the circle.



Let MN diameter be equal to $2R$.

BA is extended to cut MN at P, orthogonally

$CF \parallel MN$

$CM \parallel BP$

$MP = AD = BC = 20$

$\therefore$ $OP = OM - MP = R - 20$

and $AP = BP - AB = R - 10$ $(BP = OM = R)$

In right-angled triangle APO

$$(AP)^2 + (OP)^2 = (OA)^2$$

$\Rightarrow$ $(R - 10)^2 + (R - 20)^2 = R^2$

$\Rightarrow$ $R^2 - 20R + 100 + R^2 - 40R + 400 = R^2$

$\Rightarrow$ $R^2 - 60R + 500 = 0$

$\Rightarrow$ $(R - 50)(R - 10) = 0$

$\Rightarrow$ $R = 50, 10$

$\therefore$ $\mathbf{R = 50}$ **cm (**$\mathbf{R = 10}$**cm is not possible).**

6. Ans. (a)

Solution Area of the square $= a^2 \text{unit}^2$

In the first process: Area of each small square $= \frac{a^2}{n}$ unit^2

(for convenience we assume all n squares to be of equal area)

$\therefore$ Side of each small square $= \sqrt{\frac{a^2}{n}}$ unit

$= \frac{a}{\sqrt{n}}$ unit.

Radius of circular cloth pieces of maximum possible area from each of the small square $= \frac{a}{2\sqrt{n}}$ units.

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Sum of area of all n circular cloth pieces

$$= n \times \pi\left(\frac{a}{2\sqrt{n}}\right)^2 = n \times \frac{\pi a^2}{4n} = \frac{\pi a^2}{4}$$

The area of cloth piece remaining after cutting the circles $= a^2 - \frac{\pi a^2}{4} = \mathbf{a^2\left(1-\frac{\pi}{4}\right)\ unit^2}$

In the second process: Radius of cloth piece having maximum possible area from the given square cloth piece $= \frac{a}{2}$ units

Area of circular cloth piece $= \pi\left(\frac{a}{2}\right)^2 = \pi\frac{a^2}{4}$ unit^2

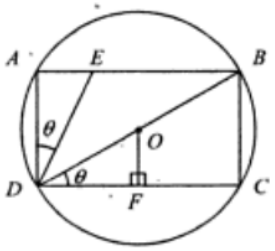
The area of cloth piece remaining after cutting the circle $= a^2 - \frac{\pi a^2}{4} = a^2\left(1-\frac{\pi}{4}\right)$

Required ratio $= \frac{a^2\left(1-\frac{\pi}{4}\right)}{a^2\left(1-\frac{\pi}{4}\right)} = \frac{1}{1} = \mathbf{1 : 1.}$

7. Ans. (a)

Solution We have to find the ratio of AE to AE, i.e. $\tan\theta = \frac{AE}{AD}$.

(Assuming $\angle ODC = \angle ADE = \theta$)



Let r be the radius of the circle

Also, let the length and the breadth of the rectangle be l and b. Then,

$$l^2 + b^2 = (2r)^2 \Rightarrow l^2 + b^2 = 4r^2 \quad ...(i)$$

According to the question,

$$\frac{\pi r^2}{lb} = \frac{\pi}{\sqrt{3}} \Rightarrow lb = \sqrt{3}r^2 \quad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{l^2}{lb} + \frac{b^2}{lb} = \frac{4r^2}{\sqrt{3}r^2} \Rightarrow \frac{l}{b} + \frac{b}{l} = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \frac{l}{b} = \sqrt{3} \Rightarrow \frac{b}{l} = \frac{1}{\sqrt{3}}$$

We have to find the value of $\frac{AE}{AD}$.

And $\frac{AE}{AD} = \tan\theta = \frac{b}{l} = \mathbf{\frac{1}{\sqrt{3}}}$.

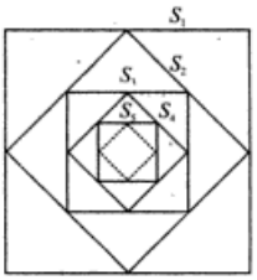
8. Ans. (c)

Solution Each side of $S_1 = a$ units

Each side of $S_2 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$ units

Each side of $S_3 = \sqrt{\left(\frac{a}{2\sqrt{2}}\right)^2 + \left(\frac{a}{2\sqrt{2}}\right)^2}$

$$= \sqrt{\frac{2a^2}{8}} = \frac{a}{2} \text{ units}$$



Each sides of $S_4 = \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{a}{4}\right)^2}$

$$= \sqrt{\frac{2a^2}{16}} = \frac{a}{2\sqrt{2}} \text{ unit}$$

and so on

$$P_1 + P_2 + P_3 + \ldots = 4a + \frac{4a}{\sqrt{2}} + \frac{4a}{2} + \frac{4a}{2\sqrt{2}}$$

$$= \frac{4a}{1-\frac{1}{\sqrt{2}}} = \frac{4a \times \sqrt{2}}{(\sqrt{2}-1)}$$

$$= \frac{4\sqrt{2} \times a}{(\sqrt{2}-1)}$$

$$A_1 + A_2 + A_3 + \ldots = a^2 + \frac{a^2}{2} + \frac{a^2}{4} + \frac{a^2}{8} + \ldots$$

$$= \frac{a^2}{1-\frac{1}{2}} = 2a^2$$

$$\therefore \frac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots} = \frac{4a\sqrt{2}}{(\sqrt{2}-1)2a^2}$$

$$= \frac{2\sqrt{2}}{(\sqrt{2}-1)a}$$

$$= \frac{2\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}+1)(\sqrt{2}-1)a}$$

$$= \frac{4+2\sqrt{2}}{a} = \mathbf{\frac{2(2+\sqrt{2})}{a}}.$$

9. Ans. (b)

Solution $ABCDEF$ is a regular hexagon. Joining the centre O with vertices A, C and E, we get

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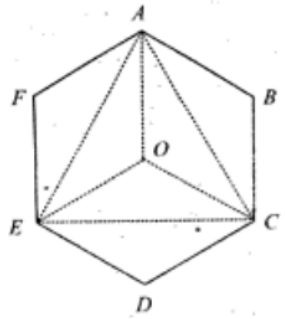


$\Delta AFE = \Delta AOE$

Similarly, $\Delta OAC = \Delta BAC$

Also, $\Delta OEC = \Delta DEC$

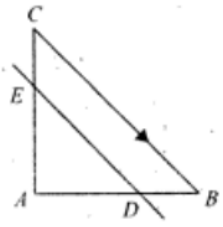
$\therefore \Delta ACE = \frac{1}{2}$ **the area of regular hexagon.**

10. Ans. (d)

Solution Given $DE \parallel BC$

$$\frac{\Delta ADE}{\Delta ABC} = \frac{DE^2}{BC^2}$$

$$DE = BC\left(1 - \frac{35}{100}\right) = BC(0.65)$$



$$\therefore \quad \frac{DE}{BC} = 0.65 \Rightarrow \left(\frac{DE}{BC}\right)^2 = 0.4225$$

$$\Rightarrow \frac{\Delta ADE}{\Delta ABC} = 0.4225$$

$$\Rightarrow \Delta ADE = 34 \times 0.4225$$

$$= \mathbf{14.365\ (inch)^2.}$$

Other Method: % change in area

$$= -35 - 35 + \frac{(-35)(-35)}{100} = -57.75$$

$\Rightarrow$ Area of smaller Δ is

(100 – 57.75) % of 34 sq inch

= 14.365 sq inch.

11. Ans. (d)

12. Ans. (d)

Solution Sides of a triangle are in ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, i.e. 6 : 4 : 3.

Let the sides be $6K$, $4K$ and $3K$ respectively.

$\therefore \quad 13K = 52 \Rightarrow K = 4$

$\therefore$ Sides of the triangle are 24 cm, 16 cm and 12 cm respectively.

13. Ans. (c)

Solution Perimeter of the circular track

$$= 2\pi r = 2 \times \frac{22}{7} \times 70 = \mathbf{440\ meters.}$$

14. Ans. (c)

Solution $\pi \times (52)^2 - \pi \times (50)^2 = \mathbf{204\pi\ sq.m.}$

15. Ans. (a)

Solution Let Length = Width = 100 metres

If length = 160 meters, then let the Width = x metres, such that $160x = 10000$

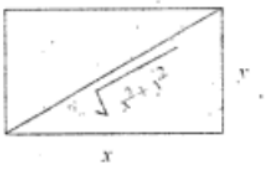
$$\Rightarrow \quad x = \frac{1000}{16} = 62\frac{1}{2}$$

$\therefore$ Width is reduced by $\mathbf{37\frac{1}{2}\%}$.

16. Ans. (b)

Solution $\therefore \sqrt{x^2 + y^2} + x = 5y$ ...(i)

Also $xy = 60$ ...(ii)



From (i)

$\Rightarrow \quad x^2 + y^2 = (5y - x)^2 = 25y^2 + x^2 - 10xy$

$\Rightarrow \quad 24y^2 = 600 \Rightarrow y = 5$

$\therefore \quad x = 12 =$ **Lenght of the carpet.**

17. Ans. (c)

Solution Percent increase in area

$$= \frac{140 \times 30 - 100 \times 100}{100 \times 100} \times 100 = \mathbf{82.}$$

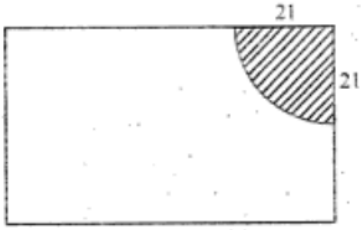
18. Ans. (a)

Solution Area of the circle formed by the path of the cow $= \pi r^2$, where r is the radius of the circle.

$\therefore \quad \pi r^2 = 9856 \Rightarrow \boldsymbol{r = 56.}$

19. Ans. (c)

Solution $\frac{1}{4} \times \pi \times 21^2 = \frac{22}{7} \times \frac{21 \times 21}{4} = \mathbf{346.5 m^2}$



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20. Ans. (d)

21. Ans. (b)

Solution Area $= \frac{1}{2} \times$ Product of the diagonals $= 2016$

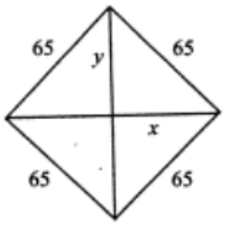
$\Rightarrow \frac{1}{2} \times$ Product of the diagonals $= \frac{1}{2}(2x \times 2y) = 2xy$

and $2xy = 2016 \Rightarrow xy = 1008$

Also, $x^2 + y^2 = 65^2 = 4225$

$\therefore \quad (x - y)^2 = x^2 + y^2 - 2xy$

$= 4225 - 2016 = 2209$



$\therefore \quad x - y = 47$

Now $(x + y)^2 = (x - y)^2 + 2xy = 6241$

$\Rightarrow \quad x + y = 79$

$\therefore \quad x = 63, y = 16$

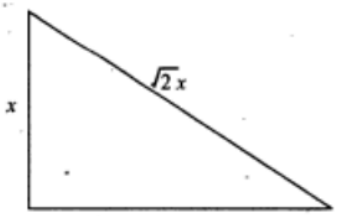
Thus the diagonals are **126** and **32.**

22. Ans. (c)

23. Ans. (a)

Solution Perimeter $= x + x + \sqrt{2}x$

$= 2x + \sqrt{2}x = 6 + 3\sqrt{2}$



$\Rightarrow \quad x = \frac{6+3\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$

$= \frac{12-6\sqrt{2}+6\sqrt{2}-6}{2} = 3$

$\therefore \quad$ Area $= \frac{1}{2} \times x \times x = \frac{1}{2}x^2 = \frac{9}{2} =$ **4.5 m².**

24. Ans. (b)

25. Ans. (b)

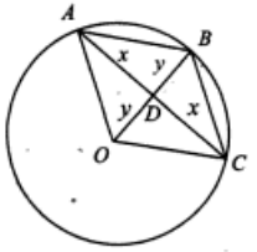
Solution Let $AC = 2x$, $OB = 2y$

$\therefore$ Radius $= OC = 2y = OB$

From ΔODC,

$OC^2 = OD^2 + CD^2$

i.e. $\quad 4y^2 = y^2 + x^2 \Rightarrow x^2 = 3y^2 \Rightarrow x = \sqrt{3}y$ ...(i)



Also, area of the ΔODC

$= \frac{1}{4} \times$ Area of the rhombus $OABC$

$\Rightarrow \quad \frac{1}{2} \times x \times y = \frac{1}{4} \times 32\sqrt{3}$ (Given)

$\Rightarrow \quad xy = 16\sqrt{3}$ ...(ii)

From (i) and (ii), we have

$y = 4, x = 4\sqrt{3}$

$\therefore$ Radius of the circle $= 2y =$ **8.**

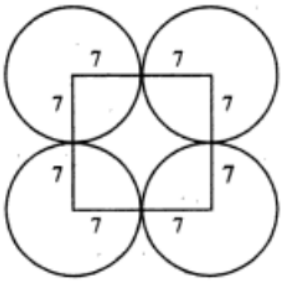
26. Ans. (b)

Solution Area of the four quadrants

= Area of the circle with radius 7

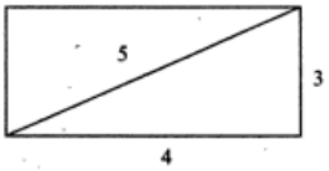
$= \pi \times (7)^2 = 154$ sq. cm.

$\therefore$ Required Area $= 196 - 154 =$ **42 sq. cm**



27. Ans. (c)

Solution



28. Ans. (c)

29. Ans. (c)

30. Ans. (d)

Solution Let r_1 be the radius of the outer circumference and r_2 be the radius of the inner circumference.

$\therefore \quad 2\pi r_2 = 352, 2\pi r_1 = 396 \Rightarrow r_2 = 56, r_1 = 63$

$\therefore \quad$ Width of the circular track $= r_1 - r_2 =$ **7 m.**

31. Ans. (a)

32. Ans. (b)

Solution Let width of the garden be x metres

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Here, $2s = 6 + 11 + 15 = 32$

i.e. $s = 16$

$$\Delta = \sqrt{16(16-6)(16-11)(16-15)}$$
$$= \sqrt{16\times10\times5\times1} = 20\sqrt{2}$$

$$\therefore \quad r = \frac{20\sqrt{2}}{16} = \frac{5\sqrt{2}}{4}.$$

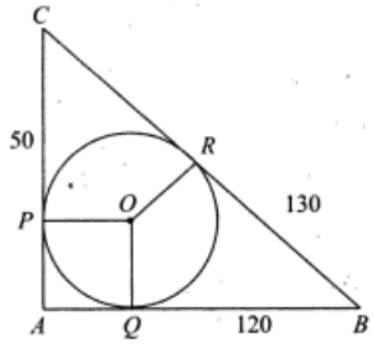
51. Ans. (a)

Solution $\frac{1}{4}\times\pi\times(14)^2 = \frac{1}{4}\times\frac{22}{7}\times14\times14$

$= \mathbf{154\ sq\ m.}$

52. Ans. (a)

Solution Let O be the centre of the circle drawn touching all the sides of the triangular field. Let OP, OQ and OR be the perpendiculars from the centre on AC, AB and BC respectively.



Area of the triangle $= \frac{1}{2}\times120\times50 = 3000$ sq. m

Perimeter of the triangle $= 2S = a + b + c$

$$\Rightarrow \quad S = \frac{130+120+50}{2} = 150$$

Let a, b, c be the sides of the given triangular field.

$\therefore\ a = 50$ m, $b = 120$ m, $c = 130$ m

Since $c^2 = a^2 + b^2$, therefore the given field is right-angled.

Area of the circle $= \pi r^2$, where $r = \frac{\Delta}{s}$

$$= \pi\frac{\Delta^2}{s^2} = \frac{\pi\times(3000)^2}{(150)^2} = 400\pi$$

$\therefore \quad r = 20.$

[$\because$ If ABC is a triangle circumscribing the circle with centre O and the sides of the triangle are a, b, and c; D, E, F are the points of contact of the sides with the circle, then $r = \frac{\Delta}{s}$, where Δ is the area of the triangle and s is the semiperimeter of the triangle]

From quadrilateral $CPOR$ and $QORB$, we find that angles ROQ and ROP are greater than 90°. Hence out of the three areas, the least will be the area of the segment $POQ = \frac{90}{360}\times400\pi = 100\pi$.

As the vegetables are cultivated in the smallest portion, therefore the required area is $\mathbf{100\pi\ m^2}$.

53. Ans. (c)

Solution Using the same figure as in Q 52, we have $\angle ACB = 72°$ and $\angle ABC = 18°$.

In quadrilateral $CPOR$,

$\angle POR = 180° - 72° = 108°$

In quadrilateral $QORB$,

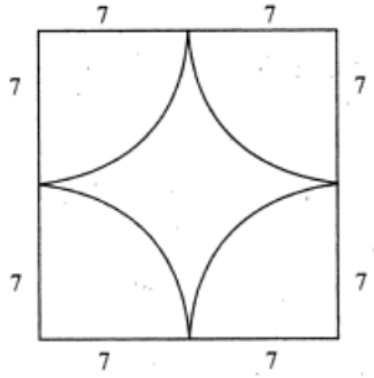
$\angle QOR = 180° - 18° = 162°$

The wheat is cultivated in the largest portion (which is equal to the segment ORQ)

$$= \frac{162}{360}\times400\pi = \mathbf{180\pi m^2}.$$

54. Ans. (a)

Solution Area of the square $= 14 \times 14 = 196$ sq.m.



Area to be grazed $= \pi\times7^2 = 49\pi$ sq.m

$\therefore$ Ungrazed area $= 196 - 49\pi$

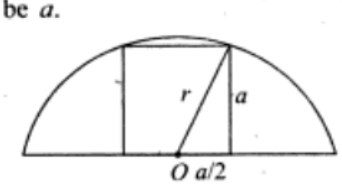
Area of 49π sq. m is sufficient to graze for 11 days.

$\therefore$ Area of $(196 - 49\pi)$ sq. m is sufficient to graze for

$$\frac{11}{49\pi}\times(196-49\pi) = \frac{44}{\pi} - 11$$
$$= 14 - 11 = \mathbf{3\ days.}$$

55. Ans. (c)

Solution Let the side of the square inscribed in a semi circle be a.



Then radius $r = \sqrt{a^2 + \frac{a^2}{4}}$

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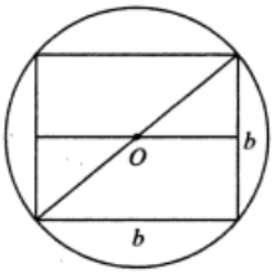
$$\Rightarrow \quad r^2 = a^2 + \frac{a^2}{4} = \frac{5a^2}{4} \qquad ...(i)$$

Let the side of the square inscribed in the circle of radius r be b. Then



$$(2r)^2 = b^2 + b^2 \Rightarrow r^2 = \frac{b^2}{2} \qquad ...(ii)$$

From (i) and (ii)

$$\frac{5a^2}{4} = \frac{b^2}{2} \Rightarrow \frac{a^2}{b^2} = \frac{2}{5}$$

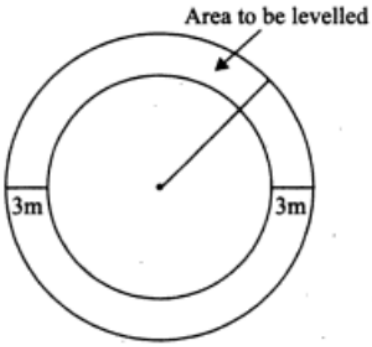
∴ The required ratio = **2 : 5.**

56. Ans. (d)

Solution 625 m^2. Each side of the square field will be **25 metres.**

57. Ans. (d)

Solution Let r be the radius of the circular ground.



∴ $2\pi r = 88 \Rightarrow r = 14$

∴ Area of the ground to be levelled

$= \pi \times 14^2 - \pi \times 11^2$

$= 196\pi - 121\pi = 75\pi$

Cost of levelling $= 75 \times \frac{22}{7} \times 7$ = **Rs 1650.**

58. Ans. (c)

59. Ans. (a)

Solution $\frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{2}{3} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{4}{9} \Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{9}$.

60. Ans. (a)

Solution Let the maximum marks be x.

∴ Pass marks = 40% of x − 10 = 50% of x − 50

$\Rightarrow$ 10% of x = 40 $\Rightarrow x$ = 400.

∴ Pass marks = **150.**

61. Ans. (b)

Solution If side is 10 cm, then area = 100 cm^2

If side is 11 cm, then area = 121 cm^2

[After increasing the side by 10%]

∴ Percent increase in area = **21.**

62. Ans. (a)

Solution $2\pi r$ = Circumference of the wheel

$= \frac{660 \times 100}{500} = 132$

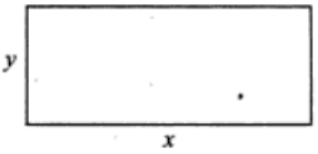
$\Rightarrow \quad r = 132 \times \frac{7}{44} = 21$

∴ Diameter = **42 cm.**

63. Ans. (b)

64. Ans. (c)

Solution Area of the given rectangle = xy



If length = $2x$, width = $\frac{1}{2}y$ then area = $2x \times \frac{1}{2}y$ = **xy.**

65. Ans. (c)

Solution Suppose the back wheel has made x revolutions.

∴ Front wheel has made $(10 + x)$ revolutions.

$\Rightarrow \quad 3\pi x = 2\pi(10 + x)$

$\Rightarrow \quad \pi x = 2\pi \times 10 \Rightarrow x = 20$

∴ The wagon has travelled $3\pi x$ = **60π.**

66. Ans. (b)

67. Ans. (c)

68. Ans. (b)

Solution $\sqrt{(100)^2 - (80)^2}$ = **60 m.**

69. Ans. (c)

Solution Length = 14 feet, Width = **9 feet.**

70. Ans. (c)

Solution Let diameter of the wheel be $2r$.

∴ $2\pi r \times 1000 = 88000$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 1000 = 88000$

$\Rightarrow \quad r = 14$

∴ Diameter of the wheel = **28 m.**

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Required area = Area of the triangle $-3\left(\frac{\pi L^2}{6}\right)$,

where $\frac{\pi L^2}{6}$ is the area of each circle inside the triangle

$$= \sqrt{3}L^2 - \frac{\pi L^2}{2} = \mathbf{1.967\ cm^2}$$

[Area of the equilateral triangle = $\frac{\sqrt{3}}{4}a^2$, a being the side].

89. Ans. (b)

Solution $\frac{2\pi r}{2r}$ = Ratio of the circumference to the diameter of the circle = π = Independent of the radius.

90. Ans. (d)

Solution Area of the road

$$= 150 \times 2 + 120 \times 2 - 2 \times 2$$
$$= 300 + 240 - 4 = \mathbf{536\ sqm.}$$

91. Ans. (a)

Solution Let x be the length and y be the breadth of the rectangle intially.

$\therefore \quad xy = (x - r\%\text{ of } x)\,[y + (r + 5)\%\text{ of } y]$

$\Rightarrow \quad xy = \frac{(100-r)x}{100} \times \frac{[100+(r+5)]y}{100}$

$\Rightarrow \quad \mathbf{r = 20.}$

92. Ans. (a)

Solution Obviously the sides of the triangle in question are 4, 5, and 6.

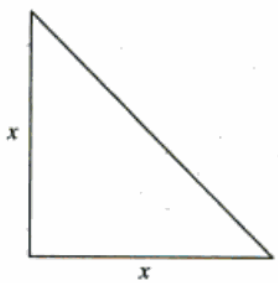
$\therefore$ Area of the triangle

$$= \sqrt{\frac{15}{2}\left(\frac{15}{2}-4\right)\left(\frac{15}{2}-5\right)\left(\frac{15}{2}-6\right)}$$
$$= \sqrt{\frac{15}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}} = \frac{\mathbf{15\sqrt{7}}}{\mathbf{4}}.$$

93. Ans. (a)

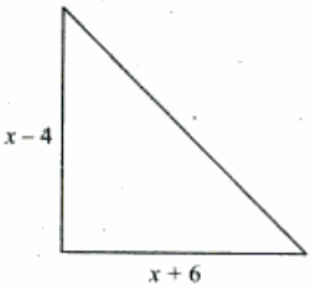
94. Ans. (e)

Solution Let x be the length of the leg of the right-angled isosceles triangle, originally.



Its area = $\frac{1}{2}x^2$

The area of the new triangle = $\frac{1}{2}(x-4)(x+6)$



$\therefore \quad \frac{1}{2}x^2 = \frac{1}{2}(x-4)(x+6) + 24$

$\Rightarrow \quad x^2 = x^2 + 2x - 24 + 48$

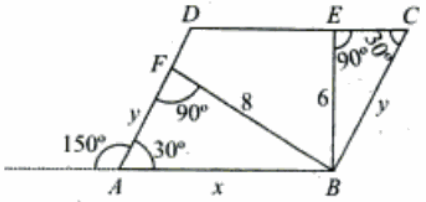
$\Rightarrow \quad \mathbf{x = -12.}$

95. Ans. (a)

96. Ans. (c)

Solution From ΔBEC,

$$\frac{6}{y} = \sin 30^\circ = \frac{1}{2} \Rightarrow y = 12$$



From ΔAFB,

$$\frac{8}{x} = \sin 30^\circ = \frac{1}{2} \Rightarrow x = 16.$$

$\therefore$ Perimeter of the $\|_{gm}$ $ABCD = 2x + 2y = \mathbf{56.}$

97. Ans. (c)

Solution $\pi r^2 = 9\pi$, where r is the radius of the circle

$\Rightarrow \quad r = 3$, $2r$ = diameter = 6,

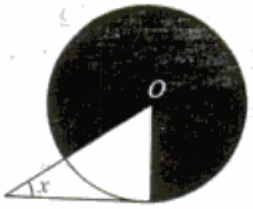
Circumference = $2\pi r = \mathbf{6\pi.}$

98. Ans. (d)

Solution Area of the circle = $\pi r^2 = 16\pi \quad (\because r = 4)$

$\therefore$ 360° covers an area of 16π

1° covers an area of $\frac{16\pi}{360^\circ} = 8$



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$\left(\frac{\pi}{2}-x\right)^{\circ}$ [unshaded region] covers an area of $8\left(\frac{\pi}{2}-x\right)$, i.e. $4\pi - 8x$.

Area of the shaded region = 14π

$\therefore$ Area of the unshaded = 2π

$\therefore$ $4\pi - 8x = 2\pi \Rightarrow 8x = 2\pi \Rightarrow x = \frac{\pi}{4} = \mathbf{45^{\circ}}$.

99. Ans. (a)

Solution Let the area of the circle be $A = \pi r^2$, where r is the radius. If r is increased to $r + 8\%$ of r,

i.e. $\frac{27r}{25}$, therefore area becomes

$$\pi \times \left(\frac{27}{25}r\right)^2 = \pi \times \frac{729}{625}r^2$$

$\therefore$ Increase in area = $\frac{104}{625}\pi r^2$.

$\Rightarrow$ Percent increase in area = $\frac{104}{625} \times 100 = \mathbf{16.64}$.

100. Ans. (b)

Solution Total area = $14 \times 14 = 196$ m^2

$$\text{Grazed area} = \left(\frac{\pi \times r^2}{4}\right) \times 4$$

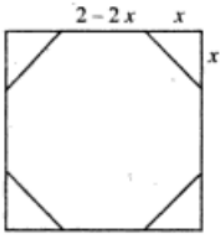
$= \pi r^2 = 22 \times 7$ $(r = 7)$

$= 154$ m^2

Ungrazed area is less than $(196 - 154) = 42$ m^2, for which there is only one option.

101. Ans. (a)

Solution Let the position of the edge cut at each corner be x m. Since the resulting figure is a regular octagon,



$\therefore$ $\sqrt{x^2 + x^2} = 2 - 2x \Rightarrow x\sqrt{2} = 2 - 2x$

$\Rightarrow \sqrt{2}x(1 + \sqrt{2}) = 2$

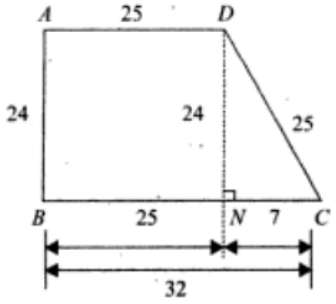
$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{2}+1}$.

102. Ans. (b)

103. Ans. (d)

Solution Area of $ABND$ $= AB \times AD = 25 \times 24$

$= 600$ sq m



Area of $\Delta DNC = \frac{1}{2} NC \times ND = \frac{1}{2} \times 7 \times 24 = 84$ m^2

$\therefore$ Area of $ABCD$ = area $(ABND)$ + area of ΔDNC

$= \mathbf{684\ m^2}$

104. Ans. (a)

Solution Let the third side be x. Then

$$s = \frac{20+10+x}{2} = \frac{35+x}{2}$$

$$\text{Area} = \sqrt{s(s-20)(s-10)(s-x)}$$

$$80 = \sqrt{\left[\frac{30+x}{2}\right]\left[\frac{x-10}{2}\right]\left[\frac{x+10}{2}\right]\left[\frac{30-x}{2}\right]}$$

$$= \sqrt{\left[\frac{30+x}{2}\right]\left[\frac{x-10}{2}\right]\left[\frac{x+10}{2}\right]\left[\frac{30-x}{2}\right]}$$

$$= \frac{1}{4}\sqrt{(30^2 - x^2)(x^2 - 10^2)}$$

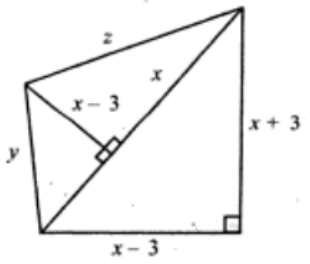
$\Rightarrow$ $320 = \sqrt{(900 - x^2)(x^2 - 100)}$

$\Rightarrow$ $102400 = (900 - x^2)(x^2 - 100) \Rightarrow x^2 = 260$

$\Rightarrow$ $x = \sqrt{260}$.

105. Ans. (b)

Solution We can find the value of x, using the answer choices given in the question. We put the choices (a), (b), (c) and (d) individually in the figure and find out the consistency of the figure. Only (b), i.e., 11 is consistent with the figure.



106. Ans. (d)

Solution Area of rhombus

$\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 24 \times 10 = 120 \text{cm}^2$

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sq m.
(a) Rs. 700 (b) Rs. 960
(c) Rs. 750 (d) Data inadequate

9. The area of a square and a rectangular field is 900 m^2. If the perimeter of the rectangular field is 2 m more than that of the square field, calculate the dimensions of the rectangular field.
(a) 24 m, 10 m (b) 30 m, 25 m
(c) 36 m, 25 m (d) 36 m, 24 m

10. The length of a room is 6.5 m. The cost of painting the walls at Rs. 56 m^2 is Rs. 4928 and the cost of carpeting the room at Rs. 224 m^2 is Rs. 6552. Find the height and width of the room.
(a) 4 m, 5 m (b) 4.5 m, 5.5 m
(c) 4 m, 4.5 m (d) Data inadequate

Test II

[Time: 10 min]

11. The cross-section of a canal is a trapezium in shape. If the canal is 10 m wide at the top and 6 m wide at the bottom and the area of cross-section is 640 m^2, the length of canal is
(a) 40 m (b) 80 m
(c) 160 m (d) 384 m

12. The expenditure incurred in carpeting a hall 28 m long with a carpet $1\frac{1}{3}$ m wide at Rs. 3.50 per m is Rs. 1176. Find the breadth of the hall.
(a) 16 m **(b) 24 m**
(c) 12 m (d) 36 m

13. The length of a rectangular floor is twice its width. If the length of a diagonal is $9\sqrt{5}$ m, then perimeter of the rectangle is
(a) 27 m (b) 54 m
(c) 81 m (d) 162 m

14. There is a regular polygon of 12 sides. Find the sum of interior angles and the value of each interior angle.
(a) $10\pi, \frac{5}{6}\pi$ (b) 12p, $\frac{8}{9}\pi$
(c) $6\pi, \frac{5}{6}\pi$ (d) Data inadequate

15. Length of a square is increased by 8 cm. Its area becomes 208 sq cm. Find its perimeter.
(a) 108 cm (b) 112 cm
(c) 104 cm (d) 116 cm

16. A path all around the inside of a rectangular park 37 m by 30 m occupies 570 sq m. Find the width of the path.
(a) 10 m (b) 15 m
(c) 20 m (d) 5 m

17. There is a rectangular field of area 60 cm^2. Sum of its diagonal and length is 5 times of its breadth. Find the breadth of the rectangular field
(a) 10 m (b) 5 cm
(c) 20 m (d) 15 m

18. A rectangular grassy plot is 112 m by 78 m. It has a gravel path 2.5 m wide all around it on the inside. Find the cost of constructing it at Rs. 2 per sq m?
(a) Rs. 1749 (b) Rs. 2065
(c) Rs. 1850 (d) Rs. 1959

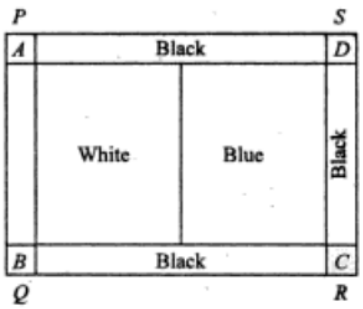
19. Two square rooms, one a m longer each way than the other, are of equal height, and cost respectively Rs. 33600 and Rs. 35280 to paper the walls at Rs. 70 per sq m. Find the height
(a) 6 m (b) 8 m
(c) 5 m (d) 4 m

20. A table cloth 4 m × 2 m is spread on a table. Calculate the cost of polishing the top of the table at Rs. 2.25 per sq m. If 25 cm of the table cloth is hanging all round the table.
(a) Rs. 13.71 (b) Rs. 15.61
(c) Rs. 11.81 (d) Rs. 14.63

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (a)

Solution Area covered by black tiles
$= (20 + 20) \times 2 + (6 + 6) \times 2 = 80 + 24$
$= 104$ sq ft.
Area of the floor $PQRS = 20 \times 10 = 200$ sq ft.
$\therefore$ Remaining area $= 200 - 104 = 96$ sq ft.
$\therefore$ Area covered by while tiles $= \frac{1}{3} \times 96 = 32$ sq ft.
$\therefore$ Area covered by blue tiles $= 96 - 32 = 64$ sq ft.



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$\therefore$ No. of blue – coloured tiles $= \frac{64}{2\times 2} =$ **16.**

3. Ans. (c)

Solution Area of the circular park $= \pi r^2 = \frac{22}{7} \times 21 \times 21$
$= 1386\ m^2$

Area of the circular park and the road
$= \pi R^2 = \pi(21 + 7)^2$
$= \frac{22}{7} \times 28 \times 28 = 2464\ m^2$

Area of the road $= (2464 - 1386)\ m^2 = 1078\ m^2$

Cost of metalling the road $=$ Rs. 1078×25.50
$=$ **Rs. 27489.**

5. Ans. (a)

Solution Let the breadth of the blackboard be x cm, then

Length $= (x + 8)$ cm

According to question,

$(x + 8 + 7)(x - 4) = (x + 8)x$

or, $(x + 15)(x - 4) = (x + 8)x$

or, $3x = 60$

$\therefore$ $x = 20$

$\therefore$ Breadth $= 20$ cm and length $= 20 + 8 =$ **28 cm.**

7. Ans. (c)

Solution The area of the shaded portion

$$= \frac{45}{360}\times\frac{22}{7}\times(8^2 - 6^2)$$
$$= \frac{45}{360}\times\frac{22}{7}\times(64 - 36)$$
$$= \frac{1}{8}\times\frac{22}{7}\times 28 = \textbf{11 m}^2.$$

9. Ans. (c)

Solution Let l and b be the length and breadth of the rectangular field, so that $l \times b = 900\ m^2$

Since the area of the square field is $900\ m^2$

$\therefore$ Its side is of length $\sqrt{900} = 30$ m

Perimeter of the square field $= 4 \times 30 = 120$ m

$\therefore$ $2(l + b) - 2 = 120$

or, $l + b = 61$ m

Now, $l - b = \sqrt{(l+b)^2 - 4lb} = \sqrt{(61)^2 - 4\times 900}$
$= \sqrt{61^2 - 60^0} = \sqrt{1\times 121} = 11$ m

$\therefore$ $l = \frac{(l+b)(l-b)}{2} = \frac{61+11}{2} = 36$ m

and, $b = \frac{(l+b)-(l-b)}{2} = \frac{61-11}{2} =$ **25 m.**

11. Ans. (b)

Solution Let length of canal be x m

Then, $\frac{1}{2}(10 + 6) \times x = 640$

$\Rightarrow$ $x = \frac{460\times 2}{16} = 80$ m.

13. Ans. (b)

Solution Let breadth $= x$ m

and, length $= 2x$ m

Then, $x^2 + (2x)^2 = (9\sqrt{5})^2$

$\Rightarrow$ $5x^2 = 405 \Rightarrow x = \sqrt{81} = 9$

$\therefore$ Perimeter $= 2(18 + 9) =$ **54 m.**

15. Ans. (c)

Solution Let the side of the square be x cm.

According to question:

$(x + 8)x = x^2 + 208$

or, $x^2 + 8x = x^2 + 208$

$\therefore$ $8x = 208$

$\Rightarrow$ $x = 26$

$\therefore$ Perimeter $= 4x = 4 \times 26$
$=$ **104 cm.**

17. Ans. (b)

Solution Let the length and breadth of the rectangular field be x cm and y cm, respectively.

According to question:

$xy = 60\ cm^2$ ...(i)

and, $\sqrt{x^2 + y^2} + x = 5y$ ...(ii)

or, $x^2 + y^2 = (5y - x)^2$

or, $x^2 + y^2 = 25y^2 + x^2 - 10xy$

or, $24y^2 = 10 \times 60$

or, $y^2 = 5 \times 5$

$\therefore$ $y = 5$ cm and $x = \frac{60}{5} =$ **12 cm.**

19. Ans. (a)

Solution Let the side of one square be x m and the other be $(x + 1)$ m

Now, as per the question,

$2h(x + x) = \frac{33600}{70} = 480$

$\therefore$ $hx = 120$ ...(i)

and, $2(x + 1) \times 2h = \frac{35280}{70} = 504$

$\therefore$ $xh + h = 126$ ...(ii)

$\therefore$ $h = 126 - 120 =$ **6 m.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (b)	**6.** (a)	**8.** (c)	**10.** (c)	**12.** (a)	**14.** (a)	**16.** (d)	**18.** (c)	**20.** (c)

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Mensuration II: Volume and Surface Area

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INTRODUCTION

Solids

A *solid* is a figure bounded by one or more surfaces. It has three dimensions namely length, breath or width and thickness or height. The plane surfaces that bind it are called its *faces*.

The *volume* of any solid figure is the amount of space enclosed within its bounding faces. It is measured in cubic units, e.g., m^3, cm^3, etc.

The area of the plane surfaces that bind the solid is called its *surface area*.

For any regular solid,

Number of faces + Number of vertices

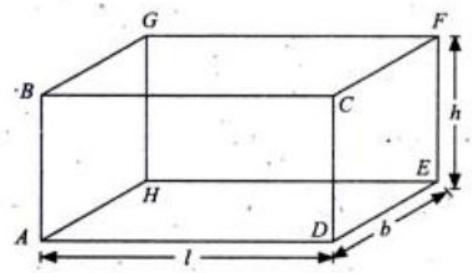
= Number of edges + 2.

We discuss below some important three dimensional figures and the formulae associated with them.

SOME BASIC FORMULAE

Cubic

It is a solid figure which has six rectangular faces. It is also called *rectangular parallelopiped.*



If l, b and h denote the length, breadth and height of the cuboid and d denotes the body diagonal (AF or BE or DG or CH), then

(i) Volume $= l \times b \times h = \sqrt{A_1 \times A_2 \times A_3}$,

where A_1 = area of base or top

A_2 = area of one side face, and

A_3 = area of other side face

(ii) Total Surface Area $= 2(lb + bh + lh)$

$= (l + b + h)^2 - d^2$

(iii) Diagonal of cuboid $= \sqrt{l^2 + b^2 + h^2}$

Note: (i) For painting the surface area of a box or to know how much tin sheet is required for making a box, we use formula (ii).

(ii) To find how much a box contains or how much space a box shall occupy, we use formula (i). To find the length of the longest pole to be placed in a room, we use formula (iii).

(iii) The rise or fall of liquid level in a container

$$= \frac{\text{Total volume of objects submerged or taken out}}{\text{Cross-sectional area of container}}$$

Illustration 1 Find the volume and the total surface area of a cuboid whose dimensions are 25 m, 10 m and 2 m.

Solution Here, $l = 25$ m, $b = 10$ m and $h = 2$ m.

Volume of the cuboid $= l \times b \times h$

$= 25 \times 10 \times 2$

$= 500\ m^3$

Total surface area of the cuboid

$= 2(lb + bh + lh)$

$= 2(25 \times 10 + 10 \times 2 + 25 \times 2)$

$= 2(250 + 20 + 50)$

$= \mathbf{640\ m^2}$.

Illustration 2 Find the length of the longest bamboo that can be placed in a room 12 m long, 9 m broad and 8 m high.

Solution Length of the bamboo

= length of the diagonal of the room

$= \sqrt{12^2 + 9^2 + 8^2}$

$= \sqrt{289} = \mathbf{17\ m}$.

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Illustration 3 The area of a side of a box is 120 sq cm. The area of the other side of the box is 27 sq cm. If the area of the upper surface of the box is 60 sq cm, then find the volume of the box.

Solution Volume to the box

$= \sqrt{\text{area of base} \times \text{area of one face} \times \text{area of the other face}}$

$= \sqrt{60 \times 120 \times 72}$

$= \sqrt{518400} = \mathbf{720\ cm^3}.$

Illustration 4 The sum of length, breadth and height of a cuboid is 12 cm long. Find the total surface area of the cuboid.

Solution Total surface area

$= (\text{Sum of all three sides})^2 - (\text{Diagonal})^2$

$= 12^2 - 8^2 = 144 - 64 = \mathbf{80\ sq\ cm.}$

Cube

It is a special type of cuboid in which each face is a square.

For a cube, length, breadth and height are equal and is called the edge of the cube.

If a be the edge of a cube, then

(i) Volume of the cube $= (\text{edge})^3 = a^3$

(ii) Total surface area of the cube $= 6\,(\text{edge})^2 = 6a^2$

(iii) Diagonal of the cube $= \sqrt{3}a$ (edge) $= \sqrt{3}a$

(iv) Volume of the cube $= \left(\frac{\text{diagonal}}{\sqrt{3}}\right)^3 = \left(\frac{d}{\sqrt{3}}\right)^2$

$= \left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$

(v) Total surface area of the cube

$= 2\,(\text{diagonal})^2 = 2d^2$

(vi) For two cubes

(a) Ratio of volumes = (ratio of sides)^3

(b) Ratio of surface areas = (Ratio of sides)^2

(c) (Ratio of surface areas)^3 = (Ratio of volumes)^2.

Illustration 5 Find the volume, surface area and the diagonal of a cube, each of whose sides measures 4 cm.

Solution Volume of the cube $= a^3 = (4)^3 = 64\ \text{cm}^3$.

Surface area of the cube $= 6a^2 = 6\,(4)^2 = 96\ \text{cm}^2$.

Diagonal of the cube $= \sqrt{3}a = \mathbf{4\sqrt{3}\ cm.}$

Illustration 6 The surface area of a cube is 216 sq cm. Find its volume.

Solution Volume of the cube

$= \left(\sqrt{\frac{\text{Surface area}}{6}}\right)^3$

$= \left(\sqrt{\frac{216}{6}}\right)^3 = (6)^3 = \mathbf{216\ cm^3}.$

Illustration 7 The diagonal of a cube is $8\sqrt{3}$ cm. Find its total surface area and volume.

Solution We have,

Diagonal of cube $= \sqrt{3}$ (edge)

$\therefore$ Edge of cube $= \frac{\text{Diagonal of cube}}{\sqrt{3}}$

$= \frac{8\sqrt{3}}{\sqrt{3}} = 8$ cm.

Total surface area $= 6\,(\text{edge})^2 = 6(8)^2$

$= 384$ sq cm.

Volume of cube $= (\text{edge})^3 = (8)^3 = \mathbf{512\ cm^3}.$

Illustration 8 If the volumes of two cubical blocks are in the ratio of 8 : 1, what will be the ratio of their edges?

Solution We have,

Ratio of volumes = (Ratio of sides)^3

Since, ratio of volumes = 8 : 1, i.e. $2^3 : 1^3$

$\therefore$ ratio of sides $= \mathbf{2 : 1.}$

Illustration 9 Volumes of the two cubes are in the ratio of 1 : 9. Find the ratio of their surface areas.

Solution (Ratio of the surface areas)^3

= (Ratio of volumes)^2

$\therefore$ Ratio of surface areas $= \sqrt[3]{1:81} = \mathbf{1: 3\ (3)^{1/3}}.$

Illustration 10 Sides of two cubes are in the ratio of 2 : 3. Find the ratio of their surface areas.

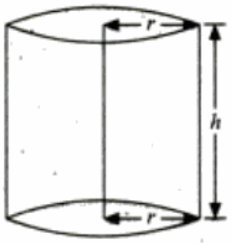
Solution Ratio of surface areas

= (Ratio of sides)^2

$= (2 : 3)^2 = \mathbf{4 : 9.}$

Right Circular Cylinder

A *right circular cylinder* is a solid with circular ends of equal radius and the line joining their centres perpendicular to them. This is called axis of the cylinder. The length of the axis is called the height of the cylinder.



Note: Take a rectangular sheet of paper and role it lengthwise or breadthwise in a round way, you will get a cylinder, i,e. a cylinder is generated by rotating a rectangle by fixing one of its sides.

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$$\Rightarrow \quad h = \frac{1232 \times 3}{\pi r^2} = \frac{1232 \times 3 \times 7}{22 \times 7 \times 7} = 24 \text{ cm.}$$

Slant height l is given by the relation

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49}$$
$$= \sqrt{625} = 25 \text{ cm.}$$

$\therefore$ Slant height of the cone is **25 cm.**

Illustration 25 A tent is of diameter 12 m at the base and its height is 8 m.

(i) Find the slant height; and

(ii) The canvas required in sq metres.

How many persons can the tent accommodate, at the most, if each person requires 18 m^3 of air?

Solution Diameter of the base of a conical tent = 12 m.

$\therefore$ Radius $(r) = \frac{12}{2} = 6$ m and its height $(h) = 8$ m

(i) Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2}$

$= \sqrt{64 + 36} = \sqrt{100} = 10$ m.

(ii) Area of canvas required

$= \pi \times r \times l$

$= \frac{22}{7} \times 6 \times 10 = 188.57$ m

(iii) Volume of conical portion

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8 = 301.71 \text{ m}^3.$

Space required for each person = 18 m^3.

$\therefore$ Number of persons that can be accommodated

$= \frac{301.71}{18} = \mathbf{16.}$

Illustration 26 The height of a cone is 21 cm and radius of its base is 28 cm. Find its total surface area.

Solution We have, $r = 28$ cm and $h = 21$ cm.

Slant height $(l) = \sqrt{r^2 + h^2} = \sqrt{(28)^2 + (21)^2}$

$= \sqrt{1225} = 35$ cm.

Total surface area $= \pi r (l + r)$

$= \frac{22}{7} \times 28 \times (35 + 28)$

$= \frac{22}{7} \times 28 \times 63$

$= \mathbf{5544 \text{ sq cm.}}$

Illustration 27 Two right circular cones of equal curved surface areas have their slant heights in the ratio of 3 : 5. Find the ratio of their radii.

Solution Ratio of radii = inverse ratio of slant heights

$= \frac{1}{3} : \frac{1}{5} = \mathbf{5 : 3.}$

Illustration 28 Two right circular cones of equal volumes have their heights in the ratio of 4 : 9. Find the ratio of their radii.

Solution Ratio of radii $= \sqrt{\text{inverse ratio of heights}}$

$= \sqrt{\frac{1}{4} : \frac{1}{9}} = \sqrt{9:4} = \mathbf{3 : 2.}$

Illustration 29 Two right circular cones of equal heights have their radii in the ratio of 1 : 3. Find the ratio of their volumes.

Solution Ratio of volumes = (Ratio of radii)^2

$= (1 : 3)^2 = \mathbf{1 : 9.}$

(v) If the ratio of volumes and the ratio of heights of two right circular cones (or cylinders) are given, then

Ratio of radii

$= \sqrt{(\text{ratio of volumes})\,(\text{inverse ratio of heights})}$

$= \sqrt{(3:2)\,(8:3)} : \sqrt{4:1} = 2 : 1.$

Illustration 30 The volumes of two cones are in the ratio 3 : 2 and their heights in the ratio 3 : 8. Find the ratio of their radii.

Solution Ratio of radii

$= \sqrt{(\text{ratio of volumes})\,(\text{inverse ratio of heights})}$

$= \sqrt{(3:2)\,(8:3)} = \sqrt{4:1} = \mathbf{2 : 1.}$

(vi) If the ratio of heights and the ratio of diameters (or radii) of two right circular cones (or cylinders) are given, then

Ratio of volumes = (ratio of radii)^2 × (ratio of heights).

Illustration 31 The heights of two cones are in the ratio of 5 : 3 and their radii in the ratio 2 : 3. Find the ratio of their volumes.

Solution Ratio of volumes

$= (\text{ratio of radii})^2 \times (\text{ratio of heights})$

$= (2 : 3)^2 \times (5 : 3)$

$= \frac{4}{9} \times \frac{5}{3} = \mathbf{20 : 27.}$

(vii) If the ratio of radii (or diameter) and the ratio of volumes of two right circular cones are given, then

Ratio of heights

= (inverse ratio of radii)^2 (ratio of volumes).

Illustration 32 The volumes of two cones are in the ratio of 1 : 4 and their diameters are in the ratio of 4 : 5. Find the ratio of their heights.

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Curved surface area $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 21 \times 21$

$= 2772$ sq cm.

Total surface area $= 3\pi r^2$

$= 3 \times \frac{22}{7} \times 21 \times 21$

$= 4158$ sq cm.

(vi) For two spheres

(a) (Ratio of radii)^2 = Ratio of surface areas

(b) (Ratio of radii)^3 = Ratio of volumes

(c) (Ratio of surface areas)^3 = (Ratio of volumes)^2.

Illustration 36 The radii of two spheres are in the ratio of 2 : 3. What is the ratio of their surface areas?

Solution Ratio of surface areas = (ratio of radii)^2

$= (2 : 3)^2 = 4 : 9.$

Illustration 37 The surface areas of two spheres are in the ratio 1 : 2. Find the ratio of their volumes.

Solution We have,

(Ratio of surface areas)^3 = (Ratio of volumes)^2

$\Rightarrow (1 : 2)^3 =$ (Ratio of volumes)^2

$\therefore$ Ratio of volumes $= \sqrt{1:8} = 1 : 2\sqrt{2}$.

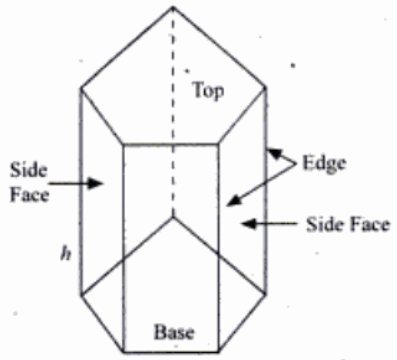
Illustration 38 The radii of two spheres are in the ratio of 2 : 45. Find the ratio of their volumes.

Solution Ratio of volumes = (Ratio of radii)^3

$= (2 : 5)^3 = 8 : 125.$

Prism

A solid having top and bottom faces identical and side faces rectangular is a prism.



In a prism with a base of n sides,

Number of vertices $= 2n$

and Number of faces $= n + 2$.

Volume of the prism = area of base × height

Lateral surface area = perimeter of base × height

Total surface area = 2 × Base area + Lateral Surface area.

Illustration 39 Find the volume and the total surface area of a triangular prism whose height is 30 m and the sides of whose base are 21 m, 20 m and 13 m, respectively.

Solution Perimeter of base $= 21 + 20 + 13 = 54$ m.

height $= 30$ m

Area of base $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27(27-21)(27-20)(27-13)}$

$= \sqrt{27 \times 6 \times 7 \times 14} = 126$ sq m.

$\therefore$ Volume of the prism = area of base × height

$= 126 \times 54 = 6804$ m^3.

Also, surface area of the prism

= 2 × Base area + lateral surface area

= 2 Base area + perimeter of base × height

$= 2 \times 126 + 54 \times 30 =$ **1872 sq m.**

SOLIDS INSCRIBED/CIRCUMSCRIBING OTHER SOLIDS

1. If a largest possible sphere is circumscribed by a cube of edge 'a' cm, then the radius of the sphere $= \frac{a}{2}$.

Illustration 40 Find the volume of largest possible sphere circumscribed by a cube of edge 8 cm.

Solution Radius of the sphere $= \frac{a}{2} = \frac{8}{2} = 4$ cm.

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4 \times 4 \times 4$

$= 26.81$ cm^3.

2. If a largest possible cube is inscribed in a sphere of radius 'a' cm, then the edge of the cube $= \frac{2a}{\sqrt{3}}$.

Illustration 41 Find the surface area of largest possible cube inscribed in a sphere of radius 4 cm.

Solution Edge of the cube $\frac{2a}{\sqrt{3}} = \frac{2 \times 4}{\sqrt{3}} = \frac{8}{\sqrt{3}}$.

$\therefore$ Surface area of the cube = $6($edge$)^2$

$= 6 \times \frac{64}{3}$

$= 128$ sq cm.

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∴ Percentage change in volume

$$= \left[3x + \frac{3x^2}{100} + \frac{x^3}{(100)^2}\right]\%$$

$$= \left[3(-10) + \frac{3(-10)^2}{100} + \frac{(-10)^3}{(100)^2}\right]\%$$

$$= (-30 + 3 - 0.1)\% = -27.1\%.$$

–ve sign indicates decrease in volume, that is, there is a decrease in volume by **27.1%.**

Illustration 48 The diameter of a sphere is increased by 20%. What is the percentage increase in its volume?

Solution Percentage increase in volume

$$= \left[3x + \frac{3x^2}{100} + \frac{x^3}{(100)^2}\right]\% \quad [\text{Here, } x = y = z]$$

$$= \left[3 \times 20 + \frac{3(20)^2}{100} + \frac{(20)^3}{(100)^2}\right]\%$$

$$= (60 + 12 + 0.8)\% = \mathbf{72.8\%.}$$

Illustration 49 The radius of a right circular cylinder is decreased by 5% but its height is increased by 10%. What is the percentage change in its volume?

Solution Here, $x = y = -15$ and $z = 10$.

∴ Percentage change in volume

$$= \left[-5 - 5 + 10 + \frac{(-5)(-5) + (-5)(10) + (-5)(10)}{100} + \frac{(-5)(-5)(10)}{(100)^2}\right]\%$$

$$= (0 - 0.75 + 0.025)\% = -0.725\%.$$

Therefore, volume decrease by **0.725%.**

Illustration 50 Each of the radius and the height of a cone is increased by 25%. Find the percentage increase in volume.

Solution Here, $x = y = 25$ and $z = 25$.

∴ Percentage increase in volume

$$= \left[25 + 25 + 25 + \frac{25 \times 25 + 25 \times 25 \times 25 \times 25}{100} + \frac{25 \times 25 \times 25}{(100)^2}\right]\%$$

$$= (75 + 18.75 + 1.56)\% = \mathbf{95.3\%.}$$

2. If the two measuring dimensions which are included in the surface area of a sphere, cuboid, cube, cylinder or cone are increased or decreased by $x\%$ and $y\%$, then the surface area of the figure will increase or decrease by

$$\left(x + y + \frac{xy}{100}\right)\%.$$

Note that in case of percentage increase, values of x, y and z are positive and in case of percentage decrease, values of x, y and z are negative.

Illustration 51 Each edge of a cube is increased by 20%. What is the percentage increase in its surface area?

Solution Here, $x = y = 20$.

∴ Percentage increase in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(20 + 20 + \frac{20 \times 20}{100}\right)\%$$

$$= (40 + 4)\% = \mathbf{44\%.}$$

Illustration 52 The radius of a hemisphere is decreased by 10%. Find the percentage change in its surface area.

Solution Here, $x = y = -10$.

∴ Percentage change in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(-10 - 10 + \frac{(-10)(-10)}{100}\right)\%$$

$$= (-20 + 1)\% = \mathbf{-19\%.}$$

Therefore, surface area of hemisphere decreases by 19%.

Illustration 53 The radius of a right circular cone is increased by 25% and slant height is decreased by 30%. Find the percentage change in curved surface area of the cone.

Solution Here, $x = 25$ and $y = -30$.

∴ Percentage change in curved surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left[25 - 30 + \frac{(25)(-30)}{100}\right]\%$$

$$= (-5 - 7.5)\% = \mathbf{-12.5\%.}$$

Therefore, curved surface area decreases by 12.5%.

Illustration 54 The radius and height of a cylinder are increased by 10% and 20%, respectively. Find the percentage increase in its surface area.

Solution Here, $x = 10$ and $y = 20$.

∴ Percentage increase in surface area

$$= \left(x + y + \frac{xy}{100}\right)\%$$

$$= \left(10 + 20 + \frac{10 \times 20}{100}\right)\% = \mathbf{32\%.}$$

3. If a sphere of radius R is melted to form smaller spheres each of radius r, then

The number of smaller spheres

$$= \frac{\text{Volume of the bigger sphere}}{\text{Volume of the smaller sphere}}$$

$$= \left(\frac{R}{r}\right)^3.$$

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Illustration 55 Find the number of lead balls of radius 1 cm each that can be made from a sphere of radius 4 cm.

Solution Number of lead balls $= \left(\frac{R}{r}\right)^3 = \left(\frac{4}{1}\right)^3 = \mathbf{64.}$

4. If by melting n spheres, each of radius r, a big sphere is made, then

   Radius of the big sphere $= r.\sqrt[3]{n}$.

Illustration 56 If by melting 8 spheres, each of radius 5 cm, a big sphere is made, what will be the radius of the big sphere?

Solution Radius of the big sphere $= r.\sqrt[3]{n}$

$= 5.\sqrt[3]{8} = 5.2 = \mathbf{19\ cm.}$

5. If a cylinder is melted to form smaller spheres each of radius r, then

   The number of small spheres $= \dfrac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$.

Illustration 57 How many bullets can be made out of a loaded cylinder 24 cm high and 5 cm diameter, each bullet being 2 cm in diameter?

Solution Number of bullets $= \dfrac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$

$= \dfrac{\pi \times \frac{5}{2} \times \frac{5}{2} \times 24}{\frac{4}{3} \times \pi \times 1 \times 1 \times 1} = \mathbf{450.}$

6. If a sphere of radius r is melted and a cone of height h is made, then

   Radius of the cone $= 2 \times \sqrt{\dfrac{r^3}{h}}$.

   or,

   If a cone of height h is melted and a sphere of radius r is made, then

   Radius of the cone $= 2 \times \sqrt{\dfrac{r^3}{h}}$.

Illustration 58 A solid cone of copper of height 3 cm is melted and a solid sphere of radius 3 cm is made. What is the diameter of the base of the cone?

Solution Radius of the base of the cone

$= 2 \times \sqrt{\dfrac{r^3}{h}} = 2 \times \sqrt{\dfrac{3^3}{3}} = 6.$

$\therefore$ Diameter of the base of the cone $= 2 \times 6$

$= \mathbf{12\ cm.}$

Illustration 59 If a solid cone of copper of height 2 cm is melted and a solid sphere of radius 2 cm is made, what is the diameter of the base of the cone?

Solution Radius of the base of the cone

$= 2 \times \sqrt{\dfrac{r^3}{h}} = 2 \times \sqrt{\dfrac{(2)^3}{2}}$

$= 2 \times 2 = 4$ cm.

$\therefore$ Diameter of the base of the cone $= 2 \times 4 = \mathbf{8\ cm.}$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. A tank 16 m long and 23 m wide contains water. How many cubic metres of water must be rushed into it to make the surface rise by $16\frac{2}{3}$ cm?
   (a) 48 m^3 (b) 40 m^3
   (c) 32 m^3 (d) 42 m^3

2. The outer dimensions of a closed box are 12 cm by 10 cm by 8 cm. If the box is made of wood 1 cm thick, find the capacity of the box.
   (a) 360 cm^3 (b) 480 cm^3
   (c) 240 cm^3 (d) 560 cm^3

3. A cistren of dimensions 2.4 m × 2.0 m × 1.5 m takes 2 hr 30 min to get filled with water. The rate at which water flows into the cistern is
   (a) 0.48000 cu.m/hr (b) 800 cu.m/min
   (c) 800 cu.m/sec (d) None of these

4. The area of three adjacent faces of a rectangular box are p, q and r square cm. The volume of the box is given by
   (a) $(p + q + r)$ cm^3 (b) $\sqrt{pqr}$ cm^3
   (c) $(pqr)^{1/3} cm^3$ (d) pqr cm^3

5. A reservoir 30 m long, 15 m broad is filled with water. How many gallons of water must be taken out to lower the level of water by 4 metres?
   (a) 342000 (b) 364200
   (c) 324000 (d) 386400

6. How many bricks each measuring 250 cm by 12.5 cm by 7.5 cm will be required to build a wall 5 m long, 3 m high and 20 cm thick?
   (a) 1480 (b) 1280
   (c) 1680 (d) 1480

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7. Find the cost of the log of wood measuring $15\frac{1}{2}$ m by $2\frac{3}{4}$ m by $1\frac{1}{3}$ m at Rs. 45 per cu m.
(a) Rs. 4257.50 (b) Rs. 4005.00
(c) Rs. 4207.50 (d) Rs. 4357.50

8. How many bricks are required to build a wall 15 m long 3 m high and 50 cm thick, if each brick measures 25 cm by 12 cm by 6 cm.
(a) 16500 (b) 14500
(c) 12500 (d) 10500

9. Find the diagonal of a cuboid whose dimensions are 12 m by 10 m by 8 m.
(a) 18 m (b) 17.5 m
(c) 17 m (d) 16.5 m

10. The outer dimensions of a closed wooden box of 1 cm thick are 12 cm by 10 cm by 8 cm. Find the cost of the wood required to make the box if 1 cm^3 of wood costs Rs. 3.00
(a) Rs. 1440 (b) Rs. 1640
(c) Rs. 1840 (d) Rs. 2040

11. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of the three cubes
(a) 3 : 5 (b) 4 : 5
(c) 6 : 7 (d) 7 : 9

12. The diagonal of a cubical box is $\sqrt{300}$ cm. Find the surface area
(a) $600\sqrt{3}$ cm^2 (b) 600 cm^2
(c) 1200 cm^2 (d) $900\sqrt{3}$ cm^2

13. An iron cube of side 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet be in the ratio 1 : 5, the sides (in cm) are
(a) 110, 50 (b) 20, 100
(c) 40, 200 (d) None of these

14. The length of a room is 12 metres, width 8 metres, height 6 metres. How many boxes will it hold if each is allowed 1.5 cubic metre of space?
(a) 864 (b) 506
(c) 384 (d) 436

15. A room 3.3 metres high is half as long again as it is wide and its volume is $123\frac{3}{4}$ cu m. Find its length and breadth.
(a) 7.5 m, 6 m (b) 8 m, 5 m
(c) 7.5 m, 5 m (d) 8.5 m, 5 m

16. A tank 3 m long, 2 m wide and 1.5 m deep is dug in a field 22 m long and 14 m wide. If the earth dug out is evenly spread out over the field, the level of the field will rise by nearly
(a) 0.299 cm (b) 0.29 mm
(c) 2.98 cm (d) 4.15 cm

17. A school room is to be built to accommodate 70 children, so as to allow 2.2 sq., metres of floor and 11 cu m of space for each child. If the room be 14 metres long, what must be its breadth and height?
(a) 12 metres, 5,5 metres
(b) 11 metres, 5 metres
(c) 13 metres, 6 metres
(d) 11 metres, 4 metres

18. If 210 cu m of sand be thrown into a tank 12 m long and 5 m wide, find how much the water will rise?
(a) 3.5 m (b) 4 m
(c) 7 m (d) Data inadequate

19. If the length, breadth and height of a rectangular parallelopiped are in the ratio 6 : 5 : 4 and if total surface area is 33,300 m^2, then the length, breadth and height of parallelopiped (in cm) respectively are
(a) 90, 85, 600 (b) 90, 75, 70
(c) 85, 75, 60 (d) 90, 75, 60

20. A cu m of metal weighing 90 kg is rolled into a square bar 9 metre long. An exact cube is cut off from the bar. How much does it weigh?
(a) $5\frac{2}{3}$ kg (b) $6\frac{1}{3}$ kg
(c) $3\frac{1}{3}$ kg (d) $4\frac{2}{3}$ kg

21. How many cubes each of surface 24 sq cm can be made out of a cube of edge measure one metre?
(a) 165000 (b) 125000
(c) 180000 (d) 155000

22. Three solid cubes whose edges are 6, 8 and 10 cm respectively, are melted and formed into a single cube. If there be no loss of metal in the process, find the edge of the new cube.
(a) 16 cm (b) 10 cm
(c) 14 cm (d) 12 cm

23. A cube of edge 6 cm is melted and smaller cubes of edge 2 cm each are formed; how many such cubes are possible?
(a) 39 (b) 24
(c) 27 (d) 21

24. How many small cubical blocks of side 5 cm can be cut from a cubical block whose each edge measures 20 cm?
(a) 56 (b) 48
(c) 64 (d) 52

25. Surface area of a cube is 600 sq. cm. Find the length of its diagonal.
(a) $15\sqrt{3}$ (b) $12\sqrt{3}$
(c) $10\sqrt{3}$ cm (d) None of these

26. A rectangular tank is 30 m long 20 m broad. Water is being flown into it through a square pipe of side

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5 cm. What is the speed of water if the level of water in the tank rises by 1 m in 8 hours?
(a) 30 km/hr (b) 36 km/hr
(c) 40 km/hr (d) None of these

27. Calculate the number of bricks, each measuring 25 cm × 15 cm × 8 cm, required to construct a wall of dimension 19 m × 4 m × 5 m, when 10% of its volume is occupied by mortar.
(a) 4000 (b) 8000
(c) 7000 (d) 6000

28. Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted into a single cube, the length of whose diagonal is $48\sqrt{3}$ m. Calculate the edges of the three cubes.
(a) 24 m, 32 m, 40 m (b) 40 m, 32 m, 24 m
(c) 30 m, 22 m, 18 m (d) 48 m, 36 m, 24 m

29. A cube of lead with edges measuring 6 cm each is melted and recasted into 27 equal cubes. The length of the edge of the new cube is
(a) 3 cm (b) 4 cm
(c) 2 cm (d) 1.5 cm

30. The volume of a cube is 729 cm^3. The total surface area of the cube is
(a) 216 cm^2 (b) 384 cm^2
(c) 486 cm^2 (d) 512 cm^2

31. Two cubes have volumes in the ratio 1 : 27. The ratio of the area of the face of one to that of the other is
(a) 1 : 2 (b) 1 : 3
(c) 1 : 6 (d) 1 : 9

32. Two cubes, each of side 12 cm, are joined end to end. The surface area of the resulting cuboid is
(a) 1240 sq cm (b) 1440 sq cm
(b) 2250 sq cm (d) 4252 sq cm

33. The perimeter of one face of a cube is 20 cm. Its volume is
(a) 1009 cm^3 (b) 525 cm^3
(c) 320 cm^3 (d) 125 cm^3

34. A cube of edge 3 cm of iron weighs 12 gm. What is the weight of a similar cube of iron whose edge is 12 cm?
(a) 768 gm (b) 678 gm
(c) 964 gm (d) 864 gm

35. The weight of the solid cube of iron of 1 cm edge is 17 gm. What should be the weight of a similar cube of edge 3 cm?
(a) 449 gm (b) 459 gm
(c) 469 gm (d) 4390 gm

36. A cubic metre of silver weighing 900 kg is rolled into a square bar 16 m long. Find the weight of an exact cube cut off from it.
(a) 14 kg $62\frac{1}{2}$ gm (b) 30 kg
(c) 10 kg (d) 7 kg 50 gm

37. A 4 cm cube is cut into 1 cm cubes. What is the ratio of surface area of small cubes to that of the large cube?
(a) 1 : 16 (b) 2 : 3
(c) 4 : 1 (d) 6 : 1

38. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cms side. What is the ratio of the total surface areas of the smaller cubes and the large cube?
(a) 2 : 1 (b) 3 : 2
(c) 25 : 18 (d) 27 : 20

39. How many small cubes, each of 96 sq cms surface area, can be formed from the material obtained by melting a larger cube of 384 cm^2 surface area?
(a) 8 (b) 5
(c) 800 (d) 8000

40. A cubical metallic tank whose each edge measures 30 cm, is completely filled with water. If 2,7 litres water is taken out of it, what will be the depth of remaining water in tank?
(a) 37 cm (b) 27 cm
(c) 17 cm (d) None of these

41. Find the weight of a hollow cylindrical lead pipe 28 cm long and $\frac{1}{2}$ cm thick. Its internal diameter is 8 cm. $\left(\text{Weight of 1 cm}^3 \text{ of lead is 11.4 g, } \pi = \frac{22}{7}\right)$
(a) 3.762 kg (b) 4.562 kg
(c) 7.462 kg (d) 6.762 kg

42. Volume of the cylinder is 1650 cu m. whereas the surface area of its base is $78\frac{4}{7}$ m^2. Find the height of the cylinder
(a) 2.1 m (b) 7.5 m
(c) 21 m (d) 14 m

43. 1496 cm^3 of a metal is used to cast a pipe of length 28 cm. If the internal radius of the pipe is 8 cm, the outer radius of the pipe is
(a) 7 cm (b) 9 cm
(c) 10 cm (d) 12 cm

44. The base of a solid cylinder of height 10 cm is a semi-circle of radius 7 cm. Its total surface (in sq cm) is $\left(\pi = \frac{22}{7}\right)$
(a) 154 (b) 176
(c) 514 (d) None of these

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62. A hemisphere of lead and of radius 6 cm, is cast into a right circular cone of height 75 cm. The radius of the base of the cone is
(a) 1.4 cm (b) 2.4 cm
(c) 1.6 cm (d) 3.2 cm

63. A solid cylinder has a total surface area of 231 sq cm. Its curved surface area is (2/3) of the total surface area. Find the volume of the cylinder
(a) 270 cu cm (b) 269.5 cu cm
(c) 256.5 cu cm (d) 289.5 cu cm

64. It is required to design a circular pipe such that water flowing through it at a speed of 7 metres per min fills a tank of capacity 440 cubic metres in 10 min. The inner radius of the pipe should be
(a) 2 m (b) $\sqrt{2}$ m
(c) $\frac{1}{2}$ m (d) $\frac{1}{\sqrt{2}}$ m

65. From a solid right circular cylinder with height 10 cm and radius of the base 6 cm; a right circular cone of the same height and base is removed. The volume (in cm^3) of the remaining solid is
(a) 377 (b) 754.3
(c) 1131 (d) None of these

66. The radii of two cylinders are in the ratio 2 : 3 and their height are in the ratio 5 : 3. Ratio of their volume is
(a) 20 : 27 (b) 10 : 9
(c) 18 : 13 (d) 9 : 20

67. The capacity of a tank in the form of a cylinder is 6160 cu m. If the diameter of its base is 28 m, find the cost of painting its inner curved surface at the rate of Rs. 2.8 per sq m. $\left(\text{Use } \pi = \frac{22}{7}\right)$
(a) 2464 (b) 2664
(c) 3064 (d) 2864

68. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m^2, then the circumference of its base and the volume of the cylinder are
(a) 68 m; 7875 m^3 (b) 52 m; 5825 m^3
(c) 44 m; 4620 m^3 (d) 30 m; 3859 m^3

69. A rectangular piece of paper is 22 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its length. The volume of the cylinder is
(a) 225 π cu cm (b) 385 cu cm
(c) 25 π cu cm (d) None of these

70. The ratio of total surface area to lateral surface area of a cylinder whose radius is 80 cm and height 20 cm, is
(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 5 : 1

71. A right cylindrical vessel is full with water. How many right cones having the same diameter and height as those of right cylinder will be needed to store that water?
(a) 2 (b) 3
(c) 4 (d) 5

72. A cylindrical bucket is 72 cm high and 29 cm in diameter and is full of water. This water is emptied in a rectangular tank whose length and breadth are 66 cm and 28 cm, respectively. What will be the height of the water level in the tank?
(a) 36 cm (b) 48 cm
(c) 24 cm (d) 22 cm

73. A cylindrical iron rod is 70 cm long and the diameter of its end is 2 cm. What is its weight, reckoning a cubic cm of iron to weigh 10 grams?
(a) 4 kg (b) 4.2 kg
(c) 2.2 kg (d) Data inadequate

74. If the radius of a cylinder is doubled and the height is halved, what is the ratio between the new curved surface area and the previous curved surface area of the cylinder
(a) 1 : 1 (b) 2 : 1
(c) 3 : 2 (d) 2 : 3

75. A cylindrical jar of diameter 24 cm contains water to a height 30 cm. A spherical steel ball is dropped into the jar and the level of the water rises by 67.5 mm. The diameter of the ball is
(a) 16 cm (b) 15 cm
(c) 20 cm (d) 18 cm

76. The material of solid cone is converted into the shape of solid cylinder of equal radius. If the height of the cylinder is 5 cm, what is the height of the cone?
(a) 25 cm (b) 15 cm
(c) 20 cm (d) 10 cm

77. The volume of solid cylinder whose diameter of the base is 14 mm and length 25 mm is 3850 cu mm. If length of the cylinder is doubled but the diameter is halved, what will be the volume of the resulting cylinder?
(a) 1172 cu mm (b) 1925 cu mm
(c) 3850 cu mm (d) 7700 cu mm
(d) None of these

78. A monument has 50 cylindrical pillars each of diameter 50 cm and height 4 m. What will be the labour charges for getting these pillars cleaned at the rate of 50 paise per sq m? (Use π = 3.14)
(a) Rs. 237 (b) Rs. 257
(c) Rs. 157 (d) Rs. 353

79. The radius of a cylinder is made twice as large. How should the height be changed so that volume

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remains the same?

(a) $\frac{1}{4}$th of original (b) $\frac{1}{3}$rd of original

(c) $\frac{1}{2}$ of original (d) $\frac{1}{8}$th of original

80. A spherical ball of lead, 3 cm in diameter is melted and re-cast into three spherical balls. The diameter of two of these are 1.5 cm and 2 cm, respectively. The diameter of the third ball is

(a) 2.66 cm (b) 2.5 cm

(c) 3 cm (d) 3.5 cm

81. A cone and a cylinder having the same area of the base have also the same area of curved surfaces. If the height of cylinder be 2 m, find the slant height of the cone

(a) 3 m (b) 3.5 m

(c) 4.5 m (d) 4 m

82. The radii of a cylinder and a cone are equal. If the height of the cylinder is equal to the slant height of the cone then the ratio of the curved surfaces of the cylinder and the cone is

(a) 1 : 1 (b) 2 : 1

(c) 3 : 1 (d) 4 : 1

83. From a cubical block of wood of side 1 m, a cylinder of the largest possible volume is curved out. The volume (in cu m) of the remaining wood is

(a) $\frac{3}{14}$ (b) $\frac{5}{14}$

(c) $\frac{1}{2}$ (d) $\frac{2}{7}$

84. The radius of the base of a solid cylinder is r cm and its height is 3 cm. It is re-casted into a cone of same radius, the height of the cone will be

(a) 3 cm (b) 6 cm

(c) 9 cm (d) 27 cm

85. Two cm of rain has fallen on a sq km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a 100 m × 10 m base, by what level would the water level in the pool have increased?

(a) 15 m (b) 20 m

(c) 10 m (d) 25 m

86. The perpendicular height of a conical tent is $4\frac{2}{3}$ m and the diameter of its base is 6 metres. If 11 persons can sleep in this tent, find how many average cu m of air each person gets?

(a) 2 cu m (b) 4 cu m

(c) 6 cu m (d) 8 cu m

87. The circumference of the base of a 9 m high conical tent is 44 m. The volume of the air contained in it is

(a) 426 m^3 (b) 452 m^3

(c) 472 m^3 (d) 512 m^3

88. A conical vessel of base radius 2 cm and height 3 cm is filled with kerosene. This liquid leaks through a hole in the bottom and collects in a cylindrical jar of radius 2 cm. The kerosene level in the jar is

(a) 1.5 cm (b) π cm

(c) 1 cm (d) 3 cm

89. A hollow cylinder of height 3 cm, is re-casted into a solid cylinder. If the external and internal radii of the hollow cylinder are 4.3 cm and 1.1 cm respectively. What will be the radius of the solid cylinder?

(a) 2.8 cm (b) 2.4 cm

(c) 3.2 cm (d) 4.8 cm

90. A solid consists of a circular cylinder with an exact fitting right circular cone placed on the top. The height of the cone is h. If the total volume of the solid is three times the volume of the cone, then the height of the cylinder is

(a) $2h$ (b) $4h$

(c) $\frac{2h}{3}$ (d) $\frac{3h}{2}$

91. A well of 11.2 m diameter is dug 8 m deep. The earth taken out has been spread all around it to a width of 7 cm to form a circular embankment. Find the height of this embankment.

(a) 304.8 m^2 (b) 400.4 m^2

(c) 408.4 m^2 (d) 412.4 m^2

92. The curved surface of a circular cylinder of height 'h' and the slant surface of the cone of slant height '$2h$' having the same circular base are in the ratio of

(a) 1 : 1 (b) 1 : 2

(c) 3 : 2 (d) 1 : 3

93. The material of a cone is converted into the shape of a cylinder of equal radius. If the height of the cylinder is 5 cm, the height of the cone is

(a) 10 cm (b) 15 cm

(c) 18 cm (d) 24 cm

94. A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 c.c, what approximately is the volume of the cone?

(a) 90 c.c. (b) 75 c.c.

(c) 80 c.c. (d) 85 c.c.

95. A solid cone is 25 cm high and radius of its base is 50 cm. It is melted and re-cast into a solid sphere. Determine the surface area of the sphere.

(a) 8757.28 sq cm (b) 5877.42 sq cm

(c) 7857.14 sq m (d) None of these

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EXPLANATORY ANSWERS

1. Ans. (c)

Solution Volume of water $= 16 \times 12 \times \frac{16\frac{2}{3}}{100}$ m^3

$= 16 \times 12 \times \frac{50}{3 \times 100}$ m^3

$=$ **32 m^3.**

2. Ans. (b)

Solution Since the wood is 1 cm thick, the inner measurements are

$l = 12$ cm $- 2$ cm $= 10$ cm

$b = 10$ cm $- 2$ cm $= 8$ cm

$h = 8$ cm $- 2$ cm $= 6$ cm

Capacity of the box $= l \times b \times h$

$= 10$ cm $\times 8$ cm $\times 6$ cm

$=$ **480 cm^3.**

3. Ans. (c)

Solution Volume of the cistern $= 2.4 \times 2 \times 1.5$ m^3

$= 7.2$ m^3

Time taken $= 2.5$ hr

$\therefore$ Rate of water flow $= \frac{7.2}{2.5}$ m^3/hr

$= \frac{7.2 \times 100 \times 100 \times 100}{2.5 \times 60 \times 60}$

$=$ **800 cu m/sec.**

4. Ans. (b)

Solution Let the sides of the box be x, y and z

Then, $p = xy$, $q = yz$, $r = zx$

Volume of the box $= xyz$

We have, $p \times q \times r = xy \times yz \times zx = x^2y^2z^2$

or, $xyz = \sqrt{pqr}$.

5. Ans. (c)

Solution Volume of water lowered

$= 30 \times 15 \times 4$ m^3 $= 1800$ m^3

$\therefore$ Number of gallons taken out

$= 1800 \times 180$ gallons

$=$ **324000 gallons**

[1 m^3 of water $= 180$ gallons]

6. Ans. (b)

Solution Number of bricks

$= \frac{\text{Volume of the wall}}{\text{Volume of the brick}}$

$= \frac{(5 \times 100) \times (3 \times 100) \times 20}{25 \times 12.5 \times 7.5}$

$=$ **1280.**

7. Ans. (c)

Solution Volume of the wood $= \frac{31}{2} \times \frac{11}{4} \times \frac{4}{3}$ m^3

$= 93.5$ m^3

Cost of the wood $=$ Rs. $45 \times 93.5 =$ **Rs. 4207.50.**

8. Ans. (a)

Solution Number of bricks

$= \frac{\text{Volume of the wall}}{\text{Volume of the brick}}$

$= \frac{1500 \text{ cm} \times 300 \text{ cm} \times 50 \text{ cm}}{12 \text{ cm} \times 12 \text{ cm} \times 6 \text{ cm}}$

$=$ **125000 cm.**

9. Ans. (b)

Solution Length of the diagonal

$= \sqrt{12^2 + 10^2 + 8^2}$ m

$= \sqrt{308}$ m

$=$ **17.5 m.**

10. Ans. (a)

Solution External dimensions

$l = 12$ cm, $b = 10$ cm, $h = 8$ cm

External volume $= 12$ cm $\times 10$ cm $\times 8$ cm

$= 960$ cm^3

Internal dimensions

$l = 12 - 2 = 10$ cm

$b = 10 - 2 = 8$ cm

$h = 8 - 2 = 6$ cm

Internal volume $= 10$ cm $\times 8$ cm $\times 6$ cm

$= 480$ cm^3

Volume of the wood $= 960$ cm$^3 - 480$ cm^3

$= 480$ cm^3

Cost of the wood $=$ Rs. $(480 \times 3) =$ **Rs. 1440.**

11. Ans. (d)

Solution Let side of the cube be x units

$\therefore$ Surface area of cube $= 6x^2$

$\therefore$ Sum of the areas of three cubes $= 18x^2$

For the cuboid, length $= 3x$ units

Breadth $= x$ units

height $= x$ units

$\therefore$ Surface area $= 2[lb + bh + hl]$

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$= 2\,[3x \times x + x \times x + x \times 3x]$

$= 2\,(7x^2) = 14x^2$

$\therefore \quad \dfrac{\text{Surface area of new cuboid}}{\text{Total surface area of three cubes}}$

$= \dfrac{14x^2}{18x^2} = \dfrac{7}{9}.$

12. Ans. (b)

13. Ans. (b)

Solution Let the sides be x, $5x$

Cubic cm of iron sheet $= 0.5 \times x \times 5x = 2.5x^2$

This should be the volume of the cube

$\therefore \quad 2.5x^2 = 10 \times 10 \times 10$

$x^2 = \dfrac{1000 \times 10}{25}$

$\therefore \quad x = 20 \quad \therefore$ Sides are 20, **100 cm.**

14. Ans. (c)

Solution Capacity of room $= 12 \times 8 \times 6$ cu m

Required number of boxes $= \dfrac{12 \times 8 \times 6}{1.5} =$ **384.**

15. Ans. (c)

Solution Length $= \dfrac{3}{2} \times$ breadth; height $= 3\dfrac{3}{10}$ metres

$\dfrac{3}{2} \times$ breadth $\times$ breadth $\times \dfrac{33}{10} = 123\dfrac{3}{4}$ cu metres or,

$(\text{breadth})^2 = \dfrac{445}{5} \times \dfrac{2}{3} \times \dfrac{10}{33} = 25$ sq m.

$\therefore$ Breadth $= \sqrt{25} = 5$ m

$\therefore$ Length $= \dfrac{3}{5} \times 5 =$ **7.5 m.**

16. Ans. (c)

Solution Volume of earth dug out $= (3 \times 2 \times 1.5)\ \text{m}^3 = 9\ \text{m}^3$

Area over which earth is spread

$= [(22 \times 141) - (3 \times 2)]\ \text{m}^2 = 302\ \text{m}^2$

$\therefore$ Increase in level $= \dfrac{9}{302}\ \text{m} = \dfrac{9 \times 100}{302}\ \text{cm}$

$=$ **2.98 cm.**

17. Ans. (b)

Solution $14 \times b = 70 \times 2.2$

$\therefore \quad b = \dfrac{70 \times 2.2}{14} = 11$ metres

and $l \times b \times h = 70 \times 11$

$\therefore \quad h = \dfrac{70 \times 11}{l \times b} = \dfrac{70 \times 11}{70 \times 2.2} =$ **5 metres.**

18. Ans. (a)

Solution Let the initial height be h and the height after sand is thrown be H metres.

We have to find $H - h$.

According to question,

$12 \times 5 \times (H - h) = 210$

$\therefore \quad H - h = \dfrac{210}{60} = \dfrac{7}{2} =$ **3.5 metres.**

19. Ans. (d)

Solution Let the length, breadth and height be $6x$, $5x$ and $4x$ metres, respectively.

Then, $2 \times [6x \times 5x + 4x \times 5x + 4x \times 6x] = 33300$

$\therefore \quad 148x^2 = 33300.$

or, $\quad x^2 = 225 \quad$ or, $\quad x = 15$

So, length $= 90$ m

breadth $= 75$ m

height $=$ **60 m.**

20. Ans. (c)

Solution (Area of the square end) $\times 9 =$ Volume $= 1\ \text{m}^3$

$\therefore$ Side of the square end $= \sqrt{\dfrac{1}{9}}\ \text{m} = \dfrac{1}{3}\ \text{m}$

$\therefore$ Volume of this cube $= \left(\dfrac{1}{3}\right)^3\ \text{m}^3 = \dfrac{1}{27}\ \text{m}^3$

$\therefore$ Weight of this cube $= \dfrac{1}{27} \times 90\ \text{kg} =$ **$3\dfrac{1}{3}$ kg.**

21. Ans. (b)

Solution $6e^2 = 24\ \text{cm}^2$, so that e (edge) $= 2$ cm

$\therefore$ Number of such cubes out of a cube of edge 1 metres (or 100 cm)

$= \left(\dfrac{100}{2}\right)^3 =$ **125000.**

22. Ans. (d)

Solution Side of the new cube

$= \sqrt[3]{\text{Sum of the cubes of sides of all the cubes}}$

$\therefore$ Side $= \sqrt[3]{6^3 + 8^3 + 10^3} = \sqrt[3]{216 + 512 + 1000}$

$= \sqrt[3]{1728} =$ **12 cm.**

23. Ans. (c)

Solution Number of cubes

$= \left(\dfrac{\text{Original length of edge}}{\text{New length of edge}}\right)^3$

$\therefore$ Number of cubes $= \left(\dfrac{6}{2}\right)^3 =$ **27.**

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24. Ans. (c)

Solution Volume of the cube of side 20 cm
$= (20)^3 = 8000 \text{ cm}^3$
Volume of the cube of side 5 cm $= 125 \text{ cm}^3$
$\therefore$ Number of smaller cubes of side 5 cm
$= \frac{8000}{125} = \mathbf{64.}$

25. Ans. (c)

Solution Surface area of a cube $= 6 \times (\text{side})^2$
$\therefore \quad 6 \times (\text{side})^2 = 600$
$\Rightarrow (\text{Sides})^2 = 100$
$\Rightarrow$ Side $= \sqrt{100} = 10$ cm
$\therefore$ Diagonal of the cube $= \sqrt{3} \times$ side
$= \sqrt{3} \times 10 = \mathbf{10\sqrt{3}}$ **cm.**

26. Ans. (a)

Solution Volume of water collected in the tank in 8 hours
$= 30 \text{ m} \times 20 \text{ m} \times 1 \text{ m}$
$= 600$ cu m.
$\therefore$ Volume of water collected in the tank in 1 hour
$= \frac{600}{8} = 75$ cu m.
Water comes through a pipe of cross-section
$= 5 \text{ cm} \times 5 \text{ cm} = \frac{25}{10000}$ sq m.
The speed of water = Distance travelled by the water in the pipe in one hour
$= \frac{75 \times 10000}{25}$ m $= \mathbf{30}$ **km/hr.**

27. Ans. (d)

Solution Volume of the wall $= 10 \times \frac{4}{10} \times 5$
$= 20$ cu m.
Volume of the mortar $= \frac{10}{100} \times 20 = 2$ cu m.
Hence, the volume occupied by the bricks
$= 20 - 2 = 18$ cu m
Volume of each brick $= \frac{25}{100} \times \frac{15}{100} \times \frac{8}{100}$ cu m.
$= \frac{3}{1000}$ cu m.
Therefore, the required number of bricks
$= 18 \div \frac{3}{1000} = \mathbf{6000.}$

28. Ans. (a)

Solution Since the edges of the cubes are in the ratio 3 : 4 : 5, let these be $3k$, $4k$, $5k$ metres, respectively.
Their volumes are $27k^3$, $64k^3$, $125k^3$ cu m.
Thus, the volume of the single cube
$= (27 + 64 + 125)\, k^3$ cu m
$= 216\, k^3 = (6k)^3$ cu m
We know that the length of the diagonal of a cube of side x is $\sqrt{3}x$. Therefore, the length of the diagonal of the single cube mentioned above is equal to $6k\sqrt{3}$. But the length of the diagonal of this cube is given to be $48\sqrt{3}$, hence
$6k\sqrt{3} = 48\sqrt{3}$ or, $k = 8$.
Therefore, the length of the edges of the three cubes are 3×8, 4×8, 5×8 m, that is **24 m, 32 m, 40 m.**

29. Ans. (c)

Solution Edge of the cube = 6 cm
$\therefore$ Volume of lead $= 6^3 \text{ cm}^3 = 216 \text{ cm}^3$
Let the edge of the new cube be x cm
Then $27x^3 = 216$
$\Rightarrow x^3 = 8$ or $\boldsymbol{x = 2}$ **cm.**

30. Ans. (c)

Solution Length of the cube $= (729)^{1/3} = 9$
Total surface area $= 6 \times (9)^2 = \mathbf{486}$ **cm^2.**

31. Ans. (d)

Solution $a_1^3 : a_2^3 = 1 : 27$
$\Rightarrow a_1 : a_2 = 1 : 3$
$\therefore$ Required ratio is $1^2 : 3^2 = \mathbf{1 : 9.}$

32. Ans. (b)

Solution $4(24 \times 12) + 2(12 \times 12) = \mathbf{1440}$ **sq cm.**

33. Ans. (d)

Solution $4a = 20 \Rightarrow a = 5$
$\therefore$ Volume $= a^3 = 5^3 = \mathbf{125}$ **cm^3.**

34. Ans. (a)

Solution Ratio of the edge of cubes
$= 3 : 12 = 1 : 4$
Ratio of their volumes $= 1^3 : 4^3 = 1 : 64$.
Because volume of the new cube is 64 times the volume of the first cube, the weight of the new cube is also 64 times the weight of the first cube.
Weight of the new cube
$= 64 \times 12 \text{ gm} = \mathbf{768}$ **gm**

35. Ans. (b)

Solution When edge is increased 3 times, the volume or weight is increased 3^3, i.e. 27 times.
$\therefore$ The weight of the other cube
$= 27 \times 17 = \mathbf{459}$ **gm.**

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65. Ans. (b)

Solution Required volume
= Volume of cylinder – Volume of cone

$$= \pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10$$

$$= \frac{2}{3} \times \pi \times 36 \times 10$$

$$= \frac{2}{3} \times \frac{22}{7} \times 360 = \frac{5280}{7}$$

$$= \mathbf{754.3 \text{ cu cm.}}$$

66. Ans. (a)

Solution $r = 2x$, $r_1 = 3x$

$h = 5y$, $h_1 = 3y$

Volume ratio $= \pi r^2 h : \pi r_1^2 h_1$

$$= \pi \times (2x)^2 \times 5y : \pi \times (3x)^2 \times 3y$$

$$= 20\pi x^2 y : 27\pi x^2 y$$

$$= \mathbf{20 : 27.}$$

67. Ans. (a)

Solution Radius of the base of the cylinder
$= r = 14$ m.

h = depth of the tank

Capacity = Volume of the tank

$$= \pi r^2 h = 6160 \text{ cu m.}$$

or, $\frac{22}{7} \times 14 \times 14 \times h = 6160 \quad \therefore \quad h = 10$ m

Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 14 \times 10$

$= 880$ sq m

$\therefore$ Cost of painting this curved surface

$$= 880 \times 2.80 = \mathbf{Rs.\ 2464.}$$

68. Ans. (c)

Solution $(h + r) = 37$, $2\pi r(h + r) = 1628$

$$\Rightarrow 2\pi r\ (37) = 1628 \Rightarrow r = \frac{1628 \times 7}{2 \times 22 \times 37} = 7$$

$\Rightarrow r = 7$ and $h = 30$

$\therefore$ Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7 = \mathbf{44\ m.}$

69. Ans. (b)

Solution Rolled along with its length, then $h = 10$ cm, and the other side = 22 cm

$$\therefore \quad V = \frac{10 \times (22)^2 \times 7}{4 \times 22} = \mathbf{385\ cm^2.}$$

70. Ans. (d)

Solution $\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi rh + 2\pi r^2}{2\pi rh}$

$$= \frac{2\pi r\,(h+r)}{2\pi rh}$$

$$= \frac{h+r}{h} = \frac{20+80}{20}$$

$$= \frac{5}{1} = \mathbf{5 : 1.}$$

71. Ans. (b)

Solution Let x cones be needed

Then, $\frac{1}{3}\pi r^2 h \times x = \pi r^2 h$ or, $\mathbf{x = 3.}$

72. Ans. (c)

Solution Radius of the bucket $= r = 14$ cm

Height of the bucket $= h = 72$ cm

Volume of the water = Volume of the bucket

$$= : \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 72$$

$$\therefore \quad 66 \times 28 \times H = \frac{22}{7} \times 14 \times 14 \times 72$$

$$\therefore \quad H = \frac{22 \times 72}{66} = \mathbf{24\ cm.}$$

73. Ans. (c)

Solution Volume of the iron rod $= \frac{22}{7} \times 1 \times 1 \times 70$

$= 220$ cu cm.

$\therefore$ Weight of the cylinder $= \frac{220 \times 10}{1000} = \mathbf{2.2\ kg.}$

74. Ans. (a)

Solution Let the initial radius and height of the cylinder be r cm and h cm, respectively.

Then, curved surface area of the original cylinder $= 2\pi rh$ and curved surface area of the new cylinder

$$= 2\pi(2r) \times \frac{h}{2} = 2\pi rh$$

$\therefore$ Required ratio

$$= \frac{\text{New curved surface area}}{\text{Previous curved surface area}}$$

$$= \frac{2\pi rh}{2\pi rh} = \mathbf{1 : 1.}$$

75. Ans. (d)

Solution Volume of the water in jar $= \pi \times 12^2 \times 30\ \text{cm}^3$

When the ball is dropped into the jar, volume of water + ball

$$= \pi \times 12^2 \times (30 + 6.75)$$

Increase in volume

$$= \pi \times 12^2 \times (30 + 6.75 - 30)\ \text{cm}^3$$

$$= \pi \times 144 \times 6.75\ \text{cm}^3$$

It r is the radius of the ball, then

$$\frac{4}{3}\pi r^3 = \pi \times 144 \times 6.75$$

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109. Ans. (d)

Solution Let radius of the cone = r, height = h

Then, volume of the cone = $\frac{1}{3}\pi r^2 h$

Increased radius = $2r$, height = $2h$

$\therefore$ Increased volume

$$= \frac{1}{3}\pi(2r)^2(2h)$$

$$= \frac{1}{3}\pi \times 8r^2h = 8\left(\frac{1}{3}\pi r^2 h\right)$$

= 8 times the original volume.

110. Ans. (c)

Solution Original volume = $\frac{4}{3}\pi r^3$

New volume = $\frac{4}{3}\pi(2r)^3 = \frac{32}{3}\pi r^3$

Increase % = $\left(\frac{28}{3}\pi r^3 \times \frac{3}{4\pi r^3} \times 100\right)\%$ **= 700%.**

111. Ans. (c)

Solution Radius of the sphere = 3 cm

Volume of the sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3 \quad \ldots(1)$$

Radius of the wire = 0.1 m

Volume of the wire of length l cm and radius 1.0 m

$$= \pi r^2 l = \pi \times 0.1 \times 0.1 \times l \quad \ldots(2)$$

Now, $36\pi = \pi \times 0.1 \times 0.1 \times l$

$\Rightarrow$ $l = \frac{36\pi}{\pi \times 0.1 \times 0.1} = 3600$ cm **= 36 m.**

112. Ans. (b)

Solution If r is the radius of the base then the circumference of the base = $2\pi r$ = 44 m

$\therefore$ $r = \frac{44 \times 7}{2 \times 22} = 7$ cm

The height of the cone = h

Then, the volume of the air in the tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 9$$

= 462 cu m.

113. Ans. (c)

Solution Original area = $2\pi r^2$

New area = $4\pi(2r)^2 = 16\pi r^2$

Increase % = $\left(\frac{12\pi r^2}{4\pi r^2} \times 100\right)\%$ **= 300 %.**

114. Ans. (c)

Solution Here, $x = y = 0$ and $z = -8$.

$\therefore$ Percentage change in volume

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right]\%$$

$$= \left[0 + 0 - 8 + \frac{0 \times 0 + 0 \times .8 + 0 \times .8}{(100)^2}\right]\%$$

$= -8\%$.

$\therefore$ Volume of the cylinder decreases by **8%.**

115. Ans. (b)

Solution Here, $x = y = 20$ and $z = 0$.

$\therefore$ Percentage increase in volume

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}\right]\%$$

$$= \left[20 + 20 + 0 + \frac{20 \times 20 + 20 \times 0 \times 20 \times 0}{100} + \frac{20 + 20 \times 0}{(100)^2}\right]\%$$

$= (40 + 4)\%$ **= 44%.**

■ QUESTIONS FROM COMPETITIVE EXAMS ■

1. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be K% lower than the volume of B. The value of K must be

(a) 85.5 (b) 92.5

(c) 90.5 (d) 87.5

[Based on CAT 2003]

Directions (Q 2 to 4): Answer the questions on the basis of the information given below. Consider a cylinder of height h cm and radius $r = \frac{2}{\pi}$ cm as shown in the figure (not drawn to scale). A string of certain length, when wound on its cylindrical surface, starting at a pint A and ending at point B, gives a maximum of n turns (in other words, the string length is the minimum length of wind n turns).

2. What is the vertical spacing in cm between consecutive turns?

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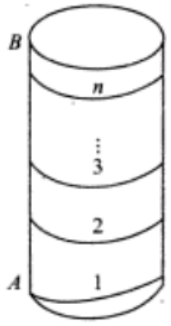
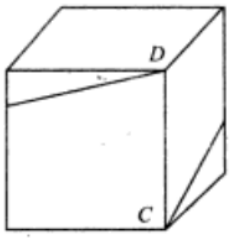


(a) $\frac{h}{n}$ (b) $\frac{h}{\sqrt{n}}$

(c) $\frac{h}{n^2}$

(d) Cannot be determined with the given information **[Based on CAT 2004]**

3. The same string, when wound on the exterior four walls of a cube of side n cms, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string, in cm, is



(a) $\sqrt{2}n$ (b) $\sqrt{17}n$

(c) n (d) $\sqrt{13}n$

[Based on CAT 2004]

4. In the setup of the previous two questions, how is h related to n?

(a) $h = \sqrt{2}n$ (b) $h = \sqrt{17}n$

(c) $h = n$ (d) $h = \sqrt{13}n$

[Based on CAT 2004]

5. A square tin sheet of side 12 inches is converted into a box with open top in the following steps: The sheet is placed horizontally. Then, equal-sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?

(a) 3 (b) 4

(c) 1 (d) 2 **[Based on CAT 2004]**

6. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube?

(a) 2 : 1 (b) 3 : 2

(c) 25 : 18 (d) 27 : 20

[Based on MAT 2004]

7. A cylinder 6cm in diameter is partially filled with water. A sphere 3cm in diameter is gently dropped into the cylinder. To what further height will the water in the cylinder rise?

(a) 6cm (b) 2cm

(c) 1/2cm (d) None of these

[Based on MAT 2004]

8. A copper sphere is drawn into a cylindrical wire of 4 metres length. If the diameter of the sphere is ten times the diameter of the wire, then what is the radius of the sphere?

(a) 3cm (b) 3mm

(c) 6cm (d) 6π mm

[Based on MAT 2004]

9. How many bricks, each measuring 24 cm × 11.5 cm × 8cm, will be needed to construct a wall 8m long, 6m high and 23cm wide, while 5% of the total wall contains mortar?

(a) 5000 (b) 5250

(c) 4750 (d) 4250

[Based on MAT 2003]

10. The dimensions of an open box are 52 cm × 40 cm × 29 cm. Its thickness is 2cm. If 1 cu cm of metal used in the box weighs 0.5g, then the weight of the box is

(a) 6.832 kg (b) 7.576 kg

(c) 7.76 kg (d) 8.56 kg

[Based on MAT 2003]

11. How many metres of cloth 2.5 m wide will be needed to make a conical tent with base radius 7 m and height 24 m?

(a) 120 m (b) 180 m

(c) 220 m (d) 550 m

[Based on MAT 2003]

12. Sixteen cylindrical cans, each with a radius of 1 unit, are placed inside a cardboard box four in a row. If the cans touch the adjacent cans and or the walls of the box, then which of the following could be the interior area of the bottom of the box in square units?

(a) 16 (b) 32

(c) 64 (d) 128

[Based on MAT 2003]

13. The length of the longest rod that can be placed in a room which is 12m long, 9m broad and 8m high is

(a) 27 m (b) 19 m

(c) 17 m (d) 13 m

[Based on MAT 2003]

14. A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. The height of the conical heap is 12 cm. The radius of the heap at the base is

(a) 63 cm (b) 53 cm

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(c) 56 cm (d) 66 cm

[Based on MAT 2003]

15. A hemispherical bowl is made of steel 0.5 cm thick. The inside radius of the bowl is 4 cm. The volume of the steel used in making the bowl is

(a) 55.83 cm^2 (b) 56.83 cm^2

(c) 57.83 cm^3 (d) 58.83 cm^3

[Based on MAT 2003]

16. A metallic sheet is of rectangular shape with dimensions 48 m × 36 m. From each of its corners, a square is cut off so as to make an open box. The volume of the box is X m^3, when the length of the square is 8 m, the value of X is

(a) 5120 (b) 8960

(c) 4830 (d) 5120

[Based on MAT 2003]

17. The sum of length, breadth and height of a room is 19m. The length of the diagonal is 11 m. The cost of painting the total surface area of the room at the rate of Rs. 10 per m^2 is

(a) Rs 240 (b) Rs 2400

(c) Rs 420 (d) Rs 4200

[Based on MAT 2003]

18. The cost of painting the walls of a room at the rate of Rs 1.35 per square metre is Rs 340.20 and the cost of matting the floor at the rate of Re 0.85 per square metre is Rs 91.80. If the length of the room is 12 m, then the height of the room is

(a) 6 m (b) 12 m

(c) 1.2 m (d) 12.6 m

[Based on MAT 2003]

19. By what percent, the volume of a cube increases, if the length of each edge was increased by 50%?

(a) 237.5% (b) 273.5%

(c) 1?5% (d) 50%

[Based on MAT 2003]

20. Five equal cubes, each of side 5 cm, are placed adjacent to each other. The volume of the new solid formed will be

(a) 15525 cm^3 (b) 625 cm^3

(c) 125 cm^3 (d) None of these

[Based on MAT 2003]

21. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The sides of the cubes are

(a) 6 cm, 8 cm, 10 cm (b) 3 cm, 4 cm, 5 cm

(c) 9 cm, 12 cm, 15 cm (d) None of these

22. A sphere and a cube have the same surface area. The ratio of the volume of the sphere to that of the cube

(a) 1 : 2 (b) $\sqrt{6} : \sqrt{\pi}$

(c) 6 : π (d) 3 : 1

[Based on MAT 2003]

23. If the height, curved surface area and the volume of a cone are h, c and v respectively, then $3\pi vh^3 - c^2h^2 + 9v^2$ will be equal to

(a) 0 (b) 1

(c) chv (d) v^2h

[Based on MAT 2003]

24. The areas of three adjacent faces of a cuboid are a, b and c. If the volume of the cuboid is V, then V^2 is equal to

(a) abc (b) $(ab + bc + ac)$

(c) $\frac{c}{ab}$ (d) None of these

[Based on MAT 2003]

25. The weight of a solid cone having diameter 14 cm and vertical height 51 cm is ... if the material of solid cone weighs 10 grams per cubic cm.

(a) 16.18 kg (2) 17.25 kg

(c) 26.18 kg (d) 71.40 kg

[Based on MAT 2002]

26. If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the surface area of each ball is

(a) 60π cm^2 (b) 50π cm^2

(c) 75π cm^2 (d) 100π cm^2.

[Based on MAT 2002]

27. The area of four walls of a room is 120 m^2. The length is twice the breadth. If the height of the room is 4 m, find the area of the floor.

(a) 48 m^2 (b) 49 m^2

(c) 50 m^2 (d) 52 m^2

[Based on MAT 2005]

28. A rectangular tank measuring 5 m × 4.5 m × 2.1 m is dug in the centre of the field measuring 13.5 m × 2.5 m. The earth dug out is spread evenly over the remaining portion of the field. How much is the level of the field raised?

(a) 4.0 m (b) 4.1 m

(c) 4.2 m (d) 4.3 m

[Based on MAT 2005]

29. Find the number of coins, 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

(a) 430 (b) 440

(c) 450 (d) 460

[Based on MAT 2005]

30. How many metres of cloth 5 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m^2. (Take $\pi = 22/7$)

(a) 108 m (b) 110 m

(c) 112 m (d) 115 m

[Based on MAT 2005]

31. A cone, a hemisphere and a cylinder have equal bases and same heights. Their volumes will be in the ratio

(a) 1 : 2 : 3 (b) 3 : 4 : 1

(c) 3 : 2 : 1 (d) None of these

[Based on MAT 2001]

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8. Ans. (b)

Solution Let the radius of the cylindrical wire be r metres.

$\therefore$ Radius of the sphere = $10r$ metres

$\therefore \quad \frac{4}{3}\pi \times (10r)^3 = \pi \times r^2 \times 4$

$\Rightarrow \quad \frac{4}{3} \times 1000r^3 = 4r^2$

$\Rightarrow \quad r = \frac{3}{1000}\text{m} = \mathbf{3mm.}$

9. Ans. (c)

Solution No. of bricks

$$= \frac{800 \times 600 \times 23 - 5\% \text{ of } (800 \times 600.23)}{24 \times 11.5 \times 8}$$

$$= \frac{11040000 - 552000}{2208} = \mathbf{4750.}$$

10. Ans. (a)

Solution Volume of the metal used in the box

$= 52 \times 40 \times 29 - 48 \times 36 \times 27$

$= 60320 - 46656$

$= 13664$ cu. cm.

$\therefore$ Weight of the box = 13664×0.5

$= 6832$ gm $= \mathbf{6.832./kg.}$

11. Ans. (d)

Solution Area of the cloth required to make a conical tent = Surface area of the cone with height 24 m and base radius 7 m

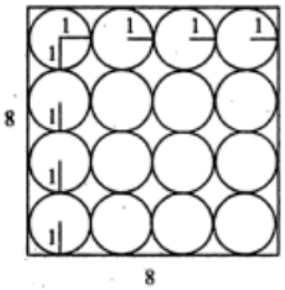
$= \pi r\sqrt{r^2 + h^2} \quad [r = 7, h = 24]$

$= \frac{22}{7} \times 7 \times \sqrt{49 + 576} = 22 \times 25 = \mathbf{550 \text{ sq. m.}}$

12. Ans. (c)

Solution Let r be the radius of the base of each can

$\Rightarrow \quad r = 1$



$\therefore$ Length of the box = 8 units

Width of the box = 8 units

$\therefore$ Interior area of the bottom of the box

$= \mathbf{64 \text{ sq.units.}}$

13. Ans. (c)

Solution $\sqrt{12^2 + 9^2 + 8^2} = \sqrt{144 + 81 + 64}$

$= \sqrt{289} = \mathbf{17.}$

14. Ans. (a)

Solution Volume of the land = $\pi \times (21)^2 \times 36$

Let r be the radius of the conical heap.

$\therefore \quad \frac{1}{3}\pi r^2 \times 12 = \pi \times (21)^2 \times 36$

$\Rightarrow \quad r^2 = (21)^2 \times 9 \Rightarrow \mathbf{r = 63.}$

15. Ans. (d)

Solution Volume of the steel = $\frac{2}{3}\pi(4.5)^3 - \frac{2}{3}\pi(4)^3$

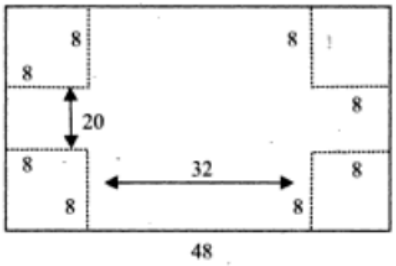
$= \frac{2}{3}\pi[(4.5)^3 - 4^3]$

$= \frac{2}{3} \times \frac{22}{7} \times (27.125)$

$= \mathbf{56.83 \text{ cm}^2.}$

16. Ans. (a)

Solution



$\therefore$ Dimensions of the box are 32, 20 and 8.

$\therefore$ Volume of the box = $32 \times 20 \times 8 = \mathbf{5120 \text{ m}^3.}$

17. Ans. (b)

Solution $L + B + H = 19$

Let $L = x$, $B = y$ and $H = z$

$\therefore \quad x^2 + y^2 + z^2 = 11^2 = 121$

Also, $x + y + z = 19$ (Given)

$\therefore \quad (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow \quad 2(xy + yz + zx) = (19)^2 - 121 = 240$

Surface are of the room

$= 2(xy + yz + zx) = 240 \text{ m}^2$

$\therefore$ Cost of painting the total surface area of the room @ Rs $10/\text{m}^2 = \mathbf{Rs\ 2400.}$

18. Ans. (a)

Solution Let the height of the room be h metres.

Area of four walls of the room

$$= 2(L + B) \times h = \frac{340.20}{1.35} = 252 \text{ m}^2$$

$\Rightarrow (L + B)h = 125$

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Also, $L \times B = \frac{91.80}{0.85} = 108$

Since $L = 12$, therefore $B = 9$

Hence, $h = \frac{126}{L+B} = \frac{126}{21} = \mathbf{6}$.

19. Ans. (a)

20. Ans. (b)

21. Ans. (a)

Solution Let the sides of the cube be $3k$, $4k$, $5k$ respectively.

$\therefore$ Volumes of these cubes are $27k^3$, $64k^3$, $125k^3$ respectively.

$\Rightarrow$ Volumes of the new bigger cube
$= 27k^3 + 64k^3 + 125k^3$
$= 216k^3$

$\therefore$ Side of the new cube $= 6k$.

Since diagonal of the new cube $= 12\sqrt{3}$, therefore

$\sqrt{(6k)^2 + (6k)^2 + (6k)^2} = 12\sqrt{3}$

$\Rightarrow 108k^2 = 432 \Rightarrow k = 2$

$\therefore$ Sides of the three cubes were 6 cm, 8 cm and 10 cm respectively.

22. Ans. (b)

Solution Let r be the radius of the sphere.

Let a be the side of the cube

$\therefore \quad 4\pi r^2 = 6a^2 \Rightarrow r^3 = \frac{6\sqrt{6}a^3}{8\pi\sqrt{\pi}}$

$\Rightarrow \frac{\frac{4}{3}\pi r^3}{a^3} =$ Required Ratio $= \frac{\sqrt{6}}{\sqrt{\pi}}$.

23. Ans. (a)

Solution Let h be the height

Volume of the cone $= v$

Curved surface area of the cone $= c$

$\therefore \quad \frac{1}{3}\pi r^2 h = v, \ \pi rl = c$

i.e. $\pi r\sqrt{r^2 + h^2} = c$ i.e. $\pi^2 r^2 (r^2 + h^2) = c^2$

Consider $3\pi v h^3 - c^2 h^2 + 9v^2$

$= 3\pi \times \frac{1}{3}\pi r^2 h \times h^3 - \pi^2 r^2 (r^2 + h^2) h + 9 \times \frac{1}{9} \times \pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = \mathbf{0}$

24. Ans. (a)

Solution Sides are $\sqrt{a}, \sqrt{b}, \sqrt{c}$

$\therefore \quad V = \sqrt{abc} \Rightarrow \mathbf{V^2 = abc.}$

25. Ans. (c)

Solution Volume of the solid cone

$= \frac{1}{3} \times \pi \times (7)^2 \times 51 = 833\pi$

$\therefore$ Weight of the solid cone $= \frac{8330}{1000}$ kg $=$ **26.18 kg.**

26. Ans. (d)

Solution Volume of the solid sphere

$= \frac{4}{3} \times \frac{22}{7} \times (10)^3 = \frac{88000}{21}$ c.c

Let radius of each spherical solid ball be r.

$\therefore \quad 8 \times \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88000}{21}$

$\Rightarrow \quad r^3 = \frac{88000}{8 \times 4 \times 22} = 125 \Rightarrow r = 5$

$\therefore$ Surface area of each ball

$= 4\pi r^2 = 4 \times \frac{22}{7} \times 25 = \mathbf{100\pi\ cm^2}$.

27. Ans. (c)

Solution Area of 4 walls $= 2(l + b) \times h = 120$
$= 2(2b + b) \times 4 = 120$

$\Rightarrow \quad 24b = 120$

$\therefore \quad b = 5$ m

$\therefore \quad l = 10$ m

$\therefore \quad$ Area of the floor $= 5 \times 10 =$ **50 m^2.**

28. Ans. (a)

Solution Volume of earth dug out $= 5 \times 4.5 \times 2.1 = 47.25$ m^3

Area over which earth is spread $= 13.5 \times 2.5 - 5 \times 4.5.$
$= 33.75 - 22.0 = 11.75$ m

Rise in level $= \frac{47.25}{11.75} =$ **4.02 m.**

29. Ans. (c)

Solution Volume of cylinder $= \pi r^2 h$
$= 2.25 \times 2.25 \times 10\ \pi = 50.625\ \pi$

Volume of coin $= \pi r^2 h = .75 \times .75 \times 0.2\ \pi = .1125\ \pi$

$\therefore \quad$ No. of coins $= \frac{50.625\pi}{.1125\pi} =$ **450.**

30. Ans. (b)

Solution $l = \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576}$
$= \sqrt{625} = 25$ m

Area of cloth needed $= \pi r l = \frac{22}{7} \times 7 \times 25 = 550$ m^2

length of cloth $= \frac{550}{5} =$ **110 m.**

31. Ans. (a)

Solution Let r be the radius of the base and h be the height in each case.

[$r = h =$ height of the hemisphere]

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{\pi r^2 h}{3} = \frac{\pi r^3}{3}$

Volume of hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

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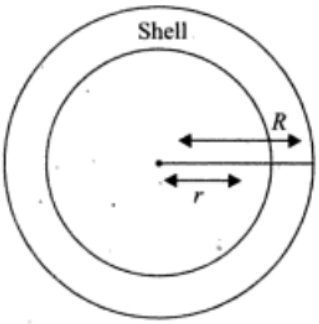
47. Ans. (a)

Solution $\dfrac{\frac{4}{3}\pi(R^3 - r^3)}{\frac{4}{3}\pi R^3} = \dfrac{7}{8} \Rightarrow 8R^3 - 8r^3 = 7R^3$

$\Rightarrow R^3 = 8r^3 \Rightarrow \dfrac{R^3}{r^3} = \dfrac{1}{8}$

$\Rightarrow \dfrac{R}{r} = \dfrac{1}{2}$.



48. Ans. (c)

Solution Volume of the new cube

$= 6^3 + 8^3 + 10^3$

$= 216 + 512 + 1000$

$= 1728$

$\therefore$ Edge of the new cube = 12

$\Rightarrow$ Diagonal of the new cube

$= \sqrt{144 + 144 + 144}$

$= \sqrt{432} = 20.78 \approx$ **20.8.**

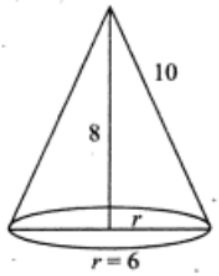
49. Ans. (c)

Solution $r = \sqrt{100 - 64} = 6$

$\therefore$ Curved surface area $= \pi rl$

$= \pi r\sqrt{r^2 + h^2} = \dfrac{22}{7} \times 6 \times \sqrt{36 + 64}$

$= \dfrac{1320}{7} =$ **188.5 m^2.**



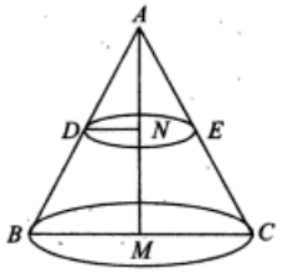
50. Ans. (a)

Solution $\dfrac{6 \times 5 \times .5 - 5\% \text{ of } (6 \times 5 \times .5)}{.25 \times .125 \times .075}$

$= \dfrac{15 - .05 \times 15}{.25 \times .125 \times .075} =$ **6080.**

51. Ans. (c)

Solution $AM = 30$ cm (Given)



Let the cone ADE is cut off from the cone ABC.

Let DN = radius of the base of the cone $ADE = r_1$ cm.

Let MC = Radius of the base of cone $ABC = r$

$\therefore \quad \dfrac{h}{30} = \dfrac{r_1}{r} \Rightarrow r_1 = \dfrac{hr}{30}$ ...(i)

Also volume of the cone ADE

$= \dfrac{1}{27} \times$ Volume of the cone ABC

$\Rightarrow \dfrac{1}{3}\pi r_1^2 h = \dfrac{1}{27} \times \dfrac{1}{3}\pi r^2 \times 30$

$\Rightarrow r_1^2 h = \dfrac{1}{27} \times 30r^2$

$\Rightarrow \dfrac{h^2 r^2}{(30)^2} \times h = \dfrac{30}{27} r^2$

$\Rightarrow h^3 = \dfrac{(30)^3}{(3)^3} \Rightarrow h = \dfrac{30}{3} =$ **10.**

52. Ans. (c)

Solution Volume of cylinder of height 5 = 45 c.c.

$\Rightarrow \pi \times r^2 \times 5 = 45$

$\Rightarrow r^2 = \dfrac{9}{\pi} \Rightarrow r = \dfrac{3}{\sqrt{\pi}}$

$\therefore$ Radius of the biggest ball can be $\dfrac{3}{\sqrt{\pi}}$.

53. Ans. (d)

Solution Surface area of a cube of side 0.5 cm

$= 6 \times (0.5)^2 = 6 \times 0.25$

$= 1.50 = \dfrac{3}{2}$ **cm^2.**

54. Ans. (e)

Solution Suppose Radius = $7K$, Height = $10K$

$\therefore$ Volume of the cylinderical tank

$= \pi \times (7K)^2 \times 10K = 12320$

$\Rightarrow 490K^3 = \dfrac{12320 \times 7}{22}$

$\Rightarrow K^3 = \dfrac{86240}{10780} = 8 \Rightarrow K = 2$

$\therefore$ Height = **20 metres.**

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55. Ans. (d)

Solution $E = \sqrt{\frac{A}{7}}$

If $A = 3087$

$E = \sqrt{\frac{3087}{7}} = \sqrt{441} = 21$

If $A = 2023$,

$E = \sqrt{\frac{2023}{7}} = \sqrt{289} = \mathbf{17.}$

56. Ans. (b)

Solution Volume of cube $= \frac{1}{2} \times$ Volume of cuboid

$= \frac{1}{2} \times 25 \times 20 \times 4 = 1000 \text{ cm}^3$

$\therefore$ Side of cube $= \sqrt[3]{1000} = 10 \text{ cm}$

Total surface area of cube $= 6 \times \text{side}^2$

$= 6 \times 10^2 = \mathbf{60 \text{ cm}^2}$

57. Ans. (c)

Solution Area of path $= 26 \times 10 - 24 \times 14$

$416 - 336 = 80 \text{ m}^2$

$\therefore$ required number of tiles $= \frac{80 \text{ m}^2}{(20 \times 20)} \text{ cm}^2$

$= \frac{80 \times 100 \times 100}{20 \times 20} = 2000$

58. Ans. (c)

Solution Required flow of water in 1 min

$= 5 \times 20 \times 60 = 6000 \text{ cm}^3$

$= \mathbf{6 \text{ litres}}$

59. Ans. (c)

60. Ans. (c)

Solution Volume of conical vessel = Volume of cylindrical jar

$\frac{1}{3}\pi \times 2^2 \times 3 = \pi \times 2^2 \times h$

$h = \frac{1 \times \pi \times 2^2 \times 3^3}{3 \times 2^2 \times \pi} = 1 \text{ cm}$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. A cistern is constructed to hold 200 litres. The base of the cistern is a square metre. What is the depth of the cistern? A cubic metre is 1000 litres
(a) 50 cm (b) 20 cm
(c) 25 cm (d) 40 cm

2. A certain cube of wood was bought for Rs. 768. If the wood costs Rs. 1500 per cu m, find the length of each edge of the cube
(a) 70 cm (b) 80 cm
(d) 90 cm (d) Cannot be determined

3. A cu m of copper weighing 9000 kg is rolled into a square bar 9 m long. An exact cube is cut off from the bar. How much does it weigh?
(a) $333\frac{1}{3}$ kg (b) $233\frac{1}{3}$ kg
(c) $334\frac{1}{3}$ kg (d) $333\frac{1}{2}$ kg

4. A cone, whose height is half of its radius, is melted to form a sphere. Find the ratio of radius of the sphere to that of the cone
(a) 2 : 1 (b) 1 : 2
(c) 3 : 2 (d) 1 : 1

5. Two cubes each of edge 10 cm are joined to form a single cuboid. What is the surface area of the new cuboid so formed?
(a) 1500 cm^2 (b) 1200 cm^2
(c) 1000 cm^2 (d) 1400 cm^2

6. A solid consists of a circular cylinder with exact fitting right circular cone placed on the top. The height of the cone is h. If total volume of the solid is 3 times the volume of cone, then the height of circular cylinder is
(a) $2h$ (b) $4h$
(c) $\frac{3h}{2}$ (d) $\frac{2h}{3}$

7. The number of solid spheres, each of diameter 5 cm, that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm; is
(a) 3 (b) 4
(c) 5 (d) 6

8. From a lead cube of edge 11 cm, how many spherical balls of diameter 0.5 cm, will be made?
(a) 2541 (b) 2415
(c) 2641 (d) 2741

9. Let S_1 and S_2 be the whole surface of a sphere and the curved surface of the circumscribed cylinder. Then, S_1 is equal to
(a) $2S_2$ (b) S_2
(c) $\frac{1}{2}S_2$ (d) $\frac{2}{3}S_2$

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SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution 1 sq m × depth = $\frac{200}{1000}$

$\therefore$ Depth = $\frac{1}{5}$ m = $\frac{1}{5}$ × 100 = **20 m.**

3. Ans. (a)

Solution (Area of the square end) × 9 = volume
= 1 cu m.

$\therefore$ Side of the square end = $\sqrt{\frac{1}{9}}$ metres = $\frac{1}{3}$ metre

$\therefore$ Volume of the cube = $\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{1}{27}$ cu m.

$\therefore$ Weight of cube = $\frac{9000}{27} = 333\frac{1}{3}$ **kg.**

5. Ans. (c)

Solution Breadth and height of the new cuboid will remain as the edge of the cube but length of the cuboid will be doubled.

Then, for the cuboid
Length (l) = 2 × 10 = 20 cm
Breadth (b) = 10 cm
Height (h) = 10 cm
$\therefore$ Surface area of cuboid
= 2 [$lb + bh + hl$]
= 2 [20 × 10 + 10 × 10 + 20 × 10]
= 2 [5000] = **1000 cm^2.**

7. Ans. (c)

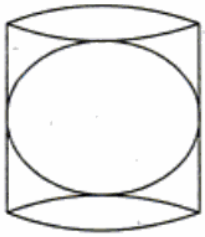
Solution Volume of one sphere = $\frac{4}{3}\pi(3)^3 = 36\pi$

Volume of one cylinder = $\pi(2)^2 \times 45 = 180\pi$

$\therefore$ Number of spheres = $\frac{180\pi}{36\pi} = 5.$

9. Ans. (b)

Solution If r is the radius of the sphere, then the radius of the cylinder = r and its height = $2r$



$\therefore$ $S_1 = 4\pi r^2$ and $S_2 = 2\pi r \times 2r = 4\pi r^2$

$\therefore$ $S_1 = S_2$.

11. Ans. (b)

Solution $6^3 + 8^3 + R^3 = 12^3$

or, $R^3 = 1728 - 216 - 512 = 1000$

$\therefore$ R = **10 cm.**

13. Ans. (d)

Solution 1 m = 1000 mm
and 1 cm = 10 mm

$$h = \frac{7\times1000\times1000}{700\times10\times10} \text{ mm}$$

= 100 mm = **10 cm.**

15. Ans. (b)

Solution Volume of the sphere
= Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7}\times\left(\frac{1.5}{2}\right)^2 \times 1 \text{ cu m.}$$

If d be the diameter of the sphere, then volume

$$= \pi\frac{d^3}{6}$$

$\therefore$ $\frac{22}{7}\times\frac{1}{6}\times d^3 = \frac{22}{7}\times\frac{3}{2}+\frac{3}{2}\times\frac{1}{4}$

or, $d^3 = \frac{27}{8}$ cu m or, $d = \frac{3}{2}$, **i.e. 1.5 m.**

17. Ans. (d)

Solution Volume of the sphere = $\frac{4}{3}\pi\left(\frac{3}{2}\right)^3$

Let h be the height through which water in the cylinder rises.

$\therefore$ $\pi(3)^2 h = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3$

$h = \frac{1}{2}$ **cm.**

19. Ans. (c)

Solution Volume of the cylinder
= 3 volume of the cone

This is valid if base radius and height is the same.

Radius is the same, so the height of cone is 3 times the height of cylinder.

$\therefore$ Height of the cone = 3 × 5 cm = **15 cm.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (b)	**6.** (d)	**8.** (a)	**10.** (a)	**12.** (a)	**14.** (c)	**16.** (d)	**18.** (b)	**20.** (c)

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PART IV

Trigonometry

CHAPTER 35 Trigonometric Ratios

CHAPTER 36 Heights and Distances

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35 Trigonometric Ratios

INTRODUCTION

The literal meaning of the word *trigonometry* is the 'science of triangle measurement'. The word *trigonometry* is derived from two Greek words *trigon* and *metron* which means measuring the sides of a triangle. It had its beginning more than two thousand years ago as a tool for astronomers. The Babylonians, Egyptians, Greeks and the Indians studied trigonometry only because it helped them in unravelling the mysteries of the universe. In modern times, it has gained wider meaning and scope. Presently, it is defined as that branch of mathematics which deals with the measurement of angles, whether of triangle or any other figure.

At present, trigonometry is used in surveying, astronomy, navigation, physics, engineering, etc.

Important Formulae and Results of Trigonometry

I. (i) $180^\circ = \pi$ radians.

(ii) $1^\circ = \dfrac{\pi}{180} = .01745$ radians (approximately).

(iii) $\pi = \dfrac{\text{circumference of a circle}}{\text{diameter of the circle}}$

$= \dfrac{22}{7} = 3.1416$ (approximately).

(iv) θ (in radian measure) $= \dfrac{l}{r}$.

(v) Each interior angle of a regular polygon of n sides $= \dfrac{n-2}{n} \times 180$ degrees.

II. (i) $\sin\theta \cdot \operatorname{cosec}\theta = 1$; $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$;

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$.

Also, $-1 \le \sin x \le 1$, $\operatorname{cosec} x \le -1$ or $\operatorname{cosec} x \ge 1$.

(ii) $\cos\theta \cdot \sec\theta = 1$; $\cos\theta = \dfrac{1}{\sec\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$.

Also, $-1 \le \cos x \le 1$, $\sec x \le -1$ or $\sec x \ge 1$.

(iii) $\tan\theta \cdot \cot\theta = 1$; $\tan\theta = \dfrac{1}{\cot\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$.

Also, $-\infty < \tan\theta < \infty$, $-\infty < \cot\theta < \infty$.

(iv) $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$; $\cos^2\theta = 1 - \sin^2\theta$

(v) $\sin^2\theta = 1 + \tan^2\theta$; $\sec^2\theta - \tan^2\theta = 1$; $\tan^2\theta = \sec^2\theta - 1$.

(vi) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$; $\operatorname{cosec}^2\theta - \cot^2\theta = 1$; $\cot^2\theta = \operatorname{cosec}^2\theta - 1$.

(vii) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

III. Values of trigonometrical ratios for particular angles

(i)

Angle	sine	cos	tan
0°	0	1	0
$30^\circ = \dfrac{\pi}{6}$	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{3}}$
$45^\circ = \dfrac{\pi}{4}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{\sqrt{2}}$	1
$60^\circ = \dfrac{\pi}{3}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{2}$	$\sqrt{3}$
$90^\circ = \dfrac{\pi}{2}$	1	0	∞
$120^\circ = \dfrac{2\pi}{3}$	$\dfrac{\sqrt{3}}{2}$	$-\dfrac{1}{2}$	$-\sqrt{3}$
$135^\circ = \dfrac{3\pi}{4}$	$\dfrac{1}{\sqrt{2}}$	$-\dfrac{1}{\sqrt{2}}$	-1
$150^\circ = \dfrac{5\pi}{6}$	$\dfrac{1}{2}$	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{1}{\sqrt{3}}$
$180^\circ = \pi$	0	-1	0
$270^\circ = \dfrac{3\pi}{2}$	-1	0	$-\infty$
$360^\circ = 2\pi$	0	1	0

(ii) $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$; $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$;

$\tan 15^\circ = 2-\sqrt{3}$.

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(iii) $\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$;

$\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72°$.

(iv) $\cos 36° = \frac{\sqrt{5}+1}{4} = \sin 54°$;

$\sin 35° = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54°$.

(v) $\tan 7\frac{1°}{2} = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$;

$\cot 7\frac{1°}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$.

IV. Signs of trigonometrical ratios

Angle	sin	cos	tan
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$90° - \theta$ or $\frac{\pi}{2} - \theta$	$\cos\theta$	$\sin\theta$	$\cot\theta$
$90° + \theta$ or $\frac{\pi}{2} + \theta$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$
$180° - \theta$ or $\pi - \theta$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$
$180° + \theta$ or $\pi + \theta$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$
$270° - \theta$ or $\frac{3\pi}{2} - \theta$	$-\cos\theta$	$-\sin\theta$	$\cot\theta$
$270° + \theta$ or $\frac{3\pi}{2} + \theta$	$-\cos\theta$	$\sin\theta$	$-\cot\theta$
$360° - \theta$ or $2\pi - \theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$360° + \theta$ or $2\pi + \theta$	$\sin\theta$	$\cos\theta$	$\tan\theta$

V. Trigonometrical ratios for sum or difference of angles

(i) $\sin(A \pm B) = \sin A \cdot \cos B \pm \cos A \cdot \sin B$.

(ii) $\cos(A \pm B) = \cos A \cdot \cos B \quad \sin A \cdot \sin B$.

(iii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A . \tan B}$.

(iv) $\cot(A \pm B) = \frac{\cot A . \cot B \mp 1}{\cot B \pm \cot A}$.

(v) $\tan(A + B + C)$

$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)}$

(vi) $\sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B$.

(vii) $\cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B$.

VI. Sum or difference of sine or cosine of angles into products

(i) $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$.

(ii) $\sin C - \sin D = 2 \cos \frac{C+D}{2} \cdot \sin \frac{C \quad D}{2}$.

(iii) $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cdot \cos \frac{C \quad D}{2}$.

(iv) $\cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D \quad C}{2}$,

VII. Product of sines and cosines of angles into sum or difference of angles

(i) $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$.

(ii) $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$.

(iii) $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$.

(iv) $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$.

VIII. Trigonometrical ratios of multiple angles

(i) $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$.

(ii) $\cos^2 A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1$

$= 1 - 2 \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$.

(iii) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$.

(iv) $\sin^2 A = \frac{1 - \cos 2A}{2}$; $\cos^2 A = \frac{1 + \cos 2A}{2}$.

(v) $\tan A = \sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}} = \frac{1 - \cos 2A}{\sin 2A}$.

(vi) $\sin 3A = 3 \sin A - 4 \sin^3 A$.

(vii) $\cos 3A = 4 \cos^3 A - 3 \cos A$.

(viii) $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$;

$\cot 3A = \frac{\cot^3 A - 3 \cot A}{3 \cot^2 A - 1}$.

IX. Trigonometrical ratios of submultiple angles

(i) $\sin A = 2 \sin = \frac{A}{2} \cdot \cos \frac{A}{2} = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$.

(ii) $\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} = 2 \cos^2 \frac{A}{2} - 1$

$= 1 - 2 \sin^2 \frac{A}{2} = \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$.

(iii) $\tan A = \frac{2 \tan \frac{A}{2}}{1 \tan^2 \frac{A}{2}}$.

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(iv) $\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$; $\cos^2\frac{A}{2} = \frac{1+\cos A}{2}$

(v) $\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A}$.

(vi) $2\sin\frac{A}{2} = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A}$.

(vii) $2\cos\frac{A}{2} = \pm\sqrt{1+\sin A} \mp \sqrt{1-\sin A}$.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. If $\frac{1+\cos A}{1-\cos A} = \frac{m^2}{n^2}$, $\tan A =$

(a) $\pm\frac{2mn}{m^2+n^2}$ (b) $\pm\frac{2nm}{m^2+n^2}$

(c) $\frac{m^2+n^2}{m^2-n^2}$ (d) None of these.

2. If $\sin 600^\circ \cos 30^\circ + \cos 120^\circ \sin 150^\circ = k$, then $k =$

(a) 0 (b) 1

(c) -1 (d) None of these.

3. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta =$

(a) $\sqrt{2}\sin\theta$ (b) $2\sin\theta$

(c) $-\sqrt{2}\sin\theta$ (d) None of these.

4. If α lies in the second quadrant, then $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} - \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} =$

(a) $\tan\alpha$ (b) $2\tan\alpha$

(c) $2\cot\alpha$ (d) $\cot\alpha$

5. If $\cot\theta + \cos\theta = p$ and $\cot\theta - \cos\theta = 9$, then $(p^2 - q^2)^2$ in terms of p and q is

(a) $16pq$ (b) $8pq$

(c) $4pq$ (d) $12pq$

6. If $x = a\,\mathrm{cosec}^n\theta$ and $y = b\cot^n\theta$, then by eliminating θ

(a) $(x/a)^{2/n} + (y/b)^{2/n} = 1$

(b) $(x/a)^{2/n} - (y/b)^{2/n} = 1$

(c) $(x/a)^2 - (y/b)^2 = 1$

(d) $(x/a)^{1/n} - (y/b)^{1/n} = 1$

7. If $\tan\theta = p/q$, then $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} =$

(a) $(p^2 + q^2) / (p^2 - q^2)$

(b) $(p^2 - q^2) / (p^2 + q^2)$

(c) $(p^2 + q^2) / (p^2 - q^2)$

(d) None of these.

8. If $\sin A = 3/5$, $\tan B = 1/2$ and $\pi/2 < A < \pi < B < 3\pi/2$, the value of $8\tan A - \sqrt{5}\sec B =$

(a) 7/2 (b) 5/2

(c) $-5/2$ (d) $-7/2$

9. If $\sec\theta - \tan\theta = \frac{a+1}{a-1}$, then $\cos\theta =$

(a) $\frac{a^2+1}{a^2-1}$ (b) $\frac{a^2-1}{a^2+1}$

(c) $\frac{2a}{a^2+1}$ (d) $\frac{2a}{a^2-1}$

10. If $\tan 20^\circ = k$, then $\frac{\tan 250^\circ + \tan 340^\circ}{\tan 200^\circ - \tan 110^\circ} =$

(a) $\frac{1+k}{1-k}$ (b) $\frac{1-k}{1+k}$

(c) $\frac{1+k^2}{1-k^2}$ (d) $\frac{1-k^2}{1+k^2}$

11. The value of $\sin 780^\circ \sin 480^\circ + \cos 240^\circ \cos 300^\circ =$

(a) 1/2 (b) 1/4

(c) 1 (d) None of these.

12. If $\tan\theta + \cot\theta = 2$, then $\sin\theta =$

(a) $\pm\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\pm\frac{1}{3}$ (d) None of these.

13. If θ is in the first quadrant and $\tan\theta = \frac{3}{4}$, then $\frac{\tan(\pi/2-\theta) - \sin(\pi-\theta)}{\sin(3\pi/2+\theta) - \cot(2\pi-\theta)} =$

(a) 8/11 (b) 6/11

(c) 11/8 (d) 11/6

14. If $\cot 20^\circ = p$, then $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ} =$

(a) $\frac{p^2-1}{2p}$ (b) $\frac{p^2+1}{2p}$

(c) $\frac{1-p^2}{2p}$ (d) $\frac{2p}{1+p^2}$

15. If A lies in the second quadrant and B lies in the third quadrant and $\cos A = -\sqrt{3}/2$, $\sin B = -3/5$, then $\frac{2\tan B + \sqrt{3}\tan A}{\cot^2 A + \cos B} =$

(a) 5/21 (b) 5/24

(c) 5/22 (d) None of these.

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16. The value of $\frac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ}$ is
(a) 2 (b) 1
(c) −1 (d) −2

17. If $f(x) = \cos^2 x + \sec^2 x$, its value always is
(a) $f(x) < 1$ (b) $f(x) = 1$
(c) $2 > f(x) > 1$ (d) $f(x) \geq 2$

18. If $\operatorname{cosec}\theta + \cot\theta = p$, then $\cos\theta =$
(a) $\frac{p^2+1}{p^2-1}$ (b) $\frac{1+p^2}{1-p^2}$
(c) $\frac{p^2-1}{p^2+1}$ (d) $\frac{1-p^2}{1+p^2}$

19. If $\sin\theta = -7/25$ and θ is in the third quadrant, then $\frac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} =$
(a) 17/31 (b) 16/31
(c) 15/31 (d) None of these.

20. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then $\frac{(m^2-n^2)^2}{mn} =$
(a) 4 (b) 3
(c) 16 (d) 9

21. If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$ then $(m^2n)^{2/3} + (mn^2)^{2/3} =$
(a) −1 (b) 1
(c) 0 (d) None of these.

22. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ =$
(a) 1 (b) −1
(c) 0 (d) None of these.

23. Without using trigonometric tables,
$\sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ =$
(a) 0 (b) 2
(c) 1 (d) None of these.

24. $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ =$
(a) −1 (b) 1
(c) 1/2 (d) None of these.

25. $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + \ldots + \cos^2 90^\circ =$
(a) $8\frac{1}{2}$ (a) $6\frac{1}{2}$
(c) $7\frac{1}{2}$ (d) None of these.

26. The value of $\log\tan 1^\circ + \log\tan 2^\circ + \log\tan 3^\circ + \ldots + \log\tan 89^\circ$ is equal to
(a) 1 (b) 0
(c) 3 (d) None of these.

27. $\log\sin 1^\circ \log\sin 2^\circ \log\sin 3^\circ \ldots \log\sin 179^\circ =$
(a) 0 (b) 1
(c) $1/\sqrt{2}$ (d) None of these.

28. The value of $\cos 24^\circ + \cos 55^\circ + \cos 155^\circ + \cos 204^\circ$ is
(a) 1 (b) −1
(c) 0 (d) None of these.

29. The value of $\cos 24^\circ + \cos 5^\circ + \cos 300^\circ + \cos 175^\circ + \cos 204^\circ$ is
(a) 0 (b) −1/2
(c) 1/2 (d) 1

30. $\sin^2\theta = \frac{(x+y)^2}{4xy}$ is possible only when
(a) $x > 0, y > 0, x \neq y$
(b) $x > 0, y > 0, x = y$
(c) None of these.

31. If $7\sin^2\theta + 3\cos^2\theta = 4$, then $\tan\theta =$
(a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{2}$
(c) $\pm\frac{1}{\sqrt{3}}$ (d) $\pm\frac{1}{\sqrt{2}}$

32. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\frac{m^2-1}{n^2-1} =$
(a) $\cos^3\alpha$ (b) $\sin^3\alpha$
(c) $\sin^2\alpha$ (d) $\cos^2\alpha$

33. If $\sec A = a + (1/4a)$, then $\sec A + \tan A =$
(a) $2a$ or $1/2a$ (b) a or $1/a$
(c) $2a$ or $1/a$ (d) a or $1/2a$

34. The value of $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$ is
(a) 0 (b) 1
(c) 2 (d) None of these.

35. The value of $\tan 20^\circ + \tan 40^\circ + \tan 60^\circ + \ldots + \tan 180^\circ$ is
(a) 1 (b) −1
(c) 0 (d) None of these.

36. If $\cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\alpha = \frac{-3}{5}$, where θ does not lie in the third quadrant and α lies in the third quadrant, $\frac{2\tan\alpha + \sqrt{3}\tan\theta}{\cot^2\theta + \cos\alpha} =$

37. The value of $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$ is
(a) 1/2 (b) −1/2
(c) 1 (d) −1

38. $\frac{\cot\theta - \operatorname{cosec}\theta + 1}{\cot\theta + \operatorname{cosec}\theta - 1}$ is equal to
(a) 1 (b) $\cot\theta + \operatorname{cosec}\theta$
(c) $\operatorname{cosec}\theta - \cot\theta$ (d) None of these.

39. If $90^\circ < \alpha < 180^\circ$, $\sin\alpha = \sqrt{3}/2$ and $180^\circ < \beta < 270^\circ$, $\sin\beta = -\sqrt{3}/2$, then $\frac{4\sin\alpha - 3\tan\beta}{\tan\alpha + \sin\beta} =$

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(a) 2/3 (b) 0
(c) –2/3 (d) None of these.

40. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=$
(a) $2\sin\theta$ (b) $2\cos\theta$
(c) $\frac{2}{|\cos\theta|}$ (d) $\frac{2}{|\sin\theta|}$

41. If $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\text{cosec}\,\alpha+\cot\alpha$, then the quadrants in which α lies are
(a) 1, 4 (b) 2, 3
(c) 1, 2 (d) 3, 4

42. If $\text{cosec}\,\theta-\cot\theta=p$, then the value of $\text{cosec}\,\theta=$
(a) $\frac{1}{2}\left(p+\frac{1}{p}\right)$ (b) $\frac{1}{2}\left(p-\frac{1}{p}\right)$
(c) $p+\frac{1}{p}$ (d) $p-\frac{1}{p}$

43. The value of $\tan 1^\circ \tan 2^\circ \ldots \tan 89^\circ$ is
(a) –1 (b) 1
(c) 0 (d) None of these.

44. If $\text{cosec}^2\,\theta=\frac{4xy}{(x+y)^2}$, then
(a) $x=-y$ (b) $x=1/y$
(c) $x=y$ (d) None of these.

45. The value of $\frac{\sin 300^\circ \tan 240^\circ \sec(-420^\circ)}{\cot(-315^\circ)\cos(210^\circ)\,\text{cosec}(-315^\circ)}$ is
(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{6}$ (d) $\sqrt{8}$

46. The length of an arc which subtends an angle 18° at the centre of the circle of radius 6 cms is
(a) $(\pi/5)$ cm (b) $(2\pi/5)$ cm
(c) $(3\pi/5)$ cm (d) None of these.

47. If x is real and $x+\frac{1}{x}=2\cos\theta$, then $\cos\theta=$
(a) $\pm\frac{1}{2}$ (b) $\pm\frac{1}{3}$
(c) ± 1 (d) None of these.

48. Which of the following is correct?
(a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ = \sin 1$
(c) $\sin 1^\circ < \sin 1$ (d) $\sin 1^\circ = \left(\frac{\pi}{180}\right)\sin 1$

49. Which one of the following is true?
(a) $\tan 1 = 1$ (b) $\tan 1 = \tan 2$
(c) $\tan 1 < \tan 2$ (d) $\tan 1 > \tan 2$

50. The value of $\cos^2\theta+\sec^2\theta$ is always
(a) less than 1
(b) equal to 1
(c) lies between 1 and 2
(d) greater than 2.

51. If $\sin\alpha=\frac{2pq}{p^2+q^2}$, then $\sec\alpha-\tan\alpha=$
(a) $\frac{p-q}{p+q}$ (b) $\frac{pq}{p^2+q^2}$
(c) $\frac{p+q}{p-q}$ (d) None of these.

52. If $13\sin A=12$, $\pi/2<A<\pi$ and $3\sec B=5$, $3\pi/2<B<2\pi$ then $5\tan A+3\tan^2 B=$
(a) 20/3 (b) –20/3
(c) 22/3 (d) –22/3

53. The value of $\sin 105^\circ+\cos 105^\circ$ is
(a) $1/\sqrt{2}$ (b) $-1/\sqrt{2}$
(c) 0 (d) None of these.

54. If $\tan A=1/2$ and $\tan B=1/3$, the value of $A+B$ is
(a) $\pi/3$ (b) $\pi/4$
(c) $\pi/2$ (d) None of these.

55. If $\tan(A-B)=7/24$ and $\tan A=4/3$ where A and B are acute, then $A+B=$
(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) None of these.

56. The value of $(\tan 69^\circ+\tan 66^\circ)/(1-\tan 69^\circ\tan 66^\circ)$ is
(a) 1 (b) 0
(c) 2 (d) –1

57. The value of $\sin^2 75^\circ-\sin^2 15^\circ$ is
(a) $\sqrt{3}/2$ (b) $-\sqrt{3}/2$
(c) 1/2 (d) None of these.

58. If $\sin\alpha=8/17$, $0<\alpha<90^\circ$ and $\tan\beta=5/12$, $0<\beta<90^\circ$, then $\cos(\alpha-\beta)$ is
(a) 210/221 (b) 171/221
(c) 220/221 (d) None of these.

59. The value of $\sin^2\theta+\sin^2(\theta+60^\circ)+\sin^2(\theta-60^\circ)=$
(a) 1/2 (b) 0
(c) 3/2 (d) None of these.

60. If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$, then $\alpha+\beta=$
(a) $\pi/3$ (b) $\pi/2$
(c) $\pi/4$ (d) None of these.

61. The value of $\frac{1}{\sin 10^\circ}-\frac{\sqrt{3}}{\cos 10^\circ}=$
(a) 1 (b) 4
(c) 3 (d) None of these.

62. The value of $\sqrt{2+\sqrt{2(1+\cos 4A)}}$ is equal to
(a) $\cos A$ (b) $\sin A$
(c) $2\cos A$ (d) $2\sin A$

63. If $\tan A=\frac{1-\cos B}{\sin B}$, then $\tan 2A=$
(a) $\tan B$ (b) $\cot B$
(c) $2\tan B$ (d) $2\cot B$

64. The value of $\frac{\cos 2\theta}{1-\sin 2\theta}=$

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16. Ans. (d)

Solution $\dfrac{\sin 150^\circ - 5\cos 300^\circ + 7\tan 225^\circ}{\tan 135^\circ + 3\sin 210^\circ}$

$= \dfrac{\sin(180^\circ - 30^\circ) - 5\cos(360^\circ - 60^\circ) + 7\tan(180^\circ + 45^\circ)}{\tan(180^\circ - 45^\circ) + 3\sin(180^\circ + 30^\circ)}$

$= \dfrac{\sin 30^\circ - 5\cos 60^\circ + 7\tan 45^\circ}{-\tan 45^\circ - 3\sin 30^\circ} = \dfrac{1/2 - 5/2 + 7}{-1 - 3/2} = -2.$

17. Ans. (d)

Solution $f(x) = \cos^2 x + \sec^2 x = (\cos x - \sec x)^2 + 2$

$\Rightarrow f(x) \geq 2.$

18. Ans. (c)

Solution Given $\operatorname{cosec}\theta + \cot\theta = p$

$\Rightarrow \operatorname{cosec}\theta - \cot\theta = 1/p$

$\Rightarrow \operatorname{cosec}\theta = \dfrac{1}{2}\left(p + \dfrac{1}{p}\right) = \dfrac{p^2+1}{2p}$

$\cot\theta = \dfrac{p^2-1}{2p}$

$\therefore \cos\theta = \dfrac{\cot\theta}{\operatorname{cosec}\theta} = \dfrac{p^2-1}{p^2+1}.$

19. ans. (a)

Solution Given $180^\circ < \theta < 270^\circ \Rightarrow \tan\theta = 7/24$

$\therefore \dfrac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} = \dfrac{7(24/7) - 24(7/24)}{7(24/7) + 24(7/24)}$

$= \dfrac{24-7}{7+24} = \dfrac{17}{31}.$

20. Ans. (c)

Solution $\dfrac{(m^2-n^2)^2}{mn} = \dfrac{(4\tan A\sin A)^2}{\tan^2 A - \sin^2 A}$

$= \dfrac{16\sin^4 A}{\cos^2 A} \cdot \dfrac{\cos^2 A}{\sin^2 A(1-\cos^2 A)}$

$= \dfrac{16\sin^4 A}{\sin^2 A \sin^2 A} = 16.$

21. Ans. (b)

Solution $m = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

$n = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$(m^2n)^{2/3} + (mn^2)^{2/3} = \left(\dfrac{\cos^4\theta}{\sin^2\theta} \cdot \dfrac{\sin^2\theta}{\cos\theta}\right)^{2/3}$

$+ \left(\dfrac{\cos^2\theta}{\sin\theta} \cdot \dfrac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$

$= \cos^2\theta + \sin^2\theta$

$= 1.$

22. Ans. (c)

Solution $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ$

$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ \ldots \cos 178^\circ \cos 179^\circ$

$= 0.$ $[\because \cos 90^\circ = 0]$

23. Ans. (b)

Solution $\sin(90^\circ - 42^\circ)\sec 42^\circ + \cos(90^\circ - 42^\circ)\operatorname{cosec} 42^\circ$

$= \cos 42^\circ \sec 42^\circ + \sin 42^\circ \operatorname{cosec} 42^\circ$

$= 1 + 1 = 2.$

24. Ans. (b)

Solution $\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$

$= \tan 5^\circ \tan 25^\circ .1. \tan(90^\circ - 25^\circ)\tan(90^\circ - 50^\circ)$

$= \tan 5^\circ \tan 25^\circ .1. \cot 25^\circ \cot 5^\circ$

$= 1.$

25. Ans. (a)

Solution $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + \ldots + \cos^2 90^\circ$

$= (\cos^2 5^\circ + \cos^2 85^\circ) + (\cos^2 10^\circ + \cos^2 80^\circ)$

$+ \ldots + (\cos^2 40^\circ + \cos^2 50^\circ) + \cos^2 45^\circ + \cos^2 90^\circ$

$= (1 + 1 + \ldots 8 \text{ times}) + \dfrac{1}{2} + 0$

$= 8\dfrac{1}{2}.$

26. Ans. (b)

Solution $\log(\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 88^\circ \tan 89^\circ)$

$= \log(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ) \ldots \tan 45^\circ$

$= \log(\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ) \ldots \tan 45^\circ$

$= \log(1.\ 1.\ 1.\ \ldots\ 1) = \log 1 = 0.$

27. Ans. (a)

Solution $\log\sin 1^\circ \log\sin 2^\circ \ldots \log\sin 90^\circ \ldots \log\sin 179^\circ$

$= \log\sin 1^\circ \log\sin 2^\circ \ldots (0) \log\sin 91^\circ \ldots \log\sin 179^\circ$

$= 0.$

28. Ans. (c)

Solution $\cos 24^\circ + \cos 55^\circ + \cos 155^\circ + \cos 204^\circ$

$= \cos 24^\circ + \cos 55^\circ + \cos(180^\circ - 25^\circ) + \cos(180^\circ + 24^\circ)$

$= \cos 24^\circ + \cos 55^\circ - \cos 25^\circ - \cos 24^\circ = 0.$

29. Ans. (c)

Solution $\cos 24^\circ + \cos 5^\circ + \cos 300^\circ + \cos 175^\circ + \cos 204^\circ$

$= \cos 24^\circ + \cos 5^\circ + \cos(360^\circ - 60^\circ)$

$+ \cos(180^\circ - 5^\circ) + \cos(180^\circ + 24^\circ)$

$= \cos 24^\circ + \cos 5^\circ + \cos 60^\circ - \cos 5^\circ - \cos 24^\circ$

$= 1/2.$

30. Ans. (b)

Solution $\sin^2\theta \geq 1 \Rightarrow \dfrac{(x+y)^2}{4xy} \geq 1$

$\Rightarrow (x+y)^2 \geq 4xy$

$\Rightarrow (x+y)^2 - 4xy \geq 0$

$\Rightarrow (x-y)^2 \geq 0$

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$(x - y)^2 > 0$ is true.

But $(x - y)^2 = 0$ is true only when $x = y$.

31. Ans. (c)

Solution Dividing by $\cos^2\theta$

$7 \tan^2\theta + 3 = 4 \sec^2\theta$

$\Rightarrow 7 \tan^2\theta + 3 = 4 (1 + \tan^2\theta)$

$\Rightarrow 3 \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = 1/3 \Rightarrow \tan\theta = \pm \frac{1}{\sqrt{3}}$.

32. Ans. (d)

Solution $m^2 - 1 = \frac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta}$

$n^2 - 1 = \frac{\tan^2\alpha - \tan^2\beta}{\tan^2\beta}$

$= \frac{\sin^2\alpha\cos^2\beta - \sin^2 13\cos^2\alpha}{\cos^2\alpha\cos^2\beta} \times \frac{\cos^2\alpha}{\sin^2\beta}$

$= \frac{\sin^2\alpha(1-\sin^2\beta) - \sin^2\beta(1-\sin^2\alpha)}{\sin^2\beta\cos^2\alpha}$

$= \frac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta\cos^2\alpha}$

$\therefore \frac{m^2-1}{n^2-1} = \frac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta} \times \frac{\cos^2\alpha\sin^2\beta}{\sin^2\alpha - \sin^2\beta}$

$= \cos^2\alpha$.

33. Ans. (a)

Solution $\tan^2 A = \sec^2 A - 1 = (a + 1/4a)^2 - 1$

$= (a - 1/4a)^2$

$\Rightarrow \tan A = \pm\left(a - \frac{1}{4a}\right)$

$\therefore \sec A + \tan A = a + \frac{1}{4a} + a - \frac{1}{4a}$

or, $a + \frac{1}{4a} - a + \frac{1}{4a} = 2a$ or $\frac{1}{2a}$.

34. Ans. (c)

Solution $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$

$= \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A + \sin A\cos A}{\sin A + \cos A}$

$+ \frac{(\cos A - \sin A)(\cos^2 A + \sin^2 A + \sin A\cos A}{\cos A - \sin A}$

$= 2(\sin^2 A + \cos^2 A) = 2$.

35. Ans. (c)

Solution $\tan 20° + \tan 40° + \tan 60° + ... + \tan 180°$

$= \tan 20° + \tan 40° + ... + \tan (180° - 40°)$

$+ \tan (180° - 20°) + \tan 180°$

$= \tan 20° + \tan 40° + ... + \tan (180° - 40°)$

$+ \tan (180° - 20°) + 0$

$= \tan 20° + \tan 40° + ... - \tan 40° - \tan 20°$

$= (\tan 20° - \tan 20°) + (\tan 40° - \tan 40°) + ...$

$= 0 + 0 + ... = 0$.

36. Ans. (a)

Solution $\cos\theta = -\sqrt{3}/2$, $90° < \theta < 180°$

$\Rightarrow \tan\theta = -1/\sqrt{3}$

and, $\sin\alpha = \frac{-3}{5}$ and $180° < \alpha < 270°$

$\Rightarrow \tan\alpha = \frac{3}{4}$, $\cos\alpha = \frac{-4}{5}$

$\therefore$ Given expression $= \frac{2(3/4) + \sqrt{3}(-1/\sqrt{3})}{(-\sqrt{3})^2 + (-4/5)}$

$= \frac{1/2}{11/5} = \frac{5}{22}$.

37. Ans. (a)

Solution Given expression

$= \cos 24° + \cos 55° - \cos 55° - \cos 24° + \cos 60°$

$= 1/2$.

38. Ans. (c)

Solution Given expression

$= \frac{(\cot\theta - \operatorname{cosec}\theta) + (\operatorname{cosec}^2\theta - \cot\theta)}{\cot\theta + \operatorname{cosec}\theta - 1}$

$= \frac{(\operatorname{cosec}\theta - \cot\theta)[\operatorname{cosec}\theta + \cot\theta - 1]}{\cot\theta + \operatorname{cosec}\theta - 1}$

$= \operatorname{cosec}\theta - \cot\theta$.

39. Ans. (a)

Solution Given $\sin\alpha = \sqrt{3}/2$, $90° < \alpha < 180°$

$\Rightarrow \tan\alpha = -\sqrt{3}$

and, $\sin\beta = -\sqrt{3}/2$, $180° < \beta < 270° \Rightarrow \tan\beta = \sqrt{3}$

Given expression $= \frac{4(\sqrt{3}/2) - 3(\sqrt{3})}{-\sqrt{3} - (\sqrt{3}/2)}$

$= \frac{-\sqrt{3}(2)}{-3\sqrt{3}} = \frac{2}{3}$.

40. Ans. (d)

Solution Given expression $= \frac{1 + \cos\theta + 1 - \cos\theta}{\sqrt{1 - \cos^2\theta}}$

$= \frac{2}{|\sin\theta|}$.

41. Ans. (c)

42. Ans. (a)

Solution Given $\operatorname{cosec}\theta - \cot\theta = p$

$\Rightarrow \operatorname{cosec}\theta + \cot\theta = 1/p$

$\Rightarrow 2\operatorname{cosec}\theta = p + \frac{1}{p}$

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$\therefore$ cosec $\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$.

43. Ans. (b)

Solution Given expression

= tan 1° tan 2° ... tan 45° ... tan 88° tan 89°

= tan 1° tan 2° ... 1 ... tan (90° – 2°) tan (90° – 1°)

= (tan 1° cot 1°) (tan 2° cot 2°) ... 1

= 1. 1. 1. ... 1 = **1.**

44. Ans. (c)

Solution We know that

$$\text{cosec}^2\theta \geq 1 \Rightarrow \frac{4xy}{(x+y)^2} \geq 1$$

$$\Rightarrow 4xy - (x+y)^2 \geq 0$$

$$\Rightarrow -(x-y)^2 \geq 0 \Rightarrow (x-y)^2 \leq 0$$

But $(x-y)^2$ cannot be negative

$\therefore$ $(x-y)^2 = 0$ is possible only when $x = y$.

45. Ans. (c)

Solution Given expression

$$= \frac{\sin(360°-60°)\tan(270°-30°)\sec(360°+60°)}{[-\cot(270°+45°)]\cos(180°+30°)[-\text{cosec}(270°+45°)]}$$

$$= \frac{(-\sin 60°)(\cot 30°)(\sec 60°)}{(\tan 45°)(-\cos 30°)(\sec 45°)}$$

$$= \frac{-(\sqrt{3}/2)(\sqrt{3})(2)}{(1)(-\sqrt{3}/2)(\sqrt{2})} = \frac{6}{\sqrt{6}} = \sqrt{6}.$$

46. Ans. (c)

Solution We have $l = r\theta$, where θ is in radians

Given: $\theta = 18° = 18° \times \pi/180° = \frac{\pi}{10}$ radians

$\therefore$ Length of the arc $= 6(\pi/10) = \frac{3\pi}{5}$.

47. Ans. (c)

Solution Given $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow$ $x^2 - 2x\cos\theta + 1 = 0$

Since x is real, discriminant ≥ 0

$\Rightarrow$ $4\cos^2\theta - 4 \geq 0$

$\Rightarrow$ $\cos^2\theta \geq 1 \Rightarrow \cos\theta = \pm 1$

As cos θ cannot be > 1 or < -1, $\cos\theta = \pm 1$.

48. Ans. (c)

Solution $1^c = \frac{180}{\pi} = \frac{180\times 7}{22} = 57°$ (approx.)

$\Rightarrow$ sin 1 = sin 57°

$\therefore \sin 45° < \sin 1 < \sin 60° \Rightarrow \frac{1}{\sqrt{2}} < \sin 1 < \sqrt{3}/2$

$\Rightarrow 0.7 < \sin 1 < 0.8$

Also, $\sin 0° < \sin 1° < \sin 30° \Rightarrow 0 < \sin 1° < 0.5$

$\therefore$ **sin 1° < sin 1.**

49. Ans. (d)

Solution $1^c = 180/\pi = 57°$ (approx.)

$\Rightarrow$ tan 1 = tan 57° > 0

Also, tan 2 = tan 114° < 0

$\therefore$ **tan 1 > tan 2.**

50. Ans. (d)

Solution $\cos^2\theta + \sec^2\theta = (\cos\theta - \sec\theta)^2 + 2\cos\theta\sec\theta$

$= (\cos\theta - \sec\theta)^2 + 2$

As $(\cos\theta - \sec\theta)^2$ being a perfect square is always positive, $\cos^2\theta + \sec^2\theta$ is always **greater than 2.**

51. Ans. (a)

Solution Given $\sin\alpha = \frac{2pq}{p^2+q^2}$

$\Rightarrow$ $\sec\theta = \frac{p^2+q^2}{p^2-q^2}$, $\tan\alpha = \frac{2pq}{p^2-q^2}$

Given expression $= \frac{p^2+q^2}{p^2-q^2} - \frac{2pq}{p^2-q^2}$

$= \frac{(p-q)^2}{p^2-q^2} = \frac{p-q}{p+q}$.

52. Ans. (b)

Solution Given: $\sin A = 12/13$, A lies in the second quadrant and $\sec B = 5/3$, B lies in the fourth quadrant.

$\Rightarrow$ $\tan A = -12/5$, $\tan B = -4/3$.

Given expression $= 5(-12/5) + 3(16/9)$

$= -12 + 16/3 = \mathbf{-20/3}$.

53. Ans. (a)

Solution Given expression

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1}{\sqrt{2}}.$$

54. Ans. (b)

Solution $\tan(A+B) = \frac{1/2+1/3}{1-(1/2)(1/3)}$

$= \frac{5/6}{5/6} = 1$

$\Rightarrow$ $A + B = \pi/4$.

55. Ans. (a)

Solution $\frac{\tan A - \tan B}{1+\tan A\ \tan B} = \frac{7}{24} \Rightarrow \frac{4/3 - \tan B}{1+\frac{4}{3}\tan B} = \frac{7}{24}$

$\Rightarrow \tan B = 3/4$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$= \frac{(3/4)(4/3)-1}{(4/3)+(3/4)} = 0$$

$\Rightarrow$ $A + B = \pi/2$.

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68. Ans. (a)

Solution We have, $5x = 3x + 2x$

$\Rightarrow \tan 5x = \dfrac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$

$\Rightarrow \tan 5x - \tan 5x \tan 3x \tan 2x = \tan 3x + \tan 2x$

$\Rightarrow \tan 5x - \tan 3x - \tan 2x = \mathbf{\tan 5x \tan 3x \tan 2x.}$

69. Ans. (b)

Solution $\tan (A + B) = \dfrac{\dfrac{n}{n+1} + \dfrac{1}{2n+1}}{1 - \dfrac{n}{n+1} \times \dfrac{1}{2n+1}}$

$= \dfrac{2n^2 + n + n + 1}{2n^2 + 2n + n + 1 - n}$

$= \dfrac{2n^2 + 2n + 1}{2n^2 + 2n + 1} = \mathbf{1.}$

70. Ans. (b)

Solution A, B are positive and each less than $90°$

$\therefore \quad \cos A = 3/\sqrt{10}, \cos B = 2/\sqrt{5}$

$\therefore \quad \sin (A + B) = \left(\dfrac{1}{\sqrt{10}}\right)\left(\dfrac{2}{\sqrt{5}}\right) + \left(\dfrac{3}{\sqrt{10}}\right)\left(\dfrac{1}{\sqrt{5}}\right)$

$= \dfrac{5}{\sqrt{50}} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\therefore \quad A + B = \mathbf{\pi/4.}$

71. Ans. (b)

Solution Given expression

$= \dfrac{2 \sin^2 \theta/2 + 2 \sin \theta/2 \cos \theta/2}{2 \cos^2 \theta/2 + 2 \sin \theta/2 \cos \theta/2}$

$= \dfrac{2 \sin \theta/2 [\sin \theta/2 + \cos \theta/2]}{2 \cos \theta/2 [\sin \theta/2 + \cos \theta/2]}$

$= \mathbf{\tan \theta/2.}$

72. Ans. (a)

Solution We have, $\tan A = \dfrac{1 - \cos 2A}{\sin 2A}$. Put $A = 7\frac{1}{2}°$

$\tan 7\frac{1}{2}° = \dfrac{1 - \cos 15°}{\sin 15°} = \dfrac{1 - \dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \dfrac{2\sqrt{2} - (\sqrt{3}+1)}{\sqrt{3}-1}.$

73. Ans. (a)

Solution $\dfrac{4\cos^3 A - 3\cos A + 3\sin A - 4\sin^3 A}{\cos A - \sin A}$

$= 1 - K \sin 2A$

$\Rightarrow \quad 1 + 2 \sin 2A = 1 - K \sin 2A$

$\Rightarrow \quad K = \mathbf{-2.}$

74. Ans. (b)

Solution $\tan 45° = \tan (57° - 12°)$

$1 = \dfrac{\tan 57° - \tan 12°}{1 + \tan 57° \tan 12°}$

$\Rightarrow \quad \tan 57° - \tan 12° = 1 + \tan 57° \tan 12°$

$\Rightarrow \quad \tan 57° - \tan 12° - \tan 57° \tan 12° = \mathbf{1.}$

75. Ans. (a)

Solution Given expression

$= \sqrt{4 \sin^4 \theta + 4 \sin^2 \theta \cos^2 \theta} + 2 [1 + \cos (\pi/2 - \theta)]$

$= \sqrt{4 \sin^2 \theta (\sin^2 \theta + \cos^2 \theta)} + 2 (1 + \sin \theta)$

$= 2 |\sin \theta| + 2 + 2 \sin \theta$

$= -2 \sin \theta + 2 + 2 \sin \theta$

$= \mathbf{2}$ (since $180° < \theta < 270° \Rightarrow |\sin \theta| = -\sin \theta$)

76. Ans. (d)

Solution $\tan 225° = \tan (100° + 125°)$

$1 = \dfrac{\tan 100° + \tan 125°}{1 - \tan 100° \tan 125°}$

$\Rightarrow \quad \tan 100° + \tan 125° + \tan 100° \tan 125° = \mathbf{1.}$

77. Ans. (d)

Solution Given expression

$= \sqrt{\dfrac{(1 + \sin \theta)^2}{(1 + \sin \theta)(1 - \sin \theta)}} = \dfrac{1 + \sin \theta}{\cos \theta}$

$= \mathbf{\sec \theta + \tan \theta.}$

78. Ans. (b)

Solution $\tan \theta = \dfrac{1 + \tan 15°}{1 - \tan 15°} = \dfrac{\tan 45° + \tan 15°}{1 - \tan 45° \tan 15°}$

$= \tan (45° + 15°)$

$= \tan 60° = \tan \pi/3$

$\therefore \quad \theta = \mathbf{\pi/3.}$

79. Ans. (c)

Solution $\tan 45° = \tan (56° - 11°)$

$1 = \dfrac{\tan 56° - \tan 11°}{1 + \tan 56° \tan 11°}$

$1 + \tan 56° \tan 11° = \tan 56° - \tan 11°$

$\therefore \quad \tan 56° - \tan 11° - \tan 56° \tan 11° = \mathbf{1.}$

80. Ans. (c)

Solution $\cot (A + B) = \cot 45°$

$\Rightarrow \quad \dfrac{\cot A \cot B - 1}{\cot B + \cot A} = 1$

$\Rightarrow \quad \cot A \cot B - \cot A - \cot B + 1 = 1 + 1$

$\Rightarrow \quad (\cot A - 1)(\cot B - 1) = 2$

$\Rightarrow \quad K = \mathbf{1/2.}$

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QUESTIONS FROM COMPETITIVE EXAMS

1. For an acule angle θ, $\sin\theta + \cos\theta$ takes the greatest value when θ is
(a) 30° (b) 45°
(c) 60° (d) 90°
[Based on MAT 2003]

2. Suresh is half his father's age. After 20 years, his father's age will be one and a half times the Suresh's age. What is his father's age now?
(a) 40 (b) 20
(c) 26 (d) 30
[Based on MAT 2003]

3. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \text{cosec}\,\theta = b$, then the value of $b(a^2 - 1)$ is equal to
(a) $2a$ (b) $3a$
(c) 0 (d) $2ab$
[Based on MAT 2003]

4. If $7\,\text{cosec}\,\theta - 3\cot\theta = 7$, then the value of $7\cot\theta - 3\,\text{cosec}\,\theta$ is equal to
(a) 5 (b) 3
(c) $\frac{7}{3}$ (d) $\frac{3}{7}$
[Based on MAT 2003]

5. If $a\sec\theta + b\tan\theta = 1$ and $a^2\sec^2\theta - b^2\tan^2\theta = 5$, then $a^2b^2 + 4a^2$ is equal to
(a) $9b^2$ (b) $\frac{9}{a^2}$
(c) $\frac{-2}{b}$ (d) 9
[Based on MAT 2003]

6. If $0 < x \le \frac{\pi}{2}$, then $\sin x + \text{cosec}\,x \ge$
(a) 0 (b) 1
(c) 2 (d) None of these
[Based on MAT 2002]

7. For an acute angle θ, $\sin\theta + \cos\theta$ takes the greatest value when 9 is
(a) 30 (b) 45
(c) 60 (d) 90
[Based on MAT 2001]

8. If $b + c = 30$, then the value of $\cot\frac{B}{2}\cot\frac{C}{2} =$
(a) 3 (b) 2
(c) 9 (d) 7
[Based on MAT 2005]

9. $\frac{3\pi}{5}$ radians is equal to
(a) 108 (b) 54
(c) 100 (d) 81
[Based on MAT 2001]

10. If $\sin A : \cos A = 4 : 7$, then the value of $\frac{7\sin A - 3\cos A}{7\sin A + 2\cos A}$ is
(a) 3/14 (b) 3/2
(c) 1/3 (d) 1/6
[Based on MAT 2001]

11. A, B and C are three angles such that
$\tan A + \tan B + \tan C = \tan A \tan B \tan C$.
Which of the following statements is always correct?
(a) ABC is a triangle, i.e. $A + B + C = \pi$
(b) $A = B = C$, i.e. ABC is an equilateral triangle
(c) $A + B = C$, i.e. ABC is a right-angled triangle
(d) None of these

12. ABC is a triangle. Then $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$
(a) > 1 (b) < 1
(c) ≥ 1 (d) ≤ 1
[Based on FMS (Delhi) 2004]

13. If the sine of an angle is 2/3, then cosine of that angle is
(a) Equal to 2/3 (b) Greater than 2/3
(c) Less than 2/3 (d) Infinite
(e) None of the above **[Based on IIFT 2003]**

14. The value of $(\sin^2\theta + \text{cosec}^2\theta)$ is always
(a) Less than 1 (b) Equal to 1
(c) Greater than or equal to 2
(d) Greater than 1 but less than 2
(e) None of the above **[Based on IIFT 2003]**

15. Simplify : $\frac{\cos 60^\circ + \sin 60^\circ}{\cos 60^\circ - \sin 60^\circ}$
(a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$
(c) $-(\sqrt{3} + 2)$ (d) 1
(e) -1 **[Based on SCMHRD 2002]**

EXPLANATORY ANSWERS

1. Ans. (b)

2. Ans. (a)

Solution $S = \frac{1}{2}F$, $\frac{3}{2}(S + 20) = F + 20$

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20. If A, B, C are acute angles such that $A = \frac{1}{2}$, $\tan B = \frac{1}{5}$ and $\tan C = \frac{1}{8}$, then $A + B + C =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (b)

Solution We have $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$
$= 2(\cos^2\theta + \sin^2\theta) = 2$
$\Rightarrow (\cos\theta + \sin\theta)^2 = 2 - 2\sin^2\theta = 2\cos^2\theta$
$\Rightarrow \cos\theta + \sin\theta = \sqrt{2}\cos\theta$.

3. Ans. (b)

Solution Given $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$
Now, $\cos^2\theta + \cos^4\theta = \sin\theta + \sin^2\theta = 1$.

5. Ans. (a)

Solution Given expression
$$= \frac{1-\sin\alpha+1+\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{2}{\sqrt{\cos^2\alpha}}$$
$$= \frac{-2}{\cos\alpha}. \qquad (\because 90° < \alpha < 180°)$$

7. Ans. (c)

Solution Given $\sin\theta = x + \frac{1}{x} \Rightarrow x^2 - x\sin\theta + 1 = 0$
Since x is real, discriminant $\geq 0 \Rightarrow \sin^2\theta - 4 \geq 0$
$\Rightarrow \sin^2\theta \geq 4$ which is impossible as $-1 \leq \sin\theta \leq 1$.
Hence, given equation is not possible for any real value of x.

9. Ans. (c)

Solution $\because \pi/2 < \alpha < \pi$, $\sec\alpha = -13/12$,
$\tan\alpha = -5/12$, $\cot\alpha = -12/5$
$$\therefore \text{Given expression} = \frac{-13/12 - 5/12}{13/5 + 12/5} = \frac{-18}{12} \times \frac{5}{25}$$
$$= \frac{-3}{10} = -0.3.$$

11. Ans. (c)

Solution Given expression
$= \tan\theta\cos\theta - \sin\theta$
$= \frac{\sin\theta}{\cos\theta} \times \cos\theta - \sin\theta$
$= 0$.

13. Ans. (c)

Solution Given expression
$$= \frac{\cot 45°\cot\theta - 1}{\cot\theta + \cot 45°} \cdot \frac{\cot 45°\cot\theta + 1}{\cot\theta - \cot 45°}$$
$$= \frac{\cot\theta - 1}{\cot\theta + 1} \cdot \frac{\cot\theta + 1}{\cot\theta - 1} = 1.$$

15. Ans. (d)

Solution We know that $\cos\theta \geq -1$ and $\cos\theta \leq 1$
$\therefore \cos 2x \geq -1 \Rightarrow 1 + \cos 2x \geq 0$
$\cos 2x \leq 1 \Rightarrow 1 + \cos 2x \leq 2$
So, minimum value of $1 + \cos 2x$ is 0
and the maximum value of $(1 + \cos 2x)$ is **2.**

17. Ans. (c)

Solution We may write, $\sin + \cos x = \sqrt{2}\sin(45° + x)$
Now, maximum value of $\sin(45° + x)$ is 1
So, the maximum value of $(\sin x + \cos x)$ is $\sqrt{2}$.

19. Ans. (d)

Solution $\sin x < 0$ when $\pi < x < 3\pi/2$
Let $\sin\theta = \frac{-\sqrt{3}}{2} = -\sin 60° = \sin 24°$.

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (c)	**6.** (a)	**8.** (b)	**10.** (a)	**12.** (a)	**14.** (d)	**16.** (b)	**18.** (c)	**20.** (a)

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Heights and Distances 36

INTRODUCTION

Solution of triangles has enormous applications to surveying, navigation, and so on. We shall now consider some simple ones from among them. For this purpose, we need to explain certain terms that are generally used in practical problems.

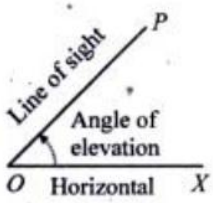


Fig. (a)

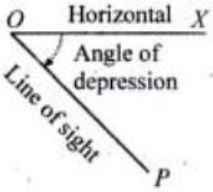


Fig. (b)

1. If OX be a horizontal line through O, the eye of the observer and P be an object in the vertical plane through OX, then $\angle XOP$ is called:

(i) **the angle of elevation,** if P is *above* OX as in Fig. (a); and

(ii) the **angle of depression,** if P is *below* OX as in Fig. (b).

The straight line OP (joining the eye of the observer to the object) is called the *line of sight* of the observer.

2. Values of the trigonometric ratios for some useful angles

angle (θ) / *t*-ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

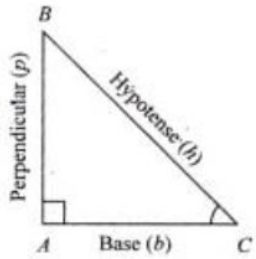
The values of cot θ, sec θ and cosec θ can be found from the above Table by using the relations $\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$.

3. Pythagoras Theorem

In a right-angled triangle the square of its hypoteuse is equal to the sum of the squares of its legs (i.e. perpendicular and base).

In other words,

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$



or, $(BC)^2 = (AB)^2 + (AC)^2$

or, $h^2 = p^2 + b^2$.

4. Few important values to memorise:

$\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

$\sqrt{5} = 2.236$.

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The angle of elevation of the sum is
(a) 30° (b) 45°
(c) 60° (d) 90°

2. The angle of elevation of a moon when the length of the shadow of a pole is equal to its height, is
(a) 30° (b) 45°
(c) 60° (d) None of these

3. A tower stands on a horizontal plane. A man on the ground 100 m from the base of the tower finds the angle of elevation of the top of the tower to be 30°. What is the height of the tower?
(a) 100 m (b) $100\sqrt{3}$
(c) $100/\sqrt{3}$ (d) None of these

4. When the sun is 30° above the horizontal, the length of shadow cast by a building 50 m high is
(a) $\frac{50}{\sqrt{3}}$ m (b) $50\sqrt{3}$ m
(c) 25 m (d) $25\sqrt{3}$ m

5. A pole being broken by the wind, the top struck the ground at an angle of 30° and at a distance of 21 m from the foot of the pole. Find the total height of the pole.
(a) 21 m (b) $21\sqrt{3}$ m
(c) $21/\sqrt{3}$ (d) None of these

6. The upper part of a tree broken by the wind makes an angle of 30° with the ground and the distance from the root to the point where the top of the tree touches the ground is 10 m. What was the height of the tree?
(a) $10\sqrt{3}$ (b) $10/\sqrt{3}$
(c) $20\sqrt{3}$ (d) None of these

7. A tower stands at the end of a straight road. The angles of elevation of the top of the tower from two points on the road 500 m apart are 45° and 60°, respectively. Find the height of the tower.
(a) $\frac{500\sqrt{3}}{\sqrt{3}-1}$ (b) $5000\sqrt{3}$
(c) $\frac{500\sqrt{3}}{\sqrt{3}+1}$ (d) None of these

8. The shadow of a tower standing on a level plane is found to be 50 m longer when sun's altitude is 30° than when it is 60°. Find the height of the tower.
(a) $20\sqrt{3}$ m (b) $25/\sqrt{3}$ m
(c) $25\sqrt{3}$ m (d) $20\sqrt{3}$ m

9. In a rectangle, if the angle between a diagonal and a side is 30° and the length of diagonal is 6 cm, the area of the rectangle is
(a) 9 cm^2 (b) $9\sqrt{3}$ cm^2
(c) 27 cm^2 (d) 36 cm^2

10. The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x metres less. The value of x is
(a) 100 m (b) $100\sqrt{3}$ m
(c) $100(\sqrt{3}-1)$ m (d) $\frac{100}{\sqrt{3}}$ m

11. An electric pole 20 m high stands upright on the ground with the help of steel wire to its top and affixed on the ground. If the steel wire makes 60° with the horizontal ground, find the length of the steel wire.
(a) $40/\sqrt{3}$ m (b) $40\sqrt{3}$ m
(c) $20/\sqrt{3}$ m (d) $20\sqrt{3}$ m

12. From the top of a light house 50 m above the sea, the angle of depression of an incoming boat is 30°. How far is the boat from the light house?
(a) $25\sqrt{3}$ m (b) $25/\sqrt{3}$ m
(c) $50/\sqrt{3}$ m (d) $50/\sqrt{3}$ m

13. From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.
(a) 40 m (b) 48 m
(c) 50 m (d) 52 m

14. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is
(a) 30° (b) 45°
(c) 60° (d) 75°

15. From the top of a light house 60 m high with its base at sea level, the angle of depression of a boat is 15°. The distance of the boat from the light house is
(a) $60\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (b) $60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m
(c) $30\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (d) $30\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m

16. On the level ground, the angle of elevation of the top of the tower is 30°. On moving 20 m nearer,

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the angle of elevation is 60°. The height of the tower is

(a) $20\sqrt{3}$ m (b) $10\sqrt{3}$ m

(c) $10(\sqrt{3}-1)$ m (d) None of these

17. The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. The height of the tower is

(a) 10 m (b) 20 m

(c) 40 m (d) $20\sqrt{3}$ m

18. At a point 15 m away from the base of a 15 m high house, the angle of elevation of the top is

(a) 45° (b) 30°

(c) 60° (d) 90°

19. Angle of depression from the top of a light house of two boats are 45° and 30° due east which are 60 m apart. The height of the light house is

(a) $60\sqrt{3}$ (b) $30(\sqrt{3}-1)$

(c) $30(\sqrt{3}+1)$ (d) None of these

20. The angle of elevation of the top of a hill from each of the vertices A, B, C of a horizontal triangle is α. The height of the hill is

(a) $b \tan\alpha \operatorname{cosec} B$ (b) $\frac{a}{2}\tan\alpha \operatorname{cosec} A$

(c) $\frac{c}{2}\tan\alpha \operatorname{cosec} C$ (d) None of these

21. A tower subtends an angle of 30° at a point on the same level as the foot of the tower. At a second point h m above the first, the depression of the foot of the tower is 60°. The horizontal distance of the tower from the point is

(a) $h \cot 60°$ (b) $h \cot 30°$

(c) $\frac{h}{3}\cot 60°$ (d) $\frac{h}{3}\cot 30°$

22. If a flag staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then the angle that the sun makes with the ground is

(a) 60° (b) 30°

(c) 45° (d) None of these

23. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 m from the bank, he finds the angle to be 30°. The breadth of the river is

(a) 40 m (b) 60 m

(c) 20 m (d) 30 m

24. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole will be

(a) 30° (b) 60°

(c) 90° (d) 45°

25. The angle of elevation of the top of a TV tower from three points A, B, C in a straight line through the foot of the tower are α, 2α, 3α, respectively. If $AB = a$, the height of the tower is

(a) $a \tan\alpha$ (b) $a \sin\alpha$

(c) $a \sin 2\alpha$ (d) $a \sin 3\alpha$

26. The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. If the elevation of the top at the same point is to be 60°, the tower must be raised to a height

(a) $120(\sqrt{3}+1)$ m (b) $120(\sqrt{3}-1)$ m

(c) $10(\sqrt{3}+1)$ m (d) None of these

27. A person walking along a straight road towards a hill observes at two points, distance $\sqrt{3}$ km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is

(a) $\frac{3}{2}$ km (b) $\sqrt{\frac{2}{3}}$ km

(c) $\frac{\sqrt{3}+1}{2}$ km (d) $\sqrt{3}$ km

28. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is

(a) 12 m (b) 10 m

(c) 8 m (d) None of these

29. A man is standing on the 8 m long shadow of a 6 m long pole. If the length of the shadow is 2.4 m, then the height of the man is

(a) 1.4 m (b) 1.6 m

(c) 1.8 m (d) 2.0 m.

30. The angle of elevation of the top of a tower at a point G on the ground is 30°. On walking 20 m towards the tower the angle of elevation becomes 60°. The height of the tower is equal to

(a) $\frac{10}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m

(b) $\frac{20}{\sqrt{3}}$ m (d) $10\sqrt{3}$ m

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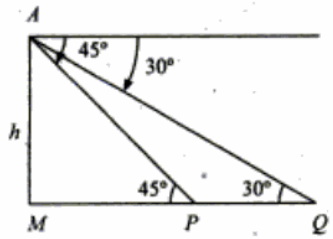


Let $h = MA$

Then, $\frac{h}{MP} = \tan 45° = 1$. $\therefore$ $h = MP$

Again, $\frac{h}{MP+60} = \tan 30° = \frac{1}{\sqrt{3}}$.

$\therefore$ $MP + 60 = \sqrt{3}h$ or, $h + 60 = \sqrt{3}h$

$\therefore$ $(\sqrt{3} - 1)h = 60$

$\therefore$ $h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{2} = \mathbf{30(\sqrt{3} + 1)\ m}$

20. Ans. (b)

Solution The distance of the foot from each vertex $= h\cot\alpha$.

$\therefore$ The foot is at the circumcentre of the triangle.

$\therefore$ $R = h\cot\alpha$

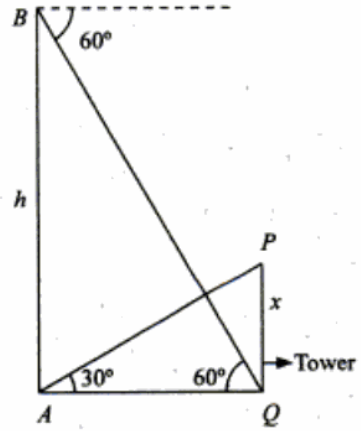
$\therefore$ $h = R\tan\alpha = \frac{a}{2\sin\alpha}\tan\alpha = \mathbf{\frac{a}{2}\tan\alpha \cdot \operatorname{cosec}\alpha}$.

21. Ans. (a)

Solution Let $PQ = x$ m denote the tower so that $\angle PAQ = 30°$. Let $BA = h$ m

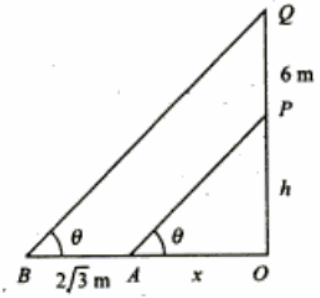
$\therefore$ $\angle BQA = 60°$.

Now, $\frac{h}{AQ} = \tan 60° = \sqrt{3}$. $\therefore$ $AQ = \frac{h}{\sqrt{3}} = \mathbf{h\cot 60°}$.



(a)

OP be the tower of height h m and PQ of height 6 m. Let the sun make an ound. Let $OA = x$ and $AB = 2\sqrt{3}$ be wer and the flagstaff, respectively.



Now, $\tan\theta = \frac{h}{x}$.

Also, $\frac{h+6}{x+2\sqrt{3}} = \tan\theta$

$\therefore$ $\frac{h}{x} = \frac{h+6}{x+2\sqrt{3}} \Rightarrow hx + 2\sqrt{3}h = hx + 6x$

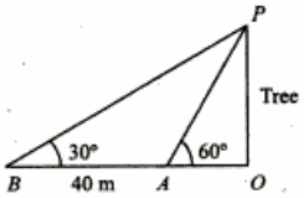
$\Rightarrow 2\sqrt{3}h = 6x$

$\Rightarrow \frac{h}{x} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3}$. $\therefore$ $\boldsymbol{\theta = 60°}$.

23. Ans. (c)

Solution Let OA denote the breadth of the river.



$\frac{OP}{OA} = \tan 60° = \sqrt{3}$

$\therefore$ $OP = \sqrt{3}OA$.

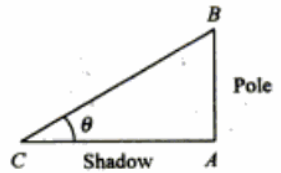
Also, $\frac{OP}{OA+40} = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ $OA + 40 = \sqrt{3}OP = \sqrt{3}(\sqrt{3}OA) = 3\ OA$.

$\therefore$ $2OA = 40 \Rightarrow \boldsymbol{OA = 20\ m}$.

24. Ans. (a)

Solution Given, $AC = \sqrt{3}AB$.



$\therefore$ $\cot\theta = \frac{AC}{AB} = \sqrt{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \boldsymbol{\theta = 30°}$.

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QUESTIONS FROM COMPETITIVE EXAMS

1. A 25 m ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4 m, then the foot of the ladder will slide
(a) 5m (b) 8m
(c) 9m (d) 15m
[Based on MAT 2004]

2. A tree 6m tall casts a 4m long shadow. At the same time, a flag pole casts a shadow 50m long. How long is the flag pole?
(a) 75m (b) 100m
(c) 150m (d) 50m
[Based on MAT 2004]

3. The angles of elevation of the top of a tower from two points at distances m and n metres are complementary. If the two points and the base of the tower are on the same straight line, then the height of the tower is
(a) $\sqrt{mn}$ (b) mn
(c) $\frac{m}{n}$ (d) None of these
[Based on MAT 2003]

4. The angles of elevation of an artificial satellite measured from two earth stations are 30° and 40° respectively. If the distance between the earth stations is 4000 km, then the height of the satellite is
(a) 2000 km (b) 6000 km
(c) 3464 km (d) 2828 km
[Based on MAT 2002]

5. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole is
(a) 30° (b) 45°
(c) 60° (d) 75°
[Based on MAT 2001]

6. The Qutab Minar casts a shadow 150 m long at the same time when the Vikas Minar casts a shadow of 120 m long on the ground. If the height of the Vikas Minar is 80 m, find the height of the Qutab Minar.
(a) 180 m (b) 100 m
(c) 150 m (d) 120 m
[Based on MAT 1999]

7. The distance between the tops of two trees 20 m and 28 m high is 17 m. The horizontal distance between the two trees is
(a) 9 m (b) 11 m
(c) 15 m (d) 31 m
[Based on MAT 2001]

8. A tree breaks due to storm and the broken part bends so that the top of the tree first touches the ground, making an angle of 30 with the horizontal. The distance from the foot of the tree to the point where the top touches the ground is 10 m. The height of the tree is
(a) $10(\sqrt{3}+1)$ m (b) $10\sqrt{3}$ m
(c) $10(\sqrt{3}-1)$ m (d) $\frac{10}{\sqrt{3}}$ m
[Based on MAT 2001]

9. A ladder is inclined to a wall making an angle of 30° with it. A man is ascending the ladder at the rate of 2 metres/second. How fast is he approaching the wall?
(a) 2 m/s (b) 1.5 m/s
(c) 1 m/s (4) None of these
[Based on FMS (Delhi) 2004]

10. One side of a parallelogram is 12 cm and its area is 60 cm^2. If the angle between the adjacent sides is 30°, then its other side is
(a) 10 cm (b) 8 cm
(c) 6 cm (d) 4 cm
[Based on IITTM, Gwalior, 2003]

11. To win an election, a candidate needs $\frac{3}{4}$ of the votes cast. If, after $\frac{2}{3}$ of the votes have been counted, a candidate has $\frac{5}{6}$ of what he needs, then what part of the remaining ratio does he still need?
(a) $\frac{1}{10}$ (b) $\frac{1}{8}$
(c) $\frac{1}{4}$ (d) $\frac{3}{8}$
[Based on REC Tiruchirapalli 2003]

12. Mr. Gidwani's Padyatra Party wanted to go from Gwalior to Bhubaneswar. The walkers travelled 150 km straight and then took a 45° turn towards Varanasi and walked straight for another 120 km. Approximately, how far was the party from the starting point?
(a) 250 km (b) 90 km
(c) 81 km (d) 30 km
[Based on FMS (Delhi) 2003]

13. A vertical pole PO is standing at the centre O of a square $ABCD$. If AC subtends an angle of 90° at the point P of the pole, then the angle subtended by a side of the square at the point P is
(a) 35° (b) 45°
(c) 30° (d) 60°
(e) None of the above **[Based on IIFT 2003]**

14. A vertical pole breaks due to storm and the broken part bends, so that the top of the pole touches the ground, making an angle of 30° with the ground. The distance from the foot of the pole to the point

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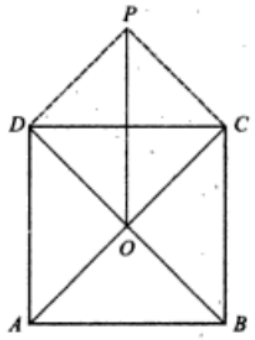
13. Ans. (d)

Solution Given $\angle APO + \angle CPO = 90°$

$\Rightarrow \angle APO = \angle CPO = 45° = \angle BPO = \angle DPO$



$$\cos \angle APB = \cos\theta = \frac{AP^2 + BP^2 - AB^2}{2AP \cdot BP}$$

$$= \frac{AO^2 + OP^2 + BO^2 + BP^2 - AB^2}{2AP \cdot BP}$$

$$= \frac{OP^2}{AP \cdot BP} = \frac{OP}{AP} \cdot \frac{OP}{BP}$$

$$= \sin 45° \times \sin 45° = \frac{1}{2}.$$

14. Ans. (d)

15. Ans. (c)

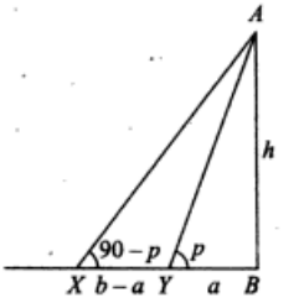
Solution Let h be the height of the tower AB.

$$\therefore \quad \frac{h}{a} = \tan p$$

$$\frac{h}{b} = \tan(90 - p) = \cot p$$

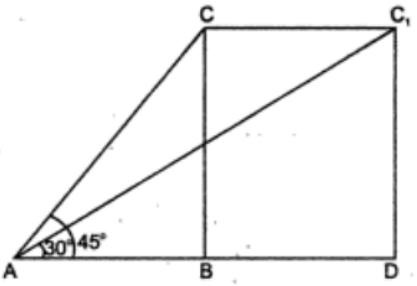
$$\therefore \quad \frac{h}{a} \times \frac{h}{b} = \tan p \times \cot p = 1$$

$$\Rightarrow \quad h^2 = ab \Rightarrow h = \sqrt{ab}.$$



16. Ans. (d)

Solution The Areroplane moves B D = $3\left(\sqrt{3}-1\right)$ in seconds



$$\therefore \text{ Speed } = \frac{3 \times \left(\sqrt{3}-1\right) \times 3600}{15} = 527 \text{ km/h}$$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. Two posts are 25 m and 15 m high and the line joining their tops makes an angle of 45° with the horizontal. The distance between these posts is

(a) 5 m (b) $\frac{10}{\sqrt{2}}$ m

(c) 10 m (d) $10\sqrt{2}$ m

2. The angle of depression of the top of a tower at a point 70 m from the base is 45°. The height of the tower is

(a) 70 m (b) $70\sqrt{2}$ m

(c) $\frac{70}{\sqrt{2}}$ m (d) 35 m

3. The angle of elevation of the top of a tower from the top and bottom of a building of height 'a' are 30° and 45°, respectively. If the tower and the building stand at the same level, the height of the tower is

(a) $\sqrt{3}a$ (b) $a\left(\sqrt{3}-1\right)$

(c) $\frac{(3+\sqrt{3})}{2}a$ (b) $a\left(\sqrt{3}+1\right)$

4. The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30°. The height of the tower is

(a) $\frac{500\sqrt{3}}{3}$m (b) $\frac{500(\sqrt{3}-1)}{3}$m

(c) $\frac{500(\sqrt{3}+1)}{3}$m (d) 500 m.

5. From the top of a house 32 m high, the angle of elevation of the top of a river is 45° and the angle of depression of the foot of the tower is 30°. The distance of the tower from the house is

(a) $30\sqrt{3}$ m (b) $32\sqrt{3}$m

(c) $35\sqrt{3}$ m (d) None of these

6. If from a point 100 m above the ground the angles of depression of two objects due to south on the ground are 60° and 45°, then the distance between the objects is

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(a) $\frac{100}{\sqrt{3}}(\sqrt{3+1})$ m (b) $\frac{100}{\sqrt{3}}(\sqrt{3-1})$ m

(c) $100\sqrt{3}$ m (d) 100 m

7. Length of the shadow of a person is x when the angle of elevation of the sun is 45°. If the length of the shadow increases by $(\sqrt{3}-1)x$, then the angle of elevation of the sun should become

(a) 15° (b) 18°

(c) 25° (d) 30°

8. A balloon is connected to a mateorological station by a cable of length 200 m, inclined at 60° to the horizontal. Find the height of the balloon from the ground.

(a) $100\sqrt{3}$ (b) $100/\sqrt{3}$

(c) $50\sqrt{3}$ (d) $50/\sqrt{3}$

9. The distance between two multistoreyed buildings is 60 m. The angle of depression of the top of the first building as seen from the top of the second building which is 150 m high is 30°. The height of the first building is

(a) $(150+20\sqrt{3})$ m (b) $(150-20\sqrt{3})$

(c) $(150+10\sqrt{3})$ m (d) None of these

10. A boot is being rowed away from a cliff 150 m high. At the top of the cliff the angle of depression of the boat changes from 60° to 45° in 2 minutes. The speed of the boat is

(a) 2 km/hr (b) 1.9 km/hr

(c) 3.4 km/hr (d) 3 km/hr

Test II

[Time: 10 min]

11. The angle of elevation of an aeroplane from a point on the ground is 45°. After 15 seconds flight, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 m, the speed of the plane in km per hour is

(a) 306.72 (b) 208.34

(c) 402.56 (d) 527

12. A balloon of radius r makes an angle α at the eye of an observer and the angle of elevation of its centre is β. The height of its centre from the ground level is given by

(a) $r\cos\frac{\beta}{2}\sec\alpha$ (b) $r\cos\beta\sec\frac{\alpha}{2}$

(c) $r\sin\frac{\beta}{2}\operatorname{cosec}\alpha$ (d) $r\sin\beta\operatorname{cosec}\frac{\alpha}{2}$

13. A landmark on a river bank is observed from two points X and Y on the opposite bank of the river. The lines of sight make equal angles with the bank of the river. If $XY = 1$ km, then the width of the river is

(a) $\frac{3}{2}$km (b) $\frac{1}{2}$km

(c) $\frac{3\sqrt{2}}{2}$km (d) $\frac{3\sqrt{2}}{2}$km

14. Two men are on the opposite sides of a tower. They measure the angle of elevation of the top of the tower as 30° and 60°, respectively. If the height of the tower is 30 m, find the distance between the men.

(a) $20\sqrt{3}$ m (b) $20/\sqrt{3}$ m

(c) $40\sqrt{3}$ m (d) $40/\sqrt{3}$ m

15. A person standing on the bank of a river finds that the angle of elevation of the top of a tower on the opposite bank is 45°, then

(a) breadth of the river is twice the height of the tower

(b) height of the river is twice the breadth of the river

(c) both are equal (d) none of the above.

16. An artist climbs a rope stretched from the top of a pole and fixed at the ground. The height of the pole is 10 m and the angle made by the rope with the ground is 30°. Calculate the length of the rope.

(a) 10 m (b) 20 m

(c) 30 m (d) 40 m

17. The length of a string between a kite and a point on the ground is 90 m. The string makes an angle of 60° with the level ground. Assuming that there is no slack in the string, then the height of the kite is

(a) $45\sqrt{3}$ m (b) $45/\sqrt{3}$ m

(c) $50\sqrt{3}$ m (d) $50/\sqrt{3}$ m

18. A ladder leaning against a vertical wall makes an angle of 30° with the ground. The foot of the ladder is 5 m from the wall. Determine the length of the ladder.

(a) $10\sqrt{3}$ m (b) $10/\sqrt{3}$ m

(c) $20\sqrt{3}$ m (d) $20/\sqrt{3}$ m

19. The angles of elevation of an aeroplane flying vertically above the ground as observed from two consecutive stones 1 km apart are 45° and 60°. The height of the aeroplane above the ground in km is

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{3+\sqrt{3}}{2}$

(c) $3+\sqrt{3}$ (d) $\sqrt{3}+1$

20. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light house be h m and the line joining the ships passes through the foot of the light house, the distance between the ships is

(a) $\frac{h(\cot\alpha+\cot\beta)}{\cot\alpha\cot\beta}$ (b) $\frac{h(\tan\alpha+\tan\beta)}{\tan\alpha\tan\beta}$

(c) $h(\tan\alpha+\tan\beta)$ (d) $\frac{h\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

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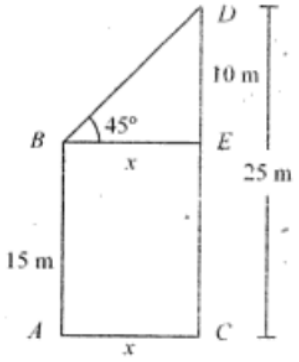
SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (c)

Solution $\frac{10}{x} = \tan 45° = 1 \Rightarrow x = 10$. Thus, $BE = 10$ m.



3. Ans. (c)

Solution $\frac{h}{x} = \tan 45° = 1$

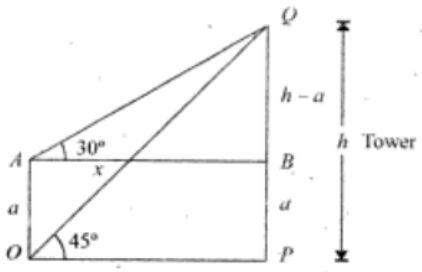
$\therefore \quad h = x.$

$$\frac{h-a}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad h - a = \frac{x}{\sqrt{3}} = \frac{h}{\sqrt{3}}$$

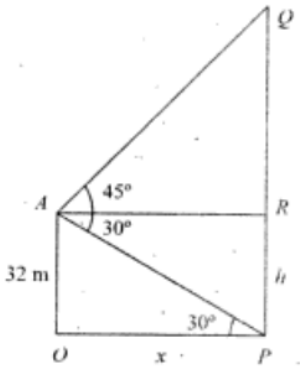
$$\therefore \quad h\left(1-\frac{1}{\sqrt{3}}\right) = a$$

$$\therefore \quad h = \frac{\sqrt{3}a}{\sqrt{3}-1} = \frac{\sqrt{3}a}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(3+\sqrt{3})a}{2}.$$



5. Ans. (b)

Solution Here, x is required.



Clearly, $\frac{x}{32} = \cot 30° = \sqrt{3}$.

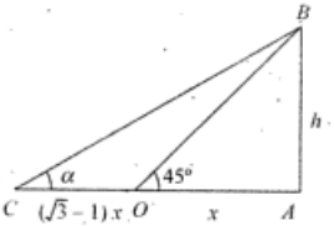
$\therefore \quad x = 32\sqrt{3}$ **m.**

7. Ans. (d)

Solution Let h be the height of the person

$$\therefore \quad \frac{h}{x} = \tan 45° = 1$$

$\therefore \quad h = x.$



Also, $\frac{h}{(\sqrt{3}-1)x+x} = \tan\alpha \Rightarrow \frac{h}{\sqrt{3}x} = \tan\alpha$

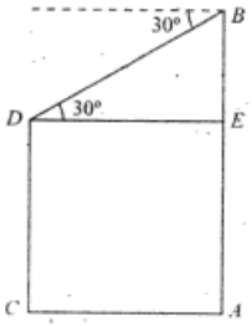
$$\Rightarrow \frac{1}{\sqrt{3}} = \tan\alpha$$

$\therefore \quad \alpha = \mathbf{30°}.$

$$\Rightarrow AB = \frac{200\times\sqrt{3}}{2} = \mathbf{100\sqrt{3}\ m}.$$

9. Ans. (b)

Solution Let AB and CD be the buildings such that $AB = 150$ m and $AC = 60$ m.



Draw $DE \perp AB$. Then, $\angle BDE = 30°$.

$$\frac{BE}{DE} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow BE = \frac{DE}{\sqrt{3}} = \frac{AC}{\sqrt{3}} = \frac{60}{\sqrt{3}}.$$

Height of the first building $= CD = AE$

$$= AB - BE = \left(150 - \frac{60}{\sqrt{3}}\right)\text{m}$$

$$= \mathbf{(150 - 20\sqrt{3})\ m}.$$

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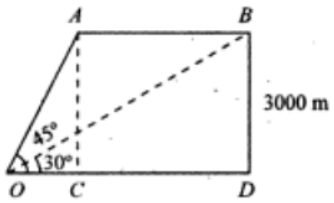
11. Ans. (d)

Solution Let A and B be the two positions of the plane and let O be the point of observation and OD be the horizontal. Draw $AC \perp OD$ and $BD \perp OD$.



$\angle DOB = 30°$, $\angle DOA = 45°$ and

$AC = BD = 3000$ m.

$\therefore \quad \frac{OD}{DB} = \cot 30° = \sqrt{3} \Rightarrow OD = 3000\sqrt{3}$ m

and, $\frac{OC}{AC} = \cot 45° = 1 \Rightarrow OC = 3000$ m.

Distance covered in 15 sec.

$= AB = CD = OD - OC$

$= (3000\sqrt{3} - 3000)$m $= 2196$ m

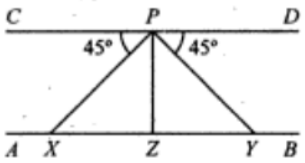
$\therefore$ Speed of the plane

$= \left(\frac{2196}{15} \times \frac{1}{1000} \times 60 \times 60\right)$km/hr

$= 527$ km/hr.

13. Ans. (b)

Solution Let AB and CD be the two banks of a river and P be the landmark. $\angle X = \angle Y = 45°$.



So, $PX = PY$.

Thus, XPY is an isosceles triangle. So, Z is the mid-point of XY.

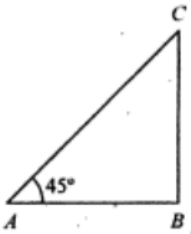
$\therefore \quad XZ = ZY = 0.5$ km.

In ΔZYP, $\angle ZYP = \angle YPZ = 45°$

$\therefore \quad PZ = YZ = \frac{1}{2}$ **km.**

15. Ans. (c)

Solution Let AB be the width of the river and BC be the height of tower, then

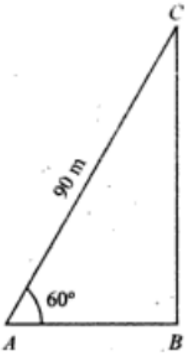


$\frac{BC}{AB} = \tan 45° = 1 \Rightarrow BC = AB$

$\therefore$ both are equal.

17. Ans. (a)

Solution $\frac{BC}{90} = \sin 60° \Rightarrow \frac{BC}{90} = \frac{\sqrt{3}}{2}$



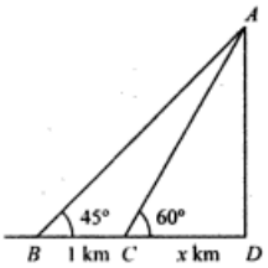
$\therefore \quad BC = \frac{90 \times \sqrt{3}}{2} =$ **$45\sqrt{3}$ m.**

19. Ans. (b)

Solution Let $CD = x$ km.

$\frac{AD}{BD} = \tan 45° = 1 \Rightarrow AD = BD = (x + 1)$ km

$\frac{AD}{CD} = \tan 60° = \sqrt{3} \Rightarrow \frac{x+1}{x} = \sqrt{3} \Rightarrow x(\sqrt{3} - 1) = 1$



$\therefore \quad x = \frac{1}{\sqrt{3}-1} = \left(\frac{\sqrt{3}+1}{2}\right)$km

So, $AD = x + 1 = \frac{\sqrt{3}+1}{2} + 1 = \left(\frac{3+\sqrt{3}}{2}\right)$ **km.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (a)	**4.** (a)	**6.** (b)	**8.** (a)	**10.** (b)	**12.** (d)	**14.** (c)	**16.** (b)	**18.** (b)	**20.** (b)

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PART V
Geometry

CHAPTER 37 Plane Geometry
CHAPTER 38 Coordinate Geometry

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Plane Geometry 37

SECTION 1
LINES AND ANGLES

Line A geometrical straight line is a set of points that extends endlessly in both the directions.

Axiom-1 A line contains infinitely many points.

Axiom-2 Through a given point, infinitely many lines pass.

Axiom-3 Given two distinct points A and B, there is one and only one line that contains both the points.

Parallel Lines If two lines have no point in common, they are said to be parallel lines

Intersecting Lines If two lines have a point in common, they are said to be *Intersecting lines*. Two lines can intersect at the most at one point.

Line Segment and Ray A part (or portion) of a line with two end points is called a *line-segment* and a part of a line with one end point is called a *ray*. A line segment $\overline{AB}$ and its length is denoted as AB. Ray AB (i.e. A towards B) is denoted as $\overrightarrow{AB}$ and ray BA (i.e. B towards A) is denoted as $\overrightarrow{BA}$.

Collinear Points Three or more than three points are said to be *collinear* if there is a line which contains them all.

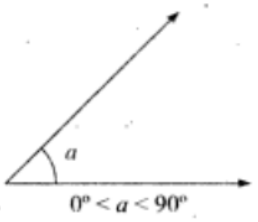
Concurrent lines Three or more than three lines are said to be concurrent if there is a point which lies on all of them.

Angle An *angle* is a figure formed by two rays with a common initial point. The two rays forming an angle are called the *arms* of the angle and the common initial point is called the *vertex* of the angle.

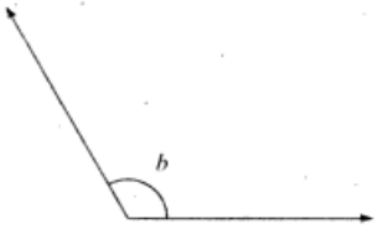
Types of Angles

An angle is said to be:

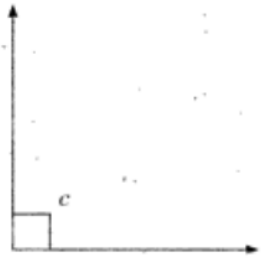
(i) *Acute*, if $a < 90°$.



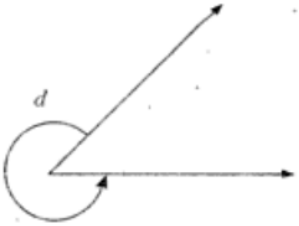
(ii) *Obtuse*, if $90° < b < 180°$.



(iii) *Right angle*, if $c = 90°$



(iv) *Reflex angle*, if $180° < d < 360°$



(v) *Straight angle*, if $e = 180°$

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(vi) Complete angle: An angle whose measure is 360°, is called a *complete angle*.

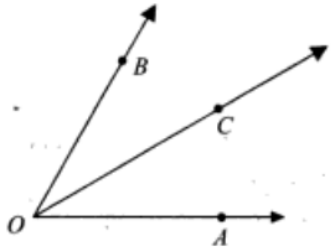
Complementary Angles Two angles, the sum of whose measures is 90° are called *complementary angles* e.g. 50° and 40° is a pair of complementary angles.

Supplementary angles Two angles, the sum of whose measures is 180°, are called *supplementary angles*. e.g. 72° and 108° is a pair of supplementary angles.

Adjacent Angles Two angles are called *adjacent angles* if

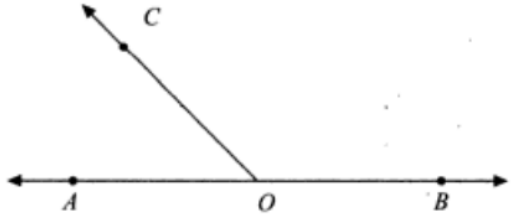
(i) they have the same vertex,
(ii) they have a common arm, and
(iii) uncommon arms are on either side of the common arm.

e.g. $\angle AOC$ and $\angle BOC$ are adjacent angles



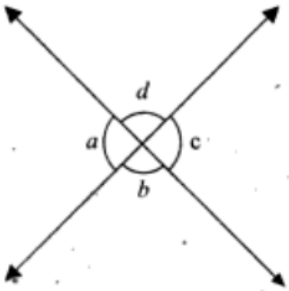
Linear Pair: Two adjacent angles are said to form a linear pair of angles if their non-common arms are two opposite rays

e.g. $\angle AOC$ and $\angle BOC$ form a linear pair.



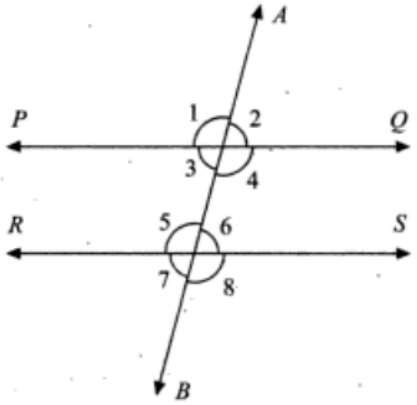
Linear Pair Axiom: If a ray stands on a line, then the sum of the two adjacent angles so formed is 180°. Conversely, if the sum of two adjacent angles is 180°; then the non-common arms of the angles are two opposite rays.

Vertically opposite angles: When two lines intersect, four angles are formed. The angles opposite to each other are called *vertically opposite angles*



a and c are vertically opposite angles, $\angle a = \angle c$
b and d are vertically opposite angles, $\angle b = \angle d$

Angles Made by a Transversal* with two parallel lines Suppose $PQ \parallel RS$ and a transversal AB cuts them, then



(a) Pair of corresponding angles are
(1 and ∠5), (∠2 and ∠6),
(∠4 and ∠8) and (∠3 and ∠7)
(b) Pair of alternate angles are
(∠3 and ∠6), (∠4 and ∠5)
(c) Pair of interior angles (consecutive interior angles or cointerior angles) on the same side of the transversal are
(∠3 and ∠5), (∠4 and ∠6)

Key Points to Remember

If two parallel lines are intersected by a transversal, then

(i) each pair of corresponding angles are equal.
(ii) each pair of alternate angles are equal.
(iii) interior angles on the same side of the transversal are supplementary.

*A line which intersects two or more lines at distinct points is called a transversal of the given lines.

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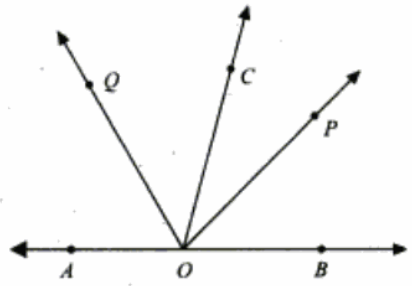
MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. An angle is equal to one-third of its supplement. Its measure is equal to
(a) 40° (b) 50°
(c) 45° (d) 55°

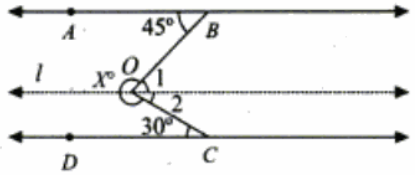
2. The complement of 30°20′ is
(a) 69°40′ (b) 59°40′
(c) 35°80′ (d) 159°40′

3. In the given figure, OP bisect $\angle BOC$ and OQ bisects $\angle AOC$. Then $\angle POQ$ is equal to



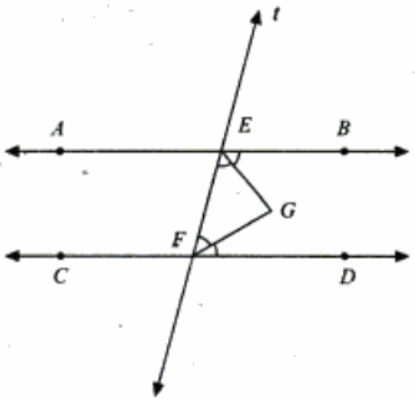
(a) 90° (b) 120°
(c) 60° (d) 100°

4. In the given, $AB \parallel CD$. Then X is equal to



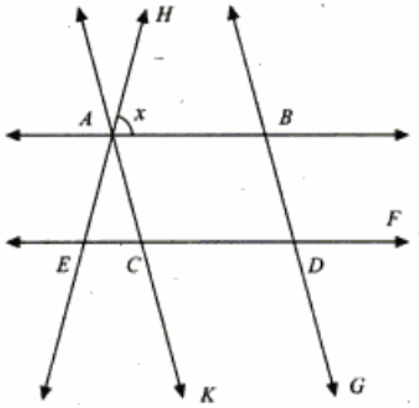
(a) 290° (b) 300°
(c) 280° (d) 285°

5. In the adjoining figure, $AB \parallel CD$, t is the transversal, EG and FG are the bisectors of $\angle BEE$ and $\angle DFE$ respectively, then $\angle EGF$ is equal to



(a) 90° (b) 75°
(c) 80° (d) 110°

6. In the given figure, $AB \parallel CD$ and $AC \parallel BD$. If $\angle EAC = 40°$, $\angle FDG = 55°$, $\angle HAB = x$; then the value of x is



(a) 95° (b) 70°
(c) 35° (d) 85°

7. Find the measure of an angle, if six times its complement is 12° less than twice its supplement
(a) 48° (b) 96°
(c) 24° (d) 58°

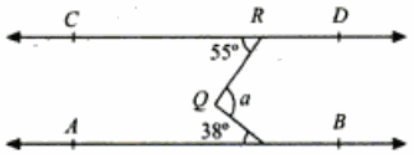
8. In a ΔABC, the sides AB and AC are produced to P and Q respectively. The bisectors of $\angle PBC$ and $\angle QCB$ intersect at a point O. Then $\angle BOC$ is equal to

(a) $90° + \frac{1}{2}\angle A$ (b) $90° - \frac{1}{2}\angle A$

(c) $120° + \frac{1}{2}\angle A$ (d) $120° - \frac{1}{2}\angle A$

17. If two parallel lines are intersected by a transversal, then the bisectors of the two pairs of interior angles enclose a
(a) Trapezium (b) Rectangle
(c) Square (d) none of these

19. In fig., $AB \parallel CD$, $\angle a$ is equal to



(a) 93° (b) 103°
(c) 83° (d) 97°

10. The complement of an angle exceeds the angle by 60°. Then the angle is equal to
(a) 25° (b) 30°
(c) 15° (d) 35°

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EXPLANATORY ANSWERS

1. Ans. (c)

Solution Let the measured of the required angle be x degree. Then, its supplement $= 180 - x$

$$\text{Now, angle} = \frac{1}{3}(\text{its supplement})$$

$$x = \frac{1}{3}(180 - x)$$

$$3x + x = 180^\circ \Rightarrow \boldsymbol{x = 45^\circ}.$$

2. Ans. (b)

Solution Complement of

$$30^\circ 20' = 90^\circ - (30^\circ 20') = 90^\circ - (30^\circ + 20')$$
$$= (89^\circ - 30^\circ) + (1^\circ - 20')$$
$$= 59^\circ + 60' - 20' \quad [\because 1^\circ = 60']$$
$$= 59^\circ + 40' = \boldsymbol{59^\circ 40'}.$$

3. Ans. (a)

Solution Since OP bisects $\angle BOC$,

$$\therefore \quad \angle BOC = 2\angle POC$$

Again, OQ bisects $\angle AOC$, $\therefore$,

$$\angle AOC = 2\angle QOC$$

Since ray OC stands on line AB, $\therefore$,

$$\angle AOC + \angle BOC = 180^\circ \Rightarrow 2\angle QOC + 2\angle POC = 180^\circ$$
$$\Rightarrow 2(\angle QOC + \angle POC = 180^\circ$$
$$\Rightarrow \angle QOC + \angle POC = 90^\circ$$
$$\Rightarrow \boldsymbol{\angle POQ = 90^\circ}.$$

The above sum can also be restated as follows: The angle between the bisectors of a linear pair of angles is a right angle.

4. Ans. (d)

Solution Through O, draw a line l parallel to both AB and CD. Then

$$\angle 1 = 45^\circ \quad \text{(alt. } \angle S\text{)}$$

and

$$\angle 2 = 30^\circ \quad \text{(alt. } \angle S\text{)}$$

$\therefore \angle BOC = \angle 1 + \angle 2 = 45^\circ + 30^\circ = 75^\circ$

So, $X = 360^\circ - \angle BOC = 360^\circ - 75^\circ = 285^\circ$

Hence $\boldsymbol{X = 285^\circ}$.

5. Ans. (a)

Solution $AB \parallel CD$ and t transversal intersects them at E and F

$$\angle BEF + \angle EFD = 180^\circ \quad \text{(co-interior angles)}$$
$$\Rightarrow \frac{1}{2}\angle BEF + \frac{1}{2}\angle EFD = 180^\circ$$
$$\Rightarrow \angle BEF + \Delta EFD = 90^\circ$$

In $\angle EFG$

$$\angle EFG + \angle FEG + \angle EGF = 180^\circ$$
$$\therefore \quad \angle EGF + 90^\circ = 180^\circ$$
$$\therefore \quad \angle EGF = 90^\circ.$$

The above result can be restated as:

If two parallel lines are cut by a transversal, then the bisectors of the interior angles on the same side of the transversal intersect each other at right angles.

6. Ans. (d)

$$\angle DCK = \angle FDG = 55^\circ \text{ (corr. } \angle s\text{)}$$
$$\therefore \quad \angle ACE = \angle DCK = 55^\circ \text{ (vert. opp. } \angle s\text{)}$$
$$\text{So,} \quad \angle AEC = 180^\circ - (40^\circ + 55^\circ) = 85^\circ$$
$$\therefore \quad \angle HAB = \angle AEC = 85^\circ \text{ (corr. } \angle s\text{)}$$

Hence, $\boldsymbol{x = 85^\circ}$.

7. Ans. (a)

Solution Let the measure of the required angle be x°.

Then, measure of its complement $= (90 - x)^\circ$

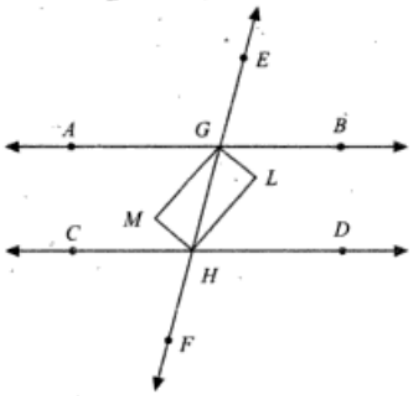
measure of its supplement $= (180 - x)^\circ$

$$6(90^\circ - x) = 2(180^\circ - x) - 12^\circ$$
$$\Rightarrow 540^\circ - 6x = 360^\circ - 2x - 12^\circ$$
$$\Rightarrow 4x = 192^\circ \Rightarrow \boldsymbol{x = 48^\circ}.$$

8. Ans. (b)

Solution $\angle AGH = \angle DHG$ (alt. int. angles)

$$\frac{1}{2}\angle AGH = \frac{1}{2}\angle DHG \Rightarrow \angle HGM = \angle GHL$$



Thus, lines GM and HL are intersected by a transversal GH at G and H respectively such that pair of alternate angles are equal, i.e. $\angle HGM = \angle GHL$

$$\therefore \quad GM \parallel HL$$

Similarly, $GL \parallel HM$

So, $GMHL$ is a $\parallel_{gm}$.

Since $AB \parallel CD$ and EF is a transversal

$$\therefore \quad \angle BGH + \angle DHG = 180^\circ \text{ [co-interior angles]}$$
$$\therefore \quad \frac{1}{2}\angle BGH + \frac{1}{2}\angle DHG = 90^\circ$$
$$\Rightarrow \quad \angle LGH + \angle LHG = 90^\circ$$

But $\angle LGH + \angle LHG + \angle GLH = 180^\circ$

$$\therefore \quad 90^\circ + \angle GLH = 180^\circ \Rightarrow \angle GLH = 90^\circ$$

Thus, in $\parallel_{gm}$ $GMHL$, we have $\angle GLH = 90^\circ$

Hence, $GMHL$ is a rectangle.

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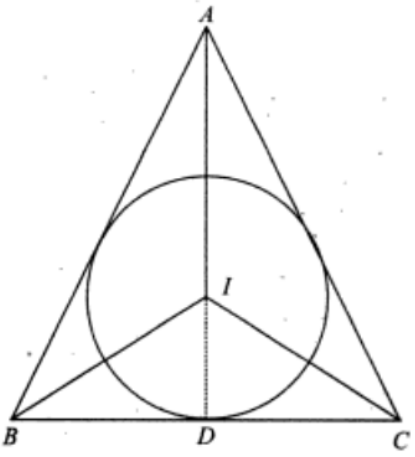
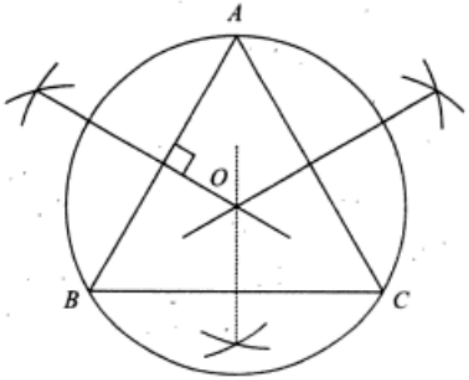


Note: The incentre of a triangle is equidistant from its sides.

6. **Circumcentre of *a* triangle** The point of intersection of the perpendicular bisectors of the sides of a triangle is called its *circumcentre*.
   In the figure given below, the right bisectors of the sides of ΔABC intersect at O.



   $\therefore$ O is the *circumcentre* of ΔABC with O as centre and radius equal to $OA = OB = OC$. We draw a circle passing through the vertices of the given Δ. This circle is called the *circumcircle* of ΔABC.

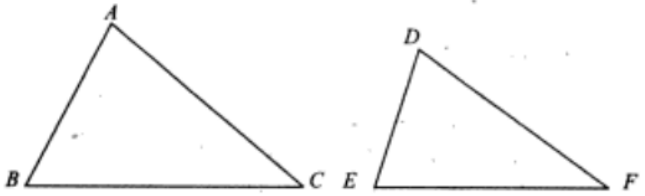
   Note: The circumcentre of a triangle is *equidistant* from its *vertices*.

CONGRUENT TRIANGLES

Two triangles are *congruent* if and only if one of them can be superposed on the other, so as to cover it exactly.

Thus, congruent triangles are exactly identical

For example, If $\Delta ABC \cong \Delta DEF$ then we have



$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$;

and $AB = DE$, $BC = EF$ and $AC = DF$.

Similar Triangles

Congruent figures Two geometric figures having the same shape and size are known as *congruent figures*

Similar figures Two figures (plane or solid) are said to be *similar* if they have the same shape irrespective of their sizes.

Note: Two similar figures may not be congruent as their size may be different.

For example,

1. any two lines segments are similar,
2. any two equilateral triangles are similar,
3. any two squares are similar,
4. any two circles are similar,
5. any two rectangles are similar.

Similar triangles Two triangles are similar if

(a) their corresponding angles are equal and

(b) their corresponding sides are proportional

Key Results to Remember

1. The sum of all the angles round a point is equal to 360°.
2. Two lines parallel to the same line are parallel to each other.
3. The sum of three angles of a triangle is 180°.
4. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles. (Exterior Angle Theorem)
5. If two sides of a triangle are unequal, the longer side has greater angle opposite to it.
6. In a triangle, the greater angle has the longer side opposite to it.
7. The sum of any two sides of a triangle is greater than the third side.
8. If a, b, c denote the sides of a triangle then
   (i) If $c^2 < a^2 + b^2$, triangle is acute angled.
   (ii) If $c^2 = a^2 + b^2$, triangle is right angled.
   (iii) If $c^2 > a^2 + b^2$, triangle is obtuse angled.
9. Two triangles are congruent if:
   (i) Any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle. (SAS congruence theorem)
   (ii) Two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle. (ASA congruence theorem)
   (iii) The three sides of one triangle are equal to the corresponding three sides of the other triangle. (SSS congruence theorem)

   Note: Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and the corresponding side of the other triangle. (RHS Congruence theorem)

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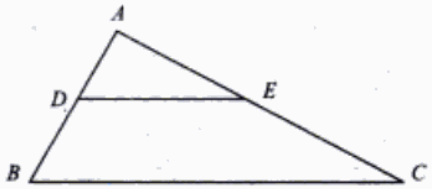
10. The line-segments joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it

11. **Basic Proportionality Theorem** If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In the figure given below, In a ΔABC



If $DE \parallel BC$

Then, $\frac{AD}{DB} = \frac{AE}{EC}$

Illustration 1 In the figure given above D and E are the points on the AB and AC respectively such that $DE \parallel BC$. If $AD = 8$ cm, $AB = 12$ cm and $AE = 12$ cm. Find CE

Solution In ΔABC, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ (Basic Proportionality Theorem)}$$

$$\Rightarrow \quad \frac{8}{12-8} = \frac{12}{EC} \Rightarrow \frac{8}{4} = \frac{12}{EC}$$

or $EC = 6$ cm

12. If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

Explanation In the figure given in 11, In ΔABC

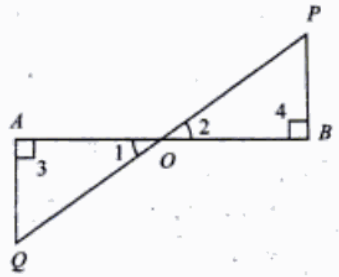
if $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$.

Similarity Theorems

13. **AAA Similarity** If in two triangles, corresponding angles are equal, then the triangles are similar.

Corollary (AA-similarity): If two angles of one triangle are respectively equal to two angles of another triangle then the two triangles are similar.

Illustration 2 In the figure given below, QA and PB are perpendiculars to AB. If $AO = 15$ cm, $BO = 9$ cm, $PB = 12$ cm, find AQ.



Solution In Δs AOQ and BOP

$\angle 1 = \angle 2$ [vertically opposite angles]

$\angle 3 = \angle 4$ [each 90°]

$\therefore$ $\Delta AOQ \sim \Delta BOP$ [AA Similarty Criterion]

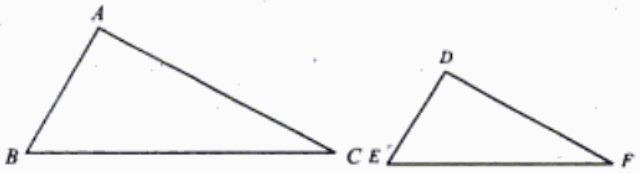
$\therefore$ $\frac{AO}{BO} = \frac{AQ}{BP}$ (corresponding sides of $\sim\Delta s$)

or $\frac{15}{9} = \frac{AQ}{12}$

or $\frac{5}{1} = \frac{AQ}{4} \Rightarrow AQ = 20$ cm.

14. **SSS-Similarity** If the corresponding sides of two triangles are proportional then they are similar.

Explanation In Δs ABC and DEF,

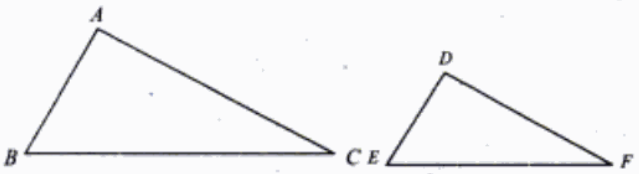


if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Then $\Delta ABC \sim \Delta DEF$ [SSS Similarity]

15. **SAS-Similarity** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Explanation In Δs ABC and DEF,



if $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

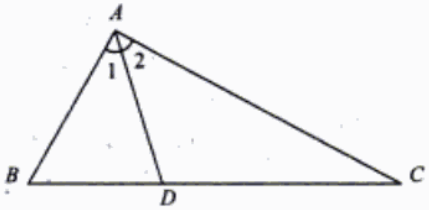
or $\angle B = \angle E$ and $\frac{AB}{DE} = \frac{BC}{EF}$

or $\angle C = \angle F$ and $\frac{AC}{DF} = \frac{BC}{EF}$,

then $\Delta ABC \sim \Delta DEF$ [SAS-Similarity]

16. **Internal Bisector Property** The internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle

Explanation In ΔABC, if $\angle 1 = \angle 2$



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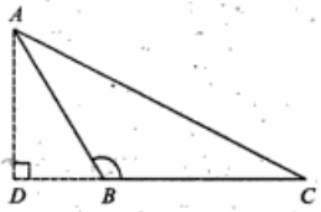
23. In two similar triangles, the ratio of two corresponding sides is same as the ratio of their perimeters.

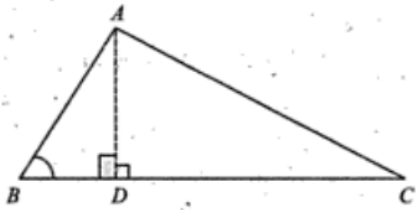
24. **Obtuse Angle Property** In a ΔABC, if $\angle B$ is obtuse then

$$AC^2 = AB^2 + BC^2 + 2\,BC \cdot BD$$

where $AD \perp BC$

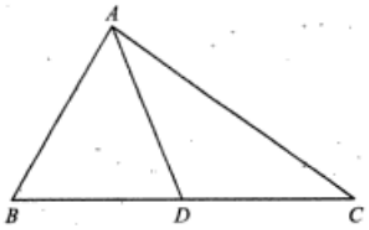


25. **Acute Angle Property** In a ΔABC, if $\angle C$ is acute, then $AB^2 = AC^2 + BC^2 - 2BC \times CD$ where $AD \perp BC$



26. **Apollonius theorem** The sum of the squares on any two sides of a triangle is equal to the sum of twice the square of the median, which bisects the third side and half the square of the third side.

Explanation In the given ΔABC,



$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

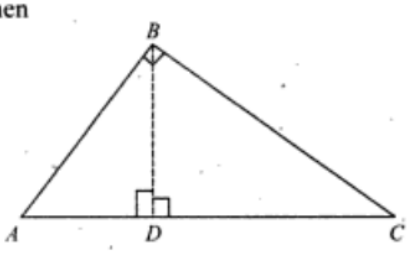
or

$$AB^2 + AC^2 = 2[AD^2 + BD^2]$$

27. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other. Also the square of the perpendicular is equal to the product of the lengths of the two parts of the hypotenuse.

Explanation In the figure given below,

ABC is a right triangle, right angled at B and $BD \perp AC$, then



(i) $\Delta ADB \sim \Delta ABC$ (AA Similarity)

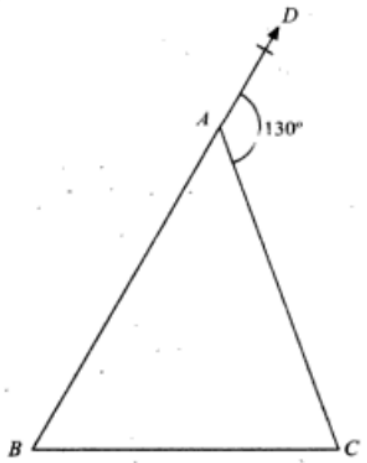
(ii) $\Delta BDC \sim \Delta ABC$ (AA Similarity)

(iii) $\Delta ADB \sim \Delta BDC$ also $BD^2 = AD \cdot CD$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

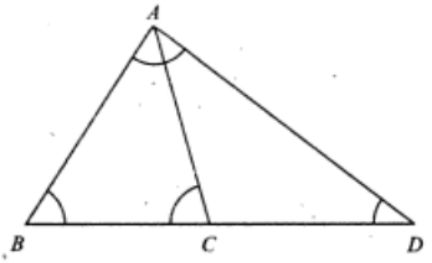
1. In the following figure, $\angle B : \angle C = 2 : 3$, find $\angle B + \angle C$



(a) 120° (b) 52°

(c) 78° (d) 130°

2. In the given figure, $\angle B = \angle C = 55°$ and $\angle D = 25°$. Then



(a) $BC < CA < CD$ (b) $BC > CA > CA$

(c) $BC < CA$, $CA > CD$ (d) $BC > CA$, $CA < CD$

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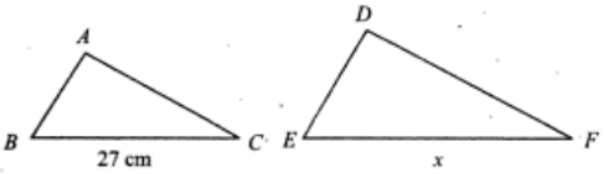
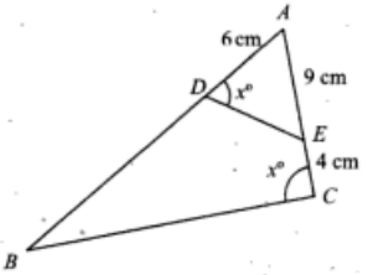
13. The areas of two similar Δs are 81 cm^2 and 144 cm^2. If the largest side of the smaller Δ is 27 cm, then the largest side of the larger Δ is:



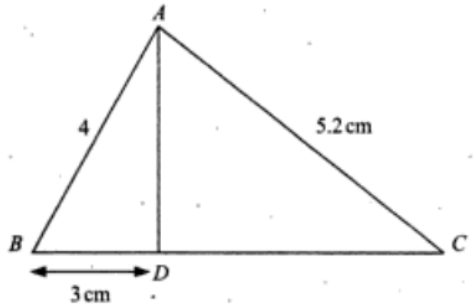
(a) 24 cm (b) 48 cm
(c) 36 cm (d) None of these

14. In the given figure, find the length of BD



(a) 13.5 cm (b) 12 cm
(c) 14.5 cm (d) 15 cm

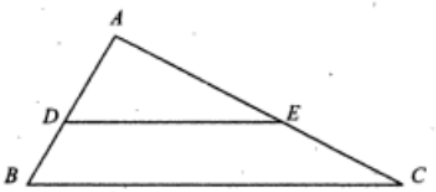
15. In the given figure $\angle BAD = \angle CAD$, $AB = 4$ cm, $AC = 5.2$ cm, $BD = 3$ cm. Find BC



(a) 6.9 cm (b) 9.6 cm
(c) 3.9 cm (d) 9.3 cm

16. A ladder 15 m long reaches a window which is 9 m above the ground on one side of street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. What is the width of the street
(a) 31 m (b) 12 m
(c) 30 m (d) 21 m

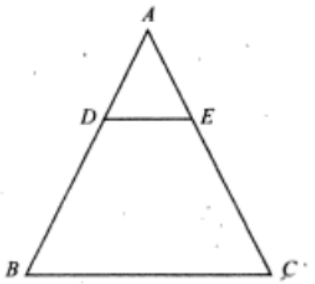
17. In, ΔABC, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC



(a) 1/2 (b) 1/4
(c) 3/4 (d) 1/8

18. A vertical stick 12 cm long casts a shadow 8 cm long on the ground. At the same time a tower casts the shadow 40 m long on the ground. Find the height of the tower.
(a) 600 m (b) 160 m
(c) 60 m (d) 52 m

19. D and E are the points on the sides AB and AC respectively of ΔABC such that $AD = 8$ cm, $BD = 12$ cm, $AE = 6$ cm and $EC = 9$ cm Then find BC/DE



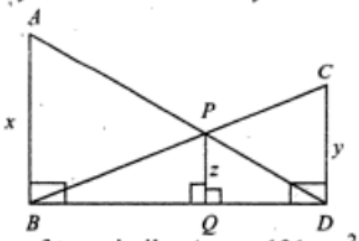
(a) $\frac{5}{2}$ (b) $\frac{2}{5}$
(c) $\frac{5}{7}$ (d) $\frac{5}{3}$

20. In an equilateral ΔABC, if $AD \perp BC$, then
(a) $3AB^2 = 2AD^2$ (b) $2AB^2 = 3AD^2$
(c) $3AB^2 = 4AD^2$ (d) $4AB^2 = 3AD^2$

21. In a right angled ΔABC, rt. angled at A, $AD \perp BC$. Then
(a) $AD^2 = BD \times CD$ (b) $AD^2 = AB \times AC$
(c) $AD^2 = BD \times AB$ (d) $AD^2 = CD \times AC$

22. If $ABCD$ is a ||gm and AC and BD be its diagonals, Then
(a) $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 - BD^2$
(b) $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$
(c) $4AD^2 = 2AC^2 + 2BD^2$
(d) $4AB^2 = 2AC^2 - 2BC^2$

23. In the given figure, $\angle ABD = \angle CDB = \angle PQB = 90°$. Then
(a) $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ (b) $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$
(c) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (d) $\frac{1}{y} - \frac{1}{x} = \frac{1}{z}$



24. The area of two similar Δs are 121 cm^2 and 81 cm^2 respectively. What is the ratio of their corresponding heights (altitudes)
(a) $\frac{11}{9}$ (b) $\frac{22}{9}$

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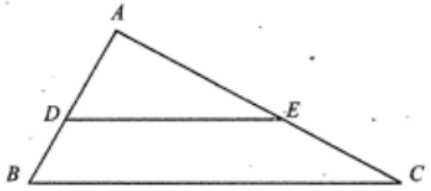
(c) $\frac{11}{18}$ (d) None of these

25. In the given figure, $DE \parallel BC$ and $DE : BC = 3 : 5$ the ratio of the areas of ΔADE and the trapezium $BCED$



(a) $\frac{9}{25}$ (b) $\frac{12}{25}$

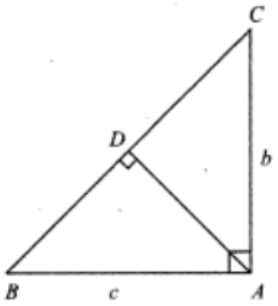
(c) $\frac{3}{4}$ (d) $\frac{9}{16}$

26. In a ΔABC, AD intersects $\angle A$ and intersects BC in If $BC = a$, $AC = b$ and $AB = c$, Then

(a) $CD = \frac{b+c}{ab}$ (b) $CD = \frac{ab}{b+c}$

(c) $CD = \frac{bc+ab}{ac}$ (d) $CD = \frac{ac}{bc+ab}$

27. In the given figure, what is the length of AD in terms of b and c



(a) $\frac{bc}{b^2+c^2}$ (b) $\frac{b^2+c^2}{bc}$

(c) $\frac{\sqrt{b^2+c^2}}{bc}$ (d) $\frac{bc}{\sqrt{b^2+c^2}}$

28. ABC is a Δ in which $AB = AC$ and D is a point on AC such that $BC^2 = AC \times CD$. Then

(a) $BD = DC$ (b) $BD = BC$

(c) $BD = AB$ (d) $BD = AD$

29. Two poles of ht. a and b metres are p metres apart $(b > a)$. The height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is

(a) $\frac{a+b}{ab}$ (b) $\frac{p}{a+b}$

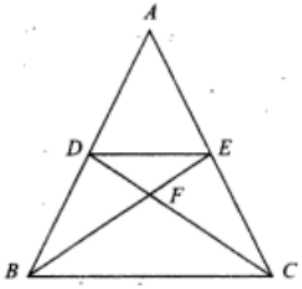
(c) $\frac{ab}{a+b}$ (d) $\frac{a+b}{p}$

30. ABC is a right Δ, right-angled at C. If $AB = c$, $BC = a$ and $CA = b$ and p is the length of the perpendicular from C on AB. Then

(a) $\frac{1}{p^2}+\frac{1}{a^2} = \frac{1}{b^2}$ (b) $\frac{1}{p^2}+\frac{1}{b^2} = \frac{1}{a^2}$

(c) $\frac{1}{a^2} = \frac{1}{b^2}+\frac{1}{p^2}$ (d) $\frac{1}{p^2} = \frac{1}{a^2}+\frac{1}{b^2}$

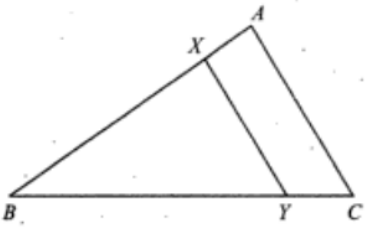
31. In the given figure $DE \parallel BC$ and $AD : DB = 5 : 4$ Then $\frac{ar(\Delta DFE)}{ar(\Delta CFB)}$



(a) $\frac{25}{81}$ (b) $\frac{25}{16}$

(c) $\frac{16}{25}$ (d) $\frac{16}{81}$

32. In the fig, $XY \parallel AC$ and XY divides triangular region ABC into two part equal in area. Then $\frac{AX}{AB}$ is equal to



(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{2}+2}{\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{2}-1}{\sqrt{2}}$

33. A point O in the interior of a rectangle $ABCD$ is joined with each of the vertices A, B, C and D. Then

(a) $OA^2 + OC^2 = OB^2 - OD^2$

(b) $OA^2 + OC^2 = OB^2 + OD^2$

(c) $OA^2 = OB^2 = OC^2 + OD^2$

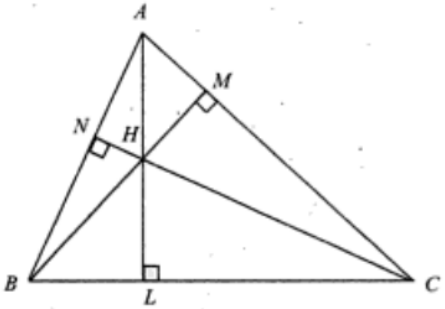
(d) $OA^2 + OD^2 = OB^2 + OC^2$

34. In ΔABC, the median BE intersects AC at E, if $BG = 6$ cm, where G is the centroid, then BE is equal to

(a) 7 cm (b) 9 cm

(c) 8 cm (d) 10 cm

35. If H is the orthocentre of ΔABC, then the orthocentre of ΔHBC is (fig. given)



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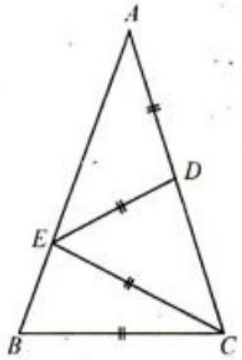
45. In fig, $AB = AC$, D is a point on AC and E on AB such that $AD = ED = EC = BC$. Then $\angle A : \angle B$



(a) 1 : 2 (b) 2 : 1
(c) 3 : 1 (d) 1 : 3

EXPLANATORY ANSWERS

1. Ans. (b)

Solution $\angle DAC = \angle B + \angle C$
(Exterior angle prop. of a Δ)

$130^\circ = 2x + 3x$

$5x = 130^\circ$

$x = 26^\circ$

$\therefore \quad \angle B = 52^\circ$; $\angle C = \mathbf{78^\circ}$.

2. Ans. (d)

Solution $\angle B = \angle C \Rightarrow AB = BC$

$\angle CAD = 30^\circ$

$\therefore \quad \angle CAD > \angle CDA \Rightarrow CD > AC$

(In a Δ, greater angle has longer side opposite to it)

$\angle BAC = 180^\circ - 110^\circ = 70^\circ > \angle ABC$

$\Rightarrow \quad BC > AB$ and $BC > AC$

$\therefore \quad BC > CA$ and $\boldsymbol{CA < CD}$.

3. Ans. (c)

Solution Let $2\angle A + 3\angle B = 6\angle C = K$

$$\therefore \quad \angle A = \frac{K}{2}, \angle B = \frac{K}{3}, \angle C = \frac{K}{6}$$

But $\angle A + \angle B + \angle C = 180^\circ$

$$\therefore \quad \frac{K}{2} + \frac{K}{3} + \frac{K}{6} = 180^\circ$$

$K = 180^\circ$

Hence, $\angle A = \dfrac{180^\circ}{2} = \mathbf{90^\circ}$.

4. Ans. (b)

Solution Since A, B and C are the angles of a Δ,

$\therefore \quad A + B + C = 180^\circ$

Now $A - B = 15^\circ$; $B - C = 30^\circ$; $\therefore B = C + 30^\circ$

$\therefore \quad \angle A = B + 15^\circ = C + 30^\circ + 15^\circ = C + 45^\circ$

$\therefore \quad A + B + C = (C + 45^\circ) + (C + 30^\circ) + C$

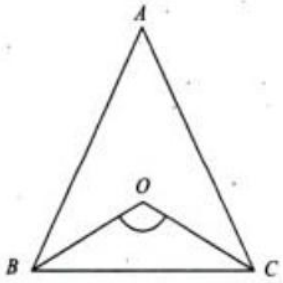
$3C = 180^\circ - 75^\circ = 105^\circ$

$C = 35^\circ$

$\therefore \quad \angle A = 35^\circ + 45^\circ = \mathbf{80^\circ}$.

5. Ans. (b)

Solution $\angle BOC = 90^\circ + \dfrac{1}{2}\angle A$

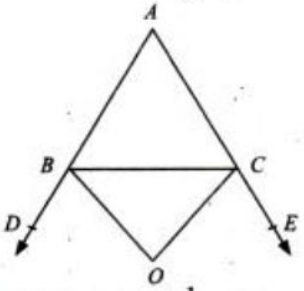


$$\therefore \quad \angle BOC = 90^\circ + \frac{1}{2}(70^\circ) = 90^\circ + 35^\circ$$

$\angle BOC = \mathbf{125^\circ}$.

6. Ans. (d)

Solution $\angle BOC = 90^\circ - \dfrac{1}{2}\angle A$



$\therefore \quad \angle BOC = 90^\circ - \dfrac{1}{2}(40^\circ)$

$= 90^\circ - 20^\circ$

$\angle BOC = \mathbf{70^\circ}$.

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7. The perimeters of two similar Δs; ΔABC and ΔDEF are respectively 48 cm and 32 cm. If $DE = 10$ cm, Then AB

(a) 12 cm (b) 14 cm

(c) 16 cm, (d) 15 cm

8. In ΔABC, BF is the bisector of $\angle B$ which meets CA at F. If $AB = 24$ cm, $AF = 12$ cm and $AC = 30$. What is the perimeter of ΔABC

(a) 80 cm (b) 90 cm

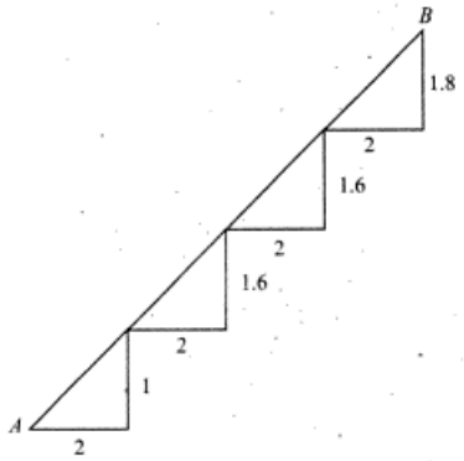
(c) 100 cm (d) 70 cm

9. In rt ΔACB, $\angle ACB = 90°$ and $CD \perp AB$. Then

(a) $\frac{CB^2}{CA^2} = \frac{BD}{AD}$ (b) $\frac{CB^2}{CA^2} = \frac{AD}{BD}$

(c) $\frac{CB^2}{CA^2} = \frac{CD}{AD}$ (d) $\frac{CB^2}{CA^2} = \frac{CD}{BD}$

10. There is a staircase as shown in the fig. Connecting points A and B. Measurements of steps are marked in the Fig. What is the straight line distance between A and B



(a) 12 cm (b) 9 cm

(c) 10 cm (d) 14 cm

EXPLANATORY ANSWERS

1. Ans. (d)

Solution Since $BC^2 = AB^2 + AC^2$ $(144 = 36 + 108)$

$\therefore$ by converse of pythagoras theorem, angle opposite to BC i.e. $\angle A = \mathbf{90°}$.

2. Ans. (c)

Solution Area theorem

3. Ans. (a)

Solution $\Delta KNP \sim \Delta KML$ ($\because NP \parallel ML$) AA Similarity Criterion

$$\therefore \quad \frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow \boldsymbol{x = \frac{ac}{b+c}}.$$

4. Ans. (b)

Solution Since AD bisects $\angle BAC$

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{CD} \quad \text{(Internal bisector Prop.)}$$

$$\frac{8}{AC} = \frac{5}{4}$$

$$AC = \frac{32}{5} = \mathbf{6.4\ cm.}$$

5. Ans. (c)

Solution Since $AB \parallel CD$

$\therefore$ $\Delta AOB \sim \Delta COD$ (AA Similarity Criterion)

Now $\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2} = \frac{4CD^2}{CD^2} = \frac{4}{1}$

6. Ans. (a)

Solution $\Delta DBA \sim \Delta DCB$

$$\therefore \quad \frac{BD}{CD} = \frac{AD}{BD}$$

$$\Rightarrow \quad CD = \frac{BD^2}{AD} = \frac{8^2}{4} = \mathbf{16\ cm.}$$

7. Ans. (d)

Solution When the Δs are similar, then the ratio of the perimeters is same as the ratio of their corresponding sides.

$$\Delta ABC \sim \Delta DEF$$

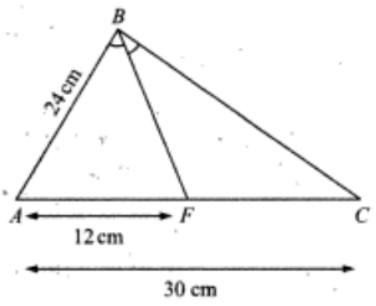
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AB+BC+AC}{DE+EF+DF} = \frac{48}{32} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{AB}{10} = \frac{3}{2}$$

$$\Rightarrow \quad \boldsymbol{AB = 15\ cm.}$$

8. Ans. (b)

Solution Since BF bisects $\angle B$



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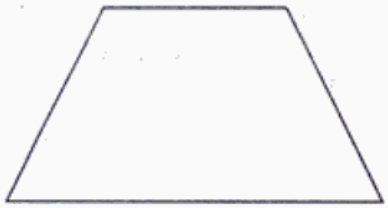
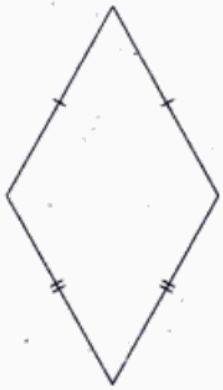
(v) **Trapezium** A quadrilateral in which two opposite sides are parallel and two opposite sides are non-parallel, is called a *trapezium*



Trapezium

(iv) **Kite** A quadrilateral in which pairs of adjacent sides are equal is known as *kite*.



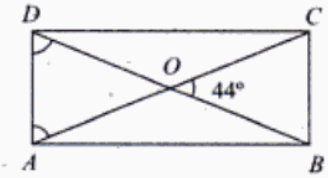
Key Results to Remember

1. The sum of all the four angles of a quadrilateral is 360°.
2. In a parallelogram
   (i) opposite sides are equal.
   (ii) opposite angles are equal.
   (iii) each diagonal bisects the parallelogram
   (iv) the diagonal bisect each other.
3. A quadrilateral is a $\|_{gm}$
   (i) if both pairs of opposite sides are equal
   or (ii) if both pairs of opposite angles are equal
   or (iii) if the diagonals bisect each other.
   or (iv) if a pair of opposite sides are equal and parallel
4. The diagonals of a rectangle are equal.
5. If the diagonals of a $\|_{gm}$ are equal, it is a rectangle.
6. Diagonals of a rhombus are perpendicular to each other.
7. Diagonals of a square are equal and perpendicular to each other.
8. The figure formed by joining the mid-points of the pairs of consecutive sides of a quadrilateral is a $\|_{gm}$.
9. The quadrilateral formed by joining the mid-points of the consecutive sides of a rectangle is a rhombus
10. The quadrilateral formed by joining the mid-points of the consecutive sides of a rhombus is a rectangle
11. If the diagonals of a quadrilateral are perpendicular to each other, then the quadrilateral formed by joining the mid-points of its sides, is a rectangle.
12. The quadrilateral formed by joining the mid-points of the sides of a square, is also a square.

MULTIPLE CHOICE QUESTIONS

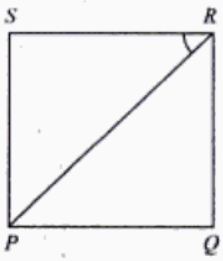
In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The diagonals of a rectangle $ABCD$ meet at 0. If $\angle BOC = 44°$, then $\angle OAD$ is equal to

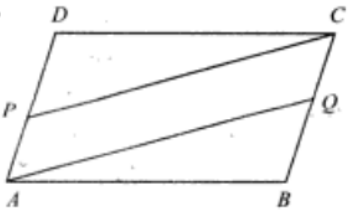


(a) 90° (b) 60°
(c) 100° (d) 68°

2. $PQRS$ is a square. The $\angle SRP$ is equal to

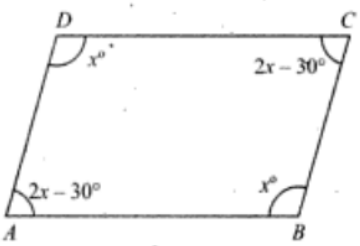


(a) 45° (b) 90°
(c) 100° (d) 60°



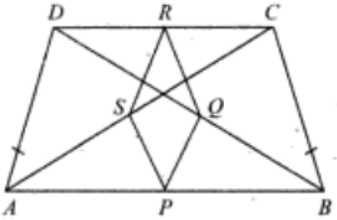
(a) parallelogram (b) Rhombus
(c) rectangle (d) square

11. Find the measure of each angle of a parallelogram, if one of its angles is 30° less than twice the smallest angle.



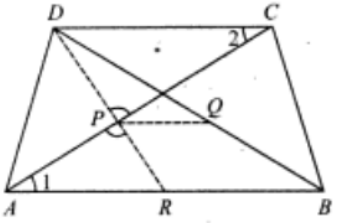
(a) 60°, 100°, 90°; 20° (b) 80°, 40°, 120°, 90°
(c) 100°, 90°, 90°, 80° (d) 70°, 110°, 70°, 110°.

12. *ABCD* is trapezium in which $AB \parallel DC$ and $AD = BC$. If *P*, *Q*, *R*, *S* be respectively the mid-point of *BA*, *BD* and *CD*, *CA*. The *PQRS* is a



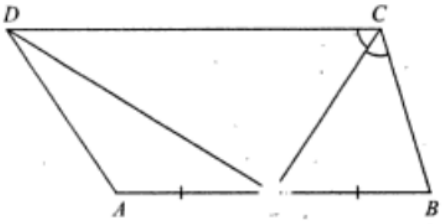
(a) Rhombus (b) rectangle
(c) parallelogram (d) square

13. *ABCD* is a trapezium and *P*, *Q* are the mid-points of the diagonals *AC* and *BD*. Then *PQ* is equal to



(a) $\frac{1}{2}(AB)$ (b) $\frac{1}{2}(CD)$
(c) $\frac{1}{2}(AB - CD)$ (d) $\frac{1}{2}(AB + CD)$.

14. *ABCD* is a parallelogram, *E* is the mid-point of *AB* and *CE* bisects $\angle BCD$. The $\angle DEC$ is



(a) 60° (b) 90°
(c) 100° (d) 120°

EXPLANATORY ANSWERS

1. And. (d)

Solution In the rectangle *ABCD*.

$AB = CD$ and $AD = BC$.

Since the diagonals of a rectangle bisect each other,

$\therefore \quad OA = OD$ i.e. $\angle ODA = \angle OAD$.

But, $\angle AOD = 44°$ (Vertically opposite angle to $\angle BOC$)

$\therefore \quad \angle OAD = \frac{1}{2}(180° - 44°) = \frac{1}{2}(136°) = 68°$

Hence, $\angle OAD = \mathbf{68°}$.

2. And. (a)

Solution *PQRS* is a square

$SP = SR$ and $\angle S = 90°$

and $\angle SRP = \angle SPR = \frac{1}{2}(90°) = 45°$

Hence, $\angle SRP = \mathbf{45°}$.

3. Ans. (d)

Solution Since $AB = BC$

$\therefore \quad \angle BAC = \angle BCA = \frac{1}{2}(180° - 56°) = 62°$

Also as $AB \parallel CD$ and *AC* transversal

So $\angle BAC = 62° = \angle ACD$

(Alternate interior angles)

$\therefore \quad \angle ACD = \mathbf{62°}$.

4. Ans. (a)

Solution Since *X* and *Y* are the mid-points of *AB* and *DC* respectively.

$\therefore \quad AX = \frac{1}{2}AB$ and $CY = \frac{1}{2}DC$

But $AB = DC \Rightarrow \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AX = CY$

Also, $AB \parallel DC$ [$\because$ *ABCD* is a parallelgram]

$\Rightarrow \quad AX \parallel YC$

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Thus, in quadrilateral $AXCY$,

$AX \parallel YC$ and $AX = YC$

Hence, quad. $AXCY$ is a parallelogram.

5. Ans. (c)

Solution Proceeding as in Q. No. 4, we can prove that $AXCY$ is a parallelogram.

Similarly, $BXDY$ is a parallelogram.

Now, $AXCY$ is a parallelogram

$\Rightarrow AY \parallel CX$

[$\because$ Opposite sides of a parallelogram are parallel].

$\Rightarrow PY \parallel QX$ ...(i)

Also, $BXDY$ is a parallelogram

$\Rightarrow DX \parallel BY$

[$\because$ Opposite sides of a parallelogram are parallel]

$\Rightarrow PX \parallel QY$ ...(ii)

Thus, in a quadrilateral $PXQY$, we have

$PY \parallel QX$ and $PX \parallel QY$ [From (i) and (ii)]

$\Rightarrow$ $PXQY$ is a parallelogram.

6. Ans. (c)

Solution In ΔARB, P is the mid-point of AB and $PD \parallel BR$.

$\Rightarrow$ D is the mid-point of AR.

$\because$ $ABCD$ is a parallelogram

$\Rightarrow DC \parallel AB \Rightarrow DQ \parallel AB$

Thus, in ΔRAB, D is the mid-point of AR and $DQ \parallel AB$.

$\therefore$ Q is the mid-point of $RB \Rightarrow \boldsymbol{BR = 2BQ}$.

7. Ans. (d)

Solution $ABCD$ is a trapezium in which $AB \parallel DC$ and M, N are the mid-points of AD and BC.

Hence, $MN \parallel AB$ and $MN \parallel DC$.

In ΔACB,

ON passes through the mid point N of BC and $ON \parallel AB$

$$\therefore \quad ON = \frac{1}{2}AB = \frac{1}{2}(12\text{cm}) = 6 \text{ cm}$$

But $MO = MN - ON = (14 - 6)\text{cm} = 8$ cm

Again MO passes through the mid point M of AD and $MO \parallel DC$

$$\therefore \quad MO = \frac{1}{2}DC = \frac{1}{2}CD$$

Hence, $CD = 2(MO) = 2(8) = \mathbf{16\ cm.}$

8. Ans. (a)

Solution Consider Δs PSX and QRY, in which $\angle X = \angle Y = 90^\circ$ [$\because$ $PX \perp SR$ and $QY \perp SR$.]

and $SX = RY$

[$\because SX = SY - XY$ and $RY = SY - SR = SY - PQ = SY - XY$]

and $PS = QR$ [Sides of a parallelogram]

$\therefore$ $\Delta PSX \cong \Delta QRY$ [R.H.S. axiom]

$\therefore$ $PX = QY$

[Corresponding parts of congruent Δs are congruent]

9. Ans. (c)

Solution ABC is a Δ and P, Q, R are the mid-points of sides BC, CA and AB resp.

$\therefore$ $PQ \parallel AB$

and $$PQ = \frac{1}{2}AB = \frac{1}{2}(30) = 15 \text{ cm.}$$

Similarly, $RP \parallel AC$

and $$RP = \frac{1}{2}AC = \frac{1}{2}(21) = 10.5 \text{ cm.}$$

$\therefore$ Perimeter of $ARPQ = (AR + RP + PQ + QA)$ cm.

$= (15.0 + 10.5 + 15.0 + 10.5)$ cm.

$= \mathbf{51\ cm.}$

10. Ans. (a)

Solution $ABCD$ is a parallelogram.

$\Rightarrow AD = BC$ and $AD \parallel BC$.

$\Rightarrow \frac{1}{3}AD = \frac{1}{3}BC$ and $AD \parallel BC$.

$\Rightarrow AP = CQ$ and $AP \parallel CQ$.

Thus, $APCQ$ is a quad. Such that one pair of opposite side AP and CQ are parallel and equal.

Hence, $APCQ$ is a parallelogram.

11. Ans. (d)

Solution In a parallelogram $ABCD$,

$\angle A + \angle D = 180^\circ$

Let $\angle D = x^\circ$, $\angle A = 2x - 30^\circ$

$\therefore$ $(2x^\circ - 30^\circ) + x^\circ = 180^\circ$

$\Rightarrow$ $3x^\circ = 180^\circ + 30^\circ$

$\Rightarrow$ $3x^\circ = 210^\circ$ or $x = \frac{210^\circ}{3}$

$\therefore$ $x^\circ = 70^\circ$

$\therefore$ $\angle D = 70^\circ = \angle B$

and $\angle A = 2x - 30^\circ = 110^\circ = \boldsymbol{\angle C}$.

12. Ans. (a)

Solution In ΔBDC, Q and R are the mid-points of BD and CD respectively.

$\therefore$ $QR \parallel BC$ and $QR = \frac{1}{2}BC$.

Similarly, $PS \parallel BC$ and $PS = \frac{1}{2}BC$.

$\therefore$ $PS \parallel QR$ and $PS = QR$ $\left[\text{each equal to } \frac{1}{2}BC\right]$.

Similarly $PQ \parallel SR$ and $PQ = SR$ [each equal to 1/2 AD]

$\therefore$ $PS = QR = SR = PQ$ [$\because$ $AD = BC$]

Hence, $PQRS$ is a rhombus.

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13. Ans. (c)

Solution Since $AB \parallel DC$ and transversal AC cuts them at A and C cresp.

$\therefore \quad \angle 1 = \angle 2.$ ...(i)

[$\because$ Alternate angles are equal.]

Now, in ΔAPR and ΔDPC, $\angle 1 = \angle 2$

$AP = CP$ [$\because$ P is the mid-point of AC]

and $\angle 3 = \angle 4$. [Vertically opposite angles].

So, $\Delta APR \cong \Delta DPC$ [ASA].

$\Rightarrow \quad AR = DC$ and $PR = DP$ ...(ii)

Again, P and Q are the mid-points of sides DR and DB respectively in ΔDRB.

$\therefore \quad PQ = \frac{1}{2}RB = \frac{1}{2}(AB - AR)$. [$\because$ $AR = DC$].

$\therefore \quad \mathbf{PQ = \frac{1}{2}(AB - DC).}$

14. Ans. (b)

Solution $AB \parallel DC$ and EC cuts them

$\Rightarrow \quad \angle BEC = \angle ECD.$

$\Rightarrow \quad \angle BEC = \angle ECB$ [$\because$ $\angle ECD = \angle ECB$].

$\Rightarrow \quad EB = BC \Rightarrow AE = AD.$

Now, $AE = AD \Rightarrow \angle ADE = \angle AED$

$\Rightarrow \angle ADE = \angle EDC$

[$\because$ Alternate Int. angles].

$\therefore \quad DE$ bisects $\angle ADC$.

Again, $\angle ADC + \angle BCD = 180°$ [Co. Int. angles].

$\Rightarrow \quad \frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = 90°$

$\Rightarrow \quad \angle EDC + \angle DCE = 90°$

But, $\angle EDC + \angle DEC + \angle DCE = 180°$

[$\because$ sum of the $\angle$s of a Δ is 180°]

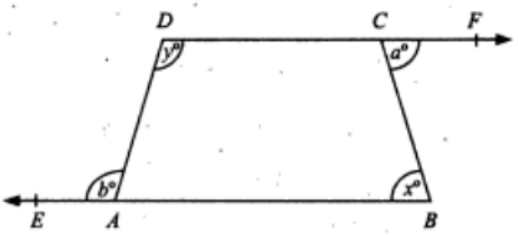
$\therefore \quad \angle DEC = 180 - 90°$

$\therefore \quad \mathbf{\angle DEC = 90°.}$

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

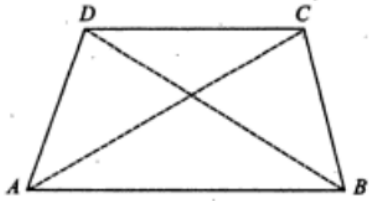
1. The sides BA and DC of quadrilateral $ABCD$ are produced as shown in the figure given below. Then $x + y$ is equal to



(a) $a + b$ (b) $a - b$

(c) $\frac{a}{b}$ (d) $\frac{b}{a}$

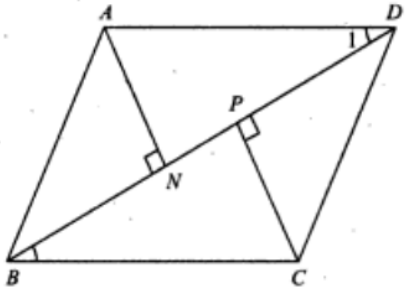
2. In the figure given below, $ABCD$ is a quadrilateral in which AB is the longest side and CD is the shortest side. Then



(a) $\angle C < \angle A$ (b) $\angle C > \angle A$

(c) $\angle C = \angle A$ (d) None of these

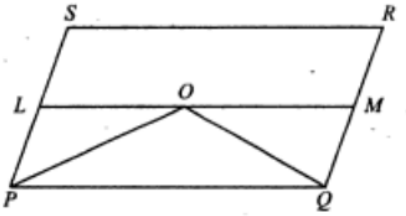
3. In the figure given below AN and CP are perpendiculars to the diagonal BD of a parallelogram. Then,



(a) $AN = CP$ (b) $AN > CP$

(c) $AN < CP$ (d) None of these

4. In the given figure, $PQRS$ is a parallelogram. PO and QO are respectively, the bisectors of $\angle P$ and $\angle Q$. Line $\angle LOM$ is drawn parallel to PQ. Then PL is equal to



(a) PQ (b) QR

(c) QM (d) OQ

5. The diagonals of a parallelogram $ABCD$ intersect at O. A line through O intersect AB at X and DC at Y. Then

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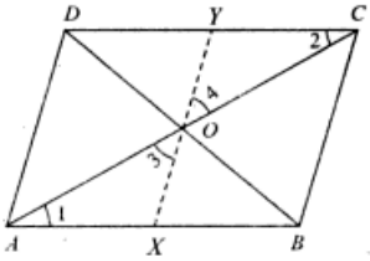
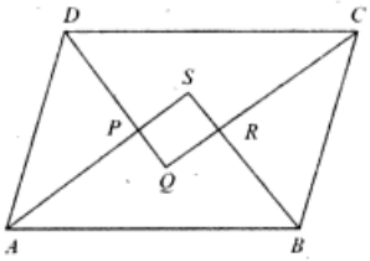


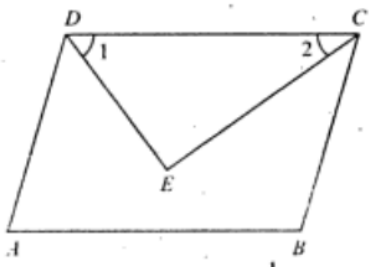
(a) $QX = OY$ (b) $OX > OY$
(c) $OX < OY$ (d) None of these

6. The bisectors of the angles of a parallelogram enclose a



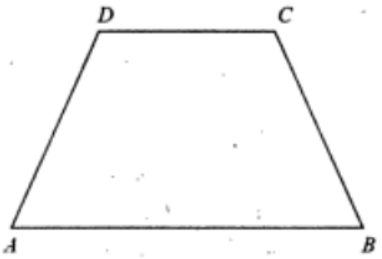
(a) Rhombus (b) Rectangle.
(c) Square (d) None of these

7. In a quad. $ABCD$, the line segments bisecting $\angle C$ and $\angle D$ meet at E. Then $\angle A + \angle B$ is equal to



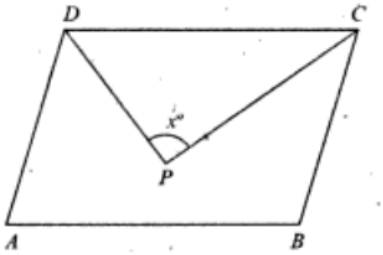
(a) $\angle CED$ (b) $\frac{1}{2}\angle CED$
(c) $2\angle CED$ (d) None of these

8. The measures of the angles of a quadrilateral $ABCD$ are respectively in the ratio 1 : 2 : 3 : 4. Then which of the following is true.



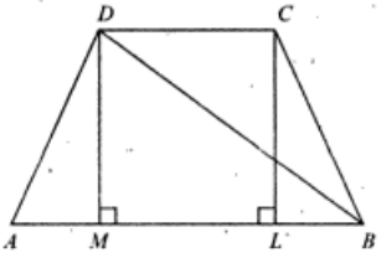
(a) $AC = BD$ (b) $AD \parallel BC$
(c) $AB \parallel DC$ (d) None of these

9. The measure of an angle between the internal bisectors of any two adjacent angles of a parallelogram is



(a) 60° (b) 80°
(c) 100° (d) 90°

10. $ABCD$ is a trapezium in which $AB \parallel CD$. Then $AC^2 + BD^2$ is equal to



(a) $AD^2 + BC^2 - 2AB \cdot CD$.
(b) $AD^2 + BC^2 + 2AB \cdot CD$.
(c) $AD^2 - BC^2 + 2AB \cdot CD$.
(d) $AD^2 - BC^2 - 2AB \cdot CD$.

EXPLANATORY ANSWERS

1. Ans. (a)

Solution $\angle A + b^\circ = 180^\circ$ [Linear pair]
$\Rightarrow \quad \angle A = 180^\circ - b^\circ$.
Also, $\angle C + a^\circ = 180^\circ$ [Linear pair]
$\Rightarrow \quad \angle C = 180^\circ - a^\circ$
Now, $\angle A + \angle B + \angle C + \angle D = 360^\circ$.
[$\because$ Sum of the $\angle s$ of a quad. is 360°].
$\Rightarrow \quad (180^\circ - b^\circ) + x^\circ + (180^\circ - a^\circ) + y = 360^\circ$
$\Rightarrow \quad \boldsymbol{x + y = a + b.}$

2. Ans. (b)

Solution Join AC and BD.
In ΔABC, $AB > BC \Rightarrow \angle ACB > \angle BAC$.
In ΔADC, $AD > DC \Rightarrow \angle ACD > \angle CAD$.
$\therefore \quad \angle ACB + \angle ACD > \angle BAC + \angle CAD$.
$\therefore \quad \boldsymbol{\angle C > \angle A.}$

3. Ans. (a)

Solution Since $ABCD$ is a parallelogram.
$\therefore \quad AD \parallel BC$

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EXPLANATORY ANSWERS

1. Ans. (c)

Solution $100° + 98° + 92° + x° = 360$ (sum of the angles of a quad.)

$\Rightarrow \quad 290° + x = 360 \quad$ or $\quad x = 360° - 290° = \mathbf{70°}.$

2. Ans. (d)

Solution Let the measure of each angle be $x°$. Then, sum of all the angles $= 6x°$

We have,

Sum of all interior angles of a polygon $= (2n - 4)$ rt. angle

$\therefore$ Sum of all interior angles of a hexagon

$= (2 \times 6 - 4)$ right angles

$= 8$ rt. angles $= 720°$

$\therefore \quad 6x° = 720° \quad$ or $\quad x = \mathbf{120°}.$

3. Ans. (d)

Solution Let there be n sides of the polygon. Then, each interior angles is of measure $\left(\frac{2n-4}{n} \times 90\right)^\circ$

$\therefore \frac{2n-4}{n} \times 90 = 108 \Rightarrow 180n - 360 = 108n$

$\Rightarrow 72n = 360° \Rightarrow n = 5$

So, the polygon has **5 sides.**

4. Ans. (a)

Solution We know the no. of diagonals of a polygon of n sides is $\frac{n(n-1)}{2} - n$

$\therefore$ for a hexagon, $n = 6$

$\frac{6(6-1)}{2} - 6 = \frac{6 \times 5}{2} - 6 = 15 - 6 = \mathbf{9}.$

5. Ans. (b)

Solution Since the no. of diagonals of a polygon of n sides is $\frac{n(n-1)}{2} - n$

$\therefore \quad \frac{n(n-1)}{2} - n = 27 \Rightarrow n^2 - n - 2n = 54$

$\Rightarrow n^2 - 3n - 54 = 0$

$\Rightarrow (n - 9)(n + 6) = 0$

$\Rightarrow n = 9$ or $n = -6$

$\therefore \quad \mathbf{n = 9}.$

6. Ans. (c)

Solution One angle of the pentagon is $140°$

Since the remaining angles are in the ratio $1 : 2 : 3 : 4$, $\therefore$, let the remaining angles be $x°$, $(2x)°$, $(3x)°$ and $(4x)°$

But the sum of interior angles of a pentagon $(2.5 - 4) \times 90 = 6 \times 90° = 540°$

$\therefore \quad 140 + x + 2x + 2x + 3x + 4x = 540$

$\Rightarrow \quad 10x = 400 \Rightarrow x = 40$

$\therefore$ The angles of the pentagon are $140°$, $40°$, $80°$, $120°$ and $160°$

Hence the size of the greatest angle = **160°.**

7. Ans. (b)

Solution Let there be n sides of the polygon. Then, each exterior angle $= \left(\frac{360}{n}\right)^\circ$ and each interior angle $= \left(\frac{2n-4}{n} \times 90\right)^\circ$

We have

Exterior angle $= \frac{1}{3}$ (interior angle)

$\Rightarrow \quad \frac{360}{n} = \frac{1}{3}\left(\frac{2n-4}{n} \times 90\right)$

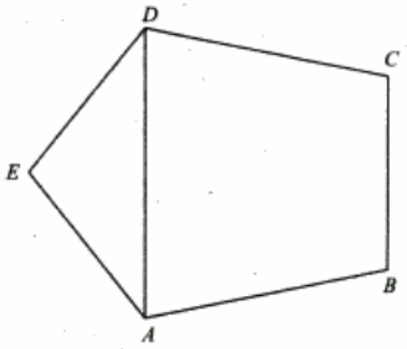
$\Rightarrow \quad 360 = 60\,(n - 2) \Rightarrow 6 = n - 2 \Rightarrow n = 8$

Thus the polygon has **8 sides.**

8. Ans. (a)

Solution Each exterior angle of a regular octagon

$= \frac{360°}{8} = 45°$



Each exterior angle of a regular octagon

$= (180 - 45°) = 135°$

$\therefore$ Required ratio $= \frac{135}{45} = \mathbf{3 : 1}.$

9. Ans. (a)

Solution Here number of sides = 5

$\therefore$ Each interior angle $= \left(\frac{2n-4}{n}\right)$ rt. angles

$= \frac{2 \times 5 - 4}{5} \times 90° = 108°$

In ΔAEA, $\angle AED = 108°$ and $AE = ED$

$\therefore \quad \angle EDA = \angle EAD = \frac{180 - 108}{2} = 36°$

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$\therefore$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 4$ right angles.

and $\angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 4$ right angles.

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 = 8$ right angles.

The sum of the angles of a Δ is 2 right angles.

$\therefore$ $(\angle P + \angle 5 + \angle 10) + (\angle Q + \angle 1 + \angle 9) + (\angle R + \angle 2 + \angle 8) + (\angle S + \angle 3 + \angle 7) + (\angle T + \angle 4 + \angle 6)$

$= 5 \times 2$ right angles.

$\Rightarrow$ $(\angle P + \angle Q + \angle R + \angle S + \angle T) + (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 4 + \angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10)$

$= 10$ right angles.

$\Rightarrow$ $(\angle P + \angle Q + \angle R + \angle S + \angle T) + 8$ right angles $= 10$ right angles

$\Rightarrow$ $\angle P + \angle Q + \angle R + \angle S + \angle T$ **$= 2$ right angles.**

10. Ans. (a)

Solution Let the number of sides of the regular polygon be n and $2n$.

$\therefore$ Their interior angles are

$$\left[\frac{(2n-4)}{n} \times 90^\circ\right] \text{ and } \left[\frac{2(2n)-4}{2n} \times 90^\circ\right]$$

The ratio of measures of interior angles is 3 : 4.

$$\therefore \frac{\left(\frac{2n-4}{n}\right) \times 90^\circ}{\left(\frac{2(2n)-4}{2n}\right) \times 90^\circ} = \frac{3}{4} \Rightarrow \frac{\frac{2n-4}{n}}{\frac{4n-4}{2n}} = \frac{3}{4}$$

$$\Rightarrow \frac{2n-4}{n} \times \frac{2}{4n-4} = \frac{3}{4}$$

$$\Rightarrow \frac{2(n-2)}{1} \times \frac{2}{4(n-1)} = \frac{3}{4}$$

$$\Rightarrow \frac{n-2}{n-1} = \frac{3}{4}$$

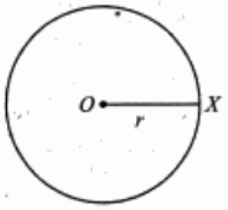
$$\Rightarrow 4n - 8 = 3n - 3$$

$$\Rightarrow 4n - 3n = 8 - 3$$

$$\Rightarrow \mathbf{n = 5.}$$

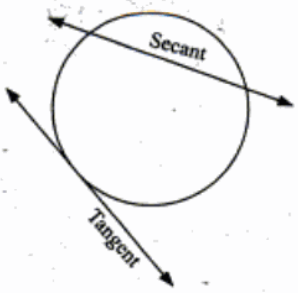
SECTION 5
CIRCLES AND TANGENTS

Circle A circle is a set of all those points in a plane, each one of which is at given constant distance from a given fixed point in the plane.



The fixed point is called the *centre* and the given constant distantance is called the *radius* of the circle. A circle with centre O and radius r is usually denoted by $C(O, r)$

Tangent A line meeting a circle in only one point is called a *tangent* to the circle. The point at which the tangent line meets the circle is called the *point of contact*.

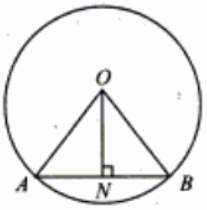


Secant A line which intersects a circle in two distinct points is called a *secant line*.

Key Results to Remember

1. The perpendicular from the centre of a circle to a chord bisects the chord.

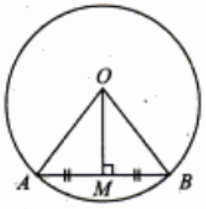
Explanation If $ON \perp AB$, then $AN = NB$.



Note: The converse of above theorem is true and can be stated as.

2. The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord

Explanation If $AM = MB$, then $OM \perp AB$.



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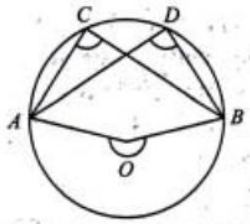
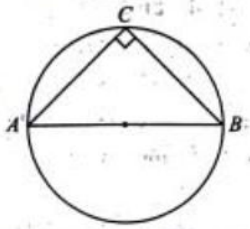


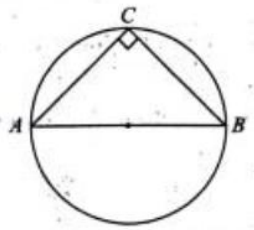
11. The angle in a semicircle is a right angle

Explanation In the figure given below $\angle ACB = 90°$

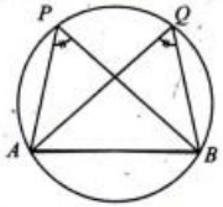


12. (Converse of above theorem) The circle drawn with hypotenuse of a right triangle as diameter passes through its opposite vertex.

Explanation The circle drawn with the hypotenuse AB of a right triangle ACB as diameter passes through its opposite vertex C.

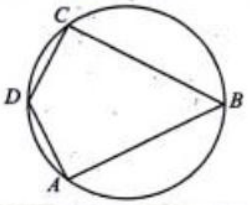


13. If $\angle APB = \angle AQB$, and if P, Q are on the same side of AB, then A, B, Q, P are concyclic i.e. lie on the same circle.



14. The sum of the either pair of the opposite angles of a cyclic quadrilateral is 180°

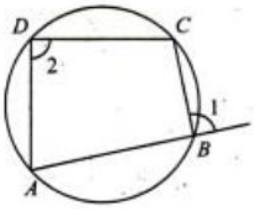
Explanation If $ABCD$ is a cyclic quadrilateral, then $\angle A + \angle C = \angle B + \angle D = 180°$



15. (converse of above theorem) If the two angles of a pair of opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.

16. If a side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle

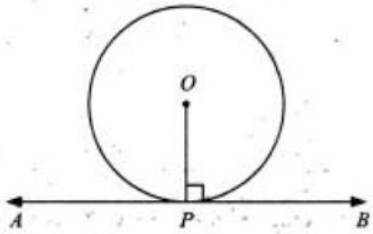
Explanation If the side AB of a cyclic quadrilateral $ABCD$ is produced then $\angle 1 = \angle 2$.



THEOREMS ON TANGENTS

17. A tangent at any point of a circle is perpendicular to the radius through the point of contact.

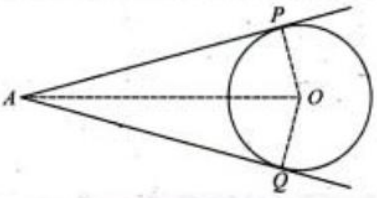
Explanation If AB is a tangent at a point P to a circle $C(O, r)$ then $PO \perp AB$



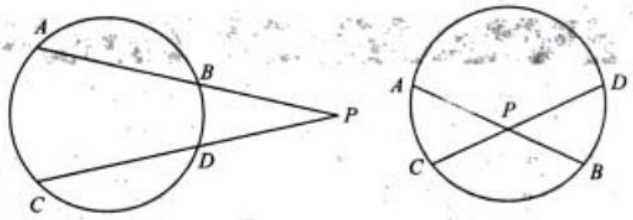
18. (converse of above theorem) A line drawn through the end of a radius and perpendicular to it, is a tangent to the circle.

19. The lengths of two tangents drawn from an external point to a circle are equal.

Explanation If two tangents AP and AQ are drawn from a point A to a circle $C(O, r)$, then $AP = AQ$



20. If two chords AB and CD intersect internally (ii) or externally (i) at a point P then $PA \times PB = PC \times PD$



(i) (ii)

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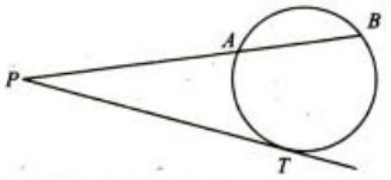
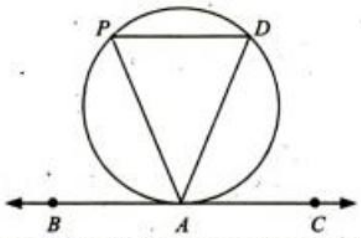
21. If PAB is a secant to a circle intersecting the circle at A and B is a tangent segment then $PA \times PB = PT^2$ (refer the figure below)
(popularly known as Tangent-Secant theorem)



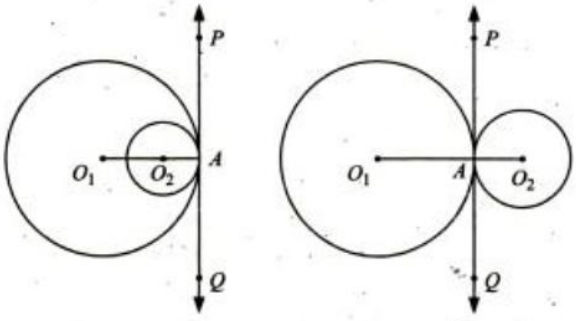
22. Alternate Segment Theorem:
In the figure below, if BAC is the tangent at A to a circle and if AD is any chord, then
$\angle DAC = \angle APD$ and
$\angle PAB = \angle PDA$
(Angles in alternate segment)



Note: **The converse of the above theorem is true**

23. If two circles touch each other internally or externally, the point of contact lies on the line joining their centres.

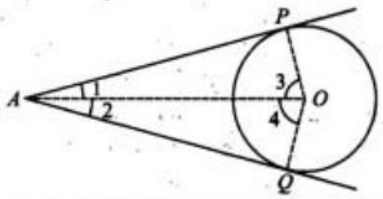
Explanation If two circles with centre O_1 and O_2 which touch each other internally (i) or externally (ii), at a point A then the point A lies on the line $O_1 O_2$, i.e. three points A, O_1 and O_2 are collinear.



SOME USEFUL RESULTS

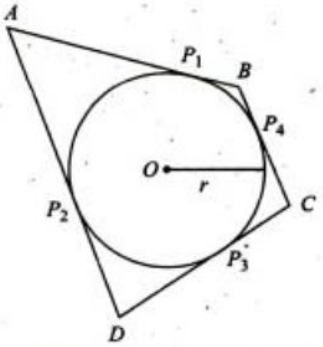
1. There is one and only one circle passing through three non-collinear points.
2. Two circles are congruent if and only if they have equal radii.
3. Of any two chords of a circle, the one which is greater is nearer to the centre.
4. Of any two chords of a circle, the one which is nearer to the centre is greater.
5. If two circles intersect in two points, then the line through the centres is the perpendicular bisector of the common chord.
6. Angle in a major segment of a circle is acute and angle in a minor segment is obtuse.
7. If two tangents are drawn to a circle from an external point then
   (i) they subtend equal angles at the centre.
   (ii) they are equally inclined to the segment, joining the centre to that point.

Explanation **In a circle $C(O,r)$, A is a point outside it and AP and AQ are the tangents drawn to the circle**
Then, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

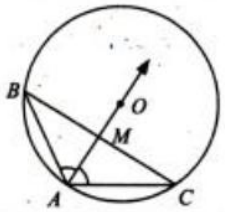


8. If a circle touches all the four sides of a quadrilateral then the sum of opposite pair of sides are equal

Explanation If $ABCD$ is a circumscribed quadrilateral, then
Then, $AB + CD = AD + BC$



9. If two chords AB and AC of a circle are equal. then the bisector of $\angle BAC$ passes through the centre O of the circle



10. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.

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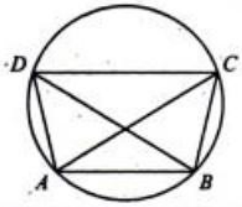
Explanation If $ABCD$ is a cyclic quadrilateral in which AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively then quadrilateral $PQRS$ is also cyclic.

11. A cyclic trapezium is isoceles and its diagonals are equal.

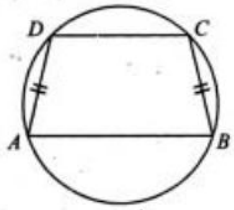
Explanation If ABC cyclic trapezium such that $AB \parallel DC$, then $AD = BC$ and $AC = BD$



12. If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel

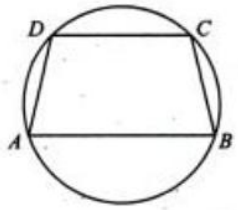
Explanation A cyclic quadrilateral $ABCD$ in which $AD = BC$

Then, $AB \parallel CD$



14. An isoceles trapezium is always cyclic.

Explanation A trapezium $ABCD$ in which $AB \parallel CD$ and $AD = BC$



Then $ABCD$ is a cyclic trapezium.

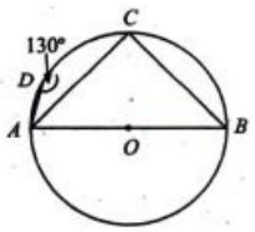
15. Any four vertices of a regular pentagon are *concyclic* (lie on the same circle).

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

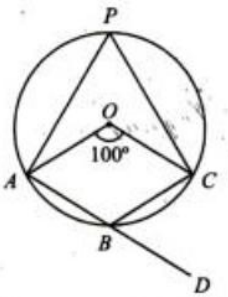
1. The radius of a circle is 13 cm and the length of one its chords is 10 cm. What is the distance of the chord from the centre
   (a) 10 cm (b) 15 cm
   (c) 12 cm (d) 9 cm

2. In the given figure, $ABCD$ is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B and C. If $\angle ADC = 130°$, find $\angle CAB$.



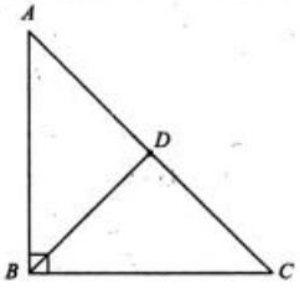
   (a) 140° (b) 50°
   (c) 30° (d) 130°

3. In the given figure, O is the centre of the circle, Find $\angle CBD$



   (a) 140° (b) 50°
   (c) 40° (d) 130°

4. In a ΔABC, $\angle B$ is a right angle, $AC = 6$ cm and D is the mid-point of AC. Find the length of BD



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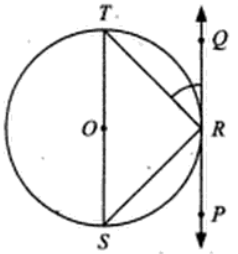
(a) 4 cm (b) $\sqrt{6}$ cm
(c) 3 cm (d) 4.5 cm

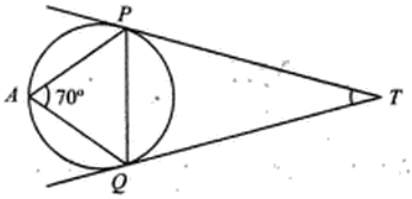
5. A cyclic quadirateral whose opposite angles are equal, is a:
(a) parallelogram but not a rhombus
(b) rhombus
(c) rectangle
(d) Square but not a rectangle

6. In the given figure, ST is a diameter of the circle with centre O and PQ is the tangent at a point R. If $\angle TRQ = 40$, what is $\angle RTS$



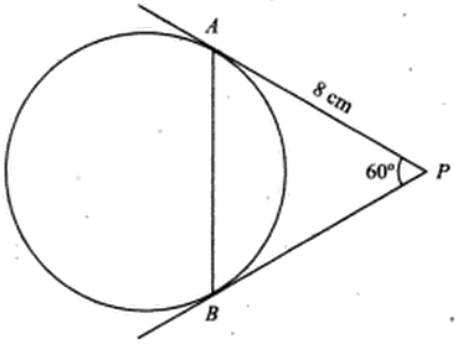
(a) 40° (b) 50°
(c) 60° (d) 30°

7. In the given figure, TP and TQ are tangents to the circle. If $\angle PAQ = 70°$, what is $\angle PTQ$?



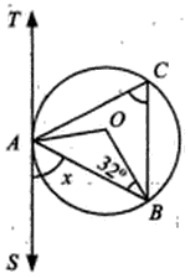
(a) 30° (b) 45°
(c) 60° (d) 40°

8. In the given figure, PA and PB are tangents from a point P to a circle such that $PA = 8$ cm and $\angle APB = 60°$. what is the length of the chord AB?



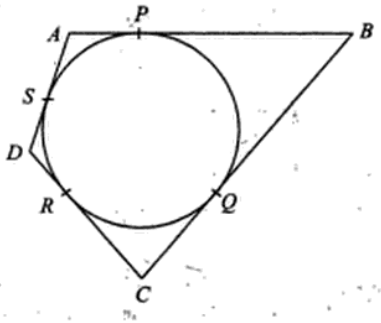
(a) 8 cm (b) 10 cm
(c) 6 cm (d) 12 cm

9. In the given figure, TAS is a tangent to the circle at the point A. If $\angle OBA = 32°$, what is the value of x



(a) 64° (b) 40°
(c) 58 (d) 50°

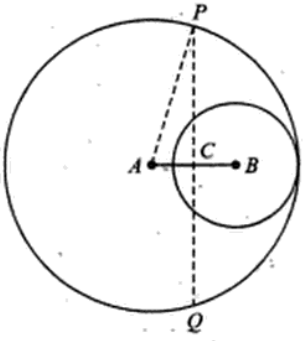
10. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ whose sides $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm. Find AD.



(a) 5 cm (b) 4 cm
(c) 3 cm (d) 2 cm

11. AB and CD are two parallel chords of a circle such that $AB = 10$ cm and $CD = 24$ cm. If the chords are on opposite sides of the centre and the distance between them is 17 cm, what is the radius of the circle
(a) 14 cm (b) 10 cm
(c) 13 cm (d) 15 cm

12. In the given figure, two circle with centres A and B of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of segment AB meets the bigger circle in P and Q, find the length of PQ



(a) $4\sqrt{6}$ cm (b) $\sqrt{24}$
(c) $8\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

13. In the given figure O is the centre of the circle and PT is a tangent at T. If $PC = 3$ cm, $PT = 6$ cm, calculate the radius of the circle.

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(a) 100° (b) 120°
(c) 90° (d) 110°

24. If an equilateral triangle ABC be inscribed in a circle, then the tangents at their vertices will form another Δ.
(a) scalene (b) equilateral
(c) Isoceles (d) Right Δ

25. The radius of the incircle of a Δ is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8cm, then the length of the shortest side of the Δ is
(a) 12 cm (b) 15 cm
(c) 13 cm (d) 14 cm

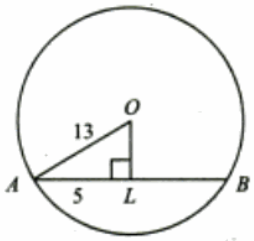
EXPLANATORY ANSWERS

1. Ans. (c)

Solution $OA = 13$ cm, $AB = 10$ cm

From O, draw $OL \perp AB$

We know that the perpendicular from the centre of a circle to a chord bisects the chord.



$\therefore \quad AL = \frac{1}{2}AB = 5$ cm

In right ΔOLA,

$OA^2 = OL^2 + LA^2$ (pythagoras them.)

$169 - 25 = OL^2$

$\therefore \quad OL = \sqrt{144} = \mathbf{12\ cm.}$

2. Ans. (a)

Solution Since $ABCD$ is a cyclic quadrilateral

$\therefore \quad \angle ADC + \angle ABC = 180°$

$\Rightarrow \quad 130° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 50°$

Also, $\angle ACB = 90°$

$\therefore$ In ΔABC,

$\angle ACB + \angle ABC + \angle CAB = 180°$ (ASP)

$\Rightarrow \quad 90° + 50° + \angle CAB = 180° \Rightarrow \mathbf{\angle CAB = 40°.}$

3. Ans. (d)

Solution $\angle AOC = 2\angle APC$

$\therefore \quad \angle APC = 50°$

Also, $ABCP$ is a cyclic quad.

$\therefore \quad \angle ABC = \angle APC$

$\therefore \quad \angle ABC = 50°$

$\therefore \quad \angle CBD = 180 - 50 = \mathbf{130°.}$

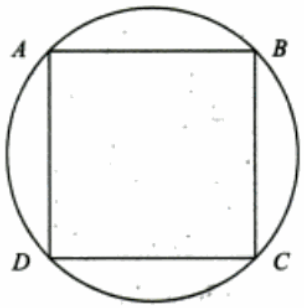
4. Ans. (c)

Solution $BD = DA = DC$

$\therefore \quad BD = \mathbf{3\ cm.}$

5. Ans. (c)

Solution A cyclic quadrilateral whose opposite angles are equal is a rectangle



$\because$ In a cyclic quad.

$\angle A + \angle C = 180°$

But $\angle A = \angle C$

$\therefore \quad \angle A = \angle C = 90°$

Similarly, $\angle B = \angle D = 90°$

and hence $ABCD$ is a rectangle.

6. Ans. (b)

Solution Since ST is a diameter

$\therefore \quad \angle TRS = 90°$

Also, $\angle TRQ = \angle TSR$ (angles in alternate segments.)

$\therefore \quad \angle TSR = 40°$

Hence, $\angle STR = \mathbf{50°.}$

7. Ans. (d)

Solution $\angle TPQ = \angle PAQ = 70°$

($\angle$s in the alternate segments)

$TP = TQ \Rightarrow \angle TQP = \angle TPQ = 70°$

$\therefore \quad \angle PTQ = 180° - 70° - 70° = \mathbf{40°.}$

8. Ans. (a)

Solution $PA = PB$

$\therefore \quad \angle PAB = \angle PBA$

Also, $\angle PAB + \angle PBA = 180° - \angle APB$

$= 180° - 60 = 120°$

$\therefore \quad \angle PAB = \angle PBA = 60$

i.e., ΔPAB is an equilateral triangle

$\therefore \quad AB = \mathbf{8\ cm.}$

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9. Ans. (c)

Solution $OA = OB \Rightarrow \angle OAB = \angle OBA = 32°$

$\therefore \quad \angle OAB + \angle OBA = 32° + 32° = 64°$

$\therefore \quad \angle AOB = 180 - 64 = 116°$

$\Rightarrow \quad \angle ACB = \frac{1}{2}\angle AOB = 58°$

(Degree Measure Thm.)

Also, $\angle ACB = \angle BAS$

(angles in alternate segments)

$\therefore \quad \angle BAS = x = \mathbf{58°}.$

10. Ans. (c)

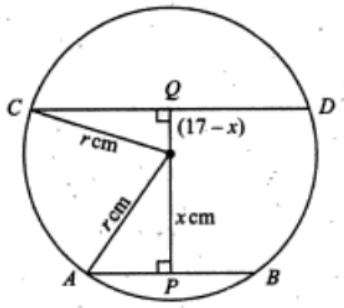
Solution Since $ABCD$ is a *circumscribed* quadrilateral

$\therefore \quad AB + CD = BC + AD \Rightarrow 6 + 4 = 7 + AD$

$\therefore \quad AD = 10 - 7 = \mathbf{3cm}.$

11. Ans. (c)

Solution



In right Δs OAP and OQC, we have

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$r^2 = x^2 + 5^2$ and $r^2 = (17 - x)^2 + 12^2$

$\Rightarrow \quad x^2 + 25 = 289 + x^2 - 34x + 144$

$\Rightarrow \quad 34x = 408 \Rightarrow x = 12$ cm

$\therefore \quad r^2 = 122 + 52 = 169$

$\therefore \quad r = \mathbf{13\ cm}.$

12. Ans. (a)

Solution If two circles touch internally then distance between their centres is equal to the difference of their radii.

$\therefore \quad AB = (5 - 3)$ cm $= 2$ cm

Also, the common chord PQ is the $\perp$ bisector of AB

$\therefore \quad AC = CB = 1$ cm

In rt. ΔACP, we have

$AP^2 = AC^2 + CP^2$

$\Rightarrow \quad 25 - 1 = CP^2$

$\therefore \quad CP = \sqrt{24}$ cm

Hence, $PQ = 2CP = 2\sqrt{24} = \mathbf{4\sqrt{6}\ cm}.$

13. Ans. (b)

Solution Since PT is a tangent and PCB is a secant to the circle

$\therefore \quad PC \times PB = PT^2$

$\Rightarrow \quad 3 \times PB = 6^2 \Rightarrow PB = 12$ cm

$\Rightarrow \quad 3 + BC = 12 \Rightarrow BC = 9$ cm

$\therefore$ radius of the circle $= \frac{1}{2}BC = \mathbf{4.5\ cm}.$

14. Ans. (d)

Solution Let $AB = 9$ cm, $BC = 7$ cm and $CA = 6$ cm

Then, $x + y = 9$ cm

$y + z = 7$ cm

$z + x = 6$ cm

Adding, we get $2(x + y + z) = 22°$

$\Rightarrow \quad x + y + z = 11$

$\therefore \quad z = (11 - 9) = 2,\ x = (11 - 7) = 4$

and $\quad y = (11 - 6) = 5$

Hence, the radii of the given circles are 4 cm., 5 cm and 2 cm respectively

15. Ans. (a)

Solution $OR = OS$, $OR \perp DR$ and $OS \perp DS$

$\therefore \quad ORDS$ is a square

Also, $BP = BQ$, $CQ = CR$ and $DR = DS$

$\therefore\ BQ = BP = 27$ cm $\Rightarrow BC - CQ = 27$ cm

$\Rightarrow 38 - CQ = 27$

$\Rightarrow CQ = 11$ cm

$\Rightarrow CR = 11$ cm

$\Rightarrow CD - DR = 11$

$\Rightarrow 25 - DR = 11$

$\Rightarrow DR = 14$ cm

$\Rightarrow \mathbf{r = 14\ cm}.$

16. Ans. (d)

Solution Since AD is the tangent to the circle from A and APB is a secant

$\therefore \quad AP \times AB = AD^2$

$\Rightarrow \quad AP \times AB = \left(\frac{1}{2}AC\right)^2 = \frac{1}{4}AC^2$

$\Rightarrow \quad AP \times AB = \frac{1}{4}AB^2 \qquad (\because AC = AB)$

$\Rightarrow \quad \mathbf{AP = \frac{1}{4}AB}.$

17. Ans. (c)

Solution Draw $OD \perp AB$,

$OE \perp BC$ and $OF \perp AC$.

Let $OD = OE = OF = x$

Then, $AF = AD = (8 - x)$; $CF = CE = (6 - x)$

$\therefore \quad AC = \sqrt{AB^2 + BC^2} = \sqrt{8^2 + 6^2} = 10$ cm

So, $AC = AF + FC \Rightarrow (8 - x) + (6 - x) = 10$

or $\quad \mathbf{x = 2}.$

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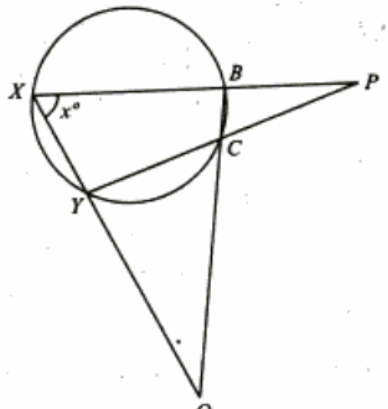
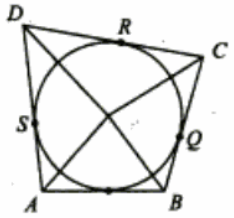


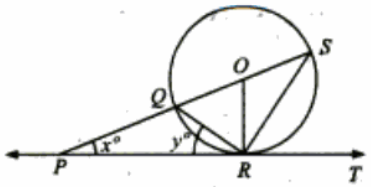
(a) 30° (b) 20°
(c) 60° (d) 45°

5. A circle touches the sides of a quadrilateral $ABCD$ at P, Q, R, S respectively. The angles subtended at the centre by a pair of opposite sides are:



(a) Supplementary (b) Complementary
(c) Equal (d) Unequal

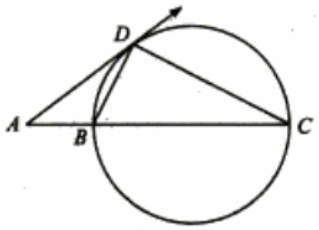
6. In the given fig, PT touches the circle whose centre is O, at R. Diameter SQ when produced meets PT at P. If $\angle SPR = x°$ and $m\angle QRP = y°$, then $x + 2y =$



(a) 100° (b) 120°
(c) 80° (d) 90°

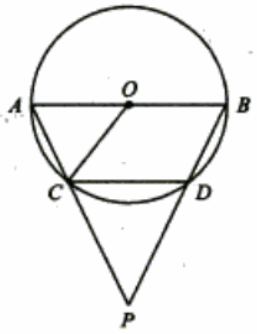
7. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres.
(a) 15 cm (b) 12.5c m
(c) 13.29 cm (d) 14.5 cm

8. In the given fig, $AB = BD$ and AD is the tangent to the circle, then ΔACD is



(a) Isoceles (b) Scalene
(c) Right Δ (d) Equilateral

9. In the given, fig, AB is a diameter of a circle $C(0, r)$. Chord CD is equal to radius OC. If AC and BD when produced intersect at P, the $\angle APB$ is equal to



(a) 90° (b) 60°
(c) 100 (d) 70°

10. The sum of the angles in the four segments exterior to a cyclic quadrilateral is equal to
(a) 4 right angles (b) 5 right angles
(c) 6 right angles (d) None

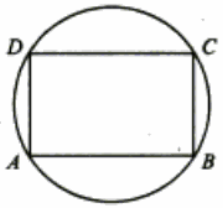
EXPLANATORY ANSWERS

1. Ans. (c)

Solution $ABCD$ is a $\|_{gm}$

$\therefore \quad \angle A = \angle C$ (opp. angles of a $\|_{gm}$)

Also, $ABCD$ is a cyclic quadrilateral



$\Rightarrow \quad \angle A + \angle C = 180°$

$\therefore \quad 2\angle A = 180$

$\Rightarrow \quad \angle A = 90°$

Thus, $\angle A = \angle C = 90°$

Hence, $ABCD$ is a rectnalge

2. Ans. (b)

Solution Arc AD subtends $\angle AOD$ at the centre and $\angle ACD$ at C (on the circumference)

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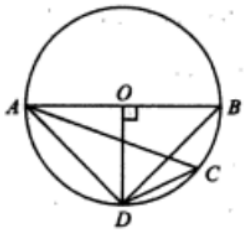


$\therefore \quad \angle AOD = 2\angle ACD$

$\Rightarrow \quad 90 = 2\angle ACD$

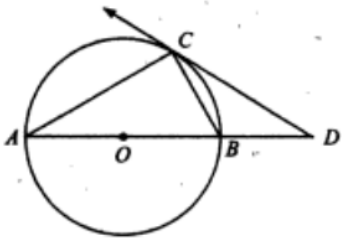
$\therefore \quad \angle ACD = \mathbf{45°}$

3. Ans. (a)

Solution Since CD is tangent to the circle at C

$\therefore \quad \angle BCD = \angle BAC = 30°$

Also, AB is diameter



$\therefore \quad \angle ACB = 90°$

$\Rightarrow \quad \angle DBC = \angle ACB + \angle BAC$ (Ext. angle of a Δ)

$\therefore \quad \angle DBC = 120°$

In ΔBCD,

$\angle CDB = 180° - (120° + 30°) = 30°$

$\therefore \quad \angle CDB = \angle BCD = 30°$

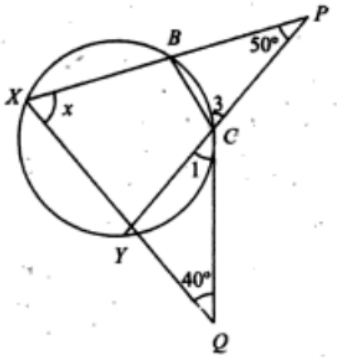
Hence $BC = \mathbf{BD}$.

4. Ans. (d)

Solution $\angle 1 = \angle 2 = x°$

Also $\angle Y = \angle 1 + 40°$ and $\angle B = \angle 2 + 50°$

(Ext. angle of a Δ = sum of int. opp. angles)



Now $\angle Y + \angle B = 180°$

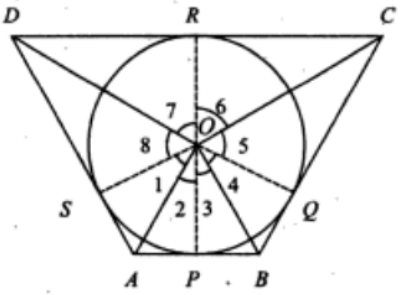
$\Rightarrow \quad \angle 1 + 40° + \angle 2 + 50° = 180°$

$2\angle 1 = 90$

$\angle 1 = \mathbf{45°}$.

5. Ans. (a)

Solution AP and AS are the tangents drawn from pt. A to the same circle



$\therefore \quad \angle 1 = \angle 2$

Similarly, $\angle 3 = \angle 4$

(Refer Some Useful Results–7)

$\angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

and $2(\angle 1 + \angle 4 + \angle 5) = 360°$

$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$

and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

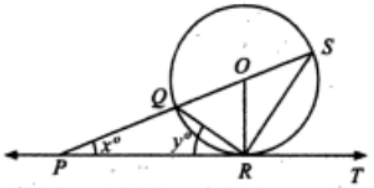
$\Rightarrow \quad \angle AOB + \angle COD = 180°$

and $\angle AOD + \angle BOC = \mathbf{180°}$.

6. Ans. (d)

Solution $\angle QRP = \angle QSR$

Also, $\angle QRS = 90°$



Now $\angle PRS = \angle QRP + \angle QRS \Rightarrow \angle PRS = y° + 90°$

In ΔPRS, we have

$\angle SPE + \angle PRS + \angle PSR = 180°$

$\Rightarrow \quad x + y° + 90° + y° = 180°$

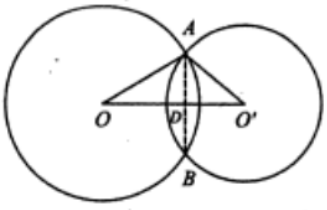
or $\quad x° + 2y° = \mathbf{90}$.

7. Ans. (c)

Solution $OA = 10$ cm, $AD = \frac{1}{2}AB = 6$ cm

$\therefore \quad OD = \sqrt{OA^2 - AD^2} = \sqrt{64} = 8$cm

$O'A = 8$ cm, $AD = \frac{1}{2}AB = 6$ cm



$\therefore \quad O'D = \sqrt{O'A^2 - AD^2} = \sqrt{28} = 5.29$ cm

$OO' = OD + O'D = 8 + 5.29 = \mathbf{13.29}$ cm.

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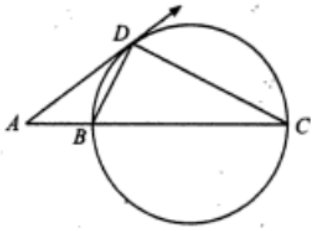
8. Ans. (a)

Solution Since $AB = BD$

$\therefore \quad \angle BAD = \angle BDA$



But $\angle BAD = \angle ACD$

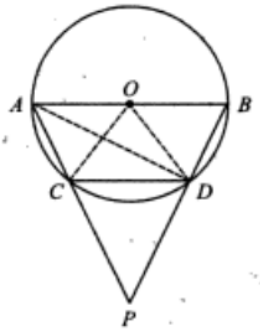
Hence, ΔDAC is isoceles.

9. Ans. (b)

Solution Join OD and AD

In ΔOCD, $OC = OD = CD$

$\Rightarrow$ ΔOCD is equidlateral $\Rightarrow \angle COD = 60°$



Arc CD subtends $\angle COD$ at the centre and $\angle CAD$ at A

$\therefore \quad \angle CAD = \frac{1}{2}\angle COD = \frac{1}{2}(60°) = 30°$

Now $\angle ADB = 90°$

and BDP is a straight line

$\therefore \quad \angle ADP = 90°$

Thus, in ΔADP, we have

$\therefore \quad \angle CPB = 180° - (\angle CAD + \angle ADP)$

$= 180° - (30° + 90°) = 60°$

$\therefore \quad \angle APB = \angle CPB = 60°$ **= constant.**

10. Ans. (c)

Solution Joint SB and SC

Since $APBS$ is a cyclic quadrilateral

$\therefore \quad \angle 1 + \angle P = 180°$

Similarly, $\angle 2 + \angle Q = 180°$

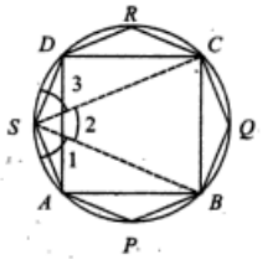
$\angle 3 + \angle R = 180°$

Adding,

$\angle 1 + \angle 2 + \angle 3 + \angle P + \angle Q + \angle R = 540°$

$\Rightarrow \quad \angle P + \angle Q + \angle R + \angle S = 6 \times 90°$

$(\because \quad \angle 1 + \angle 2 + \angle 3 = \angle S)$



$\Rightarrow \quad \angle P + \angle Q + \angle R + \angle S =$ **6 right angles.**

QUESTIONS FROM COMPETITIVE EXAMS

1. The two sides of a right triangle containing the right angle measure 3cm and 4cm. The radius of the incircle of the triangle is

(a) 3.5 cm (b) 1.75 cm

(c) 1cm (d) 0.875 cm

[Based on MAT 2003]

2. If two diameters of a circle intersect each other at right angles, then the quadrilateral formed by joining their end points is a

(a) Rhombus (b) Rectangle

(c) Square (d) Parallelogram

[Based on MAT 2003]

3. Of all the chords of a circle passing through a given point in it, the smallest is that which

(a) Is trisected at the point

(b) Is bisected at the point

(c) Passes through the centre

(d) None of these **[Based on MAT 2003]**

4. In a circular lawn, there is a 16 m long path in the form of a chord. If the path is 6 m away from the centre of the lawn, then find the radius of the circular lawn.

(a) 16m (b) 6m

(c) 10m (d) 8m

[Based on MAT 2003]

5. The number of tangents that can be drawn to two non-intersecting circles

(a) 4 (b) 3

(c) 2 (d) 13

[Based on MAT 2003]

6. In a triangle ABC, the lengths of the sides AB, AC and BC are 3, 5 and 6 cm respectively. If a point D on BC is drawn such that the line AD bisects the angle A internally, then what is the length of BD?

(a) 2 cm (b) 2.25 cm

(c) 2.5 cm (d) 3 cm

[Based on MAT 2003]

7. In a triangle ABC, $\angle A = x°$, $\angle B = y°$ and $\angle C = (y + 20)°$. If $4x - y = 10$, then the triangle is

(a) Right-angled (b) Obtuse-angled

(c) Equilateral (d) None of these

[Based on MAT 2003]

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8. If the sides of a right triangle are x, $x + 1$ and $x - 1$, then the hypotenuse
(a) 5 (b) 4
(c) 1 (d) 0

9. If one of the diagonals of a rhombus is equal to its side, then the diagonals of the rhombus are in the ratio
(a) $\sqrt{3} : 1$ (b) $\sqrt{2} : 1$
(c) $3 : 1$ (d) $2 : 1$

[Based on MAT 2003]

10. If P and Q are the mid points of the sides CA and GB respectively of a triangle ABC, right-angled at C. Then the value of $4(AQ^2 + BP^2)$ is equal to
(a) $4BC^2$ (b) $5AB^2$
(c) $2AC^2$ (d) $2BC^2$

[Based on MAT 2003]

11. In a quadrilateral $ABCD$, $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$, then $\angle ACD$ is equal to
(a) 90° (b) 60°
(c) 30° (d) None of these

[Based on MAT 2003]

12. $ABCD$ is a square, F is mid point of AB and E is a point on BC such that BE is one-third of BC. If area of $\Delta FBE = 108$ m^2, then the length of AC is
(a) 63 m (b) $36\sqrt{2}$ m
(c) $63\sqrt{2}$ m (d) $72\sqrt{2}$ m

[Based on MAT 2003]

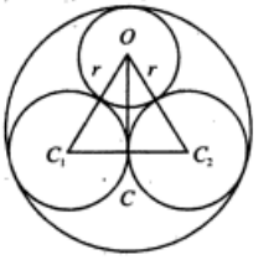
13. We have an angle of $2\frac{1}{2}°$. How big will it look through a glass that magnifies things three times?
(a) $2\frac{1}{2}° \times 4$ (b) $2\frac{1}{2}° \times 3$
(c) $2\frac{1}{2}° \times 2$ (d) None of these

[Based on MAT 2002]

14. Two circles with radii 'a' and 'b' respectively touch each other externally. Let 'c' be the radius of a circle that touches these two circles as well as a common tagent to the two circles. Then
(a) $\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$ (b) $\frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$
(c) $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$ (d) None of these

[Based on MAT 2002]

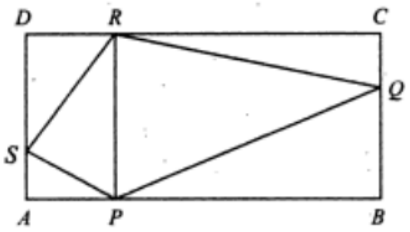
15. Two circles of unit radius touch each other and each of them touches internally a circle of radius two, as shown in the following figure. The radius of the circle which touches all the three circles



(a) 5 (b) $\frac{3}{2}$
(c) $\frac{2}{3}$ (d) None of these

[Based on MAT 2002]

16. $ABCD$ is a parallelogram P, Q, R and S are points on sides AB, BC, CD and DA respectively such that $AP = DR$. If the area of the parallelogram $ABCD$ is 16 cm^2, then the area of the quadrilateral $PQRS$ is



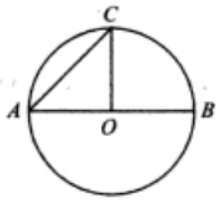
(a) 6 cm^2 (b) 6.4 cm^2
(c) 4 cm^2 (d) 8 cm^2

[Based on MAT 2002]

17. Let ABC be an acute-angled triangle and CD be the altitude through C. If $AB = 8$ and $CD = 6$, then the distance between the mid-points of AD and BC is
(a) 36 (b) 25
(c) 27 (d) 5

[Based on MAT 2002]

18. In the accompanying figure, AB is one of the diameters of the circle and OC is perpendicular to it through the center O. If AC is $7\sqrt{2}$ cm, then what is the area of the circle in sq. cm.?



(a) 24.5 (b) 49
(c) 98 (d) 154

[Based on MAT 2001]

19. The circumcentre of a triangle is always the point of intersection of the
(a) Medians
(b) Bisectors
(c) Perpendiculars
(d) Perpendiculars dropped from the, vertices on the opposite sides of the triangle

[Based on MAT 2001]

20. In the following figure, if $BC = 8$ cm, $AB = 6$ cm, $AC = 9$ cm, then DC is equal to

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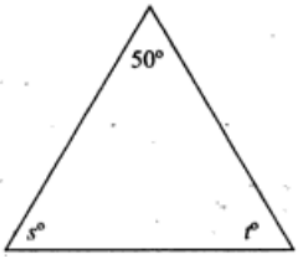
(a) 1 : 4 (b) 1 : 3
(c) 1 : 2 (d) 2 : 1

Note: The diagram is not drawn to scale

[Based on REC Tiruchiraplli 2003]

35. In the figure above, if $s < 50° < t$, then



(a) $t < 80$ (b) $s + t < 130$
(c) $50 < t < 80$ (d) $t > 80$

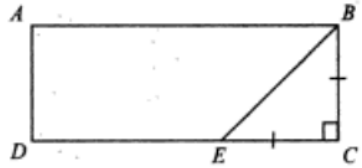
Note: The diagram is not drawn to scale

[Based on REC Tiruchiraplli 2003]

36. The radius of the circumcircle of an equilateral triangle of side 12 cm is

(a) $(4/3)\sqrt{3}$ (b) $4\sqrt{2}$
(c) $4\sqrt{3}$ (d) 4
(e) None of the above **[Based on IIFT 2003]**

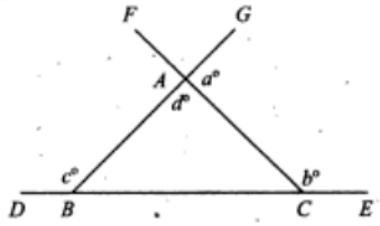
37. In the diagram below, $ABCD$ is a rectangle. The area of isosceles right triangle BCE is 14, and $DE = 3EC$. What is the area of $ABCD$?



(a) 112 (b) 56
(c) 84 (d) $3\sqrt{28}$

[Based on SCMHRD Ent. Exam. 2003]

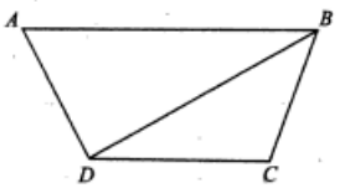
38. It is given that $d° = 70°$, $b° = 120°$. Then



(a) $c° = 130°$ (b) $a° = 110°$
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong.

[Based on IMT Ghaziabad, 2002]

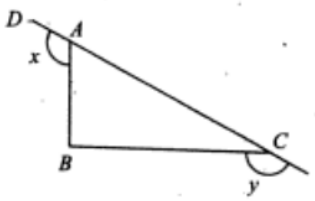
39. In the quadrilateral $ABCD$
$AB + BC + CD + DA$ is



(a) Greater than $2BD$ (b) Less than $2BD$
(c) Equal to $2BD$ (d) None of these

[Based on IMT Ghaziabad, 2002]

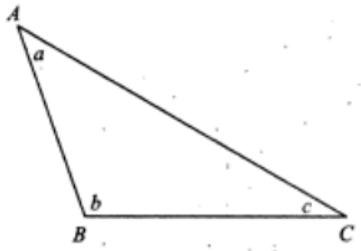
40. It is given that $AB \perp BC$. Which one of the following is true?



(a) $x + y = 180°$ (b) $x + y = 270°$
(c) $x + y = 300°$ (d) Cannot be said

[Based on IMT Ghaziabad, 2002]

41. ABC is a triangle. It is given that $a + c > 90°$, then b is



(a) Greater than 90° (b) Less than 90°
(c) Equal to 90° (d) Cannot be said

[Based on IMT Ghaziabad, 2002]

42. Which of the following are possible measures for the angles of a parallelogram?

(a) 90, 90, 90, 90 (b) 40, 70, 50, 150
(c) 50, 130, 50, 130
(1) (a) only (2) (b) only
(3) (c) only (4) (b) and (c) only

[Based on Narsee Monjee Inst. of Man. Studies 2002]

43. Find the distance of a-perpandicular from the centre of a circle to the chord if the diameter of the circle is 30 cm and its chord is 24 cm.

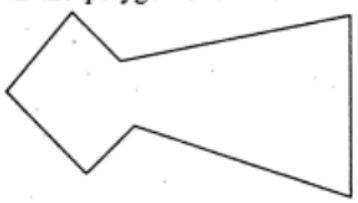
(a) 6 cm (b) 7 cm
(c) 9 cm (d) 10 cm

[Based on I.P. Univ. 2002]

44. The medians AD, BE and CF of a triangle ABC intersect in G. Which of the following is true for any ΔABC?

(a) $GB + GC = 2GA$ (b) $GB + GC < GA$
(c) $GB + GC > GA$ (d) $GB + GC = GA$
(e) None of these **[Based on SCMHRD 2002]**

45. Find the sum of the degree measures of the internal angles in the polygon shown below.



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(a) 600° (b) 720°
(c) 900° (d) 1080°
(e) 750° **[Based on SCMHRD 2002]**

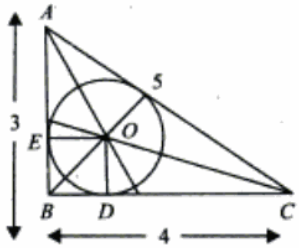
46. A semi-circle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is draw meeting the circumference of the semi-circle at D. Given that AC = 2 cm and CD = 6 cm, the area of the semi-circle (in cm^2) will be:
(a) $32\ \pi\ cm^2$ (b) $50\ \pi\ cm^2$
(c) $40.5\ \pi\ cm^2$ (d) $81\ \pi\ cm^2$
(e) undeterminable **[Based on CAT 2006]**

47. An equilateral triangle BPC is drawn inside a square ABCD. What is the value of the angle APD in degrees?
(a) 75 (b) 90
(c) 12 (d) 135
(e) 150 **[Based on CAT, 2006]**

EXPLANATORY ANSWERS

1. Ans. (c)

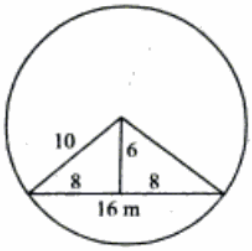
Solution If the in circle of a triagle ABC touches BC at D, then $|BD - CD| = |AB - AC|$



In our case, $AC = 5$, $AB = 3$
$\Rightarrow$ $AC - AB = 2$
$\therefore$ $CD - BD = 2$
In our case, $BC = 4$
$\Rightarrow$ $BD + DC = 4$ and $-BD + DC = 22$
$\Rightarrow$ $CD = 3$
$\Rightarrow$ $BD = 1 = OE$ = Radius of the in circle.

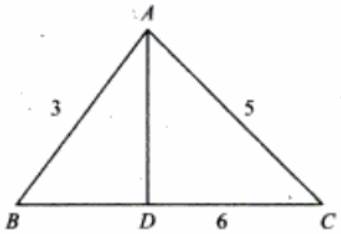
2. Ans. (c)
3. Ans. (d)
4. Ans. (c)

Solution



5. Ans. (a)
6. Ans. (b)

Solution



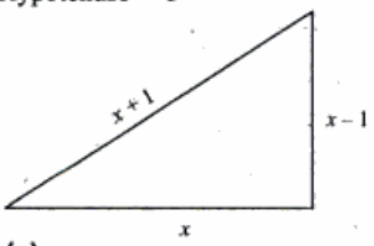
$BD : DC = 3 : 5$
$\therefore$ Divided $BC = 6$ in the ratio 3 : 5
$\Rightarrow$ $BD = 2.25$, $CD = 3.75$.

7. Ans. (a)

Solution $x + y + (y + 20) = 5 \Rightarrow x + 2y = 160$
$4x - y = 10 \Rightarrow y = 70,\ x = 20$
$\therefore$ The angles of the triangle are **20°, 70°, 90°.**

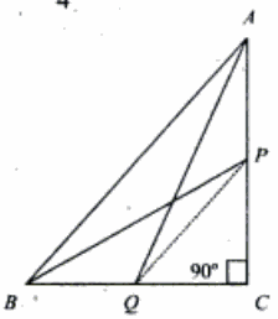
8. Ans. (a)

Solution Let $(x + 1)$ be the hypotenuse
$\therefore$ $(x + 1)^2 = x^2 + (x - 1)^2 \Rightarrow x = 4$
$\therefore$ Hypotenuse = 5



9. Ans. (a)
10. Ans. (b)

Solution
$$AQ^2 = AC^2 + QC^2$$
$$BP^2 = BC^2 + CP^2$$
$$AQ^2 = BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2)$$
$$= AB^2 + PQ^2$$
$$= AB^2 + \left(\frac{1}{2} \cdot AB\right)^2 \quad \left[\because PQ = \frac{1}{2} AB\right]$$
$$= \frac{5}{4} AB^2 = 4\ (AQ^2 + BP^2) = \mathbf{5AB^2}.$$



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18. Ans. (d)

Solution Let $OA = OC$ = Radius = x

$\therefore \quad AO^2 + OC^2 = AC^2 \Rightarrow 2x^2 = 98 \Rightarrow x = 7$

$\therefore$ Area of the circle $= \pi x^2 = \frac{22}{7} \times 49$

$= \mathbf{154}$ **sq.cm.**

19. Ans. (b)

20. Ans. (b)

Solution Using the sine fomula, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore \quad \frac{\sin 40^\circ}{BD} = \frac{\sin(40^\circ + C)}{6}$$

$$\Rightarrow \quad \frac{\sin 40^\circ}{B - DC} = \frac{\sin[180 - (40 + C)]}{6}$$

$$= \frac{\sin(140 - C)}{6} \qquad \ldots(i)$$

Also $\quad \frac{\sin 40^\circ}{DC} = \frac{\sin(140^\circ - C)}{9} \qquad \ldots(ii)$

$\therefore$ From (i) and (ii) give

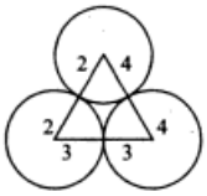
$$\frac{DC}{8 - DC} = \frac{9}{6} = \frac{3}{2} \Rightarrow 5DC = 24 \Rightarrow \boldsymbol{DC = 4.8.}$$

21. Ans. (d)

22. Ans. (d)

23. Ans. (d)

Solution



Sum of radii = 2 + 3 + 4 = **9 cm.**

24. Ans. (b)

25. Ans. (d)

26. Ans. (b)

27. Ans. (b)

Solution The sum of the interior angles of a polygon of n sides $= (2n - 4) \times \frac{\pi}{2}$

$$\therefore \quad (2n + 4) \times \frac{\pi}{2} = 1620 \Rightarrow n = \mathbf{11.}$$

28. Ans. (c)

Solution Let n be the number of sides of the polygon.

$\therefore$ Interior angle = 8 × Exterior angle.

$$\Rightarrow \quad \frac{(2n - 4) \times \pi/2}{n} = 8 \times \frac{2\pi}{n}$$

$\Rightarrow \quad n = 18.$

29. Ans. (a)

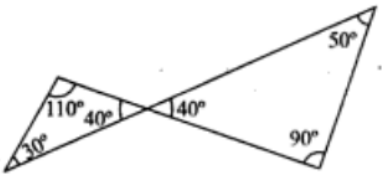
30. Ans. (a)

31. Ans. (d)

32. Ans. (d)

33. Ans. (b)

Solution

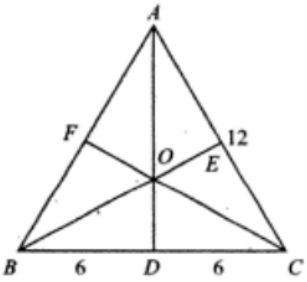


34. Ans. (c)

35. Ans. (d)

36. Ans. (c)

Solution Circumcircle of a triangle is the point of intersection of the perpendicular bisectors of the sides of the triangle.



O is the circumcentre of the ΔABC, whose sides

$$AB = BC = CA = 12 \text{ cm}$$

$\therefore$ From ΔADC

$$AC^2 = AD^2 + CD^2 \Rightarrow AD = \sqrt{(12)^2 - 6^2} = 6\sqrt{3}$$

Since, triangle is equilateral, therefore circumcentre = centroid

$\therefore \quad AO : OD = 2 : 1$

i.e. $\quad AO = 4\sqrt{3},$

$\quad OD = 2\sqrt{3} \qquad [\because AD = 6\sqrt{3}]$

$\therefore$ Radius of the circumcircle

$= 4\sqrt{3} = OA = OB = \boldsymbol{OC.}$

37. Ans. (a)

Solution Area of $\Delta BCE = \frac{1}{2} \times b \times b \Rightarrow b^2 = 28$

Area of rectangle $ABCD = (DE + EC) \times b$

$= 4EC \times b = 4b^2 = \mathbf{112.}$

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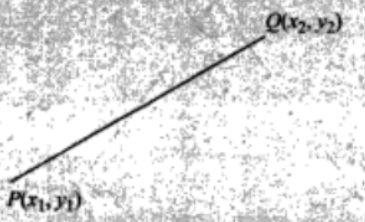
Coordinate Geometry 38

INTRODUCTION

Geometry begins with a point and straight line. Uptil now, we have studied geometry without any use of algebra. In 1637, Descartes used algebra in the study of geometrical relationships. Thus, a new type of geometry was introduced which was given the name analytical geometry or coordinate geometry. Thus, coordinate geometry is that branch of mathematics in which geometry is studied algebraically, i.e. geometrical figures are studied with the help of equations.

SOME BASIC FORMULAE

1. Distance Formula Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by



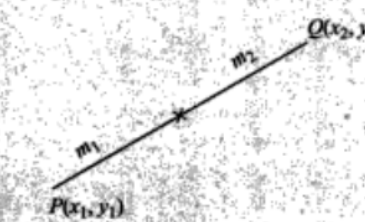
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Illustration 1 Find the distance between the pair of points $A(2, 5)$ and $B(-3, 7)$.

Solution $AB = \sqrt{(-3-2)^2 + (7-5)^2} = \sqrt{25+4} = \sqrt{29}$.

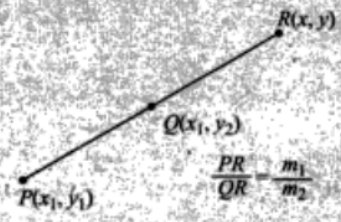
2. Section Formulae

(a) *Formula for internal division* The coordinates of the point $R(x, y)$ which divides the join of two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are given by



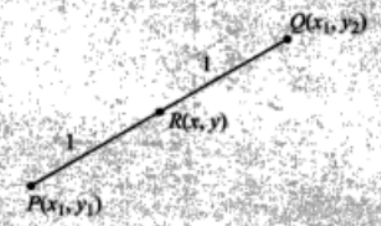
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

(b) *Formula for external division* The coordinates of the point $R(x, y)$ which divides the join of two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio $m_1 : m_2$ are given by



$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right).$$

(c) *Mid-point Formula* If R is the mid point of $P\,Q$, then its coordinates are given by



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Illustration 2 Find the coordinates of the point which divides:

(i) the join of (2, 3) and (5, –3) internally in the ratio 1 : 2

(ii) the join of (2, 1) and (3, 5) externally in the ratio 2 : 3

Solution Let (x, y) be the coordinates of the point of division. Then,

$$x = \frac{1(5) + 2(2)}{1+2} = \frac{5+4}{3} = 3$$

$$y = \frac{1(-3) + 2(3)}{1+2} = \frac{-3+6}{3} = 1.$$

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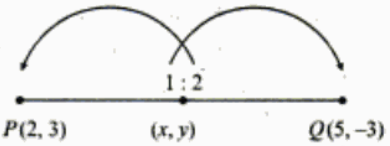
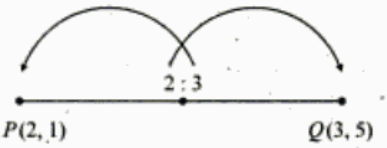
$\therefore$ Coordinates of the point of division are (3, 1).



(ii) Let (x, y) be the coordinates of the point of division.



Then, $x = \dfrac{2(3)-3(2)}{2-3} = \dfrac{6-6}{-1} = 0$

$y = \dfrac{2(5)-3(1)}{2-3} = \dfrac{10-3}{-1} = -7$

$\therefore$ Coordinates of the point of division are (10, –7).

Illustration 3 Find the coordinates of the mid point of the join of points $P(2, -1)$ and $Q(-3, 4)$.

Solution The coordinates of the mid-point are

$$x = \frac{2-3}{2} = -\frac{1}{2}$$

$$y = \frac{-1+4}{2} = \frac{3}{2}.$$

$\therefore$ Coordinates of the mid point are $\left(-\dfrac{1}{2}, \dfrac{3}{2}\right)$.

Note: If the point R is given and we are required to find the ratio in which R divides the line segment PQ, it is convenient to take the ratio $k : 1$.

Then, the coordinates of R are

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{x+1}\right).$$

Illustration 4 In what ratio does the point (6, – 6) divide the join of (1, 4) and (9, –12)?

Solution Let the point R (6, – 6) divides the join of $P(1, 4)$ and Q (9, –12) in the ration $k : 1$.

By section formula, the coordinates of R are

$\left(\dfrac{k(9)+1(1)}{k+1}, \dfrac{k(-12)+1(4)}{k+1}\right)$, i.e. $\left(\dfrac{9k+1}{k+1}, \dfrac{-12k+4}{k+1}\right)$.

But the coordinates of R are given to be (6, – 6).

$\therefore \quad \dfrac{9k+1}{k+1} = 6$ and $\dfrac{-12k+4}{k+1} = -6$

$\Rightarrow \quad 9k + 1 = 6k + 6$ and $- 12k + 4 = -6k - 6$

$\Rightarrow \quad 3k = 5$ and $- 6k = -10$

In either case, $k = \dfrac{5}{3}$ (+ve)

$\therefore$ R divides PQ internally in the ratio $\dfrac{5}{3} : 1$ i.e. 5 : 3.

3. Centroid of a Triangle The point of concurrence of the medians of a triangle is called the centroid of triangle. It divides the median in the ratio 2 : 1.

The coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

Illustration 5 Find the centroid of the triangle whose angular points are (3, – 5), (–7, 4) and (10, –2), respectively.

Solution The coordinates of centroid are

$$\left(\frac{3-7+10}{3}, \frac{-5+4-2}{3}\right) = (2, -1).$$

4. Incentre of a Triangle Incentre of a triangle is the point of concurrence of the internal bisectors of the angles of a triangle.

The coordinates of the incentre of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right).$$

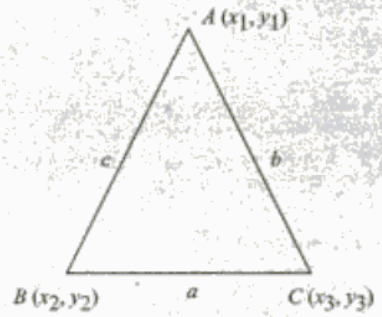


Illustration 6 Find the coordinates of incentre of a triangle having vertices $A(0, 0)$, $B(20, 15)$ and $C(-36, 15)$.

Solution We have,

$a = BC = \sqrt{(20+36)^2 + (15-15)^2} = 56$

$b = AC = \sqrt{(36)^2 + (15)^2} = 39$

$c = AB = \sqrt{(20)^2 + (15)^2} = 25.$

$\therefore$ Coordinates of incentre are

$$x = \frac{ax_1+bx_2+cx_3}{a+b+c} = \frac{56.0+39.20+25.(-36)}{56+39+25} = -1.$$

$$y = \frac{ay_1+by_2+cy_3}{a+b+c} = \frac{56.0+39.15+25.15}{56+39+25} = 8$$

Thus, $I \equiv (-1, 8)$.

5. Area of a Triangle The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$A = \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

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Solution Let l_1 be the line joining the points (2, –3) and (–5, 1).

$\therefore$ Slope of $l_1 = \dfrac{1-(-3)}{-5-2} = -\dfrac{4}{7}$.

(a) Let l_2 be the line joining the points (7, –1) and (0, 3).

$\therefore$ Slope of $l_2 = \dfrac{3-(-1)}{0-7} = -\dfrac{4}{7}$.

$\because$ Slope of l_1 = slope of l_2 (each = – 4/7).

$\therefore$ Lines l_1 and l_2 are parallel.

(b) Let l_3 be the line joining the points (4, 5) and (0, –2).

$\therefore$ Slope of $l_3 = \dfrac{-2-5}{0-4} = \dfrac{-7}{-4} = \dfrac{7}{4}$.

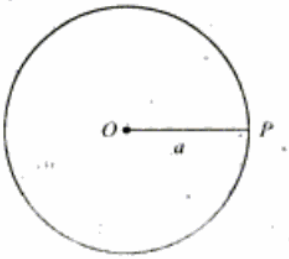
$\because$ Slope of l_1 × slope of $l_3 = -\dfrac{4}{7} \times \dfrac{7}{4} = -1$,

$\therefore$ the lines l_1 and l_3 are perpendicular.

Locus

When a point moves so that it always satisfies a given condition or conditions, the path traced out by it is called its locus under these conditions.

Illustration 12 Let O be a given point in the plane of the paper and let a point P move on the paper so that its distance from O is constant and is equal to a. All the positions of the moving point must lie on a circle whose centre is O and radius is a. This circle is, therefore, the *locus* of P when it moves under the condition that its distance from O is equal to a constant a.



Shortcut Method to Find the Locus

1. Take a point on the locus and suppose that its coordinates are (x, y).
2. Apply the given conditions(s) to (x, y) and simplify the algebraic equation so formed.
3. The simplified equation is the required equation of the locus.

Illustration 13 A point moves so that its distance from (3, 0) is twice its distance from (–3, 0). Find the equation of its locus.

Solution Let $P(x, y)$ be any point on the locus. And, A (3, 0) and B (–3, 0) be the given points.

By the given condition, $PA = 2\,PB$.

$\Rightarrow \sqrt{(x-3)^2+(y-0)^2} = 2\sqrt{(x+3)^2+(y-0)^2}$

Squaring both sides,

$x^2 + y^2 - 6x + 9 = 4\,(x^2 + y^2 + 6x + 9)$

or $3x^2 + 3y^2 + 30x + 27 = 0$

or $x^2 + y^2 + 10x + 9 = 0$,

which is the required equation of the locus.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The distance between the points (7, 9) and (3, –7) is
(a) $4\sqrt{15}$ (b) $4\sqrt{17}$
(c) $17\sqrt{4}$ (d) $17\sqrt{5}$

2. The distance between (cos θ, sin θ) and (sin θ, –cos θ) is
(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) 1 (d) 0

3. The distance between the points (4, p) and (1, 0) is 5 then p =
(a) ± 4 (b) 4
(c) –4 (d) 0

4. The distance between the points (a sin 60°, 0) and (0, a sin 30°) is
(a) $a/\sqrt{2}$ (b) $a\sqrt{2}$
(b) $a/\sqrt{3}$ (d) None of these.

5. The distance between the two points is 5. One of them is (3, 2) and the ordinate of the second is – 1, then its x-coordinate is
(a) 7, –1 (b) –7, 1
(c) –7, –1 (d) 7, 1

6. A line is of length 10 and one end is (2, –3). If the abscissa of the other end is 10 then its ordinate is
(a) 3 or 9 (b) –3 or –9
(c) 3 or –9 (d) –3 or 9

7. The nearest point from origin is
(a) (2, –3) (b) (5, 0)
(c) (2, –1) (d) (1, 3)

8. The vertices of a triangle are A (2, 2), B (–4, –4), C(5, –8). Then, the length of the median through C is
(a) $\sqrt{65}$ (b) $\sqrt{117}$
(c) $\sqrt{85}$ (d) $\sqrt{113}$

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(a) $-2 : 3$ (b) $-3 : 2$
(c) $3 : 2$ (d) $2 : 3$

34. The line segment joining $(-3, -4)$ and $(1, -2)$ is divided by y-axis in the ratio.
(a) $1 : 3$ (b) $2 : 3$
(c) $3 : 1$ (d) $3 : 2$

EXPLANATORY ANSWERS

1. Ans. (b)

Solution Distance between $(7, 9)$ and $(3, -7)$ is
$\sqrt{(7-3)^2+(9+7)^2} = \sqrt{16+256} = 4\sqrt{17}$.

2. Ans. (b)

Solution $\sqrt{(\cos\theta-\sin\theta)^2+(\sin\theta+\cos\theta)^2}$
$= \sqrt{1+1} = \sqrt{2}$.

3. Ans. (a)

Solution $9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4$.

4. Ans. (d)

Solution Distance between $(a \sin 60°, 0)$ and $(0, a \sin 30°)$ is $\sqrt{(a\sin 60°)^2+(a\sin 30°)^2}$

$= \sqrt{a^2\left(\frac{3}{4}+\frac{1}{4}\right)} = a$.

5. Ans. (a)

Solution $\sqrt{(3-x)^2+(2+1)^2} = 5$
$\Rightarrow (3-x)^2 + 9 = 25 \Rightarrow 9 + x^2 - 6x + 9 - 25 = 0$
$\Rightarrow x^2 - 6x - 7 = 0 \Rightarrow x = 7, -1$.

6. Ans. (c)

Solution $\sqrt{(10-2)^2+(y+3)^2} = 10$
$\Rightarrow 64 + y^2 + 9 + 6y - 100 = 0$
$\Rightarrow y^2 + 6y - 27 = 0 \Rightarrow y = -9, 3$.

7. Ans. (c)

Solution $\sqrt{(2-0)^2+(-1-0)^2} = \sqrt{5}$
$\Rightarrow$ $(2, -1)$ is the nearest point.

8. Ans. (c)

Solution Mid-point of $AB = \left(\frac{2-4}{2}, \frac{2-4}{2}\right)$
$= (-1, -1) = D$

Now, $CD = \sqrt{(5+1)^2+(-8+1)^2}$
$= \sqrt{36+49} = \sqrt{85}$.

9. Ans. (c)

10. Ans. (c)

Solution Third vertex
$= \left\{\frac{(2+2)\pm\sqrt{3}(6-4)}{2}, \frac{6+4\pm\sqrt{3}(2-2)}{2}\right\}$
$= (2 \pm \sqrt{3}, 5)$.

11. Ans. (d)

Solution Third vertex
$= \left\{\frac{(3+0)\pm\sqrt{3}(\sqrt{3}-0)}{2}, \frac{(\sqrt{3}+0)\mp\sqrt{3}(3-0)}{2}\right\}$
$= \left\{\frac{3\pm3}{2}, \frac{\sqrt{3}\mp3\sqrt{3}}{2}\right\} = (3, -\sqrt{3})$ or $(0, 2\sqrt{3})$.

12. Ans. (d)

Solution Perimeter $= 1 + 1 + \sqrt{2} = 2 + \sqrt{2}$.

13. Ans. (a)

Solution $AB^2 = \left(1-\frac{5}{6}\right)^2 + (3-6)^2 = \frac{1}{36} + 9 = \frac{325}{36}$.

14. Ans. (c)

Solution Area of $\Delta = \frac{1}{2}\begin{vmatrix} a & a & 1 \\ a+1 & a+1 & 1 \\ a+2 & a & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} a & a & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix}$ $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= \frac{1}{2}|(-2)| = 1$.

15. Ans. (a)

Solution Let the ratio be $l : m$, then,
$\frac{7l+m}{l+m} = -3 \Rightarrow \frac{l}{m} = -\frac{2}{5}$.

16. Ans. (c)

Solution Let $D = (3, 5)$, $E = (-3, -3)$, then
$DE = \sqrt{(3+3)^2+(5+3)^2} = \sqrt{36+64}$
$= \sqrt{100} = 10 \Rightarrow BC = 2\,DE = 2 \times 10 = 20$.

17. Ans. (b)

Solution $\frac{2t-6}{t+2} = \frac{6-1}{-2-3} \Rightarrow 15t = 20 \Rightarrow t = 4/3$.

18. Ans. (b)

Solution $A = (2, 4)$, $B = (4, 2)$, then C
$= \left\{\frac{(2+4)\pm(4-2)}{2}, \frac{(4+2)\mp(2-4)}{2}\right\}$

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Since $BC = AC$, therefore, the given straight lines from an isosceles triangle.

7. Ans. (a)

Solution $2x + 3y = 5,\ x - y = 10 \Rightarrow x = 7,\ y = -3.$

8. Ans. (b)

Solution The condition that the three straight lines

$$a_1x + b_1y + c_1 = 0,$$
$$a_2x + b_2y + c_2 = 0,$$
$$a_3x + b_2y + c_3 = 0$$

may meet in a point is given by

$$a_1(b_2c_3 - b_3c_2) + b_1(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0 \quad ...(i)$$

Put $a_1 = a,\ b_1 = b,\ c_1 = c$

$a_2 = b,\ b_2 = c,\ c_2 = a$

$a_3 = c,\ b_3 = a,\ c_3 = b$

$\therefore$ (i) gives

$$a(cb - a^2) + b(ac - b^2) + c(ab - c^2) = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc.$$

9. Ans. (c)

Solution General equation of the second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines

$$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \quad ...(i)$$

In our case,

$a = 1,\ 2h = -3,\ b = \lambda,\ 2g = 3,\ 2f = -5,\ c = 2$

$\therefore$ From (i)

$$\Rightarrow 2\lambda + 2\times\left(-\frac{5}{2}\right)\times\left(\frac{3}{2}\right)\times\left(-\frac{3}{2}\right) - \frac{25}{4} - \lambda\times\frac{9}{4} - 2\times\frac{9}{4} = 0$$

$$\Rightarrow -\frac{\lambda}{4} + \frac{45}{4} - \frac{25}{4} - \frac{18}{4} = 0 \Rightarrow \frac{\lambda}{4} = \frac{1}{2} \Rightarrow \lambda = 2.$$

10. Ans. (c)

Solution The given equation can be written as

$$x(x+\sqrt{2}y+4) + \sqrt{2}y(x+\sqrt{2}y+4) + 1 = 0$$

or $(x+\sqrt{2}y)(x+\sqrt{2}y+4) + 1 = 0$

Let $(x+\sqrt{2}y) = z \Rightarrow z(z+4) + 1 = 0$

$$\Rightarrow z^2 + 4z + 1 = 0$$

$$\Rightarrow z = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

Thus, the two lines are $(x + \sqrt{2} + 2 - \sqrt{3}) = 0$ and $(x + \sqrt{2} + 2 + \sqrt{3}) = 0$, which are parallel, because their equations differ by a constant only.

$\therefore$ The distance between the parallel lines is

$$\frac{(2+\sqrt{3}) - (2-\sqrt{3})}{\sqrt{1^2 + (\sqrt{2})^2}} = \frac{2\sqrt{3}}{\sqrt{1+2}} = 2.$$

11. Ans. (a)

Solution Eliminating t from the given equations, we get $x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$, which is an equation of the pair of straight lines.

EXERCISES FOR SELF-PRACTICE

Test I **[Time: 10 min]**

1. The distance between the points $(\sqrt{3} + 1, \sqrt{2} - 1)$ and $(\sqrt{3} - 1, \sqrt{2} + 1)$ is
(a) $\sqrt{3}$ (b) $2\sqrt{3}$
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

2. The distance between $(x, 2)$ and $(3, 4)$ is 8 then $x =$
(a) $2 \pm 3\sqrt{15}$ (b) $3 \pm 2\sqrt{15}$
(c) $3 \pm 5\sqrt{2}$ (d) $2 \pm 3\sqrt{5}$

3. Two opposite vertices of a square are $(1, -2)$ and $(-5, 6)$, then the other two vertices are
(a) $(2, 5), (-6, -1)$ (b) $(-2, 5), (6, 1)$
(c) $(2, -5), (6, -1)$ (d) None of these.

4. If $(0, 0), (3, \sqrt{3})$ are two vertices of an equilateral triangle then its third vertex is
(a) $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$
(b) $(2\sqrt{3}, 0)$ or $(3, -\sqrt{3})$
(c) $(0, 2\sqrt{3}), (-\sqrt{3}, 3)$
(d) $(2\sqrt{3}, 0)$ or $(-3\sqrt{3}, 3)$

5. The mid-points of AB, BC and CA of ΔABC are $(6, -1), (-4, -3), (2, -5)$. The centroid of the ΔABC is
(a) $(4, -9)$ (b) $(4/3, -3)$
(c) $(4, -12)$ (d) None of these.

6. If $A(-1, 3)$, $B(1, -1)$ and $C(5, 1)$ are the vertices of a triangle then the length of the median through A is
(a) 20 (b) 15
(c) 10 (d) 5

7. If $A = (-3, 4)$ and $B = (2, 1)$ then the coordinates of the point C on AB produced such that $AC = 2BC$ is
(a) $(2, 4)$ (b) $(3, 7)$
(c) $(7, -2)$ (d) $(3, -2)$

8. The points $(1, 2), (-5, 6)$ and $(a, -2)$ are collinear then $a =$
(a) -3 (b) 7
(c) 2 (d) -2

9. P and Q are points on the line joining $A(-2, 5)$ and $B(3, 1)$ such that $AP = PQ = QB$. Then the

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mid-point of PQ is
(a) (1/2, 3) (b) (–1/2, 4)
(c) (2, 3) (d) (–1, 4)

10. The incentre of the triangle formed by (0, 0), (5, 12) and (16, 12) is
(a) (–7, –9) (b) (7, –9)
(c) (–7, 9) (d) (7, 9)

Test II **[Time: 10 min]**

11. The locus of the point which is at a constant distance of 5 from the point (2, 3) is
(a) $x^2 + y^2 + 2x + 3y + 5 = 0$
(b) $x^2 + y^2 + 4x + 6y + 5 = 0$
(c) $x^2 + y^2 - 4x - 6y + 8 = 0$
(d) None of these.

12. In a parallelogram $PQRS$, $P = (-1, -1)$. $Q = (8, 0)$, $R = (7, 5)$, then s =
(a) (–1, 4) (b) (–2, 4)
(c) (–2, 7/2) (d) (–3/2, 4)

16. The point of trisection of the line joining the points (–2, –19) and (5, 4) is
(a) (2, –3) (b) (1, 2)
(c) (1/3, –34/3) (d) (8/3, 11/3)

14. The ratio in which (6, –6) divides the line joining the points (1, 4) and (9, –12) is
(a) 3 : 5 (b) 5 : 3
(c) –3 : 5 (d) –5 : 3

15. If $A = (a, 0)$, $B = (-a, 0)$ and $C = (c, 0)$ then the locus of P such that $PA^2 + PB^2 = 2PC^2$ is
(a) $2x\,(c - a)^2$ (b) $2cx = c^2 - a^2$
(c) $2y = c^2 + a^2$ (d) None of these.

13. The locus of P such that $PA^2 + PB^2 = 10$ where $A = (2, 0)$ and $B = (4, 0)$ is
(a) $x^2 + y^2 + 6x + 5 = 0$
(b) $x^2 - y^2 - 6x + 5 = 0$
(c) $x^2 + y^2 - 6x + 5 = 0$
(d) None of these.

17. The locus of the point equidistant from (1, –1) and (–1, 1) is.
(a) $x + y = 0$ (b) $x - y = 0$
(c) $2y - x = 0$ (d) None of these.

18. A straight line segment AB of length a moves with its ends on the axes. The locus of the point P which divides the line in the ratio 1 : 2 is
(a) $9x^2 + 4y^2 = a^2$ (b) $9(y^2 + 4x^2) = 4a^2$
(c) $9(x^2 + 4y^2) = 4a^2$ (d) $9x^2 + 9y^2 = 4a^2$

19. The locus of the point P from which the distance to (4, 0) is double the distance from P to the x-axis is
(a) $x^2 - 3y^2 - 8x + 16 = 0$
(b) $x^2 + 3y^2 - 8x - 16 = 0$
(c) $3x^2 - y^2 - 8y + 16 = 0$
(d) None of these.

20. If the point $P(x, y)$ is equidistant from $A\,(a + b, b - a)$ and $B\,(a - b, a + b)$, then
(a) $x = y$ (b) $ax = by$
(c) $bx = ay$ (d) $xy = ab$

SOLUTIONS: ODD NUMBERED PROBLEMS

1. Ans. (d)

Solution

$$\sqrt{[(\sqrt{3}+1)-(\sqrt{3}-1)]^2+[(\sqrt{2}-1)-(\sqrt{2}+1)]^2}$$
$$= \sqrt{(2)^2+(2)^2} = 2\sqrt{2}.$$

3. Ans. (a)

Solution Mid-point of (1, –2) and (–5, 6) = (–2, 2)
Mid-point of (a) is $\left(\frac{2-6}{2}, \frac{5-1}{2}\right)$ = **(–2, 2).**

5. Ans. (b)

Solution Centroid $= \left(\frac{6-4+2}{3}, \frac{-1-3-5}{3}\right) = \left(\frac{4}{3}, -3\right)$.

7. Ans. (c)

Solution B will be mid point of AC
$\Rightarrow$ $C = \{2\,(2) + 3,\ 2\,(1) - 4\}$ = **(7, –2).**

9. Ans. (a)

Solution Mid-point of PQ = Mid-point of AB = **(1/2, 3).**

11. Ans. (c)

Solution $\sqrt{(x-2)^2+(y-3)^2} = 5$
$\Rightarrow$ $x^2 + y^2 - 4x - 6y + 8 = 0$.

13. Ans. (c)

Solution $[(x - 2)^2 + y^2] + [(x - 4)^2 + y^2] = 10$
$\Rightarrow$ $x^2 + y^2 - 6x + 5 = 0$.

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15. Ans. (b)

Solution $PA^2 + PB^2 = (x - a)^2 + y^2 + (x + a)^2 + y^2$
$= 2(x^2 + y^2 + a^2)$
$PC^2 = (x - c)^2 + y^2 = x^2 + y^2 - 2cx + c^2$
Now, $PA^2 + PB^2 = 2PC^2$
$\Rightarrow 2(x^2 + y^2 + a^2) = 2[x^2 + y^2 - 2cx + c^2]$
$\Rightarrow$ **$c^2 - a^2 = 2cx$.**

17. Ans. (b)

Solution $(x - 1)^2 + (y + 1)^2 = (x + 1)^2 + (y - 1)^2$
$\Rightarrow$ **$x - y = 0$.**

19. Ans. (a)

Solution $\sqrt{(x-4)^2 + y^2} = 2y \Rightarrow$ **$x^2 - 3y^2 - 8x + 16 = 0$.**

ANSWERS: EVEN NUMBERED PROBLEMS

2. (b)	**4.** (a)	**6.** (b)	**8.** (c)	**10.** (d)	**12.** (b)	**14.** (c)	**16.** (c)	**18.** (c)	**20.** (c)

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PART VI
Miscellany

CHAPTER 39 Data Interpretation
CHAPTER 40 Data Sufficiency
CHAPTER 41 Miscellaneous Topics

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39 Data Interpretation

INTRODUCTION

Data interpretation deals with careful reading, understanding, organising and interpreting the data provided so as to derive meaningful conclusions. It is an important section today in all competitive examinations. The data representation can be broadly classified as tables, graphs and charts.

Types of Data Representation

1. Tables It is the systematic and scientific presentation of numerical data. It helps the person to make comparisons and draw quick conclusions. Tabular presentation makes complicated information easier to understand. In a table, data is arranged systematically in columns and rows.

Illustration 1 In the Table below are given daily wages of male and female workers (in rupees) of two different factories:

YEAR	*FACTORY A*		*FACTORY B*	
	Male	*Female*	*Male*	*Female*
1997	120	120	240	200
1998	100	108	170	160
1999	80	110	120	130
2000	90	108	140	160
2001	170	130	240	260
2002	120	170	250	270

2. Line Graph A line graph depicts the variation of a quantity with respect to the two parameters calibrated on the x and y axes, respectively. In most of the cases the quantity is measured as a function of time, that is, the variation in the quantity as time changes.

In the line graph:

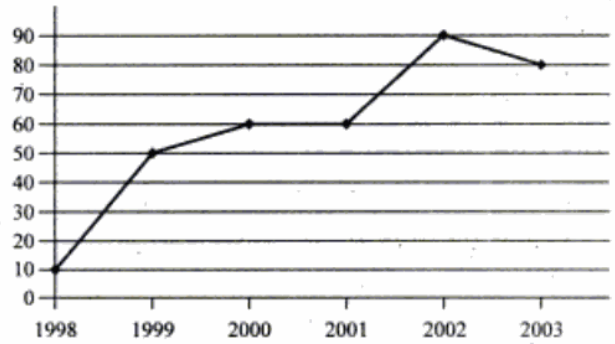
(a) x-axis represents the time parameter (may be year or month) and y-axis represents any other variable parameter which have different values with respect to time.

(b) the line going up indicates increase in the quantity with time.

(c) the line going down indicates decrease in the quantity with time.

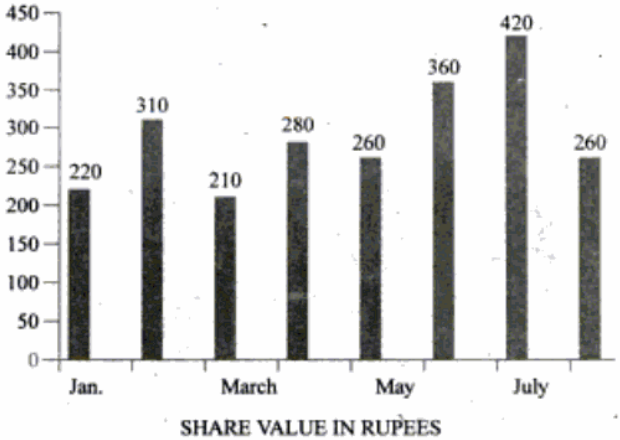
(d) a horizontal line indicates no change in the quantity over that period.

Illustration 2 The following line graph shows the number of children suffering from liver disorders in a State (in thousands).



3. Bar Graph A bar is a thick line whose width is shown merely for attention. A bar graph consists of bars. The height of the bar is a measure of the quantity that it represents. Therefore, quantities can be compared by the height of bars in the graph. Bars may be horizontal or vertical. They may be placed adjacent to each other or may be separated from each other by spaces depending upon the problem.

Illustration 3 The graph given below shows the share price (in rupees) of a company during first eight months of a year.



SHARE VALUE IN RUPEES

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Illustration 4 The graph given below shows the yearly production and sale of a company in lakh tons.

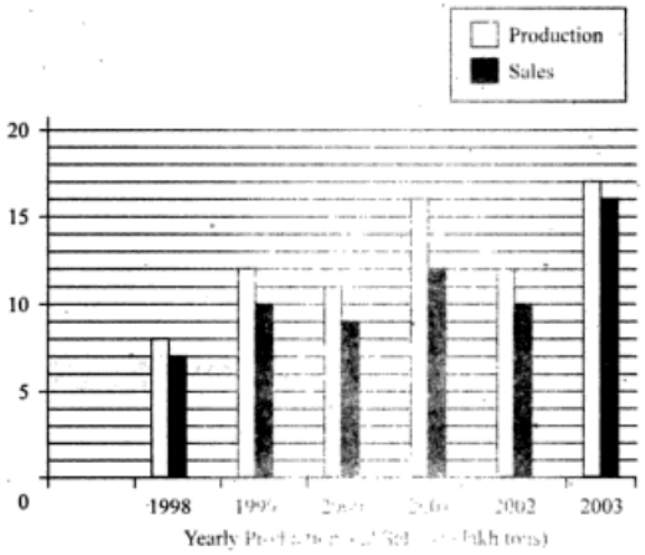
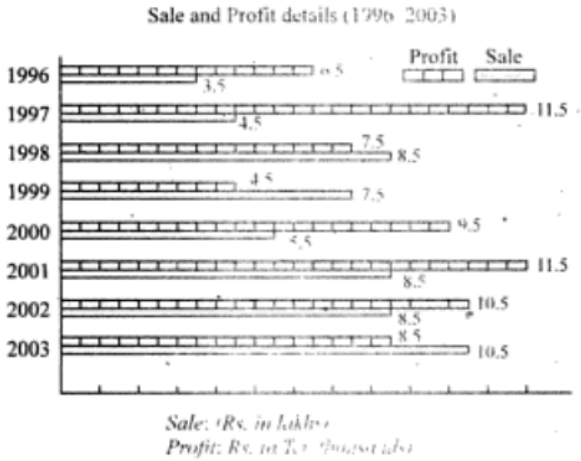


Illustration 5 The graph given below gives information about the sale and profit details of a departmental store during the years from 1996 to 2003.



4. Cumulative Bar Graph In a cumulative bar graph, the length of the bar is divided proportionately among various quantities represented in the graph. Thus, it may be conveniently used for making comparisons.

Illustration 6

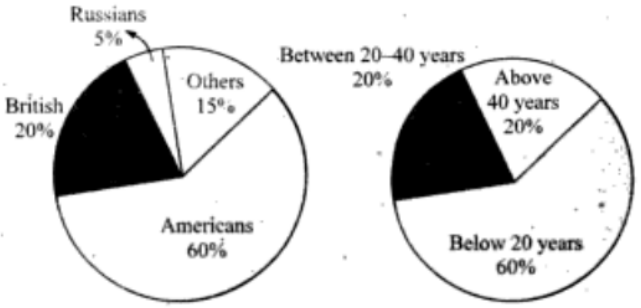
5. Pie Charts In a pie chart, the given data is distributed over a circle. Each part of the data makes a certain central angle.

For example, if from all the questions asked in a Bank PO exam, 25% are on data interpretation, then central angle made by this item

$$= \left(\frac{25}{100} \times 360\right) = 90°.$$

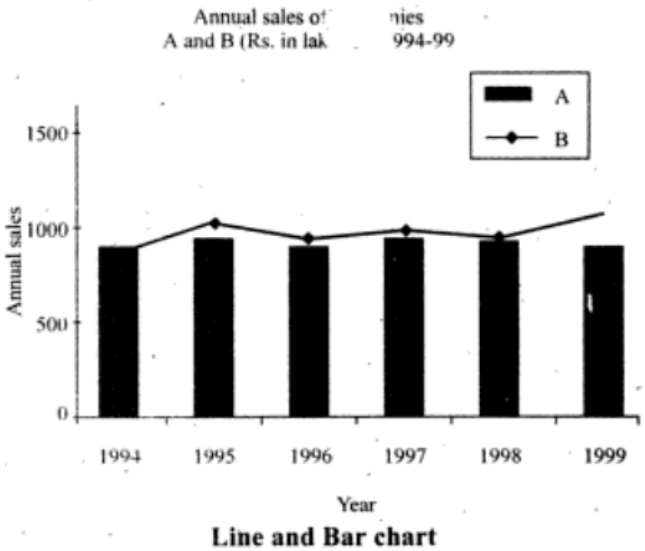
Pie charts are useful for representing percentages or proportions of various elements with respect to the total quantity. They also represent shares of various parts of a particular quantity.

Illustration 7 The following pie charts describe the characteristics of foreign tourists visiting India in a particular year.



6. Combination graph Quite often, data interpretation question is based not on a single graph but on a combination of two or more graphs. It can be line and bar, line and pie, bar and pie, cumulative and pie charts.

Illustration 8 The following graph shows annual sales of companies *A* and *B* (Rs. in lakh) for 1994–1999.



Line and Bar chart

Some Useful Instructions

1. Do not waste time memorising a Table. You can refer to it as many times as you want during the examination.

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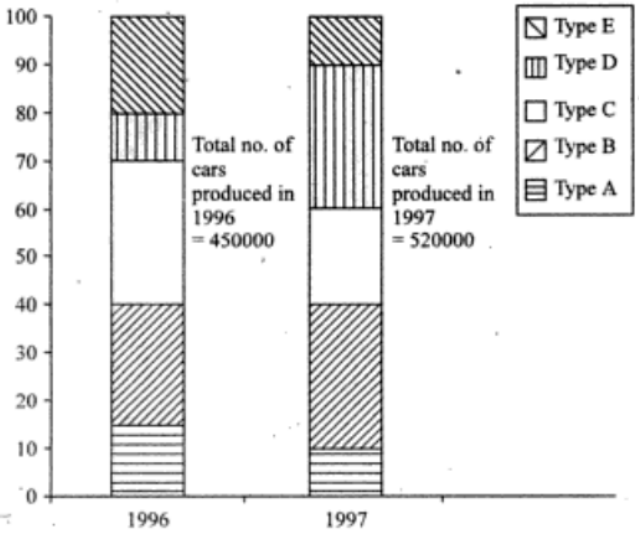
(ii) **Approximately** what was the average expenditure of the given years?
(a) Rs. 110 lakh (b) Rs. 130 lakh
(c) Rs. 120 lakh (d) Rs. 140 lakh
(e) Data inadequate.

(iii) In which of the following years was the increase/ decrease in per cent profit from the previous year the minimum?
(a) 1994 (b) 1996
(c) 1997 (d) 1995
(e) None of these.

(iv) **Approximately** what was the expenditure in 1994?
(a) Rs. 120 lakh (b) Rs. 160 lakh
(c) Rs. 140 lakh (d) Rs. 180 lakh
(e) Data inadequate.

(v) If the profit percentage in 1997 was 25, what would have been the expenditure in that year?
(a) Rs. 130 lakh (b) Rs. 148 lakh
(c) Rs. 120 lakh (d) Rs. 152 lakh
(e) None of these.

3. Directions: Study the following graph carefully and then answer the questions based on it. The percentage of five different types of cars produced by a company during two years is given below.



(i) What was the difference in the production of *C* type cars between 1996 and 1997?
(a) 5000 (b) 7500
(c) 10000 (d) 2500
(e) None of these.

(ii) If 85% of *E* type cars produced during 1996 and 1997 are being sold by the company, then how many *E* type cars are left unsold by the company?
(a) 142800 (b) 21825
(c) 29100 (d) 25200
(e) None of these.

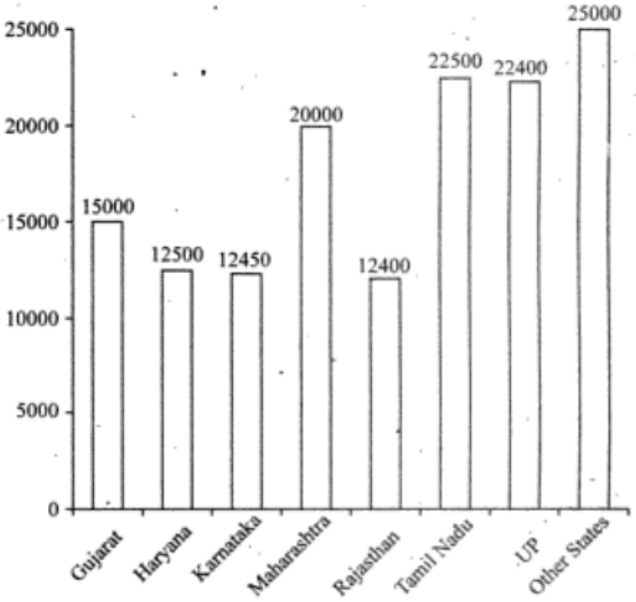
(iii) If the number of *A* type cars manufactured in 1997 was the same as that of 1996, what would have been its **approximate** percentage share in the total production of 1997?
(a) 11 (b) 13
(c) 15 (d) 9
(e) None of these.

(iv) In the case of which of the following types of cars was the percentage increase from 1996 to 1997 the maximum?
(a) A (b) E
(c) D (d) B
(e) C

(v) If the percentage production of *B* type cars in 1997 was the same as that of 1996, what would have been the number of cars produced in 1997?
(a) 112500 (b) 120000
(c) 130000 (d) Data inadequate.
(e) None of these.

4. Directions: Study the following graph carefully and answer the questions given below it.

Rose production



(i) Which of the following State(s) contribute(s) less than 10 per cent in the total rose production?
(a) Only Rajasthan
(b) Rajasthan, Karnataka
(c) Rajasthan, Karnataka, Haryana
(d) Rajasthan, Karnataka, Haryana and Gujarat
(e) None of these.

(ii) By what percentage rose production of other States is more than that of the Maharashtra?
(a) 25 (b) 30
(c) 20 (d) 15
(e) None of these.

(iii) What is the **approximate** average production of roses (in thousands) across all the states?
(a) 21 (b) 20

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(i) What is the difference between the production of company *C* in 1991 and the production of company *A* in 1996?
(a) 50000 tonnes (b) 50000000 tonnes
(c) 5000000 tonnes (d) 500000 tonnes
(e) None of these.

(ii) What is the percentage increase in production of company *A* from 1992 to 1993?
(a) 37.5 (b) 38.25
(c) 35 (d) 36
(e) None of these.

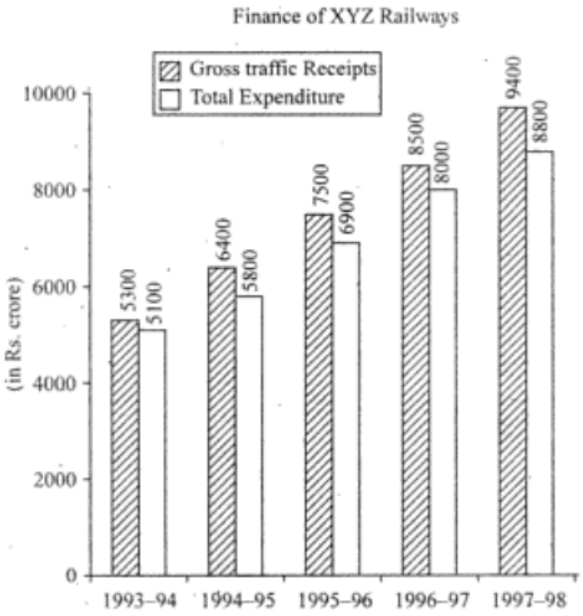
(iii) For which of the following years the percentage of rise/fall in production from the previous year the maximum for company *B*?
(a) 1992 (b) 1993
(c) 1994 (d) 1995
(e) 1996.

(iv) The total production of company *C* in 1993 and 1994 is what percentage of the total production of company *A* in 1991 and 1992?
(a) 95 (b) 90
(c) 110 (d) 115
(e) None of these.

(v) What is the difference between the average production per year of the company with highest average production and that of the company with lowest average production in lakh tonnes?
(a) 3.17 (b) 4.33
(c) 4.17 (d) 3.33
(e) None of these.

12. Directions: These questions are based on the following bar graph. Read the graph and answer the questions.

Finances of XYZ Railways



(i) What is the percentage increase in the gross traffic receipts in 1995–96 as compared to 1993–94?
(a) 33.9% (b) 41.5%
(c) 20.7% (d) 17%

(ii) If profit = gross traffic receipts – total expenditure, then in 1996–97, what percentage of gross traffic receipts is the profit made?
(a) 5.9% (b) 6.4%
(c) 7.2% (d) 8%

(iii) In which year was the profit as a percentage of gross traffic receipts the highest?
(a) 1997–98 (b) 1996–97
(c) 1995–96 (d) 1994–95

(iv) In order to make a profit of 10%, what should have been the gross traffic receipts (in Rs. crore) in 1994–95, total expenditure remaining the same?
(a) 5667 (b) 5876
(c) 6444 (d) 7667

(v) By what amount (in Rs. crore) has the expenditure increased over the period 1993–94 to 1997–98?
(a) 4100 (b) 3900
(c) 3850 (d) 3700

13. Directions: Study the following table carefully and answer the question given below it.

Number of candidates from different locations appeared and passed in a competitive examination over the years

Year	*Rural*		*Semi-urban*		*State capitals*		*Metropolises*	
	App.	*Passed*	*App.*	*Passed*	*App.*	*Passed*	*App.*	*Passed*
1990	1652	208	7894	2513	5054	1468	9538	3214
1991	1839	317	8562	2933	7164	3248	10158	4018
1992	2153	932	8139	2468	8258	3159	9695	3038
1993	5032	1798	9432	3528	8529	3628	11247	5158
1994	4915	1668	9784	4015	9015	4311	12518	6328
1995	5628	2392	9969	4263	1725	4526	13624	6419

(i) For the candidates from which of the following locations was there continuous increase both in appeared and passed?
(a) Semi-urban
(b) State capital
(c) State capital and Rural
(d) Metropolises
(e) None of these.

(ii) In which of the following years was the percentage passed to appeared candidates from semi-urban area the least?
(a) 1991 (b) 1993
(c) 1990 (d) 1992
(e) None of these.

(iii) What approximate value was the percentage drop in the number of semi-urban candidates appeared from 1991 to 1992?
(a) 5 (b) 10

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(c) 15 (d) 8
(e) 12.

(iv) In 1993 percentage of candidates passed to appeared was **approximately** 35 from which location?
(a) Rural
(b) Rural and Metropolises
(c) Semi-urban and Metropolises
(d) Rural and Semi-urban
(e) None of these.

(v) The total number of candidates passed from Rural in 1993 and Semi-urban in 1990 was exactly equal to the total number of candidates passed from State capital in which of the following years?
(a) 1990 (b) 1993
(c) 1994 (d) 1992
(e) None of these.

14. Directions: Study the following table carefully and answer the questions given below it.

Fare in rupees for three different types of vehicles

Fare for distance up to	*Vehicle*		
	Type *A*	Type *B*	Type *C*
2 km	Rs. 5.00	Rs. 7.50	Rs. 10.00
4 km	Rs. 900	Rs. 14.50	Rs. 19.00
7 km	Rs. 13.50	Rs. 24.25	Rs. 31.00
10 km	Rs. 17.25	Rs. 33.25	Rs. 41.50
15 km	Rs. 22.25	Rs. 45.75	Rs. 56.50
20 km	Rs. 26.00	Rs. 55.75	Rs. 69.00

Note: Fare per km for intermittent distance is the same.

(i) Shiv Kumar has to travel a distance of 15 km in all. He decides to travel equal distance by each of the three types of vehicles. How much money is to be spent as fare?
(a) Rs. 51.75 (b) Rs. 47.50
(c) Rs. 47.25 (d) Rs. 51.25
(e) None of these.

(ii) Ajit Singh wants to travel a distance of 15 km. He starts his journey by Type *A* vehicle. After travelling 6 km, he changes the vehicle to Type *B* for the remaining distance. How much money will he be spending in all?
(a) Rs. 42.25 (b) Rs. 36.75
(c) Rs. 40.25 (d) Rs. 42.75
(e) None of these.

(iii) Mr. *X* wants to travel a distance of 8 km by Type *A* vehicle. How much more money will be required to be spent if he decides to travel by Type *B* vehicle instead of Type *A*?
(a) Rs. 16 (b) Rs. 12.50
(c) Rs. 14 (d) Rs. 13.50
(e) None of these.

(iv) Rita hired a Type *B* vehicle for travelling a distance of 18 km. After travelling 5 km, she changed the vehicle to type A. Again, after travelling 9 km by Type A vehicle, she changed the vehicle to Type *C* and completed her journey. How much money did she spend in all?
(a) Rs. 50 (b) Rs. 45.50
(c) Rs. 55 (d) Rs. 50.50
(e) None of these.

(v) Fare for 14th km by Type *C* vehicle is equal to the fare for which of the following?
(a) Type *B*—11th km (b) Type *B*—9th km
(c) Type *A*—4th km (d) Type *C*—8th km
(e) None of these.

15. Directions: Read the following table carefully and answer the questions given below it.

Details of leading openers' performance in 20 one-day cricket matches

Openers	Total Runs	Highest Runs	No. of matches with runs		
			100 or more	50–99	0's
A	994	141	5	3	1
B	751	130	1	8	2
C	414	52	—	2	2
D	653	94	—	4	1
E	772	85	—	7	—

(i) What is the difference between the **average runs** of top two openers in terms of **highest runs**, if matches having 0's were ignored?
(a) 4.7 (b) 13.7
(c) 11.1 (d) 16.62
(e) None of these.

(ii) If matches having zero runs and the one with highest runs is ignored, what will be the average runs for opener *C*?
(a) 21.29 (b) 21.79
(c) 20.7 (d) 21.17
(e) 20.19.

(iii) By how much the difference between the two highest total runs differs from the difference between the two lowest total runs?
(a) Lower by 18 (b) More by 18
(c) Lower by 4 (d) More by 4
(e) None of these.

(iv) Which of the given pairs of openers have ratio 3 : 2 in their highest runs?
(a) *B* and *D* (b) *B* and *C*
(c) *A* and *D* (d) *D* and *C*
(e) None of these.

(v) Excluding the match with the highest runs and matches with 50–99 runs, what will be the **approximate** average runs for opener *B*?

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Type	Metro H				
	Colour				
	Black	Red	Blue	White	Silver
A	45	32	40	60	20
B	30	37	39	81	35
C	40	42	41	6	37
D	35	39	37	90	42
E	50	44	43	77	22
F	47	34	45	87	17

(i) The difference between the white-coloured cars sold in the two metros of which of the following models is the **minimum**?
(a) *A* (b) *C*
(c) *D* (d) *F*
(e) None of these.

(ii) The total number of blue-coloured cars of Model *E* and *D* sold in Metro *H* is exactly equal to the number of white-coloured cars of which model in Metro *M*?
(a) *B* (b) *F*
(c) *C* (d) *A*
(e) None of these.

(iii) What is the difference between the number of blue-colour cars of model '*C*" sold in Metro *M* and number of red colour cars of model '*F*" sold in Metro *H*?
(a) 8,000 (b) 10,000
(c) 12,000 (d) 15,000
(e) None of these.

(iv) The total number of silver-coloured cars sold in Metro *H* is approximately what percentage of that in Metro *M*?
(a) 130 (b) 140
(c) 90 (d) 100
(e) 110.

(v) In Metro *M* the number of cars sold was maximum for which of the colour-model combinations?
(a) White-*C* (b) Blue-*B*
(c) Silver-*B* (d) White-*D*
(e) None of these.

21. Directions: Study the following Table to answer the given questions.
Number of students of different classes of a school playing different games

Class → Games ↓	XII	XI	X	IX	VIII	VII	VI
Chess	11	12	5	4	2	2	1
Cricket	38	40	12	17	25	18	20
Basketball	11	9	7	6	0	0	0
Table Tennis	9	9	21	19	11	9	0
Football	40	27	18	19	12	16	14
Carrom	16	15	8	19	12	16	14
Tennis	8	9	11	5	6	0	0
Badminton	47	39	33	21	19	0	0

(i) **Approximately** what per cent of Class VIII students play Cricket out of the total students playing Cricket?
(a) 13 (b) 4
(c) 25 (d) 15
(e) 17.

(ii) What is the ratio of the students playing Football in Class XI to those in Class *X*?
(a) 1 : 2 (b) 2 : 5
(c) 2 : 3 (d) 3 : 2
(e) None of these.

(iii) Which game is the most popular?
(a) Badminton (b) Football
(c) Carrom (d) Table Tennis
(e) Cricket.

(iv) **Approximately** what per cent of Class *X* students play Table Tennis out of the total Class *X* students playing the different given games?
(a) 20 (b) 21
(c) 27 (d) 26
(e) 18.

(v) Which game has ascending number of students from class IX to XII?
(a) Basketball Only
(b) Badminton Only
(c) Chess and Badmi.on
(d) No game
(e) None of these.

22. Directions: A Table showing the percentages of the total population of a State by age groups for the year 1991 is given below. Answer the questions given below it.

Age Group (in years)	*Per cent*
up to 15	30.00
16–25	17.75
26–35	17.25
36–45	14.50
46–55	14.25
56–65	5.12
66 and above	1.13
Total	100.0

(i) Which age group accounts for the maximum population in the State?
(a) 16–25 (b) 26–35
(c) 36–45 (d) 56–65

(ii) Out of every 4200 persons, the number of persons below 26 years is
(a) 2006 approx. (b) 1260 approx.
(c) 746 approx. (d) 515 approx.

(iii) There are 200 million people below 36 years. How many million (approx) people are in the age group 56–65?
(a) 30.07 (b) 15.75
(c) 12.72 (d) 59.30

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(i) What is the ratio between the number of students studying Arts from Institute *E* and the number of students studying Commerce from Institute *D*, respectively?
(a) 17 : 19 (b) 19 : 27
(c) 14 : 19 (d) 19 : 21
(e) None of these.

(ii) What is the total number of students studying Arts from Institutes *A* and *G* together?
(a) 1102 (b) 918
(c) 966 (d) 1130
(e) None of these.

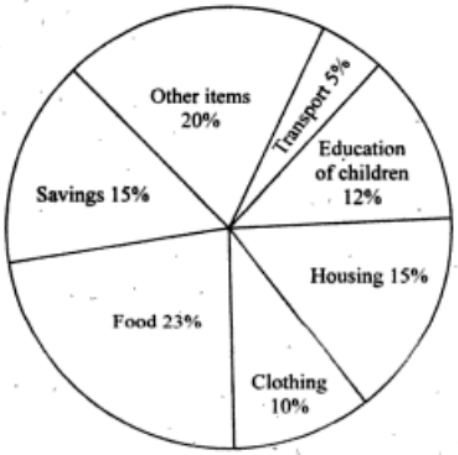
(iii) How many students are studying Commerce from Institutes *B* and *D* together?
(a) 1158 (b) 1302
(b) 1232 (d) 1272
(e) None of these.

(iv) How many students are studying Art and Commerce from Institute '*B*'?
(a) 1418 (b) 2000
(c) 1018 (d) 1208
(e) None of these.

(v) What is the ratio between the number of students studying Arts and Commerce, respectively from Institute '*E*'?
(A) 19 : 27 (b) 17 : 29
(c) 19 : 29 (d) 17 : 27
(e) None of these

29. Directions: The pie-chart drawn below shows the expenses of a family on various items and its savings during the year 2001. Study the graph and answer the questions given below:

Percentage of Money Spent on Various Items and Savings by a Family during 2001



(i) Maximum expenditure of the family was on
(a) Food (b) Housing
(c) Education of Children (d) Other items

(ii) The total savings of the family for the year were equal to the expenditure on
(a) Food (b) Clothing
(c) Housing
(d) Other items including transport.

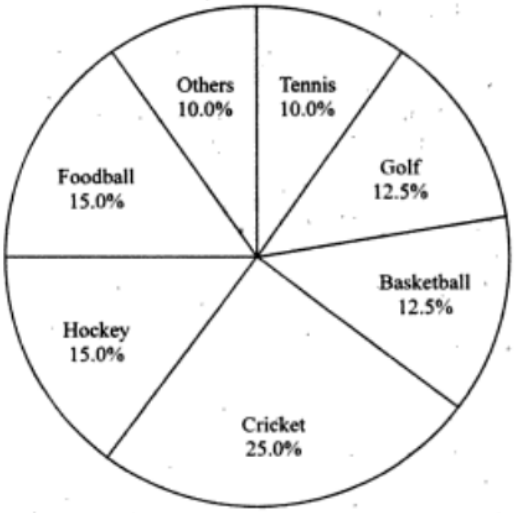
(iii) What per cent of the income was spent on transport and other items together?
(a) 25% (b) 20%
(c) 30% (d) 32%

(iv) If the total income of the family was Rs. 100000, how much money was spent on the eduction of children?
(a) Rs. 10000 (b) Rs. 12000
(c) Rs. 15000 (d) Rs. 23000

(v) If the total income for the year was Rs. 100000, the difference of the expenses (in rupees) between housing and transport was
(a) 15000 (b) 12000
(c) 7000 (d) 10000

30. Directions: The circle graph given here shows the spendings of a country on various sports during a year. Study the graph carefully to answer these questions.

Percent of Money Spent on Various Sports for One Year



(i) If the total amount spent on sports during the year was Rs. 15000000, then the amount spent on Cricket and Hockey together was
(a) Rs. 6000000 (b) Rs. 5000000
(c) Rs. 3750000 (d) Rs. 7500000

(ii) If the total amount spent during the year was Rs. 1,20,00,000 how much was spent on basketball?
(a) Rs. 12,50,000 (b) Rs. 10,00,000
(c) Rs. 12,00,000 (d) Rs. 15,00,000

(iii) The ratio of the total amount spent on football to that spent on Hockey was
(a) 1 : 15 (b) 1 : 1
(c) 15 : 1 (d) 3 : 2

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(a) Rs. 9.78 cr (b) Rs. 97.8 lakhs
(c) Rs. 52.8 lakhs (d) Rs. 5.28 cr
(e) None of these.

(vii) The cost of production of both items together by company E is equal to the total cost of production of both items together by which of the two companies?
(a) C and D (b) B and G
(c) A and D (d) C and F
(e) A and B.

(viii) What is the total of the cost of production of item I by company A and the cost of production of item II by company B?
(a) Rs. 2.6 cr (b) Rs. 26 lakh
(c) Rs. 3. 35 cr (d) Rs. 33. 65 lakh
(e) None of these.

33. Directions: Study the following information to answer the given questions.
Percentage of students in various courses (A, B, C, D, E, F) in pie chart I and percentage of girls in pie chart II.
Total students: 1200 (800 girls + 4000 boys)

Chart I

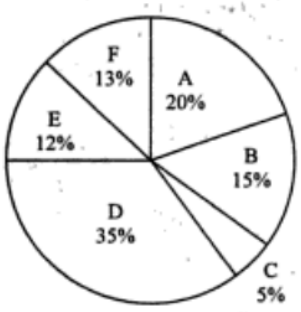
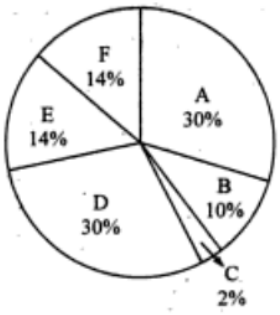


Chart II



(i) For course D, what is the respective ratio of boys and girls?
(a) 3 : 4 (b) 4 : 5
(c) 3 : 5 (d) 5 : 6
(e) None of these.

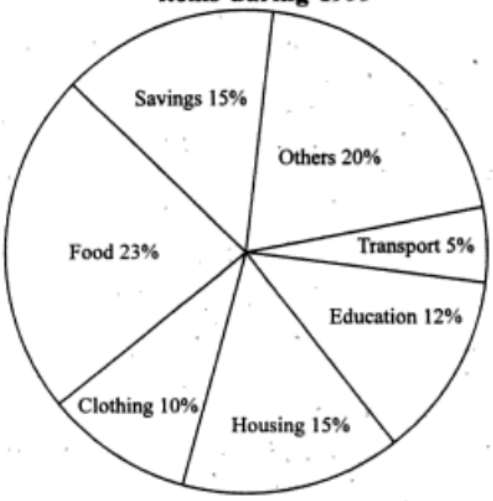
(ii) For which pair of courses is the number of boys the same?
(a) E and F (b) A and D
(c) C and F (d) B and D
(e) None of these.

(iii) For course E, the number of girls is how much per cent more than the number of boys for course E?
(a) 250 (b) 350
(c) 150 (d) 80
(e) None of these.

(iv) For which course is the number of boys the minimum?
(a) E (b) F
(c) C (d) A
(e) None of these.

(v) How many girls are there in course C?
(a) 44 (b) 16
(d) 40 (e) 160
(e) None of these.

34. Directions: The circle graph given here shows the spendings by a family on various items during the year 1999. Study the graph and answer these questions.
Per cent of money spent by a family on various items during 1999



(i) If the total amount spent during the year 1999 was Rs. 46000, the amount spent on food was
(a) Rs. 2000 (b) Rs. 10580
(c) Rs. 23000 (d) Rs. 2300

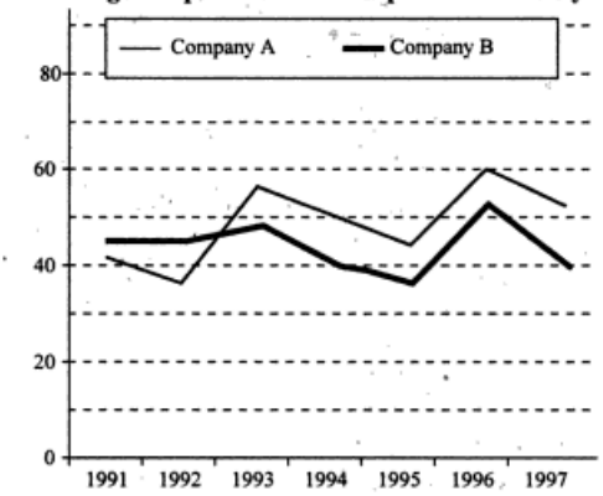
(ii) If the total amount spent was Rs 46000, how much money was spent on clothing and housing together?
(a) Rs. 11500 (b) Rs. 1150
(c) Rs. 10000 (d) Rs. 15000

(iii) The ratio of the total amount of money spent on housing to that spent on education was
(a) 5 : 2 (b) 2 : 5
(c) 4 : 5 (d) 5 : 4

(iv) Graph shows that the maximum amount was spent on
(a) Food (b) Housing
(c) Clothing (d) Others

(v) If the total expenditure of the family for the year 1999 was Rs. 46000, the family saved during the year
(a) Rs. 1500 (b) Rs. 15000
(c) Rs. 6900 (d) Rs. 3067 approx.

35. Directions: Study the following graph carefully and answer the questions given below.
Percentage net profit of two companies over the years



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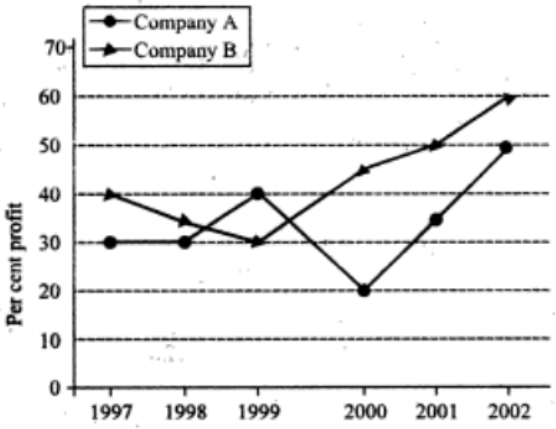
(a) Rs. 14.5 lakh (b) Rs. 15.4 lakh
(c) Rs. 15.8 lakh (d) Rs. 14.6 lakh
(e) None of these.

(iv) Income of company '*A*' in 1995 is equal to the investment of the company '*B*' in 1996. What is the ratio of the investment of company '*A*' in 1995 to the investment of company '*B*' in 1996?
(a) 31 : 36 (b) 31 : 20
(c) 20 : 31 (d) Data inadequate
(e) None of these.

(v) Investment of company '*B*' in 1993 was Rs. 1540000. What was its income in that year?
(a) Rs. 23. 33 lakh (b) Rs. 22.33 lakh
(d) Rs. 22. 23 lakh (d) Rs. 23.23 lakh
(e) None of these.

38. Directions: Study the following graph to answer the given questions.

Percent profit earned by two companies over the given years



(i) If the expenditure of Company *B* in 2000 was Rs 200 crore, what was its income?
(a) Rs. 240 crore (b) Rs. 220 crore
(c) Rs. 160 crore (d) Cannot be determined
(e) None of these.

(ii) If the income of Company *A* in 2002 was Rs 600 crore, what was its expenditure?
(a) Rs. 360 corer (b) Rs. 480 corer
(c) Rs. 375 corer (d) Cannot be determined
(e) None of these.

(iii) If the income of Company *B* in 1998 was Rs. 200 crores, what was its profit in 1999?
(a) Rs. 21.5 corer (b) Rs. 153 crore
(c) Rs. 46.15 crore (d) Cannot be determined
(e) None of these.

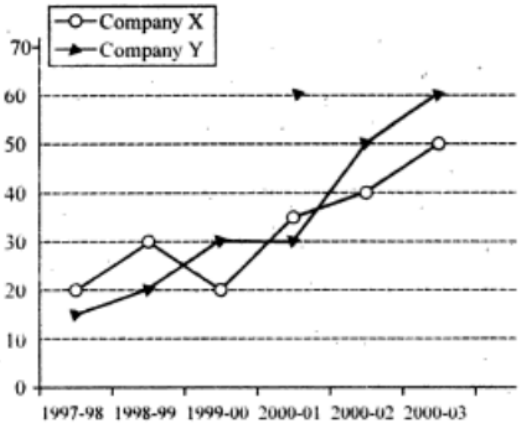
(iv) If the income of the two companies in 1998 were equal, what was the ratio of their expenditures?
(a) 1 : 2 (b) 26 : 27
(c) 100 : 67 (d) Cannot be determined
(e) None of these.

(v) What is the percent increase in profit for company *B* from year 2000 to 2001?
(a) 75 (b) 175
(c) 42.86 (d) Cannot be determined
(e) None of these.

39. Directions: Study the following graph to answer the given questions.

Per cent profit earned by two companies over the given years

$$\textbf{\% profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$



(i) If the income of Company *X* in 1998–99 was equal to the expenditure of Company *Y* in 2001–2002, what was the ratio of their respective profits?
(a) 13 : 15 (b) 15 : 26
(c) 13 : 26 (d) Cannot be determined
(e) None of these.

(ii) For Company *X*, its income in 2001–2002 was equal to its expenditure in 2002–2003, what was the ratio of its respective incomes in these two years?
(a) 4 : 5 (b) 3 : 4
(c) 2 : 3 (d) Cannot be determined
(e) None of these.

(iii) For Company *Y*, in which year is the per cent of increase in per cent profit over that of previous year the highest?
(a) 2002–03 (b) 1999–2000
(c) 2001–02 (d) Cannot be determined
(e) None of these.

(iv) In 1997-98, the expenditure of Company *X* was Rs. 40 crores. What was its income in that year?
(a) Rs. 50 crore (b) Rs. 48 crore
(c) Rs. 46 crore (d) Cannot be determined
(e) None of these.

(v) What was the difference in the expenditures of the two companies in 1999–2000?
(a) 10 (b) 100
(c) 1000 (d) Cannot be determined
(e) None of these.

(vi) In 2002–03 the income of Company *Y* was Rs. 128 crores. What was its expenditure in that year?

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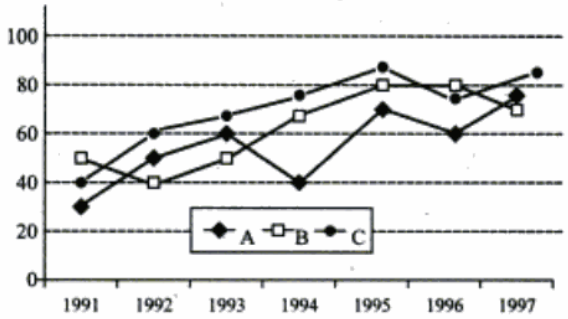
(a) Rs. 76.8 crore (b) Rs. 64 crore
(c) Rs. 48 crore (d) Cannot be determined
(e) None of these.

40. Directions: Study the following graph carefully and answer the questions given below it.

Imports of 3 companies over the years
(Rs in crore)



(i) In which of the following years, the imports made by Company *A* was exactly equal to average imports made by it over the given years?
(a) 1992 (b) 1993
(c) 1994 (d) 1995
(e) None of these.

(ii) In which of the following years was the difference between the imports made by Company *B* and *C* the maximum?
(a) 1995 (b) 1994
(c) 1991 (d) 1992
(e) None of these.

(iii) In which of the following years was the imports made by Company *A* exactly half of the total imports made by Company *B* and *C* together in that year?
(a) 1992 only (b) 1993 only
(c) 1992 and 1993 (d) 1995 only
(e) None of these.

(iv) What was the percentage increase in imports by Company *B* from 1992 to 1993?
(a) 10 (b) 25
(c) 40 (d) 20
(e) None of these.

(v) In which of the following years was the total imports made by all the three companies together the maximum?
(a) 1996 only (b) 1997 only
(c) 1995 only (d) 1995 and 1997 only
(e) None of these.

EXPLANATORY ANSWERS

1. (i) (e)

(ii) (a) Required difference
$= 60 - 50 = 10{,}000$ tonne.

(iii) (d) Percentage increase in production
$= \frac{15}{25} \times 100 = 60\%$

(iv) (c) Average production
$= \frac{25+40+60+45+65+50+75+80}{8}$
$= \frac{440}{8} = 55.$

(v) (c) Required percentage drop
$= \frac{60-45}{60} \times 100 = \mathbf{25\%.}$

2. (i) (e)

We can use the direct formula for

$$\text{Profit} = \text{Income}\left[1 - \frac{100}{100 + \%\ \text{Profit}}\right]$$

We see that the profit is maximum in 1998.

(ii) (b) Total expenditure
$= 120 \times \frac{100}{107.5} + 160 \times \frac{100}{115} + 130 \times \frac{100}{122.5}$
$+ 170 \times \frac{100}{117.5} + 190 \times \frac{100}{120} + 150 \times \frac{100}{127.5}$
= Rs. 777.51 lakh

$\therefore$ Average $= \frac{777.51}{6} \approx$ Rs 130 lakh.

(iii) (a) Per cent profit increase/decrease from the previous year

1994	1995	1996	1997	1998
100	50	(–) 22.22	14.28	37.5

(iv) (c) Expenditure in 1994 $= 160 \times \frac{100}{115} \approx 140$ lakh.

(v) (d) Expenditure in 1997 $= 110 \times \frac{100}{125} =$ **Rs. 152 lakh.**

3. (i) (a)

Production of *C* type cars in 1996
$= (70 - 40)\%$ of 450000
$= 30\%$ of $450000 = 135000$

Production of *C* type cars in 1997
$= (65 - 40)\%$ of 520000
$= 25\%$ of $520000 = 130000$

$\therefore$ Required difference = 5000.

(ii) (e) Production of *E* type cars in 1996
$= (100 - 80)\%$ of 450000
$= 20\%$ of $450000 = 90000$

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And in 1997 = 10% of 520000 = 52000

∴ Total production = 90000 + 52000 = 142000

∴ Required no. of cars = 15% of 142000 = 21300

(iii) (b) Production of *A* type cars in 1997 = production of *A* type cars in 1996 (given) = (100 – 85) 15% of 450000 = 67500

∴ Reqd. percentage = $\frac{67500}{520000} \times 100 \approx 13$.

(iv) (c) Clearly, by visual inspection *D* is the desired option.

(v) (c) Percentage production of *B* type cars in 1997 = that in 1996 (given)

= (40 – 15) 25% of 520000 = **130000.**

4. (i) (c)

Total rose production

= (15 + 12.5 + 12.45 + 20 + 12.4 + 22.5 + 22.4 + 25) × 1000

= 142250

Percentage production of rose in the States (the lowest four states)

Rajasthan	Karnataka	Haryana	Gujarat
8.71	8.75	8.78	10.54

(ii) (a) Required percentage

$$= \frac{25-20}{20} \times 100 = 25\% \text{ (more)}$$

(iii) (d) Total production of rose by all the States = 142250

∴ Average = $\frac{142250}{8\times1000} \approx 18$ thousand.

(iv) (b) Required percentage

$$= \frac{25}{142.25} \times 100 \approx 20\%.$$

(v) (e) It is 36.8% approximately.

5. (i) (a)

Reqd. % = $\frac{5.1}{12.5} \times 100 = 40.8\%$.

(ii) (e) Percentage increase

$$= \frac{38.8-11.8}{11.8} \times 100 \approx 225\%.$$

(iii) (b)

(iv) (d) Reqd. ratio = $\frac{7.4}{16.8} = 37 : 84$.

(v) (a) Reqd. % = $\frac{38.8}{63.9} \times 100 \approx 60\%$.

(vi) (e) Reqd. % = $\frac{101.80}{138.50} \times 100 \approx 75\%$.

(vii) (c) Reqd. % = $\frac{1.8}{74.6} \times 100 = \mathbf{2.41\%}$.

6. (i) (e)

Sale of Pep-up was the maximum in the year 1989.

(ii) (a) Avg. annual sale of Dew-drop

$$= \frac{10+15+25+15+30+25}{6} = 20 \text{ lakh}$$

Avg. annual sale of Cool-sip

$$= \frac{25+7+20+20+25+30}{6} = 21.16 \text{ lakh}$$

Avg. annual sale of Pep-up

$$= \frac{30+35+30+25+20+20}{6}$$

= 26.66 lakh.

(iii) (c) Reqd. % = $\frac{25-20}{20} \times 100 = 25\%$.

(iv) (e) Reqd. no. = 30 – 20 = 1000000.

(v) (c) Reqd. % drop = $\frac{35-30}{35} \times 100 \approx \mathbf{14\%}$.

7. (i) (b)

Obvious from the chart.

(ii) (d) Total imports in the given years

= 35 + 30 + 40 + 50 + 55 + 60 + 45

= 315 crore

Total exports in the given years

= 40 + 45 + 35 + 40 + 60 + 50 + 55

= 325 crore

Hence, required ratio = $\frac{315}{325} = \frac{63}{65}$.

(iii) (c) Obvious from the chart.

(iv) (e) Total exports in the years 1995,1996 and 1999

= 35 + 40 + 55 = 130 crore

Total imports in the years 1995, 1996 and 1999

= 40 + 50 + 45 = 135 crore

Now required % = $\frac{130\times100}{135} = 96.29\%$.

(v) (a) If you calculate approximate value you reject (b), (c) and (d). Now check (*a*).

In 1996,% increase in export

$$= \frac{5}{35}\ 100 = \frac{100}{7} = \mathbf{14.29\%}.$$

8. (i) (d)

$$\frac{(110+60+110+100+105+85)=570}{6}$$

= 95 lakh tons

(ii) (c) Average production of units *A*, *B* and *C* in 2001 [use white bars]

$$= \frac{(90+75+100)=265}{3}$$

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For 1990: $\frac{7894}{2513} \Rightarrow$ Quotient = 3 and Remainder ≈ 300

For 1991: $\frac{8562}{2933} \Rightarrow Q = 3$ and $R \approx 400$

For 1992: $\frac{8139}{2468} \Rightarrow Q = 3$ and $R \approx 800$

Similarly, for 1993, 1994, 1995, Q is 2.

So, 1992 gives the highest value.

(iii) (a) $\frac{8562-8139}{8562} \times 100 = \frac{423}{8562} \times 100 \approx \frac{42}{84} \times 10 = 5$

(iv) We do not need to calculate the values for each year. Follow as:

For Rural area: 35% of 5032 ≈ 35 × 50 ≈ 1750 ≈ 1798

For Semi-urban area: 35% of 9500 ≈ 35 × 95 ≈ 3300

Which cannot be approximated to 3500.

For State capitals: 35 × 85 ≈ 3000

For Metropolises: 35 × 110 ≈ 3850

(v) (c) 1798 + 2513 = **4311.**

14. (i) (d)

Distance to be travelled by each type of vehicle

$$= \frac{15}{3} = 5 \text{ km}$$

Since, to travel 5 km by vehicle A, he will pay Rs. 9 for 4 km and for the next 1 km he will have to pay Rs. $\frac{13.5-9.00}{(7-4)} \times 1$.

Similarly, for other cases.

Fare by A = Rs. $9 + \frac{13.50-9}{7-4} = 9 + 1.50 =$ Rs. 10.50

Fare by $B = 14.50 + \frac{24.25-14.50}{7-4}$

$= 14.50 + 3.25 = 17.75$

Fare by $C = 19 + \frac{31-19}{3} = 19 + 4 = 23$

Total fare = 10.50 + 17.75 + 23 = Rs. 51.25.

(ii) (a) Fare by $A = 9 + \frac{4.50}{3} \times 2 =$ Rs. 12

Fare by $B = 24.25 + \frac{33.25-24.25}{3} \times 2 =$ Rs. 30.25

Total fare = 30.25 + 12 = Rs. 42.25.

(iii) (b) Fare, for 8 km by $A = 13.50 + \frac{17.25-13.50}{10-7}$

$= 13.50 + \frac{3.75}{3} =$ Rs. 14.75

Fare by $B = 24.25 + \frac{33.25-24.25}{3} =$ Rs. 27.25

Difference = 27.25 – 14.75 = Rs. 12.50.

(iv) (e) Fare by B for 5 km = 14.50 + 3.25 = Rs. 17.75

Fare by A for 8 km $= 13.50 + \frac{17.25-13.50}{3}$

= Rs. 14.75

Fare by C for 5 km $= 19 + \frac{31-19}{3} =$ Rs. 23

Total fare = 17.75 + 14.75 + 23 = 55.50.

(v) (b) Fare for 14th km by $C = \frac{56.50-41.50}{15-10} =$ Rs. 3

Fare for 9th km by $B = \frac{33.25-24.25}{10-7} =$ **Rs. 3.**

15. (i) (e)

Avg. runs of $A = \frac{994}{19} = 52.31$

Avg. runs of $B = \frac{751}{18} = 41.72$

Difference = 52.31 – 41.72 = 10.59.

(ii) (a) Avg. runs of $C = \frac{414-52}{17} = 21.29$.

(iii) (e) Difference between two highest runs

= 994 – 772 = 222

Difference between two lowest runs

= 653 – 414 = 239

Difference = 239 – 222 = 17.

(iv) (c) Ratio of A and D = 141 : 94 = 3 : 2.

(v) (e) Without knowing the individual runs of 8 openers, we cannot find the average runs of remaining batsmen.

16. (i) (a)

Average $= \frac{52+66+64+75+58}{5} = \frac{315}{5} = 63$.

(ii) (a) The difference is 9.

(iii) (a) Percentage increase $= \frac{55-46}{46} \times 100 \approx 20\%$.

(iv) (e) Average highest marks $= \frac{85+80+75}{3} = \frac{240}{3} = 80$.

(v) (e)

(vi) (c) Required percentage $= \frac{80}{64} \times 100 = 125\%$.

(vii) (b) Marks obtained by students = 50 × 60 = 3000.

(viii) (e) The maximum difference is in the years 1992 and 1997, since the least value is in 1992 and the highest value is in 1997.

17. (i) (a)

Total no. of students studying in all schools in 1992

= (1025 + 230 + 190 + 950 + 350 + 225 + 1100 + 320 + 300 + 1500 + 340 + 300 + 1450 + 250 + 280) – (120 + 110 + 150 + 115 + 130 + 150 + 150 + 160 + 125 + 130)

= 8810 – 1340 = 7470

$\therefore$ Average $= \frac{7470}{5} =$ **1494.**

(ii) (c) Number of students studying in school B in 1994

= 950 + (350 – 150) + (225 – 115) + (185 – 110) + (200 – 90)

= 950 + 200 + 110 + 75 + 110 = 1445.

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1990 = 351.6 – 155.5 = 196.1;
1991 = 407.9 – 134.3 = 273.6
1992 = 380.1 – 149.9 = 230.2;
1993 = 439.7 – 160.5 = 279.2
1994 = 485.9 – 203.3 = 282.6

(ii) (d) Let 439.7 = K% of 351.6 $\Rightarrow K = 125$.

(iii) (a)

(iv) (d) Let 42.6 = K% of 439.7 $\Rightarrow K = \frac{4260}{439.7} = 9.7$.

(v) (d) Percent increase in the gross profit:

From 1991 to 1992: $\frac{15.6}{134.3} \times 100 = 100 = 11.6\%$

From 1992 to 1993: $\frac{10.6}{149.9} \times 100 = 7.07\%$

From 1993 to 1994: $\frac{42.8}{160.5} \times 100$ 26.67%.

24. (i) (b)

(ii) (c) Total production of rice
= 45 + 48 + 59 + 41 + 37 + 68 + 57 + 38
= 393 lakh tons

Total production of wheat
= 103 + 86 + 32 + 37 + 22 + 15 + 8 + 28
= 331 lakh tons

∴ Required proportion = 393 : 331 ≈ **12 : 1.**

(iii) (d)

(iv) (d) Total wheat production in the country = 331 lakh tons and state P alone produces 103 lakh tons of wheat

∴ Required percentage = $\frac{103}{331} \times 100 \approx 30\%$.

(v) (c) Total rice production in the country = 393 lakh tons

Yield per hectare = 30 tons

∴ Area under rice cultivation = $\frac{393}{30} \approx 13$ lakh hectares.

25. (i) (d)

146947 – (65104 + 60387) = 21456.

(ii) (b) 146947 + 11630 = 158577.

(iii) (a) 158577 – (70391 + 62516) = 25670.

(iv) (a) 153922 – (5337 +21560 + 69395) = 57630.

(v) (b) 160998 – (153922 – 5337) = **12413.**

26. (i) (c)

Ave. of the total prod. during the period = $\frac{476}{6} \approx 80$.

(ii) (d) Answer will be 1993.

(iii) (d)

(iv) (d) 25% of 80 = 20 = production of S's car in 1993.

(v) (b) Required per cent increase = $\frac{90-75}{75} \times 100 = \mathbf{20\%}$.

27. (i) (a)

Reqd. ratio = $\frac{8+9}{15+18} = 17 : 33$.

(ii) (e) Here, do not find the ratio of the no. of qualified candidates to that of the appeared. Simply check the ratio of % qualified candidates with respect to the appeared is the least for which state. Ans. = G.

(iii) (d) Reqd. difference = (21 – 13) % of 9000 = 720.

(iv) (b) Reqd. % = $\frac{(16+7)\% \text{ of } 9000}{(11+8)\% \text{ of } 45000} \times 100 = 24.21\%$.

(v) (c) Reqd. ratio = $\mathbf{\frac{(16+21)\% \text{ of } 9000}{8\% \text{ of } 45000}}$.

28. (i) (d)

Reqd. ratio = $\frac{14\% \text{ of } 3800}{14\% \text{ of } 4200} = 19 : 21$.

(ii) (e) Reqd. no = 27% of 3800 = 1026.

(iii) (b) Reqd. no. = 31% of 4200 = 1302.

(iv) (c) Reqd. no. = 8% of 3800 + 17% of 4200
= 304 + 714 = 1018.

(v) (e) Reqd. ratio = $\frac{14\% \text{ of } 3800}{17\% \text{ of } 4200} = \mathbf{19 : 17.}$

29. (i) (c)
(ii) (a)
(iii) (d)
(iv) (a)
(v) (d)

30. (i) (a)
40% of 15000000 = 6000000.
(ii) (d) 12.5% of 12000000 = 1500000.
(iii) (b) 15 : 15 = 1 : 1.
(iv) (c)
(v) (b) 12.5% in each.

31. (i) (c)
(ii) (b)
(iii) (a)
(iv) (e)
(v) (d)

32. (i) (b)

Total cost = $\frac{2}{5} \times \frac{15}{100} \times 25 + \frac{4}{5} \times \frac{22}{100} \times 25$
= 1.5 + 4.4 = 5.9 cr.

(ii) (d) Amount of profit earned by company D on item II
= $\frac{5}{8} \times \frac{8}{100} \times 25 \times \frac{25}{100} = 31.25$ lakh

(iii) (e) Cost of production of item I by company F

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$$= \frac{1}{5} \times \frac{5}{100} \times 25 = 0.25 \text{ cr}$$

Cost of production of item II by company D

$$= \frac{5}{8} \times \frac{8}{100} \times 25 = 1.25 \text{ cr}$$

$\therefore$ Reqd. % $= \frac{0.25}{1.25} \times 100 =$ **20%.**

(iv) (a) Total profit earned by company G

$$= \frac{1}{3} \times \frac{12}{100} \times 25 \times \frac{30}{100} + \frac{2}{3} \times \frac{12}{100} \times 25 \times \frac{24}{100}$$

$= 0.3 + 0.48 =$ Rs. 78 lakh.

(v) (c) Reqd. ratio $= \frac{\frac{2}{5} \times \frac{15}{100} \times 25}{\frac{3}{8} \times \frac{8}{100} \times 25} = 2 : 1.$

(vi) (b) Reqd. profit

$$= \frac{3}{5} \times \frac{11}{100} \times 25 \times \frac{32}{100} + \frac{3}{5} \times \frac{15}{100} \times 25 \times \frac{20}{100}$$

$= 0.528 + 0.450 =$ Rs. 97.8 lakh.

(vii) (d)

(viii) (a).

33. (i) (a)

Total no. of students for course D = 35% of 1200 = 420

No. of girl students for course D = 30% of 800 = 240

No. of boy students for course D = 420 – 240 = 180

Reqd. ratio = 180 : 240 = 3 : 4.

(ii) (c) No. of boys for difference courses are

$A = 0$; $B = 100$; $C = 44$; $D = 180$; $E = 32$; $F = 44$.

(iii) (a) No. of girls for course E = 14% of 800 = 112

No. of boys for course E = 32

Reqd. more % $= \frac{112-32}{32} \times 100 = 250\%$.

(iv) (d) Using the information given in Q. No. (ii)

(v) (b) No. of girls in course C = 2 % of 800 = **16.**

34. (i) (b)

23% of 46000 = Rs. 10580.

(ii) (a) 25% of 46000 = Rs. 11500.

(iii) (d) Required ratio = 15 : 12 = 5 : 4.

(iv) (a) 23%.

(v) (c) 15% of 46000 = **Rs. 6900.**

35. (i) (e)

$$\% \text{ Profit} = \frac{\text{Income} - \text{Expenditure}}{\text{Expenditure}} \times 100$$

or, $45 = \frac{140 - E}{E} \times 100$

or, $\frac{140}{E} = \frac{45}{100} + 1 = \frac{9}{20} + 1 = \frac{29}{20}$

$\therefore E = 140 \left(\frac{20}{29}\right) = 96.6$ cr.

(ii) (d) $I_{93} = E_{94}\left(\frac{100+50}{100}\right) = \frac{3}{2} E_{93}$

$I_{94} = E_{94}\left[\frac{100+40}{100}\right] = \frac{7}{5} E_{94}$

$E_{93} + E_{94} = 279$

But we cannot find $\frac{3}{2} E_{93} + \frac{7}{5} E_{94}$.

Hence, we cannot solve it.

(iii) (e) $E = I\left(\frac{100}{100+P}\right)$

or, $\frac{E}{I} = \frac{100}{100+P}$ ....(1)

We require $\frac{E}{I} \le 50\%$ or, $\frac{E}{I} \le \frac{1}{2}$

Now, from (1), $\frac{100}{100+P} \le \frac{1}{2}$

So, the value of P should be more than 100, which is not correct for any of the given years.

(iv) (b) $I = E\left[\frac{100 + \%\,\text{Pr}\,ofit}{100}\right]$

$$= 200\left(\frac{100+40}{100}\right) \text{ cr} = 280 \text{ cr.}$$

(v) (e) $I > 2E$

$\Rightarrow$ Profit % is more than 100, which is not correct for any of the given years.

36. (i) (b)

(ii) (c) Average percentage increase of the commodities

Jan	Feb	March	April	June
16.25%	23.75%	28.75%	35%	41.25%

(iii) (d)

(iv) (e) Sale of commodity C in July $= 100 \times \frac{95}{100} = 95.$

(v) (e)

37. (i) (b)

Let the investment of company B in 1996 be Rs x lakh.

$\therefore$ Investment of company B in 1997 = Rs. $\frac{7}{5}x$

Income of company B in 1997 $= \frac{9}{5} \times \frac{7}{5}x = \frac{63}{25}x$

(ii) (d) $\therefore$ Reqd. % $= \frac{63}{25} \times 100 = 252\%$

(iii) (e) Investment of company A in 1995

$$= 21.7 \times \frac{100}{155} = 14 \text{ lakh}$$

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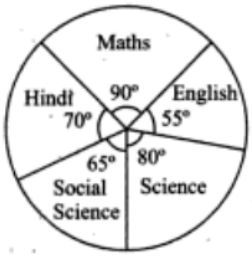
(a) Rs. 4,54,000 (b) Rs. 4,74,000
(c) Rs. 5,67,000 (d) None of these
[Based on MAT 2003]

Directions (Qs. 13 to 19): These questions are to be answered on the basis of the following pie chart which gives marks scored by a student in different subjects - English, Hindi, Mathematics, Science and Social Science in an examination. Assuming that the total marks obtained for the examination are 540, answer the following questions:



13. The marks scored by the students in Hindi and Mathematics exceed the marks scored in English and Social Science by
(a) 60 (b) 75
(c) 40 (d) 30
[Based on MAT 1999]

14. The subject in which the student scored 22.2% marks is
(a) Hindi (b) Science
(c) Social Science (d) English
[Based on MAT 1999]

15. The subject in which the student scored 105 marks is
(a) Mathematics (b) Hindi
(c) Science (d) Maths
[Based on MAT 1999]

16. The marks obtained in the three subjects : English, Science and Social Science are what percentage of the total?
(a) 45% (b) $44\frac{4}{9}\%$
(c) 55% (d) $55\frac{5}{9}\%$
[Based on MAT 1999]

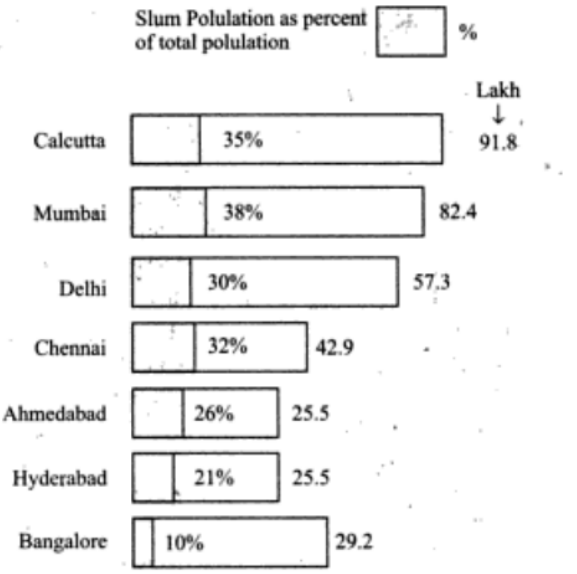
17. The marks obtained in Mathematics are what percentage of the total marks?
(a) 20% (b) 30%
(c) 35% (d) 25%
[Based on MAT 1999]

18. The difference of marks between English and Social Science is the same as between
(a) Science and Hindi
(b) Hindi and Social Science
(c) English and Hindi
(d) Hindi and Science **[Based on MAT 1999]**

19. The aggregate marks obtained by the students averaged over the 5 subjects is
(a) 72 (b) 108
(c) 90 (d) 75
[Based on MAT 1999]

Directions (Qs. 20 to 27): Study the following chart to answer these questions.

Slum Polulation in Metropolis: 1991



20. The total slum population of Calcutta in 1991 was approximately
(a) 30 lakh (b) 31 lakh
(c) 32 lakh (d) 33 lakh
[Based on MAT 2000]

21. The difference in the slum populations of Bangalore and Hyderabad was
(a) 4.1 lakh (b) 3.71 lakh
(c) 2.43 lakh (d) 2 lakh
[Based on MAT 2000]

22. The city with the highest slum population was
(a) Mumbai (b) Calcutta
(c) Delhi (d) Chennai
[Based on MAT 2000]

23. Two cities with nearly equal slum population were
(a) Ahmedabad and Hyderabad
(b) Delhi and Chennai
(c) Hyderabad and Bangalore
(d) Mumbai and Calcutta **[Based on MAT 2000]**

24. The slum population of Delhi was more than 3 times the slum population of
(a) Hyderabad (b) Ahmedabad
(c) Bangalore (d) Chennai
[Based on MAT 2000]

25. The slum population of all the seven cities nearly equalled the total population of
(a) Calcutta and Bangalore
(b) Delhi and Chennai
(c) Delhi and Hyderabad
(d) Mumbai and Ahmedabad
[Based on MAT 2000]

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45. Which company made more than 25 percent of the total investment made by all the companies in 2001?
(a) I (b) II
(c) IV (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

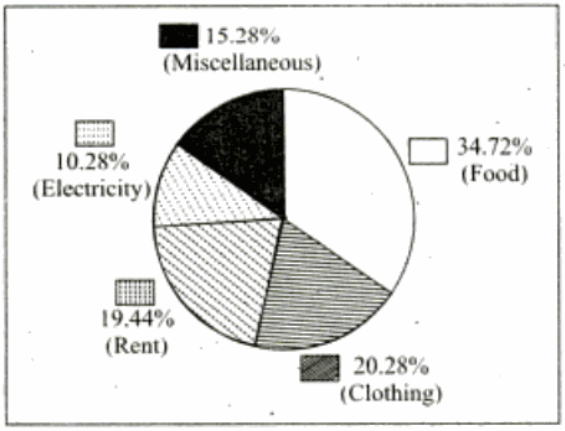
46. For which company, has the amount of investment made increased continuously over the years?
(a) I (b) II
(c) III (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

47. For which company, has the amount of investment made decreased continuously over the years?
(a) V (b) IV
(c) III (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

48. In which year, the amount of investment made was the least as compared to average investment made by all the companies over the years?
(a) 1998 (b) 1999
(c) 2000 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

49. In which year, the percentage change in the total amount of investment made is the highest over its immediately preceding year?
(a) 2002 (b) 2000
(c) 2001 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

Directions (Qs. 50 to 54): These questions are based on the following pie chart showing percentage of money spent on household items by a representative family:



50. The ratio between the money spent on Rent and Food is
(a) 1 : 2 (b) 7:5
(c) 14 : 25 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

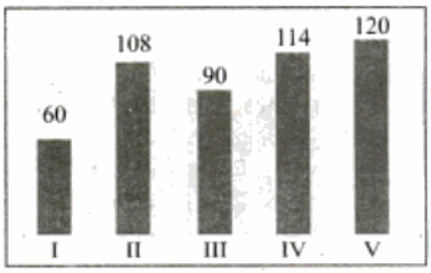
51. If the income of a family is Rs 12000 p.m., then the difference between the expenditure (in rupees) on Clothing and Electricity is nearly
(a) 8700 (b) 4400
(c) 3300 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

52. If the income of a family is Rs 20,000 p.m., then the ratio of the expenditure on Rent and Miscellaneous items is
(a) 3 : 2 (b) 7 : 5
(c) 7 : 6 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

53. If the income of a family increases by Rs 1000 p.m., then the, amount spent (in rupees) on Electricity should increase by nearly
(a) 100 (b) 200
(c) 300 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

54. If the income of a family is Rs 8000 p.m., then the approximate difference of the expenditure on Electricity and Miscellaneous items is (in percent)
(a) 18 (b) 12
(c) 5 (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

Directions (Qs. 55 to 58): These questions are based on the following bar graph showing number of employees working in five different companies:



55. Which company has the maximum number of employees?
(a) V (b) II
(c) III (d) IV
[Based on IIT Joint Man. Ent. Test 2004]

56. Which company has the minimum number of employees?
(a) I (b) II
(c) III (d) IV
[Based on IIT Joint Man. Ent. Test 2004]

57. Which company has one-and-a-half times the number of employees as compared to another company ?
(a) I (b) III
(c) V (d) None of these
[Based on IIT Joint Man. Ent. Test 2004]

58. Which company has nine-tenths of the number of employees as compared to another company?
(a) I (b) II
(c) III (d) IV
[Based on IIT Joint Man. Ent. Test 2004]

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68. The ratio expressed in decimals for weight of the brain to the weight of the muscles is
(a) 0.50 (b) 0.15
(c) 0.20 (d) 0.05
(e) None of these **[Based on IIFT 2003]**

69. The ratio expressed in decimals of the weight of the brain to the weight of the lungs is
(a) 1.5 (b) 0.15
(c) 15.0 (d) 5.1
[Based on IIFT 2003]

Directions (Qs. 70 to 74): The figures for a country's Foreign Trade for the years 1990-91 to 1996-97 are given in the following table. Answer these questions on the basis of the information given:

A Country's Foreign Trade
(Rupees in Crores)

Year	Exports	Imports	Trade Deficit
1990-91	6711	12549	5838
1991-92	7806	13608	5802
1992-93	8803	14293	5490
1993-94	9771	15831	6060
1994-95	11855	17173	5318
1995-96	10420	18371	7951
1996-97	12550	20063	7513

70. Which of the following showed an increase every year?
(a) Exports (b) Imports
(c) Trade deficit (d) All of these
[Based on FMS (Delhi) 2002]

71. The ratio of imports to exports was maximum in the year
(a) 1990-91 (b) 1996-97
(c) 1995-96 (d) 1992-93

72. The percentage increase in exports was maximum in the year
(a) 1990-91 (b) 1996-97
(c) 1994-95 (d) 1993-94
[Based on FMS (Delhi) 2002]

73. The total trade deficit for the last five years is
(a) Rs 28,508 crore (b) Rs. 32,332 crore
(c) Rs 44,322 crore (d) Rs 33,232 crore
[Based on FMS (Delhi) 2002]

74. The difference between imports and exports was maximum in the year
(a) 1995-96 (b) 1996-97
(c) 1994-95 (d) 1993-94
[Based on FMS (Delhi) 2002]

Directions (Q. 75 to 79): Study the table to answer these questions.

World Bank Aided Development Programmes
Targets, Achievements and Investments (2001–06)

Period	Number of Families Assisted (in lakh)		Funds (in Rs crores)		Credit Mobilized in Rs crores
	Target	Achievement	Allocation	Utilization	
2001–02	44	46	204	220	500
2002–03	55	57	448	460	800
2003–04	59	62	424	440	850
2004–05	51	57	395	490	900
2005–06	40	41	190	350	950

Note:
Investment = Funds Utilized + Credit Mobilized = Assistance Provided

75. When was the achievement closest to the target?
(a) 2003–04 (b) 2002–03
(c) 2004–05 (d) 2005–06
[Based on MAT 2008]

76. In which year was the allocated funds per family, minimum?
(a) 2003–04 (b) 2001–02
(c) 2004–05 (d) 2005–06
[Based on MAT 2008]

77. The achievement crosses Rs 2 cr mark in the year
(a) 2002–03 (b) 2001–02
(c) 2003–04 (d) 2004–05
[Based on MAT 2008]

78. What amount of assistance (approximately) did each family receive in 2004–05?
(a) Rs 2440 (b) Rs 1220
(c) Rs 3460 (d) Rs 4810
[Based on MAT 2008]

79. The amount of credit mobilized constituted the maximum part of investment in the year
(a) 2002–03 (b) 2001–02
(c) 2005–06 (d) 2004–05
[Based on MAT 2008]

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41. Ans. (c)

Solution $\frac{20}{100} \times 360^\circ = \mathbf{72^\circ}$.

42. Ans. (d)

Solution $\frac{24\% \text{ of } 2499.4 \text{ billion}}{20\% \text{ of } 2499.4 \text{ billion}} = \frac{\mathbf{6}}{\mathbf{5}}$.

43. Ans. (b)

Solution Total personal income in 2002 = 2499.4 billion

Total personal income in 2000 = x, say

$\therefore \quad x + 20\% \text{ of } x = 2499.4$

$\Rightarrow \quad x = \frac{5}{6} \times 2499.4 = 2082.83 = \mathbf{2100}.$

44. Ans. (d)

Solution Total income projected in the year 2007 = 150, when the income in the year 2002 was 100.

$\therefore$ Annual compond rate of growth is **8.5%.**

45. Ans. (b)

Solution $\frac{46}{179} \times 100 = \mathbf{25.70\%}.$

46. Ans. (d)

Solution For company **V**

47. Ans. (b)

Solution For company **III.**

48. Ans. (b)

Solution Average investment made by all the companies over the years $= \frac{812}{5} = \mathbf{162.4}.$

49. Ans. (b)

Solution Percentage change in the total amount of investment made in 2000 over 1999.

$$= \frac{53}{112} \times 100 = \mathbf{47.32\%}.$$

50. Ans. (c)

Solution Ratio between the money spent on rent and food

$$= \frac{19.44}{34.72} = \frac{1944}{3472} = \frac{\mathbf{14}}{\mathbf{25}}.$$

51. Ans. (d)

Solution Expenditure on clothing

= 20.28% of Rs 12000 = Rs 2433.60

Expenditure on electricity

= 10.28% of 12000 = Rs 1233.60

$\therefore$ Required difference = **Rs 1200.**

52. Ans. (d)

Solution Required ratio $= \frac{19.44}{15.28} = 1.27,$

whaterer the family ineome may be.

53. Ans. (a)

Solution Amount on electricity will increase by Rs 102.80 = **Rs 100.**

54. Ans. (c)

Solution Required difference will be 15.28% – 10.28% = 5%, whatever the family income may be.

55. Ans. (a)

Solution 120.

56. Ans. (a)

Solution 60.

57. Ans. (b)

Solution $1\frac{1}{2}$ time of company 1 = **90.**

58. Ans. (b)

Solution $\frac{9}{10}$ of company V = **108.**

59. Ans. (b)

60. Ans. (c)

Solution Percent increase in per capita income in:

1973-74 over 1972-73 $= \frac{20}{600} \times 100 = 3.3\%$

1975-76 over 1974-75 $= \frac{41.4}{617.9} \times 100 = 6.7\%$

1977-78 over 1975-76 $= \frac{34.8}{655.2} \times 100 = \mathbf{5.3\%}.$

61. Ans. (d)

Solution 1970-71 to 1971-72 (–)
1972-73 to 1973-74 (+)
1973-74 to 1974-75 (–)
1974-75 to 1975-76 (+)
1975-76 to 1976-77 (–)
1976-77 to 1977-78 (+).

62. Ans. (a)

Solution $\frac{43395 - 34412}{34412} \times 100 = \frac{898300}{34412} = \mathbf{26\%}.$

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63. Ans. (d)

Solution Total number of shares held by the shareholders = 11000470

∴ Paid up capital = Rs 110004700 crore

No. of sharesholders = 9219 ⇒ (3)

8645 shareholders have invested less than Rs 1 lakh, i.e. 93.77% of the shareholders ⇒ (2)

68 shareholders hold 7817528 shares, i.e. less than 1% of the shareholders hold 71.06% of the shares ⇒ (1).

64. Ans. (d)

Solution 8645 shareholders hold equity capital worth Rs 17764540 crore, which is 16.15% ⇒ (4) is not true.

65. Ans. (b)

Solution 51200 gm. = 51 kg 200 gm.

66. Ans. (b)

Solution $S = 10000$

∴ Weight of Liver $= 1700$

$$= \frac{S}{10000} \times 1700 = \mathbf{0.17S.}$$

67. Ans. (c)

Solution $\frac{5}{2} = \mathbf{2.5.}$

68. Ans. (d)

Solution $\frac{1500}{30000} = \frac{1}{20} = \mathbf{.05.}$

69. Ans. (a)

Solution $\frac{1500}{1000} = \frac{3}{2} = 1.5.$

70. Ans. (b)

71. Ans. (a)

Solution The ratio of imports to exports in the year

$$1990\text{-}91 = \frac{12549}{6711} = 1.87$$

$$1996\text{-}97 = \frac{20063}{12550} = 1.60$$

$$1995\text{-}96 = \frac{18311}{10420} = 1.76$$

$$1992\text{-}93 = \frac{14293}{8803} = \mathbf{1.62.}$$

72. Ans. (c)

Solution Percentage increase in exports in

$$1996\text{-}97 = \frac{12520 - 10420}{10420} \times 100 = 20.15\%$$

$$1994\text{-}95 = \frac{11855 - 9771}{9771} \times 100 = 21.33\%$$

$$1993\text{-}94 = \frac{9771 - 8803}{8803} \times 100 = 11\%.$$

73. Ans. (b)

Solution 5490 + 6060 + 5318 + 7951 + 7513

= Rs 32332 crore.

74. Ans. (a)

Solution The difference between the imports in

1995-96 = Rs 7951 crore

1996-97 = Rs 7513 crore

1994-95 = Rs 5318 crore

1993-94 = **Rs 6060 crore.**

75. Ans. (d)

76. Ans. (b)

77. Ans. (d)

78. Ans. (a)

79. Ans. (c)

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(i) What was the average marks of the five subjects of student *Q* in the 1st periodical?
(a) 32 (b) 34
(c) 40 (d) 30
(e) None of these.

(ii) What was the total of marks of student *T* in Science in all the periodicals together?
(a) 160 (b) 180
(c) 190 (d) 140
(e) None of these

(iii) The average percentage of marks obtained by student *P* in Maths in the five periodicals was exactly equal to the average percentage of marks obtained by student *R* in the five periodicals in which of the following subjects?
(a) English
(b) Geography
(c) Science and Geography
(d) Maths
(e) None of these.

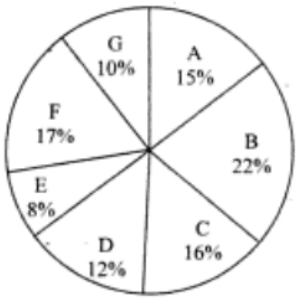
(iv) In which of the following subjects was the average percentage of marks obtained by student S the highest?
(a) Maths (b) Science
(c) History (d) Geography
(e) English

(v) In which of the periodicals the students P obtained highest percentage of marks in Geography?
(a) I (b) II
(c) III (d) IV
(e) V

3. Directions: Study the following graph carefully and answer the questions given below it.

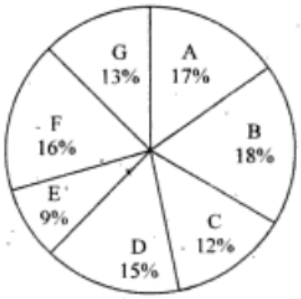
Distribution of candidates enrolled for MBA (Chart I) and the candidates who have successfully completed the course (Chart II) from different institutes

Chart I



Candidates enrolled = 2500

Chart II



Candidates who have successfully completed the course = 1900

(i) Which institute has the highest percentage of students successful to enrolled?
(a) *D* (b) *E*
(c) *A* (d) *B*
(e) None of these.

(ii) What per cent of candidates from institute *D* have come out to be successful out of the total candidates enrolled in this institute?
(a) 115 (b) 95
(c) 80 (d) 85
(e) None of these.

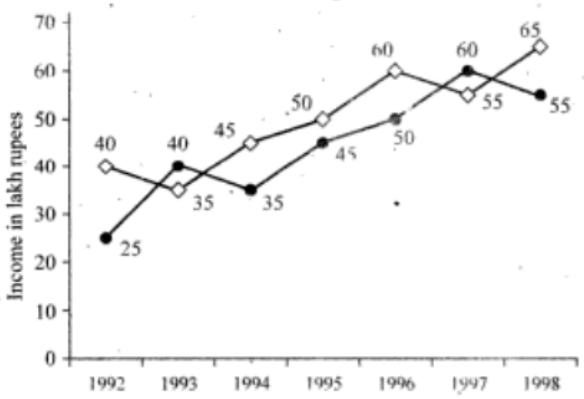
(iii) What is the approximate percentage of students successful to enrolled from institute *A*?
(a) 92 (b) 89
(c) 86 (d) 96
(e) 88.

(iv) What is ratio of candidates successful to enrolled from the institutes *B* and *C* together?
(a) 55 : 12 (b) 5 : 7
(c) 135 : 271 (d) 3 : 5
(e) None of these.

(v) What is the percentage of students successful from institute *E* and *F* together, over the students enrolled in these institutes together?
(a) 76 (b) 105
(c) 80 (d) 74
(e) None of these.

4. Directions: Study the following graph carefully and answer the questions given below.

Income of two plastic manufacturing Companies *A* and *B* over the years (in lakh rupees)



(i) If the per cent profit earned by both the companies *A* and *B* in 1997 is equal and the expenditure of company *B* in 1997 is Rs. 50 lakh, what approximately is the amount of profit earned by company *A* in 1997?
(a) Rs. 4.5 lakh (b) Rs. 5 lakh
(c) Rs. 6.2 lakh (d) Data inadequate
(e) Rs. 5.5 lakh.

(ii) For which of the following combinations of company and year is the percentage increase in income from the previous year the maximum among all such given combinations?
(a) Company *B* — 1994 (b) Company *B* — 1996
(c) Company *A* — 1993 (d) Company *A* — 1994
(e) Company *A* — 1997

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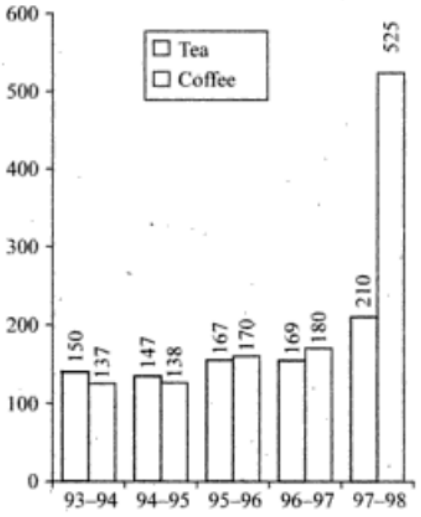
(iii) If company A had a loss of 15% in the year 1992, what approximately was its expenditure in that year?
(a) Rs. 22 lakh (b) Rs. 29 lakh
(c) Rs. 21 lakh (d) Rs. 28 lakh
(e) Rs. 23 lakh

(iv) Average income of company A per year **approximately** what percentage of the average income of company B per year?
(a) 80% (b) 110%
(c) 115% (d) 90%
(e) 75%.

(v) Income of company B in 1994 is what percent of income of company A in 1997?
(a) 75% (b) 133.33%
(c) 63.64% (d) 150%
(e) None of these.

5. Directions: Study the following graph carefully and answer the questions given below it.

Export ot Tea and Coffee (in million kg)



(i) By what **approximate percentage** the export of coffee increased from 1996 to 1998?
(a) 205 (b) 185
(c) 195 (d) 200
(e) 190.

(ii) What is the ratio of the export of coffee in 1994-95 to that in 1996-97?
(a) 69 : 85 (b) 30 : 23
(c) 85 : 69 (d) 23 : 30
(e) None of these.

(iii) What was the percentage increase in the export of tea in 1997–98 from that in 1993–94?
(a) 40 (b) 90
(c) 20 (d) 35
(e) None of these.

(iv) By what per cent did the export of tea fall in 1994–95 from that in the previous duration?
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these.

(v) What was the ratio between export of coffee to tea in 1997–98?
(a) 5 : 14 (b) 2 : 5
(c) 5 : 2 (d) 14 : 5
(e) 14 : 35.

SOLUTIONS: ODD NUMBERED PROBLEMS

1. (i) (b)

$4200 = k \times 2800 \Rightarrow k = \dfrac{4200}{2800} = \dfrac{3}{2} = 1.5$

(ii) (a) $4200 = k\%$ of $2100 \Rightarrow k = 200$.

(iii) (d) Per cent increase in deficit in 1992–93

$$= \frac{1000}{2600} \times 100 = \frac{500}{13} = 38\frac{6}{3}\%$$

In 1990–91, $\dfrac{700}{2100} \times 100 = 33\dfrac{1}{3}\%$

In 1993–94, $\dfrac{600}{3600} \times 100 = 16\dfrac{2}{3}\%$

In 1988–89, $\dfrac{900}{2200} \times 100 = 40\dfrac{10}{11}\%$.

(iv) (a) Average deficit

$$= \frac{2200+3100+2100+2800}{\dfrac{2600+3600+4200+2600}{8}}$$

$$= \frac{23200}{8} = 2900$$

$\therefore$ Required ratio = 3 : 5.

(v) (c) $3600 = k\%$ of 2900, say

$\Rightarrow k = \dfrac{3600}{29} = 124.13 \approx$ **125.**

3. (i) (e) G (98.8%).

(ii) (b)

(iii) (c)

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(iv) (d) $\frac{(342+228)}{(550+400)} = \frac{570}{950} = \frac{3}{5}$.

(v) (a) $\frac{(171+304)}{(200+425)} \times 100 = \frac{475\times100}{625} = \mathbf{76\%}$.

5. (i) (e)

Percentage increase in export of coffee in 1997–98

$$= \frac{525-180}{180} \times 100 \approx 190\%.$$

(ii) (d) Ratio of export of coffee in '94–95 and 96–97 = 138 : 180 = 23 : 30.

(iii) (a) Percentage increase in export of tea in 97–98

$$= \frac{210-150}{150} \times 100 = 40\%$$

(iv) (b) Fall in export of tea in 94–95

$$= \frac{150-147}{150} \times 100 = 2\%$$

(v) (c) Ratio of coffee to tea in 92 – 98

= 525 : 210 = **5 : 2.**

ANSWERS: EVEN NUMBERED PROBLEMS

2.	(i) (a)	(ii) (c)	(iii) (b)	(iv) (c)	(v) (b)
4.	(i) (e)	(ii) (c)	(iii) (b)	(iv) (d)	(v) (a)
6.	(i) (a)	(ii) (a)	(iii) (d)	(iv) (b)	(v) (b)
8.	(i) (c)	(ii) (e)	(iii) (a)	(iv) (a)	
10.	(i) (e)	(ii) (d)	(iii) (a)	(iv) (e)	(v) (c)

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40 Data Sufficiency

INTRODUCTION

Data sufficiency problems as the name suggests test the ability of the students to judge whether the data given in the form of statements is sufficient to answer the question asked. There is no need to solve the problem. All you need to determine is whether it would be possible to answer the question. As soon as you can tell that an answer would be obtainable, you can stop working.

In each of these problems, a question is asked followed by two or three statements. You have to study the question and all the statements given and decide whether any information provided in the statement (s) is/are redundant and can be dispensed with while answering the questions. You have to decide whether the question can be answered with any one or two of the statements or all the three statements are required to answer the question. The answer number bearing the combination of statements or single statement which is necessary to answer the question is your answer.

Illustration 1 What is the perimeter of a retangular garden?

A. The area of the garden is 2400 sq. m.
B. The diagonal of the garden is 50 m.
C. The ratio between the length and the breadth of the garden is 3 : 2.

(a) All *A*, *B* and *C* together are required
(b) Any two of *A*, *B* and *C* are sufficient
(c) Only *A* and *B* are required
(d) Only *B* and *C* are required
(e) None of these.

Ans. (b)

Solution Let the length and breadth of rectangle be l and b, respectively.

$A \rightarrow l \times b = 2400.$

$B \rightarrow l^2 + b^2 = 2500$

$C \rightarrow l = \frac{3}{2}b.$

Solving any two of the above equations, we get the values of l and b.

Illustration 2 Two friends Anu and Manu earned profit in a business. Find out their shares.

A. Anu had invested her capital for 9 months and Manu for 1 year
B. The ratio of their capitals was 4 : 3
C. The total profit was Rs. 27500

(a) Only *A* and *B* together are sufficient
(b) Only *B* and *C* together are sufficient
(c) All together are necessary
(d) Either *B* or *A* and *C* together are sufficient
(e) All even together are not sufficient

Ans. (c)

Solution From (A) and (B), we have

Ratio of profits = 9 × 4 : 12 × 3 = 36 : 36 = 1 : 1.

Now, with the help of (C), share of each of them

$$= \frac{27500}{1+1} \times 1 = \textbf{Rs. 13750.}$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

Directions (1–50): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and...

Give answer (a) If the data in statement I alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

Give answer (b) If the data in statement II alone

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are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

Give answer (c) If the data either in statement I alone or in statement II alone are sufficient to answer the question.

Give answer (d) If the data even in both statements I and II together are not sufficient to answer the question.

1. What will be the cost of the second necklace?

I. The cost of the first necklace is $\frac{1}{5}$ more than the second and the cost of the third necklace is $\frac{2}{5}$ more than the second. The total cost of all the three necklaces is Rs.120000.

II. The cost of the first necklace is $\frac{2}{5}$ more than the second. The cost of the third necklace is the least and total cost of all the three necklaces is Rs. 120000.

2. How many items did the distributor purchase?

I. The distributor purchased all the items for Rs 4500.

II. If the distributor had given Rs 5 more for each item he would have purchased 10 items less.

3. How long will it take to fill a tank?

I. One pipe can fill the tank completely in 3 hours.

II. Second pipe can empty that tank in 2 hours.

4. What will be the area of a plot in sq. metres?

I. The length of that plot is $1\frac{2}{3}$ times the breadth of that plot.

II. The diagonal of that plot is 30 metres.

5. How much minimum marks will be required to pass an examination?

I. Student *A* secured 32% marks in that examination and he failed by 1 mark. Student *B* secured 36% marks in the same examination and his marks were 1 more than the minimum pass marks.

II. Student *A* secured 30% of full marks in the examination and he failed by 2 marks. If he had secured 5 more marks his percentage of marks would have been 40%.

6. What will be the cost of painting of the inner wall of a room if the rate of painting is Rs 20 per sq.m?

I. Perimeter of the floor is 44 feet.

II. Height of the wall of the room is 12 feet.

7. What is the ratio of the number of boys and girls in a school?

I. Number of boys is 40 more than the girls.

II. Number of girls is 80 per cent of the number of boys.

8. What is the difference between two numbers?

I. First number is 60 per cent of the other number.

II. 50 per cent of the sum of first and second numbers is 24.

9. What was the speed of the running train?

I. Length of the train was 120 m.

II. The train crossed the other train whose length was 180 m in 4 sec.

10. What will be the compound interest after 3 years?

I. Rate of interest is 5 per cent.

II. The difference between the total simple interest and the total compound interest after two years is Rs 20.

11. How my boys are there in the class?

I. The class has total 45 children and ratio of boys to girls is 4 : 5.

II. The ratio of girls to boys is 4 : 5 and boys are nine more than the girls.

12. What is the average monthly income per family member?

I. Each male earns Rs 1250 a month and each female earns Rs 1050 a month.

II. Ratio of males to females in the family is 2 : 1.

13. What is the value of $m - n \div 37$?

I. m is the largest possible six-digit number and n is the smallest possible six-digit number.

II. The difference between m and n is known.

14. What selling price should be marked on the article?

I. Discount of 5% is to be given and profit percentage should be double the discount. Purchase cost is in the range of Rs 300 Rs 400.

II. 10% discount is to be allowed and 15% profit is to be obtained on the purchase cost of Rs 200 of the article.

15. What is the cost of polishing the rectangular floor?

I. Room is 9 m long and 7 m wide.

II. Cost of polishing the floor of 10 m by 5 m is Rs 112.50.

16. How many marks did Prakash obtain in Mathematics?

I. Prakash secured on an average 55 per cent marks in Mathematics, Physics and Chemistry together.

II. Prakash secured 10 per cent more than the average in Mathematics.

17. What is the rate of compound interest on a sum of money?

I. The total compound interest at the end of two years is Rs 820.

II. The total simple interest at the same rate on

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$a = \frac{6}{10}b$, II $\rightarrow$ $(a + b)$ 50% = 24 $\therefore$ $a + b = 48$

After combining these two statements we get the difference between two numbers as **12.**

9. Ans. (e)

Solution Combining both the statements, we get the speed of

$$\text{train} = \frac{180+120}{4} \times \frac{18}{5} = \textbf{270 km/hr.}$$

10. Ans. (e)

Solution I $\rightarrow$ $R = 5\%$

II $\rightarrow$ (CI – SI) for two yrs = Rs 20

Combining I and II and using

$$\text{Sum} = \frac{\text{Diff} \times 100 \times 100}{\text{Rate} \times \text{Rate}} = \frac{20 \times 100 \times 100}{25}$$

$$= \text{Rs } 8000$$

$$\text{So, CI} = 8000\left(1+\frac{5}{100}\right)^3 - 8000 = \textbf{Rs 1261.}$$

11. Ans. (c)

Solution I $\rightarrow$ No. of boys in the class $= \frac{4}{9} \times 45 = 20$

II $\rightarrow$ $\frac{B}{G} = \frac{5}{4}$ and $B - G = 9$

Solving the above two, we get $B = \textbf{45.}$

12. Ans. (e)

Solution I $\rightarrow$ $M = 1{,}250$ and $F = 1050$

II $\rightarrow$ $M : F = 2 : 1$

On combining both the statements,

$$\text{Average} = \frac{2K \times 1250 + K \times 1050}{3K} = \frac{2500+1050}{3}$$

$$= \textbf{1183.33.}$$

13. Ans. (a)

Solution I $\rightarrow$ $m = 999999, n = 1000000$

$\therefore$ $m - n + 37 = 999999 - 1000000 + 37$

$= 999999 - 2702.70 = 997296.30$

II $\rightarrow$ $m - n$ = known, but neither the value of 'm' is known nor the value of 'n' is known. So, we cannot find the value of $m - n + 37$ by this statement.

14. Ans. (b)

Solution I $\rightarrow$ The fixed value of CP is not given, so SP of the article cannot be determined.

II $\rightarrow$ Let x be SP of an article

$$x \times \frac{90}{100} = \frac{200 \times 115}{100} \therefore x = \frac{200 \times 115}{90} = \textbf{Rs 255.55.}$$

15. Ans. (e)

Solution I $\rightarrow$ Area of the room $= 9 \times 7 = 63 \text{ m}^2$

II $\rightarrow$ $\text{Rate} = \frac{\text{Cost of polishing the floor}}{\text{Area of floor}}$

$$= \frac{112.50}{10 \times 5} = \text{Rs } 2.25 \text{ per m}^2$$

$\therefore$ Combining both the statements, cost of polishing the rectangular floor = $63 \times 2.25 = \textbf{Rs 141.75.}$

16. Ans. (d)

Solution I. $M + ph + ch = 165\%$

II. $Pr \rightarrow M + 10\%$ (average)

17. Ans. (b)

Solution I. CI in two years = Rs 820

II. $\text{Rate} = \frac{250}{5000} \times 100 = \textbf{5\%.}$

18. Ans. (e)

Solution I. $x - y = \frac{1}{3}x$, or, $2x - 3y = 0$

II. $x + y = 30$

By combining I and II, we get $y = \textbf{12.}$

19. Ans. (d)

Solution I. Area of right-angled triangle $= 12 \times L$

II. $L = 18$ cm.

$\therefore$ By combing I and II we can find the area of right-angled triangle, but the height cannot be determined in absence of the base of the triangle.

20. Ans. (c)

Solution

21. Ans. (d)

Solution For solving this question, we want two equations in terms of P and Q.

22. Ans. (e)

Solution Combing both the statements together,

Let the labelled price be Rs 100.

Now, SP of the suitcase = 125% of 100 = Rs 125

$\therefore$ Labelled price $= \frac{2000}{125} \times 100 = \text{Rs } 1600$

$\therefore$ CP of the suitcase $= 1600 \times \frac{3}{4} = \textbf{Rs 1200.}$

23. Ans. (d)

Solution Here, we do not know the type of triangle. If the triangle is right-angled, then the height can be determined with the help of statement I alone.

24. Ans. (e)

Solution Combining both the statements together,

$$\text{Rate of interest} = \frac{800}{2 \times 8000} \times 100 = \textbf{5\%.}$$

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can be subtracted in the calculated area and then multiplied with the cost given in (A).

81. Ans. (d)

Solution Investment ratio or amount is not given, hence even all statements together are not sufficient.

82. Ans. (b)

Solution Let the base, height and hypotenuse of a right-angled triangle be b, p and h, respectively.

From A, $b + p + h = 30$ ...(i)

From B, $b : p = 5 : 12$ ...(ii)

We know that

$$h^2 = p^2 + b^2 = 25x^2 + 144x^2 = 169x^2$$

$\therefore \quad h = 13x$ ...(iii)

Combining equations (i), (ii) and (iii), we get

$$5x + 12x + 13x = 30 \Rightarrow x = 1$$

$\therefore$ Area of triangle $= \frac{1}{2} \times 5 \times 12 = 30\ \text{cm}^2$

Hence, only A and B together are sufficient.

83. Ans. (e)

Solution Let the bigger and smaller nos. be x and y, respectively.

From A, $x - y = 6$ ...(i)

From B, 40% of y = 30% of x or, $4y = 3x$...(ii)

From C, $\frac{x}{3} : \frac{y}{3} = 2 : 1$ or, $3x = 4y$ ...(iii)

We see that equations (ii) and (iii) are same. Hence, A and either B or C is required.

84. Ans. (e)

85. Ans. (e)

Solution **From A and B,**

Labelled price $= \frac{3{,}060 \times 100}{85} = \text{Rs } 36000$ ...(i)

Combining (i) and Statement C,

Profit $= 36000 \times \frac{2}{100} = \text{Rs } 72$ ...(ii)

Combining (ii) and Statement B,

CP = 3,060 – 72 = Rs 2988

$\therefore$ % profit $= \frac{72}{2988} \times 100 = 2.40\%$

Hence, all statements are required.

QUESTIONS FROM COMPETITIVE EXAMS

Directions (Questions 1 to 5): Each of these items consist of a question and the two statements labelled (A) and (B). Use the data given in statement (A) and (B) together with other known facts to decide whether the two statements are sufficient to answer the given question. Mark you answer as

(a) if you can data the answer from statement (A) alone but not from (B) alone.

(b) if you can get the answer from statement (B) alone but not from (A) alone.

(c) if you can get the answer from both the statements (A) alone.

(d) if you cannot get the answer from both the statements (A) and (B) together, but need even more data.

1. A car drives around a circular track once. A second car drives from point A to point B in a straight line. Which car travels faster?

(A) The car driving around the circular track takes a longer time to complete its trip than the car travelling in a straight line.

(B) The straight line from A to B is $1\frac{1}{2}$ times as long as the diameter of the circular track.

[Based on MAT 2003]

2. Is y larger than x?

(A) $x + y = 2$ (B) $\frac{x}{y} = 2$

[Based on MAT 2003]

3. Mary must work 12 hours to make, in wages, the cost of a set of luggage. How many dollars does the set of luggage cost?

(A) Jim must work 15 hours to make, in wages, the cost of the set of luggage.

(B) Jim's hourly wages are 80% of the Mary's hourly wages. **[Based on MAT 2003]**

4. A group of 49 consumers were offered a chance to subscribe to 3 magazines: A, B, and C. 38 of the consumers subscribed to at least one of the magazines. How many of the 49 consumers subscribed to exactly two of the magazines?

(A) Twelve of the 49 consumers subscribed to all the three magazines.

(B) Twenty of the 49 consumers subscribed to magazine A. **[Based on MAT 2003]**

5. Which of the four numbers w, x, y and z is the largest?

(A) The average of w, x, y and z is 25.

(B) The numbers w, x and y are each less than 24

[Based on MAT 2003]

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Directions (Q 6 to Q 10): Each question below has two statements, I and II. Mark your answer as

(a) If statement I is true, but not the other one
(b) If statement II is true, but not the other one
(c) If both the statements are true
(d) If neither of the statements is true.

6. For an equation $ax^2 + bx + c = 0$, its roots are
I. Real and different if $b^2 > 4ac$
II. Imaginary and equal if $b^2 < 4ac$
[Based on MAT 2003]

7. For an equation
$$ax^3 + bx^2 + ex + d = 0,$$
if its roots are α, β and γ, then
I. $\alpha + \beta + \gamma = c/a$
II. $\alpha\beta\gamma = d$ **[Based on MAT 2003]**

8. For a differential expression
I. $d/dx\,(\sin^2(3x)) = 2\cos(3x)$
II. $d/dx\,(a^v) = a^v(\log a)\,du/dx$
[Based on MAT 2003]

9. If $y = 2x$, then
I. $\sin y = \dfrac{2\tan x}{1+\tan^2 x}$ II. $\cos y = \dfrac{2\tan x}{1-\tan^2 x}$
[Based on MAT 2003]

10. If $z = x + iy$, where $i = (\sqrt{1})$, then
I. $1 = 0$, when $x = 0$, $y \neq 0$
II. If $a + bi = c + di$, then $a = c$, $b = d$
[Based on MAT 2003]

Directions (Qs. 11 to 15): In each of these questions, two statements I and II follow. Mark your answer as

(a) If the question can be answered by using first statement alone.
(b) If the question can be answered by using either of the statements alone.
(c) If the question can be answered by using both the statements together, but it cannot be answered by using either of the statements alone.
(d) If the question cannot be answered even by using both the statements together.

11. There are three sets A, B and C. Find $A \cap (B \cap C)$.
I. $A \cup B$ and $A \cup C$ are known.
II. $A \cap B$ and $A \cap C$ are known.
[Based on MAT 2003]

12. A moving train moves Y metres in t seconds. Find its acceleration.
I. $Y = t^3 - 4t^2 + 16t - 2$
II. Velocity at that moment was 20 m/sec.
[Based on MAT 2003]

13. Find the sum of a Geometric series 1, 3, 9, 27, 81... for N terms.
I. Nth term is 729.
II. Next term after the Nth term is thrice of it.
[Based on MAT 2003]

14. Find ${}^{25}C_{10}$.
I. ${}^{24}C_{14} = a$ II. ${}^{24}C_{9} = b$
[Based on MAT 2003]

15. Meena wants to find $\log_{70}96$.
I. She knows the value of $\log_{96}70$.
II. She knows the value of $\log_{10}70$.
[Based on MAT 2003]

Directions (Q. 16 to 20): Each of the questions below consists of a question and two statements numbered I and II given below it. Decide whether the data provided in the statements is sufficient to answer the question. Mark answer as

(a) if the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.
(b) if the data in statement I alone is sufficient to answer the question, while the date in statement II alone is not sufficient to answer the question.
(c) if the data in both statements I and II together is not sufficient to answer the question.
(d) if the data in both statements I and II together is necessary to answer the question.

16. What is the absolute difference between two numbers?
I. 60% of the bigger number is equal to the smaller number.
II. half of the bigger number is 5 less than the smaller number.
[Based on MAT 2008]

17. What is the ratio between the present ages of father and his son?
I. Ratio of the ages of father 20 years before and son 10 years before was 2 : 1.
II. Sum of the present ages of father and son is 60 years.
[Based on MAT 2008]

18. Which newspaper has the maximum circulation in Delhi?
I. 2 lakh copies of the newspaper X are sold in Delhi while the circulation of the newspapers is estimated at 3 lakh.
II. The circulation of newspaper Y is 55 per cent of the total circulation of newspapers.
[Based on MAT 2008]

19. What is the profit per cent earned by shopkeeper on selling the article in his shop?
I. Labelled price of the article sold is 110% of the cost price.
II. Cost price of the article is Rs 1200.
[Based on MAT 2008]

20. How many hours a day must 4 pumps works to empty a conical tank in 1 day?
I. 3 pumps working 8 hours a day can empty another tank in 2 days.
II. The other tank has twice the floor area and one and-a-half times the depth of the original tank.
[Based on MAT 2008]

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$$= \sin 2x = \sin y$$

II : $$\frac{2\tan x}{1-\tan^2 x} = \frac{\frac{2\sin x}{\cos x}}{\frac{\cos 2x}{\cos^2 x}} = \frac{2\sin x\cos x}{\cos 2x}$$

$$= \frac{\sin 2x}{\cos 2x} = \tan 2x = \tan y.$$

10. Ans. (b)

11. Ans. (a)

Solution $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$.

12. Ans. (c)

Solution Velocity $= v = \frac{dy}{dt} = 20$

$\Rightarrow \quad 3t^2 - 8t + 16 = 20$

$\Rightarrow \quad 3t^2 - 8t - 4 = 0$ ...(i)

Acceleration $= \frac{d^2y}{dt^2} = 6t - 8$

For values of t given by (i) Acceleration can be found.

13. Ans. (a)

Solution 7th term = 729

The series in G.P. is 1, 3, 9, 27, 81, 243, 729

$\therefore$ Sum of 7 terms of the G.P. $= S_7 = \frac{a(r^N - 1)}{r-1}$

when a = 1st term = 1

r = common ratio = 3

$\therefore \quad S_7 = \frac{3^7 - 1}{3-1} = \frac{3^7 - 1}{2} = 1093.$

14. Ans. (b)

Solution $^{25}C_{10} = \frac{25\times 24\times \ldots \times 16}{10\times 9\times \ldots \times 1} = \frac{25}{10}\left[\frac{24\times \ldots \times 16}{9\times \ldots \times 1}\right]$

$$= \frac{25}{10} \times {}^{24}C_9 = \frac{5}{2}b$$

Again $^{25}C_{10} = \frac{25\times 24\times \ldots \times 16}{10\times 9\times \ldots \times 1}$

$$= \frac{25\times(24\times 23\times \ldots \times 16)\times 15}{10\times 9\times 8\times \ldots \times 1\times 15}$$

$$= \frac{25\times(24\times 23\times \ldots \times 15)}{15\times(10\times 9\times 8\times \ldots \times 1)} = \frac{25a}{15} = \frac{5a}{3}$$

$$\left[{}^{24}C_{14} = a \Rightarrow \frac{24\times 23\times \ldots \times 15}{10\times 9\times 8\times \ldots \times 1} = a\right]$$

Hence the question can be answered by using either of the statements alone.

15. Ans. (a)

Solution $\log_{70}96 = \frac{\log 96}{\log 70} = \frac{1}{\frac{\log 70}{\log 96}} = \frac{1}{\log_{96}70}$

By (I), $\log_{96}70$ is known.

16. Ans. (d)

17. Ans. (b)

18. Ans. (c)

19. Ans. (b)

20. Ans. (d)

21. Ans. (c)

Solution $A \Rightarrow y > 0$

$B \Rightarrow y > 2$ ($y = -2$ is ruled out by A).

22. Ans. (c)

Solution 23 is not divisible by 3.

If $x + 5$ is divisible by 6, then x is not divisible by 3, e.g. 36 is divisible by 6, but 31 is not divisible by 3.

23. Ans. (d)

Solution Mohan = 2 Sohan

Sohan = 40% of (Ram + Mohan)

$$= \frac{2}{5} \text{ of (Ram + Mohan)}$$

$$= \frac{2}{5}\text{ Ram} + \frac{2}{5}\text{ Mohan}$$

$$= \frac{2}{5}\text{ Ram} + \frac{4}{5}\text{ Sohan}$$

$\Rightarrow \quad \frac{1}{5}$ Sohan $= \frac{2}{5}$ Ram

$\Rightarrow$ Sohan = 2 × Ram

$\therefore$ Mohan = 4 × Ram.

24. Ans. (b)

25. Ans. (a)

Solution $1\frac{7}{9}$ hours from A to B and $2\frac{2}{9}$ hours from B to A.

26. Ans. (c)

27. Ans. (d)

Solution 11 consumers did not subscribe to any magazine.

28. Ans. (c)

Solution 51 itself is the number.

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29. Ans. (c)

Solution $\frac{a+k}{b+k} > \frac{a}{b}$ only if a, b and k are all positive integers.

30. Ans. (a)

Solution Let r be the radius of the front wheel.

$\therefore \quad 2\pi r \times 240 = 2\pi \times 0.6 \times 80$

$\Rightarrow \quad r = \frac{0.6 \times 80}{240} = 0.2$

$\therefore$ Diameter of the front wheel is $2r = 0.4$.

31. Ans. (d)

32. Ans. (a)

Solution Year 1 is leap year

(Feb 1 of year 1 is Monday)

(May 1 of year 1 is Sunday).

33. Ans. (d)

34. Ans. (b)

Solution SQ = Diagonal of the square

= Diameter of the circle

= $2r$, where r is the radius of the circle.

(B) $\Rightarrow \pi r^2 = 2 \times SQ = 4r$

$\Rightarrow r = \frac{4}{\pi} \Rightarrow SQ = \frac{8}{\pi}$.

35. Ans. (a)

Solution Let $P = 2x$, $Q = 2x + 2$,

$R = 2x + 4$. $S = 2x + 6$.

$\therefore \quad \frac{8x+12}{4} = 11 \Rightarrow 2x + 3 = 11 \quad \Rightarrow \quad x = 4$

$\therefore$ Largest number $= 2x + 6 = 14$.

36. Ans. (d)

37. Ans. (c)

Solution R is the point of intersection of ST and RQ.

$\therefore$ (A) & (B) $\Rightarrow 3x - 4 = 17 - 6x$

$\Rightarrow 9x = 21 \Rightarrow x = \frac{7}{3}$

$\therefore \quad y = 17 - 6x = 17 - 6 \times \frac{7}{3} = 17 - 14 = 3$

$\therefore$ Point of intesection of ST and RQ is given by $R\left(\frac{7}{3}, 3\right)$.

38. Ans. (c)

Solution With three weighings, counterfeit coin can be identified.

39. Ans. (b)

40. Ans. (d)

41. Ans. (b)

Solution The third number is 9.

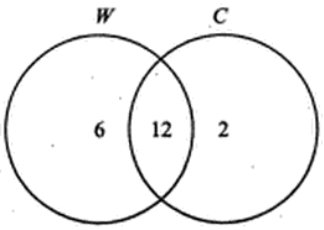
42. Ans. (b)

43. Ans. (c)

Solution Let the secretary spoke for x minutes.

$\therefore \quad 50x = 46(x + 10) \Rightarrow x = 115.$

44. Ans. (c)



45. Ans. (d)

46. Ans. (c)

Solution Using 3 distinct symbols, the number of code words which can be formed by using all the possible distinct horizontal arrangements

$= {}^3P_1 + {}^3P_2 + {}^3P_3 = 3 + 6 + 6 = 15.$

47. Ans. (b)

Solution $N = 21 = ab$

$N^2 = 441 = ccb.$

48. Ans. (a)

Solution $20 \times 22 \times 25 = 11000$ cu. ft.

49. Ans. (c)

Solution For $x = 110, 121, 132, \ldots$, infinite number of integers are of the form $\frac{7x}{11}$.

50. Ans. (a)

Solution $\frac{12}{3} \times \frac{10}{3} \times 20 = \text{Rs } \frac{800}{3}$.

51. Ans. (b)

Solution $8x = 6y \Rightarrow x = \frac{3}{4}y = 75\%$ of y.

52. Ans. (d)

Solution Let present age be x years

$\therefore \quad x - 18 = \frac{x}{4} \Rightarrow x = 24.$

OR

$x + 12 = \frac{3x}{2}$

$\Rightarrow \quad 2x + 24 = 3x \Rightarrow \quad x = 24.$

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Miscellaneous Topics 41

BINOMIAL THEOREM

1. $(x + a)^n = x^n + {}^nC_1\, x^{n-1}\, a + {}^nC_2\, x^{n-2}\, a^2 + \ldots + {}^nC_r\, x^{n-r}\, a^r + \ldots + a^n$, where n is a positive integer. This is called Binomial Theorem for +ve integral index.
2. Total number of terms in this expansion = $n + 1$ i.e. one more than the power of the Binomial.
3. General term ($(r + 1)$th term) of this expansion is $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$.
4. If the index n is even, then there is only one middle term in the expansion of $(x + a)^n$ and that term is $\left(T_{\frac{n}{2}+1}\right)$th term.
5. If the index n is odd, then there are two middle terms in the expansion of $(x + a)^n$ and those are $T_{\frac{n+1}{2}}$ and $T_{\frac{n+1}{2}+1}$.
6. If the index n is even, the greatest coefficient in the expansion of $(x+a)^n$ is ${}^nC_{\frac{n}{2}}$ and if n is odd, the greatest coefficient is ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$, both being equal.
7. If $\frac{n+1}{\frac{x}{a}+1}$ is an integer say p, then T_p, T_{p+1} are the two greatest terms and they are equal. If $\frac{n+1}{\frac{x}{a}+1}$ is not an integer and m is its integral part, then T_{m+1} is the greatest term.
8. In the Binomial expansion of $(x + a)^n$, the $(m + 1)$th term from the end = $(n - m + 1)$th term from the beginning = T_{n-m+1}.
9. To find the coefficient of x^m $(m \le n)$ in $(x + a)^n$.

   Step I. Find $T_{r+1} = {}^nC_r\, x^{n-r}\, a^r$

   Step 2. Put $n-r = m$ i.e. $r = n - m$

   Step 3. If $n - m$ is positive integer or 0 then required coefficient = ${}^nC_r\, a^r = {}^nC_{n-m}\, a^{n-m}$.

   Step 4. If $n - m$ is a negative integer, or fraction, then coefficient of $x^m = 0$.
10. Term independent of x in Binomial Expansion means the coefficient of x^0 in the expansion.

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. If n is a positive integer, then the number of terms in the expansion of $(x + a)^n$ is
   (a) n (b) $n + 1$
   (b) $n - 1$ (c) None of these
2. In the expansion of $(1 + x)^n$, T_{r+1} is
   (a) $c\,(n, r + 1)\, x^{n-1}$ (b) $c\,(n, r)\, x^{n+1}$
   (c) $c\,(n, r)\, x^r$ (d) $c\,(n, r - 1)\, x^{r+1}$
3. $(\sqrt{3} + 1)^4 + (\sqrt{3} - 1)^4$ is equal to
   (a) a rational number
   (b) an irrational number
   (c) a negative integer
   (d) None of these
4. The coefficient of x^4 in $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is

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(a) $\frac{405}{256}$ (b) $\frac{504}{259}$

(c) $\frac{450}{263}$ (d) None of these

5. The co-efficient of x^7 in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^9$ is

(a) –56 (b) 14

(c) –14 (d) None of these

6. The 4th term in the expansion of $\left(-x-\frac{1}{x}\right)^5$ (for $x > 0$) is

(a) independent of x (b) positive

(c) negative (d) the only middle term

7. The Binomial co-efficient of the 4th term in the expansion of $(x - q)^5$ is

(a) 5 (b) 20

(c) 10 (d) 15

8. If the co-efficient of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the same, then the value of a is

(a) $\frac{9}{7}$ (b) $\frac{7}{9}$

(c) $\frac{-9}{7}$ (d) $\frac{-7}{9}$

9. The terms containing x^3 in the expansion of $(x - 2y)^7$ is

(a) 3 rd (b) 4th

(c) 5 th (d) 6th

10. If the 21st and 22nd terms in the expansion of $(1 + x)^{44}$ are equal, then x =

(a) $\frac{7}{8}$ (b) $\frac{8}{7}$

(c) $\frac{-7}{8}$ (d) $\frac{-8}{7}$

11. If the coefficient of r th term in the expansion of $(1 + x)^{20}$ = co-efficient of $(r + 2)$ th term, then r equals

(a) 12 (b) 8

(c) 10 (d) 9

12. The term independent of x in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$

(c) $\frac{11}{18}$ (d) $\frac{13}{18}$

13. The term independent of y in the expansion of $(y^{1/6} - y^{-1/3})^9$ is

(a) 84 (b) 8.4

(c) –0.84 (d) –84

14. The term independent of x in the expansion of $\left(x-\frac{3}{x^2}\right)^{18}$ is

(a) ${}^{18}C_6$ (b) ${}^{18}C_6\ 3^6$

(c) ${}^{18}C_{12}\ 3^{-6}$ (d) 3^6

15. Constant term in the expansion of $\left(x-\frac{1}{x}\right)^{10}$ is

(a) –252 (b) 152

(c) 252 (d) –152

16. The middle term in the expansion of $\left(1+\frac{1}{x}\right)^{10}$ is

(a) $-{}^{10}C_5\frac{1}{x^5}$ (b) ${}^{10}C_6\frac{1}{x^6}$

(c) ${}^{10}C_5\frac{1}{x^5}$ (d) $-{}^{10}C_6\frac{1}{x^6}$

17. The fifth term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^{12}$ is

(a) $\frac{7920}{x^4}$ (b) $\frac{-7920}{x^4}$

(c) $7220\ x^{-4}$ (d) $-7520\ x^4$

18. The total number of terms in the expansion of $(x + y)^{100} + (x - y)^{100}$ after simplification is

(a) 50 (b) 51

(c) 202 (d) None of these

19. The number of terms in the expansion of $[(a + 4b)^3\ (a - 4b)^3]^2$ are

(a) 6 (b) 7

(c) 8 (d) 32

20. The sum of the co-efficients in the expansion of $(1 - x)^{10}$ is

(a) 0 (b) 1

(c) 10^2 (d) 1024

21. If the sum of the co-efficients in the expansion of $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ vanishes, then α is equal to

(a) 2 (b) –1

(c) 1 (d) –2

22. The sum of the co-efficients in $(1 + x - 3x^2)^{2143}$ is

(a) 2^{2143} (b) 0

(c) 1 (d) –1

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14. Given $f(x) = 4x^8$, then

(a) $f\left(\frac{1}{2}\right) = f'\left(\frac{-1}{2}\right)$ (b) $f\left(\frac{-1}{2}\right) = f\left(\frac{1}{2}\right)$

(c) $f'(x) = f'\left(\frac{-1}{2}\right)$ (d) $f'\left(\frac{1}{2}\right) = f'\left(\frac{-1}{2}\right)$

15. Let $f(x) = e^x g(x)$, $g(0) = 4$, $g'(0) = 2$, then $f'(0)$ is

(a) 1 (b) 3
(c) 2 (d) 6

16. If $f(x) = x^n$ and $f'(1) = 10$, find the value of n

(a) 9 (b) 10
(c) 7 (d) 6

17. If $f(x) = \alpha x^n$, then $\alpha =$

(a) $\frac{f'(1)}{n}$ (b) $\frac{f'(1)}{2n}$

(c) $-\frac{f'(1)}{2n}$ (d) None of these

18. If $f(x) = mx + c$ and $f(0) = f'(0) = 1$, find the value of $f(2)$

(a) 2 (b) –2
(c) 1 (d) 3

19. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!}$, then $\frac{dy}{dx} + \frac{x^n}{n!} =$

(a) $2y$ (b) $-2y$
(c) y (d) $3y$

20. Given $u = 7t^4 - 2t^2 - 8t - 5$, find $\frac{du}{dt}$ at $t = 2$

(a) 193 (b) 192
(c) 190 (d) None of these

21. Given $f(t) = \frac{2}{9}t^4 - \frac{5}{3}t^3 + 2t - 1$, find $f'(-3)$

(a) 67 (b) –67
(c) 66 (d) –66

22. If $f(x) = x^6 - 3x^4 + 2\cos x$, then $f'(x)$ is

(a) even (b) Neither even nor odd
(c) odd (d) can't say

23. If $f(x) = 3x^7 - 4x^3 + 10 \sin x$, then $f'(x)$ is

(a) even (b) odd
(c) Neither even nor odd
(d) can't say

24. If $f(x) = \lambda x^2 + \mu x + 12$, $f'(4) = 15$ and $f'(2) = 11$, then λ and μ are

(a) $\lambda = -1$, $\mu = 7$ (b) $\lambda = 1$, $\mu = 7$
(c) $\lambda = 1$, $\mu = 7$ (d) None of these

25. If $f(x) = x^2 - 9x + 20$, then $f'(100) =$

(a) 191 (b) –190
(c) 192 (d) None of these

26. Find a if $f'(a) = 0$, where $f(x) = x^3 - 3x^2 + 3x - 1$

(a) 1 (b) –1
(c) 2 (d) –2

27. If $f(x) = \left(\frac{\sin^m x}{\sin^n x}\right)^{m+n} \left(\frac{\sin^n x}{\sin^p x}\right)^{n+p} \left(\frac{\sin^p x}{\sin^m x}\right)^{p+m}$ then $f'(x) =$

(a) 0 (b) –1
(c) 2 (d) 1

28. If $f(x) = \left(\frac{x^a}{x^b}\right)^{a+b} \left(\frac{x^b}{x^c}\right)^{b+c} \left(\frac{x^c}{x^a}\right)^{c+a}$, find $f'(x)$

(a) 0 (b) 1
(c) –1 (d) 2

29. If $y = \frac{1}{1 + x^{\beta-\alpha} + x^{\gamma-\alpha}} + \frac{1}{1 + x^{\alpha-\beta} + x^{\gamma-\beta}} + \frac{1}{1 + x^{\alpha-\gamma} + x^{\beta-\gamma}}$ find $\frac{dy}{dx}$

(a) 1 (b) 0
(c) –1 (d) 2

30. If $y = Ax\,e^x + x + 2$, where $\frac{dy}{dx} = -1$ when $x = 1$, find A

(a) e (b) $\frac{-1}{e}$

(c) $\frac{1}{e}$ (d) None of these

31. If $y = A\,x^2\,e^x - x + 8$, where $\frac{dy}{dx} = 2$ when $x = 1$, find A

(a) $-\frac{1}{e}$ (b) e

(c) $\frac{1}{e}$ (d) None of these

32. If $f(x) = x \cos x + \sin x - 3$, find $f'(\pi/2)$

(a) $-\pi/2$ (b) $\frac{\pi}{2}$

(c) π (d) None of these

33. If $f(x) = \frac{1 - \cos x}{1 + \cos x}$, find $f'(\pi/2)$

(a) –1 (b) 1
(c) 2 (d) None of these

34. If $f(x) = \frac{x \sin x}{\cos x - \sin x}$, find $f'(0)$

(a) –1 (b) 1
(c) 0 (d) None of these

35. If $f(t) = \frac{e^t \sec t - \tan t}{1 + \tan t}$, find $f'(\pi/4)$

(a) 0 (b) 1

(c) $\frac{1}{2}\left(2\sqrt{2}\,e^{\pi/4} - \sqrt{2}\,e^{\pi/4} - 1\right)$

(d) None of these

36. If $f(x) = \frac{x+2}{x^2 - 2}$, find $f'(0)$

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14. Ans. (b)

Solution $f\left(\frac{-1}{2}\right) = 4\left(\frac{-1}{2}\right)^8 = 4\left(\frac{1}{2}\right)^8 = f\left(\frac{1}{2}\right)$.

15. Ans. (d)

Solution $f'(x) = e^x g'(x) + e^x g(x)$

$\Rightarrow \quad f'(0) = e^0 g'(0) + e^0 g(0)$

$= g'(0) + g(0) = 4 + 2 = \mathbf{6}.$

16. Ans. (b)

Solution Given $f(x) = x^n \Rightarrow f'(x) = n\,x^{n-1}$

$\Rightarrow f'(1) = n\,(1)^{n-1}$

$\Rightarrow 10 = n$

$\therefore \quad \boldsymbol{n = 10.}$

17. Ans. (a)

Solution $f(x) = \alpha x^n \Rightarrow f'(x) = n\alpha x^{n-1}$

$\Rightarrow f'(1) = n.\alpha\,(1)^{n-1}$

$\Rightarrow \boldsymbol{\alpha = \frac{f'(1)}{n}}.$

18. Ans. (d)

Solution We have, $f(x) = mx + c \Rightarrow f'(x) = m$

Given $f(0) = 1 \Rightarrow m\,(0) + c = 1 \Rightarrow c = 1$

Also, given $f'(0) = 1 \Rightarrow m = 1$

Now $f(x) = mx + c$

$\therefore \quad f(x) = x + 1 \Rightarrow f(2) = 2 + 1 = \mathbf{3}.$

19. Ans. (c)

Solution We have $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$

$\therefore \quad \frac{dy}{dx} = 1 + \frac{2x}{2} + \frac{3x^2}{3(2!)} + \ldots + \frac{nx^{n-1}}{n(n-1)!}$

$= 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^{n-1}}{(n-1)!}$

$\Rightarrow \quad \frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$

$\Rightarrow \quad \boldsymbol{\frac{dy}{dx} + \frac{x^n}{n!} = y.}$

20. Ans. (b)

Solution We have $u = 7t^4 - 2t^3 - 8t - 5$

$\Rightarrow \quad \frac{du}{dt} = 7\,(4t^3) - 2\,(3t^2) - 8\,(1) - 0$

$= 28t^3 - 6t^2 - 8$

$\therefore \quad \left.\frac{du}{dt}\right]_{t=2} = 28(2)^3 - 6\,(2)^2 - 8 = \mathbf{192}.$

21. Ans. (b)

Solution We have, $f(t) = \frac{2}{9}t^4 - \frac{5}{3}t^3 + 2t - 1$

$\Rightarrow \quad f'(t) = \frac{2}{9}(4t^3) - \frac{5}{3}(3t^2) + 2\,(1) - 0$

$= \frac{8}{9}t^3 - 5t^2 + 2$

$\therefore \quad f'(-3) = \frac{8}{9}(-3)^3 - 5\,(-3)^2 + 2$

$= -24 - 45 + 2 = \mathbf{-67}.$

22. Ans. (c)

Solution $f'(x) = 6x^5 - 12x^3 - 2\sin x$

Replacing x by $-x$,

$f'(-x) = 6\,(-x)^5 - 12\,(-x)^3 - 2\sin\,(-x)$

$= -6x^5 + 12x^3 + 2\sin x = -f'(x).$

$\therefore \quad f'(x)$ is odd.

23. Ans. (a)

24. Ans. (c)

Solution We have, $f(x) = \lambda x^2 + \mu x + 12$

$\therefore \quad f'(x) = \lambda \frac{d}{dx}(x^2) + \mu \frac{d}{dx}(x) + \frac{d}{dx}(12)$

$= \lambda\,(2x) + \mu\,(1) + 0 = 2\lambda x + \mu$

Now $f'(4) = 15 \Rightarrow 8\lambda + \mu = 15 \quad \ldots(1)$

and $f'(2) = 11 \Rightarrow 4\lambda + \mu = 11 \quad \ldots(2)$

(1) – (2) gives, $4\lambda = 4 \Rightarrow \lambda = 1$

Putting $\lambda = 1$ in (1), we get

$8\,(1) + \mu = 15 \Rightarrow \mu = 15 - 8$

$\therefore \quad \mu = \mathbf{7}.$

25. Ans. (a)

26. Ans (a)

27. Ans. (a)

Solution $f(x) = (\sin^{m-n} x)^{m+n} (\sin^{n-p} x)^{n+p} (\sin^{p-m} x)^{p+m}$

$= \sin^{(m^2 - n^2 + n^2 - p^2 + p^2 - m^2)x}$

$= \sin^0 x = \mathbf{1}.$

28. Ans. (a)

29. Ans. (b)

Solution $y = \frac{x^\alpha}{x^\alpha + x^\beta + x^\gamma} + \frac{x^\beta}{x^\alpha + x^\beta + x^\gamma} + \frac{x^\gamma}{x^\alpha + x^\beta + x^\gamma} = 1$

30. Ans. (b)

31. Ans. (c)

32. Ans. (a)

33. Ans. (c)

34. Ans. (c)

35. Ans. (c)

36. Ans. (b)

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37. Ans. (a)

Solution Let m be the slope of the curve $y = (2x + 1)^3$

$$\therefore \quad m = \frac{dy}{dx} = 3\,(2x + 1)^2.2 = 6\,(2x + 1)^2$$

Differentiating with respect to x, we get

$$\frac{dm}{dx} = 6 \times 2\,(2x + 1) \times 2 = 24(2x + 1)$$

$\therefore$ The rate of change of slope at $(1, 27)$

$$= \left(\frac{dm}{dx}\right) \text{ at } (1, 27) = 24\,(2 \times 1 + 1) = 24\,(3) = \mathbf{72}.$$

38. Ans. (c)

Solution We have, g = inverse of $f = f^{-1}$

$\Rightarrow \quad g(x) = f^{-1}(x) \Rightarrow f[g(x)] = x$

Differentiating w.r.t. x, we get

$$f'[g(x)] \cdot g'(x) = 1$$

$$\therefore \quad g'(x) = \frac{1}{f'[g(x)]} = 1 + [g(x)]^3$$

$$\left[\because f'(x) = \frac{1}{1+x^3}, f'[g(x)] = \frac{1}{1+[g(x)]^3}\right]$$

39. Ans. (d)

Solution We have $[f(x)]^n = f(nx)$

Differentiating with respect to x, we get

$n\,[f(x)]^{n-1} \cdot f'(x) = f'(nx) \cdot n$

$\Rightarrow \quad [f(x)]^{n-1} \cdot f'(x) = f'(nx)$

$\Rightarrow \quad [f(x)]^n \cdot f'(x) = f'(nx) \cdot f(x)$ [Multiplying both sides by $f(a)$]

$\Rightarrow \quad f(nx) \cdot f'(x) = f'(nx) \cdot f(x)\ [\because [f(x)]^n = f(nx)]$

40. Ans. (b)

Solution Since, $1 \le x < 2 \ \therefore [x] = 1$

$$\therefore \quad f(x) = \sin\left(\frac{\pi}{2} - x^5\right) = \cos x^5$$

$$\Rightarrow \quad f'(x) = -\sin x^5 \cdot 5x^4$$

$$\Rightarrow \quad f'\sqrt[5]{\frac{\pi}{2}} = -5\,(\pi/2)^{4/5} \cdot \sin \pi/2 = \mathbf{-5\,(\pi/2)^{4/5}}.$$

APPLICATIONS OF DERIVATIVES

The derivatives have a wide range of applications in science, engineering and social sciences. One of the most useful and interesting applications of the derivative is determination of maximum and minimum values of the given function.

Tangents and Normals

1. **Slope of the tangent** Let $y = f(x)$ be a continuous curve and let $P(x_1, y_1)$ be a point on it.

   Then $\left(\frac{dy}{dx}\right)$ at the point P i.e. $\left(\frac{dy}{dx}\right)_P$ represents the slope of the tangent to the curve $y = f(x)$ at the point P.

   We write $\left(\frac{dy}{dx}\right)_P = \tan\psi$, where ψ is the angle which the tangent at the point p makes with the positive direction of the x-axis.

2. Tangent at (x_1, y_1) to the curve $y = f(x)$ is given by

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1).$$

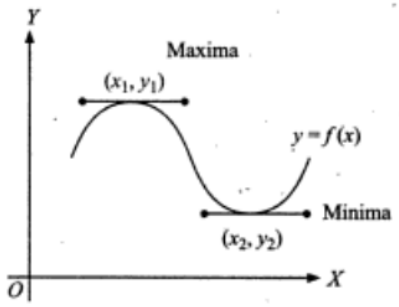
3. Normal at (x_1, y_1) is given by

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}} (x - x_1).$$

$$\left[\text{slope of the normal at } P = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}\right].$$

Maximum and Minimum Values of a function

If a function $f(x)$ has maximum or minimum values, we can find for what values of x the function f is maximum or minimum and also find the maximum or minimum values of f at a particular value of x. If $f'(x) = 0$ when solved gives $x = c$, and if $f''(c) > 0$, then f is minimum at $x = c$ and the minimum value is $f(c)$. If $f''(c) < 0$, then f is maximum at $x = c$ and the maximum value is $f(c)$.



Velocity and Acceleration

Velocity or speed is the rate of change of displacement with respect to time. Acceleration is the rate of change of velocity with respect to time. If s is the displacement and t is the time in seconds.

$$\text{Velocity} = \frac{ds}{dt} \text{ and Acceleration} = \frac{d^2s}{dt^2}$$

Marginal Cost, Average Cost and Total Cost

Marginal Cost = Derivative of total cost w.r.t the quantity

$$\text{Average cost} = \frac{\text{Total cost}}{\text{Total Quantity}}.$$

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MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. The slope of the tangent line to the curve $y = \frac{x}{x+1}$ at $x = 1$ is

(a) $\frac{-1}{4}$ (b) 4

(c) $\frac{1}{4}$ (d) not defined

2. The slope of the normal line to the curve $y = \cos 2x$ at $x = \pi/6$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $-\sqrt{3}$

(c) $\sqrt{3}$ (d) $\frac{-1}{\sqrt{3}}$

3. Slope of the tangent line at $x = 2$ on $y = \frac{8}{4+x^2}$ is

(a) 1/2 (b) −1/2

(c) −2 (d) None

4. If the tangent line at (x, y) to the curve $y = x^3 - 12x + 8$ is parallel to Y – axis then $y =$

(a) ± 2 (b) ± 1

(c) 3 (d) None

5. If the normal line at (x, y) to the curve $y = 2x^2 - 4x + 5$ is parallel to the Y – axis then $y =$

(a) 1 (b) 3

(c) −3 (d) −1

6. The tangent line at (3, −2) to the curve $y = x^3 - 3x^2 - 9x - 5$ is

(a) parallel to X – axis

(b) bisector of coordinate axes

(c) parallel to Y – axis

(d) None

7. Equation of the normal line at (1, 1) on the curve $2y = 3 - x^2$ is

(a) $x + y = 2$ (b) $x + y + 1 = 0$

(c) $x - y + 1 = 0$ (d) $x - y = 0$

8. Equation of the tangent line to the curve $y(x + 1) = 4$ at the point (2, 4/3) is

(a) $4x + 9y = 20$ (b) $4x - 9y = 20$

(c) $9x + 4y = 20$ (d) $9x - 4y = 20$

9. $f(x) = x^3 - 3x^2 + 6x + 7$ has

(a) a maximum

(b) a minimum

(c) a maximum and a minimum

(d) neither maximum nor minimum

10. $f(x) = ax^2 + bx + c$ has minimum value when

(a) $a < 0$ (b) $a > 0$

(c) $b^2 = 4ac$ (d) $x = \frac{b}{2a}$

11. The minimum value of $y = x (\log x)^2$ is

(a) $4e^{-2}$ (b) 1

(c) e (d) 0

12. If $a > b$, the minimum value of $a \sin^2 x + b\cos^2 x$ is

(a) a (b) b

(c) $a + b$ (d) $\sqrt{a^2 + b^2}$

13. Maximum value of $f(x) = (x - 2)(x - 3)^2$ is

(a) 0 (b) 3

(c) 4/27 (d) 7/3

14. Minimum value of $f(x) = x \log x$ is

(a) $-1/e$ (b) $1/e$

(c) $-e$ (d) e

15. The maximum value of $\frac{x^2 - x + 1}{x^2 + x + 1}$ is

(a) 1 (b) 3

(c) 1/3 (d) None

16. Maximum value of $f(x) = (2 + x)^{1/3}(1 - x)^{2/3}$ is

(a) −1 (b) $\sqrt[3]{2}$

(c) $\sqrt[3]{4}$ (d) 0

17. If a manufactures total cost function $C = 1500 + 30x + x^2$. Find the marginal cost when 20 units are produced

(a) Rs 40 (b) Rs 65

(c) Rs 70 (d) Rs 75

18. The average cost function (AC) for a product is given by $AC = 0.0002x^2 - 0.05x + 7 + \frac{8000}{x}$, where x is the output. What is the marginal cost when 100 units are produced?

(a) Rs 5 (b) Rs 3

(c) Rs 7 (d) Rs 2

19. Given the total cost function for x units of a commodity as $c(x) = \frac{1}{3}x^3 + 3x^2 - 7x + 16$. Find the average cost

(a) $\frac{1}{3}x^3 + 3x - 7 + \frac{16}{x}$

(b) $-\frac{1}{3}x^3 + 3x - 7 + \frac{16}{x}$

(c) $\frac{1}{3}x^3 + 3x + 7 + \frac{16}{x}$

(d) None of these

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Real Function

If $f : X \to Y$ be a function from a non-empty set X to another non-empty set Y, where $X, Y \subseteq R$ (set of all real numbers), we say that f is a real valued function or in short a real function.

Features of a Map $f: X \to Y$

(i) For each element $x \in X$, there is unique element $y \in Y$.

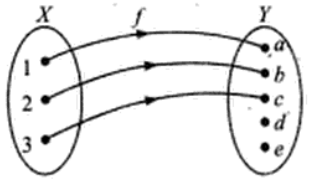
(ii) The element $y \in Y$ is called the image of x under the map f.

(iii) $f: X \in Y$ is not a function if there is an element in X which has more than one image in Y. But distinct elements of X may be associated to the same element of Y.

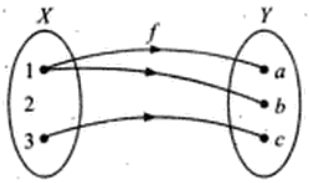
(iv) $f: X \to Y$ is not a function if there is an element in X which does not have an image in Y.

Note: Throughout this chapter function will mean a real function.

Illustration 1 (i) *If* $X = \{1, 2, 3\}$, $Y = \{a, b, c, d, e\}$, then $f(1) = a, f(2) = b, f(3) = c$, gives a map $f: X \to Y$.



(ii) *If* $X = \{1, 2\}$, $Y = \{a, b, c\}$, then $f(1) = a, f(1) = b$, $f(2) = c$ do not give map $f: X \to Y$.



Value of a Function

The value of a function $y = f(x)$ at $x = a$ is denoted by $f(a)$ and is obtained by putting a in place of x in $f(x)$.

Illustration 2 If $f(x) = \log\left(\frac{1-x}{1+x}\right)$, $x \in (-1, 1)$, then

$$f(0) = [f(x)]_{x=0} = \log\left(\frac{1-0}{1+0}\right) = \log 1 = 0,$$

$$f(1/2) = \log\left(\frac{1-1/2}{1+1/2}\right) = \log\frac{1}{3} \text{ etc.}$$

Note:

(i) If for some value of x say $x = a$, the denominator vanishes, we say that $f(a)$ is undefind (or does not exist).

Illustration 3 If $f(x) = \frac{x}{x-1}$, then $f(1) = \frac{1}{1-1} = \frac{1}{0}$.

$\therefore$ $f(1)$ is undefined.

(ii) If for some value of x say $x = a$, $f(a)$ takes the form $\frac{0}{0}$, we say that $f(a)$ is indeterminate.

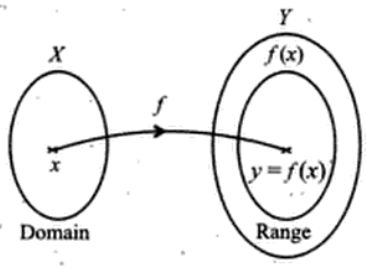
Illustration 4 If $f(x) = \frac{\sin x}{x}$, then $f(0) = \frac{\sin 0}{0} = \frac{0}{0}$.

Domain and Range of a Function

If $f: X \to Y$ be a function, then domain of $f = X$,

and range of $f = f(X) = \{f(x) : f(x) \in Y; x \in X\}$

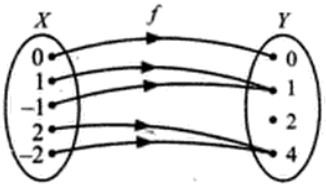
= set of all image points in Y under the map f.



Y is called the co-domain of f.

Clearly $f(X) \subseteq Y$.

Illustration 5 If $X = \{0, \pm 1, \pm 2\}$, $Y = \{0, 1, 2, 4\}$, then the rule $f: X \to Y$ given by



$f(x) = x^2$ is a map from X to Y.

$\therefore$ domain of $f = X$, range of $f = \{0, 1, 4\}$.

Intervals in R

The set of all numbers lying between two given real numbers is called an interval in R.

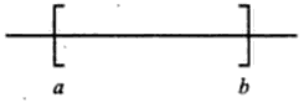
Let a and b be any two real numbers such that $a < b$, then we define the following types of intervals.

(i) Closed interval $[a, b]$

(a, b) = closed interval from a to b

$= \{x : x \in R; a \le x \le b\}$

= set of all real numbers lying between a and b including the end points a and b.



(ii) Open interval (a, b) or $]a, b[$

= open interval from a to b

$= \{x : x \in R; a < x < b\}$

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Note:

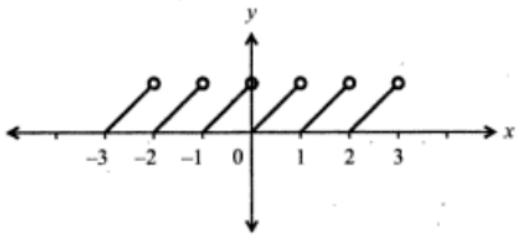
(a) $[x] \le x < [x] + 1$

(b) $[x + y] = \begin{Bmatrix} (x)+(y), & \text{if } (x)+(y) < 1 \\ (x)+(y)+1, & \text{if } (x)+(y) \ge 1 \end{Bmatrix}$,

where $\{x\}$ denotes the fractional part of x.

5. Fractional-part Function

The function $f: R \to R$ defined by $f(x) = x - [x]$ or $f(x) = \{x\}$, where $\{x\}$ denotes the fractional part of x, is called the fractional-part function. Its domain is R and range is $[0, 1)$. The graph of the fractional part function is as shown below:

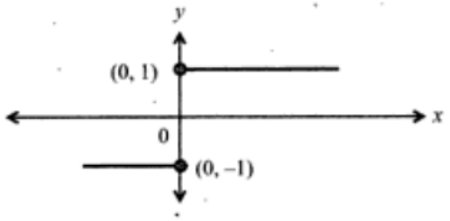


6. Signum Function

The function $f: R \to R$ defined by,

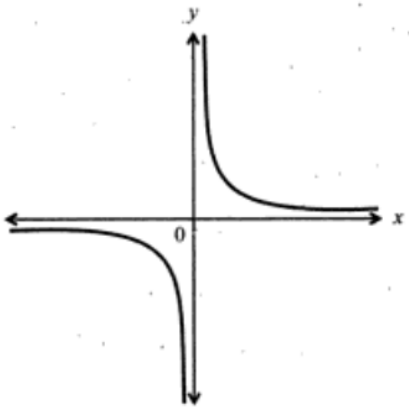
$f(x) = \begin{Bmatrix} \frac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{Bmatrix}$, is called the signum function.

Its domain is R and range is the set $\{-1, 0, 1\}$. The graph of the signum function is as shown in the figure below:



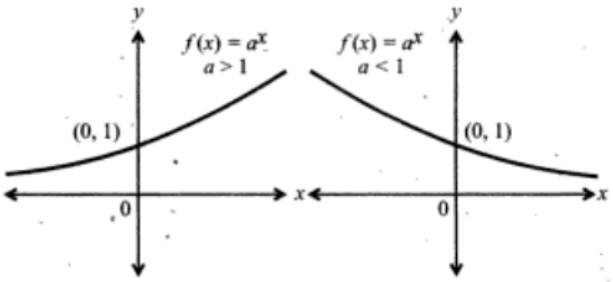
7. Reciprocal Function

The function $f: R - \{0\} \to R$ defined by $f(x) = \frac{1}{x}$, is called the *reciprocal function*. Its domain as well as range is $R - \{0\}$. The graph of the reciprocal function is as shown below:



8. Exponential Function

Let a ($\ne 1$) be a positive real number. Then the function $f: R \to R$, defined by $f(x) = a^x$, is called the *exponential function*. Its domain is R and range is $(0, \infty)$. The graph of the exponential function is as shown below:

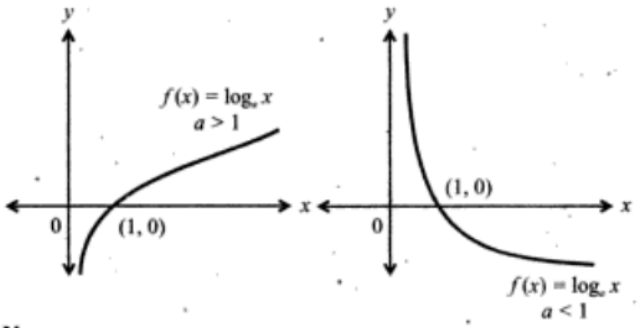


Note: (i) $a^x = e^{x \log_e a}$, $(a > 0)$,

(ii) $a^{\log_a x} = x$, $(a > 0, a \ne 1)$.

9. Logarithmic Function

Let $a(\ne 1)$ be a positive real number. Then the function $f: (0, \infty) \to R$, defined by $f(x) = \log_a x$, is called the logarithmic function. Its domain is $(0, \infty)$ and range is R. The graph of the logarithmic function is as shown below:



Note:

(i) $\log_a a = 1$, $\log_a 1 = 0$

(ii) $\log_a 0 = -\infty$, if $a > 1 = +\infty$, if $0 < a < 1$

(iii) We denote $\log_e x$ as $\ln x$.

10. Polynomial Function

A function $f: R \to R$, defined by $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where $n \in N$, $a_0, a_1, a_2, \ldots, a_n \in R$, is called a *polynomial function.*

If $a_n \ne 0$, then n is called the degree of the polynomial. The domain of a polynomial function is R.

11. Rational Function

A function of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials over the set of real numbers and $q(x) \ne 0$, is called a *rational function.* Its domain is $R \setminus \{x \mid q(x) \ne 0\}$.

Odd and Even Functions

Odd function: A function $f(x)$ is said to be *odd* if $f(-x) = -f(x)$ for all x.

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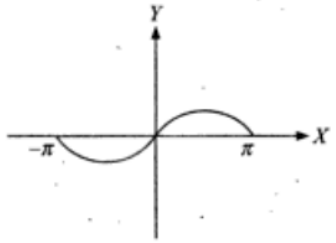


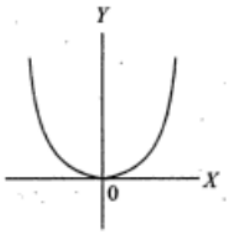
Note:

(i) Here replacing x by $-x$ in $f(x)$, we get a negative sign in its value.

(ii) The graph of an odd function is placed in the first and third quadrants or the second and fourth quadrants as shown in the figure.

Illustration 7 $y = f(x) = \sin x$ is odd.

Even function: A function $f(x)$ is said to be *even* if $f(-x) = f(x)$ for all x.



Note:

(i) Here replacing x by $-x$ in $f(x)$, we get no change in its value.

(ii) The graph of an even function $f(x)$ is symmetrical about the y-axis.

Illustration 8 $y = f(x) = x^2$ is even.

OPERATIONS ON FUNCTIONS

Let f and g be real functions with domain D_1 and D_2 respectively. Then,

(i) The sum function $(f + g)$ is defined by

$$(f + g)(x) = f(x) + g(x), \forall x \in D_1 \cap D_2.$$

The domain of $f + g$ is $D_1 \cap D_2$.

(ii) The difference function $(f - g)$ is defined by

$$(f - g)(x) = f(x) - g(x), \forall x \in D_1 \cap D_2.$$

The domain of $f - g$ is $D_1 \cap D_2$.

(iii) The product function fg is defined by

$$(fg)(x) = f(x) \cdot g(x), \forall x \in D_1 \cap D_2.$$

The domain of fg is $D_1 \cap D_2$.

(iv) The quotient function $\left(\frac{f}{g}\right)$ is defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \forall x \in D_1 \cap D_2 \backslash \{x : g(x) = 0\}.$$

The domain of $\frac{f}{g}$ is $D_1 \cap D_2 \backslash \{x : g(x) = 0\}$.

(v) The scalar multiple function cf is defined by

$$(cf)(x) = c \cdot f(x), \forall x \in D_1.$$

The domain of cf is D_1.

COMPOSITION OF FUNCTIONS

Let f and g be real functions with domain D_1 and D_2 respectively.

If range $(f) \subseteq$ domain (g), we define $(g \circ f)$ by

$$(g \circ f)(x) = g(f(x)), \forall x \in D_1$$

Also, if range $(g) \subseteq$ domain (f), we define $(f \circ g)$ by

$$(f \circ g)(x) = f(g(x)), \forall x \in D_2.$$

MULTIPLE CHOICE QUESTIONS

In each of the following questions a number of possible answers are given, out of which one answer is correct. Find out the correct answer.

1. Find $f(4)$ if $f(x) = x^4 - 3x^3 + 6x^2 - 10x + 16$

(a) 135 (b) 137

(c) 136 (d) None of these

2. Find $f(-1.5)$, if $f(z) = 2^{z-2}$

(a) $-\frac{1}{8\sqrt{2}}$ (b) $\frac{1}{8\sqrt{2}}$

(c) $8\sqrt{2}$ (d) $-8\sqrt{2}$

3. $f(t) = 2t^2 + \frac{2}{t^2} + \frac{5}{t} + 5t$. Find $f\left(\frac{1}{t}\right)$

(a) $-f(t)$ (b) $1/f(t)$

(c) $f(t)$ (d) None of these

4. If $f(x) = ax^2 + bx + 2$. $f(1) = 3$ and $f(4) = 42$, find 'b'

(a) -2 (b) 2

(c) 1 (d) 3

5. If $f(x, y) = 3x^2 - 2xy - y^2 + 4$, find $f(1, -1)$

(a) -8 (b) 7

(c) 8 (d) 5

6. Function $f(x) = 2x^2 - 4$ is

(a) even (b) odd

(c) neither even nor odd

(d) can't say

7. If $f(x) = \begin{cases} 2x^2 + 1, & x \geq 1 \\ \frac{2}{x} + 1, & 0 < x < 1 \\ 3|x|, & x \leq 0 \end{cases}$, then

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(a) $f\left(\frac{1}{2}\right) < f(1)$ (c) $f(-2) = f(2)$

(c) $f(-3) = f(2)$ (d) None of these

8. If $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$, find $f(1 + h)$

(a) $\frac{h^2 - h + 1}{h^2 + 3h + 3}$ (b) $\frac{h^2 + h + 1}{h^2 + 3h + 3}$

(c) $\frac{h^2 + h + 1}{h^2 - 3h + 3}$ (d) None of these

9. If $f(x) = \frac{\sin x - \cos x}{\sin x + \cos x}$, find $f\left(\frac{\pi}{3}\right)$.

(a) $2 - \sqrt{3}$ (b) $2 + \sqrt{3}$

(c) $-2 + \sqrt{3}$ (d) None of these

10. If $f(x) = \frac{x+1}{x-1}$, find $f(x^2)$.

(a) $\frac{x^2 - 1}{x^2 + 1}$ (b) $\frac{x^2 - 2}{x^2 + 1}$

(c) $\frac{x^2 + 1}{x^2 - 1}$ (d) None of these

11. Find a polynomial function $f(x)$ of second degree when $f(0) = 5$, $f(-1) = 10$ and $f(1) = 6$

(a) $-3x^2 - 2x + 5$ (b) $3x^2 + 2x + 5$

(c) $3x^2 - 2x + 5$ (d) None of these

12. If $f(x) = \begin{cases} 1 + x, & -1 \le x < 0 \\ x^2 - 1, & 0 < x < 2, \\ 2x, & x \ge 2 \end{cases}$ then

(a) $f(3) = 2$

(b) $f(-1 + h) = 1 - h$

(c) $f(2 - h) = h^2 + 4h + 3$

(d) $f(1/2) = -3/4$

13. If $f(x) = x - [x]$ then

(a) $f(-1.5) = 1$ (c) $f(1) = 0.5$

(c) $f\left(\sin\frac{\pi}{6}\right) = \frac{1}{4}$ (d) None of these

14. If $f(x) = \log\frac{1-x}{1+x}$, $-1 < x < 1$, then $f\left(\frac{2x}{1+x^2}\right) =$

(a) $f(x)$ (b) $2\,f(x)$

(c) $3\,f(x)$ (d) $4\,f(x)$

15. If $f(x) = \cos(\log x)$, then

$$2\,f(x)\cdot f(y) - \left[f(xy) + f\left(\frac{x}{y}\right)\right] =$$

(a) 0 (b) 1

(c) 2 (d) None of these

16. If $f(t) = \frac{1}{e^t - 1} + \frac{t}{2} + 1$, then $f(t)$ is

(a) even (b) odd

(c) neither even nor odd

(d) can't say

17. If $f(x) = \log\left(x + \sqrt{1+x^2}\right)$, then $f(x)$ is

(a) even (b) odd

(c) neither even nor odd

(d) can't say

18. If $f(x) = e^{2x} + \sin x$, then $f(x)$ is

(a) even (b) odd

(c) neither odd nor even

(d) can't say

19. Let f be defined by $f(x) = x - 4$ and g be defined by $g(x) = \begin{cases} \frac{x^2 - 16}{x+4}, & x \ne -4 \\ K, & x \ne -4 \end{cases}$

Find K such that $f(x) = g(x)$ for all x

(a) 8 (b) -8

(c) 7 (d) None of these

20. Let f be the exponential function and let g be the logarithmic function then

(a) $(f + g)(1) = e^2$ (b) $(f \circ g)(1) = 1$

(c) $(fg)(1) = 1$ (d) $(g \circ f)(1) = -1$

21. Let f be the greatest integer function and g be the modulus function then

(a) $(g \circ f)\left(\frac{-5}{3}\right) - (f \circ g)\left(\frac{-5}{3}\right) = 1$

(b) $(g \circ f)\left(\frac{-5}{3}\right) - (f \circ g)\left(\frac{-5}{3}\right) = -1$

(c) $(f \circ g)\left(\frac{5}{2}\right) - (g \circ f)\left(\frac{-5}{2}\right) = 0$

(d) $(f \circ g)\left(\frac{5}{2}\right) - (g \circ f)\left(\frac{-5}{2}\right) = 1$

22. Let f be the greatest integer function and g be the modulus function then

(a) $(g \circ f)(5/3) - (f \circ g)(5/3) = 1$

(b) $(g \circ f)\left(\frac{5}{3}\right) - (f \circ g)(5/3) = 0$

(c) $(f + 2g)(-1) = 0$

(d) $(f + 2g)(-1) = -1$

23. If $f(x) = 2x$ and $g(x) = x + 1$, find $(g \circ f)(x)$

(a) $2x - 1$ (b) $-2x + 1$

(c) $2x + 1$ (d) None of these

24. If $f(x) = 3x - 5$ and $f[g(x)] = 2x$, find $g(x)$

(a) $-\frac{2x+5}{3}$ (b) $\frac{2x-5}{3}$

(c) $\frac{2x+5}{3}$ (d) None of these

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25. If $f(x) = \begin{cases} 2x^2+1, & x \geq 1 \\ \frac{2}{x}+1, & 0 < x < 1 \\ 3|x|, & x \leq 0 \end{cases}$, find $f(f(x))$ if $x < 0$

(a) $-18x^2 + 1$ (b) $18x^2 + 1$
(c) $18x^2 - 1$ (d) None of these

EXPLANATORY ANSWERS

1. Ans. (c)

Solution $f(x) = x^4 - 3x^3 + 6x^2 - 10x + 16$

$f(4) = (4)^4 - 3(4)^3 + 6(4)^2 - 10(4) + 16$

$= 256 - 192 + 96 - 40 + 16 = \mathbf{136}.$

2. Ans. (b)

Solution $f(z) = 2^{z-2}$

$$f(-1.5) = 2^{-1.5-2} = 2^{-7/2} = \frac{1}{2^{7/2}}$$

$$= \frac{1}{\sqrt{128}} = \frac{\mathbf{1}}{\mathbf{8\sqrt{2}}}.$$

3. Ans. (c)

Solution $f(t) = 2t^2 + \frac{2}{t^2} + \frac{5}{t} + 5t$

$$f\left(\frac{1}{t}\right) = 2\left(\frac{1}{t}\right)^2 + \frac{2}{\left(\frac{1}{t}\right)^2} + \frac{5}{\left(\frac{1}{t}\right)} + 5\left(\frac{1}{t}\right)$$

$$= \frac{2}{t^2} + 2t^2 + 5t + \frac{5}{t} = \boldsymbol{f(t)}.$$

4. Ans. (a)

Solution $f(x) = ax^2 + bx + 2$

$f(1) = a(1)^2 + b(1) + 2 = 3$

$\therefore \quad a + b + 2 = 3$

$\therefore \quad a + b = 1 \qquad ...(i)$

$f(4) = a(4)^2 + b(4) + 2 = 42$

$\therefore \quad 16a + 4b + 2 = 42$

$16a + 4b = 40 \qquad ...(ii)$

Solving (i) and (ii) simultaneously, we get

$b = \mathbf{-2}.$

5. Ans. (c)

Solution $f(x, y) = 3x^2 - 2xy - y^2 + 4$

$f(1, -1) = 3(1)^2 - 2(1)(-1) - (-1)^2 + 4$

$= 3 + 2 - 1 + 4 = \mathbf{8}.$

6. Ans. (a)

Solution $f(x) = 2x^2 - 4$

$f(-x) = 2(-x)^2 - 4 = 2x^2 - 4$

$\therefore \quad f(x) = f(-x).$

Hence, $f(x) = 2x^2 - 4$ is an even function.

If $f(x) = 2x^2 + 1$ if $x \geq 1 = \frac{2}{x} + 1$ if $0 < x < 1$

$= \mathbf{3|x|}$ **if** $\boldsymbol{x \leq 0}.$

7. Ans. (c)

Solution $f\left(\frac{1}{2}\right) = \frac{2}{\frac{1}{2}} + 1 = 5$

$f(1) = 2(1)^2 + 1 = 3$

$f(2) = 2(2)^2 + 1 = 9$

$f(-2) = 3|-2| = 6$

$f(-3) = 3|-3| = 9$

$\therefore \quad f(-3) = \boldsymbol{f(2)}$ **is true.**

8. Ans. (b)

Solution $f(1 + h) = \frac{(1+h)^2 - (1+h) + 1}{(1+h)^2 + (1+h) + 1}$

$$= \frac{\boldsymbol{h^2 + h + 1}}{\boldsymbol{h^2 + 3h + 3}}.$$

9. Ans. (a)

Solution $f(\pi/3) = \frac{\sin \pi/3 - \cos \pi/3}{\sin \pi/3 + \cos \pi/3}$

$$= \frac{\sqrt{3}/2 - 1/2}{\sqrt{3}/2 + 1/2} = \mathbf{2 - \sqrt{3}}.$$

10. Ans. (c)

Solution $f(x^2) = \frac{\boldsymbol{x^2 + 1}}{\boldsymbol{x^2 - 1}}.$

11. Ans. (c)

Solution Let $f(x) = ax^2 + bx + c$

$\therefore \quad f(0) = a \cdot 0^2 + b \cdot 0 + c = 5 \Rightarrow c = 5$

Also, $f(-1) = a(-1)^2 + b(-1) + c = 10$

$\Rightarrow \quad a - b + c = 10$

$\Rightarrow \quad a - b + 5 = 10 \Rightarrow a - b = 5 \qquad ...(i)$

Also, $f(1) = a(1)^2 + b(1) + c = 6$

$\Rightarrow \quad a + b + c = 6 \Rightarrow a + b + 5 = 6$

$\Rightarrow \quad a + b = 1 \qquad ...(ii)$

Solving (i) and (ii), we get

$a = 3, \ b = -2$

$\therefore \quad f(x) = \mathbf{3x^2 - 2x + 5}.$

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$$(f + 2g)(-1) = f(-1) + 2\,g(-1)$$
$$= [-1] + 2\,|-1|$$
$$= -1 + 2(1)$$
$$= -1 + 2 = \mathbf{1}.$$

23. Ans. (c)

Solution $(g \circ f)(x) = g(f(x)) = g(2x) = \mathbf{2x + 1}.$

24. Ans. (c)

Solution $f(x) = 3x - 5$

$$f[g(x)] = 3\,g(x) - 5 = 2x$$
$$\therefore \quad 3g(x) = 2x + 5$$
$$\therefore \quad g(x) = \frac{2x+5}{3}$$

25. Ans. (b)

Solution $f(x) = 3|x|$ as $x < 0$

Now $3|x| > 3$

$$\therefore \quad f[f(x)] = 2\,[3|x|]^2 + 1$$
$$= 2 \times 9x^2 + 1 = \mathbf{18x^2 + 1}.$$

QUESTIONS FROM COMPETITIVE EXAMS

1. If $x + y > 5$ and $x - y > 3$, then which of the following gives all possible values of x?

(a) $x > 3$ (b) $x > 4$

(c) $x > 5$ (d) $x < 5$

[Based on MAT 2004]

Directions (Qs. 2 to 4): Calulate the value of each word by the formulae:

Consonants = (2 × Position of the letterin the alphabet) – 1

Vowels = (3 × Position in the alphabet) + 1

2. BREAD

(a) 67 (b) 62

(c) 59 (d) 65

[Based on MAT 2004]

3. FLUTE

(a) 153 (b) 157

(c) 151 (d) 149

[Based on MAT 2004]

4. CHANGE

(a) 77 (b) 79

(c) 83 (d) 80

[Based on MAT 2004]

5. Which values of x are satisfied by the inequality $2x^2 + x - 3 < 0$?

(a) $-\frac{3}{2} < x < 1$ (b) $-1 < x < \frac{2}{3}$

(c) $x > 1$ (d) $x < \frac{-2}{5}$

[Based on MAT 2003]

6. The cost function at production x is defined as $C(x) = 3x^3 - x + 2$ and sale function at cost x is defined as $S(x) = \frac{A}{x^{1/3}}$. Which of the following is true?

(a) Min sales $= \left(\frac{3}{4}\right)^{2/3} A$

(b) Min sales $= \left(\frac{9}{2}\right)^{2/3} A$

(c) Max sales $= \left(\frac{3}{4}\right)^{2/3} A$

(d) Max sales $= \left(\frac{9}{2}\right)^{2/3} A$ **[Based on MAT 2003]**

7. How can the relationship between x and y be best defined, if the values of x and y are as follows?

x	2	3	4	5	16
y	0	2	6	12	20

(a) $y = x^2 - 3x + 2$ (b) $y = 2x - 4$

(c) $y = x^2 - 4x$ (d) $y = x - 4$

[Based on MAT 2003]

8. I forgot the last digit of a 7-digit telephone number. If I randomly dial the final 3 digits after correctly dialling the first four, then what is the chance of dialling the correct number?

(a) $\frac{1}{1001}$ (b) $\frac{1}{990}$

(c) $\frac{1}{999}$ (d) $\frac{1}{1000}$

[Based on MAT 2003]

9. What is the quotient if $x^{-1} - 1$ is divided by $x - 1$?

(a) $\frac{1}{(x-1)}$ (b) x

(c) $-\frac{1}{(x-1)}$ (d) $-\frac{1}{x}$

[Based on MAT 2003]

10. Hari is taller than Ganesh but he is shorter than Mohan. If heights of Hari, Ganesh and Mohan are x, y and z respectively, then which of the following accurately expresses the relationship of their heights?

(a) $x > y > z$ (b) $y > x > z$

(c) $z > x < y$ (d) $z > x > y$

[Based on MAT 2003]

11. The smallest positive integer 'n', for which

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51. A crate of apples contains one bruised apple for every 30 apples in the crate. If 3 out of every 4 bruised apples are considered unsalable, and there are 12 unsalable apples in the crate, then how many apples are there in the crate?
(a) 480 (b) 270
(c) 600 (d) 740
[Based on REC Tiruchirapalli 2003]

52. If $x - y = 8$, then which of the following must be true?
I. Both x and y are positive.
II. If x is positive, y must be positive.
III. If x is negative, y must be negative.
(a) I only (b) II only
(c) I and II (d) III only
[Based on REC Tiruchirapalli 2003]

53. An hour can be divided into 60 (1-minute) periods or 1 (60-minute) period. Into how many other same size periods can we divide an hour if each period must be an integer number of minutes?
(a) 4 (b) 6
(c) 8 (d) 10
[Based on SCMHRD Ent. Exam. 2003]

54. If $6a = 4b$ and $2a = 0$, then
(a) $a = 4$ (b) a and b are equal
(c) $b/a = 3/2$ (d) $a = 4, b = 6$
[Based on SCMHRD Ent. Exam. 2003]

55. If x is less than 2, then which of the following statements is always true?
(a) x is negative
(b) $2x$ is greater than or equal to x
(c) x^2 is greater than or equal to x
(d) None of these
[Based on SCMHRD Ent. Exam. 2003]

Directions (Qs. 56 to 60): In each of these questions, a question is followed by the information given in three statements I, II and III. You have to study the question along with the information given in the statements to decide which of the statements is/are necessary and sufficient to answer the respective questions :

56. What is the total number of employees in the company?
I. Number of male employees is more than the number of female employees by 25.
II. Total number of officers is three-fourth of the clerks,
III. Number of male officers is 126.
(a) All, I, II and III (b) I and II only
(c) I and III only (d) II and III only
(e) Question cannot be answered even with the information in all the three statements
[Based on IRMA, 2002]

57. What is the area of the right-angled triangular garden?
I. Perimeter of the garden is Y cm.
II. Length of the diagonal side is X cm.
III. Perpendicular sides of the garden are in the ratio of 5 : 12
(a) Only I and III or only II and III
(b) All I, II and III
(c) Any two of the three
(d) Only I and III
(e) None of the above **[Based on IRMA, 2002]**

58. What is Sangita's present age?
I. Five years ago, Sangita's age was double that of her son's age that time
II. Present ages of Sangita and her son are in the ratio of 11 : 6 respectively.
III. Five years hence, the respective ratio of Sangita's age and her son's age will become 12 : 7.
(a) Only I and III (b) Only II and III
(c) Only I and II (d) Any two of the three
(e) None of these **[Based on IRMA, 2002]**

59. What is the two digit number?
I. Sum of the digits is 17.
II. Difference between the number and the number obtained by interchanging the digits is 9.
III. Digit in the unit's place is bigger than the digit in the ten's place by 1.
(a) Only I and II (b) Only I and III
(c) Only II and III (d) All, I, II and III
(d) None of these **[Based on IRMA, 2002]**

60. A manufacturer of batteries wishes to give a guarantee for the free replacement of the batteries whose life is less than a certain time period. If he desires to replace no more than 5% of the batteries, what should be the guarantee period (it has to be lesser than the mean life) if the lives of batteries are normally distributed with a mean of 1200 hours and standard deviation of 100 hours. (Note: For 95% population, normal variate = 1.645)
(a) 1035.5 hours (b) 2845 hours
(c) 1195 hours (d) Data is inadequate
[Based on FMS (Delhi) 2007]

61. If $\frac{a}{b}=\frac{4}{5}$ and $\frac{b}{c}=\frac{15}{16}$, then $\frac{c^2-a^2}{c^2+a^2}$ would be:
(a) $\frac{1}{7}$ (b) $\frac{7}{25}$
(c) $\frac{3}{4}$ (d) None of these
[Based on FMS (Delhi) 2007]

62. If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$, the value of $\frac{a+b+c}{c}$ is
(a) 7 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{7}$
[Based on FMS (Delhi) 2007]

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63. If $z=\frac{x^2}{y}$ and x, y both are increased in value by 10% then the value of z

(a) remains unchanged (b) increases by 10%
(c) increases by 11% (d) increases by 20%

[Based on FMS (Delhi) 2006]

64. If $x : y = 3 : 4$ then $(2x + 3y) : (3y - 2x)$ would be equal to:

(a) 2 : 1 (b) 3 : 2
(c) 3 : 1 (d) 21 : 1

[Based on FMS (Delhi) 2006]

65. If $a=\sqrt{3+\sqrt{3+\sqrt{3+....}}}$, then which of the following is true:

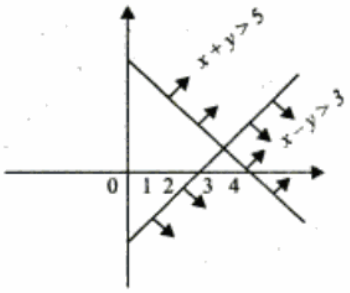
(a) $2 < a < 3$ (b) $a > 3$
(c) $3 < a < 4$ (d) $a = 3$

[Based on FMS (Delhi) 2006]

EXPLANATORY ANSWERS

1. Ans. (b)

Solution For all values of x satisfying $x + y > 5$ and $x - y > 3$, $x > 4$.



2. Ans. (d)

Solution $B = 3$, $R = 35$, $E = 16$, $A = 4$, $D = 7$.

3. Ans. (a)

Solution $F = 11$, $L = 23$, $U = 64$, $T = 39$, $E = 16$.

4. Ans. (d)

Solution $C = 5$, $H = 15$, $A = 4$, $N = 27$, $G = 13$, $E = 16$.

5. Ans. (a)

Solution $2x^2 + x - 3 < 0 \Rightarrow (x - 1)(2x + 3) < 0$

$\Rightarrow x < 1, x > -\frac{3}{2}$

$\Rightarrow -\frac{3}{2} < x < 1.$

6. Ans. (c)

Solution When cost is minimum, the sales is maximum.

For min cost, $\frac{dc}{dx} = 0 \Rightarrow 9x^2 - 1 = 0$

$\Rightarrow x = \frac{1}{3}$ (x is production)

$\left(\frac{d^2C}{dx^2}\right)_{x=\frac{1}{3}} = (18x)_{x=\frac{1}{3}} > 0$

Hence, cost is min. at $x = \frac{1}{3}$

Min. cost $= 3\times\left(\frac{1}{3}\right)^3 - \frac{1}{3} + 2 = \frac{16}{9}$

Sales at cost $x = \frac{A}{x^{1/2}}$ $\left(\text{Here, } x=\frac{16}{9}\right)$

$\Rightarrow$ Max Sales $= \left(\frac{9}{16}\right)^{1/3} A$ $\left[\text{at cost} = \frac{16}{9}\right]$

$\Rightarrow$ Max Sales $= \left(\frac{3}{4}\right)^{2/3} A.$

7. Ans. (a)

8. Ans. (d)

9. Ans. (d)

Solution $\frac{x^{-1}-1}{x-1} = \frac{1-x}{x(x-1)} = \frac{-1}{x}.$

10. Ans. (d)

Solution Hari > Ganesh $\Rightarrow x > y$
Mohan > Hari $\Rightarrow z > x.$

11. Ans. (d)

Solution For $n = 4$

$$\left(\frac{1+i}{1-i}\right)^n = \left(\frac{1+i}{1-i}\right)^4 = \left(\frac{(1+i)^2}{1-i^2}\right)^4$$

$$= \left(\frac{1+i+2i}{1+1}\right)^4 = i^4 = 1.$$

12. Ans. (b)

Solution $2 - 2x - x^2$ be +ve
or $x^2 + 2x - 2$ is –ve
or $(x + 1)^2 - 3$ is –ve
$\Rightarrow -\sqrt{3} \le (x+1) \le \sqrt{3}$
$\Rightarrow -1 - \sqrt{3} \le x \le -1 + \sqrt{3}.$

13. Ans. (a)

Solution $\bar{x} = \frac{\Sigma x_i}{n} = \frac{200}{20} = 10$

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