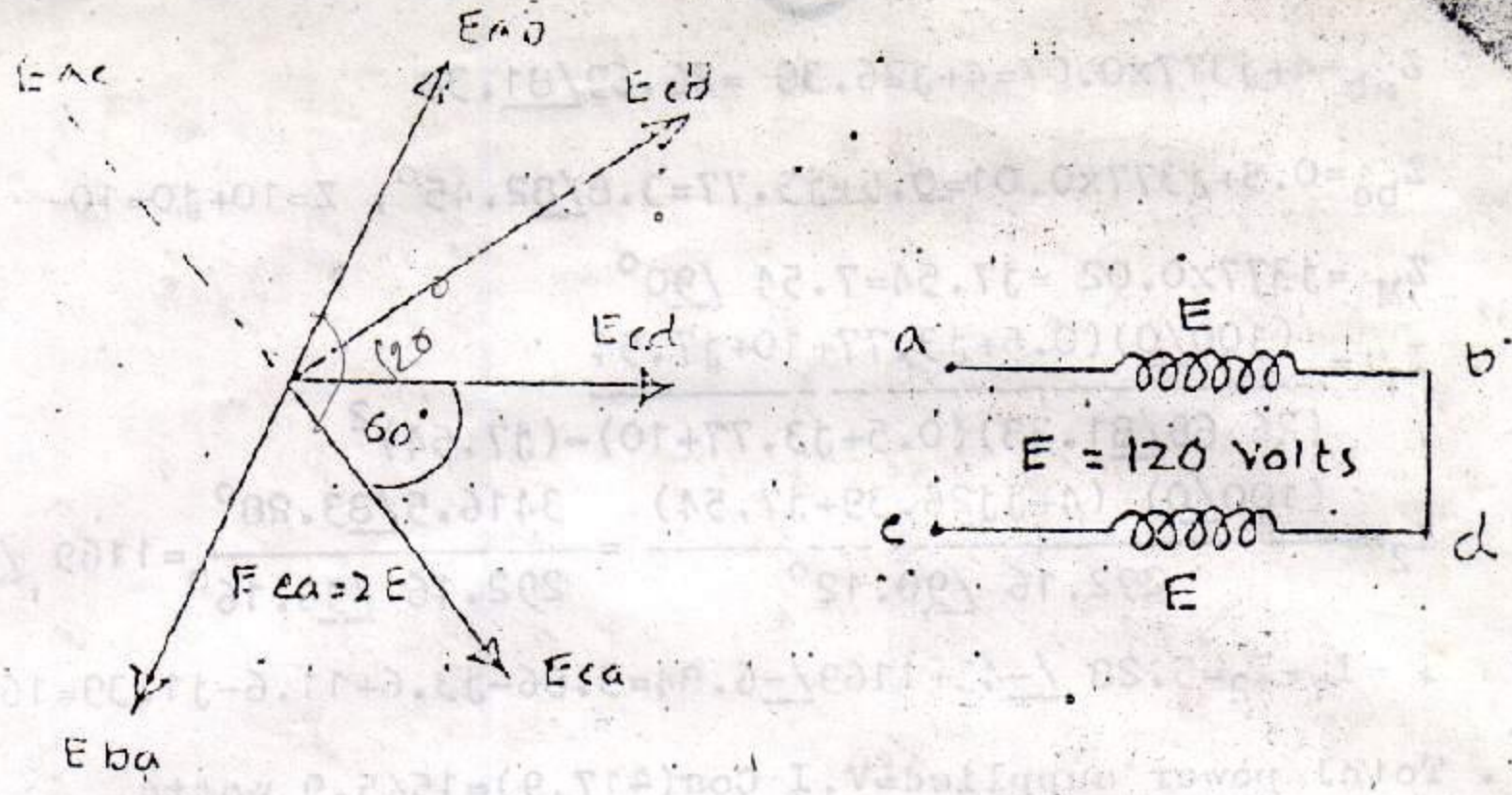
2



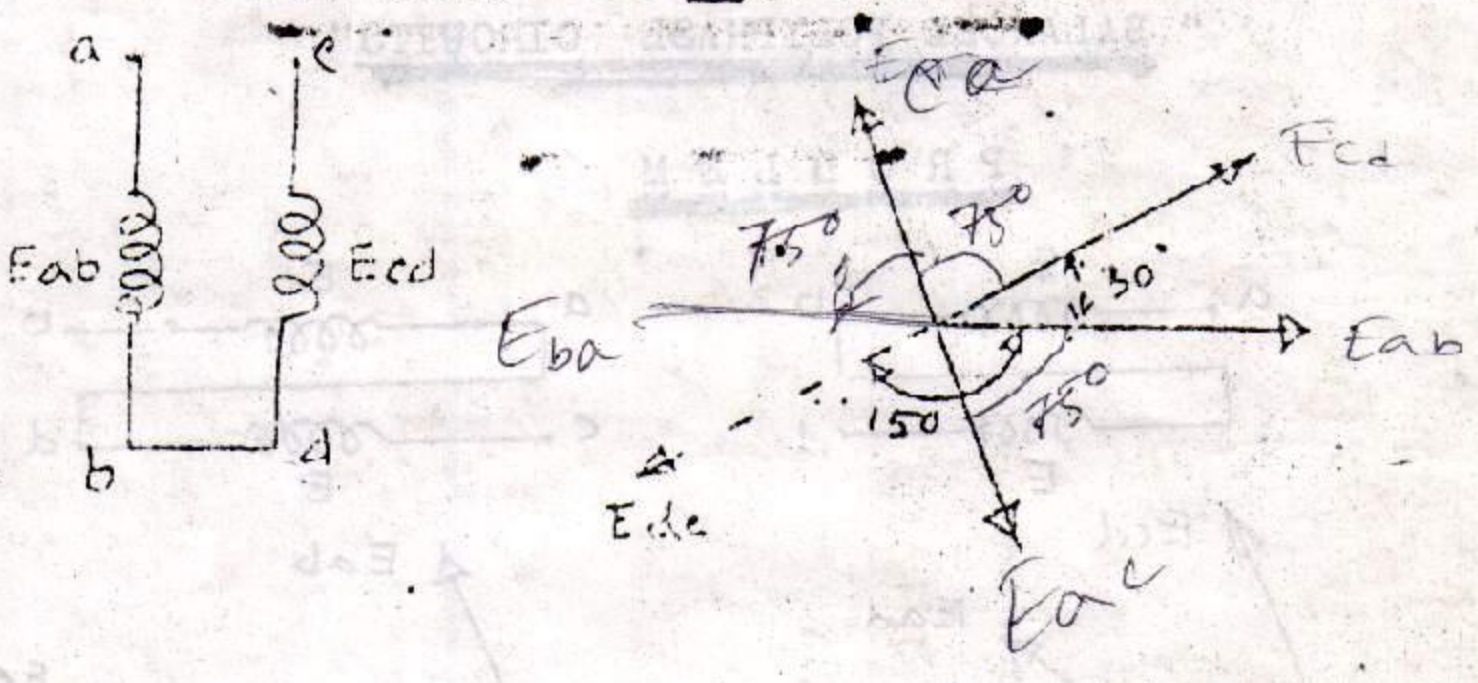
(a) $\vec{E}_{ca} = \vec{E}_{cd} + \vec{E}_{ba} = E_{cd} - E_{ab} = 120\ \angle 0^\circ - 120\ \angle 60^\circ$

$= 120 + j0 - 60 - j103.923 = 60 - j103.923 = 120\ \angle -60^\circ$

(b) $E_{ac} = E_{ab} + E_{dc} = E_{ab} - E_{cd} = 120\ \angle 60^\circ - 120\ \angle 0^\circ = 60 + j103.923 - 120$

$= -60 + j103.923 = 120\ \angle 120^\circ$ Ans.

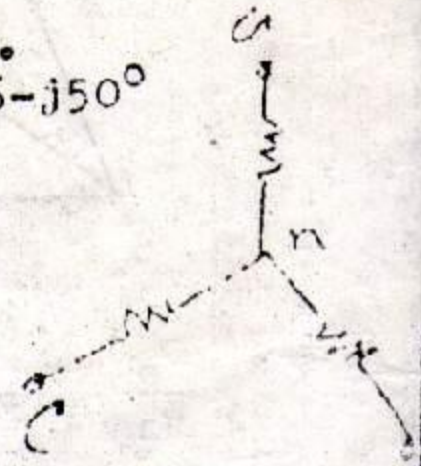
3.



$\vec{E}_{ac} = \vec{E}_{ab} + \vec{E}_{dc} = 100\ \angle 0^\circ + 100\ \angle -150^\circ = 100 + j0 + 86.6 - j50^\circ$

$= 13.397 - j50 = 51.76\ \angle -75^\circ$

$\therefore \vec{E}_{cn} = 51.76\ \angle 105^\circ$ volts.



4. See Oscillogram 1. Phase Sequence: a -b -c

$(V_{ab})_{max} = 141$ volts. $= \sqrt{3}$ (max. voltage per phase)

$= \sqrt{3}\ V_m$

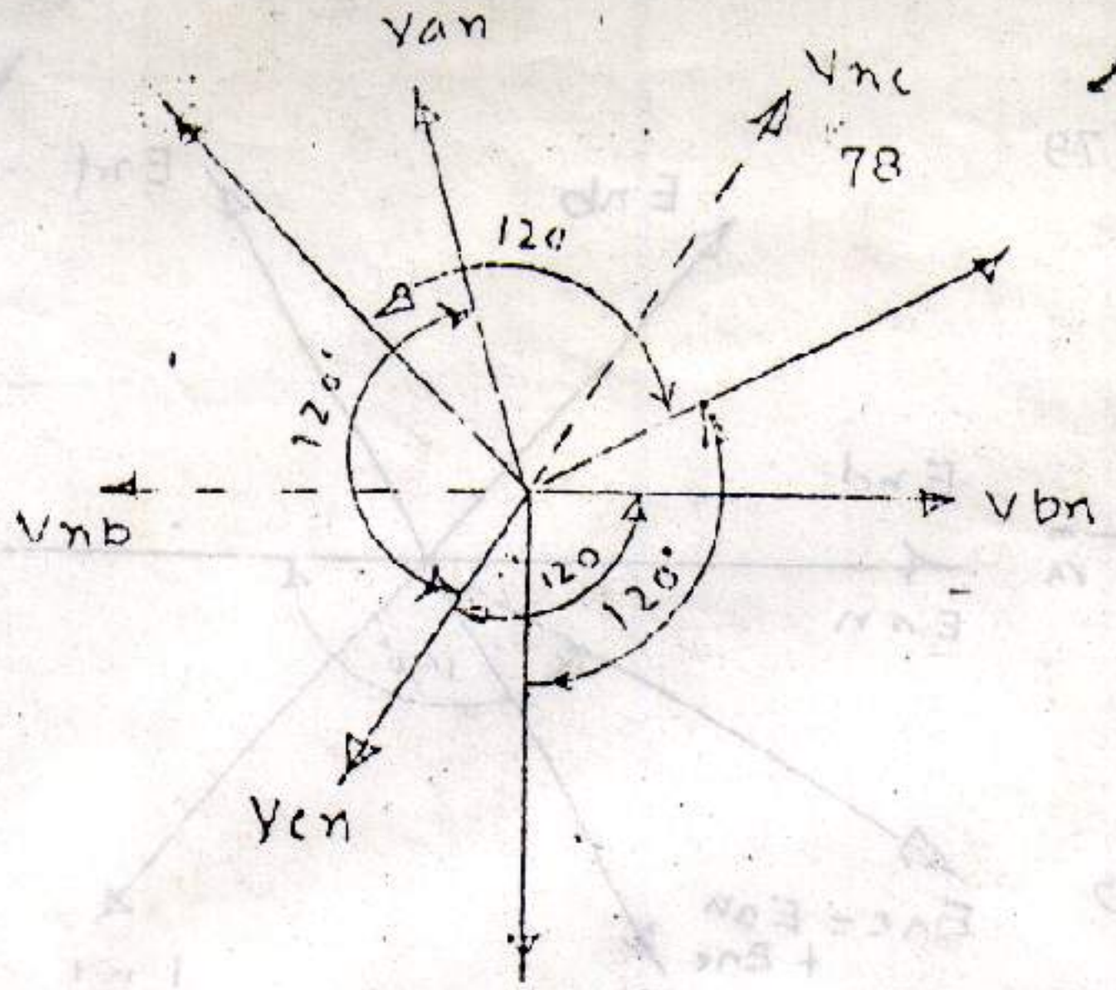
$\therefore V_m = 141/\sqrt{3} = 81.406$ volts., $V_m = 81.406$ volts (magnitude only).

$\therefore V_{rms} = V_m/\sqrt{2} = 57.6$ volts.

$\therefore V_{an} = V_{bn} = V_{cn} = 57.6$ volts (magnitude only) phase voltage

Sequence : an-bn-cn.

POLY PHASE



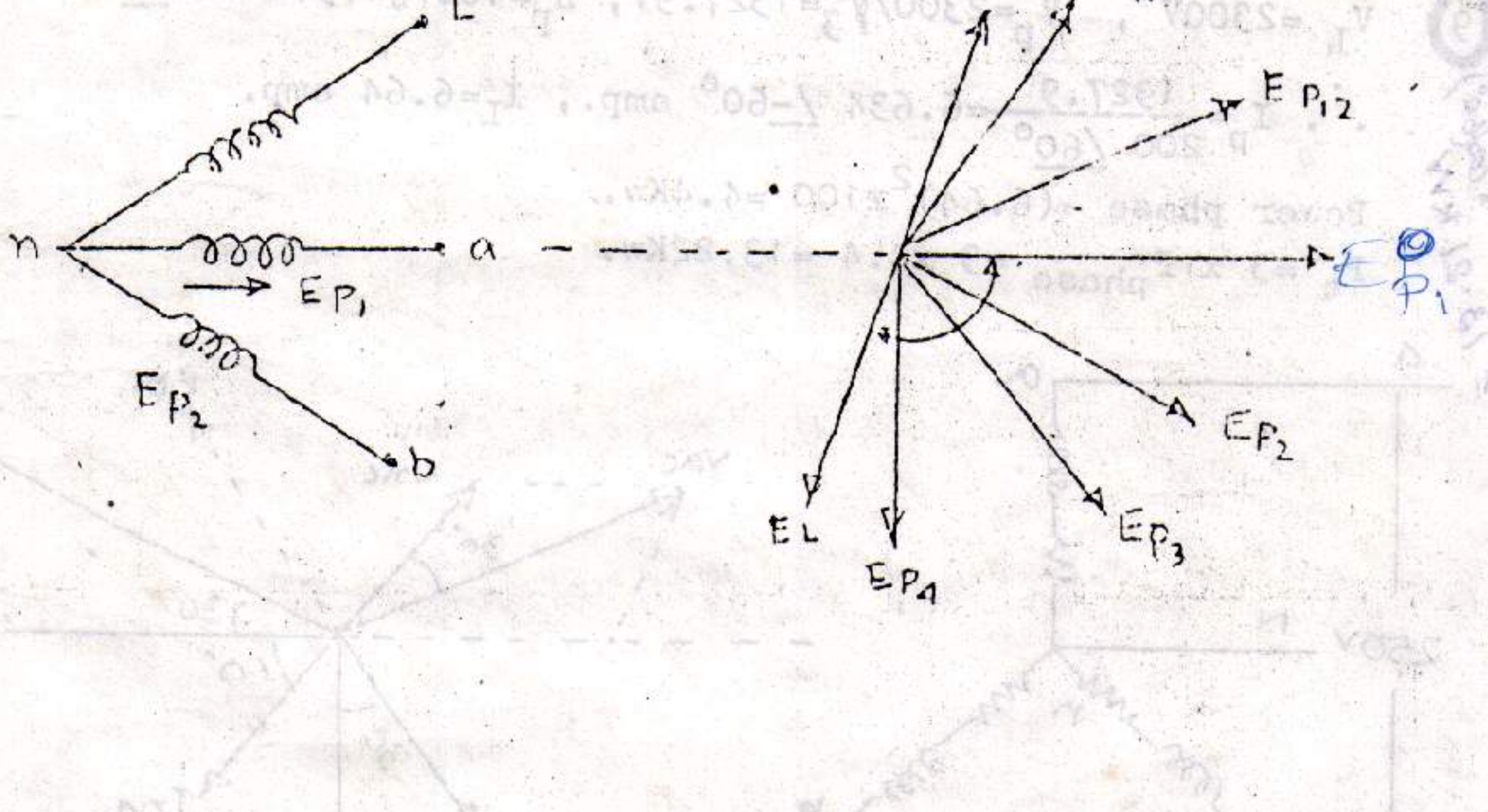
(6) $n = 6$, $I_p = 100$ amp.

$$I_L = 2I_p \sin\frac{180^\circ}{n} = 2 \times 100 \times \sin 180/6$$

$= 100$ amp.

$\therefore I_p = I_L = 100$ amp.

(7)



$E_{ab} = E_{an} + E_{nb}$

1.1cm = 50 volts.

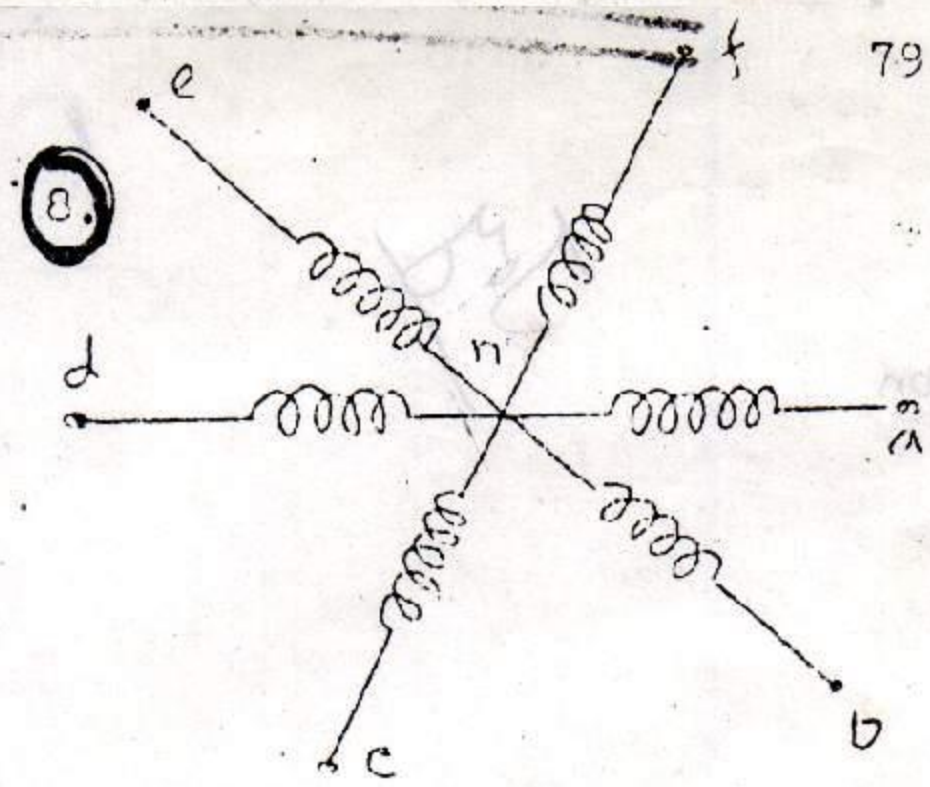
2.1cm = 25.61 volts.

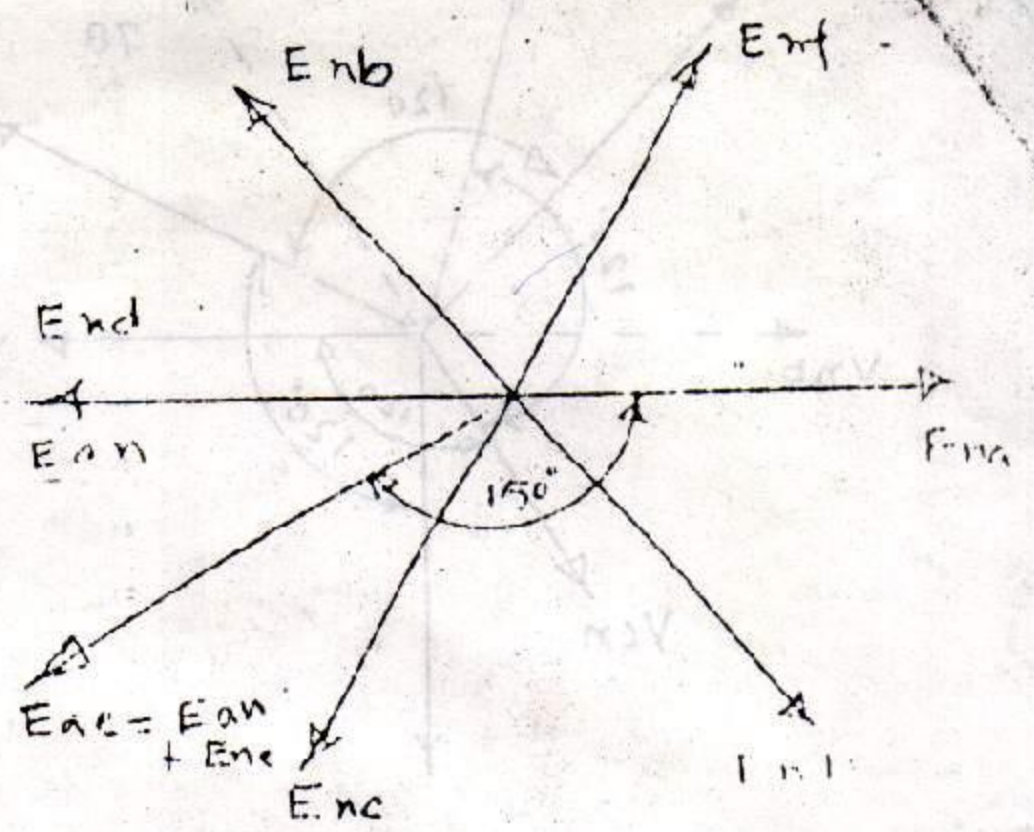
$E_L = 25.61$ Volts.

$E_L = 2 \times 50 \times \sin\frac{180}{12} = 25.88$ volts.

$E_{ab} = 25.61 \angle -105^\circ$ volts.

(8)





$E_{na} = E_{nb} = E_{nc} = E_{nd} = E_{ne} = E_{nf} = E_p = 132.8$ volts.

$E_{ac} = 2E_p \sin\frac{360^\circ}{n} = 2 \times 132.8 \times \sin\frac{360^\circ}{6}$

$= 230$ volts.

AC = 33.2 volts

$\frac{360^\circ}{6} = 60^\circ$

$E_{ac} = 6.9$ cm $= 230$ volts.

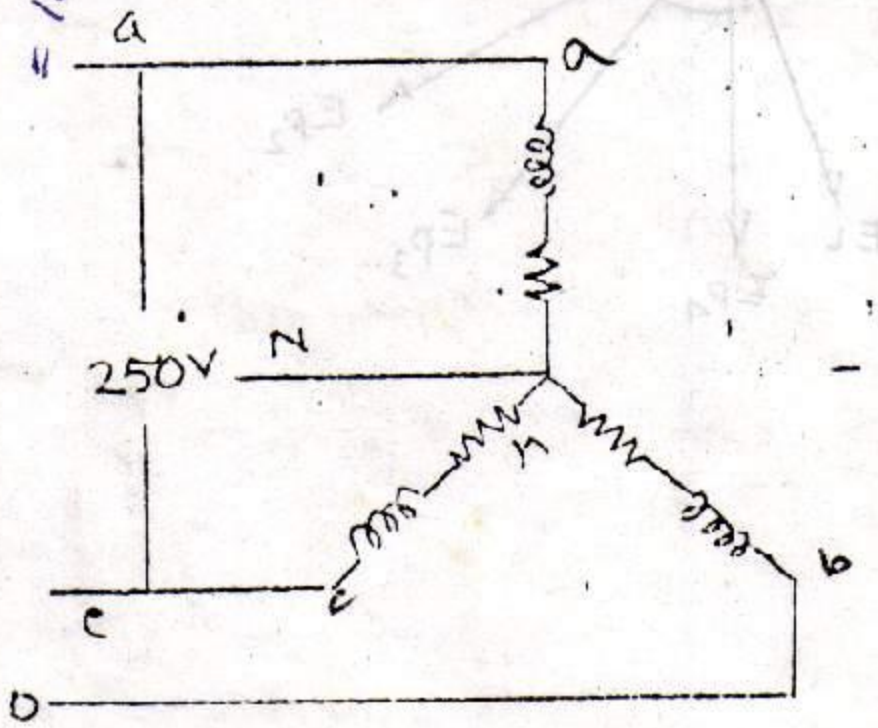
(9) $V_L = 2300V$, $V_p = 2300/\sqrt{3} = 1327.9V$, $Z_p = 100 + j173.2 = 200\angle 60^\circ$

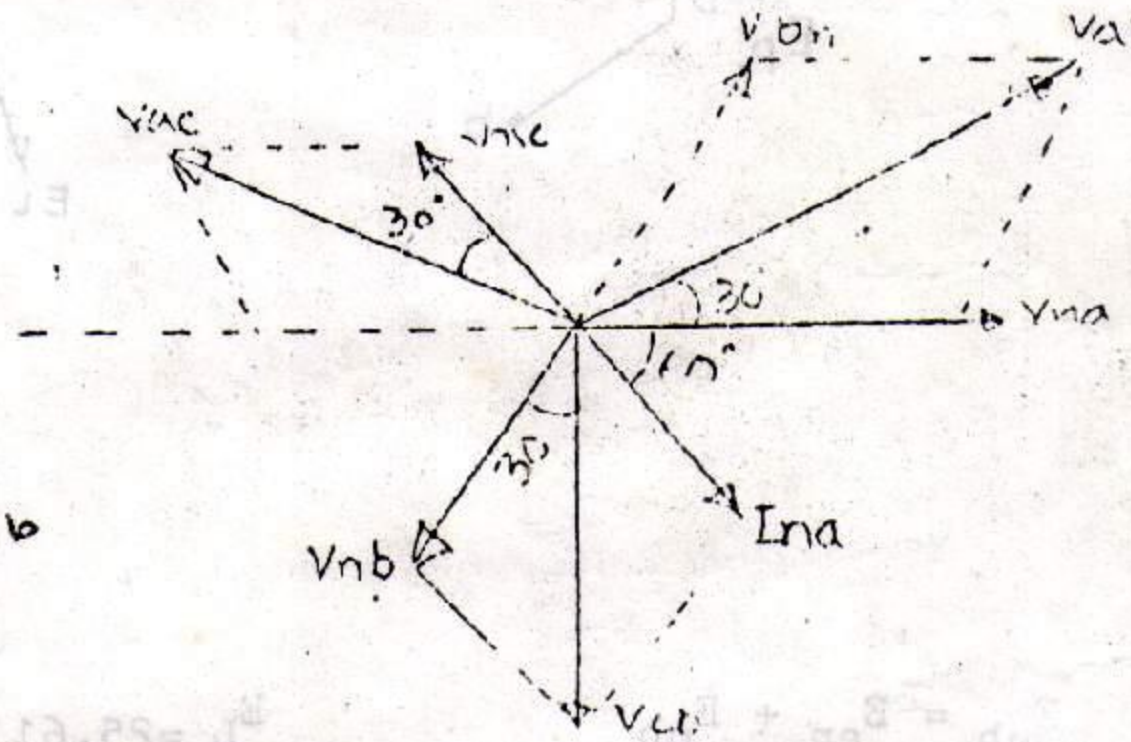
$\therefore I_p = \frac{1327.9}{200\angle 60^\circ} = 6.634 \angle -60^\circ$ amp., $I_L = 6.64$ amp.

Power phase $= (6.64)^2 \times 100 = 4.4$Kw.

$P_t = 3 \times P_{phase} = 3 \times 4.4 = 13.22$Kw.

$\sqrt{3} V_p I_p \cos\theta$
$= \sqrt{3} \times 1327.9 \times 6.634 \times \cos(60^\circ)$
$= 13.21$ KW





by vector-

$V_{na} = 1327.7 \angle 0^\circ = 1327.9 + j0$, $V_{nb} = 1327.9 \angle -120 = 663.96 - j1150$

$V_{nc} = 1327.9 \cdot \angle \pm 120 = -663 + j1150$, $V_{ba} = V_{bn} + V_{na} = -V_{nb} + V_{na}$

$= +603.36 + 1150 + 1327.9 + j0 = 1991.87 + j1150 = 2300\angle 30^\circ$

$I_{na} = \frac{V_{na}}{Z_{na}} = \frac{1327.9 \angle 0^\circ}{200 \angle 60^\circ} = 6.64 \angle -60$, $I_L = I_{na} = 6.64$ amp.

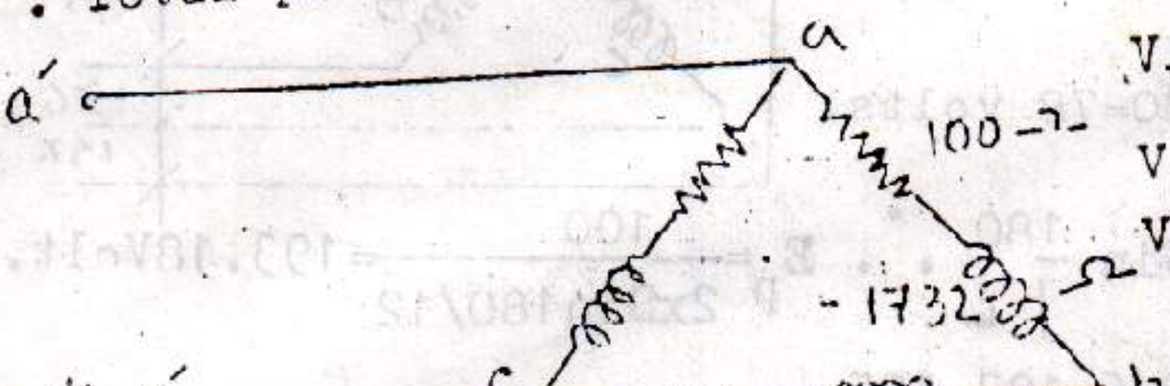
$$I_L = I_{nb} = \frac{V_{nb}}{Z} = \frac{1327.9/-120^{\circ}}{200/60^{\circ}} = 6.64/-180^{\circ}$$

$$I_L = I_{nc} = \frac{V_{nc}}{Z} = \frac{1327.9/120^{\circ}}{200/60} = 6.64/60^{\circ}$$

$P_{no} = V_1 + V'i' = 1327.9 \times 3.32 + j \times 05.75 = 4.4Kw.$

$\therefore$ Total power $= 4.41 \times 3 = 13.225Kw.$

(10)



$V_{ba} = 2300/0^{\circ}$

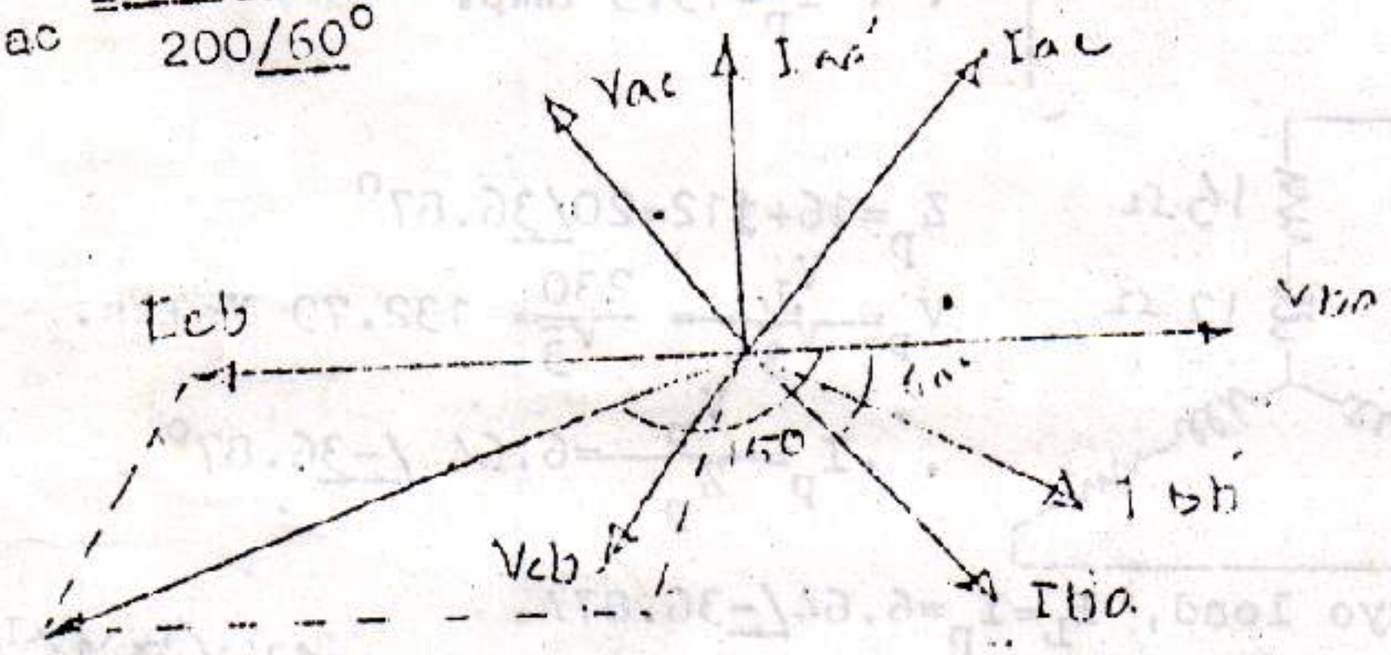
$V_{cb} = 2300/-120 = 1150 - j1991.80$

$V_{ac} = 2300/-240 = -1156 + j1991.80$

$$I_{ba} = \frac{2300/0^{\circ}}{200/60^{\circ}} = 11.5/-60 = 5.75 - j9.96.$$

$$I_{cb} = \frac{V_{ab}}{Z} = \frac{2300/-120}{200/60} = 11.5/-180 = -11.5 + j0$$

$$I_{ac} = \frac{2300/-240}{200/60^{\circ}} = 11.5/-300 = 575 + j9.96$$



$\therefore |I_P| = 11.4A$, $I_L = \sqrt{3} \times 11.5 = 19.92$ Amp.

$I_{b'b} = I_{bc} + I_{ba} = -I_{cb} + I_{ba} = 11.5 - j0 + 5.75 - j9.96 = 19.92/-30^{\circ}$ Amp.

$I_{a'a} = I_{ab} + I_{ac} = I_{ba} + I_{ac} = -5.75 + j9.96 + 5.75 + j9.96 = j19.92 = 19.92/90^{\circ}$ amp.

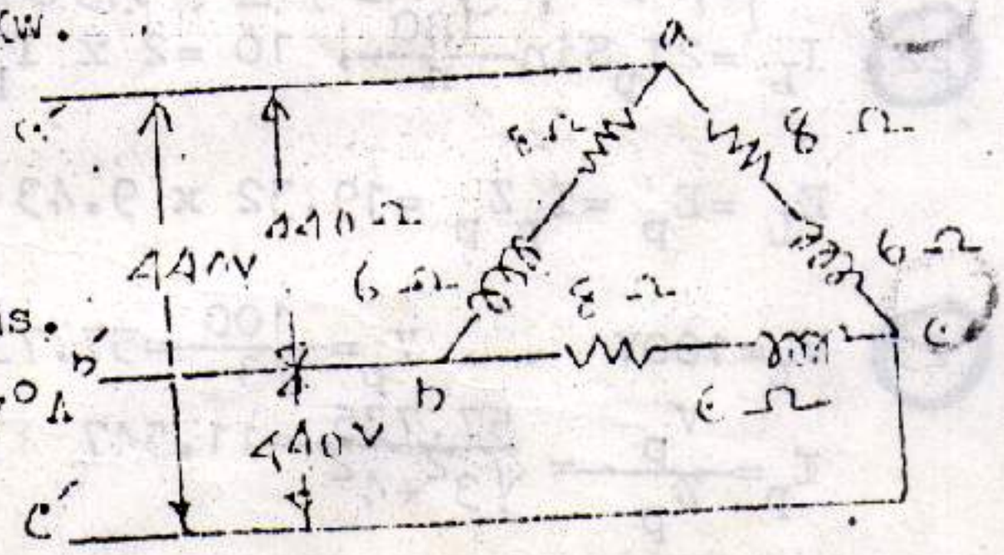
$I_{c'c} = I_{cb} + I_{ca} = I_{cb} - I_{ac} = -11.5 + j6 - 5.75 - j9.96 = -17.25 - j9.96 = 19.92/-150$

$P_{ba} = |V_{ba}| |I_{ba}| \cos(0V - 01) = 2300 \times 11.5 \cos[0 - (-60)]$

$= 2300 \times 11.5 \cos 60^{\circ} = 13225$ Watt $= 13.23Kw.$

P_t of three phase $= 13.23 \times 3 = 39.69Kw.$

(11)



$VA_p = V_p I_p$, $V_L = 440/0^{\circ}$ volts.

Let, $V_{ab} = 440/0^{\circ}$ volts.

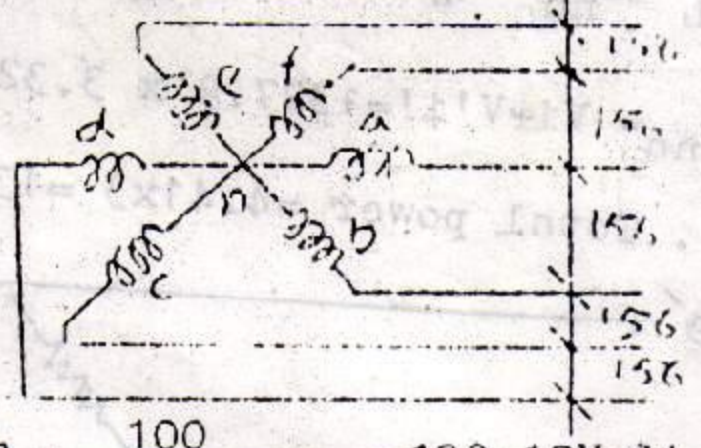
$Z_p = 8 + j6$ ohms $= 10/36.87^{\circ}$ ohms.

$$I_p = I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{440/0}{10/36.87} = 44/-36.87^{\circ} A$$

$VA_p = 440 \times 44 = 19360$ r.f $= \sin\theta_p = \sin 36.87 = 0.6$

$rV_{ap} = 440 \times 44 \times 0.6 = 11616$, $VA_t = VA_p = 58080$, $rV_{at} = 3 \times 11616 = 34848$

r.f. of the system remains the same.



14. $V_{an} + V_{nb} = 156$ Volt.

$2V_p = 156$ Volts. $V_p = 78$ volts.

$V_L = 2V_p \times \sin\frac{180^p}{n} = 2 \times 78 \sin 30 = 78$ volts.

15. $E_L = 2E_p \sin\frac{180}{n}$ or, $100 = 2E_p \sin\frac{180}{12}$ $\therefore E_p = \frac{100}{2 \times \sin 180/12} = 193.18$ Volt.

$E_E = 2E_p \sin\frac{360}{n} = 2 \times 193 \sin 180/6 = 193.185$

Greatest voltage $= 2E_p = 2 \times 193.18 = 386.37$ volts.

16. $I_L = 2I_p \sin\frac{180}{n}$

or, $10 = 2I_p \sin\frac{180}{6}$

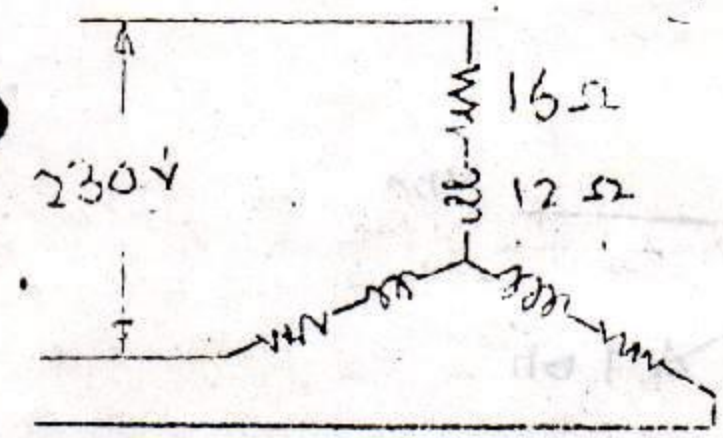
$\therefore I_p = 10$

$I_L = 2I_p \sin\frac{180}{12}$

$10 = 2 \times I_p \sin 15^\circ$

$\therefore I_p = 19.3$ amp.

21.



$Z_p = 16 + j12 = 20\angle 36.87^\circ$

$V_p = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79$ Volts.

$\therefore I_p = \frac{V_p}{Z_p} = 6.64\ \angle -36.87^\circ$

For balanced wye load, $I_L = I_p = 6.64\angle -36.87$A.

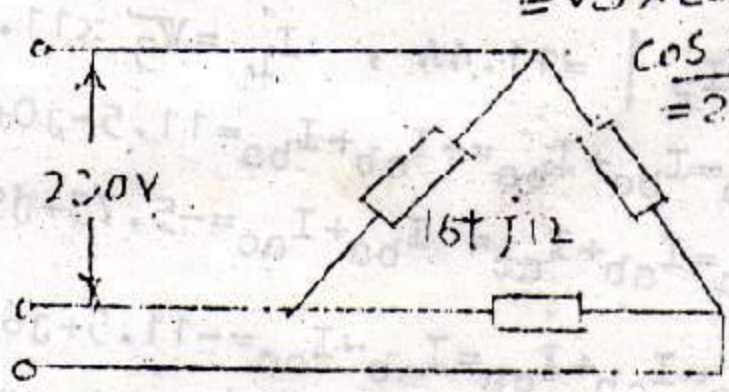
Total power, $P_t = 3V_p I_p \cos\theta_p = 2116.14$ watts. Ans. $= \sqrt{3} \times 230 \times 6.64 \cos(36.87^\circ) = 2116.14$

For Delta Connection

Here, $V_L = V_p = 230$V

$\therefore I_p = \frac{V_p}{Z_p} = \frac{230}{20\angle 30.87}$

$= 11.5\angle -36.87^\circ$



$\therefore I_L = \sqrt{3} I_p = \sqrt{3} \times 11.5 = 19.922$ A $\therefore$

$P_T = \sqrt{3} V_L I_L \cos\theta_p = 6348.44$ W

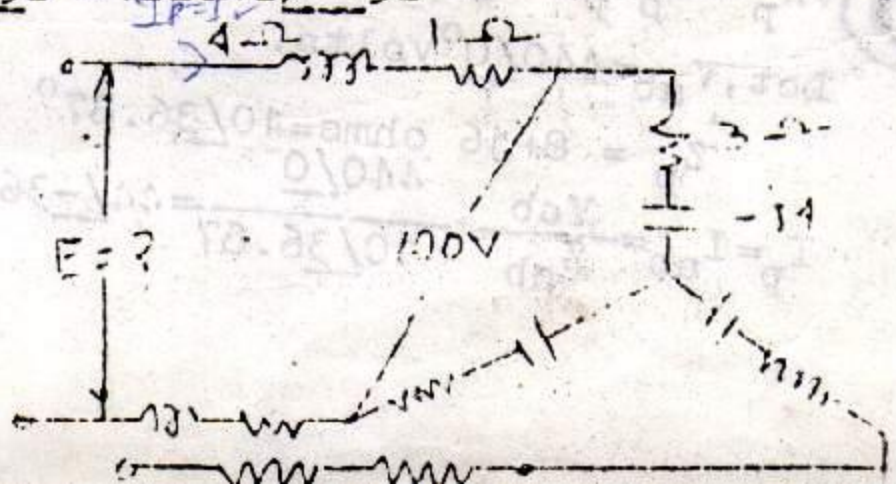
22. $I_L = 2I_p \sin\frac{180}{n}$, $10 = 2 \times I_p \sin\frac{180}{12}$ $\therefore I_p = 19.32$ Amp.

$Z_p = 5 - j8 = 9.43\angle -58$

$E_L = E_p = I_p Z_p = 19.32 \times 9.43\ \angle -58 = 182.18\angle -58^\circ$

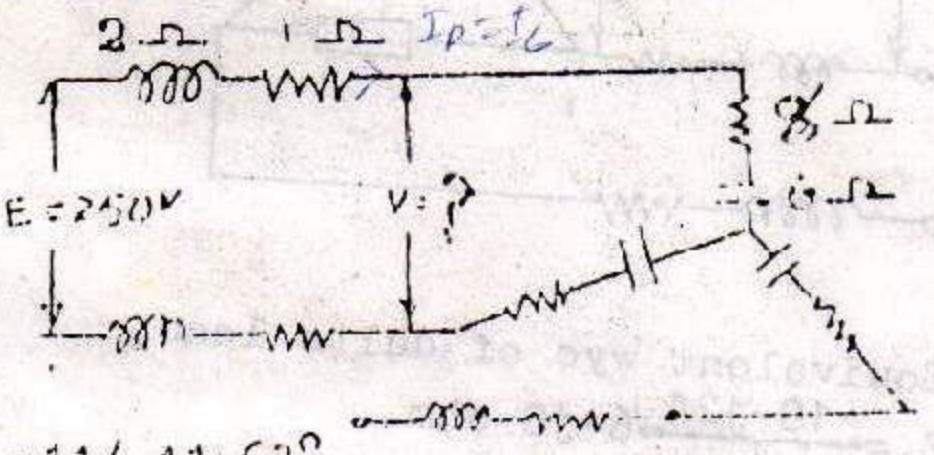
23. $V_L = 100$V $\therefore V_p = \frac{100}{\sqrt{3}} = 57.735$V

$I_p = \frac{V_p}{Z_p} = \frac{57.735}{\sqrt{3^2+4^2}} = 11.547$



Now, $E_p = I_p(j4+1+3-j4) = 11.547 \times 4 = 16.188$ Volt.

$\therefore E_L = \sqrt{3}E_p = 80$ Volts.



(24) Sending voltage $E_L = 250V$

$\therefore E_p = \frac{250}{\sqrt{3}} = 144.39$ volts.

$\therefore I_p = \frac{E_p}{1+j2+8+j6} = \frac{144.39}{12.042/41.63} = 11/-41.63°$

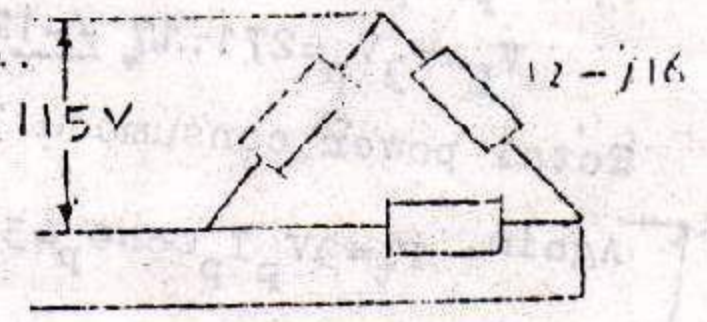
$\therefore V_p = I_p x(8+j6) = (12/-41.63)(10/36.87) = 120/-4.76°$

$\therefore V_L = \sqrt{3}\, V_p = 207.85$ volts. Ans.

(25) Here, $V_p = V_L = 115V$

$\therefore I_p = \frac{V_p}{Z_p} = \frac{115}{12-j16} = 5.75/-53.13°$

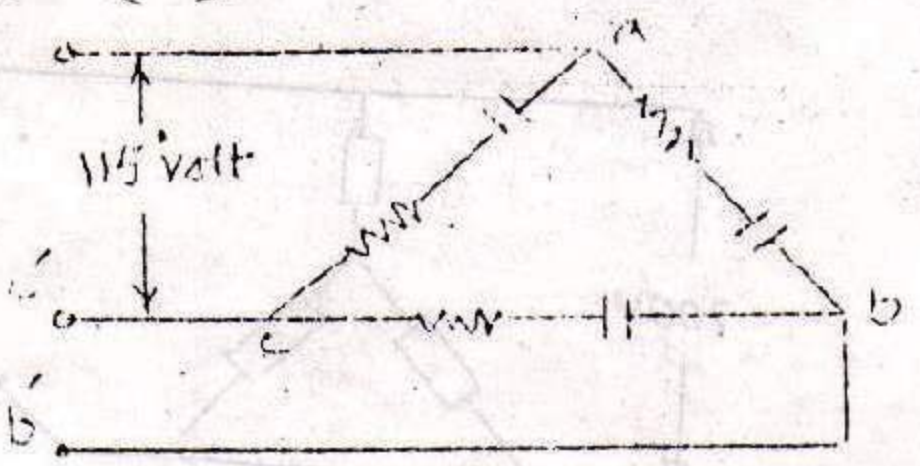
$\therefore I_L = \sqrt{3}I_p = 9.96/53.13°$ Ans. (OR)



Z/phase = 12-j16 = 20/-53.13°

$V_p = 115$ volts.

$I_p = \frac{115}{20} = 5.75$ amp.



$V_{ab} = 115/0°$, $V_{bc} = 115/-120$

$V_{ca} = 115/-240$

$I_{ab} = \frac{Vab\ /0}{Z} = \frac{115/0}{20/-53.13} = 5.75/53.13 = 3.45+j4-6$

$I_{bc} = \frac{V_{bc}/-120}{Z} = \frac{115/-120}{20/-53.13} = 5.75/-66.87 = 2.26-j5.29$

$I_{ca} = \frac{V_{ca}}{Z} = \frac{115/-240}{20/-53.13} = 5.75\ /-186.87 = -5.71+j0.69$

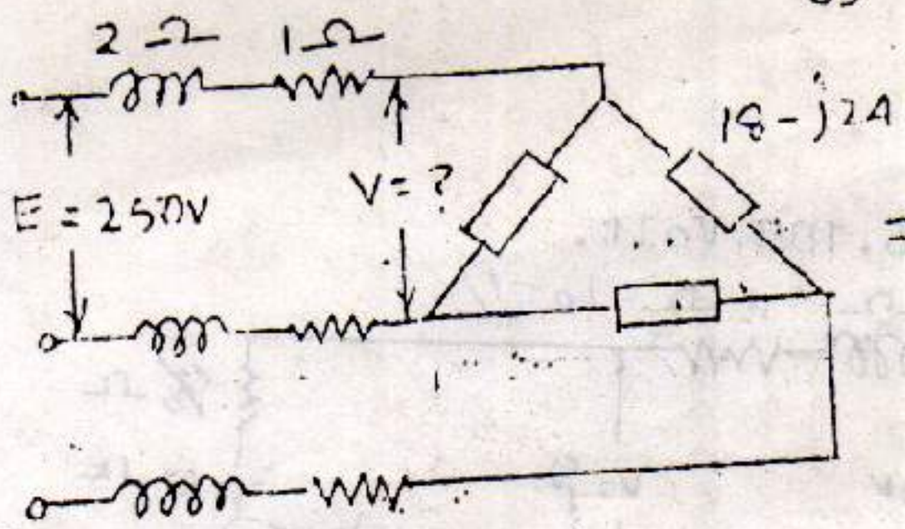
$Ia'a = I_{ab}+I_{ac} = I_{ab}-I_{ca} = 3.45+j4.6+j5.71-j0.69 = 9.16+j3.91 = 9.96/23.11$

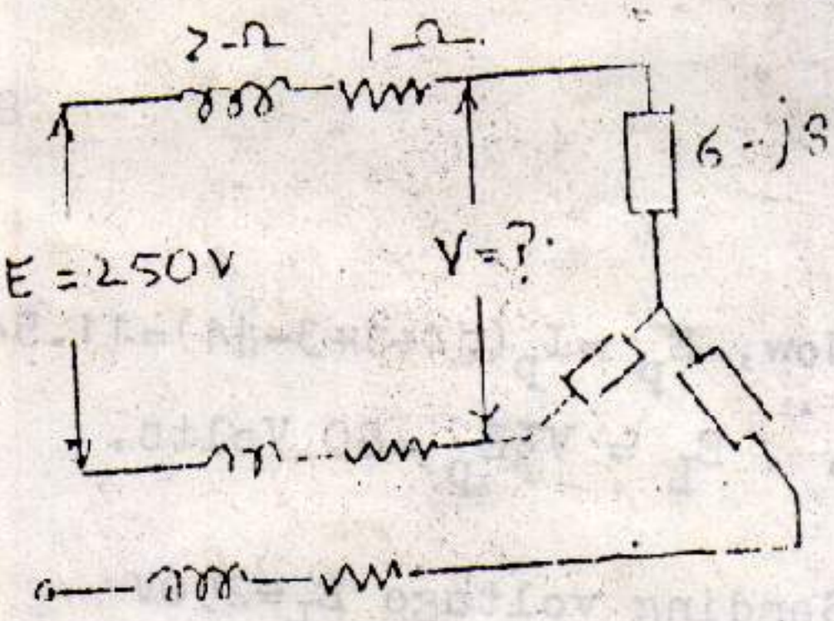
$Ib'b = I_{ba}+I_{bc} = I_{bc}-I_{ab} = 2.26-j5.29-3.45+j4.6 = 1.19-j9.89 = 9.96/-96.93$

$Ic'c = I_{ca}+I_{cb} = I_{ca}-I_{bc} = -5.71+j0.69-2.26+5.29 = -7.97+j5+j5.98$
$= 9.96\ /113.118$

$\therefore$ Line to line $(I_L) = 9.96$.

(25)





Equivalent wye of delta load,

$Z_p = \frac{18-j24}{3} = 6-j8$ ohm.

Now, $E_p = \frac{E_L}{\sqrt{3}} = \frac{250}{\sqrt{3}} = 144.34$ volts.

$\therefore I_p = \frac{E_p}{Z_p} = \frac{144.34}{1+j2+6-j8} = 15.656 \angle 40.6°$

$\therefore V_p = I_p(6-j8) = 156.56 \angle -12.53°$

$V_L = \sqrt{3} V_p = 271.17 \angle -12.53°$

$Z_A = \frac{Z_1 Z_2}{Z_1+Z_2+Z_3} = \frac{Z_1 * Z_2}{3Z_1}$

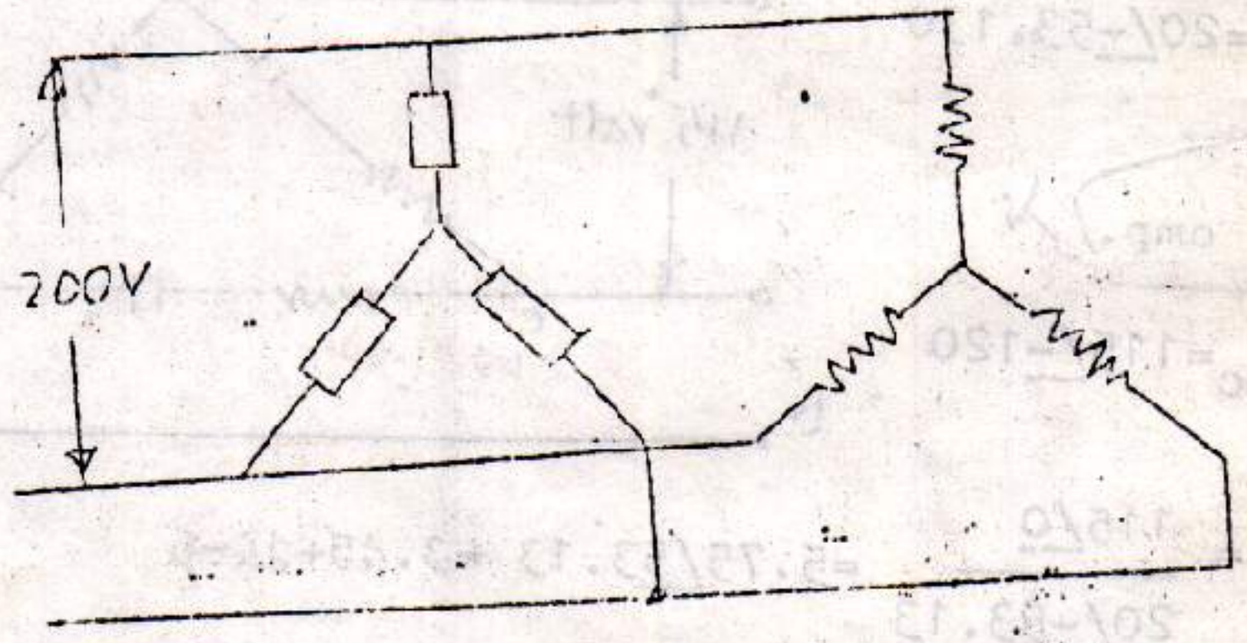
$= Z/3$ [$Z_1 = Z_2 = Z_3$]

Total power consumed $= 3I_p^2 R_p = 3 \cdot (15.656)^2 \times 6 = 4412$ watts.

Again, $P_t = 3V_p I_p \cos\theta_p = 3 \times 156.56 \times 15.656 \times \cos\{40.6-(-12.53)\}$

$= 4412$ watts.

(27)



1st Method:

For balanced inductive load, $P = \sqrt{3} V_L I_L \cos\theta_p$

or, $5.4 \times 10^3 = \sqrt{3} \times 200 \times I_L \times 0.6$ $\therefore I_{L1} = 25.98$ Amp.

Since p.f. is 0.6 and the load in inductive.

$\therefore I_{L1} = 25.98 \angle -53.13°$ $\therefore \cos\theta = 0.6$, $\therefore \theta = 53.13$

For valanced resistive load. $P = \sqrt{3} V_L I_L \cos\theta_p$ ($\theta = 0°$)

or, $5 \times 10^3 = \sqrt{3} \times 200 \times I_L 2 \times 1$ $\therefore I_{L2} = 14.43$

Since the load is resistive $\therefore I_{L2} = 14.43 \angle 0°$

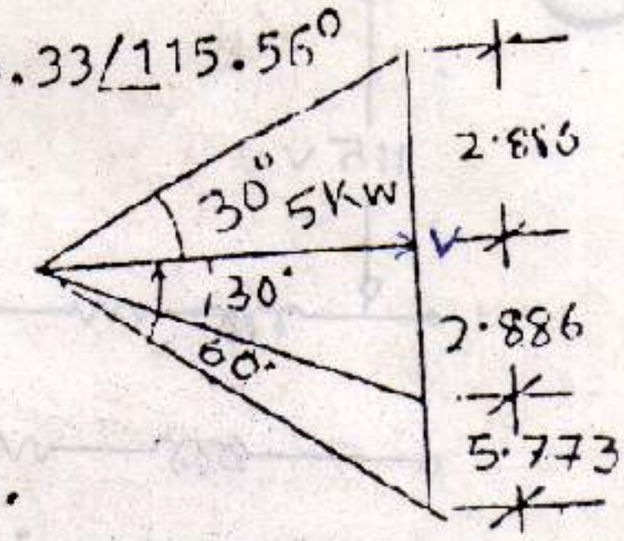
$\therefore$ Resultant current $= 25.98 \angle -53.13° + 14.43 \angle 0°$

$= 15.588 - j20.784 + 14.43 = 36.5 \angle -34.7°$ Ans.

$\therefore I_L = 10\angle 36.87$ . . Line drop per phase, is $V_p = 10\angle 36.87(1+j5)$
$= 51\angle 115.56^\circ$

$\therefore V_{Line} = \sqrt{3} \times 51\angle 115.56 = 88.33\angle 115.56^\circ$

$\therefore$ Voltage at the load terminal is $-230\angle 0^\circ - 88.33\angle 115.56^\circ$
$= 230 + 38.11 - j79.69 = 279.7\ \angle -16.55$ Ans.



(32) For the load, $P = \sqrt{3} V_L I_L \cos\theta = 10 \times 0.5 = 5$Kw.

$\therefore P_x = 10 \sin\theta = 10 \times \sin 60^\circ = 8.66$K vars.

For capacitive bank, if $P = 0$, $P_x = x$(say)

Since the p.f. is reduced to 0.866(lagging)from 0.5(lagging)
we have- $\tan(\cos^{-1}0.866) = \frac{8.66 - x}{5}$ . . $x = 5.773$K vars. → $\tan\theta = \frac{P_x}{P_T}$

Since the p.f. is reduced to 0.866(load) from 0.5 (lag)
We have-

$\tan(\cos^{-1} 0.866) = \frac{x - 8.66}{5}$ . . $x = 11.547$K vars. Ans.

(33) When they are Delta connected and p.f. is 0.866 lagging-

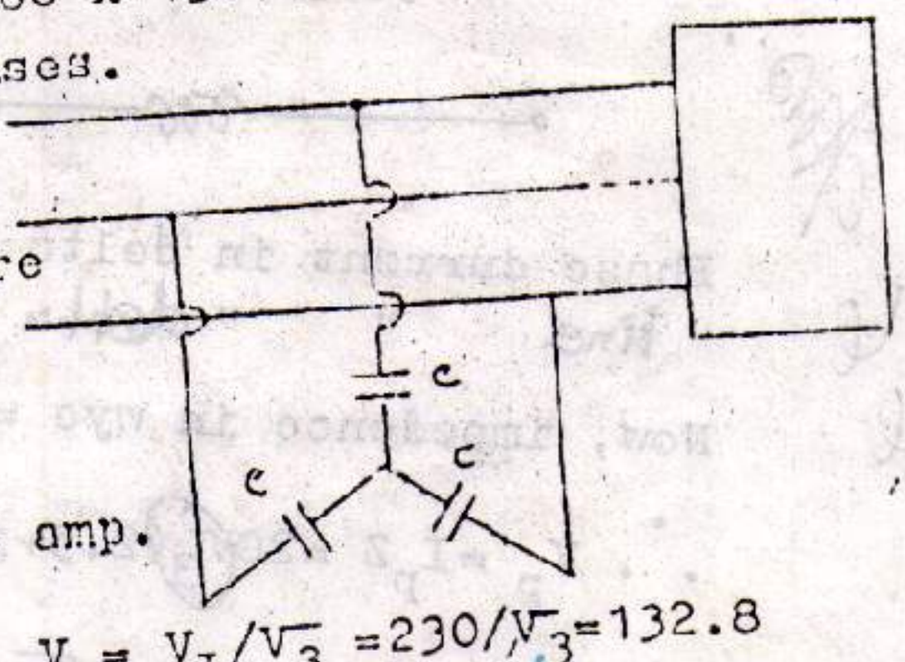
$P_x = \sqrt{3} V_L I_L \sin\theta_p$, or $5.773 \times 10^3 = \sqrt{3} \times 230 \times I_L \times \sin(\cos^{-1} 0.866)$

$\therefore I_L = 28.98$ . . $I_p = \frac{28.98}{\sqrt{3}}$

Now, $Z_p = \frac{V_p}{I_p} = \frac{230}{\frac{28.98}{\sqrt{3}}} = 13.746$ ohm.

$\therefore Z_p = \frac{1}{\omega c}$ or, $c = \frac{1}{\omega Z_p} = \frac{1}{2\pi \times 60 \times 13.746} = 193\mu F.$

Similarly calculated for other cases.



For Star

When p.f. 0.866 load we get before

$I_L = 14.484$ Amp.

Since it is star connection,
I_p is the same of I_L i.e. 14.484 amp.

$V_p = V_L/\sqrt{3} = 230/\sqrt{3} = 132.8$

$X_{cp} = \frac{V_p}{I_p} = \frac{132.8}{14.484} = 9.14$ ohm.

$\frac{1}{2\pi fc} = X_{cp} = 9.14$ . . $c = 289\mu F$

When p.f. leading- $\sqrt{3} V_L I_L = 11.55 \times 10^3$ . . $I_L = \frac{11.55 \times 10^3}{\sqrt{3} \times 236}$

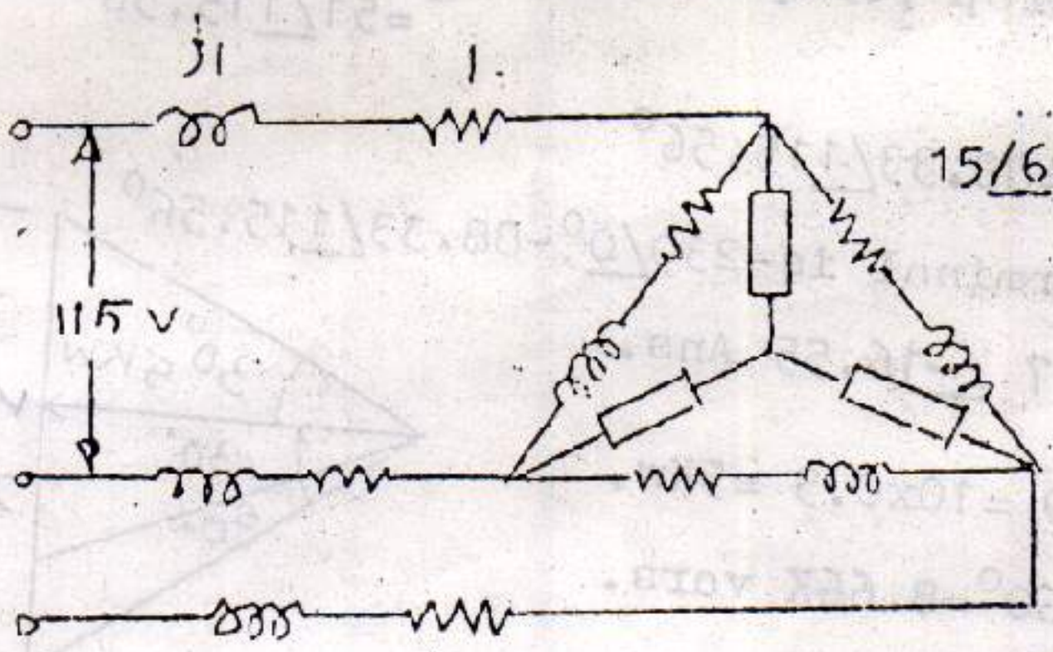
$= 29$ amp.

. . $I_p = 29$ amp.

$X_{cp} = \frac{V_p}{I_p} = \frac{132.8}{29} = 4.58$ ohm. $\frac{1}{2\pi fc} = 4.58$ . . $c = \frac{1}{2\pi f\ 4.58}$

$= 5.79 \times 10^{-7} = 579.11\mu F.$

34.



$15\angle 60^\circ = 7.5 + j13$

Converting $\triangle$ to wye, $Z_p = \frac{7.5 + j13}{3} = 2.5 + j\,4.33$

Now, current, $I_p = \frac{V_p}{Z_{op}} = \frac{\frac{115}{\sqrt{3}}}{1 + j1 + 2.5 + j4.33} = 10.4\angle -56.7^\circ$

Voltage across the load is, $V_p' = (10.4\ \angle -56.7)(2.5 + j4.33)$

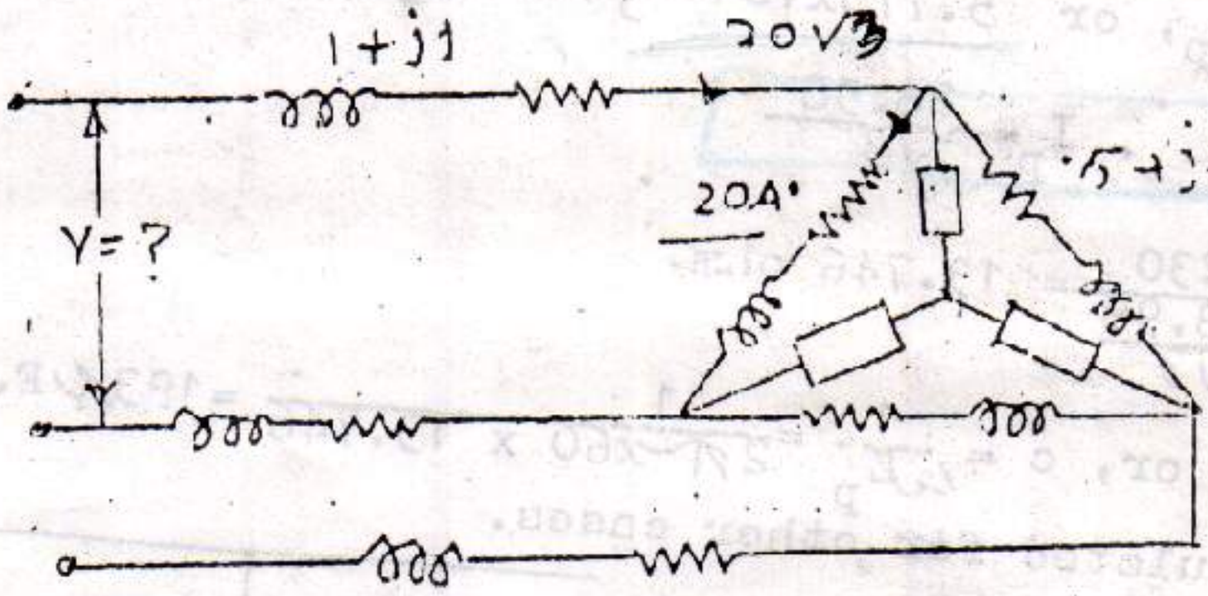
$= 52\angle 3.3^\circ$

Line to line Voltage $= \sqrt{3} \times 52 = 90.07$ Volts.

Power loss in the supply lines $= 3I_L^2 R_L = 3 \times (10.4)^2 \times 1 = 325$ watts. Ans.

Power dissipted in the load $= 3 \times (10.41)^2 \times 2.5 = 812.76$ watts. Ans.

35.



Phase current in delta $= 20A$

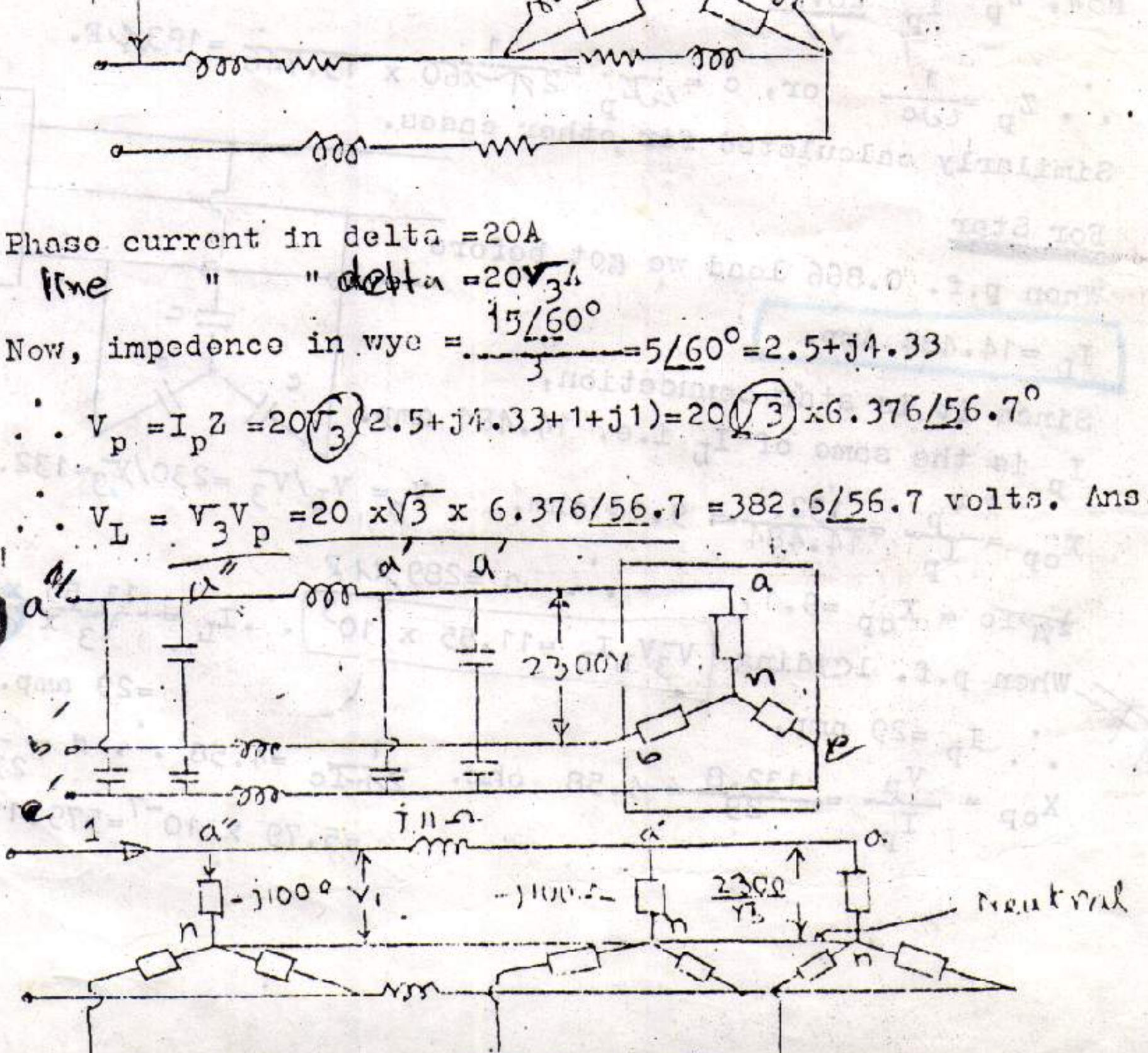
line " " delta $= 20\sqrt{3}A$

Now, impedence in wye $= \frac{15\angle 60^\circ}{3} = 5\angle 60^\circ = 2.5 + j4.33$

$\therefore V_p = I_p Z = 20\sqrt{3}(2.5 + j4.33 + 1 + j1) = 20\sqrt{3} \times 6.376\angle 56.7^\circ$

$\therefore V_L = \sqrt{3} V_p = 20 \times \sqrt{3} \times 6.376\angle 56.7 = 382.6\angle 56.7$ volts. Ans.

36.



Let the load is connected in delta form. The balanced load has V_L=2300V.

$\therefore \sqrt{3}\ V_L I_L = 100 \times 10^3 \quad \therefore I_L = \frac{100 \times 10^3}{\sqrt{3} \times 2300} = 25.1$ Amp.

Since the p.f. is 0.6 (log) $\therefore I_L = 25.1 \angle -53.13^\circ$

$\therefore Z_p = \frac{V_p}{I_p} = \frac{2300/\sqrt{3}}{25.1\angle -53.13} = 52.9 \angle 53.13^\circ = 31.74 + j42.32$

$I_{an} = I_p = I_L = 25.1 \angle -53.13^\circ$. Now, $Ia'n = \frac{2300/\sqrt{3}}{-j100} = 13.28\angle 90^\circ$ Amp.

$I_{a''a} = I_{an} + I_{a'n} = 25.1\angle -53.13^\circ + 13.28\angle 90^\circ = 16.54\angle -24.4^\circ$

$\therefore V_1 = \frac{2300}{\sqrt{3}} + (16.54\angle -24.4)(j10) = 1327.9 + 165.4\angle 65.6^\circ = 1404.33\angle 6.16^\circ$

$\therefore$ Line to line voltage $= 1404.33 \times \sqrt{3} = 2432.37$ volts. Ans.

Now, Current, $Ia''n = \frac{V_1}{-j100} = 14.043\angle 96.16^\circ$

$\therefore I = Ia''a + Ia''n = 16.54\angle -24.4 + 14.043\angle 96.16$

$= 15.063 - j6.833 - 1.507 + j13.96 = 15.316\angle 27.74^\circ$

Voltage to the line input $= \sqrt{3} \times 1404.33\angle 6.16^\circ$

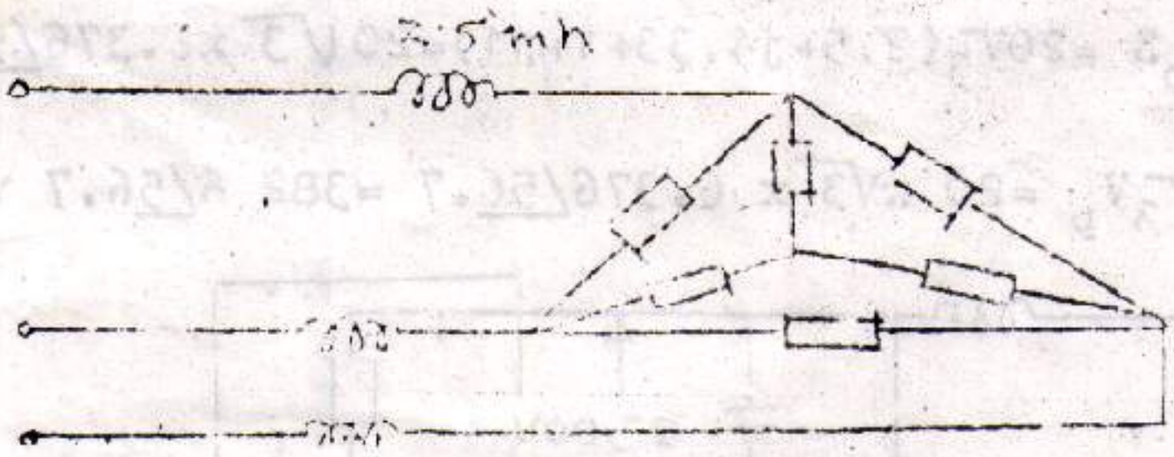
$\therefore$ Power input $= \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times \sqrt{3} \times 1404.33 \times 15.316 \cos(6.16 - 27.74)$

$= 60103.19w = 60Kw$.

Power factor $= \cos(6.16 - 27.74) = 0.93$(load)

37. 38. $\therefore$ More or less same as 36 Do yourself.

39.



Since the shunt consists of three 10KVa. 2300 volt. capacitance in delta.

$\therefore \sqrt{3} V_L I_L = 3 \times 10 \times 10^3 \quad \therefore I_L = \frac{3 \times 10^4}{\sqrt{3} \times 2300} = 7.53$ Amp.

$\therefore V_L = V_p = 2300$, $I_p = \frac{I_L}{\sqrt{3}} = 4.3478A$ $\therefore Z_p = \frac{V_p}{I_p} = 529$ ohm.

Converting Delta to wye, $Z_{pwye} = \frac{529}{3} = 176.333$ ohm.

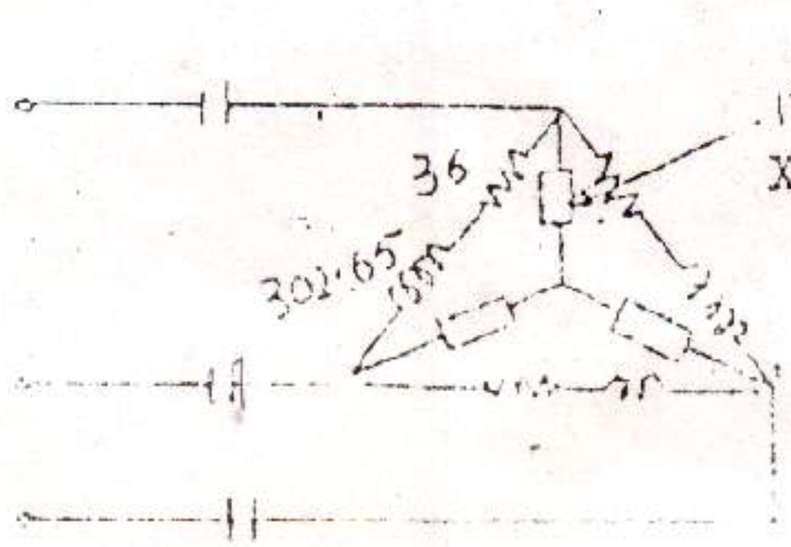
This capacitive reactance is at 60 cycles.

$\therefore \frac{1}{2\pi \times 60 \times c} = 176.33$ $\therefore$ $C = 15 \times 10^{-6}$ farad.

Now, for resonance- $w_L = \frac{1}{wc}$ or $f = \frac{1}{2\pi\sqrt{L_c}} = \frac{1}{2\pi\sqrt{2.5\times10^{-3}\times15\times10^{-6}}}$

$= 822$Hz.

10. (a)



$X_L = wL = 2\pi \times 800 \times 100 \times 10^{-3}$

$= 502.65$ ohm.

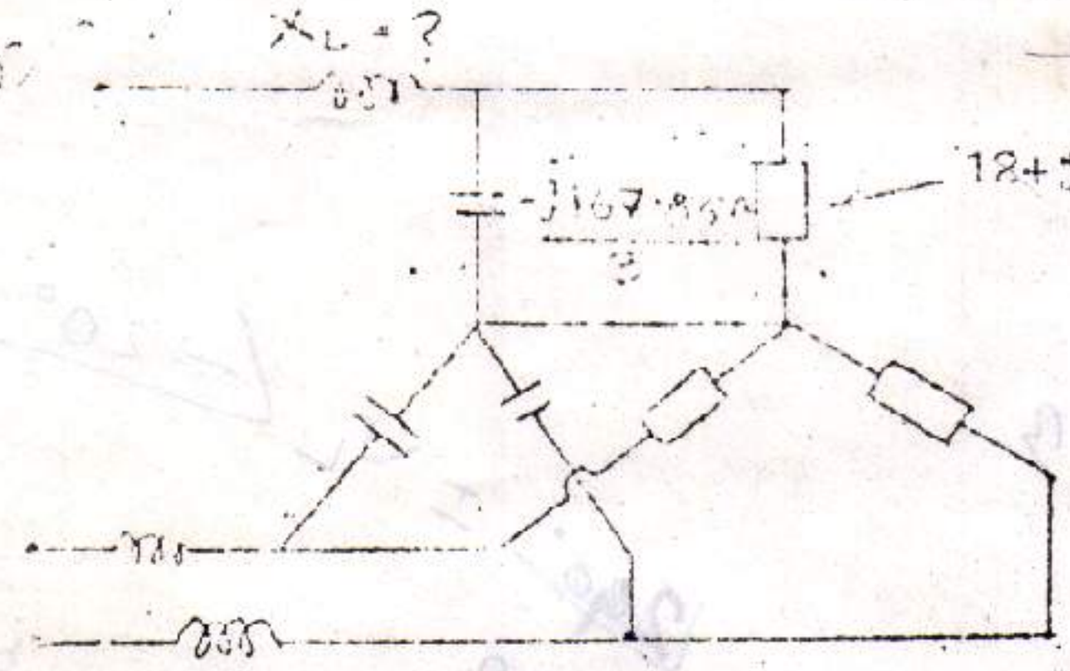
When connected in Delta $Z_p = 36 + j502.65$

" " " wye $Z_{pwye} = 12 + j167.55$

$\therefore$ For resonance the capacitive reactance will be 167.55 ohm.

$\therefore X_c = \frac{1}{wc} = 167.55$ $\therefore$ $C = \frac{1}{2\pi \times 800 \times 167.55} = 1.187$ [illegible] farad.

(b)

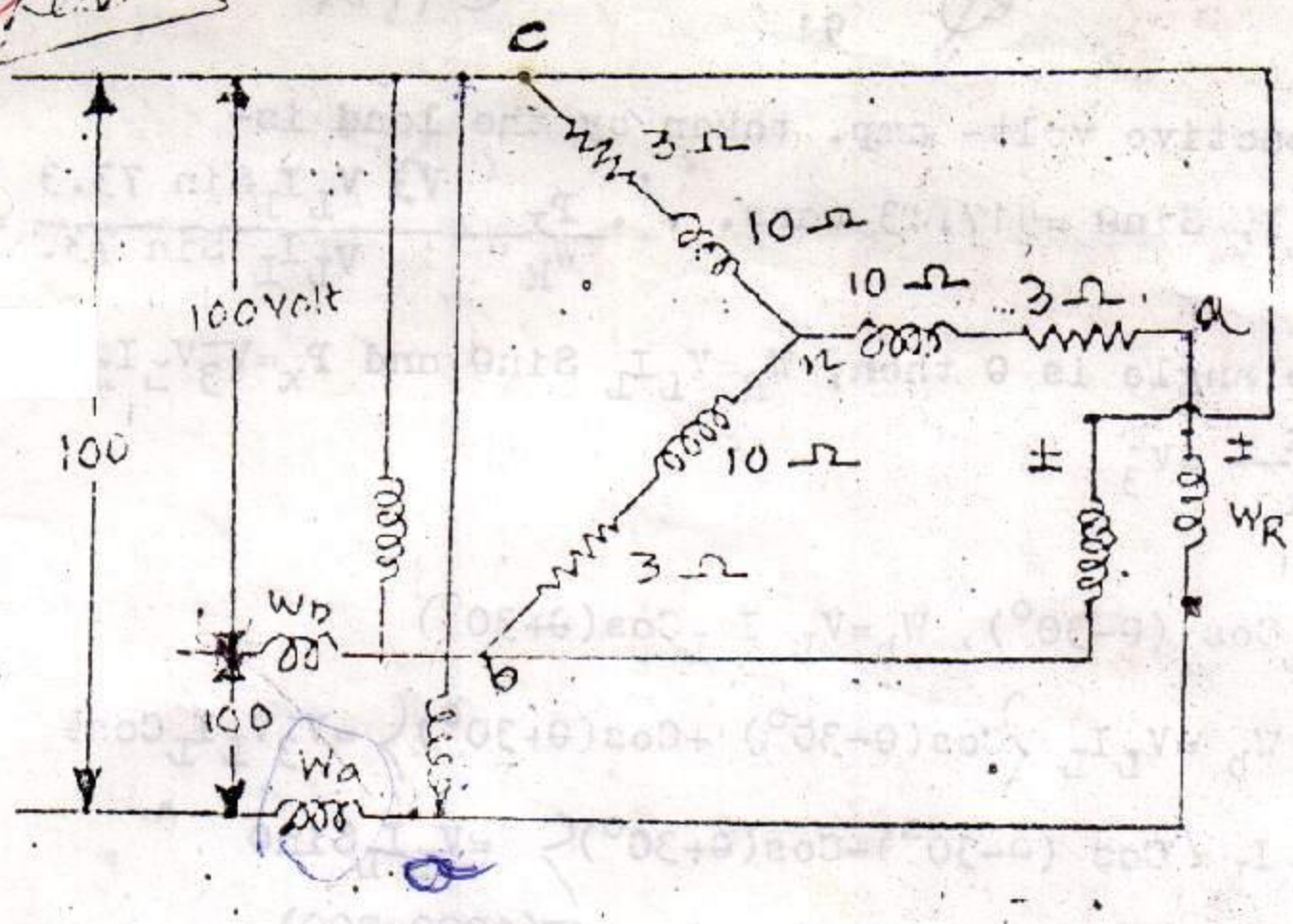


Now, equivalent parallel impedence per phase=

$$= \frac{(12+j167.55)(-j\frac{167.55}{3})}{12 + j167.55 - j\frac{167.55}{3}} = \frac{(12+j167.55)(-j55.85)}{12+j167.55-j55.85}$$

$$= \frac{9981.6368 \angle -4.1}{112.3427 \angle 83.87} = 88.85 \angle -87.97^\circ = 3.15 - j88.794.$$

$\therefore$ For resonance $X_L = 88.794$ ohm. $\therefore$ $L = \frac{88.794}{2\pi \times 800} = 17.665$mh. Ans.



Phase sequence V_{na}, V_{nc}, V_{nb} (a,c,b) V_{ac}, V_{cb}, V_{ba} -

$V_{ac} = 100 \angle 0^\circ V$

$V_{cb} = 100 \angle -120^\circ V$

$V_{ba} = 100 \angle +120^\circ V$

$V_p = \frac{100}{\sqrt{3}} = 57.74 V$

$V_{na} = 57.74 \angle 0^\circ$ volts.

$V_{nc} = 57.74 \angle -120^\circ$

$V_{nb} = 57.74 \angle 120^\circ$, $Z_p = 3 + j10 = 10.44 \angle 73.3^\circ$

$I_p = V_p / Z_p = 57.74 / 10.44 \angle \pm 73.3^\circ = 5.53 \angle -73.3^\circ$

$W_a = (100 \angle -150) \times (5.53) \cos -103.3 = -127.22$ watts.

$W_b = 100 \times 5.53 \cos 43.3 = 402.46$ watts.

Power in each phase $= (W_a + W_b)/3 = 91.75$ W, Alternatively,

$I^2 R_p = (5.53)^2 \times 3 = 91.75$ W

(a2) Power, $W_a + W_b = 5000$ reactive power, $\sqrt{3}(W_a - W_b) = 20{,}000$

or $W_a - 5000 + W_b = \frac{20000}{\sqrt{3}}$ . . $W_a = 8273.5$ watts. Ans.

. . $W_b = -3273.5$ watts. Ans.

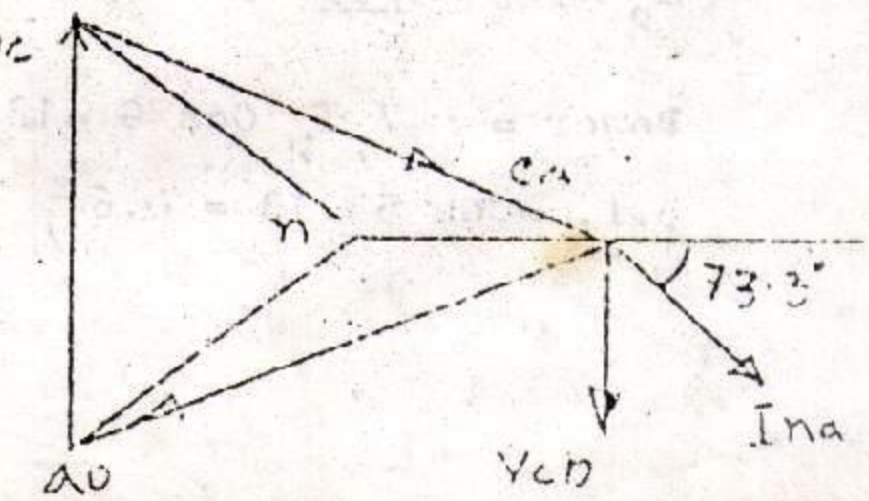
(b) The reading of W_R is

$W_R = V_{cb} I_{na} \cos(V_{cb} I_{na})$

Ans. $= V_L I_L \cos(90^\circ - 73.3^\circ)$

$= V_L I_L \sin 73.3^\circ = 100 \times 5.53 \times \sin 73.3^\circ$

$= 529$ watts.



Total reactive volt- amp. taken by the load is-

$P_x=\sqrt{3}\ V_L I_L \sin\theta = 917.43$ vars. $\therefore \dfrac{P_x}{W_R} = \dfrac{\sqrt{3}\ V_L I_L \sin 73.3}{V_L I_L \sin 73.3}$ =

44. If phase angle is θ then, $W_R=V_L I_L \sin\theta$ and $P_x=\sqrt{3}V_L I_L \sin\theta$

$\therefore \dfrac{P_x}{W_R} = \sqrt{3}$

45. $W_a=V_L I_L \cos(\theta-30^\circ)$, $W_b=V_L I_L \cos(\theta+30^\circ)$

$\therefore W_a + W_b = V_L I_L \{\cos(\theta-30^\circ)+\cos(\theta+30^\circ)\} = \sqrt{3}V_L I_L \cos\theta$

$W_a - W_b = V_L I_L \{\cos(\theta-30^\circ)-\cos(\theta+30^\circ)\} = V_L I_L \sin\theta$

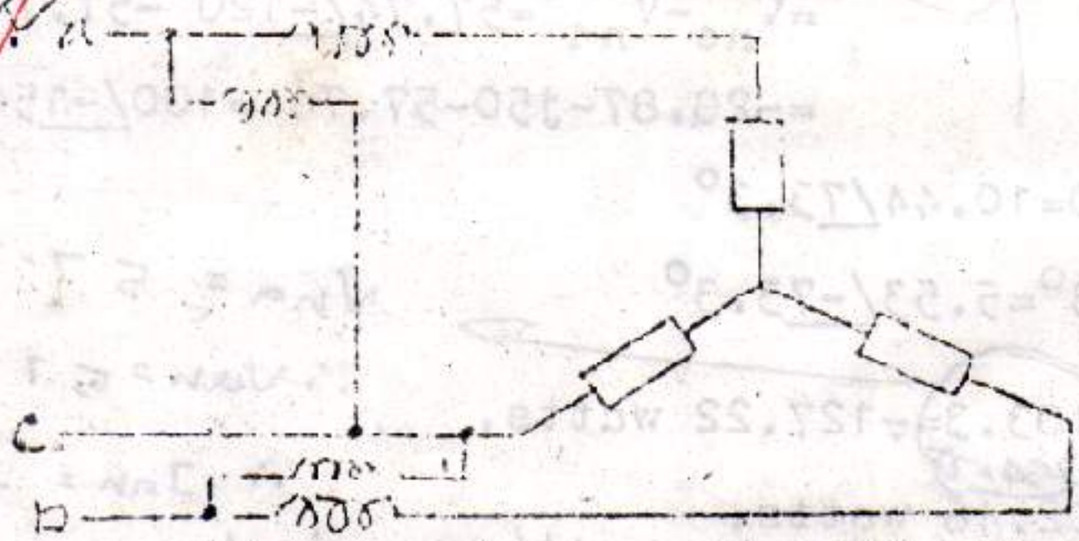
$\therefore \dfrac{W_a - W_b}{W_a + W_b} = \dfrac{\sin\theta}{\sqrt{3}\cos\theta}$ or $\tan\theta = \dfrac{\sqrt{3}(1000-800)}{1000+800}$ $\therefore \theta = 10.9^\circ$

(b) Here, $\tan\theta = \dfrac{\sqrt{3}(100+800)}{(1000-800)}$ $\therefore \theta = 66.33^\circ$

46. $\tan\theta = \dfrac{\sqrt{3}(1200+400)}{1200-400}$ $\therefore \theta = 73.9^\circ$ $\therefore$ p.f. $=\cos\theta = 0.277$ Ans.

Power, $P = W_a + W_b = 800$ watts. Now, $P = \sqrt{3}V_L I_L \cos\theta$

or, $I_x = P_x \cos\theta$ $\therefore P_x = \dfrac{P}{\cos\theta} = 2888$ Va Ans.



$W_a = +1000$ watts.

$W_b = +400$ watts.

$\therefore P = 1400$ watts.

$P_x = \sqrt{3}(W_a - W_b) = 1039.23$ vars.

p.f. $= \dfrac{P}{\sqrt{P^2 + P_x^2}} = 0.8029$

$\therefore W_a = \sqrt{3}V_L I_L \cos(\theta-30^\circ)$, $W_b = \sqrt{3}V_L I_L \cos(\theta+30^\circ)$

$\therefore$ The voltage sequence is ACB.

48. $V_L = 51.76V$, $V_L = \sqrt{3}V_p \sin\dfrac{180^\circ}{12}$ or, $51.76 = \sqrt{3}V_p \sin\dfrac{180^\circ}{12}$ $\therefore V_p = 100V$

$Z_p = 3+j4 = 5\angle 53.13^\circ$ $\therefore I_p = \dfrac{V_p}{Z_p} = \dfrac{100}{5\angle 53.13} = 20\angle -53.13^\circ$

Power $= 12 \times V_p I_p \cos\theta = 12 \times 100 \times 20 \times \cos(53.13) = 14400$ watts.

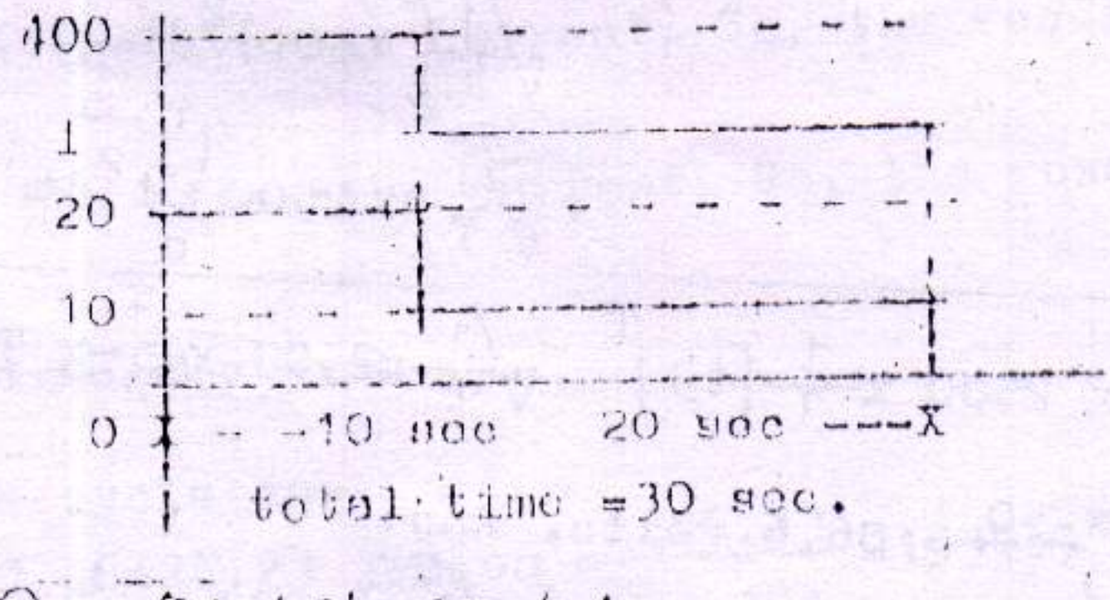
p.f. $= \cos 53.13 = 0.6$, $I_L = I_p = 20$ Amp.

PROBLEMS (Chapter : 3)

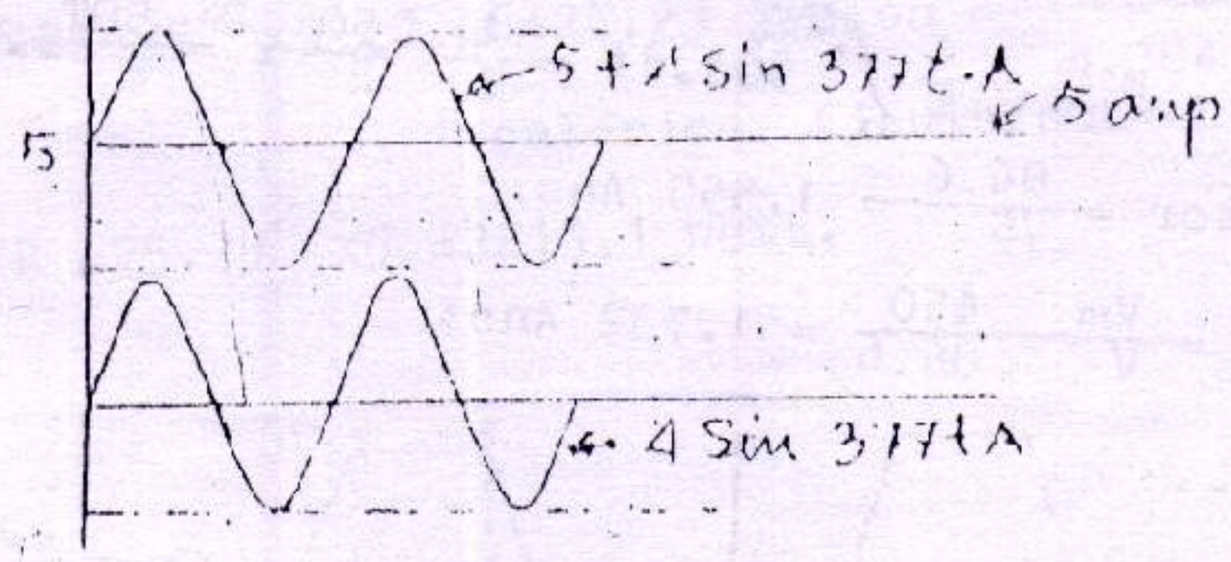
1. $I = \sqrt{\text{average } I^2}$

$= \sqrt{\frac{400 \times 10 + 100 \times 200}{30}}$

$= 14.14$ amp. Ans:



2. (a)



(b) $I = 5 + 4 \sin 377t$ A Ans:

(c) Since the average value of a wave which is symmetrical about x axis is zero.

∴ Average Value (I_{av}) $= 5 + 0 = 5$ amp Ans:

(d) $I_{eff} = \sqrt{\frac{1}{T}\int_0^T (5 + 4 \sin 377t)^2 dt}$ | $16\sin^2 377t$, $-8.2\sin^2 377t$, $8(1+\cos 2\times 377t)$

$= \sqrt{\frac{1}{T}\int_0^T \{25 + 40 \sin 377t + 8(1+\cos 754)\} dt}$

$= \sqrt{\frac{1}{T}\left[33t - \frac{40}{377}\cos 377t + \frac{8}{754}\sin t\right]_0^T}$

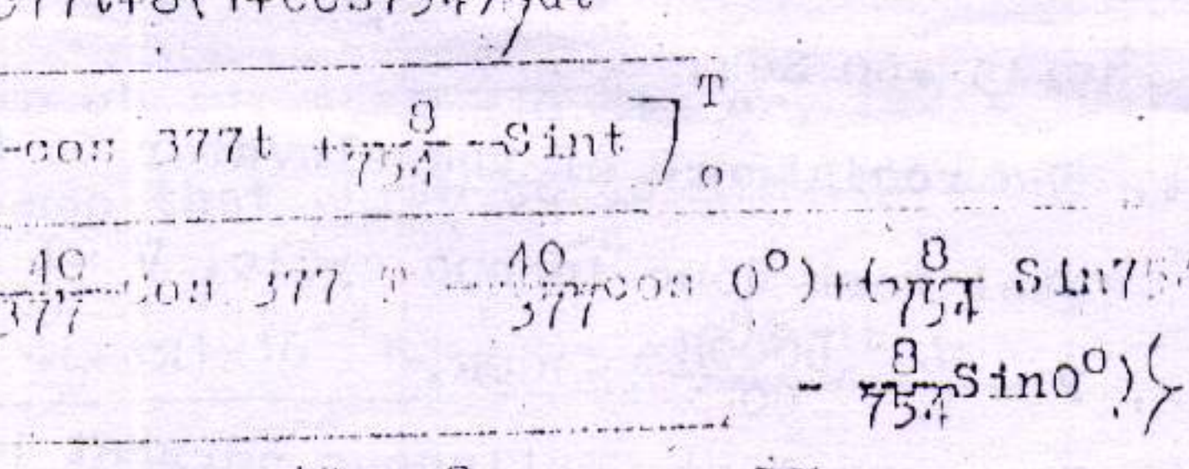
$= \sqrt{\frac{1}{T}\left\{(33T - 0) - \left(\frac{40}{377}\cos 377T - \frac{40}{377}\cos 0^\circ\right) + \left(\frac{8}{754}\sin 754T - \frac{8}{754}\sin 0^\circ\right)\right\}}$

$= \sqrt{\frac{1}{T}\left[33T - \frac{40}{377}\cos 377T + \frac{40}{377} + \frac{8}{754}\sin 754T\right]}$ | $\cos 377T = 1$, $\cos 377\times T = 1$, $\cos 0^\circ = 1$, $\sin 754T = 0$, $\sin 0^\circ = 0$

$= \sqrt{\frac{1}{T}\left[33T - \frac{40}{377}\times 1 + \frac{40}{377} + 0\right]} = \sqrt{33 - \left(\frac{1}{T} - \frac{1}{T}\right)\frac{40}{377}}$

$= \sqrt{33 - \frac{40}{377}} = 5.$ [illegible] amp. Ans:

3.



Now, $e = \frac{50}{3}t$

or, $e = 50t$ Volts.

∴ Effective Value, $E = \sqrt{\frac{1}{T}\int_0^T e^2 dt}$ | We know, $y = mx + c$ hence $c = 0$

$= \sqrt{\frac{1}{T}\int_0^T (50t)^2 dt}$

$= \sqrt{\frac{1}{T} \times 2500 \int_0^T t^2 dt}$

$= \sqrt{\frac{1}{T} \times 2500 \times \frac{1}{3}\left[t^3\right]_0^T} \quad \sqrt{\frac{1}{T} \times 2500 \times \frac{1}{3} \times T^3} \quad \sqrt{\frac{2500T^2}{3}} = \frac{T \times 50}{\sqrt{3}}$

$= \frac{3 \times 50}{3} = 86.6$ volts. Ans:

Average Value, $E_{av} = \frac{1}{T}\int_0^T 50t.dt = \frac{1}{T} \cdot \frac{50}{2} T^2 = \frac{50T}{2} = 25 \times 3 = 75$ Volts.

∴ Form Factor $= \frac{86.6}{75} = 1.555$ Ans:

4. Crest Factor $= \frac{Vm}{V} = \frac{150}{86.6} = 1.732$ Ans:

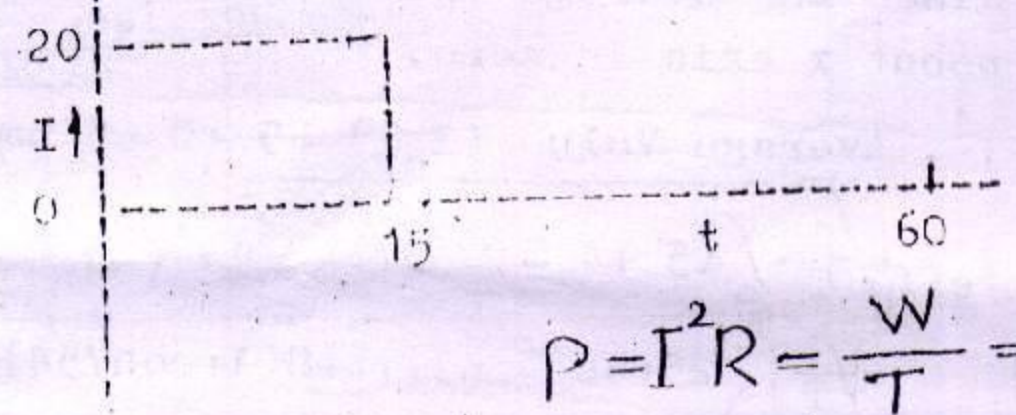
(8.)

$P = I^2 R = \frac{W}{T} = \frac{I^2 R t}{T}$

Average current,

$I = \sqrt{\frac{1}{60}(20^2 \times 15)}$

$= 10$ amp. Ans:



Total time period,

$T = 15+45 = 60$ Sec.

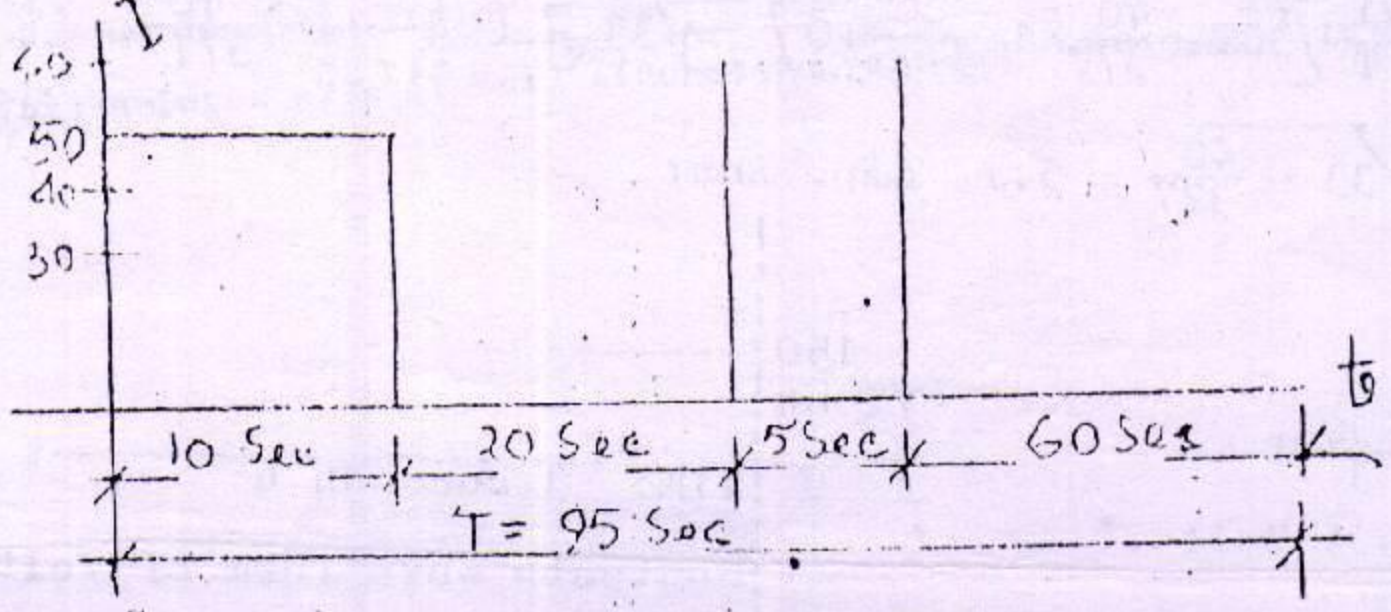
Let, the resistance of the elevator be R ∴ $I^2 = \frac{i^2 t}{T} = \frac{(20)^2 \times 15}{60}$

∴ Total work done in one cycle, $W = I^2 Rt = 20^2 . R . 15 = 6000R$ ∴ $I = 10$ amp

Now, $P = \frac{W}{t} = \frac{6000R}{60} = 100R.$

Let, The equvalent Continoous current be I_1

∴ $I = I_1^2 R = 100R$ or, $I_1 = 10$ amp. Ans:



$= \sqrt{\frac{50^2 \times 10 + 60^2 \times 5}{95}} = 21.274$ Amp:

10. The period, T = 0.3 + 0.5 + 0.4 + 0.5 + 0.3 = 2 Sec.

(a) Average curfent, $I_{ave} = \frac{10\times0.3+15\times0.5+50\times0.4+15\times0.5+10\times0.3}{2}$ = 20.5A Ans:

(b) $I = \sqrt{\frac{10^2\times0.3+15^2\times0.5+50^2\times0.4+10^2\times0.3+15^2\times0.5}{2}}$ = 25.35A Ans:

11.(a) d.c. ammeter shows the average current. So, the reading will be 20.5A.

a-c ammeter shows the effective current. So, the reading will be 25.35A.

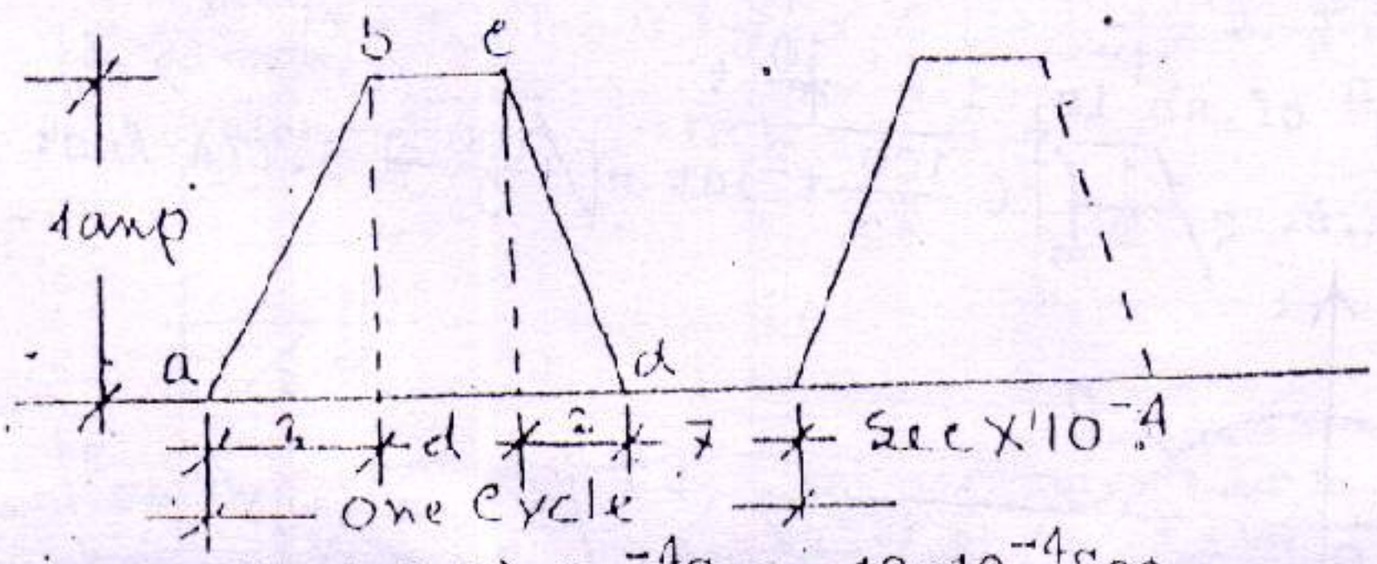
(b) a-c reading should be employed.

12. Energy produced in one cycle is-

(a) $I^2Rt = 25.35^2 \times 5 \times 2 = 6426.23$ joules $= \frac{6426.23}{4.184} = 1535.9$ gm. — calories. (joule/sec=watt).

(b) $P = I^2R = 25.35^2 \times 5 = 3213.1$ watt.

13.



Time period $T = (2+1+2+7)\times10^{-4}$ Sec. $= 12\times10^{-4}$ Sec.

(a) Frequency $n = \frac{1}{T} = \frac{1}{12\times10^{-4}} = 833.33$ cps.

Eq. of the curve ab is, $i = \frac{4}{2\times10^{-4}}t$ or, $i = 2\times10^4 t$.

From figure we see that curve cd is similar to ab.

(b) $I_{av} = \frac{\int_0^{2\times10^{-4}} 2\times10^4 t\ dt + 4\times1\times10^{-4} + \int_0^{2\times10^{-4}} 2\times10^4 t.dt}{12\times10^{-4}}$

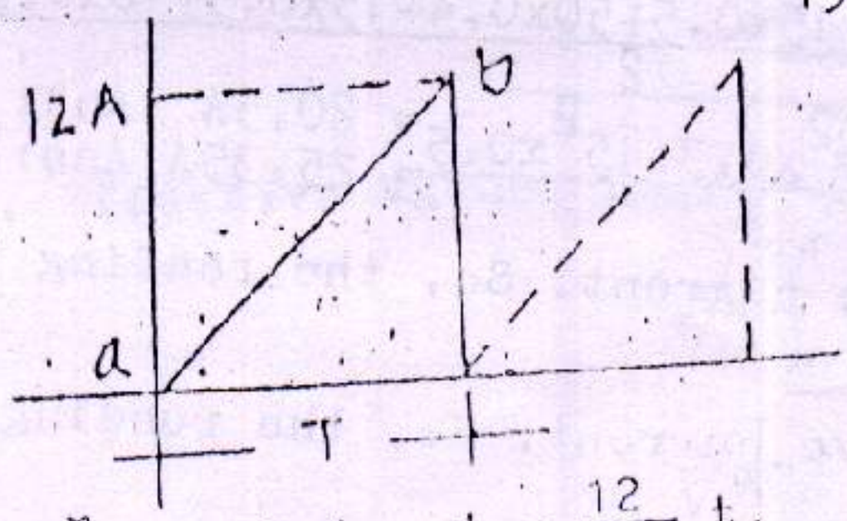
$= \frac{2\times2\times10^4\int_0^{2\times10^{-4}} t.dt + 4\times10^{-4}}{12\times10^{-4}} = \frac{4\times10^4\times\frac{1}{2}(2\times10^{-4})^2+4\times10^{-4}}{12\times10^{-4}}$

$= \frac{2\times10^4\times4\times10^{-8}+4\times10^{-4}}{12\times10^{-4}} = \frac{12\times10^{-4}}{12\times10^{-4}} = 1$ Ans:

(c) $I = \sqrt{\frac{2\times\int_0^{2\times10^{-4}}(2\times10^4 t)^2 dt + 4^2\times10^{-4}}{12\times10^{-4}}} = \sqrt{\frac{2\times\frac{8}{3}\times10^8\times(2\times10^{-4})^3+16\times10^{-4}}{12\times10^{-4}}}$

$= \sqrt{\frac{(\frac{64}{3}+16)\times10^{-4}}{12\times10^{-4}}} = 1.76$ Amp. Ans:

11. 12A



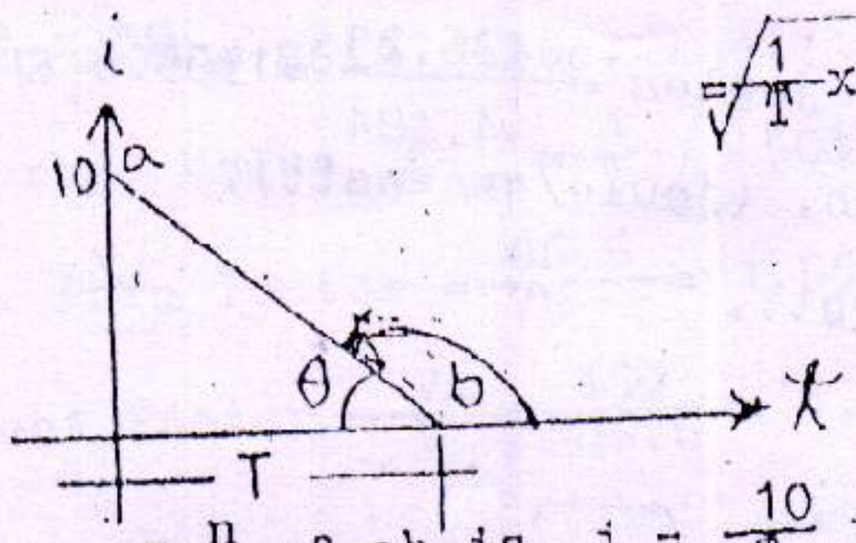
Equ^{n} of ab is $i = \frac{12}{T}t$.

Let time period be T.

$\therefore$ a-c meter reading, $I = \sqrt{\frac{1}{T}\int_0^T (\frac{12}{T}t)^2\,dt} = \sqrt{\frac{1}{T}\left[\frac{144}{T^2} \times \frac{t^3}{3}\right]_0^T}$

$= \sqrt{\frac{1}{T} \times \frac{144}{T^2} \times \frac{T^3}{3}} = \frac{12}{\sqrt{3}} = 6.934$ Ans:

15.



Slop = $\tan(\pi - \theta) = -\tan\theta$.

Eq^{n} of ab $\Rightarrow \frac{i-0}{0-10} = \frac{t-T}{T-0}$

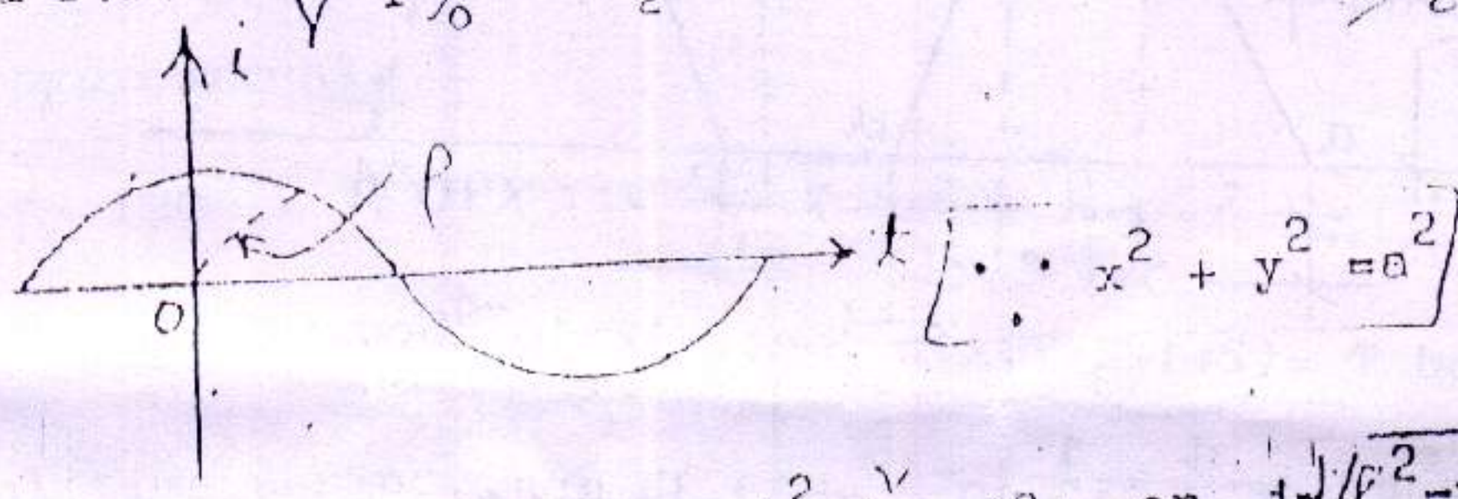
$\Rightarrow \frac{i}{10} = \frac{t-T}{T}$

$\Rightarrow i = 10 - \frac{10t}{T}$

Eq^{n} of ab is, $i = \frac{10}{T}t$

$\therefore$ I r.m.s. $= \sqrt{\frac{1}{T}\int_0^T (\frac{100}{T^2}t^2)\,dt} = \sqrt{\frac{100}{3}} = 5.77$A Ans:

16.



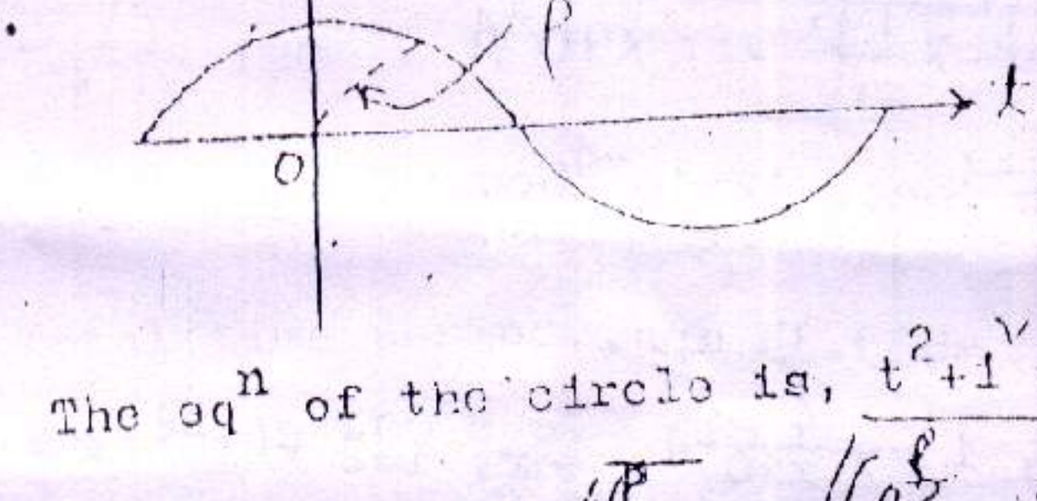
$[\because x^2 + y^2 = a^2]$

The eq^{n} of the circle is, $t^2 + i^2 = \rho^2$, or $i = \sqrt{\rho^2 - t^2}$

$\therefore$ I r.m.s. $= \sqrt{\int_{-\rho}^{\rho} \frac{1}{2}\,dt} = \sqrt{\int_{-\rho}^{\rho} \frac{\rho^2 - t^2}{2\rho}\,dt} = \sqrt{\frac{1}{2\rho}\left[\rho^2 t - \frac{t^3}{3}\right]_{-\rho}^{\rho}}$

$= \sqrt{\frac{1}{2\rho}\left[\rho^2(\rho + \rho) - \frac{\rho^3 + \rho^3}{3}\right]} = \sqrt{\frac{4}{6}\rho^2} = 0.816\rho$ Ans:

17.



eq^{n} of the semicircle is $i^2 + t^2 = 1$

or $i = \sqrt{1 - t^2}$

$\therefore$ Eq^{n} of the resultant current $= 1 + \sqrt{1 - t^2}$

$\therefore$ I r.m.s. $= \int_{-1}^{1} \frac{1 + \sqrt{1-t^2}}{1+1}\,dt = \sqrt{\frac{1}{2}(1 + 2\sqrt{1-t^2} + 1 - t^2)\,dt}$

$= \sqrt{\int_{-1}^{1} (2 + 2\sqrt{1-t^2} - t^2)\,dt}$

$= \sqrt{\left[t + t\sqrt{1-t^2} + \sin^{-1}t - \frac{t^3}{3}\right]_{-1}^{1}}$

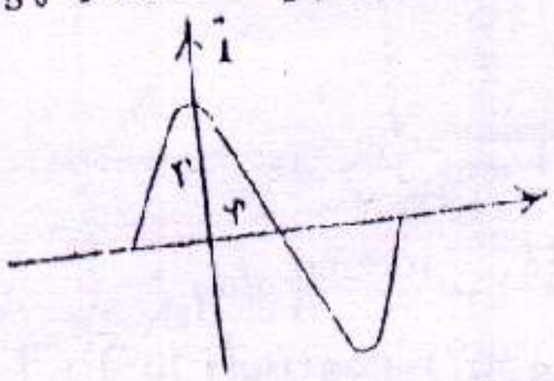
$[\because \int \sqrt{a^2 - t^2}\,dt = \frac{t\sqrt{a^2-t^2}}{2} + \frac{1}{2}\sin^{-1}\frac{t}{a}]$

Again, form factor = $I_{rms}/I_{av} = I_m/(I_m/2) = 2$ Ans.

∴ Peak Factor = $(I_m/\sqrt{2})/(I_m/2) = \sqrt{2} = 1.414$ Ans.

25. Crest Factor = $\frac{\text{max}^m \text{ value}}{\text{r.m.s. value}} = \frac{50}{25.35} = 1.972$ Ans. from 10(b)

26.



Let, the radius be r.

From problem (16), we have $I_{r.m.s.} = 0.816\ r$

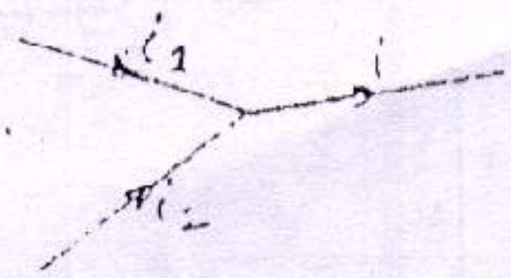
$I_{av} = \frac{1}{2r}\int_{-r}^{r}\sqrt{(r^2-t^2)}\,dt = \frac{1}{2r}\left[\frac{t\sqrt{r^2-t^2}}{2} + \frac{r^2}{2}\sin^{-1}\frac{t}{r}\right]_{-r}^{r}$

$= \frac{1}{2r}\left[0 + \frac{r^2}{2}\sin^{-1}1 - 0 - \frac{r^2}{2}\sin^{-1}(-1)\right] = \frac{1}{2r}\cdot r^2\cdot\frac{\pi}{2} = \frac{\pi r}{4} = 0.785r.$

∴ Crest Factor = $\frac{I_{max}}{I_{r.m.s}} = \frac{r}{0.816r} = 1.225$ Ans.

From Factor = $\frac{I_{r.m.s.}}{I_{av}} = \frac{0.816r}{0.785r}$ ← From problem (16) = 1.04 Ans.

27.



$i_1 = 30\sin(\omega t + 60^\circ) = 30\ \angle 60^\circ$

$i_2 = 20\sin(\omega t - 20^\circ) = 20\ \angle -20$

∴ $i = i_1 + i_2 = 30\ \angle 60^\circ + 20\ \angle -20$

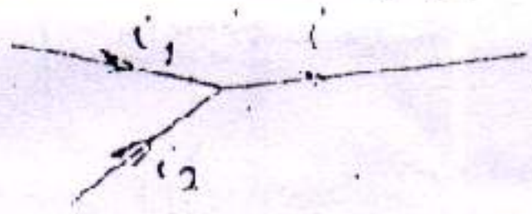
$= 15 + j25.98 + 18.79 - j6.84 = 38.83\ \angle 29.53$

∴ $i = 38.83\sin(\omega t + 29.53^\circ)$ A Ans.

∴ $i = 38.83\sin(\omega t + 29.53^\circ)$ A Ans.

Effective current = $\frac{38.83}{\sqrt{2}} = 27.46$A Ans.

28.



$i = 40\sin(\quad -40^\circ) = 40\ \angle -40^\circ$

$i = 50\sin(\omega t + 80^\circ) = 50\ \angle 80^\circ$

$i_2 = ?$

$i = i_1 + i_2$ or, $-i_2 = i - i_1 = 50\ \angle 80 - 40\ \angle -40$

$= 8.68 + j49.24 - 30.64 + j25.71 = 78\ \angle 106.05^\circ$

∴ $i_2 = 78\sin(\omega t + 106.05)$ A Ans.

∴ Effective current, $I = \frac{78}{\sqrt{2}} = 55.15$A Ans.

29. I = 25A, V = 220V, Cosθ = 0.88

∴ Power = $VI\cos\theta = 25 \times 220 \times 0.88 = 4840$ watt.

Vars = $VI\sin\theta = 25 \times 220 \times \sqrt{1-(.88)^2} = 2612.36$ vars.

Reactive Factor, $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(0.88)^2} = 0.475$ Ans.

Volt -amp. = VI = 25x220 = 5500 Va Ans.

30. I = 10A, V = 220V, Cosθ = 0.8

∴ Power = $VI\cos\theta = 10 \times 220 \times 0.8 = 1760$ watts.

Reactive volt-amperes = $VI\sin\theta = 10 \times 220 \times \sqrt{1-(0.8)^2} = 1320$ vars.

Reacti

Volt

31. V

∴ Impe

Now, V

Average po

Volt-ampe

Power fact

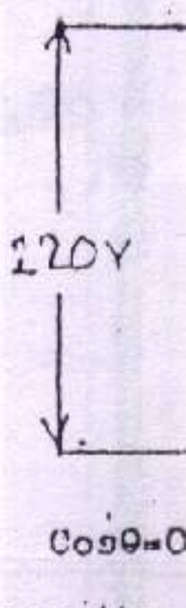
32. I = 15A

P = VIcosθ =

Reactive p

Volt-ampe

33.



Cosθ = 0

Since the

" "

∴ I = I_1+

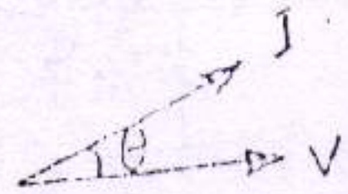
= 22-

∴ Volt-a

Line c

Power,

34.



$\cos\theta_1$

$\cos\theta_2$

Reactive Factor, $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(0.8)^2} = 0.6$

Volt-amperes = VI = 10x220 = 2200Va Ans:

31. $v = 200 \sin(\omega t + 30^\circ) = 200 \angle 30^\circ$, $i = 50 \sin(\omega t + 60^\circ) = 50 \angle 60^\circ$

$\therefore$ Impedance $= \frac{200 \angle 30^\circ}{50 \angle 60^\circ} = 4 \angle -30^\circ$

Now, $V = \frac{200}{\sqrt{2}} = 141.42V$, $I = \frac{50}{\sqrt{2}} = 35.36A$

Average power, $p = VI\cos\theta = 141.42 \times 35.36 \times \cos(-30^\circ) = 4330.6W$ Ans.

Volt-amperes $= VI = 141.42 \times 35.36 = 5000Va$ Ans.

Power factor $= \cos(-30^\circ) = 0.866$ Ans.

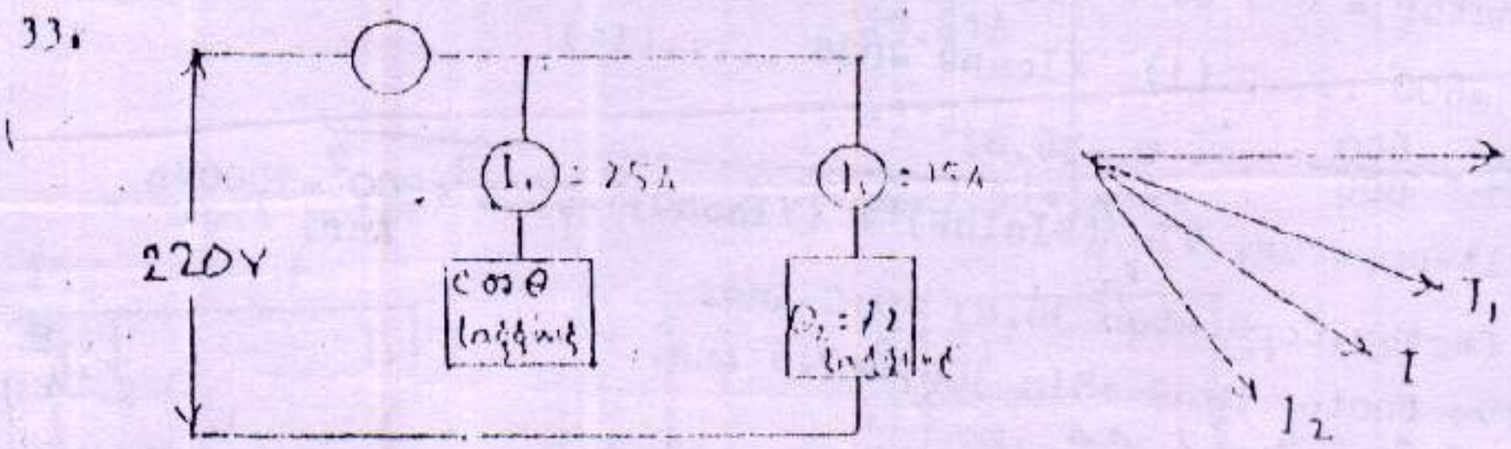
32. I = 15A, V = 220V, $\theta = 72^\circ$ (When current is lagging, the cosθ is lagging power factor and vice versa).

$P = VI\cos\theta = 15 \times 220 \cos 72^\circ = 109.76W$ Ans; ×1019.76 watt

Reactive power $= VI\sin\theta = 15 \times 220 \times \sin 72^\circ = 3138.49$ vars.

Volt-amperes $= VI = 15 \times 220 = 3300Va$ Ans:

33.



$\cos\theta = 0.88$, $\therefore \theta_1 = 28.36^\circ$

Since the p.f. is lagging power factor, $\therefore I_1 = 25 \angle -28.36$

" " " " " " " $\therefore I_2 = 15 \angle -72^\circ$

$\therefore I = I_1 + I_2 = 25 \angle -28.36 + 15 \angle -72^\circ$

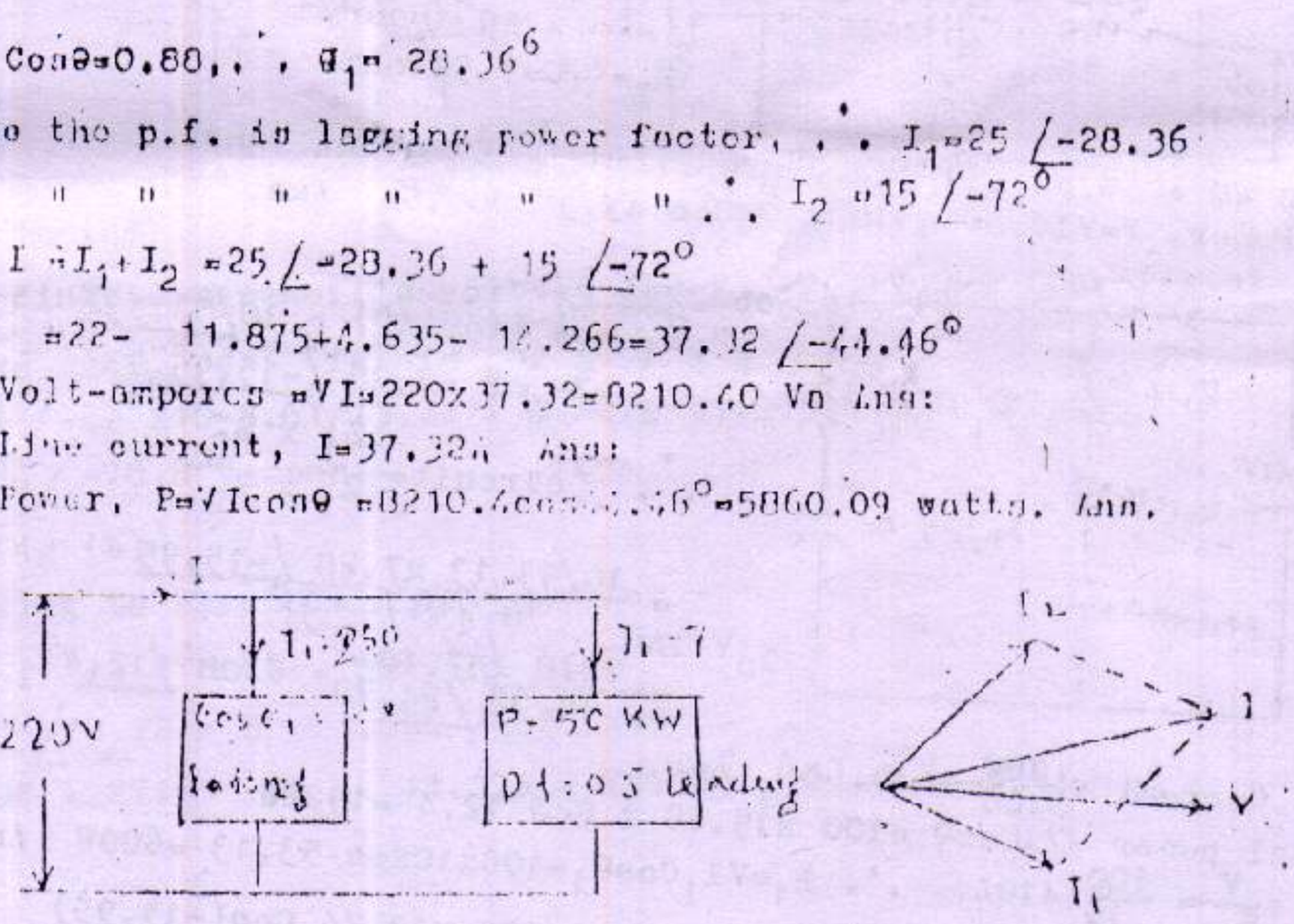
$= 22 - 11.875 + 4.635 - 14.266 = 37.32 \angle -44.46^\circ$

$\therefore$ Volt-amperes $= VI = 220 \times 37.32 = 8210.40$ Va Ans:

Line current, I = 37.32A Ans:

Power, $P = VI\cos\theta = 8210.4 \cos 44.46^\circ = 5860.09$ watts. Ans.

34.



$\cos\theta_1 = 0.8$ $\therefore \theta_1 = 36.37^\circ$

$\cos\theta_2 = 0.5$ $\therefore \theta_2 = 60^\circ$

Sine power factor is leggins, $I_1 = 250 \angle -36.87^\circ$

Now, $VI_2 \cos\theta_2 = 50 \times 10^3$ or, $220 \times I_2 \times 0.5 = 50 \times 10^3$ $\therefore I_2 = 454.55A$

Sine leading power factor, $\therefore I_2 = 454.55 \angle +60^\circ$

$I = I_1 + I_2 = 250 \angle -36.87^\circ + 454.55 \angle 60^\circ$

$= 200 - j150 + 227.275 + j393.652 = 491.86 \angle 39.70$

$\therefore$ Line current $= 491.86$ Ans.

Power current is leading $\therefore$ p.f. is leading. Ans.

35. $v = 200 \sin \omega t$

$i = 50 \sin(\omega t - 30^\circ)$

$= 50 \sin(\omega t - 30^\circ + 90^\circ)$

$= 50 \sin(\omega t + 60^\circ)$

$= \frac{50}{\sqrt{2}} \angle 60^\circ = 35.355 \angle 60^\circ$

$\therefore V = \frac{200}{\sqrt{2}} \angle 0$

$\therefore Z = \frac{141.421 \angle 0^\circ}{35.355 \angle 60^\circ} = 4 \angle -60^\circ$

$\therefore$ Average power $= VI\cos\theta = 141.421 \times 35.355 \cos 60^\circ = 2500W$ Ans.

Vars $= VI\sin\theta = 4330$ vars. Ans.

Power factor $= \cos 60^\circ = 0.5$ Ans.

36. $VI\sin\theta = 600$ .....(i) $VI\cos\theta = 800$ .....(ii)

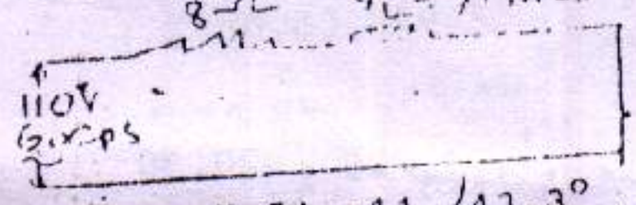
$\therefore \tan\theta = \frac{600}{800}$ $\therefore \theta = 36.87^\circ$

Now, Volt-amperes, $VI = \sqrt{(VI\sin\theta)^2 + (VI\cos\theta)^2} = \sqrt{600^2 + 800^2} = 1000Va$ Ans.

Power factor, $\cos\theta = \cos 36.87^\circ = 0.8$ Ans.

Reactive factor $\sin\theta = \sin 3687^\circ = 0.6$ Ans.

37.



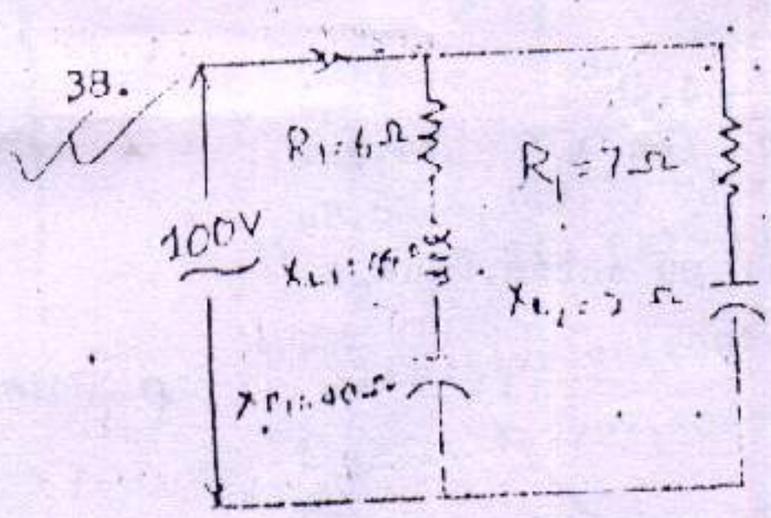
$L = 20 \times 10^{-3}$ henry

$X_L = \omega L = 7.54$ ohm.

$Z = 8 + 7.54 = 11 \angle 43.3^\circ$, current $= \frac{110}{11} = 10A$ Ans.

Power, $P = VI\cos\theta = 110 \times 10 \times \cos 43.3^\circ = 800.5A$ Ans.

38.



$Z_1 = 6 + j(48 - 40) = 10 \angle 53.13^\circ$

$Z_2 = 7 - j2 = 7.28 \angle -15.95$

$Z_{circuit} = \frac{Z_1 \times Z_2}{Z_1 \times Z_2}$

$= \frac{10 \angle 53.13 \times 7.28 \angle -15.95}{6 + j8 + 7 - j2}$

$= \frac{72.8 \angle 37.18}{12.32 \angle 24.78} = 5.08 \angle 12.4^\circ$

Current $= \frac{100}{5.08} = 19.68A$ Ans.

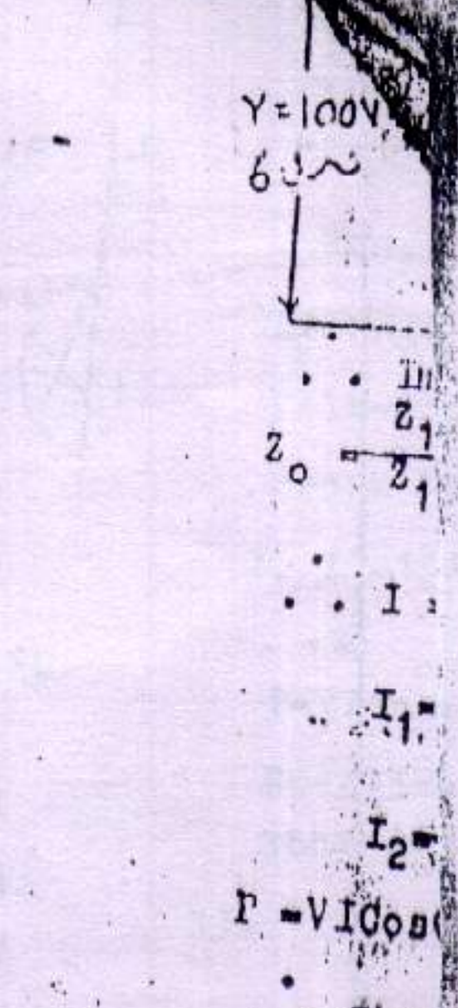
Total power $= VI\cos\theta = 100 \times 19.68 \times \cos 12.4 = 1922W$

$I_1 = \frac{V}{Z_1} = \frac{100}{10} = 10A$. $\therefore P_1 = VI_1\cos\theta_1 = 100 \times 10 \times \cos 53.13 = 600W$ Ans.

$I_2 = \frac{V}{Z_2} = \frac{100}{7.28} = 13.74A$ Ans. $\therefore P_2 = 100 \times 13.74 \cos(-15.95)$

$= 1321.1W$ Ans.

39.



Fig

(c) According

or, $V_{dc} =$

$= 4.686 \angle -1$

$= 70.364 \angle -$

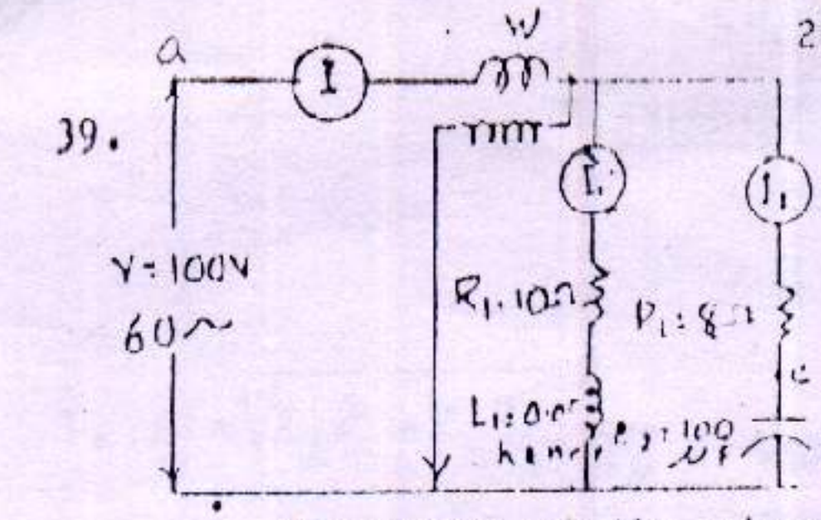
(d) for fig (

According

$V_{dc} + V_{or} + V_{fc}$

$= 3.61 \angle 73.$

$= 8.338 + 2$



$X_{L_1} = \omega L_1 = 2\pi \times 60 \times 0.05 = 18.85$ ohm.

$X_{C_2} = \frac{106}{377 \times 100} = 25.53$ ohm.

$Z_1 = 10 + j\,18.85 = 21.338 \angle 62^\circ$

$Z_2 = 8 - j\,26.53 = 27.71 \angle -73.22^\circ$

. . Impedance of the circuit is-

$$Z_o = \frac{Z_1 \times Z_2}{Z_1 \times Z_2} = \frac{21.338 \angle 62 \times 27.71 \angle -73.22^\circ}{10 + 18.85 + 0 - 26.53} = \frac{591.276 \angle -11.22^\circ}{19.57 \angle -23.1}$$

$= 30.213 \angle 11.88^\circ$

$\therefore I = \frac{V}{Z_o} = \frac{100 \angle 0}{30.213 \angle 11.88} = 3.31 \angle -11.88^\circ$

$\therefore I_1 = \frac{V}{Z_1} = \frac{100 \angle 0}{21.338 \angle 62^\circ} = 4.686 \angle -62^\circ$

$I_2 = \frac{V}{Z_2} = \frac{100 \angle 0}{27.71 \angle -73.22^\circ} = 3.61 \angle 73.22^\circ$

$P = VI\cos\theta = 100 \times 3.31 \times \cos 11.88 = 323.91$W. Ans.

. . The ammeter readings will be $I_1 = 4.686$ A

$I_2 = 3.61$A

$I = 3.31$A

" watt meter " " " $P = 323.91$W Ans.

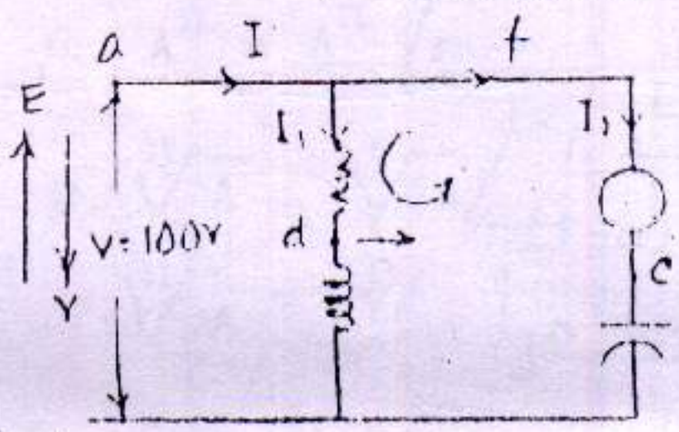
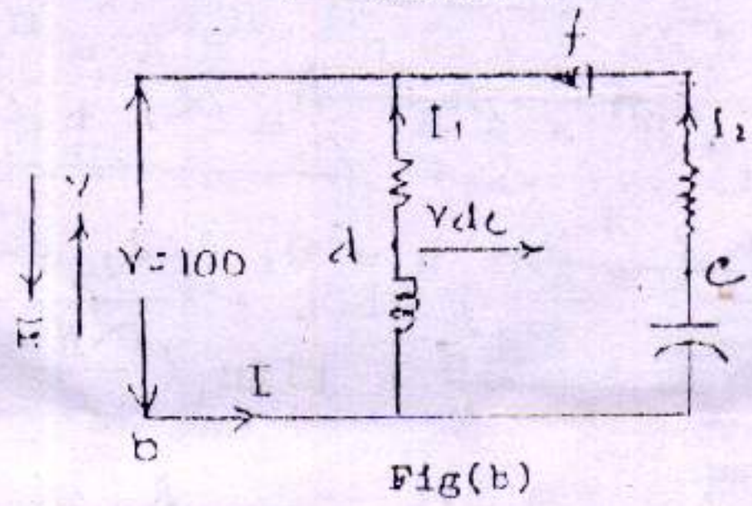


Fig (a)



Fig(b)

(c) According to kirchoff's voltage law $-V_{dc}+V_{cf}+V_{fd}=0$ (for fig.a)

or, $V_{dc} = -V_{cf} - V_{fd} = V_{fc} - V_{fd} = I_1R_1 - I_1R_1$.

$= 4.686 \angle -62^\circ \times 10 - 3.61 \angle 73.22 \times 8 = 22 - 41.375 - 8.338 - 27.65$

$= 70.364 \angle -78.8^\circ$V Ans.

(d) for fig (b)

According to kirchoff's voltage law -

$V_{dc}+V_{cf}+V_{fd}=0$ or, $V_{dc} = -V_{cf} - V_{fd} = Vdf - V_{cf}$

$= 3.61 \angle 73.22 \times 8 - 4.686 \angle -62^\circ \times 10$

$= 8.338 + 27.65 - 22 + 41.375 = 70.364 \angle 101.2^\circ$V Ans.

CHAPTER IV

" PHASOR ALGEBRA "

F O R M U L A S

1. $A = \sqrt{X_A^2 + Y_A^2}$, $\theta_A = \tan^{-1} \frac{Y_A}{X_A}$

2. $A = \pm a \pm ja'$ $\therefore A = \sqrt{a^2 + a'^2}$

$\theta_{1st} = \tan^{-1} \frac{(+a')}{(+a)}$, $\theta_{4th} = \tan^{-1} \frac{(-a')}{(+a)}$

3. A (initially) $= a + ja' = A(\cos\alpha + j\sin\alpha)$

4. $e^{j\theta} = \cos\theta + j\sin\theta$, $e^{-j\theta} = \cos\theta - j\sin\theta$, $Ae^{j\theta} = A(\cos\theta + j\sin\theta)$

5. $\frac{A}{B} = \frac{Ae^{j\alpha_A}}{Be^{j\alpha_B}} = \frac{A}{B} e^{j\alpha_A - j\alpha_B} = \frac{A}{B} e^{j(\alpha_A - \alpha_B)}$

6. $\frac{A}{B} = \frac{A \angle \alpha_A}{B \angle \alpha_B} = \frac{A}{B} \angle \alpha_A - \alpha_B$

7. $A^n = A^n \angle n\alpha_A$, $A^n B^n = A^n B^n \angle n\alpha_A + \alpha_B$

8. $\sqrt[n]{A} = \sqrt[n]{A} \angle \frac{\alpha_A + 2\pi q}{n}$ $q = 0, 1, 2, \ldots (n-1)$

$\sqrt[n]{A} = \sqrt[n]{A} \left[\cos \frac{\alpha_A + 2\pi q}{n} + j\sin \left(\frac{\alpha_A + 2\pi q}{n}\right)\right.$

9. $\log e^{j\theta} = \log^{e^{A}} + \log e^{jA} = \log e^{A} + j\theta \log e^{e} = \log^{e} A + j\theta$

10. $V = v + jv'$

$I = i + ji'$

$P_x = VI\sin\theta$

$= v'i - vi'$

11. $Va = \sqrt{P^2 + P_x^2}$

PROBLEM (Chaptor : IV)

20. (a) $A' = (8.66 + j5.0)\ 1 \angle 40^\circ = (8.66+j5.0)(0.766+j0.6427)$

$= 10 \angle 30 \times 1 \angle 40^\circ = 10 \angle 70^\circ = 3.42 + j9.396.$

21. (a) $(8-j6)\ (10 \angle -120^\circ)\ (\cos 36.87^\circ - j\sin 36.87^\circ)(0.1 e^{+j60^\circ})$

$= 10 \angle 36.85 \times 10 \angle -120 \times (0.7999-j6)(0.1 \angle 60^\circ)$

$= 100 \angle -83.15 \times 0.1 \angle 60^\circ \times 0.999 \angle -36.9 = 10 \angle -60^\circ$ $\therefore$ Ans: $10 \angle 60^\circ$

22. (a) $\sqrt{4.5-j7.79} + \log e^{10 \angle 172^\circ} = \sqrt{8.99 \angle 60^\circ} + \log e^{10} + j\ 172/57.30$

$= 4.49 \angle -30^\circ + \log e^{10} + j3 = 3.89 - j2.245 + \log e^{10} + j3$

(c)

(d)

23.

24.

25. V =

26. Z_1

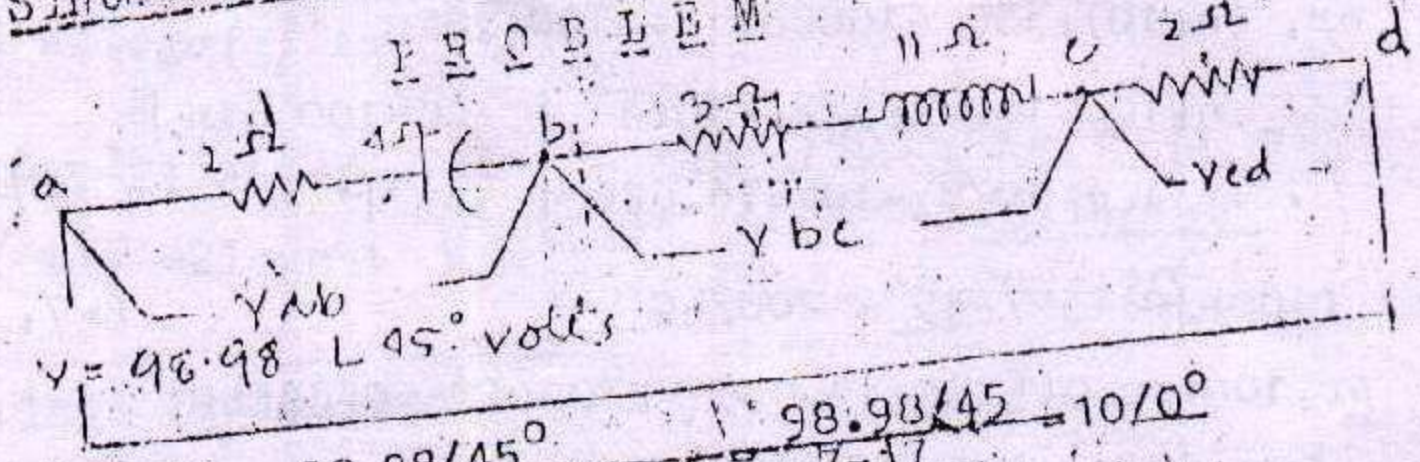
27. Z_0

CHAPTER V

" SINUSOIDAL SINGLE - PHASE CIRCUIT ANALYSIS "

P R O B L E M

1.



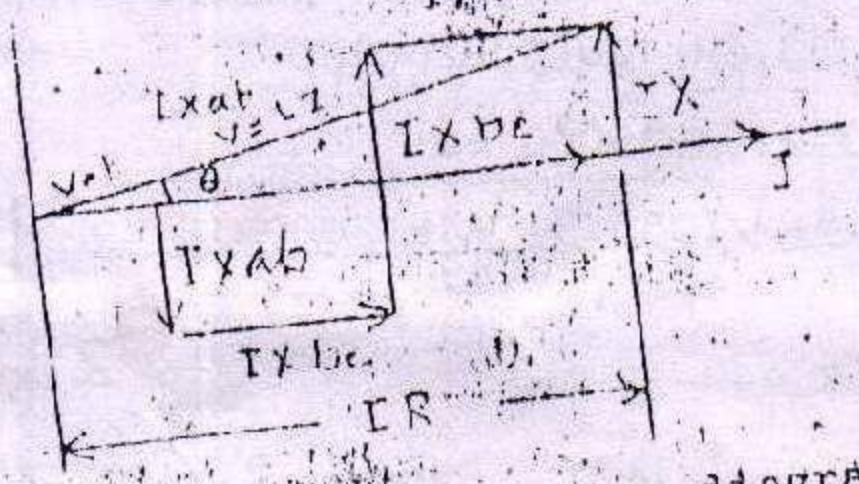
(a) $I = \frac{V}{Z} = \frac{98.98/45^\circ}{2-j4+3+j11+2+j0} = \frac{98.98/45}{7-j7} = 10/0^\circ$

$V_{ab} = I Z_{ab} = (10+j0)(2-j4) = 44.7/-63.45^\circ$ volts.

$V_{bc} = I Z_{bc} = (10+j0)(3+j11) = 30+j110 = 114/74.75^\circ$ volts.

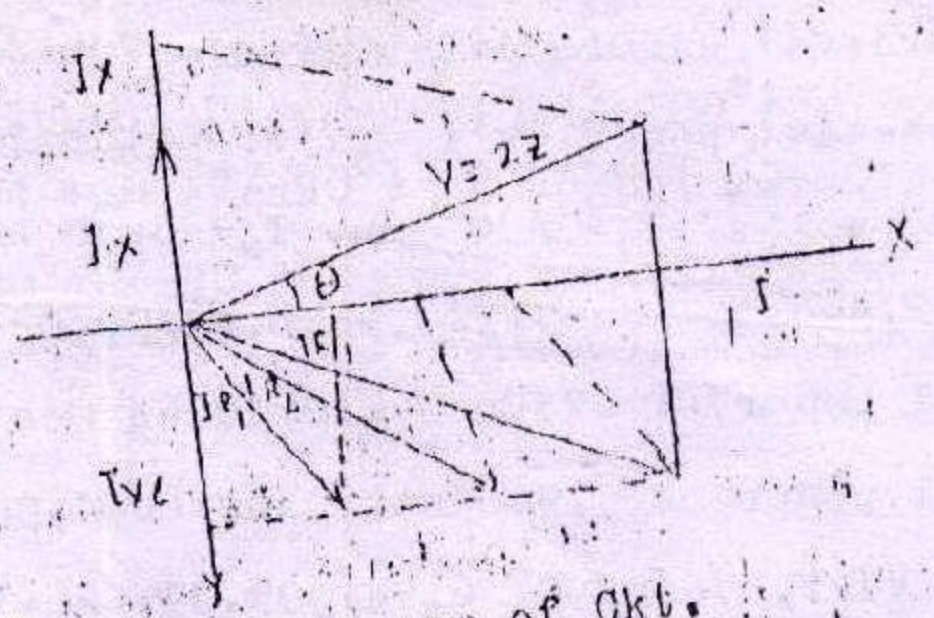
$V_{cd} = I Z_{cd} = (10+j0)(2+j0) = 20+j0 = 20/0^\circ$ volts.

(b)



Funicular or String vector diagram of Ckt.

(c)



Polar Vector Diagram of Ckt.

2. $P = I^2R = 10^2 \times (2+3+2) = 700$ watts., $P = VI\cos\theta = 98.98/45^\circ \times 10/0$

$= 989.8 \times \text{Cos } 45^\circ \ 989.89429 = 700$ watts.

$V = 95.5/45^\circ$

$P = I \cdot V'I'$

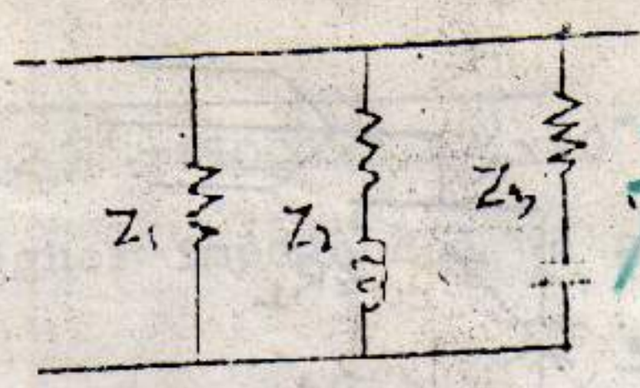
$= 69.989429 \times 10 + 69.989429 \times 0$

$= 69.989429 = 700$ watts.

$= 69.989419 + j69.989429$

$I = 10/0^\circ = 10 + j0$

#Secmi
00006103



Y = Admittance

$Y_1 = \frac{1}{Z} = \frac{1}{10+j0}$ 0.1+j0 mho

$Y_2 = \frac{1}{20+j10} \quad \frac{20-j20}{400+400}$ 0.025-j0.025

$\therefore g_2 = 0.025, \; b_2 = 0.025$

$Y_3 = \frac{1}{30-j40} \quad \frac{30+j40}{900+1600}$

$= 0.012 + j0.016$

$\therefore g_3 = 0.012, \; b_3 = -0.016$

$\therefore$ Resulting $g = g_1 + g_2 + g_3$

$= 0.1 + 0.025 + 0.012 = 0.137$ ohm

$b = b_1 + b_2 + b_3 = 0 + 0.025 - 0.016$

$= 0.009$ ohm.

note; $g_1 = \frac{R_1}{Z_1^2} = \frac{10}{(10+j0)^2} = 0.1$ ohm

$b_1 = \frac{X_1}{Z_1^2} = \frac{0}{100} = 0$

$g_2 = \frac{R_2}{Z_2^2} = \frac{20}{800} = 0.025$

$b_2 = \frac{20}{800} = 0.025$

$g_3 = \frac{R_3}{Z_3^2} = 0.012$

$b_3 = \frac{-40}{2500} = -0.016$

(c) Inphase component of resulting current $= V_g = 40 \times 0.137 = 5.48$ amps.

Quadraure component of the resulting current $= V_b = 40 \times 0.009 = 0.36$ amps.

(8) Given here, $R_c = 1$ ohm, $X_c = 10$ ohm, $R_L = 6$ ohm. V=100V when X_L is varried, quadrature component of current in inductive branch will be maximum. When it is equal to the radius of circle made by locus of it.

i.e $I_L \sin\theta_L = \frac{V}{2R_L} = \frac{100}{2 \times 6} = 8.33$ amps.

When $R_L = 4$ ohm, $I_L \sin\theta_L = \frac{V}{2R_L} = \frac{100}{2 \times 4} = 12.5$ amps.

We know that, when $I_c \sin\theta_c > \frac{V}{2R_L}$ Resonance cannot be obtained

" $I_c \sin\theta = \frac{V}{2R_L}$ " will occure for one time

" $I_c \sin\theta_c < \frac{V}{2R_L}$ " " " " " two "

" $R_L = 6, I_c \sin\theta_c = \frac{V}{\sqrt{R_c + (X_c)^2}} X \sin \tan^{-1} \frac{X_c}{R_c}$

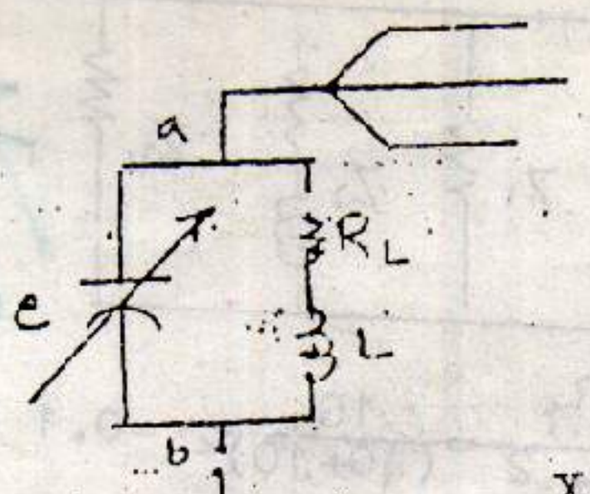
$= \frac{100}{\sqrt{1+100}} X \sin \tan^{-1} \frac{X_c}{R_c} = 9.95 \times 0.99 = 9.9$ amps.

Again, $\frac{V}{2R_L} = 8.33$ amps which is less than $I_c \sin\theta_c$

$\therefore$ resonance cannot be obtained when R=4 ohm, $\frac{V}{2R_L} = 12.5$ amps.

$\therefore$ Resonance will occure at two points.

(9.)



$L = 250 \times 10^{-6}$ henry

$f = 10^6$ HZ, $\frac{X_L}{R} = 170$

Now, $X_L = \omega L = 2\pi \times 10^6 \times 250 \times 10^{-6} = 1570.8$ ohm

$\therefore R = \frac{X_L}{170} \quad \frac{1570.8}{170} = 9.24$ ohm.

(a) For Resonance, $\frac{X_L}{R_L^2 + X_L^2} = \frac{X_c}{R_c^2 + X_c^2}$ $\left[\because R_c = 0 \right.$

$\left. \therefore X_c = \frac{1}{\omega c}\right]$

$\therefore$ or, $\frac{X_L}{R_L^2 + X_L^2} = \omega c$

$\therefore$ or, $c = \frac{1570.8}{2\pi 10^6 \{(9.24)^2 + (1570.8)^2\}} = 101.3 \times 10^{-12} F = 101.3 \mu\mu F$

(b) $\omega_m = \frac{1}{\sqrt{Lc}} \Rightarrow c = \frac{1}{L\omega_m^2} = \frac{1}{X_L \omega_m} = \frac{1}{1570.8 \times \omega_m} = 101.3 \mu\mu F$

(c) $Z = \frac{L}{CR_L} = \frac{250 \times 10^{-6}}{101.3 \times 10^{-12} \times 9.24} = 267090.6$ ohm.

(d) $Y = G - jb_L + jbc = \frac{R_L}{R_L^2 + X_L^2} - j\frac{X_L}{R_L^2 + X_L^2} + j.\omega c$

$= \frac{9.24}{(9.24)^2 + (1555)^2} - j\frac{1555}{(9.24)^2 + (1555)^2} + j\, 2\pi \times 990 \times 10^3 \times 101.3 \times 10^{-12}$

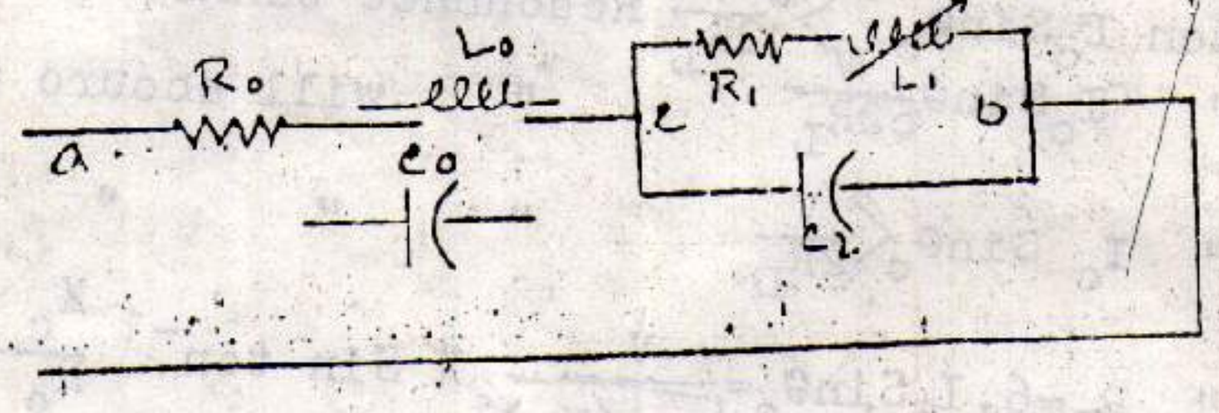
$(X_L = 2\pi \times 990 \times 10^3 \times 250 \times 10^{-6} = 1555$ ohm)

$= 3.82 \times 10^{-6} - j\, 6.43 \times 10^{-4} + j 6.3 \times 10^{-4}$

$= 3.82 \times 10^{-6} - j 1.294 \times 10^{-5}$

$Z = \frac{1}{Y} = \frac{1}{3.82 \times 10^{-6} - j1.29 \times 10^{-5}} \quad \frac{1}{1.349 \times 10^{-5} \angle -73.5^\circ} = 74105 \angle 73.5$

(11.)



$R_0 = 20$ ohm

$R_1 = 40$ ohm

$C_2 = 0.05\ \mu f$

(a) For parallel Resonance at 15,000 cycles- $b_L = bc$

or, $\frac{\omega L}{R_1^2 + \omega^2 L^2} = \omega C$ or, $\omega^2 L^2 c + R_1^2 c - L = 0$

or, $\omega^2 c\ L^2 - L + R_1^2 c = 0$ $\therefore L = \frac{+1 \pm \sqrt{1 - 4.\omega^2 c.R_1^2 c}}{2.\omega^2 c}$

$= \frac{1 \pm \sqrt{1 - 4(2\pi \times 15000 \times 0.05 \times 10^{-6} \times 40)^2}}{2 \times (2\pi \times 15000)^2 \times 0.05 \times 10^{-6}} = \frac{1 \pm 0.92622}{888.264}$

$= 2.17 \times 10^{-3}$ henry (taking +ve) $= 0.083 \times 10^{-3}$ (taking -ve)

$Y_{bc} = G_{L1} = \frac{R_1}{Z^2} \quad \frac{40}{40^2+(\omega L)^2} = \frac{40}{40^2+(2\pi \times 15000 \times 2.17 \times 10^{-3})^2}$

∴ For lower conductance (i.e higher impedance) we shall take 2.17 $\times 10^{-3}$ henry.

(b) For 45000 cycles, $X_{L1} = 2\pi \times 45000 \times 2.17 \times 10^{-3} = 613.55$ ohm

$X_{c2} = \frac{1}{2\pi \times 45000 \times 0.05 \times 10^{-6}} = 70.74$ ohm

∴ $Z_{bc} = \frac{(40+j613.55)\times(-j70.74)}{(40+j613.55)+(-j70.74)} = \frac{43402.527 - j\ 2829.6}{40 + j\ 542.81}$

$= \frac{43493.682/-3.73}{544.28/85.78} = 79.91/-89.51 = 0.68 - j\ 79.9$ bc is predominantly capacitive.

(c) An inductive reactance of 79.9 ohm should be placed in series with R_o to produce series resonance.

∴ $L_o = \frac{79.9}{2\pi \times 45000} = 0.283 \times 10^{-3}$ henry

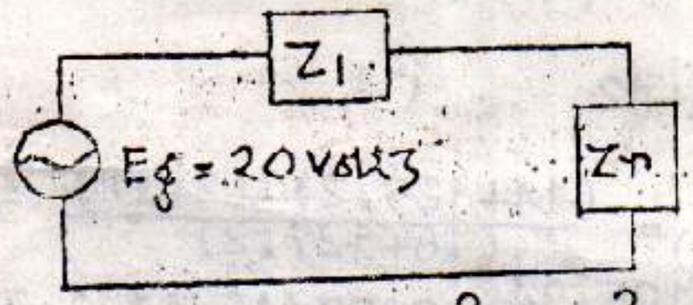
(d) Z_{ab} at 45000 cycles $= 0.68 - j79.9 + j79.9 + 20 = 20.68$ ohm.

Z_{ab} at 15000 cycles $= Z_{bc} + Z_{ac} = \frac{1}{Y_{bc}} + Z_{ac}$

$= 1085.687 + 20 + j\ 2\pi \times 15000 \times 0.283 \times 10^{-3} = 1105.687 + j26.67$

$= 1106/1.38^{\circ}$ ∴ $Z_{ab} = 1106$ ohm Ans:

12.



$Z_1 = 0.5 + j1 + 1.5 + j4 = 2 + j5$

∴ $Z_{Load} = 2 - j5$ ohms.

$P_{max\ max} = \frac{Eg^2}{4R_1} = \frac{20^2}{4 \times 2} = 50$ watts.

Current (I) $= \frac{20}{(2+j5+2-j5)} = 5$ amps.

∴ Line loss, $P_{Line\ loss} = I^2R = 5^2 \times 1.5 = 37.5$ watts.

gen. Loss, $P_{gen\ loss} = I^2R = 5^2 \times 0.5 = 12.5$ watts.

13. $K = \pm \frac{X_r}{R_r} = \pm 5$, $R_r = \frac{Z_1}{\sqrt{1+K^2}} = \frac{\{(1.5+0.5)^2+(1+4)^2\}^{\frac{1}{2}}}{\sqrt{1+(5)^2}} = 1.056$ ohm.

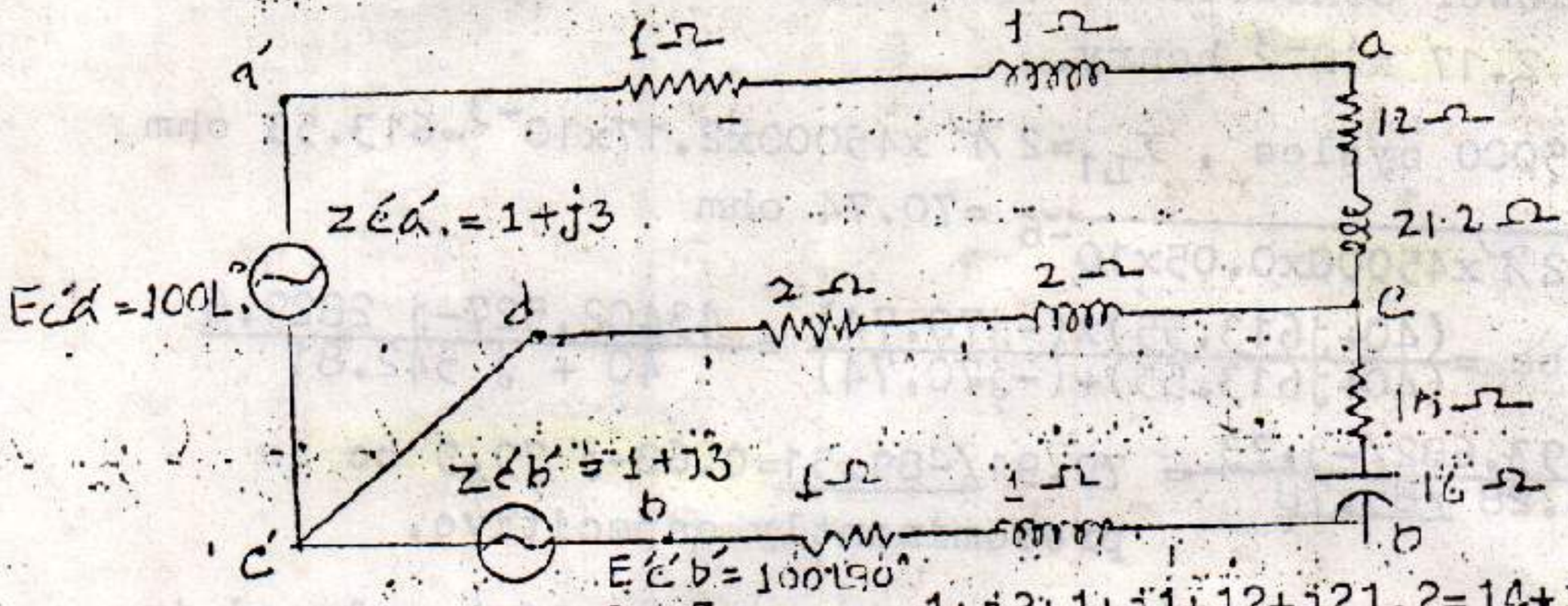
$X_r = K R_r = \pm 5 \times 1.056 = \pm 5.28$

For Positive K, $P_r = I^2R_r = \frac{Eg^2R_r}{(R_1+R_r)^2+(X_1+X_r)^2}$

$= \frac{20^2 \times 1.056}{(2+1.056)^2+(5+5.28)^2} = 3.6725$ watts.

For negative K = $P_r = \frac{20^2 \times 1.056}{(3.056)^2+(5-5.28)^2} = 44.85$

14. Calculate the current in branch ac of Fig.



Assuming $E_{c'b'} = 0$, $Z_{c'a'ac} = 1+j3+1+j1+12+j21.2 = 14+j25.2$

$Z_{c'b'bc} = 1+j3+1+j1+15-j16.6 = 17-j12.6$

$Z_{c'b'bcd} = \frac{(17-j12.6)(2+j2)}{17-j12.6+2+j2} = \frac{(17-j12.6)(2+j2)}{(19-j10.6)}$

$= \frac{21.16/-36.54 \times 2.82/45}{21.75/-29.15} = \frac{59.67/8.46}{21.75/-29.15} = 2.74/37.64 = 2.17+j1.67$

$\therefore Z_{a'acbc'} = 14+j25.2+2.17+j1.67 = 16.17+j26.87 = 31.36/58.96$

$\therefore I_{a'ac} = \frac{100/0^\circ}{31.36/58.96} = 3.18/-58.93 = 1.64-j2.72$

Assuming $E_{c'a'} = 0$

Then, $Z_{c'a'ac} = 14+j25.2$, $Z_{cc'} = 2+j2$

$\therefore Z_{a'acdc'} = \frac{(14+j25.2)(2+j2)}{(14+j25.2)+(2+j2)} = \frac{(-14+j25.2)(2+j2)}{(16+j27.2)}$

$= \frac{28.82/60.94 \times 2.82/45^\circ}{31.55/59.53} = \frac{81.27/105.24}{31.55/59.53} = 2.57/46.41 = 1.77+j1.86$

$\therefore Z_{a'acbc'a'} = 17-j12.6+1.77+j1.86 = 18.77-j10.74 = 21.67/-27.77$

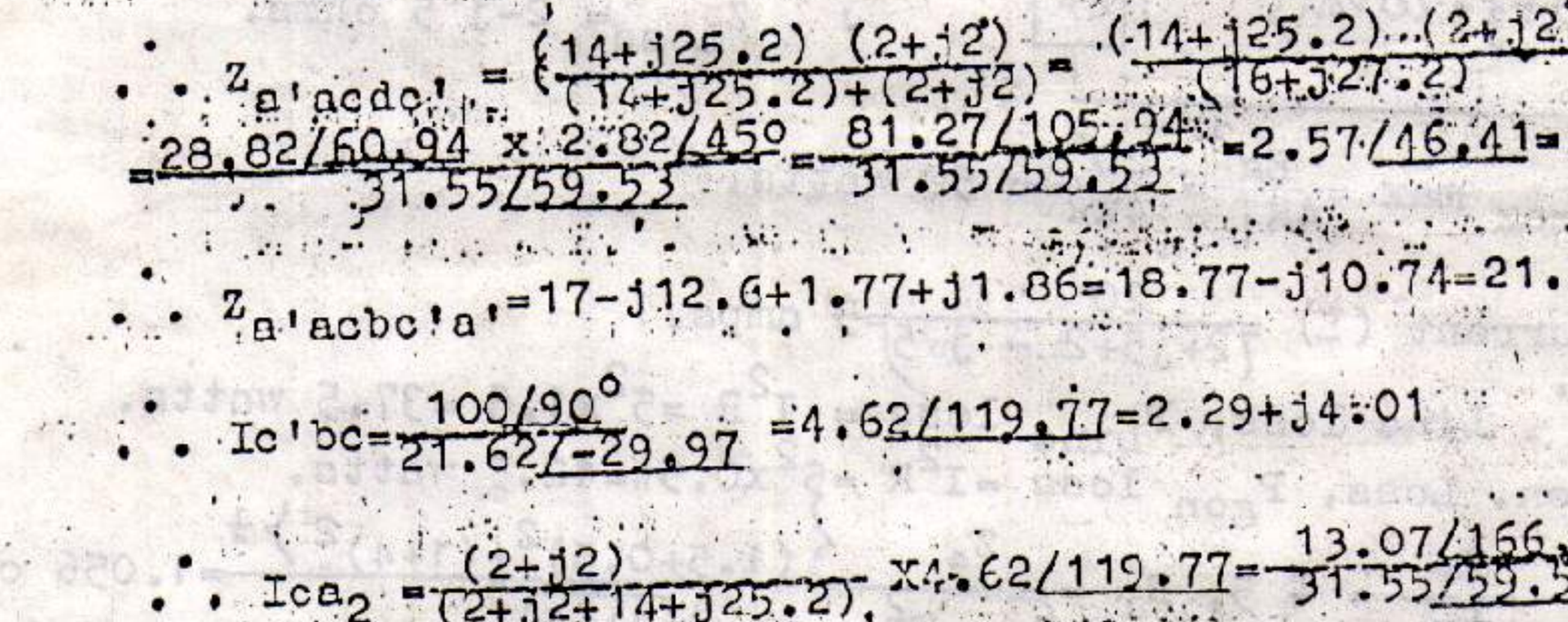
$\therefore Ic'bc = \frac{100/90^\circ}{21.62/-29.97} = 4.62/119.77 = 2.29+j4.01$

$\therefore Ica_2 = \frac{(2+j2)}{(2+j2+14+j25.2)} \times 4.62/119.77 = \frac{13.07/166.77}{31.55/59.53}$

$= 0.41/105.24 = 0.01+j.39$ $\therefore Iac = Ia'ac_1 - Ica_2$

$= 1.64-j2.72-(-j0.1+j.39) = 1.64-j2.72+j0.1-j0.39 = (1.74-j3.11)$amp.

15.



Vac = 81+j26 volts. $Iad = \frac{81+j26}{-j10} = \frac{-26+j81}{10} = -2.6+j81$

$$Z_{ec} = \frac{(2-j2)(3)j4)}{2-j2+3+j4} = \frac{6+8-j6+j8}{5+j2} = \frac{14+j2}{5+j2} = \frac{(14+j2)(5-j2)}{25+4}$$

$$= \frac{1}{29}(70+4+j10-j28) = \frac{1}{29}(7.4 - j18) = 2.55-j0.621 \text{ ohms.}$$

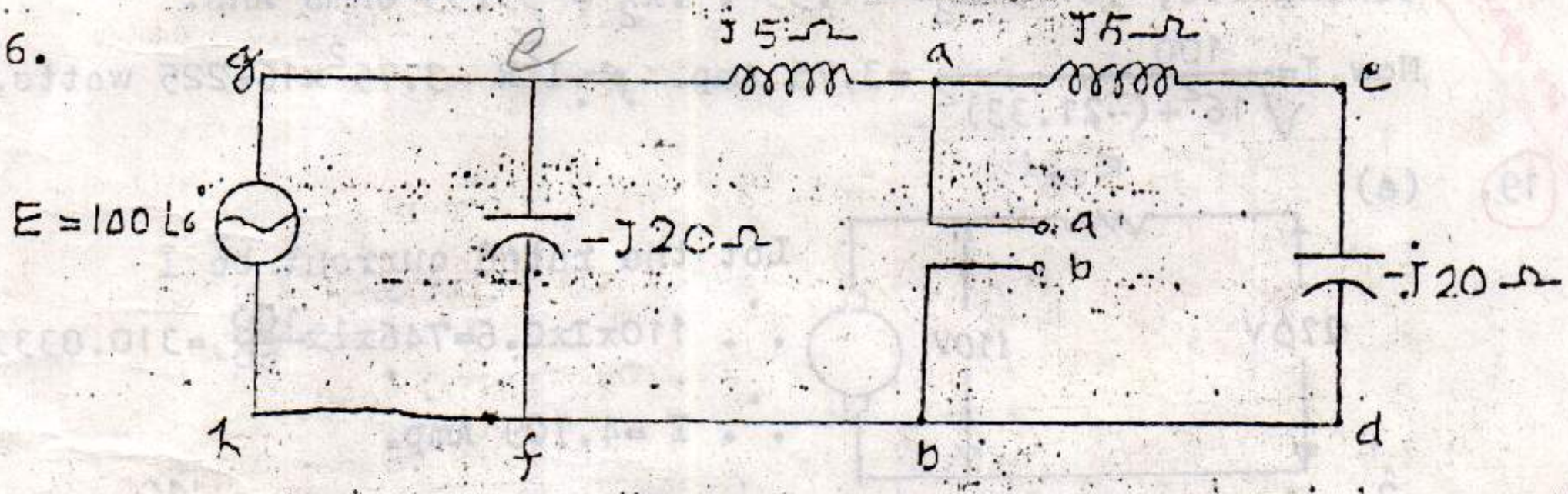
$$Z_{dce} = Z_{ec} + Z_{ad} = 2.55-j0.621-j10 = 2.55-j10.621$$

$$\therefore I_{ad} = \frac{100+j0}{2.55-j10.621} = \frac{100\angle 0^\circ}{10.923\angle 76.5} = 9.16\angle 76.5^\circ = 2.14+j8.9$$

$$I_{ef} = \frac{Vce}{Z_{af}} = \frac{(9.16\angle 76.5^\circ)(2.55-j0.621)}{2-j2} = \frac{(9.16\angle 76.5^\circ)(2.625\angle -13.69)}{(2.828\angle -45^\circ)}$$

$$= 8.5 \angle 107.81^\circ = -2.6+j8.1 \text{ amperes Ans;}$$

16.



Soln:- $Z_{eacd} = j5+j5-j20 = -j10 = 10\angle -90^\circ$

$$Zfeacd = \frac{10\angle -90^\circ \times 20\angle -90^\circ}{-j10-j20} \quad (\because Zef = -j20 = 20\angle -90^\circ)$$

$$= \frac{200\angle -180}{0\angle -90} = 6.66\angle -90. \quad \text{Total } I = \frac{100\angle 0^\circ}{6.66\angle -90} = 15\angle 90A = j15.$$

$$Ief = \frac{100\angle 0^\circ}{-j20} = \frac{100}{20\angle -90^\circ} = 5\angle 90 = j5 \quad \therefore Iea = j15-j5 = j10.$$

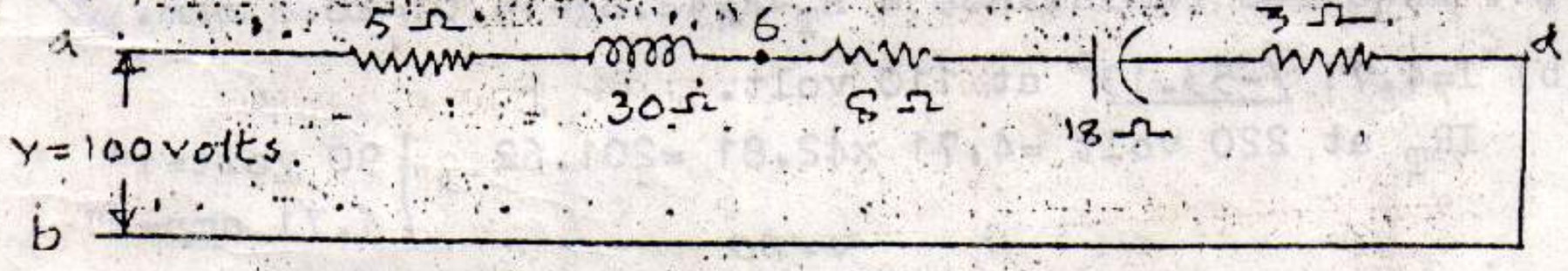
$V_{ea} = j10 \times j5 = j^2 50 = -50$ volts. $\therefore$ Voltage of ab $= 100-(-50) = 150\angle 0^\circ$ Volts.

$Z_{aef} = j5 = 5\angle 90^\circ$. ($\because$ When gh point is shorted the current will not flow through ef).

$Z_{ca} = j5-j20 = 15\angle 90^\circ$

$Z_0 = j5 \times -j15/j5-j15 = j7.5 = 7.5\angle 90$; $I_L = Vab/Z_0+Z_L = \frac{150\angle 0}{j7.5+10-j7.5} = 15\angle 0^\circ$

17.



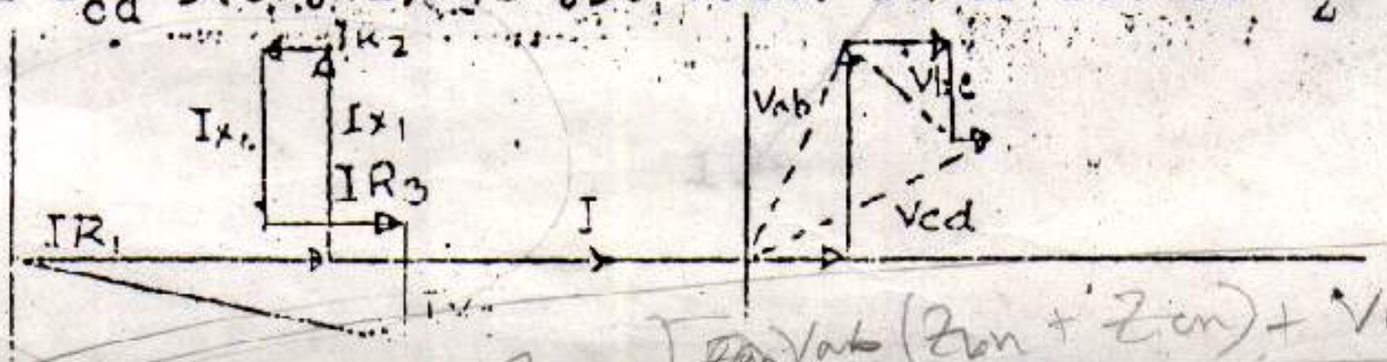
$Z = Zab+Zbc+Zcd = 5+j30+8-j18+3 = 16+j12$

$$\therefore I = \frac{100\angle 0^\circ}{16+j12} = \frac{100\angle 0^\circ}{20\angle 36.869} = 5\angle -36.86 = 16-j12 \text{ amp.}$$

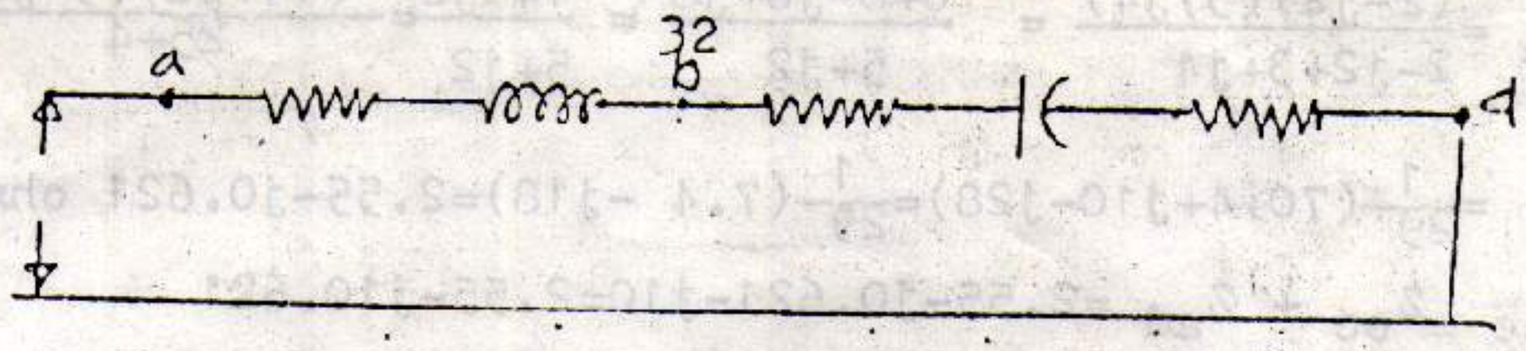
$V_{ab} = I\ Z_{ab} = 5\angle -36.869 \times (5+j30) = 152\angle 43.6686 = 109.94+j104.95$ volt.

$V_{bc} = I\ Z_{bc} = 5\angle -36.869 \times (8-j18) = 98.48\angle -102.9065 = -21.996-j95.99$ volt.

$V_{cd} = I\ Z_{cd} = 3(16-j12) = 48-j36$ volt. Power factor $= \frac{R}{Z} = \frac{16}{20} = 0.8$ logg.



18.



Total R=5+8+3=16 ohm. Now, Cosθ=0.6 ∴ θ = ± 53.13°

∴ tan(± 53.13) $=\frac{X}{16}$ or X = ± 21.33.

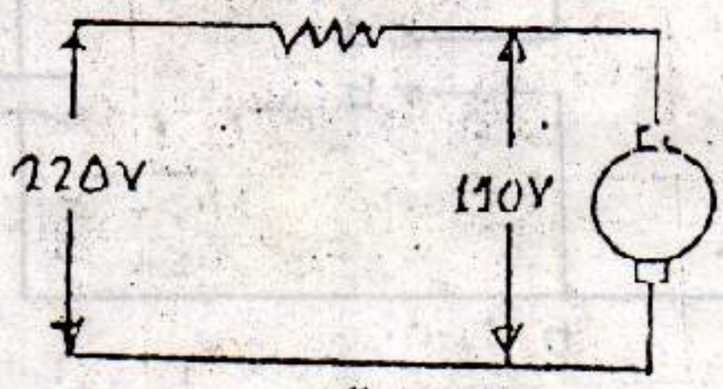
taking +ve, 30-18+x_1=21.33 ∴ x_1 =9.33 ohms Ans.

taking -ve, 30-18+x_2=-21.33 ∴ x_2 =-33.33 ohms Ans.

Need to see again

Now, $I=\frac{100}{\sqrt{16^2+(\pm 21.33)^2}}$ =3.75 Amp. ∴ $P=I^2R$ =3.75^2x16=225 watts.

19. (a)



Let the rated current be I

∴ 110xIx0.6=746xtx$\frac{100}{60}$ =310.8333

∴ I =4.709 Amp.

I^2R_m=310.8 (Motor Input) ∴ 220 =IR+110 ∴ R = $\frac{110}{4.709}$ =23.37

or, $R_m=\frac{310.83}{I^2}$ = $\frac{310.83}{(4.709)^2}$=14.01 (ohm)

∴ Total resistance of the motor is 14.01 ohm

p.f = $\frac{R_m}{Z_m}$ ∴ $Z_m=\frac{14.01}{0.6}$ =23.25 ohm

∴ X_m of the motor $X_m=\sqrt{Z^2-R^2}=\sqrt{23.35^2-14.01^2}$=18.68 ohm.

Again, Impedance at 220 volts IZ_T =220 ∴ $Z_T=\frac{220}{4.709}$ =46.70 ohm.

∴ $R_T=\sqrt{Z_T^2-X_m^2}=\sqrt{(46.71)^2-(18.68)^2}$=42.81 ohm.

∴ Required resistance = R_T-R_m=42.81-14.01=28.8 ohm.

(b) I=4.71 $\angle -53.13^\circ$ at 110 volt.

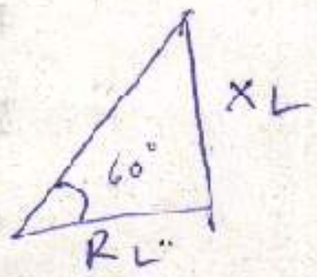
IR_T at 220 volt =4.71 x42.81 =201.62

90 volt=1"
4.71 amp=2"

20.

Power + X_L/R_L
24 Δ

20 $P = VI\cos\theta$ or, $300 = 120 \times 5 \times \cos\theta$ . . $\theta = 60^\circ$.



Since lagging p.f . . Voltage is leading

Now, $I^2R_L = 300$ or, $R_L = \frac{300}{5^2} = 12$ ohm.

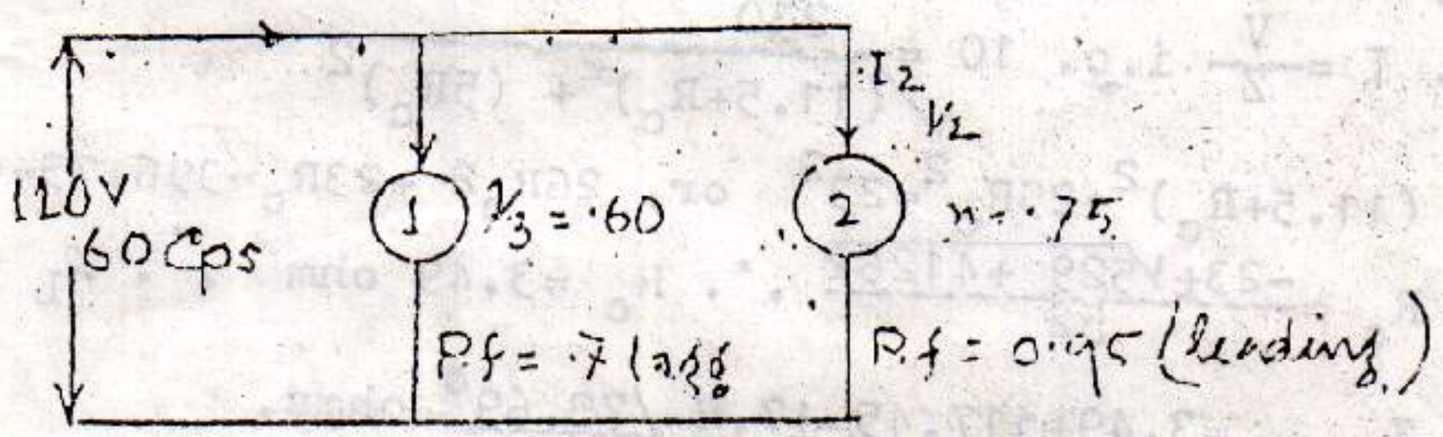
Now, $\tan 60^\circ = \frac{X_{Load}}{R_{load}}$ . . $X_{Logd} = 20.78$ ohm. $Z = \sqrt{12^2+(X_L-X_c)^2}$

Now, $I = \frac{V}{Z}$ or, $5 = \frac{240}{\sqrt{12^2+(x_L-X_c)^2}}$ or, $12^2+(x_L-X_c)^2 = (\frac{240}{5})^2$

or, $X_L-X_c = \pm\sqrt{48^2-12^2} = \pm 46.476$ or, $X_c = 20.78 \pm 46.476 = 67.256$ Ans.

or, -25.696 (it is inductive) hence

21.



$P = VI_1\cos\theta = 746 \times \frac{1}{3} \times \frac{100}{60}$ . . $I_1 = \frac{746 \times \frac{1}{3} \times \frac{100}{60}}{110 \times 0.7} = 5.382$ Amp.

$\cos\theta = 0.7$ . . $\theta_1 = 45.57^\circ$.

Since current is lagging

. . $I_1 = 5.382 \angle -45.57$ ($\cos\theta_2 = 0.95$ . . $\theta_2 = 18.19^\circ$)

$I_2 = \frac{746 \times \frac{1}{2} \times \frac{100}{75}}{110 \times 0.95} = 4.76$

Since Power factor is loading . . $I_2 = 4.76 \angle 18.19$

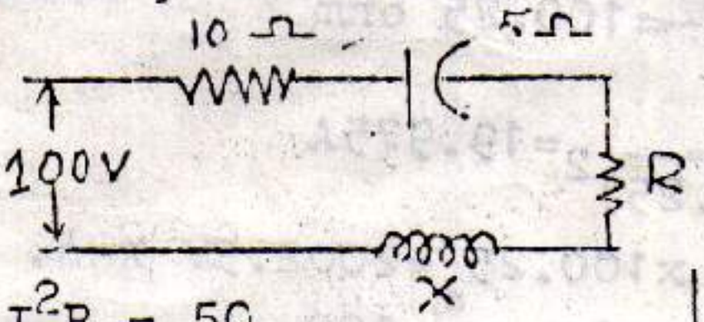
Now, $I = I_1+I_2 = 5.382 \angle -45.57 + 4.76 \angle 18.19^\circ$

$= 3.768 - j\,3.843 + 4.52 + j1.49 = 8.617 \angle -15.83^\circ$

$P = VI\cos\theta = 110 \times 8.617 \cos(-15.83^\circ) = 911.8$ watts

Resultant power factor = $\cos(-15.83^\circ) = 0.962$ lagging Ans.

22.



Now, $I^2R = 50$

or, $I^2 = \frac{50}{R}$ ..(1)

$V_x = XI$ Now, $V_xI = 100$ vars.

or, $XIxI = 100$ or, $I^2 = \frac{100}{X}$ ...(ii)

From equation(1) and (2) $\frac{50}{R} = \frac{100}{X}$ $\therefore X = 2R$

Now, $Z = \sqrt{(100+R)^2+(x-5)^2}$

Now, $V = ZI$ or, $V^2 = Z^2I^2$ $100^2 = \left\{(10+R)^2+(2R-5)^2\right\}I^2$

or, $100^2 = (100+20R+R^2+4R^2-20R+25)x\frac{50}{R}$

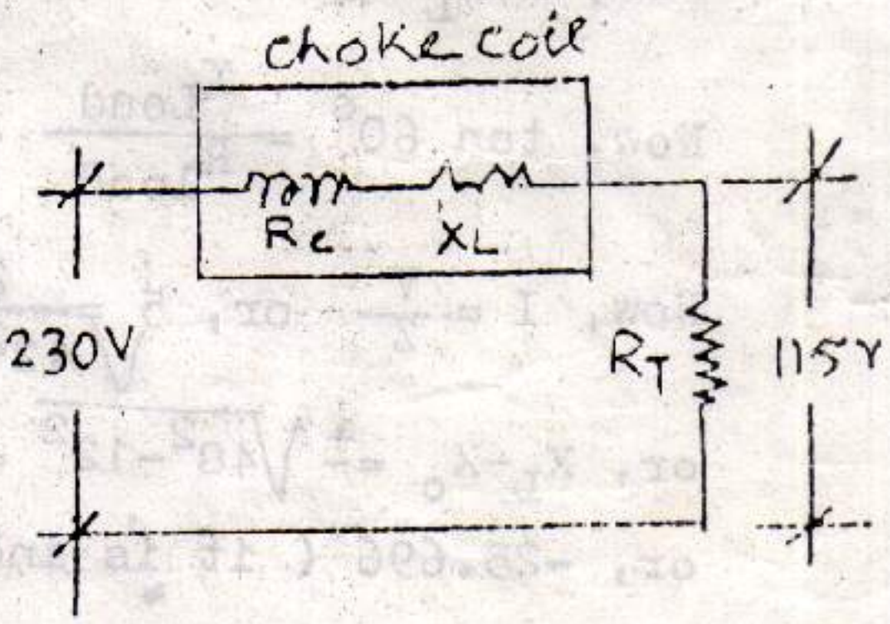
or, $250R^2-10000R+6250 = 0$ or, $R^2-40R+25=0$ $\therefore R = \frac{40 \pm \sqrt{1600-100}}{2}$

$= 39.36$ ohm or 0.635

$\therefore X = 78.72$ ohm, or, 1.72 ohm.

23. $P = VI\cos\theta$ $\therefore \cos\theta = \frac{1150}{115 \times 10} = 1$

$R = \frac{V}{I} = \frac{115}{10} = 11.5$ volts.



$\frac{X_L}{R_c} = 5$ i.e. $X_L = 5R_c$

$Z_{ckt} = R+R_c+jX_L = 11.5+j\ 0.5\ R_c$

Now, $I = \frac{V}{Z}$ i.e. $10 = \frac{230}{\sqrt{(11.5+R_c)^2+(5R_c)^2}}$

or, $(11.5+R_c)^2+25R_c^2 = 23^2$ or, $26R_c^2+23R_c-396.75=0$

or, $R_c = \frac{-23+\sqrt{529+41262}}{52}$ $\therefore R_c = 3.49$ ohm $\therefore X_L = 17.45$ ohm.

(a) $Z_{coil} = 3.49+j17.45 = 17.8 \angle 78.69^\circ$ ohms.

(c) $Z_{ckt} = (11.5+3.45)+j17.45 = 23 \angle 49.34^\circ$ ohms.

p.f. $= \cos(49.34^\circ) = 0.652$.

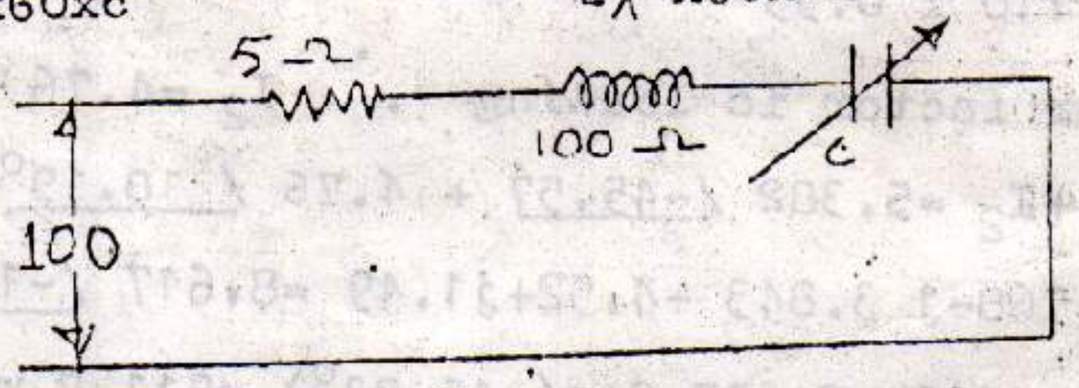
24. For series resonance, $X_L = X_C$

Reactance present in the ckt $= 30-18 = 12$ ohm

Hence we need 12 ohm capacitive reactance

Now, $\frac{1}{2\pi \times 60 \times C} = 12$ $\therefore C = \frac{10^6}{2\pi \times 60 \times 12} = 221\ \mu$f Ans.

25.



For max Voltage across C

(a) $X_C = \frac{R^2+X_L^2}{X_L} = \frac{5^2+100^2}{100} = 100.25$ ohm

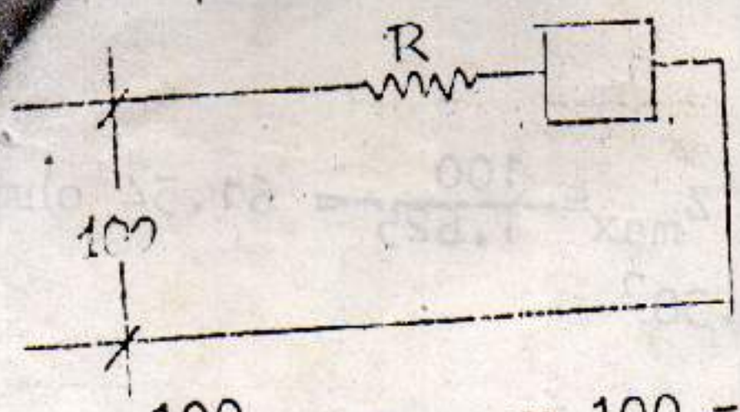
Now, $I = \frac{100}{\sqrt{5^2+(100-100.25)^2}} = 19.975$A

Now, $V_C = I\,X_C = 19.975 \times 100.25 = 2002.5$V Ans.

(b) $X_C = \frac{100^2+100^2}{100} = 200$ $\therefore I = \frac{100}{100^2+100^2} = 0.005$A

$V_C = IX_C = 200 \times 0.005 = 1$V.

35.



$VI = 1000$ or $100I = 1000$ $\therefore I = 10A$ | $I^2R = 800$ or $R = 8$ ohm.

$I = \frac{100}{\sqrt{8^2 + X^2}}$ or $100 = \frac{100^2}{8^2+X^2}$ or, $8^2 + X^2 = 100$ $\therefore X = \pm 6$ ohm. Ans.

27. $Q = \frac{fm}{\Delta f}$ or $50 = \frac{fm}{100}$ $\therefore$ fm = 5000 cycles /sec.

So the frequency limits onn $(5000 - \frac{100}{2})$ and $(5000 + \frac{100}{2})$

i.e. 4950 and 5050.

28.

(a) $fm = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-12}}} = 159154.94$ cps.

(b) $Q = \frac{1}{Rs}\sqrt{\frac{L}{C}} = \frac{1}{1000}\sqrt{\frac{100\times10^{-3}}{10\times10^{-12}}} = 100$

(c) $Q = \frac{fm}{\Delta f} = \frac{\omega_m}{\Delta\omega}$ $\because \Delta\omega = \frac{1\times10^6}{100} = 10^4$ $\therefore \omega_m = 1\times10^6$ rad/sec.

$\therefore \omega_2 = (1\times10^6 + \frac{10^4}{2}) = 1005000$ rad/sec.

$\omega_1 = (1\times10^6 - \frac{10^4}{2}) = 995000$ rad/sec. Ans.

(d) $X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi\times159\times10^3\times10\times10^{-12}} = 100097.45$ ohm.

Now, $2\pi fL = \frac{Xc^2+R^2}{Xc}$ $\therefore L = \frac{Xc^2+R^2}{Xc\times 2\pi f} = 100.2\times10^{-3}$ henry

$= 100.2$ mh. Ans.

29 (a) $b_L = bc$ or, $\frac{x_L}{RL^2 + x_L^2} = \frac{Xc}{Rc^2+Xc^2}$

or, $\frac{x_L}{x_L^2+31.25^2} = \frac{40}{900+1600}$

or $40x_L^2 + 2500x_L - 39002.5 = 0$

$\therefore x_L = \frac{-2500 \pm \sqrt{2500^2+4\times40\times39062.5}}{2\times40} = 31.25$ ohm or 12.94 ohm.

(b) $I_c = \frac{100\angle0}{30-j40} = 2\angle53.13^\circ$

$OM = I_c = 2$

$MP = \frac{V}{2R_L} = \frac{100}{2\times31.5} = 1.6$

$\angle OMP = 180^\circ - 53.13^\circ = 126.87^\circ$

$OP = \sqrt{OM^2 + MP^2 - 2OM.MP\ Cos OMP} = \sqrt{2+1.6^2-2\times2\times1.6Cos\ 126.87}$ (... MP ⊥ OD)

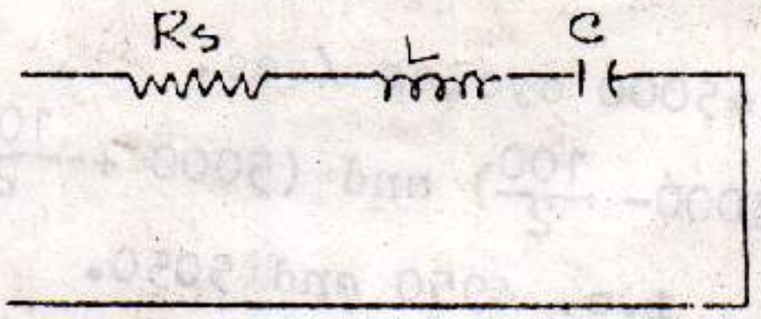
$= 3.225$

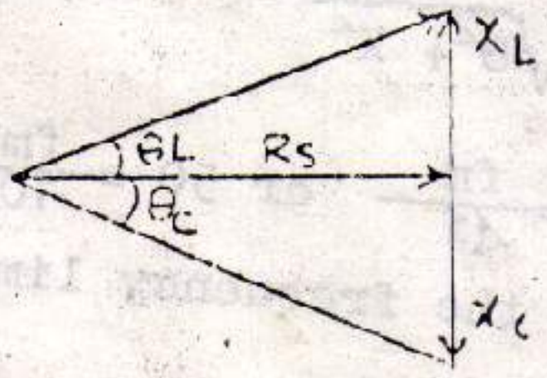
Now, Im = OP-PQ = 3.225-1.6 = 1.625 $\therefore Z_{max} = \frac{100}{1.625} = 61.54$ ohm.

Now, $\angle MOP = \cos^{-1} \frac{OM^2 + OP^2 - MP^2}{2OM.\ OP} = 23.38^\circ$

Now, $\theta = \theta_c - \angle MOP = 29.75^\circ$

30.



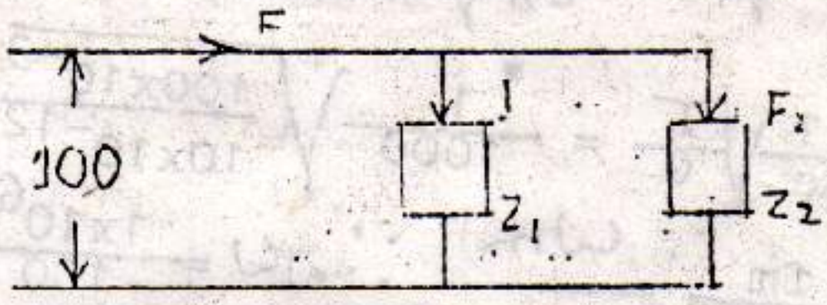


(a) Now, $Q_s = \frac{\text{Reactive factor of the coil}}{\text{Power factor of the coil}} = \frac{\sin\theta_L}{\cos\theta_L} = \tan\theta_L$

$$Q_s = \frac{X_L}{Rs} = \frac{\omega_m L}{R_s}$$

(b) Power factor of the coil, $\cos\theta_L = \frac{1}{\sec\theta_L \sqrt{1+\tan^2\theta_L}} = \frac{1}{\sqrt{1+e_s^2}}$

31.



$Z_1 = 8-j5$

$Z_2 = 3+j7$

$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{(8-j5)(3+17)}{8-j5+3+j7} = \frac{24-j15+j56+35}{11+j2} = \frac{59+j41}{11+j2}$

$= \frac{71.84\ \angle 34.796}{11.18\ \angle 10.3} = 6.43\ \angle 24.45$

$\therefore$ Over all power factor = Cos 24.796 = 0.907.

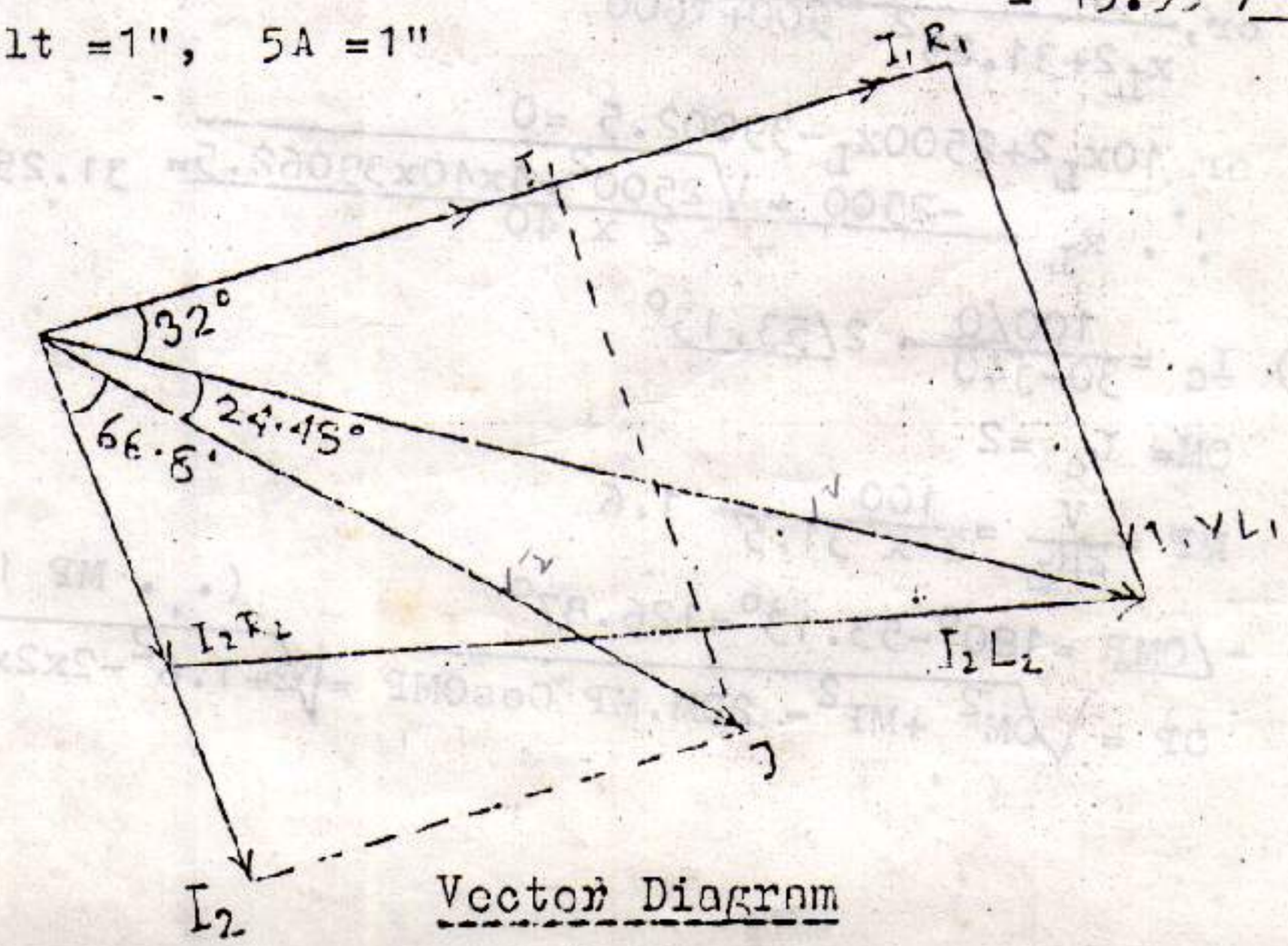
32. $I_1 = \frac{100}{8-j5} = \frac{100}{9.433\ \angle -32.605} = 106\ \angle 32^\circ$

$I_2 = \frac{100}{3+j7} = \frac{100}{7.615\ \angle 66.8} = 13.63\ \angle -66.8^\circ$

$I = I_1 + \frac{1}{2} = 10.6\ \angle 32^\circ + 13.13\ \angle -66.8^\circ = 8.99+j5.61+5.17-j12.06$

$= 15.55\ \angle -24.48^\circ$

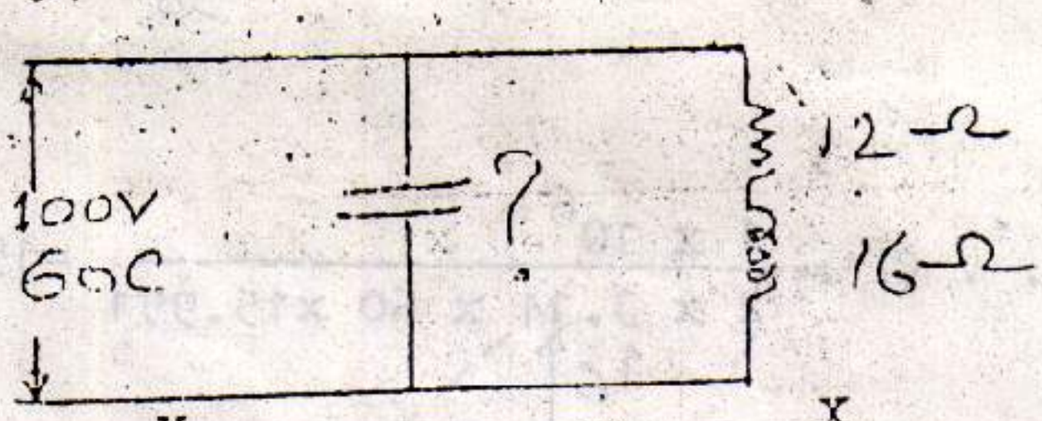
25 volt = 1", 5A = 1"



Vector Diagram

33//

(33) Here given, R_L = 12 ohm.
X_L = 16 ohm
X_c = ?
R_c = 0



At resonance, $\frac{X_L}{R_L^2 + X_L^2} = \frac{X_c}{R_c^2 + X_c^2}$ or, $\frac{16}{R^2+16^2} = \frac{X_c}{0^2+X_c^2} = \frac{1}{X_c}$

$\therefore X_c = \frac{12^2+16^2}{16} = 25$ ohm. or, $2\pi fc = C$ $\therefore C = \frac{1}{50\pi I} \times 10^6$

$= 106.1 \mu F$ Ans.

34.(1) When p.f. = 0.8 lagging

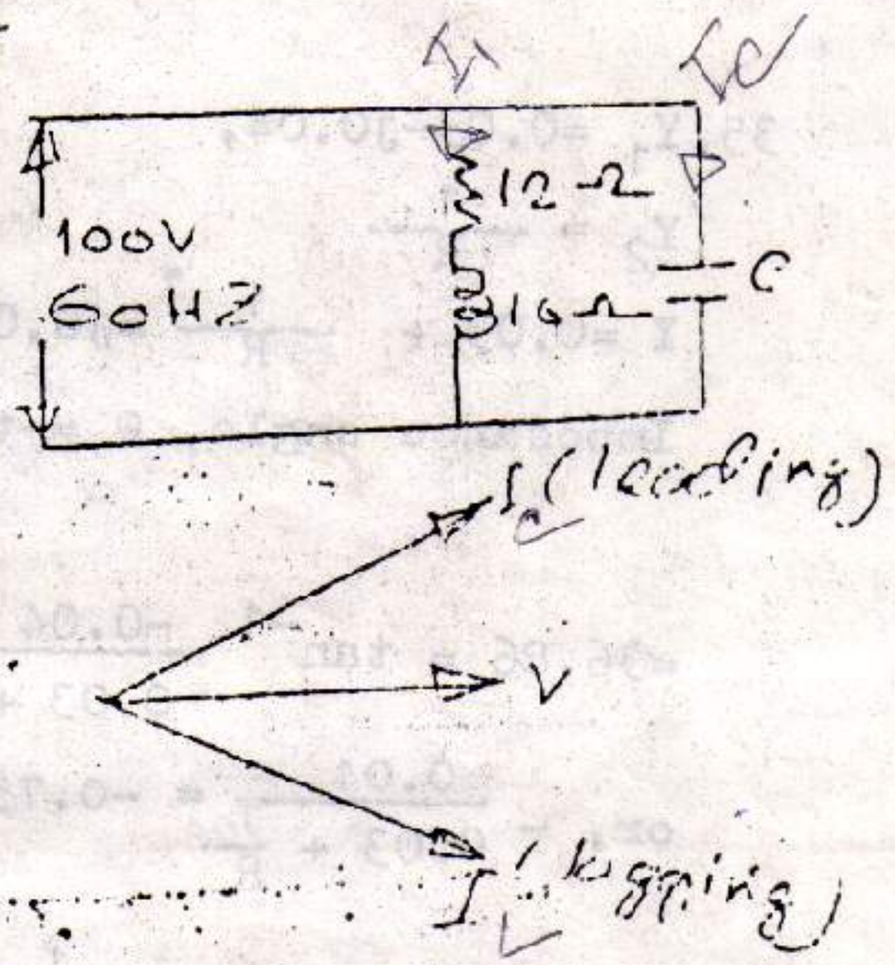
$\theta \rceil^V = \cos^{-1} 0.8 = 36.86$ lagging

$Z_L = 12 + j16 = 20 \angle 53.13°$

$X_C = 0 - jX_c = X_c \angle -90°$

$\therefore I_L = \frac{100 \angle 0°}{20 \angle 53.13°} = 5 \angle -53.13°$

$I_C = \frac{100\angle 0°}{X_c \angle -90°} = \frac{100\angle 90°}{X_c}$



From the vector we get

$IX_L = I_L \cos 53.15° = 3$ Ans.

$I_{yL} = I_L \sin 53.15° = 4$ amp.

$I' = IX_L \tan 36.88° =$

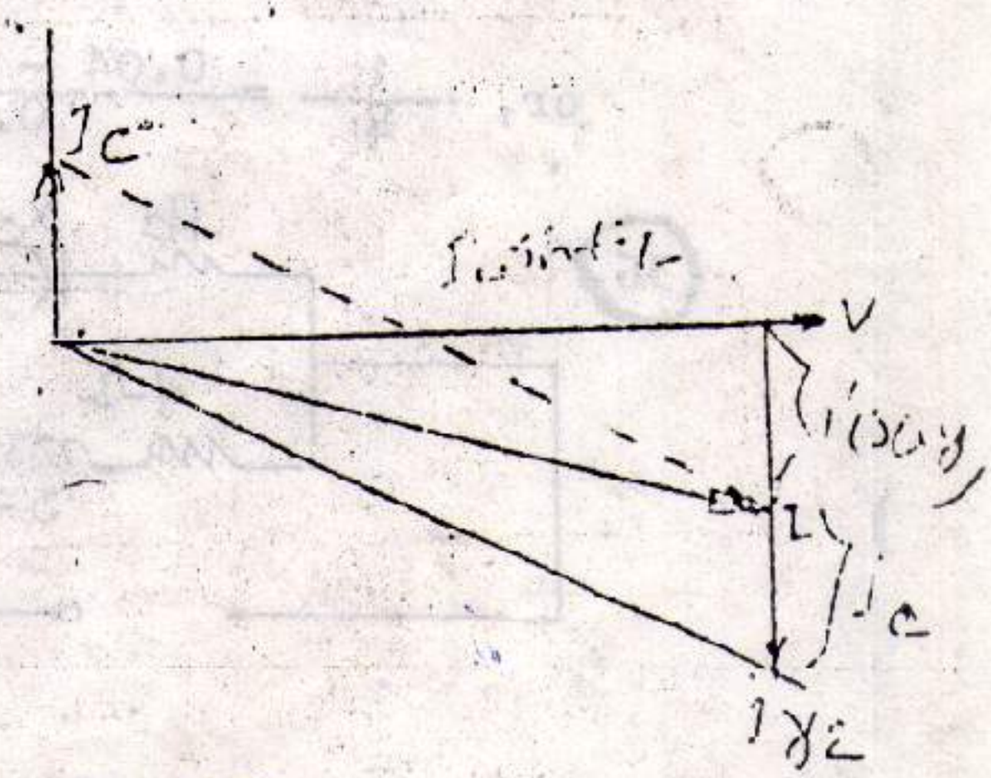
$= 3 \times \tan 36.88° = 2.2508$ Ans.

$\therefore I_C = I_{yL} - I' = 4 - 2.2508$

$= 1.75$ Amp.

$\therefore |X_C| = \frac{100}{1.75} = 57.143$

$\therefore C = \frac{1}{2\pi f X_c} \times 10^6 = 46.42 \mu f$



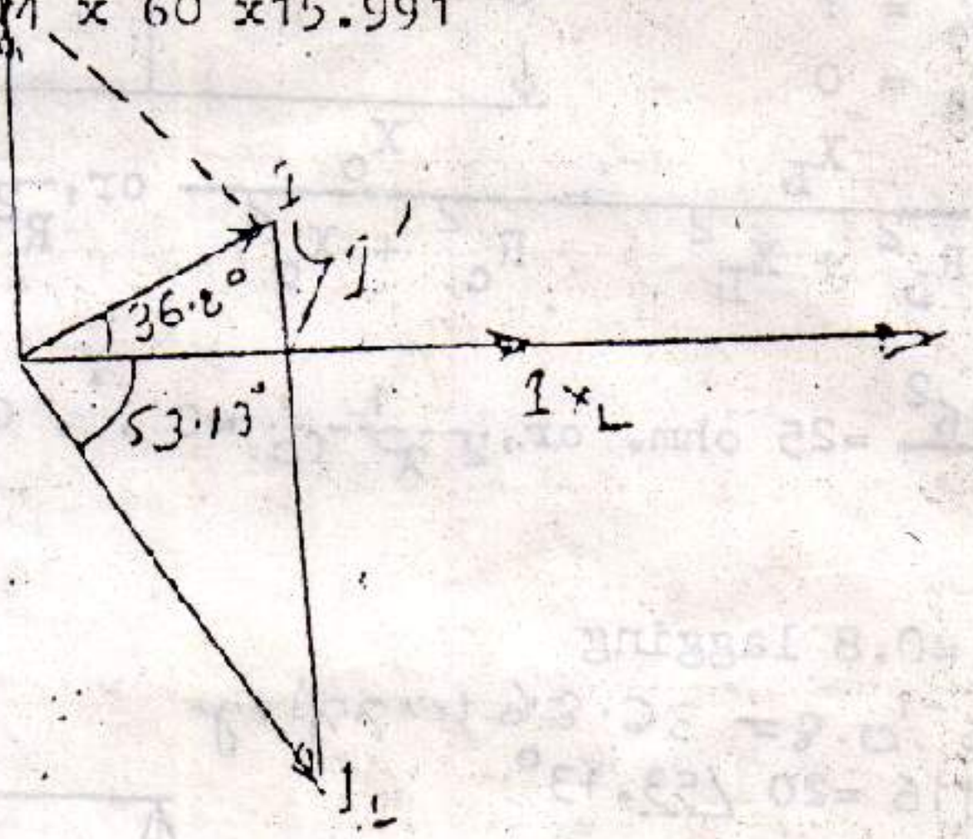
(II) When p.f. = 0.8 loading

$\theta \rceil^V_I = \cos^{-1} .0.8 = 36.87°$ loading

In this ~~case~~ $\cos\theta$ $I_c = I_{yL} + I' = 4 + 2.2508 = 6.2508$ Amp.

$\therefore X_C = \frac{100}{6.2508} = 15.991$

$$\therefore C = \frac{1 \times 10^6}{2 \times 3.14 \times 60 \times 15.991} = 156.8\ \mu f$$



35. $Y_1 = 0.03 - j0.04$,

$Y_2 = \frac{1}{R}$

$Y = 0.03 + \frac{1}{R} = j0.04$

Impedance angle, $\theta = -\tan^{-1} \frac{-0.04}{0.03 + \frac{1}{R}}$) p.f. $= \cos\theta = 0.8$

$\therefore \theta = 36.86$

$= 36.86 = \tan^{-1} \frac{-0.04}{0.03 + \frac{1}{R}}$

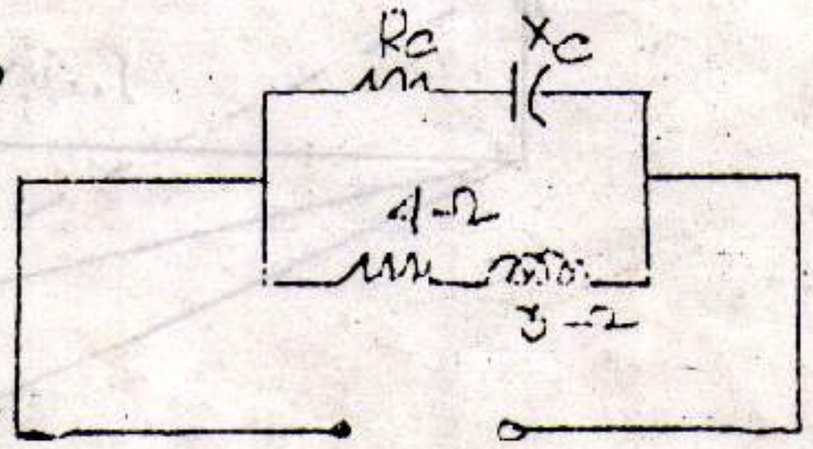
or, $-\frac{0.04}{0.03 + \frac{1}{R}} = -0.75$

or, $0.0225 + 0.75 \frac{1}{R} = 0.04$

or, $\frac{1}{R} = \frac{0.04 - 0.0225}{0.75}$

$\therefore R = 42.86$ ohm

(36)



$X_C = 5$

$\therefore X_C = 5R_C$

$$Z = \frac{(4+j3)(R - j5R)}{4+j3+R-j5R}$$

$$= \frac{(5\ \angle 36.87^\circ)(5.1R\ \angle -78.69^\circ)}{\sqrt{(4+R)^2 + (3-5R)^2}\ \tan^{-1}\frac{3-5R}{4+R}}$$

$$= \frac{25.5R}{\sqrt{(4+R)^2+(3-5R)^2}} \angle -41.82 - \tan^{-1}\frac{3-5R}{4+R}$$

Now, $I\angle\theta = \frac{V\angle 0^\circ}{Z\angle -41.82 - \tan^{-1}\frac{3-5R}{4+R}} = \frac{V}{Z}\angle 41.82 + \tan^{-1}\frac{3-5R}{4+R}$

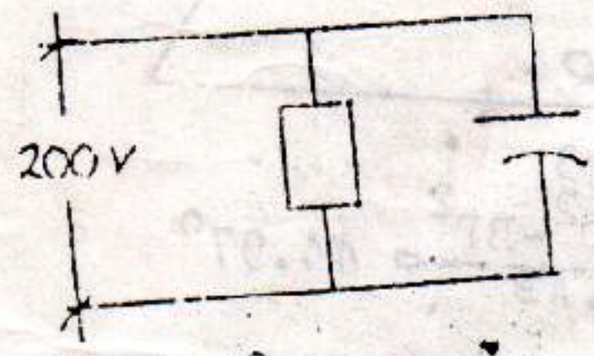
$\therefore \theta = 41.82 + \tan^{-1}\frac{3-5R}{4+R}$ Now, $\theta = \cos\theta^{-1}0.8 = 36.87$ (since leading)

$\therefore 36.87 = 41.82 + \tan^{-1}\frac{3-5R}{4+R}$ or, $\frac{3-5R}{4+R} = \tan(-4.85)$.

or, $3-5R = -0.0866\,(4+R)$ $\therefore R = 0.681$ ohm Ans.

Now, $X_c = 5 \times 0.681 = 3.405$ ohm. $\therefore C = \frac{1}{2\pi \times 60 \times 3.405} = 779\,\mu F$ Ans.

37.



$P = VI\cos\theta$

Now, $I^2R = P$ or $R = \frac{5 \times 10^3}{41.67^2} = 2.88$ ohm.

or $5 \times 10^3 = 200 \times I \times 0.6$

Now, $41.67 = \frac{200}{\sqrt{2.88^2 + X_L^2}}$

$\therefore I = 41.67\angle\theta$ $\theta = \cos^{-1}0.6 = 53.13$ or $X_L^2 = \frac{200^2}{41.67^2} - 2.88^2$ $\therefore X_L = 3.84$ ohm.

Now, For p.f. = 1 we have $b_L = b_c$ or $\frac{3.84}{2.88^2+3.84^2} = \frac{1}{X_c}$ $\therefore I = 41.67\angle -53.13^\circ$

$\therefore X_c = 6$ ohm.

Now, Now volt. amp. $= V_cI_c = \frac{V_c^2}{X_c} \cdot X_c = \frac{200^2}{6} = 6.67 \times 10^3$ volt amp.

$= 6.67$ KVA Ans.

40. $KVA = \frac{Kw}{p.f.} = \frac{2000}{0.8} = 2500$. $\therefore$ Reqd. cost $= \$2500 \times (200 \times 0.08) = \$40{,}000$

$KVA = \frac{2000}{1} = 2000$ $\therefore$ Cost $= 2000 \times 200 \times 0.08 = \$\ 32{,}000$

41.



Here given, $C = 50\,\mu F$, p.f. $= 0.6$, $R = ?$ $f = 60$Hz

We know that, $X_c = \frac{10^6}{2\pi fc\,\mu f} = \frac{10^6}{2\pi \times 60 \times 50} = 53.05$ ohm.

Here, $\cos\theta = .6$, $\theta = \cos^{-1}.6 = 53.13^\circ$ $\tan\theta = \frac{X_c}{R}$? this is true for series impedance

$\therefore \tan(53.13^\circ) = \frac{53.05}{R}$ $\therefore R = 39.78$ ohm. Ans.

42. $fm = \frac{1}{2\pi\sqrt{Lc}} = \frac{1}{2\pi\sqrt{100 \times 10^6 \times 400 \times 10^{-12}}} = 795.775 \times 10^3$ cycles/Sec.



43. For Resonance $b_L = b_c$

$\frac{20}{10^2+20^2} = \frac{X_c}{X_c^2+5^2}$

or $X_c^2 - 500X_c + 25 \times 20 = 0$, or $X_c^2 - 25X_c + 25 = 0$.

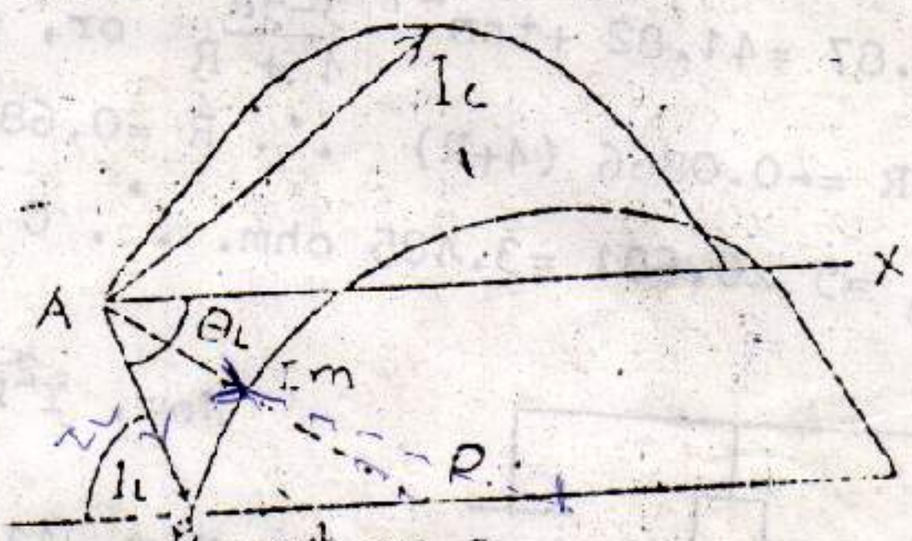
$\therefore X_c = \frac{25 \pm \sqrt{25^2 - 4.25}}{2} = 23.96$ ohm or, 1.044 ohm.

$/Ic/ = \frac{200}{\sqrt{5^2+23.96^2}} = 8.172A$ $\therefore Pc = 8.172^2 \times 5 = 333.9W$

$/Ic/ = \frac{200}{\sqrt{5^2+1.044}} = 39.156A$ $\therefore Pc = 7665.96W$

(44.) For max^m impedance current will be min. $I_L = \frac{200\ \angle 0^\circ}{10+j20} = 8.944\angle -63.43^\circ$

For triangle ABP,



$AB = /I_L/ = 8.94$

$BP = \frac{V}{2R_c} = \frac{200}{2 \times 5} = 20$

$\angle ABP = 180^\circ - 63.43 = 116.57^\circ$

Now, $AP = (AB^2+BP^2-2AB.BP \cos ABP)^{\frac{1}{2}} = 25.3$

$\therefore /Im/ = 25.3-20 = 5.3A$, $\angle PAB = \cos^{-1}\frac{AB^2+AP^2-BP^2}{2AB.AP} = 44.97^\circ$

$\therefore \angle PAX = 63.43-44.97 = 18.46^\circ$ $\therefore Im = 5.3\angle -18.46^\circ$

Now, $Ic = Im - I_L = 5.3\angle -18.46^\circ - 8.94\angle -63.43^\circ = 1.03+j6.32 = 6.4\angle 80.74$

Now, $6.4 = \frac{200}{\sqrt{5^2+X_c^2}}$ or $/Xc/ = 30.85$ ohm. $\therefore C = \frac{1}{2\pi \times 60 \times 30.85}$

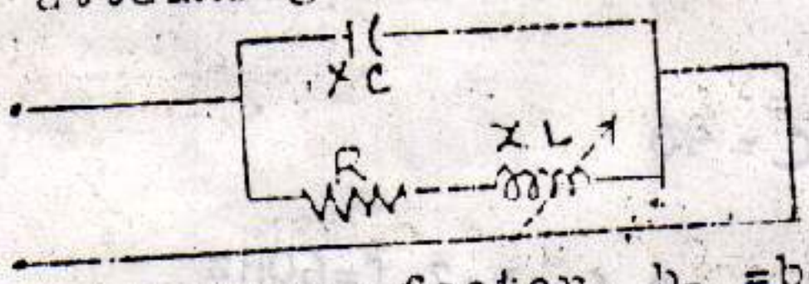
$= 85.98\ \mu F$ Ans.

(45.) From the circle diagram, the will be only one resonance when-

$I_L \sin\theta_L = \frac{V}{2R_c}$ or $R_c = \frac{200}{18.94 \sin 63.43} = 12.506$ ohm.

Hence any value of R_c over 12.506 ohm will prevent the possibility of attaining resonance.

(46.)



For unity power factor, $b_L = bc$, $\frac{X_L}{R^2+X_L^2} = \frac{1}{X_c}$

$X_L^2 - X_L X_c + R^2 = 0$ $\therefore X_L = \frac{X_c \pm \sqrt{X_c^2-4R^2}}{2} = \frac{X_c}{2} \pm \sqrt{\frac{X_c}{4}-R^2}$ proved.

(49.) When $R_L = R_c$ (a) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.05 \times 200 \times 10^{-6}}} = 50.33$ cps.

(b) $\frac{X_c}{R_c^2+X_c^2} = \frac{X_L}{R_L^2+X_L^2}$

$X_L = 2\pi \times 45 \times 0.05 = 14.14$ ohm.

$X_c = \frac{1}{2\pi \times 45 \times 100 \times 10^{-6}} = 35.37$ ohm

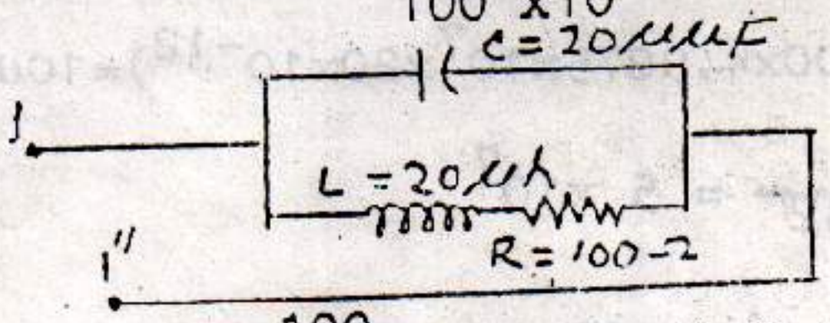
or $R_c^2 = \frac{X_c(R_L^2+X_L^2)}{X_L} - X_c^2$

$= \frac{35.37(20^2+14.14^2)}{14.14} - 35.37^2$ $\therefore R_c = 15.8$ ohm. Ans.

(c) For parallel resonance, irrespective of frequency $R_L = R_c \sqrt{\frac{L}{C}}$

or, $30^2 = \frac{L}{100 \times 10^{-6}}$ $\therefore$ L = 0.04 henry Ans.

50.



$X_L = 5\times10^7 \times 20\times10^{-6} = 1000$ ohm.

$X_c = \frac{1}{5\times10^7 \times 20 \times10^{-12}} = 1000$ ohm.

(a) $g_L = \frac{100}{100^2 + 1000^2} = 9.9 \times 10^{-5} \Longrightarrow 0.0001$

$b_L = \frac{1000}{100^2+1000^2} = 9.9\times10^{-4} \Longrightarrow 0.001$

$bc = \frac{1}{X_c} \rightarrow \frac{1}{1000} = 0.001$

(b) $V = IZ = 2\times10^{-3} \times 10049.9 \angle -5.7 = 20.09 \angle -5.7$

$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = 10019.9 \angle -5.7^\circ$

(c) $QP = \frac{1}{g}\sqrt{L/C} = \frac{1}{0.0001}\sqrt{\frac{20\times10^{-12}}{20\times10^{-6}}} = 10$

(d). $Z = \frac{Z_L Z_c}{Z_L + Z_c} = \frac{(R + j\omega L)(-j\frac{1}{\omega c})}{R + j(\omega L - 1/\omega c)} = \frac{\frac{L}{C} - j\frac{R}{\omega C}}{R + j(\omega L - 1/\omega c)}$

$= \frac{(L/C - jR/\omega c)\{R - j(\omega L - j1/\omega C)\}}{R^2 + (\omega L - 1/\omega c)^2}$

$\therefore Rz = \frac{\frac{RL}{c} - \frac{R}{\omega c}(\omega L - 1/\omega c)\{\frac{RL}{c} \cdot \frac{RL}{c} + \frac{R}{w^2 c^2}\}\omega^2 c^2 R}{R^2 + (\omega L - 1/\omega c)^2} = \frac{}{R^2 w^2 c^2 + (w^2 Lc - 1^2)\ R^2 w^2 c^2 + (w^2 LC - 1)}$

52. $R_{zn} = \frac{R}{R^2 w^2 c^2 + (w^2 Lc - 1)^2}$

R_z will be max^m when the denominator of the above expression will be max^m.

Now, $\frac{dD}{dw} = \frac{d}{dw}\{R^2 w^2 c^2 + (w^2 Lc - 1)^2\} = 0$

$\Longrightarrow 2wR^2 w + 2.2wLc(w^2 Lc - 1) = 0$

$\Longrightarrow 2w\{R^2 c^2 + 2Lc\ (w^2 Lc - 1)\} = 0$

$\therefore$ w = 0, But then line voltage will be dc.

Again, $R^2 c^2 + 2Lc\ (w^2 Lc - 1) = 0$ or $R^2 c + 2L(w^2 Lc - 1) = 0$

or $w^2 = \frac{2L - R^2 c}{2L^2 c}$ $\therefore$ $w = \sqrt{\frac{1}{Lc} - \frac{R^2}{2L^2}}$ rad/sec.

(b) $w = \sqrt{\frac{1}{20\times10^{-6}\times20\times10^{-12}} \frac{100^2}{2(20\times10^{-6})^2}} = 4.9875 \times 10^7$ rad/sec.

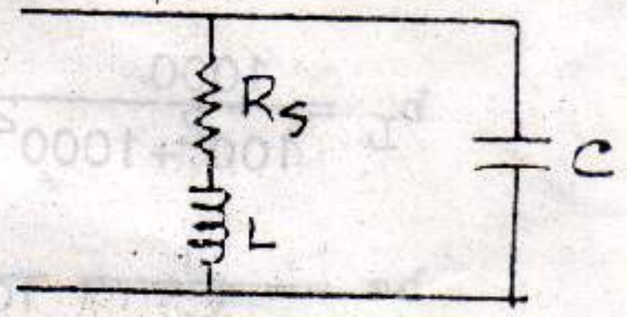
(c) $w = \frac{1}{\sqrt{LC}} = 5 \times 10^7$ rad/sec.

53. $R_{zmax} = \frac{R}{(Lcw^2-1) + R^2w^2c^2} = \frac{100}{(20x10^{-6}x20x10^{-12}x4.9875^2x10^{14}-1)^2 + (100x4.9875x10^7x20x10^{-12})} = 10025.06$ ohm.

54. $w = \frac{0.1}{\sqrt{LC}} = \frac{0.1}{\sqrt{20 \times 10^{-6} x 20 \times 10^{-12}}} = 5 \times 10^7$

$QP = \frac{1}{g\ wL} = \frac{1}{0.0001x5x10^7x20x10^{-6}} = 10$.

55. $i_L = \frac{V}{R_s^2 + (w_m L)^2}$



Since $R_s^2 << w_m^2 L^2$ $\therefore i_L = \frac{V}{w_m L}$

given that, $w = \frac{L i_L^2}{2} + \frac{Cv^2}{2} = \frac{L}{2} \cdot \frac{v^2}{w_m^2 L^2} + \frac{cv^2}{2}$

Again, $w_m = \frac{1}{\sqrt{Lc}}$ (given) or $\frac{v^2}{2}\left(\frac{1}{w_m^2 L}\right) + \frac{Cv^2}{2}$

$= \frac{v^2}{2} + \frac{Cv^2}{2} = Cv^2$ $\therefore C = \frac{w}{v^2}$

Now, given that, $QP = \frac{WmC}{G} = \frac{w_m}{G} \times \frac{w}{v^2} = \frac{w_m w}{v^2 g}$

56. (a) $R_c = 10$ ohm. $\therefore G_c = \frac{R_c}{R_c^2 + X_c^2} = \frac{R_c}{R_c^2 + \frac{1}{w_m^2 C^2}} = \frac{R_c}{R_c^2 + \frac{LC}{C^2}}$

$= \frac{R_c}{R_c^2 + \frac{L}{C}} = \frac{10}{100 + \frac{20x10^{-6}}{20x10^{-12}}} = 9.999 \times 10^{-6}$ mho.

So equivalent parallel resistance of the capacitor is=

$= \frac{1}{G_c} = 1000/0$ ohm. Ans.

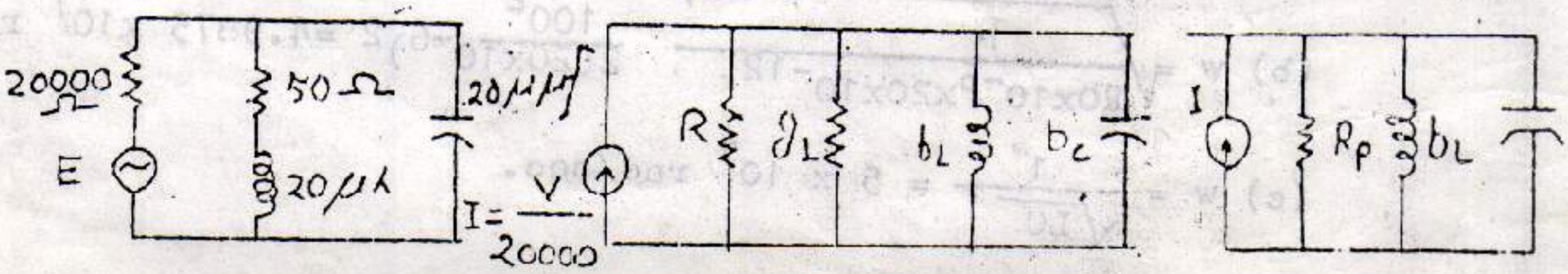
(b) Now, $G_L = \frac{100}{10000 + w_m^2 L^2} = \frac{100}{10000 + \frac{1}{LC} x L^2} = \frac{1000}{1000 + \frac{L}{C}}$

$= \frac{100}{10000 + \frac{20x10^{-6}}{20 \times 10^{-12}}} = 9.9 \times 10^{-5}$ mho.

$\therefore$ Eq. parallel resistance of the inductor $= \frac{1}{G_L} = 10100$ ohm.

Hence eq. resistance of the parallel ckt. is $\frac{100010x10100}{100010+10100} = 9173.56$ ohm.

57.



$$G_L = \frac{X_L}{R^2 + X_L^2} = \frac{20\times10^{-6}\times5\times10^{7}}{50^2+(1000)^2} = \frac{1000}{1002500} = 9.975\times10^{-4}\ \text{mho}$$

$$b_C = \frac{1}{X_C} = \frac{1}{5\times10^{7}\times20\times10^{-12}} = 0.001\ \text{mho}. \quad R = 20{,}000$$

$$g_L = \frac{50}{50^2+(1000)^2}, \quad R_L = \frac{1}{G_L} = \frac{50^2+1000^2}{50} = 20050\ \text{ohm}.$$

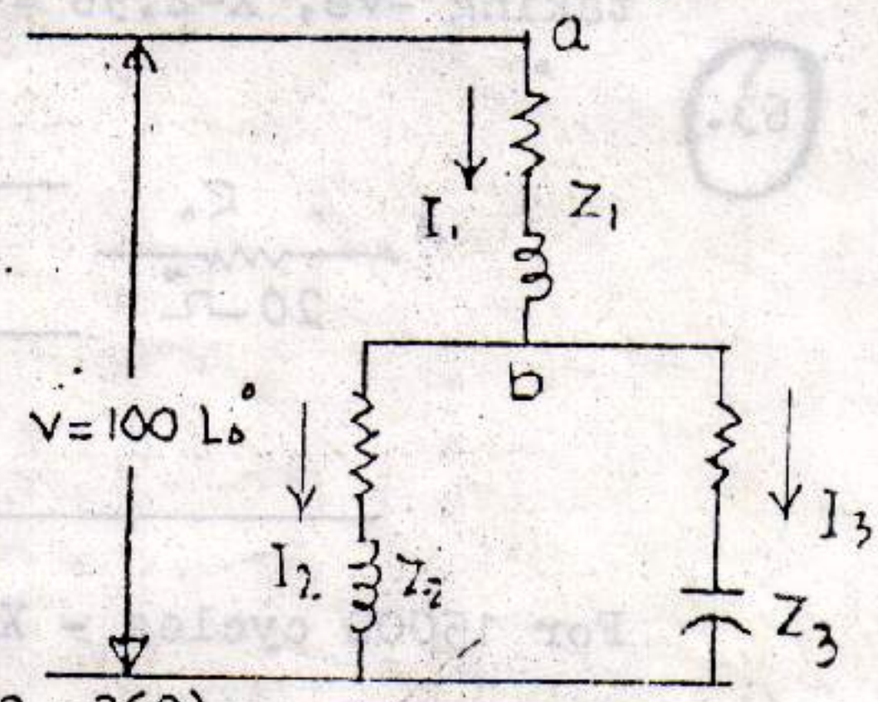
Now, $R_p = \frac{20000 \times 20050}{20000 + 20050} = 10012.5$ ohm.

(b) $Q_p = \frac{1}{g}\sqrt{L/C} = 10012.5\sqrt{\frac{20\times10^{-12}}{20\times10^{-6}}} = 10.01$

(c) $Q_s = \frac{\omega_m L}{R_s} = \frac{5\times10^{7}\times20\times10^{-6}}{50} = 20$

58. (a) $I = \frac{E}{R_s} = \frac{200}{20000} = 0.01A$ (b) $V = IR_p = 0.01\times10012.5 = 100.125V.$

60. $Z_1 = 10+j30$ ohms.
$Z_2 = 5+j\ 10$ ohms.
$Z_3 = 4-j16$ ohms.



$$Z_{bc} = \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= \frac{(5+j10)(4-j6)}{5+j10+4-j6} = \frac{20+160+j(40-80)}{9-j6}$$

$$= \frac{180-j40}{9-j6}\,\frac{9+j6}{9+j6} = \frac{1620+240+j(1080-360)}{81+36}$$

$= 15.89 + j6.15 = 17.04\ \angle 21.16^\circ$

$Z_{ac} = Z_{bc} + Z_{ab} = 15.89+j6.15+10+j30 = 25.89+j36.15 = 44.46\ \angle 54.39^\circ$ ohm.

(a) $I_1 = \frac{V}{Z_{ac}} = \frac{100}{44.46\ \angle 54.39} = 2.25\ \angle -54.4^\circ$ amp.

$V_{ab} = I_1 Z_1 = (2.25\ \angle -54.4)(10+j30) = (2.25\angle -54.4)(31.62\angle 71.6^\circ)$

$\therefore V_1 = 71.15\ \angle\ 17.2^\circ = 67.98 + j20.99$

$\therefore V_{bc} = 100 - 67.98 - j20.99 = 32.02 - j20.99 = 38.29\ \angle -33.25^\circ V$

$I_2 = \frac{38.29\ \angle -33.25^\circ}{5+j10} = \frac{38.29\ \angle -33.25}{11.18\ \angle 63.43} = 3.43\ \angle -96.7^\circ$ amp.

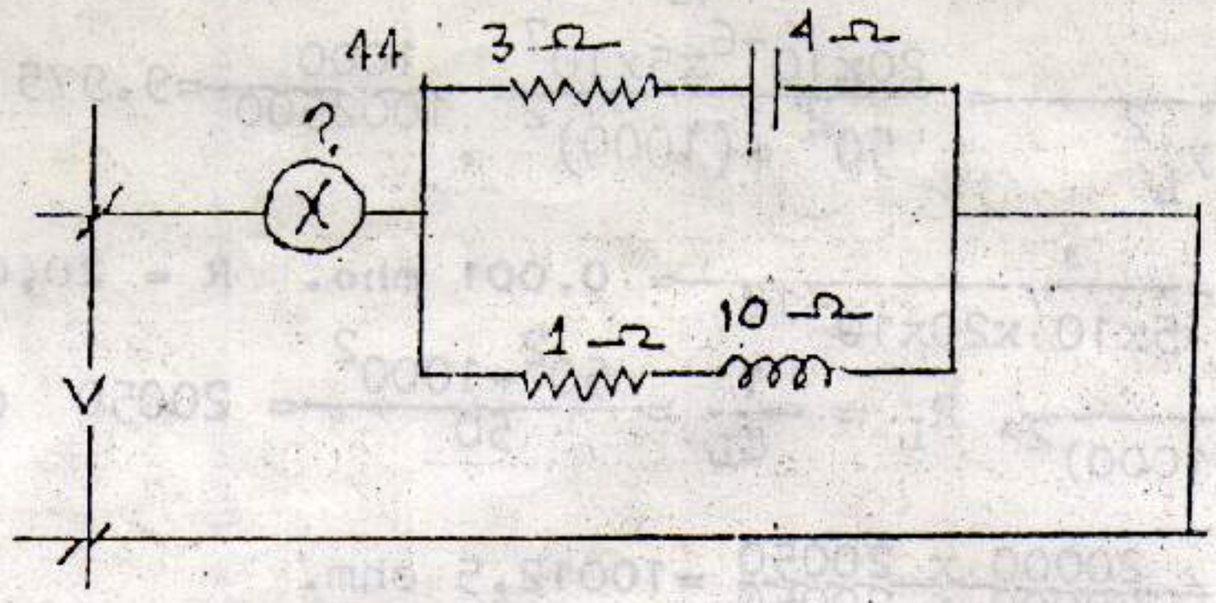
Similarly, $I_3 = 2.33\ \angle\ 42.7^\circ$

(c) Watts $= VI\cos\theta = 100 \times 2.25 \cos(54.39) = 131$

Vars $= VI\sin\theta = 100 \times 2.25 \sin 54.39 = 183$

61. $P_1 = I_1^2 R_1 = (2.25)^2(10) = 50.625$ Watts
$P_2 = I_2^2 R_2 = (3.43)^2(5) = 58.8$ watts.
$P_3 = I_3^2 R_3 = (2.33)^2(4) = 21.7$ watts.

62.



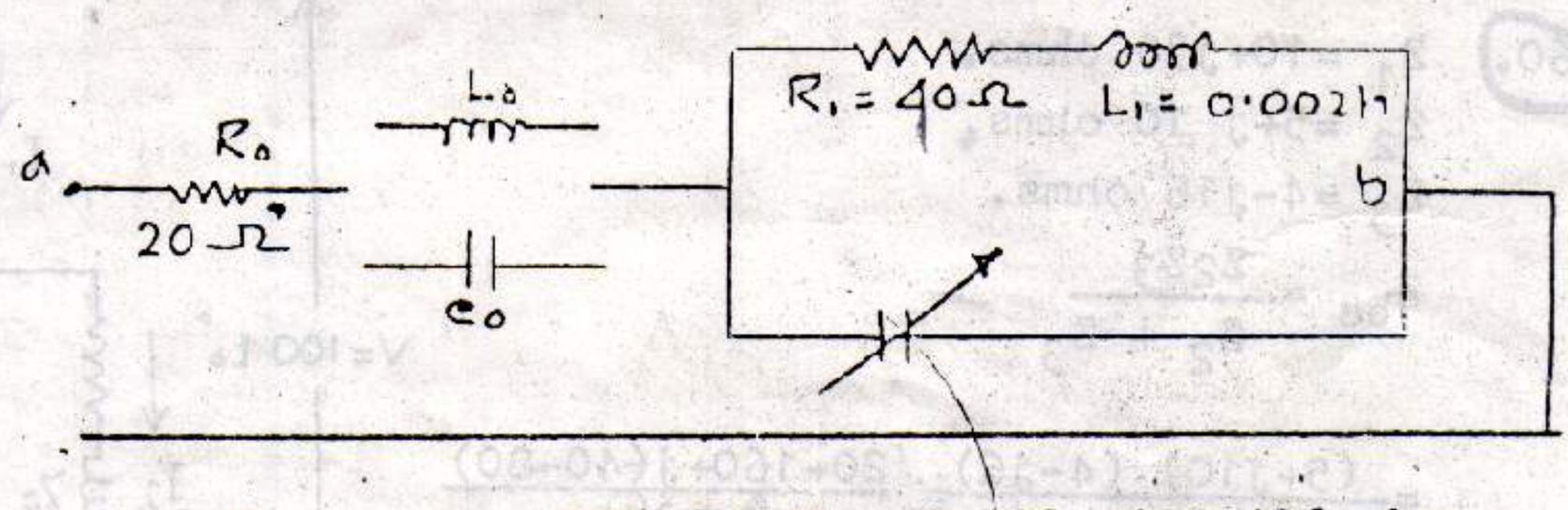
$$Z_{ab} = \frac{(3-j4)\ (1-j10)}{3-j4+1+j10} = 9.969\ \angle -25.12 = 6.31-j2.96$$

Now, $\tan \cos^{-1} 0.707 = \frac{X-2.96}{6.31}$ or, $\pm 1 = \frac{X-2.96}{6.31}$

taking +ve, $X-2.96=6.31$ $\therefore X = 9.27$ ohm (inductive)

taking -ve, $X-2.96 = -6.31$ $\therefore X = 3.35$ (capacitive)

63.



For 15000 cycles - $X_{L1} = 2\pi \times 15000 \times 0.002 = 188.496$ ohm.

(a) For parallel resonance, $b_L = b_C$ or $\frac{188.496}{40^2+188.496} = \omega C_2$

$\therefore C_2 = 5.39 \times 10^{-8} = 0.0539\ \mu F.$

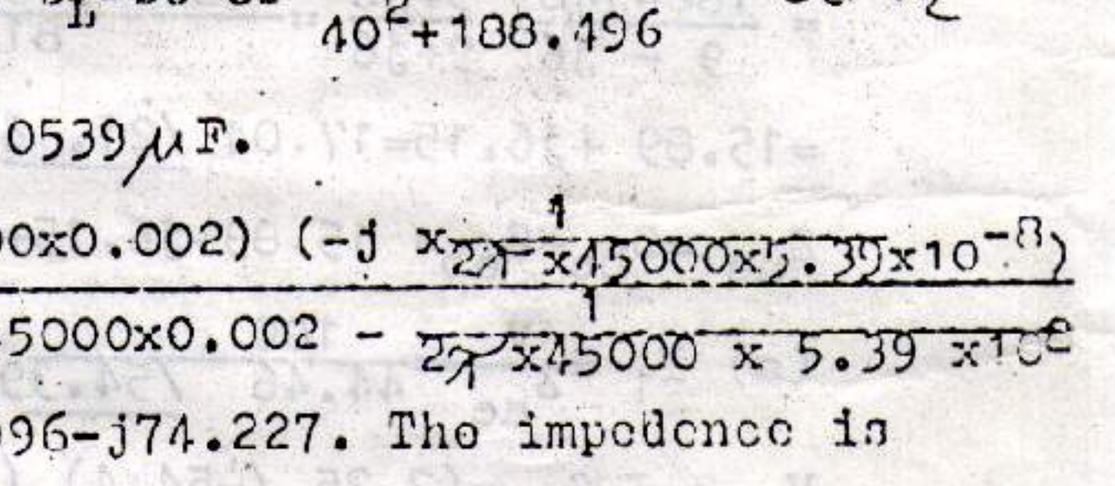
(b) Now, $Z_{bc} = \frac{(40+j2\pi \times 45000 \times 0.002)\ (-j \times \frac{1}{2\pi \times 45000 \times 5.39 \times 10^{-8}})}{40 + j(2\pi \times 45000 \times 0.002 - \frac{1}{2\pi \times 45000 \times 5.39 \times 10^{-8}})}$

$= 74.23\ \angle -89.46^\circ = 0.6996 - j74.227$. The impedence is predominanity capacitive.

(c) The inductive reactance is - $X_{Lo} = 74.227$

$\therefore L_o = \frac{74.227}{2\pi \times 45000} = 0.2625$ mh.

64. $= 1592$ cycles/Sec. $\therefore \omega = 2\pi f = 10003$ rad/Sec.



$X_{L_1} = \omega L_1 = 1000.3$ ohms.

$X_{L_2} = \omega L_2 = 500.1$ ohms.

$X_c = \frac{1}{\omega C} = 1000$ ohms.

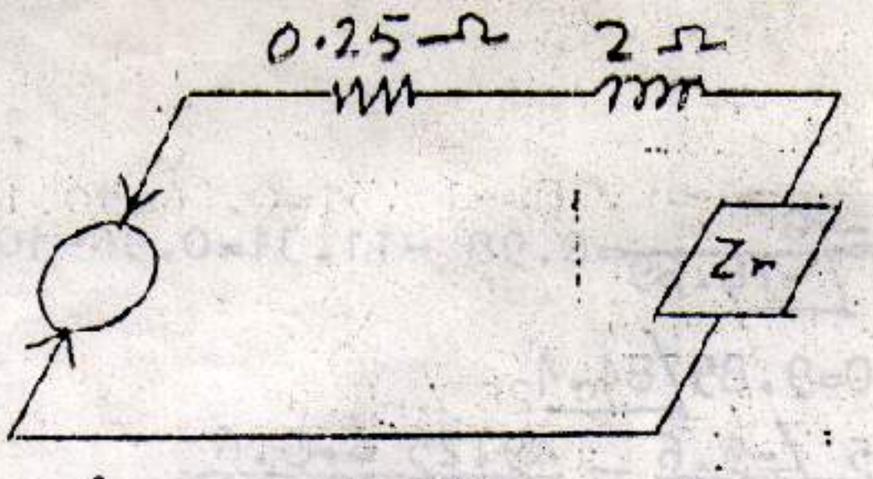
$Z_{12} = \frac{(j1000.3)\ (-j1000)}{j\ 1000.3 - j1000}$

$= \frac{1000300}{j0.583} = -j171578$

$Z_{13} = 30+j500.1-j1715780 = 30-j1715280$, $Z_{ab} = \frac{(30-j1715780)(10)}{40-j1715280}$

$= \frac{(1715280\ \angle 90^\circ)\ (10)}{(1715280\ \angle 90^\circ)} = 10\ \angle 0^\circ$ Ans.

65.



$Z_1=(0.25+.5)+j(1+2)=0.75+j3$

$\therefore$ For max. power, $Z_r=0.75-j3$, $I=\frac{20}{0.75+j3+0.75-j3}=\frac{20}{1.5}=13.33A$

$\therefore P_{load}=I^2R=13.33^2\times0.75=133.33W$

$P_{Line\ loss}=I^2\times0.25=44.44W$, $P_{gen.\ loss}=I^2\times0.5=88.89W$

66. (a) $P_r=I^2R_r=\frac{Eg^2R_r}{(R_1+R_r)^2+(X_1+X_r)^2}$. Load power would be max^m

if X_r is zero. Since all the terms in the eq^n is constant.

$\therefore$ The condition is $X_r=0$

(b) $P_r=\frac{20^2\times0.75}{(0.75+0.75)^2+3^2}=26.67W$

67. $P_r=\frac{Eg^2R_r}{(R_1+R_r)^2+X_1^2}$ Now, $\frac{dP_r}{dR_r}=0 \Rightarrow R_r^2=X_1^2+R_1^2$

Again, For max^m power. $R_r=\frac{Z_1}{\sqrt{1+K^2}}$ here, $K=\frac{X_r}{R_r}=0$

$=Z_1=\sqrt{X_1^2+R_1^2}=\sqrt{0.75^2+3^2}=3.092$ ohm.

Now, $I=\frac{20}{0.75+j3+3.092}=\frac{20}{4.87\ \underline{/37.98}}=4.1\ \underline{/-37.98^\circ}$

$P_{load}=/I^2/\times R_r=51.98$ W, $P_{line}=/I^2/\times0.25=4.2W$.

$P_{gen\ loss}=/I^2/\times0.5=8.4W$

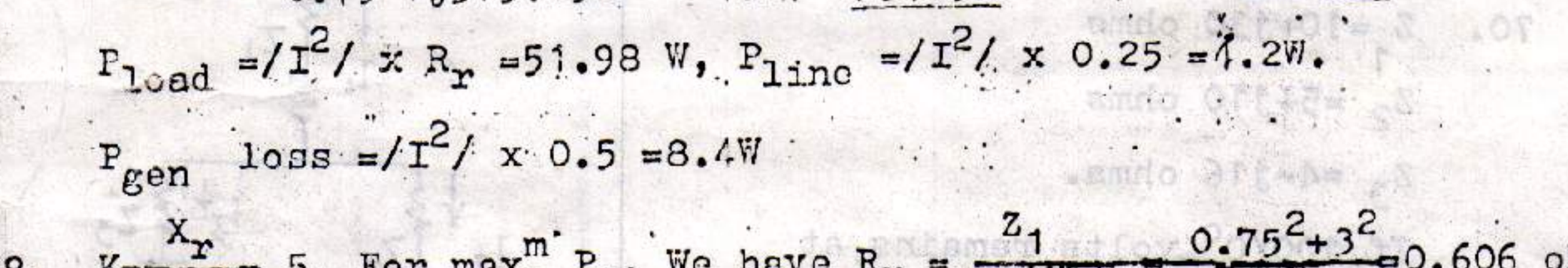
68. $K=\frac{X_r}{R_r}=5$, For max^m P_r, We have $R_r=\frac{Z_1}{\sqrt{1+K^2}}=\frac{0.75^2+3^2}{\sqrt{1+5^2}}=0.606$ ohm.

$X_r=5R_5=3.03$ ohm. $Z_{load}=0.606-j3.03$.

$I=\frac{20}{(0.75+0.606)^2+(3-3.03)^2}=14.74A$.

$\therefore P_{max}=(14.74)^2\times0.606=131.76$ Ans.

69.



$$Z_p = \frac{1\angle 0^\circ \times 5\angle -90^\circ}{1+j0+0-j5} \quad \frac{5\angle -90^\circ}{1-j5} \quad \frac{5\angle -90^\circ}{5.09\angle -78.69} = 0.98 - 11.31 = 0.96 - j0.192$$

$$Z = 0 + j10 + 0.96 - j0.192 = 0.96 + j9.80 = 9.85\angle 84.4$$

$$Z_p = \frac{5\angle -90^\circ \times 9.85\angle 84.4}{0.96+j9.80-j5} = \frac{49.25\angle -5.6}{0.96+j4.80} = \frac{49.25\angle -5.6}{4.90\angle 76.69}$$

$$= 10\angle -82.29 = 1.34 - j9.91.$$

$$Z_{eq} = 1 + 1.34 - j9.91 = 2.34 - j9.91 = 10.18\angle -76.71$$

$$I_1' = \frac{100\angle 0^\circ}{10.18\angle -76.71} = 9.82\angle 76.71.$$

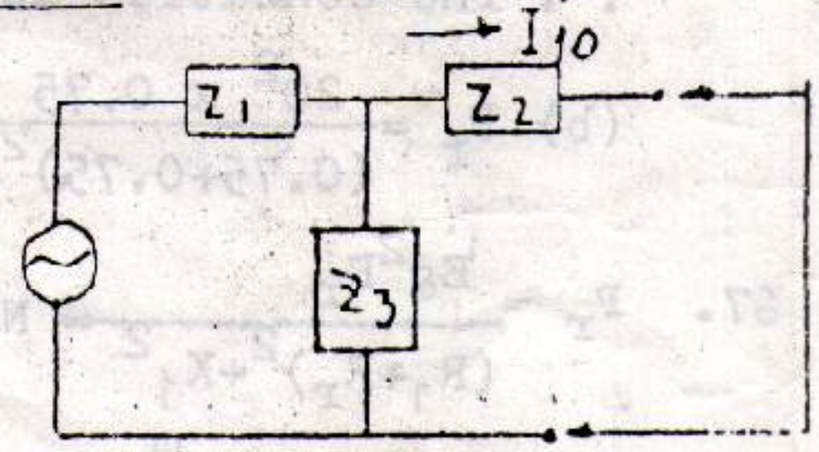
$$I_{10}' = I_1' \times \frac{5\angle -90^\circ}{0-j5+0.96+j9.80} \qquad I_1' = I_1 \times \frac{5\angle -90^\circ}{0.96+j4.80}$$

$$= \frac{9.82\angle 76.71 \times 5\angle -90^\circ}{4.90\angle 78.69} = 10.02\angle -91.98 = -0.35 - j10$$

$$I_2' = I_{10}' \times \frac{Z_3}{Z_2+Z_3} = I_{10} \times \frac{5\angle -90^\circ}{1+j0+0-j5}$$

$$= \frac{10.02\angle -91.98 \times 5\angle -90^\circ}{5.09\angle -78.69}$$

$$\therefore = 9.84\angle -103.29 = -2.26 - j9.58.$$



When $E_1 = 0$, $I_2 = \frac{50\ 60^\circ}{10.18\angle -76.71} = 4.91\angle 136.71 = 3.57 + j3.37.$

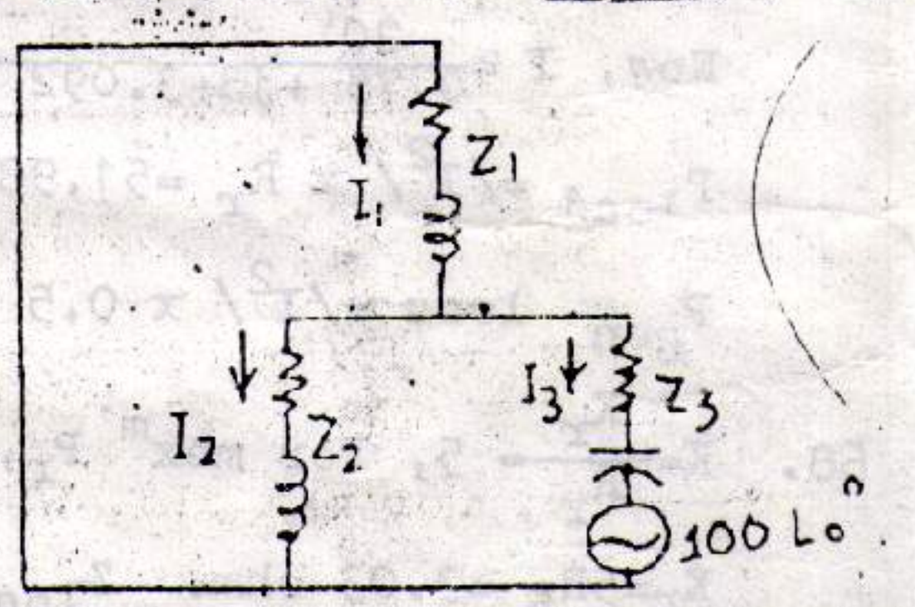
$I_2 = I_2' + I_2'' = -2.26 - j9.58 - 3.57 + j3.37 = -5.83 - j6.21 = 8.52\angle -133.19^\circ$ Ans.

70. $Z_1 = 10 + j30$ ohms

$Z_2 = 5 + j10$ ohms

$Z_3 = 4 - j16$ ohms.

If $100\angle 0^\circ$ volts remains at the branch 1. calculations of currents are as follows.



$$Z_{ckt} = Z_1 + \frac{Z_2 Z_3}{Z_2+Z_3} = (10+j30) + \frac{(5+j10)(4-j16)}{5+j10+4-j16} = 10+j10+\frac{20+160+j(40-80}{9-j6}$$

$$= 10+j30+\frac{180-j40}{9-j6} = 10+j30+\frac{1620+240+j(1080-360)}{81+36} = 10+j30+15.90+j6.$$

$$= 25.90+j36.15 = 44.47\angle 54.38^\circ \text{ ohms.} \quad \therefore I = \frac{100\ 0^\circ}{44.47\angle 54.38^\circ}$$

$$= 2.25\angle 54.38^\circ \text{ amp.} \qquad V_1 = I_1 Z_1 = (2.25\angle 54.38^\circ)(31.62\angle 71.57^\circ)$$

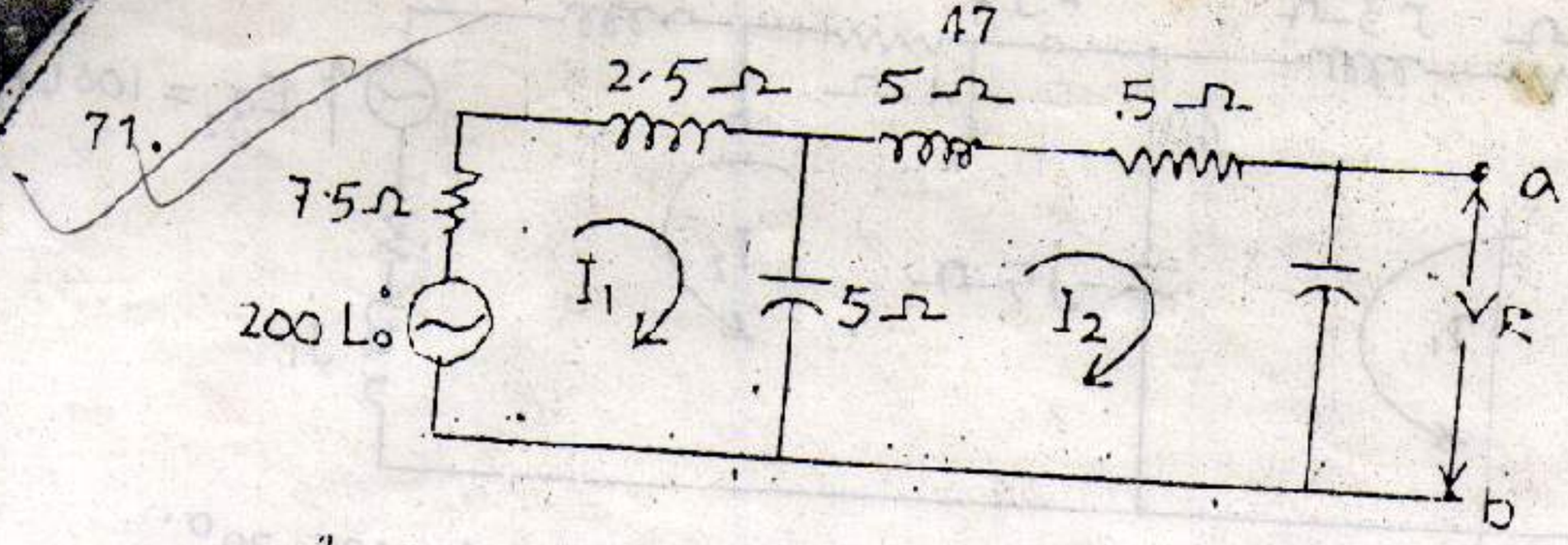
$$= 71.145\angle 17.185 = 67.969 + j21, \quad V_2 = 100 + j0 - 67.969 - j21 = 32 - j21$$

$$= 38.3\angle -33.25, \quad I_3 = \frac{38.3\angle 33.25^\circ}{4-j16} = 2.32\angle +42.7^\circ \text{ amp.}$$

Now, if the $100\angle 0^\circ$ volts is inserted in branch 3, the resulting current in branch. 1 would be equal to I_3, according to the Reciprocity theorem. Therefore, in this case.

$$I_1 = 2.32\angle 42.7^\circ \text{ amp.}$$

71.



$200 = (7.5 + j(2.5-5))\, I_1 + j5 I_2$, $0 = j5 I_1 + (5 + j(5-5-5))\, I_2$

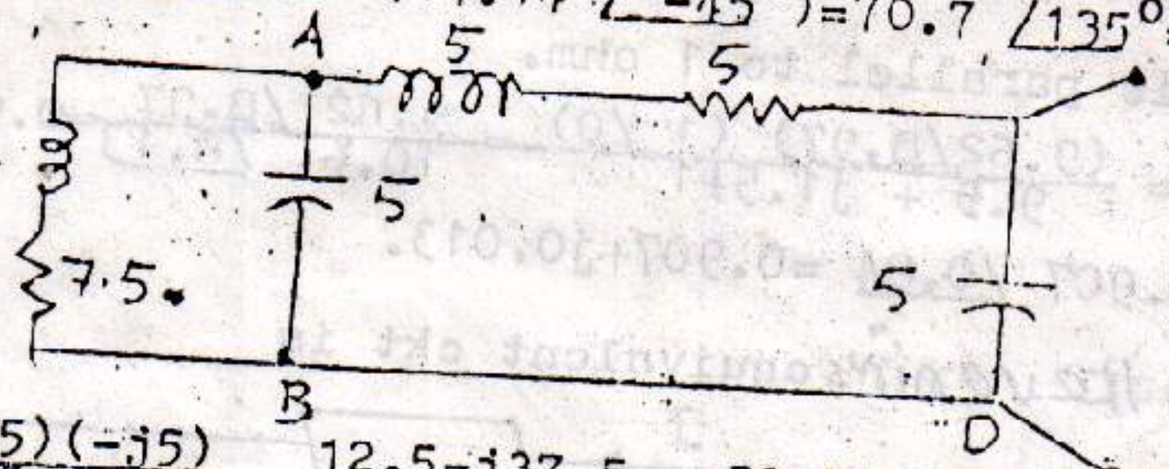
or, $I_1 = \frac{5-j5}{-j5} I_2 = (-\frac{1}{j} + 1) I_2 = (1+j) I_2$

Now, $200 = (7.5-2.5j)(1+j) I_2 + j5 I_2 = (10+5j+j5) I_2$

or $I_2 = \frac{200}{10+j10} = 14.14 \angle -45°$.

$V_R = (-j5) \times I_2 = (5 \angle -90°)(14.14 \angle -45°) = 70.7 \angle 135° = 50 - j50$

Now,



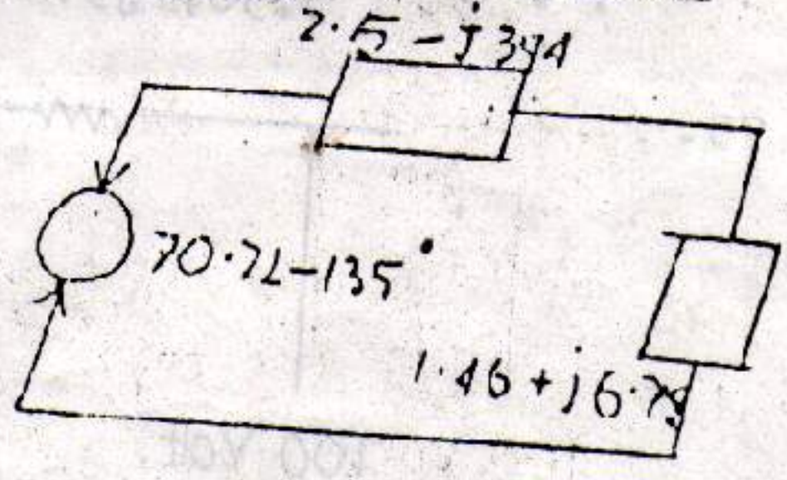
$Z_{AB} = \frac{(7.5+j2.5)(-j5)}{7.5+j2.5-j5} = \frac{12.5-j37.5}{7.5-j2.5} = \frac{39.53 \angle -71.56}{7.9 \angle -18.430} = 5 \angle -53.13°$

$= 3 - j4$.

$Z_{cAB} = 3-j4+5+j5 = 8+j1 = 8.06 \angle 7.13$

Now, $Z = \frac{(8.06 \angle 7.13)(5 \angle -90°)}{8+j1-j5} = \frac{40.30 \angle 82.87}{8.94 \angle -26.57} = 4.5 \angle -56.3°$

$= 2.5 - j3.74$



$\therefore I_{ab} = \frac{70.7 \angle -135°}{2.5+1.46+j(6.78-j3.74)}$

$= \frac{70.7 \angle -135°}{5 \angle 37.5} = 14.14 \angle -172$ Ans.

72. $Z_1 = 1+j1+2+j3 = 3+j4 = 5 \angle 53.13$

$Z_2 = 1+j3+2+j1 = 3+j4 = 5 \angle 53.13$

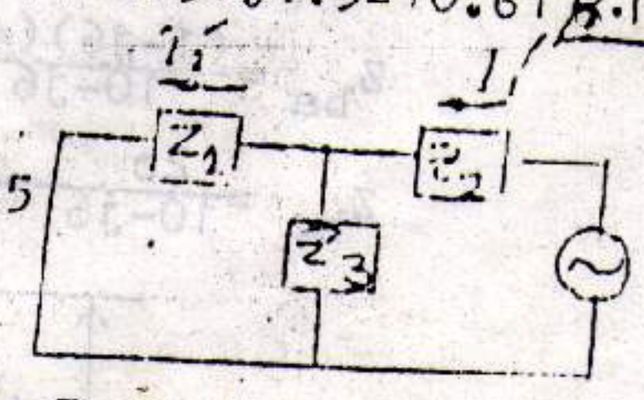
$Z_3 = -j5 = 5 \angle -90°$

$Z = \frac{Z_1 Z_3}{Z_1+Z_3} = \frac{5 \angle 53.13 \times 5 \angle -90°}{3+j4-j5} = \frac{252-36.87}{3-j1} = \frac{25 \angle -36.87}{3.16 \angle -18.43}$

$= 7.91 \angle -18.44 = 7.5 - j2.5$, $Z = 7.5-j2.5+3+j4 = 0.5+j1.5 = 10.61 \angle 8.13$

$I_1' = \frac{100 \angle 0°}{10.61 \angle 8.13} = 9.43 \angle -8.13 = 9.33 - j1.33$.

$Z = \frac{Z_2 \times Z_3}{Z_2 + Z_3} = \frac{5 \angle 53.13 \times 5 \angle -90°}{3+j4-j5} = 7.5 - j2.5$

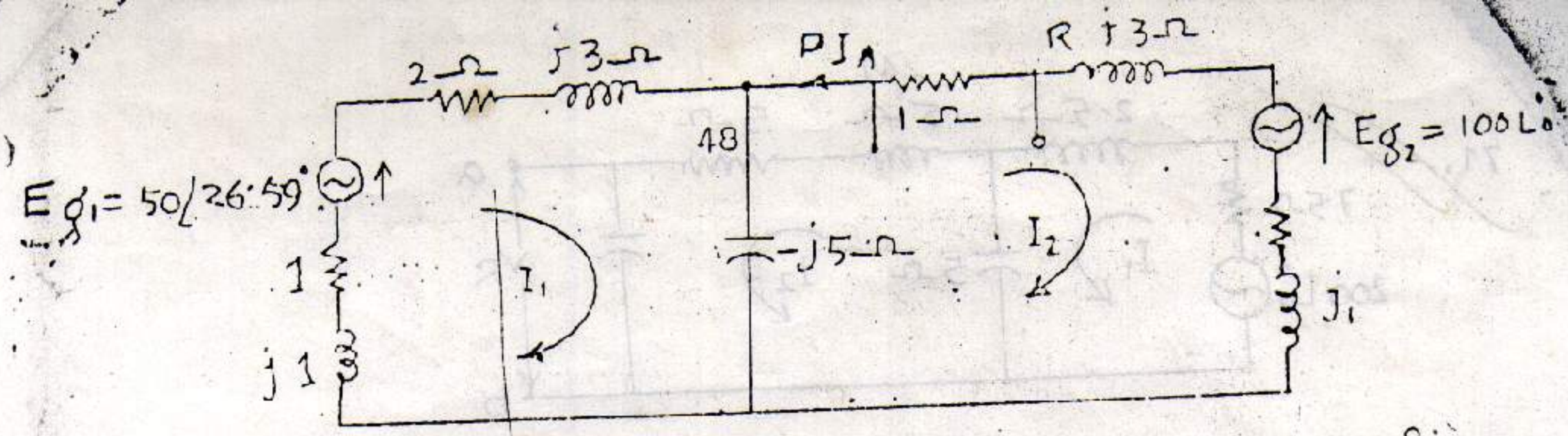


$I_2'' = \frac{50 \angle 26.59}{10.61 \angle 8.13} = 4.71 \angle 18.46$

$I_1'' = I_2'' \times \frac{Z_3}{Z_2+Z_3} = 4.71 \angle 18.46 \times \frac{5 \angle -90°}{3.16 \angle 18.43} = 7.45 \angle -53.11$

$= 4.47 - j5.96$, $I_1 = I_1'' - I_1' = 4.47 - j5.96 - 9.33 + j1.33$

$= 4.86 - j4.63 = 6.71 \angle -136.39$ . . $\boxed{6.71 \angle -136.39}$



Using loop current method, we have $\therefore I_A = 6.72 \angle -136.39°$

$\therefore$ Vab=(6.72 $\angle -136.39°$)(1 $\angle 0°$)=6.72 $\angle -136.39°$

Now, $Z_{pQ} = \frac{(3+j4)5\angle -90°}{3+j4-j5} = \frac{25\angle -36.87}{3.16\angle -18.43} = 7.91\angle -18.44 = 7.5-j2.5$

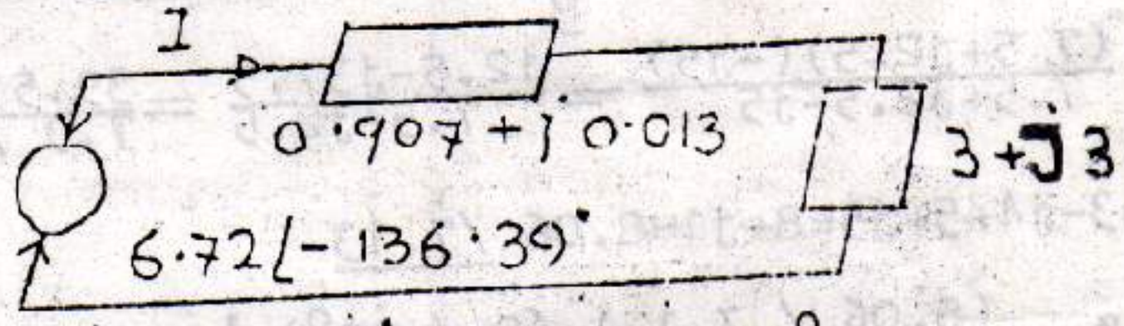
$Z_{pQR} = 7.5-j2.5+2+j(1+3) = 9.5+j1.5 = 9.62\angle 8.97°$

Which is parallel to 1 ohm.

$\therefore Z = \frac{(9.62\angle 8.97)(1\angle 0)}{9.5+j1.5+1} = \frac{9.62\angle 8.97}{10.6\angle 8.13} = 0.907\angle 0.84 = 0.907+j0.013$

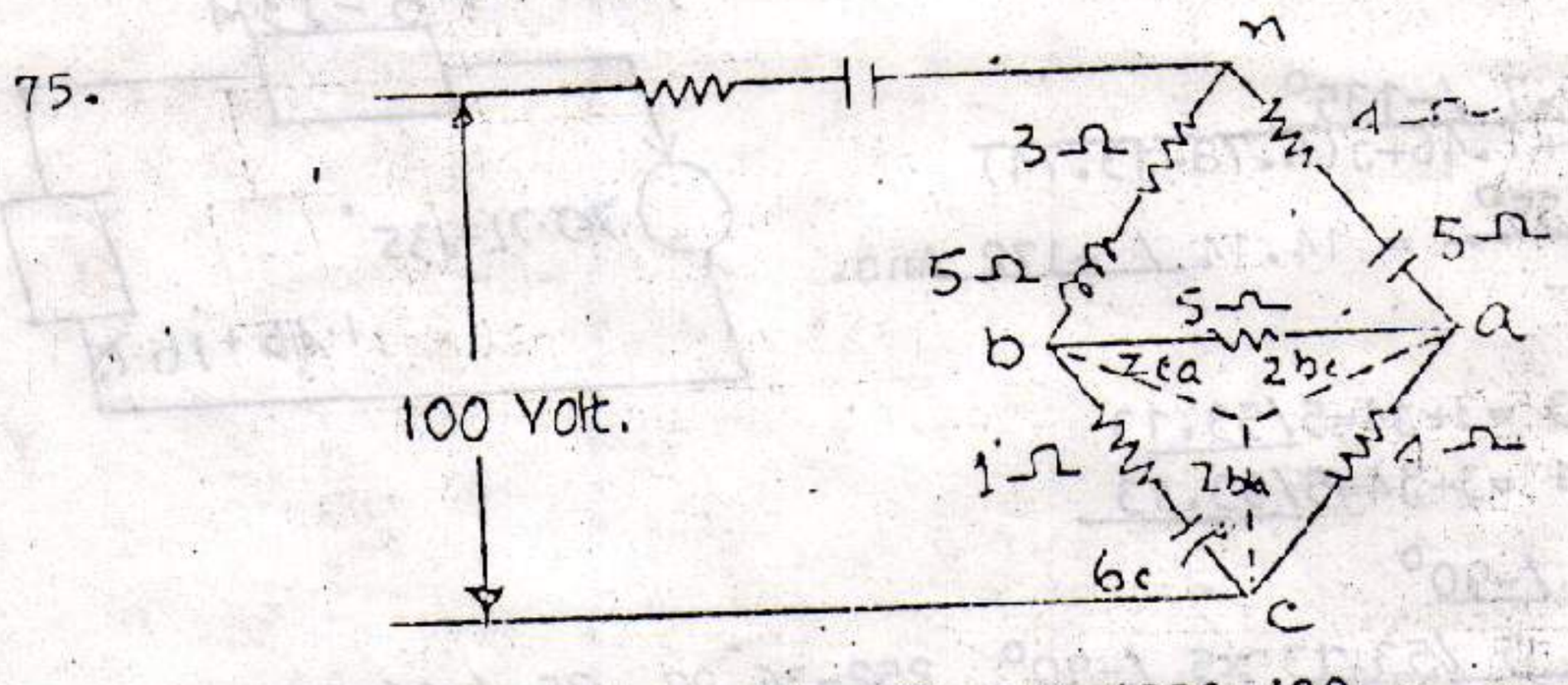
$= 0.907\angle 0.84 = 0.907+j0.013.$

So the Thevenin's equivalent ckt is



$\therefore I = \frac{6.72\angle 136.39}{3.907+j3.013} = \frac{6.72\angle -136.39°}{4.934\angle 37.64} = 1.36\angle -174°$ Ans.

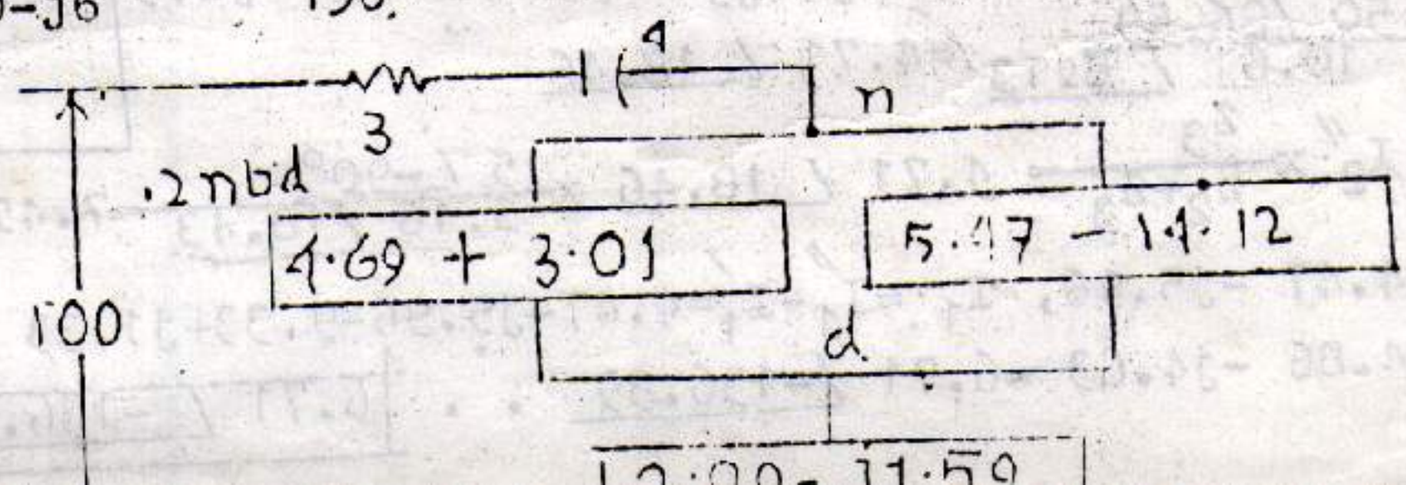
75.



$Z_{ca} = \frac{(5-j30)(10+j6)}{136} = \frac{50+180-j300+j30}{136} = 1.69-j1.99$

$Z_{ba} = \frac{(1-j6)(4)}{10-j6} = \frac{(4=j24)(10+j6)}{136} = \frac{40+j24-j240+144}{136} = 2.09-j1.59$

$Z_{bc} = \frac{20}{10-j6} = \frac{200+j120}{136} = 1.47+j0.88$



$Z_{nbd}=3+j5+1.69-j1.96=4.69+j3.01$, $Z_{nad}=4=j5+1.47+j0.88=5.47-j4.12$

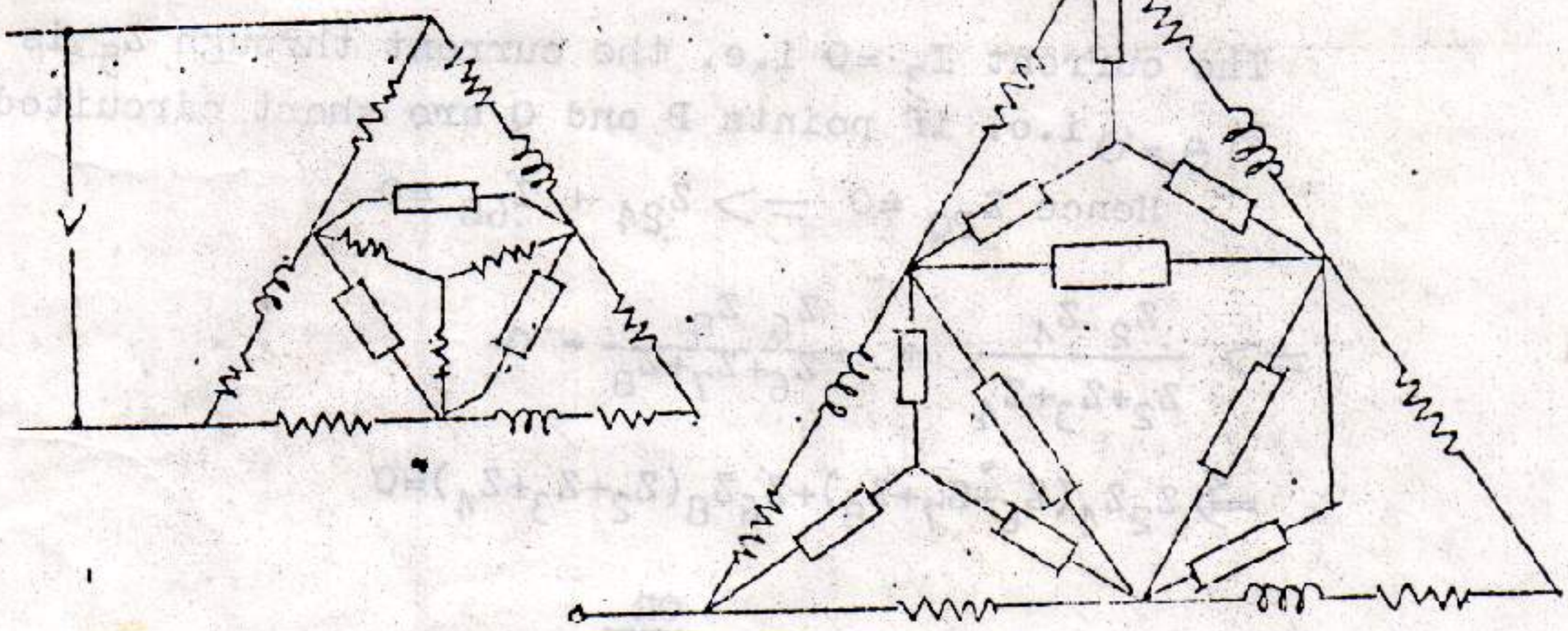
$Z_{n'd}=\dfrac{(4.69+j3.02)(5.47-j4.12)}{4.69+j3.01+5.47-j4.12}=\dfrac{(5.573\ \underline{/32.69^\circ})(6.85\underline{/-36.29^\circ})}{10.22\ \underline{/-6.23^\circ}}$

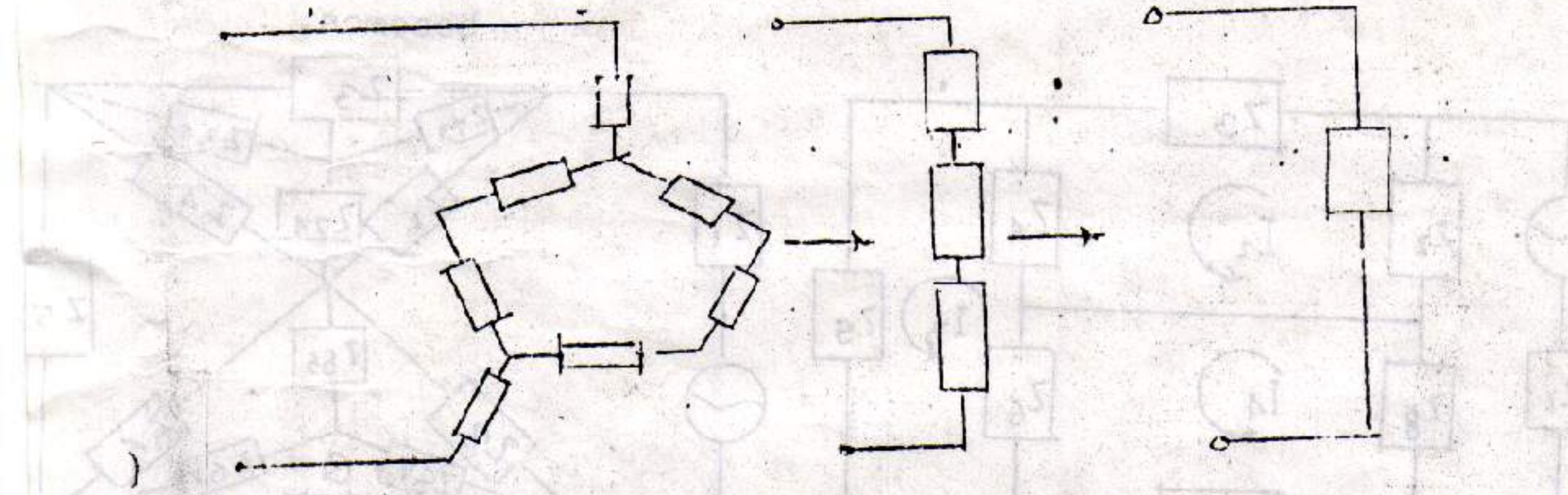
$=3.735\ \underline{/1.93^\circ}=3.73+j0.126$ $\therefore Z_{no}=3.73+j0.126+2.09-j1.59$

$=5.82-j1.464$

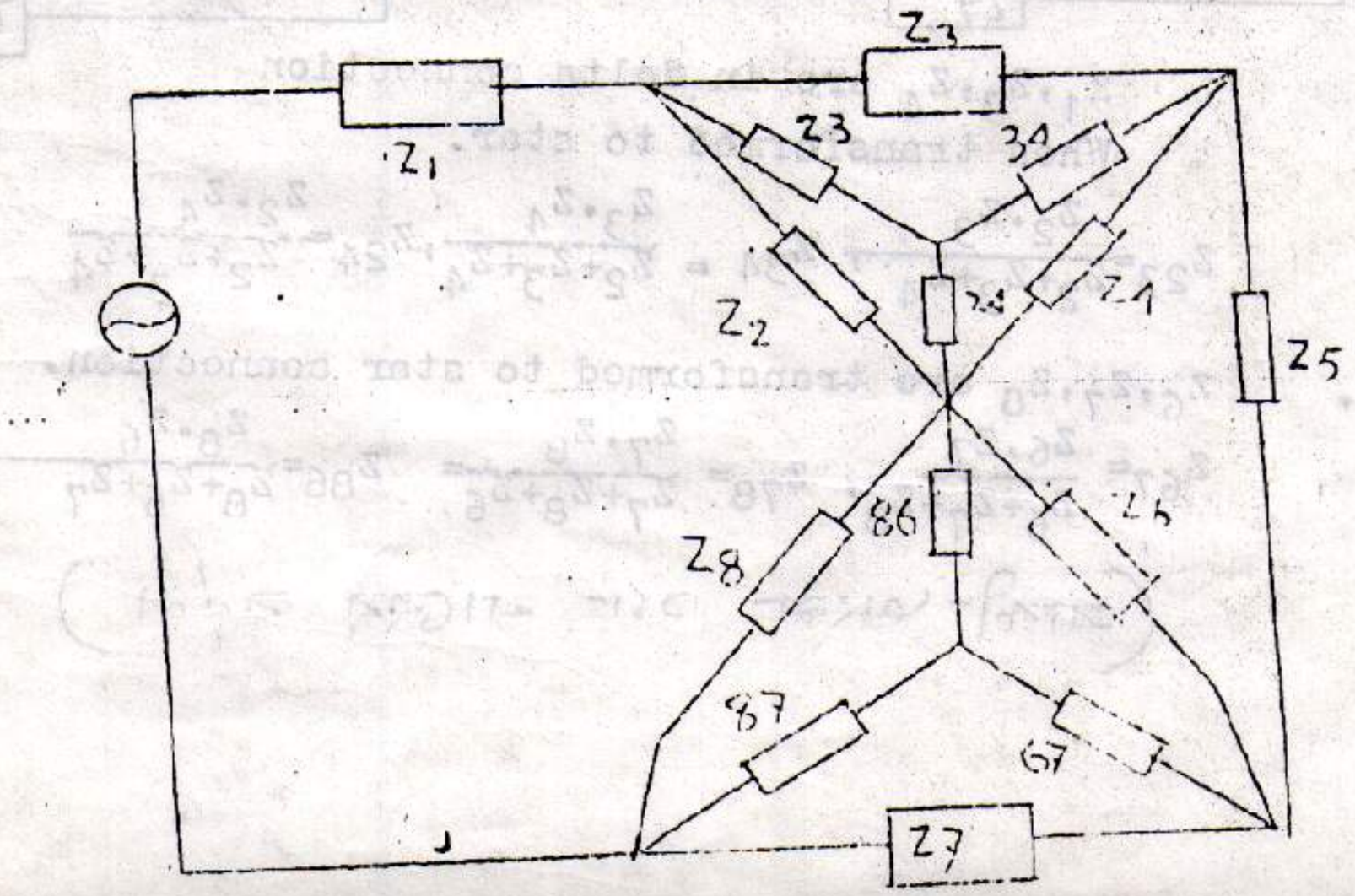
$\therefore Z_{ckt}=3=j4+5.82-j1.464=8.82-j5.464=10.375\ \underline{/31.78^\circ}$

78.



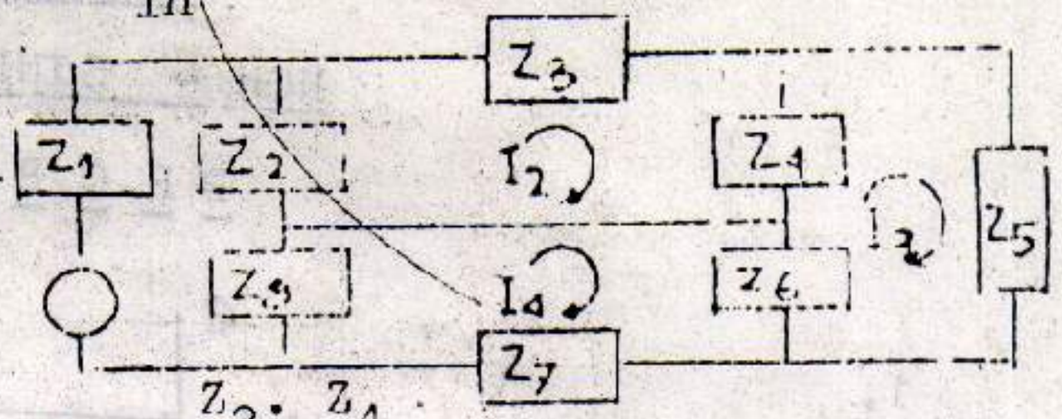


79



(OK)

What relationship but^n the Z's of fig. will make $I_3=0$ regard less of the magnitude of E_{in}?



Z_2, Z_3, Z_4 are in delta connection.

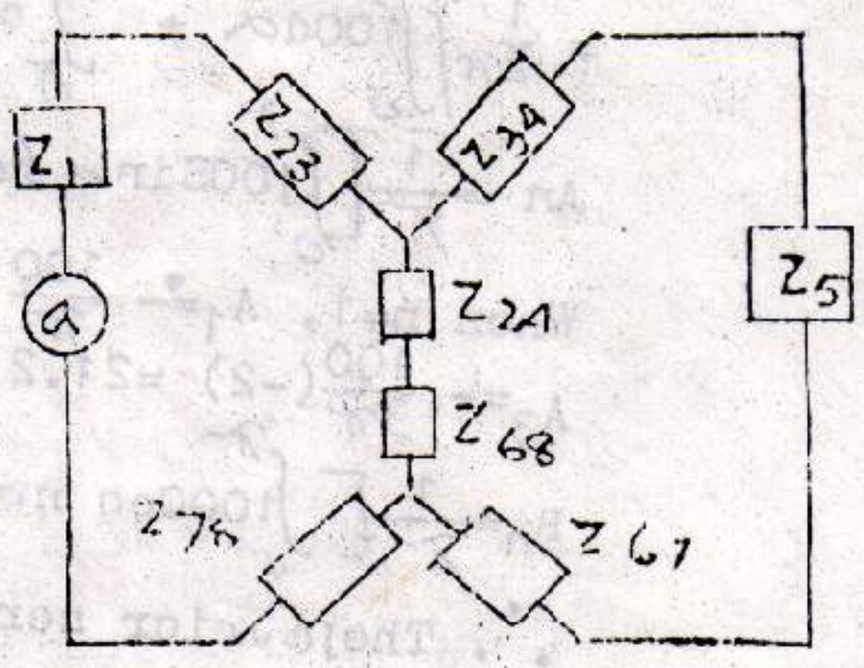
When transformed to stm.

$$Z_{23}=\frac{Z_2 \cdot Z_3}{Z_2+Z_3+Z_4},\quad Z_{34}=\frac{Z_3 \cdot Z_4}{Z_2+Z_3+Z_4},\quad Z_{24}=\frac{Z_2 \cdot Z_4}{Z_2+Z_3+Z_4}$$

Z_6, Z_7, Z_8 are transformed to star connection.

$$Z_{68}=\frac{Z_6 \cdot Z_8}{Z_6+Z_7+Z_8},\quad Z_{78}=\frac{Z_7 \cdot Z_8}{Z_6+Z_7+Z_8},\quad Z_{67}=\frac{Z_6 \cdot Z_7}{Z_6+Z_7+Z_8}$$

the figure foucuda.

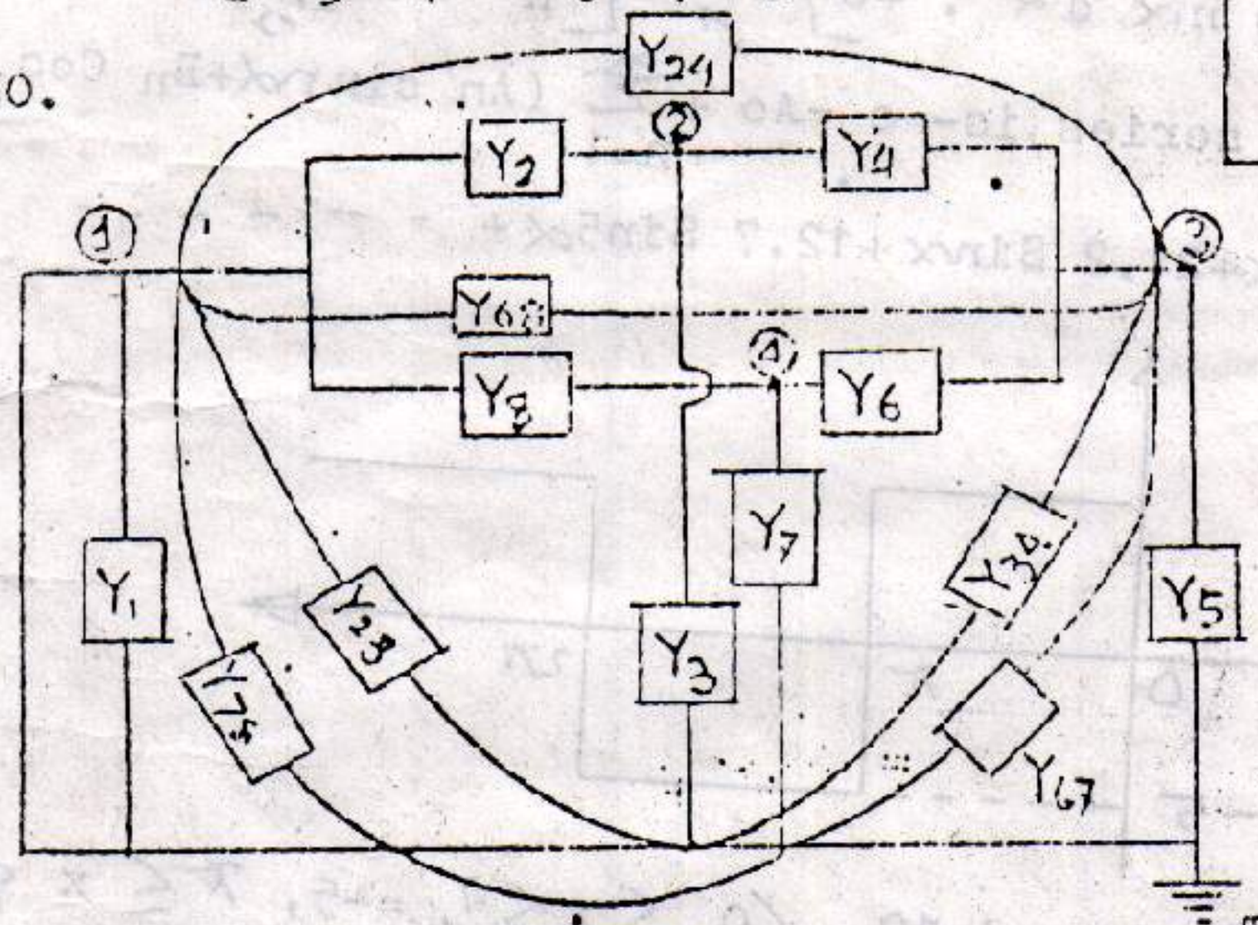


Now when $Z_{24}+Z_{68}\neq 0$ then $I_3=0$ of any magnitude of E_{in}.

The relation is- $Z_{23} + Z_{68}$

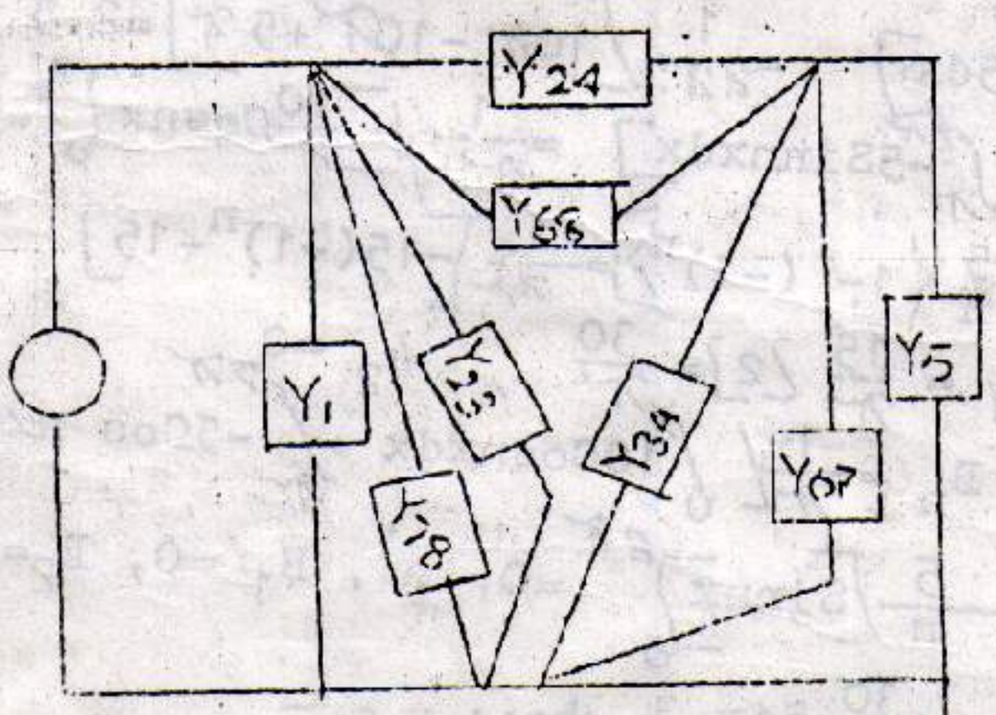
$$=\frac{Z_2 \cdot Z_4}{Z_2+Z_3+Z_4}+\frac{Z_6 \cdot Z_8}{Z_6+Z_7+Z_8}=0 \text{ Ans.}$$

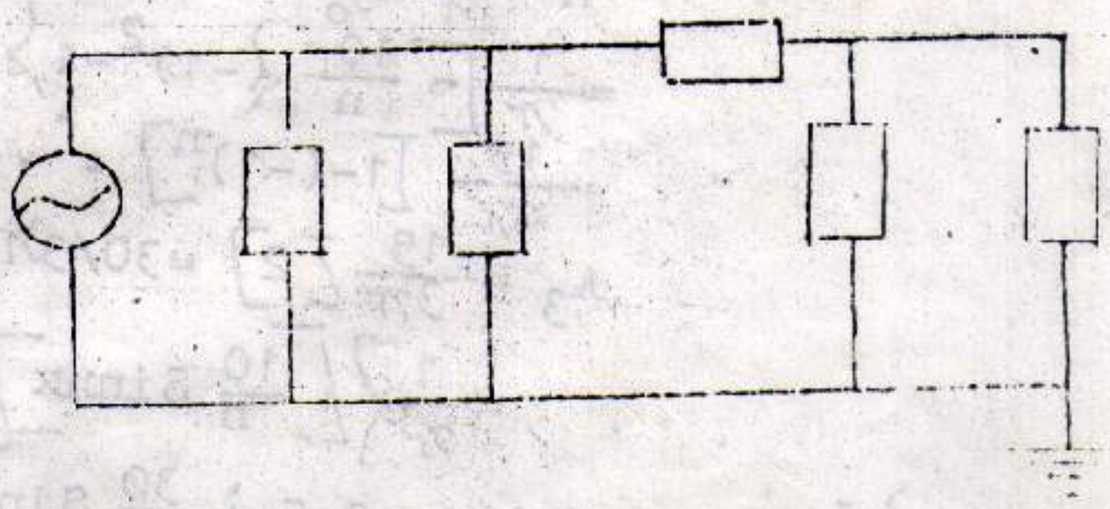
80.

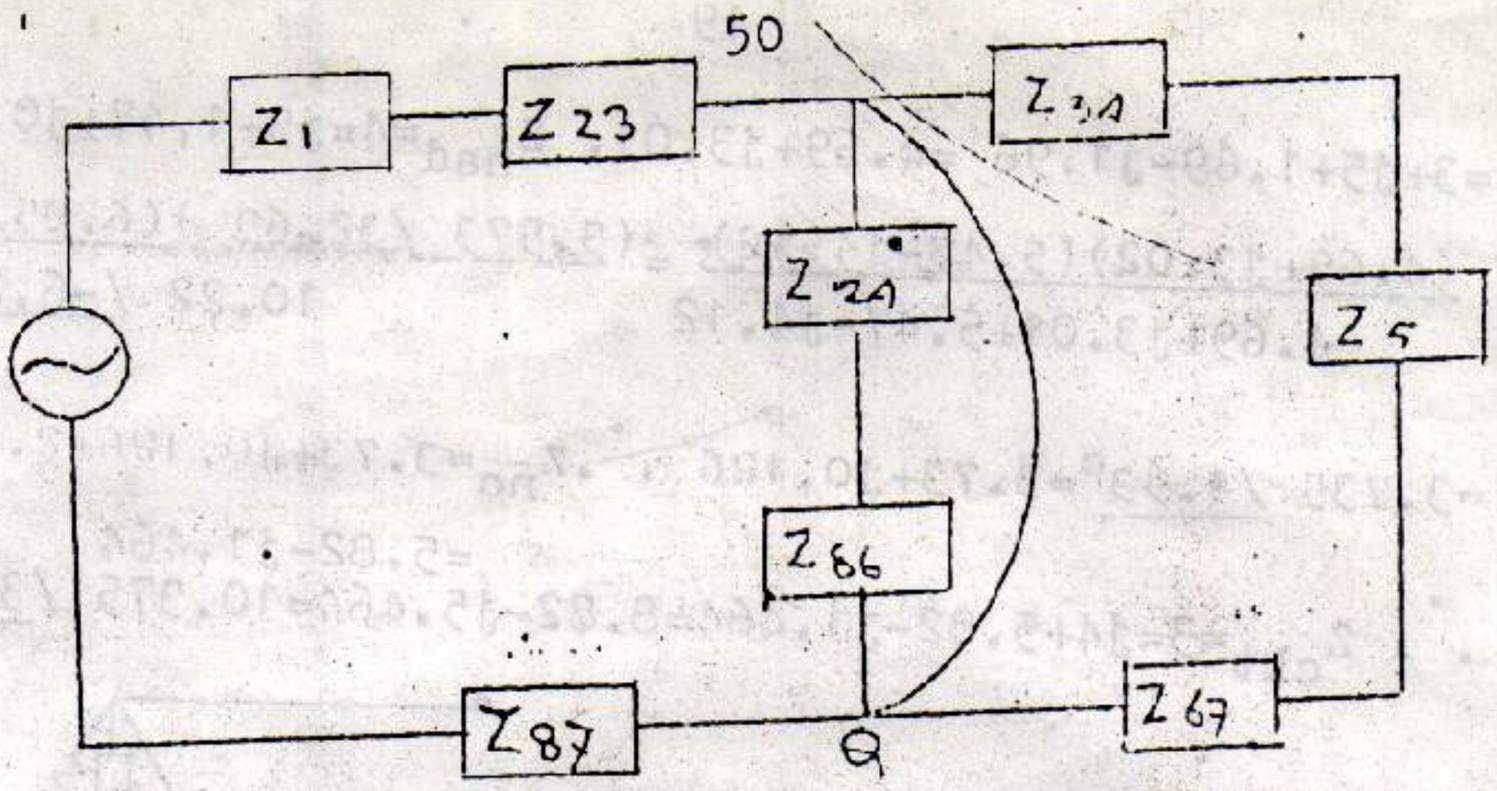


There are two wayes.

$$\frac{Y_2 \cdot Y_3 - Y_4}{Y_6 - Y_7 - Y_8}$$







The current $I_3 = 0$ i.e. the current through Z_5 is zero, if $Z_{PQ} = 0$ i.e. if points P and Q are short circuited.

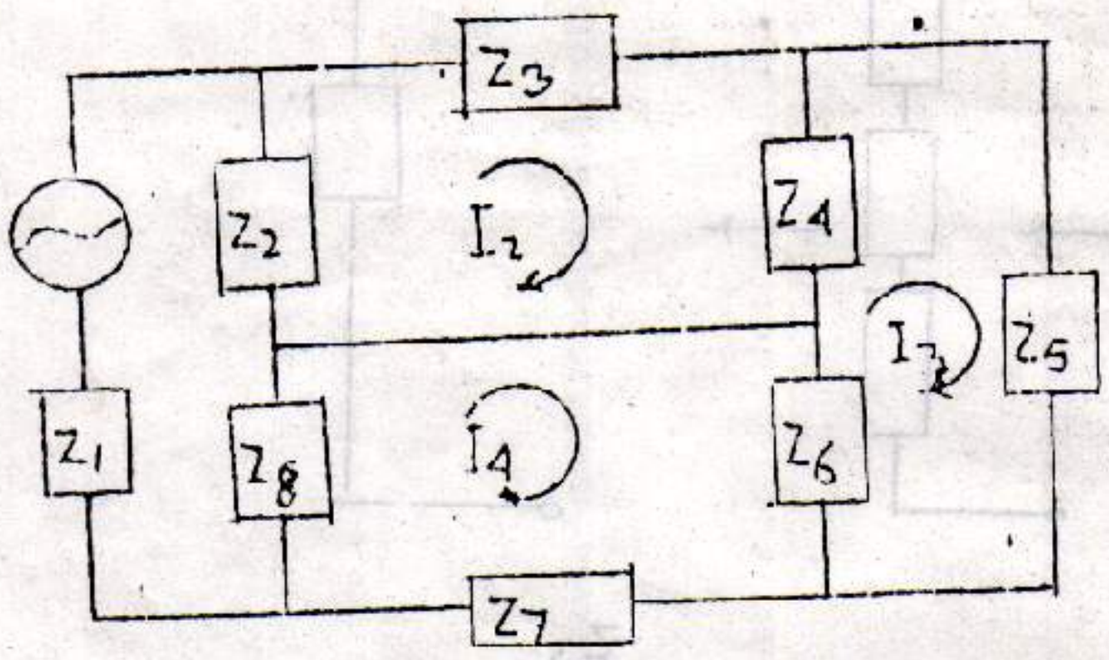
Hence $Z_{PQ} = 0 \Rightarrow Z_{24} + Z_{68} = 0$

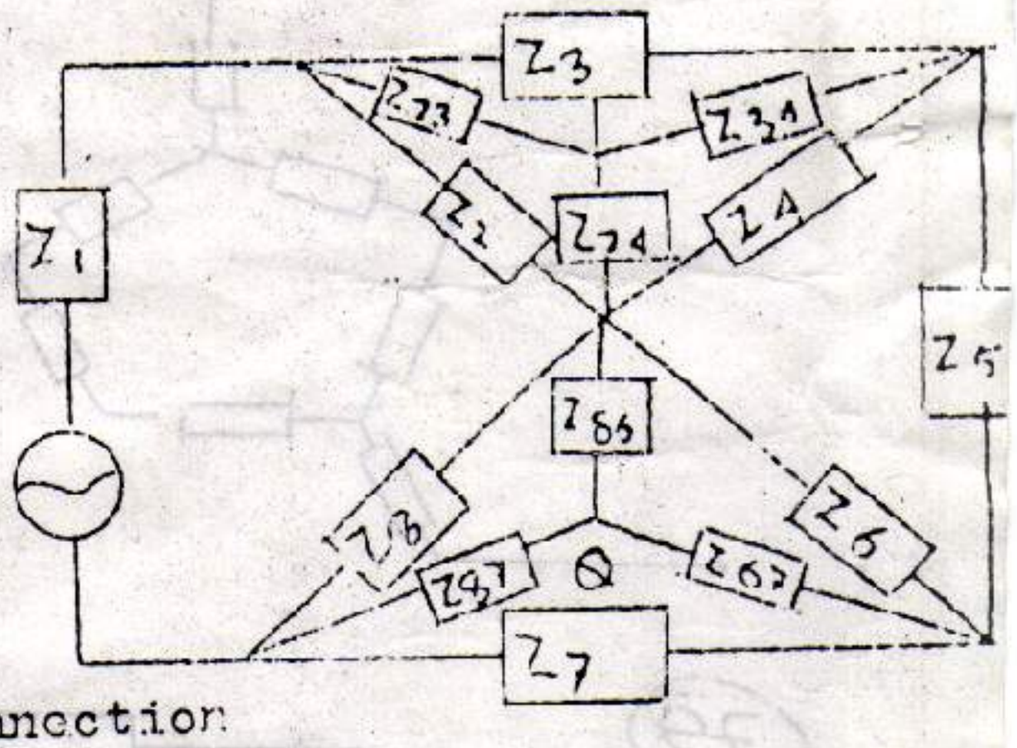
$$\Rightarrow \frac{Z_2 Z_4}{Z_2+Z_3+Z_4} + \frac{Z_6 Z_8}{Z_6+Z_7+Z_8} = 0$$

$$\Rightarrow Z_2Z_4(Z_6+Z_7+Z_8)+Z_6Z_8(Z_2+Z_3+Z_4)=0$$

OR

Y transformed to the fig becomes.





Z_1, Z_3, Z_4 are in delta connection

When transformed to star.

$$Z_{23} = \frac{Z_2 \cdot Z_3}{Z_2+Z_3+Z_4}, \quad Z_{34} = \frac{Z_3 \cdot Z_4}{Z_2+Z_3+Z_4}, \quad Z_{24} = \frac{Z_2 \cdot Z_4}{Z_2+Z_3+Z_4}$$

Z_6, Z_7, Z_8 are transformed to star connection.

$$Z_{67} = \frac{Z_6 \cdot Z_7}{Z_6+Z_7+Z_8}, \quad Z_{78} = \frac{Z_7 \cdot Z_8}{Z_7+Z_8+Z_6}, \quad Z_{86} = \frac{Z_8 \cdot Z_6}{Z_8+Z_6+Z_7}$$

(বাকী- ভাংক্ষ- ১১১- পাওয়া যায়নি)

$= \sqrt{2 + 0 + \frac{1}{2}\sin^{-1} 1 - \frac{1}{3}} = \sqrt{2 + 1.57 - \frac{1}{3}} = 1.86$ Amp:

18. Form Factor $= \frac{I_{r.m.s.}}{I_{av}} = \frac{6.93}{6} = 1.155$ Ans:

$I_{av} = \frac{1}{T}\int_0^T \frac{12}{T} \cdot t \cdot dt = \frac{1}{T} \cdot \frac{12}{T} \cdot \frac{T^2}{2} = 6$A Ans:

19. $e = 100 \sin\omega t + 60\sin(5\omega t + 30^\circ)$

$\therefore$ E r.m.s $= \frac{1}{T}\int_0^T \{100 \sin\omega t + 60\sin(5\omega t + 30^\circ)\}^2 dt$

$= \frac{1}{T}\int_0^T \left[\frac{100^2}{2}(1-\cos 2\omega t) + \frac{60^2}{2}\{1-\cos(10\omega t + 60^\circ)\} + \frac{100\times 60}{2}\cos\{(6\omega t + 30^\circ) - \cos(6\omega t + 30^\circ)\}\right]$

$= \frac{1}{T}\sqrt{(5000 + 1800)T - 5000\sin\frac{4\pi}{T} \cdot \frac{T}{4}\sin 0^\circ - 0^\circ}$ $= \sqrt{6800} = 82.46$Volts.

Ans:

(20), (21), (22) similar on(19)

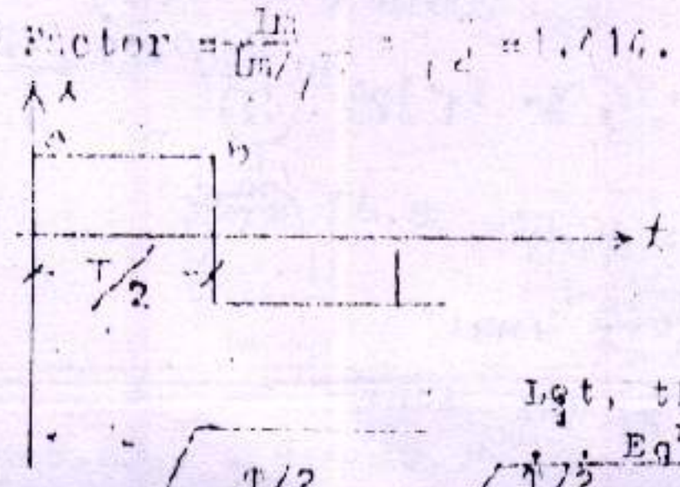
24. (a) let a sine wave, $i = I_m \sin\omega t$

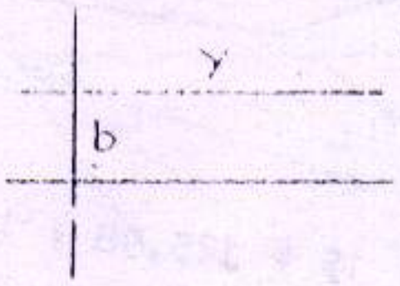
$\therefore$ I r.m.s. $= \sqrt{\frac{1}{T}\int_0^T I_m^2 \sin^2\omega t \, dt} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \times \frac{1}{2}(1-\cos 2\omega t)dt}$

$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{2}\left[T - \frac{1}{2\omega}\sin 2\omega t\right]} = \sqrt{\frac{1}{T} \times \frac{I_m^2}{2}\left[T - \frac{1}{2\omega}\sin 2 \cdot \frac{2\pi}{T} \cdot T\right]} = \frac{I_m}{\sqrt{2}}$

$\therefore$ Peak Factor $= \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414.$

(b)





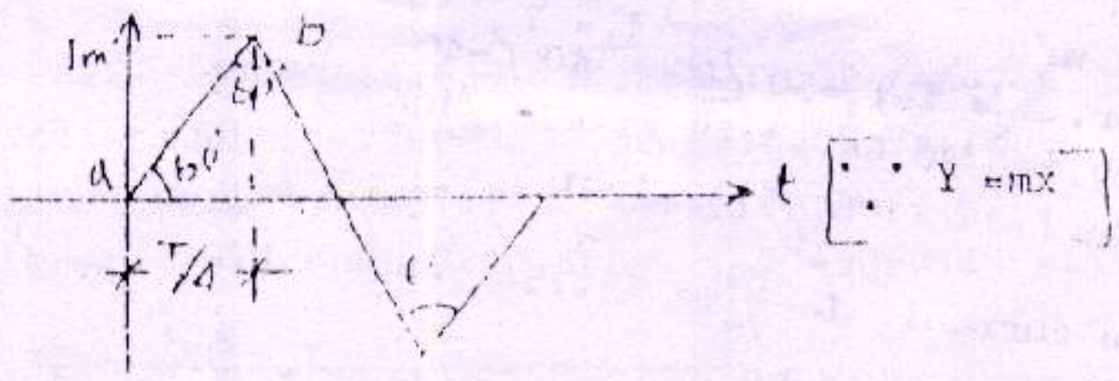
Let, the max^m current be Im

Eq^n of ab is, $i = I_m$

I r.m.s. $= \sqrt{\frac{2}{T}\int_0^{T/2} i^2 dt} = \sqrt{\frac{2}{T}\int_0^{T/2} I_m^2 \, dt} = \sqrt{\frac{2}{T} I_m^2 \cdot T/2} = I_m$

$\therefore$ Peak Factor $= \frac{I_m}{I_m} = 1$ Ans:

(c)



The eq^n of ab is, $i = \tan 60^\circ t$ or, $i = \sqrt{3}\, t$

Let, the max^m value be Im $\therefore$ I r.m.s. $= \sqrt{\frac{4}{T}\int_0^{T/4} 3t^2 dt}$

$= \sqrt{\frac{4}{T} \cdot 3 \cdot \left[\frac{t^3}{3}\right]_0^{T/4}} = \sqrt{\frac{6}{T} \times (\frac{T}{4})^3} = \frac{\sqrt{3}}{4}T$ again, $\tan 60^\circ = \frac{I_m}{T/4}$ or, $I_m = \sqrt{3} \cdot \frac{T}{4}$

$\therefore$ Peak Factor $= \frac{\sqrt{3}\cdot\frac{T}{4}}{\frac{\sqrt{3}}{4}T} = \sqrt{3} = 1.732$ Ans:

$Z''_{do} = Z''_{fg} = 4+j9-j2 = 4+j7 = 8.06 \angle 60.26^\circ$

$Z''_{bc} = \frac{4+j7}{2} = 2+j3.5 = 4.03 \angle 60.26^\circ$

$Z''_{ac} = 1.5+j6+2+j3.5 = 3.5+j9.5 = 10.124 \angle 69.8$

$V''' = \frac{50}{\sqrt{2}} = 35.355 \quad \therefore I''' = \frac{35.355 \angle 0^\circ}{10.24 \angle 69.8} = 3.49 \angle -69.8 = 1.2-j3.28$

$V'''_{bc} = I''' Z'''_{bc} = (3.49 \angle -69.8)(4.03 \angle 60.26) = 12.06 \angle -9.54^\circ$

$\therefore I'''_{do} = I'''_{fg} = \frac{12.06 \angle -9.54}{8.06 \angle 60.26} = 1.496 \angle -69.8^\circ$

$P''' = 35.355 \times 1.2 \cos 0 = 42.43 \angle -$

(a) $I = \sqrt{20^2+3.49^2} = 20.3$ amp.

(b) $I_{bc} = I_{fg} = \sqrt{10^2+1.496^2} = 10.15$

(c) $i_1 = \sqrt{2} \times 10 \sin(377t-8.13^\circ) - \sqrt{2} \times 1.496 \sin(377t+90^\circ-69.8^\circ)$

$= 14.14 \sin(377t-8.13) - 2.12 \sin(377t+20.2)$

(d) $P = P' + P''' = 1400+42.43 = 1442.43$ watts.

(e) $V = \sqrt{\frac{100^2+50^2}{2}} = 79.057$ volts.

$I = \sqrt{20^2+3.49^2} = 20.3$ amp.

$\therefore \cos\theta = \frac{P}{VI} = \frac{1442.43}{79.057 \times 20.3} = 0.89.87$ Ans.

CHAPTER VII

"COUPLED CIRCUITS"

P R O B L E M

1. Equivalent series impedance of the circuit $= Z_{ckt} = Z_1 + \frac{Z_2 Z_{12}}{Z_2+Z_{12}}$

$Z_2 Z_{12} = (4-j8)(10+j0) = 40-j80 = 89.44 \angle -63.43^\circ$

$Z_2+Z_{12} = 4-j8+10+j0 = 14-j8 = 16.12 \angle -29.74^\circ$

$\therefore \frac{Z_2 Z_{12}}{Z_2+Z_{12}} = \frac{89.44 \angle -63.43}{16.12 \angle -29.74^\circ} = 5.55 \angle -33.69^\circ = 4.62-j3.1$

$\therefore Z_{ckt} = 3+j4+4.62-j3.1 = 7.62+j0.9 = 7.67 \angle 6.74^\circ$ ohms.

$\therefore I_1 = \frac{E_1}{Z_{ckt}} = \frac{100 \angle 0^\circ}{7.67 \angle 6.74} = 13 \angle -6.64$ amp.

$I_{12} = \left(\frac{Z_2 Z_{22}}{Z_2+Z_{12}} \times I_1\right) / Z_{12} = \frac{Z_2}{Z_2 \times Z_{12}} \times I_1$

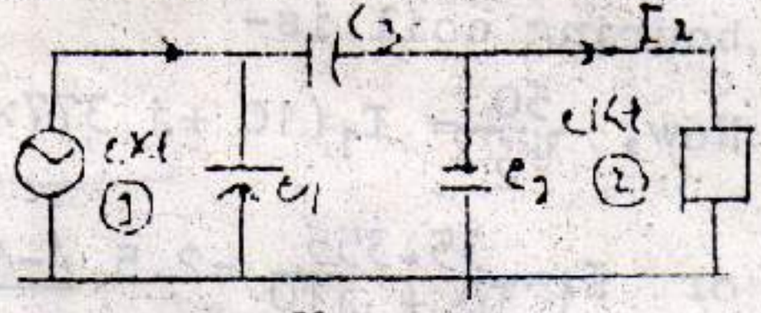
$= \frac{4-j8}{16.12 \angle -29.74} \times 13 \angle -6.74^\circ = \frac{8.94 \angle -63.43 \times 13.00 \angle -6.74^\circ}{16.12 \angle -29.74^\circ}$

$= 7.21 \angle -40$ amp.

$$I_2 = \frac{Z_{12}}{Z_2 + Z_{12}} \times I_1 = \frac{10\ \angle 0^\circ \times 13 \angle -6.74}{16.12\ \angle -29.74} = 8.06\ \angle 23^\circ \text{ amp.}$$

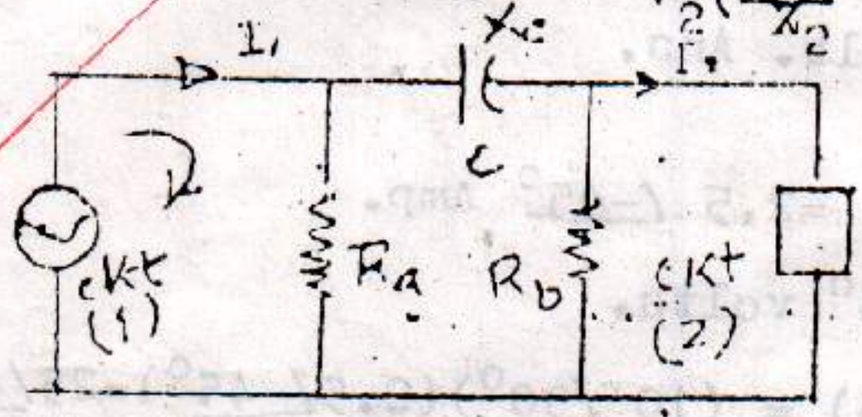
$\therefore P_{gen} = E_1 I_1 \cos\theta = 100 \times 13 \cos(-6.74) = 1290$ watts.amp.

2. Let the voltage drop across C_1 is V_1. . Voltage developed across condenser C_1 per unit current is-



$$X_{coupling} = \frac{V_1}{I_2} = \frac{\frac{V_2}{X_1 + X_3} \cdot X_1}{V_2\left(\frac{1}{X_2} + \frac{1}{X_1 + X_3}\right)} = \frac{\frac{X_1}{X_1 + X_3}}{\frac{X_1 + X_3 + X_2}{X_2(X_1 + X_3)}} = \frac{X_1 X_2}{X_1 + X_2 + X_3} \text{ Ans.}$$

3.



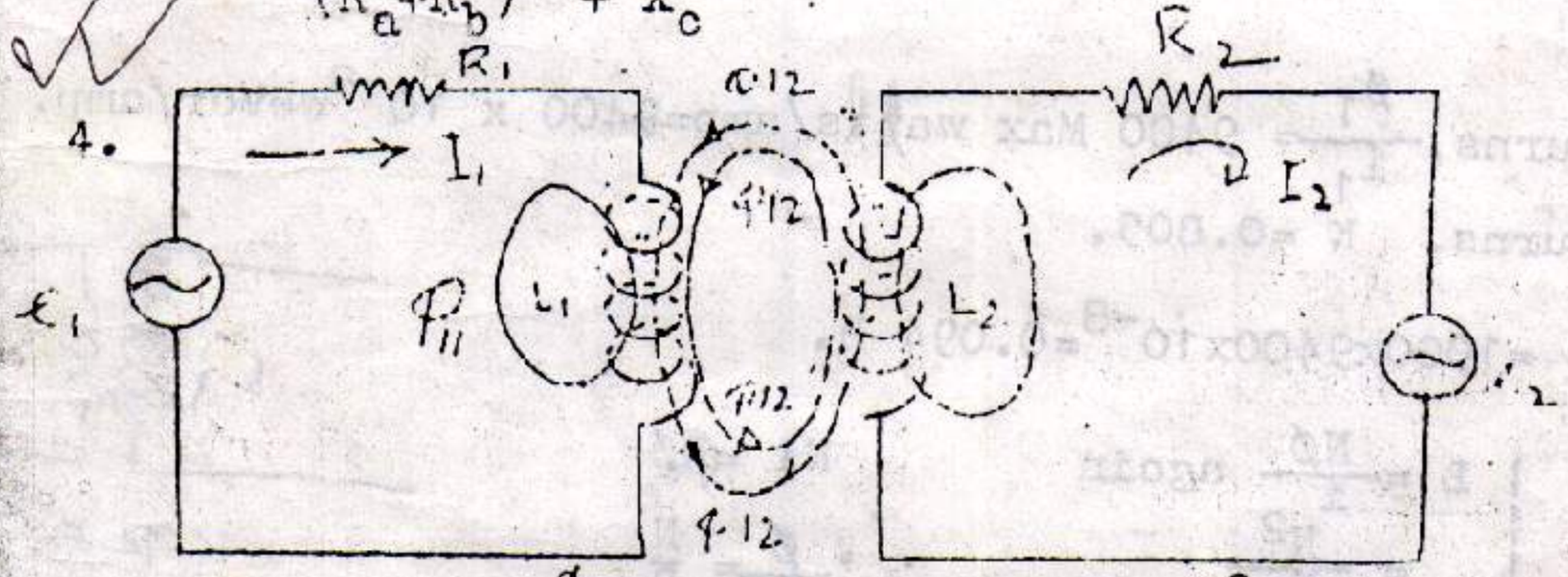
When $Z_{coupling}$ is measured then ckt-2 is to be considered as open circuit.

If V_b be the voltage developed across R_b then the coupling impedence between the two circuits is-

$$Z_{coupling} = \frac{V_b}{I_1} = \frac{\frac{V_a}{R_b - jX_c} \times R_b}{V_a\left(\frac{1}{R_a} + \frac{1}{R_b - jX_c}\right)} = \frac{\frac{R_b}{R_b - jX_c}}{\frac{R_b + R_a - jXc}{R_a(R_b - jX_c)}} = \frac{R_a R_b}{R_a + R_b - jX_c}$$

$$= \frac{R_a^2 R_b + R_a R_b^2 + jR_a R_b Xc}{(R_a + R_b)^2 + X_c^2} \text{ Ans.}$$

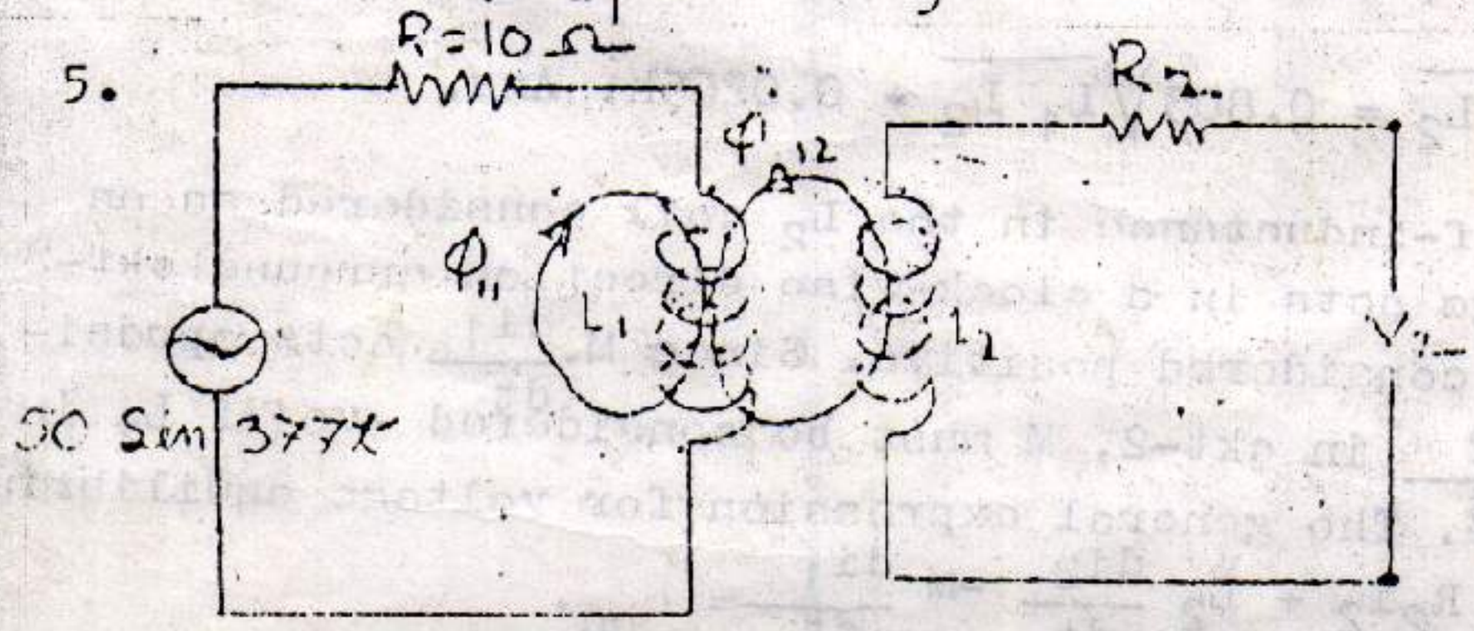
4.



(a) $M_{12} = N_2 \frac{\phi_{12}}{I_1} = \frac{500 \times 27500 \times 10^{-8}}{5} = 0.0275h = 27.5$ mh.

(b) $L_1 = N_1 \frac{\phi_1}{I_1} = \frac{50 \times 30{,}000 \times 10^{-8}}{5} = 3$mh. Ans.

5.



$v = 50\sin 3.77t$ $\therefore \omega = 377$ rad/sec.

$\therefore X_M = \omega M = 377 \times 0.02 = 7.54$ ohm. Ans.

The voltage across the open circuited terminals of neighbouring coil is-

Now, $\frac{50}{\sqrt{2}} = I_1(10 + j\ 377 \times \frac{1}{37.7}) + 0$ ($\rightarrow j\omega L$)

or $I_1 = \frac{35.355}{10 + j10} = 2.5\ \angle -45^\circ$

Now, $V_2 = I_2(j\omega L_2 + R_2) + I_1(j\omega M) = 0 + (2.5\angle -45^\circ)(j377 \times 0.02)$

$= 2.5\ \angle -45^\circ\ (7.54\ \angle 90^\circ) = 18.85\ \angle 45^\circ$

$\therefore$ The value of V_2 is 18.85 volt. Ans.

6. $V_1 = \frac{50}{\sqrt{2}} = 35.355\ \angle 0^\circ$ volts. $I_1 = 2.5\ \angle -45^\circ$ Amp.

$I_1R_1 = (2.5\ \angle -45^\circ)\ (10) = 25\ \angle -45^\circ$ volts.

$jX_{L1}I_1 = (j\ 377 \times \frac{1}{37.7})\ (2.5\ \angle -45) = (10\ \angle 90^\circ)(2.5\angle -45^\circ) = 25\angle 45^\circ$ volts.

$jX_MI_1 = (j\ 377 \times 0.02)(2.5\ \angle -45^\circ) = (7.54\angle 90^\circ)(2.5\ \angle -45^\circ) = 18.84\angle 45^\circ$

$E_{21} = -V_2 = 18.85\angle 45^\circ = -13.329 - j13.329 = 18.85\angle -135^\circ$ volts.

Now draw vector diagram by yourself.

7. $I_1 = 0.094$ h

$L_2 = 0.0108$ h

$K_m = 0.805$

$\therefore M = K\sqrt{L_1L_2}$

$= 0.0256$ h. Ans.

8. $N_1 = 1000$ turns, $\frac{\phi_1}{i_1} = 9400$ Max watts/amp $= 9400 \times 10^{-8}$ weber/amp.

$N_2 = 338$ turns. $K = 0.805$.

$L_1 = N_1\frac{\phi_1}{i_1} = 1000 \times 9400 \times 10^{-8} = 0.094$ h.

$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$ | $L = \frac{N\phi}{i}$ again $= \frac{N^2}{R}$ $\quad Ni = R\phi$ $\quad \therefore \frac{\phi}{i} = \frac{N}{R}$

$\therefore L_2 = L_1 \times \frac{N_2^2}{N_1^2} = 0.094 \times \frac{338^2}{1000^2} = 0.0107$ h Ans.

$\therefore M = K\sqrt{L_1L_2} = 0.805\sqrt{L_1\ L_2} = 0.0255$h. Ans.

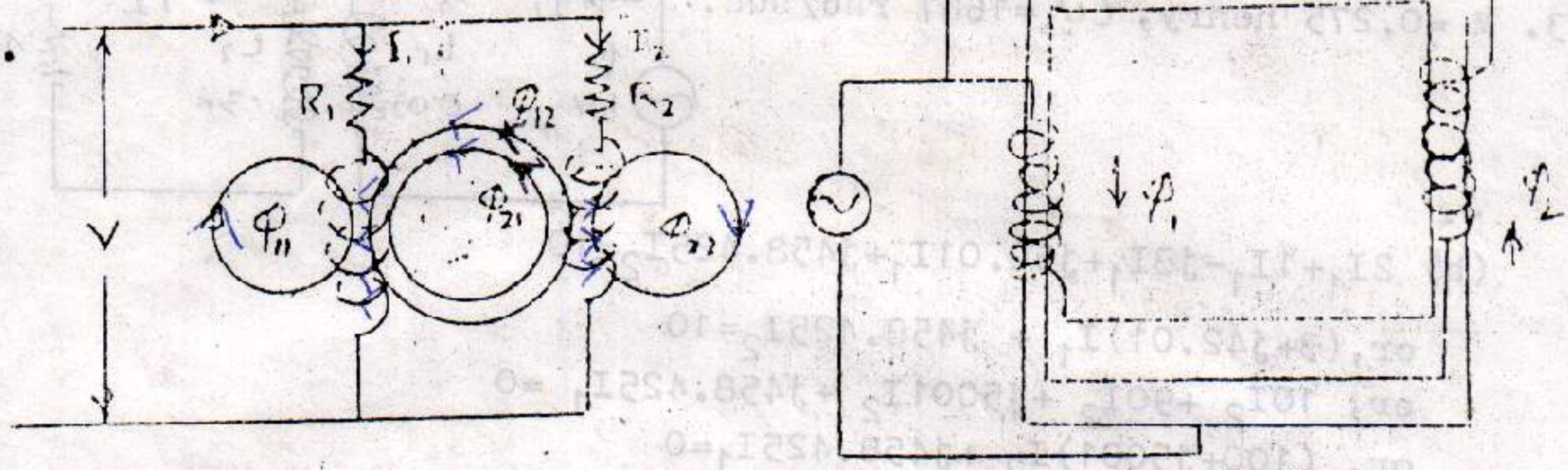
9. Voltage of self-inductance in the L_2 coil considered as an induced voltage acts in a clock-wise direction around ckt-2 when $\frac{di_2}{dt}$ is considered positive. Since $M.\frac{di_1}{dt}$ acts oppositely to $L_2\frac{di_2}{dt}$ in ckt-2, M must be considered -ve if L_2 is considered +ve. The general expression for voltage equilibrium in ckt-2 is $-R_2i_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = e_{ba}$.

10. Here, $M = -3mh$, $Z_e = (R_a+R_b) + j(L_a+L_b-2M)$

$= (1+6)+j(1000)(0.004+0.009-0.006) = 7-j7 = 9.90 \angle -45^\circ$

$$I = \frac{V}{Z_e} = \frac{40.5}{9.9} = 4.09 \text{ amp.}$$

11.



The equations are- $I_1 Z_1 + I_2 Z_M = V$, $I_1 Z_M + I_2 Z_2 = V$

$$\therefore I_1 = \frac{\begin{vmatrix} V & Z_M \\ V & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_2-Z_M)}{Z_1Z_2 - Z_M^2}, \quad I_2 = \frac{\begin{vmatrix} Z_1 & V \\ Z_M & V \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_1-Z_M)}{Z_1Z_2-Z_M^2}$$

$$I = I_1+I_2 = \frac{V(Z_1+Z_2-2Z_M)}{Z_1Z_2 - ZM^2} \quad \text{Now, } Z_e = \frac{V}{I} = \frac{Z_1Z_2 - ZM^2}{Z_1+Z_2-2ZM}$$

$$= \frac{(35.5 \angle 84.7^\circ)(4.17 \angle 79.29^\circ) - (9.65 \angle 90^\circ)^2}{3.3+j35.4 + 0.775+j4.07 - j19.30}$$

$$= \frac{-142.29 + j40.83 + 93.12}{20.978 \angle 78.58^\circ} = \frac{63.9 \angle 140.3}{20.978 \angle 78.58^\circ} = 3.1 \angle 61.7^\circ \text{ Ans.}$$

$I = V/Z_e = 50\angle 0/3.1\angle 61.7 = 16.13\angle -61.7$ $\quad I_1 = \frac{(50\angle 0)(0.775+j4.07-j9.65)}{63.9\angle 140.3} = 4.11$

$I_2 = V(Z_1-Z_M)/Z_1Z_2-Z_M^2 = (50\angle 0)(3.3+j35.4-j9.65)/63.9\angle 140.3$ $\quad 1-2.2$

12. $R_1 = 3.3$ ohms $\quad M = 0.0256$ henry $\quad P = VI\cos\theta = 50 \times 16.13 \cos 61.7 = 382.17$ W

$L_1 = 0.094$ henry, $Z = 28.15+j0$

$R_2 = 0.775$ ohms, $\omega = 377$ rad/sec.

$L_2 = 0.0108$ henry, $V_1 = 50 \angle 0^\circ$ volts.



(a) $Z_{e_1} = Z_1 - \frac{Z_M^2}{Z_2+Z}$, $Z_2+Z = 0.775+j(377)(0.0108)+28.15$

$= 28.925 + j4.07 = 29.21 \angle 8^\circ$ ohms.

$Z_{e_1} = 3.3+j35.4+\frac{(9.65)^2\angle 0^\circ}{29.21 \angle 8^\circ} = 3.3+j35.4+3.16-j0.444$

$= 6.46+j34.96 = 35.55 \angle 79.53^\circ$ ohms.

$$I_1 = \frac{50 \angle 0^\circ}{35.55 \angle 79.53^\circ} = 1.407 \angle -79.53^\circ \text{ amp.}$$

$$I_2 = \frac{-I_1 Z_M}{(Z_2+Z)} = \frac{(1.407 \angle -79.53+180)(9.65 \angle 90^\circ)}{29.21 \angle 8^\circ} = 0.465 \angle 182.47^\circ \text{ amp.}$$

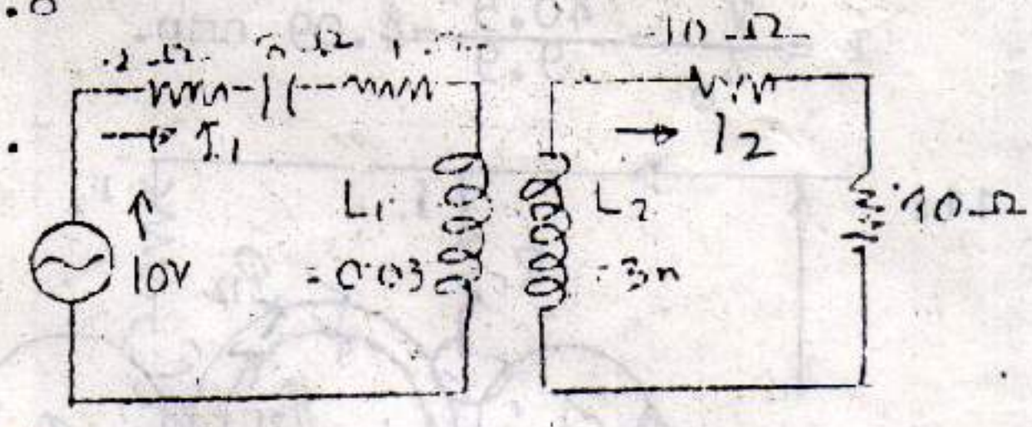
(b) $P_{in} = V_1 I_1 \cos\theta = 50 \times 1.407 \times \cos 79.53 = 12.8$ watts.

$P_{out} = V_2 I_2 \cos\theta$ $\quad V_2 = ZI_2 = 28.15 \times 0.465 \angle 182.47^\circ = 13.09 \angle 182.47^\circ$ V.

$\therefore P_{out} = 13.09 \times 0.465 \times \cos 182.47 = 6.08$ watts.

$\therefore$ Efficiency of operation $= \frac{6.08}{12.8} = 0.475$ i.e. 47.5%

13. M = 0.275 henry, ω = 1667 rad/sec.



(b) $2I_1 + 1I_1 - j8I_1 + j50.01I_1 + j458.425I_2 = 10$

or, $(3 + j42.01)I_1 + j458.425I_2 = 10$

or, $10I_2 + 90I_2 + j5001I_2 + j458.425I_1 = 0$

or, $(100 + j5001)I_2 + j458.425I_1 = 0$

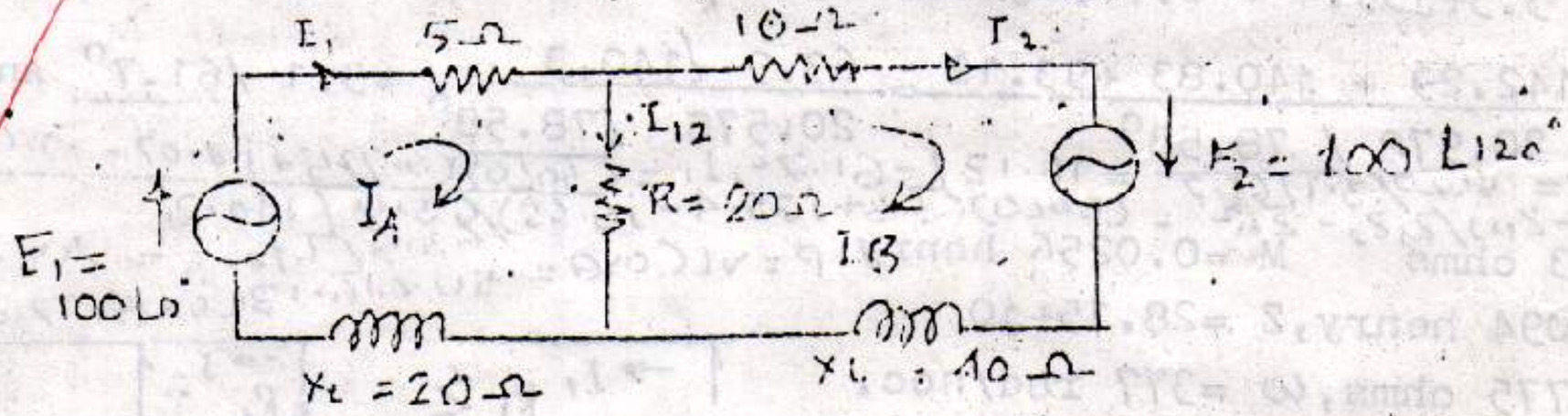
or, $j458.425I_1 + (100 + j5001)I_2 = 0$

$$I_1 = \frac{\begin{vmatrix} 10 & j458.425 \\ 0 & 100+j5001 \end{vmatrix}}{\begin{vmatrix} 3+j4201 & j458.425 \\ j458.425 & 100+j5001 \end{vmatrix}} = \frac{(5002\ \underline{/88.85^\circ})10-0}{(42.12\underline{/85.92})(5002\underline{/88}.85^\circ)+(850.425)^2}$$

$$= \frac{50020\ /88.85^\circ}{356.44+j19314.57} = \frac{50020\ \underline{/88.85}}{19317.85\ \underline{/88.94}^\circ} = 2.589\ \underline{/-0.09}^\circ$$

$\therefore I_1 = 2.589\ \underline{/-0}.09^\circ$, $I_2 = \frac{j458.425 \times 10}{19317.85\ \underline{/88.94}^\circ}$

18.



Now, $E_1 = Z_{11} I_A - Z_{12} I_B$, $E_2 = -Z_{21} I_A + Z_{22} I_B$

$$\therefore I_A = \frac{\begin{vmatrix} E_1 & -Z_{12} \\ E_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & Z_{22} \end{vmatrix}} = \frac{E_1 Z_{22} + Z_2 Z_{12}}{Z_{11} Z_{22} - Z_{12}^2} \quad (\because Z_{12} = Z_{21})$$

$$I_B = \frac{\begin{vmatrix} Z_{11} & E_1 \\ Z_{21} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & +Z_{22} \end{vmatrix}} = \frac{E_1 Z_{22} + E_2 Z_{12}}{Z_{11} Z_{22} - Z_{12}^2}$$

Where- $Z_{11} = 5 + 20 + j20 = 25 + j20 = 32.016\ \underline{/38}.66$

$Z_{22} = 10 + 20 + j40 = 30 + j40 = 50\ \underline{/50}.13^\circ$, $Z_{12} = 20$ ohm.

$$\therefore I_A = \frac{(100\ \angle 0^\circ)(50\ \angle 53.13) + (100\ \angle 120^\circ)(20)}{(32.016\ \angle 38.66)(50\ \angle 53.13) - 20^2}$$

$$= \frac{3000 + j4000 - 1000 + j1732.05}{-50 + j1600 - 400} = \frac{6070.947\ \angle 70.765}{1662.077\ \angle 105.7} = 3.653\ \angle -34.94^\circ$$

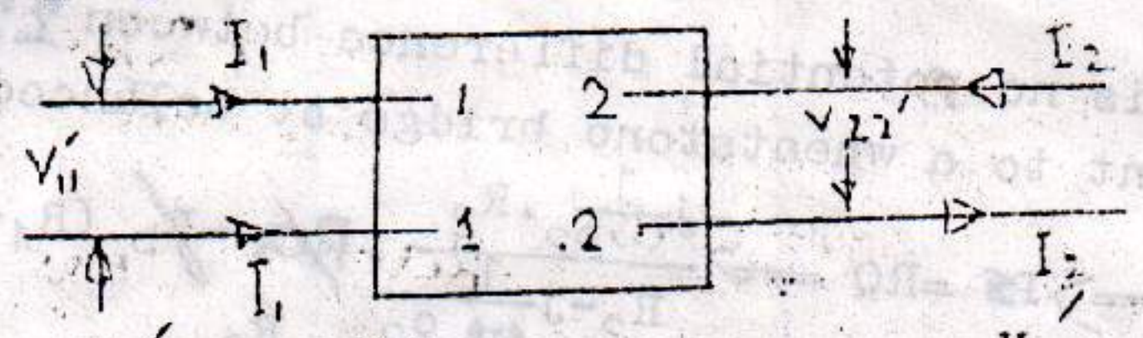
$$I_B = \frac{(32.016\ \angle 38.66)(100\ \angle 120^\circ) + (100\ \angle 0)(20)}{1662.077\ \angle 105.70}$$

$$= \frac{-2982.09 + j1165.067 + 2000}{1662.077\ \angle 105.7} = \frac{1523.772\ \angle 130.13}{1662.077\ \angle 105.7} = 0.917\ \angle 24.43^\circ$$

Now, $I_1 = I_A = 3.653\ \angle -34.94^\circ A$, $I_2 = I_B = 0.917\ \angle 24.43^\circ A$

$I_{12} = I_A - I_B = 2.995 - j2.092 - 0.335 - jj0.379 = 3.28\ \angle -48.84$ Ans.

10.

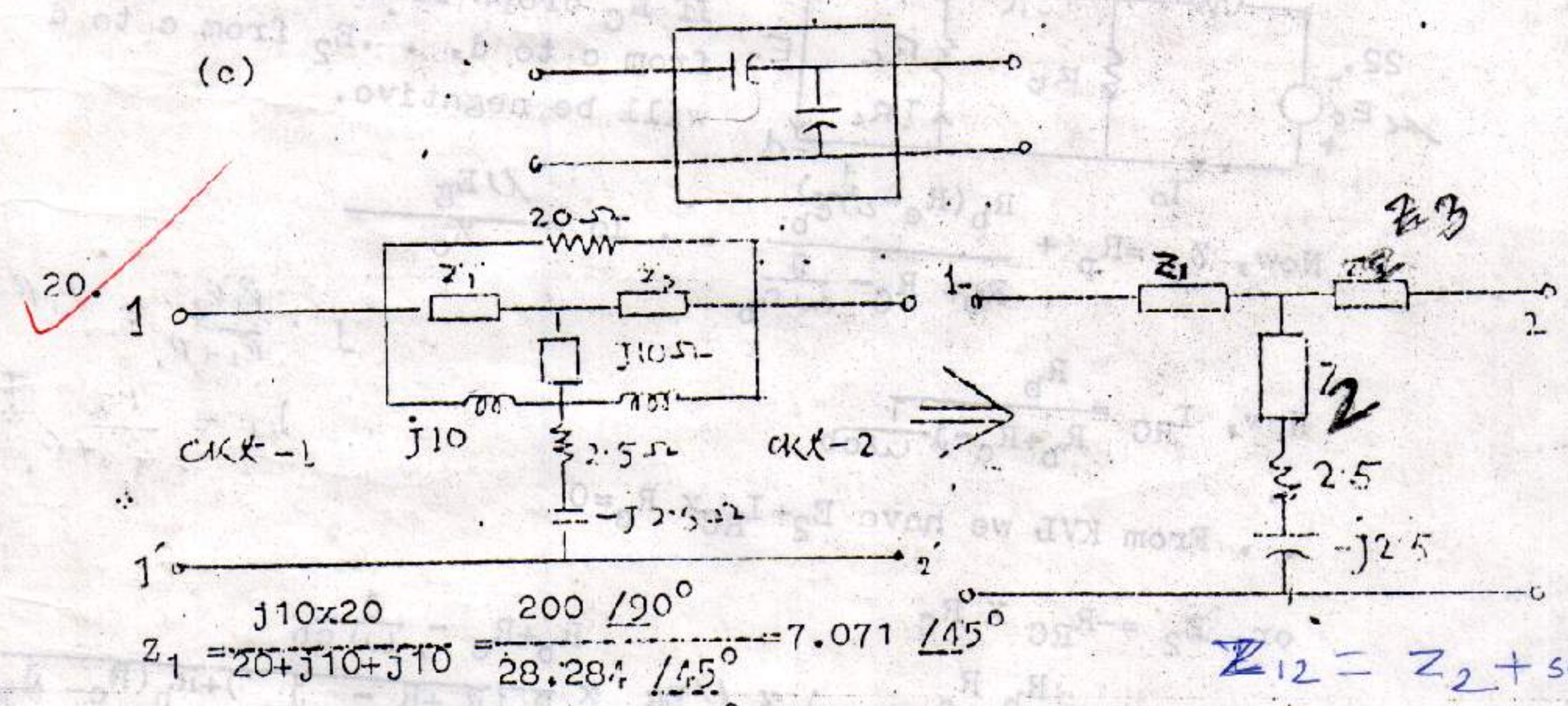


(d) $Z_{21} = \frac{V_{22}'}{I_1} = \frac{4\ \angle 10}{1\ \angle 90^\circ} = 4\ \angle -90^\circ$, $Z_{12} = \frac{V_{11}'}{I_2} = \frac{6\ \angle 0^\circ}{1.5\ \angle 90^\circ} = 4\ \angle -90^\circ$

(b) $K = \frac{Z_{12}}{\sqrt{Z_{11}'\ Z_{22}'}}$, Where, $Z_{11}' = \frac{E_1}{I_1} = \frac{6\ \angle 0}{1\ \angle 90^\circ} = 6\ \angle -90^\circ$, $Z_{22}' = \frac{E_2}{I_2} = \frac{6\ \angle 0}{1.5\ \angle 90^\circ}$
$= 4\ \angle -90^\circ$

$$\therefore K = \frac{4\ \angle -90^\circ}{\sqrt{(6\ \angle -90^\circ)(4\ \angle -90^\circ)}} = \frac{4\ \angle -90^\circ}{\sqrt{24\ \angle -90^\circ}} = 0.816$$

(c)



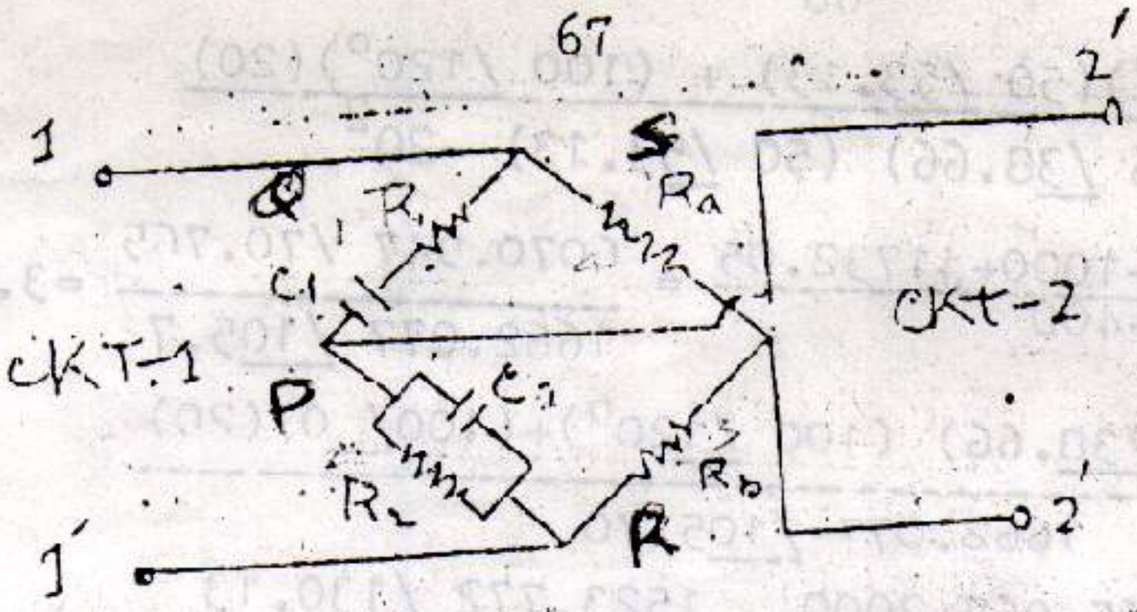
$$Z_1 = \frac{j10 \times 20}{20 + j10 + j10} = \frac{200\ \angle 90^\circ}{28.284\ \angle 45^\circ} = 7.071\ \angle 45^\circ$$

$$Z_2 = \frac{j10 \times j10}{28.284\ \angle 45^\circ} = \frac{100\ \angle 180^\circ}{28.284\ \angle 45^\circ} = 3.536\ \angle 135^\circ = -2.5 + j2.5$$

$$Z_3 = \frac{j10 \times 20}{28.284\ \angle 45^\circ} = 7.071\ \angle 45^\circ$$, Now, $Z_{12} = Z_2 + 2.5 - j2.5 = 2.5 + j2.5 + 2.5 - j2.5 = 0$

Now, $K = \frac{Z_{12}}{\sqrt{Z_{11}'\ Z_{22}'}} = 0$ Ans.

21.



Let us assume that $K = 0$, $\therefore K = \frac{Z_{12}}{\sqrt{Z_{11} Z_{22}}} = 0 \Rightarrow Z_{12} = 0$

$$\Rightarrow \frac{V_{22}}{I_1} = 0 \Rightarrow V_{22} = 0$$

i.e. there is no potential difference between 2'. The ckt. is equivalent to a wheatstone bridge at balanced condition.

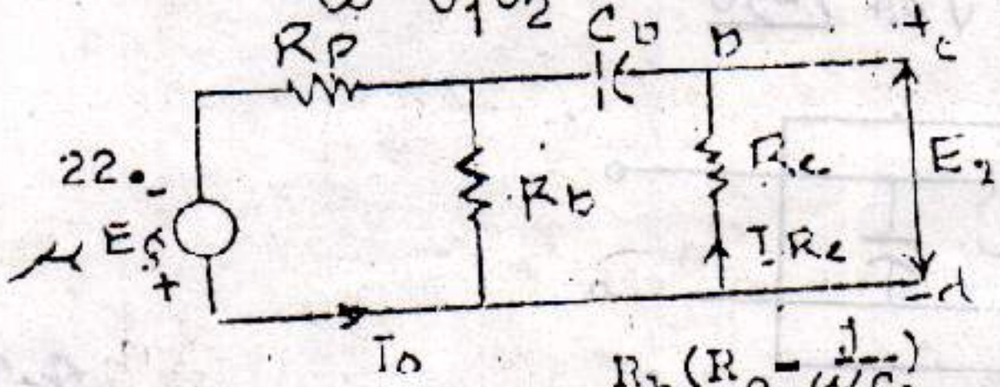
$\therefore \frac{P}{Q} = \frac{R}{S} \Rightarrow PS = RQ \Rightarrow \frac{-j\frac{1}{\omega c_2} \cdot R_2}{R_2 - j\frac{1}{\omega c_2}} \cdot R_a = R_b \left(R_1 - j\frac{1}{\omega c_1}\right) \left(\because R_a = R_b\right)$

or, $R_1 R_2 - \frac{1}{\omega^2 c_1 c_2} - j\frac{R_2}{\omega c_1} - j\frac{R_2}{\omega c_2} = -j\frac{R_2}{\omega c_2}$

or $R_1 R_2 = \frac{1}{\omega^2 c_1 c_2} - j\frac{2R_1}{\omega c_1} - j\frac{2R_1}{\omega c_1} = -j\frac{2R_1 \times 2}{\omega c_1}$

or $\frac{1}{\omega^2 c_1 c_2} = R_1 R_2 \quad \therefore \omega = \frac{1}{\sqrt{R_1 R_2 c_1 c_2}}$ proved.

22.



If R_c drops from a to b V_2 drops from c to d. $\therefore E_2$ from c to d will be negative.

Now, $Z_o = R_p + \frac{R_b\left(R_c - \frac{j}{\omega c_b}\right)}{R_b + R_c - \frac{j}{\omega c_b}}$ $\therefore I_o = \frac{\mu E_g}{Z_o}$

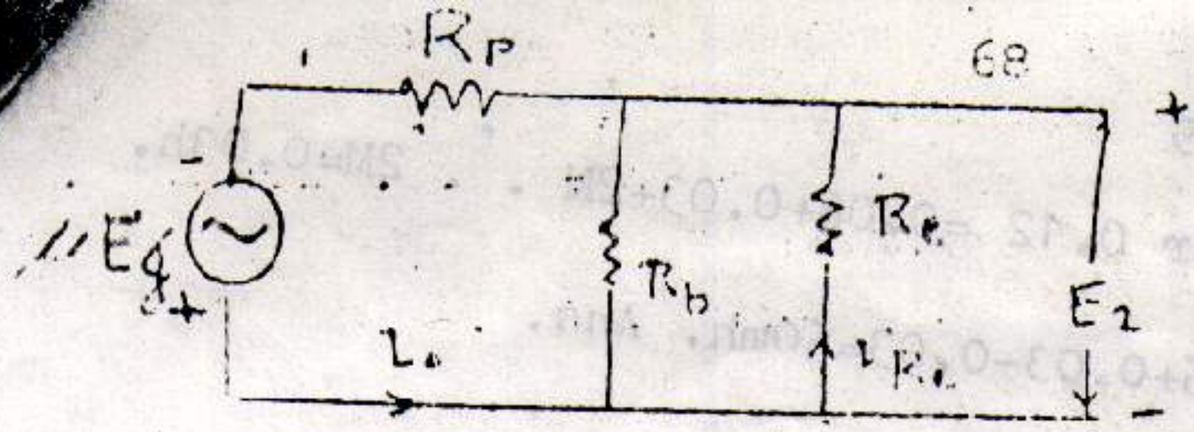
Now, $I_{RC} = \frac{R_b}{R_b + R_c - j\frac{1}{\omega c_b}}$

$\therefore$ From KVL we have $E_2 + I_{RC} \times R_c = 0$

or, $E_2 = -R_{RC} \times R_c$

or, $E_2 = -\left(\frac{R_b R_c}{R_b + R_c \frac{j}{\omega c_b}}\right) \times \frac{\mu E_g}{R_p\left(R_b + R_c - \frac{j}{\omega c_b}\right) + R_b\left(R_c - \frac{j}{\omega c_b}\right)} \times R_b + R_c - \frac{j}{\omega c_b}$

$= -\frac{\mu E_g R_b R_c}{R_p R_b + R_p R_c + R_b R_c - j\frac{R_p + R_b}{\omega c_b}}$ proved.

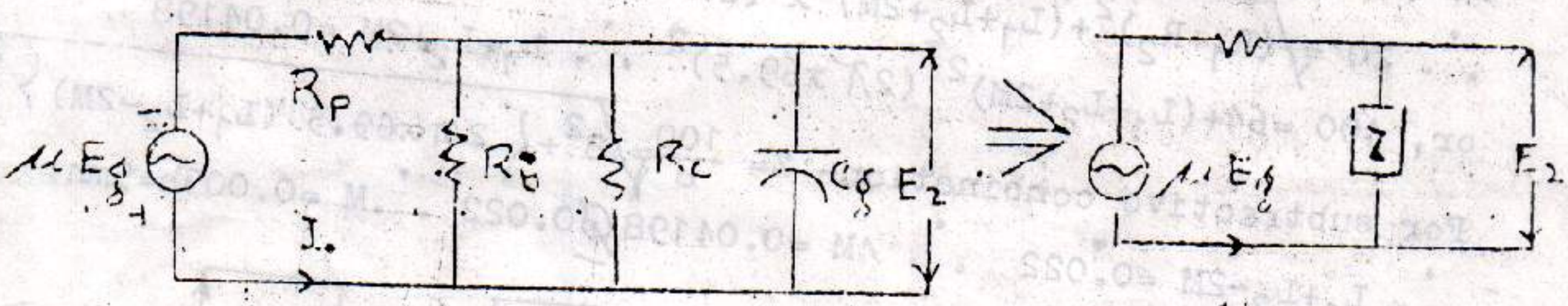


$$Z_o = R_p + \frac{R_b R_c}{R_b + R_c} = \frac{R_p(R_b+R_c) + R_b R_c}{R_b + R_c} \quad \therefore I_o = \frac{\mu E_g}{Z_o}$$

$$\therefore I_{RC} = \frac{R_b}{R_b+R_c} I_o, \text{ Now from KVL we have, } I_{RC} + E_2 = 0$$

$$\text{or, } E_2 = -R_c \frac{R_b}{R_b + R_c} \cdot \mu E_g \times \frac{R_b + R_c}{R_p(R_b+R_c)+R_b R_c} = -\frac{\mu E_g R_b R_c}{R_p(R_b+R_c)+R_b R_c}$$

$$= -\frac{\mu E_g}{R_p \frac{R_b + R_c}{R_b R_c} + 1} \quad \text{proved.}$$



$$\frac{1}{Z} = \frac{1}{R_b} + \frac{1}{R_c} + \frac{1}{-j\frac{1}{\omega C_g}} \quad \therefore Z_o = R_p + \frac{R_b R_c (-j\frac{1}{\omega C_g})}{R_b R_c + R_c(-j\frac{1}{\omega C_g}) + R_b(-j\frac{1}{\omega C_g})}$$

$$= R_p + Z \quad \therefore I_o = \frac{\mu E_g}{Z_o} = \frac{\mu E_g}{R_p + Z}. \quad \text{From KVL we have-}$$

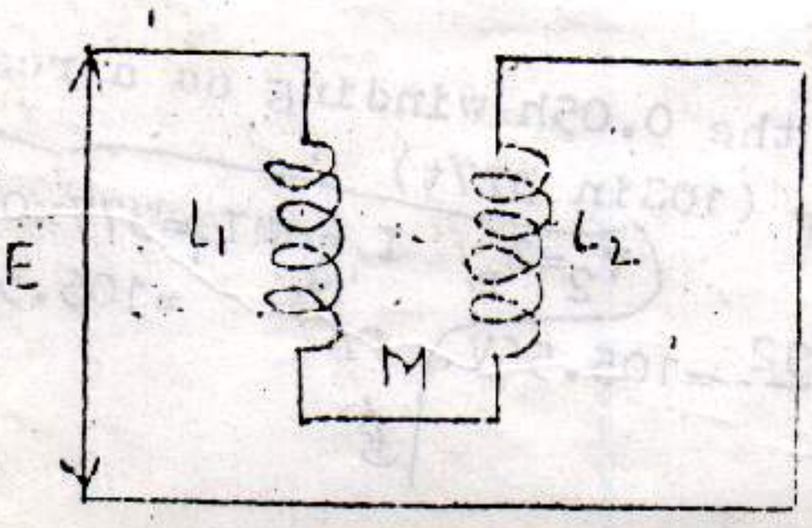
$$E_2 = -Z I_o = -\frac{R_b R_c (-j\frac{1}{\omega C_g})}{R_b R_c + R_b(-j\frac{1}{\omega C_g}) + R_c(-j\frac{1}{\omega C_g})} \times \frac{\mu E_g}{R_p + \frac{R_b R_c (-j\frac{1}{\omega C_g})}{R_b R_c + R_b(-j\frac{1}{\omega C_g}) + R_c}}$$

$$= -\frac{\mu E_g R_b R_c (-j\frac{1}{\omega C_g})}{R_p\left(R_b R_c + R_b(-j\frac{1}{\omega C_g}) + R_c(-j\frac{1}{\omega C_g})\right) + R_b R_c(-j\frac{1}{\omega C_g})} \qquad (j = \frac{1}{\omega C_g})$$

$$= -\frac{\mu E_g R_b R_c (-j\frac{1}{\omega C_g})}{R_p R_b R_c + R_p(-j\frac{R_b + R_c}{\omega C_g}) + R_b R_c(-j\frac{1}{\omega C_g})}$$

$$= -\frac{\mu E_g}{\frac{R_p}{-j\frac{1}{\omega C_g}} + R_p\frac{R_b + R_c}{R_b R_c} + 1} = -\frac{\mu E_g}{jR_p\omega C_g + R_p\frac{R_b+R_c}{R_b+R_c} + 1} \quad \text{Ans.}$$

23.



$L_1 = 60 \text{ mh} = 0.06\text{h}.$

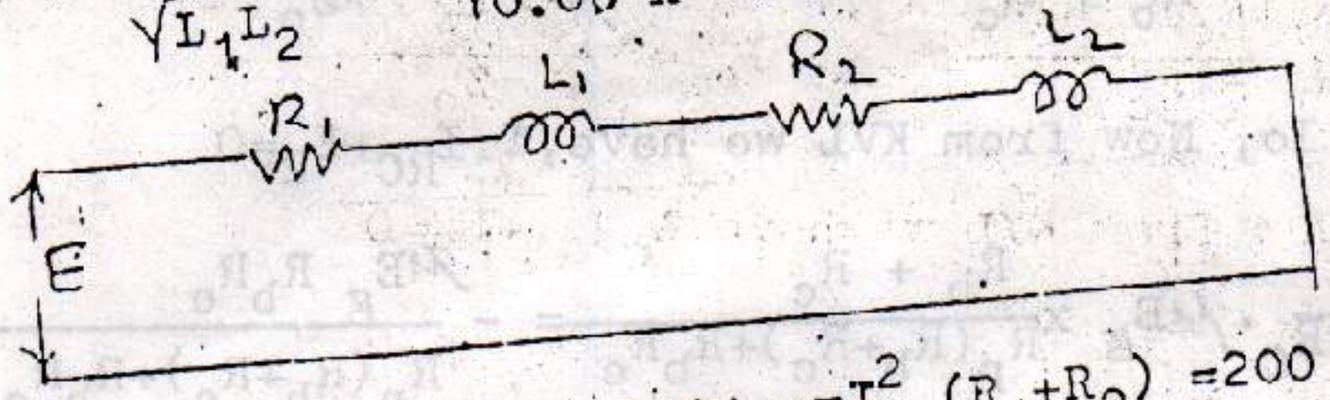
$L_2 = 30\text{mh} = 0.03\text{h}.$

Now, L_C (add) $=L_1+L_2+2M$, or $0.12 = 0.06+0.03+2M$ $\therefore$ $2M=0.03h$.

(a) L_C(Sub) $=L_1+L_2-2M = 0.06+0.03-0.03 = 60mh$. Ans.

(b) $K = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{0.03/2}{\sqrt{0.06 \times 0.03}} = 0.354$ Ans.

24.



For additive combination- $I^2 (R_1+R_2) = 200$ $\therefore$ $R_1+R_2 = \dfrac{200}{5^2} = 8$ohm.

Now, magnitude of Z_0 is $= \dfrac{100}{5} = 20$

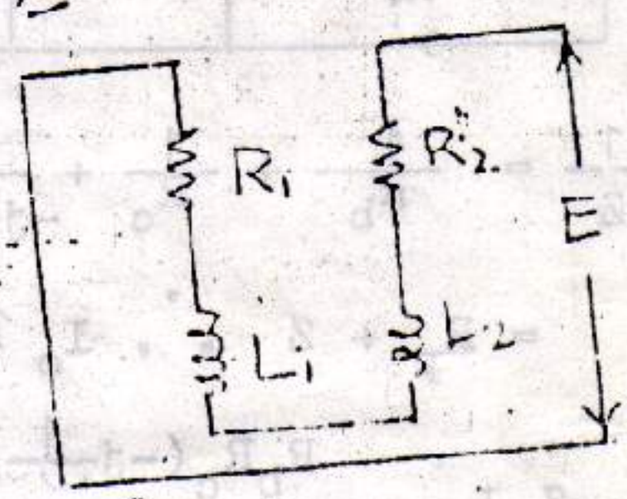
$\therefore$ $20 = \sqrt{(R_1+R_2)^2+(L_1+L_2+2M)^2 \times (2\pi 69.5)^2}$ ($\because$ $Z=\sqrt{R^2+X_L^2}$)

or, $400 = 64+(L_1+L_2+2M)^2 (2\pi \times 69.5)^2$ $\therefore$ $L_1+L_2+2M = 0.04198$

For subtractive combination- $Z = \dfrac{100}{8} = \sqrt{8^2+\{2\pi \times 69.5 (L_1+L_2-2M)\}^2}$

$\therefore$ $L_1+L_2-2M = 0.022$ $\therefore$ $4M = 0.04198 - 0.022$ $\therefore$ $M = 0.005h = 5mh$.

25. $R_1=R_2=4$ ohm.



$V = IR_1+jwL_1I+jwMI$

$\Rightarrow$ $V/I = R_1+j(L_1+M)\omega$

or, $\dfrac{26.05}{5} = 4+jw(L_1+0.005)$

or, $(7.21)^2 = 4^2+w^2(L_1+0.005)^2$

or, $(7.21)^2 = 4^2+(2 \times 69.5)^2(L_1+0.005)^2$

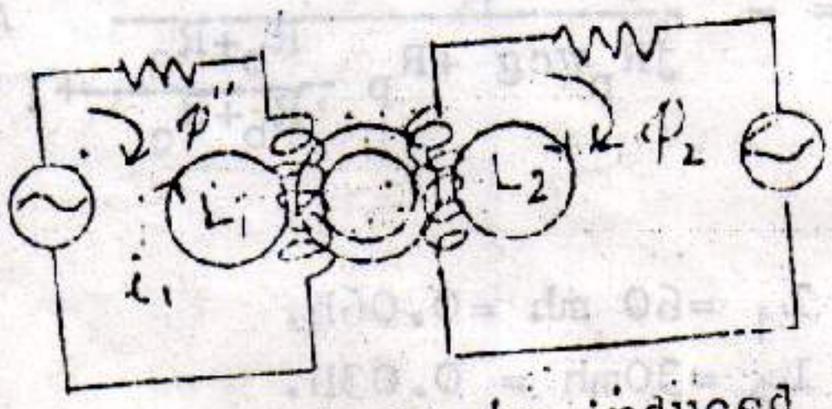
$\therefore$ $L_1 = 8.737$ mh Ans.

Now, $L_1+L_2+2M = 0.04198$ $\therefore$ $L_2 = 0.04198-2\times0.005-0.008737 = 23.249$mh.

Now, $Z_2 = \sqrt{4^2+(L_2+M)^2(w)^2} = 12.96$ ohm. $\therefore$ $V_2 = IZ_2 = 5\times12.96 = 64.8$ volt.

(b) $K = \dfrac{M}{\sqrt{L_1L_2}} = 0.35$ Ans.

26.



$L_1=0.1h$, $L_2=0.05h$, $M = K\sqrt{L_1L_2}$

$= 0.56\sqrt{0.1 \times 0.05} = 0.0396h$.

Now the voltage induced in the 0.05h winding as a result of i_1 is- $M\dfrac{di}{dt} = 0.0396\dfrac{d}{dt}$ (10Sin 377t)

$= 0.0396 \times 10 \times 377 \cos 377t$

$\therefore$ effective value $= \dfrac{149.292}{\sqrt{2}} = 105.56V$.

($V_2 = Z_m$ $I_1 = wMI_1 = 377 \times 0.0396 \times \dfrac{10}{\sqrt{2}} = 105.56V$.)

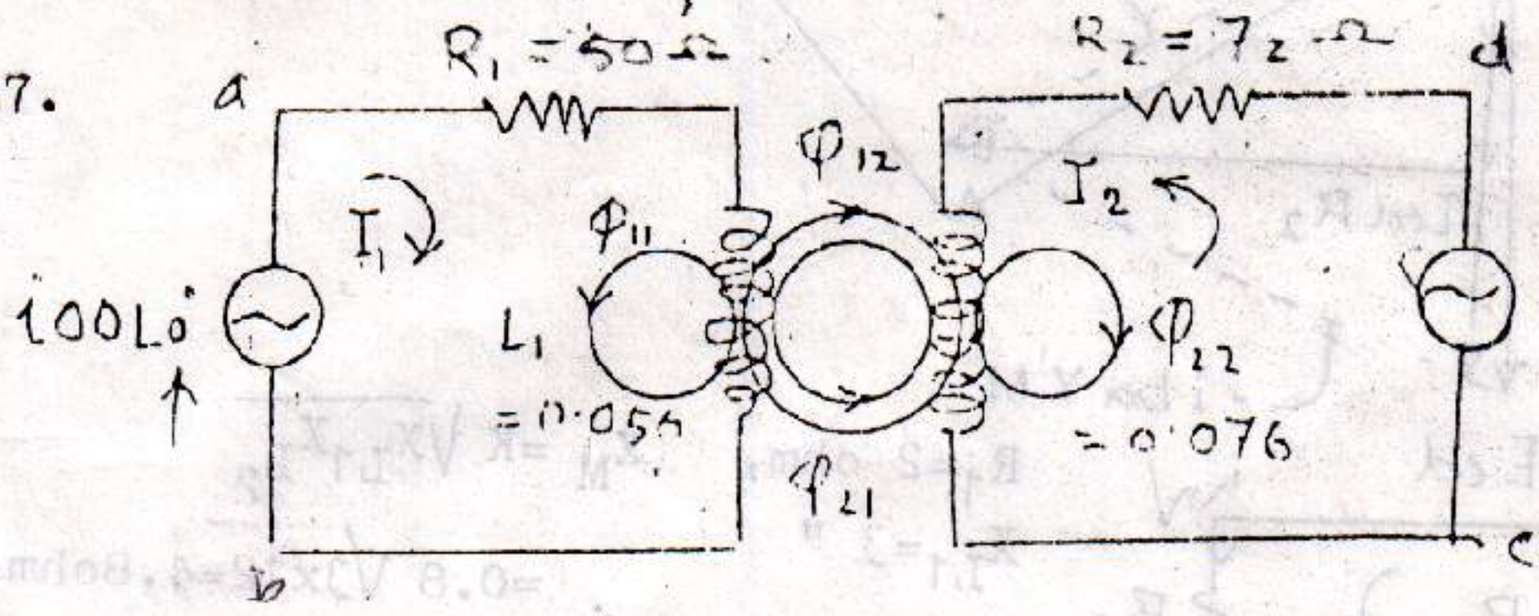
Now, the induced voltage in 0.1 henry winding is-

$L_1 = \frac{di_1}{dt} = 0.1 \times 10 \times 377 \cos 377t = 377 \cos 377t.$

$\therefore$ r.m.s. value $= \frac{377}{\sqrt{2}} = 266.58V$ (OR)

Again, $V_1 = I_1 X_{L1} = \frac{10}{\sqrt{2}} \times 377 \times 0.1 = 266.58V.$

27.



$M = K_M \sqrt{L_1 L_2} = 0.4 \sqrt{0.056 \times 0.07} = 0.025h$, M will be negative

$Z_1 = 50 + j.113 \times 0.056$

$Z_{12} = 72 + j\ 113 \times 0.07$

$Z_1 I_1 - Z_M I_2 = E_{ba}, \quad -ZMI_1 + Z_2 I_2 = E_{cd}$

$$I_1 = I_{ba} = \frac{\begin{vmatrix} E_{ba} & -ZM \\ E_{cd} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{E_{ba}Z_2 + E_{cd}Z_M}{Z_1 Z_2 - ZM^2}$$

$$= \frac{100\ \angle 0^\circ)(72+j0.07\times 1131)+(50\ \angle -90^\circ)(j1131\times 0.025)}{(50+j\ 0.056\times 1131)\ (72+j0.07\times 1131)-(j1131\times 0.025)^2}$$

$$= \frac{7199.846 + j\ 7917.528 + 1413.75}{(80.694\ \angle 51.71^\circ)(107.013\angle 47.72)+28.275^2}$$

$$= \frac{11699.26\ \angle 42.59}{1414.83+j8518.614+28.275^2} = \frac{116999.26\ \angle 42.59}{8540.811\ \angle 94.312} = 1.37\ \angle -51.54$$ Ans.

$$I_2 = I_{cd} = \frac{\begin{vmatrix} Z_1 & E_{ba} \\ -Z_M & E_{cd} \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{Z_1 E_{cd} + Z_M Z_{ba}}{Z_1 Z_2 - Z_M^2}$$

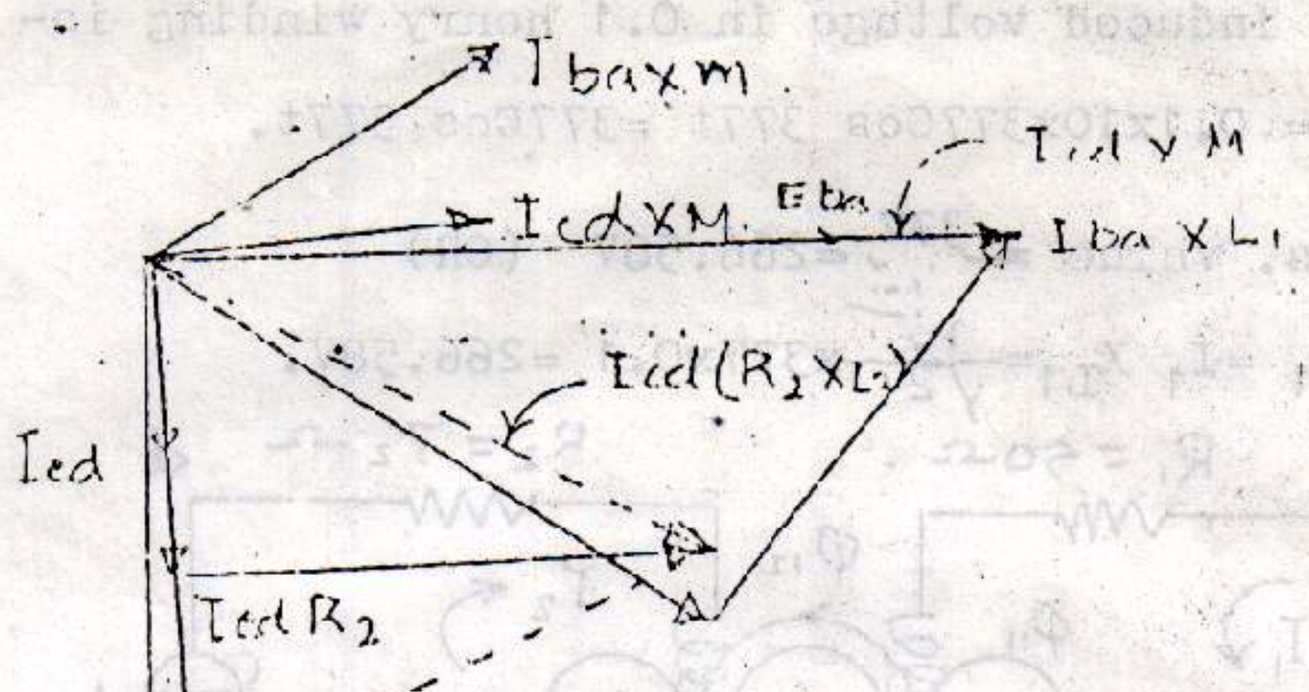
$$= \frac{(80.694\ \angle 51.71)\ (50\ \angle -90^\circ) + (28.275\ \angle 90^\circ)(100\angle 0)}{8540.811\ \angle 94.312^\circ}$$

$$= \frac{3166.774 - j2500 + j2827.5}{8540.81\ \angle 94.312} = \frac{3183.656\ \angle 5.9}{8540.81\ \angle 94.312} = 0.373 \angle -88.4^\circ$$ Ans.

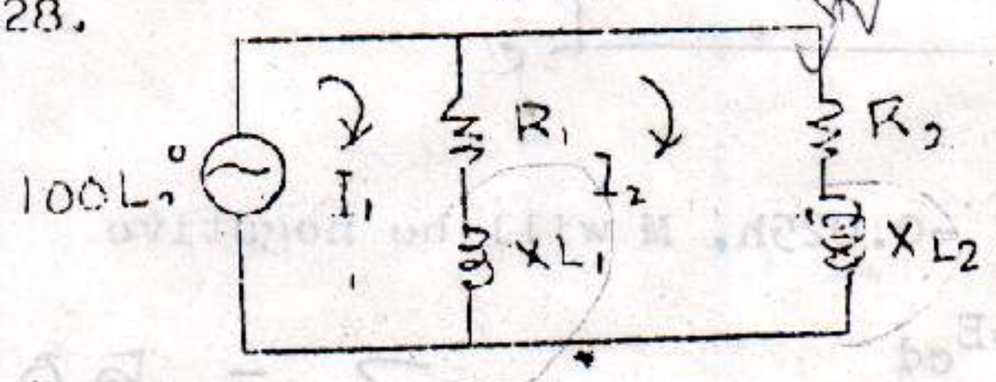
(b) $P_{ba} = V_{ba} I_1 \cos\theta = 100 \times 1.37 \times \cos 51.54 = 85.21$ watt.

$P_{cd} = V_{cd} I_2 \cos\theta = 50 \times 0.373 \cos\{-90 - (-88.4)\} = 18.64$ watts.

(c)



28.



R_1=2 ohm, X_{L1}=3 " R_2=5 " X_{L2}= 12 ohms.

$Z_M = K\sqrt{X_{L1}X_{L_2}}$

$=0.8\sqrt{3\times12}=4.8$ohm.

Since the mmf's are additive . . M will be positive.

$Z_1 = 2+j3 = 3.606\ \underline{/56.31}$, $Z_2 = 5+j12 = 13\ \underline{/67.38^\circ}$

$Z_M = j4.8 = 4.8\ \underline{/90^\circ}$

Now, $I_1Z_1+I_2Z_M=V$, $I_1Z_M+I_2Z_2=V$

$$\therefore I_1 = \frac{\begin{vmatrix} V & Z_M \\ V & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_2-Z_M)}{Z_1Z_2-Z_M^2} = \frac{(100\underline{/0^\circ})(5+j12-j4.8)}{(3.606\underline{/56.31})(13\underline{/67.38})-(4.8\underline{/90^\circ})^2}$$

$$= \frac{876.584\ \underline{/55.22}}{-22+j39+23.04} = \frac{876.584\ \underline{/55.22}}{39.112\ \underline{/94.34}} = 22.41\ \underline{/-39.12}\ \text{Amp.}$$

$$I_2 = \frac{\begin{vmatrix} Z_1 & V \\ Z_M & V \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_1-Z_M)}{Z_1Z_2-Z_M^2}$$

$$= \frac{(100\underline{/0^\circ})(2+j3-j4.8)}{39.112\ \underline{/94.34}} = \frac{269.07\ \underline{/-42^\circ}}{39.112\ \underline{/94.34}} = 6.879\ \underline{/-136.34^\circ}\ \text{A Ans.}$$

Power supplied conductively branch(2) is $=VI_2\cos\theta = 100\times6.879\cos(-136.34) = -497.66$ watts.

Power dissipited in branch (2) is $= I_2^2R_2 = (6.879)^2\times5 = 236.6$ watts.

. . Power supplied electromagnetically is-

$= -P_{\text{conductively}} + P$ dissipited $= +497.66+236.6 = +734.20$ watts.

Again, power supplied electromagnetically in branch (1) is-

$P_{elec.} = -P_{conductively.} + P_{dissipited} = -VICos\theta + I_1^2R$

$= -1738.63 + 1004.42 = -734.21$ watts.

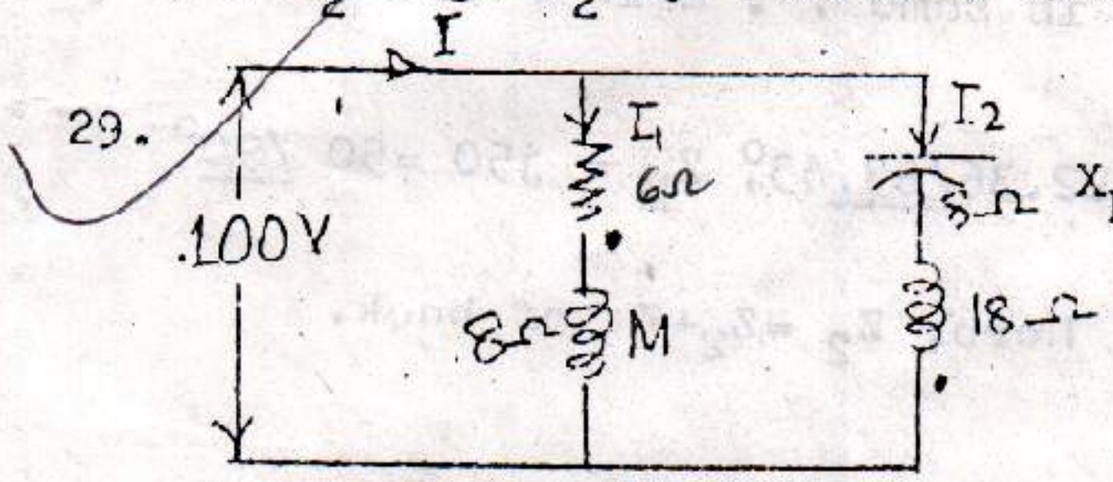
The voltage drop accross only the inductance of brandh(2) is,

$V_2 = X_{L_2} I_2 + I_1 Z_M$, $V_2 = (12 \angle 90^\circ)(6.879 \angle -136.34) + (22.41 \angle -39.12)(4.8 \angle 90^\circ)$

$= 57 - j59.72 + 67.87 + j83.454 = 127.1 \angle 10.76^\circ$

$\therefore$ I_2 lags V_2 by $(10.76 - (-136.34) = 147.1^\circ$ Ans.

29.



$X_M = K\sqrt{X_{L1} X_{L2}} = 0.5\sqrt{8 \times 18} = 6$ ohm.

Here, M will be negative

$Z_1 = 6 + j8$, $Z_2 = -j8 + j18$, $Z_M = -j6$

Now, $Z_1 I_1 - Z_M I_2 = V$

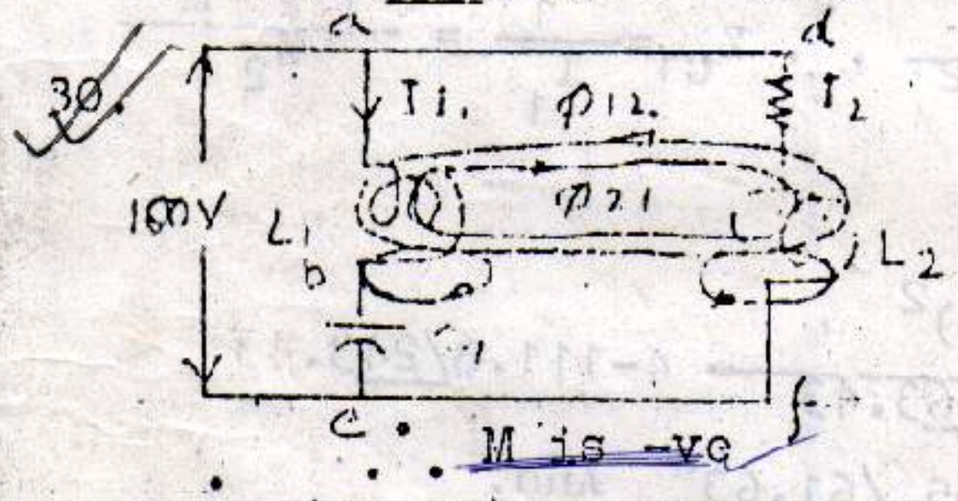
$-Z_M I_1 + Z_2 I_2 = V$

$$\therefore I_1 = \frac{\begin{vmatrix} V & -Z_M \\ V & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{V(Z_2 + Z_M)}{Z_1 Z_2 - Z_M^2}$$

$$= \frac{(100 \angle 0^\circ)\{(-j8 + j18) + j6\}}{(6 + j8)(-j8 + j18) - (j6)^2} = \frac{1600 \angle 90^\circ}{j60 - 80 + 36} = \frac{1600 \angle 70^\circ}{74.404 \angle 126.25}$$

$= 21.5 \angle -36.25^\circ$ A Ans.

30.



Now, $X_{L1} = 5$ ohm., $X_{C1} = 5$ ohm.

$X_{L2} = 5$ ohm., $X_M = 4$ ohm.

$R = 10$

M is -ve

$\therefore$ $(j5 - j5) I_1 - j4 I_2 = 100 \angle 0^\circ$ or, $(4 \angle -90^\circ) I_2 = 100 \angle 0$ $\therefore$ $I_2 = 25 \angle 90^\circ$

Again, $(10 + j5) I_2 - j4 I_1 = 100 \angle 0^\circ$ or, $(10 + j5)(j25) - j4 I_1 = 100 + j0$

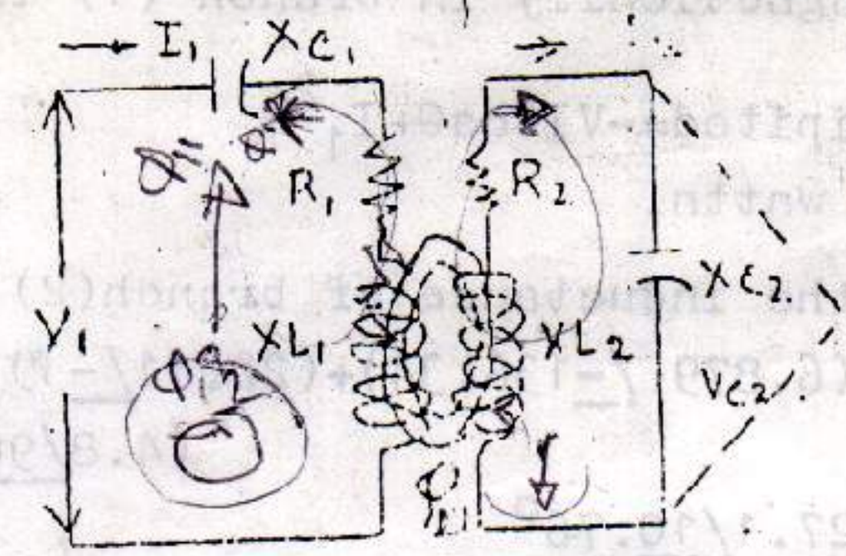
or, $j4 I_1 = 350i - 125 - 100$ $\therefore$ $I_1 = 84.085 \angle 42^\circ$

From KVL we have Now, ~~$V_{bc} + V_{ca} + V_{ab} = 0$~~

or, $V_{be} = V_{ae} - V_{ab} = R_2 I_2 - (X_{L1} I_1 - X_M I_2) = 10 \times (25 \angle 90^\circ) - \{j5(84.085 \angle 42^\circ) - j4(25 \angle 90^\circ)\}$

$= 250 \angle 90^\circ - \{-281.319 + j312.437 + 100 + 0\} = 191.8 \angle -19^\circ$ Volt. Ans.

31.



$R_1 = 4$ ohms $\quad R_2 = 100$ ohms.
$X_{L1} = 40$ ohms $\quad X_{L2} = 100$ ohms.
$XC_1 = 40$ ohms. $\quad X_{C2} = 120$ ohms.
$X_M = 500$ ohms. $\quad V_1 = 100$ volts.

Since the direction of the mutual flux in the secondary c
circuit, ϕ_{12} and current I_2 is same . . M is + ve

Now, $Z_1 = 4 + j40 - j40 = 4$ ohm.

$Z_2 = 10 + j100 - j120 = 10 - j20 = 22.36 \angle 63.43^\circ$, $Z_M = j50 = 50 \angle 90^\circ$

Now, $Z_1 I_1 + Z_M I_2 = V_1$ Here, $Z_2 = Z_2 + Z$ of book.

$Z_M I_1 + Z_2 I_2 = 0$

$$\therefore I_2 = \frac{\begin{vmatrix} Z_1 & V_1 \\ Z_M & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{-V_1 Z_M}{Z_1 Z_2 - Z_M^2} = \frac{-(100 \angle 0^\circ)(50 \angle 90^\circ)}{(4)(22.36 \angle -63.43) - (50 \angle 90^\circ)^2}$$

$$= \frac{5000 \angle 90^\circ}{40 - j80 + 2500} = \frac{-5000 \angle 90^\circ}{2541.26 \angle -1.8} = -1.9675 \angle 91.8^\circ$$

Now, $V_{C2} = I_2 X_{C2} = (1.9675 \angle 91.8)(120 \angle -20^\circ) = 236.1 \angle 1.8^\circ$

32. From the previous problem we have -

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_M \\ 0 & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{V_1 Z_2}{Z_1 Z_2 - Z_M^2} \quad \therefore Z_{C1} = \frac{V_1}{I_1} = \frac{Z_1 Z_2 - Z_M^2}{Z_2}$$

$$= Z_1 - \frac{Z_M^2}{Z_2} = 4 - \frac{(50 \angle 90^\circ)^2}{22.36 \angle -63.43} = 4 - 111.8 \angle 243.43$$

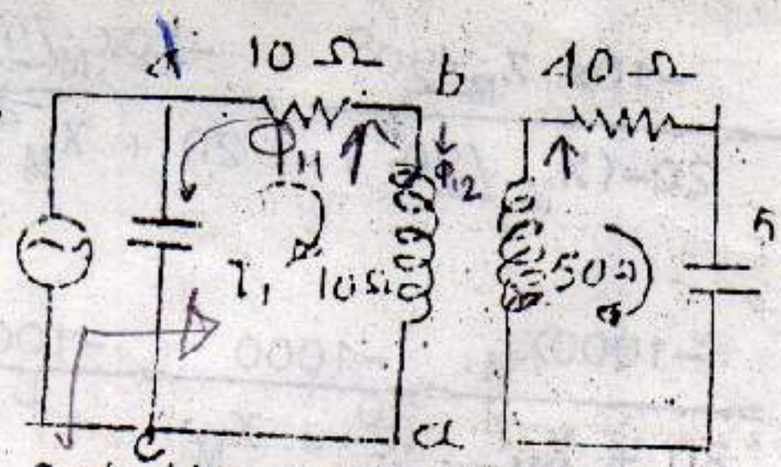
$= 4 + 50 + j100 = 54 + j100 = 113.65 \angle 61.63^\circ$ Ans.

The reflected impedence into the primary $= \frac{Z_M^2}{Z_2} = 111.8 \angle 243.43$
$= 50 + j100 = 111.8 \angle 63.43.$

Now, the value of reflected reactance into the primary $= j100$ which is indictive.

(33) and (34) For resonance, hence .

35.



$X_{L11}=100 \times 0.01=10$ ohm.

$X_{L22}=100 \times 0.05=50$ ohm.

$X_M = 1000 \times 0.02=20$ ohm.

$X_{C22}=\frac{106}{20 \times 1000}=50$ ohm.

Let the capacitive reactance of C_{11} be X.

Now, $Z_{cabd} = Z_1 - \frac{Z_M^2}{Z_2} = 10+j10-\frac{(j20)^2}{40+j50-j50} = 10+j10+10$

$=20+j10=22.36 \angle 20.57^\circ$

Now, $Z_{e1}=\frac{(-j\,X)Zcabd}{-jx + Z_{cabd}} = \frac{(X \angle -90^\circ)\,(22.36 \angle 26.57^\circ)}{-jx+20+j10}$

$= \frac{22.36 \times \angle -63.43}{20+j(10-x)} = \frac{22.36 \times \angle -63.43}{\sqrt{20^2+(10-x)^2}\tan^{-1}\frac{10-x}{20}} = \frac{22.36x}{\sqrt{20^2+(10-x)^2}} \times$

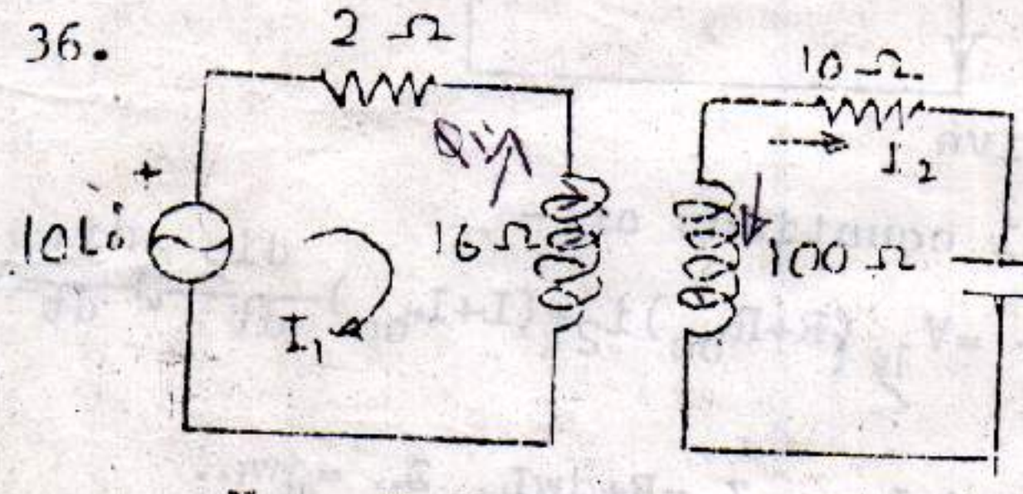
$\angle -63.43 . -\tan^{-1}\frac{10-x}{20}$ For pure resistance phase angle will zero . . $\tan^{-1}=\frac{10-x}{20}=-63.43$ or, $\frac{10-x}{20}=\tan(-63.43)$

or $10-x=20x(-2)$ . . $x=50$ ohm.

. . $C_{11}=\frac{1}{1000 \times 50}=20 \mu$ F Ans.

. . The value of the pure resistance $=\frac{22.36 \times 50}{\sqrt{20^2+(10-50)^2}}=25$ ohm. Ans.

36.



$X_{L1}=16$ ohm., $X_{L2}=100$

$X_M = K\sqrt{X_{L1}X_{L2}}=0.05 \quad 16\times100=2$ ohm.

Now, $Z_1I_1 + Z_MI_2 = 10 \angle 0^\circ$, $Z_MI_1 + Z_2I_2 = 0$

$$\therefore I_2 = \frac{\begin{vmatrix} Z_1 & 10\angle 0^\circ \\ Z_M & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{-Z_M\,(10\angle 0^\circ)}{Z_1Z_2 - Z_M^2} = \frac{-j\,2(10)}{(2+j16)(10+j100-j100)-(j2)^2}$$

$$= \frac{-j20}{20+j160+1} = \frac{20\angle -90^\circ}{161.79\angle 81.47} = 0.1236 \angle -171.47 \text{ A.}$$

. . $V_{C22}=X_{C22} \cdot I_2 = 100(0.1236)=12.36$ volt.

(b) At resonance, the impedance of ckt-1 will be,

$Z_1=2+j16-j16=2$ ohm. . . $I_2=\frac{-j2(10)}{2x10-(j2)^2}=\frac{20\angle -90^\circ}{24}=0.833\angle -90^\circ$

. . $V_{C_{22}}=100 \times 0.833=83.3$ volt.

(c) At resonance the equation are- $ZI_1 + Z_M I_2 = 10$, $Z_M I_1 + 10 I_2 = 0$

$$I_2 = \begin{vmatrix} Z & 10 \\ Z_M & 0 \end{vmatrix} \Big/ \begin{vmatrix} Z & Z_M \\ Z_M & 10 \end{vmatrix} = \frac{-10Z_M}{20 - Z_M^2} \quad \frac{-10 \times Z_M \angle 90^\circ}{20 - (X_M \angle 90^\circ)} \quad \frac{-10X_M \angle 90^\circ}{20 + X_M^2}$$

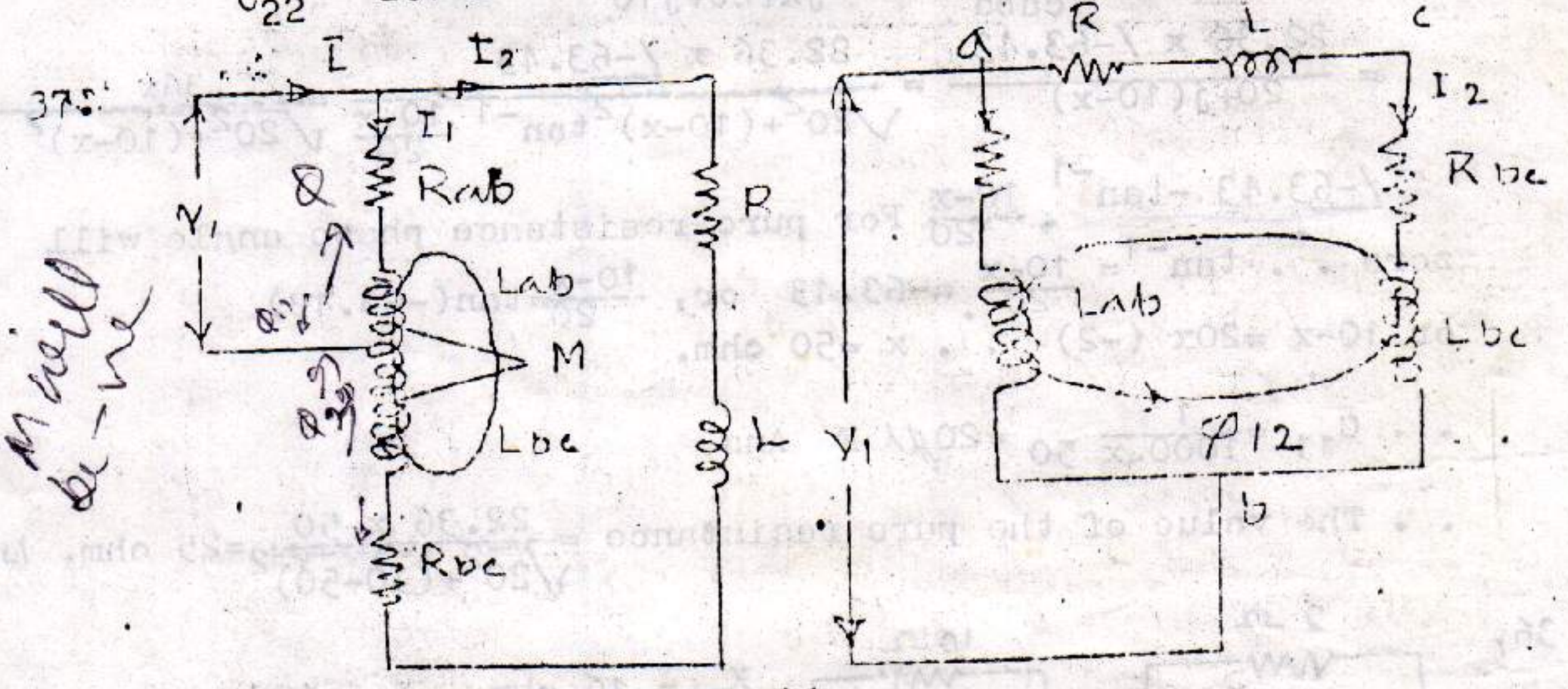
$$\therefore V_{C_{22}} = (100 \angle -90^\circ)\ \frac{(-10X_M \angle 90^\circ)}{20 + X_M^2} \Big/ \frac{-1000X_M}{20 + X_M^2} = \frac{-1000}{\frac{20}{X_M} + X_M} = \frac{-1000}{D},$$

When, Denoininator $D = \frac{20}{X_M} + X_M$

$V_{C_{22}}$ will be maximum it Dis minimum $\therefore \frac{dD}{dX_M} = -\frac{20}{X_M^2} + 1 = 0$

or, $X_M^2 = 20$ or, $X_M = \sqrt{20} = 4.472$ ohm.

$$\therefore V_{C_{22}} = \frac{1000 \times 4.472}{20+20} = 111.8 \text{ volt. Ans.}$$



Here M will be negative

$\therefore$ The general differential equations are-

$$R_{ab} i_1 + L_{ab} \frac{di_1}{dt} - M \frac{di_2}{dt} = V_1, \quad (R+R_{bc}) i_2 + (L+L_{bc}) \frac{di_2}{dt} - M \frac{di_1}{dt} = V_1$$

38. Let $Z_{ab} = R_{ab} + jwL_{ab}$, $Z_{bc} = R_{bc} + jwL_{bc}$, $Z = R + jwL$, $Z_M = jw_M$.

Now The equations become- $Z_{ab} I_1 - Z_M I_2 = V_1$, $-Z_M I_1 + (Z_{bc} + Z) I_2 = V_1$

$$\therefore I_1 = \frac{\begin{vmatrix} V_1 & -Z_M \\ V_1 & Z_{bc}+Z \end{vmatrix}}{\begin{vmatrix} Z_{ab} & -Z_M \\ -Z_M & Z_{bc}+Z \end{vmatrix}} = \frac{V_1 (Z_{bc} + Z + Z_M)}{Z_{ab}(Z_{bc}+Z) - Z_M^2}$$

$$I_2 = \frac{\begin{vmatrix} Z_{ab} & V_1 \\ -Z_M & V_1 \end{vmatrix}}{\begin{vmatrix} Z_{ab} & -Z_M \\ -Z_M & Z_{bc}+Z \end{vmatrix}} = \frac{V_1 (Z_{ab} + Z_M)}{Z_{ab}(Z_{bc}+Z) - Z_M^2}.$$

Ramon. Khotha-2.

$Z_{ab}=4+j377\times0.07=4+j26.39=26.69\angle 81.38$

$Z_{bc}=0.5+j377\times0.01=0.5+j3.77=3.8\angle 82.45^\circ$, $Z=10+j0=10$

$Z_M=j377\times0.02=j7.54=7.54\angle 90^\circ$

$$I_1=\frac{(100\angle 0)(0.5+j3.77+10+j7.54}{(26.69\angle 81.38)(0.5+j3.77+10)-(j7.54)^2}$$

$$I_2=\frac{(100\angle 0)\ (4+jj26.39+j7.54)}{292.16\ \angle 90.12^\circ}=\frac{3416.5\angle 83.28^\circ}{292.16\ \angle 90.16^\circ}=1169\ \angle -6.84$$

$I=I_1+I_2=5.28\ \angle -43+1169\angle -6.84=3.86-j3.6+11.6-j1.39=16.25\angle -17.9^\circ$

Total power supplied$=V_1 I\ Cos(417.9)=1545.9$ watts.

Power dissipated incircuit 1 $=I_1^2\ R_{ab}=111.5$ watts.

" " " 2 $=I_2^2(R_{bc}+R)=(11.69)^2(0.5+10)$.
$=1443.9$ W

Next proble,s are reject because they are for resonance.

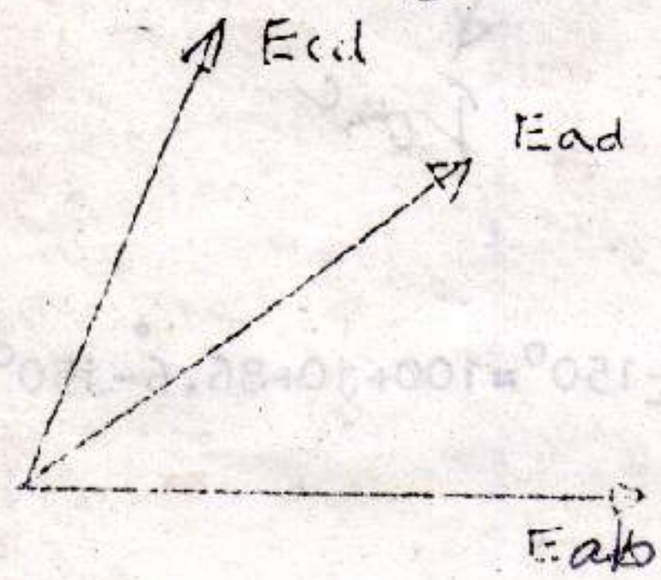
CHAPTER VIII

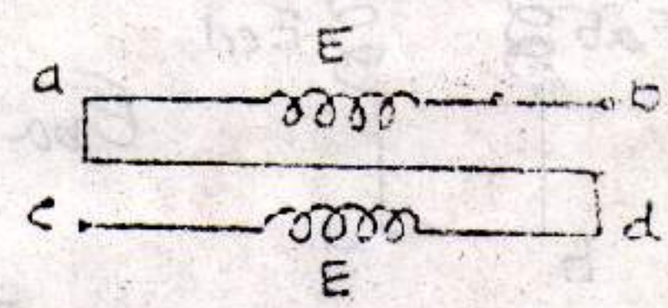
" BALANCED POLYPHASE CIRCUITS"

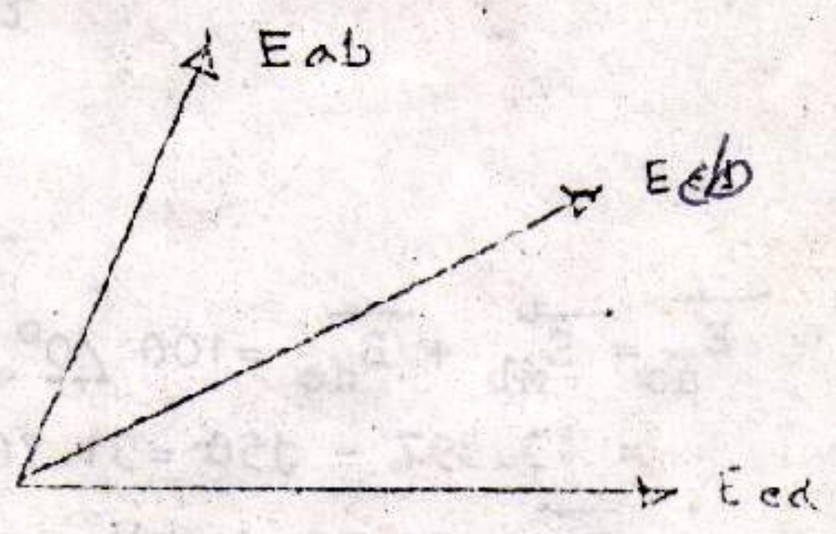
P R O B L E M

1.









$E_{ad}^2 = E^2+E^2+2E^2Cos60^\circ = 2E^2(1+Cos60^\circ)=2E^2.2Cos^2 30^\circ$

$E_{cb} = 2E\ Cos30^\circ = 1.732E.$

$E_{ad} = 2E\ Cos30^\circ = 1.732E.$

$\therefore E_{ad} = E_{cd}$

CHAPTER XI

" ELECTRIC WAVE FILTERS "

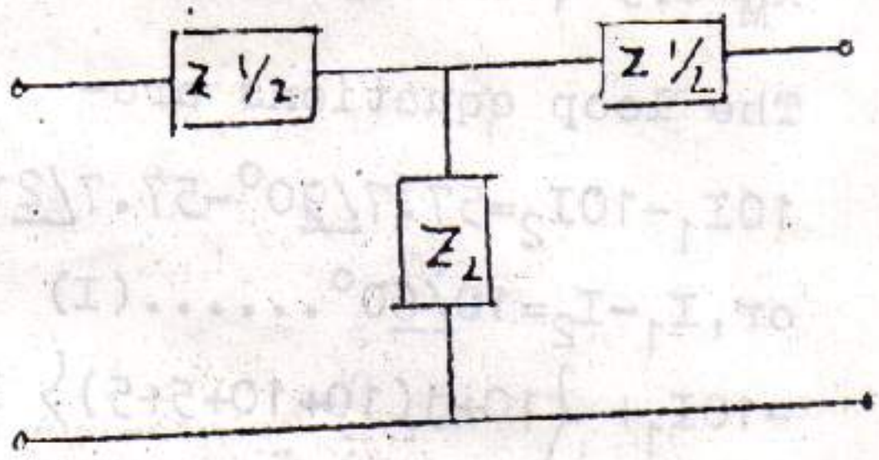
P R O B L E M

1. Here,

$L_1 = 0.047$ h. ; $Z_{oT} = ?$

$R_1 = 0$; At 50 cycle

$C_2 = 300\ \mu F$; At 100 cycle



$\frac{Z_1}{2} = 0 + j2\pi fL_1 = 0 + j2\pi \times 50 \times 0.047 = 14.77 \angle 90^\circ$ $\therefore Z_1 = 29.54 \angle 90^\circ$

$Z_2 = \frac{1}{j\omega C_2} = -j\frac{10^6}{2\pi \times 50 \times 300} = 10.61 \angle -90^\circ$

$Z_1 Z_2 = 29.54 \angle 90^\circ \times 10.61 \angle -90^\circ = 313.42 \angle 0^\circ = 313.42 + j0$

$\frac{Z_1^2}{4} = \frac{(29.54 \angle 90^\circ)^2}{4} = 218.153 \angle 180^\circ = -218.153 + j0$

$Z_{oT} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \sqrt{313.42 + j0 - 218.153 + j0}$

$= 95.267 \angle 0^\circ = 9.76 \angle 0^\circ$ $\therefore Z_{oT} = 9.76 \angle 0^\circ$ ohm

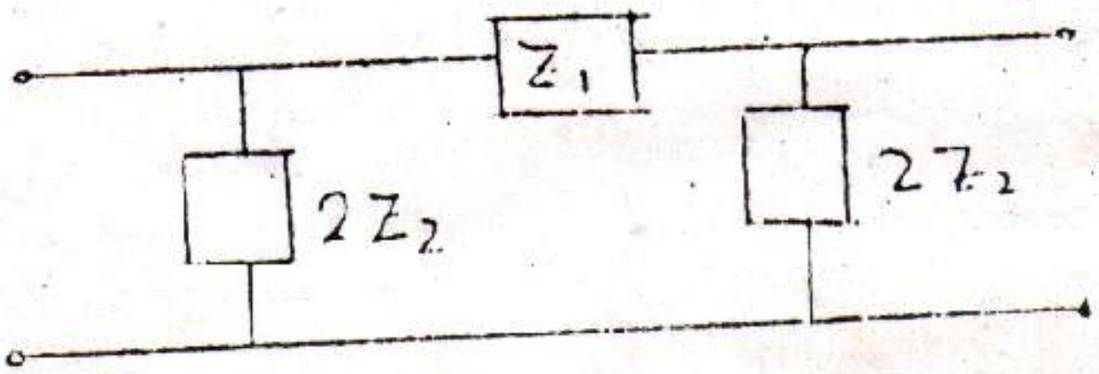
Similarly, At 100 cycle. $Z_{oT} = 23.65 \angle 90^\circ$ ohm

$\therefore Z_2 = \frac{1}{\omega C_2} = -\frac{jx10^6}{2\pi \times 100 \times 300} = 5.31 \angle -90^\circ$ ohm.

Now, $Z_{oT} = \sqrt{Z_1 Z_2 + \frac{Z_1}{4}} = \sqrt{(59.1 \angle 88.06)(5.31 \angle -90^\circ) + 873.20 \angle 176.12)}$

$\therefore Z_{oT} = 23.7 \angle 87.52^\circ$

2.



At 200 C/Sec.

$Z_1 = j\omega L_1 = jx2\pi x200x0.02 = 25.13 \angle 90^\circ$

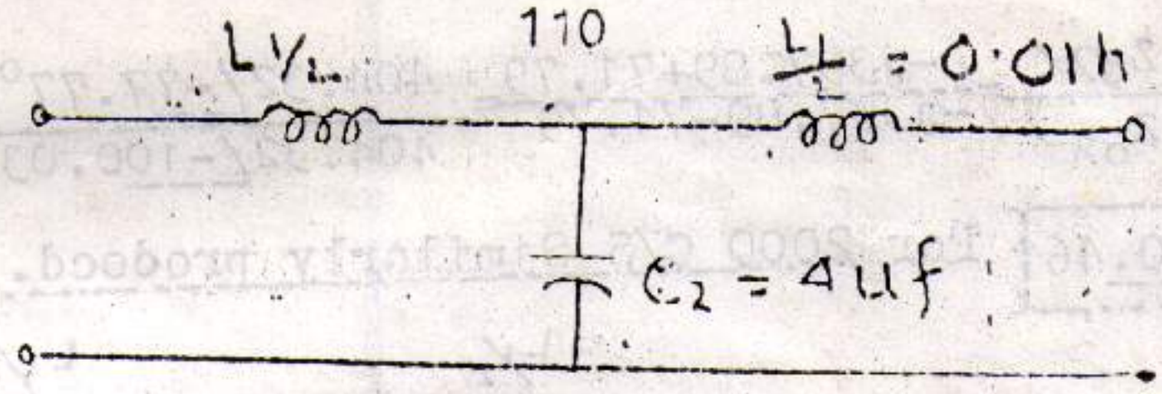
$2Z_2 = \frac{1}{j\omega C_2/2} = -j\frac{1}{2\pi fC_2/2} = \frac{jx10^6}{2 \times 200 \times 2} = 397.89 \angle -90^\circ$ $\therefore Z_2 = 198.94 \angle -90^\circ$

Now, $1 + \frac{Z_1}{4Z_2} = 1 + \frac{25.13 \angle 90^\circ}{4 \times 198.94 \angle 90^\circ} = 1 + 0.0315 \angle 180^\circ = 0.9685 \angle 0^\circ$

$Z_1 Z_2 = 4999.36 \angle 0^\circ$

$Z_{oT} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \sqrt{\frac{4999.36 \angle 0^\circ}{0.9685 \angle 0^\circ}} = 71.85 \angle 0^\circ$ ohm.

3.



$\frac{Z_1}{2} = jwL_1/2 = j\ 2\pi x200x0.01 = 12.57\angle 90^\circ$ ohm. $\therefore Z_1 = 25.14\angle 90^\circ$ ohm.

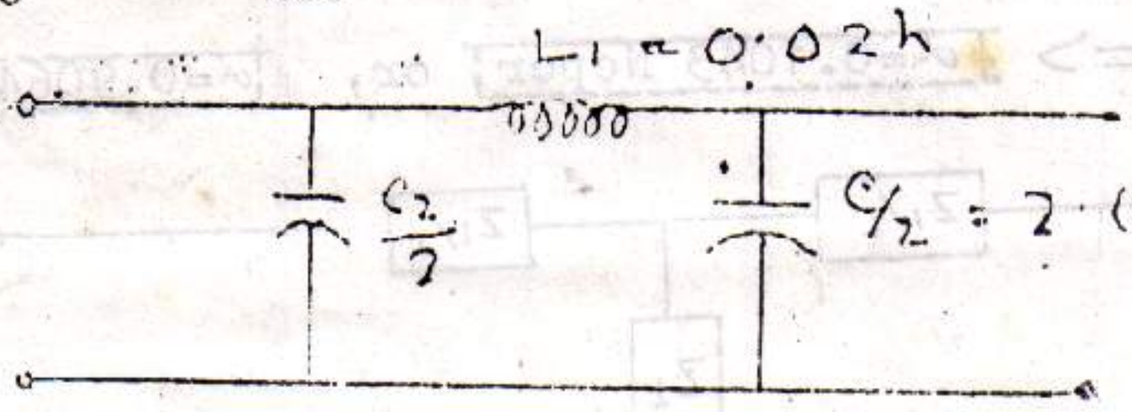
$\therefore Z_2 = \frac{1}{jwc_2} = \frac{10^6}{jx2\pi x200x7} = 198.94\ \angle -90^\circ$ ohm.

$Z_{oc} = \frac{Z_1}{2} + Z_2 = 12.57\ \angle 90^\circ + 198.94\ \angle -90^\circ$ $\boxed{Z_{oc} = 186.37\ \angle -90^\circ}$

$Z_{sc} = \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2} + \frac{Z_1}{2} = \frac{j\ 12.57x(-j198\angle 94)}{j\ 12.57 - j198.94} + j12.57\angle 90^\circ$

$= \frac{2500.6758}{186.37\ \angle -90^\circ} + j\ 12.57 = 25.99\ \angle 90^\circ$ $\boxed{\therefore Z_{sc} = 26\ \angle 90^\circ \text{ ohm.}}$

$Z_o = \sqrt{Z_{oc}\ Z_{sc}} = 69.61\ \angle 0^\circ$ ohm.



$Z_1 = jwL_1 = jx2\pi x200x0.02 = 25.13\ \angle 90^\circ$ $\therefore Z_1 = 25.13\angle 90^\circ$ ohm.

$2Z_2 = \frac{1}{jwC_2/2} = \frac{jx10^6}{2\pi x200 x 2.0} = 397.89\ \angle -90^\circ$ $\therefore Z_2 = 198.94\angle -90^\circ$ ohm.

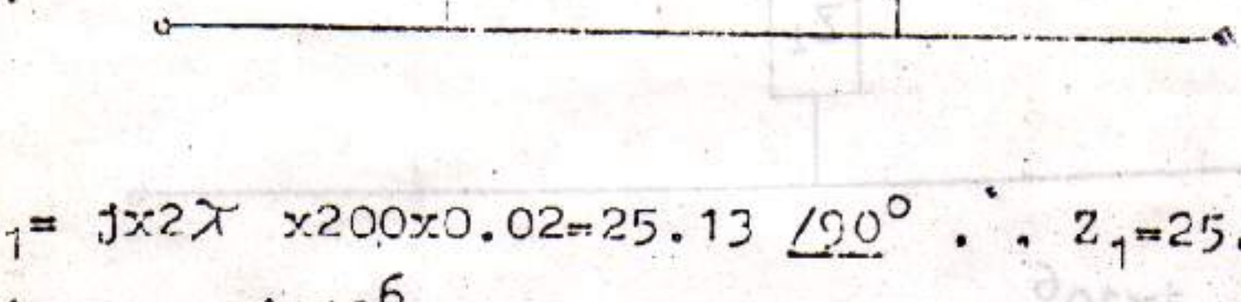
$Z_{sc} = \frac{Z_1 x 2Z_2}{Z_1 + 2Z_2} = \frac{(j25.13)x(-j397.89)}{j25.13 - j\ 397.89}$ $\therefore Z_{sc} = \frac{(2Z_2 + Z_1)x2Z_2}{Z_1 + 4Z_2}$

$Z_{sc} = 26.82\ \angle 90^\circ$ ohm

$= \frac{-j372.76x-j397.89}{j25.13 - j795.76} = 192.46\angle -90^\circ$ $\boxed{\therefore Z_{oc} = 192.46\angle -90^\circ}$

$Z_{oc} = \sqrt{Z_{oc}\ Z_{sc}} = 71.85\ \angle 0^\circ\ \Omega$

5.



At 200 C/S:

$Z_1 = jwL_1 = j2\pi x200x0.02 = j\ 25.13$, $2Z_2 = \frac{-j}{2\pi\ I\ C_2/2}$

$= \frac{-j\ x10^6}{2\pi\ x200x2.0} = -j\ 397.89$

Now, $1 + Z_1/4Z_2 = 1 + \frac{25.13\angle 90^\circ}{2x397.89\angle -90^\circ} = 1 - 0.0315 = 0.97$

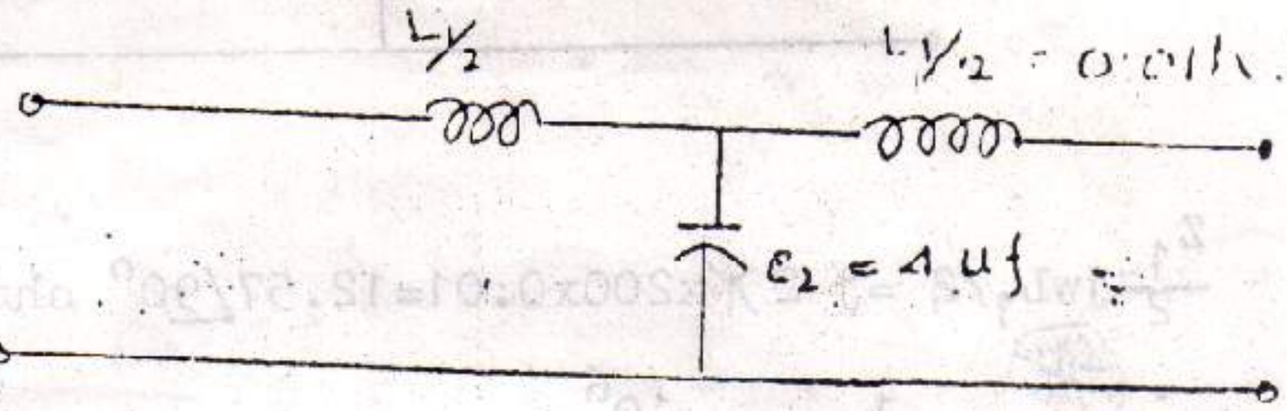
$Z_1 Z_2 = (25.13\angle 90^\circ)(198.95\angle -90^\circ) = 1999.45\ \angle 0^\circ$

$Z_{sc} = \sqrt{Z_1 Z_2}$ = [illegible]

Now, $\frac{I_1}{I_2} = \frac{2Z_2+Z_{oT}}{2Z_2-Z_{oT}} = \frac{-j397.89+71.79}{-j397.89-71.79} = \frac{404.32/-77.77^\circ}{404.32/-100.03} = 1/20.46^\circ$

$\therefore I_1/I_2 = 1/20.46$ For 2000 C/S Similarly prodeed.

6. At 200 C/S:



$\frac{YZ_1}{2} = 12.57/90^\circ$ ohm.

$Z_2 = 198.94/-90^\circ$ ohm.

$Z_{oT} = 69.61/0^\circ$ ohm.

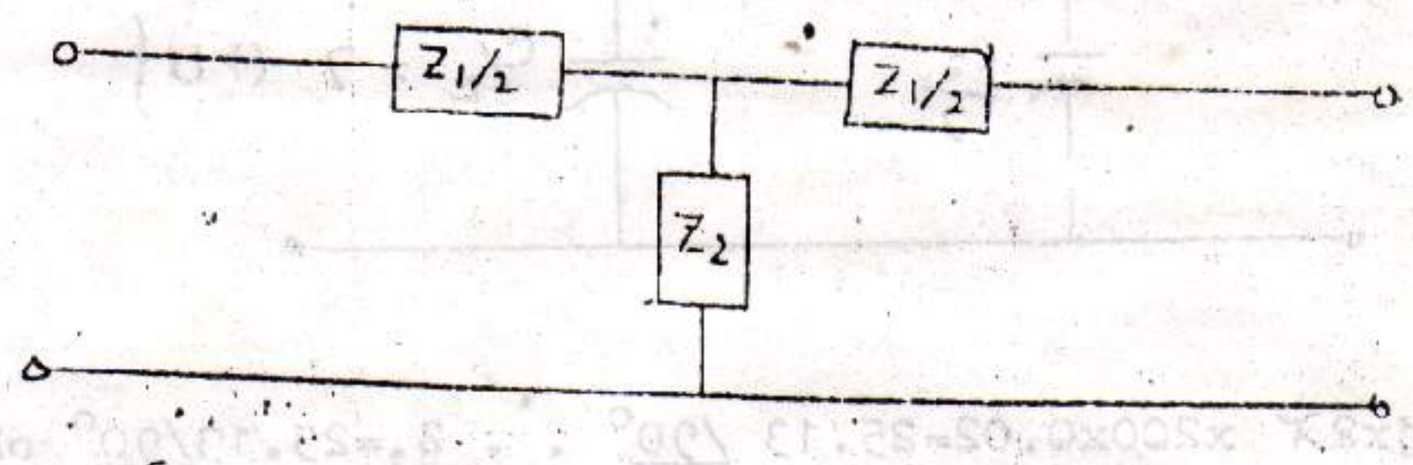
$\frac{I_1}{I_2} = \frac{Z_{oT} + \frac{Z_1}{2}}{Z_{oT} - \frac{Z_1}{2}} = \frac{69.61+j12.57}{69.61-j12.57} = \frac{70.74/10.24^\circ}{70.74/-10.24} = 1.0/20.48^\circ$

$\therefore I_1 / I_2 = 1.0/20.48$

7. $\frac{I_1}{I_2} = 1.11/113^\circ \Rightarrow \frac{I_1}{I_2} = e^{\alpha+j\beta} = e^{\alpha} \cdot e^{j\beta} = 1.11/113^\circ$

$\therefore e^{\alpha} = 1.11 \Rightarrow \alpha = 0.1043$ Nepar or, $\alpha = 0.9064$ decibel

9.



$\frac{Z_1}{2} = -\frac{jx10^6}{wC_2} = -\frac{jx10^6}{1257 \times 7.96}$, $Z_2 = 4 + jwL_1 = 4 + j1257x0.0159 = 20.38/78.68$

At 200 C/S: $Z_1 = 200/-90^\circ$, $Z_2 = 20.4/78.7^\circ$

$\frac{Z_1}{4Z_2} = \frac{200/-90^\circ}{4 \times 20.4/78.7^\circ} = 2.45/-168.7^\circ$ $\sqrt{\frac{Z_1}{4Z_2}} = 1.57/-84.35 = 0.15 - j1.56$

$1 + \frac{Z_1}{4Z_2} = 1 + 2.45/-168.7^\circ = 1 - 2.40 = j0.48 = -1.40 - j0.48 = 1.48/-161^\circ$

$\sqrt{1 + \frac{Z_1}{4Z_2}} = 1.22/-80.54^\circ = 0.2 - j1.20$ $\therefore \sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} = 0.2 - j1.20$

$+0.15 - j1.56 = 2.78/-82.77^\circ$

We know, $\alpha + j\beta = 2 \log_e (2.78/-82.77^\circ)$

$\alpha + j\beta = 2\log_e(2.78) - 2j\frac{82.77^\circ}{57.3}$, $\alpha + j\beta = 2.05 - j2.89$

$\therefore \alpha = 2.05$ Nopar $= 2.05 \times 8.686 = 17.8063$ db and $\beta = -2.89$ rad. $= -165.56^\circ$

10. Here $Z_1 = 200 \angle 90^\circ$, $Z_2 = 50 \angle -90^\circ$

$$\sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{200\angle 90^\circ}{200\angle -90^\circ}} = 1\angle 90^\circ; \quad \sqrt{1+\frac{Z_1}{4Z_2}} = \sqrt{1+1\angle 180^\circ} = \sqrt{1-1} = 0$$

Now, $\alpha + j\beta = 2 \log (1\angle 0^\circ) = 2 \log o(1) + 2j \times \frac{90}{57.3}$

$\therefore \alpha + j\beta = 0 + j3.141$ Hence $\boxed{\alpha = 0, \beta = \pi}$

11. $Z_1 = jwL_1$, $Z_2 = -j\frac{1}{wC_2}$ $\therefore \frac{Z_1}{4Z_2} = \frac{w^2 L_1 C_2}{4} \angle 180^\circ$ if $w = 1.5/\sqrt{L_1 C_2}$

$\therefore \frac{Z_1}{4Z_2} = \frac{L_1 C_2}{4} \times \frac{1.5^2}{L_1 C_2} \angle 180^\circ = 0.5625 \angle 180^\circ.5 = -5625 + j0$

for, $w = \frac{3}{\sqrt{L_1 C_2}}$, $\frac{Z_1}{4Z_2} = \frac{L_1 C_2}{4} \times \frac{9}{L_1 C_2} = 2.25 \angle 180^\circ$ ohm. $= -2.25 + j0$

(a) $\sqrt{\frac{Z_1}{4Z_2}} = \sqrt{0.5625 \angle 180^\circ} = 0.75 \angle 90^\circ = 0 + j0.75$ $\therefore \sqrt{1+\frac{Z_1}{4Z_2}} = \sqrt{1-0.5625+j0}$

$= \sqrt{0.4375+j0} = \sqrt{0.4375\angle 0^\circ} = 0.6614 \angle 0^\circ = 0.6614 + j0.$

$\therefore \alpha + j\beta = 2 \log_e [0.6614 + j0 + 0 + j0.75] = 2 \log [0.6614 + j0.75]$

$= 2 \log_e [1 \angle 48.59^\circ] = 2 \log 1 + j\frac{2 \times 48.59}{57.3} = 0 + j1.7$ $\therefore \alpha = 0, \beta = 1.7$ rad.

not if $\left|\frac{Z_1}{4Z_2}\right| \leq 1$, $\alpha = 0$. (b) Try yourself.

12. $\frac{W_1}{W_2} = \frac{I_1^2}{I_2^2} = (e^{\alpha} \cdot e^{j\beta})^2 = e^{2\alpha} \cdot e^{j2\beta} \Rightarrow 1 = e^{2\alpha}, \alpha = 0.$

Similarly proceed in the same way.

13. Given, $R_k = 600$ ohm. $f_c = 940$ Hz

$R_k^2 = \frac{L_1}{C_2}$

$\therefore C_2 = \frac{L_1}{R_k^2}$

$f_c = \frac{1}{\pi\sqrt{L_1 C_2}}$

$= \frac{1}{\pi \sqrt{\frac{L_1}{R_k^2}} \cdot L_1} = \frac{R_k}{\pi L_1}$

$\therefore L_1 = \frac{R_k}{\pi f_c} = \frac{600}{\pi \times 940}$

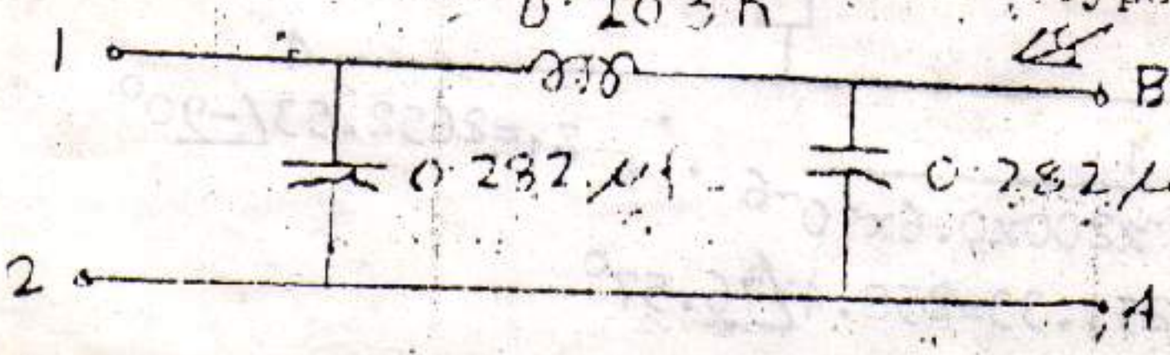
$= 0.203$ h.

$\therefore C_2 = \frac{L_1}{R_k^2} = \frac{1}{\pi R_k f_c}$

$= \frac{1}{\pi \times 600 \times 940}$

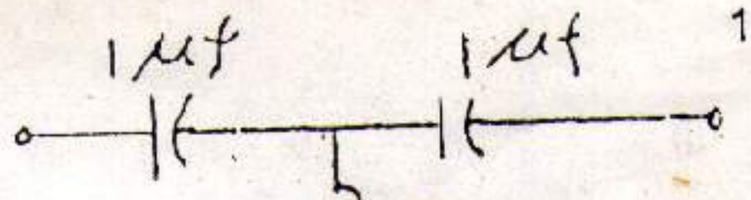
$= 0.564 \mu f.$

$\therefore$ T section K type low pass filter is :



↗ Section K type low pass filter is:

14.



$$f_c = \frac{1}{4\pi\sqrt{L_2 C_{1k}}}$$

$$= \frac{1}{4\pi\sqrt{0.015 \times 0.5 \times 10^{-6}}}$$

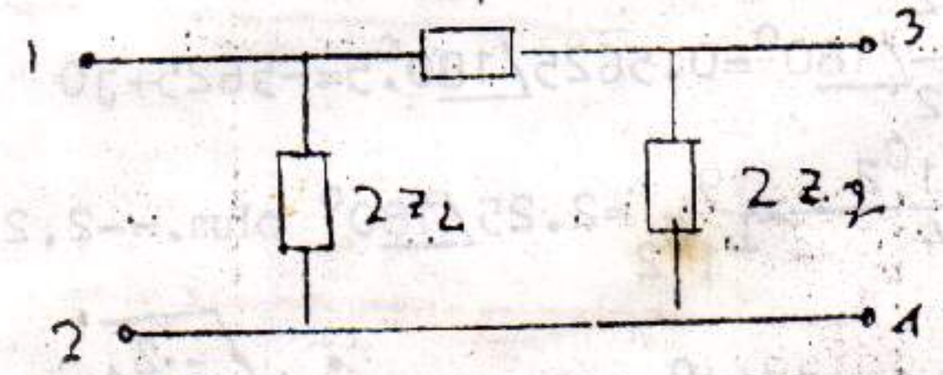
$= 919\ H_z$ Ans.

$C_{1k} = 1\mu f$ | $L_{2k} = 15$ mh.

$C_{1k} = 0.5\mu f$ | $= 0.015$ h.

$$R_k = \sqrt{\frac{L_{2k}}{C_{1k}}} = \sqrt{\frac{0.015}{0.5 \times 10^{-6}}} = 173.2 \text{ ohm. Ans.}$$

17.



(a) at 500 cycles:

$Z_1 = 50 + j2\pi \times 500 \times 100 \times 10^{-3} = 50 + j314.16 = 318.11\angle 80.96^\circ$

$2Z_2 = -j\frac{1}{2\pi \times 500 \times 0.3 \times 10^{-6}} = -j1061 = 1061\angle -90^\circ$

$\therefore Z_2 = -j530.5 = 530.5\angle -90^\circ$

$$Z_{oc} = \frac{(Z_1 + 2Z_2)2Z_2}{Z_1 + 2Z_2 + 2Z_2} = \frac{(50 + j314.16 - j1061)(1061\angle -90^\circ)}{50 + j314.16 - j2122}$$

$$= \frac{794171.07\angle -176.17^\circ}{1808.53\angle -88.42^\circ} = 439.12\angle -87.75^\circ$$

$$Z_{sc} = \frac{Z_1 \cdot 2Z_2}{Z_1 + 2Z_2} = \frac{(318.11\angle 80.96^\circ)(1061\angle -90^\circ)}{50 + j314.16 - j1061} = \frac{337514.71\angle -9.04^\circ}{748.51\angle -86.17^\circ}$$

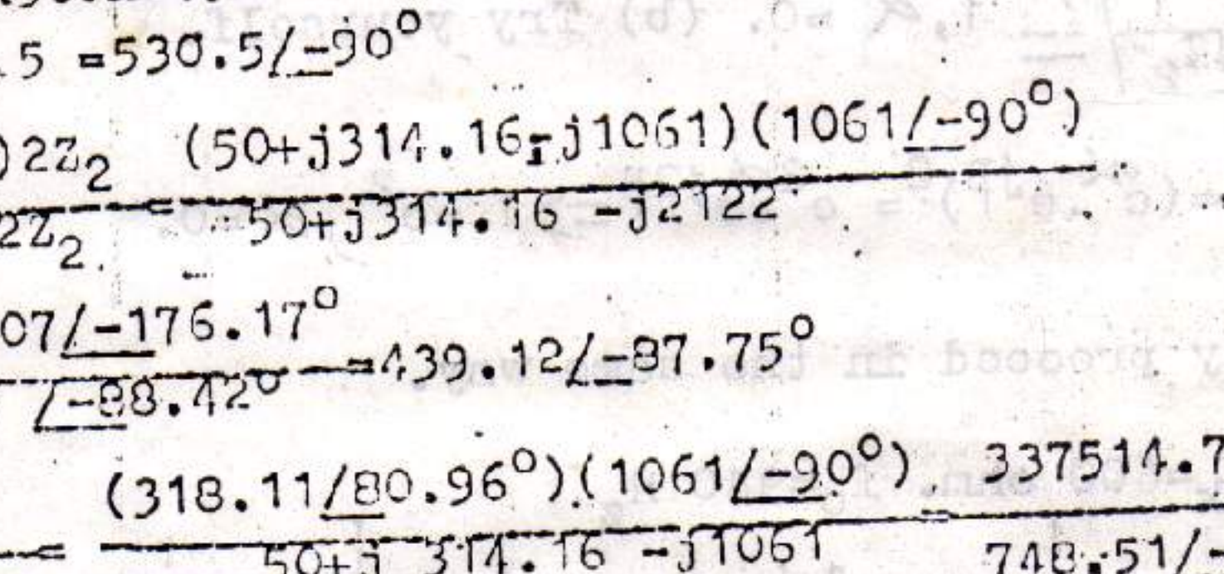
$= 450.9\angle 77.13^\circ$

(b) Characteristics impedence, $Z_{oT} = \sqrt{Z_{oc} \cdot Z_{sc}}$

$= \sqrt{(439.12\angle -87.75)(450.9\angle 77.13^\circ)} = 444.97\angle -5.31^\circ$ Ans.

Similar calculations will be for other cycles.

18.



(a) $\frac{Z_1}{2} = -j\frac{1}{2\pi \times 200 \times 0.6 \times 10^{-6}}$ $\therefore Z_1 = 2652.53\angle -90^\circ$

$Z_2 = j\omega L + R = 60 + j251.33 = 258.4\angle 76.57^\circ$

$\frac{Z_1}{4Z_2} = \frac{2652.58\angle -90^\circ}{4\times 258.4\angle 76.57^\circ} = 2.566\angle -166.57^\circ$

Characteristics impedence, $Z_{oT} = \sqrt{Z_1 Z_2 (1+\frac{Z_1}{4Z_2})}$

$= \{(2652.58\angle 90^\circ)(2584\angle 76.57)(1+2.566\angle -166.57)\}^{\frac{1}{2}} = 1050.55\angle -85.35^\circ$ Ans.

Propagation constant is $\gamma = \alpha + j\beta = 2\log_e\{\sqrt{1+\frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\}$

$= 2\log_e\{\sqrt{1-2.496-j0.596} + \sqrt{2.566\angle -166.57}\}$

$= 2\log_e\{1.269\angle -79.14 + 1.6\angle -33.235^\circ\}$

$= 2\log_e\{0.239-j1.25 + 0.187-j1.589\} = 2\log_e\{2.871\angle -81.47^\circ\}$

$= 2.11 - j162.94^\circ$ $\therefore \alpha = 2.11$ nepers, $\beta = -162.94^\circ$ Ans.

19. When the output terminals open circuited-

$R = 10$ ohm., $L = -190\times 10^{-3}$ h. $\therefore Z_{oc} = 10 + j2\pi\times 400(-190\times 10^3)$

$= 10 - j477.52 = 477.63\angle -88.8^\circ$

Similarly,

$Z_{sc} = 20 + j2\pi\times 400(250\times 10^{-3}) = 20 + j628.32 = 628.64\angle 88.2^\circ$

$\therefore Z_{oT} = \sqrt{Z_{oc}\, Z_{sc}} = 547.96\angle -0.3^\circ$ Ans.

20. 1 o—$Z_1/2$—[ ]—$Z_1/2$—3 (a) $\frac{Z_1}{2} = -j100$ $\therefore Z_1 = 200\angle -90^\circ$

$Z_2 = j100 = 100\angle 90^\circ$

2 o——————o 4

$Z_{oT} = \sqrt{Z_1 Z_2 (1+\frac{Z_1}{4Z_2})} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \sqrt{(200\angle -90^\circ)(100\angle 90^\circ) + \frac{(200\angle 90)^2}{4}}$

$= \sqrt{20000 - 10000 + j0} = 100\angle 0^\circ$ Ans.

(b) at half frequency- $Z_1 = -j400 = 400\angle -90^\circ$, $Z_2 = 50\angle 90^\circ$

$Z_{oT} = \sqrt{(50\angle 90^\circ)(400\angle -90^\circ) + \frac{(400\angle -90^\circ)^2}{4}} = \sqrt{20000-40000+j0}$

$= 141.42\angle -90^\circ$

(For high pass filter (i.e. if c is in series arm) phase shift will be negative).

(c) $\frac{Z_1}{4Z_2} = \frac{-j200}{4\times j100} = -0.5$ Since $0 > \frac{Z_1}{4Z_2} > -1$

$\therefore$ The frequency is within the pass band.

(d) $\frac{Z_1}{4Z_2} = \frac{-j400}{4\times j50} = -2$, Since $\frac{Z_1}{4Z_2} < -1$

$\therefore$ =The frequency is within the stop band.

(e) $\gamma = 2\log_e\{\sqrt{1+\frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\}$ or, $\alpha + j\beta = 2\log_2\{\sqrt{-0.5} + \sqrt{-0.5}\}$

$= 2\log_e\{0.707 + j0.707\} = 2\log_e\{1\angle 45^\circ\} = 0 + j90$ $\therefore \alpha = 0$ Ans.

For half frequency :

$\gamma = 2 \log_e \{\sqrt{1 - 1.1 + \sqrt{-2}}\}$

$\alpha + j\beta = 2 \log_e \{-j - 0.414\}$

$= 2 \log_e \{2.414 \angle -90^\circ\}$

$= 1.76 - j180^\circ$

$\alpha = 1.76$ nepers. Ans.

($\sqrt{-2} = \pm j2$ For high pass filter -ve root will be taken).

(f) From (a) & (b) we can say that the characteristics impedence in the pass band s pure resistive and in the attenuation band (i.e. stop band) is pure reactive.

(g) Yes.

21. $R_k = \sqrt{\frac{L_{1k}}{C_{2k}}}$ or, $C_{2k} = \frac{L_{1k}}{R_k^2}$ | $f_c = \frac{1}{\pi\sqrt{L_{1k} C_{2k}}}$ | $C_{2k} = \frac{L_{1k}}{R_k^2} = \frac{0.0255}{(800)^2} = 0.04 \times 10^{-6}$ f

$= \frac{1}{\pi\sqrt{L_{1k} \cdot \frac{L_{1k}}{R_k^2}}}$

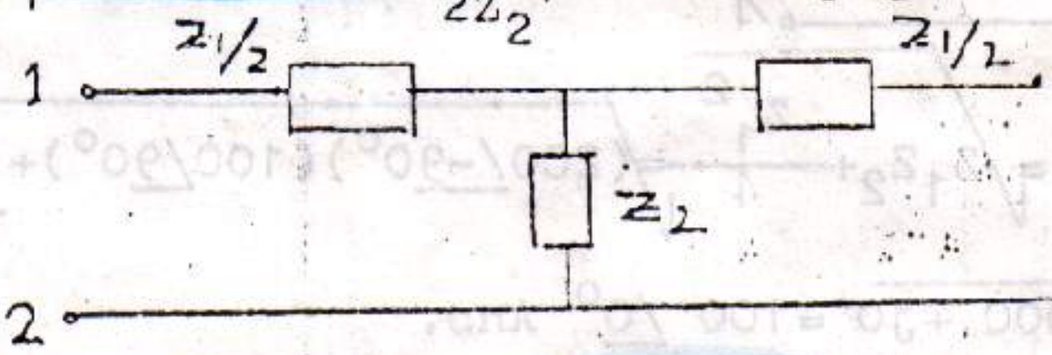
or, $L_{1k} = \frac{R_k}{\pi f_c}$

$= \frac{800}{\pi \times 10.000}$

$= 0.0255$ h.

1000, 4000, 7000, 10,000 cycles are within pass band phase shift, $\beta = \cos^{-1}(1 + \frac{Z_1}{2Z_2})$ (See page 473)

Now,



Now, $Z_1 = j2\pi fx0.0255 = j0.1602f$, $Z_2 = -j\frac{1}{2\pi fx0.04x10^{-6}} = -j\frac{3.98x10^6}{f}$

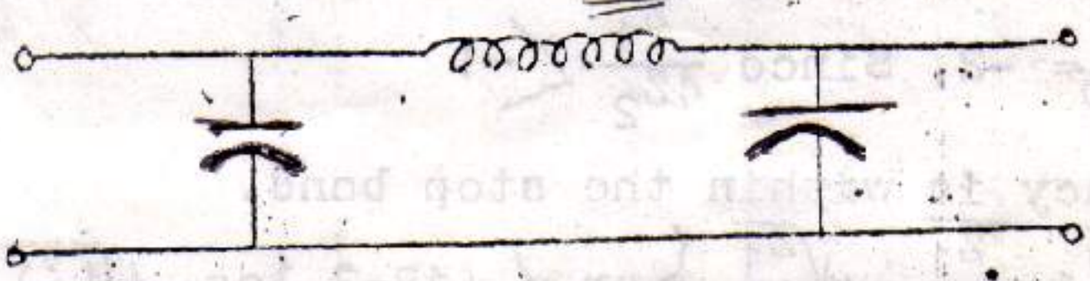
$\therefore \frac{Z_1}{2Z_2} = \frac{j\,0.1602f}{2(-j\frac{3.98x10^6}{f})} = -2.013 \times 10^{-8} f^2$

$\therefore \beta_{1000} = \cos^{-1}\{1 - 2.013 \times 10^{-8}(1000)^2\} = 11.5^\circ$

$\beta_{4000} = 47.32^\circ$, $\beta_{7000} = 89.2^\circ$, $\beta_{10000} = 180^\circ$ Ans.

Attention $\alpha = \cosh^{-1}(-1 - \frac{Z_1}{2Z_2})$

22.



Here, $Z_1 = j\omega L_1 = 2\pi f L_1 xJ = J\ 2\pi fx0.10$ $\therefore Z_1 = J.628f$ $C_1 = \frac{C}{2}$

$2Z_2 = \frac{-J}{2\pi fC_1} = \frac{-Jx10^6}{2\pi fx0.3} = \frac{-J530516.48}{f}$ $\therefore Z_2 = \frac{-J265258.24}{f}$

Now, $\sqrt{\frac{Z_1}{4Z_2}} = \sqrt{5.92 \times 10^{-7} f^2}\ \angle 180^\circ = 7.694 \times 10^{-4} f\ \angle 90^\circ$

$\sqrt{1+\frac{Z_1}{4Z_2}} = \sqrt{1+5.92 \times 10^{-7} f^2}\ \angle 180^\circ$

We know, $\alpha + J\beta = 2 \log_e \left(\sqrt{\frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right)$

$\alpha + J\beta = 2 \log_e \left(\sqrt{1+5.92 \times 10^{-7} f^2}\ \angle 180^\circ + 7.694 \times 10^{-4} f \angle 90^\circ\right)$

At 500 C/Sec.

$\therefore \alpha + J\beta = 2 \log_e \left(\sqrt{1+5.92 \times 10^{-7} \times (500)^2}\ \angle 180^\circ + 7.694 \times 10^{-4} \times 500 \angle 90^\circ\right)$

$\alpha + j\beta = 2 \log_e (0.923 + j0.3847)$

$\alpha + j\beta = 2 \log_e (1\ \angle 22.63^\circ)$, $\alpha + j\beta = 2 \log_e 1 + 2J\left(\frac{22.63}{57.3}\right) = 0 + 0.79J.$

$\therefore \alpha = 0$ And $\beta = 0.79$ rad $= 45.26^\circ$ For others similar way.

23. (a) Decibel $= 10 \log_{10} \frac{W_{(general)}}{W_{(reference)}}$

$\Rightarrow$ db $= 10 \log_{10} \frac{0.00001}{1 \times 10^{-3}} = -20$ $\boxed{db = 20}$

(b) db $= 10 \log_{10} \frac{W_{general}}{W_{reference}} = 10 \log_{10} \frac{6}{1 \times 10^{-3}}$ $\boxed{db = 37.78}$

24. Here $L_{1k} = 20h$ $\frac{C_{2k}}{2} = 8uf.$ $\therefore C_{2k} = 16 \times 10^{-6} f.$

We know for Low pass filter-

$fc = \frac{1}{2\pi\sqrt{L_{1k} C_{2k}}} = \frac{1}{\pi\sqrt{20 \times 16 \times 10^{-6}}} = 17.79$ C/S. $\boxed{\therefore f_c = 17.79 \text{ C/S}}$

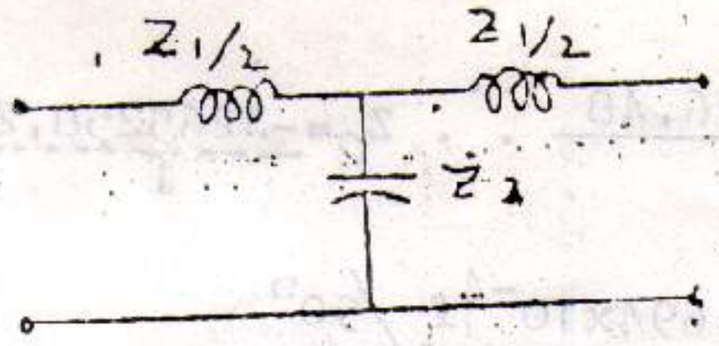
At 200 C/S

$Z_{1k} = J\ 2\pi f L_{1k} = J.\ 2\pi \times 200 \times 20 = 25132.74 \angle 90^\circ$ ohm.

$2Z_{2k} = \frac{-Jx10^6}{2\pi fx8} = 99.4 \angle -90^\circ$, $Z_{2k} = 49.74 \angle -90^\circ$ ohm.

Now, $Z_{onk} = \sqrt{\frac{Z_{1k} Z_{2k}}{(1 + \frac{Z_{1k}}{4 Z_{2k}})}} = \sqrt{\frac{1250102.488\ \angle 0^\circ}{1 + 126 \angle 180^\circ}} = \sqrt{-9974.23} = 99.87J$

$\boxed{\therefore Z_{onk} = 99.87 \angle 90^\circ}$

25. 

$\frac{Z_1}{2} = j\,100$ ⋮ $Z_2 = -j\,1000 = 1000\angle -90^\circ$

$\therefore Z_1 = 200\angle 90^\circ$

$\frac{Z_1}{4Z_2} = \frac{200\angle 90^\circ}{4 \times 1000\angle -90^\circ} = 0.05\angle 180^\circ = -0.05$

$Z_{oI} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{(200\angle 90^\circ)(1000\angle -90^\circ)(1-0.05)} = 435.89\angle 0$ Ans.

$\gamma = 2\log_e \left\{ \sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} \right\}$, $\alpha + j\beta = 2\log_e \left\{ \sqrt{0.95} + \sqrt{-0.05} \right\}$

$= 2\log_e \left\{ 0.97468 + j0.2236 \right\} = 2\log_e \left\{ 1\angle 12.92^\circ \right\} = 0 + j\,25.84^\circ$

$\therefore \alpha = 0, \quad \beta = 25.84^\circ$

(c) Since attention is Zero.

$\therefore$ The frequency is within the pass band.

(d) for 5 times frequency- $Z_1 = 1000\angle 90^\circ$, $Z_2 = 200\angle -90^\circ$.

$\frac{Z_1}{4Z_2} = \frac{1000\angle 90^\circ}{4 \times 200\angle -90^\circ} = 1.25\angle 180^\circ = -1.25$

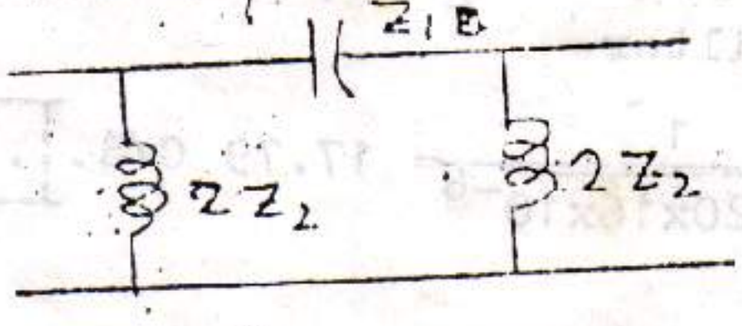
$\therefore Z_{oT} = \sqrt{(1000\angle 90^\circ)(200\angle -90^\circ)\left\{1 - 1.25\right\}} = 223.6\angle 90^\circ$

(c) $\gamma = \alpha + j\beta = 2\log_e \left\{ \sqrt{1-1.25} + \sqrt{-1.25} \right\} = 2\log_e \left\{ j0.5 + j1.118 \right\}$

$= 2\log_e \left\{ 1.618\angle 90^\circ \right\} = 0.96238 + j180^\circ$

$\therefore$ Attenuation, $\alpha = 0.96238$ neper $= 8.359$ db.

Phase shift, $\beta = 180^\circ$

26. 

$Z_1 = -j100 = 100\angle -90^\circ$

$2Z_2 = j\,500 \quad \therefore Z_2 = 250\angle 90^\circ = -0.1$

$\frac{Z_1}{4Z_2} = \frac{100\angle -90^\circ}{4 \times 250\angle 90^\circ} = 0.1\angle -180^\circ = -0.1$

(a) $Z_{o\pi} = \sqrt{Z_1 Z_2 / \left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{(100\angle -90)(250\angle 90^\circ)/(1-0.1)} = 166.667\angle 0^\circ$ Ans.

(b) $\gamma = \alpha + j\beta = 2\log_e \left\{ \sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} \right\}$ ... (-root, it is high pass filter).

$= 2\log_e \left\{ \sqrt{1-0.1} + \sqrt{-0.1} \right\}$

$= 2\log_e \left\{ 0.94868 - j0.316228 \right\} = 2\log_e \left\{ 1\angle -18.435^\circ \right\} = 0 - j36.87^\circ$

$\therefore \alpha = 0, \quad \beta = -36.87^\circ$

(c) Since attenuation is Zero. $\therefore$ The reactances given are in pass band.

(d) For one fifth frequency- $Z_1=500\angle -90^\circ$, $Z_2=50\angle 90^\circ$

$\frac{Z_1}{4Z_2}=\frac{500\angle -90}{4\times 50\angle 90}=2.5\angle -180^\circ=-2.5$ (∵ high pass)

$Z_{oT}=\sqrt{(500\angle 90^\circ)(50\angle 90^\circ)/(1-2.5)}=\sqrt{-16666.667}=-j129.1=129.1\angle 90^\circ$

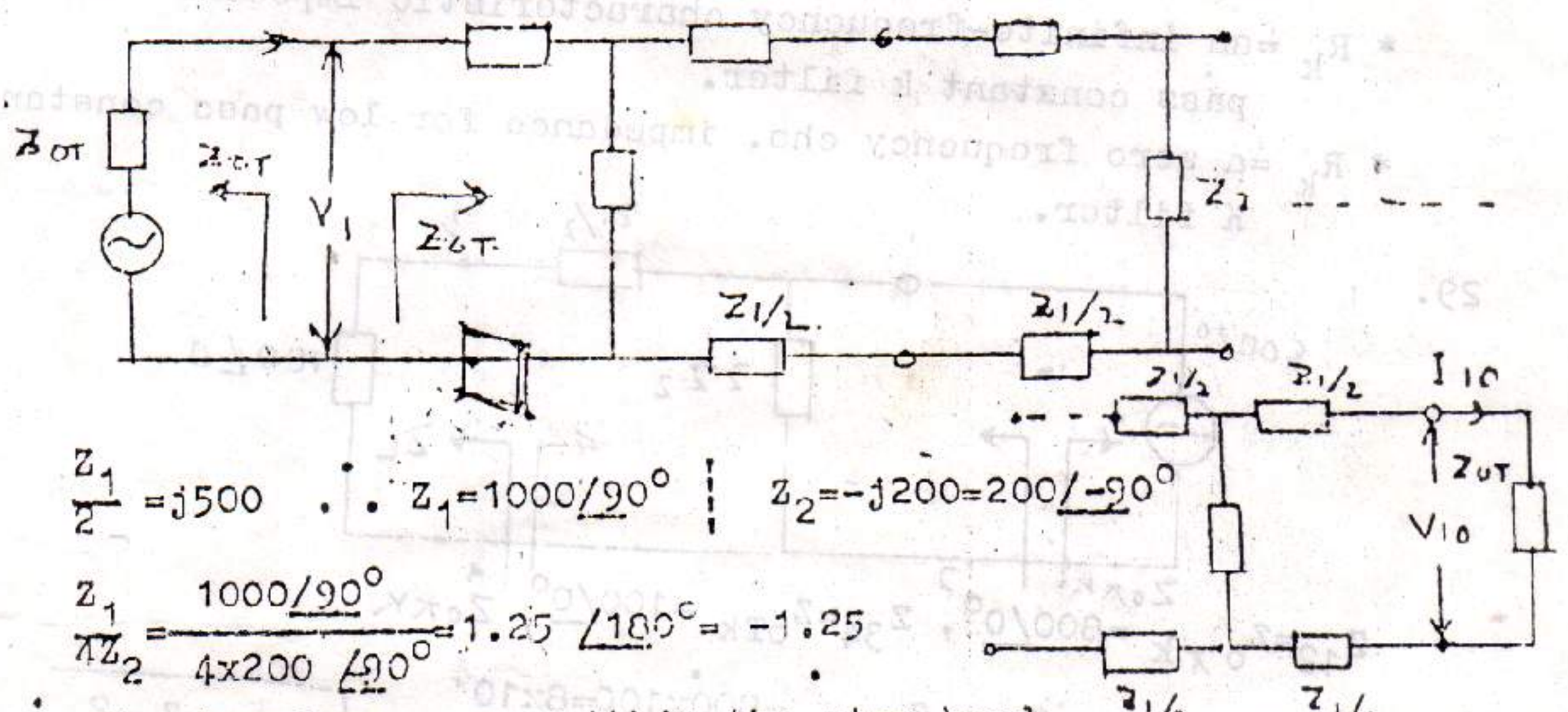
$\gamma=\alpha+j\beta=2\log_e\{\sqrt{1-2.5}+\sqrt{-2.5}\}=2\log_e\{-j1.2247-j1.581\}$

$=2\log_e\{2.8059\angle -90^\circ\}=2.06344-180^\circ$

∴ $\alpha=2.06344$ neper$=2.06344\times 8.686=17.923$ db. $\beta=-180^\circ$

Within stop band.

27.



$\frac{Z_1}{Z}=j500$ ∴ $Z_1=1000\angle 90^\circ$ | $Z_2=-j200=200\angle -90^\circ$

$\frac{Z_1}{4Z_2}=\frac{1000\angle 90^\circ}{4\times 200\angle 90^\circ}=1.25\angle 180^\circ=-1.25$

∴ The impedences one within the stop band.

$Z_{oT}=\sqrt{Z_1Z_2(1+\frac{Z_1}{4Z_2})}=\sqrt{(1000\angle 90^\circ)(200\angle -90)(1-1.25)}=223.61\angle 90^\circ$

$\gamma=2\log_e\{\sqrt{1-1.25}\ \sqrt{-1.25}\}=2\log_e\{1.618\angle 90^\circ\}^2$

$1.618^2=2.618$ strange.

∴ $e^\gamma=2.618\angle 180^\circ$

Now, $\frac{I_1}{I_2}=\frac{I_2}{I_3}=\cdots\cdots=\frac{I_9}{I_{10}}=e^\gamma$ ∴ $\frac{I_1}{I_{10}}=e^{9\gamma}$ Similarly, $\frac{V_1}{V_{10}}=e^{9\gamma}$

$V_1=100\angle 0^\circ$ Volt, $I_1=\frac{V_1}{Z_{oT}}=\frac{100\angle 0}{223.61\angle 90^\circ}=0.4472\angle -90^\circ$

∴ $I_{10}=\frac{0.4472\angle -90^\circ}{(2.618\angle 180^\circ)^9}=\frac{0.4471\angle -90^\circ}{2.618^9\angle 9\times 180^\circ}=7.74\times 10^{-5}\angle 90^\circ$

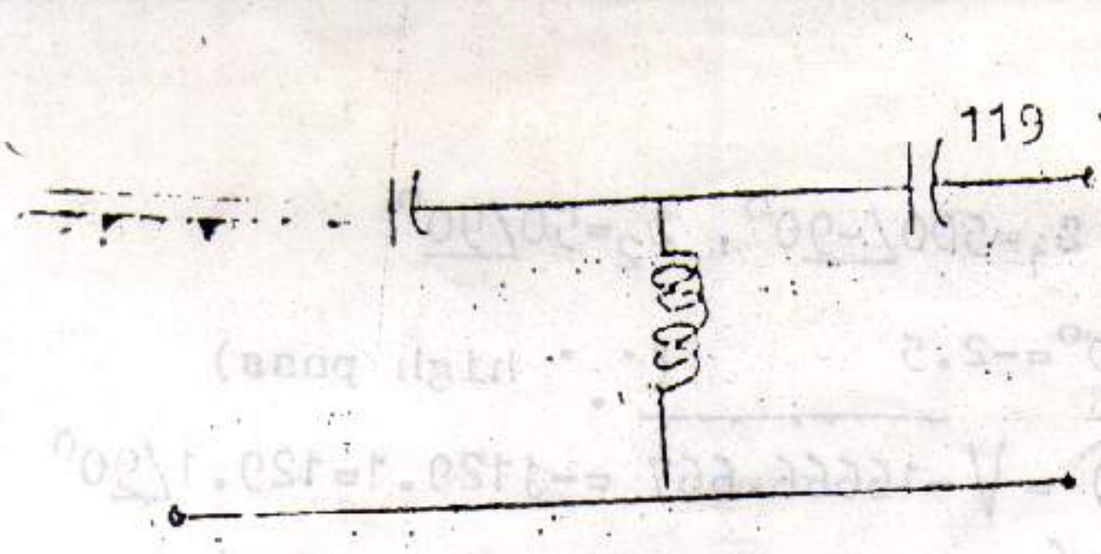
$V_{10}=\frac{100\angle 0}{(2.618\angle 180^\circ)^9}=17.31\times 10^{-3}\angle -180^\circ$ Ans.

$\frac{V_1}{V_{10}}=e^{9\gamma}$

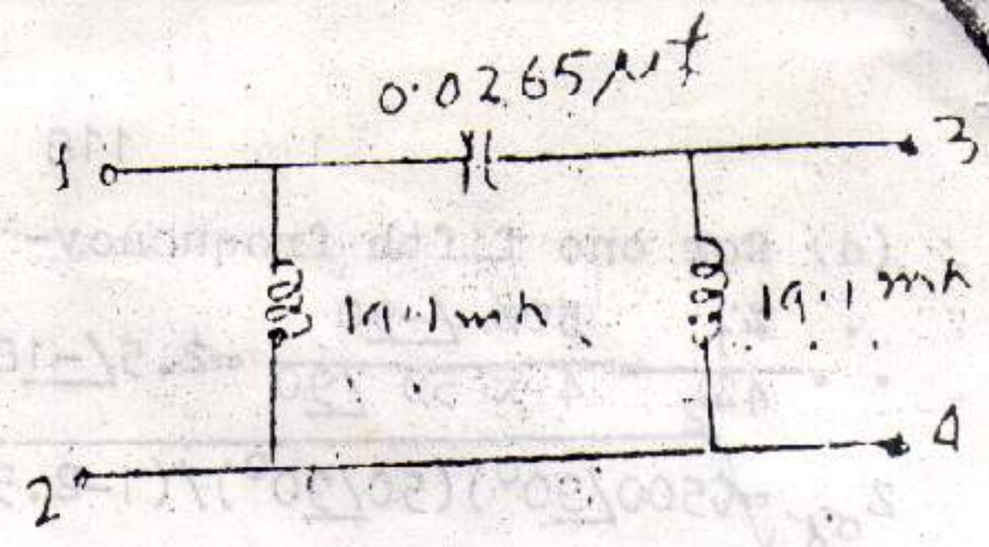
28. $f_{cn}=\frac{1}{4\pi\sqrt{L_{2k}C_{1k}}}=\frac{1}{4\pi\sqrt{L_{2k}\cdot\frac{L_{2k}}{R_k^2}}}=\frac{R_k}{4\pi L_{2k}}$ | $R_k^2=\frac{L_{2k}}{C_{1k}}$ or, $C_{1k}=\frac{L_{2k}}{R_k^2}$

$L_{2k}=\frac{R_k}{4\pi f_{cn}}=\frac{600}{4\pi\times 5000}=9.55$ mh.

$C_{1k}=\frac{L_{2k}}{R_k^2}=0.0265$ μf.



T-Section



π -Section

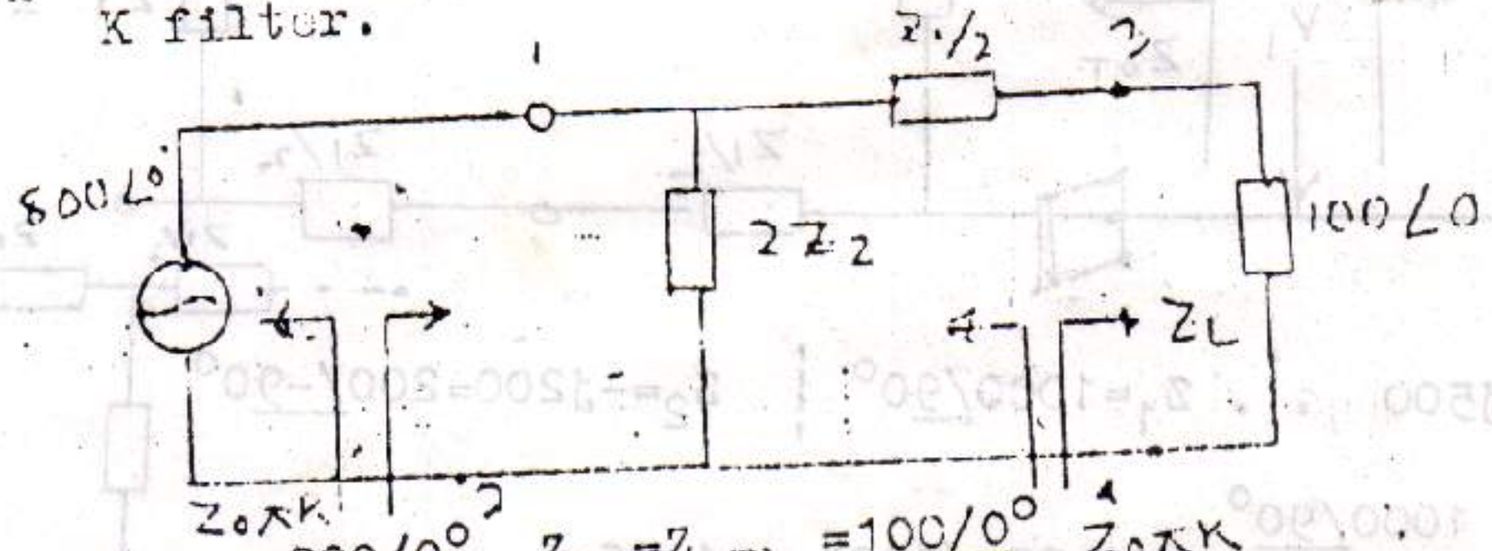
∴ In series arm,
$2C_{1k}=2\times0.0265=0.053\mu f$.

In shunt arm,
$2L_{2k}=2\times9.55=19.1$ mh.

* R_k =an infinite-frequency characteristic impedence for high pass constant k filter.

* R_k =n zero frequency cha. impedance for low pass constant K filter.

29.



$Z_{12}=Z_{0\pi k}=800\underline{/0^\circ}$, $Z_{34}=Z_{0Tk}=100\underline{/0^\circ}$

Now, $Z_{1k}\,Z_{2k}=Z_{0Tk}\,Z_{0\pi k}=800\times100=8\times10^4$

Again, $Z_{0Tk}=\sqrt{Z_{1k}\,Z_{2k}+\frac{Z_{1k}^2}{4}}$ or, $100\underline{/0^\circ}=\sqrt{8\times10^4+\frac{Z_{1k}^2}{4}}$

or, $\frac{Z_{1k}^2}{4}=(100\underline{/0})^2-8\times10^4$ or, $Z_{1k}=j529.15$ (+ve npot in tnkon

or, $\frac{Z_{1k}}{2}=264.575\underline{/90^\circ}$ Ans. ∴ $\frac{Z_{1k}}{2}$ in tnkon nn inductivo).

Now, $Z_{1k}\,Z_{2k}=8\times10^4$ or, $Z_{2k}=\frac{8\times10^4}{Z_{1k}}$

or, $2Z_{2k}=\frac{2\times8\times10^4}{529.15\underline{/90^\circ}}=302.37\underline{/-90^\circ}$ Ans.

30. given, $f_{cn}=5000$ cycles, $f_\infty=4500$ cycles, $R_k=600$ ohm.

∴ $m=\sqrt{1-\frac{f_\infty^2}{f_s^2}}=\sqrt{1-(\frac{4500}{5000})^2}$ (for high pass section)

$=0.436$.

Now, $R_k^2=\frac{L_{2k}}{C_{1k}}$ or, $C_{1k}=\frac{L_{2k}}{R_k^2}$

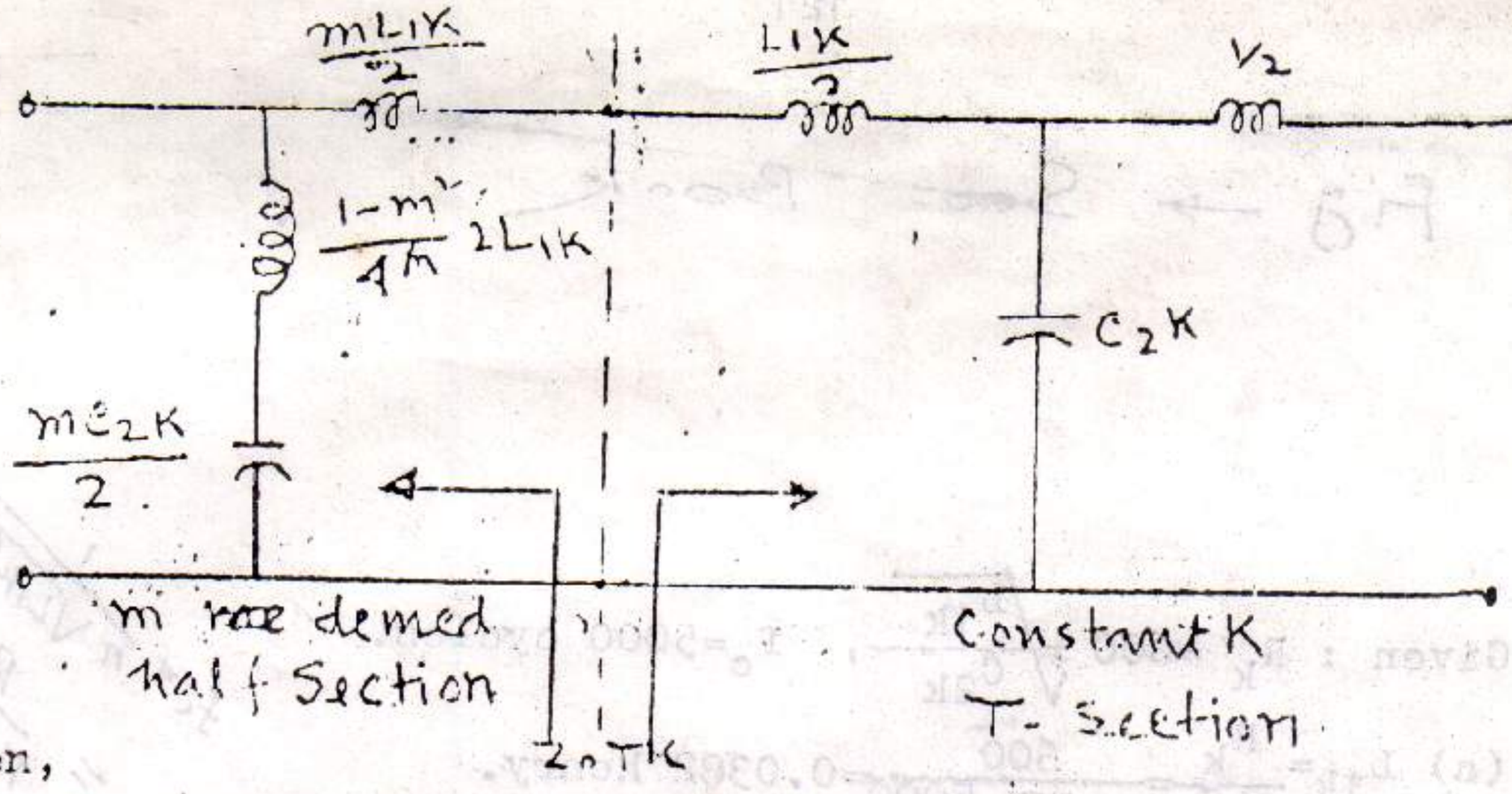
Again, $f_{cn}=\frac{1}{4\sqrt{L_{2k}\,C_{1k}}}$ ∴ $L_{2k}=\frac{R_k}{4f_{cn}}=\frac{600}{4\times5000}=9.55$ mh.

∴ $C_{1k}=\frac{9.55\times10^{-3}}{600^2}=0.0263\ \mu f$.

each series condenser $\frac{2C_{1k}}{m}=0.12064\ \mu f$. $\frac{L_{2k}}{m}=21.9$ mh

shint condenser $\frac{4m}{1-m^2}C_{1k}=0.05663\ \mu f$. shunt inductano.

31.



Given,

L_{1k}=0.3 henry, C_{2k}= 0.03 μf., m=0.6

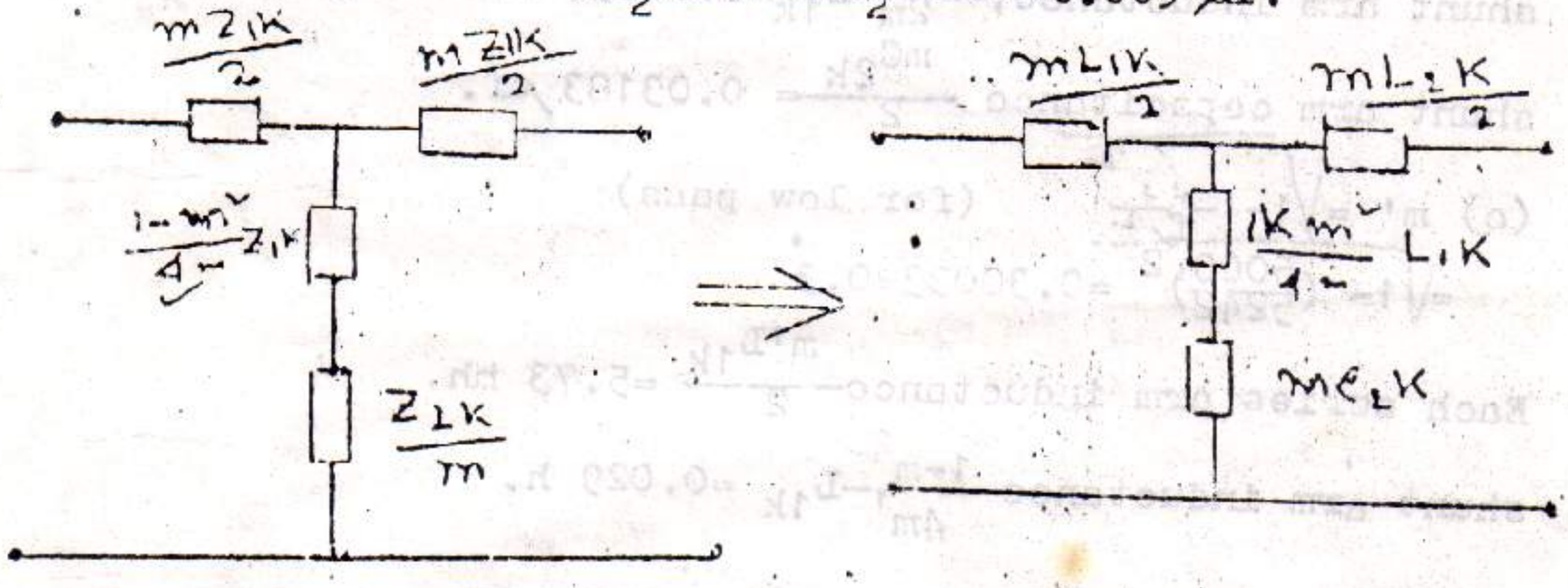
For m derived half section :

series arm inductance $\frac{mL_{1k}}{2} = \frac{0.6\times0.3}{2} = 0.09$ h.

shunt arm inductance $\frac{1-m^2}{4m} 2L_{1k} = \frac{(1-0.6)^2}{2\times 0.6} = 0.16$ h.

shunt arm capacitance $\frac{mC_{2k}}{2} = \frac{0.6\times0.03}{2} = 0.009$ μf.

32.



given:

$mL_{1k} = L_{1m}$; $mC_{2k} = C_{2m}$

$\therefore L_{1k} = \frac{L_{1m}}{m}$; or, $C_{2k} = \frac{C_{2m}}{m}$

Now,

$f_c = \frac{1}{\pi\sqrt{L_{1k}C_{2k}}}$ (for low pass)

$= \frac{1}{\pi\sqrt{\frac{L_{1m}}{m}\cdot\frac{C_{2m}}{m}}}$ or, $f_c = \frac{1}{\pi\sqrt{\frac{L_{1m}}{m^2}\cdot C_{2m}}}$ .....(1)

Now,

$\frac{1-m^2}{4m} L_{1k} = L_{2m}$ or, $\frac{1-m^2}{4m}\cdot\frac{L_{1m}}{m} = L_{2m}$ or, $(1-m^2)L_{1m} = 4m^2 L_{2m}$

or, $L_{1m} = (L_{1m}+4L_{2m})m^2$ or, $m^2 = \frac{L_{1m}}{L_{1m}+4L_{2m}}$ or, $\frac{L_{1m}}{m^2} = L_{1m}+4L_{2m}$

From(1)

$\Rightarrow f_c = \frac{1}{\pi\sqrt{(L_{1m}+4L_{2m})C_{2m}}}$ (Proved).

$L_{1m} = m^2 Z_{1m} + 4m^2 L_{2m}$

33.

Fig → See Book

Given : $R_k = 600\sqrt{\frac{L_{1k}}{C_{2k}}}$, $f_c = 5000$ cycles.

$f_c = \frac{1}{\pi\sqrt{L_{1k}C_{1k}}}$ // $\frac{R_k}{\pi L_{1k}}$

(a) $L_{1k} = \frac{R_k}{\pi f_c} = \frac{600}{\pi \times 5000} = 0.0382$ henry.

$C_{2k} = \frac{1}{\pi f_c R_k} = \frac{1}{\pi \times 5000 \times 600} = 0.1061$ μf.

(b) Given: m = 0.6

Series arm inductance, $\frac{mL_{1k}}{2} = \frac{0.6 \times 0.0382}{2} = 0.01146$ h.

shunt arm inductance, $\frac{1-m^2}{2m} L_{1k} = 0.02037$ h.

shunt arm capacitance $\frac{mC_{2k}}{2} = 0.03183$ μf.

(c) $m' = \sqrt{1 - \frac{f_c^2}{f_{c1}^2}}$ (for low pass)

$= \sqrt{1 - \left(\frac{5000}{5242}\right)^2} = 0.30033 = 0.3$

Each series arm inductance $\frac{m'L_{1k}}{2} = 5.73$ mh.

shunt arm inductance $\frac{1-m'^2}{4m'} L_{1k} = 0.029$ h.

shunt arm capacitance $m'C_{2k} = 0.03183$ μf.

CHAPTER XIV

TRANSIENT CONDITIONS

F O R M U L A S

1. Dynamic voltage equations' are:
   across inuction is $L\frac{di}{dt}$
   " capacitor is $\frac{q}{C} = \frac{1}{C}\int i\,dt$.
   " resistor is Ri.

2. Dynamic current equations' are:
   through resistor is $\frac{v}{R}$ this v is across that R.
   " inductor is $\frac{1}{L}\int v\,dt$ " " " " L.
   " capacitor is $C\frac{dv}{dt}$ " " " " C.

3. When switch is closed-
   (i) The induction acts as open ckt. or as current generator having the value of current flowing at $t=0^+$

(b) $\sqrt{\frac{940+j3.2}{1.6\angle 10^\circ}} = \sqrt{\frac{940\angle 0.2}{10\angle 10^\circ} + \frac{1000.28\angle 20^\circ}{10\angle 10^\circ}} = \sqrt{100.28\angle 10^\circ} = 10.01\angle 5^\circ$ Ans.

(c) $\frac{(-8.66+j5.0)(50-100)(28\angle 70^\circ)}{j^5} = 10\angle 150^\circ \times 50\angle -100 \times 2\angle 70^\circ / j^5$

$= 1000\angle 120/5\angle 90^\circ = 200\angle 30^\circ = 173.2 + j100$

(d) $50e^{-j\omega t} = 5(e^{-j0.785})$, $\omega t = 377 \times \frac{1}{480}$ rad $= 377 \times \frac{1}{480} \times \frac{180^\circ}{3.14} = 45^\circ$

$\therefore 5e^{-j\omega t} = 50\angle -\omega t = 50\angle -45^\circ = 35.35 - j35.35.$

23. $\sqrt[3]{\frac{10\angle 45^\circ 5\angle 60^\circ (-4.047-j2.94)}{1-j1.732}} = \sqrt[3]{10\angle 45^\circ 5\angle 60^\circ (5\angle -144)/2.0\angle -60^\circ}$

$= \sqrt[3]{5\angle 165^\circ \times 25\angle -144^\circ} = \sqrt[3]{125\angle 21^\circ} = 5\angle \frac{21+2\pi q}{3} = 5\angle 7 + \frac{2\pi}{3}$

$q=0$, $\Lambda = 5\angle 7$, $q=1$, $\Lambda = 5\angle 122.33$

$q=2$, $\Lambda = 5\angle 362.33^\circ = 5\angle 2.33^\circ$, $q=6$, $\Lambda = 5\angle 2.33^\circ$, $q=4$, $\Lambda = 5\angle 206.71^\circ$

$q=5$, $\Lambda = 5\angle 152.33^\circ$

24. $Z_a = 10\angle 68^\circ$ ohms, $Z_b = 25000\angle -90^\circ$ ohms, $Z_o = \sqrt{Z_a Z_b} = \sqrt{10\angle 68^\circ \times 25000\angle -90^\circ}$

$= \sqrt{250000\angle -22^\circ} = 500\angle -11 + 2\pi q/2.$

Where, $q = 0, 1, 2, 3 \ldots\ldots$

Where, $q = 0$, $Z_o = 500\angle -11$ ohms

$q = 1$, $Z_o = 500\angle$ .5 ohms. $q=2$, $Z_o = 500/5.5$. ohms.

25. $V = 125\angle 40^\circ$, R = 2.0 ohms, $\omega L = 8.0$ ohms, $Z = 2.0 + j8 = 8.24\angle 75.96^\circ$

$\therefore I = \frac{V}{Z} = 125\angle 40^\circ / 8.24\angle 75.96 = 15.16\angle -35.96$

$\therefore I = 15.16\angle -35.96$ Ans.

26. $Z_1 = (1-j3) = 3.162\angle 71.56$, $I_1 = 10\angle 0^\circ$, $V = IZ = 10\angle 0^\circ \times 3.162\angle -71.56$

$= 31.62\angle -71.56$

$\therefore V = 31.62\angle -71.56^\circ$, $Z_2 = (3+j6) = 6.70\angle 63.43$

$\therefore I_2 = 31.62\angle -71.56/6.7\angle 63.43 = 4.71\angle -8.12^\circ$

$\therefore I_2 = 4.17\angle -8.12$ amp.

27. $Z_{OT} = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} = \sqrt{30\angle 86.0^\circ \times 10.0\angle -90^\circ + (30\angle 86^\circ)}$

$= \sqrt{300\angle -04.0^\circ + 900\angle 172^\circ} = \sqrt{299.26 - j20.92 + 225\angle 172^\circ}$

$= \sqrt{299.26 - j20.92 - 222.8 + j31.31} = \sqrt{76.46 + j10.39}$

$= \sqrt{77.16\angle 7.73} = 8.78\angle 3.865 + 2\pi q/2.$

Where, $q=0$, $Z_{OT} = 8.76\angle 3.865$ ohms.

$q=1$: $Z_{OT} = 8.78\angle 181.93$ "

28. $\text{Log}_e\sqrt{\frac{125/-90^\circ}{5/90^\circ}} = \text{Log}\ 25/-180^\circ + 2\pi q/2 = \log^5/\frac{-180+2\pi q}{2}$

when, $q=0 \longrightarrow = \log^5 -j90/573 = 1.61 - j\pi/2$

∴ The value of the given $= 1.61 \pm j\pi/2$ Ans.

29. $\alpha + j\beta = 2\log_e\left(\sqrt{1+\frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}}\right)$

Here, $Z_1 = 25.14/-90^\circ$ ohm, $4Z_2 = 795/+90^\circ$, $\alpha = ?$ $\beta = ?$

$\therefore \alpha + j\beta = 2\log_e\left(\sqrt{1+\frac{25.14/-90^\circ}{795/+90^\circ}} + \sqrt{\frac{25.14/-90^\circ}{795/+90^\circ}}\right)$

$= 2\ \log_e\ (\sqrt{1+0.031/-180^\circ} + \sqrt{0.031/-180^\circ})$

$= 2\ \log_e\ (\sqrt{1-0.0316+j0}) + \sqrt{0.03166/-180^\circ})$

$= 2\ \log_e\ (\sqrt{0.9684+j0} + 0.01583/-90)$

$= 2\ \log_e \{\sqrt{0.9684/0^\circ} + 0.01583/-90^\circ\}$

$= 2\ \log_e\ (0.4872/0^\circ + 0.01583/-90^\circ)$

$= 2\ \log_e{}^{0.}.4872/0^\circ + 2\ \log_e{}^{0}.01583/-90^\circ$

$= 2\ [\log_e\ 0.4872+j0+\log\ 0.01583\ -j\pi/2]$

$= 2\ [0.7190+j0 - 4.14 - j1.14] = 2\ [3.421-j1.14]$

$\therefore \alpha + j\beta = 6.842 - j2.28$; $\therefore \alpha = 6.842$, $\beta = 2.28$ Ans.

30. $\alpha + j\beta = 2\ \log_e\ (\sqrt{1+\frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}})$, $Z_1 = 4\times10^3/-90^\circ$ ohms.

$4Z_2 = 1000/90^\circ$ ohms.

$\frac{Z_1}{4Z_2} = \frac{4\times10^3/-90^\circ}{1000/90^\circ} = 4/-180^\circ$

$\therefore \alpha + j\beta = 2\ \log_e\ (\sqrt{1+4/-180^\circ} + \sqrt{4/-180^\circ})$

$= 2\ \log_e\ (\sqrt{1-4+j0} + 2/-90^\circ) = 2\ \log_e\ (\sqrt{-3+j0} + 2/-90^\circ)$

Pr. $= 2\ \log_e\ (\sqrt{3/180^\circ} + 2/-90^\circ) = 2\ [\log_e\ 1.5+j1.57+\log\ 2-j1.57]$

$= 2\ [\log_e\ 3.0 + j0]$ ∴ $\alpha + j\beta = 2.19 + j0$

∴ $\alpha = 2.19$, $\beta = 0$ Ans.

31. $V_m = V - IZ$.

Where, $V = 100/0^\circ$ volts, $Z = 15/80^\circ$ ohms, $I = 10/-3^\circ$

∴ $V_m = 100/0^\circ - 15/80^\circ \times 10/+3^\circ$

$= 100/0^\circ - 150/77^\circ = 100+j0-33.74-j\ 146.155 = 66.26 - j146.155$

∴ $V_m = 160.47/-65.61$ volts. Ans.

2. (a) $(12+a)+jb = 20+j\,10$

or, $12+a = 20$ $\therefore$ $\boxed{a = 8}$ Ans.

$jb = j\,10$ $\therefore$ $\boxed{b = 10}$ Ans.

(b) $(a+10)+j50 = 100(\cos\beta + j\sin\beta)$

or, $(a+10)+j50 = 100\cos\beta + 100\,j\sin\beta$

$a+10 = 100 \times \sqrt{3}/2 = 50\sqrt{3}$ | $50 = 100\sin\beta$

$\therefore$ $A = 50\sqrt{3} - 10 = 76.666$ | $\sin\beta = .5$ $\therefore$ $\beta = 30°$

(c) $(100+j0)+5R\angle -45° = 200\angle -\theta°$ R=?, θ=?

or, $100+j0+3.535R-j3.535$ $R=200(\cos\theta - j\sin\theta)$

or, $(100+3.535R)-j(3.535)R = 200\cos\theta - 200\,j\sin\theta$

$\therefore$ $3.535R = 200\sin\theta$ ....(i), $(100+3.535R) = 200\cos\theta$....(ii)

Squaring(i) and (ii) and adding.

$12.49R^2+(100)^2+707.000R+12.49R^2=(200)^2.1$

or, $12.49R^2+353.5R+\frac{100\times100}{2} = 200 \times 100$; [Dividing 2]

or, $12.49R^2+353.5R+50\times100-200\times100=0$

or, $12.49R^2+353.5R+100(50-200)=0$

or, $12.49R^2+353.5R-15000=0$

$\therefore$ $R = \frac{-353.5 \pm \sqrt{(353.5)^2+4\times15000\times12.49}}{2\times12.49}$ | $P = -14.16+j37.43$

34. $V = 100-j\,50$, $I = -2-j8$ Amp. $= 23.28$ ✓

We know, $V=v+jv$, $I=i+ji$, here, $V = 100-j50$

Comparing; $v' = -50$ volts, $v=100$, $i'=-8$ Amp, $i=-2$

$\therefore$ $I=vi+v'i' = 100(-2)+(-50)(-8) = -200+400 = 200$ watts. absorbed Pow.

P_x = Reactiv power $= v'i - vi' = (-50)(-2)-100(-8) = 100+800 = 900$ var.

37. $V = 40\angle 80°$ volts. $I_1 = 5\angle 30°$ amp, $I_2 = -6+j8$ Amp, $I_o = I_1+I_2 = 5\angle 30° + (-6+j8)$

$= 4.33+j2.5-6+j8 = -1.67+j10.5 = 10.63\angle 99.03°$

$\therefore$ Real power, $P = VI\cos\theta = 40\times10.63\cos(-19.03)$

$\therefore$ Real power $= 401.96$ watt. Ans: 401.96 watt.

$I = 4.7\angle 57.83$

$V = 40\angle 80$

$P_{real} = 174$ watt

Check:- $VI=P$, $V = 40\angle 80° = 6.94+j39.39$, $I_o = 10.63\angle 99.03° = -1681+j10.49$

$P = VI = (6.9459+j39.39)(-1.67+j10.499)$

$= -413.44+j72.86-11.58-j65.78) = -426.02-j7.0787$

$P_{reactive} = 70.94$ var

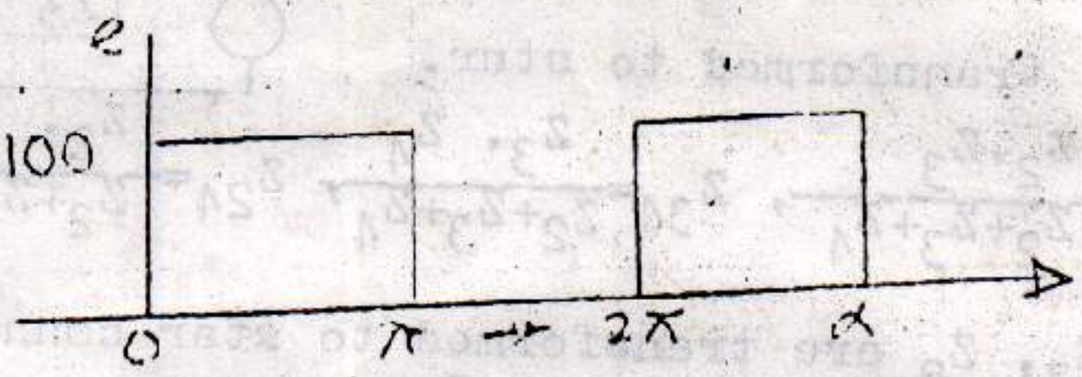
$\therefore$ $P = \sqrt{(425.02)^2+(7.07)^2} = 425.07$ watt.

CHAPTER VI.

NON - SINUSOIDAL WAVES

P R O B L E M

1.



The equation is $e = 100 \quad 0 \le \alpha \le \pi, e = 0 \quad \pi \le \alpha \le 2\pi$

$\therefore A_o = \frac{1}{2\pi}\left[\int_0^{\pi} e\,d\alpha + \int_{\pi}^{2\pi} 0\,d\alpha\right]$

$= \frac{1}{2\pi}\left[\int_0^{\pi} 100\,d\alpha + \int_{\pi}^{2\pi} 0\,d\alpha\right] = \frac{1}{2\pi}\left[100\alpha\right]_0^{\pi} = \frac{1}{2\pi} \times 100\pi = 50.$

$A_n = \frac{1}{\pi}\left[\int_0^{\pi} 100\sin n\alpha\,d\alpha + \int_{\pi}^{2\pi} 0\,d\alpha\right] = \frac{10}{\pi}\left[-\frac{1}{n}\cos n\alpha\right]_0^{\pi} = \frac{100}{n\pi}\left[(-1)^n - 1\right]$

When $n=1$, $A_1 = -\frac{100}{\pi}(-2) = 63.7$, When $n=2$, $A_2 = 0$, $n=3$,

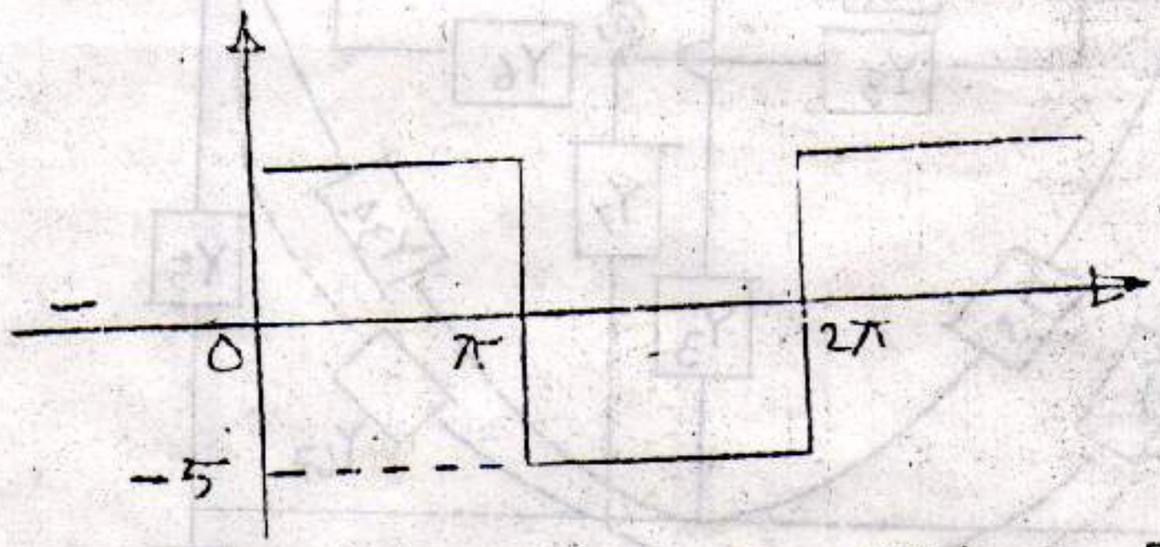
$A_3 = -\frac{100}{3\pi}(-2) = 21.2$, $n=4$, $A_4 = 0$, $n=5$, $A_5 = -\frac{100}{5\pi}(-2) = 12.7$

$B_n = \frac{1}{\pi}\left[\int_0^{\pi} 100\cos n\alpha\,d\alpha + 0\right] = \frac{1}{\pi}\left[\frac{100}{n}\sin n\alpha\right]_0^{\pi} = 0$

$\therefore$ The Fourier series is- $e = A_o + \sum_{n=1}^{\infty} (A_n \sin n\alpha + B_n \cos n\alpha)$

$= 50 + 63.7 \sin\alpha + 21.2 \sin 3\alpha + 12.7 \sin 5\alpha + \cdots$

9. (a)



The equation is $i = 10, \quad 0 < x \le \pi, i = -5, \quad \pi \le x \le 2\pi$

$\therefore A_o = \frac{1}{2\pi}\left[\int_0^{\pi} 10\,dx + \int_{\pi}^{2\pi} -5\,dx\right] = \frac{1}{2\pi}\left[10\pi - 10\pi + 5\pi\right] = 2.5$

$A_n = \frac{1}{\pi}\left[\int_0^{\pi} 10\sin nx\,dx + \int_{\pi}^{2\pi} -5\sin nx\,dx\right] = \frac{1}{\pi}\left[-\frac{10}{n}\cos nx\right]_0^{\pi} + \frac{5}{n}\left[\cos nx\right]_{\pi}^{2\pi}$

$= \frac{1}{\pi}\left[-\frac{10}{n}\left\{(-1)^2 - 1\right\} + \frac{5}{n}\left\{1 - (-1)^n\right\}\right] = \frac{1}{n\pi}\left[-15(-1)^n + 15\right]$

$= \frac{15}{n\pi}\left[1 - (-1)^n\right] \quad \therefore A_1 = \frac{15}{\pi}[2] = \frac{30}{\pi}, \quad A_2 = 0$

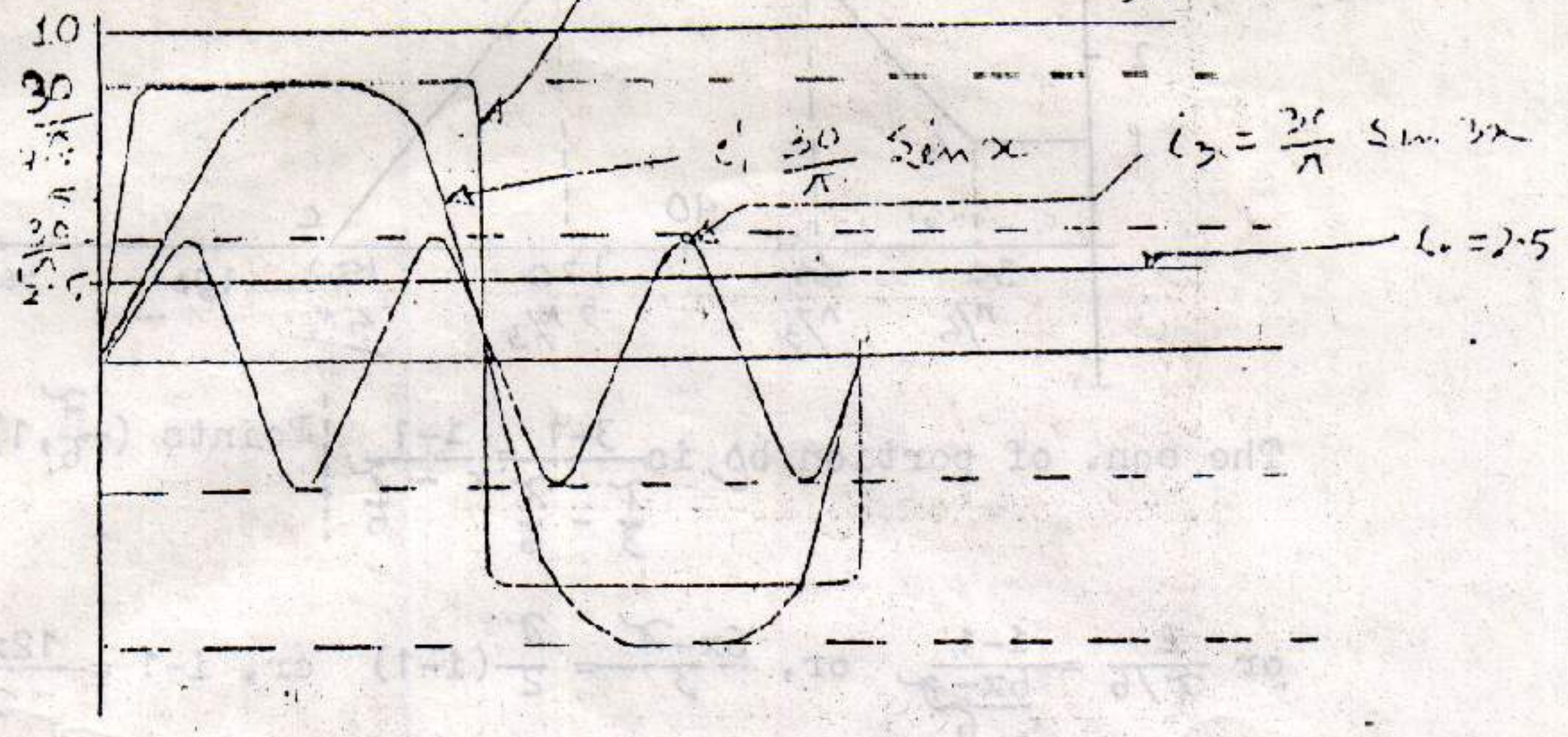
$A_3 = \frac{15}{3\pi}[2] = 30/3\pi, \quad B_n = \frac{1}{\pi}\left[\int_0^{\pi} 10\cos nx\,dx + \int_{\pi}^{2\pi} -5\cos nx\,dx\right]$

$= \frac{1}{\pi}\left[\frac{10}{n}\sin nx\right]_0^{\pi} - \frac{5}{n}\left[\sin nx\right]_{\pi}^{2\pi} = 0, \quad \therefore B_1 = 0, \; B_2 = 0, \; B_3 = 0$

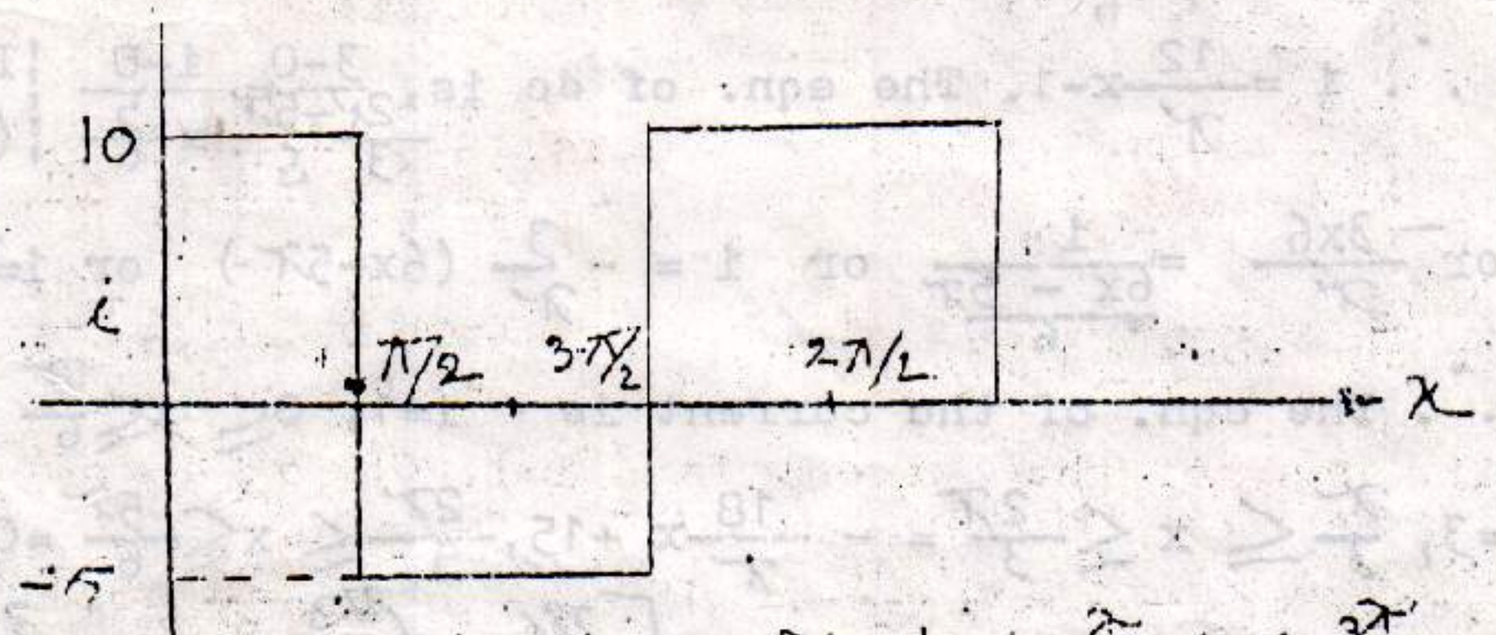
(b) $i = 2.5 + \frac{30}{\pi}\sin x + \frac{30}{3\pi}\sin 3 + \cdots$

53 $i = 2.5 + \frac{30}{\pi}\sin x + \frac{30}{3\pi}\sin 3x$

(a)



10. (a)



The equation is, $i=10;\ 0 \le x \le \pi/2, i=-5, \frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$i = 10, \frac{3\pi}{2} \le x \le 2\pi$

$$\therefore A_0 = \frac{1}{2\pi}\left[\int_0^{\pi/2} 10\,dx + \int_{\pi/2}^{3\pi/2} -5\,dx + \int_{3\pi/2}^{2\pi} 10\,dx\right]$$

$$= \frac{1}{2\pi}\left[10.\frac{\pi}{2} - 5\left(\frac{3\pi}{2} - \pi/2\right) + 10\,(2\pi - 3\pi/2)\right]$$

$$= \frac{1}{2\pi}\left[5\pi\right] = 2.5$$

$$A_n = \frac{1}{\pi}\left[\int_0^{\pi/2} 10\sin nx\,dx + \int_{\pi/2}^{3\pi/2} -5\sin nx\,dx + \int_{3\pi/2}^{2\pi} 10\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{10}{n}\cos nx\right]_0^{\pi/2} + \frac{1}{\pi}\left[\frac{5}{n}\cos nx\right]_{\pi/2}^{3\pi/2} + \frac{1}{\pi}\left[-\frac{10}{n}\cos nx\right]_{3\pi/2}^{2\pi}$$

$$= \frac{1}{\pi}\left[0 + \frac{10}{n}\right] + 0 + \frac{1}{\pi}\left[-\frac{10}{n} - 0\right] = 0 \quad \therefore A_1 = A_2 = A_3 = A_4 = A_5 = 0$$

$$B_n = \frac{1}{\pi}\left[\int_0^{\pi/2} 10\cos nx\,dx + \int_{\pi/2}^{3\pi/2} -5\cos nx\,dx + \int_{3\pi/2}^{2\pi} 10\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{10}{n}\sin nx\right]_0^{\pi/2} + \frac{1}{\pi}\left[-\frac{5}{n}\sin nx\right]_{\pi/2}^{3\pi/2} + \frac{1}{\pi}\left[\frac{10}{n}\sin nx\right]_{3\pi/2}^{2\pi}$$

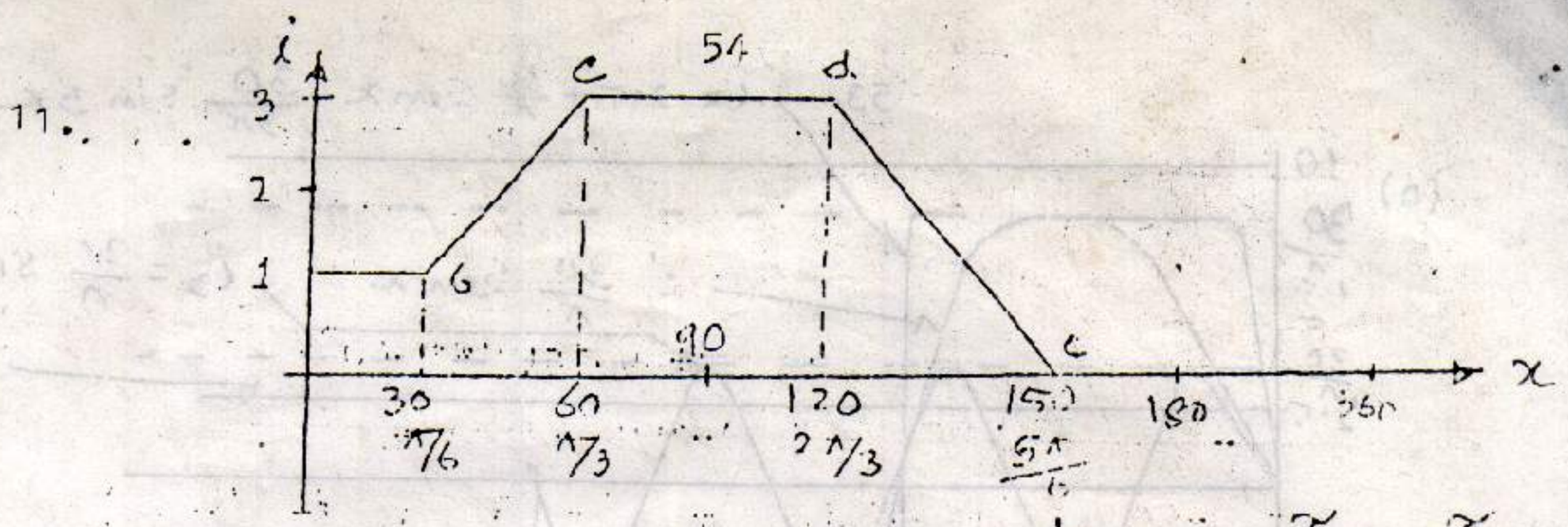
$$= \frac{1}{\pi}\left[\frac{10}{n}\sin\frac{n\pi}{2} - 0\right] + \frac{1}{\pi}\left[-\frac{5}{n}\left(\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right)\right] + \frac{1}{\pi}\left[-\frac{10}{n}\sin 2n\pi + \frac{10}{n}\sin\frac{3n\pi}{2}\right]$$

$$= \frac{1}{\pi n}\left[10\sin\frac{n\pi}{2} + 5\sin n\pi/2 + 5\sin\frac{n\pi}{2} + 10\sin n\pi/2\right] = \frac{30}{n\pi}\left\{\sin\frac{n\pi}{2}\right\}$$

$$\therefore B_1 = \frac{30}{\pi},\ B_2 = 0,\ B_3 = -\frac{30}{3\pi},\ B_4 = 0,\ B_5 = -\frac{10}{5\pi}$$

(b) $i = 2.5 + \frac{30}{\pi}\cos x - \frac{30}{3\pi}\cos 3x + \frac{30}{5\pi}\cos 5x$ Ans.

11.



The eqn. of portion bc is $\frac{3-1}{\frac{\pi}{3}-\frac{\pi}{6}} = \frac{i-1}{x-\frac{\pi}{6}}$ | Points $(\frac{\pi}{6},1),(\frac{\pi}{3},3)$

or $\frac{2}{\pi/6} = \frac{i-1}{\frac{6x-\pi}{6}}$ or, $\frac{6x-\pi}{3} = \frac{\pi}{2}(i-1)$ or, $i-1 = \frac{12x-2\pi}{\pi}$

$\therefore i = \frac{12}{\pi}x-1$. The eqn. of de is, $\frac{3-0}{2\pi-5\pi} = \frac{i-0}{x-\frac{5}{6}}$ | Points are $(\frac{2\pi}{3},3)(\frac{5\pi}{6},0)$

or $\frac{-3x6}{\pi} = \frac{i}{6x-5\pi}$ or $i = -\frac{3}{\pi}(6x-5\pi)$ or $i = \frac{18}{\pi}x+15$

$\therefore$ The eqn. of the current is $i=1,\ 0 \le x \le \frac{\pi}{6} = \frac{12x-1}{\pi}, \frac{\pi}{6} \le x \le \frac{\pi}{3}$

$=3,\ \frac{\pi}{3} \le x \le \frac{2\pi}{3} = -\frac{18}{\pi}x+15, \frac{2\pi}{3} \le x \le \frac{5\pi}{6} = 0,\ \frac{5\pi}{6} \le x \le 2\pi$

$$\therefore A_0 = \frac{1}{2\pi}\left[\int_0^{\pi/6} 1\,dx + \int_{\pi/6}^{\pi/3}\left(\frac{x}{\pi}-1\right)dx + \int_{\pi/3}^{2\pi/3} 3\,dx + \int_{\pi/3}^{5\pi/6}\left(\frac{-18}{\pi}x+15\right)dx + \int_{5\pi/6}^{2\pi} 0\,dx\right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{6} + \frac{6}{\pi}\left\{\left(\frac{\pi}{3}\right)^2-\left(\frac{\pi}{6}\right)^2\right\} - \left(\frac{\pi}{3}-\frac{\pi}{6}\right) + 3\left(\frac{2\pi}{3}-\frac{\pi}{3}\right) - \frac{9}{\pi}\left\{\left(\frac{5\pi}{6}\right)^2 - \left(\frac{2\pi}{3}\right)^2 + 15\left(\frac{5\pi}{6}-\frac{2\pi}{3}\right)\right\}\right] = \frac{1}{2\pi}\times\frac{7\pi}{4} = 0.875 \text{ Ans.}$$

$$A_1 = = \frac{1}{\pi}\left[\int_0^{\pi/6} \text{Sinn}\,dx + \int_{\pi/6}^{\pi/3}\left(\frac{12}{\pi}x-1\right)\text{Sinn}.\,dx + \int_{\pi/3}^{5\pi/6} 3\,\text{Sinn}.\,dx + \int_{5\pi/6}^{2\pi} 0.\,\text{Sinn}.\,dx\right] = \text{Evolute yourself. Ans.}$$

$$B_1 = \frac{1}{\pi}\left[\int_0^{\pi/6} \text{Cosx}\,dx + \int_{\pi/6}^{\pi/3}\left(\frac{12}{\pi}x-1\right)\text{Cosx}\,dx + \int_{\pi/3}^{2\pi/3} 3\,\text{Cosx}\,dx + \int_{2\pi/3}^{5\pi/6}\left(\frac{-18}{x}+15\right)\text{Cosx}\,dx + 0\right]$$

= Do yourself Ans.

14. $v = 4.0\text{Sin}\omega t - 3.0\text{Cos}\omega t - 7.66\text{Sin}2\omega t + 6.43\text{Cos}2\omega t - 2\text{Sin}3\omega t - 1.5\text{Cos}3\omega t$

$$= 5\left\{\frac{4}{5}\text{Sin}\omega t - \frac{3}{5}\text{Cos}\omega t\right\} - \sqrt{(7.66)^2+(6.43)^2}\left\{\frac{7.66}{10}\text{Sin}2\omega t - \frac{6.43}{10}\text{Cos}2\omega t\right\} - \sqrt{2^2+1.5^2}\left\{\frac{2}{2.5}\text{Sin}3\omega t + \frac{1.5}{2.5}\text{Cos}3\omega t\right\}$$

$$= 5\left\{\text{Sin}\omega t\,\text{Cos}36.87^\circ - \text{Sin}36.87^\circ\text{Cos}\omega t\right\} - 10\left\{\text{Sin}2\omega t\,\text{Cos}40^\circ - \text{Cos}2\omega t\,\text{Sin}40^\circ\right\} - 2.5\left\{\text{Sin}3\omega t\,\text{Cos}36.87 + \text{Cos}3\omega t\,\text{Sin}36.87\right)$$

$$= 5\text{Sin}(\omega t - 36.87^\circ) - 10\text{Sin}(2\omega t - 40^\circ) - 2.5\text{Sin}(3\omega t + 35.87^\circ)$$

C. Neither of the voltage and current waves are symmetry w.r.t. the positive and negative loops, because each contains a second harmonic.

20. $v = 100\sin(\omega t+70^\circ)-60\sin(2\omega t--30^\circ)+30\sin(3\omega t--60^\circ)$

$i = 50\cos(\omega t--60^\circ)+30\sin(2\omega t+70^\circ)-15\cos(3\omega t--90^\circ)$

$= 50\sin(\omega t--60^\circ+90^\circ)+30\sin(2\omega t+70^\circ)-15\sin(3\omega t-90^\circ+90^\circ)$

$= 50\sin(\omega t+30^\circ)+30\sin(2\omega t+70^\circ)-15\sin 3\omega t$

Now, the fundamentals should be brought into phase by shifting the current wave forward 40°. Shifting the fundamentals by 40° corresponding in shifting the nth harmonic by nx40.

$\therefore i = 50\sin(\omega t+70^\circ)+30\sin(2\omega t+70^\circ+80^\circ)-15\sin(3\omega t+120^\circ)$

$= 50\sin(\omega t+70^\circ)+30\sin\{180^\circ+(2\omega t-30^\circ)\}-15\sin\{180^\circ+(3\omega t-60^\circ)\}$

$= 50\sin(\omega t+70^\circ)-30\sin(2\omega t-30^\circ)+15\sin(3\omega t-60^\circ)$

$\therefore$ The corresponding harmonics of the current and voltage waves are hence in phase.

Again all harmonic of the voltage wave are double of the corresponding current wave. $\therefore$ The waves are the same shape.

21. $e = 100\sin(\omega t-20)+50\sin(3\omega t+60^\circ)-25\cos(5\omega t-30^\circ)$

$= 100\sin(\omega t-20^\circ)+50\sin(3\omega t+60^\circ)-25\sin(5\omega t+60^\circ)$

$i = 20\cos(\omega t-60^\circ)-10\sin(3\omega t++15)+5\sin(5\omega t-70^\circ)$

$= 20\sin(\omega t+30^\circ)-10\sin(3\omega t+15^\circ)+5\sin(5\omega t-70^\circ)$

Shifting the fundamental of the voltage wave forward by 50° which corresponding to shift the nth harmonic by $n50^\circ$.

$\therefore V = 100\sin(\omega t-20+50)+50\sin(3\omega t+60^\circ+150)-25\sin(5\omega t+60^\circ+250)$

$= 100\sin(\omega t+30^\circ)-50\sin(3\omega t+30^\circ)+25\sin(5\omega t-50^\circ)$

All harmonics of the voltage wave are 5 times of the corresponding current wave. which fulfilled the first condition.

But the third and fifth harmonics of the current and voltage wave are not in phase.

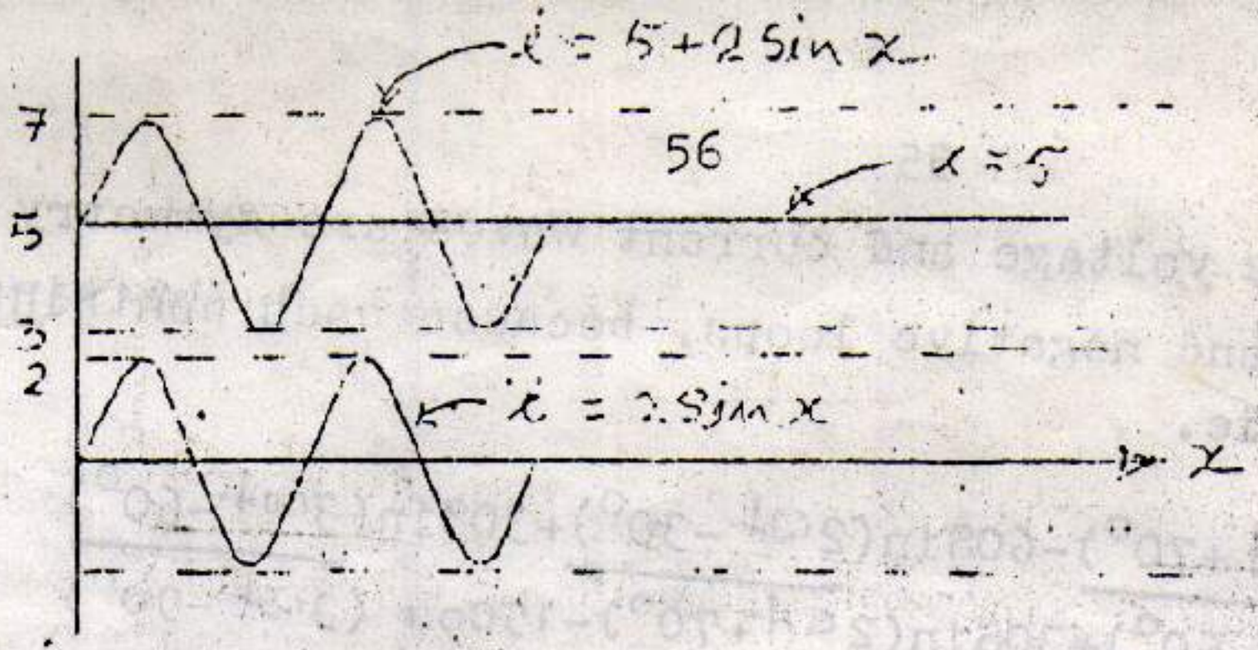
$\therefore$ The waves are not the same shape.

23. $V = \sqrt{\dfrac{100^2 + 50^2 + 40^2 + 20^2}{2}} = 82.46$ volts.

24. Volt-meter will read the effective value.

$\therefore V = \sqrt{100^2 + 70^2 + 50^2} = 131.9$ volts.

26.



(a) $i = 5+2\sin x$ $\therefore$ $I_{max} = 5+2 = 7$ amp. $I_{min} = 5-2 = 3$ amp.

$I_{av} = 5$ amp ($\because$ $i = 5 + 2\sin x$)

(b) The max^m value of ac component $I_{m(ac)} = 2$ amp. ($\because$ $i = 5+2\sin x$)

Now, $0.5\ (I_{max} - I_{min}) = 0.5\ (7-3) = 2$ amp.

$\therefore$ $I_{m(ac)} = 0.5\ (I_{max} - I_{min})$

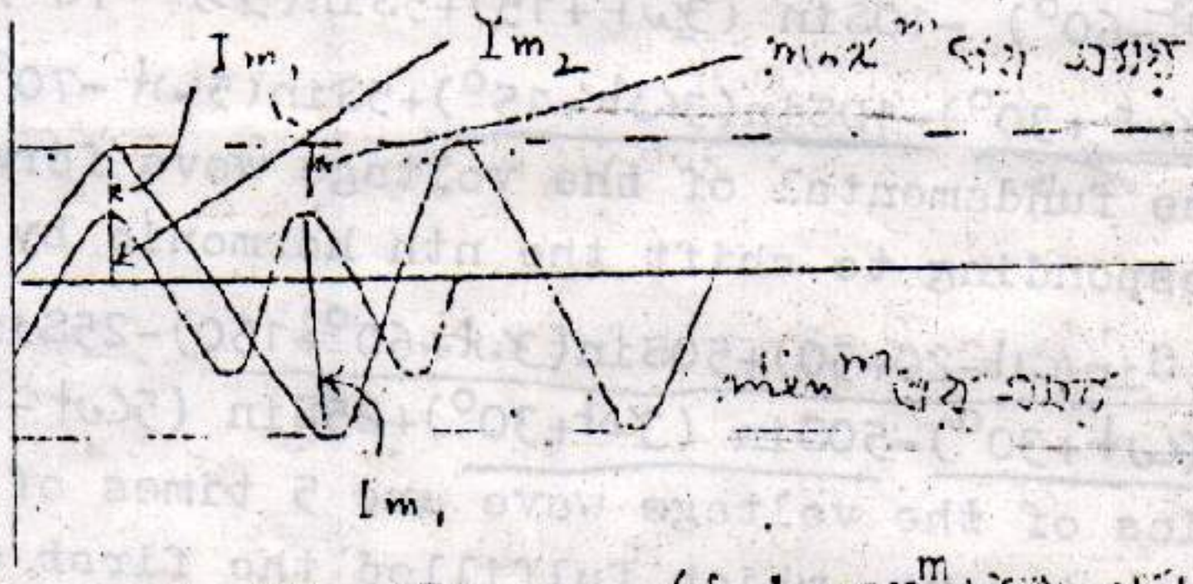
(c) $I_{effective} = \sqrt{5^2 + \frac{2^2}{2}} = \sqrt{27} = 5.2$ amp [$\because$ $I_{eff} = \sqrt{I_0^2 + \frac{I_m^2}{2}}$]

27. (a) yes. (b) The average value is 4 amp. $\therefore$ Idc = 4.

The effective value is, $5 = \sqrt{(I_{dc})^2 + \frac{I_{m(ac)}^2}{2}}$

or $25 = 16 + \frac{I_m^2}{2}$ $\therefore$ $I_m(ac) = 4.243$ amp. Ans.

28. (a) $I_o = I_b + I_{m_2}$ $\therefore$ $I_b = I_o - I_{m_2} = 0.2 - 0.01 = 0.19$ amp.



$\therefore$ $I_{max} = I_o + I_{m1} + I_{m_2}$ ($\because$ max^m [illegible])

$= 0.2 + 0.1 + 0.01 = 0.31$ amp.

$I_{min} = I_o - I_{m_1} + I_{m_2}$

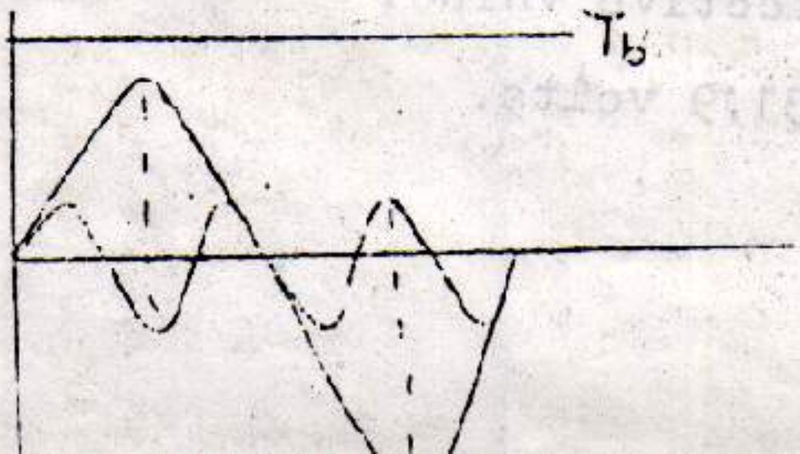
$= 0.2 - 0.1 + 0.01 = 0.11$ amp.

[$\because$ min^m [illegible] I_o [illegible] I_{m_2}]

$i = 0.2 + 0.1\sin x - 0.01\cos 2x$ $\therefore$ $I_{av} = 0.2$

$0.5\ (I_{max} + I_{min}) = 0.5(0.31+0.11) = 0.21$

30.



(d.c. value is always average value, so no calculation is required.)

$\therefore I_{max} = 0.2 + 0.07 - 0.005 = 0.265$ amp.

$I_{min} = 0.2 - 0.07 + 0.005 = 0.135$ amp. $I_{av} = 0.2$

31. $e = 100\sin(\omega t+30^\circ) - 50\sin(2\omega t+90^\circ) + 25\sin(3\omega t+150^\circ)$ volts.
$i = 20\sin(\omega t+40^\circ) + 10\sin(2\omega t+30^\circ) - 5\sin(3\omega t-30^\circ)$ amp.

$\therefore P = \frac{100\times20}{2}\cos(30^\circ-40^\circ) + \frac{-50\times10}{2}\cos(90^\circ-30^\circ) + \frac{25\times-5}{2}\cos(150^\circ+30^\circ)$

$= 918.54$ watts.

32. $e = 100\sin(4\omega t-20^\circ) + 50\sin(3\omega t+60^\circ) - 25\cos(5\omega t-30^\circ)$
$= 100\sin(\omega t-20) + 50\sin(3\omega t+60^\circ) - 25\sin(5\omega t+60^\circ)$
$i = 20\cos(\omega t-60^\circ) - 10\sin(3\omega t+15^\circ) + 5\sin(5\omega t-70^\circ)$
$= 20\sin(\omega t+30^\circ) - 10\sin(3\omega t+15^\circ) + 5\sin(5\omega t-70^\circ)$

$\therefore P = \frac{100 \times 20}{2}\cos(-20-30) - \frac{50\times10}{2}\cos(60-15) - \frac{25\times5}{2}\cos(60^\circ+70^\circ)$

$= 506.2$ watts.

33. $E = \sqrt{\frac{100^2+50^2+25^2}{2}} = 81$ volt. $I = \sqrt{\frac{20^2+10^2+5^2}{2}} = 16.2$ amp.

$\therefore P = E\,I\cos\theta$ $\therefore \cos\theta = \frac{918.54 \text{ From } 31}{81 \times 16.2} = 0.7$ Ans.

34. $E = \sqrt{\frac{100^2+50^2+25^2}{2}} = 81$. $I = \sqrt{\frac{20^2+10^2+5^2}{2}} = 16.2$

$\therefore \cos\theta = \frac{506.2}{81 \times 16.2} = 0.386$ Ans.

35. $v = 100\sin(\omega t+60^\circ) - 50\sin(3\omega t-30^\circ)$
$i = 10\sin(\omega t+60^\circ) + 5\cos(3\omega t+60^\circ) = 10\sin(\omega t+60^\circ) + 5\sin(3\omega t+150)$

(a) $P = \frac{100 \times 10}{2}\cos(60-60^\circ) - \frac{50\times5}{2}\cos(-30-150^\circ) = 625$ watts.

$V = \sqrt{\frac{100^2+50^2}{2}} = 79.057$, $I = \sqrt{\frac{10^2+5^2}{2}} = 7.9057$.

$\therefore \cos\theta = \frac{P}{VI} = \frac{625}{79.057 \times 7.9057} = 1$ Ans.

(b) Let it be Im_3 $\therefore i = 10\sin(\omega t+60^\circ) + Im_3\sin(3\omega t+150^\circ)$

$\therefore P = \frac{100 \times 10}{2}\cos(60^\circ-60^\circ) - \frac{50 \times Im_3}{2}\cos(-30-150^\circ)$

$= 500 + 25\ Im_3$, $I = \sqrt{\frac{10^2+Im_3^2}{2}}$ $\therefore 1.8 = \frac{500+25\ Im_3}{79\times\sqrt{\frac{10^2+Im_3}{2}}}$

or, $0.64 \times 79^2 \times \frac{100+Im_3^2}{2} = (500+25\ Im_3)^2$

or, $200000 + 2000\ Im_3^2 = 250000 + 25000\ Im_3 + 625\ Im_3$

or, $11\ Im_3^2 - 200\ Im_3 - 400 = 0$

$\therefore Im_3 = \frac{200 \pm \sqrt{200^2 + 4 \times 11 \times 400}}{2 \times 11} = \frac{200 \pm 240}{22}$

$\therefore Im_3 = 20$ amp. Ans.

36. From problem (33) E=81 volts. I=16.2 amp. Cosθ=0.7

$\therefore \theta = 45.57^\circ$

Sinc the fundamental of current loads the fundamental of voltage, the equavalent sinee wave of current must leads the equivalent sine wave ofvoltage.

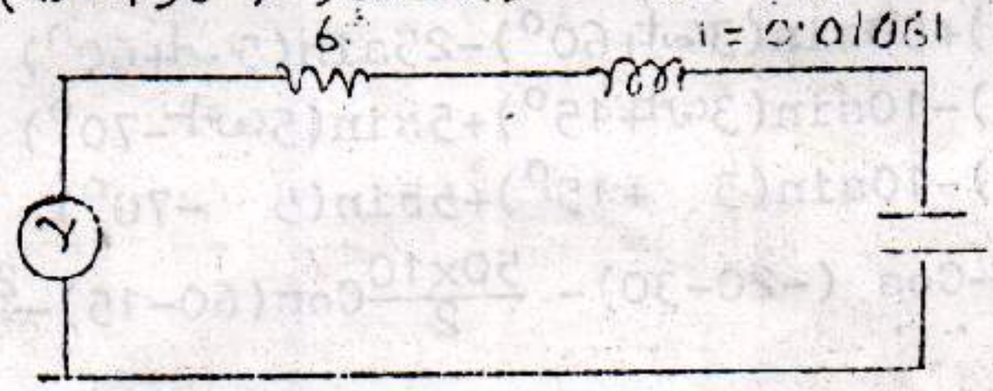
$\therefore$ The equavalent wave of voltage and current are-

$e = \sqrt{2} \times 81 \sin\omega t$ or, $e = 114.55 \sin\omega t$ Ans.

$i = \sqrt{2} \times 16.2 \sin(\omega t + 45.57^\circ) = 22.91 \sin(\omega t + 45.57^\circ)$ Ans.

33. $v = 100\sin(\omega t + 30^\circ) - 50\sin(3\omega t + 60^\circ) + 30\cos\omega t$

$= 100\sin(\omega t + 30^\circ) - 50\sin(3\omega t + 60^\circ) + 30\sin(5\omega t + 90^\circ)$



Fundamental

$V_1 = \frac{100}{\sqrt{2}} \angle -70.71^\circ$ volts., $R_1 = 6$ ohm. $X_{L1} = \omega L_1 = 377 \times 0.01061 = 4$ ohm.

$X_{c1} = \frac{10^6}{377 \times 88.4} = 30$ ohm. $\therefore Z_1 = 6 + j4 - j30 = 6 - j26 = 26.68 \angle -77^\circ$

$= 2.65 \angle 77^\circ$

$\therefore I_1 = \frac{V}{Z_1} = \frac{70.71}{26.68 \angle -77}$, I_1 leads by 77°

$P_1 = I_1^2 R_1 = 42.135$ watts., $V_{c1} = I_1 X_{c1} = 2.65 \times 30 = 79.5$ volts.

Third harmonic

$V_1 = \frac{50}{\sqrt{2}} = 35.355$ volts. $R_3 = 6$ ohm.

$X_{L3} = 3 \times 377 \times 0.01061 = 12$ ohm. $X_{c_3} = \frac{10^6}{377 \times 3 \times 88.4} = 10$ ohm.

$\therefore Z_3 = 6 + j12 - j1 \quad 6 + i2 = 6.325 \angle 18.43^\circ$

$\therefore I_3 = \frac{V_3}{Z_3 \angle 18.43} = 5.59 \angle -18.43$. I_3 lags V_3 by 18.43°

$P_3 = I_3^2 R_3 = 5.59^2 \times 6 = 187.49$ watts.

$V_{c3} = I_3 X_{c3} = 5.59 \times 10 = 55.9$ volts.

Fifth harmonic

$V_5 = \frac{30}{\sqrt{2}} = 21.21$ $R_5 = 6$ ohm. $X_{L5} = X_{L1} \times 5 = 20$ ohm.

$X_{c5} = \frac{X_{c3}}{5} = 6$ ohm. $Z_5 = 6 + j20 - j6 = 6 + j14 = 15.23 \angle 66.8^\circ$

$\therefore I_5 = \frac{V_5}{Z_5 \ 66.8} = 1.39 \angle -66.8$ $\therefore$ I_5 lags V_5 by 66.8°

$P_5 = I_5^2 R_5 = 1.39^2 \times 6 = 11.63$ watts.

$V_{c5} = I_5 \times X_{c5} = 1.39 \times 6 = 8.34$ volts.

$\therefore I = \sqrt{I_1^2 + I_3^2 + I_5^2} = 6.34$ amp.

$P = P_1 + P_3 + P_5 = 241.25$ watts.

$V_c = \sqrt{Vc_1^2 + Vc_3^2 + Vc_5^2} = 97.54$ volts.

the eqn. of the current wave is -

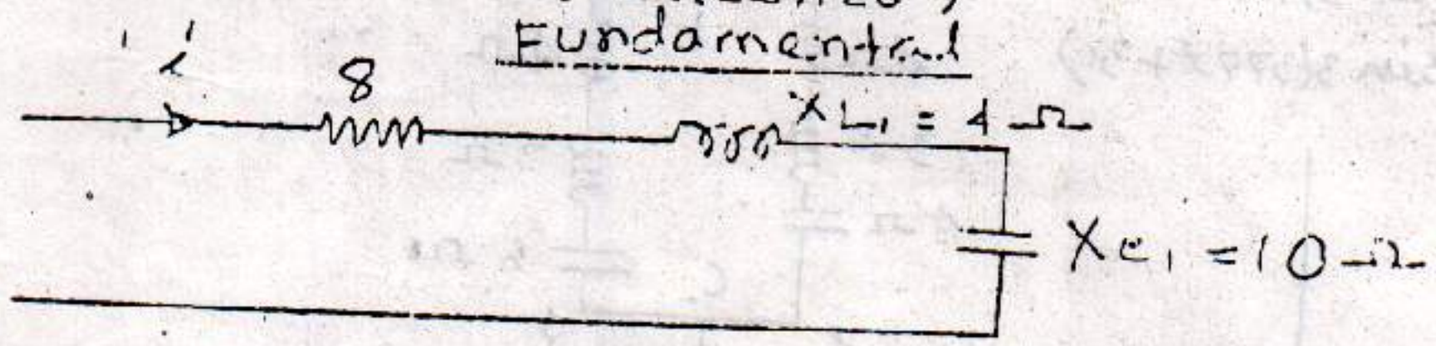
$i = \sqrt{2} \times 2.55 \sin(\omega t + 30^\circ + 77^\circ) - \sqrt{2} \times 5.59 \sin(3\omega t + 60^\circ - 18.43^\circ) + \sqrt{2} \times 1.39 \sin(5\omega t + 90^\circ - 66.8) = 3.75 \sin(\omega t + 107^\circ) - 7.9 \sin(3\omega t + 41.57) + 1.97 \sin(5\omega t + 23.2^\circ)$

$\therefore P = \frac{100 \times 3.75}{2} \cos(30 - 107) + \frac{50 \times 7.9}{2} \cos(60 - 41.57) + \frac{30 \times 1.97}{2} \cos(90 - 23.2) = 241.2$ watts.

$V = \frac{\sqrt{100^2 + 50^2 + 30^2}}{2} = 81.85$ $\therefore$ p.f. $\cos\theta = \frac{P}{VI} = 0.405$ lags.

39. $i = 10\sin(\omega t - 60^\circ) + 5\sin(2\omega t + 20^\circ)$

Fundamental



Since 60 cycle capacitive reactance is 10 $\therefore X_{c_1} = 10$ ohm.

" 60 " inductive " 4, $\therefore X_{L_1} = 4$ ohm.

$\therefore Z_1 = 8 + j4 - j10 = 8 - j6 = 10 \angle -36.87^\circ$

$\therefore I_1 = \frac{10}{\sqrt{2}} = 7.071$, $V_1 = I_1 Z_1 = 7.071 \times 10 = 70.71$ volts.

V_1 = lags I_1 by 36.87°

2nd harmonic

$R_2 = 8$ ohm. $X_{L_2} = 2 \times X_{L_1} = 2 \times 4 = 8$ ohm. $X_{c_2} = \frac{X_{c1}}{2} = 5$ ohm.

$\therefore Z_2 = 8 + j8 - j5 = 8 + j3 = 8.544 \angle 20.5^\circ$, $I_2 = \frac{5}{V_2} = 3.535$

$\therefore V_2 = I_2 Z_2 = 30.2$. V_2 load I_2 by 20.6°

$\therefore v = \sqrt{2} \times 70.71 \sin(\omega t - 60^\circ - 96.87) + \sqrt{2} \times 30.2 \sin(2\omega t + 20^\circ + 20.6)$

$= 100 \sin(\omega t - 96.87) + 42.7 \sin(2\omega t + 40.6)$ Ans.

40 - 43 ====> same as example -13.

44. $i = 20\sin(\omega t + 30^\circ) - 10\sin(2\omega t - 30^\circ) + 5\sin(3\omega t - 40^\circ)$

$= 20\sin(\omega t + 30^\circ) + 10\sin(2\omega t + 150^\circ) + 5\sin(3\omega t - 40^\circ)$

$i_2 = 15\cos\omega t + 10\cos(2\omega t - 60^\circ) + 10\cos(3\omega t + 50)$

$= 15\sin(\omega t + 90^\circ) + 10\sin(2\omega t + 30^\circ) + 10\sin(3\omega t + 140^\circ)$

Fundamental

A wave of the phase sin is taken as reference.

$\therefore \overline{Im_3} = \overline{Im_1} + \overline{Im_2} = 20 \angle 30^\circ + 15 \angle 90^\circ = 17.32 + j10 + 0 + j15$

$= 30.41 \angle 55.3^\circ$

2nd harmonic

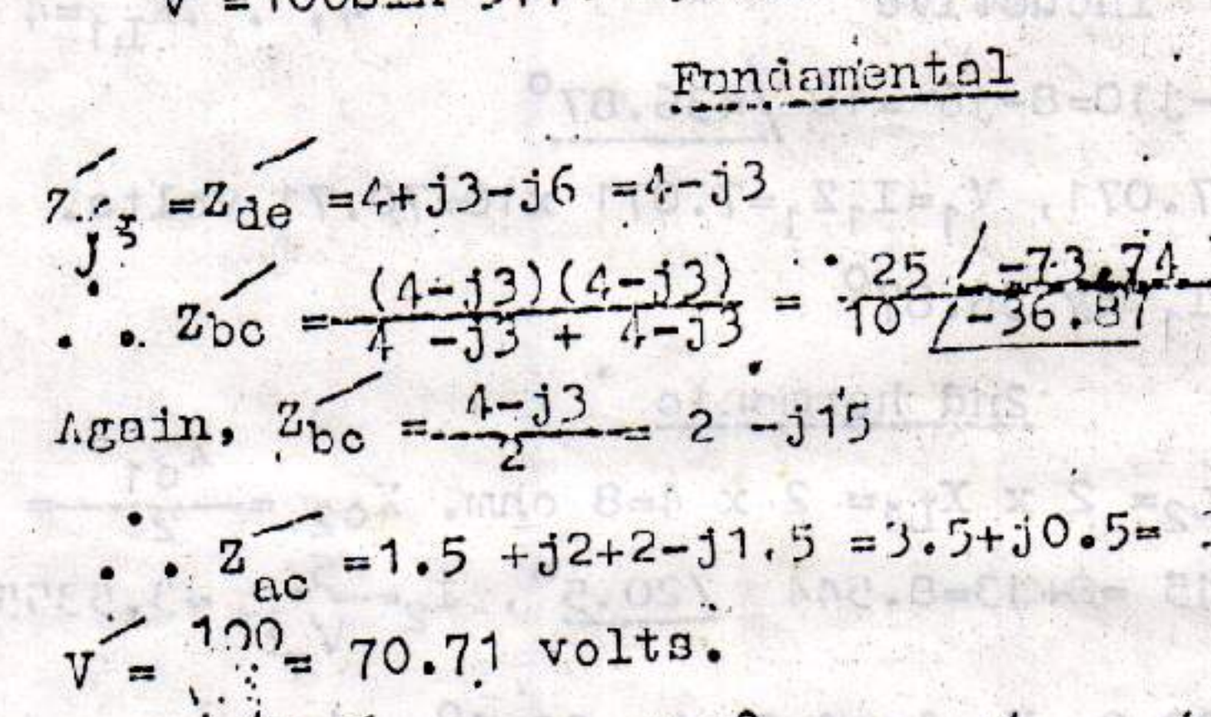
$I_{m_3}'' = I_{m_1}'' + I_{m_2}'' = 10 \angle 150^\circ + 10 \angle 30^\circ = -8.66 + j5 + 8.66 + j5 = 10 \angle 90^\circ$

3rd harmonic

$I_{m_3}''' = I_{m_1}''' + I_{m_2}''' = 5 \angle 40^\circ + 10 \angle 140^\circ = 3.83 - j3.214 - 7.66 + j6.43 = 5 \angle 140^\circ$

$i_3 = 30.41 \sin(\omega t + 55.3) + 10 \sin(2\omega t + 90^\circ) + 5 \sin(3\omega t + 140^\circ)$

46.



$v = 100 \sin 377t - 50 \sin (3 \times 377t + 90^\circ)$

Fondamental

$Z_{fg}' = Z_{de}' = 4 + j3 - j6 = 4 - j3$

$\therefore Z_{bc}' = \dfrac{(4-j3)(4-j3)}{4-j3+4-j3} = \dfrac{25 \angle -73.74}{10 \angle -36.87} = 2.5 \angle -36.87 = 2 - j1.5$

Again, $Z_{bc}' = \dfrac{4-j3}{2} = 2 - j1.5$

$\therefore Z_{ac}' = 1.5 + j2 + 2 - j1.5 = 3.5 + j0.5 = 3.536 \angle 8.13^\circ$

$V' = \dfrac{100}{\sqrt{2}} = 70.71$ volts.

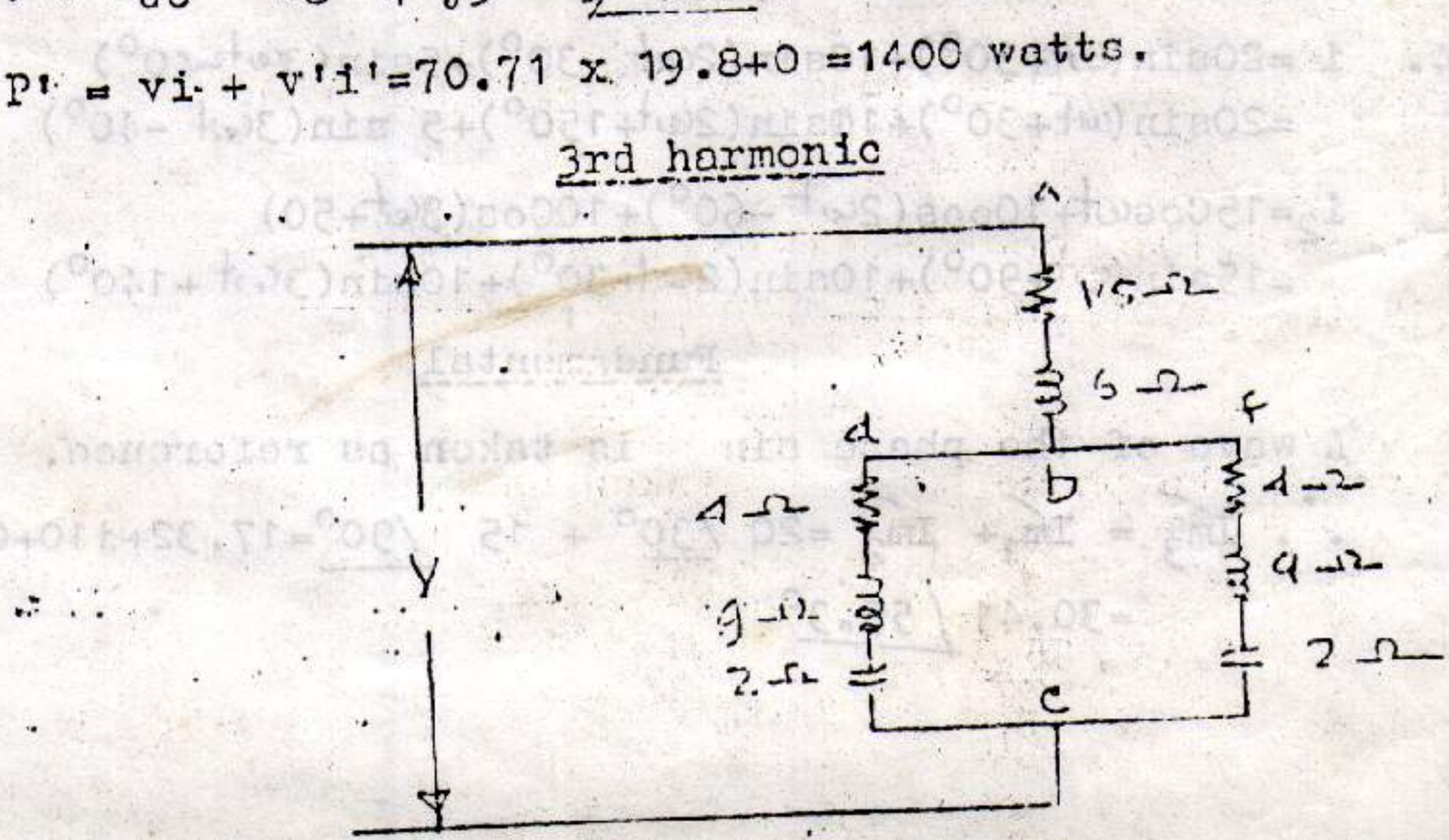
$\therefore I' = \dfrac{V'}{Z_{ac}'} = \dfrac{70.71 \angle 0^\circ}{3.536 \angle 8.13} = 20 \angle -8.13^\circ = 19.8 - j2.83$

$V_{bc}' = I' Z_{bc}' = (20 \angle -8.13)(2 - j1.5) = 50 \angle -45^\circ$

$\therefore I_{dc}' = I_{fg}' = \dfrac{V_{bc}'}{4-j3} = \dfrac{30 \angle -45^\circ}{5 \angle -36.87} = 10 \angle -8.13^\circ$

$p' = vi + v'i' = 70.71 \times 19.8 + 0 = 1400$ watts.

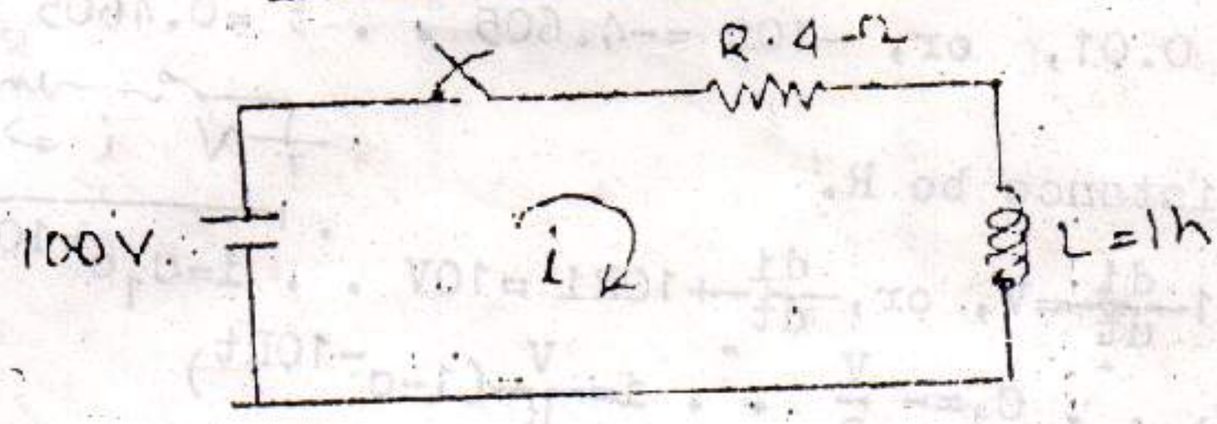
3rd harmonic



(2) The capacitor acts as short ckt. if there is no initial charge or as a voltage source of value $V_o=\frac{q_o}{C}$ if there is an initial charge q_o.

PROBLEM (Chapter:XIV)

3.



$0.4i + 1.\frac{di}{dt} = 10$ or, $\frac{di}{dt} + 0.4i = 10$

$\therefore$ The complete solution is $i = i_c + i_p = C_1 e^{-0.4t} + \frac{10}{0.4}$

$= C_1 e^{-0.4t} + 25$

at $t=0^+$, $i=0$ $\therefore 0 = C_1 + 25$ $\therefore C_1 = -25$ $\therefore i = -25e^{-0.4t} = 25$

$= 25(1-e^{-0.4t})$

(a) at t =1 sec., $i = 25(1-e^{-0.4}) = 8.242$ amp.

(b) at t =2.5 sec., $i = 25(1-e^{-0.4x2.5}) = 15.8$ amp.

(c) $V_L = L\frac{di}{dt} = 1.\frac{d}{dt}\left\{25(1-e^{-0.4t})\right\} = 10e^{-0.4t}$

accelerating voltage at t=1 sec. is $10e^{-0.4} = 6.7$ volt.

" " t=2.5" " $10e^{-0.4x2.5} = 3.68$ volt.

OR

(a) Herem L=1 henry, R=0.4 ohm. t=1 sec., E=10 volt.

We know that,

$i = \frac{E}{R}(1-e^{-0.4x1/1}) = 8.2,1$ Amp.

(b) after 2.5 sec. $i = \frac{E}{R}(1-e^{-Rt/L}) \quad \frac{10}{0.7}(1-e^{\frac{-0.4x2.5}{1}})$

$\therefore$ i =15.803 amp.

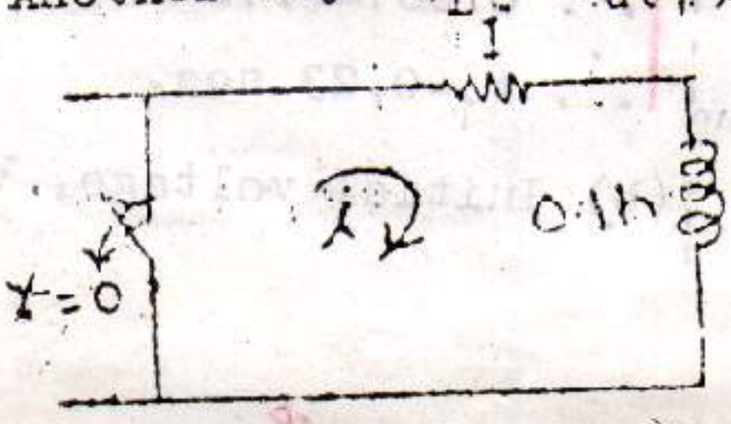
(c) After 1 sec. the value of voltage accelerating the current across the resistor=Ri=0.4x8.242 =3.2960 volts.

Across the inductance, $V_L = -L\frac{di}{dt} = -L\left[-\frac{E}{R} - \frac{R}{L}e^{-Rt/L}\right] = 6.703$volts.

$\therefore$ The accelerating voltage is, $E - L.\frac{di}{dt} = -3.2968$ volts.

Another way- $V_L = -L.\frac{di}{dt} = -L.\frac{R}{L} x\ i_t = -0.4 \times 8.242 = -3.2968$ v.

4.



$1xi + 0.1\frac{di}{dt} = 0$ or, $\frac{di}{dt} + 10i = 0$

$\therefore i = C_1 e^{-10t}$, at $t=0^+$, $i=10$ amp.

$\therefore 10 = C_1$.

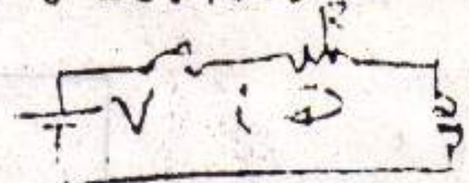
$\therefore i=10e^{-10t}$

$\therefore$ After 0.1 sec, $i=10e^{-10\times1}=3.68$ amp.

Let the time be t_1 $\therefore 0.1=10e^{-10t_1}$ or, $e^{-10t}=0.01$

or, $-10t=\ln 0.01$, or, $-10t=-4.605$ $\therefore t=0.4605$ sec.

5. Let the resistance be R.



$\therefore Ri+0.1\frac{di}{dt}=V$, or, $\frac{di}{dt}+10Ri=10V$ $\therefore i=C_1e^{-10Rt}+\frac{10V}{10R}$

at $t=0$, $i=0$, $\therefore C_1=-\frac{V}{R}$ $\therefore i=\frac{V}{R}(1-e^{-10Rt})$

The final value of current $=\frac{V}{R}$. $\therefore$ after 2 sec., $i=0.632\frac{V}{R}$

$\therefore 0.632\frac{V}{R}=\frac{V}{R}(1-e^{-10R\times2})$ or, $e^{-10R}=1-.632$ or, $-20R=\ln 0.368$

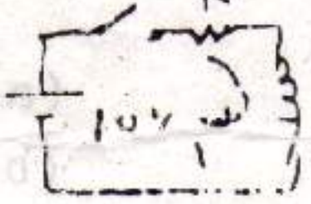
or, $R=0.05$ ohm. Ans. 0.05 ohm.

6. $\therefore 0.1i+\frac{di}{dt}=10$, or, $\frac{di}{dt}+0.1i=10$ $\therefore i=C_1e^{-0.1t}+\frac{10}{0.1}$

$=C_1e^{-0.1t}+100$, at $t=0$, $i=0$, $\therefore C_1=-100$ $\therefore i=100(1-e^{-0.1t})$

$V_L=L\frac{di}{dt}=1.\frac{d}{dt}\{100(1-e^{-0.1t})\}=100\times0.1e^{-0.1t}=10e^{-0.1t}$.

$P_L=V_Li=10e^{-0.1t}\times100(1-e^{-0.1t})=1000(e^{-0.1t}-e^{-0.2t})$



(a) at $t=10$ sec. $i=100(1-e^{-0.1\times10})=63.2$ amp.

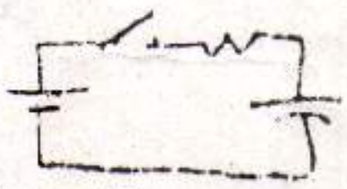
Stored energy $=\frac{1}{2}xLI^2=\frac{1}{2}x1x(63.2)^2=1998$ joule..

(b) En. dissipated $=iR^2=R^2-\frac{E}{R}(1-e^{-Rt/L})=ER(1-e^{-Rt/L})$ joule.

$E_R=\int_0^t i^2Rdt=R\int_0^t\frac{E^2}{R^2}(1-e^{-Rt/L})^2dt=\frac{E^2}{R}\int_0^t(1-e^{-Rt/L})^2dt$

$=\frac{E^2}{R}\int_0^t(1-2e^{-Rt/L}-\frac{2Rt}{L})dt=\frac{E^2}{R}\left[t+\frac{2L}{R}e^{-Rt/L}-\frac{L}{2R}e^{-2Rt/L}\right]_0^t$

$=\frac{E^2}{R}\left[t=2\frac{L}{R}e^{-Rt/L}-\frac{1}{2}R\ e^{-2Rt/L}-2\frac{L}{R}+\frac{L}{R}\right]$ Joules.



7. $Ri+\frac{1}{C}\int idt=V$, or, $R\frac{dq}{dt}+\frac{q}{C}=V$, or $1x10^6\frac{dq}{dt}+\frac{q}{50x10^{-6}}=V$

or, $\frac{dq}{dt}+0.02q=10^{-6}V$ $\therefore q=C_1e^{-0.02t}+\frac{10^{-6}V}{0.02}=50x10^{-6}V(1-e^{-0.02t})$

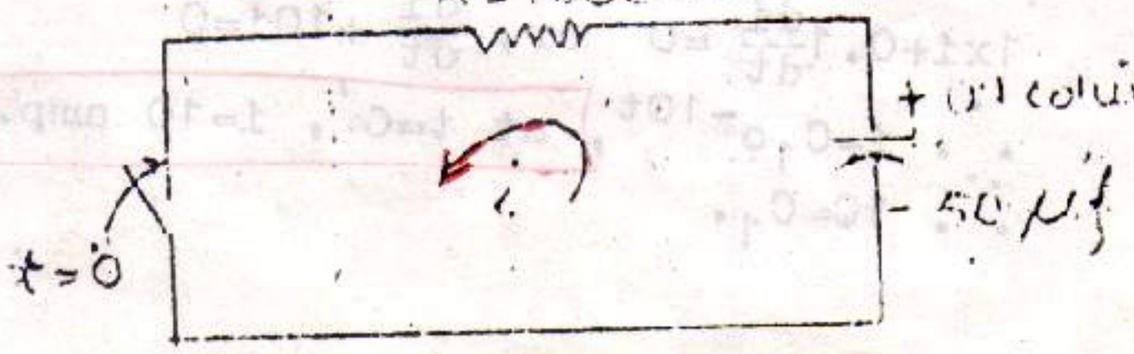
Let the time be t_1 $\therefore 0.632x50x10^{-6}V=50x10^{-6}V(1-e^{-0.02t})$

or, $e^{-0.02t_1}=1-0.632$, or, $-0.02t_1=\ln 0.368$ $\therefore t_1=50$ sec. Ans.

8. (a) Let the time be t_1

$\therefore 0.001=0.1e^{-20t_1}$ or, $-20t_1=\ln 0.01$

$\therefore t_1=0.23$ sec.



(b) Initial voltage, $V_c=\frac{0.1}{50\ x10^{-6}}$

$R_1 + \frac{q}{C} = 0$

or, $1000\frac{dq}{dt} + \frac{q}{50\times10^{-6}} = 0$

or, $\frac{dq}{dt} + 20q = 0$

$\therefore q = C_1 e^{-20t}$

at $t=0$, $q_0 = 0.1$ coumb.

$\therefore 0.1 = C_1$, $\therefore q = 0.1e^{-20t}$

$\therefore i_0 = \frac{2000}{1000} = 2$ amp.

(c) Then $V_c = \frac{0.001}{50\times10^{-6}} = 20V$.

$\therefore i = \frac{20}{1000} = 0.02$ amp.

$V_c = i_0 \times R$

9. Sol^n:- $Q_0 = 0.1$ coulomb, $C = 100\times10^{-6}$f, $R = 10{,}000$ ohm.

Decay of charge in 1 sec. $q = CE\, e^{-t/Rc}$

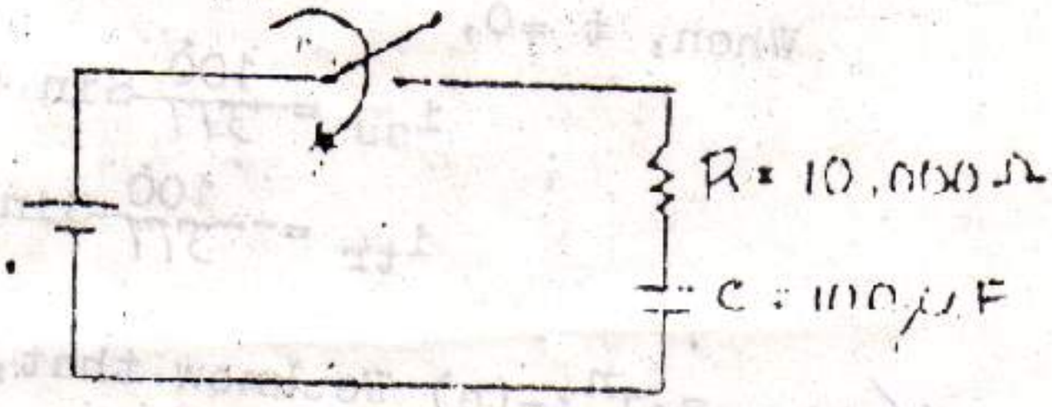
$q = 100\times10^{-6} \times \frac{0.1}{100\times10^{-6}} \times e^{-\frac{1}{10{,}000\times100\times10^{-6}}} = 0.0367$ coulomb.

We know, $W_C = \int_0^\infty P_c\,dt = \int_0^\infty v_c\, i\,dt = \int_0^\infty \frac{q}{C}\cdot\frac{dq}{dt}\,dt$

$= \int_0^q \frac{q}{C}\,dq = \frac{1}{2}q^2$ But Here,

$W_C = \frac{(Q_0-q)^2}{2C} = \frac{(0.1-0.0367)^2}{2\times100\times10^{-6}} = 20$ Joules Ans.

10. Sol^n:- $R = 10{,}000$ ohm.
$C = 100\times10^{-6}$f
$t = 1$ Sec.
$W_C = 19.98$ Joule in 1 sec.
$Q_0 = 0$, $E = ?$



We know, $W_C = \int_0^\infty P_c dt = \int_0^\infty v_c\, i\,dt = \int_0^q \frac{q}{c}\cdot\frac{dq}{dt}\,dt$, $W_C = \int_0^q \frac{q}{C}\,dq = \frac{q^2}{2C}$

$\therefore 19.98 = \frac{q^2}{2\times100\times10^{-6}} \Longrightarrow q = 0.063$ coulomb.

$E = \frac{q}{c} = \frac{0.063}{100\times10^{-6}} = 632.139$ volts.

11. (a) $q = CE - (CE - Q_0)\, e^{-t/Rc}$

$= CE\,(1-e^{-t/Rc})$

$= 10^{-6}\times100\,(1-e^{-1/10^6\times10^{-6}})$

$= 10^{-4}(1-e^{-1}) = 6.32\times10^{-5}$

Here Given :
$R = 1\times10^6$ ohm.
$C = 1\times10^6$ Farad
$E = 100$ volt.

Energy stored, $E_S = \frac{Q^2}{2C} = \frac{(6.32\times10^{-5})^2}{2\times10^{-6}} = 1.99712\times10^{-3}$ watt-sec or joule.

(b) $E_R = \int_0^t i^2R\,dt = \int_0^t R\cdot\frac{E^2}{R^2}\cdot e^{-\frac{2t}{R_c}}\,dt$

$= \frac{E^2}{R}\int_0^t e^{-2t/R_c}\,dt = \frac{E^2}{R}\times\frac{-R_c}{2}\left[e^{-2t/R_c}\right]_0^t$

$$= -\frac{E^2C}{2}\left[e^{-2t/Rc}\right] + \frac{E^2C}{2} = -\frac{(100)^2x10^{-6}}{2}\left[e^{\frac{-20x1}{10^6x10^{-6}}}\right] + \frac{(100)^2x10^{-6}}{2}$$

$$= 4323 \times 10^{-3} \text{ watt-Sec.}$$

(c) E_R full $= \int_0^\infty i^2R\,dt = \int_0^\infty R.\frac{E^2}{R^2}.e^{-2t/Rc}dt = \frac{E^2}{R} x \frac{-RC}{2}\left[e^{-2t/Rc}\right]_0^\infty$

$$= \frac{-E^2C}{2}\left[e^{-2t/Rc}\right] = \frac{-E^2C}{2}\left[e^{-2t/Rc}\right]_0^\infty = \frac{E^2C}{2}.\ e^{\frac{-2x0}{Rc}} = \frac{E^2C}{2} x1$$

$$= \frac{E^2C}{2} = \frac{(100)^2x10^{-6}}{2} = 5x10^{-3} \text{ watt-Sec.}$$

13. Sol^n:-

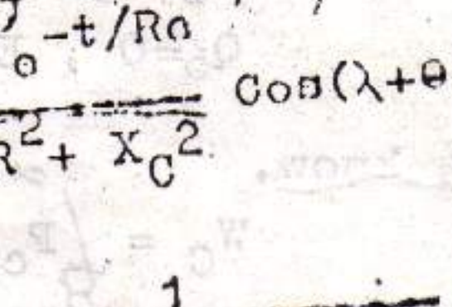
We know, $i = \frac{E_m}{Z}\text{Sin}(wt+\lambda-\theta) - \frac{E_m}{Z}\text{Sin}(\lambda-\theta)\ e^{-Rt/L}$

Steady State term — Transient term

Here, $E_m = 100$, $wt = 377t$, $\lambda = \pi/4$

$\theta = \tan^{-1}\frac{wL}{R} = \tan^{-1}\frac{377x1}{1} = 89.848^0$

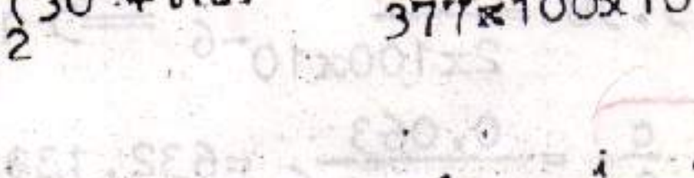
$L = 1$ henry, $R = 1$ ohm., $Z = \sqrt{R^2+(wL)^2} = \sqrt{(1)^2+(377x1)^2} = 377$ ohm.



When, $t = 0$,

$i_{ss} = \frac{100}{377}\text{Sin}\left(\frac{\pi}{4} - 89.848\right) = -0.187$ Amp. Ans.

$i_{tr} = \frac{100}{377}\text{Sin}\left(\frac{\pi}{4} - 89.848\right)e^0 = -0.187$ Amp. Ans.



14. Sol^n:-(a) We know that, $q = -\frac{E_m}{w\sqrt{R^2+(X_C)^2}}\text{Cos}(wt+\lambda+\theta) + \frac{E_m}{w\sqrt{R^2+X_C^2}}\text{Cos}(\lambda+\theta)\ e^{-t/Rc}$

$$q = -\frac{100}{377\sqrt{1^2+\left(\frac{1}{377x100x10^{-6}}\right)^2}}\text{Cos}\left(30^0+\tan^{-1}\frac{1}{377x100x10^{-6}x1}\right) +$$

$$= \frac{100xe^0}{277\sqrt{(1)^2+\left(\frac{1}{377x100x10^{-6}}\right)^2}}\text{Cos}\left(30^0+\tan^{-1}\frac{1}{377x100x10^{-6}x1}\right)$$

$= 0.00499 - 0.00499$ $\therefore$ $q_{ss} = 0.00499$ coulomb and

$\therefore$ $q_{tr} = -0.00499$ coulomb

(b) $i = \frac{E_m}{Z}\text{Sin}(wt+\lambda'+\theta) - \frac{E_m}{wCR^2}\text{Cos}(\lambda+\theta)\ e^{\frac{-t}{R_C}}$

at $t=0$, $Z = \sqrt{I^2+\left(\frac{1}{377x100x10^{-6}}\right)^2} = 26.544$

$i = \frac{100}{26.544}\text{Sin}(30^0+87.848^0) - \frac{100}{26.544}\text{Cos}(30^0+87.848^0)\ e^0 x \frac{1}{377x100x10}$

$= 3.33 + 46.67$ $\therefore$ $i_{ss} = 3.33$ and $i_{tr} = 46.67$ amp. Ans.

15. R=100 ohm, L=0.1 henry, $C=200\times10^{-6}$, $Q_o=0$, E=50V, t=0.01 sec.

$a=\frac{R}{2L}=500$

$b=\sqrt{(R/2L)^2-\frac{1}{LC}}=447.213$

$(-a+b)=477.213-500=-52.787$

$\therefore (-a+b)t = e^{-52.787\times0.01}=0.589$

$(-a-b) = -947.213 \quad \therefore e^{(-a-b)t}=e^{-947.213\times0.01}=7.696\times10^{-5}$

$\therefore \left[e^{(-a+b)t}-e^{(-a-b)t}\right]=0.588$

$CE=200\times10^{-6}\times50=0.01, \quad \sqrt{R^2C^2-4LC}=0.0178$

$RC=100\times200\times10^{-6}=0.02$

We know that,

$i=\frac{CE-Q_o}{\sqrt{R^2C^2-4LC}}\left[e^{(-a+b)t}-e^{(-a-b)t}\right]=\frac{0.01}{0.0178}\times0.588=0.330$amp.

$q=Ce-(CE-Q_o)\left[\frac{Rc+\sqrt{R^2C^2-4LC}}{2\sqrt{R^2C^2-4LC}}e^{(-a+b)t}-\frac{RC-\sqrt{R^2C^2-4LC}}{2\sqrt{R^2C^2-4LC}}e^{(-a-b)t}\right]$

$=0.01-0.01\left[\frac{0.02+0.0178}{2\times0.0178}\times0.589-\frac{0.02-0.0178}{2\times0.0178}\times7.696\times10^{-5}\right]$

$=0.01-0.01\left[0.625-4.755\times10^{-6}\right]=0.00375$ coulomb.

16. According to Sir (Shaidulla).

R=4 ohm, L=0.1 h, $C=200\times10^{-6}$, t=0.01 sec. E=1000V, $Q_o=0$,

$A=\frac{R}{2L}=\frac{5}{2\times0.1}=25 \qquad B=\sqrt{\frac{1}{LC}-\frac{RL}{4L^2}}=22.204$

$\sqrt{4LC-R^2C^2}=\sqrt{4\times.1\times200\times10^{-6}-(5\times200\times10^{-6})^2}=0.00888$

(a) $i=\frac{2(CE-Q_o)\,e^{-at}}{\sqrt{4LC-R^2C^2}}\sin\beta t=\frac{2\,CE\,e^{-at}}{4LC-R^2C^2}\sin\beta t$

$=\frac{2\times200\times10^{-6}\times1000\,e^{-25\times0.01}}{0.00888}\sin(222.204\times0.01)=\frac{0.311}{0.00888}\times0.769.$

$=35\times.796=27.87$ Ans.

(b) yes.

(c) $f=\frac{B}{2\pi}=\frac{222.204}{2\pi}=35.36$ HZ.

$q=\frac{CE-2(CE-Q_o)\,LC\,e^{-at}}{\sqrt{4LC-R^2C^2}}\sin(\beta t+\theta)$

$\theta=\tan^{-1}\frac{\sqrt{4LC-(RC)^2}}{RC}$

$=\tan^{-1}\frac{.00888}{5\times200\times10^{-6}}=83.57^\circ=\frac{83.57\times\pi}{180}=1.450.$

$$q = \frac{200\times10^{-6}\times1000 - 2\times200\times10^{-6}\times1000}{.00688}\sqrt{.1\times200\times10^{-6}}\, e^{-25\times.01}\sin(222.204\times0.01 + 1.45)$$

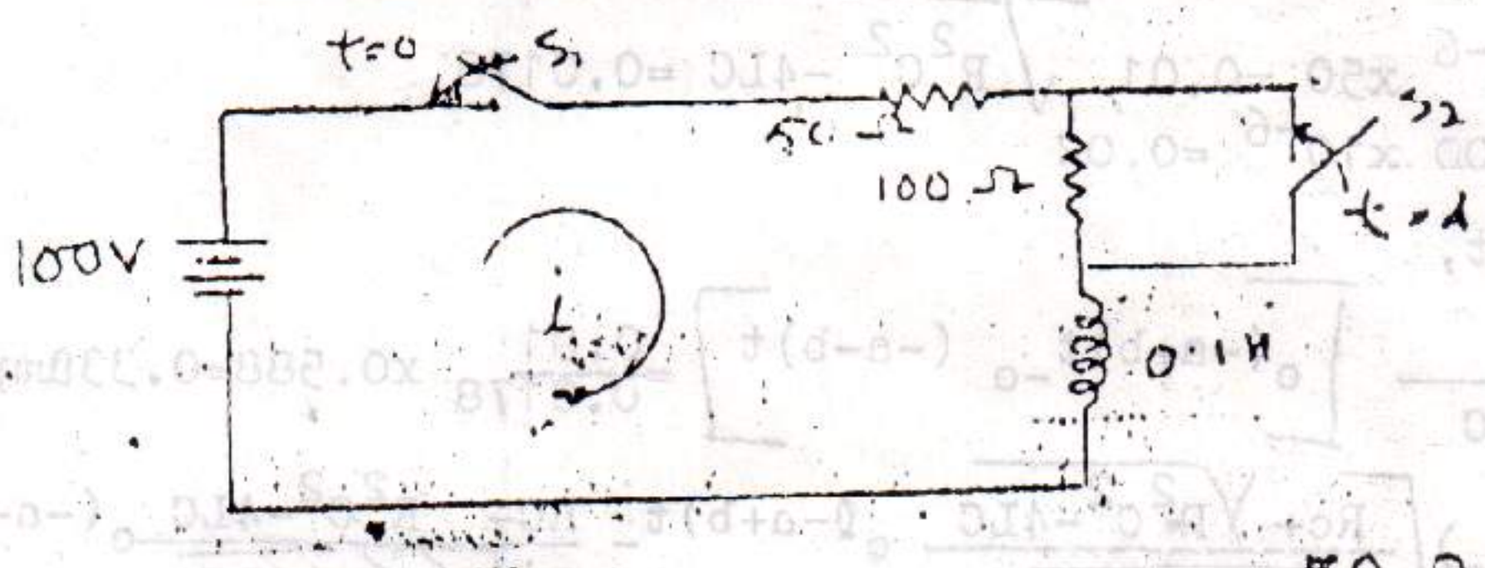
$$= .2 - \frac{1.39\times10^{-3}}{0.00688}\sin(3.67204)$$

$$= .2 - 0.1555 \times \sin\left(3.67204 \times \frac{180}{\pi}\right)$$

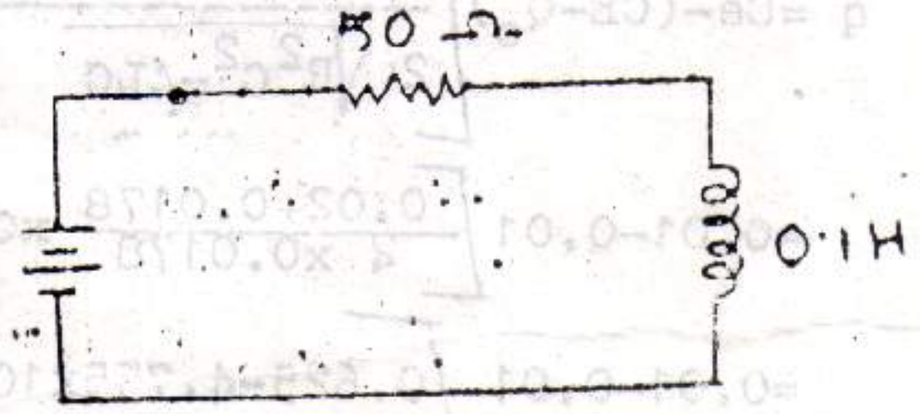
$$= 0.279 \text{ Coloumb.}$$

SCHAUM'S SERIES (BY ED MINISTER)

16-20



at $t = 0^+$ the ckt. will be →



$$\therefore 50\, i + 0.1\frac{di}{dt} = 100$$

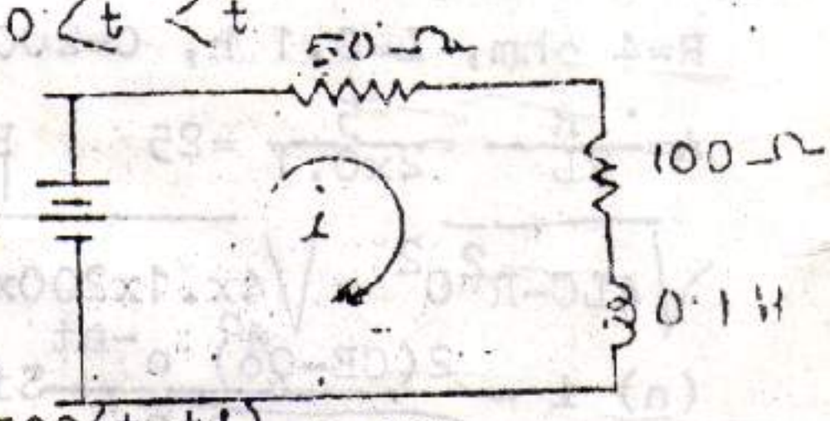
$$\text{or, } \frac{di}{dt} + 500i = 1000$$

$$\therefore i = C_1 e^{-500t} + \frac{1000}{500} = C_1 e^{-500t} + 2$$

at $t=0^+$, $i = 0$, $\therefore C_1 = -2$

$\therefore i = 2(1 - e^{-500t})$ amp. at internal $0 < t < t'$

at $t = 4^+$ ms the ckt. will be →



$$\therefore (50+100)i + 0.1\frac{di}{dt} = 100$$

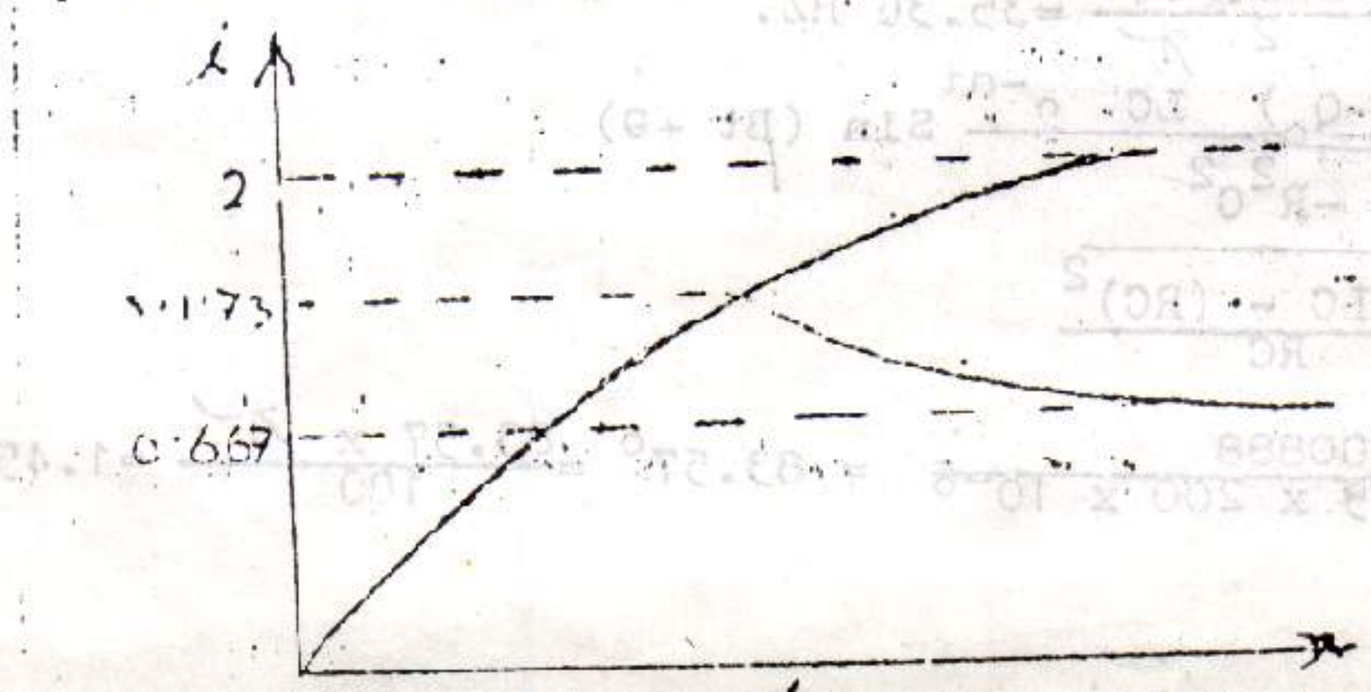
$$\text{or, } \frac{di}{dt} + 1500\, i = 1000$$

$$\therefore i = C_2 e^{-1500(t-t')} + \frac{1000}{1500} = C_2 e^{-1500(t-t')} + 0.667$$

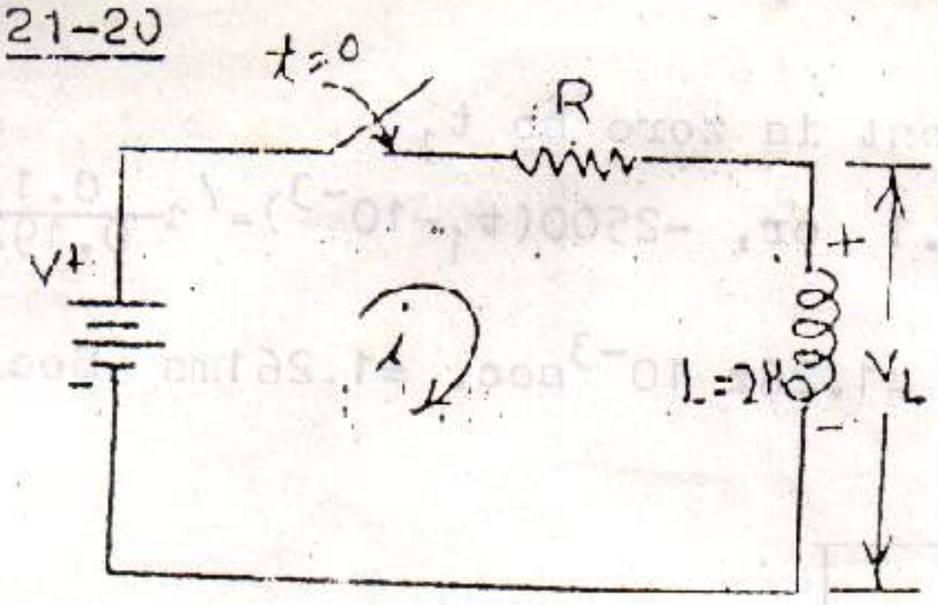
at $t = 4$ ms, $i = 2(1 - e^{-500\times4\times10^{-3}}) = 1.729$ amp.

$\therefore 1.729 = C_2(1) + 0.667 \quad \therefore C_2 = 1.06$ at internal $1^+ < t < t'$

$\therefore i = 1.06\, e^{-1500(t-t')} + 0.667$ amp.



21-20



$R_i + 2\frac{di}{dt} = V$ or, $\frac{di}{dt} + \frac{R}{2}i = \frac{V}{2}$

$\therefore i = C_1 e^{-\frac{R}{2}t} + \frac{V}{R}$

Now at t = 0, $i = \frac{V-25}{R}$

$\therefore \frac{V-25}{R} = C_1 + \frac{V}{R}$

or, $C_1 = -\frac{25}{R}$

$\therefore i = -\frac{25}{R} e^{-\frac{R}{2}t} + \frac{V}{R}$ ....(1)

Now, at t= 25ms $=25\times10^{-3}$Sec. $i = \frac{V-5}{R}$

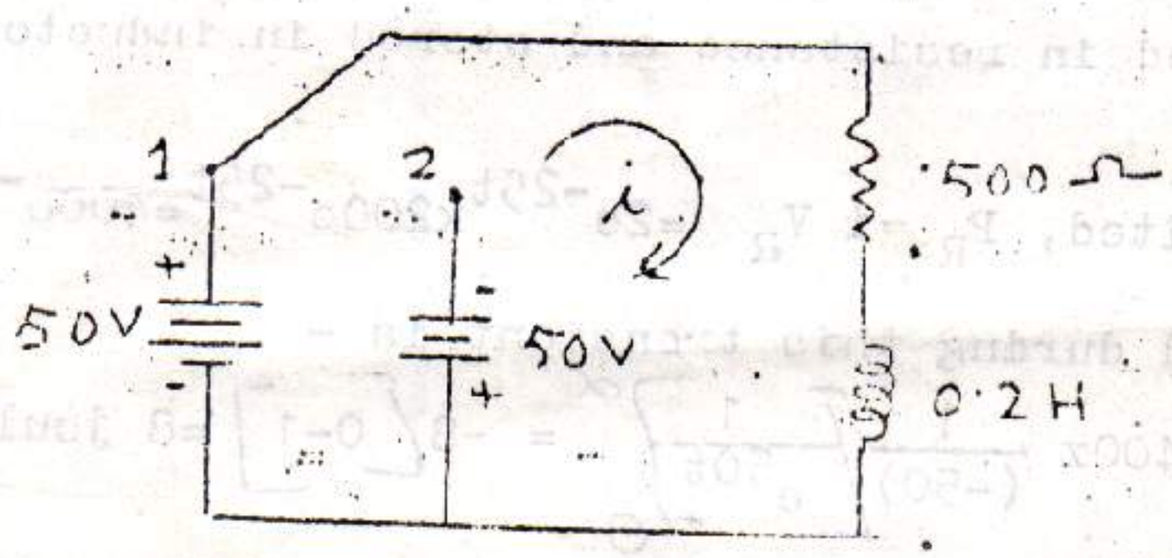
From (1) $\therefore \frac{V-5}{R} = -\frac{25}{R} e^{-\frac{R}{2}\times 25\times10^{-3}} + \frac{V}{R}$

or, $-\frac{5}{R} = -\frac{25}{R} e^{-\frac{R}{2}\times25\times10^{-3}}$ or, $1 = 5\, e^{-\frac{R}{2}\times25\times10^{-3}}$

or, $-\frac{R}{2}\times25\times10^{-3} = \ln\frac{1}{5}$ $\therefore$ R= 128.76 ohm. Ans:

22-20 Same as 20.

23-20



When switch is closed on position 1 at t=0

$500i + 0.2\frac{di}{dt} = 50$ or, $\frac{di}{dt} + 2500i = 250$

$\therefore i = C_1 e^{-2500t} + \frac{250}{2500} = C_1 e^{-2500t} + 0.1$

at t=0, current i=0, $\therefore C_1 = 0.1$

$\therefore i = 0.1(1-e^{-2500t})$

At $t=1\times10^{-3}$ sec., $i=0.1\,(1-e^{-2500\times10^{-3}}) = 0.0092$ amp

When switch is closed on position 2 at t=t' : $500i + 0.2\frac{di}{dt} = -50$

or, $\frac{di}{dt} + 2500\, i = -250$ $\therefore i = C_1 e^{-2500(t-t')} - \frac{250}{2500}$

$= C_2 e^{-2500(t-10^{-3})} - 0.1$

at t = t' =1ms. i=0.092 amp. $\therefore 0.092 = C_2 - 0.1$ $\therefore C_2 = 0.192$

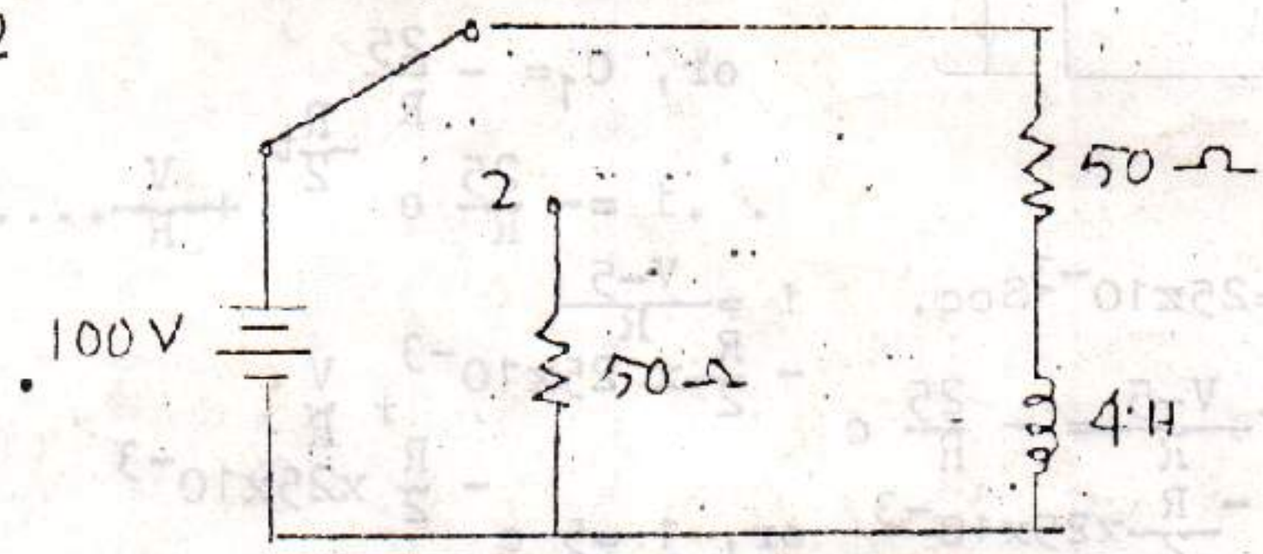
$\therefore i = 0.192e^{-2500(t-10^{-3})} - 0.1$ amp.

at the time at which the current is zero be t_1

$\therefore 0 = 0.192e^{-2500(t_1-10^{-3})} - 0.1$ or, $-2500(t_1-10^{-3}) = \ln \frac{0.1}{0.192}$

or, $t_1 - 10^{-3} = 2.61 \times 10^{-4}$ $\therefore t_1 = 1.26 \times 10^{-3}$ sec. $= 1.261$ ms Sec.

24-20



at steady state current $= \frac{100}{50} = 2$ amp. when the switch is moved to position2 - $(50+50)i + 4\frac{di}{dt} = 0$ or, $\frac{di}{dt} + 25i = 0$

$\therefore i = C_1 e^{-25t}$. Now at $t = 0^+$, $i = 2$ amp. $\therefore 2 = C_1$ $\therefore i = 2e^{-25t}$

Now $V_R = iR = (50+50) \times 2e^{-25t} = 200e^{-25t}$

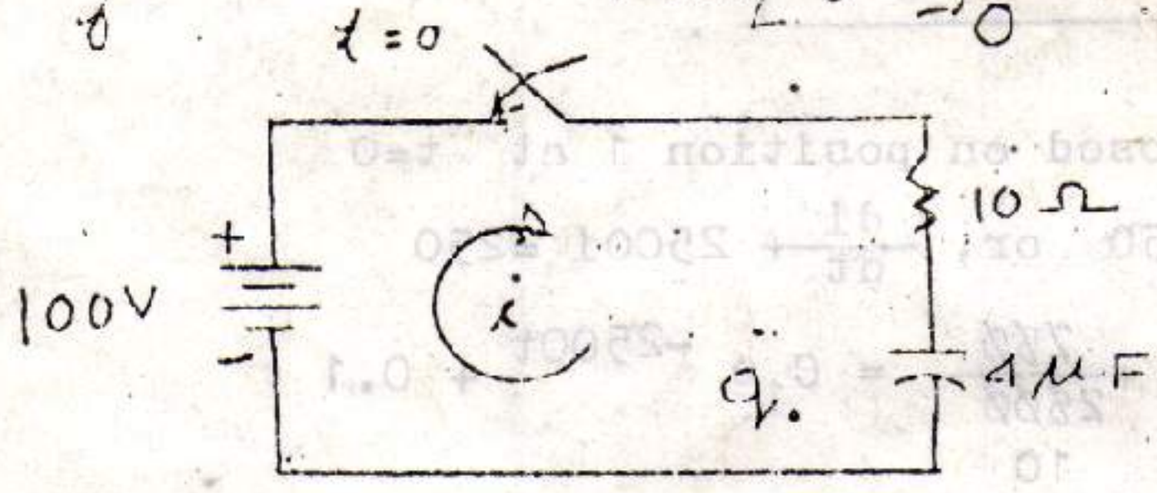
Power is dissipted in resistance and stored in inductor and capacitor.

$\therefore$ Power dissipited, $P_R = i\, V_R = 2e^{-25t} \times 200e^{-25t} = 400e^{-50t}$

Energy dissipited durdng this transjent in -

$$= \int_0^\infty 400e^{-50t} = 400 \times \frac{1}{(-50)} \left[e^{-50t} \right]_0^\infty = -8\left[0 - 1 \right] = 8 \text{ joules Ans:}$$

25-20



$10\, i + \frac{1}{4 \times 10^{-6}} \int i\, dt = 100$

or, $10\frac{di}{dt} + 250000\, i = 0$

or, $\frac{di}{dt} + 25000i = 0$

$\therefore i = C_1 e^{-25000t}$

at $t=0$, $i_o = -10$ amp.

$\therefore -10 = C_1$ $\therefore i = -10\, e^{-25000t}$ amp. Ans.

Now initial charge on the capacitor has an equivalent voltage $V_o = \frac{q_o}{C} = \frac{800 \times 10^{-6}}{4 \times 10^{-6}} = 200$V. this send current in the direction opposite to i $\therefore$ at $t=0$ current

$i_o = \frac{100-200}{10} = -10$ amp

F O R M U L A S

1. $f = \frac{PN}{120}$ cps $= \frac{w}{2\pi}$ cps | p = Pole, N = r.p.m.

2. $i = \frac{Vm}{R} \sin \omega t = Im \sin \omega t$

   $i = Im \sin (\omega t + \theta)$

   $v = Ri = Vm \sin \omega t$

   $v = Vm \sin (\omega t + \theta)$

3. $p = vi$ (absorbed power).

4. $P_{av} = \frac{Vm\ Im}{2} \cos \theta$ watts.

   Reactive voltamperes $= \frac{Vm\ Im}{2} \sin \theta$ VA or Vars.

5. $X_L = \omega L$, $Xc = \frac{10^6}{\omega C}$ ohm.

6. Energy Stored $= \frac{Vm\ Im}{2\omega}(2\pi f)$,

7. $P_{RL} = \frac{Vm\ Im}{2} \cos \theta - \frac{Vm\ Im}{2} \cos 2\omega t \cos \theta + \frac{Vm\ Im}{2} \sin 2\omega t \sin \theta$

8. $\frac{Vm}{Im} = \frac{1}{\omega C}$

9. $v = \frac{ImZ \cos (\omega t + \theta)}{} = Vm \cos (\omega t + \theta)$

   $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, $\theta = \tan^{-1} \frac{(\omega L - 1/\omega C)}{R}$

PROBLEM (Chapter : 2)

rpm = revolution per minute
cps = cycle per second

(a) $f = \frac{PN}{120} = \frac{10 \times 360}{120} = 30$ cps. | P = 10, N = 360 rpm.

(b) $f = \frac{PN}{120}$ $\therefore N = \frac{f \times 120}{6} = \frac{25 \times 120}{6} = 500$ rpm. | P = 6, f = 25

2. $P = \frac{f \times 120}{N} = \frac{50 \times 120}{300} = 20$ Poles. N=300 rpm, f=50 cps.

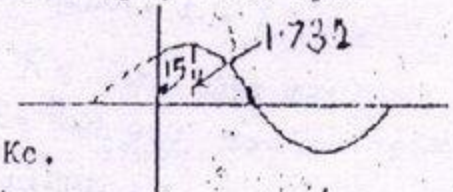
3.

4. We know that,

   $i = Im \sin (\omega t + \theta)$,

   at t=0, i =1.5A, Imax =1.732 A, f=1591 Kc.

   $\therefore$ $1.5 = 1.732 \sin (2\pi \times 1591 \times 10^3 \times 0 + \theta) = 1.732 \sin \theta$.

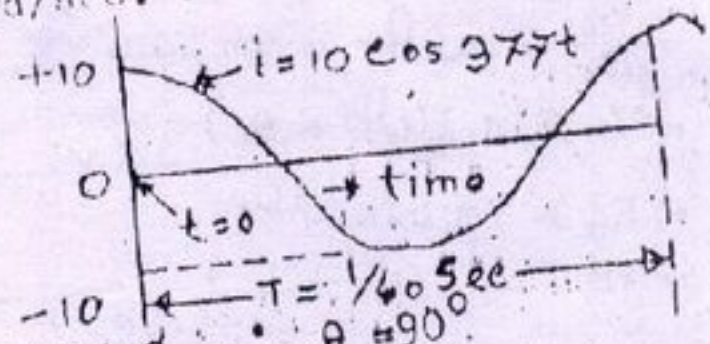


or, $\sin\theta = \frac{1.5}{1.732}$ $\therefore \theta = 60^\circ$

$\therefore$ The eqn is, $i = 1.732 \sin(2\pi \times 1591 \times 10^3 + 60^\circ)$ Ans.

5. $i = I_m \sin\omega t = 10 \sin(2\pi \times 50 \times t) = 10 \sin 314.16t$. Ans. $I_m = 10A$, $f = 50$ cps.

$\therefore$ Angular velocity $\omega = 314.16$ rad/sec. Ans.



6. $i = 10 \cos(377\ t)$

frequency $= \frac{377}{2\pi} = 60$ cps. Ans.

7. $i = I_m \sin(\omega t + \theta)$

at $t=0$

$\therefore 10 = 10\sin(2\pi 50t + \theta)$ or, $10 = 10\sin\theta$ $\therefore \theta = 90^\circ$

$\therefore$ Eq. is $i = 10\sin(2\pi 50t + 90^\circ)$ $i = 10\sin(314.16t + \theta)$

When, $i = -5$amp at $t=0$ $-5 = 10\sin\theta$, $\sin\theta = -\frac{1}{2}$ $\therefore \theta = -30, 210^\circ$

$\therefore i = 10 \sin(314.16t - 30^\circ)$, $i = 10\sin(314.16 + 210^\circ)$ Ans:-

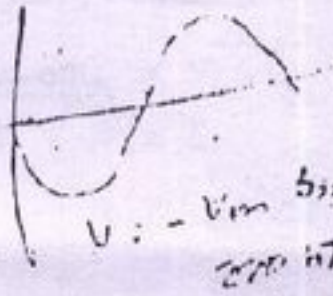
8. $e = 100 \sin 157t$.

(a) max^m voltage $(V_m) = 100$V.

(b) frequency $(f) = \frac{\omega}{2\pi} = \frac{157}{2\pi} = 25$cps.

(c) $e = 100\sin(157t + 30^\circ)$ $\therefore$ max^m voltage $= 100$V

frequency $= 25$ cps.



9. We know that, $v = V_m \sin\omega t$, $\frac{dv}{dt} = V_m\omega \cos\omega t$

$\frac{dv}{dt}$ will be max^m. when $\cos\omega t = 1$

$\therefore$ Max^m Value $= V_m\omega = 140 \times 2\pi 60 = 52778.75$ volt/sec.

$\frac{dv}{dt}$ will be mini^m when $\cos\omega t = 0$ $\therefore$ Min^m value $= 0$ Ans:

10. $i = 10 \sin(\omega t - 30^\circ)$, $\frac{di}{dt} = 10\ \omega \cos(\omega t - 30^\circ)$

(a) $3265 = 10 \times 2\pi 60 \cos(\omega t - 30^\circ)$

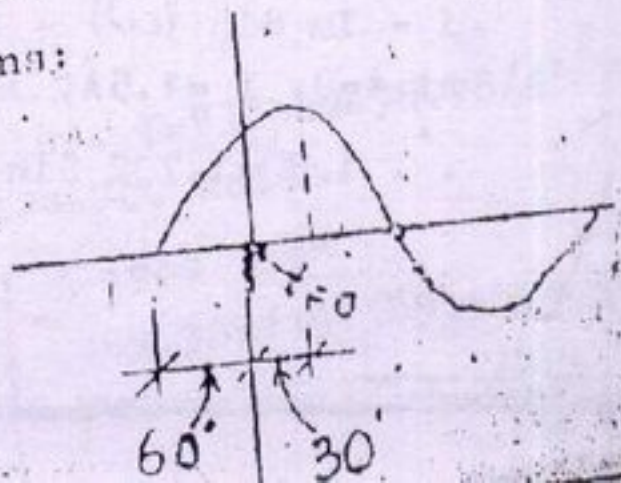
or $\cos(\omega t - 30^\circ) = 0.866$ or, $\omega t - 30^\circ = 30^\circ$ $\therefore \omega t = 60^\circ$

$\therefore i = 10\sin(60^\circ - 30^\circ) = 5$ amp Ans:

(b) $2260 = 10 \times 2\pi 60 \cos(\omega t - 30^\circ)$

or $\cos(\omega t - 30^\circ) = 0.599$ or $\omega t - 30^\circ = 53.167^\circ$

$\therefore i = 10\sin 53.167^\circ = 8$ amp Ans:



11. $i = I_m \sin(\omega t + \theta)$

or $\frac{di}{dt} = I_m \times \omega \cos(\omega t + \theta)$

or $2000 = I_m \times 2\pi 50 \cos(0 + 60^\circ)$

$\therefore I_m = 12.73$ A Ans:

$v = 100 \sin(\omega t - 30^\circ)$

$i = 10 \sin(\omega t - 60^\circ)$

$\theta = \theta_v - \theta_i = 30^\circ - (-60^\circ) = 30^\circ$

∴ Voltage leads current by 30°

13. $v = 100 \cos(\omega t - 30^\circ) = 100 \sin\{90^\circ + (\omega t - 30^\circ)\}$

$= 100 \sin(\omega t + 60^\circ)$.

$i = -10 \sin(\omega t - 30^\circ) = 10 \sin\{180^\circ + \omega t - 30^\circ\} = 10 \sin(\omega t + 120^\circ)$

Phase difference $(\theta) = \theta_v - \theta_i = +60 - (+120^\circ) = -60^\circ$

∴ Voltage lags current by 60° Ans:

14. $v = 100 \cos \omega t$, $i = 10 \cos(\omega t + \frac{2\pi}{6}) = 10 \cos(\omega t + \pi/3)$ Ans:

16.(a) $I_m = 90$ amp $\quad i = I_m \sin(\omega t = 90 \sin 60^\circ$

$\omega t = 60^\circ \quad i = 90 \cdot \sqrt{3}/2 = 77.94$ amp. $\omega t = 60^\circ$ or $\omega t = \pi/3$.

$$\text{or } t_1 = \frac{\pi}{3 \times 2 \times 50} = 0.0033 \text{ Sec.} \quad (\omega t_2 = 225^\circ)$$

$$\text{or } t_2 = \frac{225 \times \pi}{180 \times 2 \times 180 \times 50} = 0.0125 \quad \therefore t_2 = 0.0125$$

$\therefore t_2 - t_1 = \Delta t = 0.0125 - 0.0033 = 0.0092$ Sec. Ans:-0.0092Sec.

17. $i = 1.0 + 0.50 \sin 1885t - 0.10 \cos 3770t$ Amperes.

$i = 1.0 + 0.50 \sin \alpha - 0.10 \cos \alpha$ Amperes.

(a) Sine	Cosine
$\omega = 2\pi f = 1885$	$2\pi f = 3770t$
$\therefore f = 300$ cps.	$f = 600$ cps.

(b) When, $\sin \alpha = 1 \quad \therefore \alpha = \pi/2$

We have,

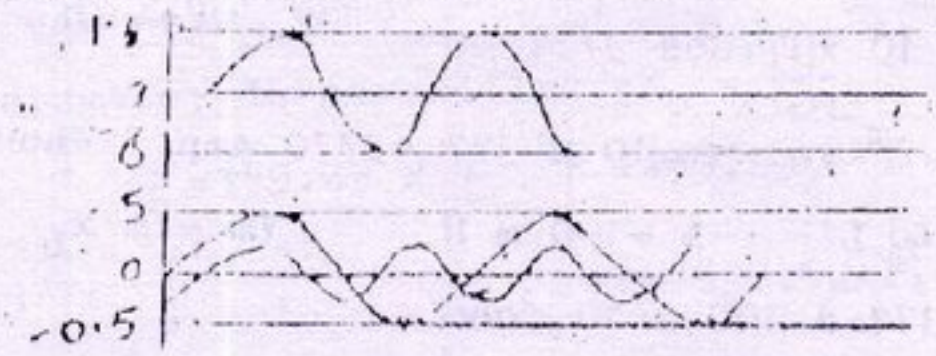
$i = 1.0 + 0.50 \sin \pi/2 - 0.10 \cos \pi = 1.6$ A Ans:

When, $\sin \alpha = -1 \quad \therefore \alpha = -\pi/2$

We have,

$i = 1.0 + 0.50 \sin(-\pi/2) - 0.10 \cos(-\pi) = 0.6$ Ans:

(c)



18. $v = 150 \cos 314t$ R = 0 ohm.

(a) $i = \frac{150}{30} \cos 314t = 5 \cos 314t$ Ans.

(b) $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50$ cps. Ans:

(c) $P = vi = 150 \cos 314t \times 5 \cos 314t = 750 \cos^2 314t$

$= 375 \times 2 \cos^2 314t = 375(1 + \cos 2 \times 314t)$

$= 375 + 375 \cos 628t$ Ans:-

(d) fp = $\frac{628}{2\pi}$ = 100 cps. Ans:

19. i = 5Sin (110t + 30°) (R = 20 ohm.)

(a) v = 5Sin (110t + 30°) x 20 = 100Sin (110t + 30°) V.

(b) fv = $\frac{110}{2\pi}$ = 17.5 Cps.

(c) P = vi = 100Sin (110t + 30°) x 5Sin (110t + 30°)
= 500Sin^2(110t + 30°) = 250 x 2Sin^2(110t + 30°)
= 250 (1 - cos 2(110t + 30°)) ∴ P = 250 - 250cos (220t + 60°).

(d) fp = $\frac{220}{2\pi}$ = 35 Cps.

20. v = 100 Cos (ωt + 60°), R = 10 ohm.

(a) i = $\frac{100}{10}$ Cos (ωt + 60°) = 10Cos (ωt + 60°)

(b) p = vi = 100 Cos (ωt + 60°) x 10 cos (ωt + 60°)
= 1000 Cos^2(ωt + 60°) = 500 [1 + Cos 2(ωt + 60°)]
= 500 + 500Cos (2ωt + 120) watts.

(c) Max^m power, When cos (2ωt + 120) = 1

∴ Pmax = 500 + 500 = 1000 watts.

(d) Mini^m power, When cos (2ωt + 120°) = -1

∴ P_{min} = 500 - 500 = 0 watts.

(e) $P_{av} = \frac{Vm\ Im}{2}$ Cosθ = $\frac{100 \times 10}{2}$ Cosθ° ∴ Phase diff: = 0°
= 500 watts.

21. i = 10 Sin 377t

(a) $\frac{di}{dt}$ = 10 x 377Cos 377t

∴ Max^m value = 10 x 377 = 3770 Amp. When, Cos 377t = 1

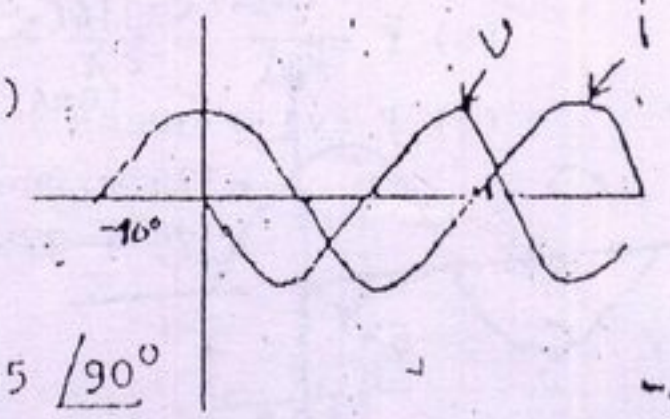
(b) X_L = ωL , L = 100 m H Vm = Im x_L = 10 x 37.7

= 377 x 100 x 10^{-3} ohm
= 37.7 ohm.

∴ v = 10 x 37.7 sin (377t + 90°) = 377sin (377t + 90°)

V_{max} = 377 volt. Ans.

22. v = -150 Sin 377t = 150 Sin(377t + 180°)
i = 10 Cos 377t = 10 Sin (377t + 90°)
As voltage leads current by 90°



∴ The circuit is inductive"

∴ impedance function, $\frac{150}{10}\angle 90° = 15\angle 90°$

Now, 377 L = 15 ∴ L = $\frac{15}{377}$ = 0.0398 henry.

23. $v = 100 \sin (377t + 30^\circ)$ $\quad \omega = 377$

$X_L = 377 \times 0.02\,654 = 10$ ohm. $\quad L = 0.02654$ henry.

(a) $\therefore i = \frac{100}{10} \sin (377t + 30^\circ - 90^\circ) = 10 \sin (377t - 60^\circ)$

(b) $P = vi = 100 \sin (377t + 30^\circ) \times 10 \sin (377t - 60^\circ)$

$= -1000 \sin (377t + 30^\circ) \cos (377t - 60^\circ + 90^\circ)$

$= -1000 \sin (377t + 30^\circ) \cos (377t + 30^\circ) = -500 \sin 2(377t + 30^\circ)$

(c) $P_{av} = \frac{V_m I_m}{2} \cos \theta = \frac{100 \times 10}{2} \cos 90^\circ = 0$ watts.

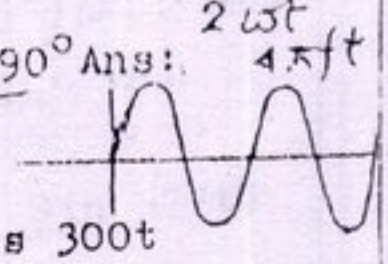
(d) $P = -500 \sin (754t + 60^\circ)$ watts.

$\frac{dp}{dt} = -500 \times 754 \cos (754t + 60^\circ)$, $dp/dt_m = 0$, $\cos (754t + 60^\circ) = 0$

(e) Energy stored $\frac{V_m I_m}{2\omega} = \frac{100 \times 10}{2 \times 377} = 1.326$ Joule.

24. $i = 5 \sin 300t$

(a) impedance function is $300 \times 0.2 \angle 90^\circ$ i.e. $60 \angle 90^\circ$ Ans:



(c) $v = 300 \sin (300t + 90^\circ) = 300 \cos 300t$.

$\therefore P = vi = 300 \cos 300t \times 5 \sin 300t = 1500 \sin 300t \cos 300t$

$= 750 \sin 2 \times 300t = 750 \sin 600t$.

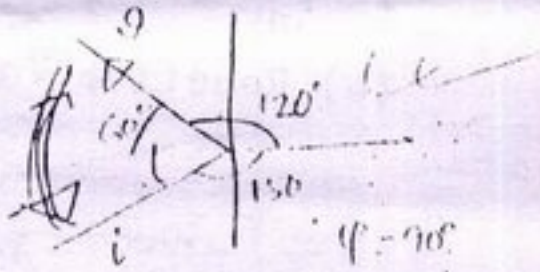
Stored Energy $(w_2) = \int_0^{0.05} +750 \sin 600t = \frac{-750}{600} \left[\cos 600t\right]_0^{0.05}$

$= -750/600 \ (\sqrt{3}/2 - 1) = 0.1675$ Joules.

25. $v = 200 \cos (157t + 30^\circ)$V, $i = 5 \sin (157t - 150^\circ)$ Amp.

$v = 200 \sin (157t + 30^\circ + 90^\circ)$. $\therefore v = 200 \sin (157t + 120^\circ)$

$i = 5 \sin (157t - 150^\circ)$.



Voltage is loading current by 270°

i.e " " loading " " -90°

i.e." " lagging " " 90°

$Z = \frac{200}{5} \angle -90^\circ = 40 \angle -90^\circ$ ohms.

The circuit parameter is capacitive $X_c = \frac{1}{\omega C}$

or $40 = \frac{1}{157c}$ $\therefore C = 159.23 \times 10^{-6}$ F $= 159.23 \mu$F Ans:

26. $V = 100 \sin 377t$ $\quad c = 530.5 \mu$F, $\omega = 377$

$X_c = \frac{10^6}{377 \times 530.5 \mu F} = 5$ ohm

(a) $i = \frac{100}{5} \sin (377t + 90^\circ) = 20 \sin (377t + 90^\circ)$ Ans.

(b) $p = vi = 100 \sin 377t \times 20 \sin (377t + 90^\circ) = 1000 \sin 754t$ Ans.

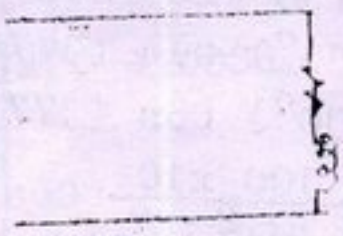
(c) $W_c = \frac{V_m I_m}{2} = \frac{100 \times 20}{2 \times 377} = 2.65$ Joules.

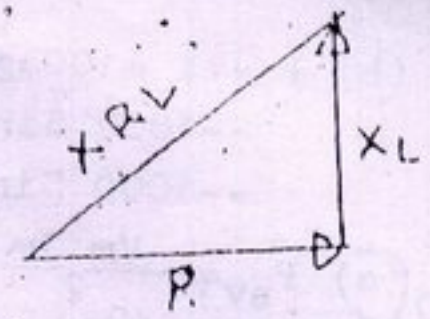
When the current is zero $I_m = 0$ $\therefore W_c = 0$

27. $v = 200 \sin 377t$, $X_L = \frac{200}{10} = 20$ ohm.

(a) $L = \frac{X_L}{\omega} = \frac{20}{377} = 53.05$ mh.

(b) $R_L = 20$ ohm.





$X_{RL} = 20$ ohm.

$X R_L = 20 \angle$

$X_L = \sqrt{20^2 - 1^2} = 19.975$ ohm.

$1 = \frac{X_L}{\omega} = 52.98$ mh.

28. $R = 10$ ohm. $X_L = 2\pi f L = 7.854$ ohm. $V_m = 150$ V

(a) $Z_{RL} = \sqrt{R^2 + X_L^2} \angle \text{Tan}^{-1} \frac{X_L}{R} = \sqrt{10^2 + 7.854^2} \angle \text{Tan}^{-1} \frac{7.854}{10}$

$= 12.71 \angle 38.15^\circ$

(b) $v = 150 \sin 157t$ volts.

(c) $i = \frac{150}{12.71} \sin (157t - 38.15^\circ) = 11.8 \sin (157t - 38.15^\circ)$ Ans:

(d) $p = \frac{11.8 \times 150}{2} \cos 38.15^\circ - 885 \cos 38.15^\circ \cos 314t + 885 \sin 38.15^\circ \sin 314t$

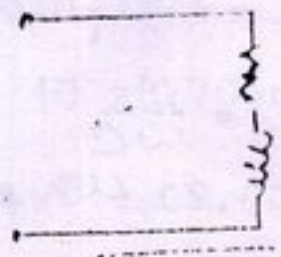
$= 695.96 - 695.96 \cos 314t + 546.68 \sin 314t$ watts Ans:

$P_{av} = \frac{V_m I_m}{2} \cos \theta = \frac{11.8 \times 150}{2} \times \cos 38.15^\circ = 695.96$ watts.

(e) Reactive Voltamperes $= \frac{V_m I_m}{2} \sin \theta = \frac{150 \times 11.8}{2} \times \sin 38.15^\circ$

$= 546.68$ VA or Vars. Ans:-

29.



$R = 10$ ohm.

$X_L = 7.85$ ohm.

(a) $Z_{RL} = \sqrt{R^2 + X_L^2} \angle \text{Tan}^{-1} \frac{X_L}{R} = 12.7 \angle 38.2^\circ$

(b) $v = 150 \sin (157t + \theta)$ at $t = 0$ $75 = 150 \sin\theta$

$\therefore \theta = \sin^{-1} \frac{1}{2} = 30^\circ$ $\therefore v = 150 \sin (157t + 30^\circ)$.

(c) As current lages voltage by 38.2°

$\therefore i = \frac{150}{12.7} \sin (157t + 30^\circ - 38.2) = 11.8 \sin (157t - 8.2^\circ)$ A Ans:

(d) $p = \frac{V_m I_m}{2} \cos\theta - \frac{V_m I_m}{2} \cos 2(\omega t - \theta) + \frac{V_m I_m}{2} \sin 2(\omega t) \cdot \sin\theta$

$= \frac{150 \times 11.8}{2} \cos 38.2^\circ - \frac{150 \times 11.8}{2} \cos (314t - 16.2^\circ)$.

$\cos 38.2^\circ + \frac{150 \times 11.8}{2} \sin (314t - 16.2^\circ) \sin 38.2^\circ$

$=885 \text{ Cos } 38.2^\circ - 885 \text{ Cos } (314t-16.2^\circ) \text{ Sin } 38.2^\circ$

$=695.48 - 695.48 \text{ Cos } (314t-16.2^\circ) + 547.29 \text{ Sin } (314t-16.2^\circ)$

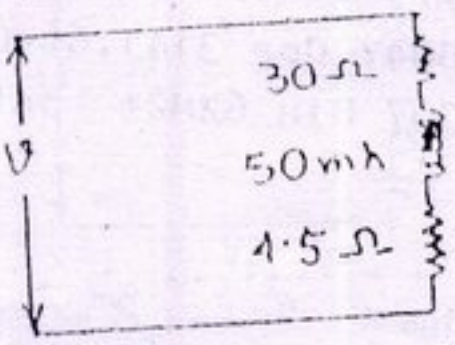
$=695.48 - 665.8 \text{ Cos } 314t - 194.03 \text{ Sin } 314t + 525.56 \text{ Sin } 314t$

$-152.69 \text{ Cos } 314t$

$=695.48 - 818.5 \text{ Cos } 314t) 331.53 \text{ Sin } 314t$ watts.

$P_{av} = \frac{Vm\ Im}{2} \text{Cos } \theta = \frac{150 \times 11.8}{2} \times \text{Cos } 38 = 695.48$ watts Ans:

30.



total R = 34.5 ohm.

$X_L = 18.85$ ohm.

$Z_{RL} = \sqrt{34.5^2 + 18.85^2} \angle \tan^{-1} \frac{18.85}{34.5}$

$= 39.31 \angle 28.65^\circ$

(a) $i = \frac{100}{39.31} \text{ Cos } (377t - 28.65^\circ) = 2.54 \text{Cos } (377t - 28.65^\circ)$ Amp. Ans.

(b) $P = vi = 100 \text{Cos } 377t \times 2.54 \text{ Cos } (377t - 28.65^\circ)$

$= 254 \text{ Cos } 377t \ (0.87756 \text{ Cos } 377t + 0.4794 \text{ Sin } 377t)$

$= \frac{229.91}{2}(1 + \text{Cos } 754t) + \frac{121.76}{2} \text{Sin } 754t$

$= 114.9 + 114.9 \text{ Cos } 754t + 60.88 \text{ Sin } 754t$ watts Ans:

(c) real power = $114.9 + 114.9 \text{ Cos } 754$ watts.

Average value = 114.9 watts.

(d) Reactive voltamperes $= 60.88 \text{ Sin } 754t$

Average value = 0 ($\because$ Sine wave)

(e) $X_L = \omega L = 18.85$ ohm Ans:

31. $i = 10 \text{ Cos } 157t$, $R = 15$ ohm, $X_L = \omega L = 157 \times 0.0637 = 10$ ohm.

$Z_{RL} = \sqrt{15^2 + 10^2} \angle \tan^{-1} \frac{10}{15} = 18.03 \angle 33.69^\circ$

(a) $v = 18.03 \times 10 \text{ Cos } (157t + 33.69) = 180.3 \text{Cos } (157t + 33.69^\circ)$

(b) $p = vi = \frac{180.3 \times 10}{2} \text{ Cos } 33.69 - 901.5 \text{ Cos } 314t \text{.Cos } 33.69^\circ$

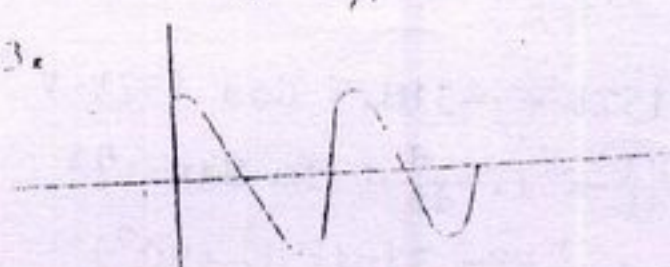
$+ 90.15 \text{ Sin } 314t \text{ Sin } 33.69^\circ$

$= 750 - 750 \text{ Cos } 314t + 500 \text{ Sin } 314t$ watts.

32. (a) $X_c = \frac{10^6}{\omega C} = \frac{10^6}{2\pi \times 60 \times 80} = 33.157$ ohm Ans:

(b) $X_c = \frac{10^6}{2\pi \times 60 \times 80} = 33.157$ ohm Ans:

33.



$r = 151$ ohm

$X_c = \frac{10^6}{2\pi \times 500 \times 4} = 79.58$ ohm.

(a) $v = 15 \cos(2\pi \times 500t) = 15 \cos 3141.6t$ V. Ans:

(b) $Z_{Rc} = \sqrt{151^2 + (-X_c)^2} \angle \tan^{-1} \frac{-X_c}{R} = 170.69 \angle -27.79^\circ$

(c) $i = \frac{15}{170.69} \cos(314.6t + 27.79^\circ)$

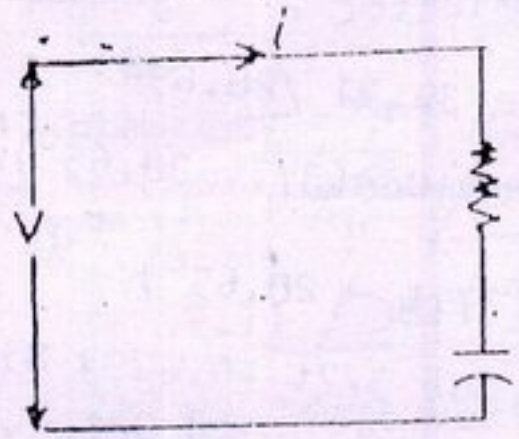
$= 0.0879 \cos(314.6t + 27.79^\circ)$ Ans:

(d) $p = vi = 15\cos 3141.6t \times 0.0879 \cos(3141.6t + 27.79)$

$= 1.3185 \cos 3141.6t \ (0.8847 \cos 3141.6t - 0.466 \sin 3141.6t)$

$= 0.583 + 0.583 \cos 6282t - 0.307 \sin 6282t$ watts.

35.



10 ohm

$X_c = \frac{10^6}{220 \times 303} = 15$ ohm.

$X_c = 15$ ohm

$v_c = 150 \sin(220t - 60^\circ)$, $i = \frac{150}{15} \sin(220t - 60^\circ + 90^\circ)$

$= 10 \sin(220t + 30^\circ)$ Amp. Ans:

Now, $Z_{tc} = \sqrt{10^2 + (-15)^2} \angle \tan^{-1} \frac{-15}{10} = 18.03 \angle -56.3^\circ$

∴ Voltage drop across the entive circuit is

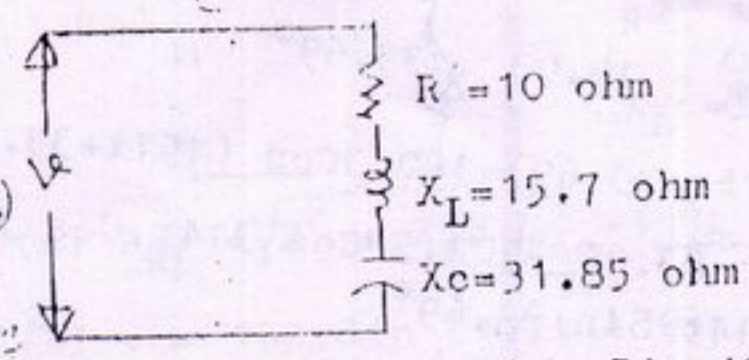
$v = 18.03 \times 10 \sin(220t + 30^\circ - 56.3^\circ)$

$= 180.3 \sin(220t - 26.3^\circ)$ Volt Ans:

36. $X_c = \frac{V_m}{I_m} = \frac{20}{0.01} = 2000$ ohm. $X_c = \frac{10^6}{\omega C_{\mu F}}$

$C = \frac{10^6}{\omega X_c} = \frac{10^6}{2\pi \times 2000 \times 2000} = 0.03978 \mu F$ Ans:-

37.



$X_L = \omega L = 157 \times 0.10 = 15.7$ ohm.

$X_c = \frac{10^6}{\omega C_{\mu F}} = \frac{10^6}{157 \times 200} = 31.85$ ohm

(a) $i = 10 \sin 157t$, $Ri = 10 \times 10 \sin 157t = 100 \sin 157t$ Volts.

(b) $V_L = L.\frac{di}{dt} = 0.10 \times \frac{d}{dt}(10 \sin 157t) = 0.1 \times 10 \times 157 \times \cos 157t$

$= 157 \cos 157t$ Ans:-

(c) $Vc = \frac{q}{c} = \int \frac{idt}{c} = \frac{10^6}{c} \int 10 \sin 157t \ dt.$

$= \frac{10^6}{200} \times \frac{1}{157} \times 10 \times -\cos 157t = -318.5 \cos 157t$ V

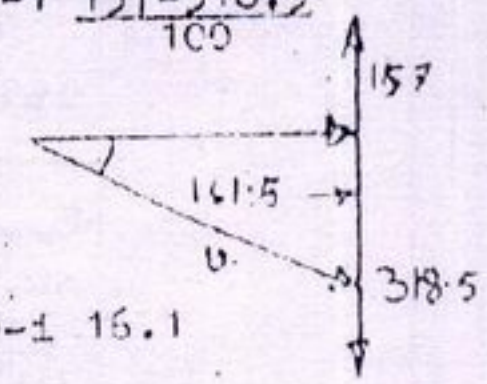
H.B.: $V = V_m \sin(\omega t + 90^\circ)$, $v = L.\frac{di}{dt} = I.\frac{d}{dt}(I_m \sin \omega t)$

$= L\omega I_m \cos \omega t = X_L I_m \cos \omega t = V_m \sin(\omega t + 90^\circ)$

(d) $V = V_R + V_L + V_c = \sqrt{100^2 + \left(\frac{157-318.5}{161.5}\right)^2} \angle \tan^{-1} \frac{157-318.5}{100}$

$= .190 \angle -58.2^\circ$

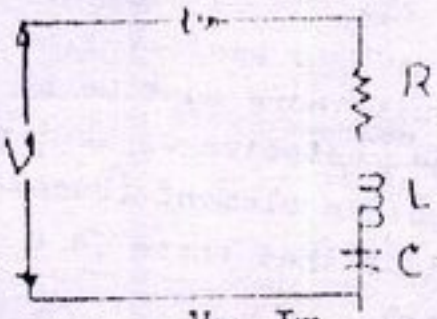
$\therefore v = 190 \sin (157t - 58.2^\circ)$ V Ans:



(e) $Z_{RLc} = R + j (X_L - X_c) = 10 + j (15.7 - 31.8) = 10 - j\ 16.1$

$= 18.79 \angle -58.2^\circ$ Ans:

(39.)



$i = Im \sin (t\sqrt{Lc})$

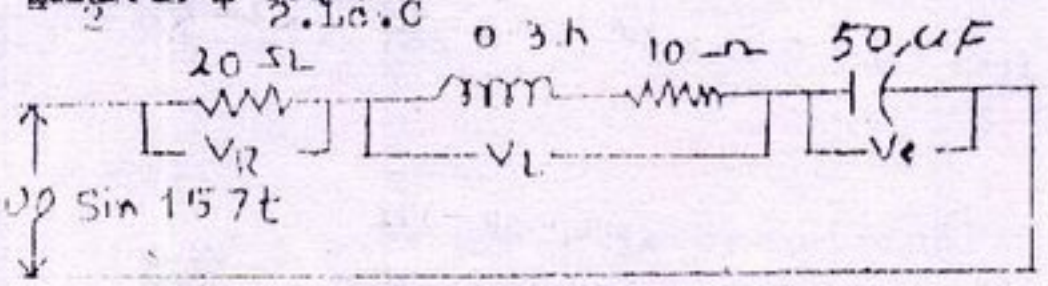
$\therefore$ here, $\omega = \sqrt{Lc}$

Now, $W_L + W_c = \frac{V_m I_m}{2w} + \frac{V_m I_m}{2w}$ $\quad \frac{V_m}{I_m} = \frac{1}{w_c}$ $\quad V_m = \frac{I_m}{w_c}$

$= \frac{w_L I_m}{2w} \times I_m + \frac{\frac{I_m}{w_c}}{2w} = \frac{L I_m^2}{2} + \frac{I_m}{2w^2 c}$

$= \frac{L I_m^2}{2} + \frac{I_m}{2.Lc.c}$

40.



total r = 20 + 10 = 30 ohm.

$X_L = \omega L = 157 \times 300 \times 10^{-2} = 47.1$ ohm

$X_c = \frac{10^6}{\omega C} = \frac{10^6}{157 \times 50} = 127.39$ ohm.

(a) $Z_{RLc} = 30 + j (47.1 - 127.39) = 85.7 \angle -69.5^\circ$ Ans:

(b) $i = \frac{100}{85.7} \sin (157t + 69.5) = 1.167 \sin (157t + 69.5)$. Amp. Ans

(c) $p = \frac{100 \times 1.167}{2} \cos (-96.5) - \frac{100 \times 1.167}{2} \cos 2 \times 157t . \cos (-69.5^\circ)$

$+ \frac{100 \times 1.167}{2} \sin 2.157t . \sin (-69.5^\circ)$

$= 20.44 - 20.48 \cos 314t - 54.65 \sin 314t$ watts. Ans:-

(d) $P_{av} = \frac{V_m I_m}{2} \cos \theta = 20.44$ watts Ans:-

(e) VA (Reactive) $= \frac{V_m I_m}{2} \sin \theta = 54.65$ W Ans:-

(f) $v_R = Ri = 20 \times 1.167 \sin (157t + 69.5)$

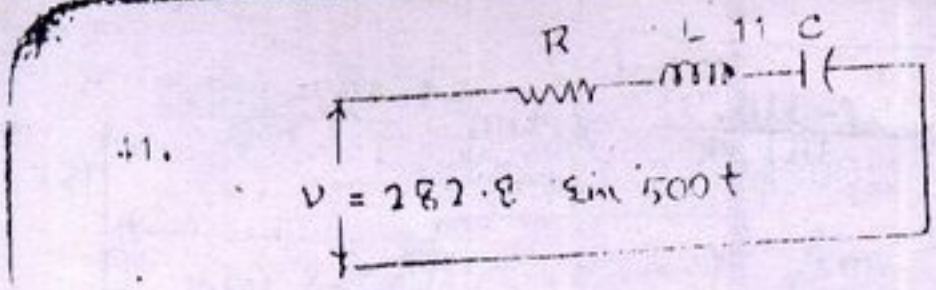
$= 23.34 \sin (157t + 69.5^\circ)$ Ans:-

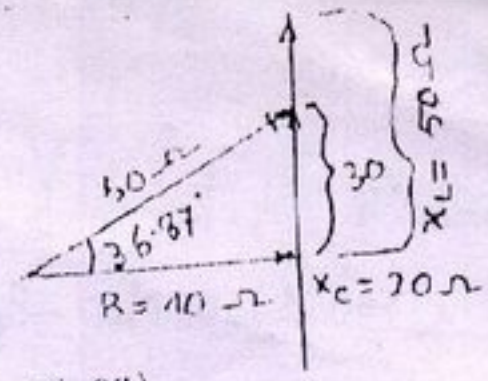
(g) $p_R = v_R i = 23.34 \sin (157t + 69.5) \times 1.167 \sin (157t + 69.5)$

$= \frac{23.34 \times 1.167}{2} (1 - \cos 2(157t + 69.5^\circ))$

$= 13.62 - 13.62 \cos (314t + 139^\circ)$ W Ans:-

41.





$v = 282.8 \sin 500t$, $i = 5.656 \sin(500t - 36.87)$

$\therefore Z = \frac{282.8}{5.656} \angle 36.87 = 50 \angle 36.87^\circ$

$C = 100\ \mu F$

$X_C = \frac{10^6}{500 \times 100} = 20$ ohm.

Now, $\tan\theta = \frac{\omega L - 1/\omega C}{R}$

or, $\tan 36.87 = \frac{500L - 20}{R}$ ..(1)

Since voltage leads current by 36.87° (which is positive), So in the circuit there must be at least one resistive element and one inductive element (because we consider that there is a capacitory).

Again, $50 = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

or, $2500 = R^2 + (500L - 20)^2$ ....(2)

From (1) $\Rightarrow 2500 = R^2 + (0.75R)^2$ $\therefore R = \sqrt{\frac{2500}{1.5625}} = 40$ ohm. Ans:

From (1) $0.75 \times 40 = 500L - 20$ $\therefore L = \frac{50}{500} = 0.1$ henry Ans:

1. $I = \sqrt{}$

$= \sqrt{100 \times 10}$ 3

$= 14.14$

2. (a)

(b) $i = 5 + 4$

(c) Since the about x a

$\therefore$ Averag

(d) $I_{eff} = \sqrt{\frac{1}{T}} \int_0^T$

$= \sqrt{\frac{1}{T}} \left[\right]_0^T$

$= \sqrt{\frac{1}{T}} \left[\right]$

$= \sqrt{\frac{1}{T}} \left[\right]$

$= \sqrt{\frac{1}{T}} \left[\right]$

$= \sqrt{33}$

3.

Now, $\omega = \frac{150}{3}$

or, $\omega = 50t$

CHAPTER III

EFFECTIVE CURRENT AND VOLTAGE
AVERAGE POWER

F O R M U L A :

1. r.m.s or effective Value, $I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$.
2. Average Value, $I_{av} = \frac{2}{T} \int_0^{T/2} i\,dt$
3. $I_{r.m.s.} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$
4. $I_{av} = \frac{2}{\pi} I_m = 0.636\ I_m$
5. Form Factor $= \frac{\text{r.m.s value}}{\text{average value}}$
6. Crest or peak or amplitud factor $= \frac{\text{Max}^m\text{. Value}}{\text{effective value}} = \sqrt{2}$ (for sine wave)
7. Power $(P_{av}) = \frac{V_m I_m}{2} \cos\theta = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\theta = VI \cos\theta$
8. Reactive power or Reactive volt-amperes $= VI \sin\theta$ (Vars)
9. p.f. $\Rightarrow$ Power Factor $= \frac{\text{Power}}{\text{Volt-amp}} = \frac{VI\cos\theta}{VI} = \cos\theta$
10. R.f $\Rightarrow$ Reactive Factor $= \frac{VI \sin\theta}{VI} = \sin\theta$
11. Volt-amperes $= VI = \sqrt{(\text{Power})^2 + (\text{Reactive volt-amp})^2} = \sqrt{(VI\cos\theta)^2 + (VI\sin\theta)^2}$

$\therefore I_{ab}=10\angle 0-60^\circ+6.67\angle 0+36.87=5-i8.66+5.34+i4=11.31\angle -24.2^\circ$

$I_{bc}=10\angle(-120-60)+6.67\angle -120+36.87+\frac{4000}{200}\times\angle -120^\circ$

$=-10+i0+0.8-j6.62-10-i17.32=30.69\angle -128.73^\circ$

$I_{ca}=10\angle 120-60^\circ+6.67\angle 120^\circ+36.87^\circ=5+i8.66-6.13+i2.62$

$=11.34\angle 95.72^\circ$

$\therefore I_{a'a}=I_{ab}-I_{ca}=10.34-i4.66+1.13-i11.28=1964\angle -51.26^\circ$

$I_{c'c}=I_{ca}-I_{bc}=-1.13+i11.28+19.2+i23.94=39.59\angle 62.84^\circ=2294.38$ watts.

$W_{c'c-cb}=200x39.59\cos(60-62.84)$ $\quad V_{cb}=-V_{bc}=-200\angle -120^\circ$

$=7908.28$ watts. $\quad =200\angle 60^\circ$

(b) $\text{Varmeter}_{a'a-ab}=200x19.64\sin(0+54.26)=3188.26$ vars(inductive)

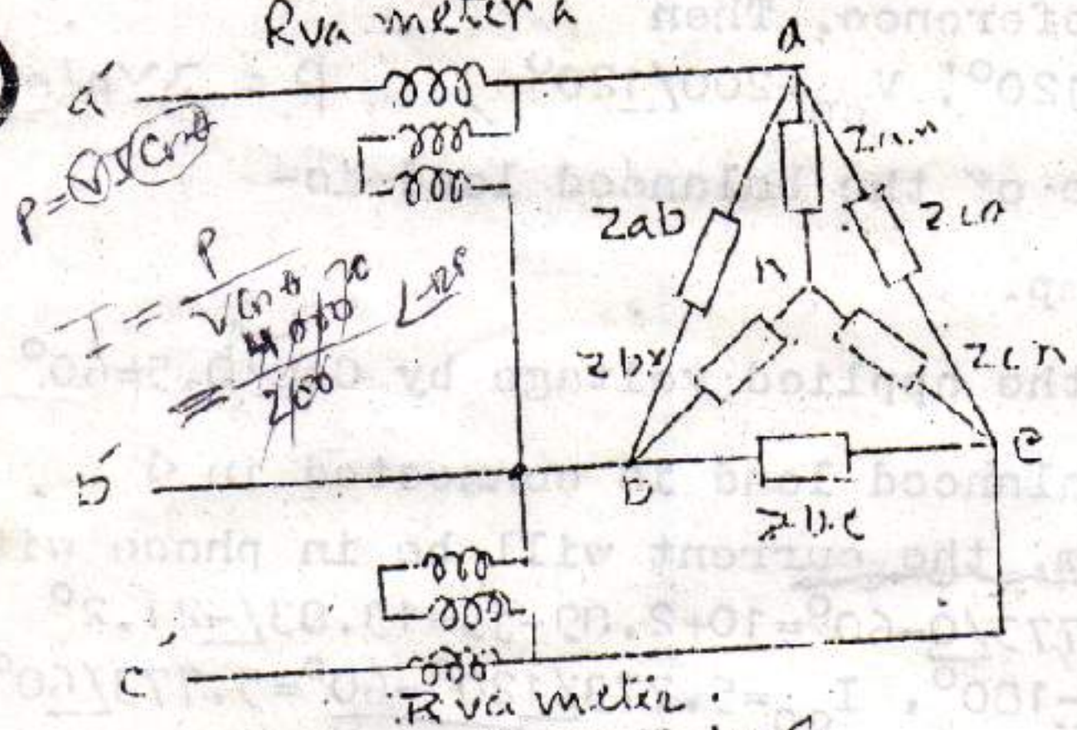
$\text{Vermeter}_{c'c-cb}=200x39.59\sin(60-62.84)=392.31$ vars(capacitive)

(c) $\Sigma VI\cos\theta=2294.38+7908.28=10202.66$ watts.

$\Sigma VI\sin\theta=3188.26-392.31=2795.95$ vars.

$\therefore$ Vector p.f. $=\frac{10202.66}{\sqrt{(10202.66)^2+(2795.95)^2}}=0.964$ Ans.

(27)



$\therefore Z_{an}=10\angle 0$, $Z_{bn}=10\angle -60^\circ$

$Z_{cn}=10\angle 90^\circ$

The voltage sequence is-

a'b'-c'a'-b'c'

$\therefore V_{ab}=V_{a'b'}=100\angle 0$

$V_{bc}=V_{b'c'}=100\angle 120^\circ$

$V_{ca}=V_{c'a'}=100\angle -120^\circ$

Transfering from Y to Δ

$Z_{ab}=\frac{(10\angle 0)(10\angle -60)+(10\angle -60)(10\angle 90^\circ)+(10\angle 90^\circ)(10\angle 0)}{10\angle 90^\circ}$

$=\frac{50-i86.6+86.6+i50+0+i100}{10\angle 90^\circ}=\frac{150.6\angle 24.9}{10\angle 90^\circ}=15.06\angle -65.1^\circ$

$Z_{bc}=\frac{150.6\angle 24.9}{10\angle 0}=15.06\angle 24.9$, $Z_{ca}=\frac{150.6\angle 24.9}{10\angle -60^\circ}=15.06\angle 84.9^\circ$

$\therefore I_{ab}=\frac{V_{ab}}{Z_{ab}}=\frac{100\angle 0}{15.06\angle -65.1}=6.64\angle 65.1$

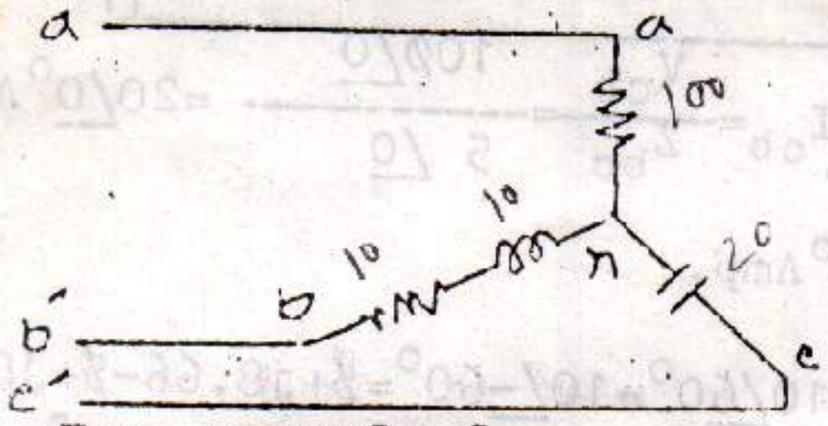
$I_{bc}=\frac{100\angle 120^\circ}{15.06\angle 24.9}=6.64\angle 95.1$, $I_{ca}=\frac{100\angle -120^\circ}{15.06\angle 84.9}=6.64\angle -204.9^\circ$

$I_{a'a}=I_{ab}-I_{ca}=2.8+i6.02+6.02-i2.8=9.39\angle 20.06$

$I_{c'c}=I_{ca}-I_{bc}=-6.02+i2.8+0.59-i6.61=6.63\angle -145^\circ$

CHAPTER IX

" UNBALANCED POLYPHASE CIRCUITS "

(1) 

From example 2.

$Z_{an} = 100+j0$ ohms.

$Z_{bn} = 10+j10$ ohms.

$Z_{cn} = 0-j20$ ohms.

$I_{an} = Ia'a = 3.66\angle 15$ Amp.

$I_{bn} = Ib'b = 14.56\angle -125.1$ amp.

$I_{cn} = Ic'c = 11.98\angle 66.7$ amp.

$\therefore V_{an} = I_{an}Z_{an} = (3.66\angle 15)(10\angle 0) = 36.6\angle 15$ Volts.

$V_{bn} = I_{bn}Z_{bn} (14.56\angle -125.1)(14.142\angle 45^\circ) = 205.9\angle -80.1$

$V_{cn} = I_{cn}Z_{cn} = (11.98\angle 66.2)(20\angle -90^\circ) = 239.6\angle -23.8$ Volts.

(2) $P_{an} = V_{an}I_{an}\cos\theta$ (V_{an}, I_{an}) $= 36.6 \times 3.66 \times \cos(15-15) = 134$ Watts.

$P_{bn} = 205.9 \times 14.56 \cos(-80.1+185.1) = 2119.84$ watts.

$P_{cn} = 239.6 \times 11.98 \cos(-23.8-66.2) = 0$ watts.

(4) $$\frac{E_{n'a}-I_{nn'}Z_n}{Z_a} + \frac{E_{n'b}-I_{nn'}Z_n}{Z_b} + \frac{E_{n'c}-I_{nn'}Z_n}{Z_c} = I_{nn'}$$

or, $E_{n'a}Z_bZ_c + E_{n'b}Z_cZ_a + E_{n'c}Z_aZ_b - I_{nn'}Z_n(Z_bZ_c+Z_cZ_a+Z_aZ_b) = I_{nn'}Z_aZ_bZ_c$

$$\therefore I_{nn'} = \frac{E_{n'a}Z_bZ_c + E_{n'b}Z_cZ_a + E_{n'c}Z_aZ_b}{Z_aZ_bZ_c + Z_n(Z_bZ_c+Z_cZ_a+Z_aZ_b)}$$ Ans.

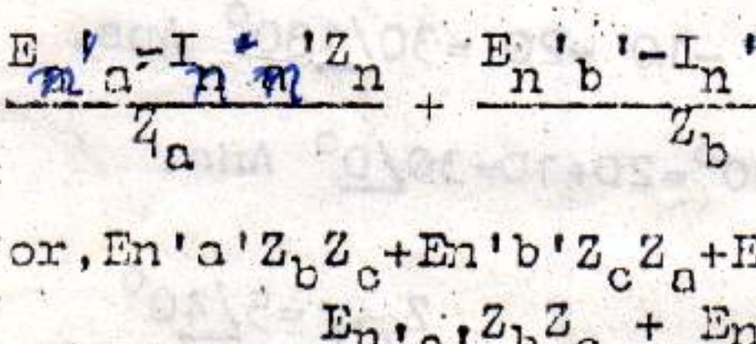
(7) Soln:- $W_{ba} = V_{ba} I_{b'b} \cos\theta$ (V_{ba}, $I_{b'b}$)

$V_{ba} = V_{ab}\angle 0+80^\circ$

$V_{b'b} = 37.55\angle 45.55^\circ$

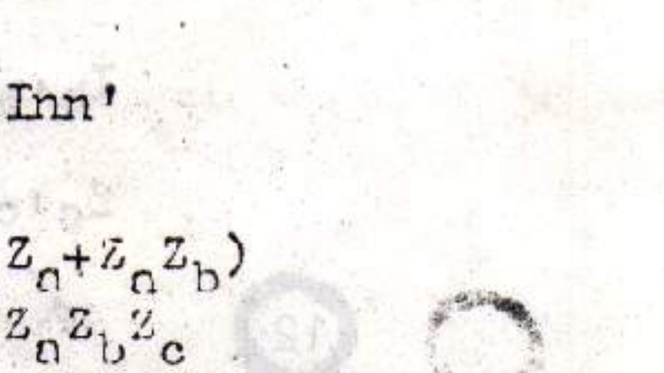
$= 200 \times 35.06 \cos(180-145) = 5742.25$

$W_{ca} = c'c = 200 \times 21.66 \cos\{120-(-126.75)\} = 4301.97.$

(11) 

$Z_{ab} = 10\angle -60^\circ$, $Z_{bc} = 5\angle 0^\circ$

$Z_{ca} = 10\angle 60^\circ$

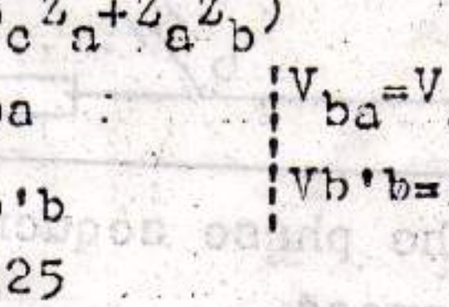


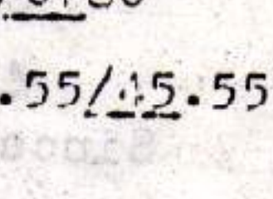
Since the three phase supply voltages are balanced and the phase sequence is cb-ba-ac.

$\therefore V_{cb} = 100\angle 0$

$V_{ba} = 100\angle -120^\circ$

$V_{ac} = 100\angle 120^\circ$





Phase sequence is always connected in the clockwise direction.

(8) $Z = 100 - j100 = 141.4\angle -45$

Now, $I_{ba} = \frac{V_{ba}}{Z_{ab}}$ [Since the impedences are bilateral. $\therefore Z_{ab} = Z_{ba}$]

$= \frac{100/120}{10/-60^\circ} = 10/-60^\circ$ Amp. $I_{cb} = \frac{V_{cb}}{Z_{bc}} = \frac{100/0}{5/0} = 20/0^\circ$ Amp.

$I_{ac} = \frac{V_{ac}}{Z_{ca}} = \frac{100/120^\circ}{10/60^\circ} = 10/60^\circ$ Amp.

Now, $Ia'a = I_{ac} + I_{ab} = I_{ac} - I_{ba} = 10/60^\circ - 10/-60^\circ = 5 + j8.66 - 5 + j8.66$

$= 17.32/90^\circ$ Ans.

$Ib'b = I_{ba} - I_{cb} = 10/-60^\circ - 20/0^\circ = 5 - j8.66 - 20 + j0 = 17.32/-150^\circ$ Ans.

$Ic'c = I_{cb} - I_{ac} = 20/0^\circ - 10/60^\circ = 20 - 5 - j8.66 = 17.32/-30^\circ$ Ans.

For opposite sequence, the sequence will be cb-ac-ba.

$\therefore V_{cb} = 100/0^\circ \quad V_{ac} = 100/-120^\circ \quad V_{ba} = 100/120^\circ$

$\therefore I_{ba} = \frac{V_{ba}}{Z_{ab}} = \frac{100/120^\circ}{10/-60} = 10/180^\circ$

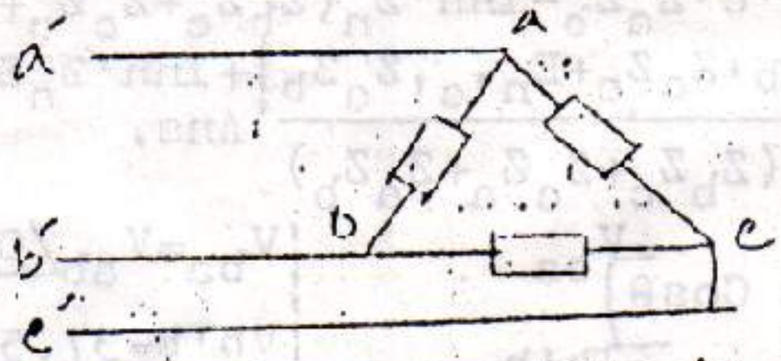
$\therefore I_{cb} = \frac{Vcb}{Z_{bc}} = \frac{100/0}{5/0} = 20/0^\circ \quad I_{ac} = \frac{V_{ac}}{Z_{ca}} = \frac{100/-120^\circ}{10/60^\circ} = 10/-180^\circ$

$\therefore Ia'a = I_{ac} - I_{ba} = 10/-180^\circ - 10/180^\circ = 10 + 10 = 0$ Ans.

$Ib'b = I_{ba} - I_{cb} = 10/180^\circ - 20/0^\circ = -10 - 20 = 30/180^\circ$ Ans.

$Zc'c = I_{cb} - I_{ac} = 20/0^\circ - 10/-180^\circ = 20 + 10 = 30/0^\circ$ Ans.

(12)



$Z_{ab} = 5/40^\circ$
$Z_{bc} = 10/-30^\circ$
$Z_{ca} = 8/45^\circ$

Since the phase sequence is cb-ac-ba and the line voltages are balanced.

$\therefore V_{cb} = 115/0^\circ, \quad V_{ac} = 115/-120^\circ, \quad V_{ba} = 115/120^\circ$

Now, $I_{cb} = \frac{V_{cb}}{Z_{bc}} = \frac{115/0}{10/-30^\circ} = 11.5/30^\circ$

$I_{ac} = \frac{V_{ac}}{Z_{ca}} = \frac{115/-120^\circ}{8/45^\circ} = 14.375/-165^\circ$

$I_{ba} = \frac{V_{ba}}{Z_{ab}} = \frac{115/120^\circ}{5/40^\circ} = 23/80^\circ$

Since the currents are leaving the terminals.

$\therefore I_{aa'} = I_{ba} + I_{ca} = I_{ba} - I_{ac} = 23/80^\circ - 14.375/-165^\circ$

$= 4 + j22.65 + 13.885 + j3.72 = 31.863/55.85^\circ$ A Ans.

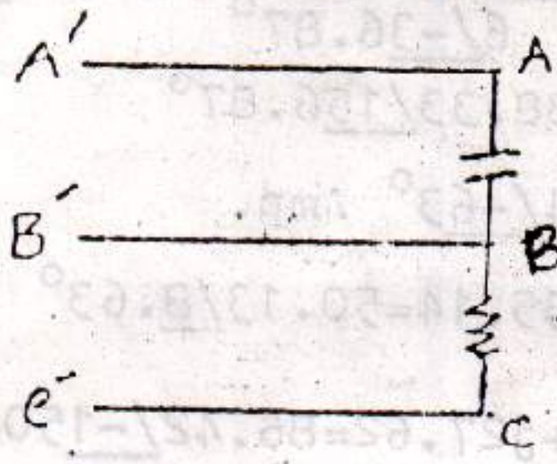
$I_{bb'} = I_{cn} - I_{ba} = 11.5\angle 30^\circ - 23\angle 80^\circ = 9.96 + j5.751 - 4 - j22.65$

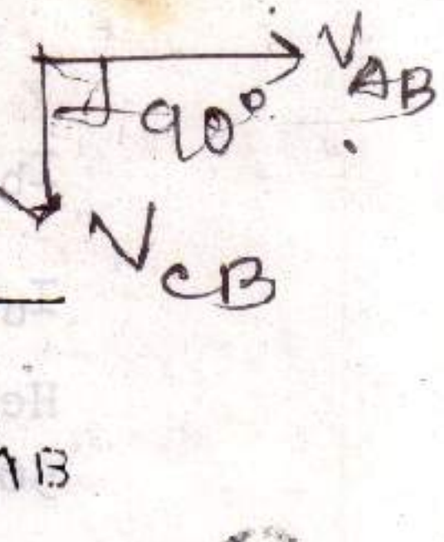
$= 17.92\angle -70.57^\circ$ Amp.

$I_{cc'} = I_{ac} - I_{cb} = 14.375\angle -165^\circ - 11.5\angle 30^\circ$

$= -13.885 - j3.72 - 9.96 - j5.75 = 25.66\angle -158.34^\circ$ Amp.

(13)





Since the voltage sequence is AB-CB

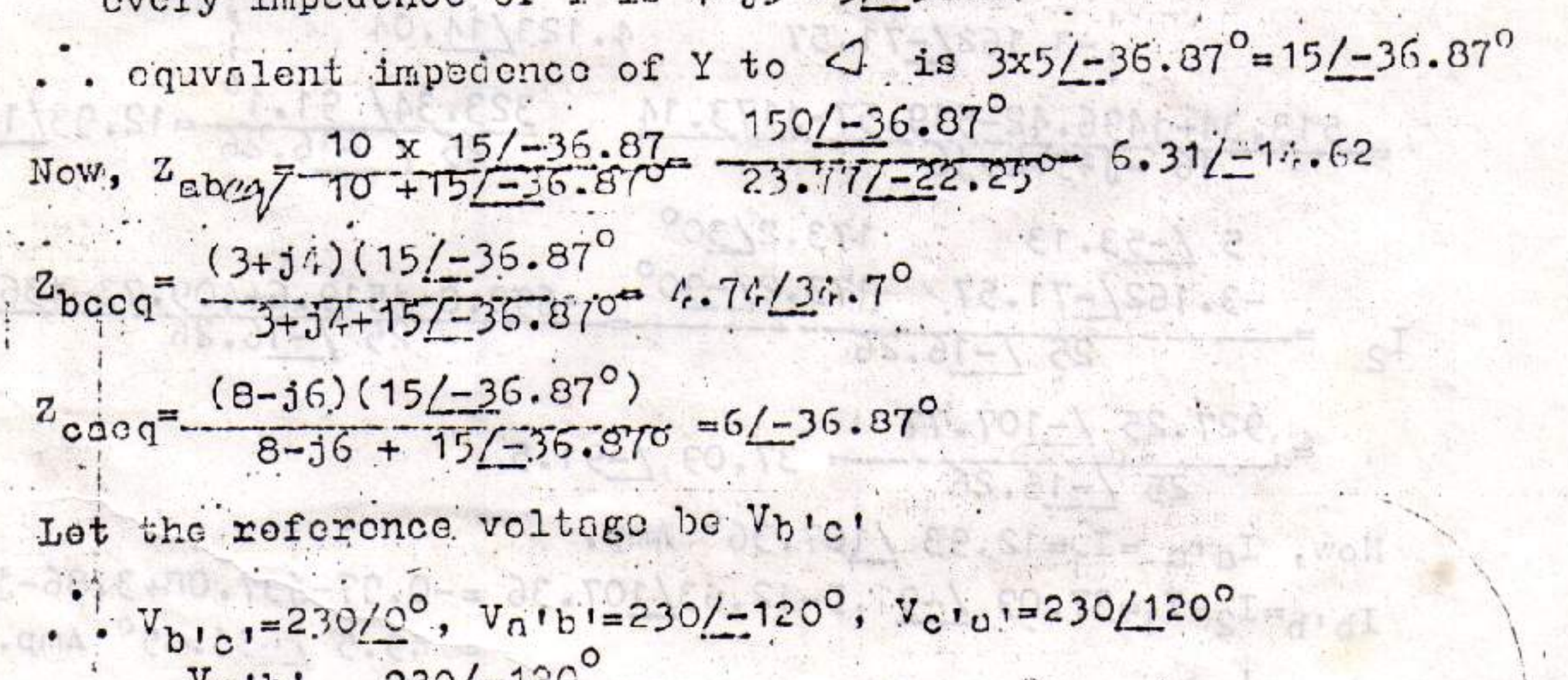
$\therefore V_{AB} = 115\angle 0$, $V_{cB} = 115\angle -180^\circ - (-90^\circ) = 115\angle -90^\circ$

$\therefore I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{115\angle 0^\circ}{23\angle -90^\circ} = 5\angle 90^\circ$ Amp.

$I_{CB} = \frac{V_{cB}}{Z_{Bc}} = \frac{115\angle -90^\circ}{23\angle 0^\circ} = 5\angle -90^\circ$ Amp.

$I_{BB'} = I_{AB} + I_{cB} = 5\angle 90^\circ + 5\angle -90^\circ = 0 + j50 + 0 - j5 = 0$ Ans.

(15)



every impedence of Y is $4 - j3 = 5\angle -36.87^\circ$

$\therefore$ equvalent impedence of Y to $\triangle$ is $3 \times 5\angle -36.87^\circ = 15\angle -36.87^\circ$

Now, $Z_{abeq} = \frac{10 \times 15\angle -36.87}{10 + 15\angle -36.87^\circ} = \frac{150\angle -36.87^\circ}{23.77\angle -22.25^\circ} = 6.31\angle -14.62$

$Z_{bceq} = \frac{(3+j4)(15\angle -36.87^\circ}{3+j4+15\angle -36.87^\circ} = 4.74\angle 34.7^\circ$

$Z_{caeq} = \frac{(8-j6)(15\angle -36.87^\circ)}{8-j6 + 15\angle -36.87^\circ} = 6\angle -36.87^\circ$

Let the reference voltage be $V_{b'c'}$

$\therefore V_{b'c'} = 230\angle 0^\circ$, $V_{a'b'} = 230\angle -120^\circ$, $V_{c'a'} = 230\angle 120^\circ$

$I_{ab} = \frac{V_{a'b'}}{Z_{abeq}} = \frac{230\angle -120^\circ}{6.31\angle -14.62} = 36.45\angle -105.38^\circ$

$I_{bc} = \frac{230/0^\circ}{4.74/34.7} = 48.52/-34.7^\circ$, $I_{ca} = \frac{230/120^\circ}{6/-36.87^\circ} = 38.33/156.87^\circ$

$\therefore I_{a'a} = I_{ab} - I_{ca} = 36.45/-105.38 - 38.33/156.87^\circ$

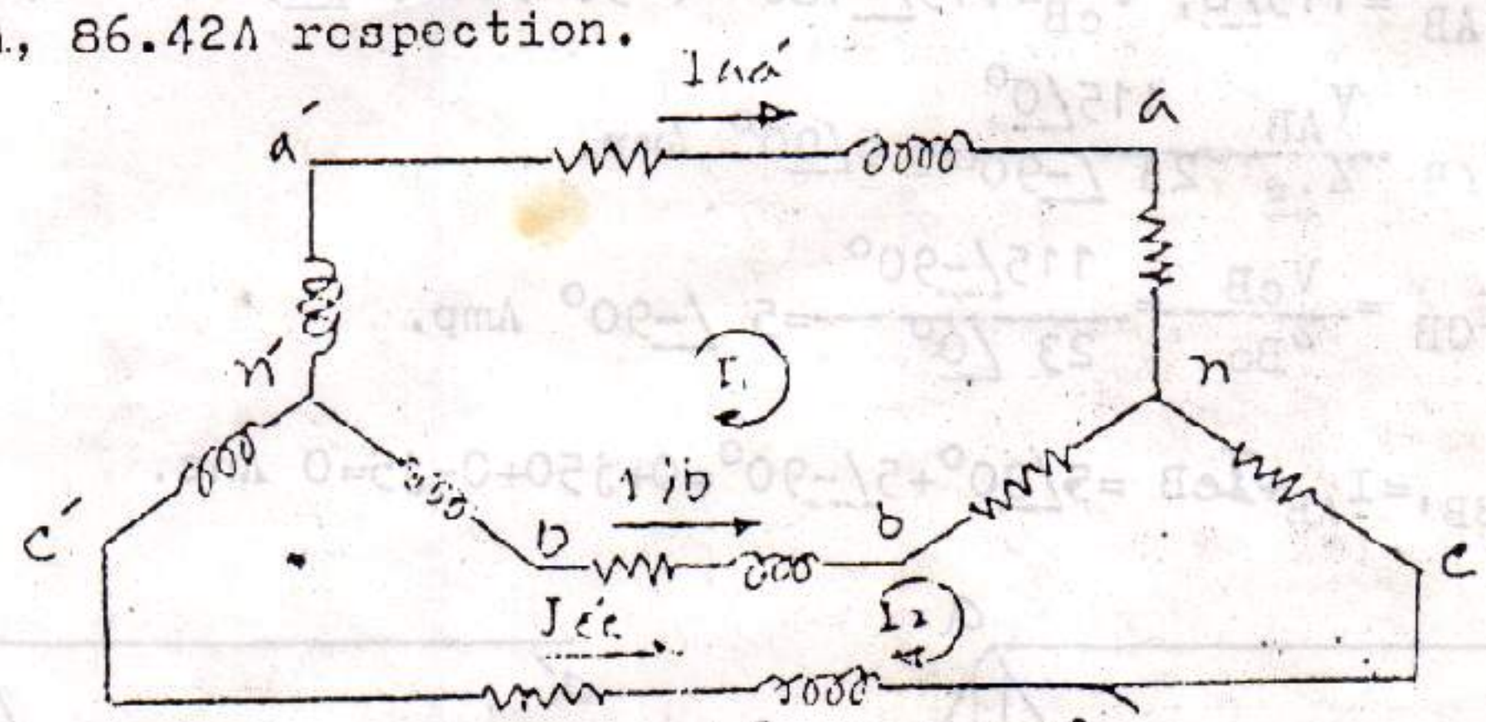
$= -9.67 - j35.14 + 35.25 - j15.06 = 5634/-63^\circ$ Amp.

$I_{b'b} = I_{bc} - I_{ab} = 39.89 - j27.62 + 9.67 + j35.14 = 50.13/8.63^\circ$

$I_{c'c} = I_{ca} - I_{bc} = -35.25 + j15.06 - 39.89 + j27.62 = 86.42/-150.4$ Ans.

Hence the meter a'a, b'b, c'c readings will be 56.34A, 50.13A, 86.42A respection.

16



$I_1(2 - j1 + 1 - j3) - (1 - j3) I_2 = 100/0^\circ - 100/-120^\circ$

or, $5/-53.13\ I_1 - 3.162/-71.57^\circ I_2 = 173.2/30^\circ - (1 - j3) I_1 + (1 - j3 + 3 + j4) I_2$

$= 100/-120^\circ - 100/-240^\circ$

or, $-3.162/-71.57\ I_1 + 4.123/14.04\ I_2 = 173.2/-90^\circ$

$$I_1 = \frac{\begin{vmatrix} 173.2/30 & -3.162/-71.57^\circ \\ 173.2/-90 & 4.123/14.04 \end{vmatrix}}{\begin{vmatrix} 5/-53.13 & -3.162/-71.57 \\ -3.162/-71.57 & 4.123/14.04 \end{vmatrix}} = \frac{513.34 + j496.42 - 519.57 - j173.14}{16 - j43 + 8 + j6} = \frac{323.34/91.1^\circ}{25/-16.26} = 12.93/107.36 \text{ Amp.}$$

$$I_2 = \frac{\begin{vmatrix} 5/-53.13 & 173.2/30^\circ \\ -3.162/-71.57 & 173.2/-90^\circ \end{vmatrix}}{25/-16.26} = \frac{-692.8 - j519.6 + 409.73 - j363.39}{25/-16.26}$$

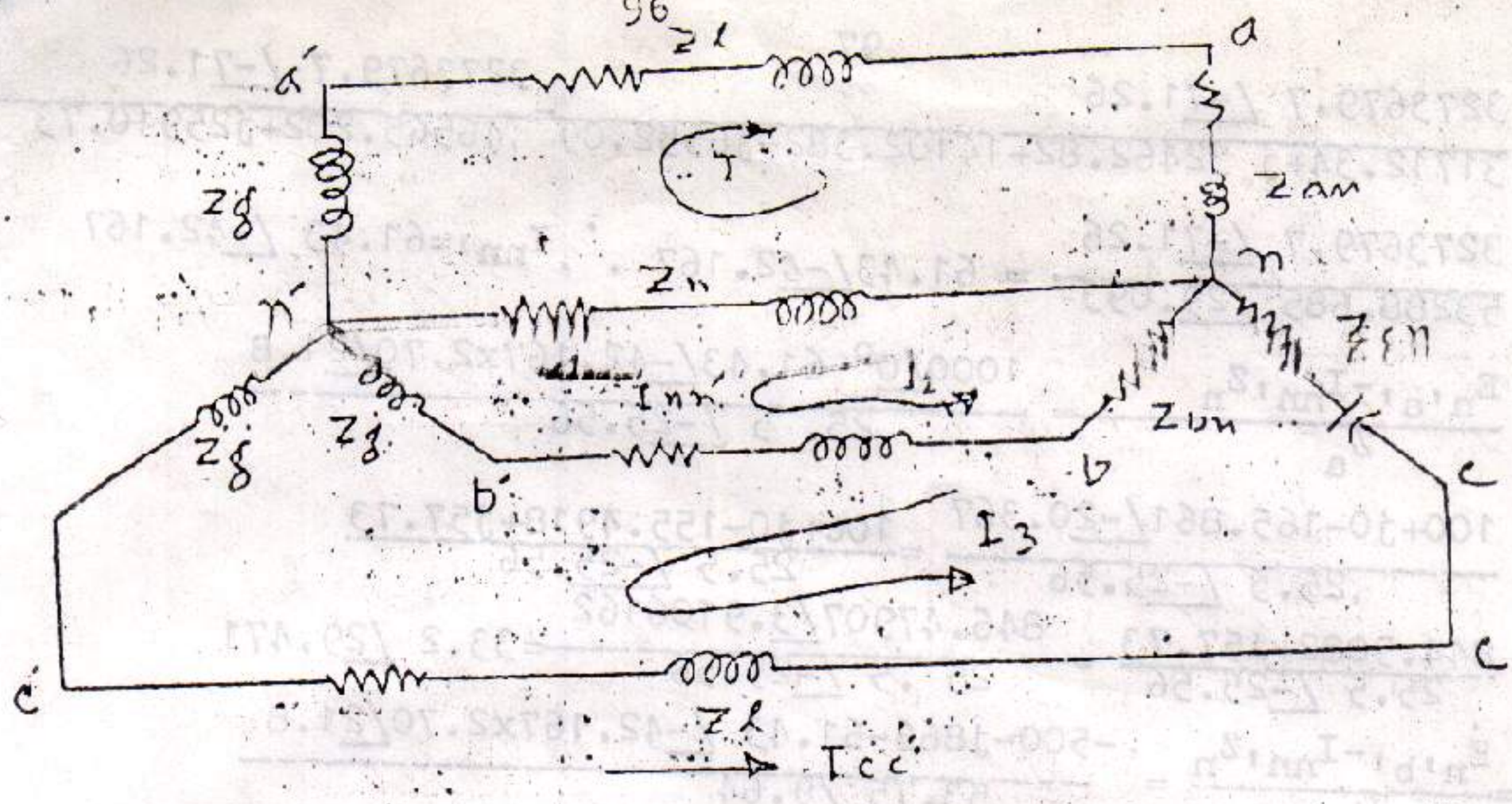
$$= \frac{927.25/-107.77}{25/-16.26} = 37.09/-91.5$$

Now, $I_{a'a} = I_1 = 12.93/107.36^\circ$ Amp.

$I_{b'b} = I_2 - I_1 = 37.09/-91.5 - 12.93/107.36 = -0.97 - j37.08 + 3.86 - j12.34$

$= 49.5/-33.65^\circ$ Amp.

$I_{c'c} = -I_2 = 37.09/-91.5 = 37.09/88.5^\circ$



A four - wire three phase system.

$E_{n'a'} = 1000+j0 = 1000\angle 0^\circ$

$E_{n'b'} = -500-j866 = 1000\angle -120^\circ$

$E_{n'c'} = -500+j866 = 100\angle 120^\circ$

$Z_g = 2 + j8 = 8.25\angle 76^\circ$

$Z_{an} = 20-j20 = 28.28\angle -45^\circ$

$Z_{bn} = 50+j0 = 50\angle 0^\circ$

$Z_{cn} = 30+j52 = 60\angle 60^\circ$

$Z_L = 1+j1 = 1.41\angle 45^\circ$

$Z_n = 2.5+j1 = 2.69\angle 21.80$

$Z_g = 2+j8 = 8.25\angle 76^\circ$

$Z_a = Z_g+Z_l+Z_{an} = 2+j8+1+j1+20-j20 = 23-j11 = 25.50\angle -25.56^\circ$

$Z_b = Z_g+Z_l+Z_{bn} = 2+j8+1+j1+50+j0 = 53+j9 = 53.76\angle 9.64$

$Z_c = Z_g+Z_l+Z_{an} = 2+j8+1+j1+30+j52 = 33+j61 = 69.35\angle 61.59$

$$I_{nn'} = \frac{E_{n'a'} Z_b Z_c + E_{n'b'} E_c Z_a + E_{n'c'} Z_a Z_b}{Z_a Z_b Z_c + Z_n (Z_b Z_c + Z_c Z_a + Z_a Z_b)}$$

$$= \frac{100\angle 0^\circ \times 53.75\angle 9.64 \times 69.35\angle 61.59 + 1000\angle 120^\circ \times 69.35\angle 61.59 \times 25.5\angle -25.56 + 100\angle 120^\circ \times 25.5\angle -25.56 \times 53.75\angle 9.64}{25.5\angle -25.56 \times 53.75\angle 9.64 \times 69.35\angle 61.59 + 2.70\angle 21.8 (53.75\angle 9.64 \times 69.35\angle 61.58 + 69.35\angle 61.58 \times 25.5\angle -25.56 + 25.5\angle -25.56 \times 53.75\angle 9.63)}$$

$$= \frac{3727562.5\angle 71.23 + 1768425\angle -83.97 + 1370625\angle 104.08}{95381.794\angle 45.67 + 2.70\angle 21.8 (3727.5625\angle 71.22 + 1768.425\angle 36.02 + 1370.625\angle -15.93)}$$

$$= \frac{1199417.7 + j3529322.8 + 185771.59 - j1758640.4 - 33440.78 + j1329447.3}{95381.794\angle 45.67 + 2.70\angle 21.8 (1200.0337 + j3529.1134 + 1430.323 + j1039.9535 + 1317.9899 - j376.1855)}$$

$$= \frac{1051748.5 - j3100129.7}{95381.794\angle 45.67 + 2.70\angle 21.8 (3948.3466 - j4192.8814)}$$

$$= \frac{3278679.7\angle -71.26}{45381.794\angle 45.67 + 2.70\angle 21.8 \times 5759.3138\angle -46.72}$$

$$= \frac{3273679.7\angle -71.26}{45381.794\angle 45.67 + 15550.147\angle -24.92}$$

$$= \frac{3273679.7\ \angle -71.26}{31712.34+j\ 32462.82+14102.382-j6552.09} = \frac{3273679.7\ \angle -71.26}{46565.202+j25910.73}$$

$$= \frac{3273679.7\ \angle -71.26}{53288.685\ \angle 29.093} = 61.43\angle -42.167 \quad \therefore I_{nn'}=61.43\ \angle -42.167$$

$$I_{rr'} = \frac{E_{n'a'}-I_{nn'}Z_n}{Z_a} = \frac{1000\angle 0^\circ - 61.43\angle -42.167 \times 2.70\angle 21.8}{25.5\ \angle -25.56}$$

$$= \frac{100+j0-165.861\angle -20.367}{25.5\ \angle -25.56} = \frac{100+j0-155.4918+j57.73}{25.5\ \angle -25.56}$$

$$= \frac{844.5082+i57.73}{25.5\ \angle -25.56} = \frac{846.47907\angle 3.9106162}{25.5\ \angle -25.56} = 33.2\ \angle 29.471$$

$$I_{b'b} = \frac{E_{n'b'}-I_{nn'}Z_n}{Z_b} = \frac{-500-i866-61.43\ \angle -42.167 \times 2.70\angle 21.8}{53.75\ \angle 9.64}$$

$$= \frac{-500-i866-155.4918+i57.73}{}$$

$$I_{c'c} = \frac{E_{n'c'}-I_{nn'}Z_n}{Z_c} = \frac{-500+j866-155.4918+j57.73}{69.35\ \angle 61.59} = 16.33\ \angle 63.77.$$

(18) Solution:-

Equivalent Y-loads of loads as follows

$$Z_{an} = \frac{Z_{ab}\,Z_{ac}}{Z_{ab}+Z_{bc}+Z_{ca}} = \frac{50\angle 0^\circ \times 5\angle -60^\circ}{5\angle 0^\circ + 5\angle 60^\circ + 5\angle -60^\circ} = 2.5\ \angle 60^\circ$$

$Z_{ab}=5\ \angle 0^\circ$, $Z_{bc}=5\ \angle 60^\circ$

$Z_{ca}=5\ \angle -60^\circ$

$Z_{ab}+Z_{bc}+Z_{ca}=10\ \angle 0^\circ$, $Z_{bn}=\frac{Z_{ab}xZ_{bc}}{10\ \angle 0^\circ} = \frac{5\angle 0^\circ x 5\angle 60^\circ}{10} = 2.5\ \angle 60^\circ$

$Z_{cn}=\frac{Z_{ca}\,Z_{bc}}{10\angle 0^\circ} = \frac{5\angle -60^\circ x 5\angle 60^\circ}{10} = 2.5\ \angle 0^\circ$

(19)

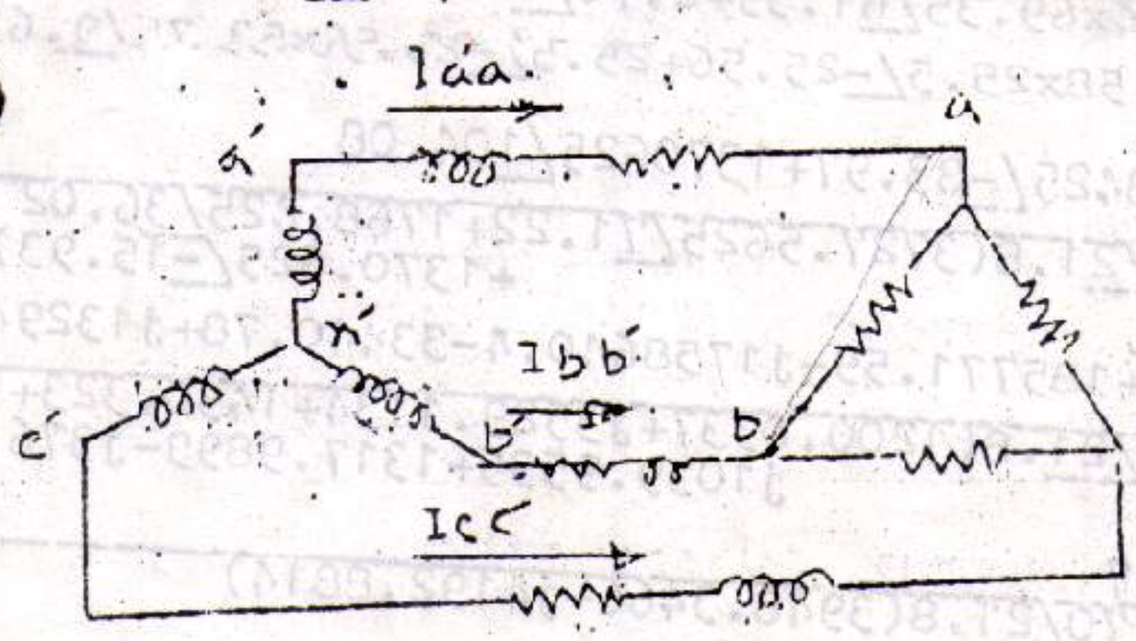


Fig.1

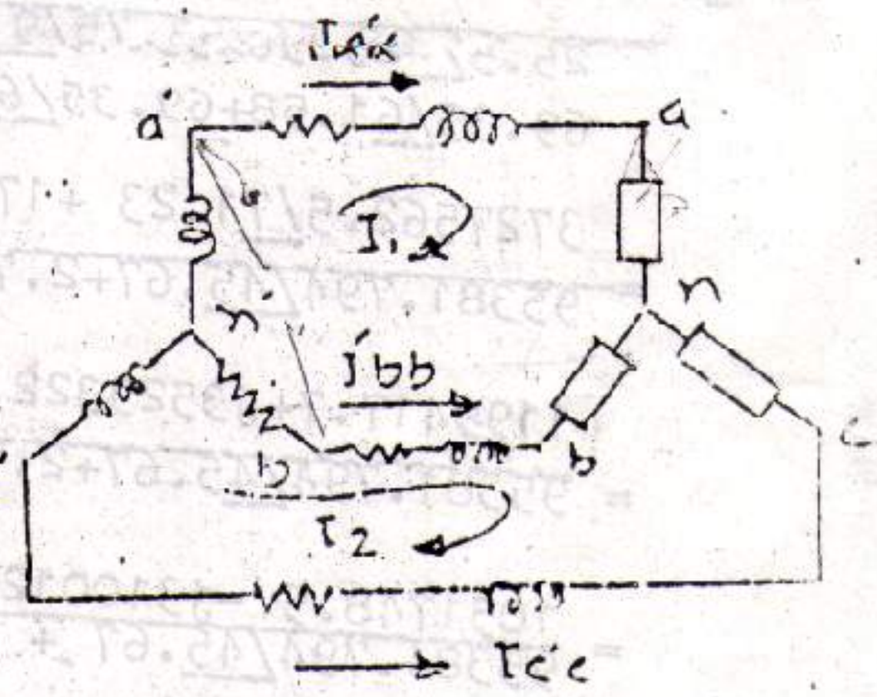


Fig.2

In Fig.1: $Z_{ab}=10\angle 0 = 10+i0$, $Z_{bc}=10\angle 60^\circ = 5+j866$, $Z_{ca}=10\angle -60^\circ = 5-i866$

In Fig.2: $Z_{an}=\frac{10\angle 0 \times 10\angle -60}{10+5+j8.66+5-j8.66}=5\angle -60^{\circ}=2.5-j4.33$

$Z_{bn}=\frac{10\angle 0 \times 10\angle 60^{\circ}}{20}=5\angle 60^{\circ}=2.5+j4.33$

$Z_{cn}=\frac{10\angle 60 \times 10\angle -60}{20}=5\angle 0^{\circ}=5+j0$

Voltage sequence is b'a' -a'c' -c'b'

$\therefore E_{b'a'}=100\angle 0$; $E_{c'b'}=100\angle 120$.

$E_{a'c'}=100\angle -120$

From Fig.2

$(0.5+j0.5+2.5-j4.33+2.5+j4.33+0.5+j0.5)I_1-(2.5+j4.33+0.5+j0.5)=E_{b'n'}$

$-(2.5+j4.33+0.5+j0.5)I_1+(0.5+j0.5+2.5+j4.33+5+j0+0.5+j0.5)I_2=E$

or, $6.08\angle 9.46\ I_1-5.69\angle 58.15\ I_2=100\angle 0^{\circ}$

$-5.69\angle 58.15\ I_1+10.03\angle 32.09\ I_2=100\angle 120^{\circ}$

$$\therefore I_1=\frac{\begin{vmatrix} 100\angle 0 & -5.69\angle 58.15 \\ 100\angle 120^{\circ} & 10.03\angle 32.09 \end{vmatrix}}{\begin{vmatrix} 6.08\angle 9.46 & -5.69\angle 58.15 \\ -5.69\angle 58.15 & 10.03\angle 32.09 \end{vmatrix}}=\frac{849.76+j532.84-568.7+j18.37}{45.64+j40.45+14.34-j29.02}=\frac{618.73\angle 62.9^{9}}{61.06\angle 10.8^{\circ}}=10.13\angle 52.1^{\circ}$$

$$I_2=\frac{\begin{vmatrix} 6.08\angle 9.46^{\circ} & 100\angle 0^{\circ} \\ -5.69\angle 58.15^{\circ} & 100\angle 120^{\circ} \end{vmatrix}}{61.06\ 0.8^{\circ}}=\frac{386.41+j469.42+300.26+j433.33}{61.06\angle 10.8^{\circ}}$$

$$=\frac{956.64\angle 95.17^{\circ}}{61.06\angle 10.8^{\circ}}=15.67\angle 84.37^{\circ}$$

Now, $I_{a'a}=I_1=10.13\angle 52.1^{\circ}$ amp.

$I_{b'b}=I_2-I_1=15.67\angle 84.37-10.13\angle 52.1=1.54+j15.59-6.22-j7.99$

$=8.93\angle 121.62^{\circ}$ amp.

Now, From Fig.1 : $V_{ab}+I_{a'a}Z_{a'a}-I_{b'b}Z_{b'b}=E_{b'a'}$

or, $V_{ab}=100+j0-(10.13\angle 52.1)(0.5+j0.5)+(8.93\angle 121.62)(0.5+j0.5)$

$=100+j0+0.89-j7.11-6.14+j1.46=94.92\angle -3.41^{\circ}$

$\therefore I_{ab}=\frac{V_{ab}}{Z_{ab}}=\frac{94.92\angle -3.41}{10\angle 0}=9.49\angle -3.41$ amp.

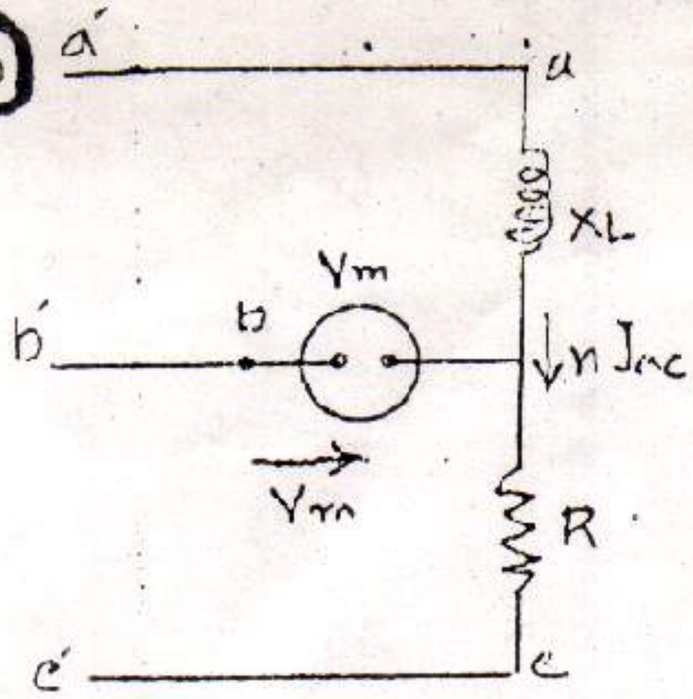
Now, $I_{ab}+I_{ac}=I_{a'a}$ or, $I_{ca}=I_{ab}-I_{a'a}=9.47-j56-6.22-j7.99$

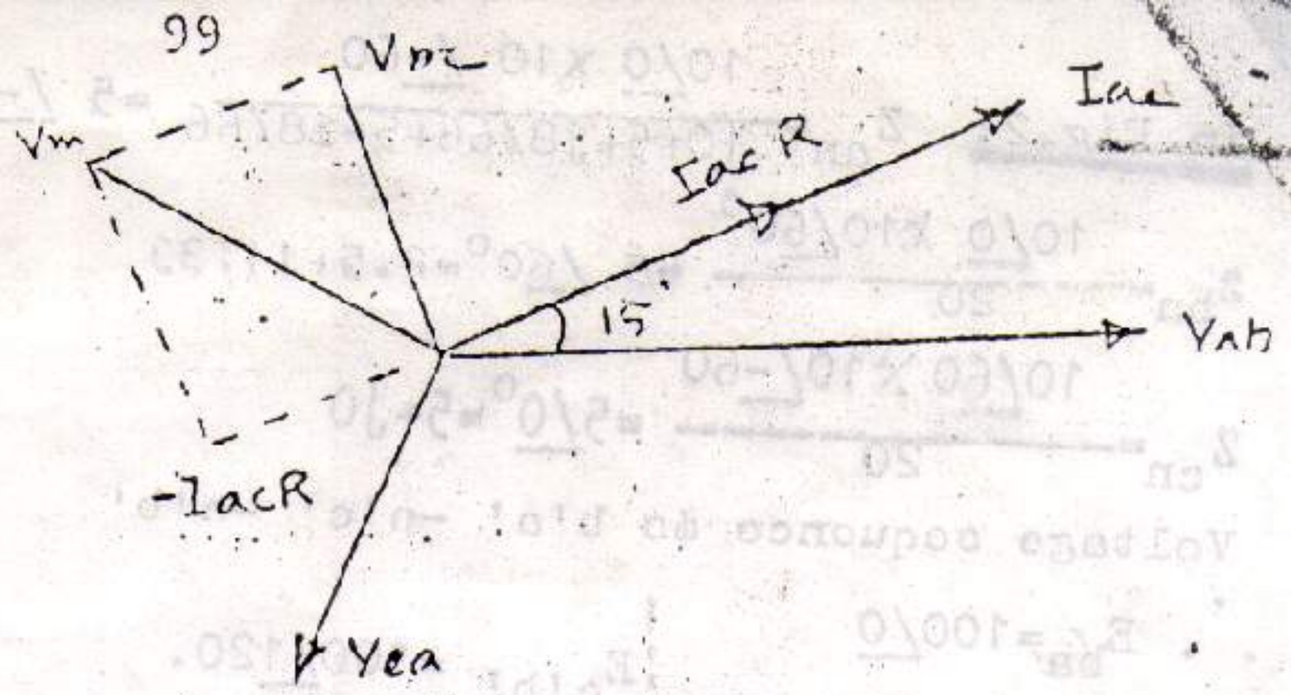
$=9.15\angle -69.19^{\circ}$ amp.

$I_{bc}+I_{ba}=I_{b'b}$ or, $I_{bc}=I_{b'b}+I_{ab}=-4.68+j7.6+9.47-j0.56$

$=8.52\angle 55.77$ amp.

(20)





The sequence is ab-ca-bc.

Let the magnitude of line voltage be 141.4 volts.

X_L=100 ohm., R=100 ohm.

$\therefore V_{ab}=141.4\ \angle 0$, $V_{ca}=141.4\ \angle -120$, $V_{bc}=141.4\ \angle 120$.

Since the current through the voltmeter is negligible, we can assume Z_L and R are connected in series across the voltage V_{ac}.

$$\therefore I_{ac}=\frac{V_{ac}}{Z_{ac}}=\frac{-141.4\ \angle -120^\circ}{100+i\,100}=\frac{141.4\ \angle 60^\circ}{141.4\ \angle 45^\circ}=1\ \angle 15^\circ$$

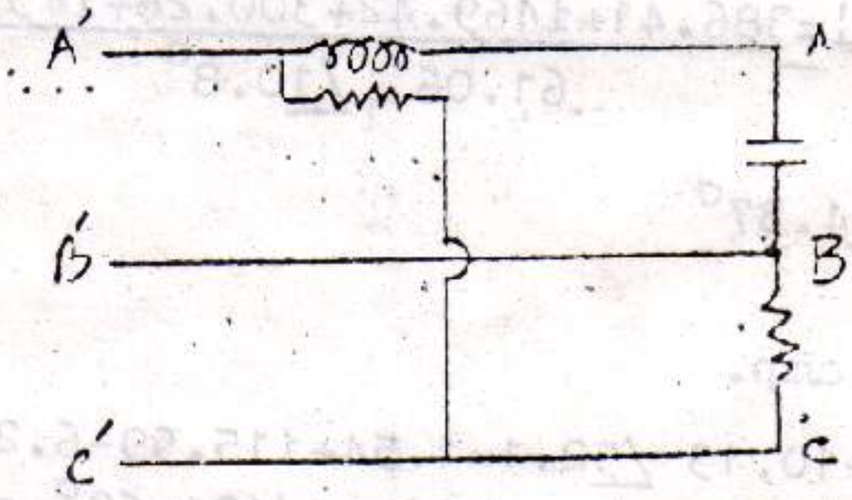
Now, From KVL- $V_{bn}+V_{rc}=V_{bc}$ or, $V_m=V_{bc}-I_{ac}R$

$=141.4\ \angle 120^\circ-1\ \angle 15^\circ\times 100=-70.7+i122.46-96.59-i25.98$

$=193.17\ \angle 150^\circ$ volts.

$\therefore$ The voltmeter read above the line voltage.

(22)



From prob. 13 $I_{A'A}=I_{AB}=5\ \angle 90^\circ$

$V_{AC}=V_{AB}+V_{BC}=V_{AB}-V_{CB}=115\angle 0-115\angle -90^\circ=115+i0+0+i115=162.63\angle 45^\circ$

$\therefore W_{AC-A'A}=V_{AC}\,I_{A'A}\cos\theta_{\overline{AC}-\overline{A'A}}=162.63\times 5\times\cos(45-90^\circ)=575$ watts. Ans.

(23)

Since the sequence is ab - bc - ca.

$\therefore V_{ab}=200\ \angle 0^\circ$

$V_{bc}=141.4\angle -120^\circ$

$V_{ca}=141.4\angle 120^\circ$

$Z_{ab}=Z_{bc}=Z_{ca}=8-16=10\ \angle -36.87^\circ$

$V_{ab}=200\ \angle 0^\circ$, $V_{bc}=141.4\angle -135^\circ$
$V_{ca}=141.4\angle 135^\circ$

?

$$I_{ab}=\frac{V_{ab}}{Z_{ab}}=\frac{200\angle 0}{10\ \angle -36.87}=20\ \angle 36.87^\circ$$

$$I_{bc}=\frac{V_{bc}}{Z_{bc}}=\frac{141.4\angle -120^\circ}{10\ \angle -36.87}=14.14\ \angle -83.13^\circ$$

$$I_{ca}=\frac{V_{ca}}{Z_{ca}}=\frac{141.4\ \angle 120^\circ}{10\ \angle -36.87}=14.14\ \angle 156.87^\circ$$

$W_{ab}=V_{ab}I_{ab}\cos\theta=200\times 20\cos(0-36.87)=3200$ watts.

$W_{bc}=V_{bc}I_{bc}\cos\theta=141.4\times 14.14\cos(-120+83.13)=1600$ watts.

$W_{ca}=141.4\times 14.14\cos(120-156.87)=1600$ watts.

Now, $I_{a'a}=I_{ab}-I_{ca}=16+i12+13-i5.55=29.7\ \angle 12.51^\circ$

$I_{c'c}=I_{ca}-I_{bc}=-13+i5.55-1.69+i14=24.45\angle 126.92^\circ$

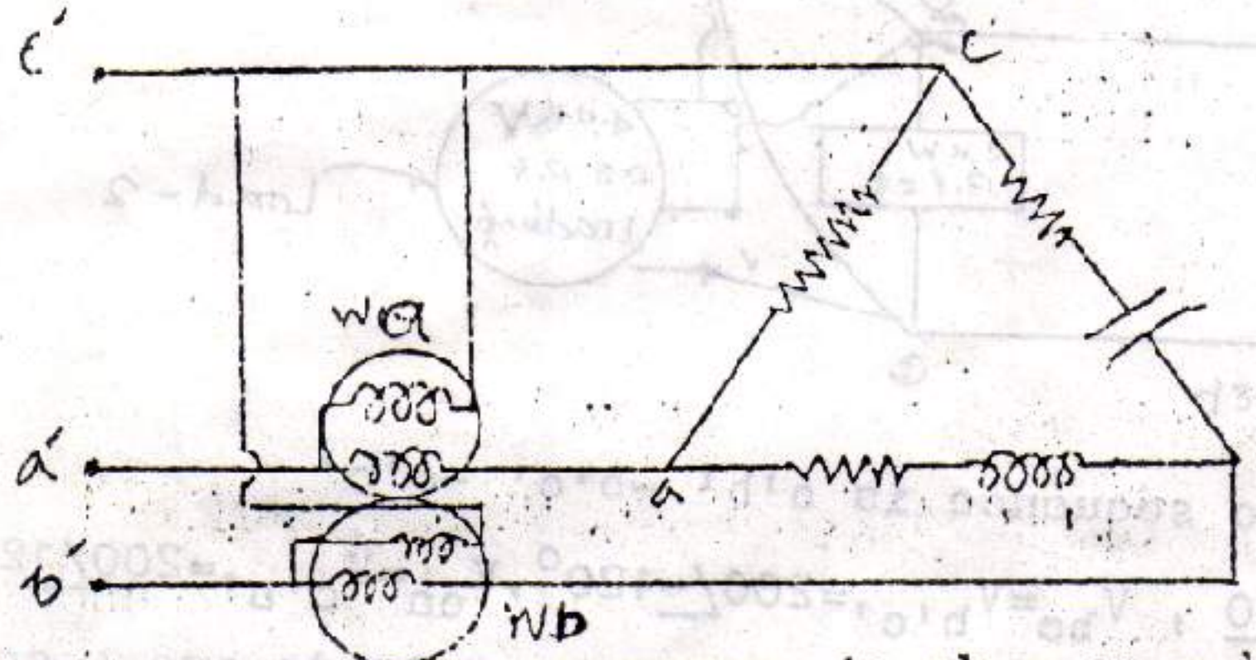
Now, $W_a=V_{ab}\ I_{a'a}\cos\theta$ (V_{ab}, $I_{a'a}$) $=200\times 29.7\cos(0-12.54)=5798.3$ watts.

$W_b=V_{cb}\ I_{c'c}\cos\theta$ (V_{cb}, $I_{c'c}$) $=141.4\times 24.45\cos(60^\circ-126.92)$,

$V_{cb}=-V_{bc}$
$=-141.4\angle -120^\circ$
$=141.4\angle 60^\circ$

=

(24)



Since the voltage sequence is ab - ca - bc, from example 1 we have-

(a) $W_a=V_{ac}\ I_{a'a}\cos\theta$ (V_{ac}, $I_{a'a}$)

$V_{ac}=-V_{ca}=-100\angle -120^\circ=100\angle 60^\circ$

$=100\times 9.26\cos(60^\circ+23.4)$

$=106.43$ watts.

$W_b=V_{bc}\ I_{b'b}\cos\theta$ (V_{bc}, $I_{b'b}$) $=100\times 2.9\cos(120-146.9)$

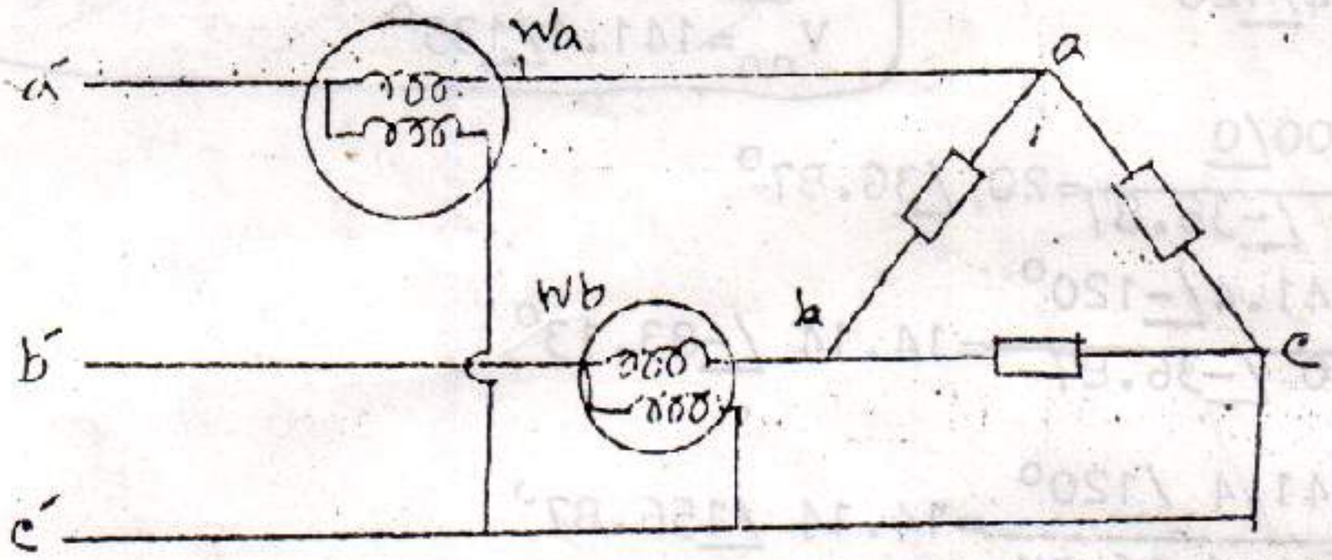
$=2586.2$ Watts.

(b) If W_a and W_b are varmeters then their readings will be-

P_{xa}=100 x9.26 Sin(60+23.4)=919.86 var (inductive).

P_{xb}=100 x29Sin (120-146.9)=1312.06 vars(capacitive).

25



Taking calculations from problem 11 we have

(a) $W_a=V_{ac}I_{a'a}\ \underline{\cos\theta}$ $\left.\begin{matrix}\\ \\ V_{ac}\end{matrix}\right\}$ =100x17.32 Cos(120°-90°)=1500 watts.

$W_b=V_{bc}I_{b'b}\ \cos\theta$ $\left.\begin{matrix}\\ \\ V_{bc}\end{matrix}\right\}$ $V_{bc}=-V_{cb}=-100\underline{/0^\circ}=100\underline{/180^\circ}$

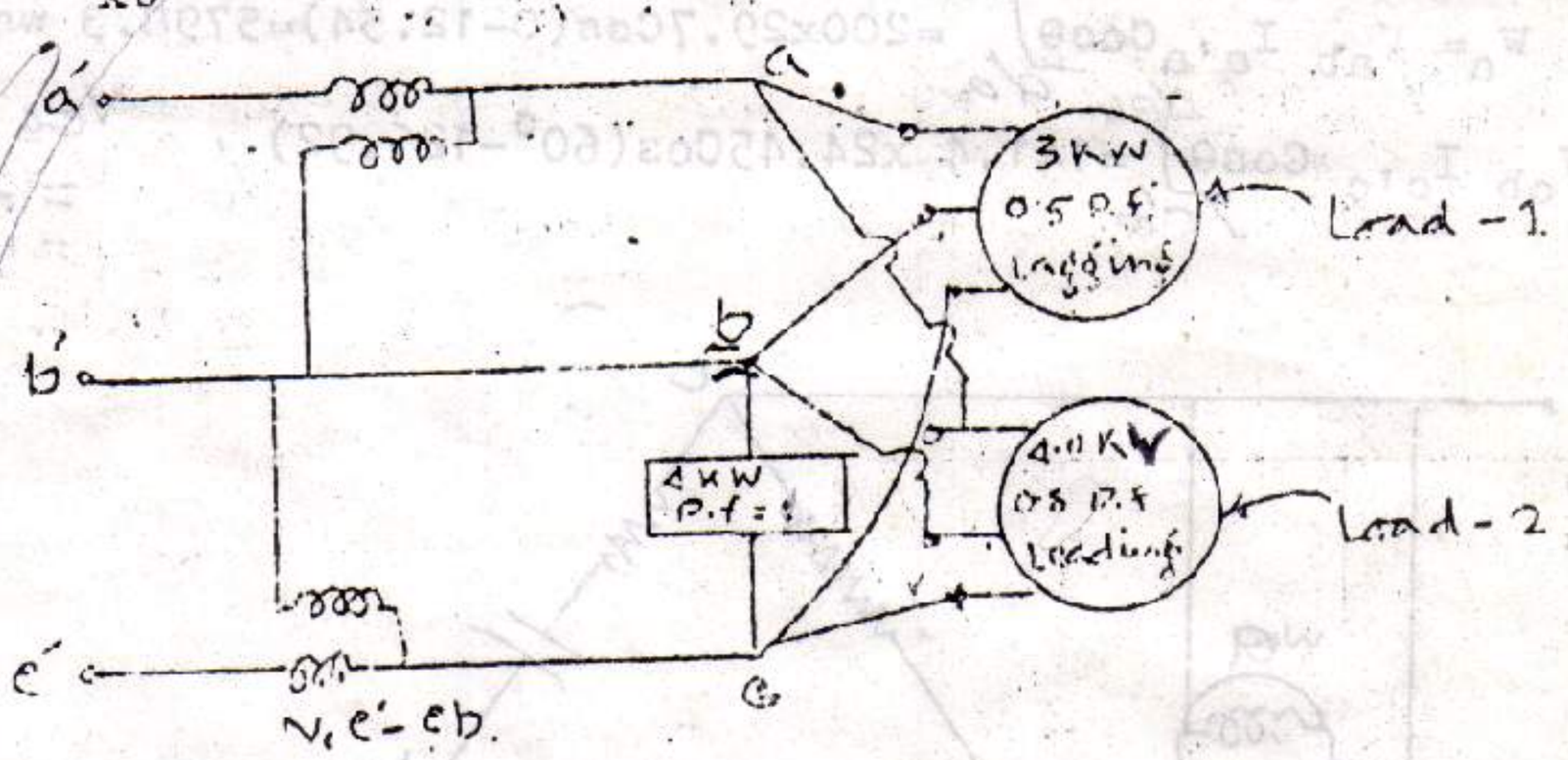
=100x17.32xCos(180°+150°)

=1500 watts.

(b) P_{xa}=100x17.32Sin(120°-90°)=866 vars (inductive).

P_{xb}=100x17.32Sin (180°+150°)=866 vars (capacitive).

26



Since the phase sequence is a'b' -b'c' -c'a'

$V_{\underline{ab}}=V_{a'b'}=200\underline{/0}$, $V_{bc}=V_{b'c'}=200\underline{/-120^\circ}$, $V_{ca}=V_{c'a'}=200\underline{/120^\circ}$

Let us assume that both the three phase loads are Δ connected.

∴ Per phase current in load 1 is, $I_{\phi 1}=\frac{3000}{\sqrt{3}x200x0.5}$=10 amp.

and these phase currents lag the applied voltage by Cos'0.5=60°

Per phase current in load 2 is $I_{\phi 2}=\frac{4000}{\sqrt{3}\ x\ 200}$=6.67 amp.

and these phase currents load the applied voltage by Cos'0.8=36.87°

The unity p.f. load current is in phase with V_{bc}.

∴ Since these loads are parallel.

. .(a) Reading of R_{va} meter a is= $V_{ab}I_{a'a}\ Sin\theta$
=100x9.39Sin(0-20.06)=322.08 var(capacitive).

$V_{cb} = -V_{bc}$
$= -100\angle 120^\circ$
$= 100\angle -60^\circ$

Varmeter $_{c'c-cb}$=100x6.63sin(-60+145°)=660.48 vars(inductive).

(b) $W_{a'a-ab}$ =100x9.39Cos(0-20.06)=882.03 watts.

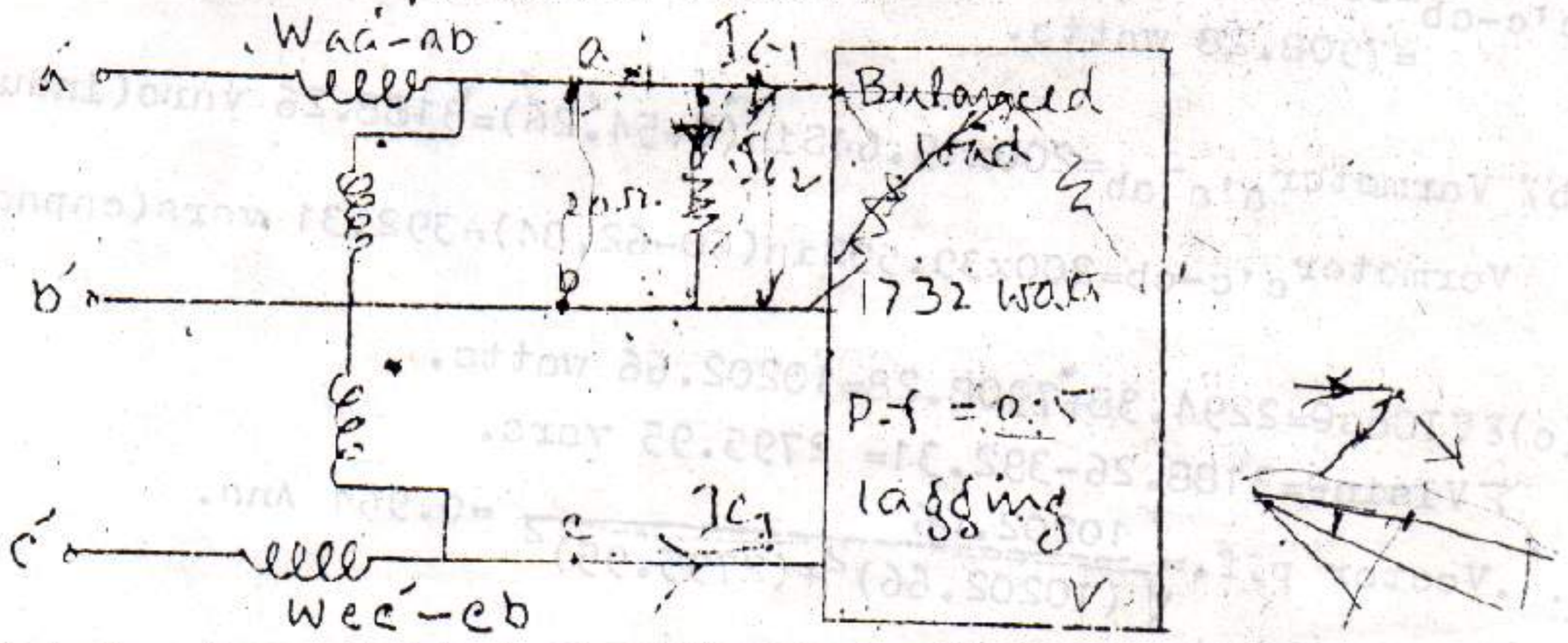
$W_{c'c-cb}$= 100x6.63Cos(-60+145°)=57.78 watts.

(c) $\Sigma VICos\theta$ =882.03+57.78 =939.81 watts.

$\Sigma VIsin\theta$=-322.08+660.48=338.4 vars.

$$\therefore \text{Vector p.f} = \frac{939.81}{\sqrt{(939.81)^2+(338.4)^2}} = 0.941 \text{ Ans.}$$

(28)



Let V_{ab} be selected as reference. Then

$V_{ab}=200\angle 0^\circ$, $V_{bc}=200\angle -120^\circ$; $V_{ca}=200\angle 120^\circ$

The current in each phase of the balanced load is-

$$I_\phi = \frac{1732}{\sqrt{3} \times 200 \times 0.5} = 5.773 \text{ amp.}$$

These phase current lag the applied voltage by $\cos^{-1}0.5=60^\circ$

Let us assume that the balanced load is connected in Δ.

For the resistance 20 ohm. the current will be in phase with V_{ab}. . . $I_{ab}=\frac{200\angle 0}{20}+5.773\angle -60^\circ=10+2.89-j5=13.83\angle -21.2^\circ$

$I_{bc}=5.773\angle -120-60^\circ=5.773\angle -180^\circ$, $I_{ca}=5.773\angle 120^\circ-60^\circ=5.773\angle 60^\circ$

$I_{a'a}=I_{ab}-I_{ca}=12.89-j5-2.89-i5=14.14\angle -45^\circ$

$I_{c'c}=I_{ca}-I_{bc}=2.89+j5+5.773=10\angle 30^\circ$

$V_{cb}=-V_{bc}=-200\angle -120^\circ$
$=200\angle 60^\circ$

$I \angle 60^\circ$

. . $W_{a'a-ab}$=200x14.14Cos(0+45°)=2000 watts.

$W_{c'c-cb}$ =200x10Cos(60-30°)=1732 watts.

(29) Since the phase sequence is n'a'-n'b'-n'c'.

Let, $E_{n'a'}=115.4\angle 0$, $E_{n'b'}=115.4\angle -120^\circ$; $E_{n'c'}=115.4\angle 120^\circ$

$V_{ab}=E_{n'a'}-E_{n'b'}=115.4\angle 0-115.4\angle -120^\circ=115.4+57.7+i100=200\angle 30^\circ$

$V_{bc}=E_{n'b'}-E_{n'c'}=115.4\angle -120^\circ-115.4\angle 120^\circ=-57.7-i100+57.7-i100$
$=200\angle -90^\circ$

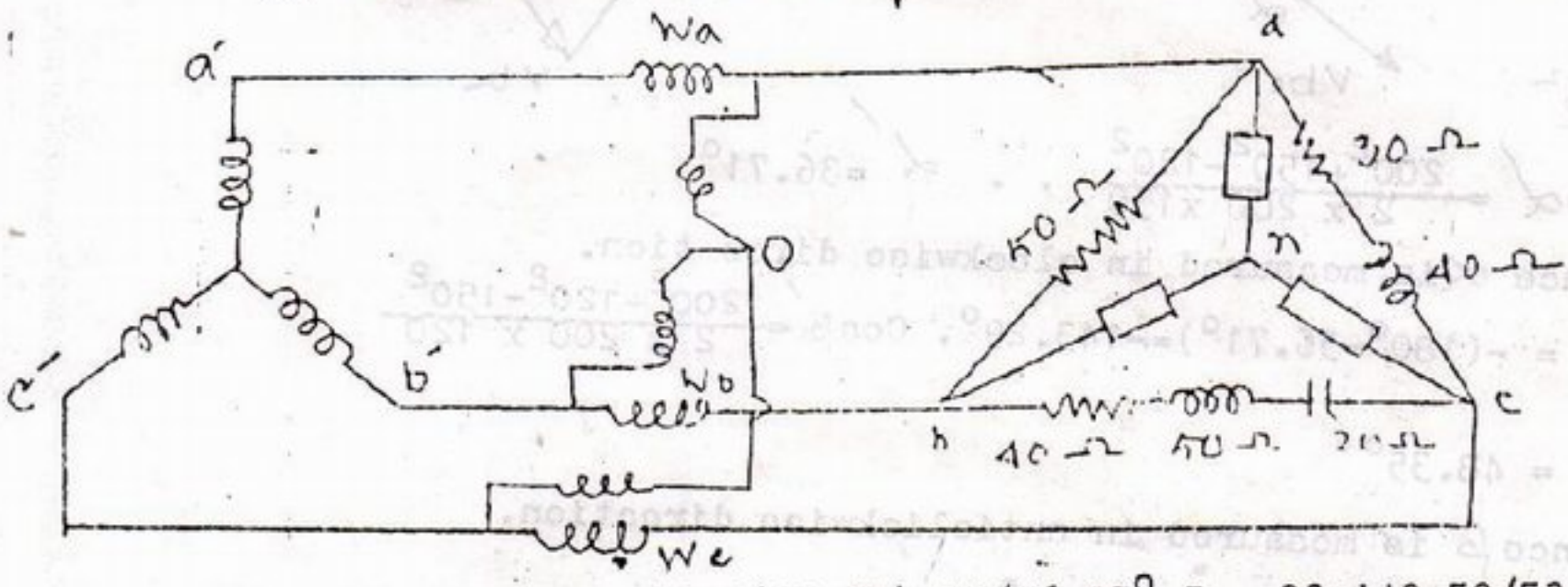
$V_{ca}=E_{n'c'}-E_{n'a'}=-57.7+i100-115.4=200\angle 150^\circ$ ...

Taking V_{ab} as reference we can write-

(a) $V_{ab}=200\angle 0^\circ$ | $E_{n'a'}=115.4\angle -30^\circ$

$V_{bc}=200\angle -90^\circ-30^\circ=200\angle -120^\circ$ | $E_{n'b'}=115.4\angle -150^\circ$

$V_{ca}=220\angle +150^\circ-30^\circ=200\angle 120^\circ$ | $E_{n'c'}=115.4\angle 90^\circ$



$Z_{ab}=50$ ohm., $Z_{bc}=40+i(50-20)=50\angle 36.87^\circ$, $Z_{ca}=30+140=50\angle 53.13^\circ$

(b) $I_{ab}=\frac{V_{ab}}{Z_{ab}}=\frac{200\angle 0}{50\angle 0}=4\angle 0^\circ$ Amp. $I_{bc}=\frac{V_{bc}}{Z_{bc}}=\frac{200\angle -120^\circ}{50\angle 36.87^\circ}=4\angle -156.87^\circ$ amp.

$I_{ca}=\frac{200\angle 120^\circ}{50\angle 53.13^\circ}=4\angle 66.87^\circ$ amp.

(c) $I_{a'a}=I_{ab}-I_{ca}=4-1.57-i3.68=4.41\angle -56.56^\circ$

$I_{b'b}=I_{bc}-I_{ab}=-3.68-i1.57-4=7.84\angle -168.4^\circ$

$I_{c'c}=I_{ca}-I_{bc}=1.57+i3.68+3.68+i1.57=7.42\angle 45^\circ$

(d) Transfering tremΔ to Y. We get

$Z_{an}=\frac{50\times 50\angle 53.13^\circ}{50+40+j30+30+j40}=\frac{50\times 50\angle 53.13^\circ}{138.92\angle 30.26}=18\angle 22.87^\circ$

$Z_{bn}=\frac{50\times 50\angle 36.87}{138.92\angle 30.26}=18\angle 6.61$

$Z_{cn}=\frac{50\angle 36.87\times 50\angle 53.13^\circ}{138.92\angle 30.26}=18\angle 59.74^\circ$

$V_{an}=I_{a'a}\ Z_{an}=79.38\angle -33.69^\circ$, $V_{bn}+I_{b'b}\ Z_{bn}=141.12\angle -6161.79^9$

$V_{cn}=I_{c'c}\ Z_{cn}=133.56\angle 104.74^\circ$

Now, $W_a+W_b+W_c=\sum V_{p\Delta}\ I_{p\Delta}\ \cos\theta$(from theory:page-388)

$=79.38\times 4.41\cos(-33.69+56.56)+14.12\times 7.84\cos(-161.79+168.4)$
$+133.56\times 7.42\cos(104.74-45)=1920.97$ watts.

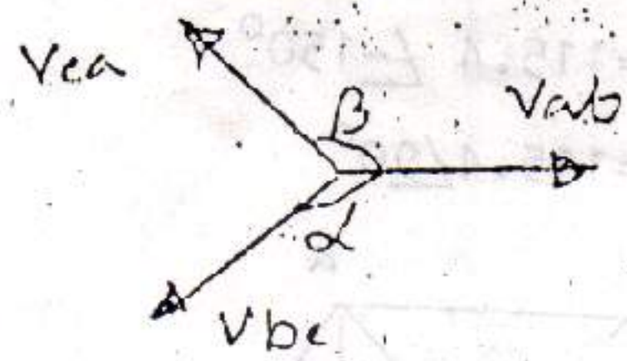
(e) $W_a=V_{ab}\ I_{a'a}\cos\theta=200\times 4.41\cos(0+56.56)=486.04$ watts.

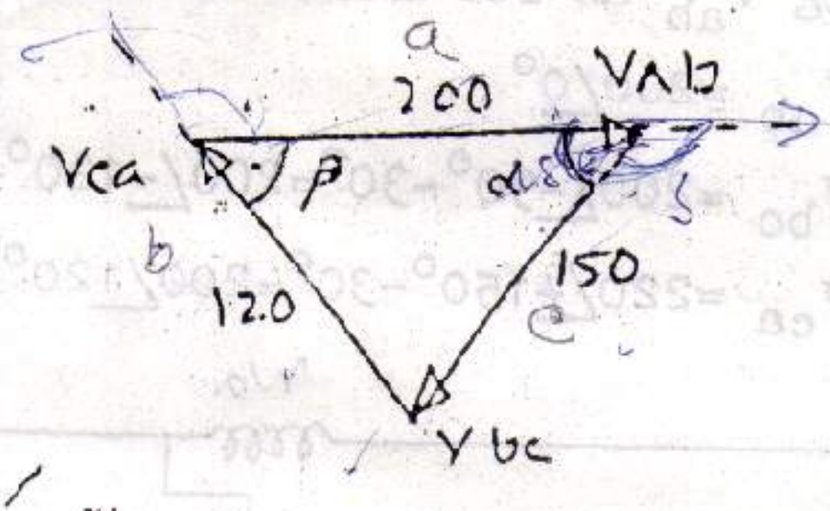
$W_b=V_{b'b}\cos\theta=0\times$ .....$=0$?

$W_c=V_{cb}\ I_{c'c}\cos\theta=200\times 7.42\cos(60^\circ-45^\circ)=1435$ watts.

(30) There may be two types of voltage sequence:
1) ab - bc -ca, (II) ab - ca - bc.

When the sequence is ab-bc-ca.





$\cos\alpha = \frac{a^2+c^2-b^2}{2ac}$

$\cos\alpha' = \frac{200^2+150^2-120^2}{2 \times 200 \times 150}$ $\therefore \alpha' = 36.71^\circ$

Since α is measured in clockwise direc tion.

$= -(180^\circ - 36.71^\circ) = -143.29^\circ$, $\cos\beta' = \frac{200^2+120^2-150^2}{2 \times 200 \times 120}$

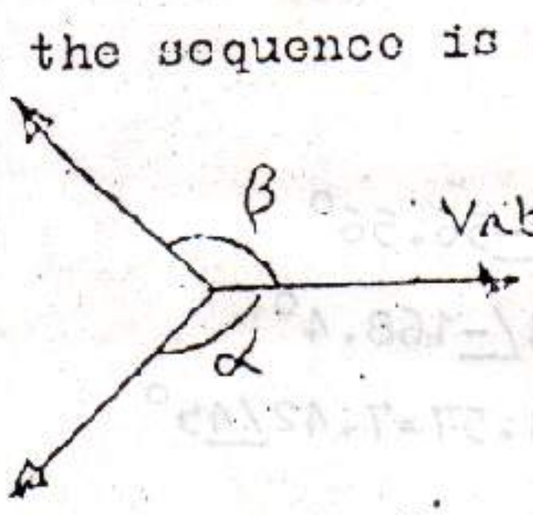
$\beta' = 48.35^\circ$

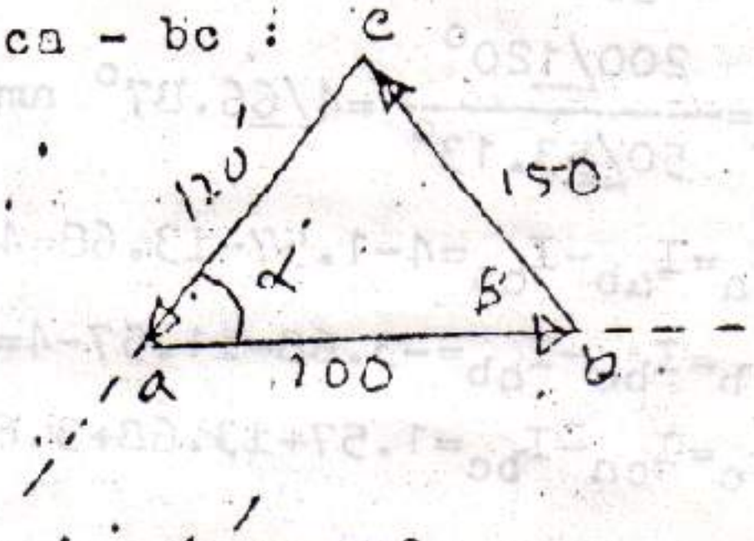
Since β is measured in anticlickwise direction.

$\therefore \beta = 180 - 48.35 = 131.65^\circ$

$\therefore V_{ab} = 200\angle 0$, $V_{bc} = 150\angle -143.29^\circ$, $V_{ca} = 120\angle 131.65^\circ$

(II) When the sequence is ab- ca - bc :





Here, $\cos\alpha' = \frac{200^2+120^2-150^2}{2 \times 200 \times 120}$ $\therefore \alpha' = 48.35^\circ$.

Since α is measured in the clockwise direction.

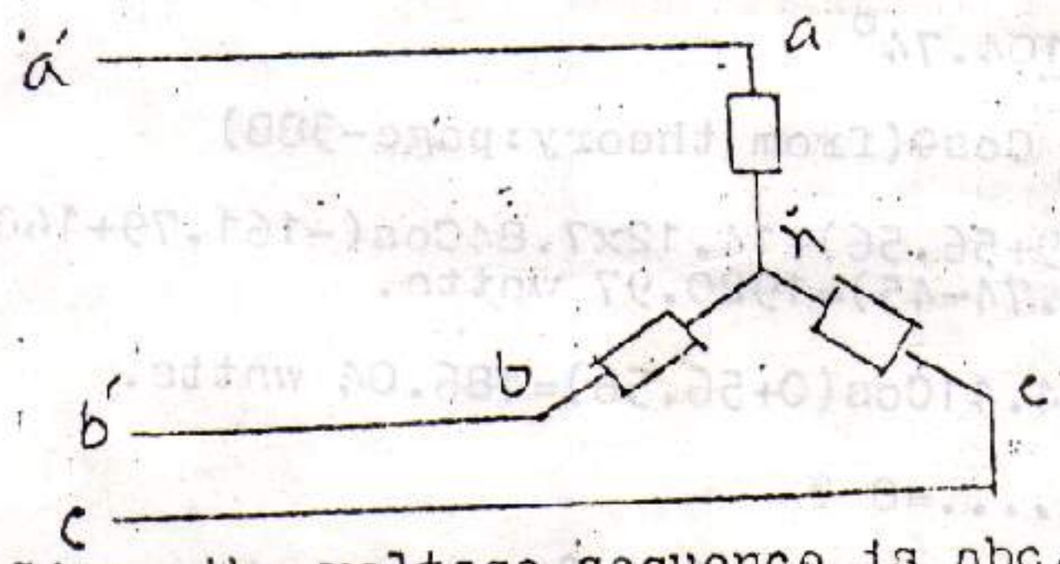
$\therefore \alpha = (180 - 48.35) = -131.65^\circ$, $\cos\beta' = \frac{220^2+150^2-120^2}{2 \times 200 \times 150}$ $\therefore \beta' = 36.71^\circ$

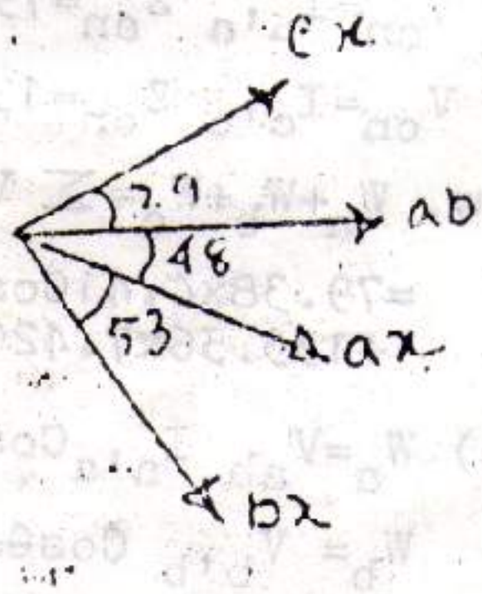
Since is measured in the anticlockwise direction.

$\therefore \beta = 180 - 36.71 = 143.29^\circ$

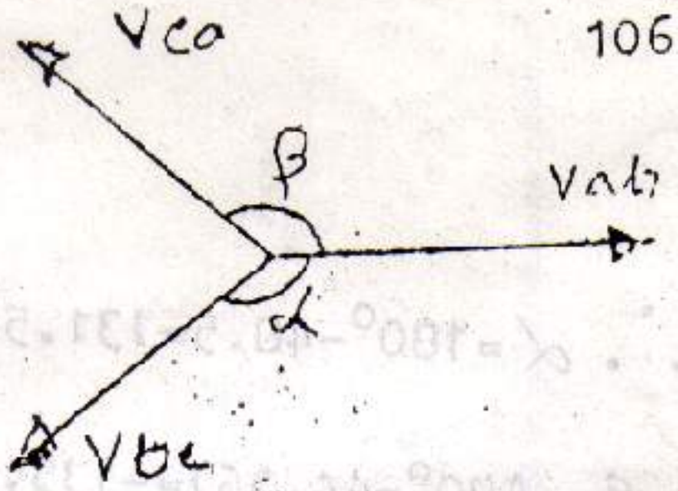
$\therefore V_{ab} = 200\angle 0$, $V_{ca} = 120\angle -131.65$, $V_{bc} = 150\angle 143.29^\circ$

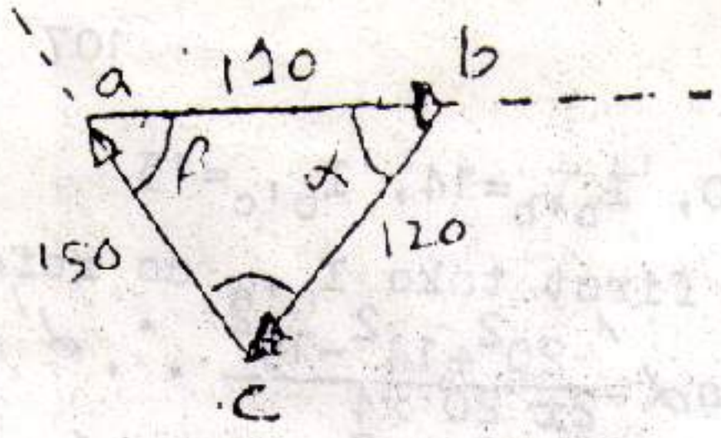
(31)





Since the voltage sequence is abc.





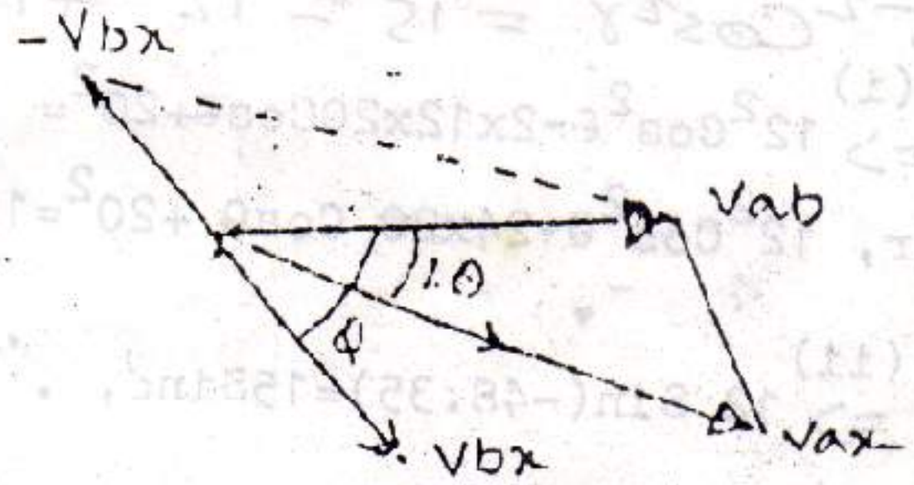
$\cos\alpha' = \frac{140^2+120^2-150^2}{2 \times 140 \times 120}$ $\therefore \alpha = 70^\circ$

Since α is measured in the clockwise direction

$\therefore \alpha = -(180^\circ - 70^\circ) = -110^\circ$ $\cos\beta' = \frac{140^2+150^2-120^2}{2 \times 140 \times 150}$ $\therefore \beta' = 48.74^\circ$

$\therefore \beta = 180 - 48.74 = 131.26^\circ$ $\therefore V_{ab} = 140\angle 0^\circ$, $V_{bc} = 120\angle -110^\circ$,

$V_{ca} = 150\angle 131.26^\circ$



Let $V_{an} = 200\angle 0$, $V_{bn} = 80\angle \phi$

$V_{cn} = 104.2\angle x$

Now, $V_{an} - V_{bn} = V_{ab}$

or, $200\angle 0 - 80\angle \phi = 140\angle 0$

$\therefore 200\cos\theta - 80\cos\phi = 140$ ...(i) (equating real parts).

$200\sin\theta - 80\sin\phi = 0$ ....(ii)(equating imaginary parts).

or, $10\sin\theta = 4\sin\phi$, or, $10^2(1-\cos^2\theta) = 4^2(1-\cos^2\phi)$, or, $4^2\cos^2\phi = 4^2 - 10^2 + 10^2\cos^2\theta$.

From (i) $\Rightarrow 10\cos\theta + 7 = 4\cos\phi$, or, $10^2\cos^2\theta - 140\cos\theta + 7 = 4^2 - 10^2 + 10^2\cos^2\theta$ or, $\theta = 18.2^\circ$

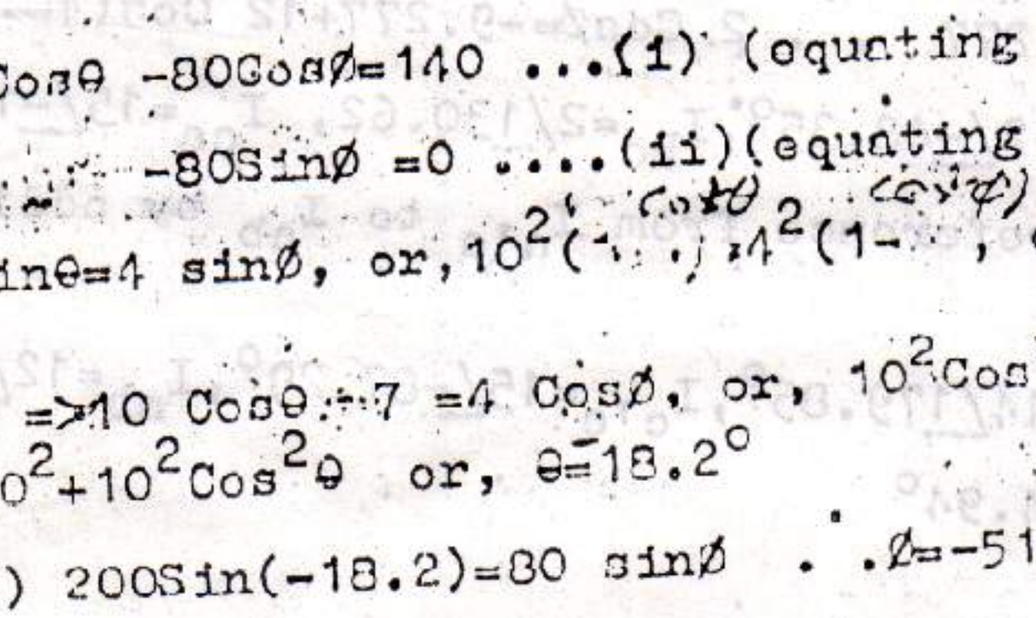
From (ii) $200\sin(-18.2) = 80\sin\phi$ $\therefore \phi = -51.34^\circ$

Again, $V_{bn} - V_{cn} = V_{bc}$

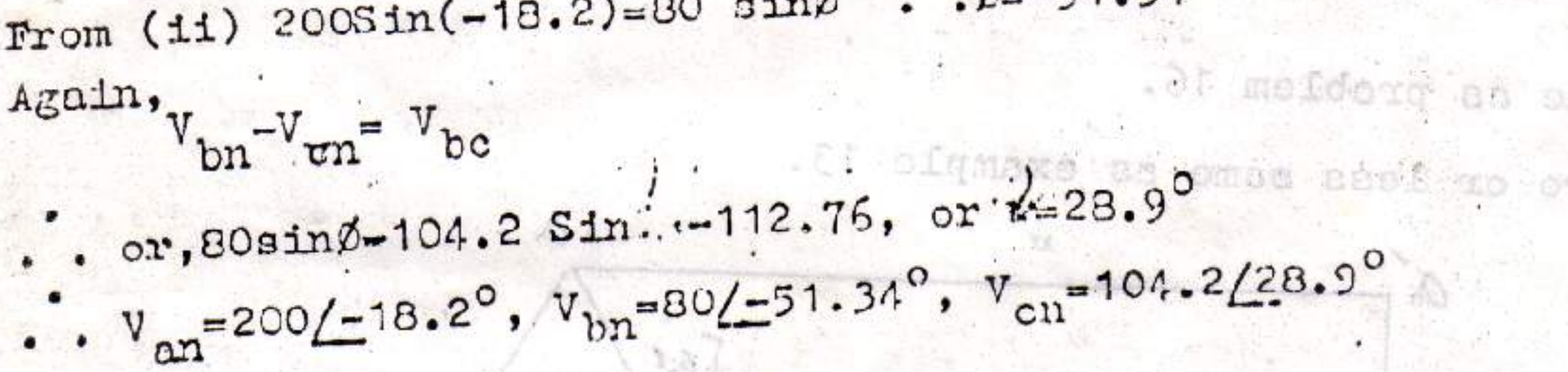
$\therefore$ or, $80\sin\phi - 104.2\sin x = -112.76$, or $x = 28.9^\circ$

$\therefore V_{an} = 200\angle -18.2^\circ$, $V_{bn} = 80\angle -51.34^\circ$, $V_{cn} = 104.2\angle 28.9^\circ$

32.



$I_{a'a} + I_{b'b} + I_{c'c} = 0$



$I_{a'a}=20$, $I_{b'b}=14$, $I_{c'c}=15$

Let us first take $I_{a'a}$ as reference.

$\therefore \cos\alpha' = \frac{20^2+14^2-15^2}{2\times 20\times 14}$ $\therefore \alpha' = 48.5^\circ$, $\therefore \alpha = 180^\circ - 48.5 = 131.5^\circ$

$\cos\beta' = \frac{20^2+15^2-14^2}{2\times 20\times 15}$ $\therefore \beta' = 44.36^\circ$ $\therefore \beta = -(180^\circ - 44.36) = -135.64^\circ$

$\therefore I_{a'a}=20\angle 0$, $I_{b'b}=14\angle 131.5^\circ$, $I_{c'c}=15\angle -135.64^\circ$

Let, $I_{ab}=12\angle 0$, $I_{bc}=2\angle\emptyset$, $I_{ca}=15\angle\theta$. $I_{ab}-I_{ca}=I_{a'a}$.

or, $12\angle 0 - 15\angle\gamma = 20\angle 0$, or, $12\cos\theta - 15\cos\gamma = 20$ .....(i)

$12\sin\theta - 15\sin\gamma = 0$ .....(ii), or, $12^2(1-\cos^2\theta) = 15^2(1-\cos^2\gamma)$

$\Rightarrow 15^2\cos^2\gamma = 15^2 - 12^2 + 12^2\cos^2\theta$

From (i) $\Rightarrow 12^2\cos^2\theta - 2\times 12\times 20\cos\theta + 20^2 = 15^2\cos^2\gamma$.

or, $12^2\cos^2\theta - 24\times 20\cos\theta + 20^2 = 15^2 - 12^2 + 12^2\cos\theta$ $\therefore \theta = -48.35^\circ$

From (ii) $\Rightarrow 12\sin(-48.35) = 15\sin\gamma$, $\therefore \gamma = -143.29$

Again, $I_{bc}-I_{ab}=I_{b'b}$, or, $2\angle\emptyset - 12\angle\theta = 14\angle 131.5$

$\therefore 2\cos\emptyset - 12\cos\theta = -9.277$ $\therefore 2\cos\emptyset = -9.277 + 12\cos(48.35)$

$\therefore \emptyset = 130.62$ $\therefore I_{ab}=12\angle -48.35^\circ$, $I_{bc}=2\angle 130.62$, $I_{ca}=15\angle -143.29^\circ$

Now, transfering the reference from $I_{a'a}$ to I_{ab} by adding 48.35° we have -

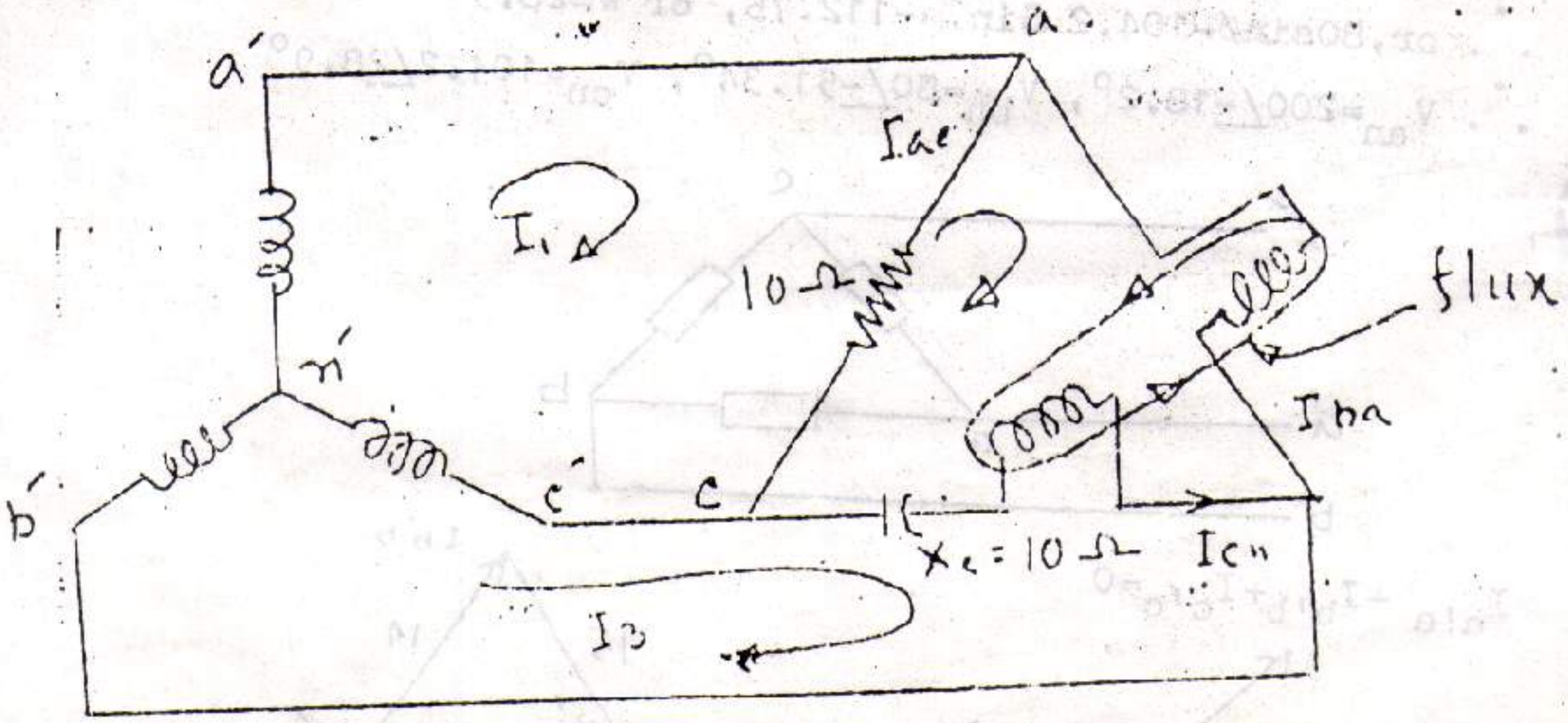
$I_{a'a}=20\angle 48.35^\circ$, $I_{b'b}=14\angle 179.85^\circ$, $I_{c'c}=15\angle -87.29^\circ$, $I_{ab}=12\angle 0^\circ$

$I_{bc}=2\angle 179^\circ$, $I_{ca}=15\angle -94.94^\circ$

33. Same as problem 16.

34. More or less same as example 13.

35.



$L_{nb}=L_{cb}=0.01$ h

$X_{Lab}=X_{Lcb}=1000\times0.01=0$ ohm.

$X_M=0.5\sqrt{10\times10}=5$ ohm.

The loop equations are-

$10I_1-10I_2=57.7\angle90^\circ-57.7\angle210^\circ$

or, $I_1-I_2=10\angle60^\circ$ .....(I)

$-10I_1+\{10+j(10+10+5+5)\}I_2-j(10-10+5)I_3=0$

or, $-10I_1+22.36\angle63.43^\circ I_2+5\angle-90^\circ I_3=0$ .....(II)

$-j(10-10+5)I_2+(j10-j10)I_3=57.7\angle210-57.7\angle-30^\circ$

or, $5\angle-90^\circ I_2=100\angle180^\circ$.....(III)

$\therefore I_2=20\angle270^\circ=20\angle-90^\circ$

From (I) $I_1=10\angle60^\circ+20\angle-90^\circ=5+j8.66+0-j20=12.39\angle-66.21^\circ$

From(II) $5\angle-90^\circ I_3=10\times12.39\angle-66.21^\circ-(22.36\angle63.43^\circ)(20\angle-90^\circ)$

$=50-j113.37-400+j200=360.56\angle166.1^\circ$ $\therefore I_3=72.11\angle256.1^\circ$

The line currents are-

$I_{a'a}=I_1=12.39\angle-66.21^\circ$, $I_{b'b}=-I_3=-72.\emptyset\angle256.1^\circ=-72.\emptyset\angle76.1^\circ$

$I_{c'c}=I_3-I_1=17.32-j70-5+j11.34=62.76\angle-110.83^\circ$

The phase currents are- $I_{ac}=I_1-I_2=5-j11.34+j20=10\angle60^\circ$

$I_{cb}=I_3-I_2=72.11\angle256.1^\circ+j20=-17.32-j70+j20=52.91\angle-109.1^\circ$

$I_{ba}=-I_2=-20\angle-90^\circ=20\angle90^\circ$

Since the sequence is

n'a'-n'b'-n'c'

$\therefore E_{n'a'}=57.7\angle90^\circ$

$E_{n'b'}=57.7\angle90^\circ-120$

$=57.7\angle-30^\circ$

$E_{n'c'}=57.7\angle90^\circ+120^\circ$

$=57.7\angle210^\circ$

Find the voltage of the neutral point n with respect to the ground, when-

$E_{n'a'}=57.7\angle-30^\circ$, $E_{n'b'}=57.7\angle-150^\circ$, $E_{n'c'}=57.7\angle90^\circ$

From example 11 we can write that- $I_{an}=I_1=0.364\angle-48.45^\circ$ amp.

Now, From KVL, we have- $V_{an}+V_{nn'}=E_{n'a'}$