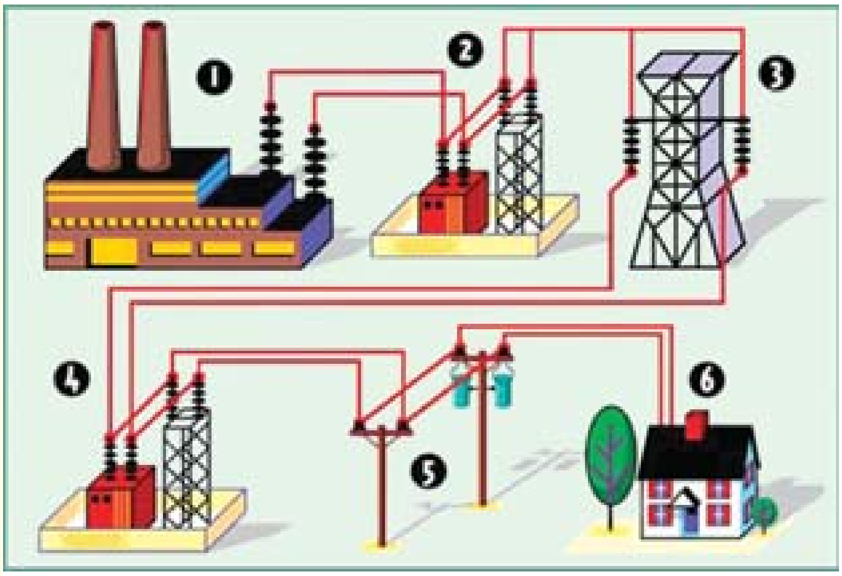
CONTENTS

CHAPTER 40

D.C. TRANSMISSION AND DISTRIBUTION

Learning Objectives

- Transmission and Distribution of D.C. Power
- Two-wire and Three-wire System
- Voltage Drop and Transmission Efficiency
- Methods of Feeding Distributor
- D.C.Distributor Fed at One End
- Uniformaly Loaded Distributor
- Distributor Fed at Both Ends with Equal Voltage
- Distributor Fed at Both ends with Unequal Voltage
- Uniform Loading with Distributor Fed at Both Ends
- Concentrated and Uniform Loading with Distributor Fed at One End
- Ring Distributor
- Current Loading and Load-point Voltage in a 3-wire System
- Three-wire System
- Balancers
- Boosters
- Comparison of 2-wire and 3-wire Distribution System



(1) Electricity leaves the power plant, (2) Its voltage is increased at a step-up transformer, (3) The electricity travels along a transmission line to the area where power is needed, (4) There, in the substation, voltage is decreased with the help of step-down transformer, (5) Again the transmission lines carry the electricity, (6) Electricity reaches the final consumption points

CONTENTS

40.1. Transmission and Distribution of D.C. Power

By transmission and distribution of electric power is meant its conveyance from the central station where it is generated to places, where it is demanded by the consumers like mills, factories, residential and commercial buildings, pumping stations etc. Electric power may be transmitted by two methods.

(i) By overhead system or *(ii)* By underground system—this being especially suited for densely-populated areas though it is somewhat costlier than the first method. In over-head system, power is conveyed by bare conductors of copper or aluminium which are strung between wooden or steel poles erected at convenient distances along a route. The bare copper or aluminium wire is fixed to an insulator which is itself fixed onto a cross-arm on the pole. The number of cross-arms carried by a pole depends on the number of wires it has to carry. Line supports consist of *(i)* pole structures and *(ii)* tower. Poles which are made of wood, reinforced concrete or steel are used up to 66 kV whereas steel towers are used for higher voltages.

The underground system employs insulated cables which may be single, double or triple-core etc.

A good system whether overhead or underground should fulfil the following requirements :

1. The voltage at the consumer's premises must be maintained within ± 4 or ± 6% of the declared voltage, the actual value depending on the type of load*.
2. The loss of power in the system itself should be a small percentage (about 10%) of the power transmitted.
3. The transmission cost should not be unduly excessive.
4. The maximum current passing through the conductor should be limited to such a value as not to overheat the conductor or damage its insulation.
5. The insulation resistance of the whole system should be very high so that there is no undue leakage or danger to human life.

It may, however, be mentioned here that these days all production of power is as a.c. power and nearly all d.c. power is obtained from large a.c. power systems by using converting machinery like synchronous or rotary converters, solid-state converters and motor-generator sets etc. There are many sound reasons for producing power in the form of alternating current rather than direct current.

(i) It is possible, in practice, to construct large high-speed a.c. generators of capacities up to 500 MW. Such generators are economical both in the matter of cost per kWh of electric energy produced as well as in operation. Unfortunately, d.c. generators cannot be built of ratings higher than 5 MW because of commutation trouble. Moreover, since they must operate at low speeds, it necessitates large and heavy machines.

(ii) A.C. voltage can be efficiently and conveniently raised or lowered for economic transmission and distribution of electric power respectively. On the other hand, d.c. power has to be generated at comparatively low voltages by units of relatively low power ratings. As yet, there is no economical method of raising the d.c. voltage for transmission and lowering it for distribution.

Fig. 40.1 shows a typical power system for obtaining d.c. power from a.c. power. Other details such as instruments, switches and circuit breakers etc. have been omitted.

Two 13.8 kV alternators run in parallel and supply power to the station bus-bars. The voltage is stepped up by 3-phase transformers to 66 kV for transmission purposes** and is again stepped down to 13.8 kV at the sub-station for distribution purposes. Fig. 40.1 shows only three methods commonly used for converting a.c. power to d.c. power at the sub-station.

---

* According to Indian Electricity Rules, voltage fluctuations should not exceed ± 5% of normal voltage for L.T. supply and $\pm 12\frac{1}{2}$ % for H.T. supply.

** Transmission voltages of upto 400 kV are also used.

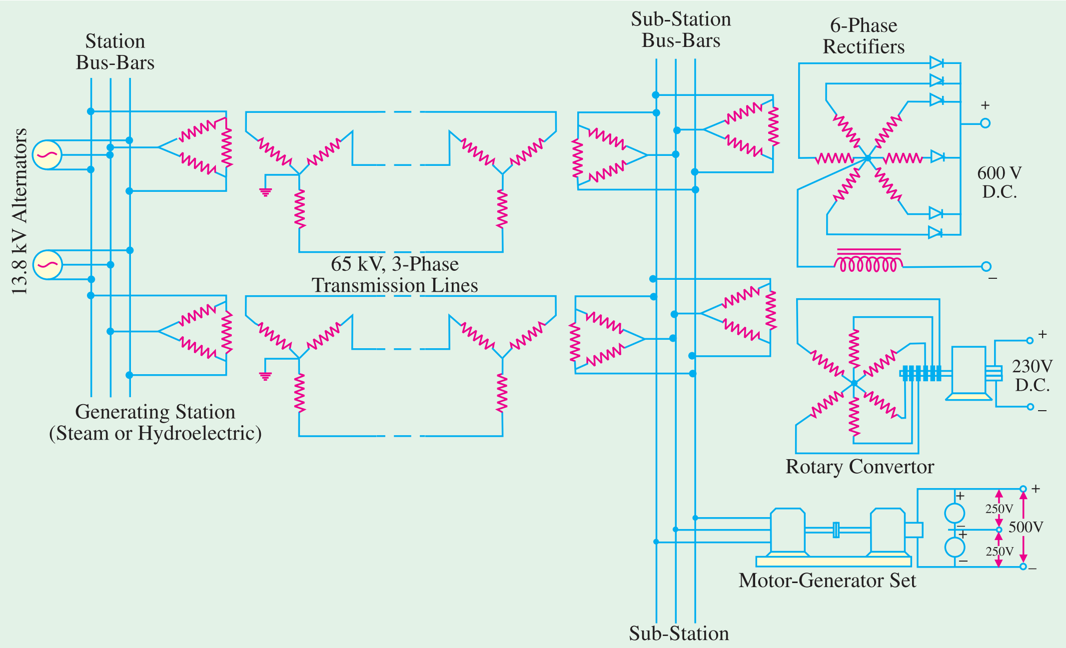


Fig. 40.1

(*a*) a 6-phase mercury-arc rectifier gives 600 V d.c. power after the voltage has been stepped down to a proper value by the transformers. This 600-V d.c. power is generally used by electric railways and for electrolytic processes.

(*b*) a rotary converter gives 230 V d.c. power.

(*c*) a motor-generator set converts a.c. power to 500/250 d.c. power for 3-wire distribution systems.

The above figure shows a motor-generator set. Nowadays, we use solid-state devices, called rectifiers, to convert standard AC to DC current. Back in the olden days, they needed a "motor dynamo" set to make the conversion as shown above. An AC motor would turn a DC Generator, as pictured above

In Fig. 40.2 is shown a schematic diagram of ***low tension*** distribution system for d.c. power. The whole system consists of a network of cables or conductors which convey power from central station to the consumer's premises. The station bus-bars are fed by a number of generators (only two shown in the figure) running in parallel. From the bus-bars, the power is carried by many ***feeders*** which radiate to various parts of a city or locality. These feeders deliver power at certain points to a distributor which runs along the various streets. The points *FF*, as shown in the figure, are known as ***feeding points***. Power connections to the various consumers are given ***from this distributor and not directly from the feeder***. The wires which convey power from the distributor to the consumer's premises are known as ***service mains (S)***. Sometimes when there is only one distributor in a locality, several sub-distributors (*SD*) branching off from the distributor are employed and service mains are now connected to them instead of distributor as shown in the figure.

Obviously, a feeder is designed on the basis of its current-carrying capacity whereas the design of distributor is based on the voltage drop occurring in it.

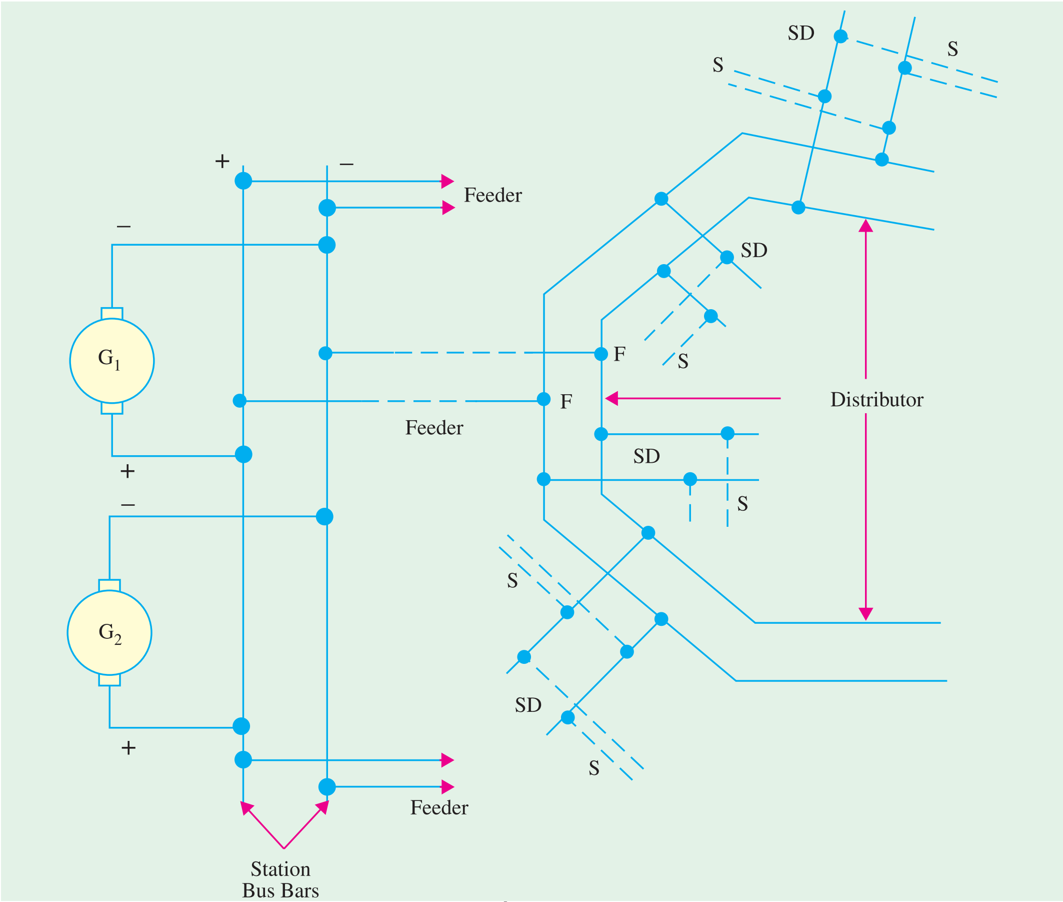


Fig. 40.2

40.2. Two-wire and Three-wire Systems

In d.c. systems, power may be fed and distributed either by *(i) 2-wire system* or *(ii) 3-wire system*.

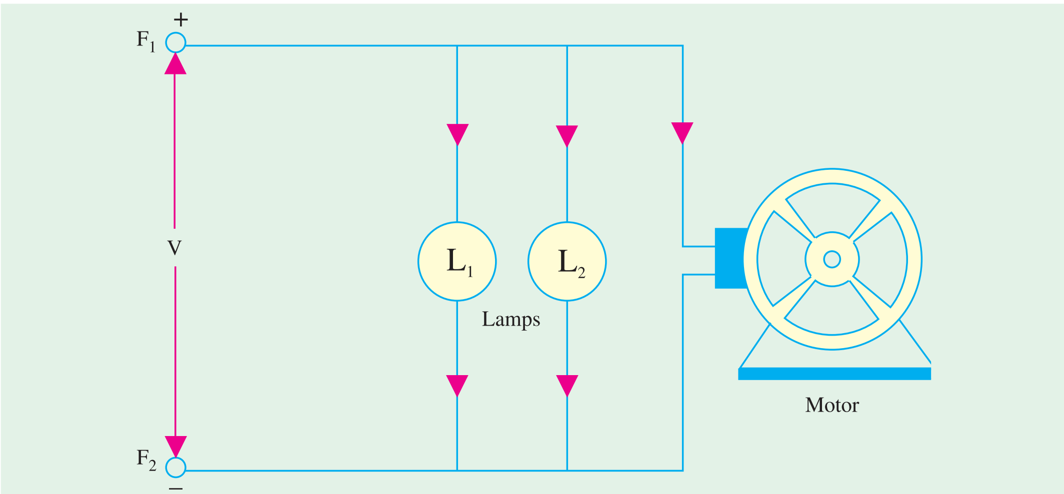


Fig. 40.3

In the 2-wire system, one is the outgoing or positive wire and the other is the return or negative wire. In the case of a 2-wire distributor, lamps, motors and other electrical apparatus are connected in parallel between the two wires as shown in Fig. 40.3. As seen, the potential difference and current have their maximum values at feeding points F_1 and F_2. The standard voltage between the conductors is 220 V.

The 2-wire system when used for transmission purposes, has much lower efficiency and economy as compared to the 3-wire system as shown later.

A 3-wire has not only a higher efficiency of transmission (Fig. 40.4) but when used for distribution purposes, it makes available two voltages at the consumer's end (Fig. 40.5). This 3-wire system consists of two '***outers***' and a ***middle*** or ***neutral*** wire which is earthed at the generator end. Its potential is midway between that of the outers *i.e.* if the p.d. between the outers is 460 V, then the p.d. of positive outer is 230 V ***above*** the neutral and that of negative outer is 230 V ***below*** the neutral. Motors requiring higher voltage are connected across the outers whereas lighting and heating circuits requiring less voltage are connected between any one of the outers and the neutral.

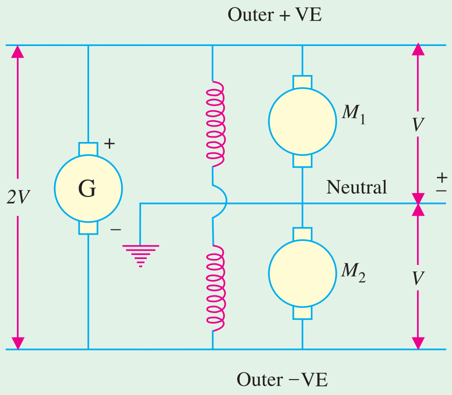


Fig. 40.4

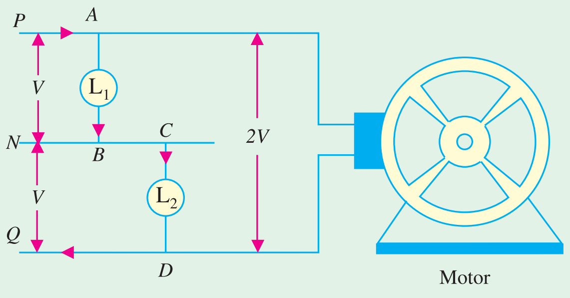


Fig. 40.5

The addition of the middle wire is made possible by the connection diagram shown in Fig. 40.4. *G* is the main generator which supplies power to the whole system. M_1 and M_2 are two identical shunt machines coupled mechanically with their armatures and shunt field winding joined in series across the outers. The junction of their armatures is earthed and the neutral wire is taken out from there.

40.3 Voltage Drop and Transmission Efficiency

Consider the case of a ***2-wire feeder*** (Fig. 40.6). *AB* is the sending end and *CD* the receiving end. Obviously, the p.d. at *AB* is higher than at *CD*. The difference in potential at the two ends is the potential drop or 'drop' in the cable. Suppose the transmitting voltage is 250 V, current in *AC* is 10 amperes, and resistance of each feeder conductor is 0.5 Ω, then drop in each feeder conductor is 10 × 0.5 = 5 volt and drop in both feeder conductor is 5 × 2 = 10 V.

$\therefore$ *P.d.* at Receiving end CD is = 250 – 10 = 240 V

Input power at *AB* = 250 × 10 = 2,500 W

Output power at *CD* = 240 × 10 = 2,400 W

$\therefore$ power lost in two feeders = 2,500 – 2,400 = 100W

The above power loss could also be found by using the formula

$$\text{Power loss} = 2I^2R = 2 \times 10^2 \times 0.5 = 100 \text{ W}$$

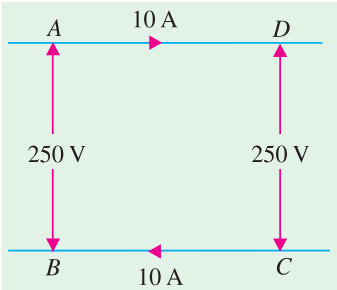


Fig. 40.6

The efficiency of transmission is defined, like any other efficiency, as the ratio of the output to input

$$\therefore \quad \text{efficiency of transmission} = \frac{\text{power delivered by the line}}{\text{power received by the line}}$$

In the present case, power delivered by the ***feeder*** is = 2500 W and power received by it as 2400 W.

$$\therefore \quad \eta = 2400 \times 100/2500 = 96\%$$

In general, if V_1 is the voltage at the sending end and V_2 at the receiving end and I the current delivered, then

$$\text{Input} = V_1 I, \qquad \text{output} = V_2 I \qquad \therefore \quad \eta = \frac{V_2 I}{V_1 I} = \frac{V_2}{V_1}$$

$$\text{Now} \quad V_2 = V_1 - \text{drop in both conductors}$$

$$= V_1 - IR, \quad \text{where } R \text{ is the resistance of both conductors}$$

$$\therefore \quad \eta = \frac{V_1 - IR}{V_1} = 1 - \frac{IR}{V_1} \qquad \ldots(i)$$

$$\text{or} \quad \%\ \text{efficiency} = 100 \times \left(1 - \frac{IR}{V_1}\right) = 100 - \left(\frac{IR}{V_1} \times 100\right)$$

Now, $(IR/V_1) \times 100$ represents the voltage drop in both conductors expressed as a percentage of the voltage at the sending end. It is known as percentage drop.

$$\therefore \quad \%\ \eta = 100 - \%\ \text{drop}$$

In the present case total drop is 10 volt which expressed as percentage becomes $(10/250) \times 100 = 4\%$. Hence $\%\ \eta = 100 - 4 = 96\%$ ...**as before**

It is seen from equation ***(i)*** above that for a given drop, transmission efficiency can be considerably increased by increasing the voltage at the transmitting end. *i.e.*, V_1. Moreover, the cross-section of copper in the cables is decreased in proportion to the increase in voltage which results in a proportionate reduction of the cost of copper in the cables.

The calculation of drop in a ***feeder*** is, as seen from above, quite easy because of the fact that current is constant throughout its length. But it is not so in the case of ***distributors*** which are tapped off at various places along their entire lengths. Hence, their different sections carry different currents over different lengths. For calculating the total voltage drop along the entire length of a distributor, following information is necessary.

A DC power distribution system consists of a network of cables or conductors which conveys power from central station to the consumer's premises.

(i) value of current tapped at each load point.

(ii) the resistance of each section of the distributor between tapped points.

Example 40.1. *A DC 2-wire feeder supplies a constant load with a sending-end voltage of 220 V. Calculate the saving in copper if this voltage is doubled with power transmitted remaining the same.*

Solution. Let $\quad l$ = length of each conductor in metre

σ = current density in A/m^2

P = power supplied in watts

(*i*) **220 V Supply**

Current per feeder conductor I_1 = $P/220$

Area of conductor required A_1 = $I_1/\sigma = P/220\,\sigma$

Volume of Cu required for both conductors is

$$V_1 = 2A_1 l = \frac{2Pl}{220\sigma} = \frac{Pl}{110\sigma}$$

(*ii*) **440 V Supply**

$$V_2 = \frac{Pl}{220\sigma}$$

$$\text{\%age saving in Cu} = \frac{V_1 - V_2}{V_1} \times 100 = \frac{\frac{Pl}{110\,\sigma} - \frac{Pl}{220\,\sigma}}{Pl/110\,\sigma} \times 100 = 50\%$$

40.4. Methods of Feeding a Distributor

Different methods of feeding a distributor are given below :

1. feeding at one end
2. feeding at both ends with equal voltages
3. feeding at both ends with unequal voltages
4. feeding at some intermediate point

In adding, the nature of loading also varies such as

(*a*) concentrated loading (*b*) uniform loading (*c*) combination of (*a*) and (*b*).

Now, we will discuss some of the important cases separately.

40.5. D.C. Distributor Fed at One End

In Fig. 40.7 is shown one conductor AB of a distributor with concentrated loads and fed at one end.

Let i_1, i_2, i_3 etc. be the currents tapped off at points C, D, E and F and I_1, I_2, I_3 etc. the currents in the various sections of the distributor. Let r_1, r_2, r_3 etc. be the ohmic resistances of these various sections and ***R_1, R_2, R_3 etc. the total resistance from the feeding end A to the successive tapping points.***

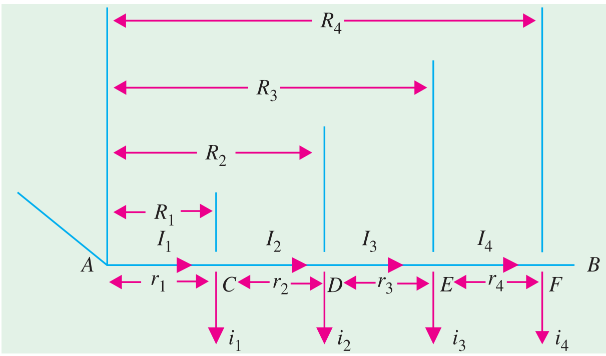


Fig. 40.7

Then, total drop in the distributor is

$$= r_1 I_1 + r_2 I_2 + r_3 I_3 + \ldots\ldots\ldots\ldots$$

Now $\quad I_1 = i_1 + i_2 + i_3 + i_4 + \ldots\ldots;\ I_2 = i_2 + i_3 + i_4 + \ldots\ldots;\ I_3 = i_3 + i_4 + \ldots\ldots$

$\therefore \quad = r_1(i_1 + i_2 + i_3 + \ldots\ldots) + r_2(i_2 + i_3 + i_4 + \ldots\ldots) + r_3(i_3 + i_4 + \ldots\ldots)$

$= i_1 r_1 + i_2(r_1 + r_2) + i_3(r_1 + r_2 + r_3) + \ldots\ldots = i_1 R_1 + i_2 R_2 + i_3 R_3 + \ldots\ldots$

= ***sum of the moments of each load current about feeding point A.***

1. Hence, the drop at ***the far end*** of a distributor fed at one end is given by the sum of the moments of various tapped currents about the feeding point *i.e.* $v = \Sigma\, i\, R$.

2. It follows from this that the total voltage drop is the same as that produced by a single load equal to the sum of the various concentrated loads, acting at the centre of gravity of the load system.

3. Let us find the drop at any intermediate point like E. The value of this drop is

$$= i_1 R_1 + i_2 R_2 + i_3 R_3 + R_3(i_4 + i_5 + i_6 + \ldots\ldots)^*$$

* The reader is advised to derive this expression himself.

$$= \begin{pmatrix}\text{sum of moments}\\ \text{upto } E\end{pmatrix} + \begin{pmatrix}\text{moment of load beyond}\\ E \text{ assumed acting at } E\end{pmatrix}$$

In general, the drop at any intermediate point is ***equal to the sum of moments of various tapped currents upto that point plus the moment of all the load currents beyond that point assumed to be acting at that point.***

The total drop over both conductors would, obviously, be twice the value calculated above.

40.6. Uniformly Loaded Distributor

In Fig. 40.8 is shown one conductor AB of a distributor fed at one end A and uniformly loaded with i amperes per unit length. Any convenient unit of length may be chosen *i.e.* a metre or 10 metres but at every such unit length, the load tapped is the same. Hence, let

i = current tapped off per unit length $\quad$ l = total length of the distributor

r = resistance per unit length of the distributor

Now, let us find the voltage drop at a point C (Fig. 40.9) which is at a distance of x units from feeding end A. The current at point C is $(il - ix) = i\,(l - x)$.

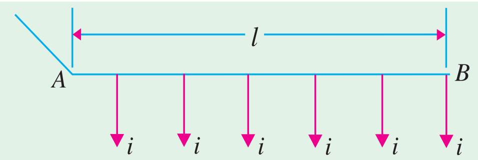


Fig. 40.8

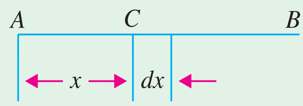


Fig. 40.9

Consider a small section of length dx near point C. Its resistance is (rdx). Hence, drop over length dx is

$$dv = i\,(l - x)\,(rdx) = (ilr - ixr)dx$$

The total drop up to point x is given by integrating the above quantity between proper limits.

$$\therefore \quad \int_0^v dx = \int_0^x (ilx - ixr)\,dx \qquad \therefore \quad v = ilrx - \frac{1}{2}irx^2 = ir\left(lx - \frac{x^2}{2}\right)$$

The drop at point B can be obtained by putting $x = l$ in the above expression.

$$\therefore \quad \text{drop at point } B = ir\left(l^2 - \frac{l^2}{2}\right) = \frac{irl^2}{2} = \frac{(i \times l) \times (r \times l)}{2} = \frac{1}{2}IR = I \times \frac{1}{2}R$$

where $i \times l = I$ – total current entering at point A; $r \times l = R$–the total resistance of distributor AB.

Total drop in the distributor $AB = \frac{1}{2}IR$

(i) It follows that in a uniformly loaded distributor, total drop is equal to that produced by the whole of the load assumed concentrated at the middle point.

(ii) Suppose that such a distributor is fed at both ends A and B with ***equal voltages***. In that case, the point of minimum potential is obviously the middle point. We can thus imagine as if the distributor were cut into two at the middle point, giving us two uniformly-loaded distributors each fed at one end with equal voltages. The resistance of each is $R/2$ and total current fed into each distributor is $I/2$. Hence, drop at the middle point is

$$= \frac{1}{2} \times \left(\frac{I}{2}\right) \times \left(\frac{R}{2}\right) = \frac{1}{8}IR$$

It is 1/4th of that of a distributor fed at one end only. The advantage of feeding a distributor at both ends, instead of at one end, is obvious.

The equation of drop at point C distant x units from feeding point $A = irlx - \frac{1}{2}irx^2$ shows that the diagram of drop of a uniformly-loaded distributor fed at one end is a parabola.

Example 40.2. *A uniform 2-wire d.c. distributor 200 metres long is loaded with 2 amperes/metre. Resistance of single wire is 0.3 ohm/kilometre. Calculate the maximum voltage drop if the distributor is fed (a) from one end (b) from both ends with equal voltages.*

(Elect. Technology ; Bombay Univ.)

Solution. **(*a*)** Total resistance of the distributor is $R = 0.3 \times (200/1000) = 0.06$ W

Total current entering the distributor is $I = 2 \times 200 = 400$ A

Total drop on the whole length of the distributor is (Art. 40.6)

$$= \frac{1}{2} IR = \frac{1}{2} \times 400 \times 0.06 = \mathbf{12\ V}$$

(*b*) Since distributor is fed from both ends, total voltage drop is

$$= \frac{1}{8} IR = \frac{1}{8} \times 400 \times 0.06 = \mathbf{3\ V}$$

As explained in Art. 40.6 the voltage drop in this case is one-fourth of that in case **(*a*)** above.

Example 40.3. *A 2-wire d.c. distributor AB is 300 metres long. It is fed at point A. The various loads and their positions are given below :*

At point	*distance from A in metres*	*concentrated load in A*
C	*40*	*30*
D	*100*	*40*
E	*150*	*100*
F	*250*	*50*

If the maximum permissible voltage drop is not to exceed 10 V, find the cross-sectional area of the distributor. Take $\rho = (1.78 \times 10^{-8})$ Ω*-m.* **(Electrical Power-I ; Bombay Univ.)**

Solution. The distributor along with its tapped currents is shown in Fig. 40.10.

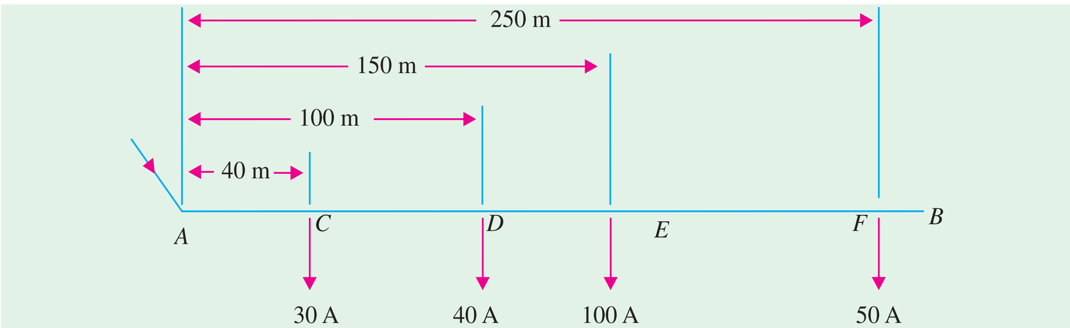


Fig. 40.10

Let 'A' be cross sectional area of distributor

As proved in Art. 40.5 total drop over the distributor is

$$v = i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4 \qquad \text{—for one conductor}$$

$$= 2(i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4) \qquad \text{—for two conductors}$$

where R_1 = resistance of $AC = 1.78 \times 10^{-8} \times 40/A$

R_2 = resistance of $AD = 1.78 \times 10^{-8} \times 100/A$

R_3 = resistance of $AE = 1.78 \times 10^{-8} \times 150/A$

R_4 = resistance of $AF = 1.78 \times 10^{-8} \times 250/A$

Since v = 10 V, we get

$$10 = \frac{2 \times 1.78 \times 10^{-8}}{A}(30 \times 40 + 40 \times 100 + 100 \times 150 + 50 \times 250)$$

$$= \frac{2 \times 1.78 \times 10^{-8}}{A} \times 32,700$$

$$\therefore \quad A = 3.56 \times 327 \times 10^{-7}\ \text{m}^2 = \mathbf{1,163\ cm^2}$$

40.7. Distributor Fed at Both Ends with Equal Voltages

It should be noted that in such cases

(i) the maximum voltage drop must always occur at one of the load points and

(ii) if both feeding ends are at the same potential, then the voltage drop between each end and this point must be the same, which in other words, means that the sum of the moments about ends must be equal.

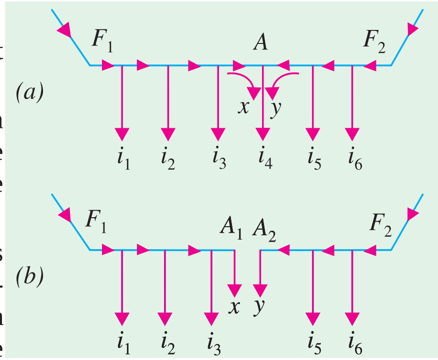


Fig. 40.11

In Fig. 40.11 (*a*) is shown a distributor fed at two points F_1 and F_2 with equal voltages. The potential of the conductor will gradually fall from F_1 onwards, reach a minimum value at one of the tapping, say, A and then rise again as the other feeding point F_2 is approached. All the currents tapped off between points F_1 and A will be supplied from F_1 while those tapped off between F_2 and A will be supplied from F_2. The current tapped at point A itself will, in general, be partly supplied by F_1 and partly by F_2. Let the values of these currents be x and y respectively. If the distributor were actually cut off into two at A—the point of minimum voltage, with x amperes tapped off from the left and y amperes tapped off from the right, then potential distribution would remain unchanged, showing that we can regard the distributor as consisting of two separate distributors each fed from one end only, as shown in Fig. 40.11 (*b*). The drop can be calculated by locating point A and then values of x and y can be calculated.

This can be done with the help of the following pair of equations :

$$x + y = i_4 \quad \text{and} \quad \text{drop from } F_1 \text{ to } A_1 = \text{drop from } F_2 \text{ to } A_2.$$

Example 40.4. *A 2-wire d.c. distributor F_1 F_2 1000 metres long is loaded as under :*

Distance from F_1 (in metres) :	*100*	*250*	*500*	*600*	*700*	*800*	*850*	*920*
Load in amperes :	*20*	*80*	*50*	*70*	*40*	*30*	*10*	*15*

The feeding points F_1 and F_2 are maintained at the same potential. Find which point will have the minimum potential and what will be the drop at this point ? Take the cross-section of the conductors as 0.35 cm² and specific resistance of copper as (1.764×10^{-6}) Ω-cm.

Solution. The distributor along with its tapped currents is shown in Fig. 40.12. The numbers along the distributor indicate length in metres.

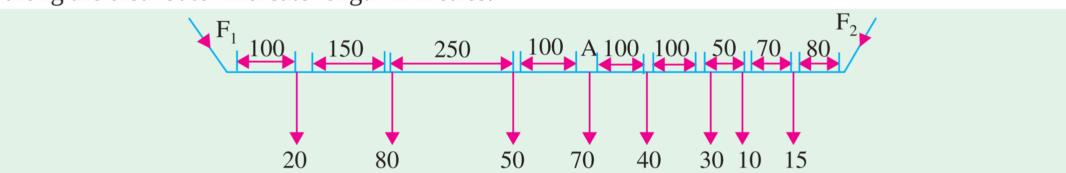


Fig. 40.12

The easiest method of locating the point of minimum potential is to take the moments about the two ends and then by comparing the two sums make a guess at the possible point. The way it is done is as follows :

It is found that 4th point from F_1 is the required point *i.e.* point of minimum potential. Using the previous two equations, we have

$$x + y = 70 \quad \text{and} \quad 47,000 + 600x = 20,700 + 400\,y$$

Solving the two equations, we get $x = 1.7$ A, $y = 70 - 1.7 = 68.3$ A

Drop at A per conductor = 47,000 + 600 × 1.7 = 48,020 ampere-metre.

$$\text{Resistance/metre} = \frac{\rho l}{A} = \frac{1.764 \times 10^{-8} \times 1}{0.35 \times 10^{-4}} = 50.4 \times 10^{-5}\ \Omega/\text{m}$$

$$\text{Hence, drop per conductor} = 48,020 \times 50.4 \times 10^{-5} = 24.2\ \text{V}$$

Reckoning both conductors, the drop at A is = **48.4 V**

Moments about F_1 in ampere-metres	*Sum*	*Moments about F_2 in ampere-metres*	*Sum*
20 × 100 = 2,000	2,000	15 × 80 = 1,200	1,200
80 × 250 = 20,000	22,000	10 × 150 = 1,500	2,700
50 × 500 = 25,000	47,000	30 × 200 = 6,000	8,700
		40 × 300 = 12,000	20,700
		70 × 400 = 28,000	48,700

Alternative Solution

The alternative method is to take the total current fed at one end, say, F_1 as x and then to find the current distribution as shown in Fig. 40.13 (*a*). The drop over the whole distributor is equal to the sum of the products of currents in the various sections and their resistances. For a distributor fed at both ends with ***equal voltages***, this drop equals zero. In this way, value of x can be found.

Resistance per metre single = $5.04 \times 10^{-4}\ \Omega$

Resistance per metre double = $10.08 \times 10^{-4}\ \Omega$

$\therefore$ $10.08 \times 10^{-4}\,[100x + 150(x - 20) + 250\,(x - 100) + 100\,(x - 150) + 100\,(x - 200) + 100(x - 260) + 50(x - 290) + 70(x - 300) + 80\,(x - 315)] = 0$ or $1000x = 151,700$

$\therefore$ $x = 151.7$ A

This gives a current distribution as shown in Fig. 40.13 (*b*). Obviously, point A is the point of minimum potential.

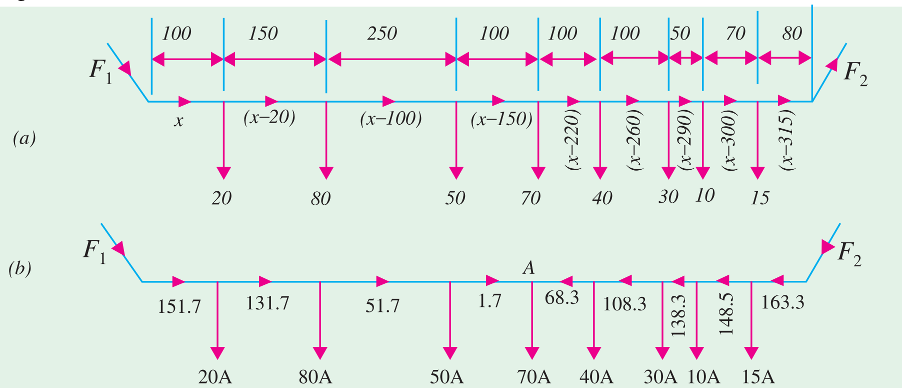


Fig. 40.13

Drop at A (considering both conductors) is

$$= 10.08 \times 10^{-4}\,(100 \times 151.7 + 150 \times 131.7 + 250 \times 51.7 + 100 \times 1.7)$$

$$= 10.08 \times 10^{-4} \times 48,020 = \mathbf{48.4\ V}$$ **—as before.**

Example 40.5. *The resistance of a cable is 0.1 Ω per 1000 metre for both conductors. It is loaded as shown in Fig. 40.14 (a). Find the current supplied at A and at B. If both the ends are supplied at 200 V.* **(Electrical Technology-II, Gwalior Univ.)**

Solution. Let the current distribution be as shown in Fig. 40.14 (*b*). Resistance for both conductors = 0.1/1000=10^{-4} Ω/m. The total drop over the whole cable is zero because it is fed at both ends by equal voltages.

∴ 10^{-4} [500 i + 700 (i – 50) + 300 (i – 150) + 250 (i – 300)] = 0

i = **88.6 A**

This gives the current distribution as shown in Fig. 40.14 (*c*).

Current in *AC* = 88.6 A

Current in *CD* = (88.6 – 50) = 38.6 A

Current in *DE* = (38.6 – 100) = – 61.4 A

Current in *EB* = (–61.4 – 150) = –211.4 A

Hence, current entering the cable at point *B* is = **211.4 A.**

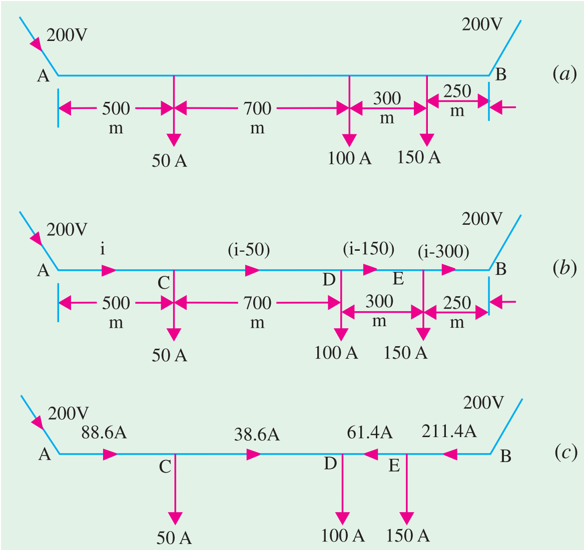


Fig. 40.14

Example 40.6. *The resistance of two conductors of a 2-conductor distributor shown in Fig. 39.15 is 0.1 Ω per 1000 m for both conductors. Find* ***(a)*** *the current supplied at A* ***(b)*** *the current supplied at B* ***(c)*** *the current in each section* ***(d)*** *the voltages at C, D and E. Both A and B are maintained at 200 V.* **(Electrical Engg. Grad. I.E.T.E.)**

Solution. The distributor along with its tapped currents is shown in Fig. 40.15. Let the current distribution be as shown. Resistance per metre double is = 0.1/1000 = 10^{-4} Ω.

The total drop over the whole distributor is zero because it is fed at both ends by equal voltages.

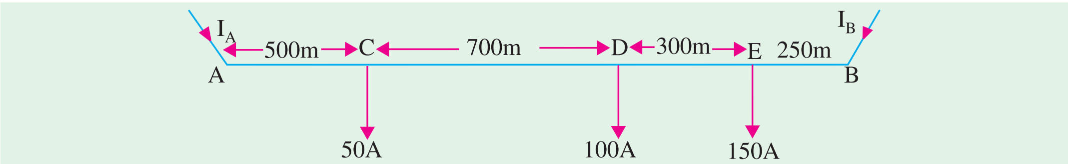


Fig. 40.15

∴ 10^{-4} [500i + 700 (i – 50) + 300 (i – 150) + 250 (i – 300)] = 0

or 1750i = 155,000 or i = 88.6 A

This gives the current distribution shown in Fig. 40.16.

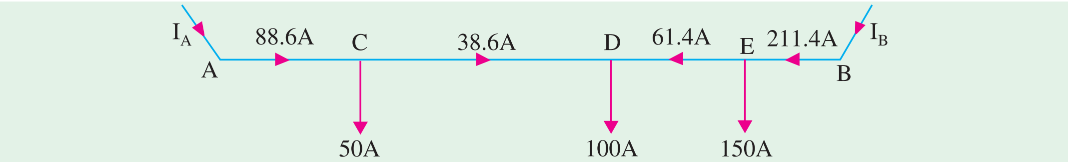


Fig. 40.16

(*a*) I_A = **88.6 A** **(*b*)** I_B = **–211.4 A**

(*c*) Current in section *AC* = **88.6 A**; Current in *CD* = (88.6 – 50) = **38.6 A**

Current in section *DE* = (31.6 – 100) = **– 61.4 A**

Current in *EB* = (– 61.4 – 150) = **– 211.4 A**

(*d*) Drop over AC = 10^{-4} × 500 × 88.6 = 4.43 V; Drop over *CD* = 10^{-4} × 700 × 38.6 = **2.7 V**

Drop over $DE = 10^{-4} \times 300 \times -61.4 = -1.84$ V $\therefore$ Voltage at $C = 200 - 4.43 =$ **195.57 V**
Voltage at $D = (195.57 - 2.7) =$ **192.87 V**, Voltage at $E = 192.87 - (-1.84) =$ **194.71 V**

Example 40.7. *A 200 m long distributor is fed from both ends A and B at the same voltage of 250 V. The concentrated loads of 50, 40, 30 and 25 A are coming on the distributor at distances of 50, 75, 100 and 150 m respectively from end A. Determine the minimum potential and locate its position. Also, determine the current in each section of the distributor. It is given that the resistance of the distributor is 0.08 Ω per 100 metres for go and return.*

(Electric Power-I (Trans & Dist) Punjab Univ. 1993)

Solution. As shown in Fig. 40.17, let current fed at point A be i. The currents in various sections are as shown. Resistance per metre of the distributor (go and return) is 0.08/100 = 0.0008 Ω.

Since the distributor is fed at both ends with equal voltages, total drop over it is zero.

Hence, $0.0008\left[50i + 25(i - 50) + 25(i - 90) + 50(i - 120) + 50(i - 145)\right] = 0$

or $200i = 16{,}750$ or $i = 83.75$ A

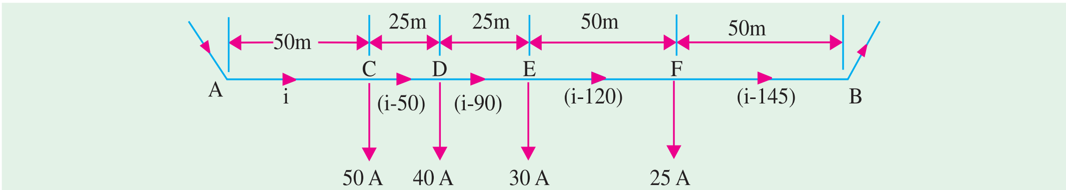


Fig. 40.17

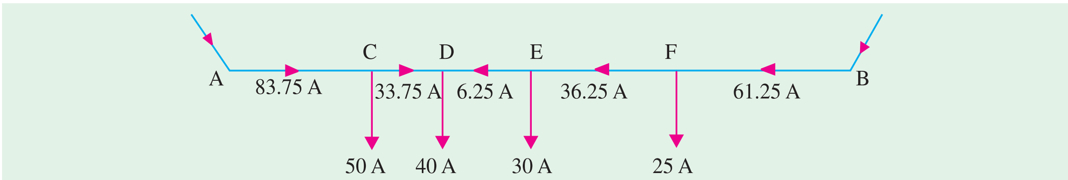


Fig 40.18

The actual current distribution is shown in Fig. 40.18. Obviously, point D is the point of minimum potential.

Drop at point D is $= 0.0008(50 \times 83.75 + 25 \times 33.75) = 4.025$ V

$\therefore$ minimum potential $= 250 - 4.025 =$ **245.98 V**

40.8. Distributor Fed at Both Ends with Unequal Voltages

This case can be dealt with either by taking moments (in amp-m) about the two ends and then making a guess about the point of minimum potential or by assuming a current x fed at one end and then finding the actual current distribution.

Consider the case already discussed in Ex. 40.4. Suppose, there is a difference of ν volts between ends F_1 and F_2 with F_1 being at higher potential. Convert ν volts into ampere-metres with the help of known value of resistance/metre. Since F_2 is at a lower potential, these ampere-metres appear in the coloumn for F_2 as initial drop.

If, for example, ν is 4 volts, then since resistance/metre is 5.04×10^{-4} Ω, initial ampere-metres for F_2 are

$$= \frac{4 \times 10^4}{5.04} = 7{,}938 \text{ amp-metres}$$

The table of respective moments will be as follows :

Moments about F_1	*Sum*	*Moments about F_2*	*Sum*
$20 \times 100 = 2000$	2,000	initial = 7,938	
$80 \times 250 = 20{,}000$	22,000	$15 \times 80 = 1{,}200$	9,138
$50 \times 500 = 25{,}000$	47,000	$10 \times 150 = 1{,}500$	10,638
		$30 \times 200 = 6{,}000$	16,638
		$40 \times 300 = 12{,}000$	28,638
		$70 \times 400 = 28{,}000$	56,638

As seen, the dividing point is the same as before.

$\therefore \quad x + y = 70 \quad$ and $\quad 47{,}000 + 600x = 28{,}638 + 400y$

Solving for x and y, we get $x = 9.64$ A and $y = 60.36$ A

After knowing the value of x, the drop at A can be calculated as before.

The alternative method of solution is illustrated in Ex. 40.12.

40.9. Uniform Loading with Distributor Fed at Both Ends

Consider a distributor PQ of length l units of length, having resistance per unit length of r ohms and with loading per unit length of i amperes. Let the difference in potentials of the two feeding points be ν volts with Q at the lower potential. The procedure for finding the point of minimum potential is as follows:

A circuit-board as shown above uses DC current

Let us assume that point of minimum potential M is situated at a distance of x units from P. Then drop from P to M is $= irx^2/2$ volts (Art. 40.6)

Since the distance of M from Q is $(l - x)$, the drop from Q to M is $= \dfrac{ir\,(l-x)^2}{2}$ volt.

Since potential of P is greater than that of Q by ν volts,

$$\therefore \quad \frac{irx^2}{2} = \frac{ir\,(l-x)^2}{2} + \nu \quad \text{or} \quad x = \frac{l}{2} + \frac{\nu}{irl}$$

40.10. Concentrated and Uniform Loading with Distributor Fed at One End

Such cases are solved in two stages. First, the drop at any point due to concentrated loading is found. To this add the voltage drop due to uniform loading as calculated from the relation.

$$ir\left(lx - \frac{x^2}{2}\right)$$

As an illustration of this method, please look up Ex. 40.9 and 40.13.

Example 40.8. *Each conductor of a 2-core distributor, 500 metres long has a cross-sectional area of 2 cm^2. The feeding point A is supplied at 255 V and the feeding point B at 250 V and load currents of 120 A and 160 A are taken at points C and D which are 150 metres and 350 metres respectively from the feeding point A. Calculate the voltage at each load. Specific resistance of copper is 1.7 × 10^{-6} Ω-cm.* **(Elect. Technology-I, Bombay Univ.)**

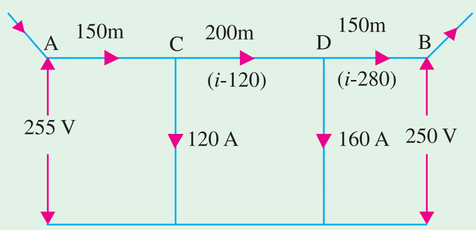


Fig. 40.19

Solution. $\rho = 1.7 \times 10^{-6}$ Ω-cm

$= 1.7 \times 10^{-8}$ Ω.m

Resistance per metre single $= \rho \dfrac{l}{A} = 1.7 \times 10^{-8}/2 \times 10^{-4} = 85 \times 10^{-6}$ m

Resistance per metre double $= 2 \times 85 \times 10^{-6} = 17 \times 10^{-5}$ Ω

The dorp over the whole distributor = 255 – 250 = 5 V

$\therefore \quad 17 \times 10^{-5}\,[150i + 200\,(i - 120) + 150\,(i - 280)] = 5$

$\therefore \quad i = 190.8$ A — **(Fig. 40.19)**

Current in section AC = 190.8 A; Current in section CD = (190.8 – 120) = 70.8 A

Current in section DB = (190.8 – 280) = – 89.2 A (from B to D)

Voltage at point C = 255 – drop over AC

$= 255 - (17 \times 10^{-5} \times 150 \times 190.8) =$ **250.13 V**

Voltage at point D = 250 – drop over BD

$= 250 - (17 \times 10^{-5} \times 150 \times 89.2) =$ **247.72 V**

Example 40.9. *A 2-wire distributor 500 metres long is fed at P at 250 V and loads of 40A, 20A, 60A, 30A are tapped off from points A, B, C and D which are at distances of 100 metres, 150 metres, 300 metres and 400 metres from P respectively. The distributor is also uniformly loaded at the rate of 0.1 A/m. If the resistance of the distributor per metre (go and return) is 0.0005 Ω, calculate the voltage at* ***(i)*** *point Q and* ***(ii)*** *point B.*

Solution. First, consider drop due to concentrated load only.

Drop in PA = 150 × (100 × 0.0005) = 7.5 V

Drop in AB = 110 × (50 × 0.0005) = 2.75 V

Drop in BC = 90 × (150 × 0.0005) = 6.75 V

Drop in CD = 30 × (100 × 0.0005) = 1.5 V

$\therefore$ total drop due to this load = 18.5 V

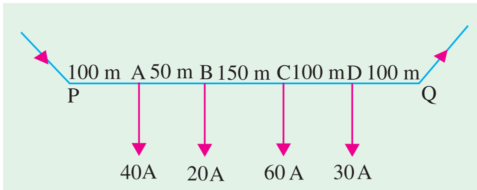


Fig. 40.20

Now, let us consider drop due to uniform load only.

Drop over length $l = ir\dfrac{l^2}{2} = 0.1 \times 0.0005 \times 500^2/2 = 6.25$ V

(i) $\therefore$ potential of point $Q = 250 - (18.5 + 6.25) =$ **225.25 V**

(ii) Consider point B at the distance of 150 metres from P.

Drop due to concentrated loading = 7.5 + 2.75 = 10.25 V.

Drop due to uniform loading $= ir\,(lx - x^2/2)$; Here l = 500 m ; x = 150 m

$$\therefore \quad \text{drop} = 0.1 \times 0.0005\left(500 \times 150 - \frac{150^2}{2}\right) = 3.1875 \text{ V}$$

$\therefore$ potential of point $P = 250 - (10.25 + 3.1875) =$ **13.44 V**

Example 40.10. *A distributor AB is fed from both ends. At feeding point A, the voltage is maintained at 236 V and at B at 237 V. The total length of the distributor is 200 metres and loads are tapped off as under :*

(i) *20 A at 50 metres from A* ***(ii)*** *40 A at 75 metres from A*

(iii) *25 A at 100 metres from A* ***(iv)*** *30 A at 150 metres from A*

The resistance per kilometre of one conductor is 0.4Ω. Calculate the currents in the various sections of the distributor, the minimum voltage and the point at which it occurs.

(Electrical Technology, Calcutta Univ.)

Solution. The distributor along with currents in its various sections is shown in Fig. 40.21.

Resistance/metre single $= 0.4/1000 = 4 \times 10^{-4}\ \Omega$

Resistance/metre double $= 8 \times 10^{-4}\ \Omega$

Voltage drop on both conductors of 200-metre long distributor is

$$= 8 \times 10^{-4} [50i + 26(i - 20) + 25(i - 60) + 50(i - 85) + 50(i - 115)]$$

$$= 8 \times 10^{-4} (200i - 12{,}000) \text{ volt}$$

This drop must be equal to the potential difference between A and B.

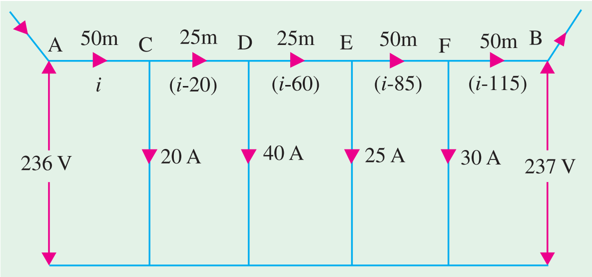


Fig. 40.21

$\therefore \quad 8 \times 10^{-4} (200i - 12{,}000) = 236 - 237 = -1$

$\therefore \quad i = 53.75$ A

Current in section AC = **53.75 A**

Current in section CD = 53.75 – 20 = **33.75 A**

Current in section DE = 53.75 – 60 = **– 6.25 A**

Current in section EF = 53.75 – 85 = **– 31.25 A**

Current in section FB = 53.75 – 115 = **– 61.25 A**

The actual current distribution is as shown in Fig. 40.22.

Obviously, minimum voltage occurs at point D *i.e.* 75 ***metre from point*** A (or 125 m from B)

Voltage drop across both conductors of the distributor over the length AD is

$$= 8 \times 10^{-4} (50 \times 53.75 + 25 \times 33.75) = 2.82 \text{ V}$$

$\therefore$ potential of point

$D = 236 - 2.82$

$=$ **233.18 V**

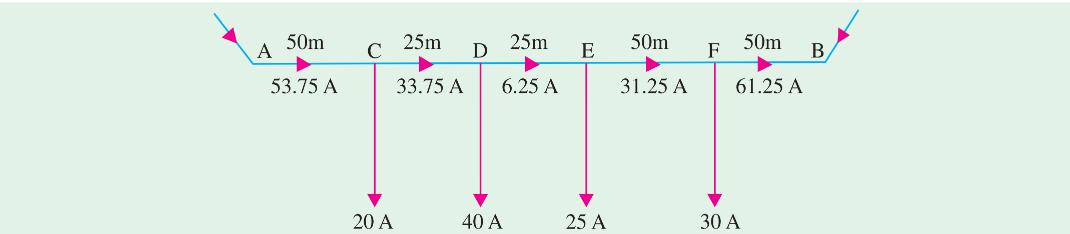


Fig. 40.22

Example 40.11. *A distributor cable AB is fed at its ends A and B. Loads of 12, 24, 72 and 48 A are taken from the cable at points C, D, E and F. The resistances of sections AC, CD, DE, EF and FB of the cable are 8, 6, 4, 10 and 5 milliohm respectively (for the go and return conductors together).*

The p.d. at point A is 240 V, the p.d. at the load F is also to be 240 V. Calculate the voltage at the feeding point B, the current supplied by each feeder and the p.d.s. at the loads C,D and E.

(Electrical Technology ; Utkal Univ.)

Solution. Let the current fed at the feeding point A be i. The current distribution in various sections becomes as shown in Fig. 40.23.

Voltage drop on both sides of the distributor over the section AF is

$$= [8i + 6(i - 12) + 4(i - 36) + 10(i - 108)] \times 10^{-3} = (28i - 1296) \times 10^{-3} \text{ V}$$

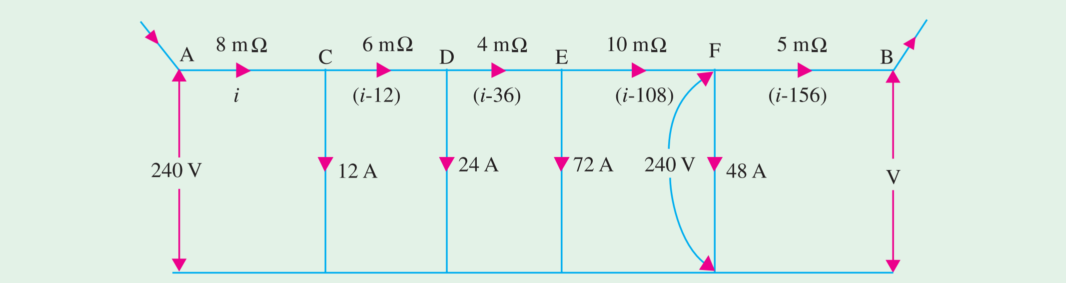


Fig. 40.23

Since points A and F are at the same potential, the p.d. between A and F is zero.

$\therefore \quad (28i - 1296) \times 10^{-3} = 0 \quad \text{or} \quad i = 46.29 \text{ A}$

Current in section AC = 46.29 A

Current in section CD = 46.29 – 12 = 34.29 A

Current in section DE = 46.29 – 36 = 10.29 A

Current in section EF = 46.29 – 108 = – 61.71 A

Current in section FB = 46.29 – 156 = –109.71 A

The actual current distribution is as shown in Fig. 40.24.

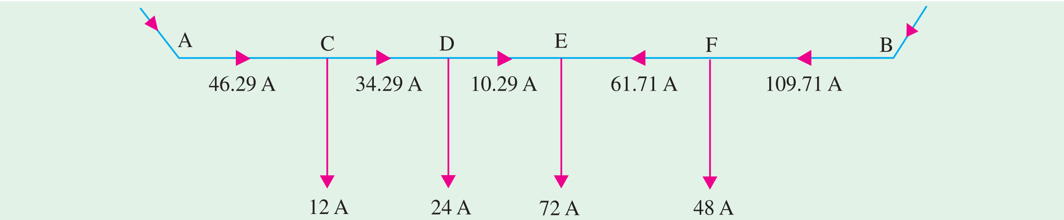


Fig. 40.24

Current applied by feeder at point A is 46.29 A and that supplied at point B is 109.71 A.

Voltage at feeding point B = 240 – drop over FB = $240 - (-5 \times 10^{-3} \times 109.71)$ = **240.55 V.**

Voltage at point C = 240 – drop over AC = $240 - (8 \times 10^{-3} \times 46.29)$ = **239.63 V**

Voltage at point D = 239.63 – drop over CD = $239.63 - (6 \times 10^{-3} \times 34.29)$ = **239.42 V**

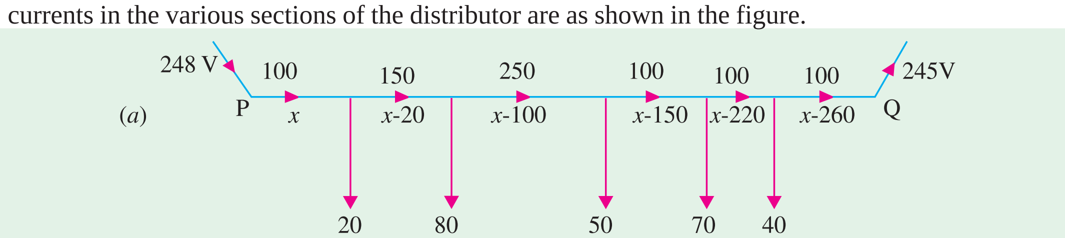
Voltage at point E = 239.42 – drop over DE = $239.42 - (4 \times 10^{-3} \times 10.29)$ = **239.38 V**

Example 40.12. *A two-wire, d.c. distributor PQ, 800 metre long is loaded as under :*

Distance from P (metres) :	*100*	*250*	*500*	*600*	*700*
Loads in amperes :	*20*	*80*	*50*	*70*	*40*

The feeding point at P is maintained at 248 V and that at Q at 245 V. The total resistance of the distributor (lead and return) is 0.1 Ω. Find **(a)** *the current supplied at P and Q and* **(b)** *the power dissipated in the distributor.*

Solution. As shown in Fig. 40.25 (a), let x be the current supplied from end P. The other currents in the various sections of the distributor are as shown in the figure.



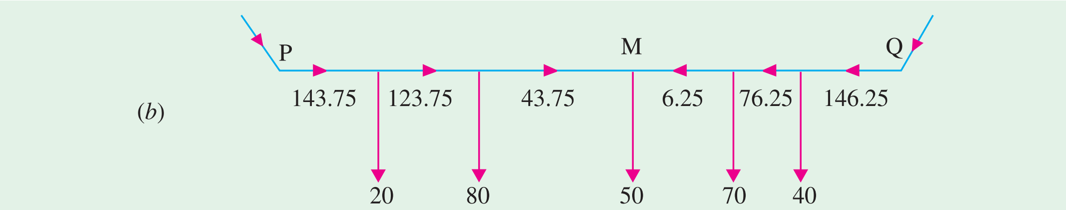


Fig. 40.25

Now, the total drop over *PQ* should equal the potential difference between ends *P* and *Q* *i.e.*, = 248 – 245 = 3.

Resistance/metre of both conductors = 0.1/800 = 1/8000 Ω

$$\therefore \quad \frac{1}{8000}[100x + 150(x-20) + 250(x-100) + 100(x-150) + 100(x-220) + 100(x-246)] = 3$$

or $\qquad 800x = 115{,}000 \qquad \therefore \quad x = 143.75$ A

The actual distribution is shown in Fig. 40.25 (*b*) from where it is seen that point *M* has the minimum potential.

I_P = **143.75 A.** $\qquad I_Q$ = **116.25 A**

$$\text{Power loss} = \Sigma I^2R = \frac{1}{8000}[143.75^2 \times 100 + 123.75^2 \times 150 + 43.75^2 \times 250 + 6.25^2 \times 100 + 76.25^2 \times 100 + 116.25^2 \times 100] = \mathbf{847.3\ W}$$

Example 40.13. *The two conductors of a d.c. distributor cable 1000 m long have a total resistance of 0.1 Ω. The ends A and B are fed at 240 V. The cable is uniformly loaded at 0.5 A per metre length and has concentrated loads of 120 A, 60 A, 100 A and 40 A at points distant 200, 400, 700 and 900 m respectively from the end A. Calculate* ***(i)*** *the point of minimum potential on the distributor* ***(ii)*** *the value of minimum potential and* ***(iii)*** *currents fed at the ends A and B.*

(Power System-I, AMIE, Sec. B, 1993)

Solution. Concentrated loads are shown in Fig. 40.26. Let us find out the point of minimum potential and currents at points *A* and *B*.

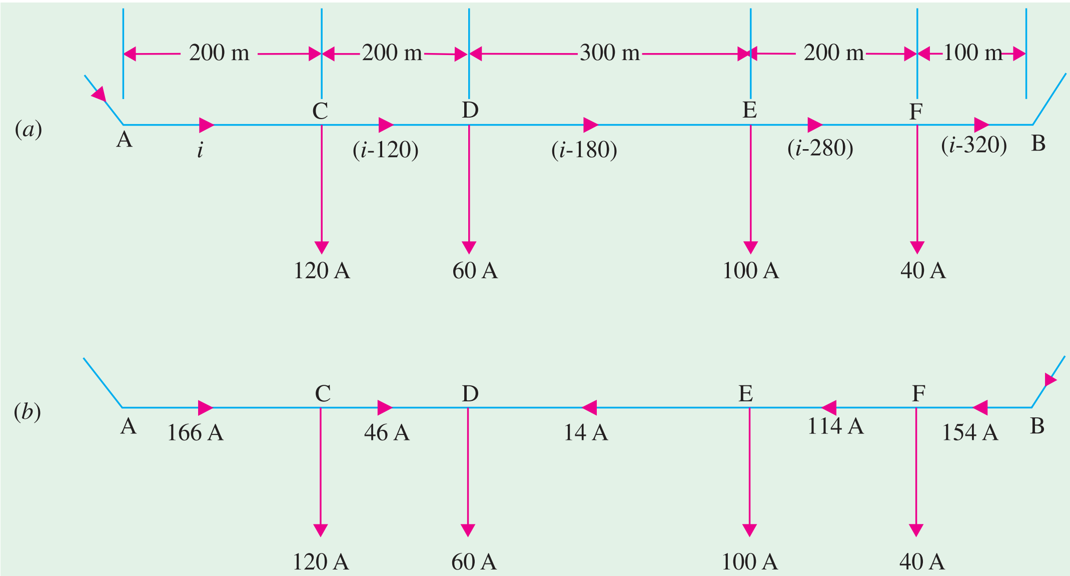


Fig. 40.26

It should be noted that location of point of minimum potential is not affected by the uniformly-spread load of 0.5 A/m. In fact, let us, for the time being, imagine that it is just not there. Then, assuming i to be the input current at A, the different currents in various sections are as shown. Since points A and B are fed at equal voltages, total drop over the distributor is zero. Distributor resistance per metre length (go and return) is $0.1/1000 = 10^{-4}\ \Omega$.

$\therefore \quad 10^{-4}\,[200i + 200(i - 120) + 300(i - 180) + 200(i - 280) + 100(i - 320)] = 0$

or $\quad 1000i = 166{,}000 \quad \therefore \quad i = 166$ A

Actual current distribution is shown in Fig. 40.26 (*b*) from where it is seen that point D is the point of minimum potential. The uniform load of 0.5 A/m upto point D will be supplied from A and the rest from point B.

Uniform load from A to $D = 400 \times 0.5 = 200$ A. Hence, $I_A = 166 + 200 =$ **366 A.**

Similarly, $\quad I_B = 154 + (600 \times 0.5) =$ **454 A.**

Drop at D due to concentrated load is $= 10^{-4}\,(166 \times 200 + 46 \times 200) =$ **4.24 V.**

Drop due to uniform load can be found by imagining that the distributor is cut into two at point D so that AD can be looked upon as a distributor fed at one end and loaded uniformly. In that case, D becomes the other end of the distributor.

$\therefore$ drop at D due to uniform load (Art. 40.6)

$$= irl^2/2 = 0.5 \times 10^{-4} \times 400^2/2 = 4 \text{ V}$$

$\therefore$ total drop at $D = 4.24 + 4 = 8.24$ V $\quad \therefore$ potential of $D = 240 - 8.24 =$ **231.76 V**

Example 40.14. *It is proposed to lay out a d.c. distribution system comprising three sections—the first section consists of a cable from the sub-station to a point distant 800 metres from which two cables are taken, one 350 metres long supplying a load of 22 kW and the other 1.5 kilometre long and supplying a load of 44 kW. Calculate the cross-sectional area of each cable so that the total weight of copper required shall be minimum if the maximum drop of voltage along the cable is not to exceed 5% of the normal voltage of 440 V at the consumer's premises. Take specific resistance of copper at working temperature equal to 2 μ Ω-cm.*

Solution. Current taken from 350-m section is $I_1 = 22{,}000/440 = 50$ A

Current taken from 1.5 km section, $I_2 = 44{,}000/440 = 100$ A

$\therefore$ Current in first section $\quad I = 100 + 50 = 150$ A

Let $\quad V$ = voltage drop across first section ; R = resistance of the first section

$\quad A$ = cross-sectional area of the first section

Then $\quad R = V/I = V/150\ \Omega$

Now, $\quad A = \dfrac{\rho l}{R} = \rho l \dfrac{I}{V} = 80{,}000 \times 2 \times 10^{-6} \times 150/V = 24/V\text{cm}^2$

Now, $\quad$ maximum allowable drop $= 5\%$ of $440 = 22$ V

$\therefore$ voltage drop along other sections $= (22 - V)$ volt

Hence, cross-sectional area of 350-m section is

$$A_1 = 35{,}000 \times 2 \times 10^{-6} \times 50/(22 - V) = 3.5/(22 - V)\text{ cm}^2$$

Also, cross-sectional area of 1500-m section is

$$A_2 = 150{,}000 \times 2 \times 10^{-6} \times 100/(22 - V) = 30/(22 - V)\text{ cm}^2$$

Now, total weight of copper required is proportional to the total volume.

$$\therefore \quad W = K[(800 \times 24/V) + 350 \times 3.5/(22 - V) + 1500 \times 30/(22 - V)]$$

$$= K[1.92/V + 4.62/(22 - V)] \times 10^4$$

Weight of copper required would be minimum when $dW/dV = 0$

$$\therefore \quad \frac{dW}{dV} = K\left[\frac{-1.92}{V^2} + \frac{4.62}{(22-V)^2}\right] \times 10^4 = 0$$

$$\text{or} \quad \frac{1.92}{V^2} = \frac{4.62}{(22-V)^2} \quad \text{or} \quad (22 - V)^2 = 2.4\,V^2$$

or $\quad V = 22/2.55 =$ **8.63 volt** $\qquad \therefore \quad A = 24/8.63 =$ **2.781 cm²**

$A_1 = 3.5/(22 - 8.63) =$ **0.2618 cm²** $\qquad A_2 = 30/(22 - 8.63) =$ **2.246 cm²**

Example 40.15. *A d.c. two-wire distributor AB is 450 m long and is fed at both ends at 250 volts. It is loaded as follows : 20A at 60 m from A, 40A at 100m from A and a uniform loading of 1.5 A/m from 200 to 450 m from A. The resistance of each conductor is 0.05 Ω/km. Find the point of minimum potential and its potential.* **(Electrical Power-II, Bangalore Univ. 1993)**

Solution. In Fig. 40.27, let D be the point of minimum potential and let i be the current in distributor section CD. Then, current supplied to load D from end B is $(40 - i)$. If r is the resistance of the distributor/metre (both go and return), then

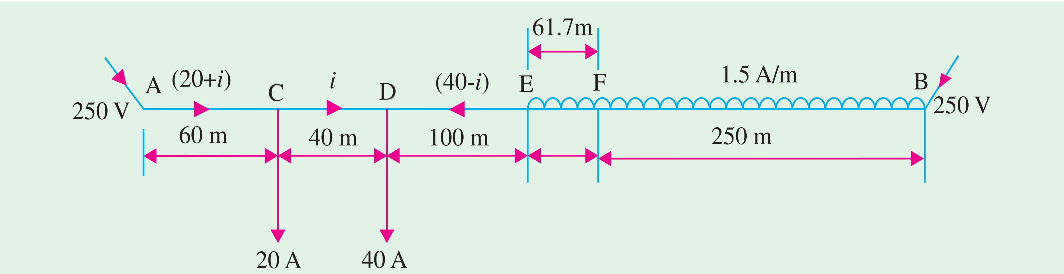


Fig. 40.27

Drop over $AD = (20 + i) \times 60r + i \times 40r$

Drop over BD = drop due to concentrated load + drop due to distributed load

$= (40 - i) \times 350r + 1.5 \times r \times 250^2/2$ —**Art. 40.6**

Since the two feeding points A and B are at the same potential of 250 V, the two drops must be equal.

$\therefore \quad (20 + i) \times 60r + i \times 40r = (40 - i) \times 350r + 1.5 \times r \times 250^2/2 \quad \therefore \quad i = 132.6$ Å

Since $(40 - i)$ comes out to be negative, it means that D is not the point of minimum potential. The required point is somewhat nearer the other end B. Let it be F. Obviously, current in section DF $= 132.6 - 40 = 92.6$ A. Hence, distance of minimum potential point F from end A is

$= 60 + 40 + 100 + 92.6/1.5 =$ **261.7 m**

Total voltage drop over section AF is

$= 2 \times 0.05 \times 10^{-3}\,(152.6 \times 60 + 132.6 \times 40 + 92.6 \times 100 + 92.6 + 61.7/4) = 2.65$ V

$\therefore$ potential of point $F = 250 - 2.65 =$ **247.35 V.**

Example 40.16. *A two-wire d.c. distributor AB, 1000 metres long, is supplied from both ends, 240 V at A and 242 V at B. There is a concentrated load of 200 A at a distance of 400 metre from A and a uniformly distributed load of 1.0 A/m between the mid-point and end B. Determine* ***(i)*** *the currents fed at A and B* ***(ii)*** *the point of minimum potential and* ***(iii)*** *voltage at this point. Take cable resistance as 0.005 Ω per 100 metre each core.*

Solution. The resistance per 100 metres of both cores $= 0.005 \times 2 = 0.01\ \Omega$.

Let us take 100 m as the unit of length. Let current fed at end B be I_B as shown in Fig. 40.28.

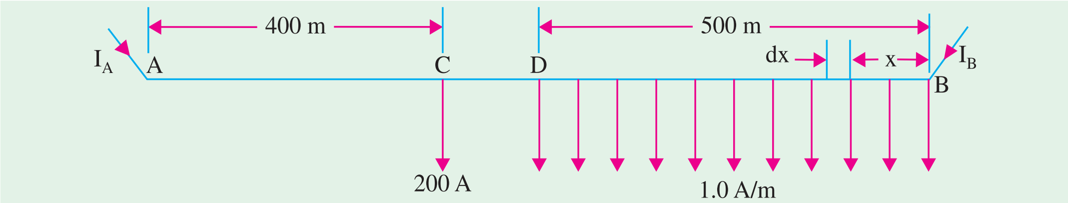


Fig. 40.28

Consider any element dx anywhere within distance BD *i.e.* distance over which uniformly distributed load is applied. Suppose it is at a distance of x units (of 100 metre each). The current through dx is $I = (I_B - 100 \times 1.0 \times x) = (I_B - 100x)$

$\therefore$ voltage drop over $dx = (I_B - 100x) \times 0.01 \times dx$

Voltage drop over distance BD

$$= \int_0^5 (I_B - 100x) \times 0.01 \times dx = 0.01 \int_0^5 (I_B - 100x)\, dx$$

$$= 0.01 \left| (I_B x - 500x^2) \right|_0^5 = (0.05\, IB - 12.5) \text{ V}$$

Voltage drop over $DC = (I_B - 500) \times 0.01$; Voltage drop over $CA = (I_B - 700) \times 0.01 \times 4$

Since total drop over $AB = 242 - 240 = 2$ volt

$\therefore$ $0.05\, I_B - 12.5 + 0.01\, I_B - 5 + 0.04\, I_B - 28 = 2$ $\qquad \therefore$ $I_B =$ **455 A**

Now, total current is $= 500 + 200 = 700$ A $\qquad \therefore$ $I_A = 700 - 455 =$ **245 A**

It is obvious that $(245 - 200) = 45$ A is fed into the distributed load at D. Hence, point of minimum potential M will be $45/1.0 = 45$ metres from D. Its distance from $B = 500 - 45 = 455$ metres or 4.55 units of length.

Voltage drop upto this point from end $B = \int_0^{4.55} (455 - 100x) \times 0.01 \times dx = 10.35$ V

$\therefore$ potential of $M = 242 - 10.35 =$ **231.65 V**

Tutorial Problem No. 40.1

1. A 2-wire direct-current distributor PQ is 500 metres long. It is supplied by three feeders entering at P, Q and R where R is midway between P and Q. The resistance of the distributor, go and return, is 0.05 Ω per 100 metres. The distributor is loaded as follows :

Point	*A*	*B*
Distance from P (metres)	*100*	*350*
Load (ampere)	*30*	*40*

In addition, a distributed load of 0.5 A/metre exists from P to Q. Calculate (a) the current supplied by each feeder if P and R are maintained at 220 V while Q is at 215 V (b) the voltage at a point between P and Q and 50 metres from P.

[(*a*) current at P = 18 A ; at Q = 28.5 A ; at R = 98.5 A (*b*) voltage at stated point = 214.8 V]

2. A section of a 2-wire distributor network is 1,200 metres long and carries a uniformly distributed load of 0.5 A/ metre. The section is supplied at each end by a feeder from a distribution centre at which the voltage is maintained constant. One feeder is 900 and the other 600 metres long and each has a cross-sectional area 50% greater than that of the distributor. Find the current in each feeder cable and the distance from one end of the distributor at which p.d. is a minimum. **(*London Univ.*)**

[Current in 900 metres: 272.7 A; current in 600 metres: 327.3 A; 545.5 metres from the 900 metre feeder point]

3. A pair of distributing mains of uniform cross-section 1,000 metres in length having a resistance of 0.15 Ω each, are loaded with currents of 50, 100, 57.5, 10 and 75 A at distances measured from one end where the voltage between mains is 211.6, of 100, 300, 540, 740 and 850 metres respectively. If the voltage of the other end is maintained at 210, calculate the total current entering the system at each end of the mains and the position of the point of minimum potential. ***(I.E.E. London)***
[160.63 A and 131.87 A ; 540 metre load]

4. A 2-core distribution cable *AB*, 400 metre long, supplies a uniformly distributed lighting load of 1 A/m. There are concentrated loads of 120, 72, 48 and 120 A at 40, 120, 200 and 320 metres respectively from end *A*. This cable has a resistance of 0.15 Ω per 1,000 m run. Calculate the voltage and the position of the lowest run lamp when the cable is fed at 250 Volts (*a*) from both ends *A* and *B* (*b*) from end *A* only. **[(*a*) 239.1 volt at 200 m (*b*) 207.6 volt at B]**

5. A 2-wire d.c. distributor *AB*, 300 m long, is fed at both ends *A* and *B*. The distributor supplies a uniformly distributed load of 0.25 A per m together with concentrated loads of 40 A at *C* and 60 A at *D*, *AC* and *BD* being 120 m each. *A* and *B* are maintained at 300 V, the loop resistance of the distributor is 0.1 ohm/100 m. Determine the current fed at *A* and *B* and also the potential of points *C* and *D*. **[85.5 A, 89.5 A, 291.54 V, 291.06 V]**

40.11. Ring Distributor

A ring distributor is a distributor which is arranged to form a closed circuit and which is fed at one or more than one points. For the purpose of calculating voltage distribution, it can be looked upon as consisting of a series of open distributors fed at both ends. By using a ring distributor fed properly, great economy in copper can be affected.

If the ring distributor is fed at one point then, for the purposes of calculation, it is equivalent to a straight distributor fed at both ends with equal voltages (Ex. 40.17).

Example 40.17. *A 400-metre ring distributor has loads as shown in Fig. 40.29 (a) where distances are in metres. The resistance of each conductor is 0.2 Ω per 1,000 metres and the loads tapped off at points B, C and D are as shown. If the distributor is fed at A, find voltages at B,C and D.*

Solution. Let us assume a current of I in section *AD* [Fig. 40.29 (*a*)] and then find the total drop which should be equated to zero.

$\therefore \quad 70\,I + 90\,(I - 50) + 80\,(I - 120) + 60\,(I - 220) = 0 \qquad \therefore \quad 300\,I = 27{,}300$ or $I = 91$ A

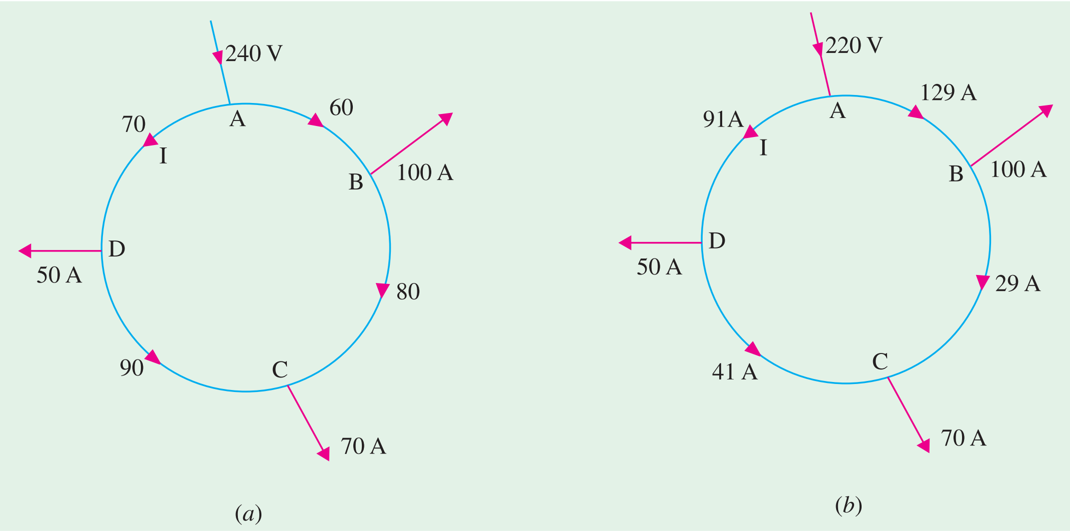


Fig. 40.29

The current distribution becomes as shown in Fig. 40.29 (*b*) from where it is seen that C is the point of minimum potential.

Drop in AD = 2(91 × 70 × 0.2/1,000) = 2.55 V ; Drop in DC = 2(41 × 90 × 0.2/1,000) = 1.48 V

Drop in CB = 2(29 × 80 × 0.2/1,000) = 0.93 V ; Drop in BA = 2(129 × 60 × 0.2/1,000) = 3.1 V

Voltage at D = 240 − 2.55 = **237.45 V** ; Voltage at C = 237.45 − 1.48 = **235.97 V**

Voltage at B = 240 − 3.1 = **236.9 V**

Example 40.18. *In a direct current ring main, a voltage of 400 V is maintained at A. At B, 500 metres away from A, a load of 150 A is taken and at C, 300 metres from B, a load of 200 A is taken. The distance between A and C is 700 metres. The resistance of each conductor of the mains is 0.03 Ω per 1,000 metres. Find the voltage at B and C and also find the current in the section BC.*
(Elect. Technology, Kerala Univ.)

Solution. Let us assume a current of I in section AB, then find the total drop round the ring main and equate it to zero. As seen from Fig. 40.30 (*a*).

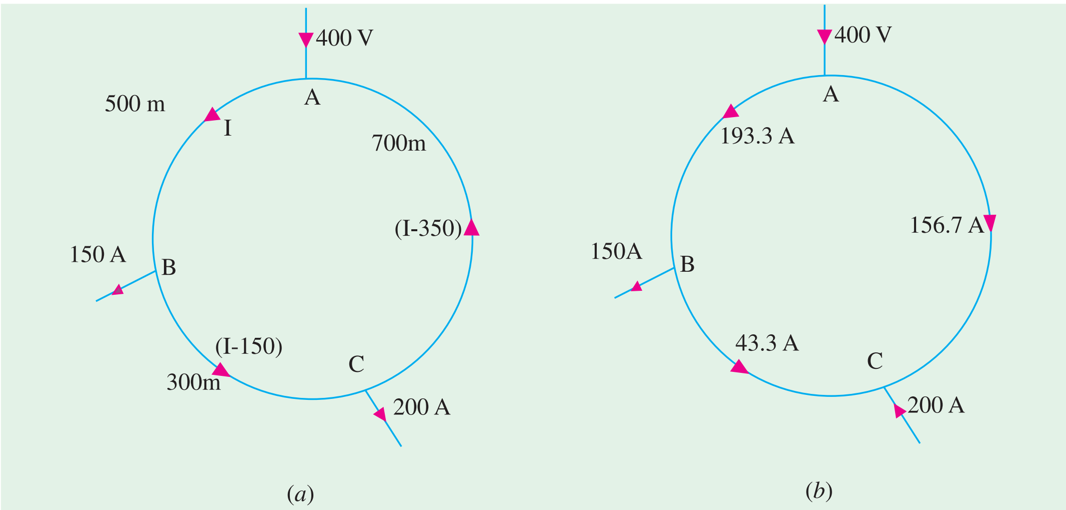


Fig. 40.30

$$500\, I + 300\,(I - 150) + 700\,(I - 350) = 0 \qquad \therefore \quad I = 193.3 \text{ A}$$

The current distribution becomes as shown in Fig. 40.30 (*b*) from where it is seen that C is the point of minimum potential.

Drop over AB = 2(193.3 × 50 × 0.03/1,000) = 5.8 V

Drop over BC = 2(43.3 × 300 × 0.03/1,000) = 0.78 V

Voltage at B = 400 − 5.8 = 394.2 V ; Voltage at C = 394.2 − 0.78 = **393.42 V**

The current in section BC is as shown in Fig. 40.30 (*b*).

Example 40.19. *A d.c. ring main ABCDE is fed at point A from a 220-V supply and the resistances (including both lead and return) of the various sections are as follows (in ohms) : AB = 0.1 ; BC = 0.05 ; CD = 0.01 ; DE = 0.025 and EA = 0.075. The main supplies loads of 10 A at B ; 20 A at C ; 30 A at D and 10 A at E. Find the magnitude and direction of the current flowing in each section and the voltage at each load point.*

If the points C and E are further linked together by a conductor of 0.05 Ω resistance and the output currents from the mains remain unchanged, find the new distribution of the current and voltage in the network.
(London Univ.)

Solution. The ring main is shown in Fig. 40.31.

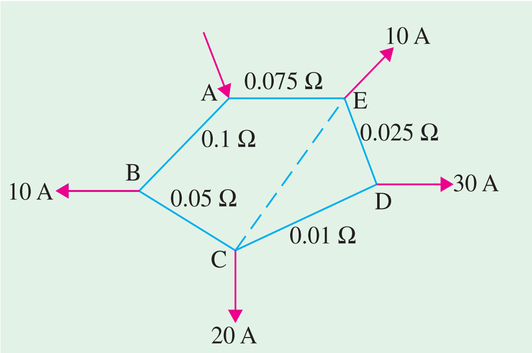


Fig. 40.31

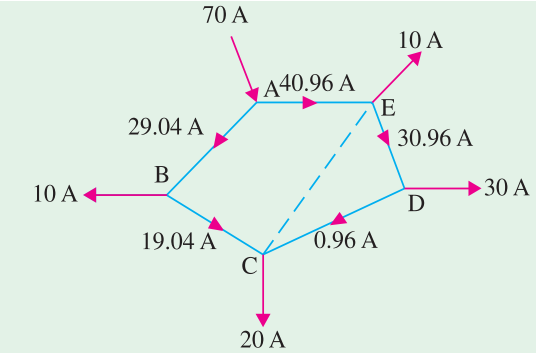


Fig. 40.32

Let us assume a current of I amperes in section AB and put the total drop round the ring equal to zero.

$\therefore$ $0.1\,I + 0.05(I - 10) + 0.01(I - 30) + 0.025(I - 60) + 0.075(I - 70) = 2$ or $I = 29.04$ A

Current distribution now becomes as shown in Fig. 40.32.

Drop in AB	=	29.04 × 0.1 = 2.9 V;	Drop in BC	=	19.04 × 0.05 = 0.95 V
Drop in ED	=	30.96 × 0.025 = 3.77V;	Drop in AE	=	40.96 × 0.075 = 3.07 V
∴ Potential of B	=	217.1 V,	Potential of C	=	**216.15 V**
Potential of E	=	216.93 V,	Potential of D	=	**216.16 V**

The interconnector between points C and E is shown in Fig. 40.31. It may be noted here that the function of the interconnector is to reduce the drop of voltage in various sections. For finding current in the interconnector, first p.d. across its ends is calculated. Then we calculate the resistance, viewed from points E and C of the network composed resistances of the distribution lines only, ignoring the load (Art, 2-22). Then current through the interconnector is given by

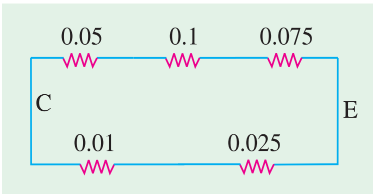


Fig. 40.33

$$I = \frac{\text{p.d. between points } E \text{ and } C}{\text{resistance of distribution network + interconnector}}$$

P.D. between points E and C = 216.93 – 216.15 = 0.78 V

To determine the resistance viewed from C and E, the network is drawn as shown in Fig. 40.33. Since the two branches are in parallel, the combined resistance is

$$= 0.225 \times 0.035/(0.225 + 0.035) = 0.03\ \Omega$$

$\therefore$ current in interconnector $= 0.78/(0.03 + 0.05) = 9.75$ A from E to C.

The currents in the other sections must be calculated again.

Let us assume a current I_1 and ED, then since the voltage round the closed mesh EDC is zero, hence

$-0.025\,I_1 - 0.01\,(I_1 - 30) + 0.05 \times 9.75 = 0$ or $0.035\,I_1 = 0.7875$ $\therefore$ $I_1 =$ **22.5 A**

Current in AE	= 10 + 22.5 + 9.75 = **42.25 A;**	Current in AB = 70 – 42.25 = **27.75 A**
Drop in AB	= 27.75 × 0.1= **2.775 V ;**	Drop in BC = 17.75 × 0.05 = **0.888 V**
Drop in ED	= 32.25 × 0.025 = **0.806 V ;**	Drop in AE = 42.25 × 0.075 = **3.169 V**
Potential of B	= 220 – 2.775=**217.225 V;**	Potential of C = 217.225 – 0.888 = **216.337 V**
Potential of E	= 220 – 3.169=**216.83 V ;**	Potential of D = 216.83 – 0.806 = **216.024 V**

Tutorial Problem No. 40.2

1. Four power loads *B*, *C*, *D* and *E* are connected in this order to a 2-core distributor cable, arranged as a ring main, and take currents of 20, 30, 25 and 30 A respectively. The ring is supplied from a substation at the point *A* between *B* and *E*. An interconnector cable joins the points *C* and *E* and from a point *F* on this inter-connector cable a current of 20 A is taken.
The total resistance of the cable between the load points is : *AB* = 0.04 Ω ; *BC* = 0.03 Ω ; *CD* = 0.02 Ω ; *DE* = 0.03 Ω ; *EA* = 0.04 Ω ; *CF* = 0.02 Ω and *EF* = 0.01 Ω. Calculate the current in each section of the ring and the interconnector. ***(London Univ.)***
[AB = 53.94 A ; BC = 33.94 A ; CD = 8.35 A ; ED = 16.65 A ; AE = 71.06 A ; FC = 4.42 A ; EF = 24.42 A]

2. A 2-core ring feeder cable *ABCDEA* is connected to a sub-station at *A* and supplies feeding points to a distribution network at *B*, *C*, *D* and *E*. The points *C* and *E* are connected by an inter-connecter *CFE* and a load is taken at *F*. The total resistance in ohms of both conductors in the several sections is *AB* = 0.05 ; *BC* = 0.4 ; *CD* = 0.03 ; *DE* = 0.04 ; *EA* = 0.05 ; *CF* = 0.02 ; *FE* = 0.1. The currents taken at the load points are *B* = 12 A ; *C* = 15 A ; *D* = 12 A; *E* = 15 A and *F* = 10 A. Calculate the current in each section of the cable and the p.d. at each load point, if the p.d. at *A* is maintained constant at 250 V. ***(City & Guilds, London)***
[Currents : AB = 27.7 A ; FC = 3.3 A ; P.D. s at B = 248.6 V ; C = 248 V ; D = 247.87 V ; E = 248.18 V ; F = 248 V]

3. A distributor cable in the form of a ring main *ABCDEA*, supplies loads of 20, 60, 30, and 40 A taken at the points *B*, *C*, *D* and *E* respectively, the feeding point being at *A*. The resistances of the sections are *AB* = 0.1 Ω, *BC* = 0.15 Ω, *CD* = 0.1 Ω, *DE* = 0.05 Ω and *EA* = 0.1 Ω. The points *E* and *C* are connected by a cable having a resistance of 0.3 Ω. Calculate the current in each section of the network. ***(City & Guilds, London)***
[A to B : 60 A ; B to C : 40 A ; E to C : 10 A ; D to C : 10 A ; E to D : 40 A ; A to E : 90 A]

40.12. Current Loading and Load-point Voltages in a 3-wire System

Consider a 3-wire, 500/250-V distributor shown in Fig. 40.34. The motor requiring 500 V is connected across the outers whereas other loads requiring lower voltage of 250 V are connected on both sides of the neutral.

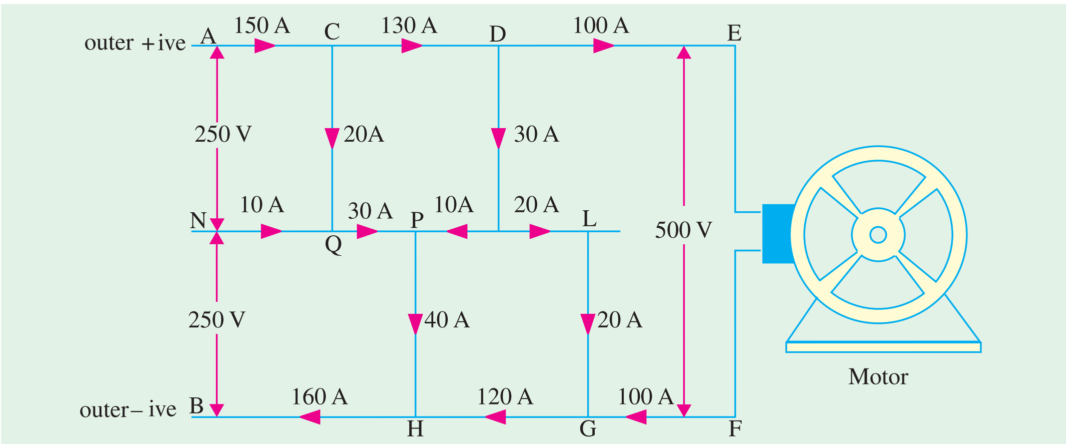


Fig. 40.34

The current in positive outer *AE* flows from the left to right and in negative outer *FB* from right to left. The current through the various sections of the neutral wire *NL* may flow in either direction depending on the load currents on the positive side and negative side but is independent of the loads connected between outers.

Since 150 A enters the +ve outer but 160 A comes out of the network, it means that a current of 10 A must flow into the neutral at point *N*. Once the direction and magnitude of current in *NQ* is known, the directions and magnitudes of currents in other sections of the neutral can be found very easily. Since *PH* takes 40 A, currents meeting at *P* should add up to 40 A. As seen, 20 A of *CQ* and 10A of *NQ* flow towards *P*, the balance of 10 A flows from point *M*. Then, 30 A current of *DM* is divided into two parts, 10 A flowing along *MP* and the other 20 A flowing along *ML* to feed the load *LG*.

Knowing the values of currents in the various conductors, voltage drops can be calculated provided resistances are known. After that, voltages at different load points can be calculated as illustrated in Ex. 40.20.

Example 40.20. *In a 3-wire distribution system, the supply voltage is 250 V on each side. The load on one side is a 3 ohm resistance and on the other, a 4 ohm resistance. The resistance of each of the 3 conductors is 0.05 Ω. Find the load voltages.*

(Elements of Elect. Engg-I, Bangalore Univ.)

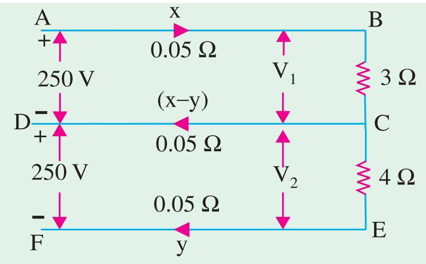


Fig. 40.35

Solution. Let the assumed directions of unknown currents in the three conductors be as shown in Fig. 40.35. Applying KVL (Art. 2.2) to closed circuit *ABCDA*, we have

$$-3.05x - 0.05(x-y) + 250 = 0$$

or $$310x - 5y = -25{,}000 \quad ...(i)$$

Similarly, circuit *DCEFD* yields

$$0.05\,(x-y) - 4.05y + 250 = 0$$

or $$5x - 410y = -25{,}000 \quad ...(ii)$$

From (*i*) and (*ii*), we get $x = 81.64$ A, $y = 61.97$ A. Since both currents come out to be positive, it means that their assumed directions of flow are correct.

$$\therefore \quad V_1 = 250 - 0.05 \times 81.64 - 0.05\,(81.64 - 61.97) = \mathbf{244.9\ V}$$

$$V_2 = 250 + 0.05(81.64 - 61.97) - 0.05 \times 61.97 = \mathbf{247.9\ V}$$

Example 40.21. *A 3-wire d.c. distributor PQ, 250 metres long, is supplied at end P at 500/250 V and is loaded as under :*

Positive side: 20 A 150 metres from P ; 30 A 250 metres from P.

Negative side: 24 A 100 metres from P ; 36 A 220 metres from P.

The resistance of each outer wire is 0.02 Ω per 100 metres and the cross-section of the middle wire is one-half that of the outer. Find the voltage across each load point.

Solution. The current loading is shown in Fig. 40.36.

The current flowing into the positive side is 30 + 20 = 50 A. Since current flowing out of the negative side is 36 + 24 = 60 A, it means that 10 A must be flowing in the section *NC* of the neutral. To make up 24 A in load *CD*. 10 A are contributed by NC and 14 A by *BC*. The balance of 6 A flows through *BF* and adds up with 30 A of *KF* to make up 36A through load *FE*.

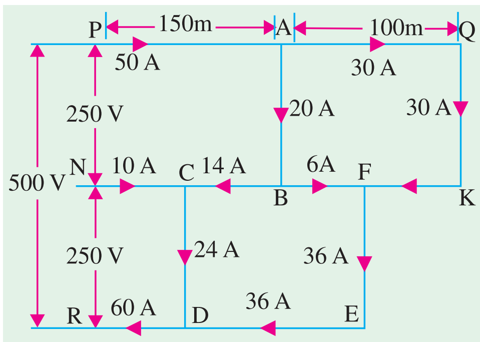


Fig. 40.36

It is given that the resistance of outers is 0.02 Ω per 100 m. Since neutral is of half the cross-section, its resistance is 0.04 Ω per 100 m. Knowing them, voltage at load points can be determined as under. Let us see how we will calculate voltage across load *A B*.

Voltage at AB = 250 – drop in PA – drop in BC + drop in CN.

It should be particularly noted that drop in CN has been added instead of being ***subtracted.*** The reason is this : as we start from P and go round $PABCN$, we go along the current over section PA *i.e.* we go 'downstream' hence drop is taken negative, but along CN we go 'upstream', hence the drop as taken as positive*. Proceeding in this way, we tabulate the currents, resistances and voltage drops in various sections as given below :

Section	*Resistance (W)*	*Current (A)*	*Drop (V)*
PA	0.03	50	1.5
AQ	0.02	30	0.6
KF	0.012	30	0.36
BC	0.02	14	0.28
BF	0.028	6	0.168
NC	0.04	10	0.4
ED	0.024	36	0.864
DR	0.02	60	1.2

P.D. across AB = 250 – 1.5 – 0.28 + 0.4 = **248.62 V**

P.D. across QK = 248.62 – 0.6 – 0.36 + 0.168 = **247.83 V**

P.D. across CD = 250 – 0.4 – 1.2 = **248.4 V**

P.D. across FE = 248.4 + 0.28 – 0.168 – 0.864 = **247.65 V**

Tutorial Problem No. 40.3

1. A 3-wire system supplies three loads (*a*) 10 (*b*) 20 and (*c*) 30 amperes situated at distances of 100, 150 and 300 metres respectively from the supply point on the positive side of the neutral wire. Connected between negative and neutral wires are two loads (*d*) 30 A and (*e*) 20A, situated 120 and 200 metres respectively from the supply point. Give a diagram showing the values and directions of the currents in various parts of the neutral wire.

If the resistance of the outers is 0.05 Ω per 1000 metres and that of the neutral 0.1 Ω per 1000 metres, calculate the potential difference at the load points (*b*) and (*e*); the pressure at the supply point being 100 V between outers and neutral. **[10A and 5A; Volts at (*b*) = 99.385 V ; at (*e*) = 99.86 V]**

2. A 3-wire d.c. distributor 400 metres long is fed at both ends at 235 volts between each outer and neutral. Two loads P and Q are connected between the positive and neutral and two loads R and S are connected between the negative outer and the neutral. The loads and their distances from one end (X) of the distributor are as follows : Load P, 50 A, 100 metres from X ; Load Q, 70 A, 300 metres from X; Load R, 60 A, 150 metres from X; Load S, 60 A,350 metres from X. Determine the p.d. at each load point and the current at each feeding point. The resistance of each outer is 0.25 Ω per 1000 metres and that of the neutral is 0.5 W per 1000 metres.

[Current into the +ve outer at X = 55 A ; other end Y = 65 A; Out from –ve outer at X = 45 A; other end Y = 75 A ; P.D. s at P = 233.125 V; Q = 232.375 V; R = 232.812 V; S = 233.812 V]

40.13. Three-wire System

As already mentioned briefly in Art. 40.2, it consists of the 'outer' conductors (between which the voltage is twice the normal value for lighting) and the third wire which is called the middle or neutral wire. It is of half the cross-section as compared to any one of the two outers and is earthed at the generator end. The voltage of the neutral is thus approximately half way between that of the outers.

* Hence, sign convention in this : While going 'upstream' take the drop as positive and while going 'down-stream' take the drop as negative. (Art. 2-3).

If the total voltage between outers is 460 V, then the positive outer is 230 V ***above*** the neutral and the negative outer 230 V ***below*** the neutral. Motor loads requiring higher voltage are connected across the outers directly whereas lighting and heating loads requiring lesser voltage of 230 V are connected between any one of the outers and the neutral. If the loads on both sides of the neutral are equal *i.e.* balanced as shown in Fig. 40.37, then there is no current in the middle wire and the effect is as if the different loads were connected in series across the outers. However, in practice, although effort is made to distribute the various loads equally on the two sides of the neutral, yet it is difficult to achieve exact balance, with the result that some out-of-balance current does flow in the neutral as shown in Fig. 40.38. In the present case as viewed from generator end, the load is equivalent to a resistance of 25/3 ohms in series with a resistance of 25/2 ohms (Fig. 40.39). Obviously, the voltages across the two will becomes unequal. The voltage across the positive side will fall to $\dfrac{500 \times 25/2}{(25/2 + 25/3)}$ = 200 V and on the negative side, it will rise to $\dfrac{500 \times 25/3}{(25/2 + 25/3)}$ = 300 V. This difficulty of unequal voltages with unbalanced loads is removed by using balancers as discussed below in Art. 40.14.

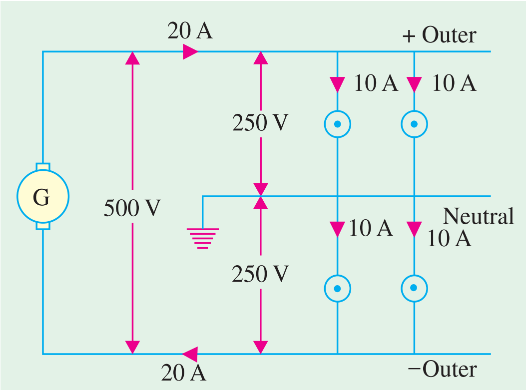


Fig. 40.37

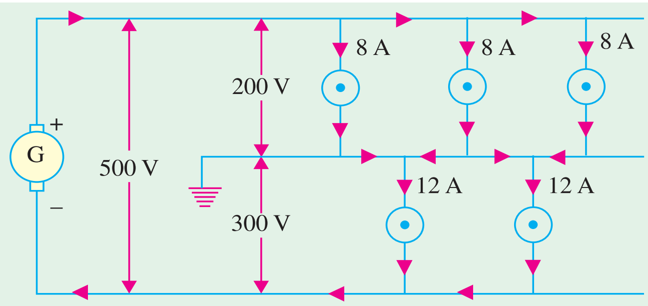


Fig. 40.38

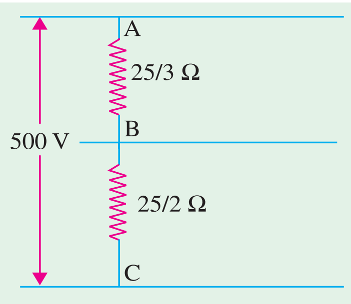


Fig. 40.39

40.14. Balancers

In order to maintain p.d.s on the two sides of neutral equal to each other, a balancer set is used. The commonest form of balancer consists of two identical shunt-wound machines which are not only coupled mechanically but have their armatures and field circuits joined in series across the outers. The neutral is connected to the junction of the armatures as shown. When the system is unloaded or when the loads on the two sides are balanced, then

1. both machines run as unloaded motors and
2. since their speeds and field currents are equal, their back e.m.fs. are the same.

When the two sides are unbalanced *i.e.* when the load supplied by + ve outer is different from that

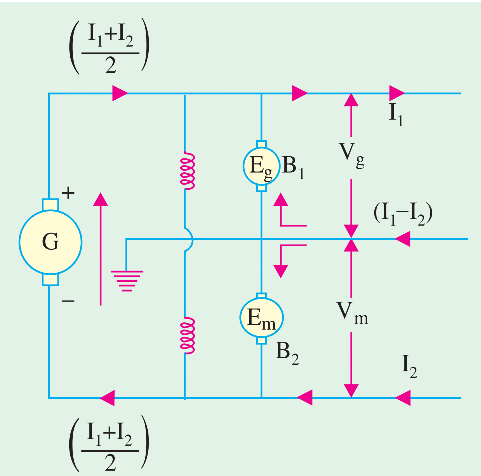


Fig. 40.40

supplied by negative outer, then out-of-balance current $(I_1 - I_2)$ flows through the mid-wire to the balancers where it divides into two halves each equal to $(I_1 - I_2)/2$ as shown in Fig. 40.40.

If I_1 is greater than I_2, then +ve side is more heavily loaded (than –ve side) hence the p.d. on this side tends to fall below the e.m.f. of the balancer on this side so that B_1 runs as a generator. However, the p.d. on the lightly-loaded side rises above the e.m.f. of the balancers on that side, hence B_2 runs as a motor. In this way, energy is transferred through balancers from the lightly-loaded side to the heavily-loaded side. In Fig. 40.40, machine B_2 is running as a motor and driving machine B_1 as a generator.

Let R_a = armature resistance of each machine

V_g = terminal p.d. of machine running as a generator *i.e.* B_1

E_g = induced e.m.f. of B_1

V_m = terminal p.d. of motoring machine *i.e.* B_2

E_m = induced e.m.f. of B_2

then $$V_g = E_g - \frac{(I_1 - I_2)}{2} R_a \quad \text{and} \quad V_m = E_m + \frac{(I_1 - I_2)}{2} R_a$$

$$\therefore \quad V_m - V_g = (E_m - E_g) + (I_1 - I_2) R_a \qquad \text{...(i)}$$

Since the speed and excitation of the two machines are equal,

$$\therefore \quad E_g = E_m \qquad \therefore \quad V_m - V_g = (I_1 - I_2) R_2 \qquad \text{...(ii)}$$

Hence, we find that the difference of voltages between the two sides of the system is proportional to —

(i) out of balance current $(I_1 - I_2)$ and *(ii)* armature resistance of the balancer.

For this reason, R_a is kept very small and effort is made to arrange the loads on the two sides such that out-of-balance current is as small as possible.

The value $(V_m - V_g)$ can be still further reduced *i.e.* the voltages on the two sides can be more closely balanced by cross-connecting the balancer fields as shown in Fig. 40.41. In this way, generator draws its excitation from the lightly-loaded side which is at a higher voltage, hence E_g is increased. The motoring machine draws its excitation from the heavily loaded side which is at a little lower voltage, hence E_m is decreased. In this way, the difference $(E_m - E_g)$ is decreased and so is $(V_m - V_g)$. Further, regulation of the voltage can be accomplished by connecting an adjustable regulator in series with the two balancer fields as shown in Fig. 40.42.

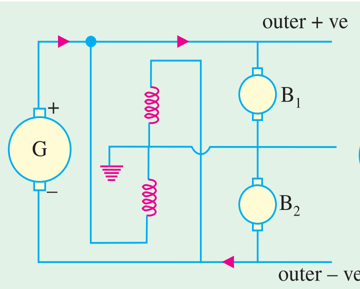


Fig. 40.41

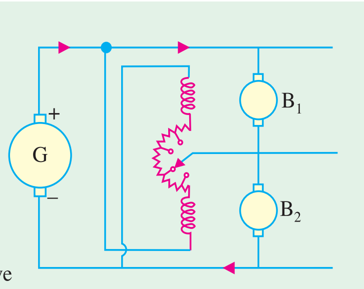


Fig. 40.42

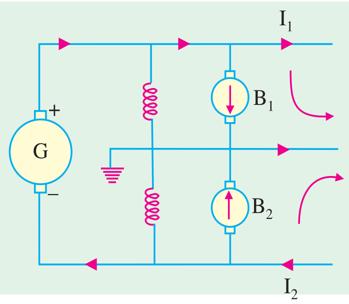


Fig. 40.43

It should be noted that since machine B_1 is running as a generator and B_2 as a motor, the directions of currents in B_1, B_2 and the neutral are as shown in Fig. 40.40. If, however, B_2 runs as a generator and B_1 as a motor *i.e.* if negative side is more heavily loaded than the +ve side, then directions of currents through B_1, B_2 and the neutral are as shown in Fig. 40.43. In particular, the change in the direction of the current through midwire should be noted.

Example 40.22. *A d.c. 3-wire system with 500-V between outers has lighting load of 100 kW on the positive and 50 kW on the negative side. If, at this loading, the balancer machines have each a loss of 2.5 kW, calculate the kW loading of each balancer machine and the total load on the system.*

Solution. The connections are shown in Fig. 40.44.

Total load on main generator $= 100 + 50 + (2 \times 2.5)$
$= \mathbf{155\ kW}$

Output current of main generator $= 155 \times 1000/500$
$= 310$ A

Load current on +ve side, $I_1 = 100 \times 1000/250$
$= 400$ A

Load current on – ve side, $I_2 = 50 \times 1000/250$
$= 200$A

Out-of-balance current $= 400 - 200 = 200$ A

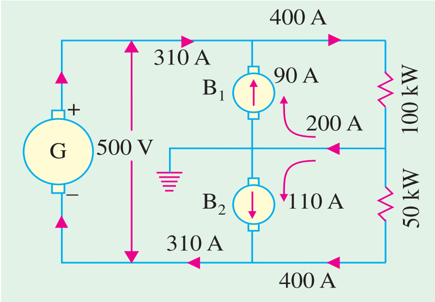


Fig. 40.44

Since +ve side is more heavily loaded , B_1 is working as a generator and B_2 as motor.

∴ current of $B_1 = 400 - 310 = 90$ A

current of $B_2 = 200 - 90 = 110$ A

Loading of $B_1 = 250 \times 90/1000 = \mathbf{22.5\ kW}$

Loading of $B_2 = 250 \times 110/1000 = \mathbf{27.5\ kW}$

Example 40.23. *In a 500/250-V d.c. 3-wire system, there is a current of 2000 A on the +ve side, 1600 A on the negative side and a load of 300 kW across the outers. The loss in each balancer set is 8 kW. Calculate the current in each armature of the balancer set and total load on the main generator.*

Solution. Connections are shown in Fig. 40.45.

It should be noted that loading across 'outers' directly in no way determines the current in the neutral.

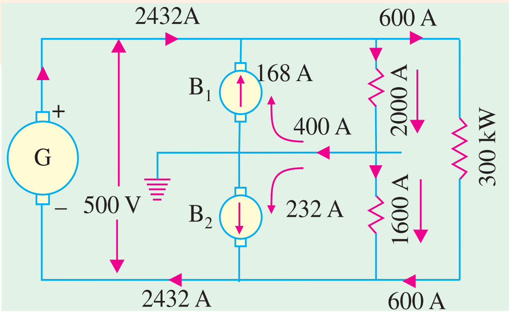


Fig. 40.45

+ve loading $= 2000 \times 250/1000$
$= 500$ kW

– ve loading $= 1600 \times 250/1000$
$= 400$ kW

Total loading on main generator is

$= 500 + 400 + 300 + (2 \times 8) = \mathbf{1216\ kW}$

∴ Current of main generator $= 1216 \times 1000/500 = 2{,}432$ A

Out-of-balance current $= 2000 - 1600 = 400$ A

Current through $B_1 = 2{,}600 - 2{,}432 = \mathbf{168\ A}$

Current through $B_2 = 400 - 168 = \mathbf{232\ A}$

Example 40.24. *On a 3-wire d.c. distribution system with 500 V between outers, there is a load of 1500 kW on the positive side and 2,000 kW on the negative side. Calculate the current in the neutral and in each of the balancer armatures and the total current supplied by the generator. Neglect losses.*

(Electrical Engineering ; Madras Univ.)

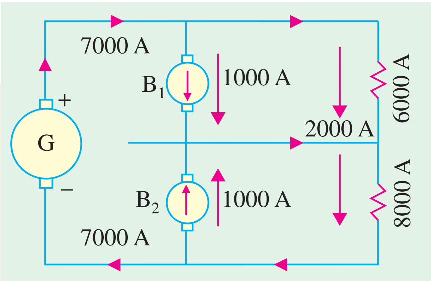


Fig. 40.46

Solution. Since negative side is more heavily loaded than the positive side, machine B_2 runs as a generator and B_1 as a motor. The directions of current through B_1 and B_2 are as shown in Fig. 40.46. Total loading on the main generator

$= 2{,}000 + 1{,}500 = 3{,}500$ kW

Current supplied by main generator $= 3{,}500 \times 1000/500 = \mathbf{7{,}000\ A}$

Current on +ve side $= 1500 \times 1000/250 = \mathbf{6{,}000\ A}$

Current on –ve side $= 2000 \times 1000/250 = \mathbf{8{,}000\ A}$

Out-of-balance current $= 8{,}000 - 6000 = \mathbf{2{,}000\ A}$

Current through the armature of each machine $= \mathbf{1000\ A}$

Example 40.25. *A 125/250 V, 3-wire distributor has an out-of-balance current of 50 A and larger load of 500 A. The balancer set has a loss of 375 W in each machine. Calculate the current in each of the balancer machines and output of main generator.*

(Electrical Technology-II, Gwalior Univ.)

Solution. As shown in Fig. 40.47, let larger load current be I_1 = 500 A. Since $(I_1 - I_2)$ = 50

$\therefore \quad I_2$ = 450 A

Larger load = 500 × 125/1000 = 62.5 kW

Smaller load = 450 × 125/1000 = 56.25 kW

Balancer loss = 2 × 375 = 0.75 kW

Output of main generator

= 62.5 + 56.25 + 0.75 = **119.5 kW**

Current of main generator

= 119.5 × 1000/250 = 478 A

As seen from Fig. 40.47, current of

B_1 = (500 – 478) = **22 A**

and current of B_2 = (50 – 22) = **28 A**

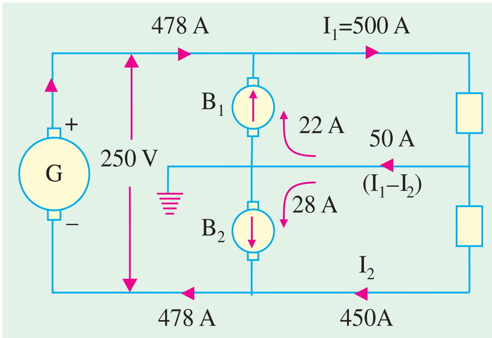


Fig. 40.47

Example 40.26. *The load on a d.c. 3-wire system with 500 V between outers consists of lighting current of 1500 A on the positive side and 1300 A on the negative side while motors connected across the outers absorb 500 kW. Assuming that at this loading, the balancer machines have each a loss of 5 kW, calculate the load on the main generator and on each of the balancer machines.*

(Electrical Engineering ; Madras Univ.)

Solution. Connections are shown in Fig. 40.48.

Positive loading = 1500 × 250/1000 = 375 kW

Negative loading = 1300 × 250/1000 = 325 kW

Total load on the main generator is

= 375 + 325 + 500 + (2 × 5) = **1210 kW**

Current supplied by the main generator is

= 1210 × 1000/500 = 2,420 A

Out-of-balance current = 1500 – 1300 = 200 A

Current through B_1 = 2500 – 2420 = 80 A

Current through B_2 = 200 – 80 = 120 A

Loading of B_1 = 80 × 250/1000 = **20 kW**

Loading of B_2 = 120 × 250/1000= **30 kW**

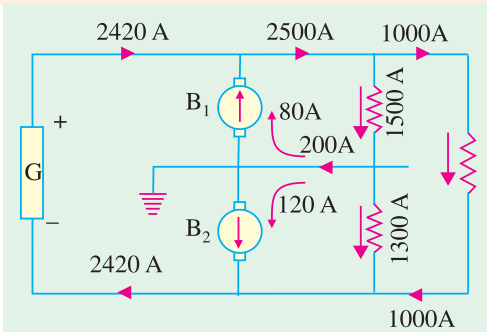


Fig. 40.48

Example 40.27. *A d.c. 3-wire system with 480 V across outers supplies 1200 A on the positive and 1000 A on the negative side. The balancer machines have each an armature resistance of 0.1W and take 10 A on no-load. Find*

(a) *the voltage across each balancer and*

(b) *the total load on the main generator and the current loading of each balancer machine.*

The balancer field windings are in series across the outers.

Solution. As shown in Fig. 40.49, B_1 is generating and B_2 is motoring.

The out-of-balance current is (1200 – 1000) = 200 A. Let current through the motoring machine be I_m, then that through the generating machine is $(200 - I_m)$. Let V_g and V_m be p.ds. of the two machines.

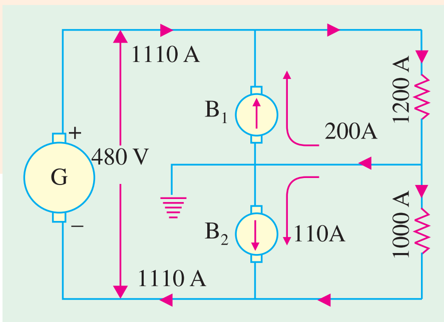


Fig. 40.49

Since B_2 is driving B_1, output of the motor supplies the losses in the set plus the output of the generator. Total losses in the set

$$= \text{no-load losses} + \text{Cu losses in two machines}$$
$$= 480 \times 10 + 0.1\, I_m^{\;2} + (200 - I_m)^2 \times 0.1$$
$$V_m I_m = V_g\,(200 - I_m) + \text{total losses}$$
$$= V_g\,(200 - I_m) + 4800 + 0.1\, I_m^2 + (200 - I_m)^2 \times 0.1$$

Now $\quad V_m = E_b + I_m R_a \quad$ and $\quad V_g = E_b - I_g R_a$

Now, back e.m.f. $E_b = (240 - 0.1 \times 10) = 239$ V

$\therefore \quad V_m = (239 + 0.1\, I_m)$ and $\quad V_g = 239 - (200 - I_m) \times 0.1$

$$\therefore \quad (239 + 0.1\, I_m)\, I_m = [239 - (200 - I_m) \times 0.1]\,(200 - I_m)^2 + 4800 + 0.1\, I_m^{\;2} + (200 - I_m)^2 \times 0.1$$

$\therefore \quad I_m = 110$ A $\quad$ and $\quad I_g = 200 - 110 = 90$ A

(a) $\therefore \quad V_m = 239 + 0.1 \times 110 =$ **250 V** $\quad$ and $\quad V_g = 239 - (0.1 \times 90) =$ **230 V**

(b) Load on main generator = 1200 − 90 = **1110 A**

Example 40.28. *A d.c. 3-wire system with 460 V between outers supplies 250 kW on the positive and 400 kW on the negative side, the voltages being balanced. Calculate the voltage on the positive and negative sides respectively, if the neutral wire becomes disconnected from the balancer set.*

(Electrical Power-III, Bangalore Univ.)

Solution. Before the disconnection of the mid-wire, voltages on both sides of the system are equal *i.e.* 230 V. The loads are 250 kW on the +ve side and 400 kW on the −ve side. If R_1 and R_2 are the resistances of the two loads, then

$$230^2/R_1 = 250{,}000\;; \quad R_1 = 0.2116\ \Omega$$

Similarly, $\quad R_2 = 2302/400{,}000 = 0.1322\ \Omega$

When the mid-wire is disconnected from the balancer set *i.e.* from the generator side, then the two resistances R_1 and R_2 are put in series across 460 V as shown in Fig. 40.50 (*b*).

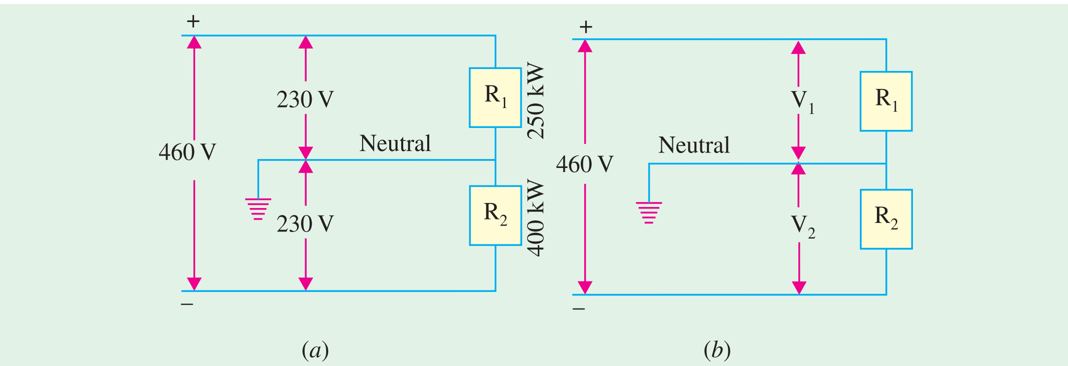


Fig. 40.50

$$\therefore \quad V_1 = \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.02116 \times 460}{(0.2116 + 0.1322)} = \mathbf{283\ V}$$

$$V_2 = \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.1322 \times 460}{0.3438} = \mathbf{177\ V}\ (\text{or } V_2 = 460 - V_1)$$

Tutorial Problem No. 40.4

1. In a 500/250-V d.c. 3-wire system there is an out-of-balance load of 125 kW on the positive side. The loss in each balancer machine is 7.5 kW and the current in the negative main is 1500 A. Calculate the total load on the generator and the current in each armature of the balancer set.

[890 kW ; 220 A ; 280 A]

2. A 460-V d.c. 2-wire supply is converted into 3-wire supply with the help of rotary balancer set, each machine having a no-load loss of 2.3 kW. If the load on the positive side of 69 kW and on the negative side 57.5 kW, calculate the currents flowing in each of the balancer machines. **[15 A ; 35 A]**
3. In a 500/250 volt 3-wire d.c. system there is an out-of-balance load of 200 kW on the positive side. The loss in each balancer is 10 kW and the current in the negative main is 2800 A. Calculate the current in each armature of the balancer set and the total load on the generators. ***(I.E.E. London)***

[Motoring machine = 440 A; Generating machine = 360 A ; Total load = 1620 kW]

40.15. Boosters

A booster is a generator whose function is to add to or inject into a circuit, a certain voltage that is sufficient to compensate for the I_R drop in the feeders etc.

In a d.c. system, it may sometimes happen that a certain feeder is much longer as compared to others and the power supplied by it is also larger. In that case, the voltage drop in this particular feeder will exceed the allowable drop of 6% from the declared voltage. This can be remedied in two ways **(*i*)** by increasing the cross-section of the feeder, so that its resistance and hence I_R drop is decreased **(*ii*)** or by increasing the voltage of the station bus-bars.

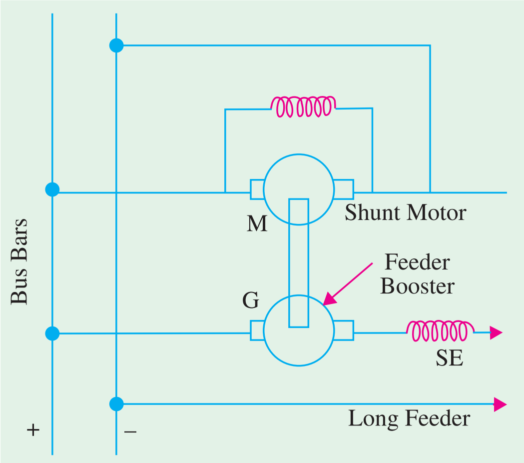


Fig. 40.51

The second method is not practicable because it will disturb the voltage of other feeders, whereas the first method will involve a large initial investment towards the cost of increased conductor material.

To avoid all these difficulties, the usual practice is to install a booster in series with this longer feeder as shown in Fig. 40.51. Since it is used for compensating drop in a feeder, it is known as feeder booster. It is a (series) generator connected in series with the feeder and driven at a constant speed by a shunt-motor working from the bus-bars. The drop in a feeder is proportional to the load current, hence the voltage injected into the feeder by the booster must also be proportional to the load current, if exact compensation is required. In other words, the booster must work on the straight or linear portion of its voltage characteristic.

Example 40.29. *A 2-wire system has the voltage at the supply end maintained at 500. The line is 3 km long. If the full-load current is 120 A, what must be the booster voltage and output in order that the far end voltage may also be 500 V.*

Take the resistance of the cable at the working temperature as 0.5 ohm/kilometre.

(Elect. Machinery-I, Calcutta Univ.)

Solution. Total resistance of the line is = $0.5 \times 3 = 1.5\ \Omega$

Full-load drop in the line is = $1.5 \times 120 = 180$ V

Hence, the terminal potential difference of the booster is **180 V** (*i.e.* 180/120 = 1.5 volt per ampere of line current).

$$\text{Booster-output} = 120 \times 180/1000 = \mathbf{21.6\ kW}$$

40.16. Comparison of 2-wire and 3-wire Distribution Systems

We will now compare the 2-wire and 3-wire systems from the point of view of economy of conductor material. For this comparison, it will be assumed that—

1. the amount of power transmitted is the same in both cases.
2. the distance of transmission is the same.

3. the efficiency of transmission (and hence losses) is the same.
4. voltage at consumer's terminals is the same.
5. the 3-wire system is balanced and
6. in the 3-wire system, the mid-wire is of half the cross-section of each outer.

Let W be the transmitted power in watts and V the voltage at the consumer's terminals. Also, let

R_2 = resistance in ohms of each wire of 2-wire system.
R_3 = resistance in ohms of each outer in 3-wire system.

The current in 2-wire system is W/V and the losses are $2\,(W/V)^2\,R_2$.

In the case of 3-wire system, voltage between outers is 2 V, so that current through outers is $(W/2V)$, because there is no current in the neutral according to our assumption (5) above. Total losses in the two outers are $2(W/2V)^2\,R_3$.

Since efficiencies are the same, it means the losses are also the same.

$$\therefore \qquad 2(W/V)^2 R_2 = 2(W/2V)^2 \times R_3 \quad \text{or} \quad \frac{R_3}{R_2} = \frac{4}{1}$$

Since the cross-section and hence the volume of a conductor of given length, is inversely proportional to its resistance,

$$\therefore \qquad \frac{\text{volume of each 3-wire conductor}}{\text{volume of each 2-wire conductor}} = \frac{1}{4}$$

Let us represent the volume of copper in the 2-wire system by 100 so that volume of each conductor is 50.

Then, volume of each outer in 3-wire system = 50/4 = 12.5

volume of neutral wire ,, ,, = 12.5/2 = 6.25

∴ total volume of copper in 3-wire system = 12.5 + 6.25 + 12.5 = 31.25

$$\therefore \qquad \frac{\text{total coper vol. in 3-wire feeder}}{\text{total copper vol. in 2-wire feeder}} = \frac{31.25}{100} = \frac{5}{6}$$

Hence, a 3-wire system requires only 5/16th (or 31.25%) as much copper as a 2-wire system.

OBJECTIVE TESTS – 40

1. If in a d.c. 2-wire feeder, drop per feeder conductor is 2%, transmission efficiency of the feeder is .... percent.
   (*a*) 98 (*b*) 94
   (*c*) 96 (*d*) 99
2. Transmitted power remaining the same, if supply voltage of a dc 2-wire feeder is increased by 100 percent, saving in copper is .... percent.
   (*a*) 50 (*b*) 25
   (*c*) 100 (*d*) 75
3. A uniformly-loaded d.c. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at middle point is
   (*a*) one-half (*b*) one-fourth
   (*c*) one-third (*d*) twice.
4. In a d.c. 3-wire distributor using balancers and having unequal loads on the two sides
   (*a*) both balancers run as motors
   (*b*) both balancers run as generators
   (*c*) balancer connected to heavily-loaded side runs as a motor
   (*d*) balancer connected to lightly-loaded side runs as a motor.
5. As compared to a dc 2-wire distributor, a 3-wire distributor with same maximum voltage to earth uses only...................... percent of copper.
   (*a*) 66.7 (*b*) 33.3
   (*c*) 31.25 (*d*) 150
6. In a d.c. 3-wire distribution system, balancer fields are cross-connected in order to
   (*a*) make both machines run as unloaded motors.
   (*b*) equalize voltages on the positive and negative outers
   (*c*) boost the generated voltage
   (*d*) balance loads on both sides of the neutral.

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*d*) **5.** (*c*) **6.** (*a*)

GO To FIRST