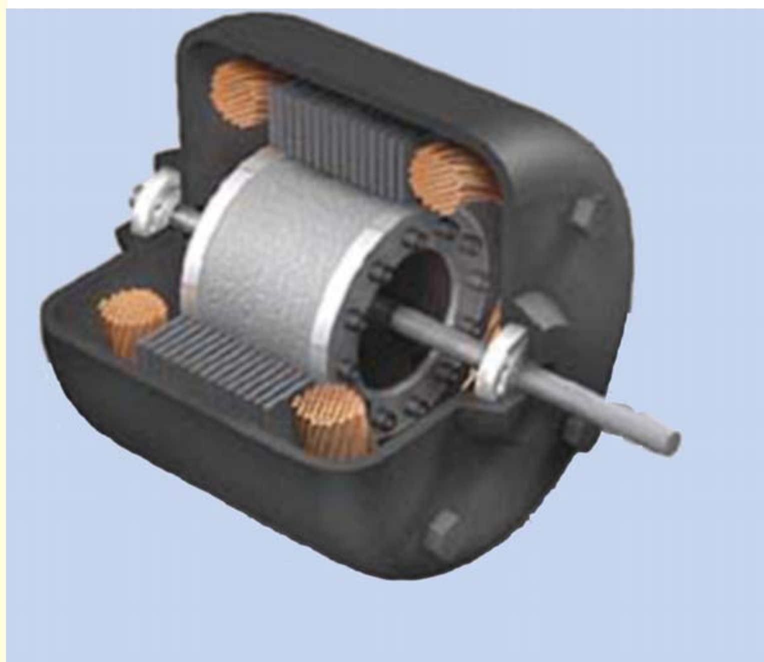
CHAPTER 11

A.C. FUNDAMENTALS

Learning Objectives

- Generation of Alternating Voltages and Currents
- Alternate Method for the Equations of Alternating Voltages and currents
- Simple Waveforms
- Cycle
- Different Forms of E.M.F. Equation
- Phase
- Phase Difference
- Root Mean Square (R.M.S.) Value
- Mid-ordinate Method
- Analytical Method
- R.M.S. Value of a Complex Wave
- Average Value
- Form Factor
- Crest or Peak Factor
- R.M.S. Value of H.W. Rectified A.C.
- Average Value
- Form Factor of H.W. Rectified
- Vector Diagrams Using R.M.S. Values
- Vector Diagrams of Sine Waves of Same Frequency
- Addition and Subtraction of Vectors
- A.C. Through Resistance, Inductance and Capacitance
- A.C. through Pure Ohmic Resistance alone
- A.C. through Pure Inductance alone
- Complex Voltage Applied to Pure Inductance
- A.C. through Capacitance alone



Alternating current circuits improves the versatility and usefulness of electrical power system. Alternating current plays a vital role in today's energy generation

11.1. Generation of Alternating Voltages and Currents

Alternating voltage may be generated by rotating a coil in a magnetic field, as shown in Fig. 11.1 (*a*) or by rotating a magnetic field within a stationary coil, as shown in Fig. 11.1 (*b*).

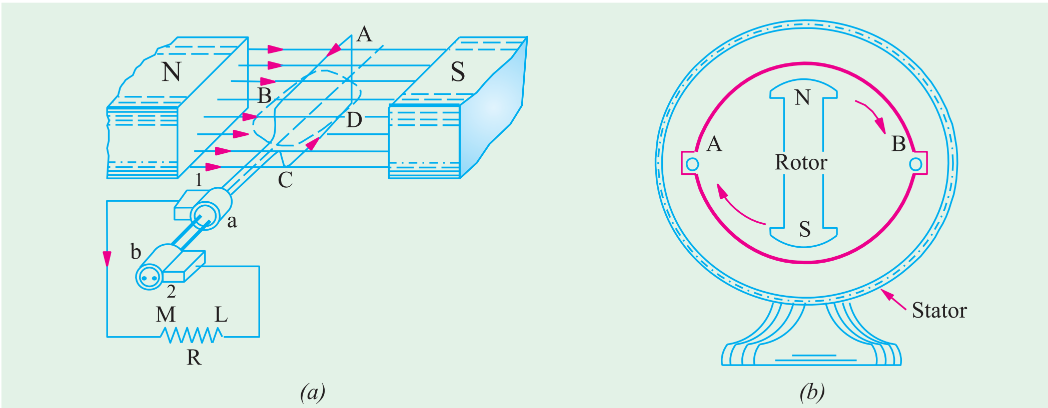


Fig. 11.1

The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. Alternating voltage may be generated in either of the two ways shown above, but rotating-field method is the one which is mostly used in practice.

11.2. Equations of the Alternating Voltages and Currents

Consider a rectangular coil, having N turns and rotating in a uniform magnetic field, with an angular velocity of ω radian/second, as shown in Fig. 11.2. Let time be measured from the X-axis. Maximum flux Φ_m is linked with the coil, when its plane coincides with the X-axis. In time t seconds, this coil rotates through an angle $\theta = \omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is $\Phi = \Phi_m \cos \omega t$. Hence, ***flux linkages*** of the coil at any time are $N\Phi = N\Phi_m \cos \omega t$.

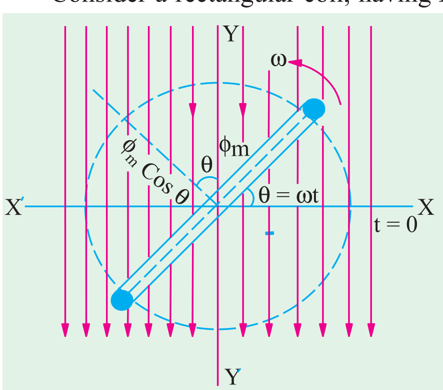


Fig. 11.2

According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil. Hence, the value of the induced e.m.f. at this instant (*i.e.* when $\theta = \omega t$) or the instantaneous value of the induced e.m.f. is

$$e = -\frac{d}{dt}(N\Phi) \text{ volt} = -N.\frac{d}{dt}(\Phi_m \cos \omega t) \text{ volt} = -N\Phi_m\,\omega(-\sin \omega t) \text{ volt}$$

$$= \omega N \Phi_m \sin \omega t \text{ volt} = \omega N \Phi_m \sin \theta \text{ volt} \qquad ...(i)$$

When the coil has turned through 90º *i.e.* when $\theta = 90°$, then $\sin \theta = 1$, hence e has maximum value, say E_m. Therefore, from Eq. (*i*) we get

$$E_m = \omega N \Phi_m = \omega N B_m A = 2\pi f N B_m A \text{ volt} \qquad ...(ii)$$

where B_m = maximum flux density in Wb/m^2 ; A = area of the coil in m^2

f = frequency of rotation of the coil in rev/second

Substituting this value of E_m in Eq. (*i*), we get $e = E_m \sin \theta = E_m \sin \omega t$ ...(*iii*)

Similarly, the equation of induced alternating current is $i = I_m \sin \omega t$ ...*(iv)*
provided the coil circuit has been closed through a resistive load.

Since $\omega = 2\pi f$, where f is the frequency of rotation of the coil, the above equations of the voltage and current can be written as

$$e = E_m \sin 2\pi f t = E_m \sin\left(\frac{2\pi}{T}\right)t \quad \text{and} \quad i = I_m \sin 2\pi ft = I_m \sin\left(\frac{2\pi}{T}\right)t$$

where T = time-period of the alternating voltage or current = $1/f$

It is seen that the induced e.m.f. varies as sine function of the time angle ωt and when e.m.f. is plotted against time, a curve similar to the one shown in Fig. 11.3 is obtained. This curve is known as sine curve and the e.m.f. which varies in this manner is known as ***sinusoidal*** e.m.f. Such a sine curve can be conveniently drawn, as shown in Fig. 11.4. A vector, equal in length to E_m is drawn. It rotates in the counter-clockwise direction with a velocity of ωradian/second, making one revolution while the generated e.m.f. makes two loops or one cycle. The projection of this vector on Y-axis gives the instantaneous value e of the induced e.m.f. *i.e.* $E_m \sin \omega t$.

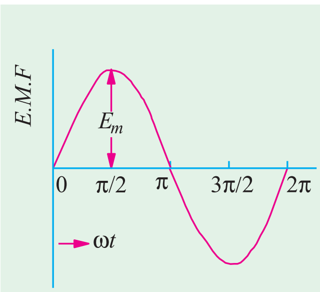


Fig. 11.3

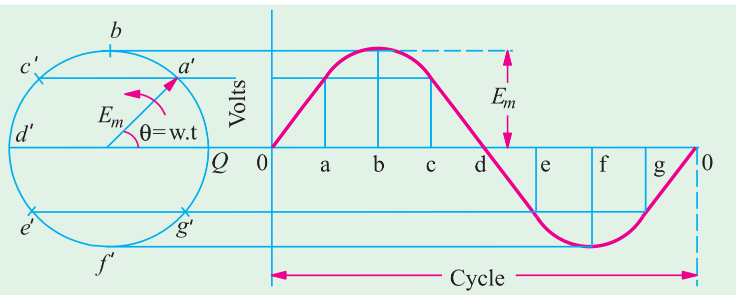


Fig. 11.4

To construct the curve, lay off along X-axis equal angular distance *oa*, *ab*, *bc*, *cd* etc. corresponding to suitable angular displacement of the rotating vector. Now, erect coordinates at the points *a*, *b*, *c* and *d* etc. (Fig. 11.4) and then project the free ends of the vector E_m at the corresponding positions a' , b' , c' , etc to meet these ordinates. Next draw a curve passing through these intersecting points. The curve so obtained is the graphic representation of equation ***(iii)*** above.

11.3. Alternate Method for the Equations of Alternating Voltages and Currents

In Fig. 11.5 is shown a rectangular coil *AC* having *N* turns and rotating in a magnetic field of flux density *B* Wb./m^2. Let the length of each of its sides *A* and *C* be *l* meters and their peripheral velocity *v* metre/second. Let angle be measured from the horizontal position *i.e.* from the *X*-axis. When in horizontal position, the two sides *A* and *C* move parallel to the lines of the magnetic flux. Hence, no flux is cut and so no e.m.f. is generated in the coil.

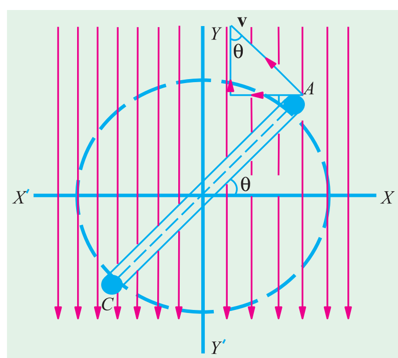


Fig. 11.5

When the coil has turned through angle θ, its velocity can be resolved into two mutually perpendicular components ***(i)*** $v \cos \theta$ component-parallel to the direction of the magnetic flux and ***(ii)*** $v \sin \theta$ component-perpendicular to the direction of the magnetic flux. The e.m.f. is generated due entirely to the perpendicular component *i.e.* $v \sin \theta$.

Hence, the e.m.f. generated in one side of the coil which contains *N* conductors, as seen from Art. 7.7, is given by, $e = N \times Bl\, v \sin \theta$.

Total e.m.f. generated in both sides of the coil is

$$e = 2BNl\,v \sin\theta \text{ volt} \quad \text{...}(i)$$

Now, e has maximum value of E_m (say) when $\theta = 90°$. Hence, from Eq. (*i*) above, we get,
$E_m = 2\,B\,N\,l\,v$ volt. Therefore Eq. (*i*) can be rewritten as $e = E_m \sin\theta$ ...as before
If b = width of the coil in meters ; f = frequency of rotation of coil in Hz, then $v = \pi\,bf$

$$\therefore \quad E_m = 2\,B\,N\,l \times \pi\,b\,f = 2\,\pi\,f\,N\,B\,A \text{ volts} \quad \text{...as before}$$

Example 11.1. *A square coil of 10 cm side and 100 turns is rotated at a uniform speed of 1000 revolutions per minute, about an axis at right angles to a uniform magnetic field of 0.5 Wb/m². Calculate the instantaneous value of the induced electromotive force, when the plane of the coil is **(i)** at right angles to the field **(ii)** in the plane of the field.*

(Electromagnetic Theory, A.M.I.E. Sec B, 1992)


square coils

Solution. Let the magnetic field lie in the vertical plane and the coil in the horizontal plane. Also, let the angle θ be measured from *X*-axis.

Maximum value of the induced e.m.f., $E_m = 2\,\pi\,f\,N\,B_m\,A$ volt.
Instantaneous value of the induced e.m.f. $e = E_m \sin\theta$
Now $f = 100/60 = (50/3)$ rps, $N = 100$, $B_m = 0.5$ Wb/m², $A = 10^{-2}$ m²

(*i*) In this case, $\theta = 0°$

$\therefore \quad e = 0$

(*ii*) Here $\theta = 90°$, $\therefore\; e = E_m \sin 90° = E_m$

Substituting the given values, we get

$$e = 2\pi \times (50/3) \times 100 \times 0.5 \times 10^{-2} = \mathbf{52.3\ V}$$

11.4. Simple Waveforms

The shape of the curve obtained by plotting the instantaneous values of voltage or current as the ordinate against time as a abscissa is called its waveform or wave-shape.

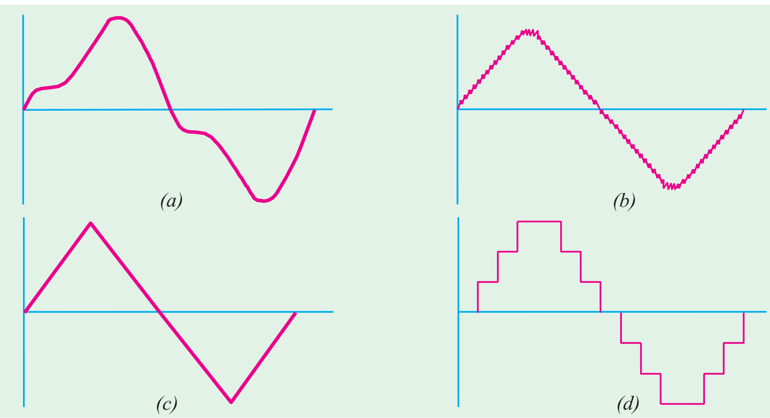


Fig. 11.6

An alternating voltage or current may not always take the form of a systematical or smooth wave such as that shown in Fig. 11.3. Thus, Fig. 11.6 also represents alternating waves. But while it is scarcely possible for the manufacturers to produce sine-wave generators or alternators, yet sine wave is the ideal form sought by the designers and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves.

In general, however, ***an alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.***

11.5. Complex Waveforms

Complex waves are those which depart from the ideal sinusoidal form of Fig. 11.4. All alternating complex waves, which are periodic and have equal positive and negative half cycles can be shown to be made up of a number of pure sine waves, having different frequencies but all these frequencies are integral multiples of that of the lowest alternating wave, called the ***fundamental*** (or first harmonic). These waves of higher frequencies are called ***harmonics.*** If the fundamental frequency is 50 Hz, then the frequency of the second ***harmonic*** is 100 Hz and of the third is 150 Hz and so on. The complex wave may be composed of the fundamental wave (or first harmonic) and any number of other harmonics.

In Fig. 11.7 is shown a complex wave which is made up of a fundamental sine wave of frequency of 50 Hz and third harmonic of frequency of 150 Hz. It is seen that

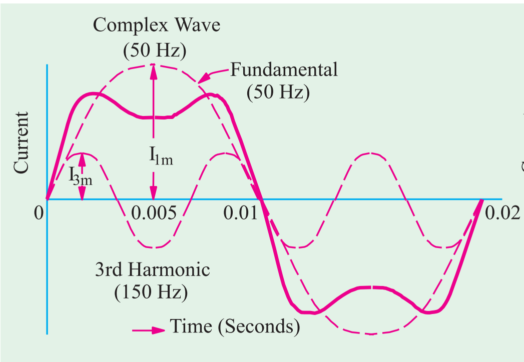


Fig. 11.7

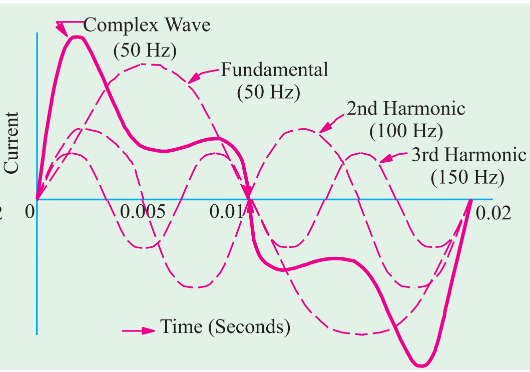


Fig. 11.8

(i) the two halves of the complex wave are identical in shape. In other words, there is no distortion. This is always the case when only ***odd*** harmonic (3rd, 5th, 7th, 9th etc.) are present.

(ii) frequency of the complex wave is 50 Hz *i.e.* the same as that of the fundamental sine wave.

In Fig. 11.8 is shown a complex wave which is a combination of fundamental sine wave of frequency 50 Hz and 2nd harmonic of frequency 100 Hz and 3rd harmonic of frequency 150 Hz. It is seen that although the frequency of the complex wave even now remains 50 Hz, yet :

(i) the two halves of the complex wave are not identical. It is always so when *even* harmonics (2nd, 4th, 6th etc.) are present.

(ii) there is distortion and greater departure of the wave shape from the purely sinusoidal shape.

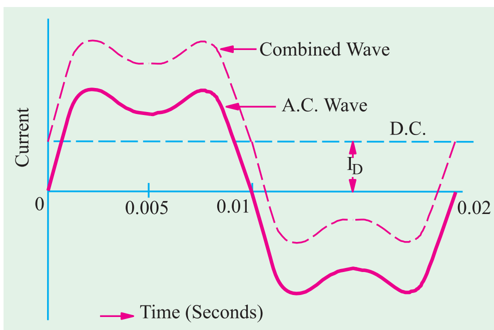


Fig. 11.9

Sometimes, a combination of an alternating and direct current flows simultaneously through a circuit In Fig. 11.9 is shown a complex wave (containing fundamental and third harmonic) combined with a direct current of value I_D. It is seen that the resultant wave remains undistorted in shape but is raised above the axis by an amount I_D. It is worth noting that with reference to the original axis, the two halves of the combined wave are not equal in area.

11.6. Cycle

One complete set of positive and negative values of alternating quantity is known as cycle. Hence, each diagram of Fig. 11.6 represents one complete cycle.

A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360º or 2π radians.

11.7. Time Period

The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 50-Hz alternating current has a time period of 1/50 second.

11.8. Frequency

The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

In the simple 2-pole alternator of Fig. 24.1 (*b*), one cycle of alternating current is generated in one revolution of the rotating field. However, if there were 4 poles, then two cycles would have been produced in each revolution. In fact, the frequency of the alternating voltage produced is a function of the speed and the number of poles of the generator. The relation connecting the above three quantities is given as

$f = PN/120$ where N = revolutions in r.p.m. and P = number of poles

For example, an alternator having 20 poles and running at 300 r.p.m. will generate alternating voltage and current whose frequency is $20 \times 300/120 = 50$ hertz (Hz).

It may be noted that the frequency is given by the reciprocal of the time period of the alternating quantity.

$$\therefore \qquad f = 1/T \quad \text{or} \quad T = 1/f$$

11.9. Amplitude

The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

11.10. Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already given in Art. 11.2, is

$$e = E_m \sin\theta = E_m \sin\omega t = E_m \sin 2\pi f t = E_m \sin \frac{2\pi}{T} t$$

By closely looking at the above equations, we find that

(i) the maximum value or peak value or amplitude of an ***alternating voltage is given by the coefficient of the sine of the time angle.***

(ii) the frequency f is given by the coefficient of time divided by 2π.

For example, if the equation of an alternating voltage is given by $e = 50 \sin 314t$ then its maximum value of 50 V and its frequency is $f = 314/2\pi = 50$ Hz.

Similarly, if the equation is of the form $e = I_m \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t$, then its maximum value is $E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$ and the frequency is $2\omega/2\pi$ or ω/π Hz.

Example 11.2. *The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are 283 V and 10 A respectively at t = 0 both increasing positively.*

(i) Write down the expression for voltage and current at time t.

(ii) Determine the power consumed in the circuit. **(Elect. Engg. Pune Univ.)**

Solution. *(i)* In general, the expression for an a.c. voltage is $v = V_m \sin(\omega t + \phi)$ where ϕ is the phase difference with respect to the point where $t = 0$.

Now, v = 283 V ; V_m = 400 V. Substituting $t = 0$ in the above equation, we get

283 = 400 (sin $\omega \times 0 + \phi$) $\therefore \sin \phi = 283/400 = 0.707$; $\therefore$ $\phi = 45°$ or $\pi/4$ radian.

Hence, general expression for voltage is

$v = 400 (\sin 2\pi \times 50 \times t + \pi/4)$

$= 400 \sin (100\,\pi\, t + \pi/4)$

Similarly, at t = 0, $10 = 20 \sin (\omega \times 0 + \phi)$

$\therefore$ $\sin \phi = 0.5$ $\therefore$ $\phi = 30°$ or $\pi/6$ radian

Hence, the general expression for the current is

$i = 20 (\sin 100\,\pi\, t + 30°) = 20 \sin (100\,\pi\, t + \pi/6)$

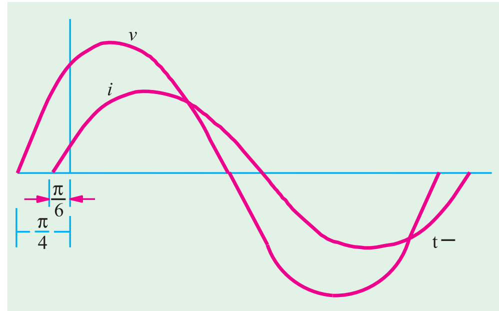


Fig. 11.10

(ii) $P = VI \cos \theta$ where V and I are rms values and θ is the phase difference between the voltage and current.

Now, $V = V_m/\sqrt{2} = 400/\sqrt{2}$; $I = 20/\sqrt{2}$; $\theta = 45° - 30° = 15°$ (See Fig. 11.10)

$\therefore$ $P = (400/\sqrt{2}) \times (20/\sqrt{2}) \times \cos 15° =$ **3864 W**

Example 11.3. *An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Reckoning time from the instant the current is zero and is becoming positive, find* ***(a)*** *the instantaneous value after 1/360 second and* ***(b)*** *the time taken to reach 96 A for the first time.*

Solution. The instantaneous current equation is

$i = 120 \sin 2\,\pi\, ft = 120 \sin 120\,\pi\, t$

Now when t = 1/360 second, then

(*a*) $i = 120 \sin (120 \times \pi \times 1/360)$ ...angle in radians

$= 120 \sin (120 \times 180 \times 1/360)$ ...angle in degree

$= 120 \sin 60° = 103.9$ A

(*b*) $96 = 120 \times \sin 2 \times 180 \times 60 \times t$ ...angle in degree

or $\sin (360 \times 60 \times t) = 96/120 = 0.8$ $\therefore$ $360 \times 60 \times t = \sin^{-1} 0.8 = 53°$ (approx)

$\therefore$ $t = \theta/2\pi f = 53/360 \times 60 =$ **0.00245 second.**

11.11. Phase

By phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference. For example, the phase of current at point A is $T/4$ second, where T is time period or expressed in terms of angle, it is $\pi/2$ radians (Fig. 11.11). Similarly, the phase of the rotating coil at the instant shown in Fig. 11.1 is ωt which is, therefore, called its phase angle.

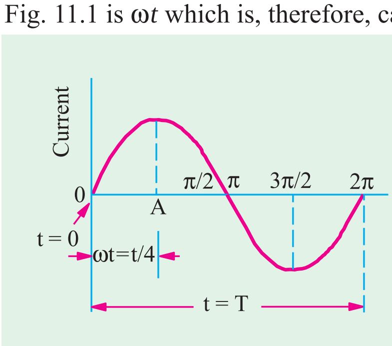


Fig. 11.11

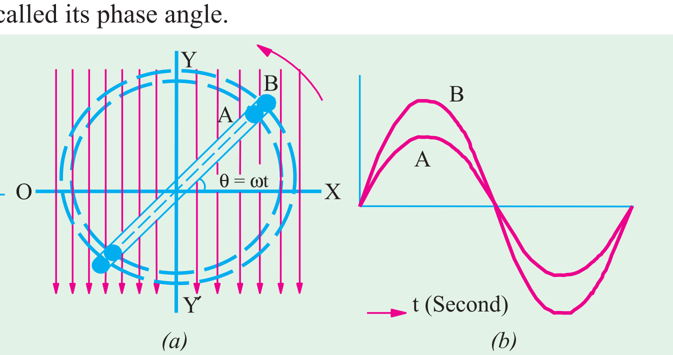


Fig. 11.12

In electrical engineering, we are, however, more concerned with relative phases or phase differences between different alternating quantities, rather than with their absolute phases. Consider two single-turn coils of different sizes [Fig. 11.12 (*a*)] arranged radially in the same plane and rotating with the same angular velocity in a common magnetic field of uniform intensity. The e.m.fs. induced in both coils will be of the same frequency and of sinusoidal shape, although the values of instantaneous e.m.fs. induced would be different. However, the two alternating e.m.fs. would reach their maximum and zero values at the same time as shown in Fig. 11.12 (*b*). Such alternating voltages (or currents) are said to be in phase with each other. The two voltages will have the equations

$$e_1 = E_{m1} \sin \omega t \quad \text{and} \quad e_2 = E_{m2} \sin \omega t$$

11.12. Phase Difference

Now, consider three similar single-turn coils displaced from each other by angles α and β and rotating in a uniform magnetic field with the same angular velocity [Fig. 11.13 (*a*)].

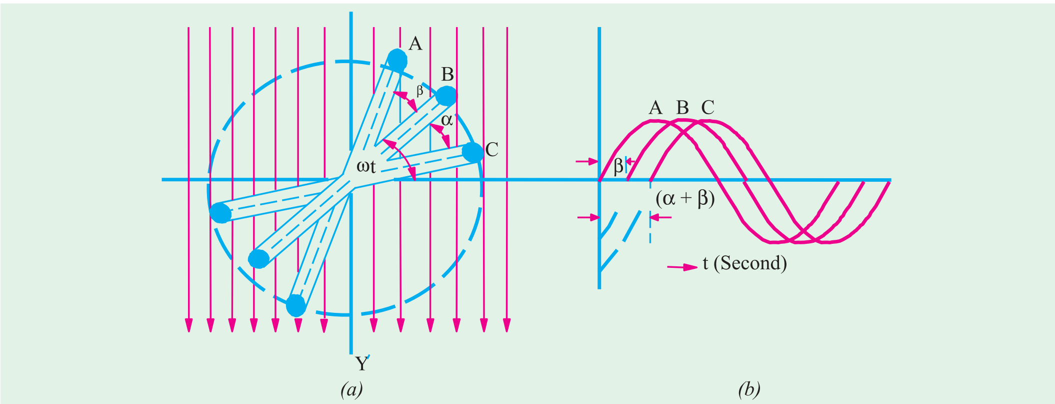


Fig. 11.13

In this case, the value of induced e.m.fs. in the three coils are the same, but there is one important difference. The e.m.fs. in these coils do not reach their maximum or zero values simultaneously but one after another. The three sinusoidal waves are shown in Fig. 11.13 (*b*). It is seen that curves *B* and *C* are displaced from curve *A* and angles β and (α + β) respectively. Hence, it means that phase difference between *A* and *B* is β and between *B* and *C* is α but between *A* and *C* is (α + β). The statement, however, does not give indication as to which e.m.f. reaches its maximum value first. This deficiency is supplied by using the terms ***'lag'*** or ***'lead'.***

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared to the other quantity.

Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity. For example, in Fig. 11.13 (*b*), *B* lags behind *A* by β and *C* lags behind *A* by (α + β) because they reach their maximum values later.

The three equations for the instantaneous induced e.m.fs. are (Fig. 11.14)

$$e_A = E_m \sin \omega t \text{ ...reference quantity}$$
$$e_B = E_m \sin (\omega t - \beta)$$
$$e_C = E_m \sin [\omega t - (\alpha + \beta)]$$

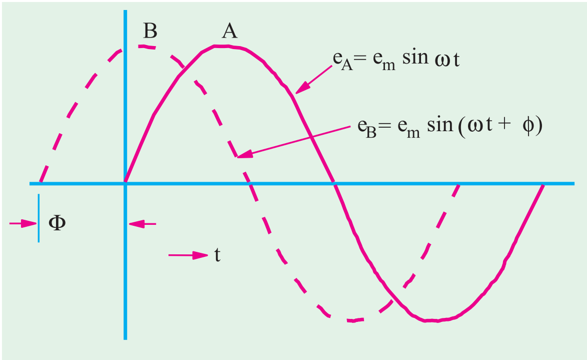


Fig. 11.14

In Fig. 11.14, quantity *B* leads *A* by an angle ϕ. Hence, their equations are

$e_A = E_m \sin \omega t$ ...reference quantity

$e_B = E_m \sin (\omega t - \phi)$

A plus (+) sign when used in connection with phase difference denotes 'lead' whereas a minus (–) sign denotes 'lag'.

11.13. Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current ***is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same* heat *as produced by the alternating current when flowing through the same circuit for the same time.***

It is also known as the ***effective*** or ***virtual*** value of the alternating current, the former term being used more extensively. For computing the r.m.s. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non-sinusoidal waves, the mid-ordinate method would be found more convenient.

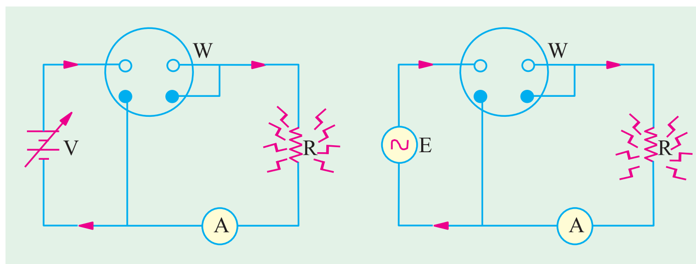


Fig. 11.15

A simple experimental arrangement for measuring the equivalent d.c. value of a sinusoidal current is shown in Fig. 11.15. The two circuits have identical resistances but one is connected to battery and the other to a sinusoidal generator. Wattmeters are used to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that heat power production in each circuit is the same. In that case, the direct current will equal $I_m/\sqrt{2}$ which is called r.m.s. value of the sinusoidal current.

11.14. Mid-ordinate Method

In Fig. 11.16 are shown the positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents. Divide time base '*t*' into *n* equal intervals of time each of duration *t*/*n* seconds. Let the average values of instantaneous currents during these intervals be respectively $i_1, i_2, i_3 .... i_n$ (*i.e.* mid-ordinates in Fig. 11.16). Suppose that this alternating current is passed through a circuit of resistance *R* ohms. Then,

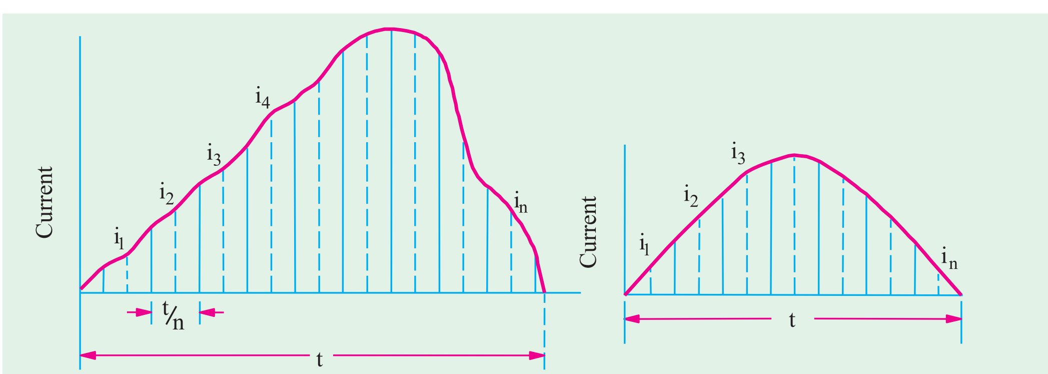


Fig. 11.16

Heat produced in 1st interval $= 0.24 \times 10^{-3}\, i_1^2\, Rt/n$ kcal $\quad (\because\ 1/J = 1/4200 = 0.24 \times 10^{-3})$

Heat produced in 2nd interval $= 0.24 \times 10^{-3}\, i_2^2\, Rt/n$ kcal

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Heat produced in *n*th interval $= 0.24 \times 10^{-3}\, i_n^2\, Rt/n$ kcal

Total heat produced in *t* seconds is $= 0.24 \times 10^{-3}\, Rt \left(\dfrac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)$ kcal

Now, suppose that a direct current of value *I* produces the same heat through the same resistance during the same time *t*. Heat produced by it is $= 0.24 \times 10^{-3}\, I^2 Rt$ kcal. By definition, the two amounts of heat produced should be equal.

$$\therefore \quad 0.24 \times 10^{-3} I^2 Rt = 0.24 \times 10^{-3}\, Rt \left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)$$

$$\therefore \quad I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \qquad \therefore \ I = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)}$$

= square root of the mean of the squares of the instantaneous currents

Similarly, the r.m.s. value of alternating voltage is given by the expression

$$V = \sqrt{\left(\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n} \right)}$$

11.15. Analytical Method

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$.

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2\pi} \frac{i^2\, d\theta}{(2\pi - 0)}$$

The square root of this value is $= \sqrt{\left(\int_0^{2\pi} \dfrac{i^2\, d\theta}{2\pi} \right)}$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta\, d\theta \right)} \qquad (\text{put } i = I_m \sin \theta)$$

Now, $\cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore\ \sin^2 \theta = \dfrac{1 - \cos 2\theta}{2}$

$$\therefore \quad I = \sqrt{\left(\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta)\, d\theta \right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{2\pi} \right)}$$

$$= \sqrt{\frac{I_m^2}{4} \quad 2} \qquad \sqrt{\frac{I_m^2}{2}} \qquad \therefore\ \mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. In electrical engineering work, ***unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.***

It should be noted that the average heating effect produced during one cycle is

$$= I^2R = (I_m/\sqrt{2})^2\,R = \frac{1}{2}\,I_m^2\,R$$

11.16. R.M.S. Value of a Complex Wave

In their case also, either the mid-ordinate method (when equation of the wave is not known) or analytical method (when equation of the wave is known) may be used. Suppose a current having the equation $i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$ flows through a resistor of R ohm. Then, in the time period T second of the wave, the effect due to each component is as follows :

Fundamental .... $(12/\sqrt{2})^2\ RT$ watt

3rd harmonic .... $(6/\sqrt{2})^2\ RT$ watt

5th harmonic .... $(4/\sqrt{2})^2\ RT$ watt

$\therefore$ Total heating effect $= RT\,[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$

If I is the r.m.s. value of the complex wave, then equivalent heating effect is I^2RT

$$\therefore \quad I^2RT = RT\,[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$$

$$\therefore \quad I = \sqrt{[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]} = 9.74 \text{ A}$$

Had there been a direct current of (say) 5 amperes flowing in the circuit also*, then the r.m.s. value would have been

$$= \sqrt{(12/\sqrt{2})^2 \quad (6/\sqrt{2})^2 \quad (4/\sqrt{2})^2 \quad 5^2} = 10.93 \text{ A}$$

Hence, for complex waves the rule is as follows : ***The r.m.s. value of a complex current wave is equal to the square root of the sum of the squares of the r.m.s. values of its individual components.***

11.17. Average Value

The average value I_a of an alternating current is expressed ***by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.***

In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. ***But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.***

(*i*) Mid-ordinate Method

With reference to Fig. 11.16, $I_{av} = \dfrac{i_1 + i_2 + \ldots + i_n}{n}$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(*ii*) Analytical Method

The standard equation of an alternating current is, $i = I_m \sin \theta$

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta \qquad \text{(putting value of } i\text{)}$$

* The equation of the complex wave, in that case, would be,
$i = 5 + 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$

$$= \frac{I_m}{\pi}\left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi}\left| +1-(-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$\therefore \quad I_{av} = I_m/\tfrac{1}{2}\pi = 0.637\, I_m \quad \therefore \text{ average value of current} = 0.637 \times \text{maximum value}$$

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

11.18. Form Factor

It is defined as the ratio, $K_f = \dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{0.707\, I_m}{0.637\, I_m} = 1.1.$ (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \dfrac{0.707\, E_m}{0.637\, E_m} = 1.11$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and ***vice-versa.***

11.19. Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \dfrac{\text{maximum value}}{\text{r.m.s. value}} = \dfrac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \dfrac{E_m}{E_m/\sqrt{2}} = 1.414$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

Example 11.4. *An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value **(a)** 0.0025 second **(b)** 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A ?*

(Elect. Science-I Allahabad Univ. 1992)

Solution. $I_m = 20\sqrt{2} = 28.2$ A, $\omega = 2\pi \times 50 = 100\,\pi$ rad/s.

The equation of the sinusoidal current wave with reference to point O (Fig. 11.17) as zero time point is $\quad \boldsymbol{i = 28.2 \sin 100\,\pi t}$ **ampere**

Since time values are given from point *A* where voltage has positive and maximum value, the equation may itself be referred to point *A*. In the case, the equation becomes :

$$i = 28.2 \cos 100\,\pi t$$

(*i*) When *t* = 0.0025 second

$i = 28.2 \cos 100\pi \times 0.0025$ ...angle in radian

$= 28.2 \cos 100 \times 180 \times 0.0025$ ...angle in degrees

$= 28.2 \cos 45^\circ = 20$ A ...point *B*

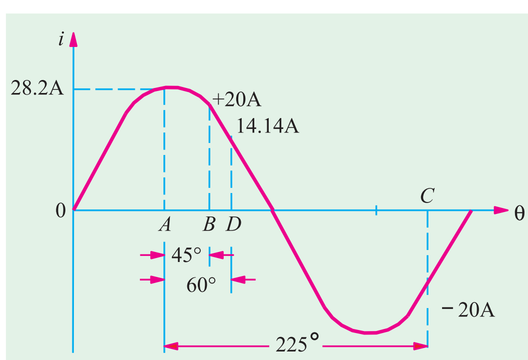

Fig. 11.17

(*ii*) When $t = 0.0125$ second

$i = 28.2 \cos 100 \times 180 \times 0.0125$

$= 28.2 \cos 225^\circ = 28.2 \times (-1/\sqrt{2})$

$= -20$ A ...point *C*

(*iii*) Here $i = 14.14$ A

$\therefore \quad 14.14 = 28.2 \cos 100 \times 180\, t \quad \therefore \cos 100 \times 180\, t = \dfrac{1}{2}$

or $\quad 100 \times 180\, t = \cos^{-1}(0.5) = 60^\circ,\ t =$ **1/300 second** ...point *D*

Example 11.5. *An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate* **(*a*)** *its value 1/600 second after the instant the current is zero and its value decreasing thereafter* **(*b*)** *how many seconds after the instant the current is zero (increasing thereafter wards) will the current attain the value of 86.6 A ?* **(Elect. Technology. Allahabad Univ. 1991)**

Solution. The equation of the alternating current (assumed sinusoidal) with respect to the origin *O* (Fig. 11.18) is

$$i = 100 \sin 2\pi \times 50t = 100 \sin 100\,\pi t$$

(*a*) It should be noted that, in this case, time is being measured from point *A* (where current is zero and decreasing thereafter) and not from point *O*.

If the above equation is to be utilized, then, this time must be referred to point *O*. For this purpose, half time-period *i.e.* 1/100 second has to be added to 1/600 second. The given time as referred to point *O* becomes

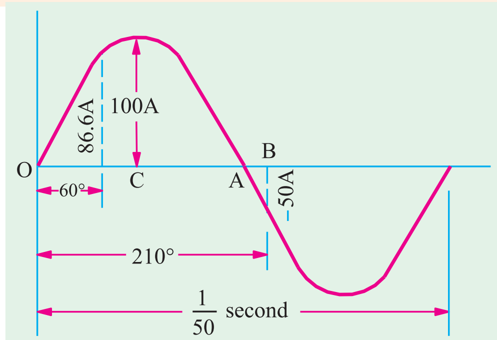

Fig. 11.18

$$= \frac{1}{100} + \frac{1}{600} = \frac{7}{600} \text{ second}$$

$\therefore \quad i = 100 \sin 100 \times 180 \times 7/600 = 100 \sin 210^\circ$

$= 100 \times -1/2 =$ **−50 A** ...point *B*

(*b*) In this case, the reference point is *O*.

$\therefore \quad 86.6 = 100 \sin 100 \times 180\, t \quad$ or $\quad \sin 18{,}000\, t = 0.866$

or $\quad 18{,}000\, t = \sin^{-1}(0.866) = 60^\circ \quad \therefore\ t = 60/18{,}000 =$ **1/300 second**

Example 11.6. *Calculate the r.m.s. value, the form factor and peak factor of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, −5, −10 V etc. What would be the r.m.s value of sine wave having the same peak value ?*

Solution. The waveform of the alternating voltage is shown in Fig. 11.19. Obviously, it is not sinusoidal but it is symmetrical. Hence, though r.m.s value may be full one cycle, the average value has necessarily to be considered for half-cycle only, otherwise the symmetrical negative and positive half-cycles will cancel each other out.

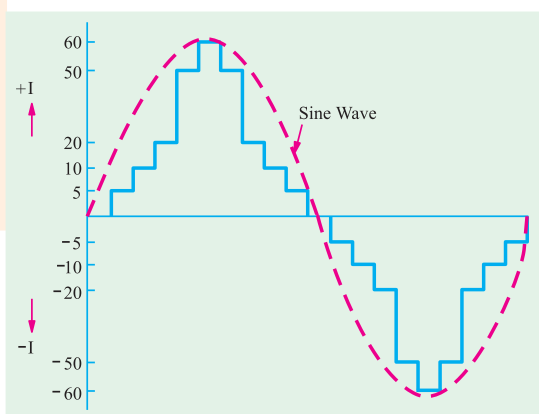

Fig. 11.19

Mean value of $v^2 = \dfrac{0^2 + 5^2 + 10^2 + 20^2 + 50^2 + 60^2 + 50^2 + 20^2 + 10^2 + 5^2}{10}$ = **965 V**

$\therefore$ r.m.s. value = $\sqrt{965}$ = **31 V (approx.)**

Average value (half-cycle) = $\dfrac{0 + 5 + 10 + 20 + 50 + 60 + 50 + 20 + 10 + 5}{10}$ = **23 V**

Form factor = $\dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{31}{23}$ = **1.35.** Peak factor = 60/31 = **2 (approx.)**

R.M.S. value of a sine wave of the same peak value = 0.707 × 60 = **42.2 V.**

Alternative Solution

If 't' be the regular time interval, then area of the half-cycle is

= $(5t + 10t + 20t + 50t)\,2 + 60t = 230t$, Base = $10t$ $\therefore$ Mean value = $230t/10t$ = 23 V.

Area when ordinates are squared = $(25t + 100t + 400t + 2500t)\,2 + 3600t = 9650t$, Base = $10t$

$\therefore$ Mean height of the squared curve = $9650t/10\,t = 965$

$\therefore$ r.m.s. value = $\sqrt{965}$ = 31 V

Further solution is as before.

Example 11.7. *Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by*

$$v = 200 \sin \omega t + 100 \sin 3\omega t + 50 \sin 5\omega t$$

Solution. Since hot-wire voltmeter reads only r.m.s value, we will have to find the r.m.s. value of the given voltage. Considering one complete cycle,

R.M.S. value $\quad V = \sqrt{\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} v^2 d\theta}$ where $\theta = \omega t$

or

$$V^2 = \frac{2}{2\pi}\int_0^{2\pi} (200 \sin \theta + 100 \sin 3\theta + 50 \sin 5\theta)^2\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (200^2 \sin^2\theta + 100^2 \sin^2 3\theta + 50^2 \sin^2 5\theta$$

$$+ 2 \times 200.100 \sin\theta.\sin 3\theta + 2 \times 100.50.\sin 3\theta.\sin 5\theta$$

$$+ 2 \times 50.200.\sin 5\theta\,.\sin\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left(\frac{200^2}{2} + \frac{100^2}{2} + \frac{50^2}{2}\right) 2\pi = 26{,}250$$

$\therefore$ $V = \sqrt{26{,}250}$ = **162 V**

Alternative Solution

The r.m.s. value of individual components are $(200/\sqrt{2})$, $(100/\sqrt{2})$ and $(50/\sqrt{2})$. Hence, as stated in Art. 11.16,

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(200/\sqrt{2})^2 + (100/\sqrt{2})^2 + (50/\sqrt{2})^2} = \mathbf{162\ V}$$

11.20. R.M.S. Value of H.W. Rectified Alternating Current

Half-wave (H.W.) rectified alternating current is one whose one half-cycle has been suppressed *i.e.* one which flows for half the time during one cycle. It is shown in Fig. 11.20 where suppressed half-cycle is shown dotted.

As said earlier, for finding r.m.s. value of such an alternating current, summation would be carried over the period for which current ***actually*** flows *i.e.* from 0 to π, though it would be averaged for the whole cycle *i.e.* from 0 to 2π.

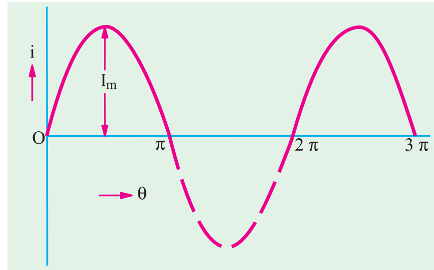

Fig. 11.20

∴ R.M.S. current

$$I = \sqrt{\left(\int_0^{\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{\pi}\sin^2\theta\, d\theta\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1-\cos 2\theta)\, d\theta}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi}\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\times\pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)} \qquad \therefore I = \frac{I_m}{2} = \mathbf{0.5 I_m}$$

11.21. Average Value of H.W. Rectified Alternating Current

For the same reasons as given in Art. 11.20, integration would be carried over from 0 –π

$$\therefore \qquad I_{av} = \int_0^{\pi}\frac{i d\theta}{2\pi} = \frac{I_m}{2\pi}\int_0^{\pi}\sin\theta\, d\theta \qquad (\because \; i = I_m \sin\theta)$$

$$= \frac{I_m}{2\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{2\pi}\times 2 = \frac{I_m}{\pi}$$

11.22. Form Factor of H.W. Rectified Alternating Current

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$$

Example 11.8. *An alternating voltage e = 200 sin 314t is applied to a device which offers an ohmic resistance of 20 Ω to the flow of current in one direction, while preventing the flow of current in opposite direction. Calculate RMS value, average value and form factor for the current over one cycle.* **(Elect. Engg. Nagpur Univ. 1992)**

Solution. Comparing the given voltage equation with the standard form of alternating voltage equation, we find that V_m = 200 V, R = 20 Ω, I_m = 200/20 = 10 A. For such a half-wave rectified current, RMS value = I_m/2 = 10/2 = **5A.**

Average current = I_m/π = 10/π = **3.18 A ;** Form factor = 5/3.18 = **1.57**

Example 11.9. *Compute the average and effective values of the square voltage wave shown in Fig. 11.21.*

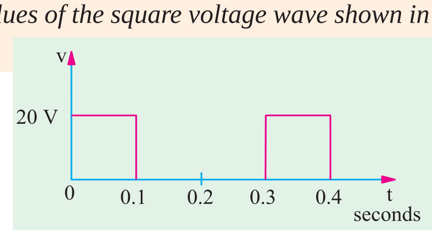

Fig. 11.21

Solution. As seen, for 0 < t < 0.1 *i.e.* for the time interval 0 to 0.1 second, v = 20 V. Similarly, for 0.1 < t < 0.3, v = 0. Also time-period of the voltage wave is 0.3 second.

$$\therefore \qquad V_{av} = \frac{1}{T}\int_0^T v\; dt = \frac{1}{0.3}\int_0^{0.1} 20\, dt$$

$$= \frac{1}{0.3}(20\times 0.1) = \mathbf{6.67\ V}$$

$$V^2 = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{0.3}\int_0^{0.1} 20^2\, dt = \frac{1}{0.3}(400\times 0.1) = 133.3;\; V = \mathbf{11.5\ V}$$

Example 11.10. *Calculate the RMS value of the function shown in Fig. 11.22 if it is given that for $0 < t < 0.1$, $y = 10(1 - e^{-100t})$ and $0.1 < t < 0.2$, $y = 10\, e^{-50(t-0.1)}$*

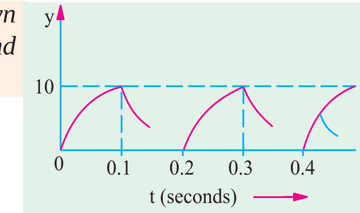

Fig. 11.22

Solution.

$$Y^2 = \frac{1}{0.2}\left\{\int_0^{0.1} y^2 dt + \int_{0.1}^{0.2} y^2 dt\right\}$$

$$= \frac{1}{0.2}\left\{\int_0^{0.1} 10^2\,(1 - e^{-100t})^2 dt + \int_{0.1}^{0.2} (10e^{-50(t-0.1)})^2 dt\right\}$$

$$= \frac{1}{0.2}\left\{\int_0^{0.1} 100\,(1 + e^{-200t} - 2e^{-100t})dt + \int_{0.1}^{0.2} 100\, e^{-100(t-0.1)} dt\right\}$$

$$= 500\left\{\left[t - 0.005e^{-200t} + 0.02\, e^{-100t}\right]_0^{0.1} + \left[-0.0\, e^{-100(t-0.1)}\right]_{0.1}^{0.2}\right\}$$

$$= 500\left\{\left[\left(0.1 - 0.005e^{-20} + 0.02e^{-10}\right) - (0 - 0.005 + 0.02)\right] + \left[\left(-0.01e^{-10}\right) - (-0.01)\right]\right\}$$

$$= 500 \times 0.095 = 47.5 \quad \therefore \quad Y = \sqrt{47.5} = \mathbf{6.9}$$

Example 11.11. *The half cycle of an alternating signal is as follows : It increases uniformly from zero at 0° to F_m at α°, remains constant from α° (180 – α)°, decreases uniformaly from F_m at (180 –α)° to zero at 180°. Calculate the average and effective values of the signal.*

(Elect. Science-I, Allahabad Univ. 1992)

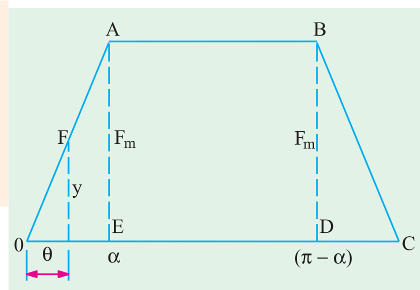

Fig. 11.23

Solution. For finding the average value, we would find the total area of the trapezium and divide it by π (Fig. 11.23).

Area = 2 × Δ *OAE* + rectangle *ABDE* = 2 × (1/2) × $F_m\alpha$ + $(\pi - 2\alpha)\, F_m = (\pi - \alpha)\, F_m$

∴ average value $= (\pi - \alpha)\, F_m/\pi$

RMS Value From similar triangles, we get $\frac{y}{\theta} = \frac{F_m}{\alpha}$ or $y^2 \ \frac{Fm^2}{2} \ 2$

This gives the equation of the signal over the two triangles *OAE* and *DBC*. The signal remains constant over the angle α to (π – α) *i.e.* over an angular distance of (π – α) –α = (π – 2α)

$$\text{Sum of the squares} = \frac{2F_m^2}{\alpha^2}\int_\theta^\alpha \theta^2\, d\,\theta + F_m^2(\pi - 2\alpha) = F_m^2(\pi - 4\alpha/3).$$

$$\text{The mean value of the squares is} = \frac{1}{\pi}F_m^2\left(\pi - \frac{4\alpha}{3}\right) = F_m^2\left(1 - \frac{4\alpha}{3\pi}\right)$$

$$\text{r.m.s. value} = \mathbf{F_m\sqrt{\left(1 - \frac{4\,\alpha}{3\pi}\right)}}$$

Example 11.12. *Find the average and r.m.s values of the a.c. voltage whose waveform is given in Fig. 11.24 (a)*

Solution. It is seen [(Fig. 11.24 (a)] that the time period of the waveform is 5s. For finding the average value of the waveform, we will calculate the net area of the waveform over one period and then find its average value for one cycle.

$$A_1 = 20 \times 1 = 20 \text{ V –s}, A_2 = -5 \times 2 = -10 \text{ V –s}$$

Net area over the full cycle $= A_1 + A_2 = 20 - 10 = 10$ V –s.

Average value $= 10$ V-s/5s $= 2$ V.

Fig. 11.24 (*b*) shows a graph of v^2 (*t*). Since the negative voltage is also squared, it becomes positive.

Average value of the area $= 400\ V^2 \times 1\ \text{s} + 25\ V^2 \times 2\ \text{s} = 450\ V^2 - \text{s}$. The average value of the sum of the square $= 450\ V^2$/- s/5s $= 90V^2$ rms value $= \sqrt{90\ V^2} = 9.49$ V.

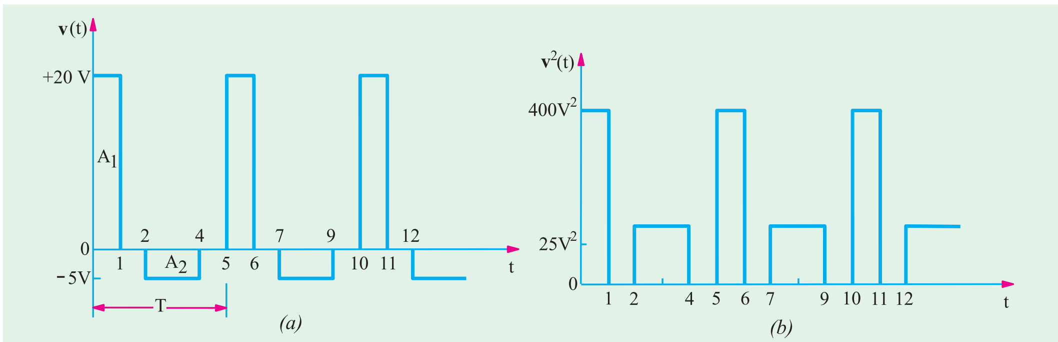

Fig. 11.24

Example 11.13. *What is the significance of the r.m.s and average values of a wave ? Determine the r.m.s. and average value of the waveform shown in Fig. 11.25*

(Elect. Technology, Indore Univ.)

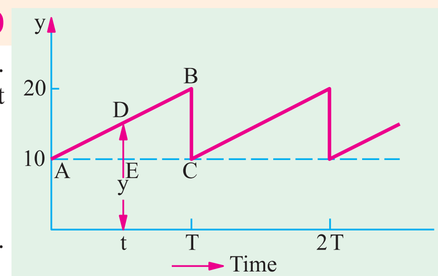

Fig. 11.25

Solution. The slope of the curve *AB* is *BC*/*AC* = 20/*T*. Next, consider the function *y* at any time *t*. It is seen that *DE*/*AE* = *BC*/*AC* = 10/*T*

or $(y - 10)/t = 10/T$

or $y = 10 + (10/T)t$

This gives us the equation for the function for one cycle.

$$Y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right) dt$$

$$= \frac{1}{T}\int_0^T \left[10.dt + \frac{10}{T}.\,t.\,dt\right] = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = 15$$

$$\text{Mean square value} = \frac{1}{T}\int_0^T y^2 dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T}\int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)dt = \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

or RMS value $= 10\sqrt{7/3} =$ **15.2**

Example 11.14. *For the trapezoidal current wave-form of Fig. 11.26, determine the effective value.*

(Elect. Technology, Vikram Univ. Ujjain , Similar Example, Nagpur Univ. 1999)

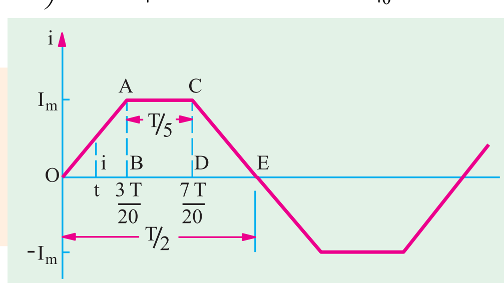

Fig. 11.26

Solution. For $0 < t < 3T/20$, equation of the current can be found from the relation

$$\frac{i}{t} = \frac{I_m}{3T/20} \quad \text{or } i = \frac{20I_m}{3T}.t$$

When $3T/20 < t < 7T/20$, equation of the current is given by $i = I_m$. Keeping in mind the fact that ΔOAB is identical with ΔCDE,

$$\text{RMS value of current} = \sqrt{\frac{1}{T/2}\left[2\int_0^{3T/20} i^2 dt + \int_{3T/20}^{7T/20} I_m^2 dt\right]}$$

$$= \sqrt{\frac{2}{T}\left[2\left(\frac{20I_m}{3T}\right)^2 \int_0^{3T/20} t^2 dt + I_m^2 \int_{3T/20}^{7T/20} dt\right]} = \frac{3}{5} I_m^2$$

$$\therefore \qquad I = \sqrt{(3/5)}.I_m = \mathbf{0.775\ I_m}$$

Incidentally, the average value is given by

$$I_{ac} = \frac{2}{T}\left\{2\int_0^{3T/20} idt + \int_{3T/20}^{7T/20} I_m dt\right\} = \frac{2}{T}\left\{2\int_0^{3T/20}\left(\frac{20I_m}{3T}\right)tdt + I_m\int_{3T/20}^{7T/20} dt\right\}$$

$$= \frac{2}{T}\left\{2\left(\frac{20I_m}{3T}\right)\left|\frac{t^2}{2}\right|_0^{3T/20} + I_m \left| t \right|_{3T/20}^{7T/20}\right\} = \frac{\mathbf{7}}{\mathbf{10}}.\mathbf{I_m}$$

Example 11.15. *A sinusoidal alternating voltage of 110 V is applied across a moving-coil ammeter, a hot-wire ammeter and a half-wave rectifier, all connected in series. The rectifier offers a resistance of 25 Ω in one direction and infinite resistance in opposite direction. Calculate **(i)** the readings on the ammeters **(ii)** the form factor and peak factor of the current wave.*

(Elect. Engg.-I Nagpur Univ. 1992)

Solution. For solving this question, it should be noted that

(*a*) Moving-coil ammeter, due to the inertia of its moving system, registers the average current for the *whole* cycle.

(*b*) The reading of hot-wire ammeter is proportional to the average heating effect over the ***whole*** cycle. It should further be noted that in a.c. circuits, the given voltage and current values, unless indicated otherwise, always refer to r.m.s values.

$$E_m = 110/0.707 = 155.5 \text{ V (approx.)} ; I_m/2 = 155.5/25 = 6.22 \text{ A}$$

Average value of current for positive half cycle = 0.637 × 6.22 = 3.96 A

Value of current in the negative half cycle is zero. But, as said earlier, due to inertia of the coil, M.C. ammeter reads the average value for the ***whole*** cycle.

(*i*) M.C. ammeter reading = 3.96/2 = **1.98 A**

Let R be the resistance of hot-wire ammeter. Average heating effect over the positive half cycle is $\frac{1}{2}I_m^2 . R$ watts. But as there is no generation of heat in the negative half cycle, the average heating effect over the whole cycle is $\frac{1}{4}I_m^2 R$ watt.

Let I be the d.c. current which produces the same heating effect, then

$$I^2R = \frac{1}{4}I_m^2 R \qquad \therefore \qquad I = I_m/2 = 6.22/2 = 3.11\ A.$$

Hence, hot-wire ammeter will read **3.11 A**

(*ii*) Form factor = $\frac{\text{r.m.s value}}{\text{average value}} = \frac{3.11}{1.98} = \mathbf{1.57}$ **;** Peak factor = $\frac{\text{max. value}}{\text{r.m.s. value}} = \frac{6.22}{3.11} = \mathbf{2}$

Example 11.16. *Find the form-factor of the wave form given in fig.*

[Nagpur University November 1991, Similar example, Sambalpur University]

Solution.

Form-factor $= \dfrac{\text{RMS value}}{\text{Average value}}$

Average value of the current

$= 1/4 \times \int_0^4 (50/4) \times t \times \text{dt} = 25$ amp

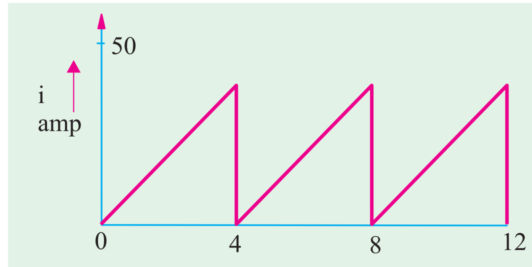

Fig. 11.27

Let RMS value of the current be I amp

$$I^2 \times 4 = \int_0^4 (12.5 \times t)^2.\text{dt}$$

$$= \left[\frac{12.5 \times 12.5 \times t^2}{3}\right]_0^4 = (1/3) \times (12.5 \times 12.5 \times 4 \times 4 \times 4)$$

Thus $I = \dfrac{50}{\sqrt{3}} = 28.87$ amp, Hence, form factor $\dfrac{28.87}{25} = 1.1548$

Example 11.17. *A half-wave rectifier which prevents current flowing in one direction is connected in series with an a.c. ammeter and a permanent-magnet moving-coil ammeter. The supply is sinusoidal. The reading on the a.c. ammeter is 10 A. Find the reading given by the other ammeter. What should be the readings on the ammeters, if the other half-wave were rectified instead of being cut off ?*

Solution. It should be noted that an a.c. ammeter reads r.m.s. value whereas the d.c. ammeter reads the average value of the rectified current.

As shown in Art. 11.20 from H.W. rectified alternating current, $I = I_m/2$ and $I_{av} = I_m/\pi$

As a.c. ammeter reads 10 A, hence r.m.s. value of the current is 10 A.

$\therefore \quad 10 = I_m/2 \quad$ or $\quad I_m = 20$ A

$\therefore \quad I_{av} = 20/\pi =$ **6.365 A** –reading of d.c. ammeter.

The full-wave rectified current wave is shown in Fig. 11.28. In this case mean value of i^2 over a complete cycle is given as

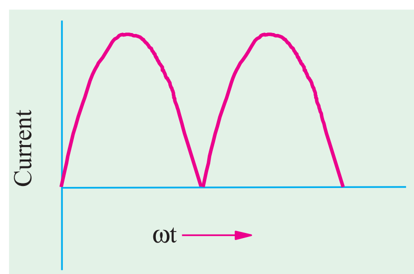

Fig. 11.28

$$= 2\int_0^{\pi} \frac{i^2\, d\theta}{2\pi - 0} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \; 2\sin^2\theta\, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi}(1 - \cos 2\theta)d\theta = \frac{I_m^2}{2\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi} = \frac{I_m^2}{2}$$

$\therefore \quad I = I_m/\sqrt{2} = 20/\sqrt{2} = 14.14$ A $\therefore$a.c. ammeter will read **14.14 A**

Now, mean value of i over a complete cycle

$$= \frac{2\int_0 I_m \sin\ \ d}{2} = \frac{I_m}{}\int_0 \sin\ \ d \quad \frac{I_m}{}|\ \cos\ |_0 \quad \frac{2I_m}{} \quad \frac{2\ \ 20}{}$$ **12.73 A**

This value, as might have been expected, is twice the value obtained in the previous case.

$\therefore \quad$ d.c. ammeter will read **12.73 A.**

Example 11.18. *A full-wave rectified sinusoidal voltage is clipped at $1/\sqrt{2}$ of its maximum value. Calculate the average and RMS values of such a voltage.*

Solution. As seen from Fig. 11.29, the rectified voltage has a period of π and is represented by the following equations during the different intervals.

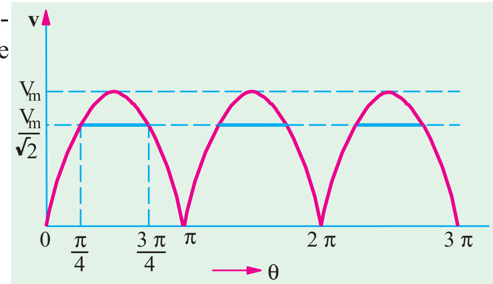

Fig. 11.29

$$0 < \theta < \pi/4 \; ; \; v = V_m \sin\theta$$

$$\pi/4 < \theta < 3\pi/4 \; ; \; v = V_m/\sqrt{2} = 0.707\, V_m$$

$$3\pi/4 < \theta < \pi \; ; \; v = V_m \sin\theta$$

$$\therefore \quad V_{av} = \frac{1}{\pi}\left\{\int_0^{\pi/4} v\,d\theta + \int_{\pi/4}^{3\pi/4} v\,d\theta + \int_{3\pi/4}^{\pi} v\,d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/4} V_m \sin\theta\,d\theta + \int_{\pi/4}^{3\pi/4} 0.707\,V_m d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\,d\theta\right\}$$

$$= \frac{V_m}{\pi}\left\{\left|-\cos\theta\right|_0^{\pi/4} + 0.707\,\left|\theta\right|_{\pi/4}^{3\pi/4} + \left|-\cos\theta\right|_{3\pi/4}^{\pi}\right\} = \frac{V_m}{\pi}(0.293 + 1.111 + 0.293) = \mathbf{0.54\ V_m}$$

$$V^2 = \frac{1}{\pi}\left\{\int_0^{\pi/4} V_m^2 \sin^2\theta\,d\theta + \int_{\pi/4}^{3\pi/4}(0.707\,V_m)^2 d\,\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\,d\theta\right\} = 0.341\,V_m^2$$

$$\therefore \quad V = \mathbf{0.584\ V_m}$$

Example 11.19. *A delayed full-wave rectified sinusoidal current has an average value equal to half its maximum value. Find the delay angle θ.* **(Basic Circuit Analysis, Nagpur 1992)**

Solution. The current waveform is shown in Fig. 11.30.

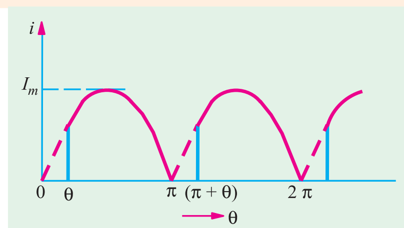

Fig. 11.30

$$I_{av} = \frac{1}{\ } \quad I_m \sin \quad d \quad \frac{I_m}{\ } \; (-\cos\pi + \cos\theta)$$

Now, $I_{av} = I_m/2$

$$\therefore \quad \frac{I_m}{\pi}(-\cos\pi + \cos\theta) = \frac{I_m}{2}$$

$$\therefore \quad \cos\theta = 0.57,\ \theta = \cos^{-1}(0.57) = \mathbf{55.25^o}$$

Example 11.20. *The waveform of an output current is as shown in Fig. 11.31. It consists of a portion of the positive half cycle of a sine wave between the angle θ and 180º. Determine the effective value for θ = 30º.*

(Elect. Technology, Vikram Univ. 1984)

Solution. The equation of the given delayed half-wave rectified sine wave is $i = I_m \sin\omega t = I_m \sin\theta$. The effective value is given by

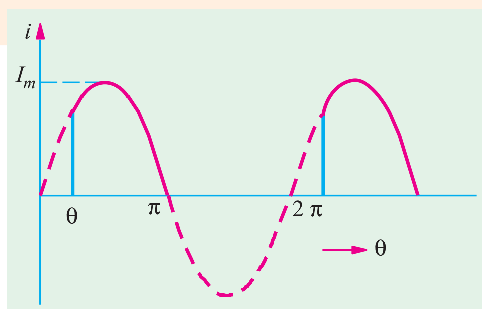

Fig. 11.31

$$I = \sqrt{\frac{1}{2\pi}\int_{\pi/6}^{\pi} i^2 d\theta} \quad \text{or } I^2 = \frac{1}{2\pi}\int_{\pi/6}^{\pi} I_m^2 \sin^2\theta\,d\theta$$

$$= \frac{I_m^2}{4\pi}\int_{\pi/6}^{\pi}(1 - \cos 2\theta)\,d\theta = \frac{I_m^2}{4\pi}\left|\left(\theta - \frac{\sin 2\theta}{2}\right)_{\pi/6}^{\pi}\right|$$

$$= 0.242\,I_m^2$$

or $\quad I = \sqrt{0.242 I_m^2} = \mathbf{0.492\ I_m}$

Example 11.21. *Calculate the "form factor" and "peak factor" of the sine wave shown in Fig. 11.32.* **(Elect. Technology-I, Gwalior Univ.)**

Solution. For $0 < \theta < \pi$, $i = 100 \sin\theta$ and for $\pi < \theta < 2\pi$, $i = 0$. The period is 2π.

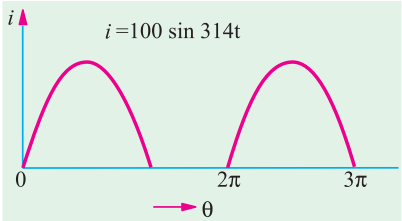

Fig. 11.32

$$\therefore 0 \quad I_{av} = \frac{1}{2\pi}\left\{\int_0^{\pi} i\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right\}$$

$$= \frac{1}{2\pi}\left\{100\int_0^{\pi}\sin\theta\,d\theta\right\} = 31.8 \text{ A}$$

$$I^2 = \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta = \frac{100^2}{2\pi}\int_0^{\pi}\sin^2\theta\,d\theta = \frac{100^2}{4} = 2500\ ;\ I = 50\text{A}$$

$\therefore$ form factor = 50/31.8 = **1.57 ;** peak factor = 100/50 = **2**

Example 11.22. *Find the average and effective values of voltage of sinusoidal waveform shown in Fig. 11.33.*

(Elect. Science-I Allahabad Univ. 1991)

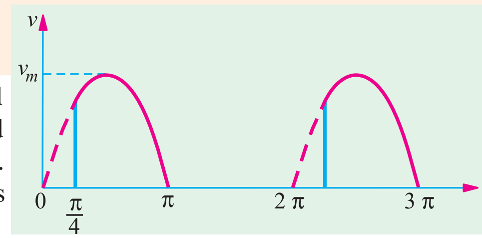

Fig. 11.33

Solution. Although, the given waveform would be integrated from π/4 to π, it would be averaged over the whole cycle because it is unsymmetrical. The equation of the given sinusoidal waveform is $v = 100 \sin\theta$.

$$\therefore \quad V_{av} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100\sin\theta\,d\theta = \frac{100}{2\pi}\left| -\cos\theta\right|_{\pi/4}^{\pi} = 27.2 \text{ V}$$

$$V^2 = \frac{1}{2}\ {}_{/4}\ 100^2\sin^2\ d\ \ \frac{100^2}{4}\ \frac{}{4}(1\ \ \cos 2\ )\,d\ \ \frac{100^2}{4}\Bigg|\ \ \frac{\sin 2}{2}\Bigg|_{/4}\ \ \frac{100^2}{4}\ \ \frac{}{4}\ \frac{1}{2}$$

$\therefore \quad V$ = **47.7 V**

Example 11.23. *Find the r.m.s. and average values of the saw tooth waveform shown in Fig. 11.34 (a).*

Solution. The required values can be found by using either graphical method or analytical method.

Graphical Method

The average value can be found by averaging the function from $t = 0$ to $t = 1$ in parts as given below :

$$\text{Average value of } (f) = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T} \times (\text{net area over one cycle})$$

Now, area of a right-angled triangle = (1/2) × (base) × (altitude).

Hence, area of the triangle during $t = 0$ to $t = 0.5$ second is

$$A_1 = \frac{1}{2}\times(\Delta t)\times(-2) = \frac{1}{2}\times\frac{1}{2}\times -2 = -\frac{1}{2}$$

Similarly, area of the triangle from $t = 0.5$ to $t = 1$ second is

$$A_2 = \frac{1}{2}\times(\Delta t)\times(+2) = \frac{1}{2}\times\frac{1}{2}\times 2 = \frac{1}{2}$$

Net area from $t = 0$ to $t = 1.0$ second is $A_1 + A_2 = -\frac{1}{2} + \frac{1}{2} = 0$

Hence, average value of $f(t)$ over one cycle is zero.

For finding the r.m.s. value, we will first square the ordinates of the given function and draw a new plot for $f^2(t)$ as shown in Fig. 11.34 (*b*). It would be seen that the squared ordinates from a parabola.

Area under parabolic curve = $\frac{1}{3}$ × base × altitude. The area under the curve from $t = 0$ to

$t = 0.5$ second is ; $A_1 = \frac{1}{3}(\Delta t) \times 2^2 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$

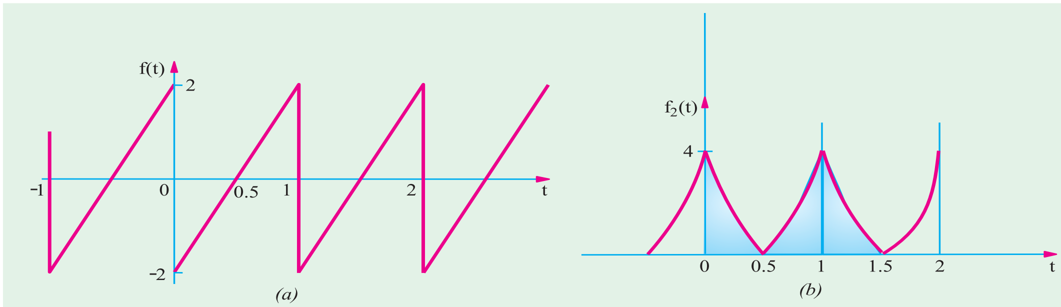

Fig. 11.34

Similarly, for $t = 0.5$ to $t = 1.0$ second $\quad A_2 = \frac{1}{3}(\Delta t) \times 4 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$

Total area $= A_1 + A_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$, r.m.s. value $= \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\text{average of } f^2(t)}$

$\therefore$ r.m.s. value $= \sqrt{4/3}$ = **1.15**

Analytical Method

The equation of the straight line from $t = 0$ to $t = 1$ in Fig. 11.34 (*a*) is

$$f(t) = 4t - 2;\ f^2(t) = 16t^2 - 16t + 4$$

$$\text{Average value} = \frac{1}{T}\int_0^T (4t - 2)\,dt = \frac{1}{T}\left|\frac{4t^2}{2} - 2t\right|_0^T = 0$$

$$\text{r.m.s value} = \sqrt{\frac{1}{T}\int_0^T (16t^2 - 16t + 4)dt} = \sqrt{\frac{1}{T}\left|\frac{16t^3}{3} - \frac{16t^2}{2} + 4t\right|_0^T} = \mathbf{1.15}$$

Example 11.24. *A circuit offers a resistance of 20 Ω in one direction and 100 Ω in the reverse direction. A sinusoidal voltage of maximum value 200 V is applied to the above circuit in series with*

(a) *a moving-iron ammeter* **(b)** *a moving-coil ammeter*

(c) *a moving-coil instrument with a full-wave rectifier* **(d)** *a moving-coil ammeter.*

Calculate the reading of each instrument.

Solution. (*a*) The deflecting torque of an *MI* instrument is proportional to (current)^2. Hence, its reading will be proportional to the average value of i^2 over the whole cycle. Therefore, the reading of such an instrument :

$$= \sqrt{\left[\frac{1}{2\pi}\left(\int_0^\pi 10^2 \sin^2\theta d\theta + \int_\pi^{2\pi} 2^2 \sin^2\theta d\theta\right)\right]}$$

$$= \sqrt{\frac{1}{2}\ \frac{100}{2}\left|\ \frac{\sin 2}{2}\right|_0\ \frac{4}{2}\left|\ \frac{\sin 2}{2}\right|^2} \qquad \sqrt{26} \qquad \mathbf{5.1\ A}$$

(*b*) An MC ammeter reads the average current over the whole cycle.

Average current over positive half-cycle is $= 10 \times 0.637 = 6.37$ A

Average current over positive half-cycle is $= -2 \times 0.637 = -1.27$ A

$\therefore$ Average value over the whole cycle is $= (6.37 - 1.27)/2 =$ **2.55 A**

(*c*) In this case, due to the full-wave rectifier, the current passing through the operating coil of the instrument would flow in the positive direction during both the positive and negative half cycles.

$\therefore$ reading $= (6.37 + 1.27)/2 =$ **3.82 A**

(*d*) Average heating effect over the positive half-cycle is $= \frac{1}{2} I_{m_1}^2 R$

Average heating effect over the negative half-cycle is = $\frac{1}{2} I_{m_2}^2 R$

where I_{m1} = 200/20 = 10 A; I_{m2} = 200/100 = 2 A

Average heating effect over the whole cycle is = $\left(\frac{1}{2} \times 10^2 R + \frac{1}{2} \times 2^2 \times R\right)/2 = 26\ R$

If I is the direct current which produces the same heating effect, then

$$I^2 R = 26\ R \qquad \therefore \quad I = \sqrt{26} = \mathbf{5.1\ A}$$

Example 11.25. *A moving coil ammeter, a hot-wire ammeter and a resistance of 100 Ω are connected in series with a rectifying device across a sinusoidal alternating supply of 200 V. If the device has a resistance of 100 Ω to the current in one direction and 5.00 Ω to current in opposite direction, calculate the readings of the two ammeters.*

(Elect. Theory and Meas. Madras University)

Solution. R.M.S. current in one direction is = 200/(100 + 100) = 1 A

Average current in the first *i.e.* positive half cycle is = 1/1.11 = 0.9 A

Similarly, r.m.s. value in the negative half-cycle is = –200/(100 + 500) = –1/3 A

Average value = (–1/3)/1.11 = –0.3 A

Average value over the whole cycle is = (0.9 –0.3)/2 = 0.3 A

Hence, M/C ammeter reads **0.3 A**

Average heating effect during the +ve half cycle = $I_{rms}^2 \times R = I^2 \times R = R$

Similarly, average heating effect during the –ve half-cycle is = $(-1/3)^2 \times R = R/9$

Here, R is the resistance of the hot-wire ammeter.

Average heating effect over the whole cycle is = $\frac{1}{2}\left(R + \frac{R}{9}\right) = \frac{5R}{9}$

If I is the direct current which produces the same heating effect, then

$$I^2 R = 5R/9 \qquad \therefore I = \sqrt{5}/3 = 0.745\ \text{A}$$

Hence, hot-wire ammeter indicates **0.745 A**

Example 11.26. *A resultant current wave is made up of two components : a 5A d.c. component and a 50-Hz a.c. component, which is of sinusoidal waveform and which has a maximum value of 5A.*

(*i*) *Draw a sketch of the resultant wave.*

(*ii*) *Write an analytical expression for the current wave, reckoning t = 0 at a point where the a.c. component is at zero value and when di/dt is positive.*

(*iii*) *What is the average value of the resultant current over a cycle ?*

(*iv*) *What is the effective or r.m.s. value of the resultant current ?*

[Similar Problem: Bombay Univ. 1996]

Solution. (*i*) The two current components and resultant current wave have been shown in Fig. 11.35.

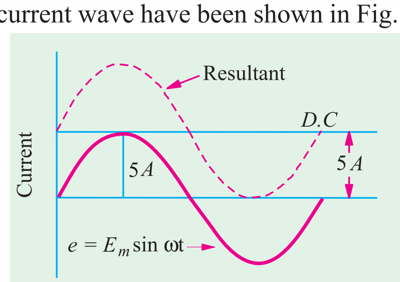

Fig. 11.35

(*ii*) Obviously, the instantaneous value of the resultant current is given by $i = (5 + 5 \sin \omega t) = (5 + 5 \sin \theta)$

(*iii*) Over one complete cycle, the average value of the alternating current is zero. Hence, the average value of the resultant current is equal to the value of d.c. component *i.e.* **5A**

(*iv*) Mean value of i^2 over complete cycle is

$$= \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} (5 + 5 \sin \theta)^2\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (25 + 50 \sin \theta + 25 \sin^2 \theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[25 + 50\sin\theta + 25\left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta = \frac{1}{2\pi}\int_0^{2\pi}(37.5 + 50\sin\theta - 12.5\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\Big|\, 37.5 \quad 50\cos \quad \frac{12.5}{2}\sin 2 \,\Big|_0^{2} \quad \frac{75}{2} \quad 37.5 \quad \therefore \text{R.M.S. value } I = \sqrt{37.5} = \mathbf{6.12\ A}$$

Note. In general, let the combined current be given by $i = A + B\sqrt{2}\sin\omega t = A + B\sqrt{2}\sin\theta$ where A represents the value of direct current and B the r.m.s. value of alternating current.

The r.m.s. value of combined current is given by

$$I_{rms} = \sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta\right)} \text{ or } I^2_{r.m.s.} = \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}(A + B\sqrt{2}\sin\theta)^2\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}(A^2 + 2B^2\sin^2\theta + 2\sqrt{2}\,AB\sin\theta)\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}(A^2 + B^2 - B^2\cos 2\theta + 2\sqrt{2}\,AB\sin\theta\, d\theta$$

$$= \frac{1}{2} \quad A^2 \quad B^2 \quad \frac{B^2\sin 2}{2} \quad 2\sqrt{2}AB\cos \Big|_0^{2} \quad \frac{1}{2} \quad 2\ A^2 \quad 2\ B^2 \quad 2\sqrt{2}\,AB \quad 2\sqrt{2}AB$$

$$= A^2 + B^2 \qquad \therefore I_{rms} = \sqrt{(A^2 + B^2)}$$

The above example could be easily solved by putting $A = 5$ and $B = 5/\sqrt{2}$ (because $B_{max} = 5$)

$$\therefore \qquad I_{rms} = \sqrt{5^2 + (5/\sqrt{2})^2} = \mathbf{6.12\ A}$$

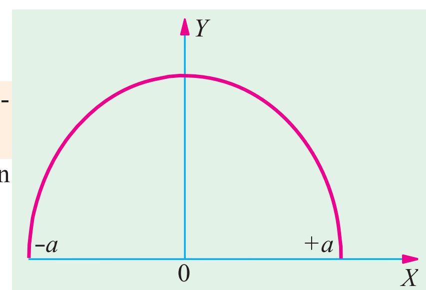

Fig. 11.36

Example 11.27. *Determine the r.m.s. value of a semi-circular current wave which has a maximum value of a.*

Solution. The equation of a semi-circular wave (shown in Fig. 11.36) is

$x^2 + y^2 = a^2$ or $y^2 = a^2 - x^2$

$$\therefore \quad I_{rms} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx} \text{ or } I^2_{rms} = \frac{1}{2a}\int_{-a}^{+a}(a^2 - x^2)\, dx$$

$$= \frac{1}{2a}\int_{-a}^{+a}(a^2\, dx - x^2\, dx) = \frac{1}{2a}\left|a^2x - \frac{x^3}{3}\right|_{-a}^{+a} = \frac{1}{2a}\left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right) = \frac{2a^2}{3}$$

$$\therefore \qquad I_{rms} = \sqrt{2a^2/3} = \mathbf{0.816\ a}$$

Example 11.28. *Calculate the r.m.s. and average value of the voltage wave shown in Fig. 11.37.*

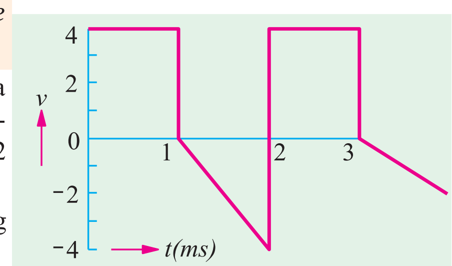

Fig. 11.37

Solution. In such cases, it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 and an other form 1 to 2 millisecond.

For t lying between 0 and 1 ms, $v_1 = 4$, For t lying between 1 and 2 ms, $v_2 = -4t + 4$

$$\therefore v_{rms} = \sqrt{\frac{1}{2}\left(\int_0^1 v_1^2\,dt + \int_1^2 v_2^2\,dt\right)}$$

$$V^2_{rms} = \frac{1}{2}\left[\int_0^1 4^2\,dt + \int_1^2 (-4t+4)^2\,dt\right]$$

$$= \frac{1}{2}\left[\left|16\,t\right|_0^1 + \left|\frac{16\,t^3}{3}\right|_1^2 + \left|16\,t\right|_1^2 - \left|\frac{32\,t^2}{2}\right|_1^2\right]$$

$$= \frac{1}{2}\left[16 + \frac{16\times 8}{3} - \frac{16}{3} + 16\times 2 - 16\times 1 - \frac{32\times 4}{2} + \frac{32\times 1}{2}\right] = \frac{32}{3} \quad \therefore V_{rms} = \sqrt{32/3} = \textbf{3.265 volt}$$

$$V_{av} = \frac{1}{2}\left[\int_0^1 v_1 dt + \int_1^2 v_2\,dt\right] = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-4t+4)\,dt\right] = \frac{1}{2}\left[\left|4t\right|_0^1 + \left|-\frac{4t^2}{2} + 4t\right|_1^2\right] = \textbf{1 volt}$$

Tutorial Problems No. 11.1

1. Calculate the maximum value of the e.m.f. generated in a coil which is rotating at 50 rev/s in a uniform magnetic field of 0.8 Wb/m^3. The coil is wound on a square former having sides 5 cm in length and is wound with 300 turns. **[188.5 V]**

2. (*a*) What is the peak value of a sinusoidal alternating current of 4.78 r.m.s. amperes ?

(*b*) What is the r.m.s. value of a rectangular voltage wave with an amplitude of 9.87 V ?

(*c*) What is the average value of a sinusoidal alternating current of 31 A maximum value ?

(*d*) An alternating current has a periodic time of 0.03 second. What is its frequency ?

(*e*) An alternating current is represented by i = 70.7 sin 520 *t*. Determine (*i*) the frequency (*ii*) the current 0.0015 second after passing through zero, increasing positively.

[6.76 A ; 9.87 V ; 19.75 A ; 33.3 Hz ; 82.8 Hz ; 49.7 A]

3. A sinusoidal alternating voltage has an r.m.s. value of 200 V and a frequency of 50 Hz. It crosses the zero axis in a positive direction when *t* = 0. Determine (*i*) the time when voltage first reaches the instantaneous value of 200 V and (*ii*) the time when voltage after passing through its maximum positive value reaches the value of 141.4 V.

[(*i*) (0.0025 second (*ii*) 1/300 second)]

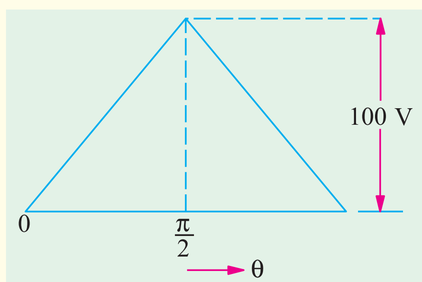

Fig. 11.38

4. Find the form factor and peak factor of the triangular wave shown in Fig. 11.38 **[1.155; 1.732]**

5. An alternating voltage of 200 sin 471 *t* is applied to a h.w. rectifier which is in series with a resistance of 40 Ω. If the resistance of the rectifier is infinite in one direction and zero in the other, find the r.m.s. value of the current drawn from the supply source. **[2.5 A]**

6. A sinusoidally varying alternating current has an average value of 127.4 A. When its value is zero, then its rate change is 62,800 A/s. Find an analytical expression for the sine wave. **[i = 200 sin 100 πt]**

7. A resistor carries two alternating currents having the same frequency and phase and having the same value of maximum current *i.e.* 10 A. One is sinusoidal and the other is rectangular in waveform. Find the r.m.s. value of the resultant current. **[12.24 A]**

8. A copper-oxide rectifier and a non-inductive resistance of 20 Ω are connected in series across a sinusoidal a.c. supply of 230 V (r.m.s.). The resistance of the rectifier is 2.5 Ω in forward direction and 3,000 Ω in the reverse direction. Calculate the r.m.s. and average values of the current.

[r.m.s. value = 5.1 A, average value = 3.22 A]

9. Find the average and effective values for the waveshape shown in Fig. 11.39 if the curves are parts of a sine wave. **[27.2 V, 47.7V] (*Elect. Technology, Indore Univ.*)**

10. Find the effective value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 15 A.

[14.58A] (*Elect. Technology, Vikram Univ. Ujjain*)

11. Determine the r.m.s. value of the voltage defined by $e = 5 + 5 \sin (314\, t + \pi/6)$

[6.12 V] (*Elect. Technology, Indore Univ.*)

12. Find the r.m.s. value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 10 A. **[12.25 A]** **(*Elect. Technology-I; Delhi Univ.*)**

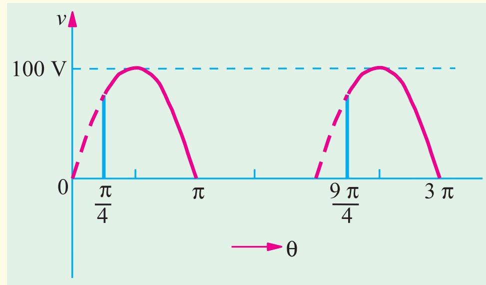

Fig. 11.39

13. An alternating voltage given by $e = 150 \sin 100\pi t$ is applied to a circuit which offers a resistance of 50 ohms to the current in one direction and completely prevents the flow of current in the opposite direction. Find the r.m.s. and average values of this current and its form factor.

[1.5 A, 0.95 A, 1.57] (*Elect. Technology, Indore Univ.*)

14. Find the relative heating effects of three current waves of equal maximum value, one rectangular, the second semi-circular and the third sinusoidal in waveform **[1: 2/2, 1/2] (*Sheffield Univ. U.K.*)**

15. Calculate the average and root mean-square value, the form factor and peak factor of a periodic current wave have the following values for equal time intervals over half-cycle, changing suddenly from one value of the next. **[0, 40, 60, 80, 100, 80, 60, 40, 0] (*A.M.I.E.*)**

16. A sinusoidal alternating voltage of amplitude 100 V is applied across a circuit containing a rectifying device which entirely prevents current flowing in one direction and offers a resistance of 10 ohm to the flow of current in the other direction. A hot wire ammeter is used for measuring the current. Find the reading of instrument. **(*Elect. Technology. Punjab Univ.*)**

11.23. Representation of Alternating Quantities

It has already been pointed out that an attempt is made to obtain alternating voltages and currents having sine waveform. In any case, a.c. computations are based on the assumption of sinusoidal voltages and currents. It is, however, cumbersome to continuously handle the instantaneous values in the form of equations of waves like $e = E_m \sin \omega t$ etc. A conventional method is to employ vector method of representing these sine waves. These vectors may then be manipulated instead of the sine functions to achieve the desired result. ***In fact, vectors are a shorthand for the represen-tation of alternating voltages and currents and their use greatly simplifies the problems in a.c. work.***

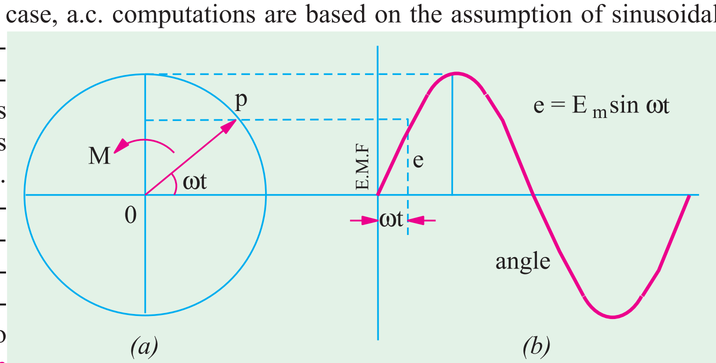

Fig. 11.40

A vector is a physical quantity which has magnitude as well as direction. Such vector quantities are completely known when particulars of their magnitude, direction and the sense in which they act, are given. They are graphically represented by straight lines called vectors. The length of the line represents the magnitude of the alternating quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of the alternating quantity. In Fig. 11.40 (*a*), *OP* is such a vector which represents the maximum value of the alternating current and its angle with *X* axis gives its phase. Let the alternating current be represented by the equation $e = E_m \sin \omega t$. It will be seen that the projection of *OP* and *Y*-axis at any instant gives the instantaneous value of that alternating current.

$$\therefore \quad OM = OP \sin \omega t \text{ or } e = OP \sin \omega t = E_m \sin \omega t$$

It should be noted that a line like *OP* can be made to represent an alternating voltage of current if it satisfies the following conditions :

(i) Its length should be equal to the peak or maximum value of the sinusoidal alternating current to a suitable scale. ***(ii)*** It should be in the horizontal position at the same instant as the alternating quantity is zero and increasing. ***(iii)*** Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

11.24. Vector Diagram using R.M.S. Values

Instead of using maximum values as above, it is very common practice to draw vector diagrams using r.m.s. values of alternating quantities. But it should be understood that in that case, the projection of the rotating vector on the *Y*-axis does not give the instantaneous value of that alternating quantity.

11.25. Vector Diagrams of Sine Waves of Same Frequency

Two or more sine waves of the same frequency can be shown on the same vector diagram because the various vectors representing different waves all rotate counter-clockwise at the same frequency and maintain a fixed position relative to each other. This is illustrated in Fig. 11.41 where a voltage *e* and current *i* of the same frequency are shown. The current wave is supposed to pass upward through zero at the instant when $t = 0$ while at the same time the voltage wave has already advanced an angle α from its zero value. Hence, their equations can be written as

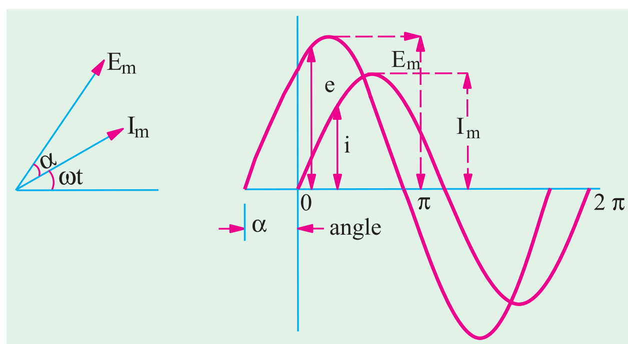

Fig. 11.41

$$i = I_m \sin \omega t$$

and $$e = E_m \sin (\omega t + \alpha)$$

Sine wave of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.

11.26. Addition of Two Alternating Quantities

In Fig. 11.42 (*a*) are shown two rotating vectors representing the maximum values of two sinusoidal voltage waves represented by $e_1 = E_{m1}$ $\sin \omega t$ and $e_2 = E_{m2} \sin (\omega t - \phi)$. It is seen that the sum of the two sine waves of the same frequency is another sine wave of the same frequency but of a different maximum value and phase. The value of

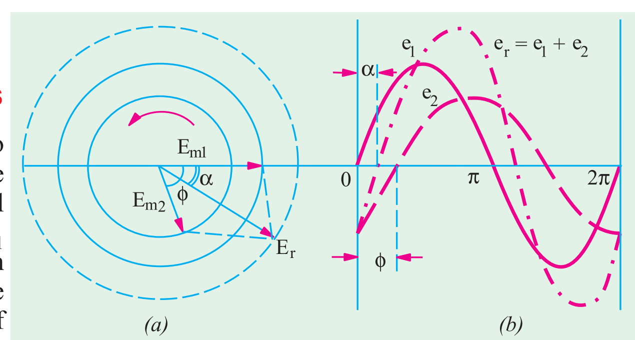

Fig. 11.42

the instantaneous resultant voltage e_r, at any instant is obtained by algebraically adding the projections of the two vectors on the Y-axis. If these projections are e_1 and e_2, then, $e_r = e_1 + e_2$ at that time. The resultant curve is drawn in this way by adding the ordinates. It is found that the resultant wave is a sine wave of the same frequency as the component waves but lagging behind E_{m1} by an angle α. The vector diagram of Fig. 11.42 (*a*) can be very easily drawn. Lay off E_{m2} lagging $\phi°$ behind E_{m1} and then complete the parallelogram to get E_r.

Example 11.29. *Add the following currents as waves and as vectors.*

$$i_1 = 7 \sin \omega t \text{ and } i_2 = 10 \sin (\omega t + \pi/3)$$

Solution. As Waves

$$i_r = i_1 + i_2 = 7 \sin \omega t + 10 \sin (\omega t + 60°)$$
$$= 7 \sin \omega t + 10 \sin \omega t \cos 60° + 10 \cos \omega t \sin 60°$$
$$= 12 \sin \omega t + 8.66 \cos \omega t$$

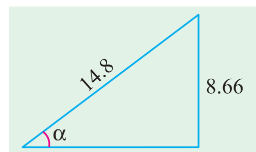

Fig. 11.43

Dividing both sides by $\sqrt{(12^2 + 8.66^2)} = 14.8$, we get

$$i_r = 14.8 \left(\frac{12}{14.8} \sin \omega t + \frac{8.66}{14.8} \cos \omega t \right)$$
$$= 14.8 (\cos \alpha \sin \omega t + \sin \alpha \cos \omega t)$$

where $\cos \alpha = 12/14.8$ and $\alpha = 8.66/14.8$ —as shown in Fig. 11.43

$\therefore$ $i_r = 14.8 \sin (\omega t + \alpha)$

where $\tan \alpha = 8.66/12$ or $\alpha = \tan^{-1}(8.66/12) = 35.8°$

$\therefore$ $i_r = 14.8 \sin (\omega t + 35.8°)$

As Vectors

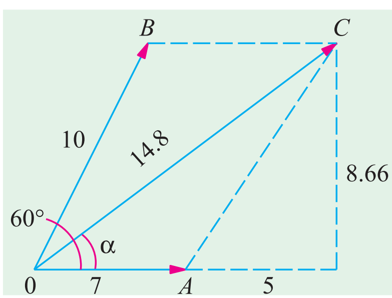

Fig. 11.44

Vector diagram is shown in Fig. 11.44. Resolving the vectors into their horizontal and vertical components, we have

X – component $= 7 + 10 \cos 60° = 12$

Y – component $= 0 + 10 \sin 60° = 8.66$

Resultant $= \sqrt{(12^2 + 8.66^2)} = 14.8$ A

and $\alpha = \tan^{-1} (8.66/12) = 35.8°$

Hence, the resultant equation can be written as

$$\mathbf{i_r = 14.8 \sin (\omega t + 35.8°)}$$

11.27. Addition and Subtraction of Vectors

(*i*) Addition. In a.c. circuit problems we may be concerned with a number of alternating voltages or currents of the same frequency but of different phases and it may be required to obtain the resultant voltage or current. As explained earlier (Art. 11.23) if the quantities are sinusoidal, they may be represented by a number of rotating vectors having a common axis of rotation and displaced from one another by fixed angles which are equal to the phase differences between the respective alternating quantities. The instantaneous value of the resultant voltage is given by the algebraic sum of the projections of the different vectors on Y-axis. The maximum value (or r.m.s. value if the vectors represent that value) is obtained by compounding the several vectors by using the parallelogram and polygon laws of vector addition.

However, another easier method is to resolve the various vectors into their X-and Y-components and then to add them up as shown in Example 11.30 and 31.

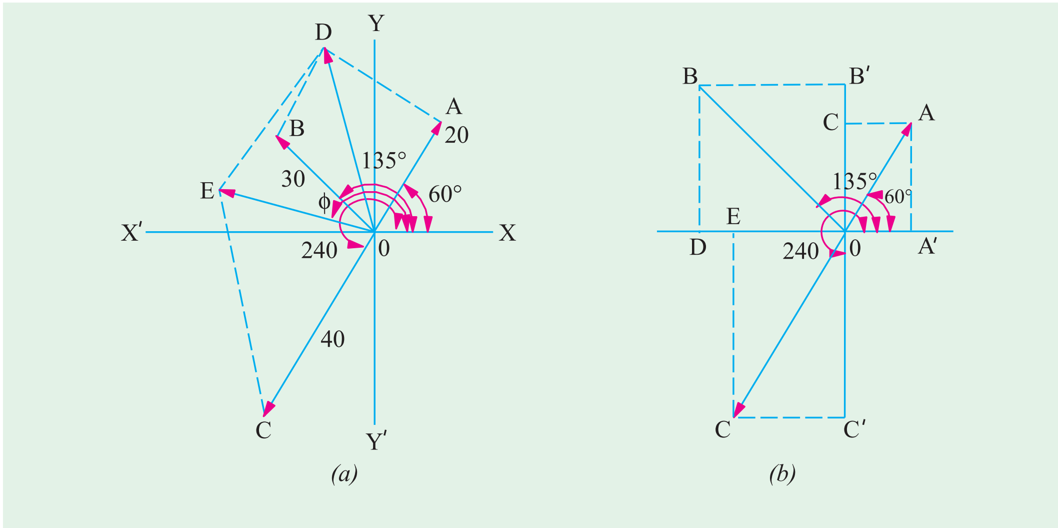

Fig. 11.45

Suppose we are given the following three alternating e.m.fs. and it is required to find the equation of the resultant e.m.f.

$$e_1 = 20 \sin (\omega t + \pi/3)$$
$$e_2 = 30 \sin (\omega t + 3\pi/4)$$
$$e_3 = 40 \sin (\omega t + 4\pi/3)$$

Then the vector diagram can be drawn as explained before and solved in any of the following three ways :

(i) By compounding according to parallelogram law as in Fig. 11.45 (*a*)

(ii) By resolving the various vectors into their *X*-and *Y*-components as in Fig. 11.45 (*b*).

(iii) By laying off various vectors end-on end at their proper phase angles and then measuring the closing vector as shown in Fig. 11.46.

Knowing the magnitude of the resultant vector and its inclination ϕ with *X* axis, the equation of the resultant e.m.f. can be written as $e = E_m \sin (\omega t + \phi)$.

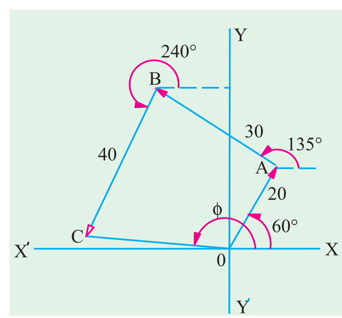

Fig. 11.46

Example 11.30. *Represent the following quantities by vectors :*

5 sin (2π ft –1) ; 3 cos (2π ft + 1) ; 2 sin (2π ft + 2.5) and 4 sin (2π ft –1)

Add the vectors and express the result in the form : A sin (2π ft ± Φ)

Solution. It should be noted that all quantities have the same frequency *f*, hence they can be represented vertically on the same vector diagram and added as outlined in Art. 11.27. But before doing this, it would be helpful to express all the quantities as sine functions. Therefore, the second expression 3 cos (2π *ft* + 1) can be written as

$$3 \sin \left(2\pi ft + 1 + \frac{\pi}{2}\right) = 3 \sin (2\pi\, ft + 1 + 1.57) = 3 \sin (2\pi\, ft + 2.57)$$

The maximum value of each quantity, its phase with respect to the quantity of reference *i.e.* *X* sin 2π*ft*, its horizontal and vertical components are given in the table on next page :

Quantity	Max. value	Phase radians	Phase angles	Horizontal component	Vertical component
(*i*) 5 sin (2π*ft* −1)	5	−1	−57.3°	5 × cos (−57.3°) = 2.7	5 sin (−57.3°) = −4.21
(*ii*) 3 sin (2π*ft* + 2.57)	3	+2.57	147.2°	3 × cos 147.2° = −2.52	3 sin 147.2° = 1.63
(*iii*) 2 sin (2π*ft* + 2.5)	2	+ 2.5	143.2°	2 cos 143.2° = −1.6	2 sin 143.2° = 1.2
(*iv*) 4 sin (2π*ft* −1)	4	− 1	− 57.3°	4 cos (−57.3°) = 2.16	4 sin (−57.3°) = −3.07
Total				0.74	−4.75

The vector diagram is shown in Fig. 11.47 in which *OA, OB, OC* and *OD* represent quantities **(*i*), (*ii*), (*iii*)** and **(*iv*)** given in the table.

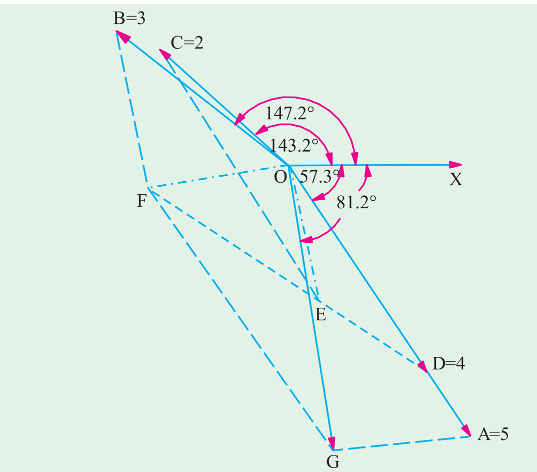

Fig. 11.47

Fig. 11.48

Their resultant is given by *OG* and the net horizontal and vertical components are shown in Fig. 11.48. Resultant $= \sqrt{[0.74^2 + (-4.75)^2]}$ = 4.81 and tan θ = −4.75 / 0.74

∴ $\theta = \tan^{-1}(-4.75/0.74) = -81.2° = -1.43$ radians

The equation of the resultant quantity is **4.81 sin (2π*ft* −1.43)**

Example 11.31. *Three voltages represented by*

$$e_1 = 20 \sin \omega t;\ e_2 = 30 \sin (\omega t - \pi/4) \text{ and } e_3 = 40 \cos (\omega t + \pi/6)$$

act together in a circuit. Find an expression for the resultant voltage. Represent them by appropriate vectors. **(Electro-technics Madras Univ.) (*Elec. Circuit Nagpur Unvi. 1991*)**

Solution. First, let us draw the three vectors representing the maximum values of the given alternating voltages.

$e_1 = 20$ sing ωt –here phase angle with *X*-axis is zero, hence the vector will be drawn parallel to the *X*-axis

$e_2 = 30 \sin (\omega t - \pi/4)$ –its vector will be below *OX* by 45°

$e_3 = 40 \cos (\omega t + \pi/6) = 40 \sin (90° + \omega t + \pi/6)$*

$= 40 \sin (\omega t + 120°)$ –its vector will be at 120° with respect to *OX* in counter clockwise direction.

These vectors are shown in Fig. 11.49 (*a*). Resolving them into *X*-and *Y*-components, we get

X - component = 20 + 30 cos 45° – 40 cos 60°= 21.2 V

Y - component = 40 sin 60° –30 sin 45° = 13.4 V

* cos θ = sin (90° + θ)

As seen from Fig. 11.49 (*b*), the maximum value of the resultant voltage is

$$OD = \sqrt{21.2^2 + 13.4^2} = 25.1 \text{ V}$$

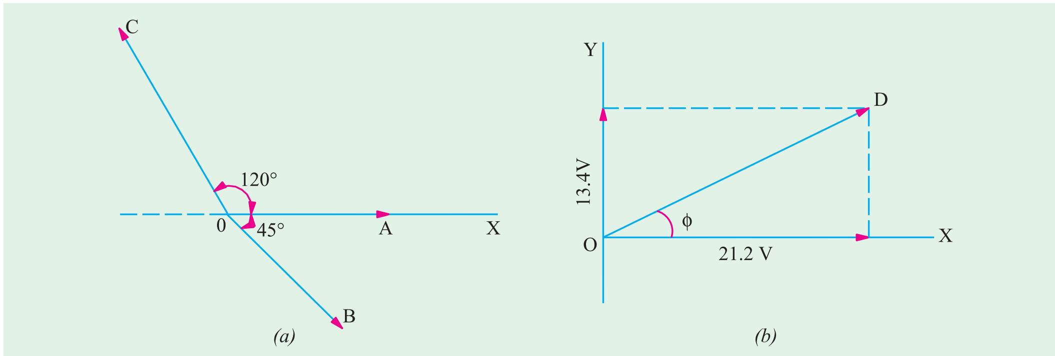

Fig. 11.49

The phase angle of the resultant voltage is given by $\tan\phi = \dfrac{\text{Y-component}}{\text{X-component}} \quad \dfrac{13.4}{27.2} \quad$ **0.632**

$\therefore \quad \phi = \tan^{-1} 0.632 = 32.3° =$ **0.564 radian**

The equation of the resultant voltage wave is $e = 25.1 \sin(\omega t + 32.3°)$ or $e = 25.1 \sin(\omega t + 0.564)$

Example 11.32. *Four circuits A, B, C and D are connected in series across a 240 V, 50-Hz supply. The voltages across three of the circuits and their phase angles relative to the current through them are, V_A, 80 V at 50° leading, V_B, 120 V at 65° lagging : V_θ, 135 V at 80° leading. If the supply voltage leads the current by 15°, find from a vector diagram drawn to scale the voltage V_D across the circuit D and its phase angle.*

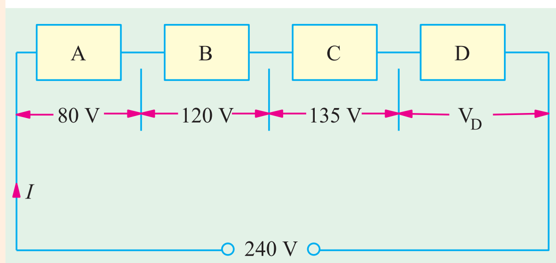

Fig. 11.50

Solution. The circuit is shown in Fig. 11.50.

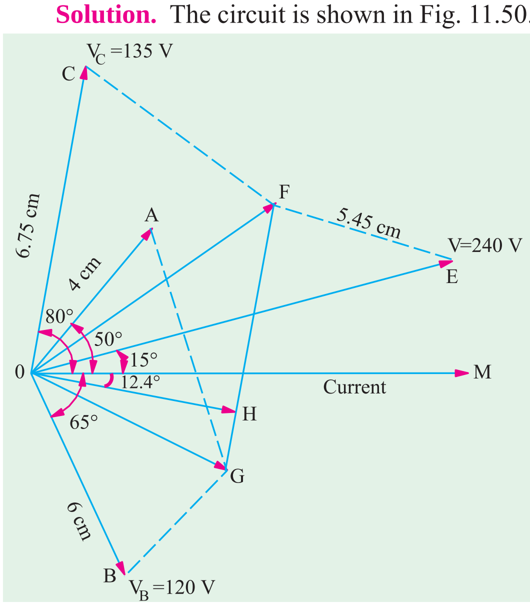

Fig. 11.51

(*a*) The vector diagram is shown in Fig. 11.50. The current vector *OM* is drawn horizontally and is taken as reference vector. Taking a scale of 1 cm = 20 V, vector *OA* is drawn 4 cm in length and leading *OM* by an angle of 50°. Vector *OB* represents 120 V and is drawn lagging behind *OM* by 65°. Their vector sum, as found by Parallelogram Law of Vectors, is given by vector *OG*.

Next, vector *OC* is drawn ahead of *OM* by 80° representing 135 V. Vector *OF* represents the vector sum of *OG* and *OC*. Vector *OE* represents the applied voltage of 240 V and is drawn 15° ahead of current vector *OM*. The vector difference of *OE* and *OF* gives the required voltage V_D. It is equal to *FE*. It measures 5.45 cm which means that it represents 20 × 5.45 = 109 V. This vector is transferred to position

OH by drawing *OH* parallel to *FE*. It is seen that *OH* lags behind the current vector *OM* by 12.4°.

Hence, V_D' = **109** volts lagging behind the current by **12.4°.**

(*ii*) Subtraction of Vectors

If difference of two vectors is required, then one of the vectors is reversed and this reversed vector is then compounded with the other vector as usual.

Suppose it is required to subtract vector *OB* from vector *OA*. Then *OB* is reversed as shown in Fig. 11.52 (*a*) and compounded with *OA* according to parallelogram law. The vector difference **(A-B)** is given by vector *OC*.

Similarly, the vector *OC* in Fig. 12.52 (*b*) represents (**B–A**) *i.e.* the subtraction of **OA** form **OB**.

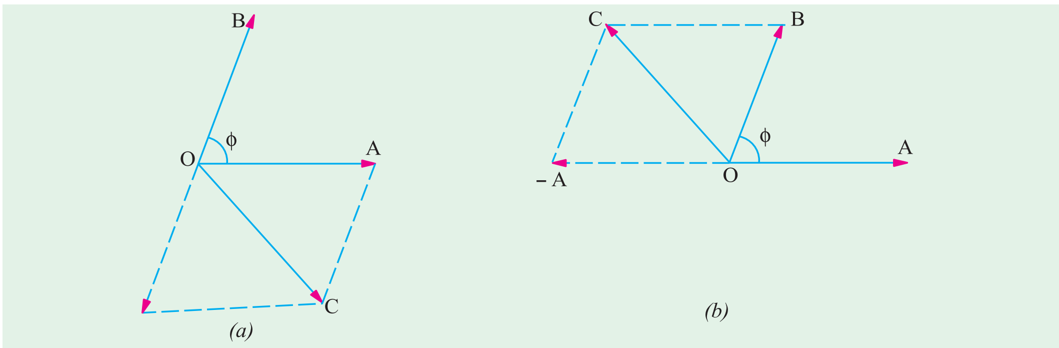

Fig 11.52

Example 11.33. *Two currents i_1 and i_2 are given by the expressions*

$$i_1 = 10 \sin (314t + \pi/4) \text{ amperes and } i_2 = 8 \sin (313\, t - \pi/3) \text{ amperes}$$

Find **(a)** $i_1 + i_2$ *and* **(b)** $i_1 - i_2$. *Express the answer in the form* $i = I_m \sin (314\, t \pm \phi)$

Solution. **(*a*)** The current vectors representing maximum values of the two currents are shown in Fig. 11.53 (*a*). Resolving the currents into their *X*-and *Y*-components, we get

X- component = 10 cos 45° + 8 cos 60° = $10/\sqrt{2} + 8/2 = 11.07$ A

Y- component = 10 sin 45° –8 sin 60 ° = 0.14 A

$$\therefore \quad I_m = \sqrt{11.07^2 + 0.14^2} = 11.08 \text{ A}$$

tan φ = (0.14/11.07) = 0.01265 ∴ φ = 44′

Hence, the equation for the resultant current is ***i* = 11.08 sin (314 *t* + 44′) amperes**

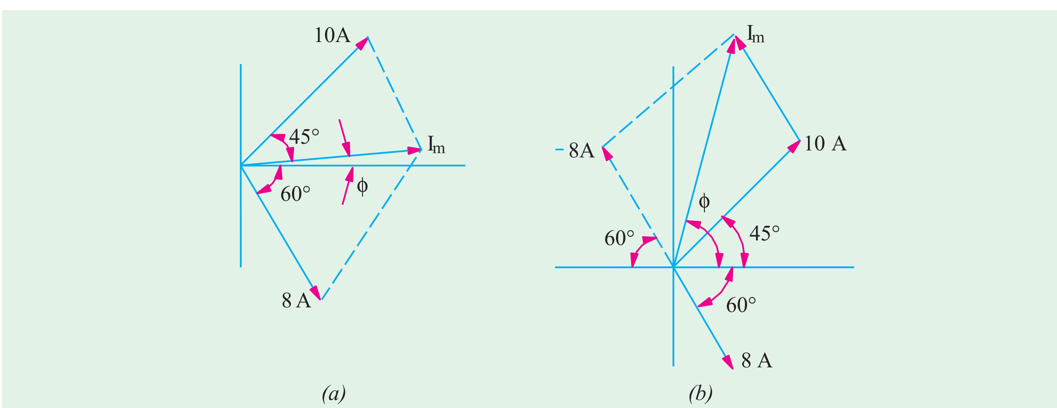

Fig. 11.53

(b) X − component = 10 cos 45° −8 cos 60° = 3.07 A

Y − component = 10 sin 45° + 8 sin 60° = 14 A

$\therefore \quad I_m = \sqrt{3.07^2 + 14^2} = 14.33$ A ...Fig. 11.53 (b)

$\phi = \tan^{-1}(14/3.07) = 77° \, 38'$

Hence, the equation of the resultant current is

$$i = 14.33 \sin (314 + 77° \, 38') \text{ amperes}$$

Example 11.34. *The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to a 50 Hz supply. The instantaneous values of voltage and current are 283 Vand 10 A respectively at time t = 0, both increasing positively.*

(i) *Write down the expression for voltage and current at time t.*

(ii) *Determine the power consumed in the circuit.*

Take the voltage and current to be sinusoidal **[Nagpur University]**

Solution. $V_m = 400$, $I_m = 20$, $\omega = 314$ rad./sec

(i) Let the expressions be as follows :

$v(t) = V_m \sin(\omega t + \theta_1) = 400 \sin(314\,t + \theta_1)$

$I(t) = I_m \sin(\omega t + \theta_2) = 20 \sin(314\,t + \theta_2)$

where θ_1 and θ_2 indicate the concerned phase-shifts with respect to some reference. Substituting the given instantaneous values at $t = 0$,

$\theta_1 = 45°$ and $\theta_2 = 30°$

The required expressions are :

$V(t) = 400 \sin(314\,t + 45°)$

$i(t) = 20 \sin(314\,t + 30°)$

Thus, the voltage leads the current by 15°.

V = RMS voltage = 400/1.414 t = 283 V

I = RMS voltage = 20/1.414 = 14.14 A

Power-factor, cos ϕ = cos 15° = 0.966 lagging, since current lags behind the voltage.

(ii) Power = V I cos ϕ = 3865 watts

Additional Hint : Draw these two wave forms

Example 11.35. *Voltage and current for a circuit with two elements in series are experssed as follows :*

$$v(t) = 170 \sin(6280\,t + \pi/3) \text{ Volts}$$
$$i(t) = 8.5 \sin(6280\,t + \pi/2) \text{ Amps}$$

(i) *Plot the two waveforms.* **(ii)** *Determine the frequency in Hz.* **(iii)** *Determine the power factor stating its nature.* **(iv)** *What are the values of the elements ?*

[Nagpur University, April 1996]

Solution. **(ii)** ω = 6280 radiation/sec, $f = \omega/2\pi$ = 1000 Hz

(i) Two sinusoidal waveforms with a phase-difference of 30° (= $\pi/2 - \pi/3$) are to be drawn. Each waveform completes a cycle in 1 milli-second, since f = 1000 Hz.

The waveform for current leads that for the voltage by 30°. At $\omega t = 0$, the current is at its positive peak, while the voltage will be at its positive peak for $\omega t = \pi/6 = 30°$. Peak value are 170 volts and 8.5 amp.

(iii) RMS value of voltage = $170/\sqrt{2} = 120$ volts

RMS value of current = $8.5/\sqrt{2} = 6$ amp.

Impedance = V/I = 120/6 = 20 ohms

Power factor = cos 30°, Leading = 0.866,

Since the current leads the voltage, the two elements must be R and C.

$R \;= Z \cos\phi = 20 \times 0.866 = 17.32$ ohms

$X_c = Z \sin\phi = 20 \times 0.50 = 10$ ohms

$$C = 1/(\omega X_c) = \frac{1000 \times 1000}{6280 \times 10} = 15.92 \text{ mF}$$

Example 11.36. *Three sinusoidally alternating currents of rms values 5,7.5, and 10 A are having same frequency of 50 Hz, with phase angles of 30°, –60° and 45°.*

(i) *Find their average values,* ***(ii)*** *Write equations for their instantaneous values,* ***(iii)*** *Draw waveforms and phasor diagrams taking first current as the reference,* ***(iv)*** *Find their instantaneous values at 100 mSec from the original reference.* **[Nagpur University, Nov. 1996]**

Solution. ***(i)*** Average value of alternating quantity in case of sinusoidal nature of variation = (RMS Value)/1.11

Average value of first current = 5/1.11 = 4.50 A

Average value of second current = 7.5/1.11 = 6.76 A

Average value of third current = 10/1.11 = 9.00 A

(ii) Instantaneous Values : $\omega = 2\pi \times 50 = 314$ rad/sec

$i_1\,(t) = 5\sqrt{2}\;\sin\,(314\,t \quad 30\;)$

$i_2\,(t) = 7.5\sqrt{2}\;\sin\,(314\,t \quad 60\;)$

$i_3\,(t) = 10\sqrt{2}\;\sin\,(314\,t \quad 45\;)$

(iii) First current is to be taken as a reference, now. From the expressions, second current lags behind the first current by 90°. Third current leads the first current by 15°. Waveforms with this description are drawn in Fig. 11.54 (*a*) and the phasor diagrams, in Fig. 11.54 (*b*).

(iv) A 50 Hz a.c. quantity completes a cycle in 20 m sec. In 100 m sec, it completes five cycles. Original reference is the starting point required for this purpose. Hence, at 100 m sec from the reference.

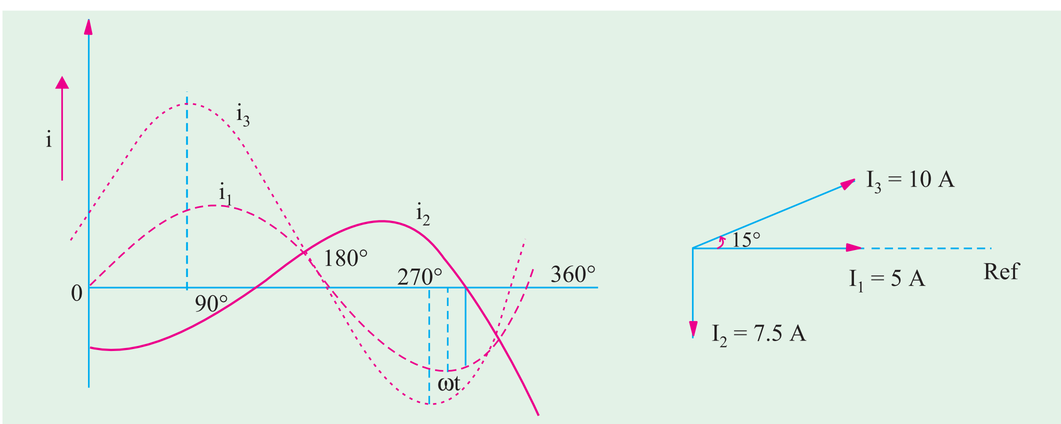

Fig. 11.54 (*a*) **Fig. 11.54 (*b*)**

(v) instantaneous value of $i_1\,(t) = 5\sqrt{2}\sin 30 \quad 3.53$ A instantaneous value of $i_2(t) = 7.5\sqrt{2}\sin(\quad 60\ ) \qquad 9.816$ A

instantaneous value of $i_3(t) = 10\sqrt{2}\sin(45\ ) \quad 10$ A

Example 11.37. *Determine the form factor and peak factor for the unshaded waveform,in Fig. 11.55.* **[Bombay University, 2000]**

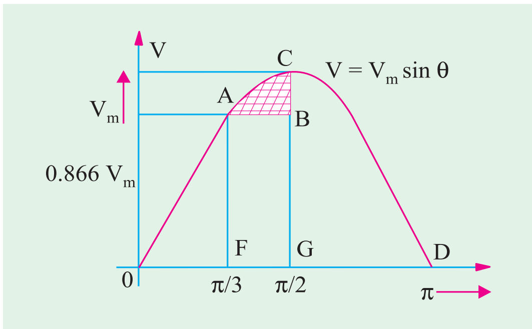

Fig. 11.55

Solution. $v(\theta) = V_m \sin\theta$, except for the region between

$\theta = 60°$ to $\theta = 90°$, wherein $v = 0.866\ V_m$.

Area under the curve will be worked out first, for calculating the average value.

$$\text{Area OAF} = V_m \int_0^{/3} \sin\ \ d \qquad 0.5\,V_m$$

Area FABG = $0.866\ V_m\ (\pi/2 - \pi/3) = 0.4532\ V_m$

Area GCD = V_m. Total area under the curve = V_m (1 + 0.432 + 0.5)

Average value, $V_{av} = (1.9532\ V_m)/3.14$

$= 0.622\ V_m$

For evaluating rms value, the square of the function is to be taken, its mean value calculated and square-root of the mean value found out.

Area under the squared function :

For Portion OF : $V_m^2 \int_0^{\pi/2} \sin^2\theta d\theta = V_m^2/2 \int_0^{\pi/2} (1-\cos 2\theta)\,d\theta = 0.307\,V_m^2$

For Portion FG : $(0.866\ V_m)^2 \times \pi \times (1/2 - 1/3) = 0.3925\,V_m^2$

For Portion GD : $V_m^2 \int_0^{\pi/2} \sin^2\theta\,d\theta = 0.785\,V_m^2$

Total area = $V_m^2\,[0.307 + 0.3925 + 0.785] = 1.4845\,V_m^2$

Let R.M.S. Value be V_c

$V_m^2\,\pi = 1.4845\,V_m^2$, or $V_c = 0.688 V_m$

Form factor = RMS Value/Average Value = 0.688/0.622 = 1.106

Peak factor = Peak Value/RMS Value = 1.0/0.688 = 1.4535

Tutorial Problems No. 11.2

1. The values of the instantaneous currents in the branches of a parallel circuit are as follows :
$i_1 = 5\sin 346\,t$; $i_2 = 10\sin(346t + \pi/4)$; $i_3 = 7.5\sin(346\,t + \pi/2)$; $i_4 = 8\sin(346\,t - \pi/3)$
Express the resultant line current in the same form as the original expression and determine the r.m.s. value and the frequency of this current. **[12.5 A; 55 Hz]**

2. Four coils are connected in series. Each has induced in it a sinusoidal e.m.f. of 100 V, 50 Hz and there is a phase difference of 14 electrical degrees between one coil and the next. What is the total e.m.f. generated in the circuit ? **[384 V]**

3. The instantaneous voltage across each of the four coils connected in series is given by
$v_1 = 100\sin 471\,t$; $v_2 = 250\cos 471\,t$; $v_3 = 150\sin(471\,t + \pi/6)$; $v_4 = 200\sin(471\,t - \pi/4)$
Determine the total p.d. expressed in similar form to those given. What will be the resultant p.d. If v_2 is reversed in sign ? **[$v = 414\sin(471\,t + 26.5°)$; $v = 486\sin(471\,t - 40°)$]**

4. An alternating voltage of $v = 100\sin 376.8\,t$ is applied to a circuit consisting of a coil having a resistance of 6Ω and an inductance of 21.22 mH.

(a) Express the current flowing in the circuit in the form $i = I_n \sin(376.8\, t \pm \phi)$
(b) If a moving-iron voltmeter, a wattmeter and a frequency meter are connected in the circuit, what would be the respective readings on the instruments ?
[i = 10 sin (376.8 t –53.1°); 70.7 V; 300 W; 60 Hz]

5. Three circuits A, B and C are connected in series across a 200-V supply. The voltage across circuit A is 50 V lagging the supply voltage by 45° and the voltage across circuit C is 100 V leading the supply voltage by 30°. Determine graphically or by calculation, the voltage across circuit B and its phase displacement from the supply voltage. **[79.4 V ; 10° 38′ lagging]**

6. Three alternating currents are given by
$i_1 = 141 \sin(\omega t + \pi/4)$ $i_2 = 30 \sin(\omega t + \pi/2)$ $i_3 = 20 \sin(\omega t - \pi/6)$
and are fed into a common conductor. Find graphically or otherwise the equation of the resultant current and its r.m.s. value. **[i = 167.4 sin (ωt + 0.797), I_{rms} = 118.4 A]**

7. Four e.m.fs $e_1 = 100 \sin \omega t$, $e_2 = 80 \sin(\omega t - \pi/6)$, $e_3 = 120 \sin(\omega t + \pi/4)$ and $e_4 = 100 \sin(\omega t - 2\pi/3)$ are induced in four coils connected in series so that the vector sum of four e.m.fs. is obtained. Find graphically or by calculation the resultant e.m.f. and its phase difference with (a) e_1 and (b) e_2. If the connections to the coil in which the e.m.f. e_2 is induced are reversed, find the new resultant e.m.f . **[208 sin (ωt – 0.202) (a) 11°34′ lag (b) 18′ 26′ lead; 76 sin (ωt + 0.528)]**

8. Draw to scale a vector diagram showing the following voltages :
$v_1 = 100 \sin 500\,t$; $v_2 = 200 \sin(500\,t + \pi/3)$; $v_3 = -50 \cos 500\,t$; $v_4 = 150 \sin(500\,t - \pi/4)$
Obtain graphically or otherwise, their vector sum and express this in the form $V_m \sin(500\,t \pm \phi)$, using v_1 as the reference vector. Give the r.m.s. value and frequency of the resultant voltage.
[360.5 sin (500 t + 0.056); 217 V; 79.6 Hz]

11.28. A.C. Through Resistance, Inductance and Capacitance

We will now consider the phase angle introduced between an alternating voltage and current when the circuit contains resistance only, inductance only and capacitance only. ***In each case, we will assume that we are given the alternating voltage of equation $e = E_m \sin \omega t$*** and will proceed to find the equation and the phase of the alternating current produced in each case.

11.29. A.C. Through Pure Ohmic Resistance Alone

The circuit is shown in Fig. 11.56. Let the applied voltage be given by the equation.

$v = V_m \sin \theta = V_m \sin \omega t$ ...(i)

Let R = ohmic resistance ; i = instantaneous current

Obviously, the applied voltage has to supply ohmic voltage drop only. Hence for equilibrium

$$v = iR;$$

Putting the value of 'v' from above, we get $V_m \sin \omega t = iR$; $i = \dfrac{V_m}{R} \sin \omega t$ ...(ii)

Current 'i' is maximum when sin ωt is unity $\therefore\ I_m = V_m/R$ Hence, equation (ii) becomes, $i = I_m \sin \omega t$ ...(iii)

Comparing (i) and (ii), we find that the alternating voltage and current are in phase with each other as shown in Fig. 11.57. It is also shown vectorially by vectors V_R and I in Fig. 11.54.

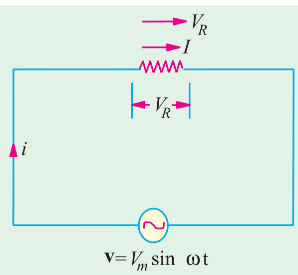

Fig. 11.56

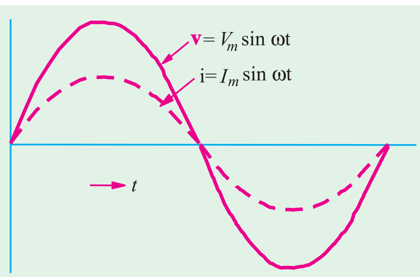

Fig. 11.57

Power. Instantaneous power, $p = vi = V_m I_m \sin^2 \omega t$ ...(Fig. 11.58)

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\,\omega t$$

Power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}$ cos 2 ωt of frequency double that of voltage and current waves. For a complete cycle. the average value of $\frac{V_m I_m}{2}$ cos 2ωt is zero.

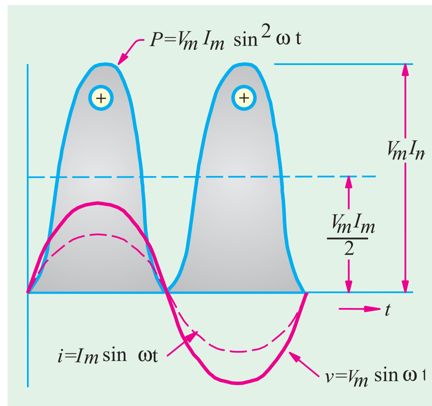

Fig. 11.58

Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $P = V \times I$ watt

where V = r.m.s. value of applied voltage.

I = r.m.s. value of the current.

It is seen from Fig. 11.58 that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

Example 11.38. *A 60-Hz voltage of 115 V (r.m.s.) is impressed on a 100 ohm resistance :*
(*i*) *Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0* **(*ii*)** *Show the voltage and current on a time diagram.* **(*iii*)** *Show the voltage and current on a phasor diagram.*

[Elect Technology. Hyderabad Univ. 1992, Similar Example, U.P. Technical Univ. 2001]

Solution. (*i*) $V_{max} = \sqrt{2}\ V = \sqrt{2} \times 115 = 163$ V

$I_{max} = V_{max}/R = 163/100 = 1.63$ A; $\phi = 0$; $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s

The required equations are : $v(t) = 1.63 \sin 377\ t$ and $i(t) = 1.63\ \sin 377\ t$

(*ii*) and **(*iii*)** These are similar to those shown in Fig. 11.56 and 11.57

11.30. A.C. Through Pure Inductance Alone

Whenever an alternating voltage is applied to a purely inductive coil*, a back e.m.f. is produced due to the self-inductance of the coil. The back e.m.f., at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only. So at every step

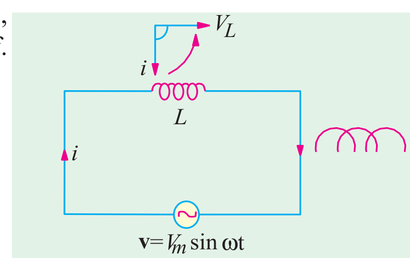

Fig. 11.59

$$v = L\frac{di}{dt}$$

Now $v = V_m \sin \omega t$

$$\therefore \quad V_m \sin \omega t = L\frac{di}{dt} \quad \therefore \quad di = \frac{V_m}{L}\sin \omega t\ dt$$

Integrating both sides, we get $i = \frac{V_m}{L}\int \sin \omega t\ dt$

$$\frac{V_m}{L}(\quad \cos \quad t)$$ ...(constant of integration = 0)

* By purely inductive coil is meant one that has no ohmic resistance and hence no I^2R loss. Pure inductance is actually no attainable, though it is very nearly approached by a coil wound with such thick wire that its resistance is negligible. If it has some actual resistance, then it is represented by a separate equivalent resistance joined in series with it.

$\therefore \quad = \dfrac{V_m}{\omega L} \sin\left(\omega t - \dfrac{\pi}{2}\right) = \dfrac{V_m}{X_L} \sin(\omega t - \pi/2)$ ...*(ii)*

Max. value of i is $I_m = \dfrac{V_m}{\omega L}$ when $\sin\left(\omega t - \dfrac{\pi}{2}\right)$ is unity.

Hence, the equation of the current becomes $i = I_m \sin(\omega t - \pi/2)$.

So, we find that if applied voltage is represented by $v = V_m \sin \omega t$, then current flowing in a *purely* inductive circuit is given by $i = I_m \sin\left(\omega t - \dfrac{\pi}{2}\right)$

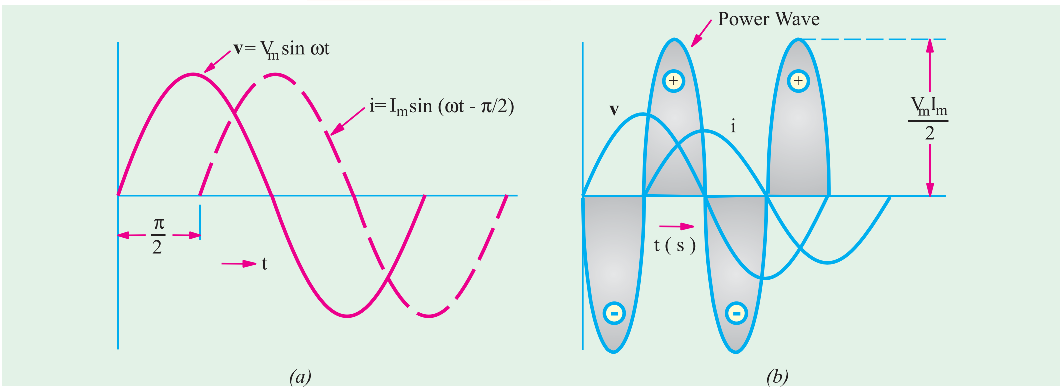

Fig. 11.60

Clearly, the current lags behind the applied voltage by a quarter cycle (Fig. 11.60) or the phase difference between the two is $\pi/2$ with voltage leading. Vectors are shown in Fig. 11.59 where voltage has been taken along the reference axis. We have seen that $I_m = V_m/\omega L = V_m/X_L$. Here '$\omega L$' plays the part of 'resistance'. It is called the (inductive) ***reactance*** X_L of the coil and is given in ohms if L is in henry and ω is in radian/second.

Now, $X_L = \omega L = 2\pi f L$ ohm. It is seen that X_L depends directly on frequency of the voltage. Higher the value of f, greater the reactance offered and ***vice-versa***.

Power

Instantaneous power $= v_i = V_m I_m \sin \quad t \sin \quad t \quad \overline{2} \qquad V_m I_m \sin \quad t . \cos \quad t$* $\quad \dfrac{V_m I_m}{2} \sin 2 \quad t$

Power for whole cycle is $P = -\dfrac{V_m I_m}{2} \displaystyle\int_0^{2\pi} \sin 2\omega t \, dt = 0$

It is also clear from Fig. 11.60 (*b*) that the average demand of power from the supply for a complete cycle is zero. Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

Example 11.39. *Through a coil of inductance 1 henry, a current of the wave-form shown in Fig. 11.61 (a) is flowing. Sketch the wave form of the voltage across the inductance and calculate the r.m.s. value of the voltage.* **(Elect. Technology, Indore Univ.)**

Solution. The instantaneous current $i(t)$ is given by

(i) $0 < t < 1$ second, here slope of the curve is $1/1 = 1$.

$\therefore \quad i = 1 \times t = t$ ampere

(ii) $1 < t < 3$ second, here slope is $\dfrac{1-(-1)}{2} = 1$

* Or $p = \frac{1}{2} E_m I_m [\cos 90° - \cos(2\omega t - 90°)]$. The constant component $= \frac{1}{2} E_m I_m \cos 90° = 0$. The pulsating component is $-\frac{1}{2} E_m I_m \cos(2\omega t - 90°)$ whose average value over one complete cycle is zero.

∴ $t = 1 - (1)\,(t - 1) = 1 - (t - 1) = (2 - t)$ ampere

(iii) $3 < t < 4$ second, here slope is $\dfrac{1-0}{1} = -1$

(b) $i = -1 - (-1)\,(t - 3) = (t - 4)$ ampere

The corresponding voltage are (*i*) $v_1 = Ldi/dt = 1 \times 1 = 1$ V (*ii*) $v_2 = Ldi/dt = 1 \times \dfrac{d}{dt}(2-t) = -1$ V

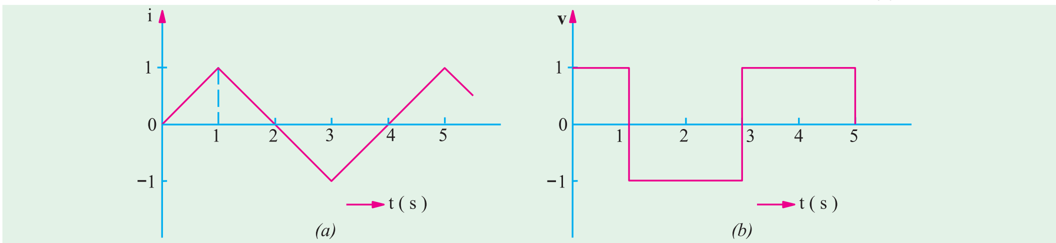

Fig. 11.61

(iii) $v_3 = Ldi/dt = 1 \times \dfrac{d}{dt}(t-4) = 1$ V

The voltage waveform is sketched in Fig. 11.61. Obviously, the r.m.s. value of the symmetrical square voltage waveform is **1 V.**

Example 11.40. *A 60-Hz voltage of 230-V effective value is impressed on an inductance of 0.265 H.*

(i) *Write the time equation for the voltage and the resulting current. Let the zero axis of the voltage wave be at t = 0.* ***(ii)*** *Show the voltage and current on a phasor diagram.* ***(iii)*** *Find the maximum energy stored in the inductance.* **(Elect. Engineering, Bhagalpur Univ.)**

Solution. $V_{max}\ \ \sqrt{2}\,V\ \ \sqrt{2}\ \ 230\,V, f\ \ 60\ Hz.$

$\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s, $X_L = \omega L = 377 \times 0.265 = 100\ \Omega$

(i) The time equation for voltage is $v(t) = 230\sqrt{2}\sin 377\,t$;

$I_{max}\ \ V_{max}/X_L\ \ 230\sqrt{2}/100\ \ 2.3\sqrt{2},\ \ \ 90\ \ (\text{lag})$

∴ Current equation is $i(t) = 2.3\sqrt{2}\sin(377\,t - \pi/2)$ or $= 2.3\sqrt{2}\cos 377\,t$.

(ii) It is shown in Fig. 11.56. ***(iii)*** $E_{max} = \dfrac{1}{2}LI_{max}^2 = \dfrac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2 =$ **1.4 J**

11.31. Complex Voltage Applied to Pure Inductance

In Art. 11.30, the applied voltage was a pure sine wave (*i.e.* without harmonics) given by

$$v = V_m \sin \omega t.$$

The current was given by $i = I_m \sin(\omega t - \pi/2)$

Now, it is applied voltage has a complex form and is given by *

$$v = V_{1m}\sin\omega t + V_{3m}\sin 3\omega t + V_{5m}\sin 5\omega t$$

then the reactances offered to the fundamental voltage wave and the harmonics would be different.

For the fundamental wave, $X_1 = \omega L$. For 3rd harmonic ; $X_3 = 3\omega L$. For 5th harmonic ; $x_5 = 5\omega L$.

Hence, the current would be given by the equation.

$$i = \frac{V_{1m}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_{3m}}{3\omega L}\sin\left(3\omega t - \frac{\pi}{2}\right) + \frac{V_{5m}}{5\omega L}\sin\left(5\omega t - \frac{\pi}{2}\right)$$

* It is assumed that the harmonics have no individual phase differences.

Obviously, the harmonics in the current wave are much smaller than in the voltage wave. For example, the 5th harmonic of the current wave is of only 1/5th of the harmonic in the voltage wave. It means that the self-inductance of a coil has the effect of 'smoothening' current waveform when the voltage waveform is complex *i.e.* contains harmonics.

Example 11.41. *The voltage applied to a purely inductive coil of self-inductance 15.9 mH is given by the equation, v = 100 sin 314 t + 75 sin 942 t + 50 sin 1570 t. Find the equation of the resulting current wave.*

Solution. Here $\omega = 314$ rad/s $\quad \therefore X_1 = \omega L = (15.9 \times 10^{-3}) \times 314 = 5\ \Omega$

$$X_3 = 3\omega L = 3 \times 5 = 15\ \Omega;\ X_5 = 5\omega L = 5 \times 5 = 25\ \Omega$$

Hence, the current equation is

$$i = (100/5)\sin(314\,t - \pi/2) + (75/15)\sin(942t - \pi/2) + (50/25)\sin(1570t - \pi/2)$$

or $\quad i = \mathbf{20\sin(314}t\mathbf{ - \pi/2) + 5\sin(942}t\mathbf{ - \pi/2) + 2\sin(1570}\,t\mathbf{ - \pi/2)}$

11.32. A.C. Through Pure Capacitance Alone

When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction. When reference to Fig. 11.62, let

v = p.d. developed between plates at any instant

q = Charge on plates at that instant.

Then $\quad q = Cv \quad$ ...where C is the capacitance

$\quad = C\,V_m \sin \omega t \quad$ ...putting the value of v.

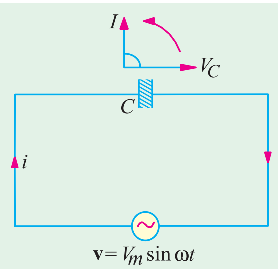

Fig. 11.62

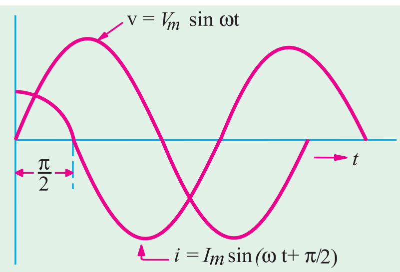

Fig. 11.63

Now, current i is given by the rate of flow of charge.

$$\therefore \quad i \quad \frac{dq}{dt} \quad \frac{d}{dt}(CV_m \sin \quad t) \quad CV_m \cos \quad t \text{ or } i \quad \frac{V_m}{1/\ \ C}\cos \quad t \quad \frac{V_m}{1/\ \ C}\sin \quad t \quad \frac{}{2}$$

Obviously, $I_m \quad \frac{V_m}{1/\ \ C} \quad \frac{V_m}{X_C} \qquad i \quad I_m \sin \quad t \quad \frac{}{2}$

The denominator $X_C = 1/\omega C$ is known as capacitive *reactance* and is in ohms if C is in farad and ω in radian/second. It is seen that if the applied voltage is given by $v = V_m \sin \omega t$, then the current is given by $i = I_m \sin(\omega t + \pi/2)$.

Hence, we find that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 11.63 or phase difference between its voltage and current is $\pi/2$ with the current leading. Vector representation is given in Fig. 11.63. Note that V_c is taken along the reference axis.

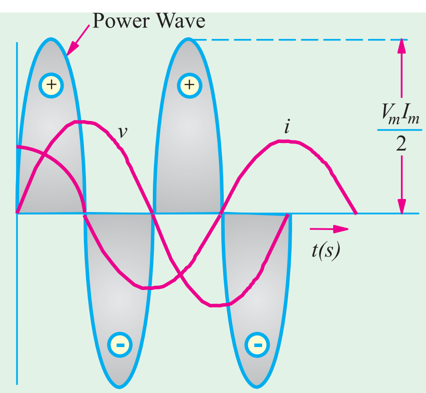

Fig. 11.64

Power. Instantaneous power

$$p = vi = V_m \sin \omega t .\ I_m \sin(\omega t + 90°)$$

$$= V_m I_m \sin \quad t \cos \quad t^{*} \quad \frac{1}{2} V_m I_m \sin 2 \quad t$$

Power for the whole cycle

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

This fact is graphically illustrated in Fig. 11.64. We find that in a purely capacitive circuit**, the average demand of power from supply is zero (as in a purely inductive circuit). Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

Example 11.42. *A 50-Hz voltage of 230 volts effective value is impressed on a capacitance of 26.5 μF. **(a)** Write the time equations for the voltage and the resulting current. Let the zero axis of the voltage wave be at t = 0. **(b)** Show the voltage and current on a time diagram.*

***(c)** Show the voltage and current on a phasor diagram. **(d)** Find the maximum energy stored in the capacitance. Find the relative heating effects of two current waves of equal peak value, the one sinusoidal and the other rectangular in waveform.* **(Elect. Technology, Allahabad Univ. 1991)**

Solution.

$$V_{max} = 230\sqrt{2} \quad 325 \text{ V}$$

$$\omega = 2\pi \times 50 = 314 \text{ rad/s}; \; X_C = 1/\omega C = 10^6/314 \times 26.5 = 120\ \Omega$$

$$I_{max} = V_{max}/X_C = 325/120 = 2.71 \text{ A}, \; \phi = 90° \text{ (lead)}$$

(a) $v(t) = 325 \sin 314\, t;\ i(t) = 2.71 \sin (314t + \pi/2) = 2.71 \cos 314\, t.$

(b) and (*c*) These are shown in Fig. 11.59.

(d) $E_{max} = \frac{1}{2} C V^2{}_{max} = \frac{1}{2}(26.5 \times 10^{-6}) \times 325^2 =$ **1.4 J**

(e) Let I_m be the peak value of both waves.

For sinusoidal wave : $H \propto I^2 R \propto (I_m/\sqrt{2})^2\, R \propto I_m^2\, R/2$. For rectangular wave : $H \propto I_m^2\, R$ - Art. 12.15.

$$\frac{H \text{ rectangular}}{H \text{ sinusoidal}} = \frac{I_m^2\, R}{I_m^2\, R/2} = \mathbf{2}$$

Example 11.43. *A 50-μF capacitor is connected across a 230-V, 50-Hz supply. Calculate **(a)** the reactance offered by the capacitor **(b)** the maximum current and **(c)** the r.m.s. value of the current drawn by the capacitor.*

Solution. (a) $X_C \quad \frac{1}{\quad C} \quad \frac{1}{2 \quad f C} \quad \frac{1}{2 \quad 50 \quad 50 \quad 10^{\quad 6}}$ **63.6 Ω**

(c) Since 230 V represents the r.m.s. value,

$\therefore \quad I_{r.m.s.} = 230/X_c = 230/63.6 =$ **3.62 A (b)** $I_m = I_{r.m.s.} \times \sqrt{2} = 3.62 \times \sqrt{2} =$ **5.11 A**

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* Or power $p = \frac{1}{2} E_m I_m [\cos 90° - \cos (2\omega t - 90°)]$. The constant component is again zero. The pulsating component averaged over one complete cycle is zero.

** By pure capacitor is meant one that has neither resistance nor dielectric loss. If there is loss in a capacitor, then it may be represented by loss in (*a*) high resistance joined in parallel with the pure capacitor or (*b*) by a comparatively low resistance joined in series with the pure capacitor. But out of the two alternatives usually. (*a*) is chosen (Art. 13.8).

Example 11.44. *The voltage applied across 3-branched circuit of Fig. 11.65 is given by v = 100 sin (5000t + π/4). Calculate the branch currents and total current.*

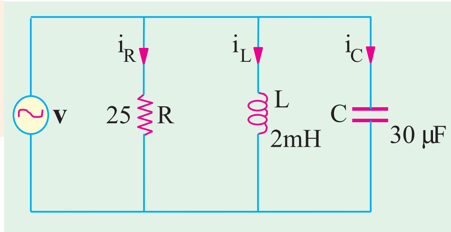

Fig. 11.65

Solution. The total instantaneous current is the vector sum of the three branch currents.

$$i_r = i_R + i_L + i_C.$$

Now $i_R = v/R = 100 \sin (5000\,t + \pi/4)/25$

$= 4 \sin (5000\,t + \pi/4)$

$$i_L = \frac{1}{L}\int v\,dt = \frac{10^3}{2}\int 100 \sin\left(5000t + \frac{\pi}{4}\right) dt$$

$$= \frac{10^3 \times 100}{2}\left[\frac{-\cos(5000\,t + \pi/4)}{5000}\right] = -10 \cos (5000\,t + \pi/4)$$

$$i_C = C\frac{dv}{dt} = C.\frac{d}{dt}[100 \sin (5000\,t + \pi/4)]$$

$= 30 \times 10^{-6} \times 100 \times 5000 \times \cos (5000\,t + \pi/4) = 15 \cos (5000\,t + \pi/4)$

$i_r = 4 \sin (5000\,t + \pi/4) - 10 \cos (5000\,t + \pi/4) + 15 \cos (5000t + \pi/4)$

$= \mathbf{4 \sin (5000\,t + \pi/4) + 5 \cos (5000\,t + \pi/4)}$

Tutorial Problems No. 11.2

1. Define r.m.s. and average value as applied to ac voltage, prove that in pure inductive circuit, current lags behind applied voltage at an angle 90^0.
(Down waveform) ***(Nagpur University, Summer 2002)***
2. Find the rms value, average value, form factor and peak factor for the waveform shown in figure. ***(Nagpur University, Winter 2003)***

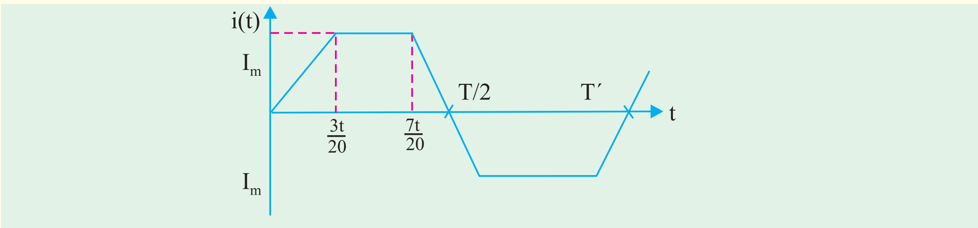

Fig. 11.66

3. Derive an expression for the instantaneous value of alternating sinusoidal e.m.f. in terms of its maximum value, angular freq. and time. ***(Gujrat University, June/July 2003)***
4. Prove that average power consumption in pure inductor is zero when a.c. voltage is applied. ***(Gujrat University, June/July 2003)***
5. Define and explain the following :
(*i*) time period (*ii*) amplitude (*iii*) phase difference. ***(Gujrat University, June/July 2003)***
6. What is the r.m.s value of an a.c. quantity? Obtain expression for the r.m.s. value of a sinusoidal current in terms of its maximum value.
(V.T.U., Belgaum Karnataka University, February 2002)
7. Deduce an expression for the average power in a single phase series R.L. circuit and therefrom explain the term power factor.
(V.T.U., Belgaum Karnataka University, February 2002)

8. Derive an expression for the RMS value of a sine wave.
(V.T.U., Belgaum Karnataka University, Summer 2002)

9. With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field. *(V.T.U., Belgaum Karnataka University, Summer 2003)*

10. Derive expressions for average value and RMS value of a sinusoidally varying AC voltage.
(V.T.U., Belgaum Karnataka University, Summer 2003)

11. A circuit having a resistance of 12Ω, an inductance of 0.15 H and a capacitance of 100μ*f* in series is connected across a 100V, 50Hz supply. Calculate the impedance, current, the phase difference between the current and supply voltage.
(V.T.U., Belgaum Karnataka University, Summer 2003)

12. Two circuits with impedances of $Z_1 = 10 + j15\Omega$ and $Z_2 = 6 - j8\Omega$ are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch?
(V.T.U., Belgaum Karnataka University, Summer 2003)

13. Show that the power consumed in a pure inductance is zero.
(U.P. TechnicalUniversity 2002) (RGPV Bhopal 2002)

14. What do you understand by the terms power factor, active power and reactive power?
(U.P. TechnicalUniversity 2002) (RGPV Bhopal 2002)

15. Current flowing through each line.
(RGPV Bhopal December 2002)

16. Distinguish between (*i*) apparent power (*ii*) active power and (*iii*) reactive power in A.C. circuits.
(U.P. TechnicalUniversity 2002) (RGPV Bhopal June 2003)

OBJECTIVE TESTS – 11

1. An a.c. current given by $i = 14.14 \sin(\omega t + \pi/6)$ has an r.m.s value of — amperes.
(*a*) 10
(*b*) 14.14
(*c*) 1.96
(*d*) 7.07
and a phase of — degrees.
(*e*) 180
(*f*) 30
(*g*) – 30
(*h*) 210

2. If $e_1 = A \sin \omega t$ and $e_2 = B \sin(\omega t - \phi)$, then
(*a*) e_1 lags e_2 by θ
(*b*) e_2 lags e_1 by θ
(*c*) e_2 leads e_2 by θ
(*d*) e_1 is in phase with e_2

3. From the two voltage equations $e_A = E_m \sin 100\pi t$ and $e_B = E_m \sin(100\pi t + \pi/6)$, it is obvious that
(*a*) *A* leads *B* by 30°
(*b*) *B* achieves its maximum value 1/600 second before *A* does.
(*c*) *B* lags behind *A*
(*d*) *A* achieves its zero value 1/600 second before *B*.

4. The r.m.s. value of a half-wave rectified current is 10A, its value for full-wave rectification would be — amperes.
(*a*) 20
(*b*) 14.14
(*c*) 20/π
(*d*) 40/π

5. A resultant current is made of two components : a 10 *A* d.c. component and a sinusoidal component of maximum value 14.14 A. The average value of the resultant current is — amperes.
(*a*) 0 (*b*) 24.14
(*c*) 10 (*d*) 4.14
and r.m.s. value is — amperes.
(*e*) 10 (*f*) 14.14
(*g*) 24.14 (*h*) 100

6. The r.m.s. value of sinusoidal a.c. current is equal to its value at an angle of — degree
(*a*) 60 (*b*) 45 (*c*) 30 (*d*) 90

7. Two sinusoidal currents are given by the equations : $i_2 = 10 \sin(\omega t + \pi/3)$ and $i_2 = 15 \sin(\omega t - \pi/4)$. The phase difference between them is — degrees.
(*a*) 105
(*b*) 75
(*c*) 15
(*d*) 60

8. As sine wave has a frequency of 50 Hz. Its angular frequency is — radian/second.

(*a*) 50/π
(*b*) 50/2 π
(*c*) 50 π
(*d*) 100 π

9. An a.c. current is given by i = 100 sin 100. It will achieve a value of 50 A after — second.

(*a*) 1/600
(*b*) 1/300
(*c*) 1/1800
(*d*) 1/900

10. The reactance offered by a capacitor to alternating current of frequency 50 Hz is 10Ω. If frequency is increased to 100 Hz reactance becomes—ohm.

(*a*) 20 (*b*) 5
(*c*) 2.5 (*d*) 40

11. A complex current wave is given by $i = 5 + 5 \sin 100\,\pi t$ ampere. Its average value is — ampere.

(*a*) 10 (*b*) 0
(*c*) $\sqrt{50}$ (*d*) 5

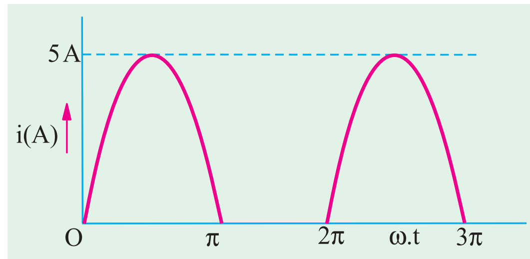

Fig. 11.67

12. The current through a resistor has a waveform as shown in Fig. 11.67. The reading shown by a moving coil ammeter will be— ampere.

(*a*) $5/\sqrt{2}$ (*b*) $2.5/\sqrt{2}$
(*c*) 5/π (*d*) 5

(Principles of Elect. Engg. Delhi Univ.)

ANSWERS

1. *a*, *f* **2.** *b* **3.** *b* **4.** *b* **5.** *c*, *f* **6.** *b* **7.** *a* **8.** *d* **9.** *a* **10.** *b*
11. *d* **12.** *c*

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