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CHAPTER 17

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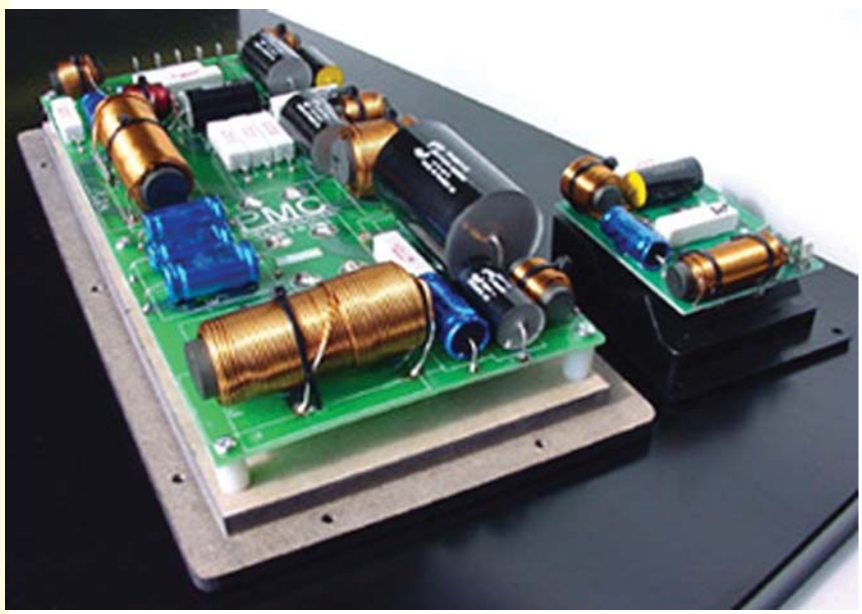
A.C. FILTER NETWORKS



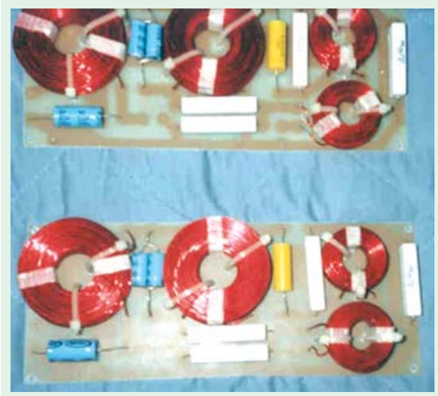
By using various combinations of resistors, inductors and capacitors, we can make circuits that have the property of passing or rejecting either low or high frequencies or bands of frequencies

17.1. Introduction

The reactances of inductors and capacitors depend on the frequency of the a.c. signal applied to them. That is why these devices are known as frequency-selective. By using various combinations of resistors, inductors and capacitors, we can make circuits that have the property of passing or rejecting either low or high frequencies or bands of frequencies. These frequency-selective networks, which alter the amplitude and phase characteristics of the input a.c. signal, are called filters. Their performance is usually expressed in terms of how much attenuation a band of frequencies experiences by passing through them. Attenuation is commonly expressed in terms of decibels (dB).

17.2. Applications

A.C. filters find application in audio systems and television etc. Bandpass filters are used to select frequency ranges corresponding to desired radio or television station channels. Similarly, bandstop filters are used to reject undesirable signals that may contaminate the desirable signal. For example, low-pass filters are used to eliminate undesirable hum in d.c. power supplies.



Closeup of a crossover network

No loudspeaker is equally efficient over the entire audible range of frequencies. That is why high-fidelity loudspeaker systems use a combination of low-pass, high-pass and bandpass filters (called crossover networks) to separate and then direct signals of appropriate frequency range to the different loudspeakers making up the system. Fig. 17.1 shows the output circuit of a high-fidelity audio amplifier, which uses three filters to separate, the low, mid-range and high frequencies, for feeding them to individual loudspeakers, best able to reproduce them.

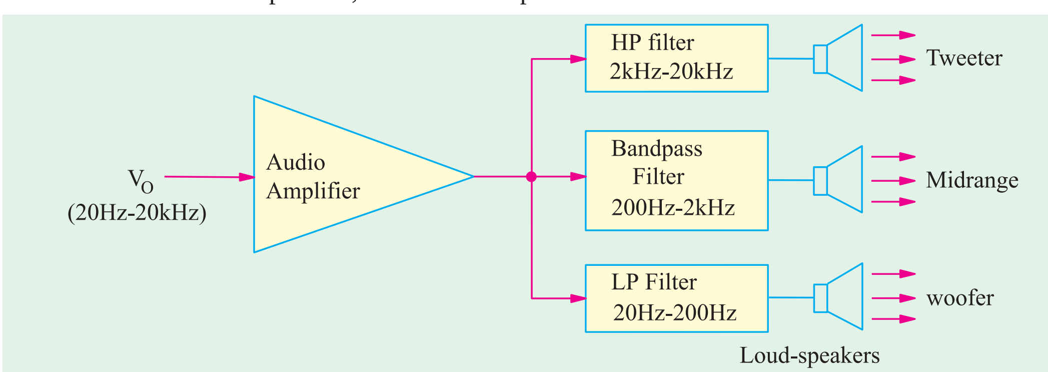


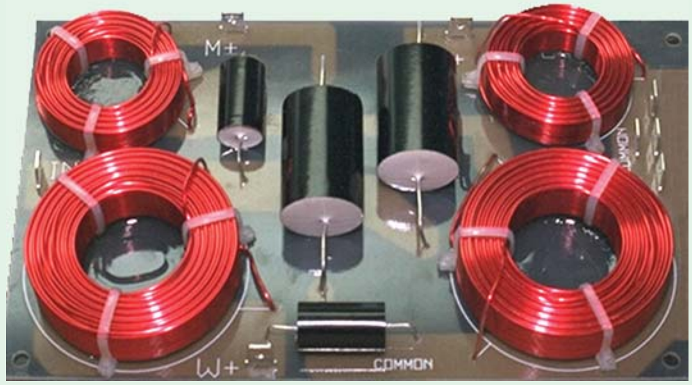
Fig. 17.1

17.3. Different Types of Filters

A.C. filter networks are divided into two major categories: ***(i)*** active networks and ***(ii)*** passive networks.

Active filter networks usually contain transistors and/or operational amplifiers in combination with *R*, *L* and *C* elements to obtain the desired filtering effect. These will not be discussed in this book. We will consider passive filter networks only which usually consist of series-parallel combinations of *R*, *L* and *C* elements. There are four types of such networks, as described below:

1. Low-Pass Filter. As the name shows, it allows only low frequencies to pass through, but attenuates (to a lesser or greater extent) all higher frequencies. The maximum frequency which it allows to pass through, is called cutoff frequency f_c (also called break frequency). There are R_L and R_C low-pass filters.



Three way passive crossover network

2. High-Pass Filter. It allows signals with higher frequencies to pass from input to output while rejecting lower frequencies. The minimum frequency it allows to pass is called cutoff frequency f_c. There are R_L and R_C high-pass filters.

High pass filter

3. Bandpass Filter. It is a resonant circuit which is tuned to pass a certain band or range of frequencies while rejecting all frequencies below and above this range (called passband).

4. Bandstop Filter. It is a resonant circuit that rejects a certain band or range of frequencies while passing all frequencies below and above the rejected band. Such filters are also called wavetraps, notch filters or band-elimination, band-separation or band-rejection filters.

17.4. Octaves and Decades of Frequency

A filter's performance is expressed in terms of the number of decibels the signal is increased or decreased per frequency octave or frequency decade. An octave means a doubling or halving of a frequency whereas a decade means tenfold increase or decrease in frequency.

17.5. The Decibel System

These system of logarithmic measurement is widely used in audio, radio, TV and instrument industry for comparing two voltages, currents or power levels. These levels are measured in a unit called bel (B) or decibel (dB) which is $1/10^{th}$ of a bel.

Suppose we want to compare the output power P_0 of a filter with its input power P_i. The power level change is

$$= 10 \log_{10}^{th} P_0/P_i \text{ dB}$$

It should be noted that dB is the unit of power change (*i.e.* increase or decrease) and not of power itself. Moreover, 20 dB is not twice as much power as 10 dB.

However, when voltage and current levels are required, then the expressions are:

$$\text{Current level} = 20 \log_{10} (I_0/I_i) \text{ dB}$$

$$\text{Similarly, voltage level} = 20 \log V_0/V_i \text{ dB}$$

Obviously, for power, we use a multiplying factor of 10 but for voltages and currents, we use a multiplying factor of 20.

17.6. Value of 1 dB

It can be proved that 1 dB represents the log of two powers, which have a ratio of 1.26.

$$1\ \text{dB} = 10\log_{10}(P_2/P_1) \text{ or } \log_{10}(P_2/P_1) \quad 0.1 \text{ or } \frac{P_2}{P_1} \quad 10^{0.1} \quad 1.26$$

Hence, it means that + 1 dB represents an increase in power of 26%.

Example 17.1. *The input and output voltages of a filter network are 16 mV and 8 mV respectively. Calculate the decibel level of the output voltage.*

Solution. Decibel level = $20\log_{10}(V_0/V_i)$dB = $-20\log_{10}(V_i/V_0)$ dB = $-20\log_{10}(16/8) = -6$ dB. Whenever voltage ratio is less than 1, its log is negative which is often difficult to handle. In such cases, it is best to invert the fraction and then make the result negative, as done above.

Example 17.2. *The output power of a filter is 100 mW when the signal frequency is 5 kHz. When the frequency is increased to 25 kHz, the output power falls to 50 mW. Calculate the dB change in power.*

Solution. The decibel change in power is

$= 10\log_{10}(50/100) = -10\log_{10}(100/50) = -10\log_{10} 2 = -10 \times 0.3 = -3$ dB

Example 17.3. *The output voltage of an amplifier is 10 V at 5 kHz and 7.07 V at 25 kHz. What is the decibel change in the output voltage?*

Solution. Decibel change = $20\log_{10}(V_0/V_i) = 20\log_{10}(7.07/10) = -20\log_{10}(10/7.07)$

$= -20\log_{10}(1.4/4)$

$= -20 \times 0.15 = -3$ dB

17.7. Low-Pass RC Filter

A simple low-pass *RC* filter is shown in Fig. 17.2 (*a*). As stated earlier, it permits signals of low frequencies upto f_c to pass through while attenuating frequencies above f_c. The range of frequencies upto f_c is called the passband of the filter. Fig. 17.2 (*b*) shows the frequency response curve of such a filter. It shows how the signal output voltage V_0 varies with the signal frequency. As seen at f_c, output signal voltage is reduced to 70.7% of the input voltage. The output is said to be – 3 dB at f_c. Signal outputs beyond f_c roll-off or attenuate at a fixed rate of – 6 dB/octave or – 20 dB/decade. As seen from the frequency-phase response curve of Fig. 17.2 (*c*), the phase angle between V_0 and V_i is 45° at cutoff frequency f_c.

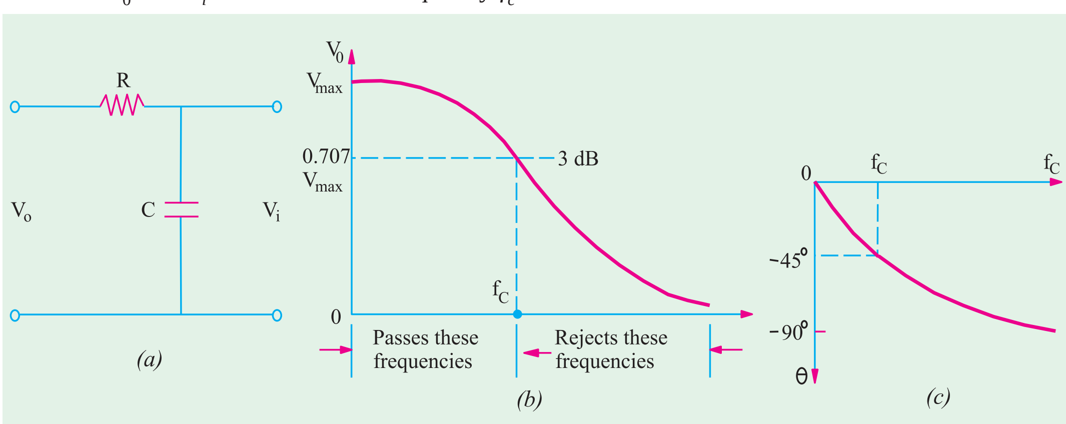


Fig. 17.2

By definition cutoff frequency f_c occurs where **(*a*)** V_0 = 70.7% V_i *i.e.* V_0 is – 3 dB down from V_i **(*b*)** $R = X_c$ and $V_R = V_C$ in magnitude. **(*c*)** The impedance phase angle θ = – 45°. The same is the angle between V_0 and V_i.

As seen, the output voltage is taken across the capacitor. Resistance *R* offers fixed opposition to frequencies but the reactance offered by capacitor *C* decreases with increase in frequency. Hence, low-frequency signal develops over *C* whereas high-frequency signals are grounded. Signal frequencies above f_c develop negligible voltage across *C*. Since *R* and *C* are in series, we can find the low-frequency output voltage V_0 developed across *C* by using the voltage-divider rule.

$$\therefore \qquad V_0 \quad V_i \frac{jX_C}{R \quad jX_C} \text{ and } f_c \quad \frac{1}{2 \quad CR}$$

17.8. Other Types of Low-Pass Filters

There are many other types of low-pass filters in which instead of pure resistance, series chokes are commonly used alongwith capacitors.

(*i*) Inverted–L Type. It is shown in Fig. 17.3 (*a*). Here, inductive reactance of the choke blocks higher frequencies and *C* shorts them to ground. Hence, only low frequencies below f_c (for which *X* is very low) are passed without significant attenuation.

(*ii*) T-Type. It is shown in Fig. 17.3 (*b*). In this case, a second choke is connected on the output side which improves the filtering action.

(*iii*) π-Type. It is shown in Fig. 17.3 (*c*). The additional capacitor further improves the filtering action by grounding higher frequencies.

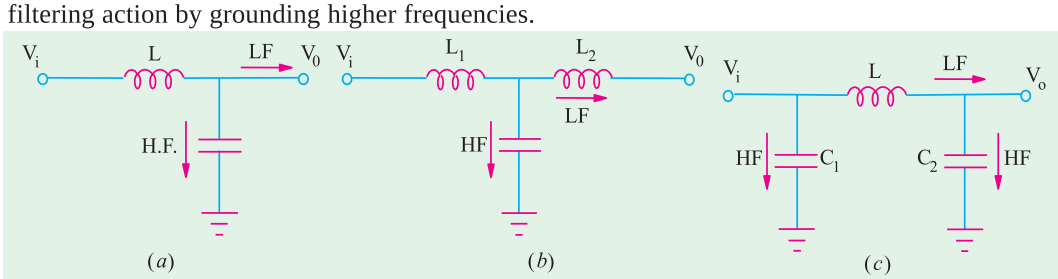


Fig. 17.3

It would be seen from the above figures that choke is always connected in series between the input and the output and capacitors are grounded in parallel. The output voltage is taken across the capacitor.

Example 17.4. *A simple low-pass RC filter having a cutoff frequency of 1 kHz is connected to a constant ac source of 10 V with variable frequency. Calculate the following :*

(*a*) value of *C* if $R = 10\,k\Omega$ **(*b*)** output voltage and its decibel level when

(*i*) $f = f_c$ **(*ii*)** $f = 2\,f_c$ and **(*iii*)** $f = 10\,f_c$.

Solution. (*a*) At f_c, $r = X_c = 1/2\pi f_c$ or $C = 1/2\pi \times 1 \times 10^3 \times 10 \times 10^3 = 15.9 \times 10^{-9} = 15.9$ nF

(*b*) (*i*) $f = f_c = 1\,\text{kHz}$. Now, $-jX_c = R = -j10 = 10\angle -90°\ \Omega$

$$\therefore \qquad V_0 = V_i \frac{-jX_c}{R - jX_c} = 10 \frac{-j10}{10 - j10} = 7.07\angle -45°$$

Output decibel level = 20 $\log_{10}$ (v_0/v_i) = –20 $\log_{10}$ (V_i/V_0) = –20 $\log_{10}$ (10/0.707)= – 3 dB

(*ii*) Here, $f = 2\,f_c$ = 2 kHz *i.e.* octave of f_c. Since capacitive reactance is inversely proportional to frequency, $\therefore X_{c2} = C_{c1}(f_1/f_2) = -j10(1/2) = -j5 = 5\angle -90°\ k\Omega$

$$\therefore \quad V_0 = \frac{5\angle -90°}{10 - j5} = \frac{5°\angle -90°}{11.18\angle -26.6°} = 4.472\angle -63.4°$$

Decibel level = –20 $\log_{10}$ (V_i/V_0) = –20 $\log_{10}$ (10/4.472) = –6.98 dB

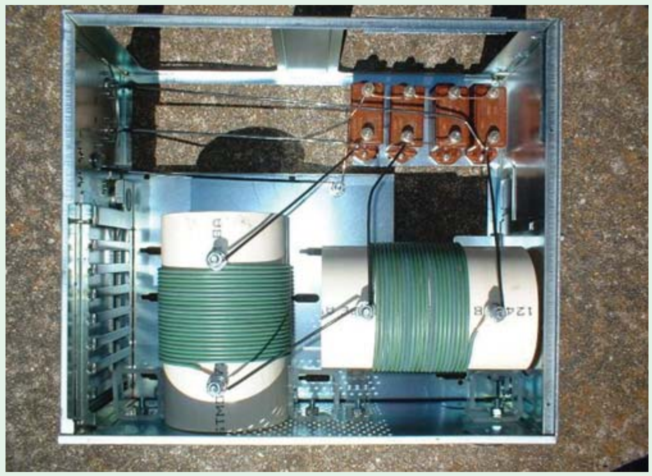
(iii) $X_{c3} = X_{c1}(f_1/f_3)$ = – j10 (1/10) = j1 = 1 ∠ –90° kΩ

$$\therefore \quad V_0 = 10\frac{10\angle -90°}{10 - j1} = 1\angle -84.3°$$

Decibel level = –20 $\log_{10}$(10/1) = –20 dB

17.9. Low-Pass RL Filter

It is shown in Fig. 17.4 (*a*). Here, coil offers high reactance to high frequencies and low reactance to low frequencies. Hence, low frequencies upto f_c can pass through the coil without much opposition. The output voltage is developed across *R*. Fig 17.4 (*b*) shows the frequency-output response curve of the filter. As seen at f_c, $V_0 = 0.707\ V_i$ and its attenuation level is –3 *dB* with respect to V_0 *i.e.* the voltage at *f* = 0.



A view of the inside of the low-pass filter assembly

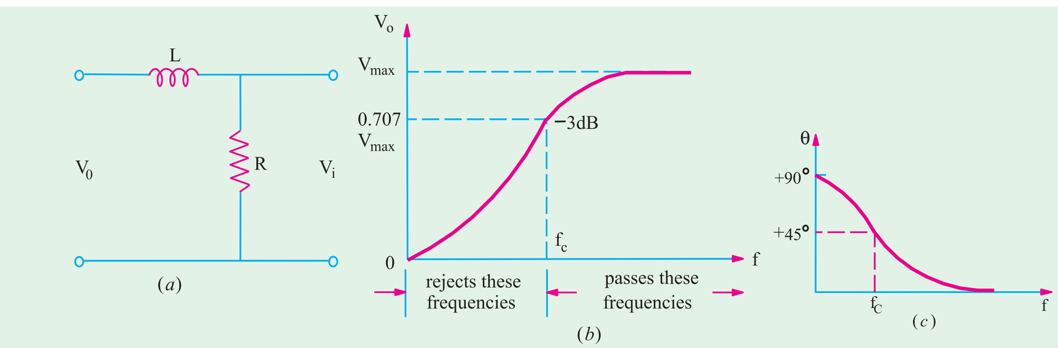


Fig. 17.4

However, it may be noted that being an *RL* circuit, the impedance phase angle is +45° (and not –45° as in low-pass *RC* filter). Again at f_c, $R = X_L$.

Using the voltage-divider rule, the output voltage developed across *R* is given by

$$V_0 = V_i \frac{R}{R + jX_L} \quad \text{and} \quad f_c = \frac{R}{2\pi L}$$

Example 17.5. *An ac signal having constant amplitude of 10 V but variable frequency is applied across a simple low-pass RL circuit with a cutoff frequency of 1 kHz. Calculate* **(a)** *value of L if R = 1 kΩ* **(b)** *output voltage and its decibel level when* **(i)** $f = f_c$ *and* **(iii)** $f = 10\ f_c$.

Solution. (a) $L \quad R/2 \quad f_c \quad 1 \quad 10^3/2 \quad 3.14 \quad 10^3 \quad 159.2$ mH

(b) (i) $f = f_c = 1k\,Hz\;;\, jX_L = R = j1; V_0 = 10\dfrac{1}{(1+j1)} = 7.07\angle -45°\,V$

Decibel decrease = –20 $\log_{10}$ (V_i / V_b) = –20 $\log_{10}$ 10/7.07 = –3 dB

(*ii*) $f = 2f_c$ = 2 kHz. Since X_L varies directly with

f, $X_{L2} = X_{L1}\,(f_2/f_1) = 1 \times 2/1 = 2\ \text{k}\Omega$

$$\therefore \qquad V_0 = 10\frac{1}{(1+j2)} = \frac{10}{2.236\angle 63.4°} = 4.472\angle -63.4°$$

Decibel decrease = –20 $\log_{10}$ (10/4.472) = – 6.98 dB

(*iii*) $f = 10\,f_c$ = 10 kHz ; $X_{L3} = 1 \times 10/1 = 10\,\Omega$, $V_0 = \dfrac{1}{(1+j10)} = 1\angle -84.3°$

Decibel decrease = – 20 $\log_{10}$(10/1) = –20 dB

17.10. High-Pass RC Filter

It is shown in Fig. 17.5 (*a*). Lower frequencies experience considerable reactance by the capacitor and are not easily passed. Higher frequencies encounter little reactance and are easily passed. The high frequencies passing through the filter develop output voltage V_0 across R. As seen from the frequency response of Fig. 13.5 (*b*), all frequencies above f_c are passed whereas those below it are attenuated. As before, f_c corresponds to –3 dB output voltage or half-power point. At f_c, $R = X_c$ and the phase angle between V_0 and V_i is +45° as shown in Fig. 17.5 (*c*). It may be noted that high-pass *RC* filter can be obtained merely by interchanging the positions of *R* and *C* in the low-pass *RC* filter of Fig. 17.5 (*a*).

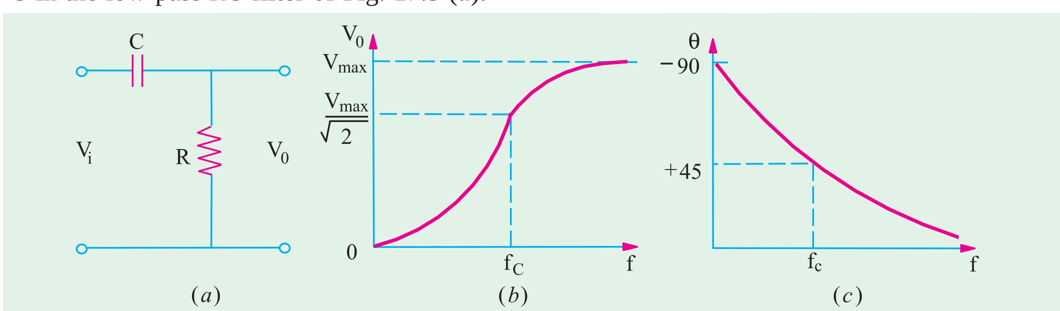


Fig. 17.5

Since *R* and *C* are in series across the input voltage, the voltage drop across *R*, as found by the voltage-divider rule, is

$$V_0 \quad V_i\frac{R}{R \quad jX_c} \text{ and } f_c \quad \frac{1}{2 \quad CR}$$

A very common application of the series capacitor high-pass filter is a coupling capacitor between two audio amplifier stages. It is used for passing the amplified audio-signal from one stage to the next and simultaneously block the constant d.c. voltage.

Other high-pass RC filter circuits exist besides the one shown in Fig. 17.5 (*a*). These are shown in Fig 17.6.

High-pass filter

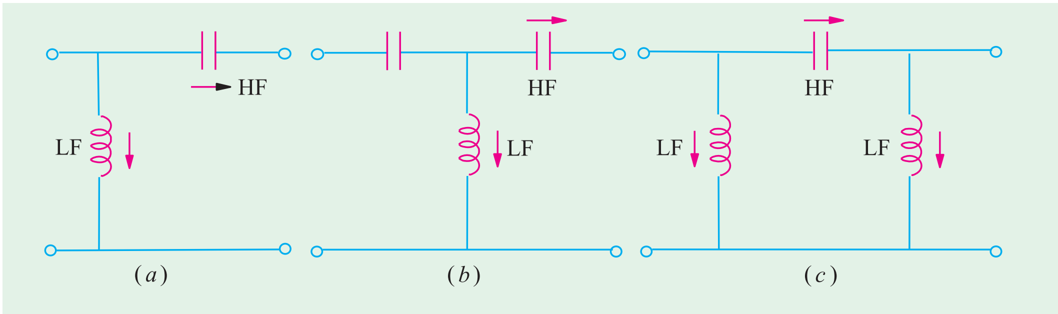


Fig. 17.6

***(i)* Inverted-L Type.** It is so called because the capacitor and inductor from an upside down *L*. It is shown in Fig. 17.6 (*a*). At lower frequencies, X_C is large but X_L is small. Hence, most of the input voltage drops across X_C and very little across X_L. However, when the frequency is increased, X_C becomes less but X_L is increased thereby causing the output voltage to increase. Consequently, high frequencies are passed while lower frequencies are attenuated.

***(ii)* T-Type.** It uses two capacitors and a choke as shown in Fig. 17.6 (*b*). The additional capacitor improves the filtering action.

***(iii)* π -Type.** It uses two inductors which shunt out the lower frequencies as shown in Fig. 17.6 (*c*).

It would be seen that in all high-pass filter circuits, capacitors are in series between the input and output and the coils are grounded. In fact, capacitors can be viewed as shorts to high frequencies but as open to low frequencies. Opposite is the case with chokes.

17.11. High-Pass RL Filter

It is shown in Fig. 17.7 and can be obtained by 'swapping' position of *R* and *L* in the low-pass *RL* circuit of Fig. 17.4 (*a*). Its response curves are the same as for high-pass *RC* circuit and are shown in Fig. 17.5 (*b*) and (*c*).

As usual, its output voltage equals the voltage which drops across X_L. It is given by

$$V_0 \quad V_i \frac{j X_L}{R \quad j X_L} \quad \text{and} \quad f_c \quad \frac{R}{2 \quad L}$$

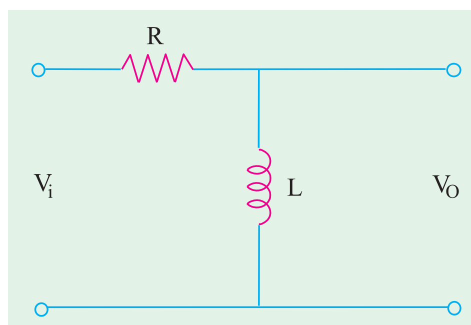


Fig. 17.7

Example 17.6. *Design a high-pass RL filter that has a cutoff frequency of 4 kHz when R = 3 kΩ. It is connected to a 10∠0° V variable frequency supply. Calculate the following :*

***(a)** Inductor of inductance L but of negligible resistance **(b)** output voltage V_0 and its decibel decrease at*

***(i)** f = 0 **(ii)** $f = f_c$ **(iii)** 8 kHz and **(iv)** 40 kHz*

Solution. (*a*) $L = R/2\pi f_c = 3/2 \quad \pi \times 4 = 119.4$ mH

(*b*) (*i*) At $f = 0$; $X_L = 0$ *i.e.* inductro acts as a short-circuit across which no voltage develops. Hence, $V_0 = 0$ V as shown in Fig. 17.74.

(*ii*) $f = f_c = 4$ kHz ; $X_L = R$. ∴ $jX_L = j3 = 3\angle 90°$ kΩ

$$\therefore \quad V_0 \quad V_i \frac{jX_L}{R \quad jX_L} \quad 10 \quad 0 \quad \frac{3 \quad 90}{3 \quad j3} \quad \frac{30 \quad 90}{4.24 \quad 45} \quad 7.07 \quad 45 \text{ V}$$

Decibel decrease = – 20 $\log_{10}(10/7.07)$ = – 3 dB

(iii) $f = 2f_c$ = 8 kHz. $X_{L2} = 2 \times j3 = j6$ kΩ

$$\therefore \quad V_0 = 10\angle 0° \frac{6\angle 90°}{3+j6} = \frac{60\angle 90°}{6.7\angle 63.4°} = 8.95\angle 26.6°\text{ V}$$

Decibel decrease = – 20 $\log_{10}(10/8.95)$ = – 0.96 dB

(iv) $f = 10f_c$ = 40 kHz; $X_{L3} = 10 \times j3 = j30$ kΩ

$$\therefore \quad B_0 = 10\frac{j30}{3+j30} = \frac{300\angle 90°}{30.15\angle 84.3°} = 19.95\angle 5.7°\text{ V}$$

Decibel decrease = – 20 $\log_{10}(10/9.95)$ = 0.04 dB

As seen from Fig. 17.7, as frequency is increased, V_0 is also increased.

17.12. R-C bandpass Filter

It is a filter that allows a certain band of frequencies to pass through and attenuates all other frequencies below and above the passband. This passband is known as the bandwidth of the filter. As seen, it is obtained by cascading a high-pass *RC* filter to a low-pass *RC* filter. It is shown in Fig. 17.8 alongwith its response curve. The passband of this filter is given by the band of frequencies lying between f_{c1} and f_{c2}. Their values are given by

Bandpass filters

$$f_{c1} = 1/2\pi\, C_1R_1 \text{ and } \quad f_{c_2} \quad 1/2 \quad C_2R_2$$

The ratio of the output and input voltages is given by

$$\frac{V_0}{V_i} = \frac{R_1}{R_1 - jX_{C1}} \quad \text{... from } f_1 \text{ to } f_{C1};$$

$$= \frac{-jX_{C2}}{R_2 - jX_{C2}} \quad \text{... from } f_{c_2} \text{ to } f_2$$

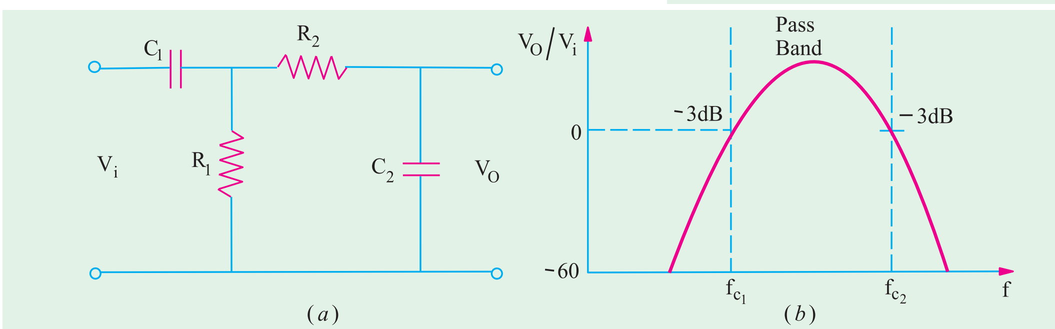


Fig. 17.8

17.13. R-C Bandstop Filter

It is a series combination of low-pass and high-pass RC filters as shown in Fig. 17.9 (*a*). In fact, it can be obtained by reversing the cascaded sequence of the RC bandpass filter. As stated earlier, this filter attenuates a single band of frequencies and allows those on either side to pass through. The stopband is represented by the group of frequencies that lie between f_1 and f_2 where response is below – 60 dB.

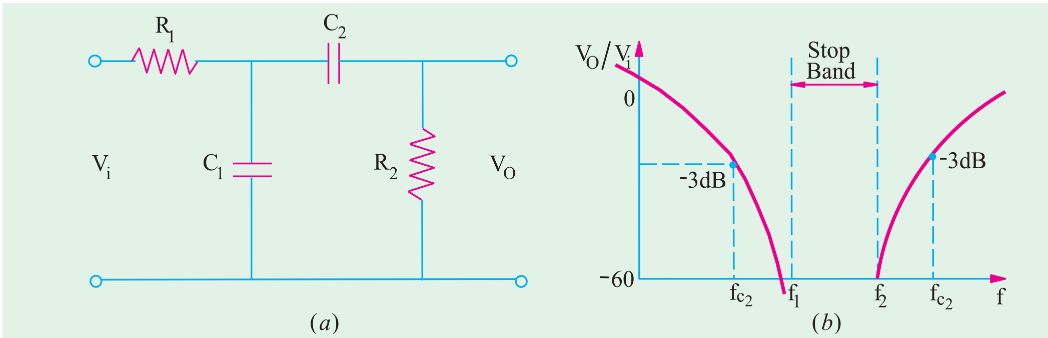


Fig. 17.9

For frequencies from f_{c1} to f_1, the following relationships hold good :

$$\frac{V_0}{V_i} \quad \frac{j\,X_{C1}}{(R_1 \quad j\,X_{C1})} \quad \text{and} \quad f_{c_1} \quad \frac{1}{2 \quad C_1 R_1}$$

For frequencies from f_2 to f_{c2}, the relationships are as under :

$$\frac{V_0}{V_i} \quad \frac{R_2}{(R_2 \quad j\,X_{C2})} \quad \text{and} \quad f_{C2} \quad \frac{1}{2 \quad C_2 R_2}$$

In practices, several low-pass *RC* filter circuits cascaded with several high-pass *RC* filter circuits which provide almost vertical roll-offs and rises. Moreover, unlike *RL* filters, *RC* filters can be produced in the form of large-scale integrated circuits. Hence, cascading is rarely done with *RL* circuits.

A window filter contains one band pass and one low-pass or one high-pass and is used for filtering out unwanted channels, in CATV reception systems or for application cablenet or other communication systems

17.14. The – 3 dB Frequencies

The output of an a.c. filter is said to be down 3 dB or –3 dB at the cutoff frequencies. Actually at this frequency, the output voltage of the circuit is 70.7% of the maximum input voltage as shown in Fig. 17.10 (*a*) for low-pass filter and in Fig. 17.10 (*b*) and (*c*) for high-pass and bandpass filters respectively. Here, maximum voltage is taken as the 0 dB reference.

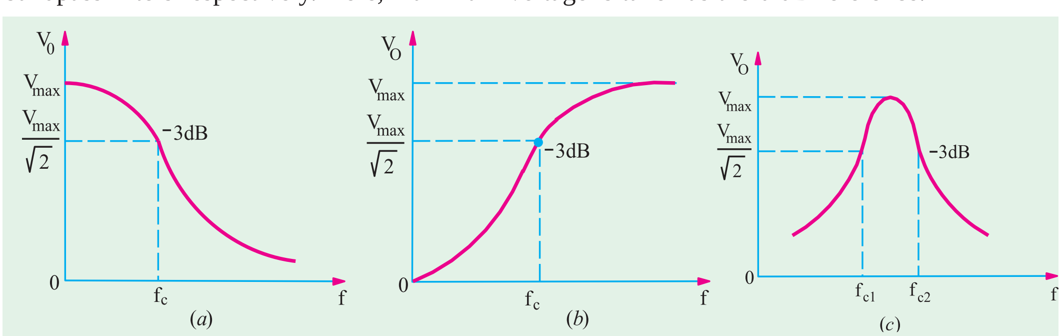


Fig. 17.10

It can also be shows that the power output at the cutoff frequency is 50% of that at zero frequency in the case of low-pass and high-pass filters and of that at f_0 in case of resonant-circuit filter.

17.15. Roll-off of the Response Curve

Gradual decreasing of the output of an a.c. filter is called roll-off. The dotted curve in Fig. 17.11 (*a*) shows an actual response curve of a low-pass *RC* filter. The maximum output is defined to be zero dB as a reference. In other words, 0 dB corresponds to the condition when $V_0 = v_i$ because 20 $\log_{10}$ V_0/V_i = 20 log 1 = dB. As seen, the output drops from 0 dB to – 3 dB at the cutoff frequency and then continues to decrease at a fixed rate. This pattern of decrease is known as the roll-off of the frequency response. The solid straight line in Fig. 17.11 (*a*) represents an ideal output response that is considered to be 'flat' and which cuts the frequency axis at f_c.

The roll-off for a basic IRC *or IRL* filter is 20 dB/decade or 6 dB/octave. Fig. 17.11 (*b*) shows the frequency response plot on a semi-log-scale where each interval on the horizontal axis represents a tenfold increase in frequency. This response curve is known as Bode plot. Fig. 17.11 (*c*) shown the Bode plot for a high-pass *RC* filter on a semi-log graph. The approximate actual response curve is shown by the dotted line. Here, the frequency is on the logarithmic scale and the filter output in decibel is alongwith the linear vertical scale. The filter output is flat beyond f_c. But as the frequency is reduced below f_c, the output drops at the rate of – 20 dB/decade.

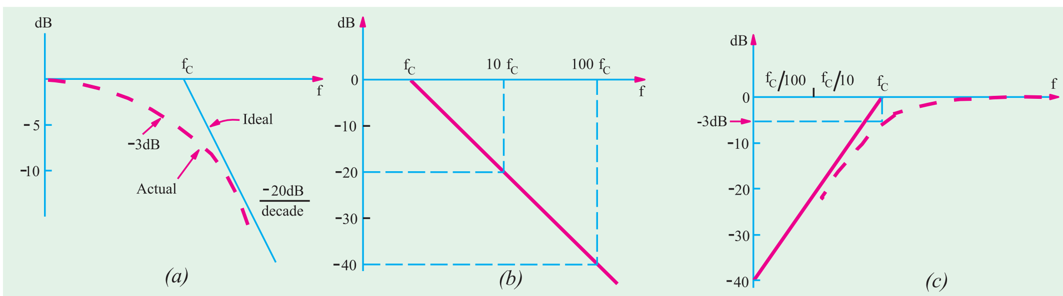


Fig. 17.11

17.16. Bandstop and Bandpass Resonant Filter Circuits

Frequency resonant circuits are used in electronic system to make either bandstop or bandpass filters because of their characteristic Q-rise to either current or voltage at the resonant frequency. Both series and parallel resonant circuits are used for the purpose. It has already been discussed in Chap. No. 7 that

(i) a series resonant circuit offers minimum impedance to input signal and provides maximum current. Minimum impedance equals *R* because $X_L = X_C$ and maximum current $I = V/R$.

(ii) a parallel circuit offers maximum impedance to the input signal and provides minimum current. Maximum impedance offered is = *L*/*CR* and minimum current $I = V/(L/CR)$.

17.17. Series-and Parallel-Resonant Bandstop Filters

The series resonant bandstop filter is shown in Fig. 17.12 (*a*) where the output is taken across the series resonant circuit. Hence, at resonant frequency f_0, the output circuit 'sees' a very low resistance *R* over which negligible output voltage V_0 is developed. That is why there is a shape resonant dip in the response curve of Fig. 17.12 (*b*). Such filters are commonly used to reject a particular frequency such as 50-cycle hum produced by transformers or inductors or turn table rumble in recording equipment.

For the series-resonant bandstop filter shown in Fig. 17.12 (*a*), the following relationships hold good :

$$\text{At } f_0, \frac{V_0}{V_i} = \frac{R_L}{(R_L + R_S)}; \; Q_0 = \frac{\omega_0 L}{(R + R_S)} \text{ and } B_{ph} = \frac{1/2\pi\sqrt{LC}}{Q_0}$$

At any other frequency f, $\frac{V_0}{V_i} = \frac{R_L + j(X_L - X_C)}{(R_L + R_S) + j(X_L - X_C)}$

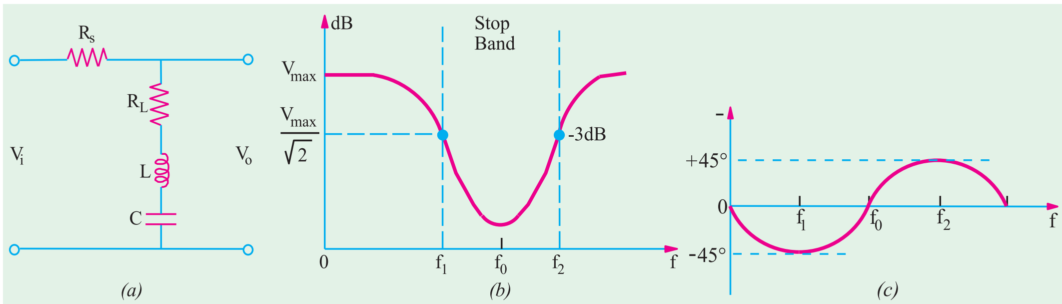


Fig. 17.12

17.18. Parallel-Resonant Bandstop Filter

In this filter, the parallel-resonant circuit is in series with the output resistor R as shown in Fig. 17.13. At resonance, the parallel circuit offers extremely high impedance to f_0 (and nearby frequencies) as compared to R. Hence the output voltage V_0 at f_0 developed across R is negligibly small as compared to that developed across the parallel-resonant circuit. Following relationships hold good for this filter :

At f_0; $\frac{V_0}{V_i} = \frac{R_0}{R_0 + Z_{p0}}$ where $Z_{p0} = Q_0^2 R_L$

At any frequency f, $\frac{V_0}{V_i} = \frac{R_0}{R_0 + Z_p}$

where $Z_p = \frac{Z_L Z_C}{R_L + j(X_L - X_C)}$

Also $Q_0 = \omega_0 L / R_L$ and $B_{hp} = (1/2\pi\sqrt{LC})/Q_0$

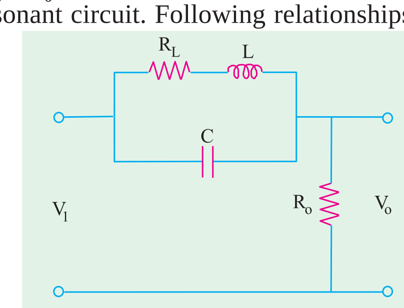


Fig. 17.13

It should be noted that the same amplitude phase response curves apply both to the series resonant and parallel-resonant bandstop filters. Since ***X_C predominates at lower frequencies, phase angle*** θ ***is negative below f_0. above f_0, X_L*** predominates and the phase current leads. At cutoff frequency f_1, θ = – 45° and at other cutoff frequency f_2, θ = + 45° as in the case of any resonant circuit.

Example 17.7. *A series-resonant bandstop filter consist of a series resistance of 2 kΩ across which is connected a series-resonant circuit consisting of a coil of resistance 10 Ω and inductance 350 mH and a capacitor of capacitance 181 pF. F if the applied signal voltage is $10\angle 0°$ of variable frequency, calculate*

(a) *resonant frequency f_0 ;* ***(b)*** *half-power bandwidth B_{hp} ;* ***(c)*** *edge frequencies f_1 and f_2 ;* ***(d)*** *output voltage at frequencies f_0, f_1 and f_2.*

Solution. We are given that R_S = 2; $k\ \Omega\ R = 10\,\Omega$; L = 350 mH; C = 181 pF.

(a) $f_0 = 1/2\pi\sqrt{LC} = 1/2\pi\sqrt{350 \times 10^{-3} \times 181 \times 10^{-12}}$ = 20 kHz

(b) $Q_0 = \omega_0 L/(R_S + R_L) = 2\pi \times 20 \times 10^3 \times 350 \times 10^{-3} / 2010 = 21.88$

$B_{hp} = f_0 / Q_0 = 0.914$ kHz

(c) $f_1 = f_0 - B_{hp}/2$ = 20 – 0.457 = **19.453 kHz**; f_2 = 20 + 0.457 = **20.457 kHz**

(d) At f_0, $V_0 = V_i \frac{R_L}{(R_L + R_S)} = 10\angle 0° \frac{10}{2110}$ = **$0.05\angle 00°$V**

At f_1, $X_{L1} = 2\pi f_1 L = 2\pi \times 19.543 \times 10^3 \times 350 \times 10^{-3} = 42{,}977\,\Omega$

$X_{C1} = 1/2\pi \times 19.543 \times 10^3 \times 181 \times 10^{-12} = 44{,}993\ \Omega$

$\therefore\ (X_L - X_C) = (42{,}977 - 44{,}993) = -2016\ \Omega$

$$V_{01} = V_i \frac{R_L + j(X_L - X_C)}{(R_S + R_L) + j(X_L - X_C)} = 10\angle 0° \times \frac{f10 - j2016}{2010 - j2016}$$

$$= \frac{20160\angle -89.7°}{2847\angle -45°} = \mathbf{7.07\angle{-44.7}°V}$$

At f_2, $X_{L2} = X_{L1}\ (f_2/f_1) = 42977 \times 20.457/19.453$

$= 44{,}987\ \Omega$; $X_{C2} = X_{C1}\ (f_1/f_2) = 44{,}993 \times 19.543/20.457$

$= 42{,}983\ \Omega$; $(X_{L2} - X_{C2}) = 44{,}987 - 42{,}983 = 2004\ \Omega$

$$\therefore\ V_0 = 10\angle 0° \frac{10 + j2004}{2010 + j2004} = \frac{2004\angle 89.7°}{2837\angle 44.9°} = \mathbf{7.07\angle 44.8°V}$$

17.19. Series-Resonant Bandpass Filter

As shown in Fig. 17.14 (*a*), it consists of a series-resonant circuit shunted by an output resistance R_0. It would be seen that this filter circuit can be produced by 'swapping' as series resonant bandstop filter. At f_0, the series resonant impedance is very small and equal R_L which is negligible as compared to R_0. Hence, output voltage is maximum at f_0 and falls to 70.7% at cutoff frequency f_1 and f_2 and shown in the response curve of Fig. 17.14 (*b*). The phase angle is positive for frequencies above f_0 and negative for frequencies below f_0 as shown in Fig 17.14 (*c*) by the solid curve.

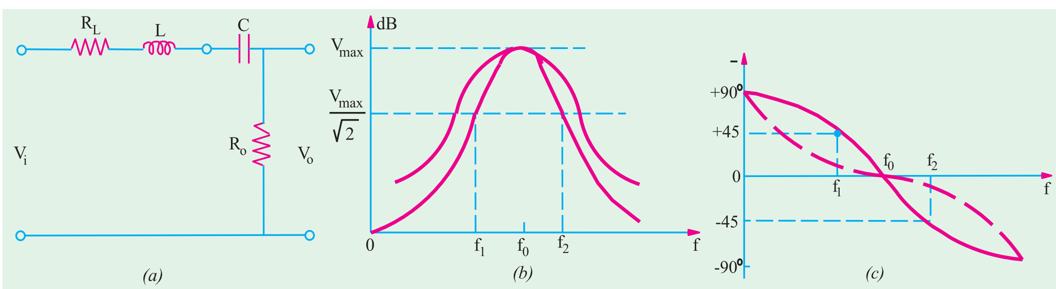


Fig. 17.14

Following relationships hold good for this filter circuit.

$$\text{At}\quad f_0,\ \frac{V_0}{V_i} = \frac{R_0}{(R_L + R_0)};\ Q_0 = \frac{\omega_0 L}{(R_L + R_0)}\ \text{and}\ B_{hp} = \frac{1/2\pi\sqrt{LC}}{Q_0}$$

17.20. Parallel-Resonant Bandpass Filter

It can be obtained by transposing the circuit elements of a bandstop a parallel-resonant filter. As shown in Fig. 17.15, the output is taken across the two-branch parallel-resonant circuit. Since this circuit offers maximum impedance at resonance, this filter produces maximum output voltage V_0 at f_0. The amplitude-response curve of this filter is similar to that of the series-resonant bandpass filter discussed above [Fig. 17.14 (*b*)]. The dotted curve in Fig. 17.14 (*c*) represents the phase relationship between the input and output voltages of this filter. The following relationships apply to this filter :

At $f_0, \dfrac{V_0}{V_i} = \dfrac{R_0}{(R_0 + Z_{p0})}$ where $Z_{p0} = R_{p0} = Q_r^2 R_L$

and $Q_0 = \dfrac{R_{p0}}{X_{CO}}$ and $B_{hp} = \dfrac{1/2\pi\sqrt{LC}}{Q_0}$

At any frequency *f*,

$$\frac{V_0}{V_i} = \frac{Z_p}{R_0 + Z_p} \text{ where } Z_p = \frac{Z_L(-jX_C)}{R_L + j(X_L - X_C)}$$

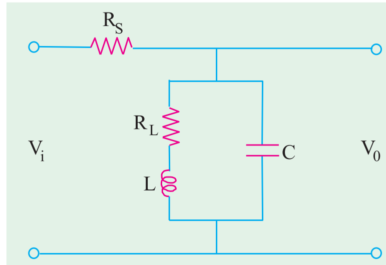


Fig. 17.15

OBJECTIVE TEST – 17

1. The decibel is a measure of
(*a*) power (*b*) voltage
(*c*) current (*d*) power level

2. When the output voltage level of a filter decreases by – 3 dB, its absolute value changes by a factor of
(*a*) $\sqrt{2}$ (*b*) $1/\sqrt{2}$
(*c*) 2 (*d*) 1/2

3. The frequency corresponding to half-power point on the response curve of a filter is known as —
(*a*) cutoff (*b*) upper
(*c*) lower (*d*) roll-off

4. In a low-pass filter, the cutoff frequency is represented by the point where the output voltage is reduced to — per cent of the input voltage.
(*a*) 50 (*b*) 70.7
(*c*) 63.2 (*d*) 33.3

5. In an *RL* low-pass filter, an attenuation of – 12 dB/octave corresponds to .... dB/decade.
(*a*) – 6 (*b*) – 12
(*c*) – 20 (*d*) – 40

6. A network which attenuate a single band of frequencies and allows those on either side to pass through is called .... filter.
(*a*) low-pass (*b*) high-pass
(*c*) bandstop (*d*) bandpass

7. In a simple high-pass RC filter, if the value of capacitance is doubled, the cutoff frequency is
(*a*) doubled (*b*) halved
(*c*) tripled (*d*) quadrupled

8. In a simple high-pass RL filter circuit, the phase difference between the output and input voltages at the cutoff frequency is .... degrees.
(*a*) – 90 (*b*) 45
(*c*) – 45 (*d*) 90

9. In a simple low-pass RC filter, attenuation is – 3 dB at f_c. At 2 f_c, attenuation is – 6 dB. At 10 f_c, the attenuation would be .... dB.
(*a*) – 30 (*b*) – 20
(*c*) – 18 (*d*) – 12

10. An a.c. signal of constant voltage 10 V and variable frequency is applied to a simple high-pass RC filter. The output voltage at ten times the cutoff frequency would be .... volt.
(*a*) 1 (*b*) 5
(*c*) $10/\sqrt{2}$ (*d*) $10\sqrt{2}$

11. When two simple low-pass filters having same values of R and C are cascaded, the combined filter will have a roll-off of ...... dB/decade.
(*a*) – 20 (*b*) – 12
(*c*) – 40 (*d*) – 36

12. An a.c. signal of constant voltage but with frequency varying from dc to 25 kHz is applied to a high-pass filter. Which of the following frequency will develop the greatest voltage at the output load resistance?
(*a*) d. (*b*) 15 kHz
(*c*) 10 kHz (*d*) 25 kHz

13. A voltage signal source of constant amplitude with frequency varying from dc to 25 kHz is applied to a low-pass filter. Which frequency will develop greatest voltage across the output load resistance?
(*a*) d.c. (*b*) 10 kHz
(*c*) 15 kl (*d*) 25 kHz

14. The output of a filter drops from 10 to 5 V as the frequency is increased from 1 to 2 kHz. The dB change in the output voltage is
(*a*) – 3 dB/decade
(*b*) – 6 dB/octave
(*c*) 6 dB/octave
(*d*) – 3 dB/octave

ANSWERS

1. *d* **2.** *b* **3.** *a* **4.** *b* **5.** *d* **6.** *c* **7.** *b* **8.** *b* **9.** *b* **10.** *a* **11.** *c* **12.** *d* **13.** *a* **14.** *b*