5

Series Circuits

5.1 INTRODUCTION

Two types of current are readily available to the consumer today. One is *direct current* (dc), in which ideally the flow of charge (current) does not change in magnitude (or direction) with time. The other is *sinusoidal alternating current* (ac), in which the flow of charge is continually changing in magnitude (and direction) with time. The next few chapters are an introduction to circuit analysis purely from a dc approach. The methods and concepts will be discussed in detail for direct current; when possible, a short discussion will suffice to cover any variations we might encounter when we consider ac in the later chapters.

The battery of Fig. 5.1, by virtue of the potential difference between its terminals, has the ability to cause (or "pressure") charge to flow through the simple circuit. The positive terminal attracts the electrons through the wire at the same rate at which electrons are supplied by the negative terminal. As long as the battery is connected in the circuit and maintains its terminal characteristics, the current (dc) through the circuit will not change in magnitude or direction.

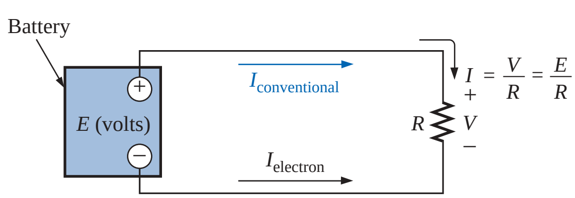


FIG. 5.1
Introducing the basic components of an electric circuit.

If we consider the wire to be an ideal conductor (that is, having no opposition to flow), the potential difference V across the resistor will equal the applied voltage of the battery: V (volts) $= E$ (volts).

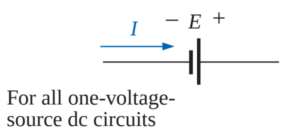


FIG. 5.2
Defining the direction of conventional flow for single-source dc circuits.

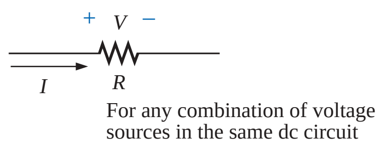


FIG. 5.3
Defining the polarity resulting from a conventional current I through a resistive element.

The current is limited only by the resistor R. The higher the resistance, the less the current, and conversely, as determined by Ohm's law.

By convention (as discussed in Chapter 2), the direction of **conventional current flow** $I_{conventional}$ as shown in Fig. 5.1 is opposite to that of **electron flow** ($I_{electron}$). Also, the uniform flow of charge dictates that the direct current I be the same everywhere in the circuit. By following the direction of conventional flow, we notice that there is a rise in potential across the battery ($-$ to $+$), and a drop in potential across the resistor ($+$ to $-$). For single-voltage-source dc circuits, conventional flow always passes from a low potential to a high potential when passing through a voltage source, as shown in Fig. 5.2. However, conventional flow always passes from a high to a low potential when passing through a resistor for any number of voltage sources in the same circuit, as shown in Fig. 5.3.

The circuit of Fig. 5.1 is the simplest possible configuration. This chapter and the chapters to follow will add elements to the system in a very specific manner to introduce a range of concepts that will form a major part of the foundation required to analyze the most complex system. Be aware that the laws, rules, and so on, introduced in Chapters 5 and 6 will be used throughout your studies of electrical, electronic, or computer systems. They will not be dropped for a more advanced set as you progress to more sophisticated material. It is therefore critical that the concepts be understood thoroughly and that the various procedures and methods be applied with confidence.

5.2 SERIES CIRCUITS

A **circuit** consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 5.4(a) has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.

Two elements are in series if

1. ***They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).***
2. ***The common point between the two elements is not connected to another current-carrying element.***

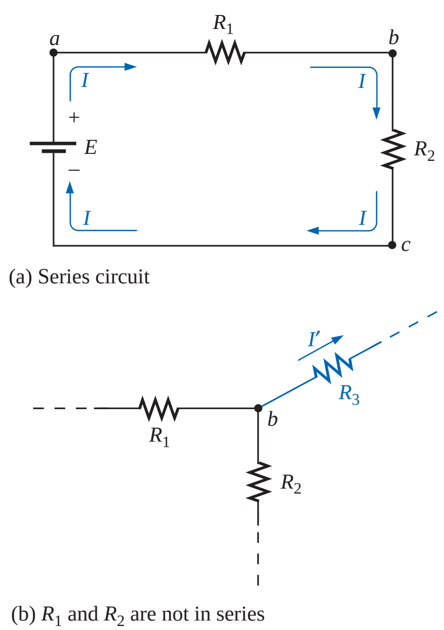


FIG. 5.4
(a) Series circuit; (b) situation in which R_1 and R_2 are not in series.

In Fig. 5.4(a), the resistors R_1 and R_2 are in series because they have *only* point b in common. The other ends of the resistors are connected elsewhere in the circuit. For the same reason, the battery E and resistor R_1 are in series (terminal a in common), and the resistor R_2 and the battery E are in series (terminal c in common). Since all the elements are in series, the network is called a **series circuit.** Two common examples of series connections include the tying of small pieces of rope together to form a longer rope and the connecting of pipes to get water from one point to another.

If the circuit of Fig. 5.4(a) is modified such that a current-carrying resistor R_3 is introduced, as shown in Fig. 5.4(b), the resistors R_1 and R_2 are no longer in series due to a violation of number 2 of the above definition of series elements.

The current is the same through series elements.

For the circuit of Fig. 5.4(a), therefore, the current I through each resistor is the same as that through the battery. The fact that the current is the same through series elements is often used as a path to determine whether two elements are in series or to confirm a conclusion.

A **branch** of a circuit is any portion of the circuit that has one or more elements in series. In Fig. 5.4(a), the resistor R_1 forms one branch of the circuit, the resistor R_2 another, and the battery E a third.

The total resistance of a series circuit is the sum of the resistance levels.

In Fig. 5.4(a), for example, the total resistance (R_T) is equal to $R_1 + R_2$. Note that the total resistance is actually the resistance "seen" by the battery as it "looks" into the series combination of elements as shown in Fig. 5.5.

In general, to find the total resistance of N resistors in series, the following equation is applied:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_N \qquad \text{(ohms, } \Omega) \qquad \textbf{(5.1)}$$

Once the total resistance is known, the circuit of Fig. 5.4(a) can be redrawn as shown in Fig. 5.6, clearly revealing that the only resistance the source "sees" is the total resistance. It is totally unaware of how the elements are connected to establish R_T. Once R_T is known, the current drawn from the source can be determined using Ohm's law, as follows:

$$I_s = \frac{E}{R_T} \qquad \text{(amperes, A)} \qquad \textbf{(5.2)}$$

Since E is fixed, the magnitude of the source current will be totally dependent on the magnitude of R_T. A larger R_T will result in a relatively small value of I_s, while lesser values of R_T will result in increased current levels.

The fact that the current is the same through each element of Fig. 5.4(a) permits a direct calculation of the voltage across each resistor using Ohm's law; that is,

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3,\ \ldots,\ V_N = IR_N \qquad \text{(volts, V)} \qquad \textbf{(5.3)}$$

The power delivered to each resistor can then be determined using any one of three equations as listed below for R_1:

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)} \qquad \textbf{(5.4)}$$

The power delivered by the source is

$$P_{del} = EI \qquad \text{(watts, W)} \qquad \textbf{(5.5)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

That is,

$$P_{del} = P_1 + P_2 + P_3 + \cdots + P_N \qquad \textbf{(5.6)}$$

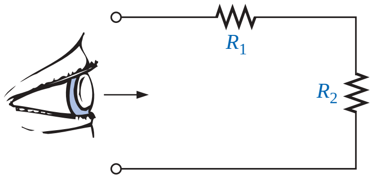


FIG. 5.5
Resistance "seen" by source.

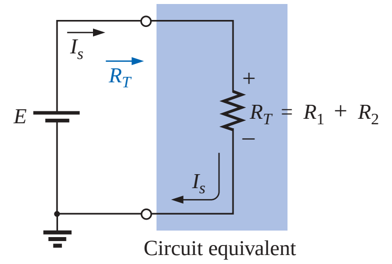


FIG. 5.6
Replacing the series resistors R_1 and R_2 of Fig. 5.5 with the total resistance.

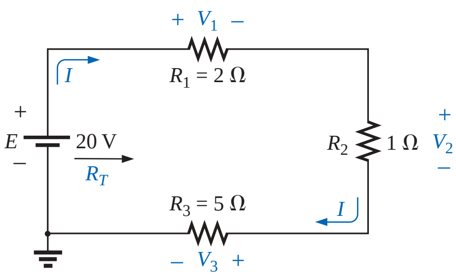


FIG. 5.7
Example 5.1.

EXAMPLE 5.1

a. Find the total resistance for the series circuit of Fig. 5.7.
b. Calculate the source current I_s.
c. Determine the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\ \text{V})(2.5\ \text{A}) = \mathbf{50\ W}$
$P_{\text{del}} = P_1 + P_2 + P_3$
$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$
$\underline{50\ \text{W} = 50\ \text{W} \quad \text{(checks)}}$

To find the total resistance of *N* resistors of the same value in series, simply multiply the value of *one* of the resistors by the number in series; that is,

$$R_T = NR \tag{5.7}$$

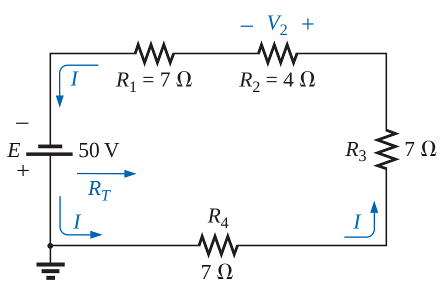


FIG. 5.8
Example 5.2.

EXAMPLE 5.2 Determine R_T, I, and V_2 for the circuit of Fig. 5.8.

Solution: Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction. Since $R_1 = R_3 = R_4$,

$$R_T = NR_1 + R_2 = (3)(7\ \Omega) + 4\ \Omega = 21\ \Omega + 4\ \Omega = \mathbf{25\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$

Examples 5.1 and 5.2 are straightforward substitution-type problems that are relatively easy to solve with some practice. Example 5.3, however, is evidence of another type of problem that requires a firm grasp of the fundamental equations and an ability to identify which equation to use first. The best preparation for this type of exercise is simply to work through as many problems of this kind as possible.

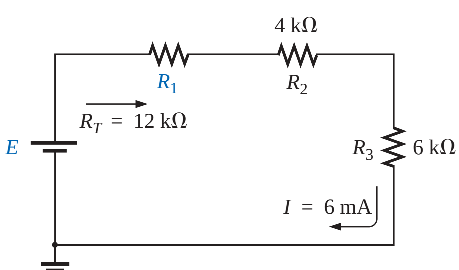


FIG. 5.9
Example 5.3.

EXAMPLE 5.3 Given R_T and I, calculate R_1 and E for the circuit of Fig. 5.9.

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$12\text{ k}\Omega = R_1 + 4\text{ k}\Omega + 6\text{ k}\Omega$$
$$R_1 = 12\text{ k}\Omega - 10\text{ k}\Omega = \mathbf{2\text{ k}\Omega}$$
$$E = IR_T = (6 \times 10^{-3}\text{ A})(12 \times 10^{3}\ \Omega) = \mathbf{72\text{ V}}$$

5.3 VOLTAGE SOURCES IN SERIES

Voltage sources can be connected in series, as shown in Fig. 5.10, to increase or decrease the total voltage applied to a system. The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite "pressure." The net polarity is the polarity of the larger sum.

In Fig. 5.10(a), for example, the sources are all "pressuring" current to the right, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10\text{ V} + 6\text{ V} + 2\text{ V} = 18\text{ V}$$

as shown in the figure. In Fig. 5.10(b), however, the greater "pressure" is to the left, with a net voltage of

$$E_T = E_2 + E_3 - E_1 = 9\text{ V} + 3\text{ V} - 4\text{ V} = 8\text{ V}$$

and the polarity shown in the figure.

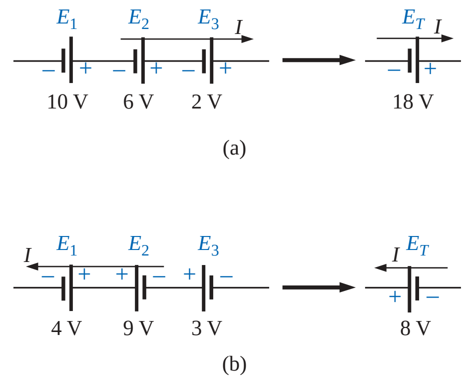


FIG. 5.10
Reducing series dc voltage sources to a single source.

German (Königsberg, Berlin)
(1824–87)
Physicist
Professor of Physics, University of Heidelberg

Courtesy of the Smithsonian Institution Photo No. 58,283

Although a contributor to a number of areas in the physics domain, he is best known for his work in the electrical area with his definition of the relationships between the currents and voltages of a network in 1847. Did extensive research with German chemist Robert Bunsen (developed the *Bunsen burner*), resulting in the discovery of the important elements of *cesium* and *rubidium*.

FIG. 5.11
Gustav Robert Kirchhoff.

5.4 KIRCHHOFF'S VOLTAGE LAW

Note Fig. 5.11.

Kirchhoff's voltage law ***(KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.***

A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit. In Fig. 5.12, by following the current, we can trace a continuous path that leaves point *a* through R_1 and returns through E without leaving the circuit. Therefore, *abcda* is a closed loop. For us to be able to apply Kirchhoff's voltage law, the summation of potential rises and drops must be made in one direction around the closed loop.

For uniformity, the clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff's voltage law. Be aware, however, that the same result will be obtained if the counterclockwise (CCW) direction is chosen and the law applied correctly.

A plus sign is assigned to a potential rise (− to +), and a minus sign to a potential drop (+ to −). If we follow the current in Fig. 5.12 from point *a*, we first encounter a potential drop V_1 (+ to −) across R_1 and then another potential drop V_2 across R_2. Continuing through the voltage source, we have a potential rise E (− to +) before returning to point *a*. In symbolic form, where Σ represents summation, ↻ the closed loop, and V the potential drops and rises, we have

$$\Sigma_{\circlearrowright} V = 0 \quad \text{(Kirchhoff's voltage law in symbolic form)} \qquad \textbf{(5.8)}$$

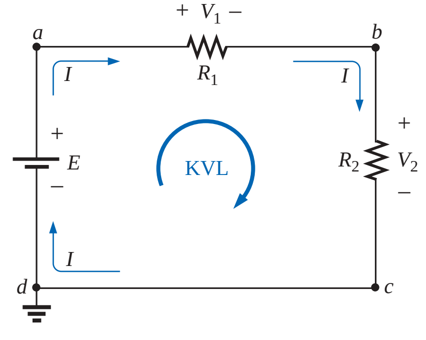


FIG. 5.12
Applying Kirchhoff's voltage law to a series dc circuit.

which for the circuit of Fig. 5.12 yields (clockwise direction, following the current I and starting at point d):

$$+E - V_1 - V_2 = 0$$

or
$$E = V_1 + V_2$$

revealing that

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

Kirchhoff's voltage law can also be stated in the following form:

$$\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}} \qquad \textbf{(5.9)}$$

which in words states that the sum of the rises around a closed loop must equal the sum of the drops in potential. The text will emphasize the use of Eq. (5.8), however.

If the loop were taken in the counterclockwise direction starting at point a, the following would result:

$$\Sigma_{\circlearrowleft} V = 0$$
$$-E + V_2 + V_1 = 0$$

or, as before,
$$E = V_1 + V_2$$

The application of Kirchhoff's voltage law need not follow a path that includes current-carrying elements.

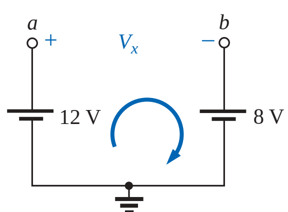


FIG. 5.13
Demonstration that a voltage can exist between two points not connected by a current-carrying conductor.

For example, in Fig. 5.13 there is a difference in potential between points a and b, even though the two points are not connected by a current-carrying element. Application of Kirchhoff's voltage law around the closed loop will result in a difference in potential of 4 V between the two points. That is, using the clockwise direction:

$$+12\text{ V} - V_x - 8\text{ V} = 0$$

and
$$V_x = \mathbf{4\ V}$$

EXAMPLE 5.4 Determine the unknown voltages for the networks of Fig. 5.14.

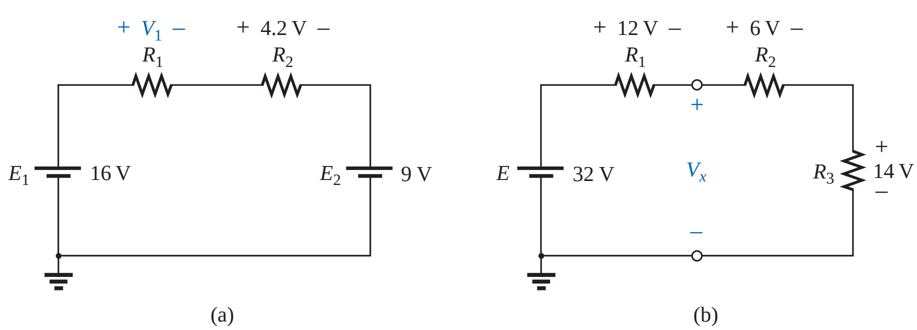


FIG. 5.14
Example 5.4.

Solution: When applying Kirchhoff's voltage law, be sure to concentrate on the polarities of the voltage rise or drop rather than on the type of element. In other words, do not treat a voltage drop across a resistive element differently from a voltage drop across a source. If the polarity dictates that a drop has occurred, that is the important fact when applying the law. In Fig. 5.14(a), for instance, if we choose the clockwise direction, we will find that there is a drop across the resistors R_1 and R_2 and a drop across the source E_2. All will therefore have a minus sign when Kirchhoff's voltage law is applied.

Application of Kirchhoff's voltage law to the circuit of Fig. 5.14(a) in the clockwise direction will result in

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and $$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V} = \mathbf{2.8\text{ V}}$$

The result clearly indicates that there was no need to know the values of the resistors or the current to determine the unknown voltage. Sufficient information was carried by the other voltage levels to permit a determination of the unknown.

In Fig. 5.14(b) the unknown voltage is not across a current-carrying element. However, as indicated in the paragraphs above, Kirchhoff's voltage law is not limited to current-carrying elements. In this case there are two possible paths for finding the unknown. Using the clockwise path, including the voltage source E, will result in

$$+E - V_1 - V_x = 0$$

and $$V_x = E - V_1 = 32\text{ V} - 12\text{ V} = \mathbf{20\text{ V}}$$

Using the clockwise direction for the other loop involving R_2 and R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

and $$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V} = \mathbf{20\text{ V}}$$

matching the result above.

EXAMPLE 5.5 Find V_1 and V_2 for the network of Fig. 5.15.

Solution: For path 1, starting at point a in a clockwise direction:

$$+25\text{ V} - V_1 + 15\text{ V} = 0$$

and $$V_1 = \mathbf{40\text{ V}}$$

For path 2, starting at point a in a clockwise direction:

$$-V_2 - 20\text{ V} = 0$$

and $$V_2 = \mathbf{-20\text{ V}}$$

The minus sign simply indicates that the actual polarities of the potential difference are opposite the assumed polarity indicated in Fig. 5.15.

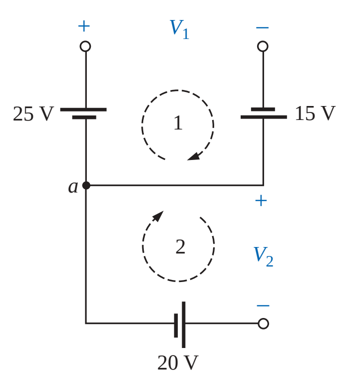


FIG. 5.15
Example 5.5.

The next example will emphasize the fact that when we are applying Kirchhoff's voltage law, it is the polarities of the voltage rise or drop that are the important parameters, and not the type of element involved.

EXAMPLE 5.6 Using Kirchhoff's voltage law, determine the unknown voltages for the network of Fig. 5.16.

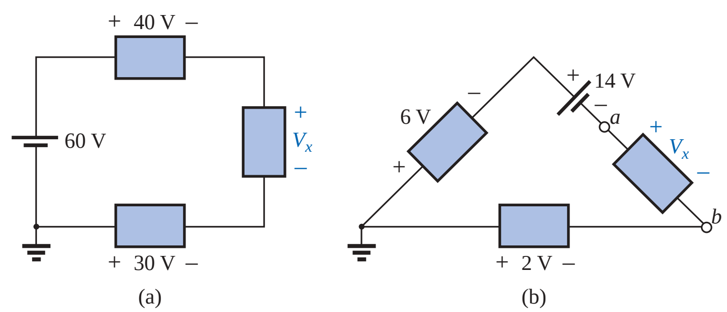


FIG. 5.16
Example 5.6.

Solution: Note in each circuit that there are various polarities across the unknown elements since they can contain any mixture of components. Applying Kirchhoff's voltage law to the network of Fig. 5.16(a) in the clockwise direction will result in

$$60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

and
$$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V} = \mathbf{50\text{ V}}$$

In Fig. 5.16(b) the polarity of the unknown voltage is not provided. In such cases, make an assumption about the polarity, and apply Kirchhoff's voltage law as before. If the result has a plus sign, the assumed polarity was correct. If it has a minus sign, the magnitude is correct, but the assumed polarity has to be reversed. In this case if we assume *a* to be positive and *b* to be negative, an application of Kirchhoff's voltage law in the clockwise direction will result in

$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$

and
$$V_x = -20\text{ V} + 2\text{ V} = \mathbf{-18\text{ V}}$$

Since the result is negative, we know that *a* should be negative and *b* should be positive, but the magnitude of 18 V is correct.

EXAMPLE 5.7 For the circuit of Fig. 5.17:

a. Find R_T.
b. Find I.
c. Find V_1 and V_2.
d. Find the power to the 4-Ω and 6-Ω resistors.
e. Find the power delivered by the battery, and compare it to that dissipated by the 4-Ω and 6-Ω resistors combined.
f. Verify Kirchhoff's voltage law (clockwise direction).

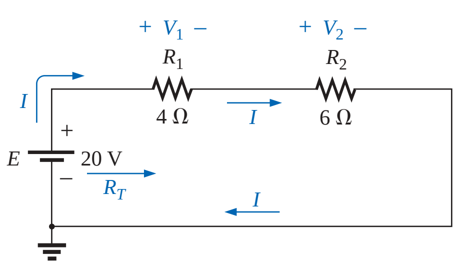


FIG. 5.17
Example 5.7.

Solutions:

a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{10\ \Omega} = \mathbf{2\text{ A}}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\ V}$

d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\ W}$

$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\ W}$

e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\ W}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

f. $\Sigma_{\circlearrowright} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

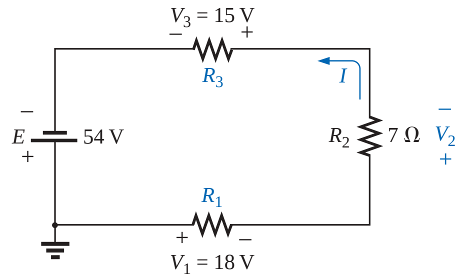


FIG. 5.18
Example 5.8.

EXAMPLE 5.8 For the circuit of Fig. 5.18:
a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.

Solutions:
a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or $$E = V_1 + V_2 + V_3$$

and $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\ V}$$

b. $I = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega} = \mathbf{3\ A}$

c. $R_1 = \dfrac{V_1}{I} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

$R_3 = \dfrac{V_3}{I} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

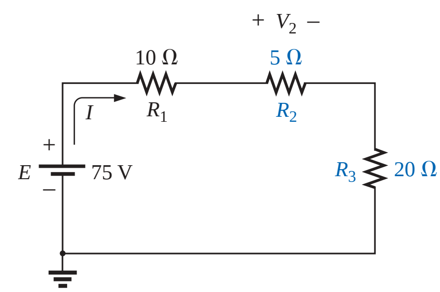


FIG. 5.19
Series dc circuit with elements to be interchanged.

5.5 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network of Fig. 5.19 can be redrawn as shown in Fig. 5.20 without affecting I or V_2. The total resistance R_T is 35 Ω in both cases, and $I = 70\text{ V}/35\ \Omega = 2\text{ A}$. The voltage $V_2 = IR_2 = (2\text{ A})(5\ \Omega) = 10\text{ V}$ for both configurations.

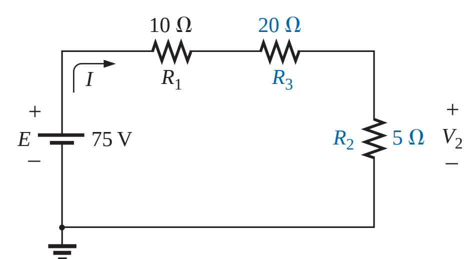


FIG. 5.20
Circuit of Fig. 5.19 with R_2 and R_3 interchanged.

EXAMPLE 5.9 Determine I and the voltage across the 7-Ω resistor for the network of Fig. 5.21.

Solution: The network is redrawn in Fig. 5.22.

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\text{ V}}{15\ \Omega} = \mathbf{2.5\ A}$$

$$V_{7\Omega} = IR = (2.5\text{ A})(7\ \Omega) = \mathbf{17.5\ V}$$

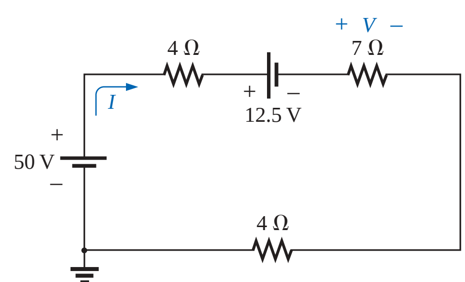


FIG. 5.21
Example 5.9.

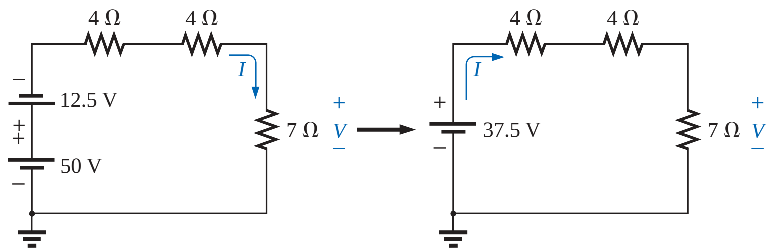


FIG. 5.22
Redrawing the circuit of Fig. 5.21.

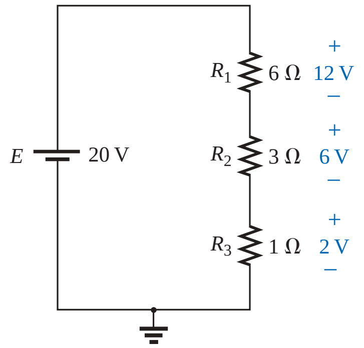


FIG. 5.23
Revealing how the voltage will divide across series resistive elements.

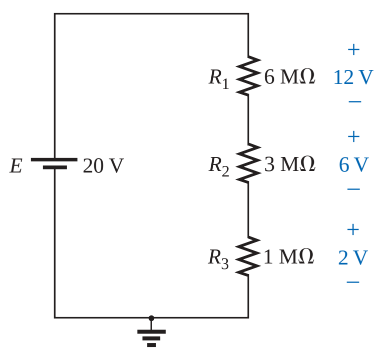


FIG. 5.24
The ratio of the resistive values determines the voltage division of a series dc circuit.

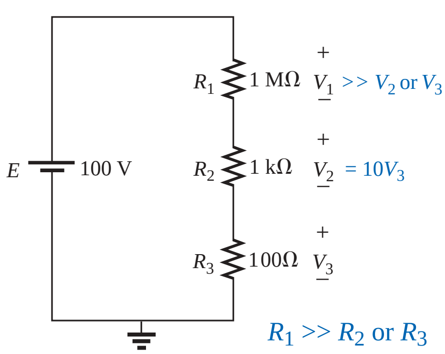


FIG. 5.25
The largest of the series resistive elements will capture the major share of the applied voltage.

5.6 VOLTAGE DIVIDER RULE

In a series circuit,

the voltage across the resistive elements will divide as the magnitude of the resistance levels.

For example, the voltages across the resistive elements of Fig. 5.23 are provided. The largest resistor of 6 Ω captures the bulk of the applied voltage, while the smallest resistor R_3 has the least. Note in addition that, since the resistance level of R_1 is 6 times that of R_3, the voltage across R_1 is 6 times that of R_3. The fact that the resistance level of R_2 is 3 times that of R_1 results in three times the voltage across R_2. Finally, since R_1 is twice R_2, the voltage across R_1 is twice that of R_2. In general, therefore, the voltage across series resistors will have the same ratio as their resistance levels.

It is particularly interesting to note that, if the resistance levels of all the resistors of Fig. 5.23 are increased by the same amount, as shown in Fig. 5.24, the voltage levels will all remain the same. In other words, even though the resistance levels were increased by a factor of 1 million, the voltage ratios remain the same. Clearly, therefore, it is the ratio of resistor values that counts when it comes to voltage division and not the relative magnitude of all the resistors. The current level of the network will be severely affected by the change in resistance level from Fig. 5.23 to Fig. 5.24, but the voltage levels will remain the same.

Based on the above, a first glance at the series network of Fig. 5.25 should suggest that the major part of the applied voltage will appear across the 1-MΩ resistor and very little across the 100-Ω resistor. In fact, $1\ \text{M}\Omega = (1000)1\ \text{k}\Omega = (10{,}000)100\ \Omega$, revealing that $V_1 = 1000V_2 = 10{,}000V_3$.

Solving for the current and then the three voltage levels will result in

$$I = \frac{E}{R_T} = \frac{100\ \text{V}}{1{,}001{,}100\ \Omega} \cong 99.89\ \mu\text{A}$$

and

$$V_1 = IR_1 = (99.89\ \mu\text{A})(1\ \text{M}\Omega) = \mathbf{99.89\ V}$$
$$V_2 = IR_2 = (99.89\ \mu\text{A})(1\ \text{k}\Omega) = \mathbf{99.89\ mV} = 0.09989\ \text{V}$$
$$V_3 = IR_3 = (99.89\ \mu\text{A})(100\ \Omega) = \mathbf{9.989\ mV} = 0.009989\ \text{V}$$

clearly substantiating the above conclusions. For the future, therefore, use this approach to estimate the share of the input voltage across series elements to act as a check against the actual calculations or to simply obtain an estimate with a minimum of effort.

In the above discussion the current was determined before the voltages of the network were determined. There is, however, a method referred to as the **voltage divider rule** (VDR) that permits determining the voltage levels without first finding the current. The rule can be derived by analyzing the network of Fig. 5.26.

$$R_T = R_1 + R_2$$

and
$$I = \frac{E}{R_T}$$

Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1 E}{R_T}$$

with
$$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2 E}{R_T}$$

Note that the format for V_1 and V_2 is

$$\boxed{V_x = \frac{R_x E}{R_T}} \qquad \text{(voltage divider rule)} \qquad \textbf{(5.10)}$$

where V_x is the voltage across R_x, E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

In words, the **voltage divider rule** states that

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

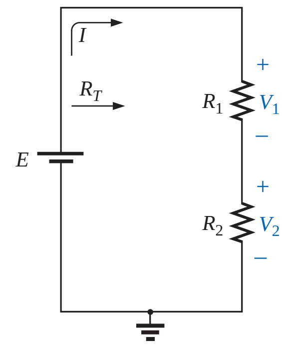


FIG. 5.26
Developing the voltage divider rule.

EXAMPLE 5.10 Determine the voltage V_1 for the network of Fig. 5.27.

Solution: Eq. (5.10):

$$V_1 = \frac{R_1 E}{R_T} = \frac{R_1 E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \textbf{16 V}$$

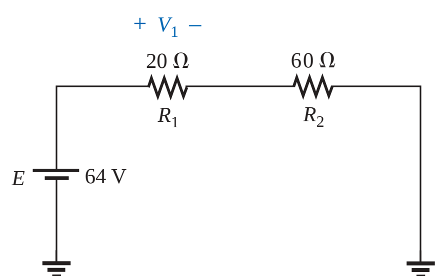


FIG. 5.27
Example 5.10.

EXAMPLE 5.11 Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 5.28.

Solution:

$$V_1 = \frac{R_1 E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \textbf{6 V}$$

$$V_3 = \frac{R_3 E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \textbf{24 V}$$

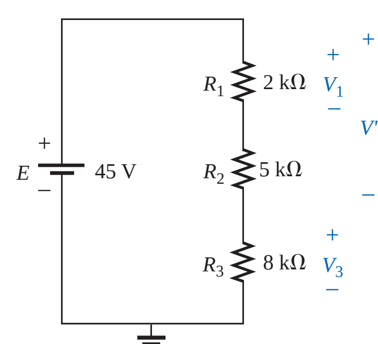


FIG. 5.28
Example 5.11.

The rule can be extended to the voltage across two or more series elements if the resistance in the numerator of Eq. (5.10) is expanded to include the total resistance of the series elements that the voltage is to be found across (R'); that is,

$$\boxed{V' = \frac{R'E}{R_T}} \quad \text{(volts)} \tag{5.11}$$

EXAMPLE 5.12 Determine the voltage V' in Fig. 5.28 across resistors R_1 and R_2.

Solution:

$$V' = \frac{R'E}{R_T} = \frac{(2\text{ k}\Omega + 5\text{ k}\Omega)(45\text{ V})}{15\text{ k}\Omega} = \frac{(7\text{ k}\Omega)(45\text{ V})}{15\text{ k}\Omega} = \mathbf{21\text{ V}}$$

There is also no need for the voltage E in the equation to be the source voltage of the network. For example, if V is the total voltage across a number of series elements such as those shown in Fig. 5.29, then

$$V_{2\Omega} = \frac{(2\ \Omega)(27\text{ V})}{4\ \Omega + 2\ \Omega + 3\ \Omega} = \frac{54\text{ V}}{9} = \mathbf{6\text{ V}}$$

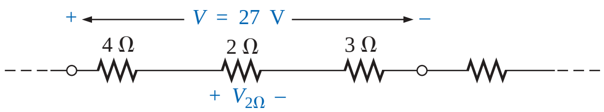


FIG. 5.29
The total voltage across series elements need not be an independent voltage source.

EXAMPLE 5.13 Design the voltage divider of Fig. 5.30 such that $V_{R_1} = 4V_{R_2}$.

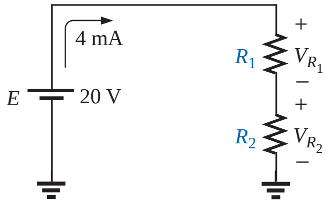


FIG. 5.30
Example 5.13.

Solution: The total resistance is defined by

$$R_T = \frac{E}{I} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

Since $V_{R_1} = 4V_{R_2}$,

$$R_1 = 4R_2$$

Thus $$R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$$

and $$5R_2 = 5\text{ k}\Omega$$
$$R_2 = \mathbf{1\text{ k}\Omega}$$

and $$R_1 = 4R_2 = \mathbf{4\text{ k}\Omega}$$

5.7 NOTATION

Notation will play an increasingly important role in the analysis to follow. It is important, therefore, that we begin to examine the notation used throughout the industry.

Voltage Sources and Ground

Except for a few special cases, electrical and electronic systems are grounded for reference and safety purposes. The symbol for the ground connection appears in Fig. 5.31 with its defined potential level—zero volts. None of the circuits discussed thus far have contained the ground connection. If Fig. 5.4(a) were redrawn with a grounded supply, it might appear as shown in Fig. 5.32(a), (b), or (c). In any case, it is understood that the negative terminal of the battery and the bottom of the resistor R_2 are at ground potential. Although Figure 5.32(c) shows no connection between the two grounds, it is recognized that such a connection exists for the continuous flow of charge. If $E = 12$ V, then point a is 12 V positive with respect to ground potential, and 12 V exist across the series combination of resistors R_1 and R_2. If a voltmeter placed from point b to ground reads 4 V, then the voltage across R_2 is 4 V, with the higher potential at point b.

On large schematics where space is at a premium and clarity is important, voltage sources may be indicated as shown in Figs. 5.33(a) and 5.34(a) rather than as illustrated in Figs. 5.33(b) and 5.34(b). In addition, potential levels may be indicated as in Fig. 5.35, to permit a rapid check of the potential levels at various points in a network with respect to ground to ensure that the system is operating properly.

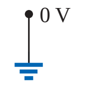


FIG. 5.31
Ground potential.

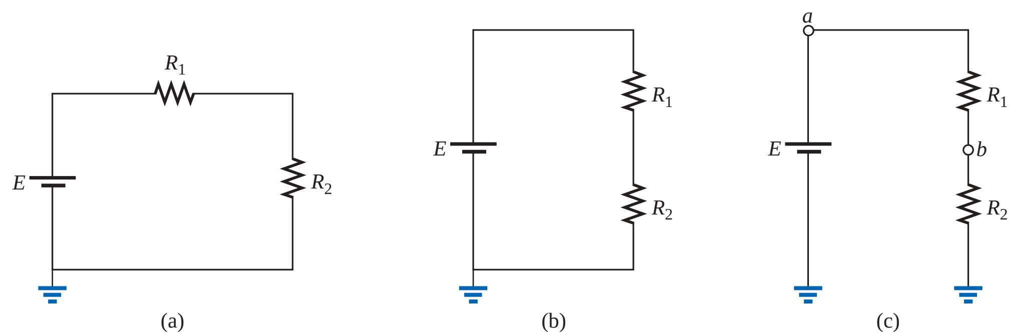


FIG. 5.32
Three ways to sketch the same series dc circuit.

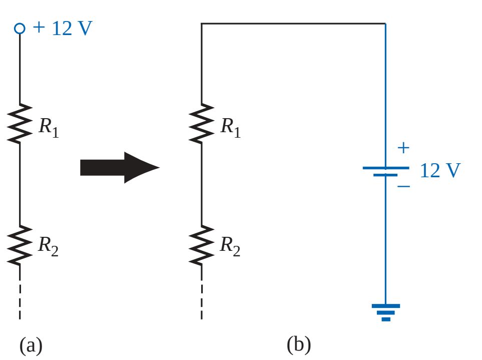


FIG. 5.33
Replacing the special notation for a dc voltage source with the standard symbol.

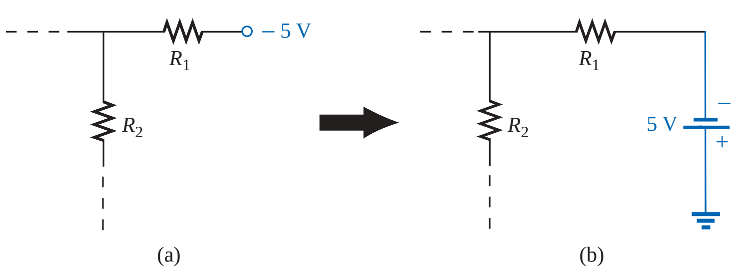


FIG. 5.34
Replacing the notation for a negative dc supply with the standard notation.

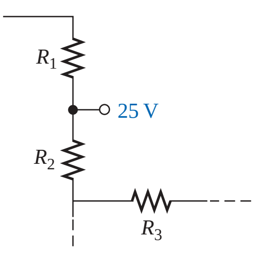


FIG. 5.35
The expected voltage level at a particular point in a network of the system is functioning properly.

Double-Subscript Notation

The fact that voltage is an *across* variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In Fig. 5.36(a), the two points that define the voltage across the resistor R are denoted by a and b. Since a is the first subscript for V_{ab}, point a must have a higher potential than point b if V_{ab} is to have a positive value. If, in fact, point b is at a higher potential than point a, V_{ab} will have a negative value, as indicated in Fig. 5.36(b).

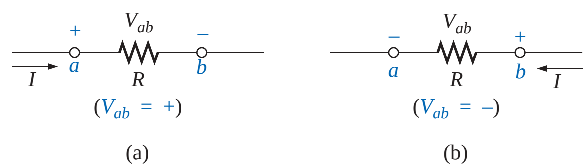


FIG. 5.36
Defining the sign for double-subscript notation.

In summary:

The double-subscript notation V_{ab} specifies point a as the higher potential. If this is not the case, a negative sign must be associated with the magnitude of V_{ab}.

In other words,

the voltage V_{ab} is the voltage at point a with respect to (w.r.t.) point b.

Single-Subscript Notation

If point b of the notation V_{ab} is specified as ground potential (zero volts), then a single-subscript notation can be employed that provides the voltage at a point with respect to ground.

In Fig. 5.37, V_a is the voltage from point a to ground. In this case it is obviously 10 V since it is right across the source voltage E. The voltage V_b is the voltage from point b to ground. Because it is directly across the 4-Ω resistor, $V_b = 4$ V.

In summary:

The single-subscript notation V_a specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of V_a.

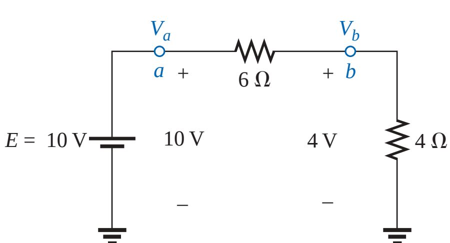


FIG. 5.37
Defining the use of single-subscript notation for voltage levels.

General Comments

A particularly useful relationship can now be established that will have extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$V_{ab} = V_a - V_b \tag{5.12}$$

In other words, if the voltage at points a and b is known with respect to ground, then the voltage V_{ab} can be determined using Eq. (5.12). In Fig. 5.37, for example,

$$\begin{aligned} V_{ab} &= V_a - V_b = 10\text{ V} - 4\text{ V} \\ &= 6\text{ V} \end{aligned}$$

EXAMPLE 5.14 Find the voltage V_{ab} for the conditions of Fig. 5.38.

Solution: Applying Eq. (5.12):

$$V_{ab} = V_a - V_b = 16\text{ V} - 20\text{ V}$$
$$= \mathbf{-4\ V}$$

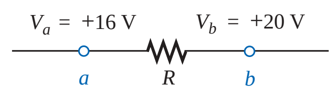


FIG. 5.38
Example 5.14.

Note the negative sign to reflect the fact that point b is at a higher potential than point a.

EXAMPLE 5.15 Find the voltage V_a for the configuration of Fig. 5.39.

Solution: Applying Eq. (5.12):

$$V_{ab} = V_a - V_b$$

and

$$V_a = V_{ab} + V_b = 5\text{ V} + 4\text{ V}$$
$$= \mathbf{9\ V}$$

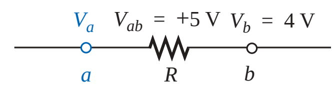


FIG. 5.39
Example 5.15.

EXAMPLE 5.16 Find the voltage V_{ab} for the configuration of Fig. 5.40.

Solution: Applying Eq. (5.12):

$$V_{ab} = V_a - V_b = 20\text{ V} - (-15\text{ V}) = 20\text{ V} + 15\text{ V}$$
$$= \mathbf{35\ V}$$

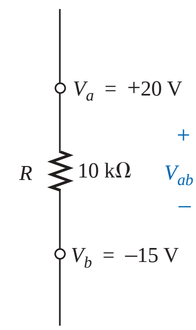


FIG. 5.40
Example 5.16.

Note in Example 5.16 the care that must be taken with the signs when applying the equation. The voltage is dropping from a high level of +20 V to a negative voltage of −15 V. As shown in Fig. 5.41, this represents a drop in voltage of 35 V. In some ways it's like going from a positive checking balance of \$20 to owing \$15; the total expenditure is \$35.

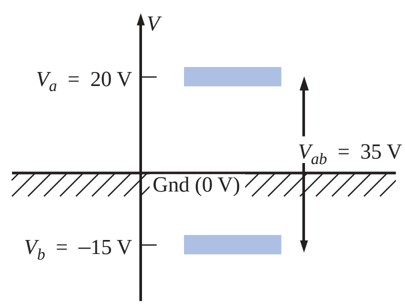


FIG. 5.41
The impact of positive and negative voltages on the total voltage drop.

EXAMPLE 5.17 Find the voltages V_b, V_c, and V_{ac} for the network of Fig. 5.42.

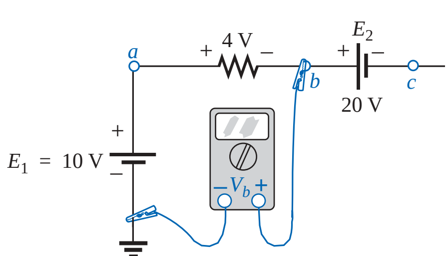


FIG. 5.42
Example 5.17.

Solution: Starting at ground potential (zero volts), we proceed through a rise of 10 V to reach point a and then pass through a drop in potential of 4 V to point b. The result is that the meter will read

$$V_b = +10\text{ V} - 4\text{ V} = \mathbf{6\ V}$$

as clearly demonstrated by Fig. 5.43.

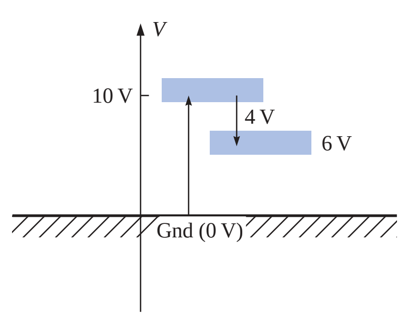


FIG. 5.43
Determining V_b using the defined voltage levels.

If we then proceed to point c, there is an additional drop of 20 V, resulting in

$$V_c = V_b - 20\text{ V} = 6\text{ V} - 20\text{ V} = \mathbf{-14\ V}$$

as shown in Fig. 5.44.

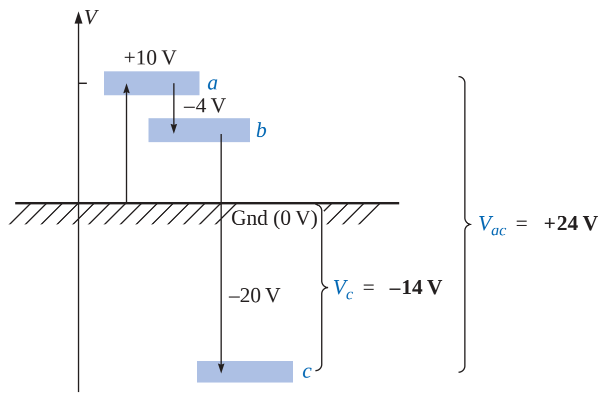


FIG. 5.44
Review of the potential levels for the circuit of Fig. 5.42.

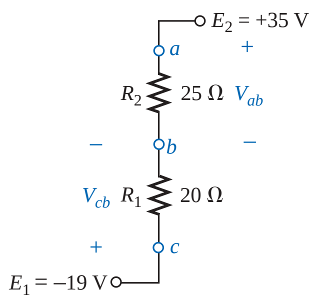


FIG. 5.45
Example 5.18.

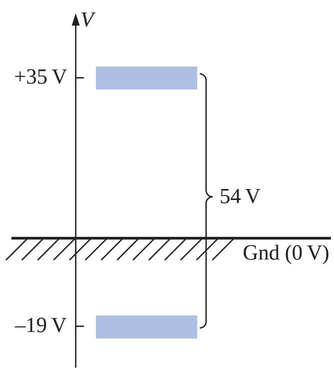


FIG. 5.46
Determining the total voltage drop across the resistive elements of Fig. 5.45.

The voltage V_{ac} can be obtained using Eq. (5.12) or by simply referring to Fig. 5.44:

$$\begin{aligned} V_{ac} &= V_a - V_c = 10\text{ V} - (-14\text{ V}) \\ &= \mathbf{24\ V} \end{aligned}$$

EXAMPLE 5.18 Determine V_{ab}, V_{cb}, and V_c for the network of Fig. 5.45.

Solution: There are two ways to approach this problem. The first is to sketch the diagram of Fig. 5.46 and note that there is a 54-V drop across the series resistors R_1 and R_2. The current can then be determined using Ohm's law and the voltage levels as follows:

$$I = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$

$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$

$$V_c = E_1 = \mathbf{-19\ V}$$

The other approach is to redraw the network as shown in Fig. 5.47 to clearly establish the aiding effect of E_1 and E_2 and then solve the resulting series circuit.

$$I = \frac{E_1 + E_2}{R_T} = \frac{19\text{ V} + 35\text{ V}}{45\ \Omega} = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

and $\quad V_{ab} = \mathbf{30\ V} \qquad V_{cb} = \mathbf{-24\ V} \qquad V_c = \mathbf{-19\ V}$

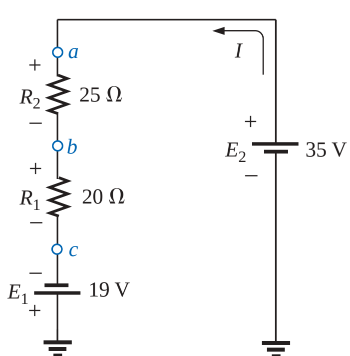


FIG. 5.47
Redrawing the circuit of Fig. 5.45 using standard dc voltage supply symbols.

EXAMPLE 5.19 Using the voltage divider rule, determine the voltages V_1 and V_2 of Fig. 5.48.

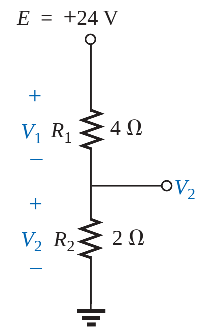


FIG. 5.48
Example 5.19.

Solution: Redrawing the network with the standard battery symbol will result in the network of Fig. 5.49. Applying the voltage divider rule,

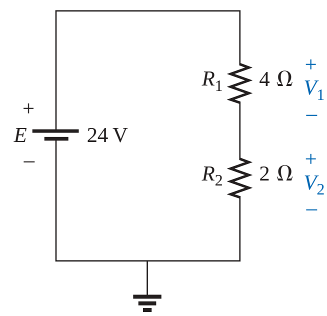


FIG. 5.49
Circuit of Fig. 5.48 redrawn.

$$V_1 = \frac{R_1E}{R_1 + R_2} = \frac{(4\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{16\ V}$$

$$V_2 = \frac{R_2E}{R_1 + R_2} = \frac{(2\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{8\ V}$$

EXAMPLE 5.20 For the network of Fig. 5.50:

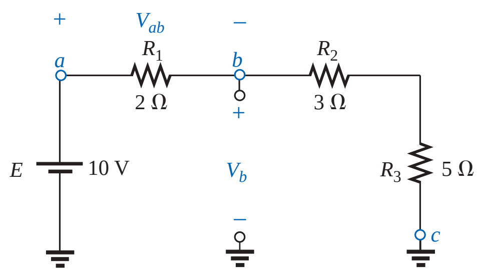


FIG. 5.50
Example 5.20.

a. Calculate V_{ab}.
b. Determine V_b.
c. Calculate V_c.

Solutions:

a. Voltage divider rule:

$$V_{ab} = \frac{R_1E}{R_T} = \frac{(2\ \Omega)(10\ \text{V})}{2\ \Omega + 3\ \Omega + 5\ \Omega} = \mathbf{+2\ V}$$

b. Voltage divider rule:

$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T} = \frac{(3\ \Omega + 5\ \Omega)(10\ \text{V})}{10\ \Omega} = \mathbf{8\ V}$$

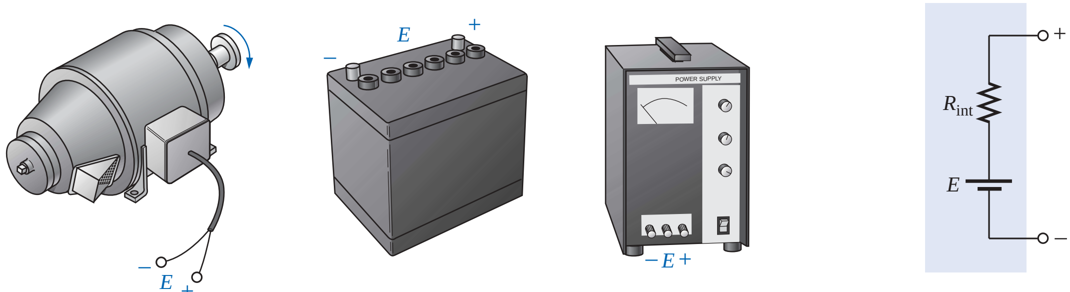
or $\quad V_b = V_a - V_{ab} = E - V_{ab} = 10\ \text{V} - 2\ \text{V} = \mathbf{8\ V}$

c. V_c = ground potential = **0 V**

5.8 INTERNAL RESISTANCE OF VOLTAGE SOURCES

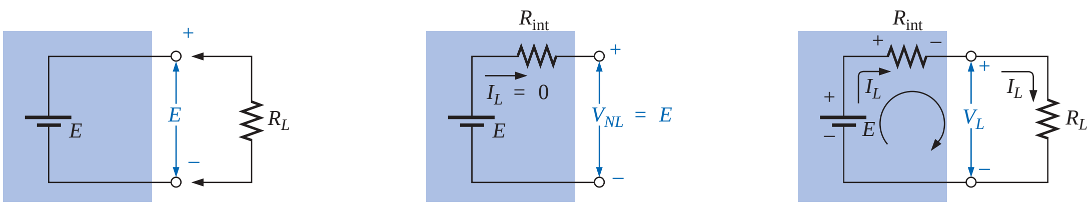
Every source of voltage, whether a generator, battery, or laboratory supply as shown in Fig. 5.51(a), will have some **internal resistance.** The equivalent circuit of any source of voltage will therefore appear as shown in Fig. 5.51(b). In this section, we will examine the effect of the internal resistance on the output voltage so that any unexpected changes in terminal characteristics can be explained.

In all the circuit analyses to this point, the ideal voltage source (no internal resistance) was used [see Fig. 5.52(a)]. The ideal voltage source has no internal resistance and an output voltage of *E* volts with no load or full load. In the practical case [Fig. 5.52(b)], where we con-



(a) (b)

FIG. 5.51
(a) Sources of dc voltage; (b) equivalent circuit.



(a) (b) (c)

FIG. 5.52
Voltage source: (a) ideal, $R_{int} = 0\ \Omega$; (b) determining V_{NL}; (c) determining R_{int}.

sider the effects of the internal resistance, the output voltage will be E volts only when no-load ($I_L = 0$) conditions exist. When a load is connected [Fig. 5.52(c)], the output voltage of the voltage source will decrease due to the voltage drop across the internal resistance.

By applying Kirchhoff's voltage law around the indicated loop of Fig. 5.52(c), we obtain

$$E - I_L R_{\text{int}} - V_L = 0$$

or, since

$$E = V_{NL}$$

we have

$$V_{NL} - I_L R_{\text{int}} - V_L = 0$$

and

$$V_L = V_{NL} - I_L R_{\text{int}} \tag{5.13}$$

If the value of R_{int} is not available, it can be found by first solving for R_{int} in the equation just derived for V_L; that is,

$$R_{\text{int}} = \frac{V_{NL} - V_L}{I_L} = \frac{V_{NL}}{I_L} - \frac{I_L R_L}{I_L}$$

and

$$R_{\text{int}} = \frac{V_{NL}}{I_L} - R_L \tag{5.14}$$

A plot of the output voltage versus current appears in Fig. 5.53 for the dc generator having the circuit representation of Fig. 5.51(b). Note that any increase in load demand, starting at any level, causes an additional drop in terminal voltage due to the increasing loss in potential across the internal resistance. At maximum current, denoted by I_{FL}, the

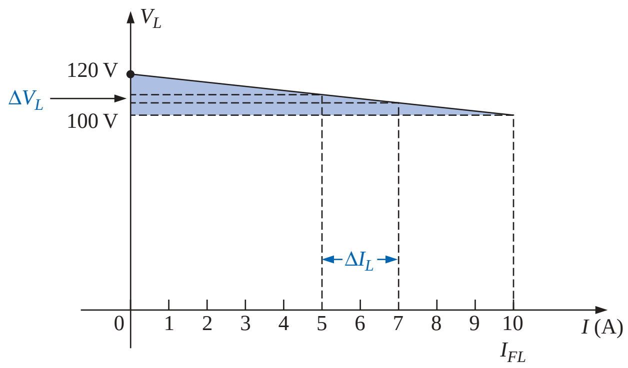


FIG. 5.53
V_L versus I_L for a dc generator with $R_{int} = 2\ \Omega$.

voltage across the internal resistance is $V_{\text{int}} = I_{FL}R_{\text{int}} = (10\text{ A})(2\ \Omega) = 20\text{ V}$, and the terminal voltage has dropped to 100 V—a significant difference when you can ideally expect a 120-V generator to provide the full 120 V if you stay below the listed full-load current. Eventually, if the load current were permitted to increase without limit, the voltage across the internal resistance would equal the supply voltage, and the terminal voltage would be zero. The larger the internal resistance, the steeper is the slope of the characteristics of Fig. 5.53. In fact, for any chosen interval of voltage or current, the magnitude of the internal resistance is given by

$$R_{\text{int}} = \frac{\Delta V_L}{\Delta I_L} \tag{5.15}$$

For the chosen interval of 5–7 A ($\Delta I_L = 2$ A) on Fig. 5.53, ΔV_L is 4 V, and $R_{\text{int}} = \Delta V_L/\Delta I_L = 4\text{ V}/2\text{ A} = 2\ \Omega$.

A direct consequence of the loss in output voltage is a loss in power delivered to the load. Multiplying both sides of Eq. (5.13) by the current I_L in the circuit, we obtain

$$\underbrace{I_LV_L}_{\substack{\text{Power}\\\text{to load}}} = \underbrace{I_LV_{NL}}_{\substack{\text{Power output}\\\text{by battery}}} - \underbrace{I_L^2R_{\text{int}}}_{\substack{\text{Power loss in}\\\text{the form of heat}}} \tag{5.16}$$

EXAMPLE 5.21 Before a load is applied, the terminal voltage of the power supply of Fig. 5.54(a) is set to 40 V. When a load of 500 Ω is attached, as shown in Fig. 5.54(b), the terminal voltage drops to 38.5 V. What happened to the remainder of the no-load voltage, and what is the internal resistance of the source?

Solution: The difference of 40 V − 38.5 V = 1.5 V now appears across the internal resistance of the source. The load current is 38.5 V/0.5 kΩ = 77 mA. Applying Eq. (5.14),

$$R_{\text{int}} = \frac{V_{NL}}{I_L} - R_L = \frac{40\text{ V}}{77\text{ mA}} - 0.5\text{ k}\Omega$$
$$= 519.48\ \Omega - 500\ \Omega = \mathbf{19.48\ \Omega}$$

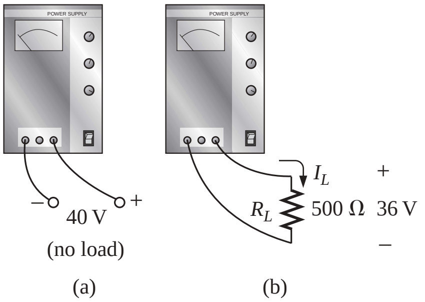


FIG. 5.54
Example 5.21.

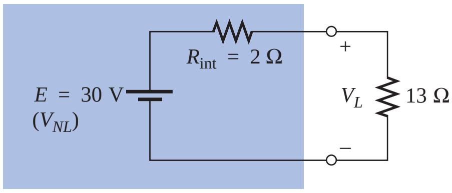


FIG. 5.55
Example 5.22.

EXAMPLE 5.22 The battery of Fig. 5.55 has an internal resistance of 2 Ω. Find the voltage V_L and the power lost to the internal resistance if the applied load is a 13-Ω resistor.

Solution:

$$I_L = \frac{30\text{ V}}{2\ \Omega + 13\ \Omega} = \frac{30\text{ V}}{15\ \Omega} = 2\text{ A}$$

$$V_L = V_{NL} - I_L R_{int} = 30\text{ V} - (2\text{ A})(2\ \Omega) = \mathbf{26\ V}$$

$$P_{lost} = I_L^2 R_{int} = (2\text{ A})^2(2\ \Omega) = (4)(2) = \mathbf{8\ W}$$

Procedures for measuring R_{int} will be described in Section 5.10.

5.9 VOLTAGE REGULATION

For any supply, ideal conditions dictate that for the range of load demand (I_L), the terminal voltage remain fixed in magnitude. In other words, if a supply is set for 12 V, it is desirable that it maintain this terminal voltage, even though the current demand on the supply may vary. A measure of how close a supply will come to ideal conditions is given by the voltage regulation characteristic. By definition, the **voltage regulation** (*VR*) of a supply between the limits of full-load and no-load conditions (Fig. 5.56) is given by the following:

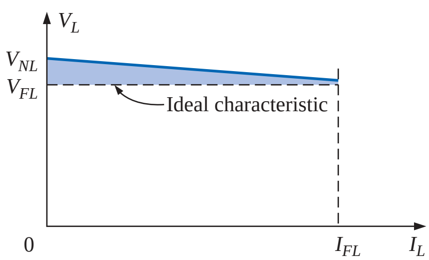


FIG. 5.56
Defining voltage regulation.

$$\text{Voltage regulation } (VR)\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% \qquad \textbf{(5.17)}$$

For ideal conditions, $V_{FL} = V_{NL}$ and $VR\% = 0$. Therefore, *the smaller the voltage regulation, the less the variation in terminal voltage with change in load.*

It can be shown with a short derivation that the voltage regulation is also given by

$$VR\% = \frac{R_{int}}{R_L} \times 100\% \qquad \textbf{(5.18)}$$

In other words, the smaller the internal resistance for the same load, the smaller the regulation and the more ideal the output.

EXAMPLE 5.23 Calculate the voltage regulation of a supply having the characteristics of Fig. 5.53.

Solution:

$$VR\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{120\text{ V} - 100\text{ V}}{100\text{ V}} \times 100\%$$

$$= \frac{20}{100} \times 100\% = \mathbf{20\%}$$

EXAMPLE 5.24 Determine the voltage regulation of the supply of Fig. 5.54.

Solution:

$$VR\% = \frac{R_{int}}{R_L} \times 100\% = \frac{19.48\ \Omega}{500\ \Omega} \times 100\% \cong \mathbf{3.9\%}$$

5.10 MEASUREMENT TECHNIQUES

In Chapter 2, it was noted that ammeters are inserted in the branch in which the current is to be measured. We now realize that such a condition specifies that

ammeters are placed in series with the branch in which the current is to be measured

as shown in Fig. 5.57.

If the ammeter is to have minimal impact on the behavior of the network, its resistance should be very small (ideally zero ohms) compared to the other series elements of the branch such as the resistor R of Fig. 5.57. If the meter resistance approaches or exceeds 10% of R, it will naturally have a significant impact on the current level it is measuring. It is also noteworthy that the resistances of the separate current scales of the same meter are usually not the same. In fact, the meter resistance normally increases with decreasing current levels. However, for the majority of situations one can simply assume that the internal ammeter resistance is small enough compared to the other circuit elements that it can be ignored.

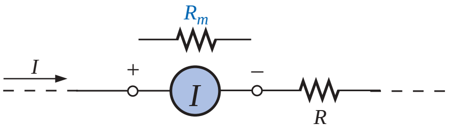


FIG. 5.57
Series connection of an ammeter.

For an up-scale (analog meter) or positive (digital meter) reading, an ammeter must be connected with current entering the positive terminal of the meter and leaving the negative terminal, as shown in Fig. 5.58. Since most meters employ a red lead for the positive terminal and a black lead for the negative, simply ensure that current enters the red lead and leaves the black one.

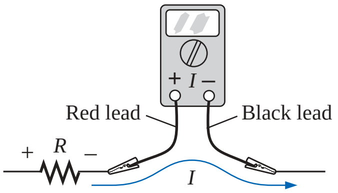


FIG. 5.58
Connecting an ammeter for an up-scale (positive) reading.

Voltmeters are always hooked up across the element for which the voltage is to be determined.

An up-scale or positive reading on a voltmeter is obtained by being sure that the positive terminal (red lead) is connected to the point of higher potential and the negative terminal (black lead) is connected to the lower potential, as shown in Fig. 5.59.

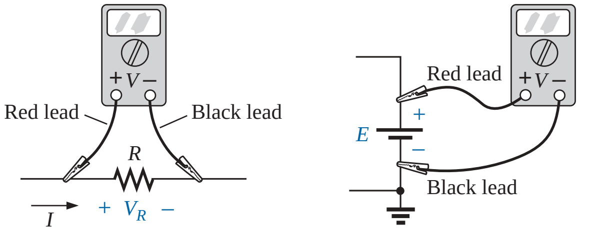


FIG. 5.59
Hooking up a voltmeter to obtain an up-scale (positive) reading.

For the double-subscript notation, always hook up the red lead to the first subscript and the black lead to the second; that is, to measure the voltage V_{ab} in Fig. 5.60, connect the red lead to point a and the black lead to point b. For single-subscript notation, hook up the red lead to the point of interest and the black lead to ground, as shown in Fig. 5.60 for V_a and V_b.

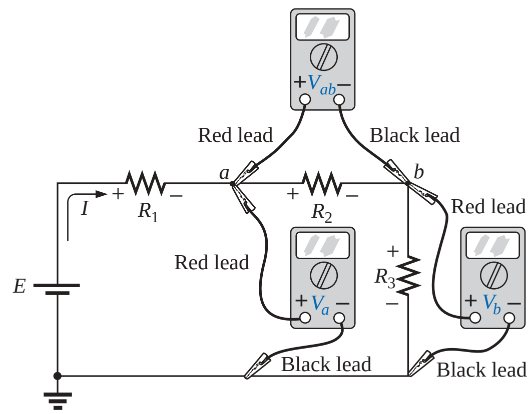


FIG. 5.60
Measuring voltages with double- and single-subscript notation.

The internal resistance of a supply cannot be measured with an ohmmeter due to the voltage present. However, the no-load voltage can be measured by simply hooking up the voltmeter as shown in Fig. 5.61(a). Do not be concerned about the apparent path for current that the meter seems to provide by completing the circuit. The internal resistance of the meter is usually sufficiently high to ensure that the resulting current is so small that it can be ignored. (Voltmeter loading effects will be discussed in detail in Section 6.9.) An ammeter could then be placed directly across the supply, as shown in Fig. 5.61(b), to measure the short-circuit current I_{sc} and R_{int} as determined by Ohm's law: $R_{int} = E_{NL}/I_{sc}$. However, since the internal resistance of the supply may be very low, performing the measurement could result in high current levels that could damage the meter and supply and possibly cause dangerous side effects. The setup of Fig. 5.61(b) is therefore *not* suggested. A better approach would be to apply a resistive load that will result in a supply current of about half the maximum rated value and measure the terminal voltage. Then use Eq. (5.14).

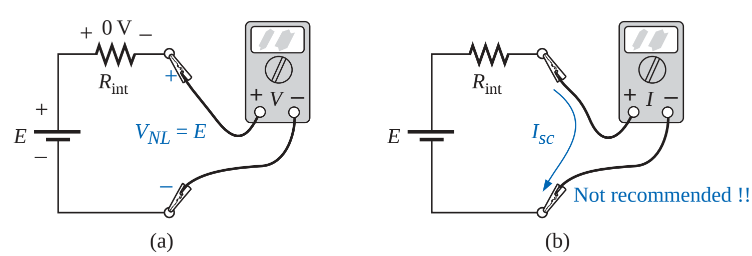


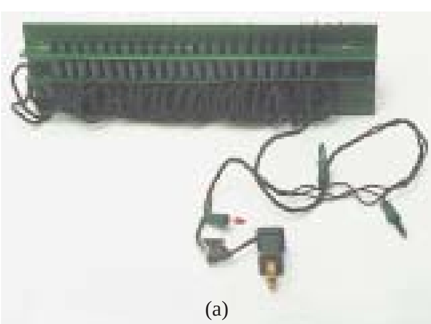
FIG. 5.61
(a) Measuring the no-load voltage E; (b) measuring the short-circuit current.

5.11 APPLICATIONS

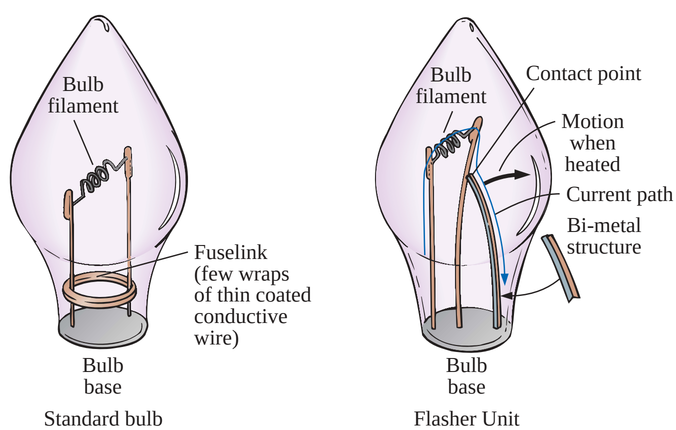
Holiday Lights

In recent years the small blinking holiday lights with as many as 50 to 100 bulbs on a string have become very popular [see Fig. 5.62(a)]. Although holiday lights can be connected in series or parallel (to be described in the next chapter), the smaller blinking light sets are normally connected in series. It is relatively easy to determine if the lights are connected in series. If one wire enters and leaves the bulb casing, they are in series. If two wires enter and leave, they are probably in parallel. Normally, when bulbs are connected in series, if one should burn out (the filament breaks and the circuit opens), all the bulbs will go out since the current path has been interrupted. However, the bulbs of Fig. 5.62(a) are specially designed, as shown in Fig. 5.62(b), to permit current to continue to flow to the other bulbs when the filament burns out. At the base of each bulb there is a fuse link wrapped around the two posts holding the filament. The fuse link of a soft conducting metal appears to be touching the two vertical posts, but in fact a coating on the posts or fuse link prevents conduction from one to the other under normal operating conditions. If a filament should burn out and create an open circuit between the posts, the current through the bulb and other bulbs would be interrupted if it were not for the fuse link. At the instant a bulb opens up, current through the circuit is zero, and the full 120 V from the outlet will appear across the bad bulb. This high voltage from post to post of a single bulb is of sufficient potential difference to establish current through the insulating coatings and spot-weld the fuse link to the two posts. The circuit is again complete, and all the bulbs will light except the one with the activated fuse link. Keep in mind, however, that each time a bulb burns out, there will be more voltage across the other bulbs of the circuit, making them burn brighter. Eventually, if too many bulbs burn out, the voltage will reach a point where the other bulbs will burn out in rapid succession. The result is that one must replace burned-out bulbs at the earliest opportunity.

The bulbs of Fig. 5.62(a) are rated 2.5 V at 0.2 A or 200 mA. Since there are 50 bulbs in series, the total voltage across the bulbs will be 50×2.5 V or 125 V which matches the voltage available at the typical



(a)



(b)

FIG. 5.62
Holiday lights: (a) 50-unit set; (b) bulb construction.

home outlet. Since the bulbs are in series, the current through each bulb will be 200 mA. The power rating of each bulb is therefore $P = VI = (2.5 \text{ V})(0.2 \text{ A}) = 0.5 \text{ W}$ with a total wattage demand of $50 \times 0.5 \text{ W} = 25 \text{ W}$.

A schematic representation for the set of Fig. 5.62(a) is provided in Fig. 5.63(a). Note that only one flasher unit is required. Since the bulbs are in series, when the flasher unit interrupts the current flow, it will turn off all the bulbs. As shown in Fig. 5.62(b), the flasher unit incorporates a bimetal thermal switch that will open when heated to a preset level by the current. As soon as it opens, it will begin to cool down and close again so that current can return to the bulbs. It will then heat up again, open up, and repeat the entire process. The result is an on-and-off action that creates the flashing pattern we are so familiar with. Naturally, in a colder climate (for example, outside in the snow and ice), it will initially take longer to heat up, so the flashing pattern will be reduced at first; but in time as the bulbs warm up, the frequency will increase.

The manufacturer specifies that no more than six sets should be connected together. The first question that then arises is, How can sets be connected together, end to end, without reducing the voltage across each bulb and making all the lights dimmer? If you look closely at the

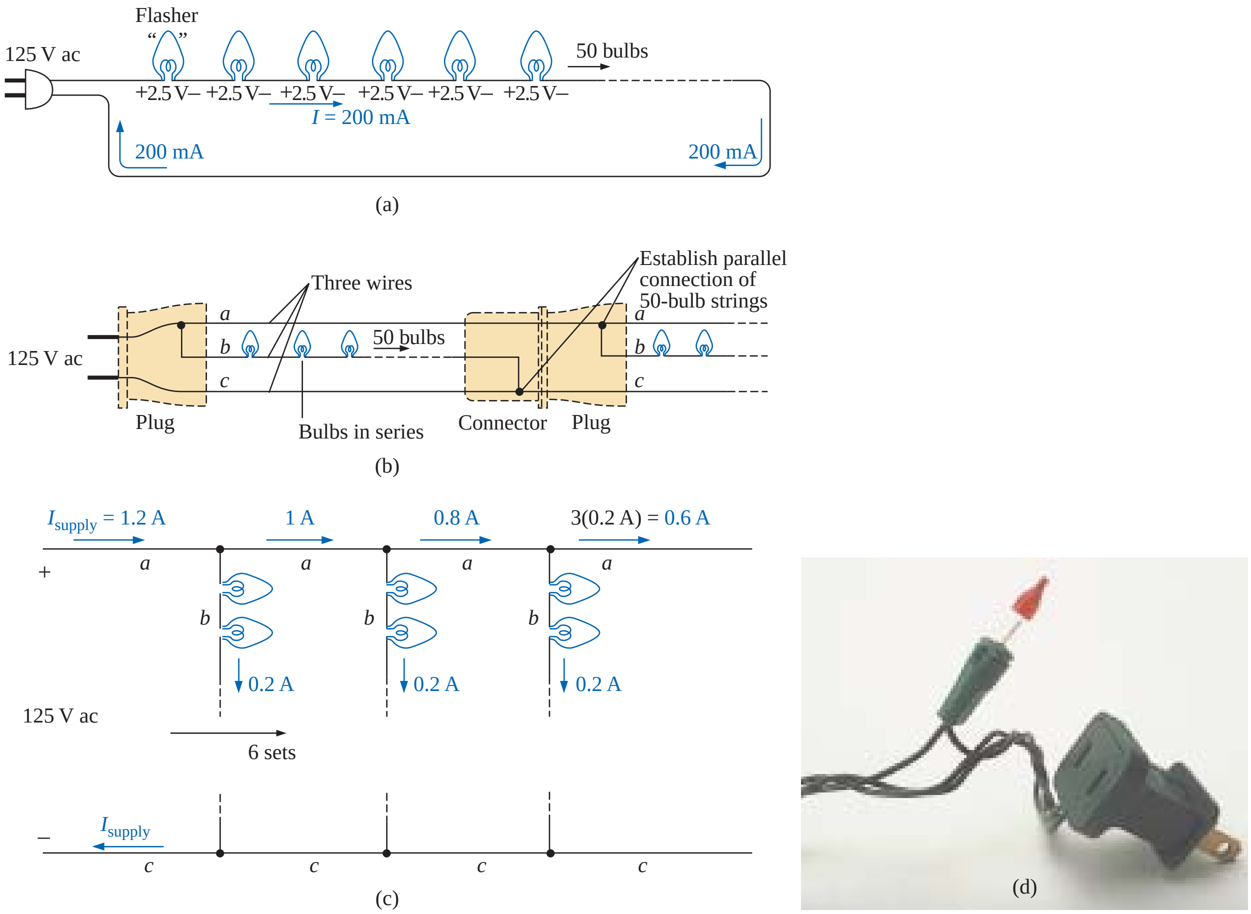


FIG. 5.63
(a) Single-set wiring diagram; (b) special wiring arrangement; (c) redrawn schematic; (d) special plug and flasher unit.

wiring, you will find that since the bulbs are connected in series, there is one wire to each bulb with additional wires from plug to plug. Why would they need two additional wires if the bulbs are connected in series? The answer lies in the fact that when each set is connected together, they will actually be in parallel (to be discussed in the next chapter) by a unique wiring arrangement shown in Fig. 5.63(b) and redrawn in Fig. 5.63(c) to clearly show the parallel arrangement. Note that the top line is the hot line to all the connected sets, and the bottom line is the return, neutral, or ground line for all the sets. Inside the plug of Fig. 5.63(d) the hot line and return are connected to each set, with the connections to the metal spades of the plug as shown in Fig. 5.63(b). We will find in the next chapter that the current drawn from the wall outlet for parallel loads is the sum of the current to each branch. The result, as shown in Fig. 5.63(c), is that the current drawn from the supply is 6 × 200 mA = 1.2 A, and the total wattage for all six sets is the product of the applied voltage and the source current or (120 V)(1.2 A) = 144 W with 144 W/6 = 24 W per set.

Microwave Oven

Series circuits can be very effective in the design of safety equipment. Although we all recognize the usefulness of the microwave oven, it can be quite dangerous if the door is not closed or sealed properly. It is not enough to test the closure at only one point around the door because the door may be bent or distorted from continual use, and leakage could result at some point distant from the test point. One common safety arrangement appears in Fig. 5.64. Note that magnetic switches are located all around the door, with the magnet in the door itself and the magnetic door switch in the main frame. Magnetic switches are simply switches where the magnet draws a magnetic conducting bar between two contacts to complete the circuit—somewhat revealed by the symbol

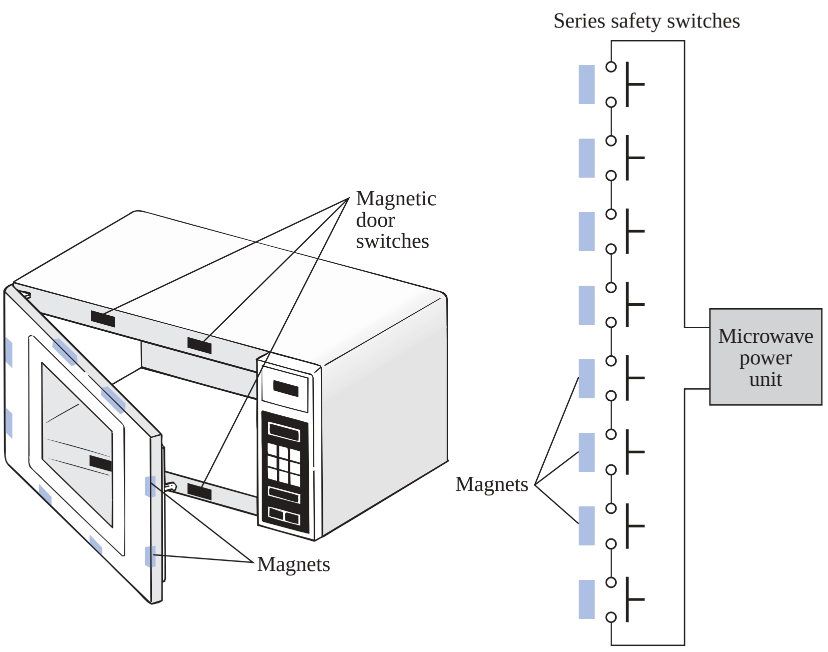


FIG. 5.64
Series safety switches in a microwave oven.

for the device in the circuit diagram of Fig. 5.64. Since the magnetic switches are all in series, they must all be closed to complete the circuit and turn on the power unit. If the door is sufficiently out of shape to prevent a single magnet from getting close enough to the switching mechanism, the circuit will not be complete, and the power cannot be turned on. Within the control unit of the power supply, either the series circuit completes a circuit for operation or a sensing current is established and monitored that controls the system operation.

Series Alarm Circuit

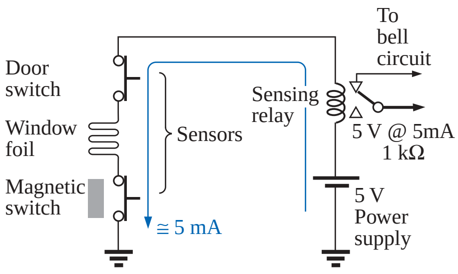


FIG. 5.65
Series alarm circuit.

The circuit of Fig. 5.65 is a simple alarm circuit. Note that every element of the design is in a series configuration. The power supply is a 5-V dc supply that can be provided through a design similar to that of Fig. 2.31, a dc battery, or a combination of an ac and a dc supply that ensures that the battery will always be at full charge. If all the sensors are closed, a current of 5 mA will result because of the terminal load of the relay of about 1 kΩ. That current energizes the relay and maintains an off position for the alarm. However, if any of the sensors are opened, the current will be interrupted, the relay will let go, and the alarm circuit will be energized. With relatively short wires and a few sensors, the system should work well since the voltage drop across each will be minimal. However, since the alarm wire is usually relatively thin, resulting in a measurable resistance level, if the wire to the sensors is too long, a sufficient voltage drop could occur across the line, reducing the voltage across the relay to a point where the alarm fails to operate properly. Thus, wire length is a factor that must be considered if a series configuration is used. Proper sensitivity to the length of the line should remove any concerns about its operation. An improved design will be described in Chapter 8.

5.12 COMPUTER ANALYSIS

PSpice

In Section 4.9, the basic procedure for setting up the PSpice folder and running the program were presented. Because of the detail provided in that section, you should review it before proceeding with this example. Because this is only the second example using PSpice, some detail will be provided, but not at the level of Section 4.9.

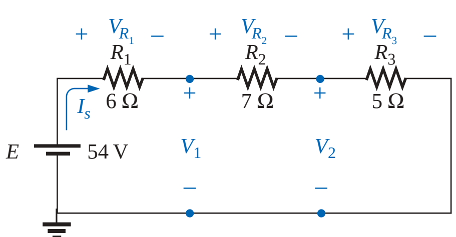


FIG. 5.66
Series dc network to be analyzed using PSpice.

The circuit to be investigated appears in Fig. 5.66. Since the **PSpice** folder was established in Section 4.9, there is no need to repeat the process here—it is immediately available. Double-clicking on the **Orcad Lite Edition** icon will generate the **Orcad Capture-Lite Edition** window. A new project is then initiated by selecting the **Create document** key at the top left of the screen (it looks like a page with a star in the upper left corner). The result is the **New Project** dialog box in which **SeriesDC** is inserted as the **Name.** The **Analog or Mixed A/D** is already selected, and **C:\PSpice** appears as the **Location**—only the **Name** had to be entered! Click **OK,** and the **Create PSpice Project** dialog box will appear. Select **Create a blank project,** click **OK,** and the working windows will appear. Grabbing the left edge of the **SCHEMATIC1:PAGE1** window will allow you to move it to the right so that you can see both screens. Clicking the + sign in the **Project Manager** window will allow you to set the sequence down to **PAGE1.** If you prefer to change the name of the **SCHEMATIC1,** just select it and right-click on the mouse. A listing will appear in which **Rename** is an option; selecting it will result in a **Rename Schematic** dialog box in which **SeriesDC** can be entered. In Fig. 5.67 it was left as **SCHEMATIC1.**

Now this next step is important! If the toolbar on the right edge does not appear, be sure to double-click on **PAGE1** in the **Project Manager** window, or select the **Schematic Window.** When the heading of the **Schematic Window** is dark blue, the toolbar will appear. To start building the circuit, select **Place a part** key (the second one down) to obtain the **Place Part** dialog box. Note that now the **SOURCE** library is already in place in the **Library** list from the efforts of Chapter 4; it does not have to be reinstalled. Selecting **SOURCE** will result in the list of sources under **Part List,** and **VDC** can be selected. Click **OK,** and the cursor can put it in place with a single left click. Right-click and select **End Mode** to end the process since the network has only one source. One more left click and the source is in place. Now the **Place a part** key is selected again, followed by **ANALOG** library to find the resistor **R.** Once the resistor has been selected, an **OK** will place it next to the cursor on the screen. This time, since three resistors need to be placed, there is no need to go to **End Mode** between depositing each. Simply click one in place, then the next, and finally the third. Then right-click to end the process with **End Mode.** Finally, a **GND** must be added by selecting the appropriate key and selecting **0/SOURCE** in the **Place Ground** dialog box. Click **OK,** and place the ground as shown in Fig. 5.67.

The elements must now be connected using the **Place a wire** key to obtain the crosshair on the screen. Start at the top of the voltage source with a left click, and draw the wire, left-clicking it at every 90° turn. When a wire is connected from one element to another, move on to the next connection to be made—there is no need to go **End Mode** between

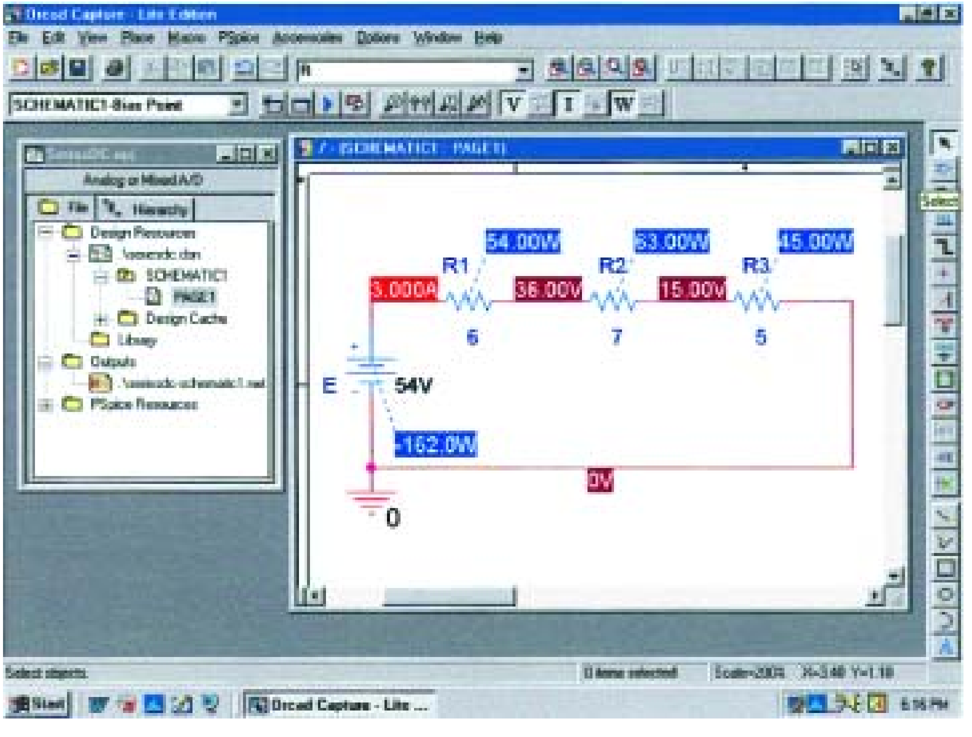


FIG. 5.67
Applying PSpice to a series dc circuit.

connections. Now the labels and values have to be set by double-clicking on each parameter to obtain a **Display Properties** dialog box. Since the dialog box appears with the quantity of interest in a blue background, simply type in the desired label or value, followed by **OK.** The network is now complete and ready to be analyzed.

Before simulation, select the **V, I,** and **W** in the toolbar at the top of the window to ensure that the voltages, currents, and power are displayed on the screen. To simulate, select the **New Simulation Profile** key (which appears as a data sheet on the second toolbar down with a star in the top left corner) to obtain the **New Simulation** dialog box. Enter **Bias Point** for a dc solution under **Name,** and hit the **Create** key. A **Simulation Settings-Bias Point** dialog box will appear in which **Analysis** is selected and **Bias Point** is found under the **Analysis type** heading. Click **OK,** and then select the **Run PSpice** key (the blue arrow) to initiate the simulation. Exit the resulting screen, and the display of Fig. 5.67 will result.

The current is clearly 3 A for the circuit with 15 V across R_3, and 36 V from a point between R_1 and R_2 to ground. The voltage across R_2 is 36 V − 15 V = 21 V, and the voltage across R_1 is 54 V − 36 V = 18 V. The power supplied or dissipated by each element is also listed. There is no question that the results of Fig. 5.67 include a very nice display of voltage, current, and power levels.

Electronics Workbench (EWB)

Since this is only the second circuit to be constructed using EWB, a detailed list of steps will be included as a review. Essentially, however, the entire circuit of Fig. 5.68 can be “drawn” using simply the construction information introduced in Chapter 4.

After you have selected the **Multisim 2001** icon, a **Multisim-Circuit 1** window will appear ready to accept the circuit elements. Select the **Sources** key at the top of the left toolbar, and a **Sources** parts bin will appear with 30 options. Selecting the top option will place the **GROUND** on the screen of Fig. 5.68, and selecting the third option down will result in **DC_VOLTAGE_SOURCE.** The resistors are obtained by choosing the second key down on the left toolbar called the **Basic** key. The result is 25 options in which **RESISTOR_VIRTUAL** is selected. We must return to the **RESISTOR_VIRTUAL** key to place each resistor on the screen. However, each new resistor is numbered in sequence, although they are all given the default value of 1 kΩ. Remember from the discussion of Chapter 4 that you should add the meters before connecting the elements together because the meters take space and must be properly oriented. The current will be determined by the **XMM1** ammeter and the voltages by **XMM2** through **XMM5.** Of particular importance, note that

in EWB the meters are connected in exactly the same way they would be placed in an active circuit in the laboratory. Ammeters are in series with the branch in which the current is to be determined, and voltmeters are connected between the two points of interest (across resistors). In addition, for positive readings, ammeters are connected so that conventional current enters the positive terminal, and voltmeters are connected so that the point of higher potential is connected to the positive terminal.

The meter settings are made by double-clicking on the meter symbol on the schematic. In each case, **V** or **I** had to be chosen, but the hori-

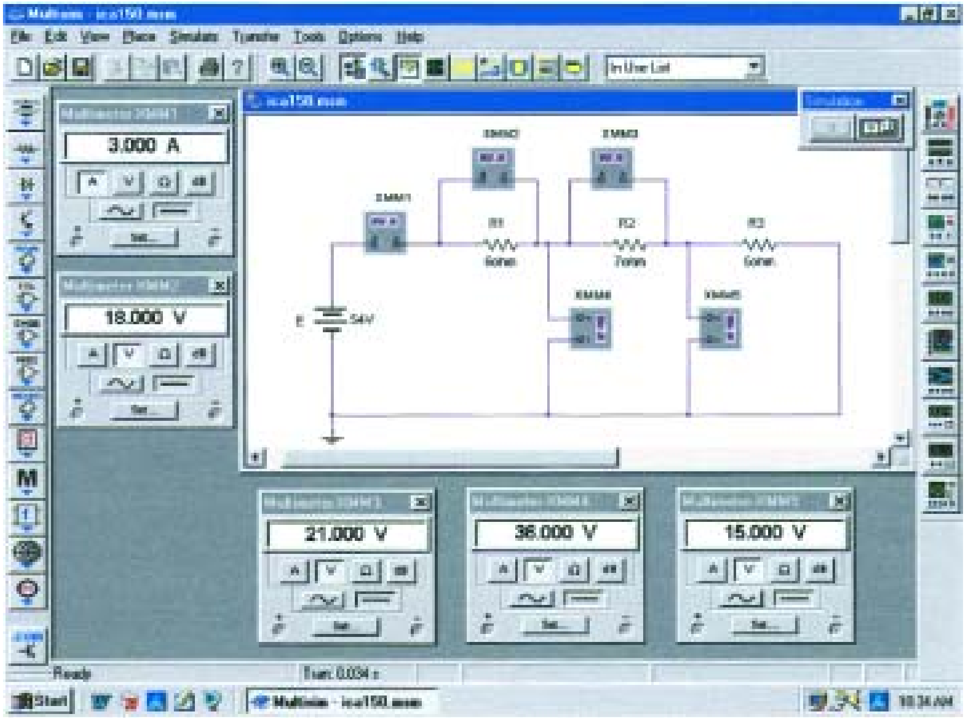


FIG. 5.68
Applying Electronics Workbench to a series dc circuit.

zontal line for dc analysis is the same for each. Again, the **Set** key can be selected to see what it controls, but the default values of meter input resistance levels will be fine for all the analyses described in this text. Leave the meters on the screen so that the various voltages and the current level will be displayed after the simulation.

Recall from Chapter 4 that elements can be moved by simply clicking on each schematic symbol and dragging it to the desired location. The same is true for labels and values. Labels and values are set by double-clicking on the label or value and entering your preference. Click **OK,** and they will appear changed on the schematic. There is no need to first select a special key to connect the elements. Simply bring the cursor to the starting point to generate the small circle and crosshair. Click on the starting point, and follow the desired path to the next connection path. When in location, click again, and the line will appear. All connecting lines can make 90° turns. However, you cannot follow a diagonal path from one point to another. To remove any element, label, or line, simply click on the quantity to obtain the four-square active status, and select the **Delete** key or the scissors key on the top menu bar.

Before simulating, be sure that the **Simulate Switch** is visible by selecting **View-Show Simulate Switch.** Then select the **1** option on the switch, and the analysis will begin. The results appearing in Fig. 5.68 verify those obtained using PSpice and the longhand solution.

C++

We will now turn to the C++ language and review a program designed to perform the same analysis just performed using PSpice and EWB. As noted in earlier chapters, do not expect to understand all the details of how the program was written and why specific paths were taken.

```
Heading          [ //C++ Series Circuit Analysis

Preprocessor     [ #include <iostream.h>          //needed for input/output
directive

                 [ class resistor {                //define resistor class
                   public:                         //allow access to variables in class
                           float value;            //resistance in ohms
                           float voltage;          //voltage across resistor
Define                     float power;            //power used by resistor
variables          };
and
data               class voltage_source {          //define voltage source class
type               public:
                           float voltage;          //source voltage
                           float current;          //source current
                           float power;            //power supplied by source
                 [ };

                   main()                          //execution begins here
                   {
Establish        [         resistor R1, R2, R3;    //create three resistor objects
objects                    float Rtotal;           //total resistance variable
and R_T                    voltage_source V1;      //create voltage source object

Assign           [         R1.value = 6;           //assign resistance values
values                     R2.value = 7;
                           R3.value = 5;
Calculate R_T    [         Rtotal = R1.value + R2.value + R3.value;  //find atotal resistance
Display R_T      [         cout << "The total resistance is " << Rtotal << " Ohms.\n";

Define E         [         V1.voltage = 54;                          //assign source voltage
I = E/R_T        [         V1.current = V1.voltage / Rtotal;         //find circuit current
Display I        [         cout << "The circuit current is " << V1.current << " Amperes.\n";

Calculate        [         R1.voltage = V1.current * R1.value;       //find resistor voltages
V_R                        R2.voltage = V1.current * R2.value;
                           R3.voltage = V1.current * R3.value;
Display          [         cout << "The voltage across R1 is " << R1.voltage << " Volts.\n";
V_R                        cout << "The voltage across R2 is " << R2.voltage << " Volts.\n";
                           cout << "The voltage across R3 is " << R3.voltage << " Volts.\n";

Calculate        [         R1.power = V1.current * R1.voltage;       //find resistor powers
P_R                        R2.power = V1.current * R2.voltage;
                           R3.power = V1.current * R3.voltage;
Display          [         cout << "The power to R1 is " << R1.power << " Watts.\n";
P_R                        cout << "The power to R2 is " << R2.power << " Watts.\n";
                           cout << "The power to R3 is " << R3.power << " Watts.\n";

Calculate P_E    [         V1.power = V1.voltage * V1.current;       //find total power
Display P_E      [         cout << "The total power is " << V1.power << " Watts.\n";
                   }

Body of program (bracket spanning main() through closing brace)
```

FIG. 5.69
C++ program designed to perform a complete analysis of the network of Fig. 5.66.

The purpose here is simply to expose the reader to the general characteristics of a program using this increasingly popular programming language.

First take note of all the double forward slashes // in the program of Fig. 5.69. They are used to identify comments in the program that will not be recognized by the compiler when the program is run. They can also be used to remind the programmer about specific objectives to be met at a particular point in the program or the reason for specific entries. In this example, however, the primary purpose was to enlighten the reader about the purpose of a particular entry or operation.

The *#include* tells the computer to include the file to follow in the C++ program. The *<iostream.h>* is a header file that sets up the input-output path between the program and the disk operating system. The *class* format defines the data type (in this case all floating points, which means that a decimal point is included), and the *public* within the { } reveals that the variables *value*, *voltage*, and *power* are accessible for operations outside the data structure.

Note that the *main* ( ) part of the program extends all the way down to the bottom, as identified by the braces { }. Within this region all the parameters of the network will be given values, all the calculations will be made, and finally all the results will be provided. Next, three resistor objects are established. *Rtotal* is defined as a floating variable, and a voltage source object is introduced. The values of the resistors are then entered, and the total resistance is calculated. Next, through *cout,* the total resistance is printed out using the *Rtotal* just calculated. The *n*” at the end of the *cout* line calls for a carriage return to prepare for the next *cout* statement.

On the next line, the magnitude of the voltage source is introduced, followed by the calculation of the source current, which is then printed out on the next line. Next the voltage across each resistor is calculated and printed out by the succeeding lines. Finally, the various powers are calculated and printed out.

When run, the output will appear as shown in Fig. 5.70 with the same results obtained using PSpice and EWB. As noted above, do not be perplexed by all the details of why certain lines appear as they do. Like everything, with proper instruction and experience, it will all become fairly obvious. Do note, however, that the first few lines set up the analysis to be performed by telling the computer the type of operations that need to be handled and the format of the data to be entered and expected. There is then a main part of the program where all the entries, calculations, and outputs are performed. When this program is run, its flow is top-down; that is, one step follows the other without looping back to certain points (an option to be described in a later program). There was no need to number the lines or to include detailed instructions. If all the comments were removed, the actual program would be quite compact and straightforward, with most of the body of the program being *cout* statements.

```
The total resistance is 18 Ohms.
The circuit current is 3 Amperes.
The voltage across R1 is 18 Volts.
The voltage across R2 is 21 Volts.
The voltage across R3 is 15 Volts.
The power to R1 is 54 Watts.
The power to R2 is 63 Watts.
The power to R3 is 45 Watts.
The total power is 162 Watts.
```

FIG. 5.70
Output results for the C++ program of Fig. 5.69.

PROBLEMS

SECTION 5.2 Series Circuits

1. Find the total resistance and current I for each circuit of Fig. 5.71.

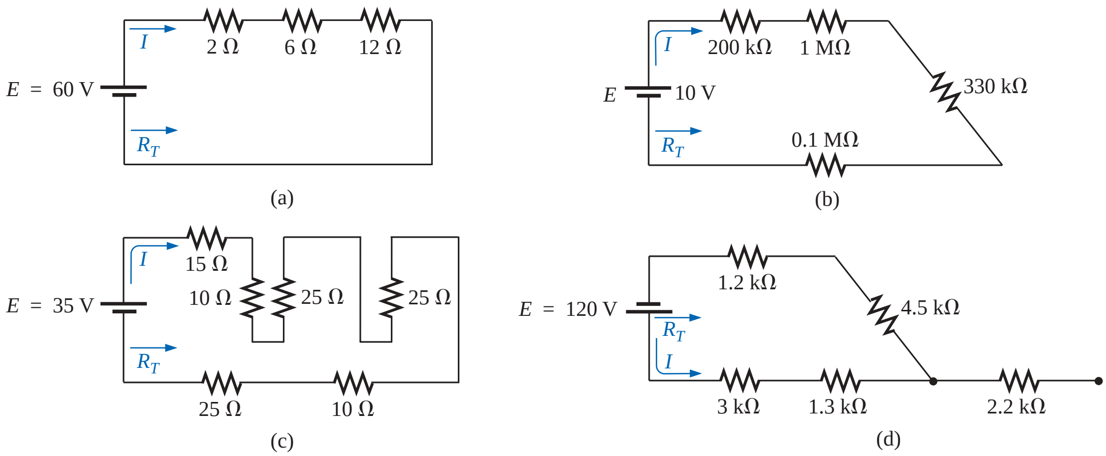


FIG. 5.71
Problems 1 and 36.

2. For the circuits of Fig. 5.72, the total resistance is specified. Find the unknown resistances and the current I for each circuit.

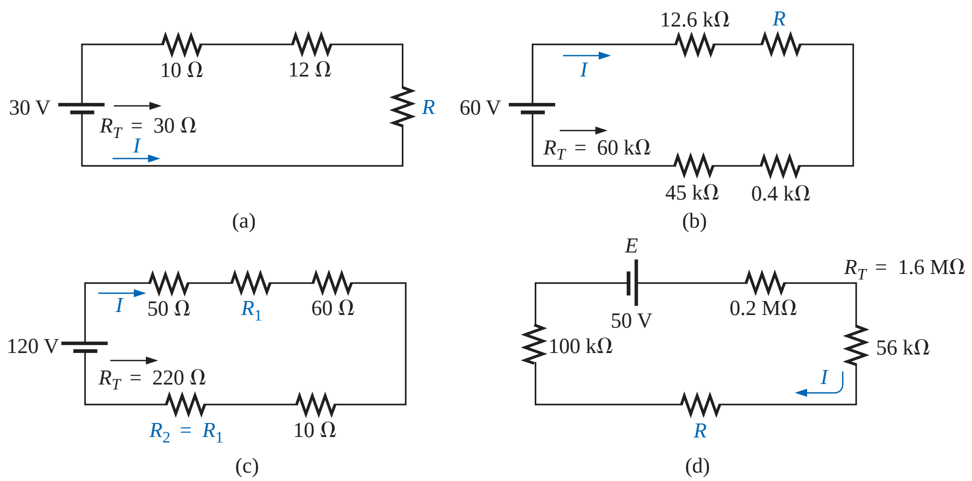


FIG. 5.72
Problem 2.

3. Find the applied voltage E necessary to develop the current specified in each network of Fig. 5.73.

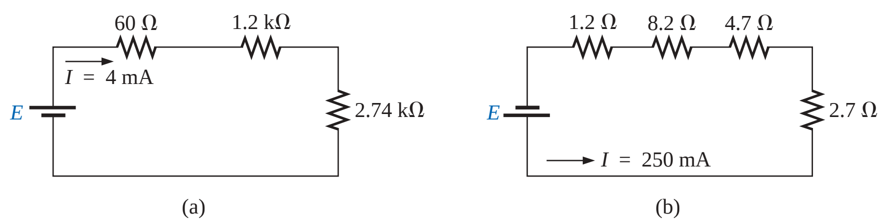


FIG. 5.73
Problem 3.

***4.** For each network of Fig. 5.74, determine the current I, the source voltage E, the unknown resistance, and the voltage across each element.

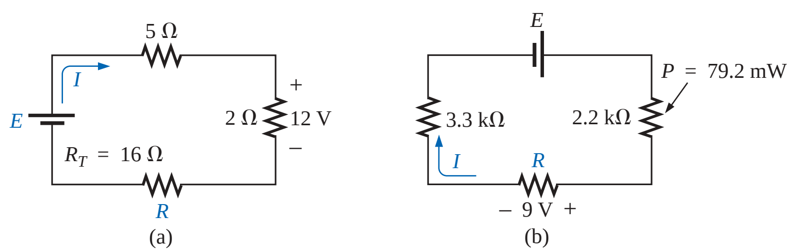


FIG. 5.74
Problem 4.

SECTION 5.3 Voltage Sources in Series

5. Determine the current I and its direction for each network of Fig. 5.75. Before solving for I, redraw each network with a single voltage source.

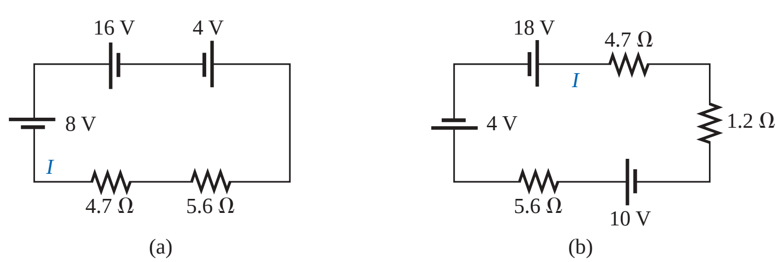


FIG. 5.75
Problem 5.

*6. Find the unknown voltage source and resistor for the networks of Fig. 5.76. Also indicate the direction of the resulting current.

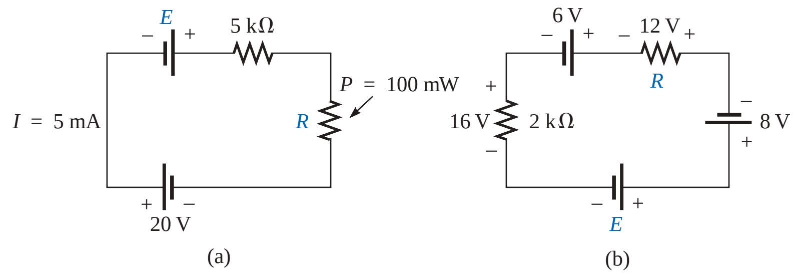


FIG. 5.76
Problem 6.

SECTION 5.4 Kirchhoff's Voltage Law

7. Find V_{ab} with polarity for the circuits of Fig. 5.77. Each box can contain a load or a power supply, or a combination of both.

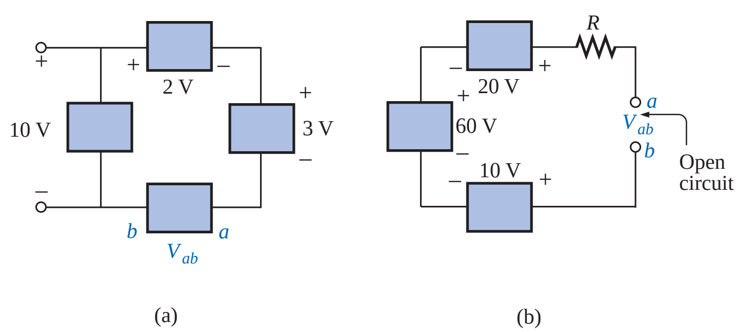


FIG. 5.77
Problem 7.

8. Although the networks of Fig. 5.78 are not simply series circuits, determine the unknown voltages using Kirchhoff's voltage law.

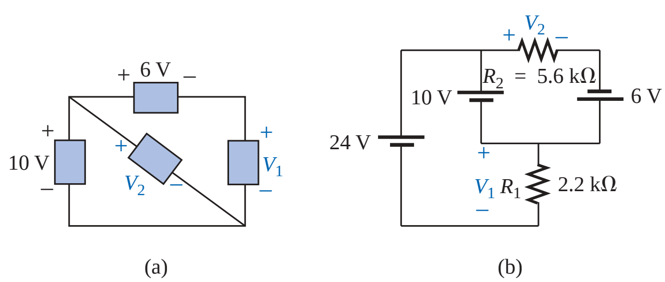


FIG. 5.78
Problem 8.

9. Determine the current I and the voltage V_1 for the network of Fig. 5.79.

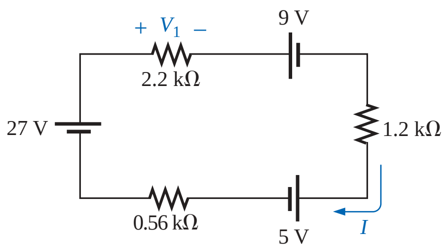


FIG. 5.79
Problem 9.

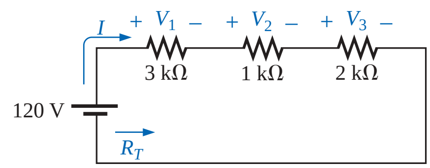


FIG. 5.80
Problem 10.

10. For the circuit of Fig. 5.80:

a. Find the total resistance, current, and unknown voltage drops.

b. Verify Kirchhoff's voltage law around the closed loop.

c. Find the power dissipated by each resistor, and note whether the power delivered is equal to the power dissipated.

d. If the resistors are available with wattage ratings of 1/2, 1, and 2 W, what minimum wattage rating can be used for each resistor in this circuit?

11. Repeat Problem 10 for the circuit of Fig. 5.81.

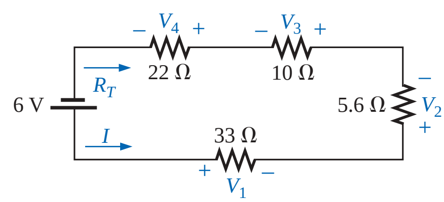


FIG. 5.81
Problem 11.

***12.** Find the unknown quantities in the circuits of Fig. 5.82 using the information provided.

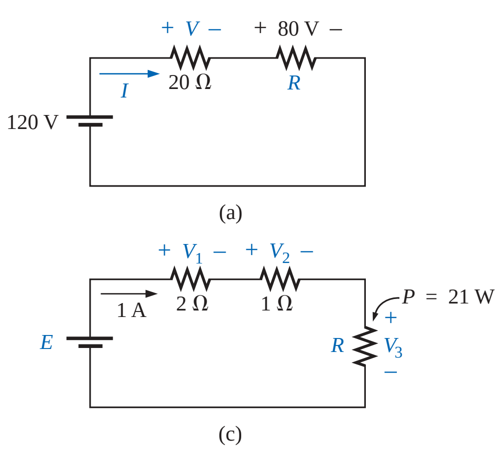


FIG. 5.82
Problem 12.

13. Eight holiday lights are connected in series as shown in Fig. 5.83.

a. If the set is connected to a 120-V source, what is the current through the bulbs if each bulb has an internal resistance of $28\frac{1}{8}\ \Omega$?

b. Determine the power delivered to each bulb.

c. Calculate the voltage drop across each bulb.

d. If one bulb burns out (that is, the filament opens), what is the effect on the remaining bulbs?

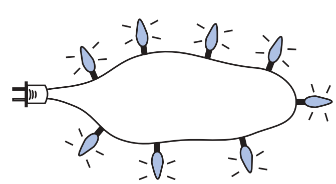


FIG. 5.83
Problem 13.

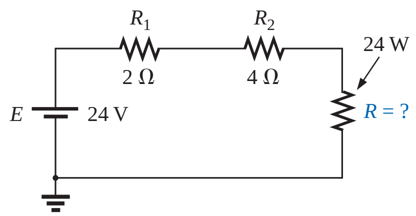


FIG. 5.84
Problem 14.

***14.** For the conditions specified in Fig. 5.84, determine the unknown resistance.

SECTION 5.6 Voltage Divider Rule

15. Using the voltage divider rule, find V_{ab} (with polarity) for the circuits of Fig. 5.85.

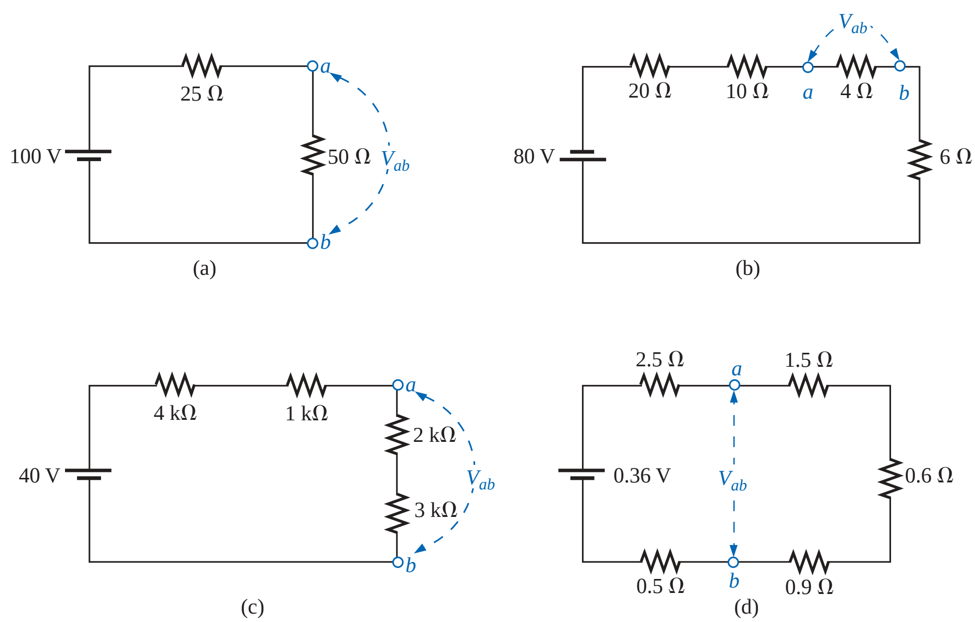


FIG. 5.85
Problems 15 and 37.

16. Find the unknown resistance using the voltage divider rule and the information provided for the circuits of Fig. 5.86.

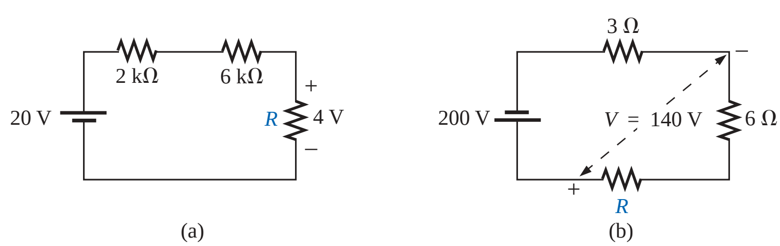


FIG. 5.86
Problem 16.

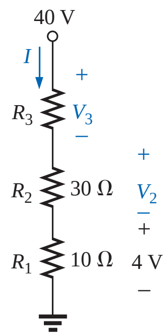


FIG. 5.87
Problem 17.

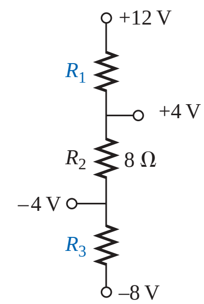


FIG. 5.88
Problem 18.

17. Referring to Fig. 5.87:
   a. Determine V_2 by simply noting that $R_2 = 3R_1$.
   b. Calculate V_3.
   c. Noting the magnitude of V_3 compared to V_2 or V_1, determine R_3 by inspection.
   d. Calculate the source current I.
   e. Calculate the resistance R_3 using Ohm's law, and compare it to the result of part (c).

18. Given the information appearing in Fig. 5.88, find the level of resistance for R_1 and R_3.

19. **a.** Design a voltage divider circuit that will permit the use of an 8-V, 50-mA bulb in an automobile with a 12-V electrical system.
   b. What is the minimum wattage rating of the chosen resistor if ¼-W, ½-W and 1-W resistors are available?

20. Determine the values of R_1, R_2, R_3, and R_4 for the voltage divider of Fig. 5.89 if the source current is 16 mA.

21. Design the voltage divider of Fig. 5.90 such that $V_{R_1} = (1/5)V_{R_2}$ if $I = 4$ mA.

22. Find the voltage across each resistor of Fig. 5.91 if $R_1 = 2R_3$ and $R_2 = 7R_3$.

23. **a.** Design the circuit of Fig. 5.92 such that $V_{R_2} = 3V_{R_1}$ and $V_{R_3} = 4V_{R_2}$.
   b. If the current I is reduced to 10 μA, what are the new values of R_1, R_2, and R_3? How do they compare to the results of part (a)?

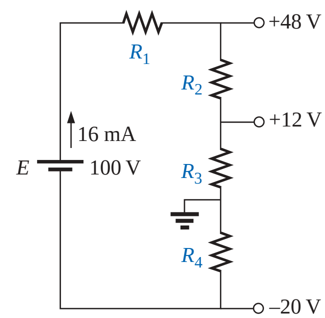


FIG. 5.89
Problem 20.

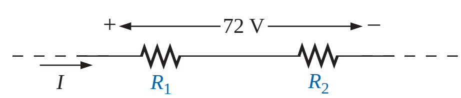


FIG. 5.90
Problem 21.

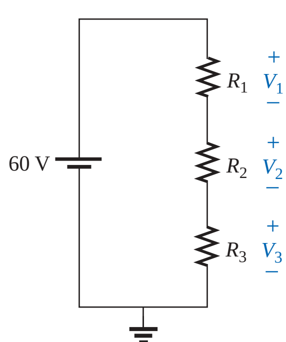


FIG. 5.91
Problem 22.

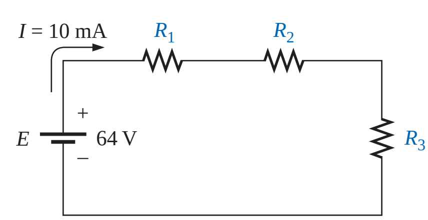


FIG. 5.92
Problem 23.

SECTION 5.7 Notation

24. Determine the voltages V_a, V_b, and V_{ab} for the networks of Fig. 5.93.

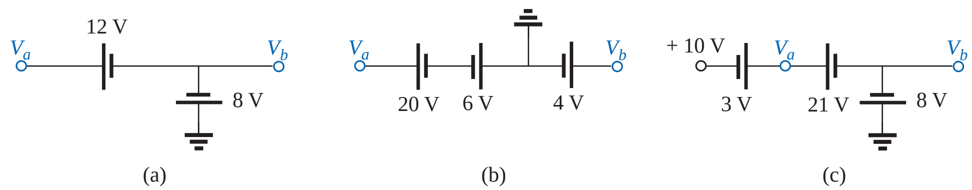


FIG. 5.93
Problem 24.

25. Determine the current I (with direction) and the voltage V (with polarity) for the networks of Fig. 5.94.

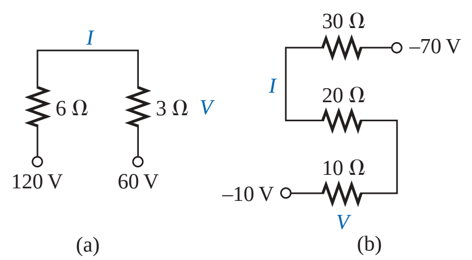


FIG. 5.94
Problem 25.

26. Determine the voltages V_a and V_1 for the networks of Fig. 5.95.

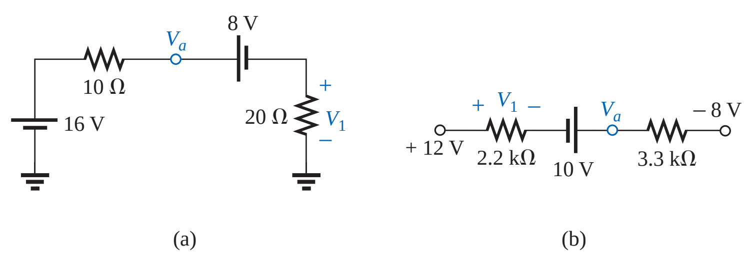


FIG. 5.95
Problem 26.

***27.** For the network of Fig. 5.96, determine the voltages:

a. V_a, V_b, V_c, V_d, V_e
b. V_{ab}, V_{dc}, V_{cb}
c. V_{ac}, V_{db}

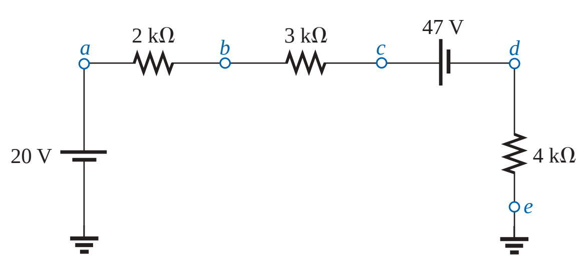


FIG. 5.96
Problem 27.

***28.** For the network of Fig. 5.97, determine the voltages:
a. V_a, V_b, V_c, V_d
b. V_{ab}, V_{cb}, V_{cd}
c. V_{ad}, V_{ca}

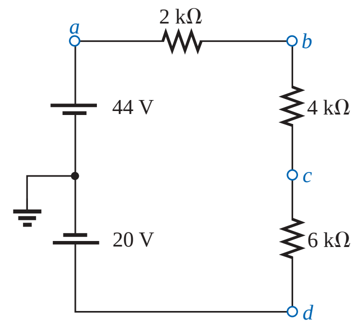


FIG. 5.97
Problem 28.

***29.** For the integrated circuit of Fig. 5.98, determine V_0, V_4, V_7, V_{10}, V_{23}, V_{30}, V_{67}, V_{56}, and I (magnitude and direction).

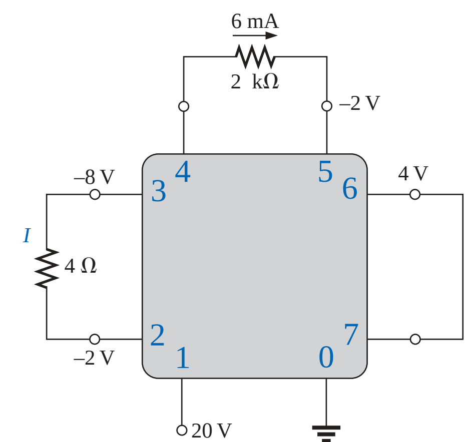


FIG. 5.98
Problem 29.

***30.** For the integrated circuit of Fig. 5.99, determine V_0, V_{03}, V_2, V_{23}, V_{12}, and I_i.

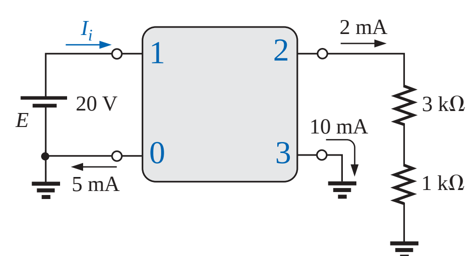


FIG. 5.99
Problem 30.

SECTION 5.8 Internal Resistance of Voltage Sources

31. Find the internal resistance of a battery that has a no-load output voltage of 60 V and that supplies a current of 2 A to a load of 28 Ω.

32. Find the voltage V_L and the power loss in the internal resistance for the configuration of Fig. 5.100.

33. Find the internal resistance of a battery that has a no-load output voltage of 6 V and supplies a current of 10 mA to a load of 1/2 kΩ.

SECTION 5.9 Voltage Regulation

34. Determine the voltage regulation for the battery of Problem 31.

35. Calculate the voltage regulation for the supply of Fig. 5.100.

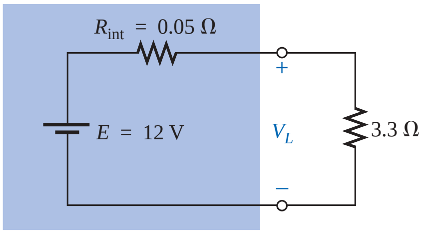


FIG. 5.100
Problems 32 and 35.

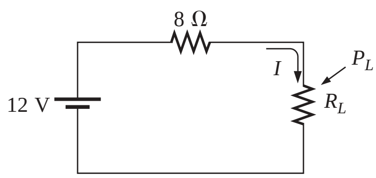


FIG. 5.101
Problem 40.

SECTION 5.12 Computer Analysis

PSpice or Electronics Workbench

36. Using schematics, determine the current I and the voltage across each resistor for the network of Fig. 5.71(a).

37. Using schematics, determine the voltage V_{ab} for the network of Fig. 5.85(d).

Programming Language (C++, QBASIC, Pascal, etc.)

38. Write a program to determine the total resistance of any number of resistors in series.

39. Write a program that will apply the voltage divider rule to either resistor of a series circuit with a single source and two series resistors.

40. Write a program to tabulate the current and power to the resistor R_L of the network of Fig. 5.101 for a range of values for R_L from 1 Ω to 20 Ω. Print out the value of R_L that results in maximum power to R_L.

GLOSSARY

Branch The portion of a circuit consisting of one or more elements in series.

Circuit A combination of a number of elements joined at terminal points providing at least one closed path through which charge can flow.

Closed loop Any continuous connection of branches that allows tracing of a path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

Conventional current flow A defined direction for the flow of charge in an electrical system that is opposite to that of the motion of electrons.

Electron flow The flow of charge in an electrical system having the same direction as the motion of electrons.

Internal resistance The inherent resistance found internal to any source of energy.

Kirchhoff's voltage law (KVL) The algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

Series circuit A circuit configuration in which the elements have only one point in common and each terminal is not connected to a third, current-carrying element.

Voltage divider rule (VDR) A method by which a voltage in a series circuit can be determined without first calculating the current in the circuit.

Voltage regulation (*VR*) A value, given as a percent, that provides an indication of the change in terminal voltage of a supply with a change in load demand.