7

Series-Parallel Networks

7.1 SERIES-PARALLEL NETWORKS

A firm understanding of the basic principles associated with series and parallel circuits is a sufficient background to begin an investigation of any single-source dc network having a combination of series and parallel elements or branches. Multisource networks are considered in detail in Chapters 8 and 9. In general,

series-parallel networks ***are networks that contain both series and parallel circuit configurations.***

One can become proficient in the analysis of series-parallel networks only through exposure, practice, and experience. In time the path to the desired unknown becomes more obvious as one recalls similar configurations and the frustration resulting from choosing the wrong approach. There are a few steps that can be helpful in getting started on the first few exercises, although the value of each will become apparent only with experience.

General Approach

1. Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use. The result may be time- and energy-saving shortcuts.
2. Next examine each region of the network independently before tying them together in series-parallel combinations. This will usually simplify the network and possibly reveal a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that might result due to the lack of a systematic approach.
3. Redraw the network as often as possible with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.

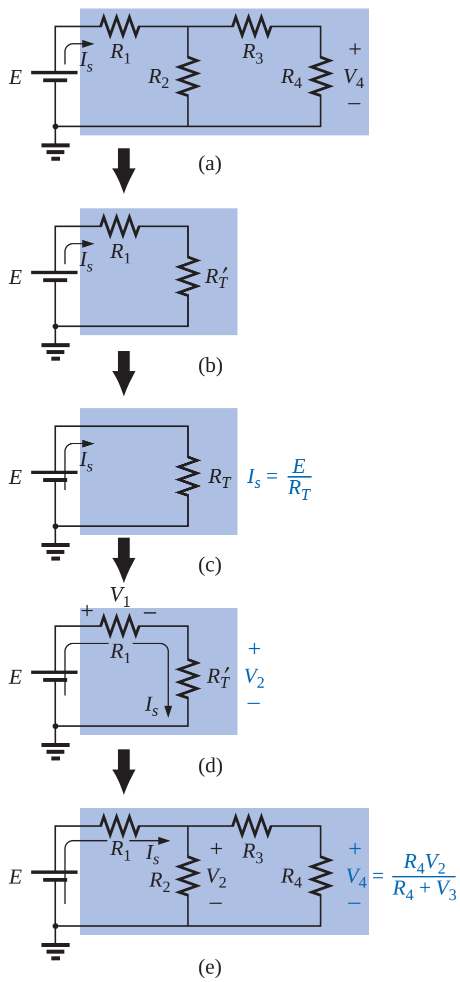

FIG. 7.1
Introducing the reduce and return approach.

4. When you have a solution, check that it is reasonable by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or check over your work very carefully.

Reduce and Return Approach

For many single-source, series-parallel networks, the analysis is one that works back to the source, determines the source current, and then finds its way to the desired unknown. In Fig. 7.1(a), for instance, the voltage V_4 is desired. The absence of a single series or parallel path to V_4 from the source immediately reveals that the methods introduced in the last two chapters cannot be applied here. First, series and parallel elements must be combined to establish the reduced circuit of Fig. 7.1(b). Then series elements are combined to form the simplest of configurations in Fig. 7.1(c). The source current can now be determined using Ohm's law, and we can proceed back through the network as shown in Fig. 7.1(d). The voltage V_2 can be determined and then the original network can be redrawn, as shown in Fig. 7.1(e). Since V_2 is now known, the voltage divider rule can be used to find the desired voltage V_4. Because of the similarities between the networks of Figs. 7.1(a) and 7.1(e), and between 7.1(b) and 7.1(d), the networks drawn during the reduction phase are often used for the return path.

Although all the details of the analysis were not described above, the general procedure for a number of series-parallel network problems employs the procedure described above: Work back for I_s and then follow the return path for the specific unknown. Not every problem will follow this path; some will have simpler, more direct solutions. However, the reduce and return approach will handle one type of problem that does surface over and over again.

Block Diagram Approach

The block diagram approach will be employed throughout to emphasize the fact that combinations of elements, not simply single resistive elements, can be in series or parallel. The approach will also reveal the number of seemingly different networks that have the same basic structure and therefore can involve similar analysis techniques.

Initially, there will be some concern about identifying series and parallel elements and branches and choosing the best procedure to follow toward a solution. However, as you progress through the examples and try a few problems, a common path toward most solutions will surface that can actually make the analysis of such systems an interesting, enjoyable experience.

In Fig. 7.2, blocks *B* and *C* are in parallel (points *b* and *c* in common), and the voltage source *E* is in series with block *A* (point *a* in common). The parallel combination of *B* and *C* is also in series with *A* and the voltage source *E* due to the common points *b* and *c*, respectively.

To ensure that the analysis to follow is as clear and uncluttered as possible, the following notation will be used for series and parallel combinations of elements. For series resistors R_1 and R_2, a comma will be inserted between their subscript notations, as shown here:

$$R_{1,2} = R_1 + R_2$$

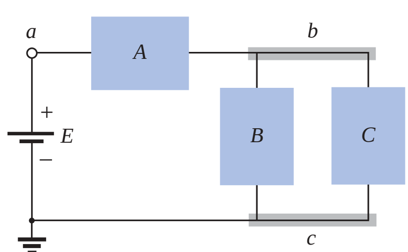

FIG. 7.2
Introducing the block diagram approach.

For parallel resistors R_1 and R_2, the parallel symbol will be inserted between their subscript notations, as follows:

$$R_{1\|2} = R_1 \,\|\, R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

EXAMPLE 7.1 If each block of Fig. 7.2 were a single resistive element, the network of Fig. 7.3 might result.

The parallel combination of R_B and R_C results in

$$R_{B\|C} = R_B \,\|\, R_C = \frac{(12\text{ k}\Omega)(6\text{ k}\Omega)}{12\text{ k}\Omega + 6\text{ k}\Omega} = 4\text{ k}\Omega$$

The equivalent resistance $R_{B\|C}$ is then in series with R_A, and the total resistance "seen" by the source is

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 2\text{ k}\Omega + 4\text{ k}\Omega = \mathbf{6\text{ k}\Omega} \end{aligned}$$

The result is an equivalent network, as shown in Fig. 7.4, permitting the determination of the source current I_s.

$$I_s = \frac{E}{R_T} = \frac{54\text{ V}}{6\text{ k}\Omega} = \mathbf{9\text{ mA}}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9\text{ mA}$$

We can then use the equivalent network of Fig. 7.5 to determine I_B and I_C using the current divider rule:

$$I_B = \frac{6\text{ k}\Omega(I_s)}{6\text{ k}\Omega + 12\text{ k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9\text{ mA}) = \mathbf{3\text{ mA}}$$

$$I_C = \frac{12\text{ k}\Omega(I_s)}{12\text{ k}\Omega + 6\text{ k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9\text{ mA}) = \mathbf{6\text{ mA}}$$

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9\text{ mA} - 3\text{ mA} = \mathbf{6\text{ mA}}$$

Note that in this solution, we worked back to the source to obtain the source current or total current supplied by the source. The remaining unknowns were then determined by working back through the network to find the other unknowns.

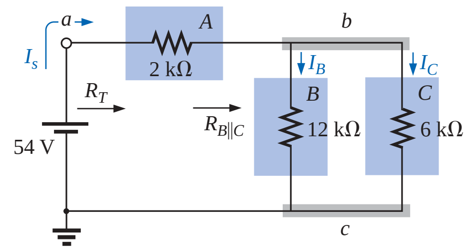

FIG. 7.3
Example 7.1.

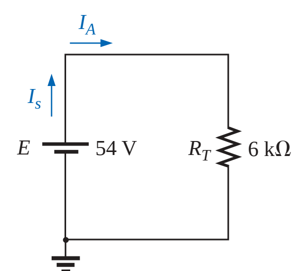

FIG. 7.4
Reduced equivalent of Fig. 7.3.

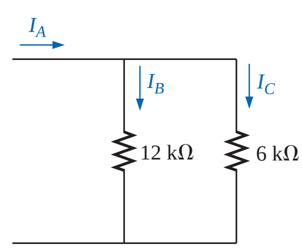

FIG. 7.5
Determining I_B and I_C for the network of Fig. 7.3.

EXAMPLE 7.2 It is also possible that the blocks A, B, and C of Fig. 7.2 contain the elements and configurations of Fig. 7.6. Working with each region:

A: $R_A = 4\ \Omega$

B: $R_B = R_2 \,\|\, R_3 = R_{2\|3} = \dfrac{R}{N} = \dfrac{4\ \Omega}{2} = 2\ \Omega$

C: $R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$

Blocks B and C are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

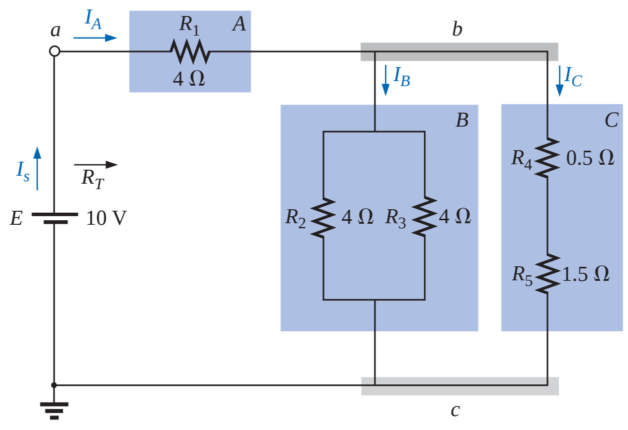

FIG. 7.6
Example 7.2.

with

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega} \end{aligned}$$

(Note the similarity between this equation and that obtained for Example 7.1.)

and
$$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

We can find the currents I_A, I_B, and I_C using the reduction of the network of Fig. 7.6 (recall Step 3) as found in Fig. 7.7. Note that I_A, I_B, and I_C are the same in Figs. 7.6 and 7.7 and therefore also appear in Fig. 7.7. In other words, the currents I_A, I_B, and I_C of Fig. 7.7 will have the same magnitude as the same currents of Fig. 7.6.

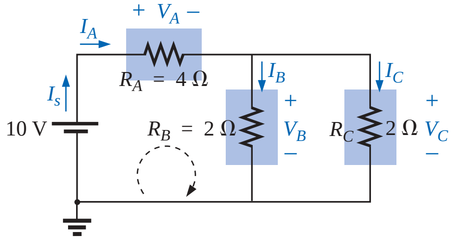

FIG. 7.7
Reduced equivalent of Fig. 7.6.

$$I_A = I_s = \mathbf{2\ A}$$

and
$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$

Returning to the network of Fig. 7.6, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$\begin{aligned} V_A &= I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V} \\ V_B &= I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V} \\ V_C &= V_B = \mathbf{2\ V} \end{aligned}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.7, we obtain

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\ \text{V} + 2\ \text{V}$$
or
$$\underline{10\ \text{V} = 10\ \text{V} \quad \text{(checks)}}$$

EXAMPLE 7.3 Another possible variation of Fig. 7.2 appears in Fig. 7.8.

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

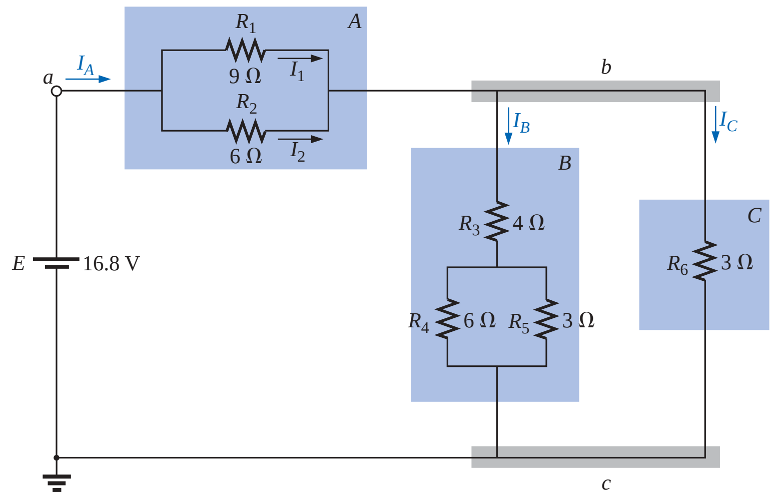

FIG. 7.8
Example 7.3.

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network of Fig. 7.8 can then be redrawn in reduced form, as shown in Fig. 7.9. Note the similarities between this circuit and the circuits of Figs. 7.3 and 7.7.

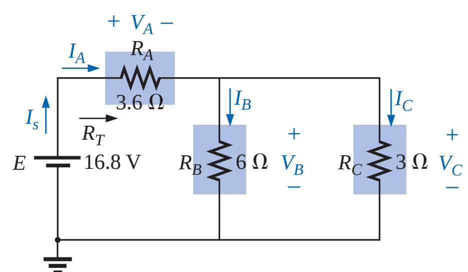

FIG. 7.9
Reduced equivalent of Fig. 7.8.

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \mathbf{10.8\ V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \mathbf{6\ V}$$

Returning to the original network (Fig. 7.8) and applying the current divider rule,

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

Figures 7.3, 7.6, and 7.8 are only a few of the infinite variety of configurations that the network can assume starting with the basic arrangement of Fig. 7.2. They were included in our discussion to emphasize the importance of considering each region of the network independently before finding the solution for the network as a whole.

The blocks of Fig. 7.2 can be arranged in a variety of ways. In fact, there is no limit on the number of series-parallel configurations that can appear within a given network. In reverse, the block diagram approach can be used effectively to reduce the apparent complexity of a system by identifying the major series and parallel components of the network. This approach will be demonstrated in the next few examples.

7.2 DESCRIPTIVE EXAMPLES

EXAMPLE 7.4 Find the current I_4 and the voltage V_2 for the network of Fig. 7.10.

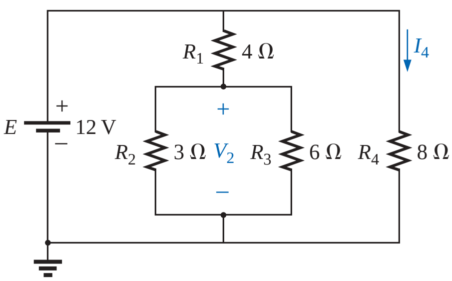

FIG. 7.10
Example 7.4.

Solution: In this case, particular unknowns are requested instead of a complete solution. It would, therefore, be a waste of time to find all the currents and voltages of the network. The method employed should concentrate on obtaining only the unknowns requested. With the block diagram approach, the network has the basic structure of Fig. 7.11, clearly indicating that the three branches are in parallel and the voltage across A and B is the supply voltage. The current I_4 is now immediately obvious as simply the supply voltage divided by the resultant resistance for B. If desired, block A could be broken down further, as shown in Fig. 7.12, to identify C and D as series elements, with the voltage V_2 capable of being determined using the voltage divider rule once the resistance of C and D is reduced to a single value. This is an example of how a mental sketch of the approach might be made before applying laws, rules, and so on, to avoid dead ends and growing frustration.

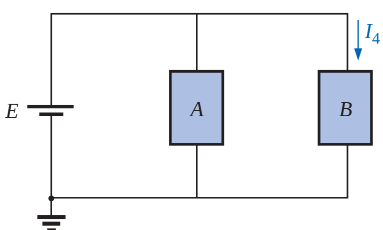

FIG. 7.11
Block diagram of Fig. 7.10.

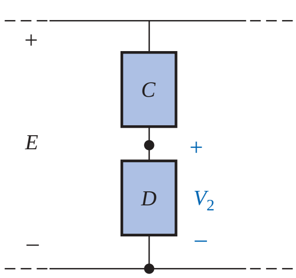

FIG. 7.12
Alternative block diagram for the first parallel branch of Fig. 7.10.

Applying Ohm's law,

$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12\text{ V}}{8\ \Omega} = \mathbf{1.5\ A}$$

Combining the resistors R_2 and R_3 of Fig. 7.10 will result in

$$R_D = R_2 \parallel R_3 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

and, applying the voltage divider rule,

$$V_2 = \frac{R_D E}{R_D + R_C} = \frac{(2\ \Omega)(12\text{ V})}{2\ \Omega + 4\ \Omega} = \frac{24\text{ V}}{6} = \mathbf{4\ V}$$

EXAMPLE 7.5 Find the indicated currents and voltages for the network of Fig. 7.13.

Solution: Again, only specific unknowns are requested. When the network is redrawn, it will be particularly important to note which unknowns are preserved and which will have to be determined using the original configuration. The block diagram of the network may appear as shown in Fig. 7.14, clearly revealing that A and B are in series. Note in this form the number of unknowns that have been preserved. The voltage V_1 will be the same across the three parallel branches of Fig. 7.13, and V_5 will be the same across R_4 and R_5. The unknown currents I_2 and I_4 are lost since they represent the currents through only one of the parallel branches. However, once V_1 and V_5 are known, the required currents can be found using Ohm's law.

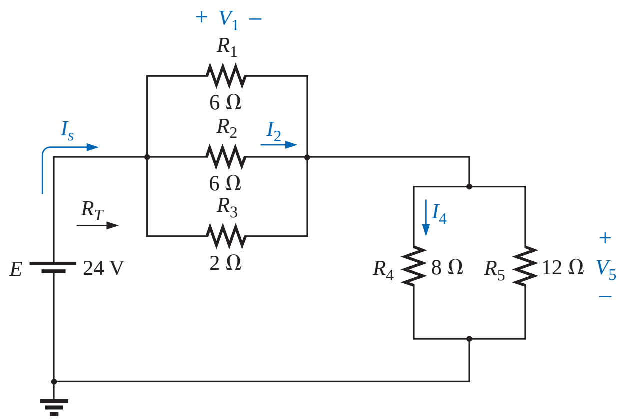

FIG. 7.13
Example 7.5.

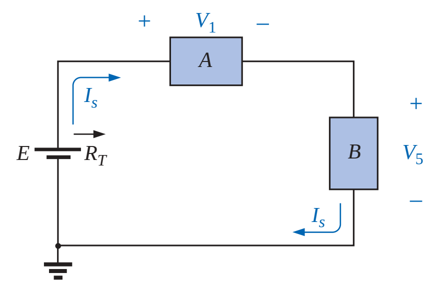

FIG. 7.14
Block diagram for Fig. 7.13.

$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig. 7.13 will then appear as shown in Fig. 7.15, and

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$

with

$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$

$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

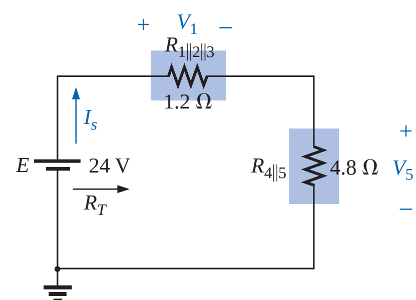

FIG. 7.15
Reduced form of Fig. 7.13.

The next example demonstrates that unknown voltages do not have to be across elements but can exist between any two points in a network. In addition, the importance of redrawing the network in a more familiar form is clearly revealed by the analysis to follow.

EXAMPLE 7.6

a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. 7.16.
b. Calculate the source current I_s.

Solutions: This is one of those situations where it might be best to redraw the network before beginning the analysis. Since combining both sources will not affect the unknowns, the network is redrawn as shown in Fig. 7.17, establishing a parallel network with the total source voltage across each parallel branch. The net source voltage is the difference between the two with the polarity of the larger.

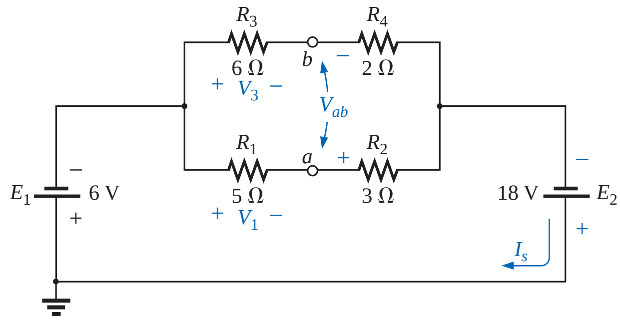

FIG. 7.16
Example 7.6.

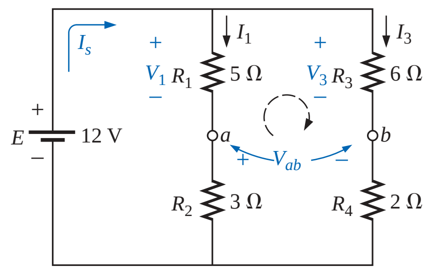

FIG. 7.17
Network of Fig. 7.16 redrawn.

a. Note the similarities with Fig. 7.12, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1E}{R_1 + R_2} = \frac{(5\,\Omega)(12\text{ V})}{5\,\Omega + 3\,\Omega} = \frac{60\text{ V}}{8} = \mathbf{7.5\text{ V}}$$

$$V_3 = \frac{R_3E}{R_3 + R_4} = \frac{(6\,\Omega)(12\text{ V})}{6\,\Omega + 2\,\Omega} = \frac{72\text{ V}}{8} = \mathbf{9\text{ V}}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop of Fig. 7.17 in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\text{ V} - 7.5\text{ V} = \mathbf{1.5\text{ V}}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\text{ V}}{5\,\Omega} = 1.5\text{ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\text{ V}}{6\,\Omega} = 1.5\text{ A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\text{ A} + 1.5\text{ A} = \mathbf{3\text{ A}}$$

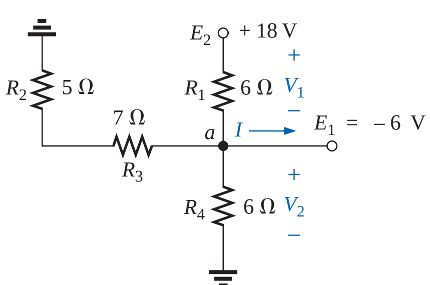

FIG. 7.18
Example 7.7.

EXAMPLE 7.7 For the network of Fig. 7.18, determine the voltages V_1 and V_2 and the current I.

Solution: It would indeed be difficult to analyze the network in the form of Fig. 7.18 with the symbolic notation for the sources and the reference or ground connection in the upper left-hand corner of the diagram. However, when the network is redrawn as shown in Fig. 7.19, the unknowns and the relationship between branches become significantly clearer. Note the common connection of the grounds and the replacing of the terminal notation by actual supplies.

It is now obvious that

$$V_2 = -E_1 = \mathbf{-6\text{ V}}$$

The minus sign simply indicates that the chosen polarity for V_2 in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain

$$-E_1 + V_1 - E_2 = 0$$

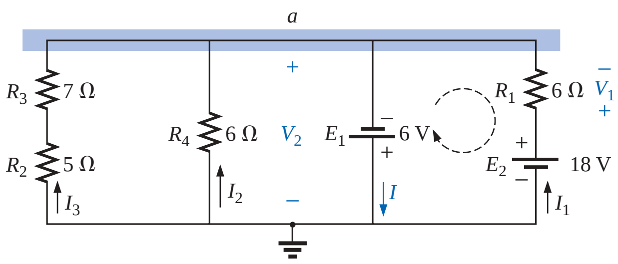

FIG. 7.19
Network of Fig. 7.18 redrawn.

and $$V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\text{ V}}$$

Applying Kirchhoff's current law to node *a* yields

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\ &= \frac{24\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{12\ \Omega} \\ &= 4\text{ A} + 1\text{ A} + 0.5\text{ A} \\ I &= \mathbf{5.5\text{ A}} \end{aligned}$$

The next example is clear evidence of the fact that techniques learned in the current chapters will have far-reaching applications and will not be dropped for improved methods. Even though the **transistor** has not been introduced in this text, the dc levels of a transistor network can be examined using the basic rules and laws introduced in the early chapters of this text.

EXAMPLE 7.8 For the transistor configuration of Fig. 7.20, in which V_B and V_{BE} have been provided:

a. Determine the voltage V_E and the current I_E.
b. Calculate V_1.
c. Determine V_{BC} using the fact that the approximation $I_C = I_E$ is often applied to transistor networks.
d. Calculate V_{CE} using the information obtained in parts (a) through (c).

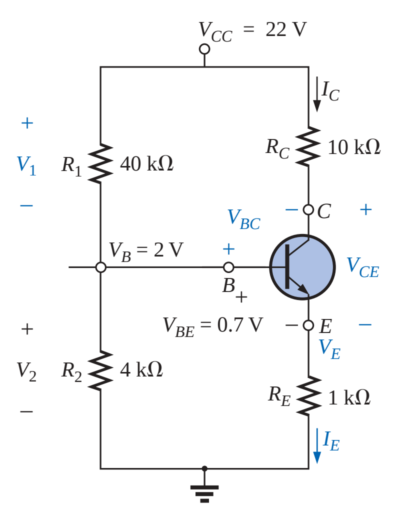

FIG. 7.20
Example 7.8.

Solutions:

a. From Fig. 7.20, we find

$$V_2 = V_B = 2\text{ V}$$

Writing Kirchhoff's voltage law around the lower loop yields

$$V_2 - V_{BE} - V_E = 0$$

or $$V_E = V_2 - V_{BE} = 2\text{ V} - 0.7\text{ V} = \mathbf{1.3\text{ V}}$$

and $$I_E = \frac{V_E}{R_E} = \frac{1.3\text{ V}}{1000\ \Omega} = \mathbf{1.3\text{ mA}}$$

b. Applying Kirchhoff's voltage law to the input side (left-hand region of the network) will result in

$$V_2 + V_1 - V_{CC} = 0$$

and $\quad V_1 = V_{CC} - V_2$

but $\quad V_2 = V_B$

and $\quad V_1 = V_{CC} - V_2 = 22\text{ V} - 2\text{ V} = \mathbf{20\ V}$

c. Redrawing the section of the network of immediate interest will result in Fig. 7.21, where Kirchhoff's voltage law yields

$$V_C + V_{R_C} - V_{CC} = 0$$

and $\quad V_C = V_{CC} - V_{R_C} = V_{CC} - I_C R_C$

but $\quad I_C = I_E$

and $\quad V_C = V_{CC} - I_E R_C = 22\text{ V} - (1.3\text{ mA})(10\text{ k}\Omega) = 9\text{ V}$

Then $\quad V_{BC} = V_B - V_C = 2\text{ V} - 9\text{ V} = \mathbf{-7\ V}$

d. $\quad V_{CE} = V_C - V_E = 9\text{ V} - 1.3\text{ V} = \mathbf{7.7\ V}$

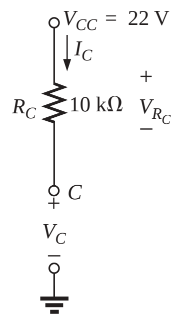

FIG. 7.21
Determining V_C for the network of Fig. 7.20.

EXAMPLE 7.9 Calculate the indicated currents and voltage of Fig. 7.22.

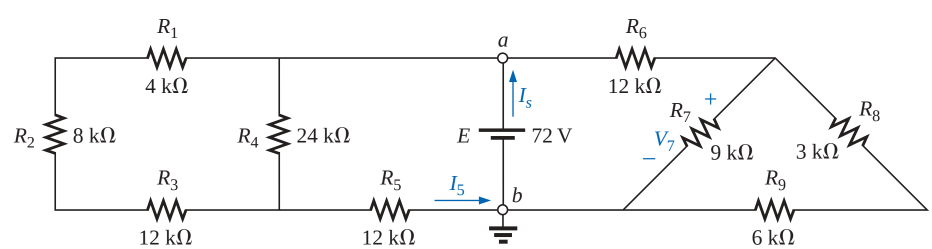

FIG. 7.22
Example 7.9.

Solution: Redrawing the network after combining series elements yields Fig. 7.23, and

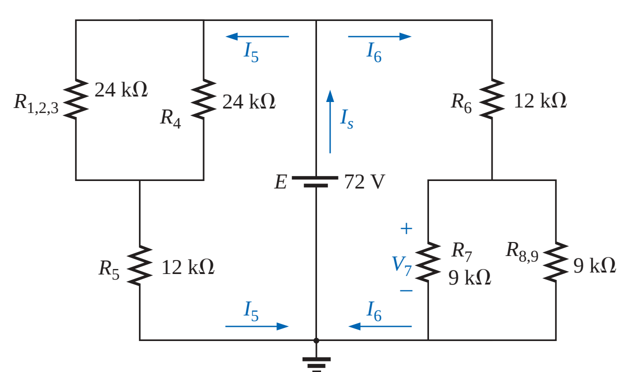

FIG. 7.23
Network of Fig. 7.22 redrawn.

$$I_5 = \frac{E}{R_{(1,2,3)\|4} + R_5} = \frac{72\text{ V}}{12\text{ k}\Omega + 12\text{ k}\Omega} = \frac{72\text{ V}}{24\text{ k}\Omega} = \mathbf{3\text{ mA}}$$

with

$$V_7 = \frac{R_{7\|(8,9)}E}{R_{7\|(8,9)} + R_6} = \frac{(4.5\text{ k}\Omega)(72\text{ V})}{4.5\text{ k}\Omega + 12\text{ k}\Omega} = \frac{324\text{ V}}{16.5} = \mathbf{19.6\text{ V}}$$

$$I_6 = \frac{V_7}{R_{7\|(8,9)}} = \frac{19.6\text{ V}}{4.5\text{ k}\Omega} = \mathbf{4.35\text{ mA}}$$

and $I_s = I_5 + I_6 = 3\text{ mA} + 4.35\text{ mA} = \mathbf{7.35\text{ mA}}$

Since the potential difference between points *a* and *b* of Fig. 7.22 is fixed at *E* volts, the circuit to the right or left is unaffected if the network is reconstructed as shown in Fig. 7.24.

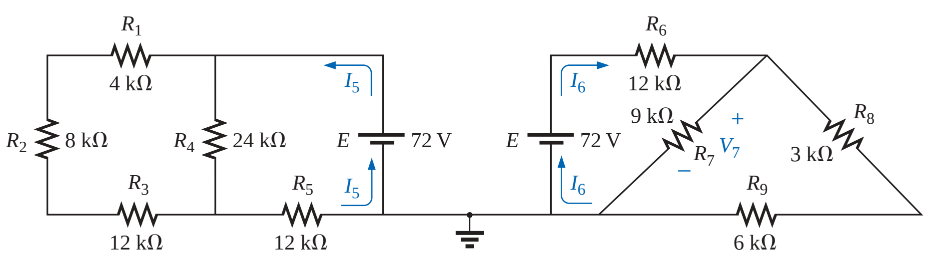

FIG. 7.24

An alternative approach to Example 7.9.

We can find each quantity required, except I_s, by analyzing each circuit independently. To find I_s, we must find the source current for each circuit and add it as in the above solution; that is, $I_s = I_5 + I_6$.

EXAMPLE 7.10 This example demonstrates the power of Kirchhoff's voltage law by determining the voltages V_1, V_2, and V_3 for the network of Fig. 7.25. For path 1 of Fig. 7.26,

$$E_1 - V_1 - E_3 = 0$$

and $V_1 = E_1 - E_3 = 20\text{ V} - 8\text{ V} = \mathbf{12\text{ V}}$

For path 2,

$$E_2 - V_1 - V_2 = 0$$

and $V_2 = E_2 - V_1 = 5\text{ V} - 12\text{ V} = \mathbf{-7\text{ V}}$

indicating that V_2 has a magnitude of 7 V but a polarity opposite to that appearing in Fig. 7.25. For path 3,

$$V_3 + V_2 - E_3 = 0$$

and $V_3 = E_3 - V_2 = 8\text{ V} - (-7\text{ V}) = 8\text{ V} + 7\text{ V} = \mathbf{15\text{ V}}$

Note that the polarity of V_2 was maintained as originally assumed, requiring that −7 V be substituted for V_2.

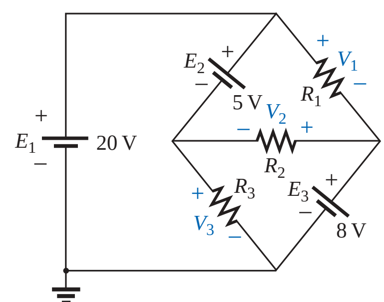

FIG. 7.25

Example 7.10.

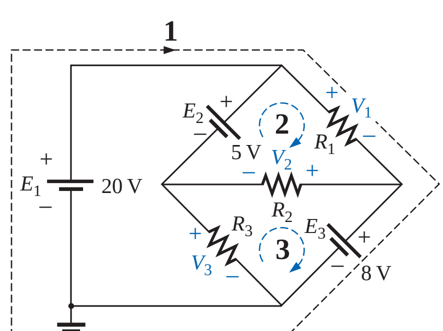

FIG. 7.26

Defining the paths for Kirchhoff's voltage law.

7.3 LADDER NETWORKS

A three-section **ladder network** appears in Fig. 7.27. The reason for the terminology is quite obvious for the repetitive structure. Basically two approaches are used to solve networks of this type.

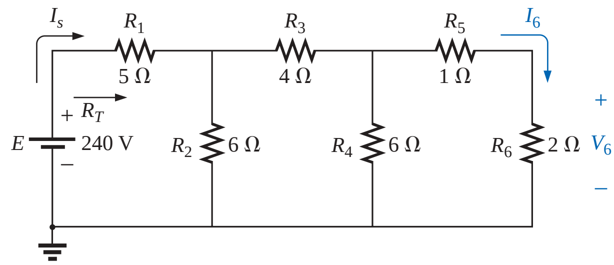

FIG. 7.27
Ladder network.

Method 1

Calculate the total resistance and resulting source current, and then work back through the ladder until the desired current or voltage is obtained. This method is now employed to determine V_6 in Fig. 7.27.

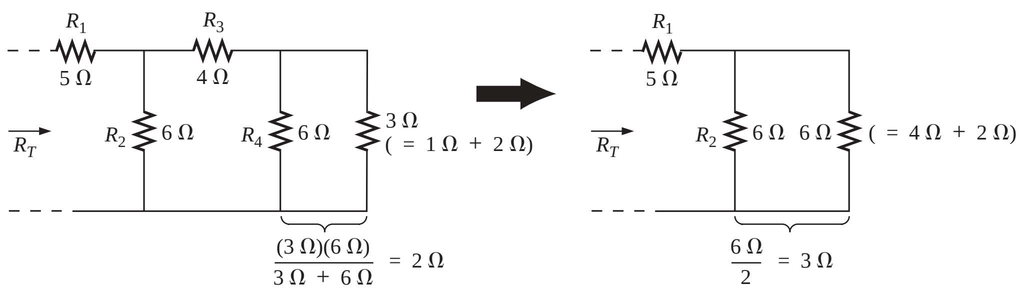

FIG. 7.28
Working back to the source to determine R_T for the network of Fig. 7.27.

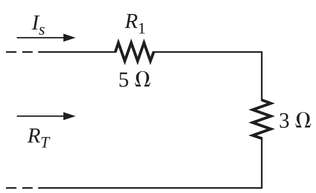

FIG. 7.29
Calculating R_T and I_s.

Combining parallel and series elements as shown in Fig. 7.28 will result in the reduced network of Fig. 7.29, and

$$R_T = 5\ \Omega + 3\ \Omega = 8\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{240\text{ V}}{8\ \Omega} = 30\text{ A}$$

Working our way back to I_6 (Fig. 7.30), we find that

$$I_1 = I_s$$

and
$$I_3 = \frac{I_s}{2} = \frac{30\text{ A}}{2} = 15\text{ A}$$

and, finally (Fig. 7.31),

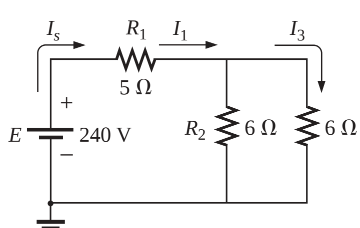

FIG. 7.30
Working back toward I_6.

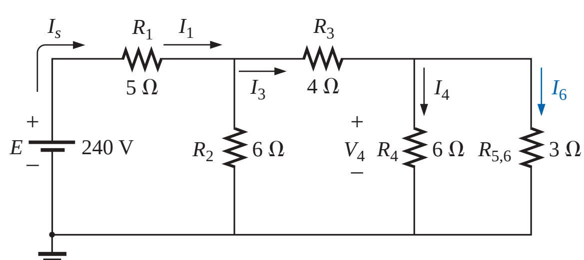

FIG. 7.31
Calculating I_6.

$$I_6 = \frac{(6\ \Omega)I_3}{6\ \Omega + 3\ \Omega} = \frac{6}{9}(15\ \text{A}) = 10\ \text{A}$$

and
$$V_6 = I_6R_6 = (10\ \text{A})(2\ \Omega) = \mathbf{20\ V}$$

Method 2

Assign a letter symbol to the last branch current and work back through the network to the source, maintaining this assigned current or other current of interest. The desired current can then be found directly. This method can best be described through the analysis of the same network considered above in Fig. 7.27, redrawn in Fig. 7.32.

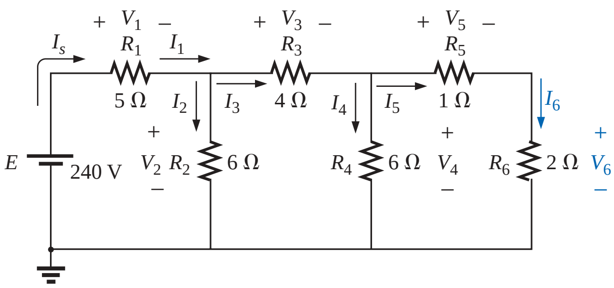

FIG. 7.32
An alternative approach for ladder networks.

The assigned notation for the current through the final branch is I_6:

$$I_6 = \frac{V_4}{R_5 + R_6} = \frac{V_4}{1\ \Omega + 2\ \Omega} = \frac{V_4}{3\ \Omega}$$

or
$$V_4 = (3\ \Omega)I_6$$

so that
$$I_4 = \frac{V_4}{R_4} = \frac{(3\ \Omega)I_6}{6\ \Omega} = 0.5I_6$$

and
$$I_3 = I_4 + I_6 = 0.5I_6 + I_6 = 1.5I_6$$
$$V_3 = I_3R_3 = (1.5I_6)(4\ \Omega) = (6\ \Omega)I_6$$

Also,
$$V_2 = V_3 + V_4 = (6\ \Omega)I_6 + (3\ \Omega)I_6 = (9\ \Omega)I_6$$

so that
$$I_2 = \frac{V_2}{R_2} = \frac{(9\ \Omega)I_6}{6\ \Omega} = 1.5I_6$$

and
$$I_s = I_2 + I_3 = 1.5I_6 + 1.5I_6 = 3I_6$$

with
$$V_1 = I_1R_1 = I_sR_1 = (5\ \Omega)I_s$$

so that
$$E = V_1 + V_2 = (5\ \Omega)I_s + (9\ \Omega)I_6$$
$$= (5\ \Omega)(3I_6) + (9\ \Omega)I_6 = (24\ \Omega)I_6$$

and
$$I_6 = \frac{E}{24\ \Omega} = \frac{240\ \text{V}}{24\ \Omega} = 10\ \text{A}$$

with
$$V_6 = I_6R_6 = (10\ \text{A})(2\ \Omega) = \mathbf{20\ V}$$

as was obtained using method 1.

Mathcad

Mathcad will now be used to analyze the ladder network of Fig. 7.27 using method 1. It will provide an excellent opportunity to practice the basic maneuvers introduced in earlier chapters.

First, as shown in Fig. 7.33, all the parameters of the network must be defined. Then the same sequence is followed as included in the text material. For Mathcad, however, we must be sure that the defining sequence for each new variable flows from left to right, as shown in Fig. 7.33, until R_{10} is defined. We are then ready to write the equation for the total resistance and display the result. All the remaining parameters are then defined and displayed as shown. The results are an exact match with the longhand solution.

The wonderful thing about Mathcad is that this sequence can be put in memory and called for as the need arises for different networks. Simply redefine the parameters of the network, and all the new values for the important parameters of the network will be displayed immediately.

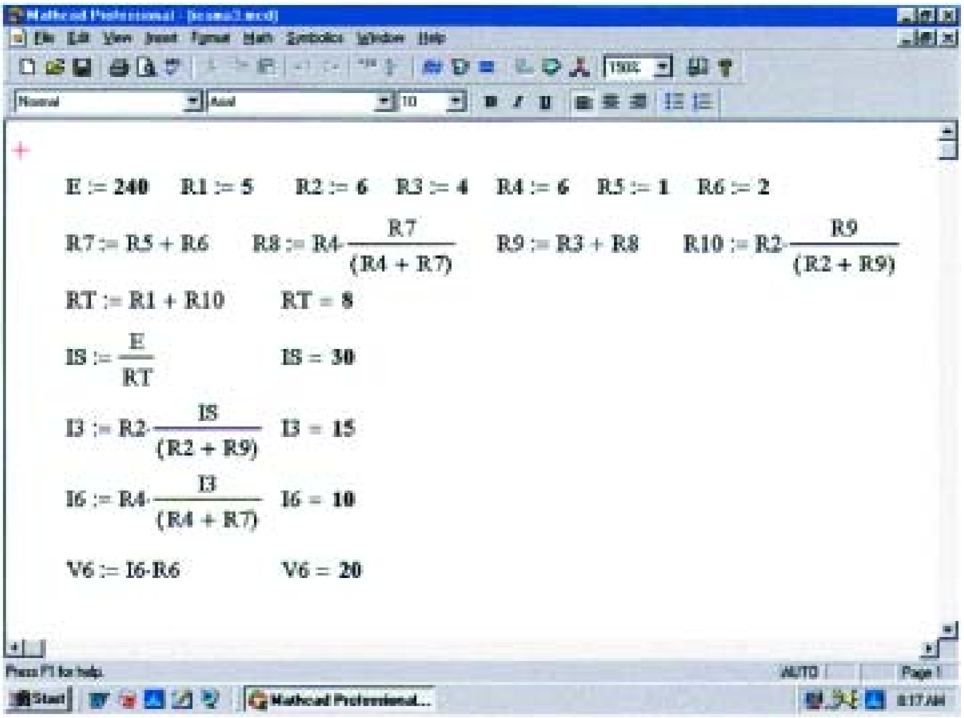

FIG. 7.33
Using Mathcad to analyze the ladder network of Fig. 7.27.

7.4 VOLTAGE DIVIDER SUPPLY (UNLOADED AND LOADED)

The term *loaded* appearing in the title of this section refers to the application of an element, network, or system to a supply that will draw current from the supply. As pointed out in Section 5.8, the application of a load can affect the terminal voltage of the supply.

Through a voltage divider network such as the one in Fig. 7.34, a number of terminal voltages can be made available from a single supply. The voltage levels shown (with respect to ground) are determined by a direct application of the voltage divider rule. Figure 7.34 reflects a no-load situation due to the absence of any current-drawing elements connected between terminals *a*, *b*, or *c* and ground.

The larger the resistance level of the applied loads compared to the resistance level of the voltage divider network, the closer the resulting terminal voltage to the no-load levels. In other words, the lower the current demand from a supply, the closer the terminal characteristics are to the no-load levels.

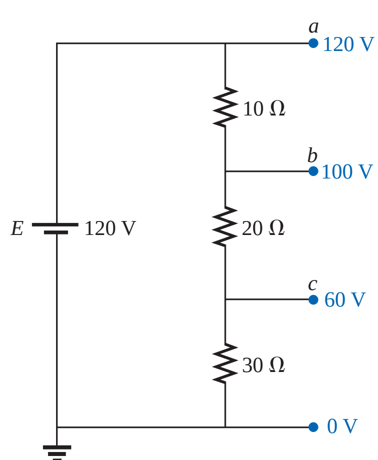

FIG. 7.34
Voltage divider supply.

To demonstrate the validity of the above statement, let us consider the network of Fig. 7.34 with resistive loads that are the average value of the resistive elements of the voltage divider network, as shown in Fig. 7.35.

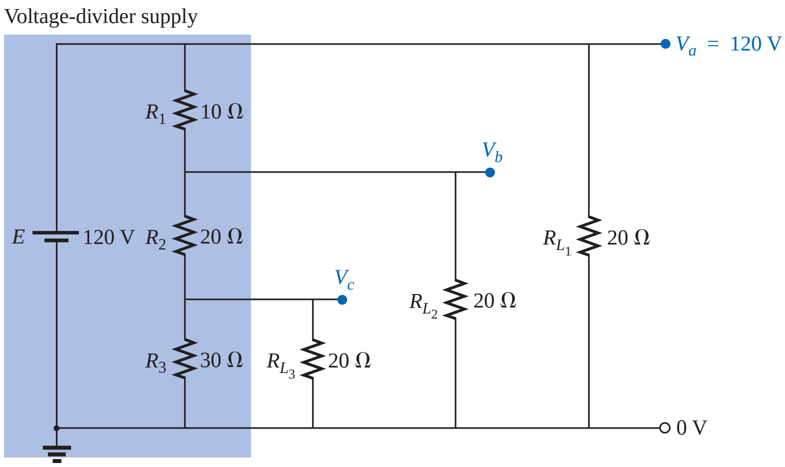

FIG. 7.35
Voltage divider supply with loads equal to the average value of the resistive elements that make up the supply.

The voltage V_a is unaffected by the load R_{L_1} since the load is in parallel with the supply voltage E. The result is $V_a = 120$ V, which is the same as the no-load level. To determine V_b, we must first note that R_3 and R_{L_3} are in parallel and $R'_3 = R_3 \parallel R_{L_3} = 30\ \Omega \parallel 20\ \Omega = 12\ \Omega$. The parallel combination $R'_2 = (R_2 + R'_3) \parallel R_{L_2} = (20\ \Omega + 12\ \Omega) \parallel 20\ \Omega = 32\ \Omega \parallel 20\ \Omega = 12.31\ \Omega$. Applying the voltage divider rule gives

$$V_b = \frac{(12.31\ \Omega)(120\ \text{V})}{12.31\ \Omega + 10\ \Omega} = 66.21\ \text{V}$$

versus 100 V under no-load conditions.

The voltage V_c is

$$V_c = \frac{(12\ \Omega)(66.21\ \text{V})}{12\ \Omega + 20\ \Omega} = 24.83\ \text{V}$$

versus 60 V under no-load conditions.

The effect of load resistors close in value to the resistor employed in the voltage divider network is, therefore, to decrease significantly some of the terminal voltages.

If the load resistors are changed to the 1-kΩ level, the terminal voltages will all be relatively close to the no-load values. The analysis is similar to the above, with the following results:

$$V_a = 120\ \text{V} \qquad V_b = 98.88\ \text{V} \qquad V_c = 58.63\ \text{V}$$

If we compare current drains established by the applied loads, we find for the network of Fig. 7.35 that

$$I_{L_2} = \frac{V_{L_2}}{R_{L_2}} = \frac{66.21\ \text{V}}{20\ \Omega} = 3.31\ \text{A}$$

and for the 1-kΩ level,

$$I_{L_2} = \frac{98.88\ \text{V}}{1\ \text{k}\Omega} = 98.88\ \text{mA} < 0.1\ \text{A}$$

As noted above in the highlighted statement, the more the current drain, the greater the change in terminal voltage with the application of the load. This is certainly verified by the fact that I_{L_2} is about 33.5 times larger with the 20-Ω loads.

The next example is a design exercise. The voltage and current ratings of each load are provided, along with the terminal ratings of the supply. The required voltage divider resistors must be found.

EXAMPLE 7.11 Determine R_1, R_2, and R_3 for the voltage divider supply of Fig. 7.36. Can 2-W resistors be used in the design?

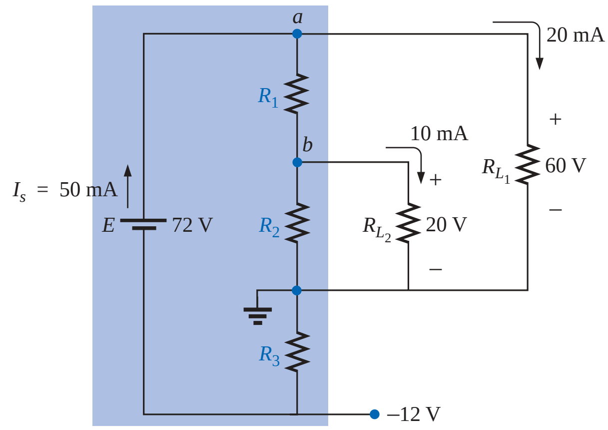

FIG. 7.36
Example 7.11.

Solution: R_3:

$$R_3 = \frac{V_{R_3}}{I_{R_3}} = \frac{V_{R_3}}{I_s} = \frac{12\text{ V}}{50\text{ mA}} = \mathbf{240\ \Omega}$$

$$P_{R_3} = (I_{R_3})^2 R_3 = (50\text{ mA})^2\,240\ \Omega = 0.6\text{ W} < 2\text{ W}$$

R_1: Applying Kirchhoff's current law to node *a*:

$$I_s - I_{R_1} - I_{L_1} = 0$$

and $$I_{R_1} = I_s - I_{L_1} = 50\text{ mA} - 20\text{ mA}$$
$$= 30\text{ mA}$$

$$R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{V_{L_1} - V_{L_2}}{I_{R_1}} = \frac{60\text{ V} - 20\text{ V}}{30\text{ mA}} = \frac{40\text{ V}}{30\text{ mA}}$$
$$= \mathbf{1.33\ k\Omega}$$

$$P_{R_1} = (I_{R_1})^2 R_1 = (30\text{ mA})^2\,1.33\text{ k}\Omega = 1.197\text{ W} < 2\text{ W}$$

R_2: Applying Kirchhoff's current law at node *b:*

$$I_{R_1} - I_{R_2} - I_{L_2} = 0$$

and $$I_{R_2} = I_{R_1} - I_{L_2} = 30\text{ mA} - 10\text{ mA}$$
$$= 20\text{ mA}$$

$$R_2 = \frac{V_{R_2}}{I_{R_2}} = \frac{20\text{ V}}{20\text{ mA}} = \mathbf{1\ k\Omega}$$

$$P_{R_2} = (I_{R_2})^2 R_2 = (20\text{ mA})^2\,1\text{ k}\Omega = 0.4\text{ W} < 2\text{ W}$$

Since P_{R_1}, P_{R_2}, and P_{R_3} are less than 2 W, 2-W resistors can be used for the design.

7.5 POTENTIOMETER LOADING

For the unloaded potentiometer of Fig. 7.37, the output voltage is determined by the voltage divider rule, with R_T in the figure representing the total resistance of the potentiometer. Too often it is assumed that the voltage across a load connected to the wiper arm is determined solely by the potentiometer, and the effect of the load can be ignored. This is definitely not the case, as is demonstrated in the next few paragraphs.

When a load is applied as shown in Fig. 7.38, the output voltage V_L is now a function of the magnitude of the load applied since R_1 is not as shown in Fig. 7.37 but is instead the parallel combination of R_1 and R_L.

The output voltage is now

$$V_L = \frac{R'E}{R' + R_2} \qquad \text{with } R' = R_1 \parallel R_L \tag{7.1}$$

If it is desired to have good control of the output voltage V_L through the controlling dial, knob, screw, or whatever, it is advisable to choose a load or potentiometer that satisfies the following relationship:

$$R_L \geq R_T \tag{7.2}$$

For example, if we disregard Eq. (7.2) and choose a 1-MΩ potentiometer with a 100-Ω load and set the wiper arm to 1/10 the total resistance, as shown in Fig. 7.39, then

$$R' = 100\text{ k}\Omega \parallel 100\ \Omega = 99.9\ \Omega$$

and
$$V_L = \frac{99.9\ \Omega(10\text{ V})}{99.9\ \Omega + 900\text{ k}\Omega} \cong 0.001\text{ V} = 1\text{ mV}$$

which is extremely small compared to the expected level of 1 V.

In fact, if we move the wiper arm to the midpoint,

$$R' = 500\text{ k}\Omega \parallel 100\ \Omega = 99.98\ \Omega$$

and
$$V_L = \frac{(99.98\ \Omega)(10\text{ V})}{99.98\ \Omega + 500\text{ k}\Omega} \cong 0.002\text{ V} = 2\text{ mV}$$

which is negligible compared to the expected level of 5 V. Even at $R_1 = 900\text{ k}\Omega$, V_L is simply 0.01 V, or 1/1000 of the available voltage.

Using the reverse situation of $R_T = 100\ \Omega$ and $R_L = 1\text{ M}\Omega$ and the wiper arm at the 1/10 position, as in Fig. 7.40, we find

$$R' = 10\ \Omega \parallel 1\text{ M}\Omega \cong 10\ \Omega$$

and
$$V_L = \frac{10\ \Omega(10\text{ V})}{10\ \Omega + 90\ \Omega} = 1\text{ V}$$

as desired.

For the lower limit (worst-case design) of $R_L = R_T = 100\ \Omega$, as defined by Eq. (7.2) and the halfway position of Fig. 7.38,

$$R' = 50\ \Omega \parallel 100\ \Omega = 33.33\ \Omega$$

and
$$V_L = \frac{33.33\ \Omega(10\text{ V})}{33.33\ \Omega + 50\ \Omega} \cong 4\text{ V}$$

It may not be the ideal level of 5 V, but at least 40% of the voltage E has been achieved at the halfway position rather than the 0.02% obtained with $R_L = 100\ \Omega$ and $R_T = 1\text{ M}\Omega$.

In general, therefore, try to establish a situation for potentiometer control in which Equation (7.2) is satisfied to the highest degree possible.

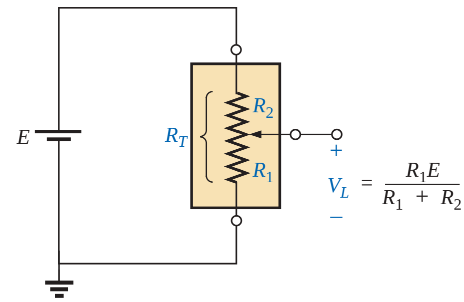

FIG. 7.37
Unloaded potentiometer.

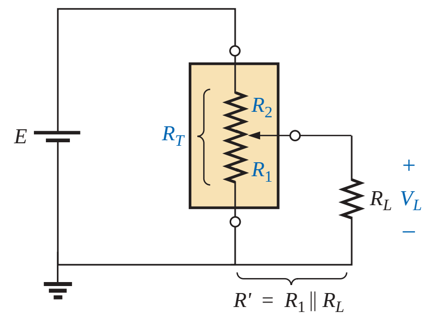

FIG. 7.38
Loaded potentiometer.

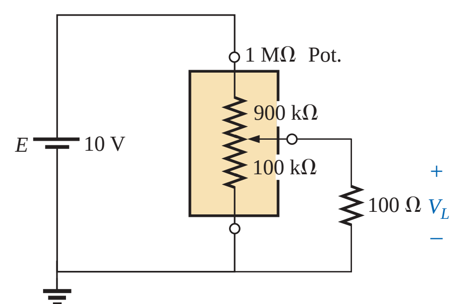

FIG. 7.39
$R_T > R_L$.

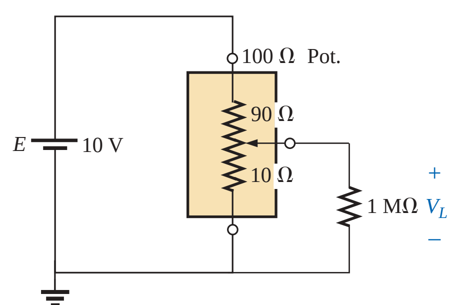

FIG. 7.40
$R_L > R_T$.

Someone might suggest that we make R_T as small as possible to bring the percent result as close to the ideal as possible. Keep in mind, however, that the potentiometer has a power rating, and for networks such as Fig. 7.40, $P_{max} \cong E^2/R_T = (10 \text{ V})^2/100 \ \Omega = 1 \text{ W}$. If R_T is reduced to 10 Ω, $P_{max} = (10 \text{ V})^2/10 \ \Omega = 10 \text{ W}$, which would require a *much larger* unit.

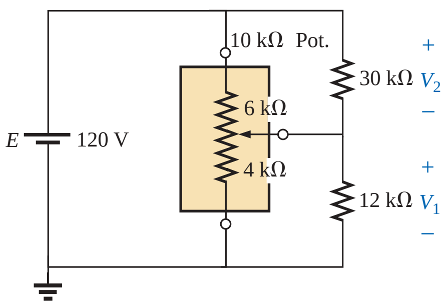

FIG. 7.41
Example 7.12.

EXAMPLE 7.12 Find the voltages V_1 and V_2 for the loaded potentiometer of Fig. 7.41.

Solution:

Ideal (no load): $$V_1 = \frac{4 \text{ k}\Omega(120 \text{ V})}{10 \text{ k}\Omega} = 48 \text{ V}$$

$$V_2 = \frac{6 \text{ k}\Omega(120 \text{ V})}{10 \text{ k}\Omega} = 72 \text{ V}$$

Loaded: $$R' = 4 \text{ k}\Omega \parallel 12 \text{ k}\Omega = 3 \text{ k}\Omega$$

$$R'' = 6 \text{ k}\Omega \parallel 30 \text{ k}\Omega = 5 \text{ k}\Omega$$

$$V_1 = \frac{3 \text{ k}\Omega(120 \text{ V})}{8 \text{ k}\Omega} = \mathbf{45 \text{ V}}$$

$$V_2 = \frac{5 \text{ k}\Omega(120 \text{ V})}{8 \text{ k}\Omega} = \mathbf{75 \text{ V}}$$

The ideal and loaded voltage levels are so close that the design can be considered a good one for the applied loads. A slight variation in the position of the wiper arm will establish the ideal voltage levels across the two loads.

7.6 AMMETER, VOLTMETER, AND OHMMETER DESIGN

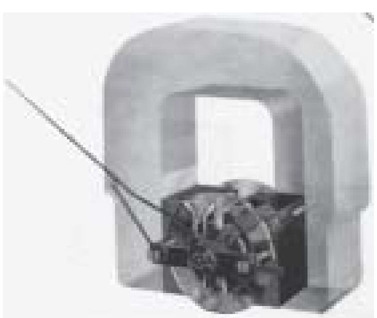

FIG. 7.42
d'Arsonval analog movement. (Courtesy of Weston Instruments, Inc.)

Now that the fundamentals of series, parallel, and series-parallel networks have been introduced, we are prepared to investigate the fundamental design of an ammeter, voltmeter, and ohmmeter. Our design of each will employ the **d'Arsonval analog movement** of Fig. 7.42. The movement consists basically of an iron-core coil mounted on bearings between a permanent magnet. The helical springs limit the turning motion of the coil and provide a path for the current to reach the coil. When a current is passed through the movable coil, the fluxes of the coil and permanent magnet will interact to develop a torque on the coil that will cause it to rotate on its bearings. The movement is adjusted to indicate zero deflection on a meter scale when the current through the coil is zero. The direction of current through the coil will then determine whether the pointer will display an up-scale or below-zero indication. For this reason, ammeters and voltmeters have an assigned polarity on their terminals to ensure an up-scale reading.

D'Arsonval movements are usually rated by current and resistance. The specifications of a typical movement might be 1 mA, 50 Ω. The 1 mA is the *current sensitivity* (*CS*) of the movement, which is the current required for a full-scale deflection. It will be denoted by the symbol I_{CS}. The 50 Ω represents the internal resistance (R_m) of the movement. A common notation for the movement and its specifications is provided in Fig. 7.43.

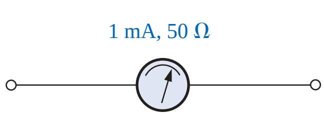

FIG. 7.43
Movement notation.

The Ammeter

The maximum current that the d'Arsonval movement can read independently is equal to the current sensitivity of the movement. However, higher currents can be measured if additional circuitry is introduced. This additional circuitry, as shown in Fig. 7.44, results in the basic construction of an ammeter.

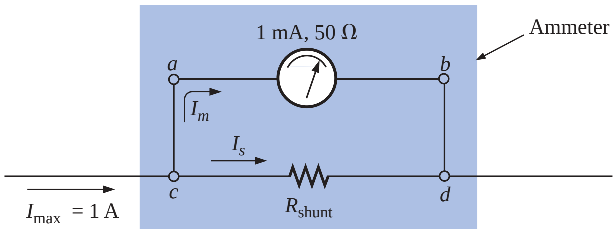

FIG. 7.44
Basic ammeter.

The resistance R_{shunt} is chosen for the ammeter of Fig. 7.44 to allow 1 mA to flow through the movement when a maximum current of 1 A enters the ammeter. If less than 1 A should flow through the ammeter, the movement will have less than 1 mA flowing through it and will indicate less than full-scale deflection.

Since the voltage across parallel elements must be the same, the potential drop across *a-b* in Fig. 7.44 must equal that across *c-d*; that is,

$$(1 \text{ mA})(50\ \Omega) = R_{shunt} I_s$$

Also, I_s must equal 1 A − 1 mA = 999 mA if the current is to be limited to 1 mA through the movement (Kirchhoff's current law). Therefore,

$$(1 \text{ mA})(50\ \Omega) = R_{shunt}(999 \text{ mA})$$

$$R_{shunt} = \frac{(1 \text{ mA})(50\ \Omega)}{999 \text{ mA}}$$

$$\cong 0.05\ \Omega$$

In general,

$$R_{shunt} = \frac{R_m I_{CS}}{I_{max} - I_{CS}} \qquad \textbf{(7.3)}$$

One method of constructing a multirange ammeter is shown in Fig. 7.45, where the rotary switch determines the R_{shunt} to be used for the

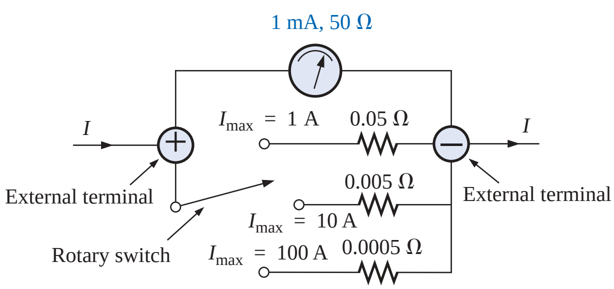

FIG. 7.45
Multirange ammeter.

maximum current indicated on the face of the meter. Most meters employ the same scale for various values of maximum current. If you read 375 on the 0–5 mA scale with the switch on the 5 setting, the current is 3.75 mA; on the 50 setting, the current is 37.5 mA; and so on.

The Voltmeter

A variation in the additional circuitry will permit the use of the d'Arsonval movement in the design of a voltmeter. The 1-mA, 50-Ω movement can also be rated as a 50-mV (1 mA × 50 Ω), 50-Ω movement, indicating that the maximum voltage that the movement can measure independently is 50 mV. The millivolt rating is sometimes referred to as the *voltage sensitivity* (*VS*). The basic construction of the voltmeter is shown in Fig. 7.46.

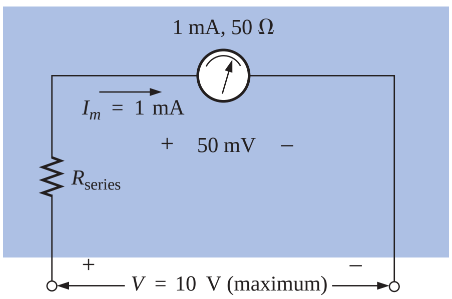

FIG. 7.46
Basic voltmeter.

The R_{series} is adjusted to limit the current through the movement to 1 mA when the maximum voltage is applied across the voltmeter. A lesser voltage would simply reduce the current in the circuit and thereby the deflection of the movement.

Applying Kirchhoff's voltage law around the closed loop of Fig. 7.46, we obtain

$$[10\text{ V} - (1\text{ mA})(R_{series})] - 50\text{ mV} = 0$$

or

$$R_{series} = \frac{10\text{ V} - (50\text{ mV})}{1\text{ mA}} = 9950\ \Omega$$

In general,

$$R_{series} = \frac{V_{max} - V_{VS}}{I_{CS}} \tag{7.4}$$

One method of constructing a multirange voltmeter is shown in Fig. 7.47. If the rotary switch is at 10 V, R_{series} = 9.950 kΩ; at 50 V, R_{series} = 40 kΩ + 9.950 kΩ = 49.950 kΩ; and at 100 V, R_{series} = 50 kΩ + 40 kΩ + 9.950 kΩ = 99.950 kΩ.

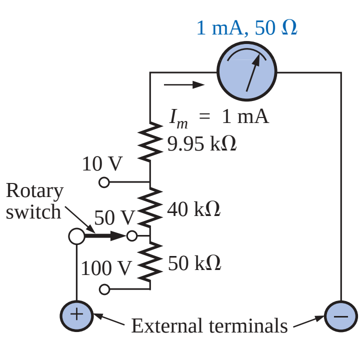

FIG. 7.47
Multirange voltmeter.

The Ohmmeter

In general, ohmmeters are designed to measure resistance in the low, mid-, or high range. The most common is the **series ohmmeter,** designed to read resistance levels in the midrange. It employs the series configuration of Fig. 7.48. The design is quite different from that of the

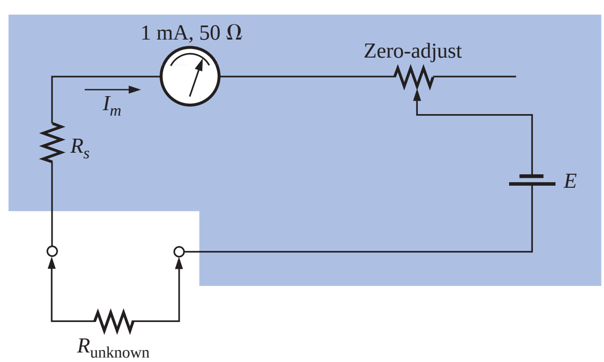

FIG. 7.48
Series ohmmeter.

ammeter or voltmeter because it will show a full-scale deflection for zero ohms and no deflection for infinite resistance.

To determine the series resistance R_s, the external terminals are shorted (a direct connection of zero ohms between the two) to simulate zero ohms, and the zero-adjust is set to half its maximum value. The resistance R_s is then adjusted to allow a current equal to the current sensitivity of the movement (1 mA) to flow in the circuit. The zero-adjust is set to half its value so that any variation in the components of the meter that may produce a current more or less than the current sensitivity can be compensated for. The current I_m is

$$I_m \text{ (full scale)} = I_{CS} = \frac{E}{R_s + R_m + \dfrac{\text{zero-adjust}}{2}} \tag{7.5}$$

and

$$R_s = \frac{E}{I_{CS}} - R_m - \frac{\text{zero-adjust}}{2} \tag{7.6}$$

If an unknown resistance is then placed between the external terminals, the current will be reduced, causing a deflection less than full scale. If the terminals are left open, simulating infinite resistance, the pointer will not deflect since the current through the circuit is zero.

An instrument designed to read very low values of resistance appears in Fig. 7.49. Because of its low-range capability, the network design must be a great deal more sophisticated than described above. It employs electronic components that eliminate the inaccuracies introduced by lead and contact resistances. It is similar to the above system in the sense that it is completely portable and does require a dc battery to establish measurement conditions. Special leads are employed to limit any introduced resistance levels. The maximum scale setting can be set as low as 0.00352 (3.52 mΩ).

FIG. 7.49
Milliohmmeter. (Courtesy of Keithley Instruments, Inc.)

The **megohmmeter** (often called a *megger*) is an instrument for measuring very high resistance values. Its primary function is to test the insulation found in power transmission systems, electrical machinery, transformers, and so on. To measure the high-resistance values, a high dc voltage is established by a hand-driven generator. If the shaft is rotated above some set value, the output of the generator will be fixed at one selectable voltage, typically 250 V, 500 V, or 1000 V. A photograph of a commercially available tester is shown in Fig. 7.50. For this instrument, the range is zero to 5000 MΩ.

FIG. 7.50
Megohmmeter. (Courtesy of AEMC Corp.)

7.7 GROUNDING

Although usually treated too lightly in most introductory electrical or electronics texts, the impact of the ground connection and how it can provide a measure of safety to a design are very important topics. Ground potential is 0 V at every point in a network that has a ground symbol. Since they are all at the same potential, they can all be connected together, but for purposes of clarity most are left isolated on a large schematic. On a schematic the voltage levels provided are always with respect to ground. A system can therefore be checked quite rapidly by simply connecting the black lead of the voltmeter to the ground connection and placing the red lead at the various points where the typical operating voltage is provided. A close match normally implies that that portion of the system is operating properly. Even though a major part of the discussion to follow includes ac systems, which will not be introduced until Chapter 13, the content is such that the background established thus far will be sufficient to understand the material to be presented. The concept of grounding is one that should be introduced at the earliest opportunity for safety and theoretical reasons.

There are various types of grounds depending on the application. An *earth ground* is one that is connected directly to the earth by a low-impedance connection. The entire surface of the earth is defined to have a potential of 0 V. It is the same at every point because there are sufficient conductive agents in the soil such as water and electrolytes to ensure that any difference in voltage on the surface is equalized by a flow of charge between the two points. Every home has an earth ground, usually established by a long conductive rod driven into the ground and connected to the power panel. The electrical code requires a direct connection from earth ground to the cold-water pipes of a home for safety reasons. A “hot” wire touching a cold-water pipe draws sufficient current because of the low-impedance ground connection to throw the breaker. Otherwise, people in the bathroom could pick up the voltage when they touch the cold-water faucet, thereby risking bodily harm. Because water is a conductive agent, any area of the home with water such as the bathroom or kitchen is of particular concern. Most electrical systems are connected to earth ground primarily for safety reasons. All the power lines in a laboratory, at industrial locations, or in the home are connected to earth ground.

A second type is referred to as a *chassis ground,* which may be *floating* or connected directly to an earth ground. A chassis ground simply stipulates that the chassis has a reference potential for all points of the network. If the chassis is not connected to earth potential (0 V), it is said to be *floating* and can have any other reference voltage for the other voltages to be compared to. For instance, if the chassis is sitting at 120 V, all measured voltages of the network will be referenced to this level. A reading of 32 V between a point in the network and the chassis ground will therefore actually be at 152 V with respect to earth potential. Most high-voltage systems are not left floating, however, because of loss of the safety factor. For instance, if someone should touch the chassis and be standing on a suitable ground, the full 120 V would fall across that individual.

Grounding can be particularly important when working with numerous pieces of measuring equipment in the laboratory. For instance, the supply and oscilloscope in Fig. 7.51(a) are each connected directly to an earth ground through the negative terminal of each. If the oscillo-

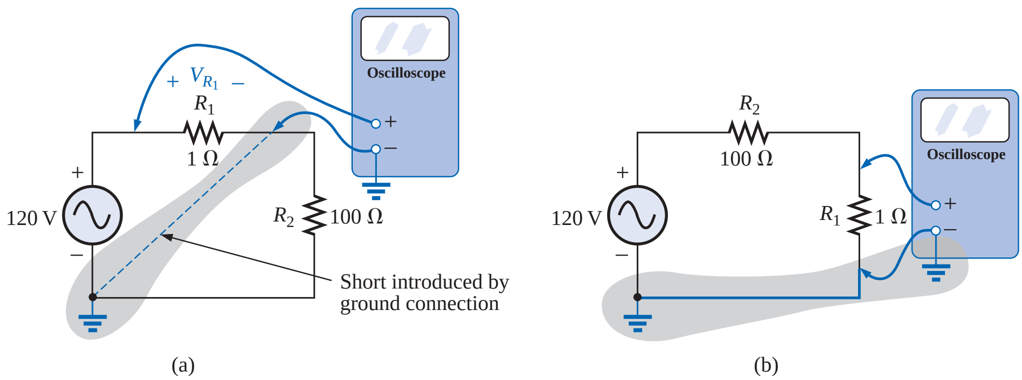

FIG. 7.51
Demonstrating the effect of the oscilloscope ground on the measurement of the voltage across the resistor R_1.

scope is connected as shown in Fig. 7.51(a) to measure the voltage V_{R_1}, a dangerous situation will develop. The grounds of each piece of equipment are connected together through the earth ground, and they effectively short out the resistor. Since the resistor is the primary current-controlling element in the network, the current will rise to a very high level and possibly damage the instruments or cause dangerous side effects. In this case the supply or scope should be used in the floating mode, or the resistors interchanged as shown in Fig. 7.51(b). In Fig. 7.51(b) the grounds have a common point and do not affect the structure of the network.

The National Electrical Code requires that the "hot" (or *feeder*) line that carries current to a load be *black,* and the line (called the *neutral*) that carries the current back to the supply be *white.* Three-wire conductors have a ground wire that must be *green* or *bare,* which will ensure a common ground but which is not designed to carry current. The components of a three-prong extension cord and wall outlet are shown in Fig. 7.52. Note that on both fixtures the connection to the hot lead is smaller than the return leg and that the ground connection is partially circular.

The complete wiring diagram for a household outlet is shown in Fig. 7.53. Note that the current through the ground wire is zero and that both the return wire and the ground wire are connected to an earth ground. The full current to the loads flows through the feeder and return lines.

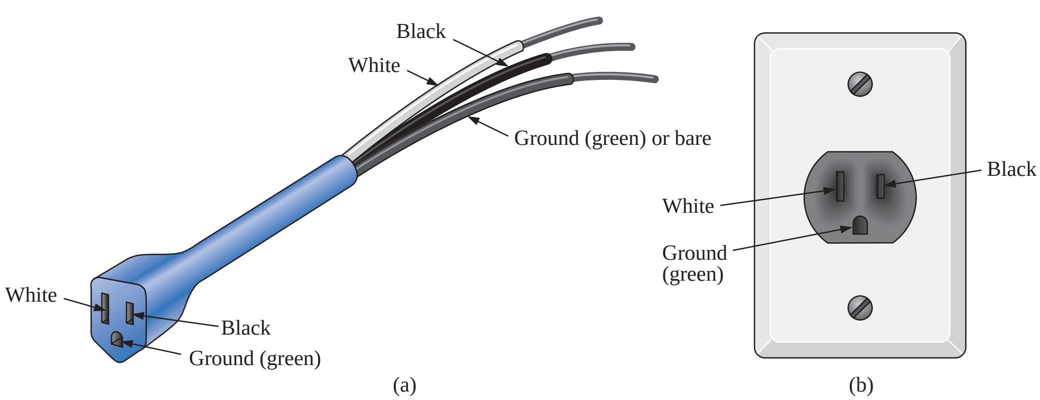

FIG. 7.52
Three-wire conductors: (a) extension cord; (b) home outlet.

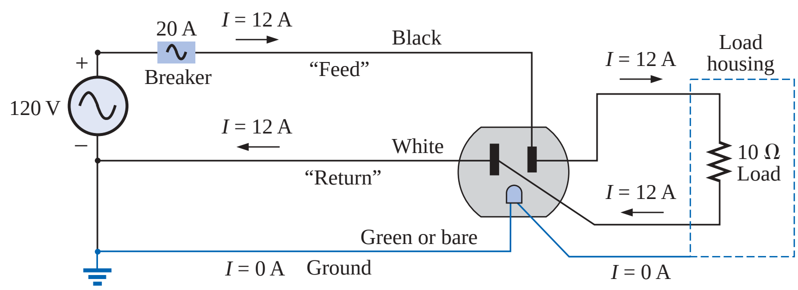

FIG. 7.53
Complete wiring diagram for a household outlet with a 10-Ω load.

The importance of the ground wire in a three-wire system can be demonstrated by the toaster in Fig. 7.54 rated 1200 W at 120 V. From the power equation $P = EI$, the current drawn under normal operating conditions is $I = P/E = 1200\text{ W}/120\text{ V} = 10\text{ A}$. If a two-wire line were employed as shown in Fig. 7.54(a), the 20-A breaker would be quite comfortable with the 10-A current, and the system would perform normally. However, if abuse to the feeder (or return line) caused it to become frayed and to touch the metal housing of the toaster, the situation depicted in Fig. 7.54(b) would result. The housing would become "hot," yet the breaker would not "pop" because the current would still be the rated 10 A. A dangerous condition would exist because anyone touching the toaster would feel the full 120 V to ground. If the ground wire were attached to the chassis as shown in Fig. 7.54(c), a low-resistance path

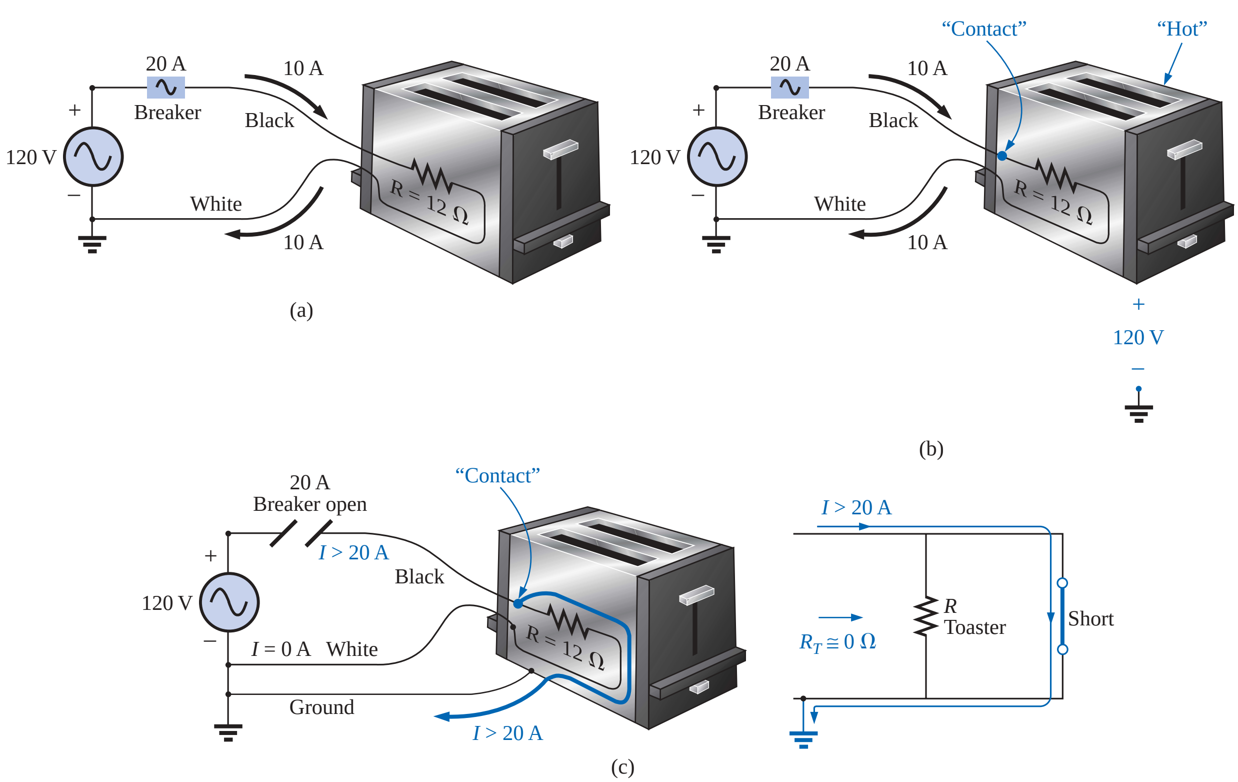

FIG. 7.54
Demonstrating the importance of a properly grounded appliance: (a) ungrounded; (b) ungrounded and undesirable contact; (c) grounded appliance with undesirable contact.

would be created between the short-circuit point and ground, and the current would jump to very high levels. The breaker would "pop," and the user would be warned that a problem exists.

Although the above discussion does not cover all possible areas of concern with proper grounding or introduce all the nuances associated with the effect of grounds on a system's performance, it should provide an awareness of the importance of understanding its impact. Additional comment on the effects of grounding will be made in the chapters to follow as the need arises.

7.8 APPLICATIONS

Boosting a Car Battery

Although boosting a car battery may initially appear to be a simple application of parallel networks, it is really a series-parallel operation that is worthy of some investigation. As indicated in Chapter 2, every dc supply has some internal resistance. For the typical 12-V lead-acid car battery, the resistance is quite small—in the milliohm range. In most cases the low internal resistance will ensure that most of the voltage (or power) is delivered to the load and not lost on the internal resistance. In Fig. 7.55, battery #2 has discharged because the lights were left on for three hours during a movie. Fortunately, a friend who made sure his own lights were out has a fully charged battery #1 and a good set of 16-ft cables with #6 gage stranded wire and well-designed clips. The investment in a good set of cables with sufficient length and heavy wire is a wise one, particularly if you live in a cold climate. Flexibility, as provided by stranded wire, is also a very desirable characteristic under some conditions. Be sure to check the gage of the wire and not just the thickness of the insulating jacket. You get what you pay for, and the copper is the most expensive part of the cables. Too often the label says "heavy-duty," but the wire is too high a gage number.

The proper sequence of events in boosting a car is often a function of whom you speak to or what information you read. For safety sake some people recommend that the car with the good battery be turned off when making the connections. This, however, can create an immediate problem if the "dead" battery is in such a bad state that when it is hooked up to the good battery, it will immediately drain it to the point that neither car will start. With this in mind, it does make some sense to leave the car running to ensure that the charging process continues until the starting of the disabled car is initiated. Because accidents do happen, it is strongly recommended that the person making the connections

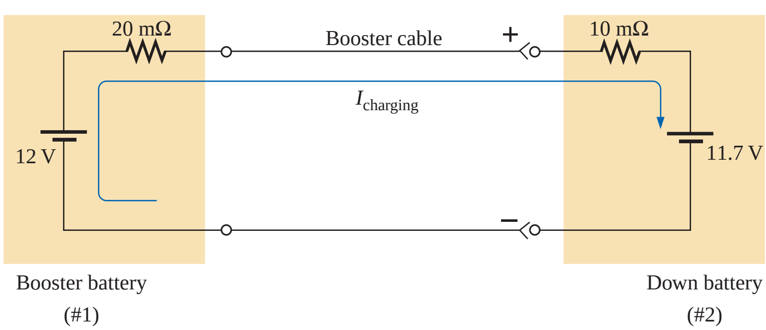

FIG. 7.55
Boosting a car battery.

wear some type of protective eye equipment even if it is just a pair of glasses. Take sufficient time to be sure that you know which are the positive and negative terminals for both cars. If it's not immediately obvious, keep in mind that the negative or ground side is usually connected to the chassis of the car with a relatively short, heavy wire. When you are sure of which are the positive and negative terminals, first connect one of the red wire clamps of the booster cables to the positive terminal of the discharged battery—all the while being sure that the other red clamp is not touching the battery or car. Then connect the other end of the red wire to the positive terminal of the fully charged battery. Next, connect one end of the black cable of the booster cables to the negative terminal of the booster battery, and finally connect the other end of the black cable to the engine block of the stalled vehicle (not the negative post of the dead battery) away from the carburetor, fuel lines, or moving parts of the car. Lastly, have someone maintain a constant idle speed in the car with the good battery as you start the car with the bad battery. After the vehicle starts, remove the cables in the reverse order starting with the cable connected to the engine block. Always be careful to ensure that clamps don't touch the battery or chassis of the car or get near any moving parts. Some people feel that the car with the good battery should charge the bad battery for 10 to 15 minutes before starting the disabled car so the disabled car will be essentially using its own battery in the starting process. Keep in mind that the instant the booster cables are connected, the booster car will be making a concerted effort to charge both its own battery and the drained battery. At starting it will then be asked to supply a heavy current to start the other car. It's a pretty heavy load to put on a single battery. For the situation of Fig. 7.55, the voltage of battery #2 is less than that of battery #1, and the charging current will flow as shown. The resistance in series with the boosting battery is more because of the long length of the booster cable to the other car. The current is limited only by the series milliohm resistors of the batteries, but the voltage difference is so small that the starting current will be in safe range for the cables involved. The initial charging current will be $I = (12\text{ V} - 11.7\text{ V})/(20\text{ m}\Omega + 10\text{ m}\Omega) = 0.3\text{ V}/30\text{ m}\Omega = 10\text{ A}$. At starting, the current levels will be as shown in Fig. 7.56 for the resistance levels and battery voltages assumed. At starting, an internal resistance for the starting circuit of $0.1\ \Omega = 100\text{ m}\Omega$ is assumed. Note that the battery of the disabled car has now charged up to 11.8 V with an associated increase in its power level. The presence of two batteries requires that the analysis wait for the methods to be introduced in the next chapter.

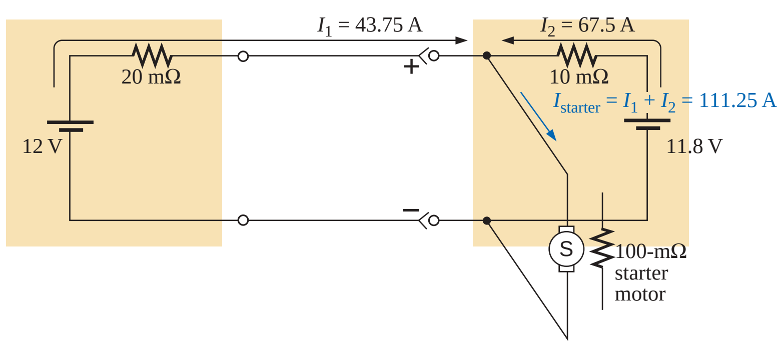

FIG. 7.56
Current levels at starting.

Note also that the current drawn from the starting circuit for the disabled car is over 100 A and that the majority of the starting current is provided by the battery being charged. In essence, therefore, the majority of the starting current is coming from the disabled car. The good battery has provided an initial charge to the bad battery and has provided the additional current necessary to start the car. But, in total, it is the battery of the disabled car that is the primary source of the starting current. For this very reason, it is advised to let the charging action continue for 10 or 15 minutes before starting the car. If the disabled car is in really bad shape with a voltage level of only 11 V, the resulting levels of current will reverse, with the good battery providing 68.75 A and the bad battery only 37.5 A. Quite obviously, therefore, the worse the condition of the dead battery, the heavier the drain on the good battery. A point can also be reached where the bad battery is in such bad shape that it cannot accept a good charge or provide its share of the starting current. The result can be continuous cranking of the down car without starting, and thus possible damage to the battery of the running car due to the enormous current drain. Once the car is started and the booster cables are removed, the car with the discharged battery will continue to run because the alternator will carry the load (charging the battery and providing the necessary dc voltage) after ignition.

The above discussion was all rather straightforward, but let's investigate what might happen if it is a dark and rainy night, you get rushed, and you hook up the cables incorrectly as shown in Fig. 7.57. The result is two series-aiding batteries and a very low resistance path. The resulting current can then theoretically be extremely high [$I = (12\text{ V} + 11.7\text{ V})/30\text{ m}\Omega = 23.7\text{ V}/30\text{ m}\Omega = 790\text{ A}$], perhaps permanently damaging the electrical system of both cars and, worst of all, causing an explosion that might seriously injure someone. It is therefore very important that you treat the process of boosting a car with great care. Find that flashlight, double-check the connections, and be sure that everyone is clear when you start that car.

Before leaving the subject, we should point out that getting a boost from a tow truck results in a somewhat different situation: The connections to the battery in the truck are very secure; the cable from the truck is a heavy wire with thick insulation; the clamps are also quite large and make an excellent connection with your battery; and the battery is heavy-duty for this type of expected load. The result is less internal resistance on the supply side and a heavier current from the truck battery. In this case, the truck is really starting the disabled car, which simply reacts to the provided surge of power.

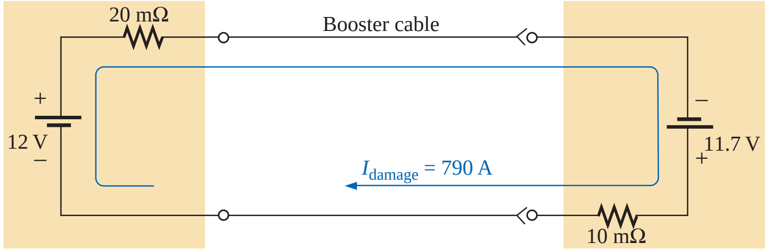

FIG. 7.57
Current levels if the booster battery is improperly connected.

Electronic Circuits

The operation of most electronic systems requires a distribution of dc voltages throughout the design. Although a full explanation of why the dc level is required (since it is an ac signal to be amplified) will have to wait for the introductory courses in electronic circuits, the dc analysis will proceed in much the same manner as described in this chapter. In other words, this chapter and the preceding chapters are sufficient background to perform the dc analysis of the majority of electronic networks you will encounter if given the dc terminal characteristics of the electronic elements. For example, the network of Fig. 7.58 employing a transistor will be covered in detail in any introductory electronics course. The dc voltage between the base (*B*) of the transistor and the emitter (*E*) is about 0.7 V under normal operating conditions, and the collector (*C*) is related to the base current by $I_C = \beta I_B = 50I_B$ (β will vary from transistor to transistor). Using these facts will enable us to determine all the dc currents and voltages of the network using simply the laws introduced in this chapter. In general, therefore, be encouraged by the fact that the content of this chapter will find numerous applications in the courses to follow.

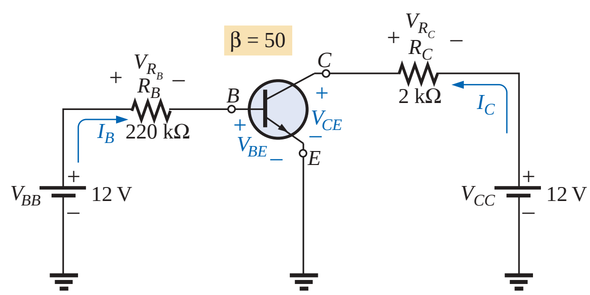

FIG. 7.58
dc bias levels of a transistor amplifier.

For the network of Fig. 7.58, we can begin our analysis by applying Kirchhoff's voltage law to the base circuit:

$$+V_{BB} - V_{R_B} - V_{BE} = 0 \qquad \text{or} \qquad V_{BB} = V_{R_B} + V_{BE}$$

and $$V_{R_B} = V_{BB} - V_{BE} = 12\text{ V} - 0.7\text{ V} = 11.3\text{ V}$$

so that $$V_{R_B} = I_B R_B = 11.3\text{ V}$$

and $$I_B = \frac{V_{R_B}}{R_B} = \frac{11.3\text{ V}}{220\text{ k}\Omega} = \mathbf{51.4\ \mu A}$$

Then $$I_C = \beta I_B = 50I_B = 50(51.4\ \mu\text{A}) = \mathbf{2.57\ mA}$$

and $$+V_{CE} + V_{R_C} - V_{CC} = 0 \qquad \text{or} \qquad V_{CC} = V_{R_C} + V_{CE}$$

with $$V_{CE} = V_{CC} - V_{R_C} = V_{CC} - I_C R_C = 12\text{ V} - (2.57\text{ mA})(2\text{ k}\Omega)$$
$$= 12\text{ V} - 5.14\text{ V} = \mathbf{6.86\ V}$$

For a typical dc analysis of a transistor, all the currents and voltages of interest are now known: I_B, V_{BE}, I_C, and V_{CE}. All the remaining voltage, current, and power levels for the other elements of the network can now be found using the basic laws applied in this chapter.

The above example is typical of the type of exercise you will be asked to perform in your first electronics course. It is now necessary for you only to be exposed to the device and to understand the reason for the relationships between the various currents and voltages of the device.

7.9 COMPUTER ANALYSIS

PSpice

Voltage Divider Supply PSpice will now be used to verify the results of Example 7.11. The calculated resistor values will be substituted and the voltage and current levels checked to see if they match the handwritten solution. The network is drawn as described in earlier chapters using only the tools described thus far (see Fig. 7.59)—in one way, a practice exercise for everything learned about the **Capture Lite Edition.** Note in this case how rotating the first resistor sets everything up for the remaining resistors. Further, it is a nice advantage that you can place one resistor after another without going to the **End Mode** option. Be especially careful with the placement of the ground, and be sure **0/SOURCE** is used. Note also that resistor R_1 of Fig. 7.59 was entered as 1.333 kΩ rather than 1.33 kΩ as in the example. When running the program, we found that the computer solutions were not a perfect match to the longhand solution to the level of accuracy desired unless this change was made.

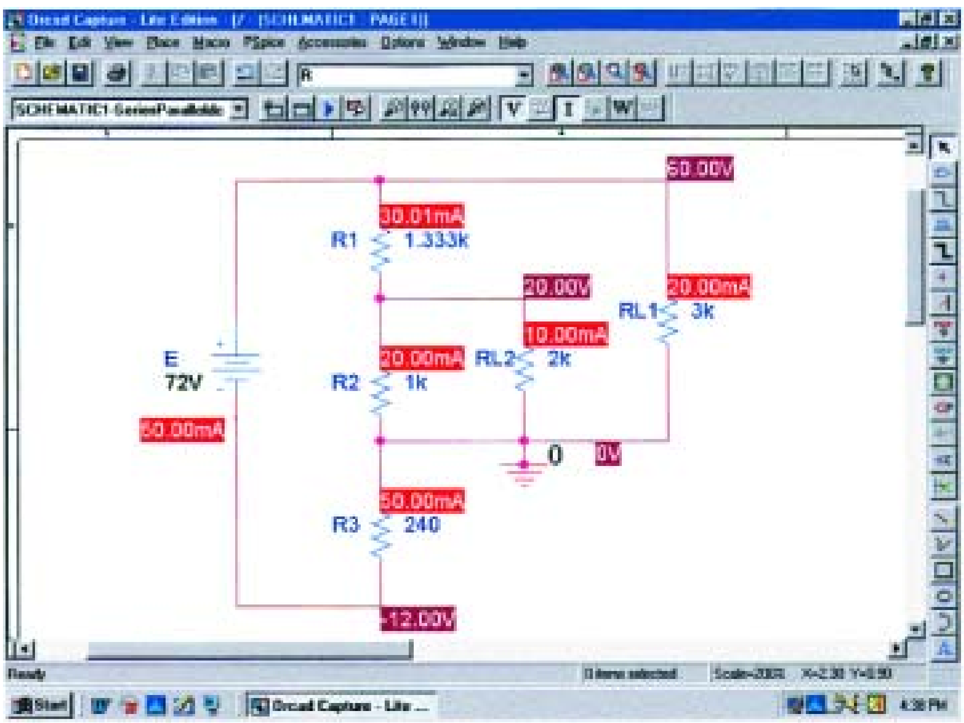

FIG. 7.59
Using PSpice to verify the results of Example 7.11.

Since all the voltages are to ground, the voltage across R_{L_1} is 60 V; across R_{L_2}, 20 V; and across R_3, -12 V. The currents are also an excellent match with the hand solution with $I_E = 50$ mA, $I_{R_1} = 30$ mA, $I_{R_2} = 20$ mA, $I_{R_3} = 50$ mA, $I_{R_{L_2}} = 10$ mA, and $I_{R_{L_1}} = 20$ mA. For the display of Fig. 7.59, the **W** option was disabled to permit concentrating on the voltage and current levels.

There is again an exact match with the longhand solution.

C++

Ladder Network The C++ program to be introduced will perform a detailed analysis of the network of Fig. 7.60 (appearing as Fig. 7.27 earlier in the text). Once all the parameters are introduced, the program will print out R_T, I_s, I_3, and I_6. The order of the program is exactly the same as that of a longhand solution. In Fig. 7.60, $R_a = R_5 + R_6$ is first determined, followed by $R_b = R_4 \parallel R_a$ and $R_c = R_3 + R_b$, with $R_d = R_2 \parallel R_c$ and $R_T = R_1 + R_d$. Then $I_s = E/R_T$ with I_3 and I_6 as determined by the current divider rule.

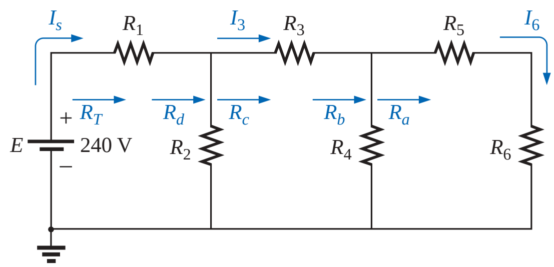

FIG. 7.60
Ladder network to be analyzed using C++.

The program begins with a heading and preprocessor directive. The <*iostream.h*> header file sets up the input-output path between the program and the disk operating system. The *main* ( ) part of the program, defined by the braces { }, includes all the remaining commands and statements. First, the network parameters and quantities to be determined are defined as floating-point variables. Next, the *cout* and *cin* commands are used to obtain the resistor values and source voltage from the user. The total resistance is then determined in the order described above, followed by a carriage return "*n*" and a printout of the value. Then the currents are determined and printed out by the last three lines.

The program (Fig. 7.61) is quite straightforward and with experience not difficult to write. In addition, consider the benefits of having a program on file that can solve any ladder network having the configuration of Fig. 7.60. For the parameter values of Fig. 7.27, the printout will appear as shown in Fig. 7.62, confirming the results of Section 7.3. If an element is missing, simply insert a short-circuit or an open-circuit equivalent, whichever is appropriate. For instance, if R_5 and R_6 are absent, leaving a two-loop network, simply plug in very large values for R_5 and R_6 compared to the other elements of the network, and they will appear as open-circuit equivalents in the analysis. This is demonstrated in the run of Fig. 7.63 with a negative supply of 60 V. The results have negative signs for the currents because the defined direction in the program has the opposite direction. The current I_6 is essentially zero amperes, as it should be if R_5 and R_6 do not exist. If R_1, R_3, R_5, or R_6 were the only resistive element to be missing, a short-circuit equivalent would be inserted. If R_2 or R_4 were the only missing element, the open-circuit equivalent would be substituted.

Heading

Preprocessor directive

Body of program

Define variables and data type

Request and obtain network parameters

Find R_T

Display R_T

Calculate I_3 and I_6

Display I_3 and I_6

```
//C++ Series-Parallel Circuit Analysis

#include <iostream.h>          //needed for input/output

main()                         //execution begins here
{
        float R1, R2, R3, R4, R5, R6;   //declare circuit resistors
        float Ra, Rb, Rc, Rd, Rt;       //declare equivalent resistances
        float E;                        //declare voltage source
        float Is, I3, I6;               //declare circuit currents

        cout << "Enter R1: ";           //get all circuit values
        cin >> R1;
        cout << "Enter R2: ";
        cin >> R2;
        cout << "Enter R3: ";
        cin >> R3;
        cout << "Enter R4: ";
        cin >> R4;
        cout << "Enter R5: ";
        cin >> R5;
        cout << "Enter R6: ";
        cin >> R6;
        cout << "Enter E: ";
        cin >> E;

        Ra = R5 + R6;                   //calculate the total resistance
        Rb = R4 * Ra / (R4 + Ra);
        Rc = R3 + Rb;
        Rd = R2 * Rc / (R2 + Rc);
        Rt = R1 + Rd;
        cout << "\n";
        cout << "The total resistance is " << Rt << " ohms.\n";

        Is = E / Rt;                    //calculate circuit currents
        I3 = Is * R2 / (R2 + Rc);
        I6 = I3 * R4 / (R4 + Ra);
        cout << "The source current is " << Is << " Amperes.\n";
        cout << "I3 equals " << I3 << " Amperes.\n";
        cout << "I6 equals " << I6 << " Amperes.\n";
}
```

FIG. 7.61

C++ program to analyze the ladder network of Fig. 7.60.

```
Enter R1: 5
Enter R2: 6
Enter R3: 4
Enter R4: 6
Enter R5: 1
Enter R6: 2
Enter E: 240

The total resistance is 8 ohms.
The source current is 30 Amperes.
I3 equals 15 Amperes.
I6 equals 10 Amperes.
```

FIG. 7.62

C++ response to an analysis of the ladder network of Fig. 7.60 with the parameter values of Fig. 7.27.

```
Enter R1: 10
Enter R2: 220
Enter R3: 12
Enter R4: 100
Enter R5: 1e30
Enter R6: 1e30
Enter E: -60

The total resistance is 84.216866 ohms.
The source current is -0.712446 Amperes.
I3 equals -0.472103 Amperes.
I6 equals -2.360515e-29 Amperes.
```

FIG. 7.63

C++ response to an analysis of the ladder network of Fig. 7.60 without the elements R_5 and R_6.

PROBLEMS

SECTION 7.2 Descriptive Examples

1. Which elements of the networks in Fig. 7.64 are in series or parallel? In other words, which elements of the given networks have the same current (series) or voltage (parallel)? Restrict your decision to single elements, and do not include combined elements.

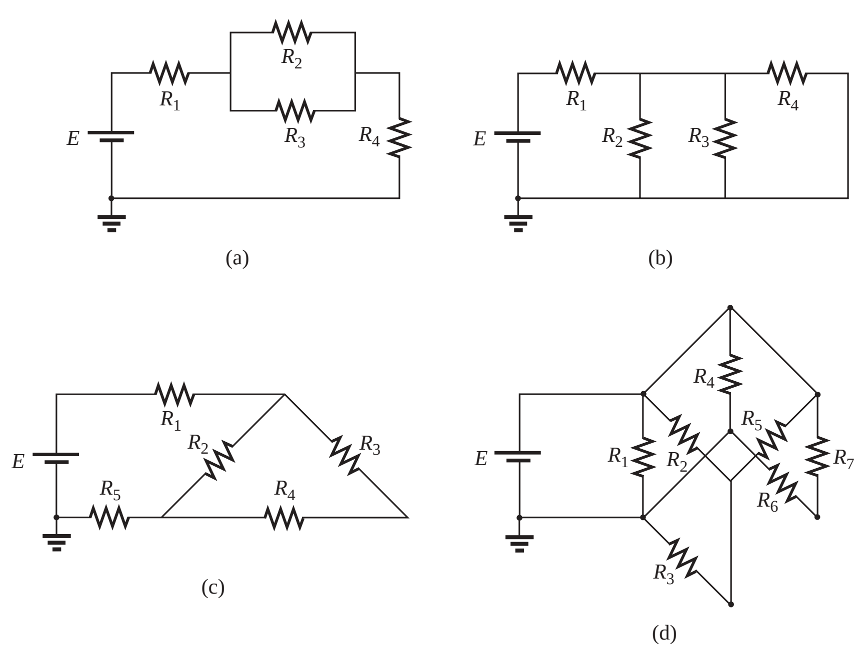

FIG. 7.64

Problem 1.

2. Determine R_T for the networks of Fig. 7.65.

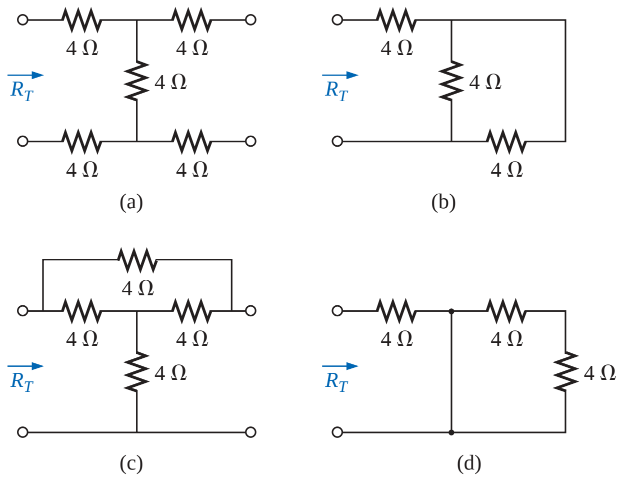

FIG. 7.65
Problem 2.

3. For the network of Fig. 7.66:
   - **a.** Does $I = I_3 = I_6$? Explain.
   - **b.** If $I = 5$ A and $I_1 = 2$ A, find I_2.
   - **c.** Does $I_1 + I_2 = I_4 + I_5$? Explain.
   - **d.** If $V_1 = 6$ V and $E = 10$ V, find V_2.
   - **e.** If $R_1 = 3\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 4\ \Omega$, and $R_4 = 1\ \Omega$, what is R_T?
   - **f.** If the resistors have the values given in part (e) and $E = 10$ V, what is the value of I in amperes?
   - **g.** Using values given in parts (e) and (f), find the power delivered by the battery E and dissipated by the resistors R_1 and R_2.

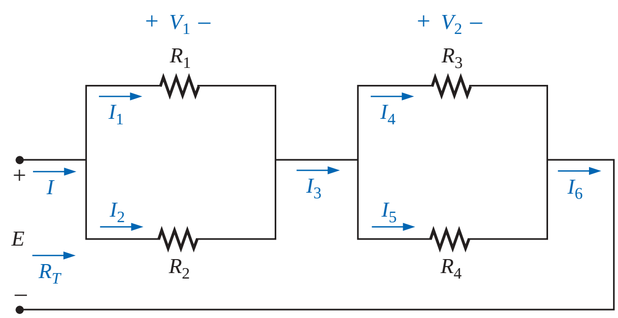

FIG. 7.66
Problem 3.

4. For the network of Fig. 7.67:
   - **a.** Calculate R_T.
   - **b.** Determine I and I_1.
   - **c.** Find V_3.
5. For the network of Fig. 7.68:
   - **a.** Determine R_T.
   - **b.** Find I_s, I_1, and I_2.
   - **c.** Calculate V_a.

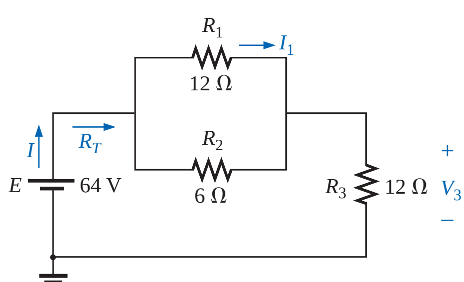

FIG. 7.67
Problem 4.

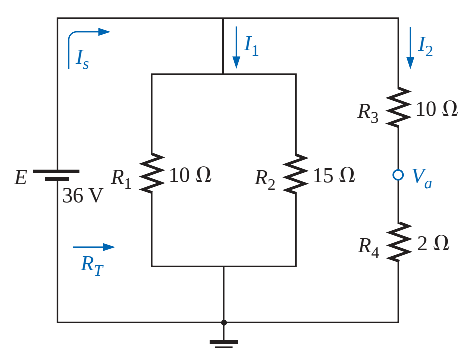

FIG. 7.68
Problem 5.

6. Determine the currents I_1 and I_2 for the network of Fig. 7.69.

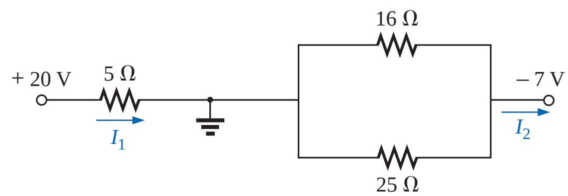

FIG. 7.69
Problem 6.

7. **a.** Find the magnitude and direction of the currents I, I_1, I_2, and I_3 for the network of Fig. 7.70.
   b. Indicate their direction on Fig. 7.70.

*8. For the network of Fig. 7.71:
   a. Determine the currents I_s, I_1, I_3, and I_4.
   b. Calculate V_a and V_{bc}.

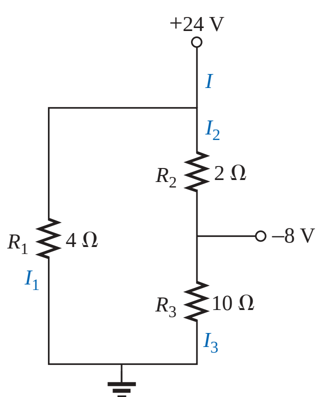

FIG. 7.70
Problem 7.

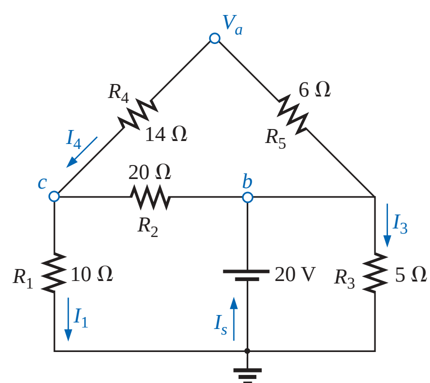

FIG. 7.71
Problem 8.

9. For the network of Fig. 7.72:
   a. Determine the current I_1.
   b. Calculate the currents I_2 and I_3.
   c. Determine the voltage levels V_a and V_b.

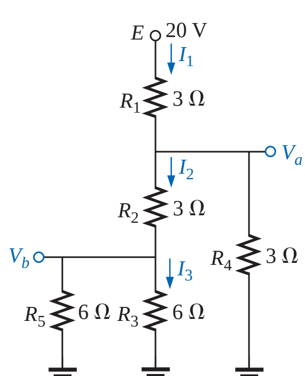

FIG. 7.72
Problem 9.

10. For the network of Fig. 7.73:
a. Find the currents I and I_6.
b. Find the voltages V_1 and V_5.
c. Find the power delivered to the 6-kΩ resistor.

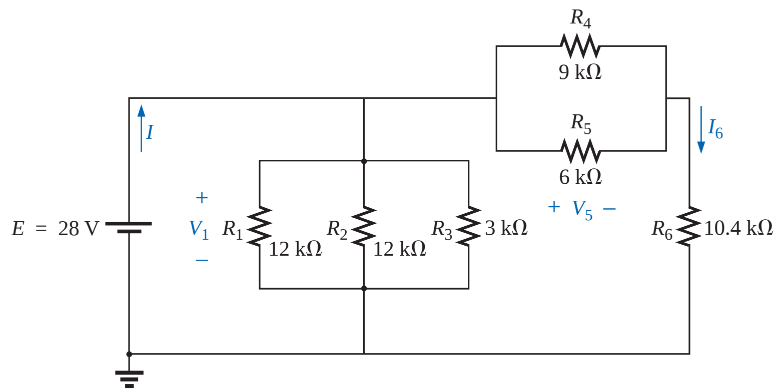

FIG. 7.73
Problem 10.

***11.** For the series-parallel network of Fig. 7.74:
a. Find the current I.
b. Find the currents I_3 and I_9.
c. Find the current I_8.
d. Find the voltage V_{ab}.

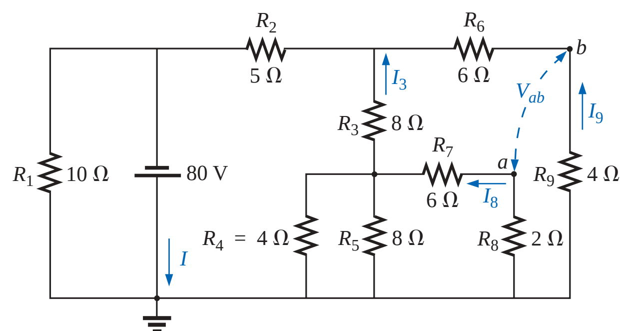

FIG. 7.74
Problem 11.

***12.** Determine the dc levels for the transistor network of Fig. 7.75 using the fact that $V_{BE} = 0.7$ V, $V_E = 2$ V, and $I_C = I_E$. That is:
a. Determine I_E and I_C.
b. Calculate I_B.
c. Determine V_B and V_C.
d. Find V_{CE} and V_{BC}.

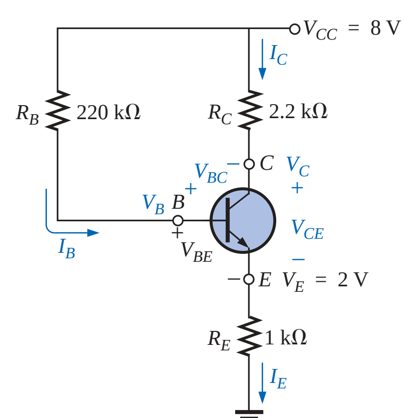

FIG. 7.75
Problem 12.

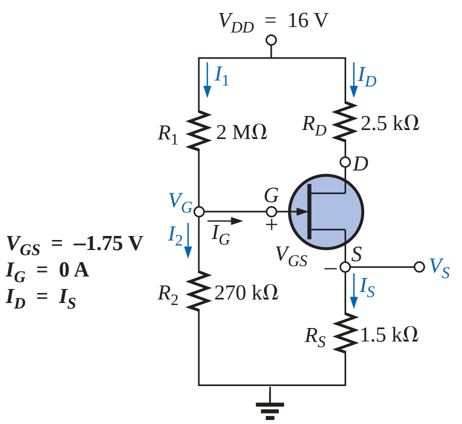

FIG. 7.76
Problem 13.

***13.** The network of Fig. 7.76 is the basic biasing arrangement for the *field-effect transistor* (FET), a device of increasing importance in electronic design. (*Biasing* simply means the application of dc levels to establish a particular set of operating conditions.) Even though you may be unfamiliar with the FET, you can perform the following analysis using only the basic laws introduced in this chapter and the information provided on the diagram.

a. Determine the voltages V_G and V_S.
b. Find the currents I_1, I_2, I_D, and I_S.
c. Determine V_{DS}.
d. Calculate V_{DG}.

***14.** For the network of Fig. 7.77:

a. Determine R_T.
b. Calculate V_a.
c. Find V_1.
d. Calculate V_2.
e. Determine I (with direction).

15. For the network of Fig. 7.78:

a. Determine the current I.
b. Find V.

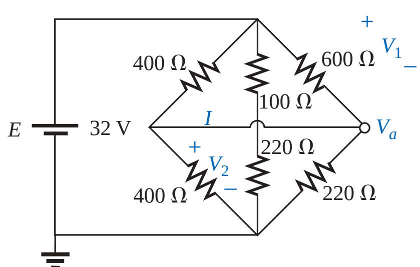

FIG. 7.77
Problem 14.

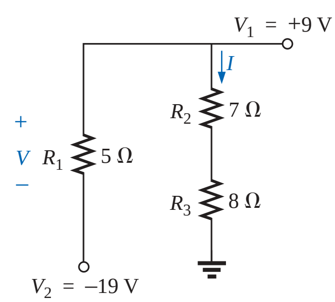

FIG. 7.78
Problem 15.

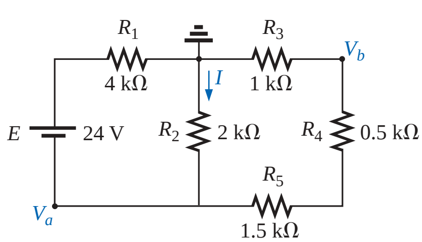

FIG. 7.79
Problem 16.

***16.** Determine the current I and the voltages V_a, V_b, and V_{ab} for the network of Fig. 7.79.

17. For the configuration of Fig. 7.80:

a. Find the currents I_2, I_6, and I_8.

b. Find the voltages V_4 and V_8.

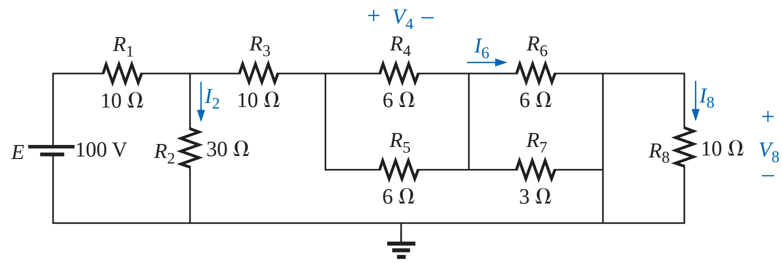

FIG. 7.80
Problem 17.

18. Determine the voltage V and the current I for the network of Fig. 7.81.

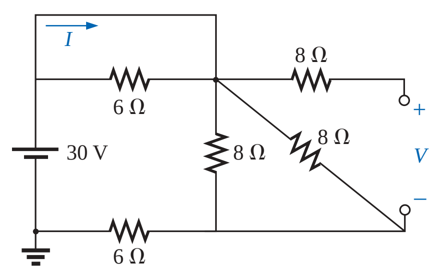

FIG. 7.81
Problem 18.

***19.** For the network of Fig. 7.82:

a. Determine R_T by combining resistive elements.

b. Find V_1 and V_4.

c. Calculate I_3 (with direction).

d. Determine I_s by finding the current through each element and then applying Kirchhoff's current law. Then calculate R_T from $R_T = E/I_s$, and compare the answer with the solution of part (a).

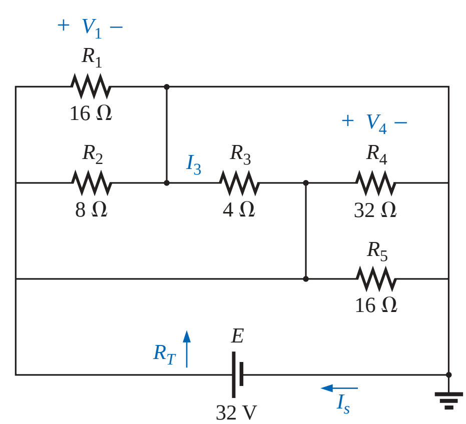

FIG. 7.82
Problem 19.

20. For the network of Fig. 7.83:

a. Determine the voltage V_{ab}. (*Hint:* Just use Kirchhoff's voltage law.)

b. Calculate the current I.

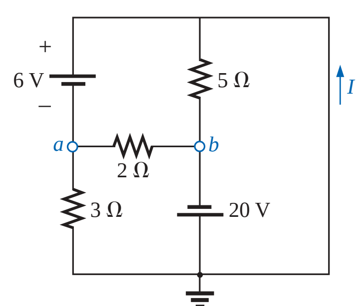

FIG. 7.83
Problem 20.

***21.** For the network of Fig. 7.84:
  a. Determine the current *I*.
  b. Calculate the open-circuit voltage *V*.

***22.** For the network of Fig. 7.85, find the resistance R_3 if the current through it is 2 A.

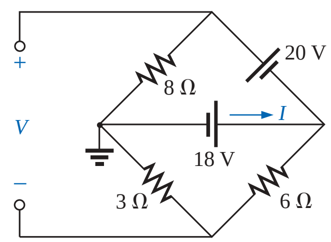

FIG. 7.84
Problem 21.

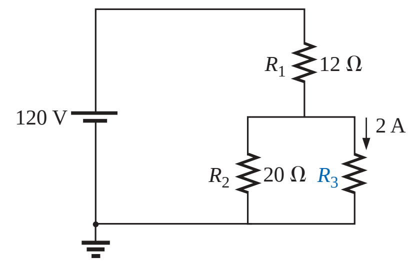

FIG. 7.85
Problem 22.

23.** If all the resistors of the cube in Fig. 7.86 are 10 Ω, what is the total resistance? **(Hint:* Make some basic assumptions about current division through the cube.)

***24.** Given the voltmeter reading $V = 27$ V in Fig. 7.87:
  a. Is the network operating properly?
  b. If not, what could be the cause of the incorrect reading?

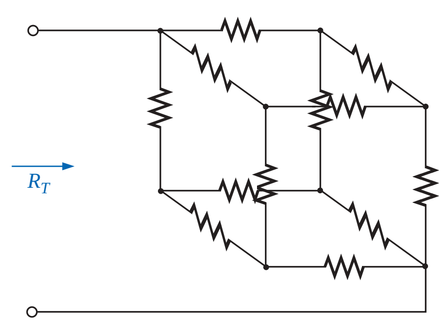

FIG. 7.86
Problem 23.

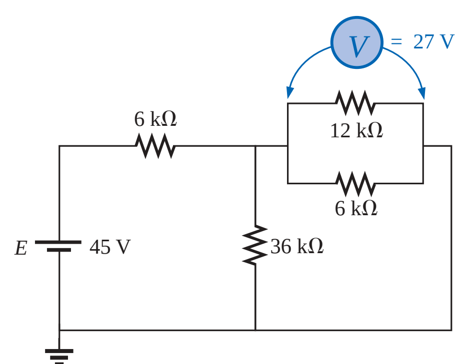

FIG. 7.87
Problem 24.

SECTION 7.3 Ladder Networks

25. For the ladder network of Fig. 7.88:
  a. Find the current *I*.
  b. Find the current I_7.
  c. Determine the voltages V_3, V_5, and V_7.
  d. Calculate the power delivered to R_7, and compare it to the power delivered by the 240-V supply.

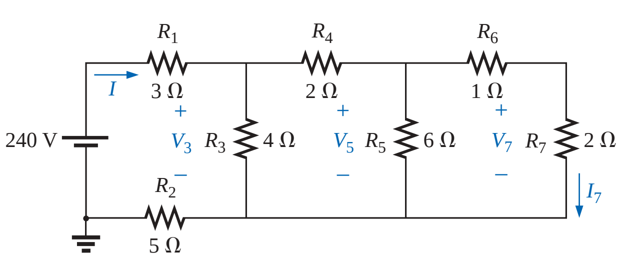

FIG. 7.88
Problem 25.

26. For the ladder network of Fig. 7.89:

a. Determine R_T.

b. Calculate I.

c. Find I_8.

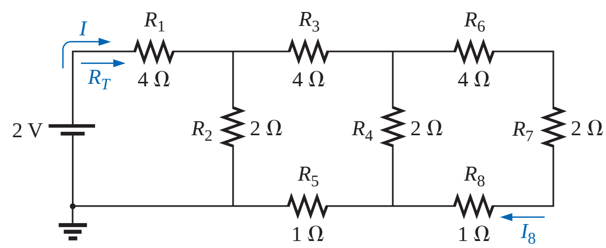

FIG. 7.89
Problem 26.

***27.** Determine the power delivered to the 10-Ω load of Fig. 7.90.

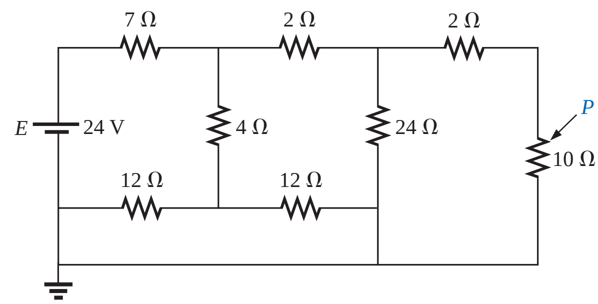

FIG. 7.90
Problem 27.

***28.** For the multiple ladder configuration of Fig. 7.91:

a. Determine I.

b. Calculate I_4.

c. Find I_6.

d. Find I_{10}.

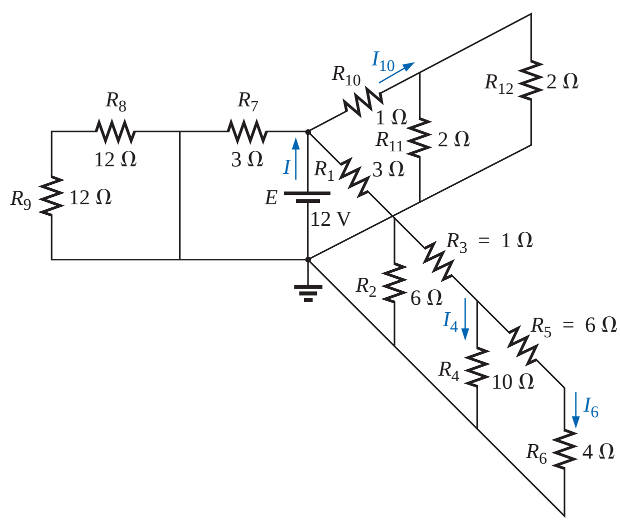

FIG. 7.91
Problem 28.

SECTION 7.4 Voltage Divider Supply (Unloaded and Loaded)

29. Given the voltage divider supply of Fig. 7.92:

a. Determine the supply voltage E.

b. Find the load resistors R_{L_2} and R_{L_3}.

c. Determine the voltage divider resistors R_1, R_2, and R_3.

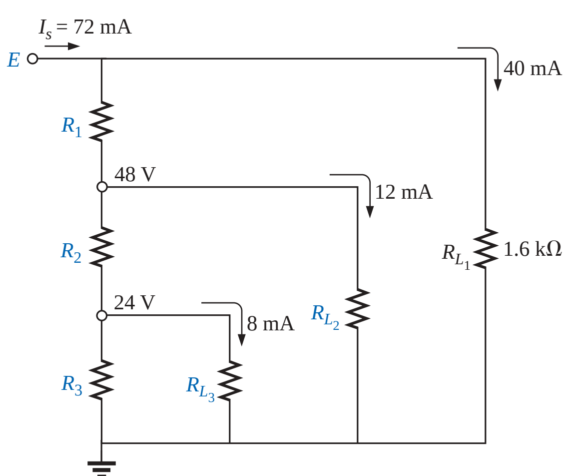

FIG. 7.92
Problem 29.

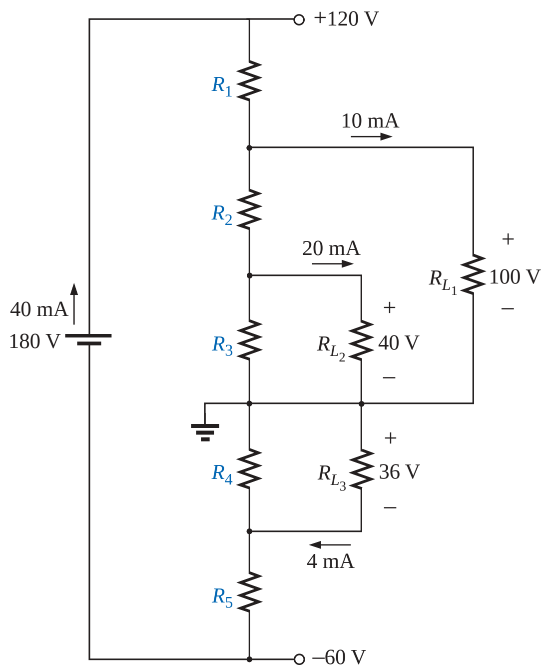

FIG. 7.93
Problem 30.

***30.** Determine the voltage divider supply resistors for the configuration of Fig. 7.93. Also determine the required wattage rating for each resistor, and compare their levels.

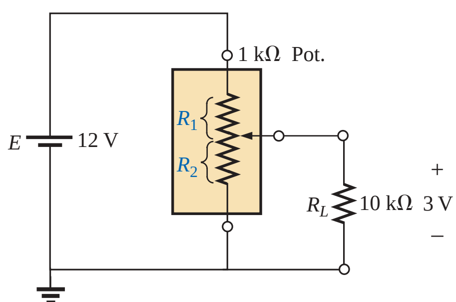

FIG. 7.94
Problem 31.

SECTION 7.5 Potentiometer Loading

***31.** For the system of Fig. 7.94:

a. At first exposure, does the design appear to be a good one?

b. In the absence of the 10-kΩ load, what are the values of R_1 and R_2 to establish 3 V across R_2?

c. Determine the values of R_1 and R_2 when the load is applied, and compare them to the results of part (b).

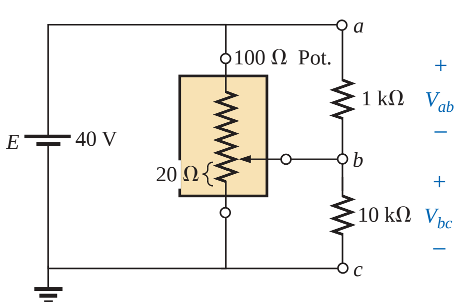

FIG. 7.95
Problem 32.

***32.** For the potentiometer of Fig. 7.95:

a. What are the voltages V_{ab} and V_{bc} with no load applied?

b. What are the voltages V_{ab} and V_{bc} with the indicated loads applied?

c. What is the power dissipated by the potentiometer under the loaded conditions of Fig. 7.95?

d. What is the power dissipated by the potentiometer with no loads applied? Compare it to the results of part (c).

SECTION 7.6 Ammeter, Voltmeter, and Ohmmeter Design

33. A d'Arsonval movement is rated 1 mA, 100 Ω.

a. What is the current sensitivity?

b. Design a 20-A ammeter using the above movement. Show the circuit and component values.

34. Using a 50-μA, 1000-Ω d'Arsonval movement, design a multirange milliammeter having scales of 25 mA, 50 mA, and 100 mA. Show the circuit and component values.

35. A d'Arsonval movement is rated 50 μA, 1000 Ω.

a. Design a 15-V dc voltmeter. Show the circuit and component values.

b. What is the ohm/volt rating of the voltmeter?

36. Using a 1-mA, 100-Ω d'Arsonval movement, design a multirange voltmeter having scales of 5 V, 50 V, and 500 V. Show the circuit and component values.

37. A digital meter has an internal resistance of 10 MΩ on its 0.5-V range. If you had to build a voltmeter with a d'Arsonval movement, what current sensitivity would you need if the meter were to have the same internal resistance on the same voltage scale?

***38. a.** Design a series ohmmeter using a 100-μA, 1000-Ω movement; a zero-adjust with a maximum value of 2 kΩ; a battery of 3 V; and a series resistor whose value is to be determined.

b. Find the resistance required for full-scale, 3/4-scale, 1/2-scale, and 1/4-scale deflection.

c. Using the results of part (b), draw the scale to be used with the ohmmeter.

39. Describe the basic construction and operation of the megohmmeter.

***40.** Determine the reading of the ohmmeter for the configuration of Fig. 7.96.

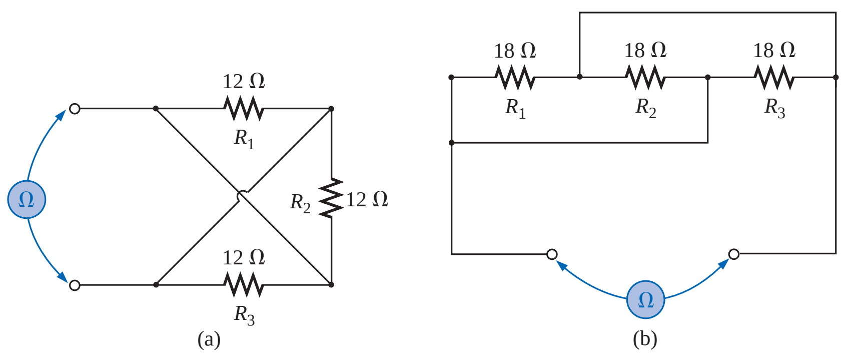

FIG. 7.96
Problem 40.

SECTION 7.9 Computer Analysis

PSpice or Electronics Workbench

41. Using schematics, determine V_1, V_3, V_{ab}, and I_s for the network of Fig. 7.16.

42. Using schematics, determine I_s, I_5, and V_7 for the network of Fig. 7.22.

Programming Language (C++, QBASIC, Pascal, etc.)

43. Write a program that will find the complete solution for the network of Fig. 7.6. That is, given all the parameters of the network, calculate the current, voltage, and power to each element.

44. Write a program to find all the quantities of Example 7.8 given the network parameters.

GLOSSARY

d'Arsonval movement An iron-core coil mounted on bearings between a permanent magnet. A pointer connected to the movable core indicates the strength of the current passing through the coil.

Ladder network A network that consists of a cascaded set of series-parallel combinations and has the appearance of a ladder.

Megohmmeter An instrument for measuring very high resistance levels, such as in the megohm range.

Series ohmmeter A resistance-measuring instrument in which the movement is placed in series with the unknown resistance.

Series-parallel network A network consisting of a combination of both series and parallel branches.

Transistor A three-terminal semiconductor electronic device that can be used for amplification and switching purposes.