# 10

# Capacitors

## 10.1 INTRODUCTION

Thus far, the only passive device appearing in the text has been the resistor. We will now consider two additional passive devices called the **capacitor** and the *inductor* (the inductor is discussed in detail in Chapter 12), which are quite different from the resistor in purpose, operation, and construction.

Unlike the resistor, both elements display their total characteristics only when a change in voltage or current is made in the circuit in which they exist. In addition, if we consider the *ideal* situation, they do not dissipate energy as does the resistor but store it in a form that can be returned to the circuit whenever required by the circuit design.

Proper treatment of each requires that we devote this entire chapter to the capacitor and, as mentioned above, Chapter 12 to the inductor. Since electromagnetic effects are a major consideration in the design of inductors, this topic will be covered in Chapter 11.

## 10.2 THE ELECTRIC FIELD

Recall from Chapter 2 that a force of attraction or repulsion exists between two charged bodies. We shall now examine this phenomenon in greater detail by considering the electric field that exists in the region around any charged body. This electric field is represented by **electric flux lines,** which are drawn to indicate the strength of the electric field at any point around the charged body; that is, the denser the lines of flux, the stronger the electric field. In Fig. 10.1, the electric field strength is stronger at position *a* than at position *b* because the flux lines are denser at *a* than at *b*. The symbol for electric flux is the Greek letter $\psi$ (psi). The flux per unit area (flux density) is represented by the capital letter $D$ and is determined by

$$D = \frac{\psi}{A} \qquad \text{(flux/unit area)} \qquad \textbf{(10.1)}$$

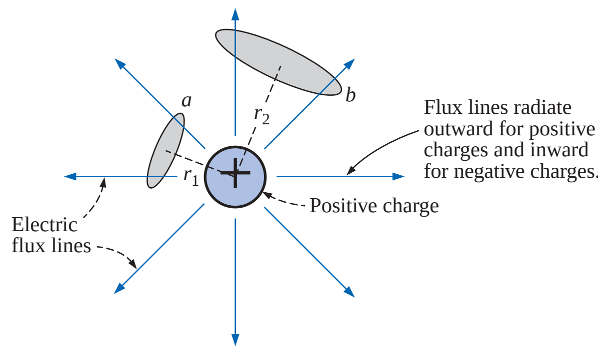



**FIG. 10.1**
*Flux distribution from an isolated positive charge.*

The larger the charge $Q$ in coulombs, the greater the number of flux lines extending or terminating per unit area, independent of the surrounding medium. Twice the charge will produce twice the flux per unit area. The two can therefore be equated:

$$\psi \equiv Q \qquad \text{(coulombs, C)} \qquad \textbf{(10.2)}$$

By definition, the **electric field strength** at a point is the force acting on a unit positive charge at that point; that is,

$$\mathscr{E} = \frac{F}{Q} \qquad \text{(newtons/coulomb, N/C)} \qquad \textbf{(10.3)}$$

The force exerted on a unit positive charge ($Q_2 = 1$ C), by a charge $Q_1$, $r$ meters away, as determined by **Coulomb's law** is

$$F = \frac{kQ_1Q_2}{r^2} = \frac{kQ_1(1)}{r^2} = \frac{kQ_1}{r^2} \qquad (k = 9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$

Substituting this force $F$ into Eq. (10.3) yields

$$\mathscr{E} = \frac{F}{Q_2} = \frac{kQ_1/r^2}{1}$$

$$\mathscr{E} = \frac{kQ_1}{r^2} \qquad \text{(N/C)} \qquad \textbf{(10.4)}$$

We can therefore conclude that the electric field strength at any point distance $r$ from a point charge of $Q$ coulombs is directly proportional to the magnitude of the charge and inversely proportional to the distance squared from the charge. The squared term in the denominator will result in a rapid decrease in the strength of the electric field with distance from the point charge. In Fig. 10.1, substituting distances $r_1$ and $r_2$ into Eq. (10.4) will verify our previous conclusion that the electric field strength is greater at $a$ than at $b$.

***Electric flux lines always extend from a positively charged body to a negatively charged body, always extend or terminate perpendicular to the charged surfaces, and never intersect.***

For two charges of similar and opposite polarities, the flux distribution would appear as shown in Fig. 10.2.

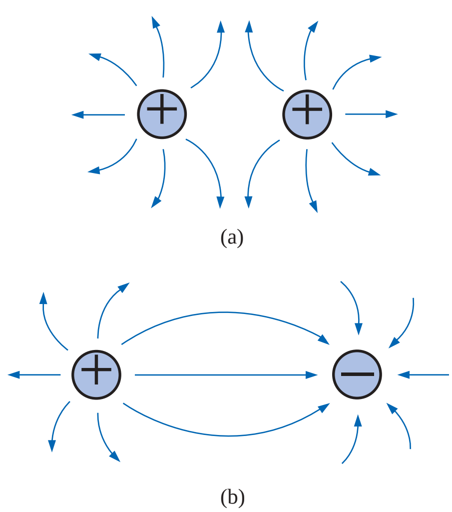



**FIG. 10.2**
*Electric flux distribution: (a) like charges; (b) opposite charges.*

The attraction and repulsion between charges can now be explained in terms of the electric field and its flux lines. In Fig. 10.2(a), the flux lines are not interlocked but tend to act as a buffer, preventing attraction and causing repulsion. Since the electric field strength is stronger (flux lines denser) for each charge the closer we are to the charge, the more we try to bring the two charges together, the stronger will be the force of repulsion between them. In Fig. 10.2(b), the flux lines extending from the positive charge are terminated at the negative charge. A basic law of physics states that electric flux lines always tend to be as short as possible. The two charges will therefore be drawn to each other. Again, the closer the two charges, the stronger the attraction between the two charges due to the increased field strengths.

## 10.3 CAPACITANCE

Up to this point we have considered only isolated positive and negative spherical charges, but the analysis can be extended to charged surfaces of any shape and size. In Fig. 10.3, for example, two parallel plates of a conducting material separated by an air gap have been connected through a switch and a resistor to a battery. If the parallel plates are initially uncharged and the switch is left open, no net positive or negative charge will exist on either plate. The instant the switch is closed, however, electrons are drawn from the upper plate through the resistor to the positive terminal of the battery. There will be a surge of current at first, limited in magnitude by the resistance present. The level of flow will then decline, as will be demonstrated in the sections to follow. This action creates a net positive charge on the top plate. Electrons are being repelled by the negative terminal through the lower conductor to the bottom plate at the same rate they are being drawn to the positive terminal. This transfer of electrons continues until the potential difference across the parallel plates is exactly equal to the battery voltage. The final result is a net positive charge on the top plate and a negative charge on the bottom plate, very similar in many respects to the two isolated charges of Fig. 10.2(b).

This element, constructed simply of two parallel conducting plates separated by an insulating material (in this case, air), is called a **capacitor. Capacitance** is a measure of a capacitor's ability to store charge on its plates—in other words, its storage capacity.

***A capacitor has a capacitance of 1 farad if 1 coulomb of charge is deposited on the plates by a potential difference of 1 volt across the plates.***

The farad is named after Michael Faraday (Fig. 10.4), a nineteenth-century English chemist and physicist. The farad, however, is generally too large a measure of capacitance for most practical applications, so the microfarad ($10^{-6}$) or picofarad ($10^{-12}$) is more commonly used. Expressed as an equation, the capacitance is determined by

$$C = \frac{Q}{V} \qquad \begin{aligned} C &= \text{farads (F)} \\ Q &= \text{coulombs (C)} \\ V &= \text{volts (V)} \end{aligned} \qquad \textbf{(10.5)}$$

Different capacitors for the same voltage across their plates will acquire greater or lesser amounts of charge on their plates. Hence the capacitors have a greater or lesser capacitance, respectively.

A cross-sectional view of the parallel plates is shown with the distribution of electric flux lines in Fig. 10.5(a). The number of flux lines per

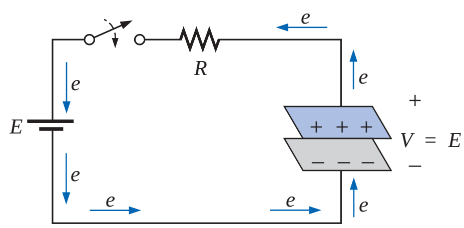



**FIG. 10.3**
*Fundamental charging network.*

**English** (London)
**(1791–1867)**
**Chemist and Electrical Experimenter**
**Honory Doctorate,** Oxford University, 1832

Courtesy of the Smithsonian Institution Photo No. 51,147

An experimenter with no formal education, he began his research career at the Royal Institute in London as a laboratory assistant. Intrigued by the interaction between electrical and magnetic effects, he discovered *electromagnetic induction*, demonstrating that electrical effects can be generated from a magnetic field (the birth of the generator as we know it today). He also discovered *self-induced currents* and introduced the concept of *lines and fields of magnetic force*. Having received over one hundred academic and scientific honors, he became a Fellow of the Royal Society in 1824 at the young age of 32.

**FIG. 10.4**
*Michael Faraday.*

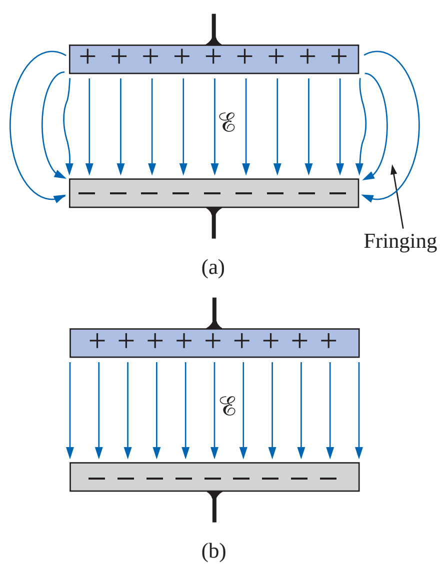



**FIG. 10.5**
*Electric flux distribution between the plates of a capacitor: (a) including fringing; (b) ideal.*

unit area ($D$) between the two plates is quite uniform. At the edges, the flux lines extend outside the common surface area of the plates, producing an effect known as **fringing.** This effect, which reduces the capacitance somewhat, can be neglected for most practical applications. For the analysis to follow, we will assume that all the flux lines leaving the positive plate will pass directly to the negative plate within the common surface area of the plates [Fig. 10.5(b)].

If a potential difference of $V$ volts is applied across the two plates separated by a distance of $d$, the electric field strength between the plates is determined by

$$\mathscr{E} = \frac{V}{d} \quad \text{(volts/meter, V/m)} \qquad \textbf{(10.6)}$$

The uniformity of the flux distribution in Fig. 10.5(b) also indicates that the electric field strength is the same at any point between the two plates.

Many values of capacitance can be obtained for the same set of parallel plates by the addition of certain insulating materials between the plates. In Fig. 10.6(a), an insulating material has been placed between a set of parallel plates having a potential difference of $V$ volts across them.

Since the material is an insulator, the electrons within the insulator are unable to leave the parent atom and travel to the positive plate. The positive components (protons) and negative components (electrons) of each atom do shift, however [as shown in Fig. 10.6(a)], to form *dipoles.*

When the dipoles align themselves as shown in Fig. 10.6(a), the material is *polarized.* A close examination within this polarized material will indicate that the positive and negative components of adjoining dipoles are neutralizing the effects of each other [note the dashed area in Fig. 10.6(a)]. The layer of positive charge on one surface and the negative charge on the other are not neutralized, however, resulting in the establishment of an electric field within the insulator [$\mathscr{E}_{\text{dielectric}}$; Fig. 10.6(b)]. The net electric field between the plates ($\mathscr{E}_{\text{resultant}} = \mathscr{E}_{\text{air}} - \mathscr{E}_{\text{dielectric}}$) would therefore be reduced due to the insertion of the dielectric.

The purpose of the dielectric, therefore, is to create an electric field to oppose the electric field set up by free charges on the parallel plates. For this reason, the insulating material is referred to as a **dielectric,** *di* for "opposing" and *electric* for "electric field."

In either case—with or without the dielectric—if the potential across the plates is kept constant and the distance between the plates is fixed, the net electric field within the plates must remain the same, as determined by the equation $\mathscr{E} = V/d$. We just ascertained, however, that the net electric field between the plates would decrease with insertion of the dielectric for a fixed amount of free charge on the plates. To compensate and keep the net electric field equal to the value determined by $V$ and $d$, more charge must be deposited on the plates. [Look ahead to Eq. (10.11).] This additional charge for the same potential across the plates increases the capacitance, as determined by the following equation:

$$C\uparrow = \frac{Q\uparrow}{V}$$

For different dielectric materials between the same two parallel plates, different amounts of charge will be deposited on the plates. But

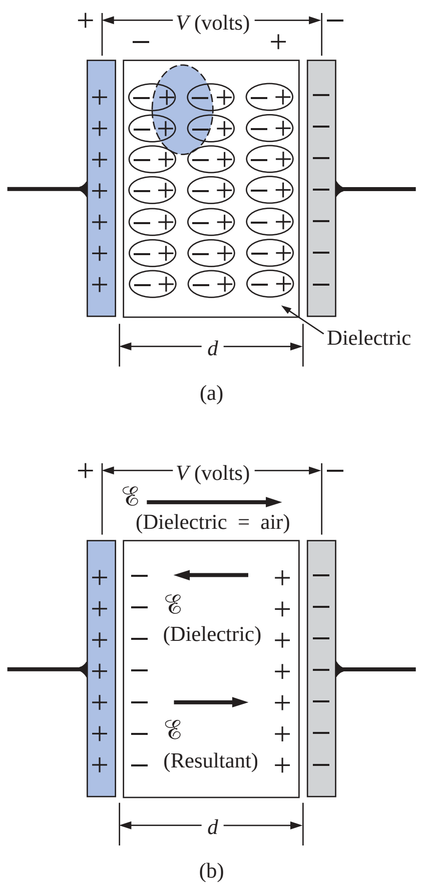



**FIG. 10.6**

*Effect of a dielectric on the field distribution between the plates of a capacitor: (a) alignment of dipoles in the dielectric; (b) electric field components between the plates of a capacitor with a dielectric present.*

$\psi \equiv Q$, so the dielectric is also determining the number of flux lines between the two plates and consequently the flux density ($D = \psi/A$) since $A$ is fixed.

The ratio of the flux density to the electric field intensity in the dielectric is called the **permittivity** of the dielectric:

$$\epsilon = \frac{D}{\mathscr{E}} \quad \text{(farads/meter, F/m)} \tag{10.7}$$

It is a measure of how easily the dielectric will "permit" the establishment of flux lines within the dielectric. The greater its value, the greater the amount of charge deposited on the plates, and, consequently, the greater the flux density for a fixed area.

For a vacuum, the value of $\epsilon$ (denoted by $\epsilon_o$) is $8.85 \times 10^{-12}$ F/m. The ratio of the permittivity of any dielectric to that of a vacuum is called the **relative permittivity,** $\epsilon_r$. It simply compares the permittivity of the dielectric to that of air. In equation form,

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} \tag{10.8}$$

The value of $\epsilon$ for any material, therefore, is

$$\epsilon = \epsilon_r \epsilon_o$$

Note that $\epsilon_r$ is a dimensionless quantity. The relative permittivity, or **dielectric constant,** as it is often called, is provided in Table 10.1 for various dielectric materials.

Substituting for $D$ and $\mathscr{E}$ in Eq. (10.7), we have

$$\epsilon = \frac{D}{\mathscr{E}} = \frac{\psi/A}{V/d} = \frac{Q/A}{V/d} = \frac{Qd}{VA}$$

But $$C = \frac{Q}{V}$$

and, therefore, $$\epsilon = \frac{Cd}{A}$$

**TABLE 10.1**

*Relative permittivity (dielectric constant) of various dielectrics.*

| Dielectric | $\epsilon_r$ (Average Values) |
|---|---|
| Vacuum | 1.0 |
| Air | 1.0006 |
| Teflon | 2.0 |
| Paper, paraffined | 2.5 |
| Rubber | 3.0 |
| Transformer oil | 4.0 |
| Mica | 5.0 |
| Porcelain | 6.0 |
| Bakelite | 7.0 |
| Glass | 7.5 |
| Distilled water | 80.0 |
| Barium-strontium titanite (ceramic) | 7500.0 |

and $$C = \epsilon \frac{A}{d} \quad \text{(F)} \tag{10.9}$$

or $$C = \epsilon_o \epsilon_r \frac{A}{d} = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d} \quad \text{(F)} \tag{10.10}$$

where $A$ is the area in square meters of the plates, $d$ is the distance in meters between the plates, and $\epsilon_r$ is the relative permittivity. The capacitance, therefore, will be greater if the area of the plates is increased, or the distance between the plates is decreased, or the dielectric is changed so that $\epsilon_r$ is increased.

Solving for the distance $d$ in Eq. (10.9), we have

$$D = \frac{\epsilon A}{C}$$

and substituting into Eq. (10.6) yields

$$\mathscr{E} = \frac{V}{d} = \frac{V}{\epsilon A/C} = \frac{CV}{\epsilon A}$$

But $Q = CV$, and therefore

$$\mathscr{E} = \frac{Q}{\epsilon A} \quad \text{(V/m)} \tag{10.11}$$

which gives the electric field intensity between the plates in terms of the permittivity $\epsilon$, the charge $Q$, and the surface area $A$ of the plates. Thus, we have the ratio

$$\frac{C = \epsilon A/d}{C_o = \epsilon_o A/d} = \frac{\epsilon}{\epsilon_o} = \epsilon_r$$

or $$C = \epsilon_r C_o \tag{10.12}$$

which, in words, states that for the same set of parallel plates, the capacitance using a dielectric (of relative permittivity $\epsilon_r$) is $\epsilon_r$ times that obtained for a vacuum (or air, approximately) between the plates. This relationship between $\epsilon_r$ and the capacitances provides an excellent experimental method for finding the value of $\epsilon_r$ for various dielectrics.

**EXAMPLE 10.1** Determine the capacitance of each capacitor on the right side of Fig. 10.7.

***Solutions:***

a. $C = 3(5\ \mu\text{F}) = \mathbf{15\ \mu F}$

b. $C = \frac{1}{2}(0.1\ \mu\text{F}) = \mathbf{0.05\ \mu F}$

c. $C = 2.5(20\ \mu\text{F}) = \mathbf{50\ \mu F}$

d. $C = (5)\frac{4}{(1/8)}(1000\ \text{pF}) = (160)(1000\ \text{pF}) = \mathbf{0.16\ \mu F}$

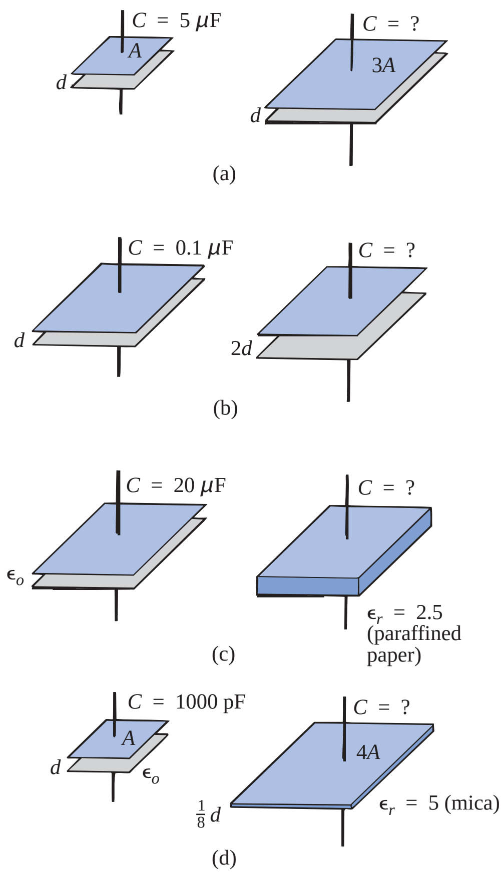



**FIG. 10.7**
*Example 10.1.*

**EXAMPLE 10.2** For the capacitor of Fig. 10.8:

a. Determine the capacitance.
b. Determine the electric field strength between the plates if 450 V are applied across the plates.
c. Find the resulting charge on each plate.

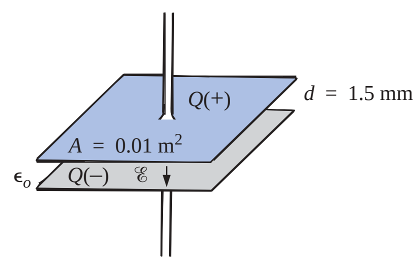



**FIG. 10.8**
*Example 10.2.*

***Solutions:***

a. $$C_o = \frac{\epsilon_o A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(0.01\ \text{m}^2)}{1.5 \times 10^{-3}\ \text{m}} = 59.0 \times 10^{-12}\ \text{F}$$
$$= \mathbf{59\ pF}$$

b. $$\mathscr{E} = \frac{V}{d} = \frac{450\ \text{V}}{1.5 \times 10^{-3}\ \text{m}}$$
$$\cong \mathbf{300 \times 10^3\ V/m}$$

c. $C = \dfrac{Q}{V}$ or

$$Q = CV = (59.0 \times 10^{-12}\ \text{F})(450\ \text{V})$$
$$= 26.550 \times 10^{-9}\ \text{C}$$
$$= \mathbf{26.55\ nC}$$

**EXAMPLE 10.3** A sheet of mica 1.5 mm thick having the same area as the plates is inserted between the plates of Example 10.2.

a. Find the electric field strength between the plates.
b. Find the charge on each plate.
c. Find the capacitance.

***Solutions:***

a. $\mathscr{E}$ is fixed by

$$\mathscr{E} = \frac{V}{d} = \frac{450\ \text{V}}{1.5 \times 10^3\ \text{m}}$$
$$\cong \mathbf{300 \times 10^3\ V/m}$$

b. $\mathscr{E} = \dfrac{Q}{\epsilon A}$ or

$$Q = \epsilon \mathscr{E} A = \epsilon_r \epsilon_o \mathscr{E} A$$
$$= (5)(8.85 \times 10^{-12}\ \text{F/m})(300 \times 10^3\ \text{V/m})(0.01\ \text{m}^2)$$
$$= 132.75 \times 10^{-9}\ \text{C} = \mathbf{132.75\ nC}$$

(five times the amount for air between the plates)

c. $C = \epsilon_r C_o$

$$= (5)(59 \times 10^{-12}\ \text{F}) = \mathbf{295\ pF}$$

## 10.4 DIELECTRIC STRENGTH

For every dielectric there is a potential that, if applied across the dielectric, will break the bonds within the dielectric and cause current to flow. The voltage required per unit length (electric field intensity) to establish conduction in a dielectric is an indication of its **dielectric strength** and is called the **breakdown voltage.** When breakdown occurs, the capacitor has characteristics very similar to those of a conductor. A typical example of breakdown is lightning, which occurs when the potential between the clouds and the earth is so high that charge can pass from one to the other through the atmosphere, which acts as the dielectric.

The average dielectric strengths for various dielectrics are tabulated in volts/mil in Table 10.2 (1 mil = 0.001 in.). The relative permittivity appears in parentheses to emphasize the importance of considering both factors in the design of capacitors. Take particular note of barium-strontium titanite and mica.

**TABLE 10.2**

*Dielectric strength of some dielectric materials.*

| Dielectric | Dielectric Strength (Average Value), in Volts/Mil | ($\epsilon_r$) |
|---|---|---|
| Air | 75 | (1.0006) |
| Barium-strontium titanite (ceramic) | 75 | (7500) |
| Porcelain | 200 | (6.0) |
| Transformer oil | 400 | (4.0) |
| Bakelite | 400 | (7.0) |
| Rubber | 700 | (3.0) |
| Paper, paraffined | 1300 | (2.5) |
| Teflon | 1500 | (2.0) |
| Glass | 3000 | (7.5) |
| Mica | 5000 | (5.0) |

**EXAMPLE 10.4** Find the maximum voltage that can be applied across a 0.2-$\mu$F capacitor having a plate area of 0.3 m$^2$. The dielectric is porcelain. Assume a linear relationship between the dielectric strength and the thickness of the dielectric.

***Solution:***

$$C = 8.85 \times 10^{-12}\epsilon_r \frac{A}{d}$$

or $$d = \frac{8.85\epsilon_r A}{10^{12}C} = \frac{(8.85)(6)(0.3\text{ m}^2)}{(10^{12})(0.2 \times 10^{-6}\text{ F})} = 7.965 \times 10^{-5}\text{ m}$$

$$\cong \mathbf{79.65\ \mu m}$$

Converting micrometers to mils, we have

$$79.65\ \mu\text{m}\left(\frac{10^{-6}\text{ m}}{\mu\text{m}}\right)\left(\frac{39.371\text{ in.}}{\text{m}}\right)\left(\frac{1000\text{ mils}}{1\text{ in.}}\right) = 3.136\text{ mils}$$

$$\text{Dielectric strength} = 200\text{ V/mil}$$

Therefore, $$\left(\frac{200\text{ V}}{\text{mil}}\right)(3.136\text{ mils}) = \mathbf{627.20\ V}$$

## 10.5 LEAKAGE CURRENT

Up to this point, we have assumed that the flow of electrons will occur in a dielectric only when the breakdown voltage is reached. This is the ideal case. In actuality, there are free electrons in every dielectric due in part to impurities in the dielectric and forces within the material itself.

When a voltage is applied across the plates of a capacitor, a **leakage current** due to the free electrons flows from one plate to the other. The current is usually so small, however, that it can be neglected for most practical applications. This effect is represented by a resistor in parallel with the capacitor, as shown in Fig. 10.9(a), whose value is typically more than 100 megohms (MΩ). Some capacitors, however, such as the electrolytic type, have high leakage currents. When charged and then disconnected from the charging circuit, these capacitors lose their charge in a matter of seconds because of the flow of charge (leakage current) from one plate to the other [Fig. 10.9(b)].

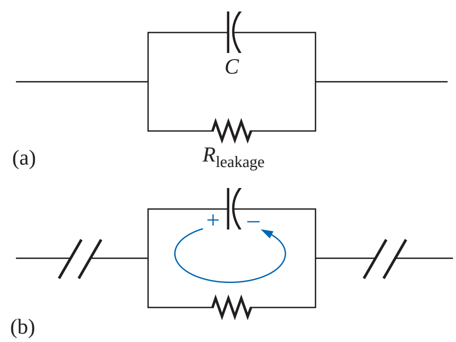



**FIG. 10.9**
*Demonstrating the effect of the leakage current.*

## 10.6 TYPES OF CAPACITORS

Like resistors, all capacitors can be included under either of two general headings: *fixed* or *variable*. The symbol for a fixed capacitor is , and for a variable capacitor, . The curved line represents the plate that is usually connected to the point of lower potential.

### Fixed Capacitors

Many types of fixed capacitors are available today. Some of the most common are the mica, ceramic, electrolytic, tantalum, and polyester-film capacitors. The typical flat *mica capacitor* consists basically of mica sheets separated by sheets of metal foil. The plates are connected to two electrodes, as shown in Fig. 10.10. The total area is the area of

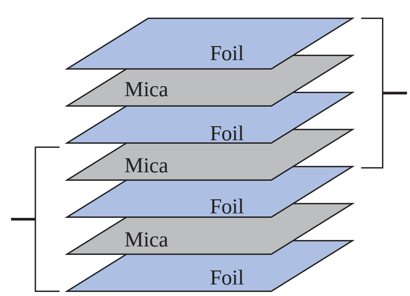



**FIG. 10.10**
*Basic structure of a stacked mica capacitor.*

**FIG. 10.11**
*Mica capacitors. (Courtesy of Custom Electronics Inc.)*

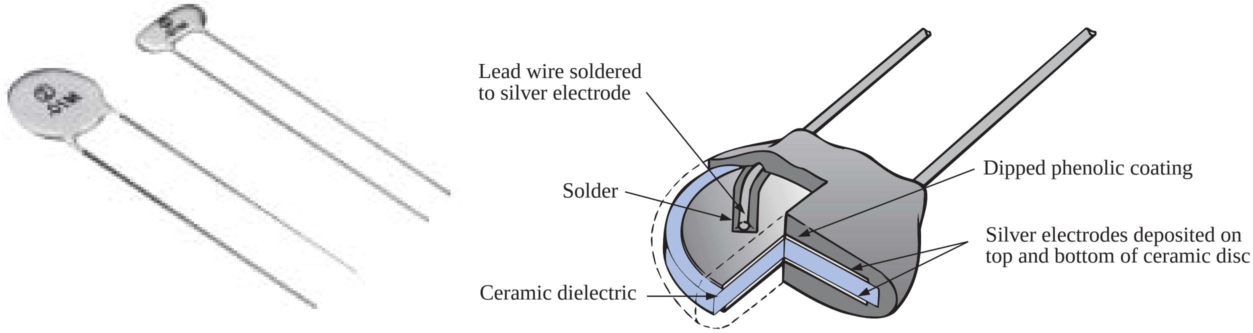
one sheet times the number of dielectric sheets. The entire system is encased in a plastic insulating material as shown for the two central units of Fig. 10.11. The mica capacitor exhibits excellent characteristics under stress of temperature variations and high voltage applications (its dielectric strength is 5000 V/mil). Its leakage current is also very small ($R_{\text{leakage}}$ about 1000 M$\Omega$). Mica capacitors are typically between a few picofarads and 0.2 $\mu$F, with voltages of 100 V or more.

The ability to "roll" the mica to form the cylindrical shapes of Fig. 10.11 is due to a process whereby the soluble contaminants in natural mica are removed, leaving a paperlike structure resulting from the cohesive forces in natural mica. It is commonly referred to as *reconstituted mica*, although the terminology does not mean "recycled" or "second-hand" mica. For some of the units in the photograph, different levels of capacitance are available between different sets of terminals.

The *ceramic capacitor* is made in many shapes and sizes, two of which are shown in Fig. 10.12. The basic construction, however, is about the same for each, as shown in Fig. 10.13. A ceramic base is coated on two sides with a metal, such as copper or silver, to act as the two plates. The leads are then attached through electrodes to the plates. An insulating coating of ceramic or plastic is then applied over the plates and dielectric. Ceramic capacitors also have a very low leakage current ($R_{\text{leakage}}$ about 1000 M$\Omega$) and can be used in both dc and ac networks. They can be found in values ranging from a few picofarads to perhaps 2 $\mu$F, with very high working voltages such as 5000 V or more.

In recent years there has been increasing interest in monolithic (single-structure) chip capacitors such as those appearing in Fig. 10.14(a) due to their application on hybrid circuitry [networks using both discrete and integrated circuit (IC) components]. There has also been increasing use of microstrip (strip-line) circuitry such as the one in Fig. 10.14(b). Note the small chips in this cutaway section. The *L* and *H* of Fig. 10.14(a) indicate the level of capacitance. For example, if in black ink, the letter *H* represents 16 units of capacitance (in picofarads), or 16 pF. If blue ink is used, a multiplier of 100 is applied, resulting in 1600 pF. Although the size is similar, the type of ceramic material controls the capacitance level.

The *electrolytic capacitor* is used most commonly in situations where capacitances of the order of one to several thousand microfarads




(a) (b)

**FIG. 10.12**
*Ceramic disc capacitors: (a) photograph; (b) construction.*

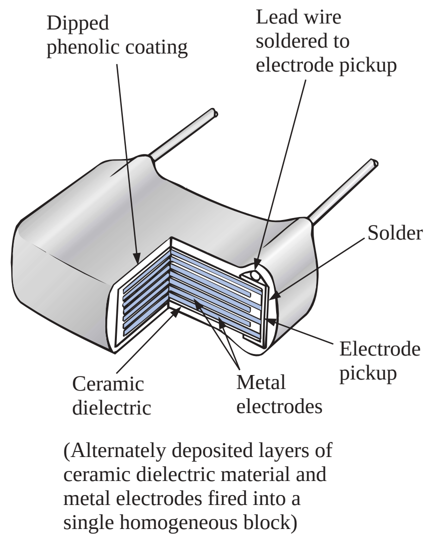



**FIG. 10.13**
*Multilayer, radial-lead ceramic capacitor.*

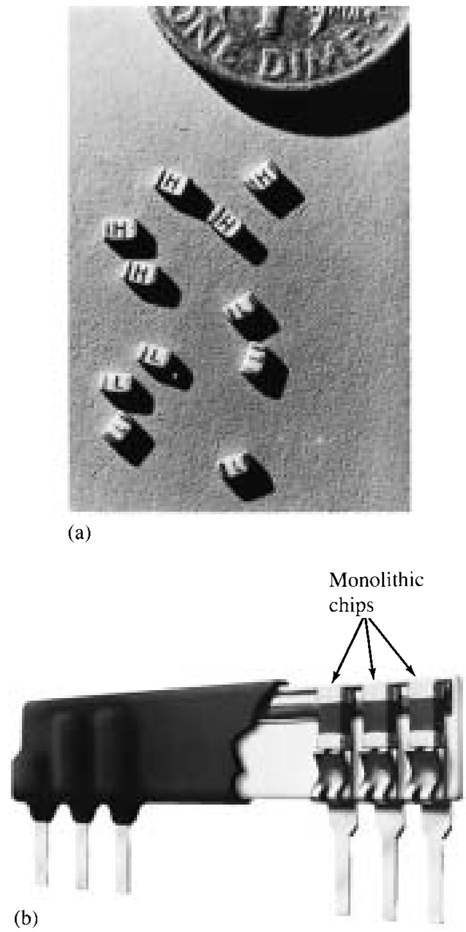
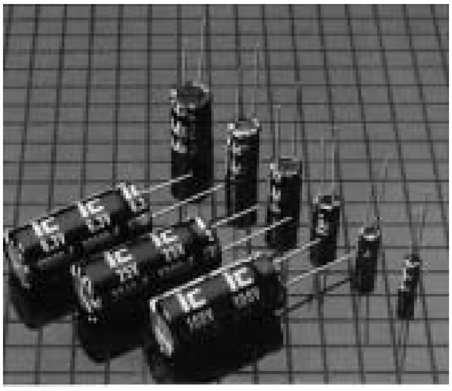



**FIG. 10.14**
*Monolithic chip capacitors. (Courtesy of Vitramon, Inc.)*

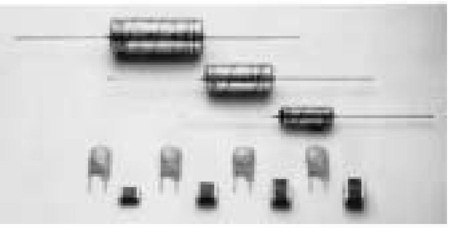
are required. They are designed primarily for use in networks where only dc voltages will be applied across the capacitor because they have good insulating characteristics (high leakage current) between the plates in one direction but take on the characteristics of a conductor in the other direction. Electrolytic capacitors are available that can be used in ac circuits (for starting motors) and in cases where the polarity of the dc voltage will reverse across the capacitor for short periods of time.

The basic construction of the electrolytic capacitor consists of a roll of aluminum foil coated on one side with an aluminum oxide, the aluminum being the positive plate and the oxide the dielectric. A layer of paper or gauze saturated with an electrolyte is placed over the aluminum oxide on the positive plate. Another layer of aluminum without the oxide coating is then placed over this layer to assume the role of the negative plate. In most cases the negative plate is connected directly to the aluminum container, which then serves as the negative terminal for external connections. Because of the size of the roll of aluminum foil, the overall area of this capacitor is large; and due to the use of an oxide as the dielectric, the distance between the plates is extremely small. The negative terminal of the electrolytic capacitor is usually the one with no visible identification on the casing. The positive is usually indicated by such designs as +, △, □, and so on. Due to the polarity requirement, the symbol for an electrolytic capacitor will normally appear as $\underset{\top}{\perp}^{+}$.

Associated with each electrolytic capacitor are the dc working voltage and the surge voltage. The **working voltage** is the voltage that can be applied across the capacitor for long periods of time without breakdown. The **surge voltage** is the maximum dc voltage that can be applied for a short period of time. Electrolytic capacitors are characterized as having low breakdown voltages and high leakage currents ($R_{leakage}$ about 1 MΩ). Various types of electrolytic capacitors are shown in Fig. 10.15. They can be found in values extending from a few microfarads to several thousand microfarads and working voltages as high as 500 V. However, increased levels of voltage are normally associated with lower values of available capacitance.




(a)



(b)

**FIG. 10.15**
*Electrolytic capacitors: (a) Radial lead with extended endurance rating of 2000 h at 85°C. Capacitance range: 0.1–15,000 μF with a voltage range of 6.3 to 250 WV dc (Courtesy of Illinois Capacitor, Inc.). (b) Solid aluminum electrolytic capacitors available in axial, resin-dipped, and surface-mount configurations to withstand harsh environmental conditions (Courtesy of Philips Components, Inc.).*

There are fundamentally two types of *tantalum capacitors:* the *solid* and the *wet-slug.* In each case, tantalum powder of high purity is pressed into a rectangular or cylindrical shape, as shown in Fig. 10.16. Next the anode (+) connection is simply pressed into the resulting structures, as shown in the figure. The resulting unit is then sintered (baked) in a vacuum at very high temperatures to establish a very porous material. The result is a structure with a very large surface area in a limited volume. Through immersion in an acid solution, a very thin manganese dioxide ($MnO_2$) coating is established on the large, porous surface area. An electrolyte is then added to establish contact between the surface area and the cathode, producing a solid tantalum capacitor. If an appropriate "wet" acid is introduced, it is called a *wet-slug* tantalum capacitor.

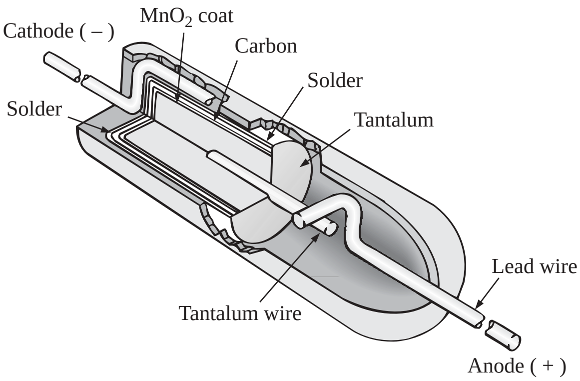



**FIG. 10.16**
*Tantalum capacitor. (Courtesy of Union Carbide Corp.)*

The last type of fixed capacitor to be introduced is the *polyester-film capacitor,* the basic construction of which is shown in Fig. 10.17. It consists simply of two metal foils separated by a strip of polyester material such as Mylar®. The outside layer of polyester is applied to act as an insulating jacket. Each metal foil is connected to a lead that extends either axially or radially from the capacitor. The rolled construction results in a large surface area, and the use of the plastic dielectric results in a very thin layer between the conducting surfaces.

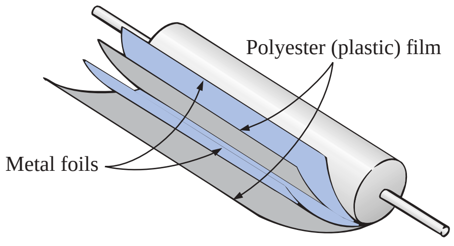
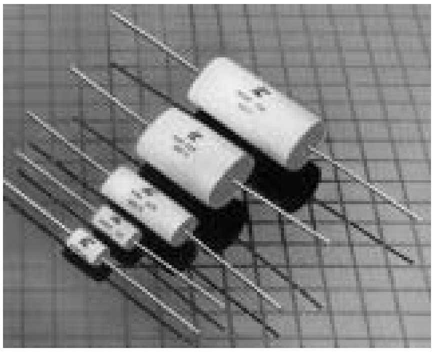



**FIG. 10.17**
*Polyester-film capacitor.*

Data such as capacitance and working voltage are printed on the outer wrapping if the polyester capacitor is large enough. Color coding is used on smaller devices (see Appendix D). A band (usually black) is sometimes printed near the lead that is connected to the outer metal foil. The lead nearest this band should always be connected to the point of lower potential. This capacitor can be used for both dc and ac networks. Its leakage resistance is of the order of 100 MΩ. An axial lead and radial lead polyester-film capacitor appear in Fig. 10.18. The axial lead variety is available with capacitance levels of 0.1 $\mu$F to 18 $\mu$F, with working voltages extending to 630 V. The radial lead variety has a capacitance range of 0.01 $\mu$F to 10 $\mu$F, with working voltages extending to 1000 V.


(a)

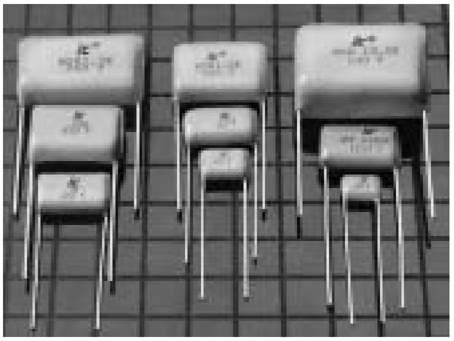
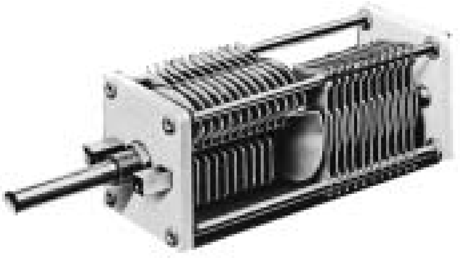

(b)

**FIG. 10.18**
*Polyester-film capacitors: (a) axial lead; (b) radial lead. (Courtesy of Illinois Capacitor, Inc.)*

## Variable Capacitors

The most common of the variable-type capacitors is shown in Fig. 10.19. The dielectric for each capacitor is air. The capacitance in Fig. 10.19(a) is changed by turning the shaft at one end to vary the common area of the movable and fixed plates. The greater the common area, the larger the capacitance, as determined by Eq. (10.10). The capacitance of the trimmer capacitor in Fig. 10.19(b) is changed by turning the screw, which will vary the distance between the plates (the common area is fixed) and thereby the capacitance.


(a)


(b)

**FIG. 10.19**
*Variable air capacitors.* [*Part (a) courtesy of James Millen Manufacturing Co.; part (b) courtesy of Johnson Manufacturing Co.*]


**FIG. 10.20**
*Digital reading capacitance meter. (Courtesy of BK PRECISION, Maxtec International Corp.)*

## Measurement and Testing

A digital reading capacitance meter appears in Fig. 10.20. Simply place the capacitor between the provided clips with the proper polarity, and the meter will display the level of capacitance.

The best check of a capacitor is to use a meter designed to perform the necessary tests. However, an ohmmeter can identify those in which the dielectric has deteriorated (especially in paper and electrolytic capacitors). As the dielectric breaks down, the insulating qualities decrease to a point where the resistance between the plates drops to a relatively low level. After ensuring that the capacitor is fully discharged, place an ohmmeter across the capacitor, as shown in Fig. 10.21. In a polarized capacitor, the polarities of the meter should match those of the capacitor. A low-resistance reading (zero ohms to a few hundred ohms) normally indicates a defective capacitor.

The above test of leakage is not all-inclusive, since some capacitors will break down only when higher voltages are applied. The test, however, does identify those capacitors that have lost the insulating quality of the dielectric between the plates.

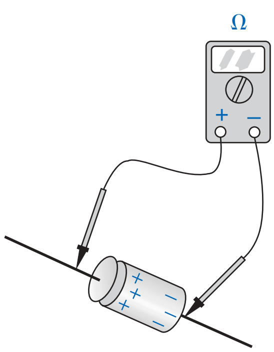



**FIG. 10.21**
*Checking the dielectric of an electrolytic capacitor.*

## Standard Values and Recognition Factor

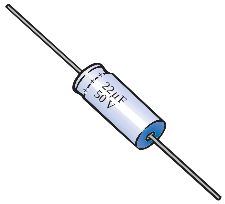
The standard values for capacitors employ the same numerical multipliers encountered for resistors. The most common have the same numerical multipliers as the most common resistors, that is, those available with the full range of tolerances (5%, 10%, and 20%) as shown in Table 3.8. They include **0.1** $\mu$F, **0.15** $\mu$F, **0.22** $\mu$F, **0.33** $\mu$F, **0.47** $\mu$F, and **0.68** $\mu$F, and then **1** $\mu$F, **1.5** $\mu$F, **2.2** $\mu$F, **3.3** $\mu$F, **4.7** $\mu$F, and so on.

Figure 10.22 was developed to establish a recognition factor when it comes to the various types of capacitors. In other words, it will help you to develop the skills to identify types of capacitors, their typical range of values, and some of the most common applications. The figure is certainly not all-inclusive, but it does offer a first step in establishing a sense for what to expect for various applications.

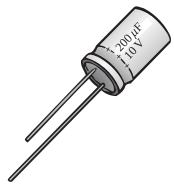
## Marking Schemes

Due to the small size of some capacitors, various marking schemes have been adopted to provide the capacitance level, the tolerance, and, if possible, the maximum working voltage. In general, however, the size of the capacitor is the first indicator of its value. The smaller units are typically in picofarads (pF) and the larger units in microfarads ($\mu$F). Keeping this simple fact in mind will usually provide an immediate indication of the expected capacitance level. On larger $\mu$F units, the value can usually be printed on the jacket with the tolerance and maximum working voltage. However, smaller units need to use some form of abbreviation as shown in Fig. 10.23. For very small units such as appearing in Fig. 10.23(a), the value is recognized immediately as in pF, with the K an indicator of a $\pm$10% tolerance level. Too often the K is read as a multiplier of $10^{+3}$, and the capacitance read as 20,000 pF or 20 nF. For the unit of Fig. 10.23(b), there was room for a lowercase “n” to represent a multiplier of $10^{-9}$. The presence of the lowercase “n” in combination with the small size is clear indication that this is a 200-nF capacitor. To avoid unnecessary confusion, the letters used for tolerance do not include N or U or P, so any form of these letters will usually sug-

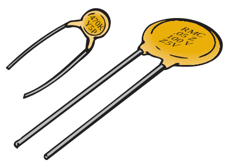
***Type:*** Miniature Axial Electrolytic
***Typical Values:*** 0.1 $\mu$F to 15,000 $\mu$F
***Typical Voltage Range:*** 5 V to 450 V
***Capacitor tolerance:*** ±20%
***Applications:*** Polarized, used in DC power supplies, bypass filters, DC blocking.



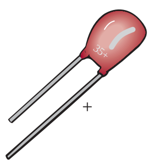

***Type:*** Miniature Radial Electrolyte
***Typical Values:*** 0.1 $\mu$F to 15,000 $\mu$F
***Typical Voltage Range:*** 5 V to 450 V
***Capacitor tolerance:*** ±20%
***Applications:*** Polarized, used in DC power supplies, bypass filters, DC blocking.



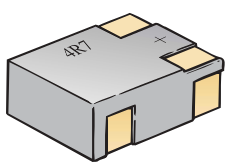

***Type:*** Ceramic Disc
***Typical Values:*** 10 pF to 0.047 $\mu$F
***Typical Voltage Range:*** 100 V to 6 kV
***Capacitor tolerance:*** ±5%, ±10%
***Applications:*** Non-polarized, NPO type, stable for a wide range of temperatures. Used in oscillators, noise filters, circuit coupling, tank circuits.



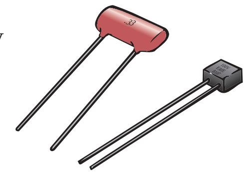
***Type:*** Dipped Tantalum (solid and wet)
***Typical Values:*** 0.047 $\mu$F to 470 $\mu$F
***Typical Voltage Range:*** 6.3 V to 50 V
***Capacitor tolerance:*** ±10%, ±20%
***Applications:*** Polarized, low leakage current, used in power supplies, high frequency noise filters, bypass filter.



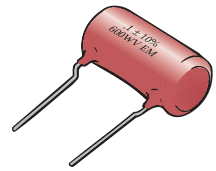

***Type:*** Surface Mount Type (SMT)
***Typical Values:*** 10 pF to 10 $\mu$F
***Typical Voltage Range:*** 6.3 V to 16 V
***Capacitor tolerance:*** ±10%
***Applications:*** Polarized and non-polarized, used in all types of circuits, requires a minimum amount of PC board real estate.




***Type:*** Silver Mica
***Typical Value:*** 10 pF to 0.001 $\mu$F
***Typical Voltage Range:*** 50 V to 500 V
***Capacitor tolerance:*** ±5%
***Applications:*** Non-polarized, used in oscillators, in circuits that require a stable component over a range of temperatures and voltages.

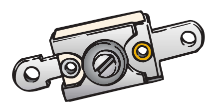


***Type:*** Mylar Paper
***Typical Value:*** 0.001 $\mu$F to 0.68 $\mu$F
***Typical Voltage Range:*** 50 V to 600 V
***Capacitor tolerance:*** ±22%
***Applications:*** Non-polarized, used in all types of circuits, moisture resistant.



***Type:*** AC/DC Motor Run
***Typical Value:*** 0.25 $\mu$F to 1200 $\mu$F
***Typical Voltage Range:*** 240 V to 660 V
***Capacitor tolerance:*** ±10%
***Applications:*** Non-polarized, used in motor run-start, high-intensity lighting supplies, AC noise filtering.


***Type:*** Trimmer Variable
***Typical Value:*** 1.5 pF to 600 pF
***Typical Voltage Range:*** 5 V to 100 V
***Capacitor tolerance:*** ±10%
***Applications:*** Non-polarized, used in oscillators, tuning circuits, AC filters.



***Type:*** Tuning variable
***Typical Value:*** 10 pF to 600 pF
***Typical Voltage Range:*** 5 V to 100 V
***Capacitor tolerance:*** ±10%
***Applications:*** Non-polarized, used in oscillators, radio tuning circuit.

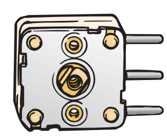


**FIG. 10.22**
*Summary of capacitive elements.*

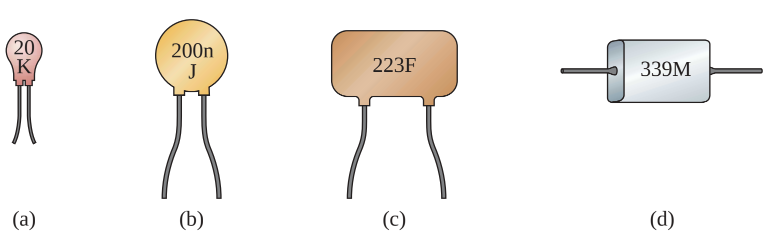



**FIG. 10.23**
*Various marking schemes for small capacitors.*

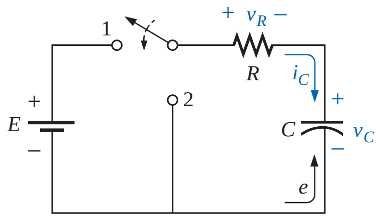



**FIG. 10.24**
*Basic charging network.*

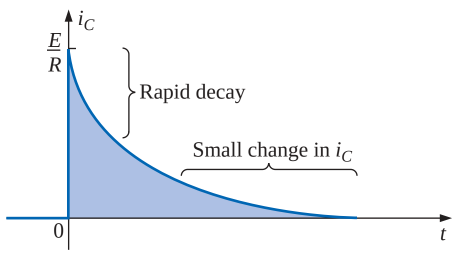



**FIG. 10.25**
*$i_C$ during the charging phase.*

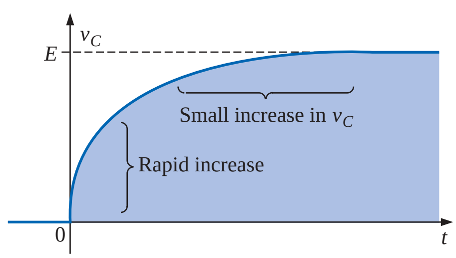



**FIG. 10.26**
*$v_C$ during the charging phase.*

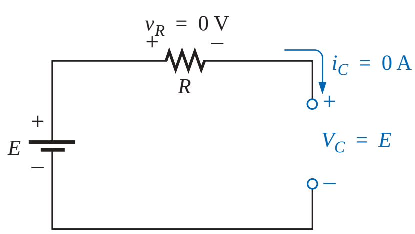



**FIG. 10.27**
*Open-circuit equivalent for a capacitor following the charging phase.*

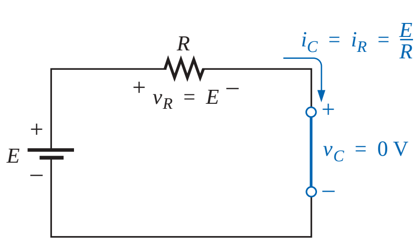



**FIG. 10.28**
*Short-circuit equivalent for a capacitor (switch closed, $t = 0$).*

gest the multiplier level. The J represents a ±5% tolerance level. For capacitors such as appearing in Fig. 10.23(c), the first two numbers are actual digits of the value, while the third number is the power of a multiplier (or number of zeros to be added). The F represents a ±1% tolerance level. Multipliers of 0.01 use an 8, while 9 is used for 0.1 as shown for the capacitor of Fig. 10.23(d) where the M represents a ±20% tolerance level.

## 10.7 TRANSIENTS IN CAPACITIVE NETWORKS: CHARGING PHASE

Section 10.3 described how a capacitor acquires its charge. Let us now extend this discussion to include the potentials and current developed within the network of Fig. 10.24 following the closing of the switch (to position 1).

You will recall that the instant the switch is closed, electrons are drawn from the top plate and deposited on the bottom plate by the battery, resulting in a net positive charge on the top plate and a negative charge on the bottom plate. The transfer of electrons is very rapid at first, slowing down as the potential across the capacitor approaches the applied voltage of the battery. When the voltage across the capacitor equals the battery voltage, the transfer of electrons will cease and the plates will have a net charge determined by $Q = CV_C = CE$.

Plots of the changing current and voltage appear in Figs. 10.25 and 10.26, respectively. When the switch is closed at $t = 0$ s, the current jumps to a value limited only by the resistance of the network and then decays to zero as the plates are charged. Note the rapid decay in current level, revealing that the amount of charge deposited on the plates per unit time is rapidly decaying also. Since the voltage across the plates is directly related to the charge on the plates by $v_C = q/C$, the rapid rate with which charge is initially deposited on the plates will result in a rapid increase in $v_C$. Obviously, as the rate of flow of charge ($I$) decreases, the rate of change in voltage will follow suit. Eventually, the flow of charge will stop, the current $I$ will be zero, and the voltage will cease to change in magnitude—the *charging phase* has passed. At this point the capacitor takes on the characteristics of an open circuit: a voltage drop across the plates without a flow of charge "between" the plates. As demonstrated in Fig. 10.27, the voltage across the capacitor is the source voltage since $i = i_C = i_R = 0$ A and $v_R = i_R R = (0)R = 0$ V. For all future analysis:

***A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.***

Looking back at the instant the switch is closed, we can also surmise that a capacitor behaves as a short circuit the moment the switch is closed in a dc charging network, as shown in Fig. 10.28. The current $i = i_C = i_R = E/R$, and the voltage $v_C = E - v_R = E - i_R R = E - (E/R)R = E - E = 0$ V at $t = 0$ s.

Through the use of calculus, the following mathematical equation for the charging current $i_C$ can be obtained:

$$i_C = \frac{E}{R} e^{-t/RC} \qquad \textbf{(10.13)}$$

The factor $e^{-t/RC}$ is an exponential function of the form $e^{-x}$, where $x = -t/RC$ and $e = 2.71828 \ldots$. A plot of $e^{-x}$ for $x \geq 0$ appears in Fig. 10.29. Exponentials are mathematical functions that all students of electrical, electronic, or computer systems must become very familiar with. They will appear throughout the analysis to follow in this course, and in succeeding courses.

Our current interest in the function $e^{-x}$ is limited to values of $x$ greater than zero, as noted by the curve of Fig. 10.25. All modern-day scientific calculators have the function $e^x$. To obtain $e^{-x}$, the sign of $x$ must be changed using the sign key before the exponential function is keyed in. The magnitude of $e^{-x}$ has been listed in Table 10.3 for a range of values of $x$. Note the rapidly decreasing magnitude of $e^{-x}$ with increasing value of $x$.

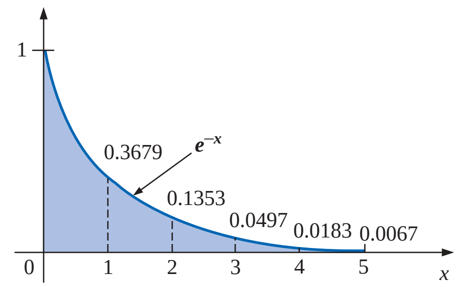



**FIG. 10.29**
*The $e^{-x}$ function ($x \geq 0$).*

**TABLE 10.3**
*Selected values of $e^{-x}$.*

| | |
|---|---|
| $x = 0$ | $e^{-x} = e^{-0} = \frac{1}{e^0} = \frac{1}{1} = 1$ |
| $x = 1$ | $e^{-1} = \frac{1}{e} = \frac{1}{2.71828 \ldots} = 0.3679$ |
| $x = 2$ | $e^{-2} = \frac{1}{e^2} = 0.1353$ |
| $x = 5$ | $e^{-5} = \frac{1}{e^5} = 0.00674$ |
| $x = 10$ | $e^{-10} = \frac{1}{e^{10}} = 0.0000454$ |
| $x = 100$ | $e^{-100} = \frac{1}{e^{100}} = 3.72 \times 10^{-44}$ |

The factor $RC$ in Eq. (10.13) is called the *time constant* of the system and has the units of time as follows:

$$RC = \left(\frac{V}{I}\right)\left(\frac{Q}{V}\right) = \left(\frac{\cancel{V}}{\cancel{Q}/t}\right)\left(\frac{\cancel{Q}}{\cancel{V}}\right) = t$$

Its symbol is the Greek letter $\tau$ (tau), and its unit of measure is the second. Thus,

$$\boxed{\tau = RC} \qquad \text{(seconds, s)} \qquad \textbf{(10.14)}$$

If we substitute $\tau = RC$ into the exponential function $e^{-t/RC}$, we obtain $e^{-t/\tau}$. In one time constant, $e^{-t/\tau} = e^{-\tau/\tau} = e^{-1} = 0.3679$, or the function equals 36.79% of its maximum value of 1. At $t = 2\tau$, $e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2} = 0.1353$, and the function has decayed to only 13.53% of its maximum value.

The magnitude of $e^{-t/\tau}$ and the percentage change between time constants have been tabulated in Tables 10.4 and 10.5, respectively. Note that the current has dropped 63.2% (100% − 36.8%) in the first time constant but only 0.4% between the fifth and sixth time constants. The rate of change of $i_C$ is therefore quite sensitive to the time constant determined by the network parameters $R$ and $C$. For this reason, the universal time constant chart of Fig. 10.30 is provided to permit a more accurate estimate of the value of the function $e^{-x}$ for specific time intervals related to the time constant. The term *universal* is used because the axes are not scaled to specific values.

**TABLE 10.4**
*$i_C$ versus $\tau$ (charging phase).*

| $t$ | Magnitude | |
|---|---|---|
| 0 | 100% | |
| $1\tau$ | 36.8% | |
| $2\tau$ | 13.5% | |
| $3\tau$ | 5.0% | |
| $4\tau$ | 1.8% | ← Less than 1% of maximum |
| **$5\tau$** | **0.67%** | |
| $6\tau$ | 0.24% | |

**TABLE 10.5**
*Change in $i_C$ between time constants.*

| | | |
|---|---|---|
| $(0 \rightarrow 1)\tau$ | 63.2% | |
| $(1 \rightarrow 2)\tau$ | 23.3% | |
| $(2 \rightarrow 3)\tau$ | 8.6% | |
| $(3 \rightarrow 4)\tau$ | 3.0% | |
| **$(4 \rightarrow 5)\tau$** | **1.2%** | |
| $(5 \rightarrow 6)\tau$ | 0.4% | ← Less than 1% |

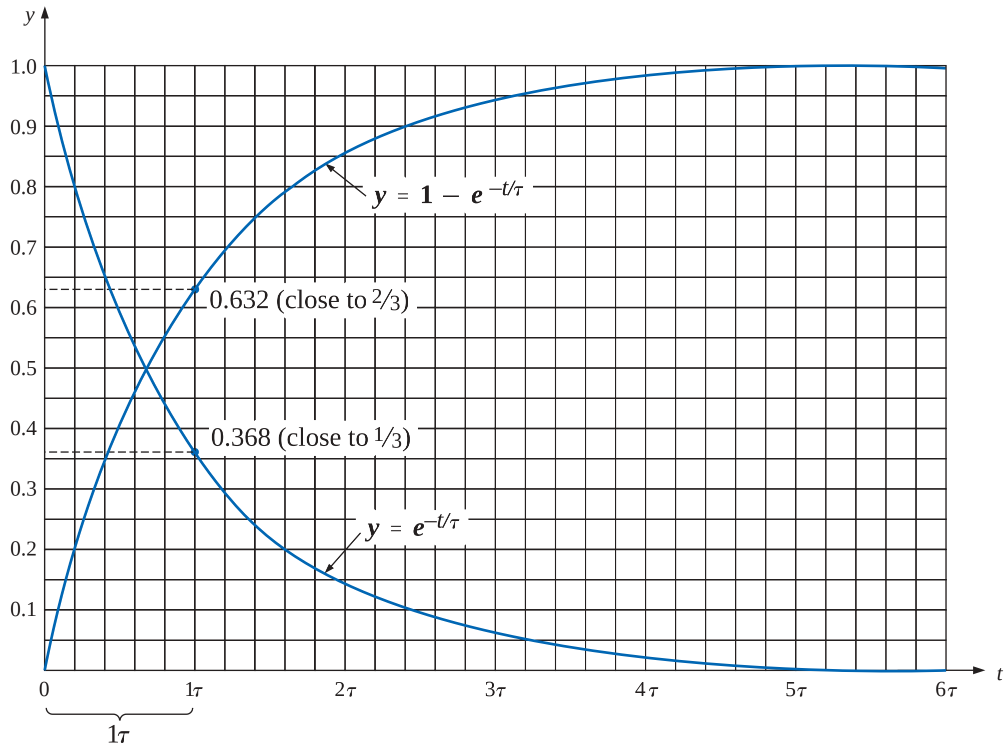



**FIG. 10.30**
*Universal time constant chart.*

Returning to Eq. (10.13), we find that the multiplying factor $E/R$ is the maximum value that the current $i_C$ can attain, as shown in Fig. 10.25. Substituting $t = 0$ s into Eq. (10.13) yields

$$i_C = \frac{E}{R}e^{-t/RC} = \frac{E}{R}e^{-0} = \frac{E}{R}$$

verifying our earlier conclusion.

For increasing values of $t$, the magnitude of $e^{-t/\tau}$, and therefore the value of $i_C$, will decrease, as shown in Fig. 10.31. Since the magnitude of $i_C$ is less than 1% of its maximum after five time constants, we will assume the following for future analysis:

***The current $i_C$ of a capacitive network is essentially zero after five time constants of the charging phase have passed in a dc network.***

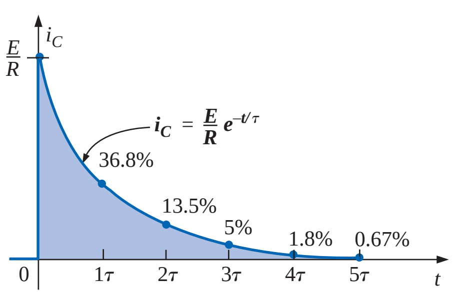



**FIG. 10.31**
*$i_C$ versus t during the charging phase.*

Since $C$ is usually found in microfarads or picofarads, the time constant $\tau = RC$ will never be greater than a few seconds unless $R$ is very large.

Let us now turn our attention to the charging voltage across the capacitor. Through further mathematical analysis, the following equation for the voltage across the capacitor can be determined:

$$v_C = E(1 - e^{-t/RC}) \qquad \textbf{(10.15)}$$

Note the presence of the same factor $e^{-t/RC}$ and the function $(1 - e^{-t/RC})$ appearing in Fig. 10.30. Since $e^{-t/\tau}$ is a decaying function, the factor $(1 - e^{-t/\tau})$ will grow toward a maximum value of 1 with time, as shown in Fig. 10.30. In addition, since $E$ is the multiplying factor, we can conclude that, for all practical purposes, the voltage $v_C$ is $E$ volts after five time constants of the charging phase. A plot of $v_C$ versus $t$ is provided in Fig. 10.32.

If we keep $R$ constant and reduce $C$, the product $RC$ will decrease, and the rise time of five time constants will decrease. The change in transient behavior of the voltage $v_C$ is plotted in Fig. 10.33 for various values of $C$. The product $RC$ will always have some numerical value, even though it may be very small in some cases. For this reason:

***The voltage across a capacitor cannot change instantaneously.***

In fact, the capacitance of a network is also a measure of how much it will oppose a change in voltage across the network. The larger the capacitance, the larger the time constant, and the longer it takes to charge up to its final value (curve of $C_3$ in Fig. 10.33). A lesser capacitance would permit the voltage to build up more quickly since the time constant is less (curve of $C_1$ in Fig. 10.33).

The rate at which charge is deposited on the plates during the charging phase can be found by substituting the following for $v_C$ in Eq. (10.15):

$$v_C = \frac{q}{C}$$

and $$q = Cv_C = CE(1 - e^{-t/\tau}) \quad \text{charging} \qquad \textbf{(10.16)}$$

indicating that the charging rate is very high during the first few time constants and less than 1% after five time constants.

The voltage across the resistor is determined by Ohm's law:

$$v_R = i_R R = Ri_C = R\frac{E}{R}e^{-t/\tau}$$

or $$v_R = Ee^{-t/\tau} \qquad \textbf{(10.17)}$$

A plot of $v_R$ appears in Fig. 10.34.

Applying Kirchhoff's voltage law to the circuit of Fig. 10.24 will result in

$$v_C = E - v_R$$

Substituting Eq. (10.17):

$$v_C = E - Ee^{-t/\tau}$$

Factoring gives $v_C = E(1 - e^{-t/\tau})$, as obtained earlier.

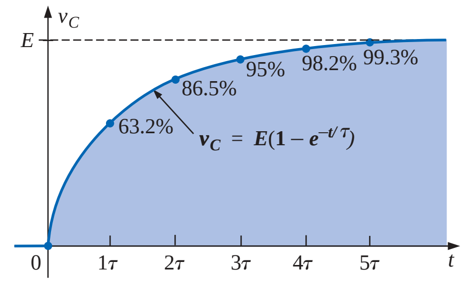



**FIG. 10.32**
*$v_C$ versus t during the charging phase.*

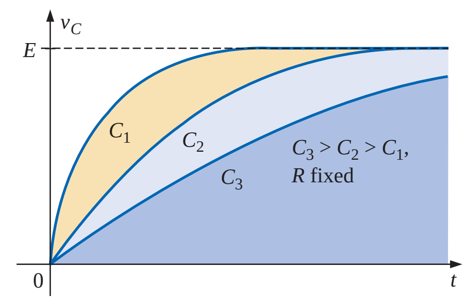



**FIG. 10.33**
*Effect of C on the charging phase.*

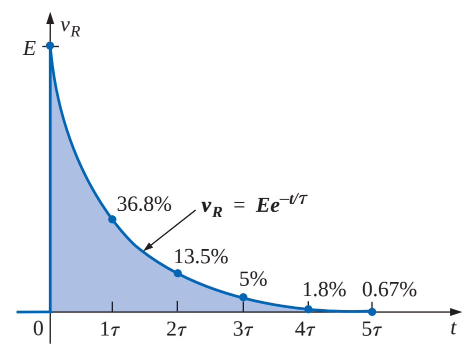



**FIG. 10.34**
*$v_R$ versus t during the charging phase.*

## EXAMPLE 10.5

a. Find the mathematical expressions for the transient behavior of $v_C$, $i_C$, and $v_R$ for the circuit of Fig. 10.35 when the switch is moved to position 1. Plot the curves of $v_C$, $i_C$, and $v_R$.
b. How much time must pass before it can be assumed, for all practical purposes, that $i_C \cong 0$ A and $v_C \cong E$ volts?

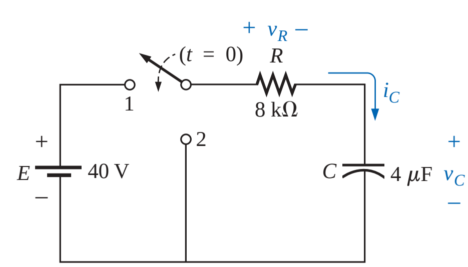



**FIG. 10.35**
*Example 10.5.*

### *Solutions:*

a. $\tau = RC = (8 \times 10^3\ \Omega)(4 \times 10^{-6}\ \text{F}) = 32 \times 10^{-3}\ \text{s} = \mathbf{32\ ms}$

By Eq. (10.15),

$$v_C = E(1 - e^{-t/\tau}) = \mathbf{40(1 - e^{-t/(32\times10^{-3})})}$$

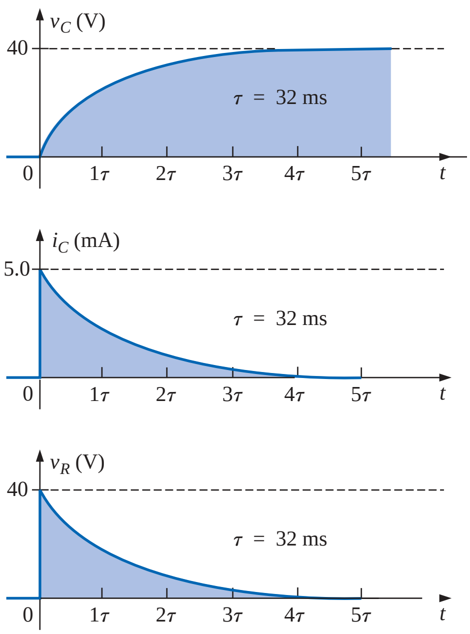



**FIG. 10.36**
*Waveforms for the network of Fig. 10.35.*

By Eq. (10.13),

$$i_C = \frac{E}{R}e^{-t/\tau} = \frac{40\text{ V}}{8\text{ k}\Omega}e^{-t/(32\times10^{-3})}$$

$$= \mathbf{(5 \times 10^{-3})e^{-t/(32\times10^{-3})}}$$

By Eq. (10.17),

$$v_R = Ee^{-t/\tau} = \mathbf{40e^{-t/(32\times10^{-3})}}$$

The curves appear in Fig. 10.36.

b. $5\tau = 5(32\text{ ms}) =$ **160 ms**

Once the voltage across the capacitor has reached the input voltage $E$, the capacitor is fully charged and will remain in this state if no further changes are made in the circuit.

If the switch of Fig. 10.24 is opened, as shown in Fig. 10.37(a), the capacitor will retain its charge for a period of time determined by its leakage current. For capacitors such as the mica and ceramic, the leakage current ($i_{leakage} = v_C/R_{leakage}$) is very small, enabling the capacitor to retain its charge, and hence the potential difference across its plates, for a long time. For electrolytic capacitors, which have very high leakage currents, the capacitor will discharge more rapidly, as shown in Fig. 10.37(b). In any event, to ensure that they are completely discharged, capacitors should be shorted by a lead or a screwdriver before they are handled.

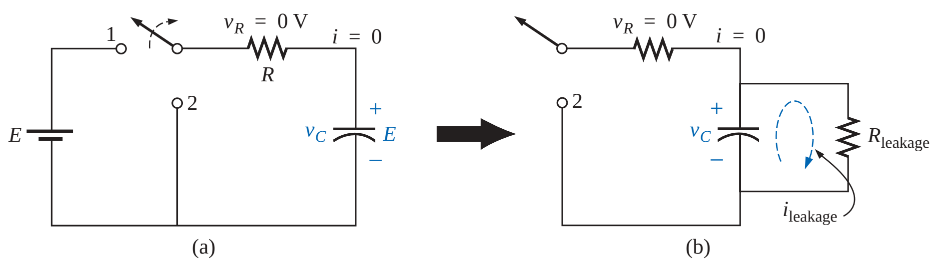



**FIG. 10.37**
*Effect of the leakage current on the steady-state behavior of a capacitor.*

## 10.8 DISCHARGE PHASE

The network of Fig. 10.24 is designed to both charge and discharge the capacitor. When the switch is placed in position 1, the capacitor will charge toward the supply voltage, as described in the last section. At any point in the charging process, if the switch is moved to position 2, the capacitor will begin to discharge at a rate sensitive to the same time constant $\tau = RC$. The established voltage across the capacitor will create a flow of charge in the closed path that will eventually discharge the capacitor completely. In essence, the capacitor functions like a battery with a decreasing terminal voltage. Note in particular that the current $i_C$ has reversed direction, changing the polarity of the voltage across $R$.

If the capacitor had charged to the full battery voltage as indicated in Fig. 10.38, the equation for the decaying voltage across the capacitor would be the following:

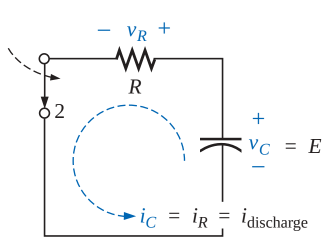



**FIG. 10.38**
*Demonstrating the discharge behavior of a capacitive network.*

$$v_C = Ee^{-t/RC} \qquad \textit{discharging} \qquad \textbf{(10.18)}$$

which employs the function $e^{-x}$ and the same time constant used above. The resulting curve will have the same shape as the curve for $i_C$ and $v_R$ in the last section. During the discharge phase, the current $i_C$ will also decrease with time, as defined by the following equation:

$$i_C = \frac{E}{R}e^{-t/RC} \qquad \textit{discharging} \tag{10.19}$$

The voltage $v_R = v_C$, and

$$v_R = Ee^{-t/RC} \qquad \textit{discharging} \tag{10.20}$$

The complete discharge will occur, for all practical purposes, in five time constants. If the switch is moved between terminals 1 and 2 every five time constants, the wave shapes of Fig. 10.39 will result for $v_C$, $i_C$, and $v_R$. For each curve, the current direction and voltage polarities were defined by Fig. 10.24. Since the polarity of $v_C$ is the same for both the charging and the discharging phases, the entire curve lies above the axis. The current $i_C$ reverses direction during the charging and discharging phases, producing a negative pulse for both the current and the voltage $v_R$. Note that the voltage $v_C$ never changes magnitude instantaneously but that the current $i_C$ has the ability to change instantaneously, as demonstrated by its vertical rises and drops to maximum values.

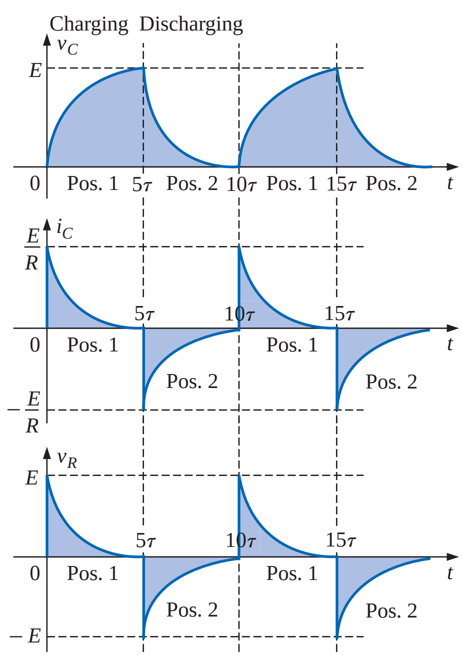



**FIG. 10.39**
*The charging and discharging cycles for the network of Fig. 10.24.*

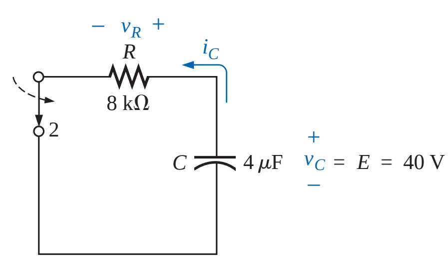



**FIG. 10.40**
*Example 10.6.*

**EXAMPLE 10.6** After $v_C$ in Example 10.5 has reached its final value of 40 V, the switch is thrown into position 2, as shown in Fig. 10.40. Find the mathematical expressions for the transient behavior of $v_C$, $i_C$,

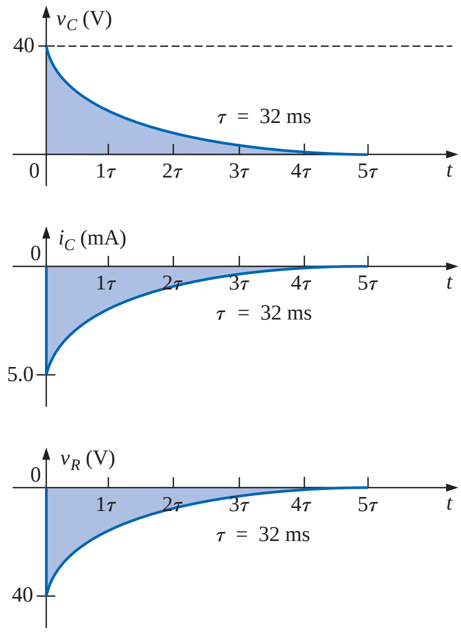



**FIG. 10.41**
*The waveforms for the network of Fig. 10.40.*

and $v_R$ after the closing of the switch. Plot the curves for $v_C$, $i_C$, and $v_R$ using the defined directions and polarities of Fig. 10.35. Assume that $t = 0$ when the switch is moved to position 2.

***Solution:***

$$\tau = 32 \text{ ms}$$

By Eq. (10.18),

$$v_C = Ee^{-t/\tau} = \mathbf{40e^{-t/(32\times10^{-3})}}$$

By Eq. (10.19),

$$i_C = -\frac{E}{R}e^{-t/\tau} = \mathbf{-(5 \times 10^{-3})e^{-t/(32\times10^{-3})}}$$

By Eq. (10.20),

$$v_R = -Ee^{-t/\tau} = \mathbf{-40e^{-t/(32\times10^{-3})}}$$

The curves appear in Fig. 10.41.

The preceding discussion and examples apply to situations in which the capacitor charges to the battery voltage. If the charging phase is disrupted before reaching the supply voltage, the capacitive voltage will be less, and the equation for the discharging voltage $v_C$ will take on the form

$$v_C = V_i e^{-t/RC} \qquad \textbf{(10.21)}$$

where $V_i$ is the starting or initial voltage for the discharge phase. The equation for the decaying current is also modified by simply substituting $V_i$ for $E$; that is,

$$i_C = \frac{V_i}{R}e^{-t/\tau} = I_i e^{-t/\tau} \qquad \textbf{(10.22)}$$

Use of the above equations will be demonstrated in Examples 10.7 and 10.8.

## EXAMPLE 10.7

a. Find the mathematical expression for the transient behavior of the voltage across the capacitor of Fig. 10.42 if the switch is thrown into position 1 at $t = 0$ s.
b. Repeat part (a) for $i_C$.
c. Find the mathematical expressions for the response of $v_C$ and $i_C$ if the switch is thrown into position 2 at 30 ms (assuming that the leakage resistance of the capacitor is infinite ohms).
d. Find the mathematical expressions for the voltage $v_C$ and current $i_C$ if the switch is thrown into position 3 at $t = 48$ ms.
e. Plot the waveforms obtained in parts (a) through (d) on the same time axis for the voltage $v_C$ and the current $i_C$ using the defined polarity and current direction of Fig. 10.42.

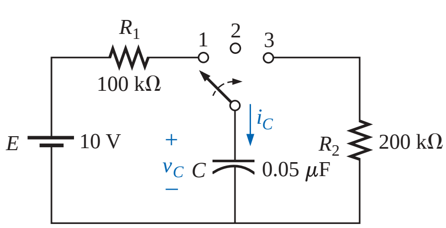



**FIG. 10.42**
*Example 10.7.*

**Solutions:**

a. *Charging phase:*

$$v_C = E(1 - e^{-t/\tau})$$
$$\tau = R_1C = (100 \times 10^3\ \Omega)(0.05 \times 10^{-6}\ \text{F}) = 5 \times 10^{-3}\ \text{s} = 5\ \text{ms}$$
$$v_C = \mathbf{10(1 - e^{-t/(5\times10^{-3})})}$$

b. $$i_C = \frac{E}{R_1}e^{-t/\tau}$$
$$= \frac{10\ \text{V}}{100 \times 10^3\ \Omega}e^{-t/(5\times10^{-3})}$$
$$i_C = \mathbf{(0.1 \times 10^{-3})e^{-t/(5\times10^{-3})}}$$

c. *Storage phase:*

$$v_C = E = \mathbf{10\ V}$$
$$i_C = \mathbf{0\ A}$$

d. *Discharge phase* (starting at 48 ms with $t = 0$ s for the following equations):

$$v_C = Ee^{-t/\tau'}$$
$$\tau' = R_2C = (200 \times 10^3\ \Omega)(0.05 \times 10^{-6}\ \text{F}) = 10 \times 10^{-3}\ \text{s} = 10\ \text{ms}$$
$$v_C = \mathbf{10e^{-t/(10\times10^{-3})}}$$
$$i_C = -\frac{E}{R_2}e^{-t/\tau'}$$
$$= -\frac{10\ \text{V}}{200 \times 10^3\ \Omega}e^{-t/(10\times10^{-3})}$$
$$i_C = \mathbf{-(0.05 \times 10^{-3})e^{-t/(10\times10^{-3})}}$$

e. See Fig. 10.43.

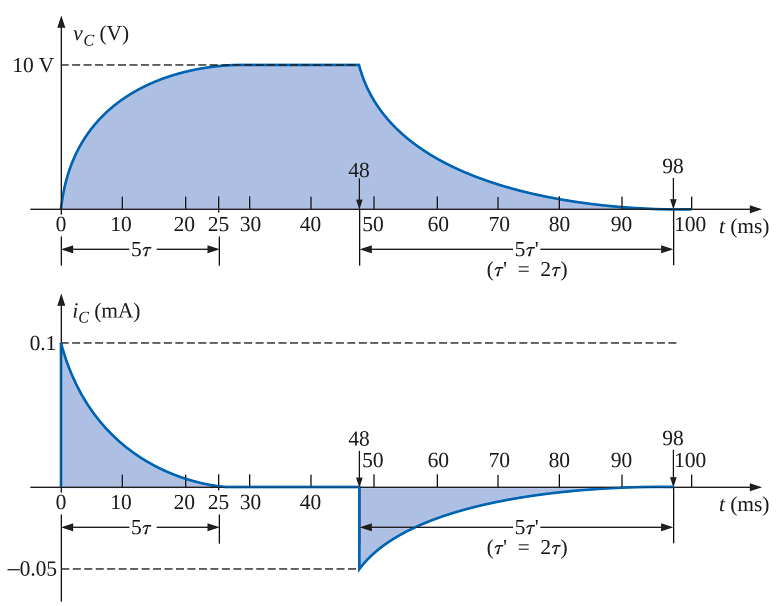



**FIG. 10.43**
*The waveforms for the network of Fig. 10.42.*

## EXAMPLE 10.8

a. Find the mathematical expression for the transient behavior of the voltage across the capacitor of Fig. 10.44 if the switch is thrown into position 1 at $t = 0$ s.

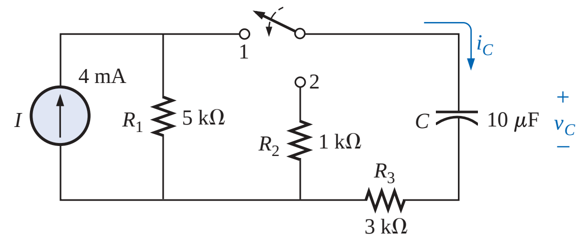



**FIG. 10.44**
*Example 10.8.*

b. Repeat part (a) for $i_C$.
c. Find the mathematical expression for the response of $v_C$ and $i_C$ if the switch is thrown into position 2 at $t = 1\tau$ of the charging phase.
d. Plot the waveforms obtained in parts (a) through (c) on the same time axis for the voltage $v_C$ and the current $i_C$ using the defined polarity and current direction of Fig. 10.44.

### *Solutions:*

a. *Charging phase:* Converting the current source to a voltage source will result in the network of Fig. 10.45.

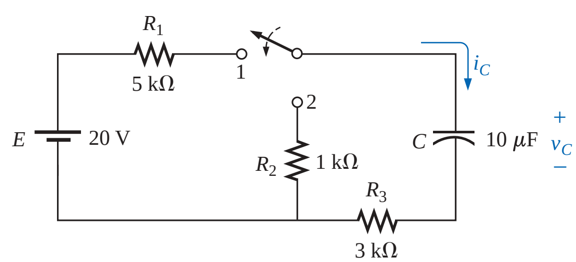



**FIG. 10.45**
*The charging phase for the network of Fig. 10.44.*

$$v_C = E(1 - e^{-t/\tau_1})$$
$$\tau_1 = (R_1 + R_3)C = (5\text{ k}\Omega + 3\text{ k}\Omega)(10 \times 10^{-6}\text{ F})$$
$$= 80\text{ ms}$$
$$v_C = \mathbf{20(1 - e^{-t/(80\times10^{-3})})}$$

b. $$i_C = \frac{E}{R_1 + R_3}e^{-t/\tau_1}$$
$$= \frac{20\text{ V}}{8\text{ k}\Omega}e^{-t/(80\times10^{-3})}$$
$$i_C = \mathbf{(2.5 \times 10^{-3})e^{-t/(80\times10^{-3})}}$$

c. At $t = 1\tau_1$, $v_C = 0.632E = 0.632(20\text{ V}) = 12.64\text{ V}$, resulting in the network of Fig. 10.46. Then $v_C = V_i e^{-t/\tau_2}$ with

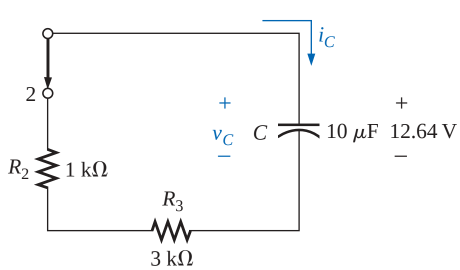



**FIG. 10.46**
*Network of Fig. 10.45 when the switch is moved to position 2 at $t = 1\tau_1$.*

$$\tau_2 = (R_2 + R_3)C = (1\ \text{k}\Omega + 3\ \text{k}\Omega)(10 \times 10^{-6}\ \text{F})$$
$$= 40\ \text{ms}$$

and $$v_C = \mathbf{12.64}e^{-t/(40\times10^{-3})}$$

At $t = 1\tau_1$, $i_C$ drops to (0.368)(2.5 mA) = 0.92 mA. Then it switches to

$$i_C = -I_i e^{-t/\tau_2}$$
$$= -\frac{V_i}{R_2 + R_3}e^{-t/\tau_2} = -\frac{12.64\ \text{V}}{1\ \text{k}\Omega + 3\ \text{k}\Omega}e^{-t/(40\times10^{-3})}$$
$$i_C = -\mathbf{3.16 \times 10^{-3}}e^{-t/(40\times10^{-3})}$$

d. See Fig. 10.47.

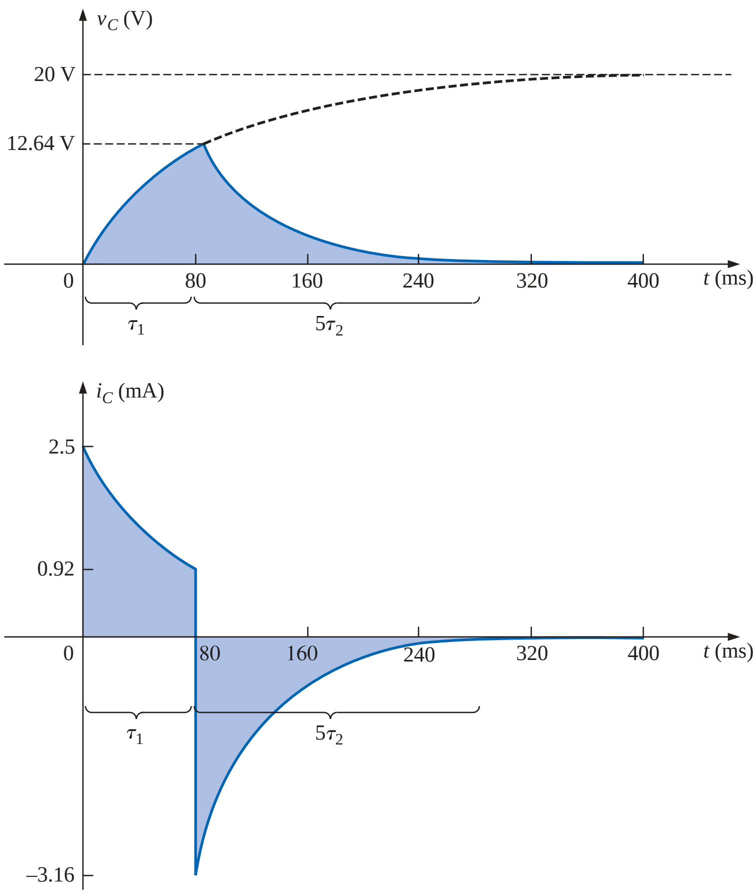



**FIG. 10.47**
*The waveforms for the network of Fig. 10.44.*

## 10.9 INITIAL VALUES

In all the examples examined in the previous sections, the capacitor was uncharged before the switch was thrown. We will now examine the effect of a charge, and therefore a voltage ($V = Q/C$), on the plates at the instant the switching action takes place. The voltage across the capacitor at this instant is called the *initial* value, as shown for the general waveform of Fig. 10.48. Once the switch is thrown, the transient

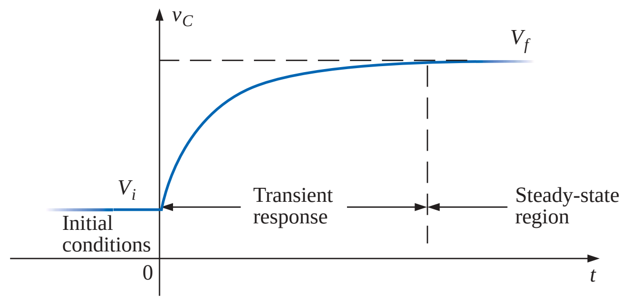



**FIG. 10.48**
*Defining the regions associated with a transient response.*

phase will commence until a leveling off occurs after five time constants. This region of relatively fixed value that follows the transient response is called the *steady-state* region, and the resulting value is called the *steady-state* or *final* value. The steady-state value is found by simply substituting the open-circuit equivalent for the capacitor and finding the voltage across the plates. Using the transient equation developed in the previous section, an equation for the voltage $v_C$ can be written for the entire time interval of Fig. 10.48; that is,

$$v_C = V_i + (V_f - V_i)(1 - e^{-t/\tau})$$

However, by multiplying through and rearranging terms:

$$\begin{aligned} v_C &= V_i + V_f - V_f e^{-t/\tau} - V_i + V_i e^{-t/\tau} \\ &= V_f - V_f e^{-t/\tau} + V_i e^{-t/\tau} \end{aligned}$$

we find

$$v_C = V_f + (V_i - V_f)e^{-t/\tau} \qquad \textbf{(10.23)}$$

If you are required to draw the waveform for the voltage $v_C$ from the initial value to the final value, start by drawing a line at the initial and steady-state levels, and then add the transient response (sensitive to the time constant) between the two levels. The example to follow will clarify the procedure.

**EXAMPLE 10.9** The capacitor of Fig. 10.49 has an initial voltage of 4 V.

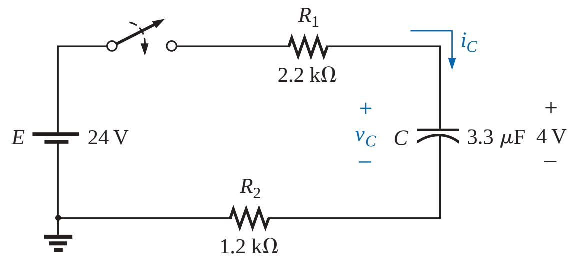



**FIG. 10.49**
*Example 10.9.*

a. Find the mathematical expression for the voltage across the capacitor once the switch is closed.
b. Find the mathematical expression for the current during the transient period.
c. Sketch the waveform for each from initial value to final value.

**Solutions:**

a. Substituting the open-circuit equivalent for the capacitor will result in a final or steady-state voltage $v_C$ of 24 V.

The time constant is determined by

$$\begin{aligned}\tau &= (R_1 + R_2)C \\ &= (2.2\ \text{k}\Omega + 1.2\ \text{k}\Omega)(3.3\ \mu\text{F}) \\ &= 11.22\ \text{ms}\end{aligned}$$

with $5\tau = 56.1$ ms

Applying Eq. (10.23):

$$\begin{aligned}v_C &= V_f + (V_i - V_f)e^{-t/\tau} \\ &= 24\ \text{V} + (4\ \text{V} - 24\ \text{V})e^{-t/11.22\text{ms}}\end{aligned}$$

and $v_C = \mathbf{24\ V - 20\ V}\boldsymbol{e}^{\mathbf{-t/11.22ms}}$

b. Since the voltage across the capacitor is constant at 4 V prior to the closing of the switch, the current (whose level is sensitive only to changes in voltage across the capacitor) must have an initial value of 0 mA. At the instant the switch is closed, the voltage across the capacitor cannot change instantaneously, so the voltage across the resistive elements at this instant is the applied voltage less the initial voltage across the capacitor. The resulting peak current is

$$I_m = \frac{E - V_C}{R_1 + R_2} = \frac{24\ \text{V} - 4\ \text{V}}{2.2\ \text{k}\Omega + 1.2\ \text{k}\Omega} = \frac{20\ \text{V}}{3.4\ \text{k}\Omega} = 5.88\ \text{mA}$$

The current will then decay (with the same time constant as the voltage $v_C$) to zero because the capacitor is approaching its open-circuit equivalence.

The equation for $i_C$ is therefore:

$$i_C = \mathbf{5.88\ mA}\boldsymbol{e}^{\mathbf{-t/11.22ms}}$$

c. See Fig. 10.50.

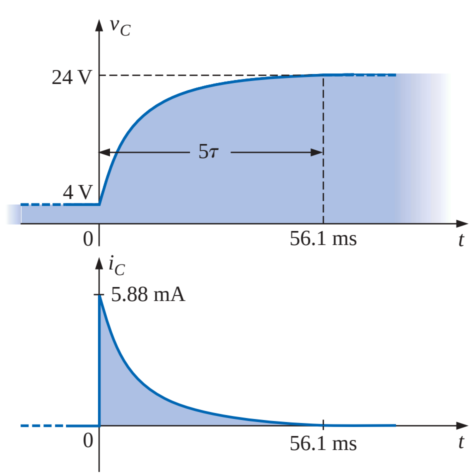



**FIG. 10.50**
*$v_C$ and $i_C$ for the network of Fig. 10.49.*

The initial and final values of the voltage were drawn first, and then the transient response was included between these levels. For the current, the waveform begins and ends at zero, with the peak value having a sign sensitive to the defined direction of $i_C$ in Fig. 10.49.

Let us now test the validity of the equation for $v_C$ by substituting $t = 0$ s to reflect the instant the switch is closed.

$$e^{-t/\tau} = e^{-0} = 1$$

and $$v_C = 24\text{ V} - 20\text{ V}e^{-t/\tau} = 24\text{ V} - 20\text{ V} = 4\text{ V}$$

When $t > 5\tau$,

$$e^{-t/\tau} \cong 0$$

and $$v_C = 24\text{ V} - 20\text{ V}e^{-t/\tau} = 24\text{ V} - 0\text{ V} = 24\text{ V}$$

## 10.10 INSTANTANEOUS VALUES

On occasion it will be necessary to determine the voltage or current at a particular instant of time that is not an integral multiple of $\tau$, as in the previous sections. For example, if

$$v_C = 20(1 - e^{-t/(2\times10^{-3})})$$

the voltage $v_C$ may be required at $t = 5$ ms, which does not correspond to a particular value of $\tau$. Figure 10.30 reveals that $(1 - e^{-t/\tau})$ is approximately 0.93 at $t = 5\text{ ms} = 2.5\tau$, resulting in $v_C = 20(0.93) = 18.6$ V. Additional accuracy can be obtained simply by substituting $t = 5$ ms into the equation and solving for $v_C$ using a calculator or table to determine $e^{-2.5}$. Thus,

$$\begin{aligned} v_C &= 20(1 - e^{-5\text{ms}/2\text{ms}}) \\ &= 20(1 - e^{-2.5}) \\ &= 20(1 - 0.082) \\ &= 20(0.918) \\ &= \mathbf{18.36\text{ V}} \end{aligned}$$

The results are close, but accuracy beyond the tenths place is suspect using Fig. 10.30. The above procedure can also be applied to any other equation introduced in this chapter for currents or other voltages.

There are also occasions when the time to reach a particular voltage or current is required. The procedure is complicated somewhat by the use of natural logs ($\log_e$, or ln), but today's calculators are equipped to handle the operation with ease. There are two forms that require some development. First, consider the following sequence:

$$v_C = E(1 - e^{-t/\tau})$$

$$\frac{v_C}{E} = 1 - e^{-t/\tau}$$

$$1 - \frac{v_C}{E} = e^{-t/\tau}$$

$$\log_e\left(1 - \frac{v_C}{E}\right) = \log_e e^{-t/\tau}$$

$$\log_e\left(1 - \frac{v_C}{E}\right) = -\frac{t}{\tau}$$

and $$t = -\tau \log_e\left(1 - \frac{v_C}{E}\right)$$

but $$-\log_e\frac{x}{y} = +\log_e\frac{y}{x}$$

Therefore, $$t = \tau \log_e\left(\frac{E}{E - v_C}\right) \quad \textbf{(10.24)}$$

The second form is as follows:

$$v_C = Ee^{-t/\tau}$$

$$\frac{v_C}{E} = e^{-t/\tau}$$

$$\log_e\frac{v_C}{E} = \log_e e^{-t/\tau}$$

$$\log_e\frac{v_C}{E} = -\frac{t}{\tau}$$

and $$t = -\tau \log_e\frac{v_C}{E}$$

or $$t = \tau \log_e\frac{E}{v_C} \quad \textbf{(10.25)}$$

For $i_C = (E/R)e^{-t/\tau}$:

$$t = \tau \log_e\frac{E}{i_C R} \quad \textbf{(10.26)}$$

For example, suppose that

$$v_C = 20(1 - e^{-t/(2\times10^{-3})})$$

and the time to reach 10 V is required. Substituting into Eq. (10.24), we have

$$t = (2\text{ ms})\log_e\left(\frac{20\text{ V}}{20\text{ V} - 10\text{ V}}\right)$$
$$= (2\text{ ms})\log_e 2$$
$$= (2\text{ ms})(0.693)$$
$$= \mathbf{1.386\ ms}$$

↑ IN key on calculator

Using Fig. 10.30, we find at $(1 - e^{-t/\tau}) = v_C/E = 0.5$ that $t \cong 0.7\tau = 0.7(2\text{ ms}) = 1.4\text{ ms}$, which is relatively close to the above.

## Mathcad

It is time to see how Mathcad can be applied to the transient analysis described in this chapter. For the first equation described in Section 10.10,

$$v_C = 20(1 - e^{-t/(2\times10^{-3})})$$

the value of $t$ must be defined before the expression is written, or the value can simply be inserted in the equation. The former approach is often better because changing the defined value of $t$ will result in an immediate change in the result. In other words, the value can be used for further calculations. In Fig. 10.51 the value of $t$ was defined as 5 ms. The equation was then entered using the $e$ function from the **Calculator** palette obtained from **View-Toolbars-Calculator.** Be sure to insert a multiplication operator between the initial 20 and the main left bracket. Also, be careful that the control bracket is in the correct place before placing the right bracket to enclose the equation. It will take some practice to ensure that the insertion bracket is in the correct place before entering a parameter, but in time you will find that it is a fairly direct procedure. The **−3** is placed using the shift operator over the number **6** on the standard keyboard. The result is displayed by simply entering $v$ again, followed by an equal sign. The result for $t = 1$ ms can now be obtained by simply changing the defined value for $t$. The result of 7.869 V will appear immediately.

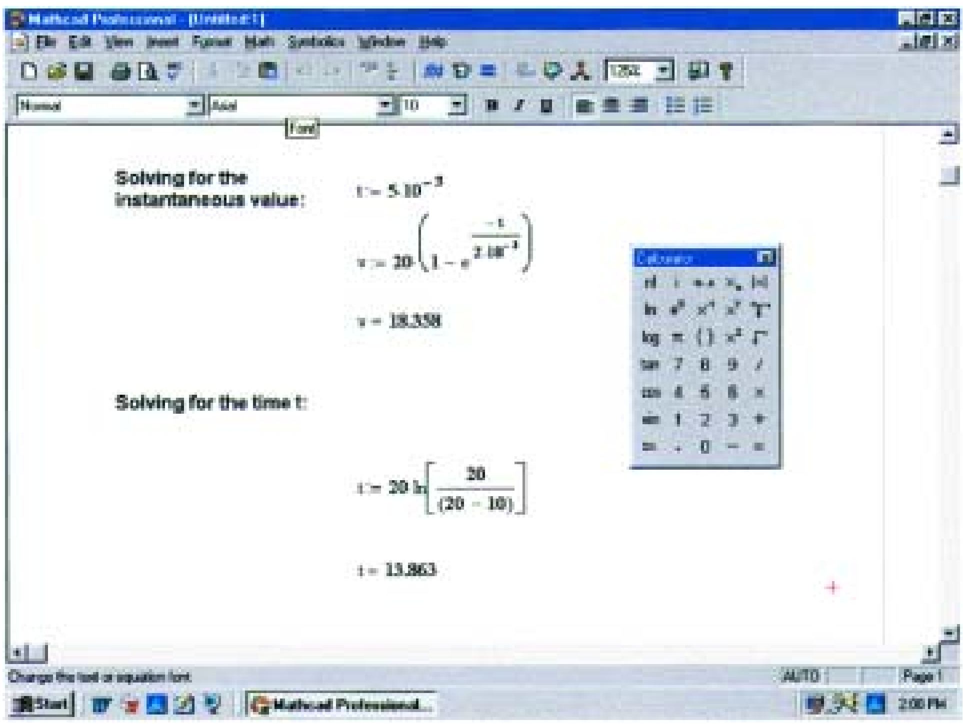



**FIG. 10.51**
*Applying Mathcad to the transient R-C equations.*

For the second equation of Section 10.10,

$$v_C = 20(1 - e^{-5\text{ms}/2\text{ms}})$$

the equation for $t$ can be entered directly as shown in the bottom of Fig. 10.51. The **ln** from the **Calculator** is for a base $e$ calculation, while **log** is for a base 10 calculation. The result will appear the instant the equal sign is placed after the $t$ on the bottom line.

The text you see on the screen to define each operation is obtained by clicking on **Insert-Text Region** and then simply typing in the text material. The boldface was obtained by simply clicking on the text material and swiping the text to establish a black background. Then select **B** from the toolbar, and the boldface will appear.

## 10.11 THÉVENIN EQUIVALENT: $\tau = R_{Th}C$

Occasions will arise in which the network does not have the simple series form of Fig. 10.24. It will then be necessary first to find the Thévenin equivalent circuit for the network external to the capacitive element. $E_{Th}$ will then be the source voltage $E$ of Eqs. (10.15) through (10.20), and $R_{Th}$ will be the resistance $R$. The time constant is then $\tau = R_{Th}C$.

**EXAMPLE 10.10** For the network of Fig. 10.52:

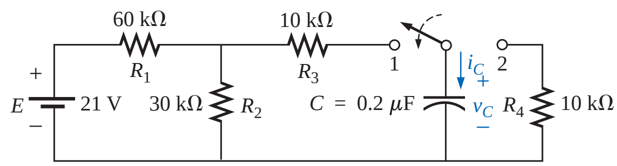



**FIG. 10.52**
*Example 10.10.*

a. Find the mathematical expression for the transient behavior of the voltage $v_C$ and the current $i_C$ following the closing of the switch (position 1 at $t = 0$ s).
b. Find the mathematical expression for the voltage $v_C$ and current $i_C$ as a function of time if the switch is thrown into position 2 at $t = 9$ ms.
c. Draw the resultant waveforms of parts (a) and (b) on the same time axis.

***Solutions:***

a. Applying Thévenin's theorem to the 0.2-μF capacitor, we obtain Fig. 10.53:

$$R_{Th} = R_1 \parallel R_2 + R_3 = \frac{(60\text{ k}\Omega)(30\text{ k}\Omega)}{90\text{ k}\Omega} + 10\text{ k}\Omega$$

$$= 20\text{ k}\Omega + 10\text{ k}\Omega$$

$$R_{Th} = 30\text{ k}\Omega$$

$$E_{Th} = \frac{R_2E}{R_2 + R_1} = \frac{(30\text{ k}\Omega)(21\text{ V})}{30\text{ k}\Omega + 60\text{ k}\Omega} = \frac{1}{3}(21\text{ V}) = 7\text{ V}$$

The resultant Thévenin equivalent circuit with the capacitor replaced is shown in Fig. 10.54. Using Eq. (10.23) with $V_f = E_{Th}$ and $V_i = 0$ V, we find that

$$v_C = V_f + (V_i - V_f)e^{-t/\tau}$$

becomes $$v_C = E_{Th} + (0\text{ V} - E_{Th})e^{-t/\tau}$$

or $$v_C = E_{Th}(1 - e^{-t/\tau})$$

with $$\tau = RC = (30\text{ k}\Omega)(0.2\ \mu\text{F}) = 6\text{ ms}$$

so that $$\mathbf{v_C = 7(1 - e^{-t/6\text{ms}})}$$

For the current: $$i_C = \frac{E_{Th}}{R}e^{-t/RC}$$

$$= \frac{7\text{ V}}{30\text{ k}\Omega}e^{-t/6\text{ms}}$$

$$i_C = \mathbf{(0.233 \times 10^{-3})e^{-t/6\text{ms}}}$$

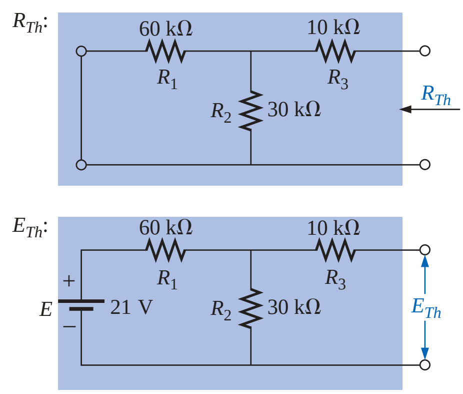



**FIG. 10.53**
*Applying Thévenin's theorem to the network of Fig. 10.52.*

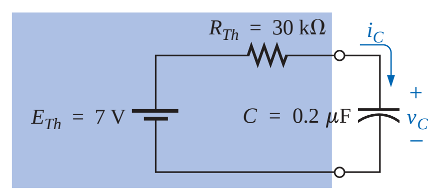



**FIG. 10.54**
*Substituting the Thévenin equivalent for the network of Fig. 10.52.*

b. At $t = 9$ ms,

$$v_C = E_{Th}(1 - e^{-t/\tau}) = 7(1 - e^{-(9\times10^{-3})/(6\times10^{-3})})$$
$$= 7(1 - e^{-1.5}) = 7(1 - 0.223)$$
$$v_C = 7(0.777) = 5.44 \text{ V}$$

and
$$i_C = \frac{E_{Th}}{R}e^{-t/\tau} = (0.233 \times 10^{-3})e^{-1.5}$$
$$= (0.233 \times 10^{-3})(0.223)$$
$$i_C = 0.052 \times 10^{-3} = 0.052 \text{ mA}$$

Using Eq. (10.23) with $V_f = 0$ V and $V_i = 5.44$ V, we find that

$$v_C = V_f + (V_i - V_f)e^{-t/\tau'}$$

becomes
$$v_C = 0 \text{ V} + (5.44 \text{ V} - 0 \text{ V})e^{-t/\tau'}$$
$$= 5.44e^{-t/\tau'}$$

with
$$\tau' = R_4C = (10 \text{ k}\Omega)(0.2\ \mu\text{F}) = 2 \text{ ms}$$

and
$$v_C = \mathbf{5.44e^{-t/2ms}}$$

By Eq. (10.22),

$$I_i = \frac{5.44 \text{ V}}{10 \text{ k}\Omega} = 0.054 \text{ mA}$$

and
$$i_C = I_ie^{-t/\tau} = \mathbf{-(0.54 \times 10^{-3})e^{-t/2ms}}$$

c. See Fig. 10.55.

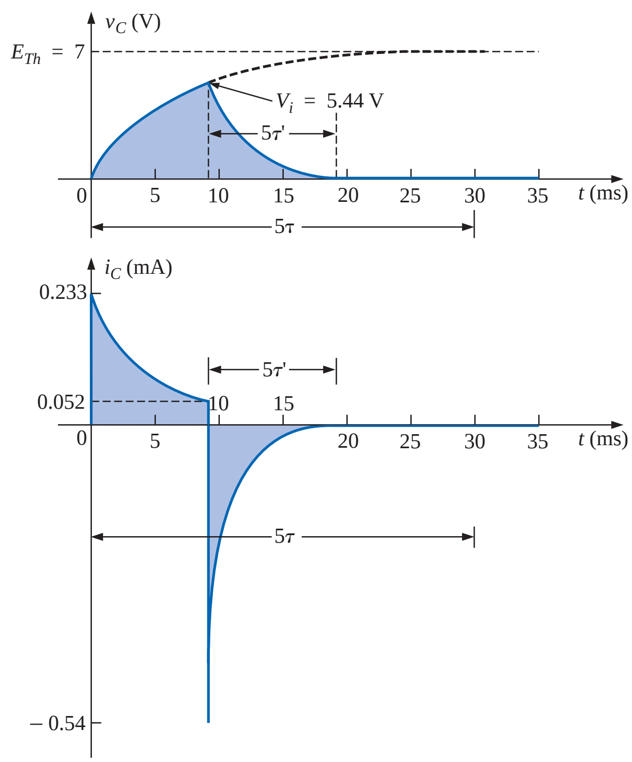



**FIG. 10.55**
*The resulting waveforms for the network of Fig. 10.52.*

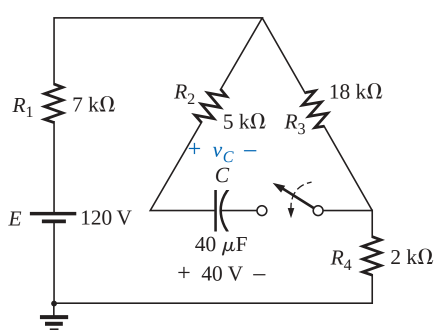



**FIG. 10.56**
*Example 10.11.*

**EXAMPLE 10.11** The capacitor of Fig. 10.56 is initially charged to 40 V. Find the mathematical expression for $v_C$ after the closing of the switch.

***Solution:*** The network is redrawn in Fig. 10.57.

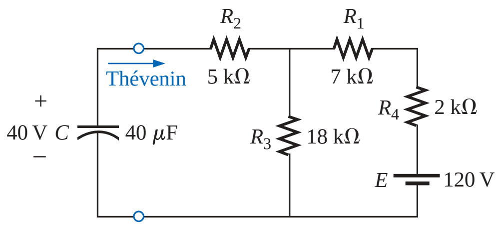



**FIG. 10.57**
*Network of Fig. 10.56 redrawn.*

$E_{Th}$:

$$E_{Th} = \frac{R_3E}{R_3 + R_1 + R_4} = \frac{18\text{ k}\Omega(120\text{ V})}{18\text{ k}\Omega + 7\text{ k}\Omega + 2\text{ k}\Omega}$$
$$= 80\text{ V}$$

$R_{Th}$:

$$\begin{aligned} R_{Th} &= 5\text{ k}\Omega + 18\text{ k}\Omega \parallel (7\text{ k}\Omega + 2\text{ k}\Omega) \\ &= 5\text{ k}\Omega + 6\text{ k}\Omega \\ &= 11\text{ k}\Omega \end{aligned}$$

Therefore, $V_i = 40\text{ V}$ and $V_f = 80\text{ V}$

and $\tau = R_{Th}C = (11\text{ k}\Omega)(40\ \mu\text{F}) = 0.44\text{ s}$

Eq. (10.23): $v_C = V_f + (V_i - V_f)e^{-t/\tau}$

$= 80\text{ V} + (40\text{ V} - 80\text{ V})e^{-t/0.44\text{s}}$

and $v_C = \mathbf{80\ V - 40\ V}\boldsymbol{e^{-t/0.44\text{s}}}$

**EXAMPLE 10.12** For the network of Fig. 10.58, find the mathematical expression for the voltage $v_C$ after the closing of the switch (at $t = 0$).

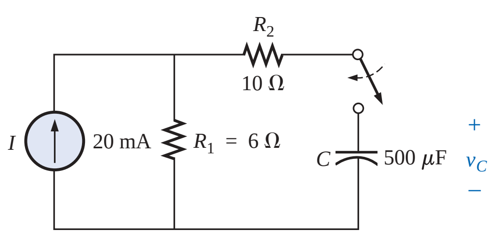



**FIG. 10.58**
*Example 10.12.*

***Solution:***

$$R_{Th} = R_1 + R_2 = 6\ \Omega + 10\ \Omega = 16\ \Omega$$
$$\begin{aligned} E_{Th} &= V_1 + V_2 = IR_1 + 0 \\ &= (20 \times 10^{-3}\text{ A})(6\ \Omega) = 120 \times 10^{-3}\text{ V} = 0.12\text{ V} \end{aligned}$$

and $\tau = R_{Th}C = (16\ \Omega)(500 \times 10^{-6}\text{ F}) = 8\text{ ms}$

so that $v_C = \mathbf{0.12(1 -}\ \boldsymbol{e^{-t/8\text{ms}}}\mathbf{)}$

## 10.12 THE CURRENT $i_C$

The current $i_C$ associated with a capacitance $C$ is related to the voltage across the capacitor by

$$i_C = C\frac{dv_C}{dt} \tag{10.27}$$

where $dv_C/dt$ is a measure of the change in $v_C$ in a vanishingly small period of time. The function $dv_C/dt$ is called the *derivative* of the voltage $v_C$ with respect to time $t$.

If the voltage fails to change at a particular instant, then

$$dv_C = 0$$

and

$$i_C = C\frac{dv_C}{dt} = 0$$

In other words, if the voltage across a capacitor fails to change with time, the current $i_C$ associated with the capacitor is zero. To take this a step further, the equation also states that the more rapid the change in voltage across the capacitor, the greater the resulting current.

In an effort to develop a clearer understanding of Eq. (10.27), let us calculate the average current associated with a capacitor for various voltages impressed across the capacitor. The average current is defined by the equation

$$i_{C\text{av}} = C\frac{\Delta v_C}{\Delta t} \tag{10.28}$$

where $\Delta$ indicates a finite (measurable) change in charge, voltage, or time. The instantaneous current can be derived from Eq. (10.28) by letting $\Delta t$ become vanishingly small; that is,

$$i_{C\text{inst}} = \lim_{\Delta t \to 0} C\frac{\Delta v_C}{\Delta t} = C\frac{dv_C}{dt}$$

In the following example, the change in voltage $\Delta v_C$ will be considered for each slope of the voltage waveform. If the voltage increases with time, the average current is the change in voltage divided by the change in time, with a positive sign. If the voltage decreases with time, the average current is again the change in voltage divided by the change in time, but with a negative sign.

**EXAMPLE 10.13** Find the waveform for the average current if the voltage across a 2-$\mu$F capacitor is as shown in Fig. 10.59.

***Solutions:***

a. From 0 ms to 2 ms, the voltage increases linearly from 0 V to 4 V, the change in voltage $\Delta v = 4\text{ V} - 0 = 4\text{ V}$ (with a positive sign since the voltage increases with time). The change in time $\Delta t = 2\text{ ms} - 0 = 2\text{ ms}$, and

$$i_{C\text{av}} = C\frac{\Delta v_C}{\Delta t} = (2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{2 \times 10^{-3}\text{ s}}\right)$$
$$= 4 \times 10^{-3}\text{ A} = 4\text{ mA}$$

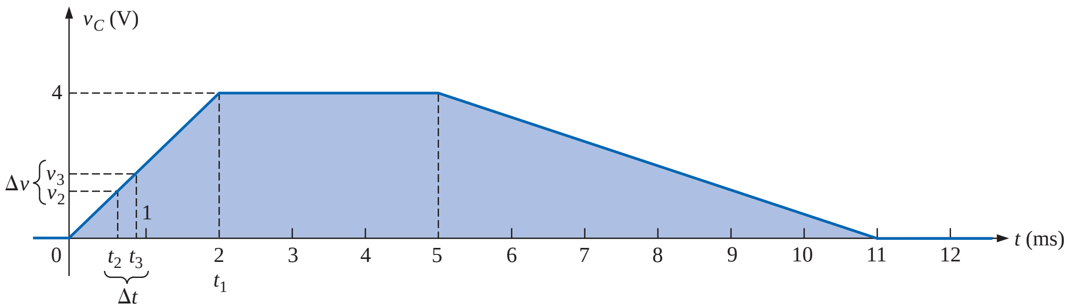



**FIG. 10.59**
*Example 10.13.*

b. From 2 ms to 5 ms, the voltage remains constant at 4 V; the change in voltage $\Delta v = 0$. The change in time $\Delta t = 3$ ms, and

$$i_{C\text{av}} = C\frac{\Delta v_C}{\Delta t} = C\frac{0}{\Delta t} = 0$$

c. From 5 ms to 11 ms, the voltage decreases from 4 V to 0 V. The change in voltage $\Delta v$ is, therefore, 4 V − 0 = 4 V (with a negative sign since the voltage is decreasing with time). The change in time $\Delta t$ = 11 ms − 5 ms = 6 ms, and

$$i_{C\text{av}} = C\frac{\Delta v_C}{\Delta t} = -(2 \times 10^{-6}\text{ F})\left(\frac{4\text{ V}}{6 \times 10^{-3}\text{ s}}\right)$$
$$= -1.33 \times 10^{-3}\text{ A} = -1.33\text{ mA}$$

d. From 11 ms on, the voltage remains constant at 0 and $\Delta v = 0$, so $i_{C\text{av}} = 0$. The waveform for the average current for the impressed voltage is as shown in Fig. 10.60.

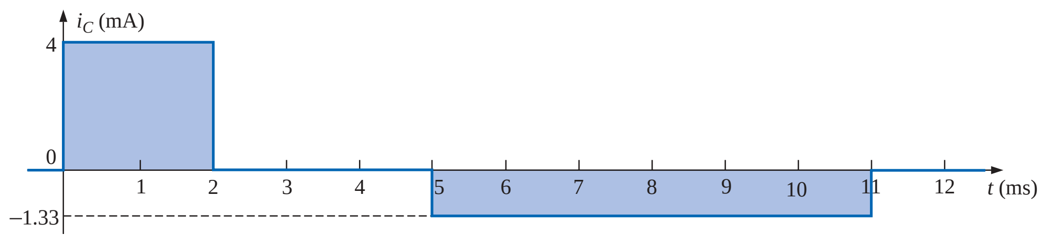



**FIG. 10.60**
*The resulting current $i_C$ for the applied voltage of Fig. 10.59.*

---

Note in Example 10.13 that, in general, the steeper the slope, the greater the current, and when the voltage fails to change, the current is zero. In addition, the average value is the same as the instantaneous value at any point along the slope over which the average value was found. For example, if the interval $\Delta t$ is reduced from $0 \rightarrow t_1$ to $t_2 - t_3$, as noted in Fig. 10.59, $\Delta v/\Delta t$ is still the same. In fact, no matter how small the interval $\Delta t$, the slope will be the same, and therefore the current $i_{C\text{av}}$ will be the same. If we consider the limit as $\Delta t \rightarrow 0$, the slope will still remain the same, and therefore $i_{C\text{av}} = i_{C\text{inst}}$ at any instant of time between 0 and $t_1$. The same can be said about any portion of the voltage waveform that has a constant slope.

An important point to be gained from this discussion is that it is not the magnitude of the voltage across a capacitor that determines the current but rather how quickly the voltage *changes* across the capacitor. An applied steady dc voltage of 10,000 V would (ideally) not create any flow of charge (current), but a change in voltage of 1 V in a very brief period of time could create a significant current.

The method described above is only for waveforms with straight-line (linear) segments. For nonlinear (curved) waveforms, a method of calculus (differentiation) must be employed.

## 10.13 CAPACITORS IN SERIES AND PARALLEL

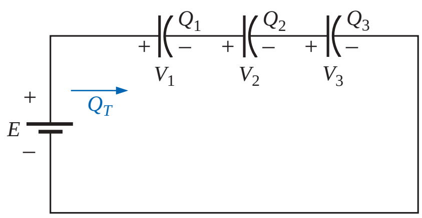



**FIG. 10.61**
*Series capacitors.*

Capacitors, like resistors, can be placed in series and in parallel. Increasing levels of capacitance can be obtained by placing capacitors in parallel, while decreasing levels can be obtained by placing capacitors in series.

For capacitors in series, the charge is the same on each capacitor (Fig. 10.61):

$$Q_T = Q_1 = Q_2 = Q_3 \qquad \textbf{(10.29)}$$

Applying Kirchhoff's voltage law around the closed loop gives

$$E = V_1 + V_2 + V_3$$

However,
$$V = \frac{Q}{C}$$

so that
$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Using Eq. (10.29) and dividing both sides by $Q$ yields

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \textbf{(10.30)}$$

which is similar to the manner in which we found the total resistance of a parallel resistive circuit. The total capacitance of two capacitors in series is

$$C_T = \frac{C_1 C_2}{C_1 + C_2} \qquad \textbf{(10.31)}$$

The voltage across each capacitor of Fig. 10.61 can be found by first recognizing that

$$Q_T = Q_1$$

or
$$C_T E = C_1 V_1$$

Solving for $V_1$:

$$V_1 = \frac{C_T E}{C_1}$$

and substituting for $C_T$:

$$V_1 = \left[\frac{1/C_1}{1/C_1 + 1/C_2 + 1/C_3}\right]E \tag{10.32}$$

A similar equation will result for each capacitor of the network.

For capacitors in parallel, as shown in Fig. 10.62, the voltage is the same across each capacitor, and the total charge is the sum of that on each capacitor:

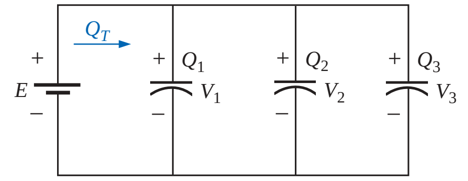



**FIG. 10.62**
*Parallel capacitors.*

$$Q_T = Q_1 + Q_2 + Q_3 \tag{10.33}$$

However, $$Q = CV$$

Therefore, $$C_TE = C_1V_1 + C_2V_2 + C_3V_3$$

but $$E = V_1 = V_2 = V_3$$

Thus, $$C_T = C_1 + C_2 + C_3 \tag{10.34}$$

which is similar to the manner in which the total resistance of a series circuit is found.

---

**EXAMPLE 10.14** For the circuit of Fig. 10.63:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the voltage across each capacitor.

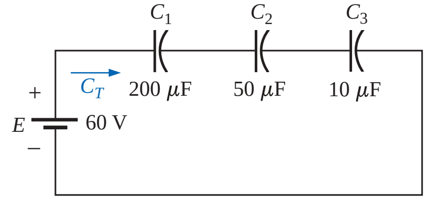



**FIG. 10.63**
*Example 10.14.*

***Solutions:***

a. $$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$= \frac{1}{200 \times 10^{-6}\,\text{F}} + \frac{1}{50 \times 10^{-6}\,\text{F}} + \frac{1}{10 \times 10^{-6}\,\text{F}}$$
$$= 0.005 \times 10^6 + 0.02 \times 10^6 + 0.1 \times 10^6$$
$$= 0.125 \times 10^6$$

and $$C_T = \frac{1}{0.125 \times 10^6} = \mathbf{8\ \mu F}$$

b. $$Q_T = Q_1 = Q_2 = Q_3$$
$$= C_TE = (8 \times 10^{-6}\,\text{F})(60\,\text{V}) = \mathbf{480\ \mu C}$$

c. $$V_1 = \frac{Q_1}{C_1} = \frac{480 \times 10^{-6}\,\text{C}}{200 \times 10^{-6}\,\text{F}} = \mathbf{2.4\ V}$$
$$V_2 = \frac{Q_2}{C_2} = \frac{480 \times 10^{-6}\,\text{C}}{50 \times 10^{-6}\,\text{F}} = \mathbf{9.6\ V}$$
$$V_3 = \frac{Q_3}{C_3} = \frac{480 \times 10^{-6}\,\text{C}}{10 \times 10^{-6}\,\text{F}} = \mathbf{48.0\ V}$$

and $$E = V_1 + V_2 + V_3 = 2.4\,\text{V} + 9.6\,\text{V} + 48\,\text{V}$$
$$= \mathbf{60\ V} \quad \text{(checks)}$$

---

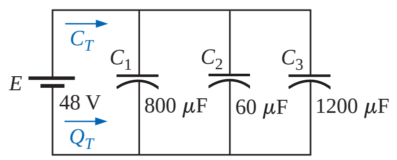



**FIG. 10.64**
*Example 10.15.*

**EXAMPLE 10.15** For the network of Fig. 10.64:

a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the total charge.

***Solutions:***

a. $C_T = C_1 + C_2 + C_3 = 800\ \mu\text{F} + 60\ \mu\text{F} + 1200\ \mu\text{F}$
$= \mathbf{2060\ \mu F}$

b. $Q_1 = C_1E = (800 \times 10^{-6}\text{ F})(48\text{ V}) = \mathbf{38.4\ mC}$
$Q_2 = C_2E = (60 \times 10^{-6}\text{ F})(48\text{ V}) = \mathbf{2.88\ mC}$
$Q_3 = C_3E = (1200 \times 10^{-6}\text{ F})(48\text{ V}) = \mathbf{57.6\ mC}$

c. $Q_T = Q_1 + Q_2 + Q_3 = 38.4\text{ mC} + 2.88\text{ mC} + 57.6\text{ mC}$
$= \mathbf{98.88\ mC}$

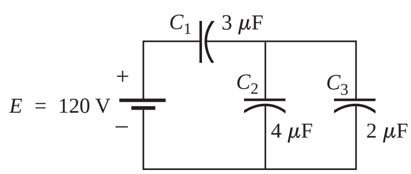



**FIG. 10.65**
*Example 10.16.*

**EXAMPLE 10.16** Find the voltage across and charge on each capacitor for the network of Fig. 10.65.

***Solution:***

$$C'_T = C_2 + C_3 = 4\ \mu\text{F} + 2\ \mu\text{F} = 6\ \mu\text{F}$$

$$C_T = \frac{C_1C'_T}{C_1 + C'_T} = \frac{(3\ \mu\text{F})(6\ \mu\text{F})}{3\ \mu\text{F} + 6\ \mu\text{F}} = 2\ \mu\text{F}$$

$$Q_T = C_TE = (2 \times 10^{-6}\text{ F})(120\text{ V})$$
$$= \mathbf{240\ \mu C}$$

An equivalent circuit (Fig. 10.66) has

$$Q_T = Q_1 = Q'_T$$

and, therefore, $Q_1 = \mathbf{240\ \mu C}$

and $$V_1 = \frac{Q_1}{C_1} = \frac{240 \times 10^{-6}\text{ C}}{3 \times 10^{-6}\text{ F}} = \mathbf{80\ V}$$

$$Q'_T = 240\ \mu\text{C}$$

and, therefore,

$$V'_T = \frac{Q'_T}{C'_T} = \frac{240 \times 10^{-6}\text{ C}}{6 \times 10^{-6}\text{ F}} = \mathbf{40\ V}$$

and $$Q_2 = C_2V'_T = (4 \times 10^{-6}\text{ F})(40\text{ V}) = \mathbf{160\ \mu C}$$
$$Q_3 = C_3V'_T = (2 \times 10^{-6}\text{ F})(40\text{ V}) = \mathbf{80\ \mu C}$$

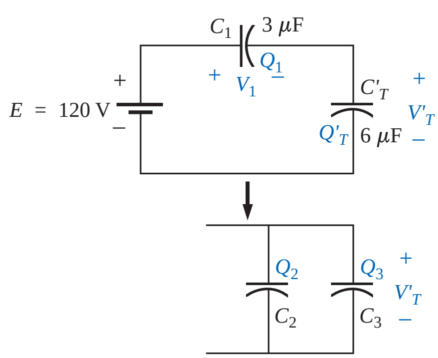



**FIG. 10.66**
*Reduced equivalent for the network of Fig. 10.65.*

**EXAMPLE 10.17** Find the voltage across and charge on capacitor $C_1$ of Fig. 10.67 after it has charged up to its final value.

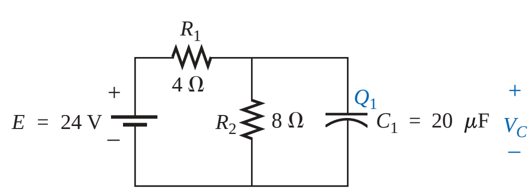



**FIG. 10.67**
*Example 10.17.*

**Solution:** As previously discussed, the capacitor is effectively an open circuit for dc after charging up to its final value (Fig. 10.68). Therefore,

$$V_C = \frac{(8\ \Omega)(24\ \text{V})}{4\ \Omega + 8\ \Omega} = \mathbf{16\ V}$$

$$Q_1 = C_1 V_C = (20 \times 10^{-6}\ \text{F})(16\ \text{V})$$
$$= \mathbf{320\ \mu C}$$

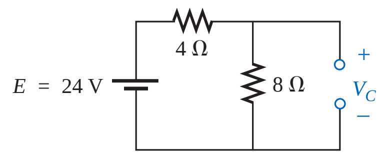



**FIG. 10.68**
*Determining the final (steady-state) value for $v_C$.*

**EXAMPLE 10.18** Find the voltage across and charge on each capacitor of the network of Fig. 10.69 after each has charged up to its final value.

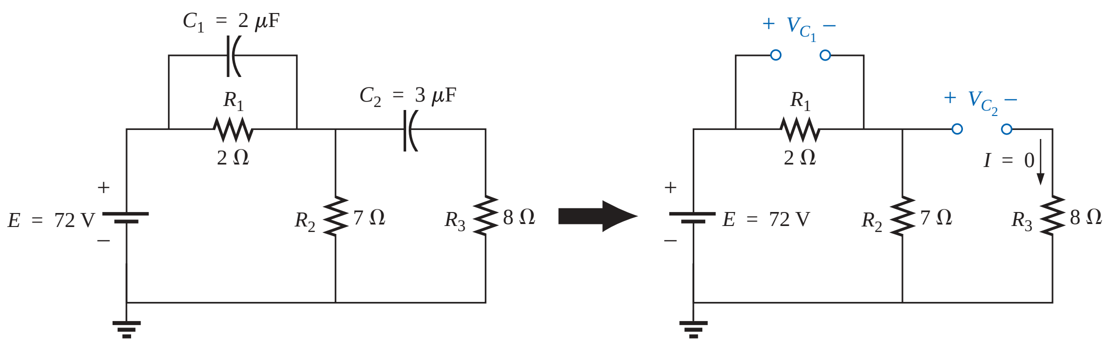



**FIG. 10.69**
*Example 10.18.*

**Solution:**

$$V_{C_2} = \frac{(7\ \Omega)(72\ \text{V})}{7\ \Omega + 2\ \Omega} = \mathbf{56\ V}$$

$$V_{C_1} = \frac{(2\ \Omega)(72\ \text{V})}{2\ \Omega + 7\ \Omega} = \mathbf{16\ V}$$

$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6}\ \text{F})(16\ \text{V}) = \mathbf{32\ \mu C}$$

$$Q_2 = C_2 V_{C_2} = (3 \times 10^{-6}\ \text{F})(56\ \text{V}) = \mathbf{168\ \mu C}$$

## 10.14 ENERGY STORED BY A CAPACITOR

The ideal capacitor does not dissipate any of the energy supplied to it. It stores the energy in the form of an electric field between the conducting surfaces. A plot of the voltage, current, and power to a capacitor during the charging phase is shown in Fig. 10.70. The power curve can be obtained by finding the product of the voltage and current at selected instants of time and connecting the points obtained. The energy stored is represented by the shaded area under the power curve. Using calculus, we can determine the area under the curve:

$$W_C = \frac{1}{2} C E^2$$

In general,

$$W_C = \frac{1}{2} C V^2 \quad \text{(J)} \qquad \textbf{(10.35)}$$

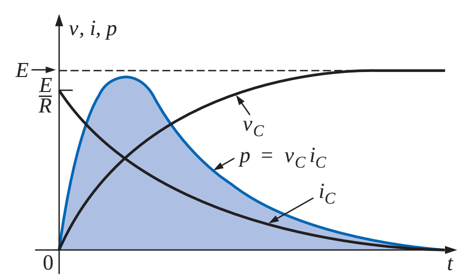



**FIG. 10.70**
*Plotting the power to a capacitive element during the transient phase.*

where $V$ is the steady-state voltage across the capacitor. In terms of $Q$ and $C$,

$$W_C = \frac{1}{2}C\left(\frac{Q}{C}\right)^2$$

or

$$W_C = \frac{Q^2}{2C} \quad \text{(J)} \qquad \textbf{(10.36)}$$

**EXAMPLE 10.19** For the network of Fig. 10.69, determine the energy stored by each capacitor.

***Solution:***

For $C_1$,

$$W_C = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(2 \times 10^{-6}\text{ F})(16\text{ V})^2 = (1 \times 10^{-6})(256)$$

$$= \mathbf{256\ \mu J}$$

For $C_2$,

$$W_C = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(3 \times 10^{-6}\text{ F})(56\text{ V})^2 = (1.5 \times 10^{-6})(3136)$$

$$= \mathbf{4704\ \mu J}$$

Due to the squared term, note the difference in energy stored because of a higher voltage.

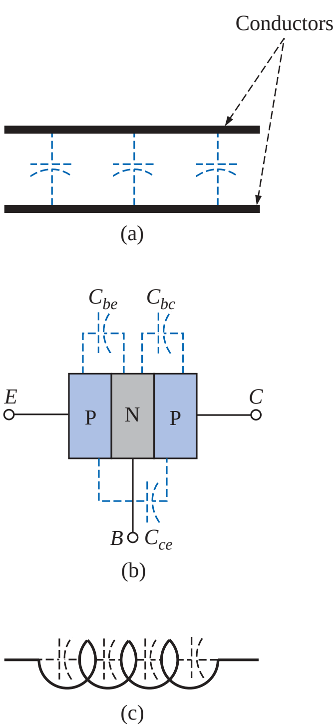



**FIG. 10.71**
*Examples of stray capacitance.*

## 10.15 STRAY CAPACITANCES

In addition to the capacitors discussed so far in this chapter, there are **stray capacitances** that exist not through design but simply because two conducting surfaces are relatively close to each other. Two conducting wires in the same network will have a capacitive effect between them, as shown in Fig. 10.71(a). In electronic circuits, capacitance levels exist between conducting surfaces of the transistor, as shown in Fig. 10.71(b). As mentioned earlier, in Chapter 12 we will discuss another element called the *inductor,* which will have capacitive effects between the windings [Fig. 10.71(c)]. Stray capacitances can often lead to serious errors in system design if they are not considered carefully.

## 10.16 APPLICATIONS

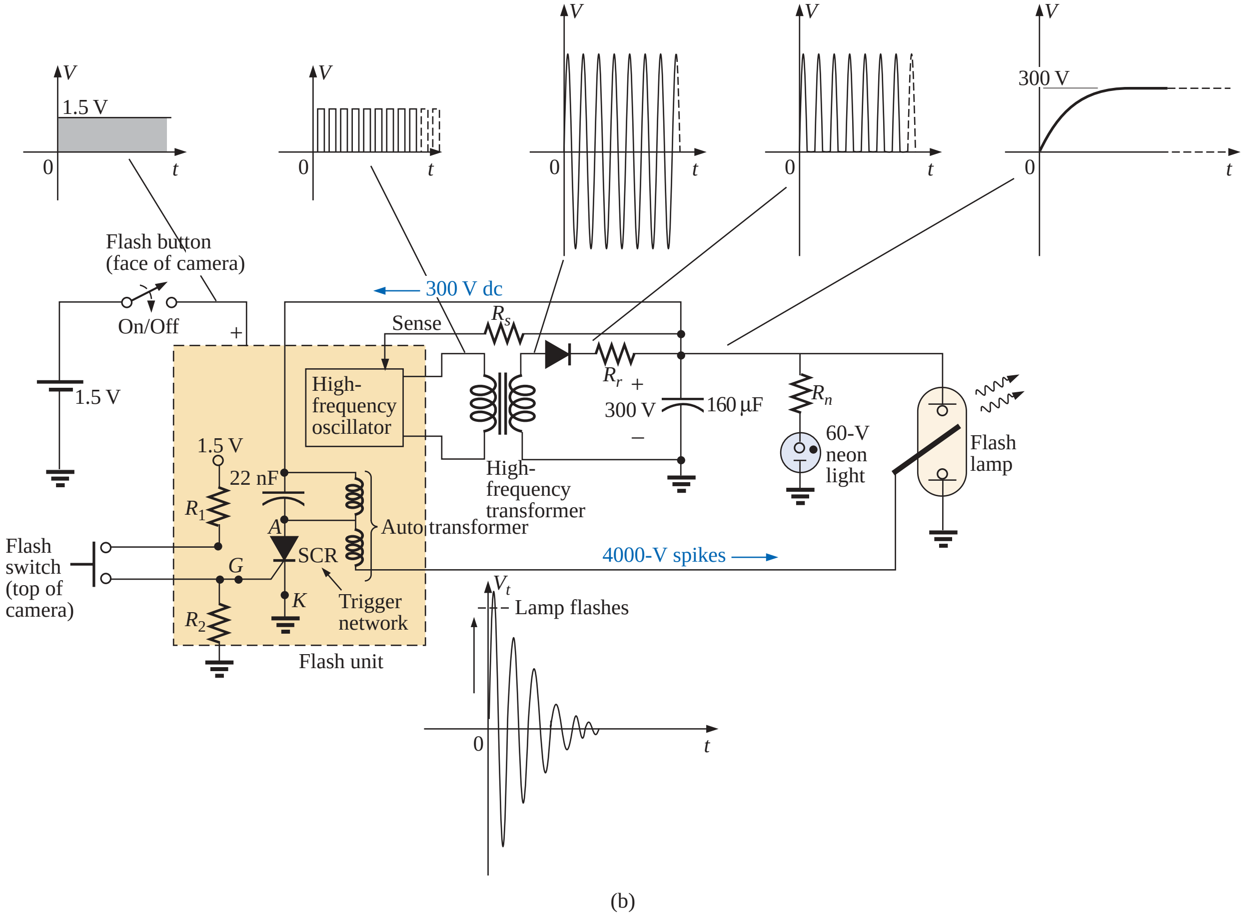
This Applications section for capacitors includes both a description of the operation of one of the less expensive, throwaway cameras that have become so popular today and a discussion of the use of capacitors in the line conditioners (surge protectors) that have found their way into most homes and throughout the business world. Additional examples of the use of capacitors will appear throughout the chapter to follow.

## Flash Lamp

The basic circuitry for the flash lamp of the popular, inexpensive, throwaway camera of Fig. 10.72(a) is provided in Fig. 10.72(b), with the physical circuitry appearing in Fig. 10.72(c). The labels added to Fig. 10.72(c) identify broad areas of the design and some individual components. The major components of the electronic circuitry include a large 160-$\mu$F, 330-V, polarized electrolytic capacitor to store the necessary charge for the flash lamp, a flash lamp to generate the required light, a dc battery of 1.5 V, a chopper network to generate a dc voltage in excess of 300 V, and a trigger network to establish a few thousand volts for a very short period of time to fire the flash lamp. There are both a 22-nF capacitor in the trigger network as shown in Fig. 10.72(b) and (c) and a third capacitor of 470 pF in the high-frequency oscillator of the chopper network. In particular, note that the size of each capacitor is directly related to its capacitance level. It should certainly be of some interest that a single source of energy of only 1.5 V dc can be converted to one of a few thousand volts (albeit for a very short period of time) to fire the flash lamp. In fact, that single, small battery has sufficient power for the entire run of film through the camera. Always keep in mind that energy is related to power and time by $W = Pt = (VI)t$.

(a)

**FIG. 10.72(a)**
*Flash camera: general appearance.*



(b)

**FIG. 10.72(b)**
*Flash camera: basic circuitry.*

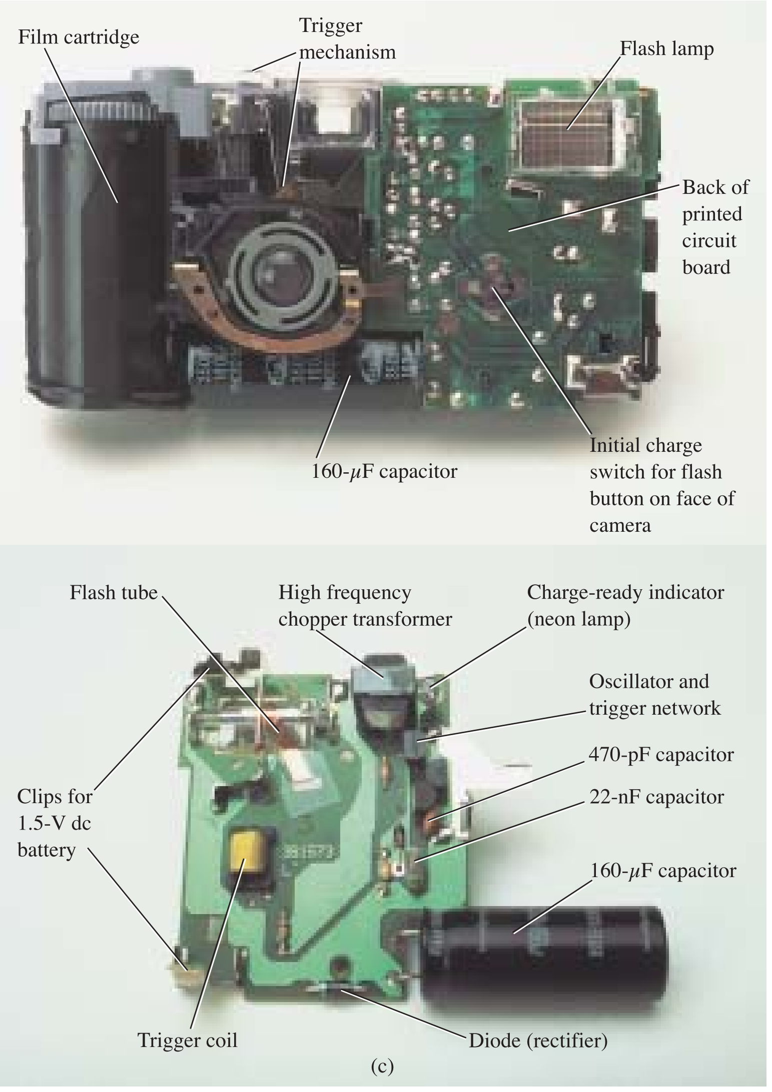



**FIG. 10.72(c)**
*Flash camera: internal construction.*

That is, a high level of voltage can be generated for a defined energy level so long as the factors $I$ and $t$ are sufficiently small.

When you first use the camera, you are directed to press the flash button on the face of the camera and wait for the flash-ready light to come on. As soon as the flash button is depressed, the full 1.5 V of the dc battery are applied to an electronic network (a variety of networks can perform the same function) that will generate an oscillating waveform of very high frequency (with a high repetitive rate) as shown in Fig. 10.72(b). The high-frequency transformer will then significantly increase the magnitude of the generated voltage and will pass it on to a half-wave rectification system (introduced in earlier chapters), resulting in a dc voltage of about 300 V across the 160-$\mu$F capacitor to charge the capacitor (as determined by $Q = CV$). Once the 300-V level is reached, the lead marked "sense" in Fig. 10.72(b) will feed the information back to the oscillator and will turn it off until the output dc voltage drops to a low threshold level. When the capacitor is fully charged, the neon light in parallel with the capacitor will turn on (labeled "flash-ready lamp" on the camera) to let you know that the camera is ready to use. The entire network from the 1.5-V dc level to the final 300-V level is called a *dc-dc converter.* The terminology *chopper network* comes from the fact that the applied dc voltage of 1.5 V was chopped up into one that changes level at a very high frequency so that the transformer can perform its function.

Even though the camera may use a 60-V neon light, the neon light and series resistor $R_n$ must have a full 300 V across the branch before the neon light will turn on. Neon lights are simply bulbs with a neon gas that will support conduction when the voltage across the terminals reaches a sufficiently high level. There is no filament, or hot wire as in a light bulb, but simply conduction through the gaseous medium. For new cameras the first charging sequence may take 12 s to 15 s. Succeeding charging cycles may only take some 7 s or 8 s because the capacitor will still have some residual charge on its plates. If the flash unit is not used, the neon light will begin to drain the 300-V dc supply with a drain current in microamperes. As the terminal voltage drops, there will come a point where the neon light will turn off. For the unit of Fig. 10.72, it takes about 15 min before the light turns off. Once off, the neon light will no longer drain the capacitor, and the terminal voltage of the capacitor will remain fairly constant. Eventually, however, the capacitor will discharge due to its own leakage current, and the terminal voltage will drop to very low levels. The discharge process is very rapid when the flash unit is used, causing the terminal voltage to drop very quickly ($V = Q/C$) and, through the feedback-sense connection signal, causing the oscillator to start up again and recharge the capacitor. You may have noticed when using a camera of this type that once the camera has its initial charge, there is no need to press the charge button between pictures—it is done automatically. However, if the camera sits for a long period of time, the charge button will have to be depressed again; but you will find that the charge time is only 3 s or 4 s due to the residual charge on the plates of the capacitor.

The 300 V across the capacitor are insufficient to fire the flash lamp. Additional circuitry, called the *trigger network*, must be incorporated to generate the few thousand volts necessary to fire the flash lamp. The resulting high voltage is one reason that there is a CAUTION note on each camera regarding the high internal voltages generated and the possibility of electrical shock if the camera is opened.

The thousands of volts required to fire the flash lamp require a discussion that introduces elements and concepts beyond the current level of the text. However, this description is sensitive to this fact and should be looked upon as simply a first exposure to some of the interesting possibilities available from the right mix of elements. When the flash switch at the bottom left of Fig. 10.72(a) is closed, it will establish a connection between the resistors $R_1$ and $R_2$. Through a voltage divider action, a dc voltage will appear at the gate ($G$) terminal of the SCR (silicon-controlled rectifier—a device whose state is controlled by the voltage at the gate terminal). This dc voltage will turn the SCR "on" and will establish a very low resistance path (like a short circuit) between its anode ($A$) and cathode ($K$) terminals. At this point the trigger capacitor, which is connected directly to the 300 V sitting across the capacitor, will rapidly charge to 300 V because it now has a direct, low-resistance path to ground through the SCR. Once it reaches 300 V, the charging current in this part of the network will drop to 0 A, and the

SCR will open up again since it is a device that needs a steady current in the anode circuit to stay on. The capacitor then sits across the parallel coil (with no connection to ground through the SCR) with its full 300 V and begins to quickly discharge through the coil because the only resistance in the circuit affecting the time constant is the resistance of the parallel coil. As a result, a rapidly changing current through the coil will generate a high voltage across the coil for reasons to be introduced in Chapter 12.

When the capacitor decays to zero volts, the current through the coil will be zero amperes, but a strong magnetic field has been established around the coil. This strong magnetic field will then quickly collapse, establishing a current in the parallel network that will recharge the capacitor again. This continual exchange between the two storage elements will continue for a period of time, depending on the resistance in the ciruit. The more the resistance, the shorter the "ringing" of the voltage at the output. This action of the energy "flying back" to the other element is the basis for the "flyback" effect that is frequently used to generate high dc voltages such as needed in TVs. In Fig. 10.72(b), you will find that the trigger coil is connected directly to a second coil to form an autotransformer (a transformer with one end connected). Through transformer action the high voltage generated across the trigger coil will be increased further, resulting in the 4000 V necessary to fire the flash lamp. Note in Fig. 10.72(c) that the 4000 V are applied to a grid that actually lies on the surface of the glass tube of the flash lamp (not internally connected or in contact with the gases). When the trigger voltage is applied, it will excite the gases in the lamp, causing a very high current to develop in the bulb for a very short period of time and producing the desired bright light. The current in the lamp is supported by the charge on the 160-$\mu$F capacitor which will be dissipated very quickly. The capacitor voltage will drop very quickly, the photo lamp will shut down, and the charging process will begin again. If the entire process didn't occur as quickly as it does, the lamp would burn out after a single use.

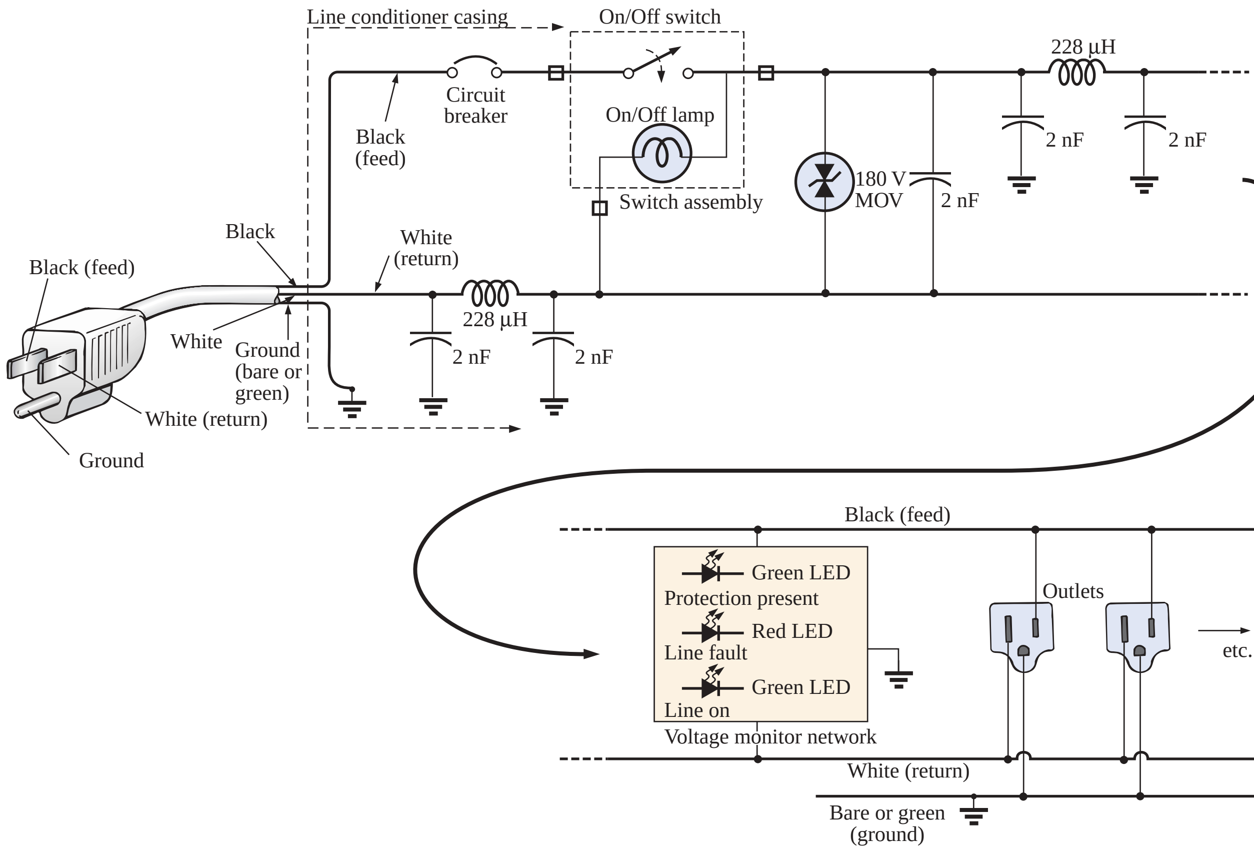
## Line Conditioner (Surge Protector)


(a)

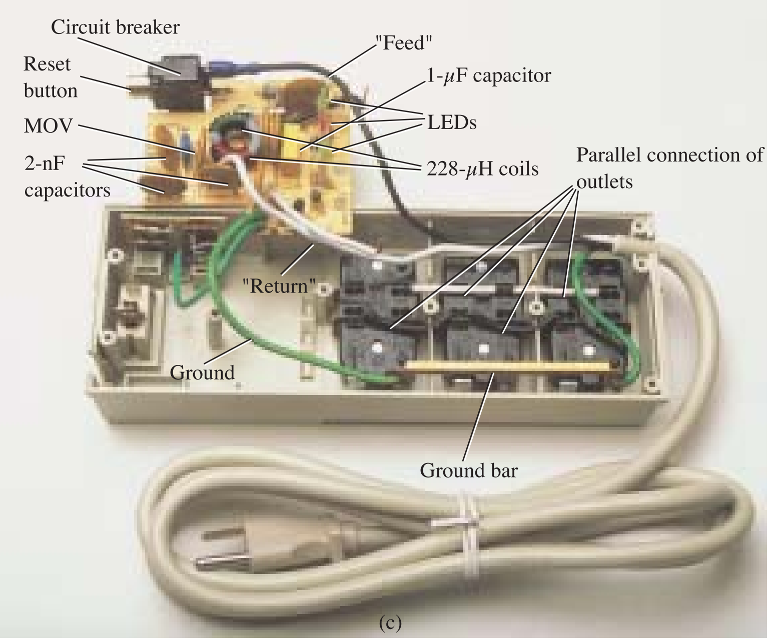
**FIG. 10.73(a)**
*Line conditioner: general appearance.*

In recent years we have all become familiar with the line conditioner as a safety measure for our computers, TVs, CD players, and other sensitive instrumentation. In addition to protecting equipment from unexpected surges in voltage and current, most quality units will also filter out (remove) electromagnetic interference (EMI) and radio-frequency interference (RFI). EMI encompasses any unwanted disturbances down the power line established by any combination of electromagnetic effects such as those generated by motors on the line, power equipment in the area emitting signals picked up by the power line acting as an antenna, and so on. RFI includes all signals in the air in the audio range and beyond which may also be picked up by power lines inside or outside the house.

The unit of Fig. 10.73 has all the design features expected in a good line conditioner. Figure 10.73(a) reveals that it can handle the power drawn by six outlets and that it is set up for FAX/MODEM protection. Also note that it has both LED (light-emitting diode) displays which reveal whether there is fault on the line or whether the line is OK and an external circuit breaker to reset the system. In addition, when the surge protector is on, a red light will be visible at the power switch.




(b)




(c)

**FIG. 10.73 (cont.)**
*Line conditioner: (b) electrical schematic; (c) internal construction.*

The schematic of Fig. 10.73(b) does not include all the details of the design, but it does include the major components that appear in most good line conditioners. First note in the photograph of Fig. 10.73(c) that the outlets are all connected in parallel, with a ground bar used to establish a ground connection for each outlet. The circuit board had to be flipped over to show the components, so it will take some adjustment to relate the position of the elements on the board to the casing. The *feed line* or *hot lead wire* (black in the actual unit) is connected directly from the line to the circuit breaker. The other end of the circuit breaker is connected to the other side of the circuit board. All the large discs that you see are 2-nF/73 capacitors [not all have been included in Fig 10.73(b) for clarity]. There are quite a few capacitors to handle all the possibilities. For instance, there are capacitors from line to return (black wire to white wire), from line to ground (black to green), and from return to ground (white to ground). Each has two functions. The first and most obvious function is to prevent any spikes in voltage that may come down the line because of external effects such as lightning from reaching the equipment plugged into the unit. Recall from this chapter that the voltage across capacitors cannot change instantaneously and in fact will act to squelch any rapid change in voltage across its terminals. The capacitor, therefore, will prevent the line to neutral voltage from changing too quickly, and any spike that tries to come down the line will have to find another point in the feed circuit to fall across. In this way the appliances to the surge protector are well protected.

The second function requires some knowledge of the reaction of capacitors to different frequencies and will be discussed in more detail in later chapters. For the moment, let it suffice to say that the capacitor will have a different impedance to different frequencies, thereby preventing undesired frequencies, such as those associated with EMI and RFI disturbances, from affecting the operation of units connected to the line conditioner. The rectangular-shaped capacitor of 1 $\mu$F near the center of the board is connected directly across the line to take the brunt of a strong voltage spike down the line. Its larger size is clear evidence that it is designed to absorb a fairly high energy level that may be established by a large voltage—significant current over a period of time that might exceed a few milliseconds.

The large, toroidal-shaped structure in the center of the circuit board of Fig. 10.73(c) has two coils (Chapter 12) of 228 $\mu$H that appear in the line and neutral of Fig. 10.73(b). Their purpose, like that of the capacitors, is twofold: to block spikes in current from coming down the line and to block unwanted EMI and RFI frequencies from getting to the connected systems. In the next chapter you will find that coils act as "chokes" to quick changes in current; that is, the current through a coil cannot change instantaneously. For increasing frequencies, such as those associated with EMI and RFI disturbances, the reactance of a coil increases and will absorb the undesired signal rather than let it pass down the line. Using a choke in both the line and the neutral makes the conditioner network balanced to ground. In total, capacitors in a line conditioner have the effect of *bypassing* the disturbances, whereas inductors *block* the disturbance.

The smaller disc (blue) between two capacitors and near the circuit breaker is an MOV (metal-oxide varistor) which is the heart of most line conditioners. It is an electronic device whose terminal characteristics will change with the voltage applied across its terminals. For the normal range of voltages down the line, its terminal resistance will be sufficiently large to be considered an open circuit, and its presence can be ignored. However, if the voltage is too large, its terminal characteristics will change from a very large resistance to a very small resistance that can essentially be considered a short circuit. This variation in resistance with applied voltage is the reason for the name *varistor.* For MOVs in North America where the line voltage is 120 V, the MOVs are 180 V or more. The reason for the 60-V difference is that the 120-V rating is an effective value related to dc voltage levels, whereas the waveform for the voltage at any 120-V outlet has a peak value of about 170 V. A great deal more will be said about this topic in Chapter 13.

Taking a look at the symbol for an MOV in Fig. 10.73(b), you will note that it has an arrow in each direction, revealing that the MOV is bidirectional and will block voltages with either polarity. In general, therefore, for normal operating conditions, the presence of the MOV can be ignored; but, if a large spike should appear down the line, exceeding the MOV rating, it will act as a short across the line to protect the connected circuitry. It is a significant improvement to simply putting a fuse in the line because it is voltage sensitive, can react much quicker than a fuse, and will display its low-resistance characteristics for only a short period of time. When the spike has passed, it will return to its normal open-circuit characteristic. If you're wondering where the spike will go if the load is protected by a short circuit, remember that all sources of disturbance, such as lightning, generators, inductive motors (such as in air conditioners, dishwashers, power saws, and so on), have their own "source resistance," and there is always some resistance down the line to absorb the disturbance.

Most line conditioners, as part of their advertising, like to mention their energy absorption level. The rating of the unit of Fig. 10.73 is 1200 J which is actually higher than most. Remembering that $W = Pt = EIt$ from the earlier discussion of cameras, we now realize that if a 5000-V spike came down the line, we would be left with the product $It = W/E = 1200\text{ J}/5000\text{ V} = 240\text{ mAs}$. Assuming a linear relationship between all quantities, the rated energy level is revealing that a current of 100 A could be sustained for $t = 240\text{ mAs}/100\text{ A} = 2.4\text{ ms}$, a current of 1000 A for 240 $\mu$s, and a current of 10,000 A for 24 $\mu$s. Obviously, the higher the power product of $E$ and $I$, the less the time element.

The technical specifications of the unit of Fig. 10.73 include an instantaneous response time of 0 ns (questionable), with a phone line protection of 5 ns. The unit is rated to dissipate surges up to 6000 V and current spikes up to 96,000 A. It has a very high noise suppression ratio (80 dB; see Chapter 23) at frequencies from 50 kHz to 1000 MHz, and (a credit to the company) it has a lifetime warranty.

## 10.17 COMPUTER ANALYSIS

### PSpice

**Transient *RC* Response** PSpice will now investigate the transient response for the voltage across the capacitor of Fig. 10.74. In all the examples in the text involving a transient response, a switch appeared in series with the source as shown in Fig. 10.75(a). When applying PSpice, we establish this instantaneous change in voltage level by

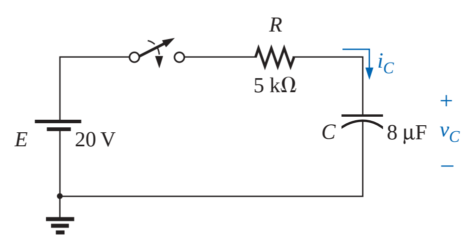



**FIG. 10.74**
*Circuit to be analyzed using PSpice.*

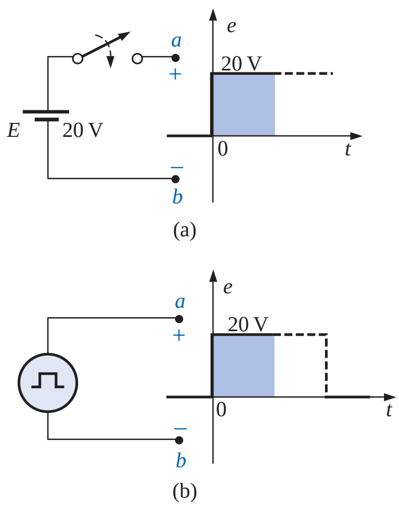



**FIG. 10.75**
*Establishing a switching dc voltage level: (a) series dc voltage-switch combination; (b) PSpice pulse option.*

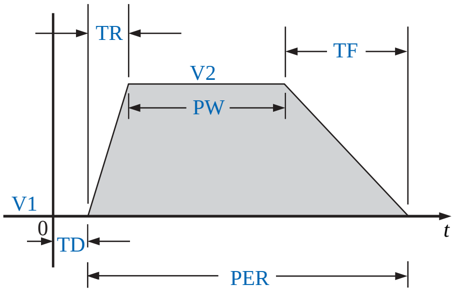



**FIG. 10.76**
*The defining parameters of PSpice* ***VPULSE.***

applying a pulse waveform as shown in Fig. 10.75(b) with a pulse width *(PW)* longer than the period ($5\tau$) of interest for the network.

A pulse source is obtained through the sequence **Place part** key-**Libraries-SOURCE-VPULSE-OK.** Once in place, the label and all the parameters can be set by simply double-clicking on each to obtain the **Display Properties** dialog box. As you scroll down the list of attributes, you will see the following parameters defined by Fig. 10.76:

**V1** is the initial value.
**V2** is the pulse level.
**TD** is the delay time.
**TR** is the rise time.
**TF** is the fall time.
**PW** is the pulse width at the $V_2$ level.
**PER** is the period of the waveform.

All the parameters have been set as shown on the schematic of Fig. 10.77 for the network of Fig. 10.74. Since a rise and fall time of 0 s is unrealistic from a practical standpoint, 0.1 ms was chosen for each in this example. Further, since $\tau = RC = (5\text{ k}\Omega) \times (8\ \mu\text{F}) = 20$ ms and $5\tau = 200$ ms, a pulse width of 500 ms was selected. The period was simply chosen as twice the pulse width.

Now for the simulation process. First the **New Simulation Profile** key is selected to obtain the **New Simulation** dialog box in which **TransientRC** is inserted for the **Name** and **Create** is chosen to leave the dialog box. The **Simulation Settings-Transient RC** dialog box will result, and under **Analysis,** the **Time Domain (Transient)** option is chosen under **Analysis type.** The **Run to time** is set at 200 ms so that only the first five time constants will be plotted. The **Start saving data after** option will be 0 s to ensure that the data are collected immediately. The **Maximum step size** is 1 ms to provide sufficient data points for a good plot. Click **OK,** and we are ready to select the **Run PSpice** key. The result will be a graph without a plot (since it has not been defined yet) and an *x*-axis that extends from 0 s to 200 ms as defined above. To obtain a plot of the voltage across the capacitor versus time, the following sequence is applied: **Add Trace** key-**Add Traces** dialog box-**V1(C)-OK,** and the plot of Fig. 10.78 will result. The color and thickness of the plot and the axis can be changed by placing the cursor on the plot line and performing a right click. A list will appear in which **Properties** should be selected; then a **Trace Properties** dialog box will appear in which the color and thickness of the line can be changed. Since the plot is against a black background, a better printout occurred when yellow was selected and the line was made thicker as shown in Fig. 10.78. Next, the cursor can be put on the axis, and another right click will allow you to make the axis yellow and thicker for a better printout. For comparison it seemed appropriate to plot the applied pulse signal also. This is accomplished by going back to **Trace** and selecting **Add Trace** followed by **V(Vpulse:+)** and **OK.** Now both waveforms appear on the same screen as shown in Fig. 10.78. In this case, the plot was left a greenish tint so it could be distinguished from the axis and the other plot. Note that it follows the left axis to the top and travels across the screen at 20 V.

If you want the magnitude of either plot at any instant, simply select the **Toggle cursor** key. Then click on **V1(C)** at the bottom left of the screen. A box will appear around **V1(C)** that will reveal the spacing between the dots of the cursor on the screen. This is important when

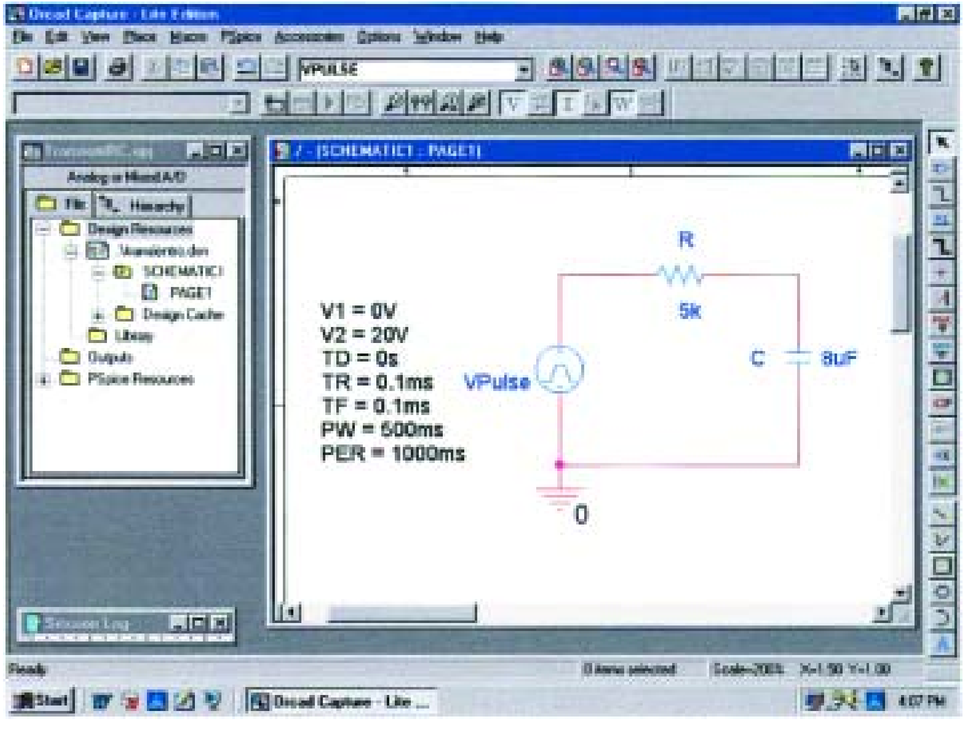



**FIG. 10.77**
*Using PSpice to investigate the transient response of the series R-C circuit of Fig. 10.74.*

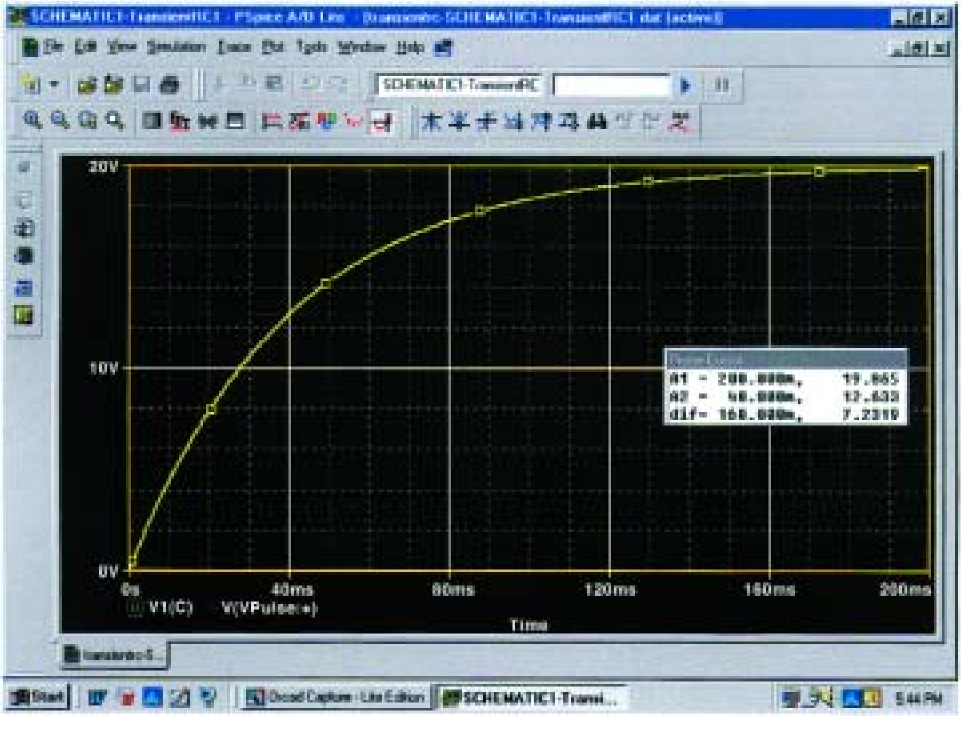



**FIG. 10.78**
*Transient response for the voltage across the capacitor of Fig. 10.74 when* ***VPulse*** *is applied.*

more than one cursor is used. By moving the cursor to 200 ms, we find that the magnitude **(A1)** is 19.865 V (in the **Probe Cursor** dialog box), clearly showing how close it is to the final value of 20 V. A second cursor can be placed on the screen with a right click and then a click on the same **V1(C)** on the bottom of the screen. The box around **V1(C)** cannot show two boxes, but the spacing and the width of the lines of the box have definitely changed. There is no box around the **Pulse** symbol since it was not selected—although it could have been selected by either cursor. If we now move the second cursor to one time constant of 40 ms, we find that the voltage is 12.633 V as shown in the **Probe Cursor** dialog box. This confirms the fact that the voltage should be 63.2% of its final value of 20 V in one time constant ($0.632 \times 20\text{ V} = 12.4\text{ V}$). Two separate plots could have been obtained by going to **Plot-Add Plot to Window** and then using the trace sequence again.

**Average Capacitive Current** As an exercise in using the pulse source and to verify our analysis of the average current for a purely capacitive network, the description to follow will verify the results of Example 10.13. For the pulse waveform of Fig. 10.59, the parameters of the pulse supply appear in Fig. 10.79. Note that the rise time is now 2 ms, starting at 0 s, and the fall time is 6 ms. The period was set at 15 ms to permit monitoring the current after the pulse had passed.

Simulation is initiated by first selecting the **New Simulation Profile** key to obtain the **New Simulation** dialog box in which **AverageIC** is entered as the **Name. Create** is then chosen to obtain the **Simulation**

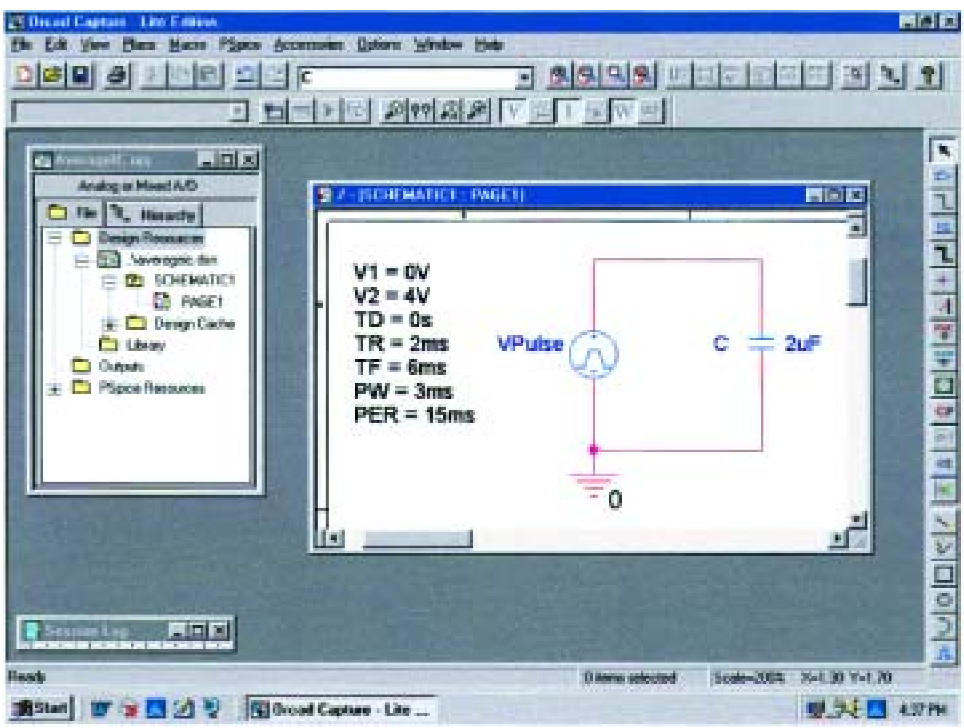



**FIG. 10.79**
*Using PSpice to verify the results of Example 10.13.*

**Settings-AverageIC** dialog box. **Analysis** is selected, and **Time Domain(Transient)** is chosen under the **Analysis type** options. The **Run to time** is set to 15 ms to encompass the period of interest, and the **Start saving data after** is set at 0 s to ensure data points starting at $t = 0$ s. The **Maximum step size** is selected from 15 ms/1000 = 15 $\mu$s to ensure 1000 data points for the plot. Click **OK,** and the **Run PSpice** key is selected. A window will appear with a horizontal scale that extends from 0 to 15 ms as defined above. Then the **Add Trace** key is selected, and **I(C)** is chosen to appear in the **Trace Expression** below. Click **OK,** and the plot of **I(C)** appears in the bottom of Fig. 10.80. This time it would be nice to see the pulse waveform in the same window but as a separate plot. Therefore, continue with **Plot-Add Plot to Window-Trace-Add Trade-V(Vpulse:+)-OK,** and both plots appear as shown in Fig. 10.80.

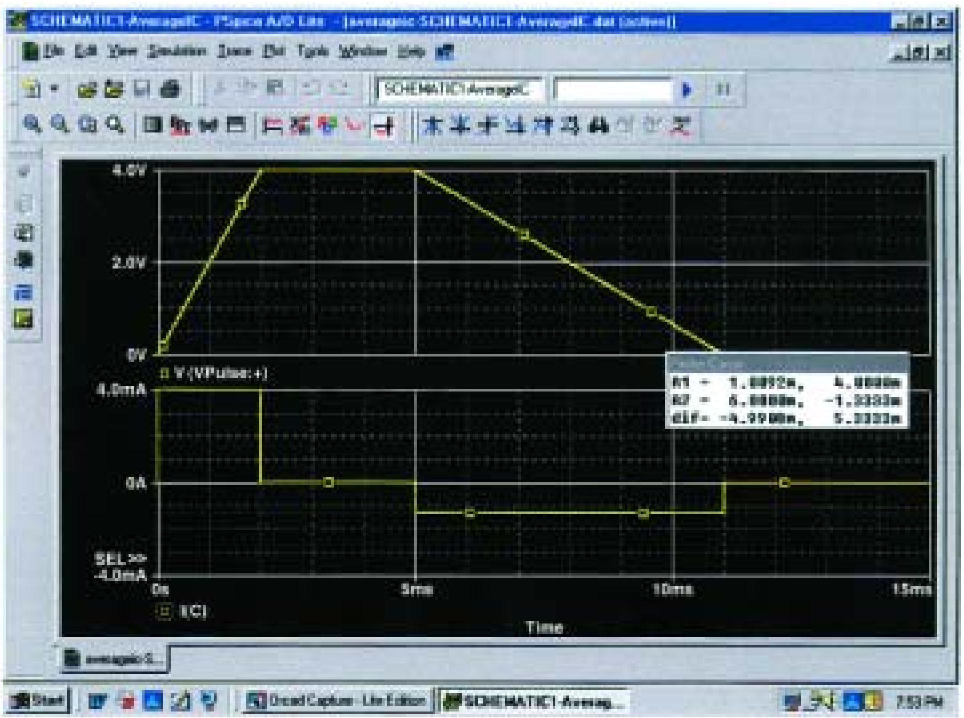



**FIG. 10.80**

*The applied pulse and resulting current for the 2-$\mu$F capacitor of Fig. 10.79.*

The cursors can now be used to measure the resulting average current levels. First, select the **I(C)** plot to move the **SEL>>** notation to the lower plot. The **SEL>>** defines which plot for multiplot screens is active. Then select the **Toggle cursor** key, and left-click on the **I(C)** plot to establish the crosshairs of the cursor. Set the value at 1 ms, and the magnitude **A1** is displayed as 4 mA. Right-click on the same plot, and a second cursor will result that can be placed at 6 ms to get a response of −1.33 mA **(A2)** as expected from Example 10.13. Both plots were again placed in the yellow color with a wider line by right-clicking on the curve and choosing **Properties.**

## PROBLEMS

### SECTION 10.2 The Electric Field

1. Find the electric field strength at a point 2 m from a charge of 4 $\mu$C.
2. The electric field strength is 36 newtons/coulomb (N/C) at a point *r* meters from a charge of 0.064 $\mu$C. Find the distance *r*.

### SECTION 10.3 Capacitance

3. Find the capacitance of a parallel plate capacitor if 1400 $\mu$C of charge are deposited on its plates when 20 V are applied across the plates.
4. How much charge is deposited on the plates of a 0.05-$\mu$F capacitor if 45 V are applied across the capacitor?
5. Find the electric field strength between the plates of a parallel plate capacitor if 100 mV are applied across the plates and the plates are 2 mm apart.
6. Repeat Problem 5 if the plates are separated by 4 mils.
7. A 4-$\mu$F parallel plate capacitor has 160 $\mu$C of charge on its plates. If the plates are 5 mm apart, find the electric field strength between the plates.
8. Find the capacitance of a parallel plate capacitor if the area of each plate is 0.075 $m^2$ and the distance between the plates is 1.77 mm. The dielectric is air.
9. Repeat Problem 8 if the dielectric is paraffin-coated paper.
10. Find the distance in mils between the plates of a 2-$\mu$F capacitor if the area of each plate is 0.09 $m^2$ and the dielectric is transformer oil.
11. The capacitance of a capacitor with a dielectric of air is 1200 pF. When a dielectric is inserted between the plates, the capacitance increases to 0.006 $\mu$F. Of what material is the dielectric made?
12. The plates of a parallel plate air capacitor are 0.2 mm apart and have an area of 0.08 $m^2$, and 200 V are applied across the plates.
    - **a.** Determine the capacitance.
    - **b.** Find the electric field intensity between the plates.
    - **c.** Find the charge on each plate if the dielectric is air.
13. A sheet of Bakelite 0.2 mm thick having an area of 0.08 $m^2$ is inserted between the plates of Problem 12.
    - **a.** Find the electric field strength between the plates.
    - **b.** Determine the charge on each plate.
    - **c.** Determine the capacitance.

### SECTION 10.4 Dielectric Strength

14. Find the maximum voltage ratings of the capacitors of Problems 12 and 13 assuming a linear relationship between the breakdown voltage and the thickness of the dielectric.
15. Find the maximum voltage that can be applied across a parallel plate capacitor of 0.006 $\mu$F. The area of one plate is 0.02 $m^2$ and the dielectric is mica. Assume a linear relationship between the dielectric strength and the thickness of the dielectric.

**16.** Find the distance in millimeters between the plates of a parallel plate capacitor if the maximum voltage that can be applied across the capacitor is 1250 V. The dielectric is mica. Assume a linear relationship between the breakdown strength and the thickness of the dielectric.

## SECTION 10.7 Transients in Capacitive Networks: Charging Phase

**17.** For the circuit of Fig. 10.81:
- **a.** Determine the time constant of the circuit.
- **b.** Write the mathematical equation for the voltage $v_C$ following the closing of the switch.
- **c.** Determine the voltage $v_C$ after one, three, and five time constants.
- **d.** Write the equations for the current $i_C$ and the voltage $v_R$.
- **e.** Sketch the waveforms for $v_C$ and $i_C$.

**18.** Repeat Problem 17 for $R = 1\ \text{M}\Omega$, and compare the results.

**19.** For the circuit of Fig. 10.82:
- **a.** Determine the time constant of the circuit.
- **b.** Write the mathematical equation for the voltage $v_C$ following the closing of the switch.
- **c.** Determine $v_C$ after one, three, and five time constants.
- **d.** Write the equations for the current $i_C$ and the voltage $v_R$.
- **e.** Sketch the waveforms for $v_C$ and $i_C$.

**20.** For the circuit of Fig. 10.83:
- **a.** Determine the time constant of the circuit.
- **b.** Write the mathematical equation for the voltage $v_C$ following the closing of the switch.
- **c.** Write the mathematical expression for the current $i_C$ following the closing of the switch.
- **d.** Sketch the waveforms of $v_C$ and $i_C$.

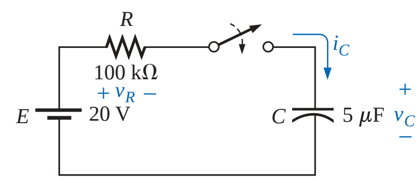



**FIG. 10.81**
*Problems 17 and 18.*

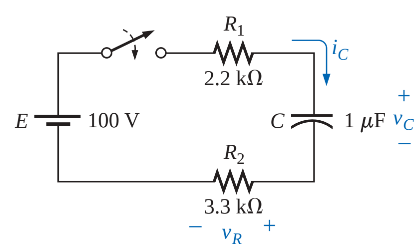



**FIG. 10.82**
*Problem 19.*

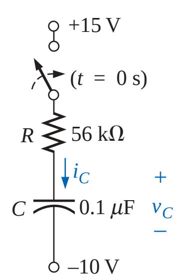



**FIG. 10.83**
*Problem 20.*

## SECTION 10.8 Discharge Phase

**21.** For the circuit of Fig. 10.84:
- **a.** Determine the time constant of the circuit when the switch is thrown into position 1.
- **b.** Find the mathematical expression for the voltage across the capacitor after the switch is thrown into position 1.
- **c.** Determine the mathematical expression for the current following the closing of the switch (position 1).
- **d.** Determine the voltage $v_C$ and the current $i_C$ if the switch is thrown into position 2 at $t = 100$ ms.
- **e.** Determine the mathematical expressions for the voltage $v_C$ and the current $i_C$ if the switch is thrown into position 3 at $t = 200$ ms.
- **f.** Plot the waveforms of $v_C$ and $i_C$ for a period of time extending from $t = 0$ to $t = 300$ ms.

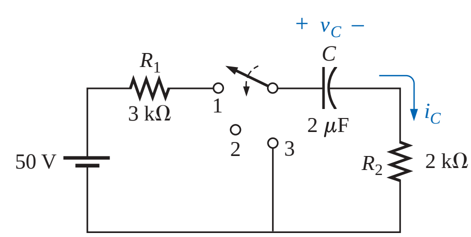



**FIG. 10.84**
*Problems 21 and 22.*

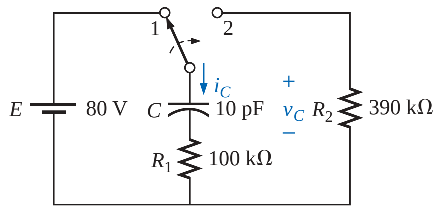



**FIG. 10.85**
*Problem 23.*

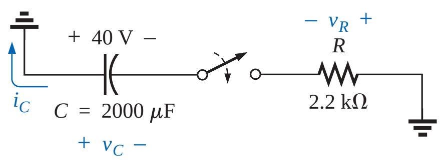



**FIG. 10.86**
*Problem 24.*

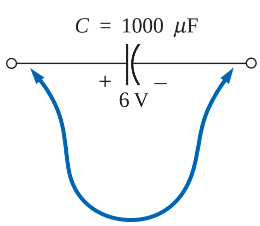



**FIG. 10.87**
*Problems 25 and 29.*

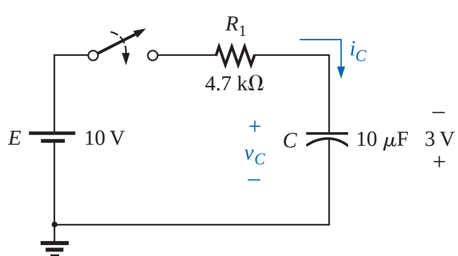



**FIG. 10.88**
*Problem 26.*

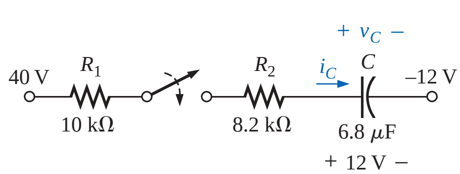



**FIG. 10.89**
*Problem 27.*

**22.** Repeat Problem 21 for a capacitance of 20 μF.

***23.** For the network of Fig. 10.85:
- **a.** Find the mathematical expression for the voltage across the capacitor after the switch is thrown into position 1.
- **b.** Repeat part (a) for the current $i_C$.
- **c.** Find the mathematical expressions for the voltage $v_C$ and current $i_C$ if the switch is thrown into position 2 at a time equal to five time constants of the charging circuit.
- **d.** Plot the waveforms of $v_C$ and $i_C$ for a period of time extending from $t = 0$ to $t = 30\ \mu s$.

**24.** The capacitor of Fig. 10.86 is initially charged to 40 V before the switch is closed. Write the expressions for the voltages $v_C$ and $v_R$ and the current $i_C$ for the decay phase.

**25.** The 1000-μF capacitor of Fig. 10.87 is charged to 6 V. To discharge the capacitor before further use, a wire with a resistance of 0.002 Ω is placed across the capacitor.
- **a.** How long will it take to discharge the capacitor?
- **b.** What is the peak value of the current?
- **c.** Based on the answer to part (b), is a spark expected when contact is made with both ends of the capacitor?

## SECTION 10.9 Initial Values

**26.** The capacitor in Fig. 10.88 is initially charged to 3 V with the polarity shown.
- **a.** Find the mathematical expressions for the voltage $v_C$ and the current $i_C$ when the switch is closed.
- **b.** Sketch the waveforms for $v_C$ and $i_C$.

***27.** The capacitor of Fig. 10.89 is initially charged to 12 V with the polarity shown.
- **a.** Find the mathematical expressions for the voltage $v_C$ and the current $i_C$ when the switch is closed.
- **b.** Sketch the waveforms for $v_C$ and $i_C$.

## SECTION 10.10 Instantaneous Values

**28.** Given the expression $v_C = 8(1 - e^{-t/(20\times10^{-6})})$:
- **a.** Determine $v_C$ after five time constants.
- **b.** Determine $v_C$ after 10 time constants.
- **c.** Determine $v_C$ at $t = 5\ \mu s$.

**29.** For the situation of Problem 25, determine when the discharge current is one-half its maximum value if contact is made at $t = 0$ s.

**30.** For the network of Fig. 10.90, $V_L$ must be 8 V before the system is activated. If the switch is closed at $t = 0$ s, how long will it take for the system to be activated?

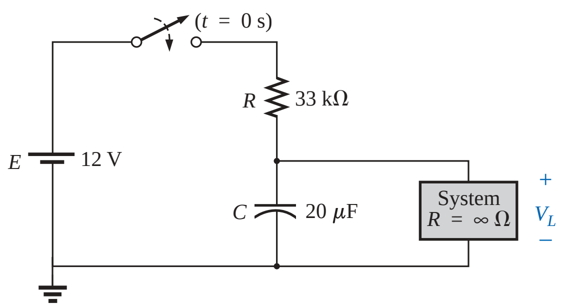



**FIG. 10.90**
*Problem 30.*

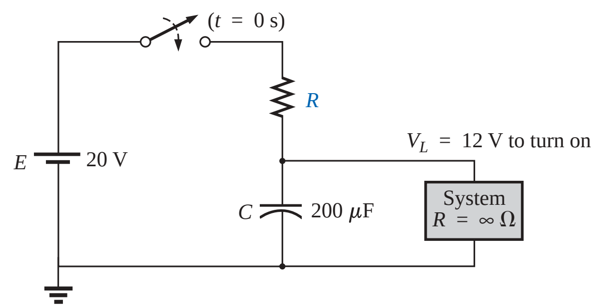



**FIG. 10.91**
*Problem 31.*

***31.** Design the network of Fig. 10.91 such that the system will turn on 10 s after the switch is closed.

**32.** For the circuit of Fig. 10.92:

**a.** Find the time required for $v_C$ to reach 60 V following the closing of the switch.

**b.** Calculate the current $i_C$ at the instant $v_C = 60$ V.

**c.** Determine the power delivered by the source at the instant $t = 2\tau$.

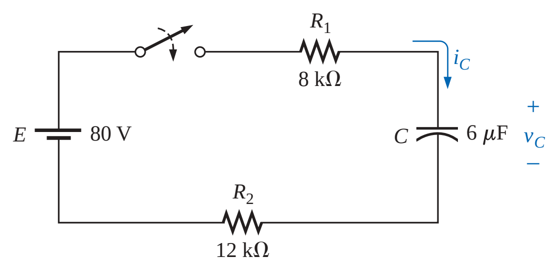



**FIG. 10.92**
*Problem 32.*

***33.** For the network of Fig. 10.93:

**a.** Calculate $v_C$, $i_C$, and $v_{R_1}$ at 0.5 s and 1 s after the switch makes contact with position 1.

**b.** The network sits in position 1 10 min before the switch is moved to position 2. How long after making contact with position 2 will it take for the current $i_C$ to drop to 8 $\mu$A? How much *longer* will it take for $v_C$ to drop to 10 V?

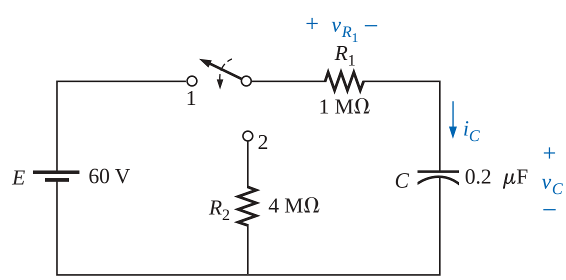



**FIG. 10.93**
*Problem 33.*

**34.** For the system of Fig. 10.94, using a DMM with a 10-MΩ internal resistance in the voltmeter mode:

**a.** Determine the voltmeter reading one time constant after the switch is closed.

**b.** Find the current $i_C$ two time constants after the switch is closed.

**c.** Calculate the time that must pass after the closing of the switch for the voltage $v_C$ to be 50 V.

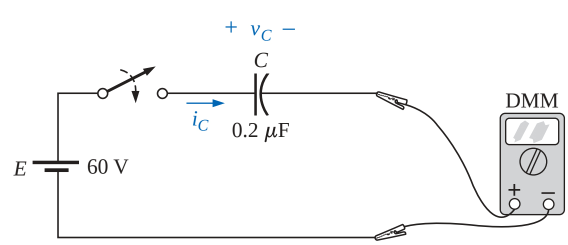



**FIG. 10.94**
*Problem 34.*

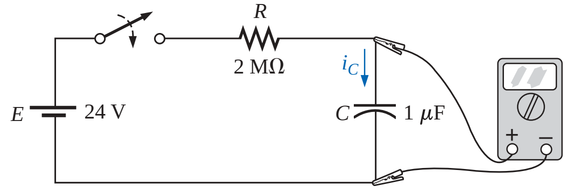



**FIG. 10.95**
*Problem 35.*

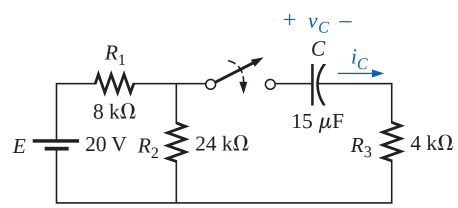



**FIG. 10.96**
*Problem 36.*

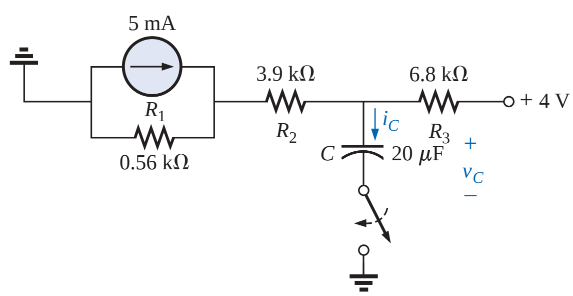



**FIG. 10.97**
*Problems 37 and 55.*

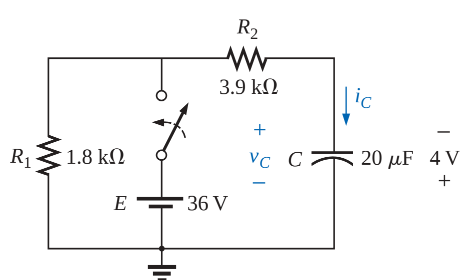



**FIG. 10.98**
*Problem 38.*

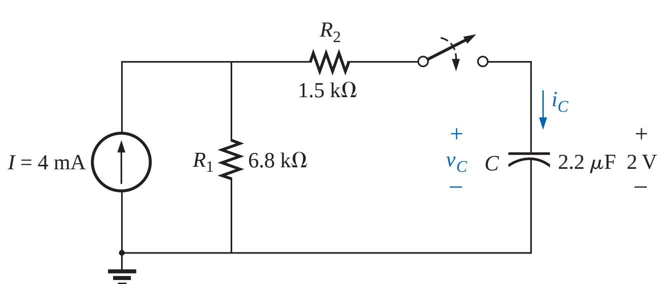



**FIG. 10.99**
*Problem 39.*

## SECTION 10.11 Thévenin Equivalent: $\tau = R_{Th}C$

**35.** For the system of Fig. 10.95, using a DMM with a 10-MΩ internal resistance in the voltmeter mode:
   **a.** Determine the voltmeter reading four time constants after the switch is closed.
   **b.** Find the time that must pass before $i_C$ drops to 3 μA.
   **c.** Find the time that must pass after the closing of the switch for the voltage across the meter to reach 10 V.

**36.** For the circuit of Fig. 10.96:
   **a.** Find the mathematical expressions for the transient behavior of the voltage $v_C$ and the current $i_C$ following the closing of the switch.
   **b.** Sketch the waveforms of $v_C$ and $i_C$.

***37.** Repeat Problem 36 for the circuit of Fig. 10.97.

**38.** The capacitor of Fig. 10.98 is initially charged to 4 V with the polarity shown.
   **a.** Write the mathematical expressions for the voltage $v_C$ and the current $i_C$ when the switch is closed.
   **b.** Sketch the waveforms of $v_C$ and $i_C$.

**39.** The capacitor of Fig. 10.99 is initially charged to 2 V with the polarity shown.
   **a.** Write the mathematical expressions for the voltage $v_C$ and the current $i_C$ when the switch is closed.
   **b.** Sketch the waveforms of $v_C$ and $i_C$.

***40.** The capacitor of Fig. 10.100 is initially charged to 3 V with the polarity shown.

**a.** Write the mathematical expressions for the voltage $v_C$ and the current $i_C$ when the switch is closed.

**b.** Sketch the waveforms of $v_C$ and $i_C$.

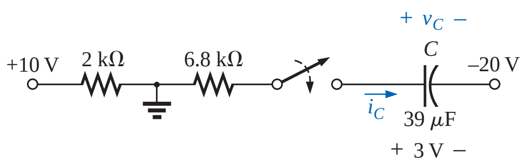



**FIG. 10.100**
*Problem 40.*

## SECTION 10.12 The Current $i_C$

**41.** Find the waveform for the average current if the voltage across a 0.06-μF capacitor is as shown in Fig. 10.101.

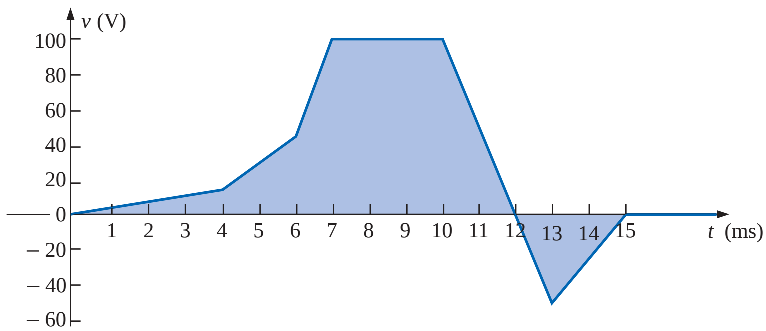



**FIG. 10.101**
*Problem 41.*

**42.** Repeat Problem 41 for the waveform of Fig. 10.102.

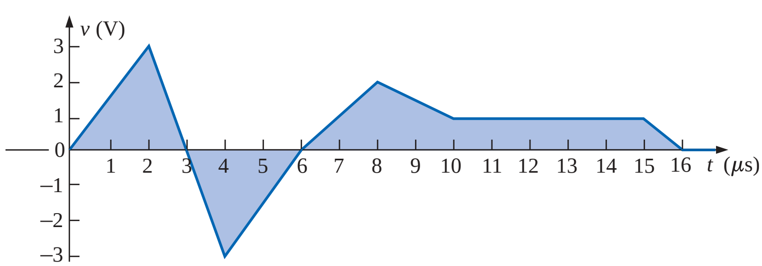



**FIG. 10.102**
*Problem 42.*

***43.** Given the waveform of Fig. 10.103 for the current of a 20-μF capacitor, sketch the waveform of the voltage $v_C$ across the capacitor if $v_C = 0$ V at $t = 0$ s.

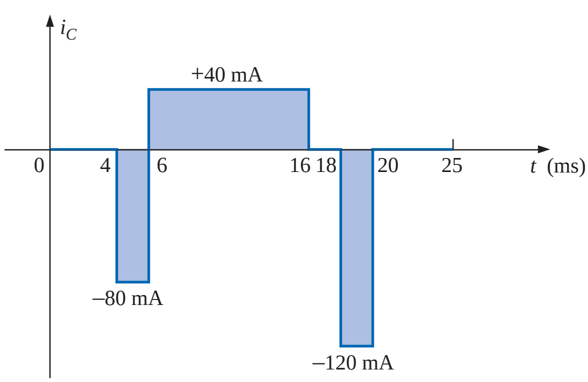



**FIG. 10.103**
*Problem 43.*

## SECTION 10.13 Capacitors in Series and Parallel

**44.** Find the total capacitance $C_T$ between points $a$ and $b$ of the circuits of Fig. 10.104.

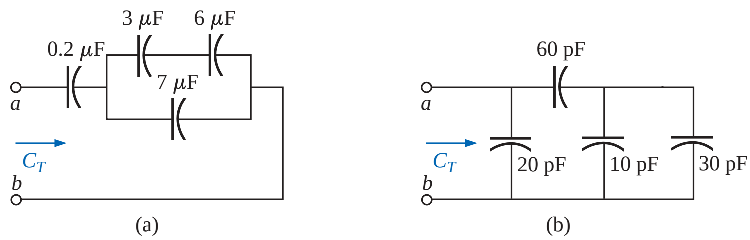



**FIG. 10.104**
*Problem 44.*

**45.** Find the voltage across and charge on each capacitor for the circuits of Fig. 10.105.

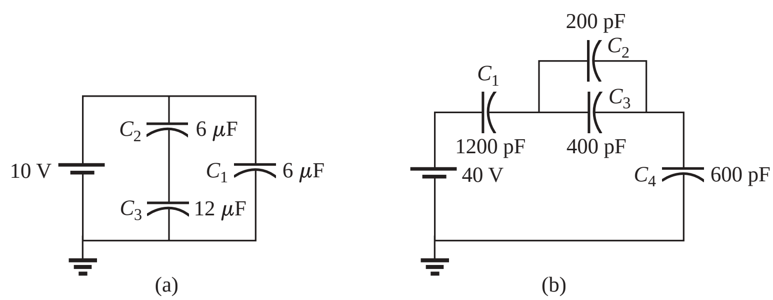



**FIG. 10.105**
*Problem 45.*

***46.** For each configuration of Fig. 10.106, determine the voltage across each capacitor and the charge on each capacitor.

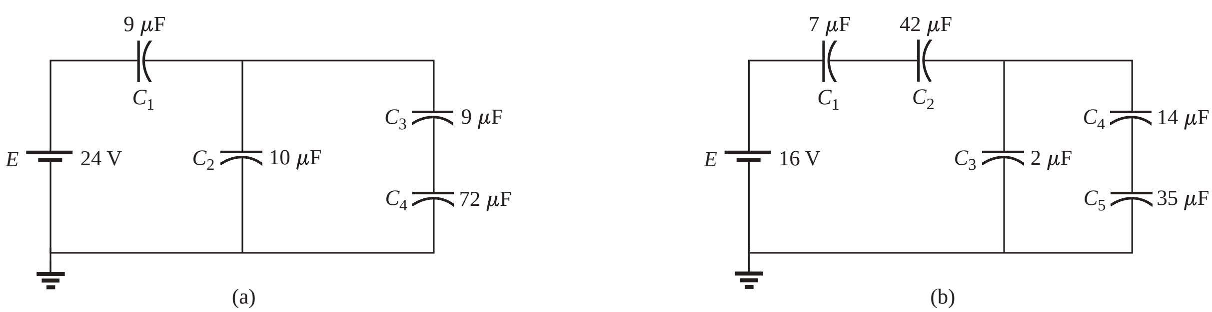



**FIG. 10.106**
*Problem 46.*

***47.** For the network of Fig. 10.107, determine the following 100 ms after the switch is closed:

**a.** $V_{ab}$
**b.** $V_{ac}$
**c.** $V_{cb}$
**d.** $V_{da}$
**e.** If the switch is moved to position 2 one hour later, find the time required for $v_{R_2}$ to drop to 20 V.

**48.** For the circuits of Fig. 10.108, find the voltage across and charge on each capacitor after each capacitor has charged to its final value.

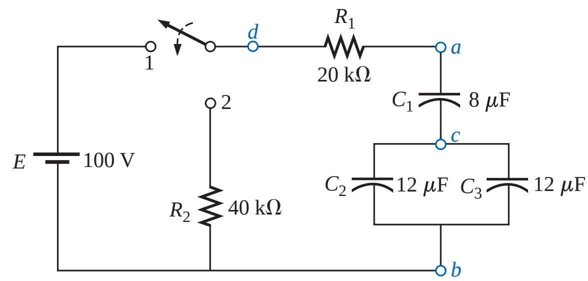



**FIG. 10.107**
*Problem 47.*

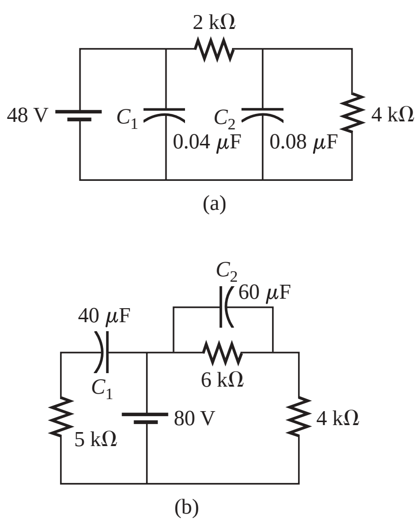



**FIG. 10.108**
*Problem 48.*

## SECTION 10.14 Energy Stored by a Capacitor

**49.** Find the energy stored by a 120-pF capacitor with 12 V across its plates.

**50.** If the energy stored by a 6-$\mu$F capacitor is 1200 J, find the charge $Q$ on each plate of the capacitor.

***51.** An electronic flashgun has a 1000-$\mu$F capacitor that is charged to 100 V.

**a.** How much energy is stored by the capacitor?
**b.** What is the charge on the capacitor?
**c.** When the photographer takes a picture, the flash fires for 1/2000 s. What is the average current through the flashtube?
**d.** Find the power delivered to the flashtube.
**e.** After a picture is taken, the capacitor has to be recharged by a power supply that delivers a maximum current of 10 mA. How long will it take to charge the capacitor?

**52.** For the network of Fig. 10.109:

**a.** Determine the energy stored by each capacitor under steady-state conditions.
**b.** Repeat part (a) if the capacitors are in series.

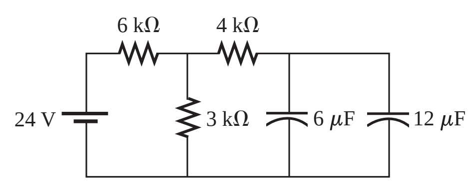



**FIG. 10.109**
*Problem 52.*

## SECTION 10.17 Computer Analysis

### PSpice or Electronics Workbench

**53.** Using schematics:

**a.** Obtain the waveforms for $v_C$ and $i_C$ versus time for the network of Fig. 10.35.

**b.** Obtain the power curve (representing the energy stored by the capacitor over the same time interval), and compare it to the plot of Fig. 10.70.

***54.** Using schematics, obtain the waveforms of $v_C$ and $i_C$ versus time for the network of Fig. 10.49 using the IC option.

**55.** Verify your solution to Problem 37 (Fig. 10.97) using schematics.

### Programming Language (C++, QBASIC, Pascal, etc.)

**56.** Write a QBASIC program to tabulate the voltage $v_C$ and current $i_C$ for the network of Fig. 10.44 for five time constants after the switch is moved to position 1 at $t = 0$ s. Use an increment of $(1/5)\tau$.

***57.** Write a program to write the mathematical expression for the voltage $v_C$ for the network of Fig. 10.52 for any element values when the switch is moved to position 1.

***58.** Given three capacitors in any series-parallel arrangement, write a program to determine the total capacitance. That is, determine the total number of possibilities, and ask the user to identify the configuration and provide the capacitor values. Then calculate the total capacitance.

## GLOSSARY

**Breakdown voltage** Another term for *dielectric strength,* listed below.

**Capacitance** A measure of a capacitor's ability to store charge; measured in farads (F).

**Capacitive time constant** The product of resistance and capacitance that establishes the required time for the charging and discharging phases of a capacitive transient.

**Capacitive transient** The waveforms for the voltage and current of a capacitor that result during the charging and discharging phases.

**Capacitor** A fundamental electrical element having two conducting surfaces separated by an insulating material and having the capacity to store charge on its plates.

**Coulomb's law** An equation relating the force between two like or unlike charges.

**Dielectric** The insulating material between the plates of a capacitor that can have a pronounced effect on the charge stored on the plates of a capacitor.

**Dielectric constant** Another term for *relative permittivity,* listed below.

**Dielectric strength** An indication of the voltage required for unit length to establish conduction in a dielectric.

**Electric field strength** The force acting on a unit positive charge in the region of interest.

**Electric flux lines** Lines drawn to indicate the strength and direction of an electric field in a particular region.

**Fringing** An effect established by flux lines that do not pass directly from one conducting surface to another.

**Leakage current** The current that will result in the total discharge of a capacitor if the capacitor is disconnected from the charging network for a sufficient length of time.

**Permittivity** A measure of how well a dielectric will *permit* the establishment of flux lines within the dielectric.

**Relative permittivity** The permittivity of a material compared to that of air.

**Stray capacitance** Capacitances that exist not through design but simply because two conducting surfaces are relatively close to each other.

**Surge voltage** The maximum voltage that can be applied across a capacitor for very short periods of time.

**Working voltage** The voltage that can be applied across a capacitor for long periods of time without concern for dielectric breakdown.