16

Series-Parallel ac Networks

16.1 INTRODUCTION

In this chapter, we shall utilize the fundamental concepts of the previous chapter to develop a technique for solving **series-parallel ac networks.** A brief review of Chapter 7 may be helpful before considering these networks since the approach here will be quite similar to that undertaken earlier. The circuits to be discussed will have only one source of energy, either potential or current. Networks with two or more sources will be considered in Chapters 17 and 18, using methods previously described for dc circuits.

In general, when working with series-parallel ac networks, consider the following approach:

1. ***Redraw the network, employing block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.***
2. ***Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time- and energy-saving shortcuts. In some cases a lengthy, drawn-out analysis may not be necessary. A single application of a fundamental law of circuit analysis may result in the desired solution.***
3. ***After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations. In most cases, work back from the obvious series and parallel combinations to the source to determine the total impedance of the network. The source current can then be determined, and the path back to specific unknowns can be defined. As you progress back to the source, continually define those unknowns that have not been lost in the reduction process. It will save time when you have to work back through the network to find specific quantities.***

4. ***When you have arrived at a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the circuit. If not, either solve the network using another approach, or check over your work very carefully. At this point a computer solution can be an invaluable asset in the validation process.***

16.2 ILLUSTRATIVE EXAMPLES

EXAMPLE 16.1 For the network of Fig. 16.1:

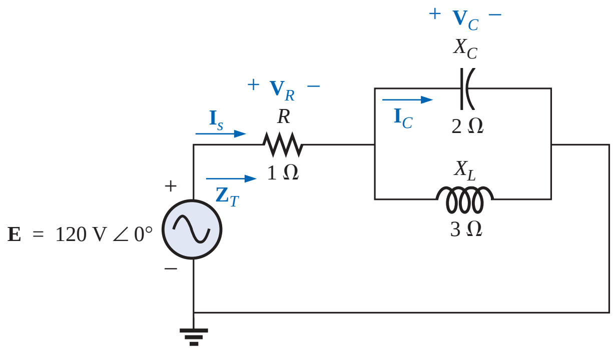


FIG. 16.1
Example 16.1.

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.
e. Compute the power delivered.
f. Find F_p of the network.

Solutions:

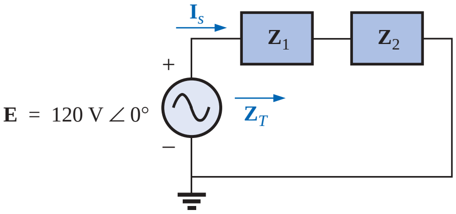


FIG. 16.2
Network of Fig. 16.1 after assigning the block impedances.

a. As suggested in the introduction, the network has been redrawn with block impedances, as shown in Fig. 16.2. The impedance $\mathbf{Z}_1$ is simply the resistor R of 1 Ω, and $\mathbf{Z}_2$ is the parallel combination of X_C and X_L. The network now clearly reveals that it is fundamentally a series circuit, suggesting a direct path toward the total impedance and the source current. As noted in the introduction, for many such problems you must work back to the source to find first the total impedance and then the source current. When the unknown quantities are found in terms of these subscripted impedances, the numerical values can then be substituted to find the magnitude and phase angle of the unknown. In other words, try to find the desired solution solely in terms of the subscripted impedances before substituting numbers. This approach will usually enhance the clarity of the chosen path toward a solution while saving time and preventing careless calculation errors. Note also in Fig. 16.2 that all the unknown quantities except $\mathbf{I}_C$ have been preserved, meaning that we can use Fig. 16.2 to determine these quantities rather than having to return to the more complex network of Fig. 16.1.

The total impedance is defined by

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

with

$$\mathbf{Z}_1 = R \angle 0° = 1\ \Omega \angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C \angle -90°)(X_L \angle 90°)}{-j\, X_C + j\, X_L} = \frac{(2\ \Omega \angle -90°)(3\ \Omega \angle 90°)}{-j\, 2\ \Omega + j\, 3\ \Omega}$$

$$= \frac{6\ \Omega \angle 0°}{j\, 1} = \frac{6\ \Omega \angle 0°}{1 \angle 90°} = 6\ \Omega \angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\, 6\ \Omega = \mathbf{6.08\ \Omega \angle -80.54°}$$

b. $\mathbf{I}_s = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{120\text{ V} \angle 0°}{6.08\ \Omega \angle -80.54°} = \mathbf{19.74\ A \angle 80.54°}$

c. Referring to Fig. 16.2, we find that $\mathbf{V}_R$ and $\mathbf{V}_C$ can be found by a direct application of Ohm's law:

$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\text{ A} \angle 80.54°)(1\ \Omega \angle 0°) = \mathbf{19.74\ V \angle 80.54°}$$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\text{ A} \angle 80.54°)(6\ \Omega \angle -90°)$$

$$= \mathbf{118.44\ V \angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\text{ V} \angle -9.46°}{2\ \Omega \angle -90°} = \mathbf{59.22\ A \angle 80.54°}$$

e. $P_{\text{del}} = I_s^2 R = (19.74\text{ A})^2(1\ \Omega) = \mathbf{389.67\ W}$

f. $F_p = \cos\theta = \cos 80.54° = \mathbf{0.164\ leading}$

The fact that the total impedance has a negative phase angle (revealing that $\mathbf{I}_s$ leads $\mathbf{E}$) is a clear indication that the network is capacitive in nature and therefore has a leading power factor. The fact that the network is capacitive can be determined from the original network by first realizing that, for the parallel *L-C* elements, the smaller impedance predominates and results in an *R-C* network.

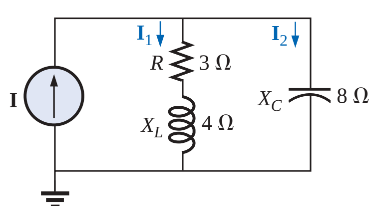


FIG. 16.3
Example 16.2.

EXAMPLE 16.2 For the network of Fig. 16.3:
a. If **I** is 50 A ∠30°, calculate $\mathbf{I}_1$ using the current divider rule.
b. Repeat part (a) for $\mathbf{I}_2$.
c. Verify Kirchhoff's current law at one node.

Solutions:

a. Redrawing the circuit as in Fig. 16.4, we have

$$\mathbf{Z}_1 = R + j\, X_L = 3\ \Omega + j\, 4\ \Omega = 5\ \Omega \angle 53.13°$$

$$\mathbf{Z}_2 = -j\, X_C = -j\, 8\ \Omega = 8\ \Omega \angle -90°$$

Using the current divider rule yields

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega \angle -90°)(50\text{ A} \angle 30°)}{(-j\, 8\ \Omega) + (3\ \Omega + j\, 4\ \Omega)} = \frac{400 \angle -60°}{3 - j\, 4}$$

$$= \frac{400 \angle -60°}{5 \angle -53.13°} = \mathbf{80\ A \angle -6.87°}$$

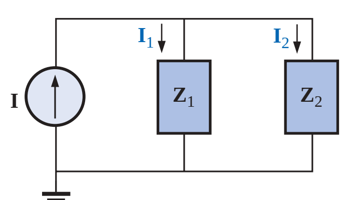


FIG. 16.4
Network of Fig. 16.3 after assigning the block impedances.

b. $\mathbf{I}_2 = \dfrac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \dfrac{(5\ \Omega\ \angle 53.13°)(50\ \text{A}\ \angle 30°)}{5\ \Omega\ \angle -53.13°} = \dfrac{250\ \angle 83.13°}{5\ \angle -53.13°}$

$= \mathbf{50\ A\ \angle 136.26°}$

c. $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$

$$\begin{aligned} 50\ \text{A}\ \angle 30° &= 80\ \text{A}\ \angle -6.87° + 50\ \text{A}\ \angle 136.26° \\ &= (79.43 - j\,9.57) + (-36.12 + j\,34.57) \\ &= 43.31 + j\,25.0 \\ 50\ \text{A}\ \angle 30° &= 50\ \text{A}\ \angle 30° \quad \text{(checks)} \end{aligned}$$

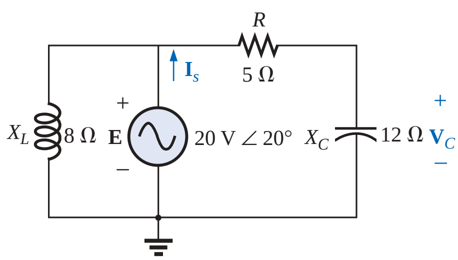


FIG. 16.5
Example 16.3.

EXAMPLE 16.3 For the network of Fig. 16.5:

a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
b. Calculate the current $\mathbf{I}_s$.

Solutions:

a. The network is redrawn as shown in Fig. 16.6, with

$$\begin{aligned} \mathbf{Z}_1 &= 5\ \Omega = 5\ \Omega\ \angle 0° \\ \mathbf{Z}_2 &= -j\,12\ \Omega = 12\ \Omega\ \angle -90° \\ \mathbf{Z}_3 &= +j\,8\ \Omega = 8\ \Omega\ \angle 90° \end{aligned}$$

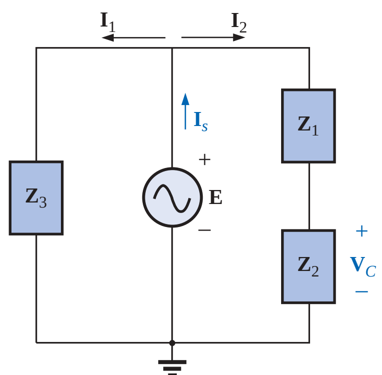


FIG. 16.6
Network of Fig. 16.5 after assigning the block impedances.

Since $\mathbf{V}_C$ is desired, we will not combine R and X_C into a single block impedance. Note also how Fig. 16.6 clearly reveals that **E** is the total voltage across the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$, permitting the use of the voltage divider rule to calculate $\mathbf{V}_C$. In addition, note that all the currents necessary to determine $\mathbf{I}_s$ have been preserved in Fig. 16.6, revealing that there is no need to ever return to the network of Fig. 16.5—everything is defined by Fig. 16.6.

$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega\ \angle -90°)(20\ \text{V}\ \angle 20°)}{5\ \Omega - j\,12\ \Omega} = \frac{240\ \text{V}\ \angle -70°}{13\ \angle -67.38°}$$

$$= \mathbf{18.46\ V\ \angle -2.62°}$$

b. $\mathbf{I}_1 = \dfrac{\mathbf{E}}{\mathbf{Z}_3} = \dfrac{20\ \text{V}\ \angle 20°}{8\ \Omega\ \angle 90°} = 2.5\ \text{A}\ \angle -70°$

$\mathbf{I}_2 = \dfrac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \dfrac{20\ \text{V}\ \angle 20°}{13\ \Omega\ \angle -67.38°} = 1.54\ \text{A}\ \angle 87.38°$

and

$$\begin{aligned} \mathbf{I}_s &= \mathbf{I}_1 + \mathbf{I}_2 \\ &= 2.5\ \text{A}\ \angle -70° + 1.54\ \text{A}\ \angle 87.38° \\ &= (0.86 - j\,2.35) + (0.07 + j\,1.54) \\ \mathbf{I}_s &= 0.93 - j\,0.81 = \mathbf{1.23\ A\ \angle -41.05°} \end{aligned}$$

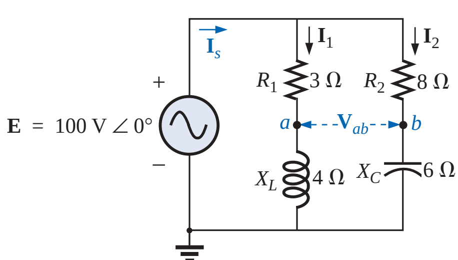


FIG. 16.7
Example 16.4.

EXAMPLE 16.4 For Fig. 16.7:

a. Calculate the current $\mathbf{I}_s$.
b. Find the voltage $\mathbf{V}_{ab}$.

Solutions:

a. Redrawing the circuit as in Fig. 16.8, we obtain

$$\begin{aligned} \mathbf{Z}_1 &= R_1 + j\,X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega\ \angle 53.13° \\ \mathbf{Z}_2 &= R_2 - j\,X_C = 8\ \Omega - j\,6\ \Omega = 10\ \Omega\ \angle -36.87° \end{aligned}$$

In this case the voltage $\mathbf{V}_{ab}$ is lost in the redrawn network, but the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ remain defined for future calculations necessary to determine $\mathbf{V}_{ab}$. Figure 16.8 clearly reveals that the total impedance can be found using the equation for two parallel impedances:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega\ \angle 53.13^\circ)(10\ \Omega\ \angle -36.87^\circ)}{(3\ \Omega + j\,4\ \Omega) + (8\ \Omega - j\,6\ \Omega)}$$

$$= \frac{50\ \Omega\ \angle 16.26^\circ}{11 - j\,2} = \frac{50\ \Omega\ \angle 16.26^\circ}{11.18\ \angle -10.30^\circ}$$

$$= \mathbf{4.472\ \Omega\ \angle 26.56^\circ}$$

and $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V}\ \angle 0^\circ}{4.472\ \Omega\ \angle 26.56^\circ} = \mathbf{22.36\ A\ \angle -26.56^\circ}$$

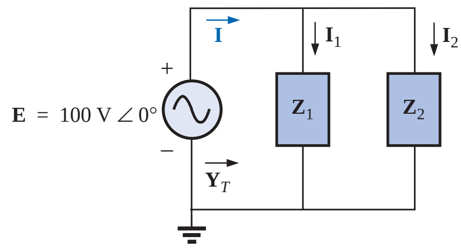


FIG. 16.8
Network of Fig. 16.7 after assigning the block impedances.

b. By Ohm's law,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{100\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = \mathbf{20\ A\ \angle -53.13^\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{100\ \text{V}\ \angle 0^\circ}{10\ \Omega\ \angle -36.87^\circ} = \mathbf{10\ A\ \angle 36.87^\circ}$$

Returning to Fig. 16.7, we have

$$\mathbf{V}_{R_1} = \mathbf{I}_1\mathbf{Z}_{R_1} = (20\ \text{A}\ \angle -53.13^\circ)(3\ \Omega\ \angle 0^\circ) = \mathbf{60\ V\ \angle -53.13^\circ}$$

$$\mathbf{V}_{R_2} = \mathbf{I}_1\mathbf{Z}_{R_2} = (10\ \text{A}\ \angle +36.87^\circ)(8\ \Omega\ \angle 0^\circ) = \mathbf{80\ V\ \angle +36.87^\circ}$$

Instead of using the two steps just shown, we could have determined $\mathbf{V}_{R_1}$ or $\mathbf{V}_{R_2}$ in one step using the voltage divider rule:

$$\mathbf{V}_{R_1} = \frac{(3\ \Omega\ \angle 0^\circ)(100\ \text{V}\ \angle 0^\circ)}{3\ \Omega\ \angle 0^\circ + 4\ \Omega\ \angle 90^\circ} = \frac{300\ \text{V}\ \angle 0^\circ}{5\ \angle 53.13^\circ} = \mathbf{60\ V\ \angle -53.13^\circ}$$

To find $\mathbf{V}_{ab}$, Kirchhoff's voltage law must be applied around the loop (Fig.16.9) consisting of the 3-Ω and 8-Ω resistors. By Kirchhoff's voltage law,

$$\mathbf{V}_{ab} + \mathbf{V}_{R_1} - \mathbf{V}_{R_2} = 0$$

or

$$\begin{aligned}\mathbf{V}_{ab} &= \mathbf{V}_{R_2} - \mathbf{V}_{R_1}\\ &= 80\ \text{V}\ \angle 36.87^\circ - 60\ \text{V}\ \angle -53.13^\circ\\ &= (64 + j\,48) - (36 - j\,48)\\ &= 28 + j\,96\\ \mathbf{V}_{ab} &= \mathbf{100\ V\ \angle 73.74^\circ}\end{aligned}$$

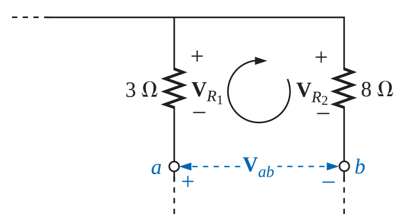


FIG. 16.9
Determining the voltage $\mathbf{V}_{ab}$ *for the network of Fig. 16.7.*

EXAMPLE 16.5 The network of Fig. 16.10 is frequently encountered in the analysis of transistor networks. The transistor equivalent circuit includes a current source **I** and an output impedance R_o. The resistor R_C is a biasing resistor to establish specific dc conditions, and the resistor R_i represents the loading of the next stage. The coupling capacitor is designed to be an open circuit for dc and to have as low an impedance as possible for the frequencies of interest to ensure that $\mathbf{V}_L$ is a maximum value. The frequency range of the example includes the entire audio (hearing) spectrum from 100 Hz to 20 kHz. The purpose of the example is to demonstrate that, for the full audio range, the effect of the capacitor can be ignored. It performs its function as a dc blocking agent but permits the ac to pass through with little disturbance.

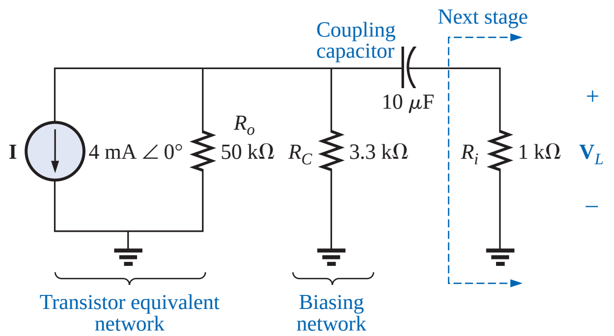


FIG. 16.10
Basic transistor amplifier.

a. Determine $\mathbf{V}_L$ for the network of Fig. 16.10 at a frequency of 100 Hz.
b. Repeat part (a) at a frequency of 20 kHz.
c. Compare the results of parts (a) and (b).

Solutions:

a. The network is redrawn with subscripted impedances in Fig. 16.11.

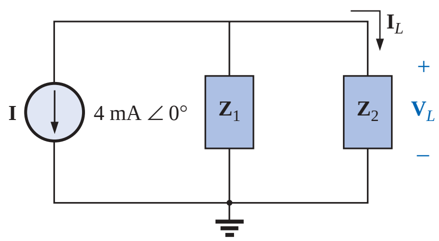


FIG. 16.11
Network of Fig. 16.10 following the assignment of the block impedances.

$$\mathbf{Z}_1 = 50\text{ k}\Omega \angle 0° \,\|\, 3.3\text{ k}\Omega \angle 0° = 3.096\text{ k}\Omega \angle 0°$$
$$\mathbf{Z}_2 = R_i - j\,X_C$$

$$\text{At } f = 100\text{ Hz: } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ Hz})(10\ \mu\text{F})} = 159.16\ \Omega$$

and $\mathbf{Z}_2 = 1\text{ k}\Omega - j\,159.16\ \Omega$

Current divider rule:

$$\mathbf{I}_L = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3.096\text{ k}\Omega \angle 0°)(4\text{ mA} \angle 0°)}{3.096\text{ k}\Omega + 1\text{ k}\Omega - j\,159.16\ \Omega}$$
$$= \frac{12.384\text{ A} \angle 0°}{4096 - j\,159.16} = \frac{12.384\text{ A} \angle 0°}{4099 \angle -2.225°}$$
$$= 3.021\text{ mA} \angle 2.225°$$

and
$$\mathbf{V}_L = \mathbf{I}_L\mathbf{Z}_R$$
$$= (3.021\text{ mA} \angle 2.225°)(1\text{ k}\Omega \angle 0°)$$
$$= \mathbf{3.021\ V \angle 2.225°}$$

b. $$\text{At } f = 20\text{ kHz: } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(20\text{ kHz})(10\ \mu\text{F})} = 0.796\ \Omega$$

Note the dramatic change in X_C with frequency. Obviously, the higher the frequency, the better the short-circuit approximation for X_C for ac conditions.

$$\mathbf{Z}_2 = 1\text{ k}\Omega - j\,0.796\ \Omega$$

Current divider rule:

$$\mathbf{I}_L = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(3.096\text{ k}\Omega \angle 0°)(4\text{ mA} \angle 0°)}{3.096\text{ k}\Omega + 1\text{ k}\Omega - j\,0.796\ \Omega}$$
$$= \frac{12.384\text{ A} \angle 0°}{4096 - j\,0.796\ \Omega} = \frac{12.384\text{ A} \angle 0°}{4096 \angle -0.011°}$$
$$= 3.023\text{ mA} \angle 0.011°$$

and $$\mathbf{V}_L = \mathbf{I}_L\mathbf{Z}_R = (3.023\text{ mA }\angle 0.011^\circ)(1\text{ k}\Omega\ \angle 0^\circ) = \mathbf{3.023\text{ V }\angle 0.011^\circ}$$

c. The results clearly indicate that the capacitor had little effect on the frequencies of interest. In addition, note that most of the supply current reached the load for the typical parameters employed.

EXAMPLE 16.6 For the network of Fig. 16.12:

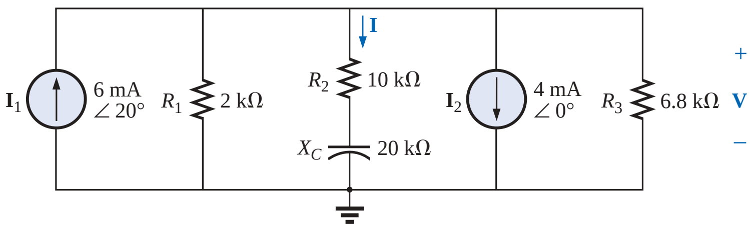


FIG. 16.12
Example 16.6.

a. Determine the current **I**.
b. Find the voltage **V**.

Solutions:

a. The rules for parallel current sources are the same for dc and ac networks. That is, the equivalent current source is their sum or difference (as phasors). Therefore,

$$\begin{aligned}\mathbf{I}_T &= 6\text{ mA }\angle 20^\circ - 4\text{ mA }\angle 0^\circ\\ &= 5.638\text{ mA} + j\,2.052\text{ mA} - 4\text{ mA}\\ &= 1.638\text{ mA} + j\,2.052\text{ mA}\\ &= 2.626\text{ mA }\angle 51.402^\circ\end{aligned}$$

Redrawing the network using block impedances will result in the network of Fig. 16.13 where

$$\mathbf{Z}_1 = 2\text{ k}\Omega\ \angle 0^\circ \parallel 6.8\text{ k}\Omega\ \angle 0^\circ = 1.545\text{ k}\Omega\ \angle 0^\circ$$

and $$\mathbf{Z}_2 = 10\text{ k}\Omega - j\,20\text{ k}\Omega = 22.361\text{ k}\Omega\ \angle -63.435^\circ$$

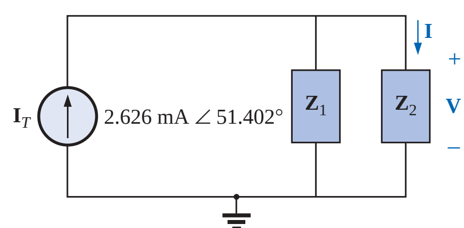


FIG. 16.13
Network of Fig. 16.12 following the assignment of the subscripted impedances.

Note that **I** and **V** are still defined in Fig. 16.13.

Current divider rule:

$$\mathbf{I} = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(1.545\text{ k}\Omega\ \angle 0^\circ)(2.626\text{ mA }\angle 51.402^\circ)}{1.545\text{ k}\Omega + 10\text{ k}\Omega - j\,20\text{ k}\Omega}$$

$$= \frac{4.057\text{ A }\angle 51.402^\circ}{11.545\times 10^3 - j\,20\times 10^3} = \frac{4.057\text{ A }\angle 51.402^\circ}{23.093\times 10^3\angle -60.004^\circ}$$

$$= \mathbf{0.176\text{ mA }\angle 111.406^\circ}$$

b. $$\mathbf{V} = \mathbf{I}\mathbf{Z}_2 = (0.176\text{ mA }\angle 111.406^\circ)(22.36\text{ k}\Omega\ \angle -63.435^\circ) = \mathbf{3.936\text{ V }\angle 47.971^\circ}$$

EXAMPLE 16.7 For the network of Fig. 16.14:

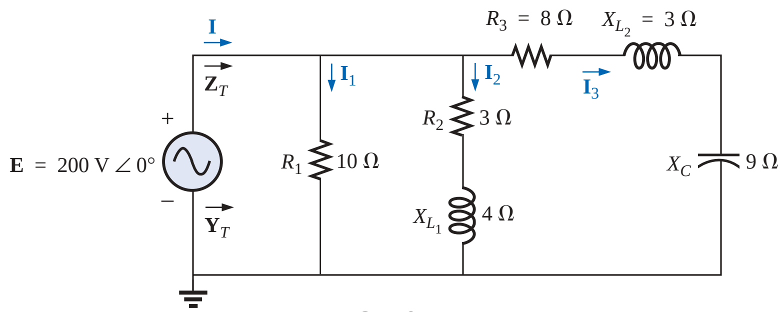


FIG. 16.14
Example 16.7.

a. Compute **I**.
b. Find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
c. Verify Kirchhoff's current law by showing that

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

d. Find the total impedance of the circuit.

Solutions:

a. Redrawing the circuit as in Fig. 16.15 reveals a strictly parallel network where

$$\mathbf{Z}_1 = R_1 = 10\ \Omega\ \angle 0°$$
$$\mathbf{Z}_2 = R_2 + j\,X_{L_1} = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_3 = R_3 + j\,X_{L_2} - j\,X_C = 8\ \Omega + j\,3\ \Omega - j\,9\ \Omega = 8\ \Omega - j\,6\ \Omega$$

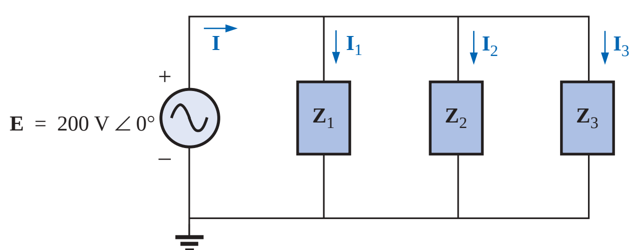


FIG. 16.15
Network of Fig. 16.14 following the assignment of the subscripted impedances.

The total admittance is

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3$$
$$= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{10\ \Omega} + \frac{1}{3\ \Omega + j\,4\ \Omega} + \frac{1}{8\ \Omega - j\,6\ \Omega}$$
$$= 0.1\ \text{S} + \frac{1}{5\ \Omega\ \angle 53.13°} + \frac{1}{10\ \Omega\ \angle -36.87°}$$
$$= 0.1\ \text{S} + 0.2\ \text{S}\ \angle -53.13° + 0.1\ \text{S}\ \angle 36.87°$$
$$= 0.1\ \text{S} + 0.12\ \text{S} - j\,0.16\ \text{S} + 0.08\ \text{S} + j\,0.06\ \text{S}$$
$$= 0.3\ \text{S} - j\,0.1\ \text{S} = 0.316\ \text{S}\ \angle -18.435°$$

Calculator The above mathematical exercise presents an excellent opportunity to demonstrate the power of some of today's calculators. Using the TI-86, the above operation would appear as follows on the display:

1/(10,0)+1/(3,4)+1/(8,−6)

with the result:

(300.000E−3,−100.000E−3)

Converting to polar form:

Ans ► Pol
(316.228E−3∠−18.435E0)

The current **I**:

$$\mathbf{I} = \mathbf{E}\mathbf{Y}_T = (200 \text{ V} \angle 0°)(0.316 \text{ S} \angle -18.435°)$$
$$= \mathbf{63.2 \text{ A} \angle -18.435°}$$

b. Since the voltage is the same across parallel branches,

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{200 \text{ V} \angle 0°}{10 \ \Omega \angle 0°} = \mathbf{20 \text{ A} \angle 0°}$$
$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \frac{200 \text{ V} \angle 0°}{5 \ \Omega \angle 53.13°} = \mathbf{40 \text{ A} \angle -53.13°}$$
$$\mathbf{I}_3 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{200 \text{ V} \angle 0°}{10 \ \Omega \angle -36.87°} = \mathbf{20 \text{ A} \angle +36.87°}$$

c. $$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$
$$60 - j\,20 = 20 \angle 0° + 40 \angle -53.13° + 20 \angle +36.87°$$
$$= (20 + j\,0) + (24 - j\,32) + (16 + j\,12)$$
$$60 - j\,20 = 60 - j\,20 \quad \text{(checks)}$$

d. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.316 \text{ S} \angle -18.435°}$$
$$= \mathbf{3.165 \ \Omega \angle 18.435°}$$

EXAMPLE 16.8 For the network of Fig. 16.16:

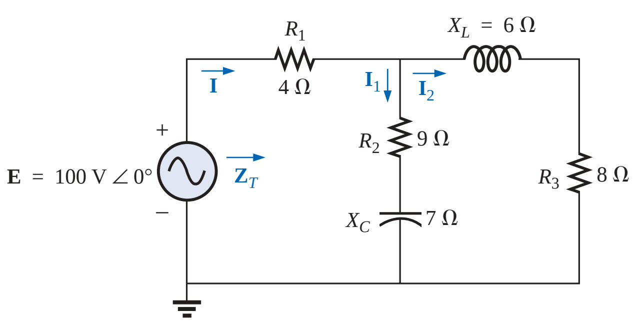


FIG. 16.16
Example 16.8.

a. Calculate the total impedance $\mathbf{Z}_T$.
b. Compute **I**.
c. Find the total power factor.
d. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$.
e. Find the average power delivered to the circuit.

Solutions:

a. Redrawing the circuit as in Fig. 16.17, we have

$$\mathbf{Z}_1 = R_1 = 4\ \Omega \angle 0°$$
$$\mathbf{Z}_2 = R_2 - j\,X_C = 9\ \Omega - j\,7\ \Omega = 11.40\ \Omega \angle -37.87°$$
$$\mathbf{Z}_3 = R_3 + j\,X_L = 8\ \Omega + j\,6\ \Omega = 10\ \Omega \angle +36.87°$$

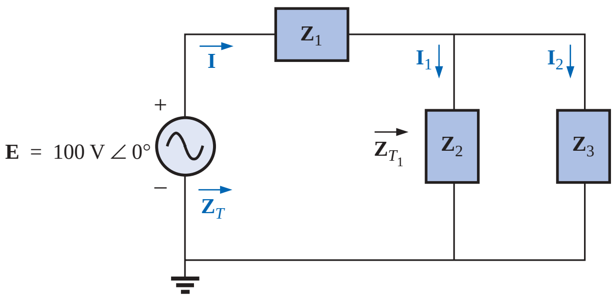


FIG. 16.17

Network of Fig. 16.16 following the assignment of the subscripted impedances.

Notice that all the desired quantities were conserved in the redrawn network. The total impedance:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_{T_1}$$
$$= \mathbf{Z}_1 + \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3}$$
$$= \frac{4\ \Omega + (11.4\ \Omega \angle -37.87°)(10\ \Omega \angle 36.87°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$
$$= 4\ \Omega + \frac{114\ \Omega \angle -1.00°}{17.03 \angle -3.37°} = 4\ \Omega + 6.69\ \Omega \angle 2.37°$$
$$= 4\ \Omega + 6.68\ \Omega + j\,0.28\ \Omega = 10.68\ \Omega + j\,0.28\ \Omega$$
$$\mathbf{Z}_T = \mathbf{10.684\ \Omega \angle 1.5°}$$

Mathcad Solution: The complex algebra just presented in detail provides an excellent opportunity to practice our Mathcad skills with complex numbers. Remember that the *j* must follow the numerical value of the imaginary part and **is not** multiplied by the numerical value. Simply type in the numerical value and then *j*. Also recall that unless you make a global change in the format, an *i* will appear with the imaginary part of the solution. As shown in Fig. 16.18, each impedance is first defined with **Shift:.** Then each impedance is entered in sequence on the same line or succeeding lines. Next, the equation for the total impedance is defined using the brackets to ensure that the bottom summation is carried out before the division and also to provide the same format to the equation as appearing above. Then enter **ZT,** select the equal sign key, and the rectangular form for the total impedance will appear as shown.

The polar form can be obtained by first going to the **Calculator** toolbar to obtain the magnitude operation and inserting **ZT** as shown in Fig. 16.18. Then selecting the equal sign will result in the magnitude of 10.693 Ω. The angle is obtained by first going to the **Greek** toolbar and picking up theta, entering **T,** and defining the variable. The **π** comes from the **Calculator** toolbar, and the **arg( )** from **Insert-*f(x)*-**

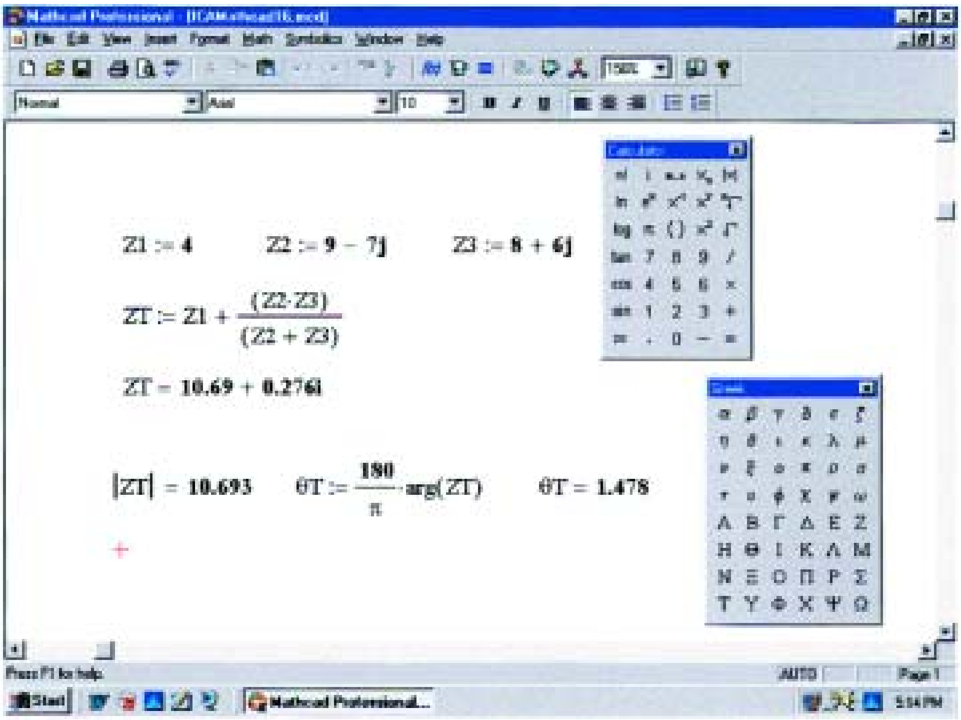


FIG. 16.18

Using Mathcad to determine the total impedance for the network of Fig.16.16.

Function Name-arg. Finally the variable is written again and the equal sign selected to obtain an angle of 1.478°. The computer solution of 10.693 Ω ∠1.478° is an excellent verification of the theoretical solution of 10.684 Ω ∠1.5°.

Calculator Another opportunity to demonstrate the versatility of the calculator! For the above operation, however, one must be aware of the priority of the mathematical operations, as demonstrated in the calculator display below. In most cases, the operations are performed in the same order they would be performed longhand.

(4,0)+((9,−7)+(8,6))$^{-1}$* (11.4∠−37.87)(10∠36.87) (ENTER)
(10.689E0,276.413E−3)
Ans ▶ Pol (ENTER)
(10.692E0∠1.481E0)

b. $\mathbf{I} = \dfrac{\mathbf{E}}{\mathbf{Z}_T} = \dfrac{100\text{ V} \angle 0°}{10.684\ \Omega \angle 1.5°} = \mathbf{9.36\text{ A} \angle -1.5°}$

c. $F_p = \cos\theta_T = \dfrac{R}{Z_T} = \dfrac{10.68\ \Omega}{10.684\ \Omega} \cong \mathbf{1}$

(essentially resistive, which is interesting, considering the complexity of the network)

d. Current divider rule:

$$\mathbf{I}_2 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(11.40\ \Omega\ \angle -37.87°)(9.36\ \text{A}\ \angle -1.5°)}{(9\ \Omega - j\,7\ \Omega) + (8\ \Omega + j\,6\ \Omega)}$$
$$= \frac{106.7\ \text{A}\ \angle -39.37°}{17 - j\,1} = \frac{106.7\ \text{A}\ \angle -39.37°}{17.03\ \angle -3.37°}$$
$$\mathbf{I}_2 = \mathbf{6.27\ A\ \angle -36°}$$

Applying Kirchhoff's current law (rather than another application of the current divider rule) yields

$$\mathbf{I}_1 = \mathbf{I} - \mathbf{I}_2$$

or

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$$
$$= (9.36\ \text{A}\ \angle -1.5°) - (6.27\ \text{A}\ \angle -36°)$$
$$= (9.36\ \text{A} - j\,0.25\ \text{A}) - (5.07\ \text{A} - j\,3.69\ \text{A})$$
$$\mathbf{I}_1 = 4.29\ \text{A} + j\,3.44\ \text{A} = \mathbf{5.5\ A\ \angle 38.72°}$$

e. $P_T = EI \cos \theta_T$
$= (100\ \text{V})(9.36\ \text{A}) \cos 1.5°$
$= (936)(0.99966)$
$P_T = \mathbf{935.68\ W}$

16.3 LADDER NETWORKS

Ladder networks were discussed in some detail in Chapter 7. This section will simply apply the first method described in Section 7.3 to the general sinusoidal ac ladder network of Fig. 16.19. The current $\mathbf{I}_6$ is desired.

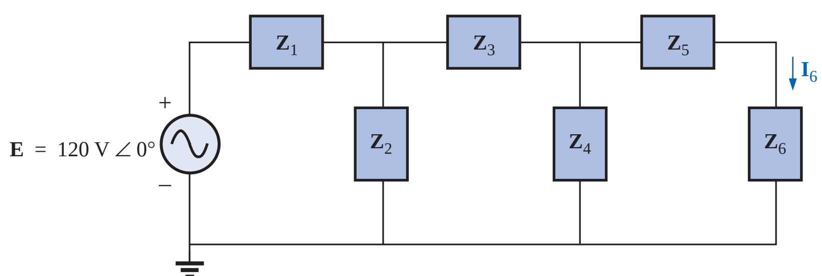


FIG. 16.19
Ladder network.

Impedances $\mathbf{Z}_T$, $\mathbf{Z}'_T$, and $\mathbf{Z}''_T$ and currents $\mathbf{I}_1$ and $\mathbf{I}_3$ are defined in Fig. 16.20:

$$\mathbf{Z}''_T = \mathbf{Z}_5 + \mathbf{Z}_6$$

and

$$\mathbf{Z}'_T = \mathbf{Z}_3 + \mathbf{Z}_4 \parallel \mathbf{Z}''_T$$

with

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}'_T$$

Then

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}$$

and

$$\mathbf{I}_3 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}'_T}$$

with

$$\mathbf{I}_6 = \frac{\mathbf{Z}_4\mathbf{I}_3}{\mathbf{Z}_4 + \mathbf{Z}''_T}$$

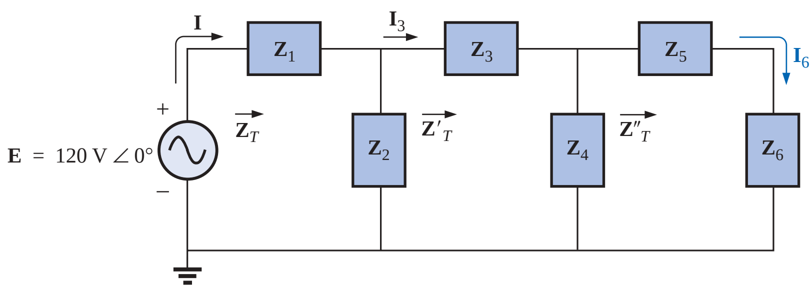


FIG. 16.20
Defining an approach to the analysis of ladder networks.

16.4 APPLICATIONS

The vast majority of the applications appearing throughout the text have been of the series-parallel variety. The following are simply two more that include series-parallel combinations of elements and systems to perform important everyday tasks. The ground fault interrupter (GFI) outlet employs series protective switches and sensing coils and a parallel control system, while the ideal equivalent circuit for the coax cable employs a series-parallel combination of inductors and capacitors.

GFI (Ground Fault Interrupter)

The National Electric Code, the "bible" for all electrical contractors, now requires that ground fault interrupter (GFI) outlets be used in any area where water and dampness could result in serious injury, such as in bathrooms, pools, marinas, and so on. The outlet looks like any other except that it has a reset button and a test button in the center of the unit as shown in Fig. 16.21(a). Its primary difference between an ordinary outlet is that it will shut the power off much more quickly than the breaker all the way down in the basement could. You may still feel a shock with a GFI outlet, but the current will cut off so quickly (in a few milliseconds) that a person in normal health should not receive a serious electrical injury. Whenever in doubt about its use, remember that the cost is such that it should be installed. It works just as a regular outlet does, but it provides an increased measure of safety.

The basic operation is best described by the simple network of Fig. 16.21(b). The protection circuit separates the power source from the outlet itself. Note in Fig. 16.21(b) the importance of grounding the protection circuit to the central ground of the establishment (a water pipe, ground bar, and so on, connected to the main panel). In general, the outlet will be grounded to the same connection. Basically, the network shown in Fig. 16.21(b) senses both the current entering (I_i) and the current leaving (I_o) and provides a direct connection to the outlet when they are equal. If a fault should develop such as caused by someone touching the hot leg while standing on a wet floor, the return current will be less than the feed current (just a few milliamperes is enough). The protection circuitry will sense this difference, establish an open circuit in the line, and cut off the power to the outlet.

In Fig. 16.22(a) you can see the feed and return lines passing through the sensing coils. The two sensing coils are separately connected to the printed circuit board. There are two pulse control switches in the line and a return to establish an open circuit under errant condi-

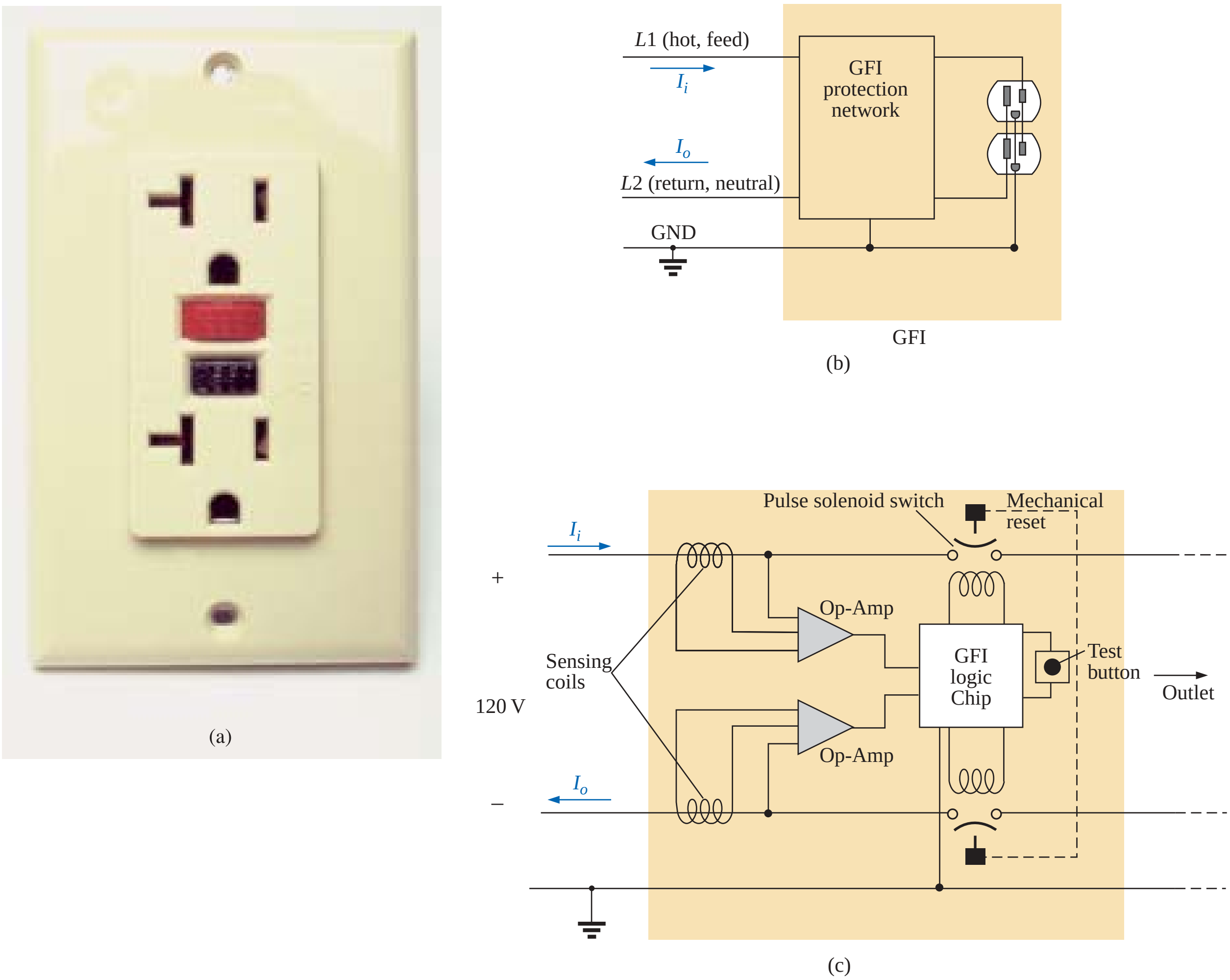


FIG. 16.21

GFI outlet: (a) wall-mounted appearance; (b) basic operation; (c) schematic.

tions. The two contacts in Fig. 16.22(a) are the contacts that provide conduction to the outlet. When a fault develops, another set of similar contacts in the housing will slide away, providing the desired open-circuit condition. The separation is created by the solenoid appearing in Fig. 16.22(b). When the solenoid is energized due to a fault condition, it will pull the plunger toward the solenoid, compressing the spring. At the same time, the slots in the lower plastic piece (connected directly to the plunger) will shift down, causing a disconnect by moving the structure inserted in the slots. The test button is connected to the brass bar across the unit in Fig. 16.22(c) below the reset button. When pressed, it will place a large resistor between the line and ground to "unbalance" the line and cause a fault condition. When the button is released, the resistor will be separated from the line, and the unbalance condition will be removed. The resistor is actually connected directly to one end of the bar and moves down with pressure on the bar as shown in Fig. 16.22(d). Note in Fig. 16.22(c) how the metal ground connection passes right through the entire unit and how it is connected to the ground terminal of an applied plug. Also note how it is separated from the rest of the network with the plastic housing. Although this unit

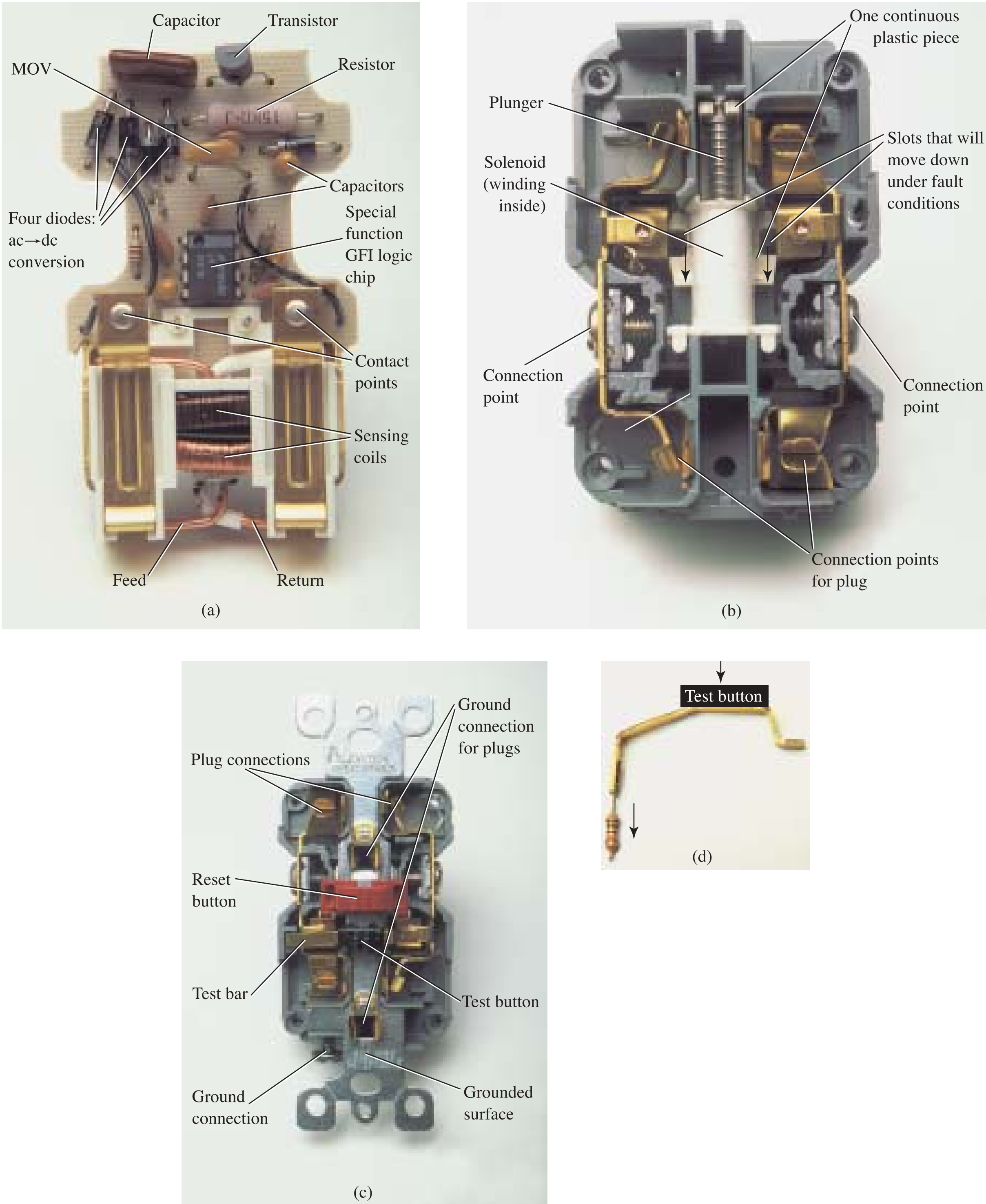


FIG. 16.22

GFI construction: (a) sensing coils; (b) solenoid control (bottom view); (c) grounding (top view); (d) test bar.

appears simple on the outside and is relatively small in size, it is beautifully designed and contains a great deal of technology and innovation.

Before leaving the subject, note the logic chip in the center of Fig. 16.22(a) and the various sizes of capacitors and resistors. Note also the four diodes in the upper left region of the circuit board used as a bridge rectifier for the ac-to-dc conversion process. The transistor is the black element with the half-circle appearance. It is part of the driver circuit for the controlling solenoid. Because of the size of the unit, there wasn't a lot of room to provide the power to quickly open the circuit. The result is the use of a pulse circuit to control the motion of the controlling solenoid. In other words, the solenoid is pulsed for a short period of time to cause the required release. If the design used a system that would hold the circuit open on a continuing basis, the power requirement would be greater and the size of the coil larger. A small coil can handle the required power pulse for a short period of time without any long-term damage.

As mentioned earlier, if unsure, then install a GFI. It provides a measure of safety—at a very reasonable cost—that should not be ignored.

Coax Cable

In recent years it appears that coax cable is everywhere, from TV connections to medical equipment, from stereos to computer connections. What makes this type of connection so special? What are its advantages over the standard two-wire connection?

The primary purpose of coax cable is to provide a channel for communication between two points without picking up noise from the surrounding medium—a direct link in its purest form. You may wonder whether noise pollution is really that bad and whether this concern is overkill, but simply think of all the signals passing through the air that we cannot see, for example, for cellular phones, pagers, and radio and TV stations. Then you start to realize that there is a lot going on out there that we can't see. None of us would like our EKG signal from our heart to be disturbed by extraneous noise or to have our stereo pick up channels other than those of interest. It is a real problem that must be solved, and it appears that the best solution is to use coax cable. Compared to standard conductors, coax displays a lower loss of signal in transmission and has much improved high-frequency transmission characteristics.

It is the construction that offers the protection we desire. The basic construction of a 75-Ω coax cable as typically used in the home appears in Fig. 16.23(a) with its terminal connection in Fig. 16.23(b). It has a

FIG. 16.23
75-Ω coax cable: (a) construction; (b) terminal connection.

solid inner conductor surrounded by a polyethylene dielectric (insulator). Copper or aluminum braid woven over the dielectric forms the outer conductor. Finally, a waterproof jacket placed over the braided wire provides protection against moisture. Since the entire outer surface of the braided wire is at the same potential, it completely isolates the solid conductor in the center of the coax cable from the outside signals—an isolation referred to as *shielding.* The question is sometimes asked, Why is the outside wire braided rather than just a flat sheet of conducting material? It is braided to reduce the effects of the fields established by any currents that pass through the outside conductor. In Fig. 16.24(a), a current in the outer conductor has established circular magnetic fields that can be additive and can create transmission problems. However, as shown in Fig. 16.24(b), if the wire is braided, the magnetic field established by one wire in the braid may be canceled by a neighboring conductor crossing the conductor on an angle. Note the opposite direction of the fields in the region between the two braided wires. Of course, the total magnetic flux may not be canceled, but the situation is certainly improved compared to that with a solid flat conductor. For added protection, a duofoil covering is sometimes added as shown in Fig. 16.23(a) to ensure 100% shielding.

Because a coax cable is most commonly referred to as an *RF* (radio-frequency) *transmission line,* most people associate the use of coax cables with high frequencies. However, this is certainly not the case, as evidenced by medical technology that deals with static dc levels and low-voltage (in microvolts or millivolts), "slow" (less than 5 Hz) ac. In general, coax cables should be used wherever there is a need to ensure that the transmitted signal is undisturbed by any surrounding noise. Coax cables are acceptable for the full range of frequencies from 0 Hz to a few hundred gigahertz, with sound frequencies extending from about 15 Hz to 20 kHz, radio frequencies from 20 kHz to 300 MHz, and microwave frequencies from 300 MHz to 300 GHz. Our discussion thus far has centered on protecting the transmitted signal from external noise. It is important to realize also that when a coax cable is used, it will not act as a transmitter for the signal that it is carrying. This fact is very important as we hook up electronic appliances such as VCRs to our TVs. If we simply used a twin lead wire between the VCR and TV, not only would the wire pick up signals by acting like an antenna, but it would also transmit channel 3 (or 4) to the surrounding medium

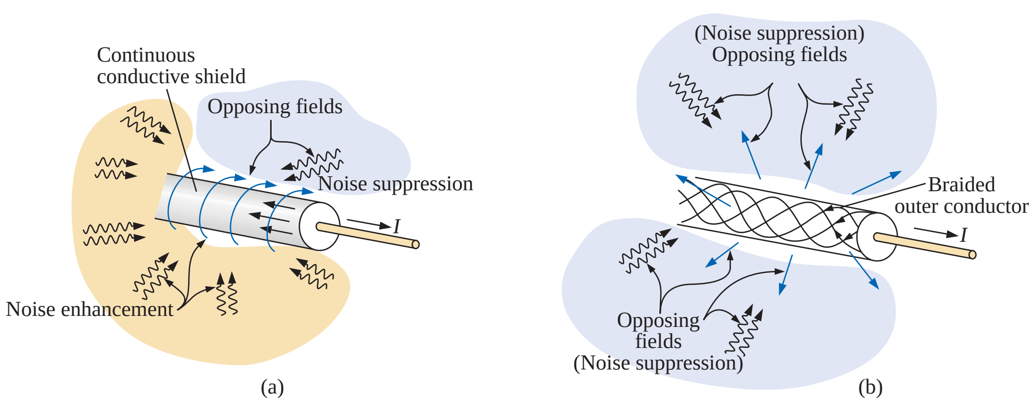


FIG. 16.24
Shielding: (a) solid outside inductor; (b) braided outside conductor.

which would affect not only your TV's response but also that of any other TV or receiver in the area.

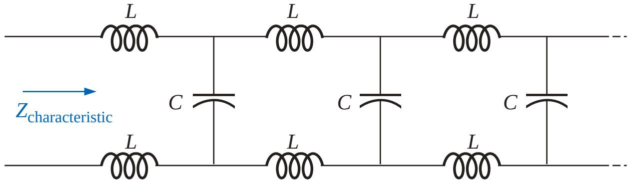
For the coupling between the systems in which coax cable is typically used, it is not the level of voltage or current that is the primary concern but whether there is a good "match" between components and the cable. Every transmission line composed of two parallel conductors will have capacitance between the conductors, and every conductor that is carrying current has a certain level of inductance. For a transmission line an equivalent model can be composed of the lumped series-parallel combination of Fig. 16.25(a), where each capacitor or inductor is for a short length of the wire. For an infinitely long chain of the elements of Fig. 16.25(a), the combination has an input impedance called the *characteristic impedance* that is proportional to $\sqrt{L/C}$ where L and C are the inductance and capacitance of a unit length of the transmission line. Although Figure 16.25(a) suggests that a transmission line is purely reactive, there is resistance in the line because of the resistance of the wire, and this resistance will absorb power. It is therefore important to realize when hooking up coax cable that the TV farthest from the source will receive the least amount of signal power, and if it is very distant, the resulting loss may be sufficient to affect the picture quality. Rearranging the equations for v_L and I_C and substituting as follows will reveal that the characteristic impedance is purely resistive and is measured in ohms:

$$v_L = L\frac{di_L}{dt} \Rightarrow L = v_L\frac{dt}{di_L}$$

$$i_C = C\frac{dv_C}{dt} \Rightarrow C = i_C\frac{dt}{dv_C}$$

so that

$$\sqrt{\frac{L}{C}} = \sqrt{\frac{v_L\dfrac{dt}{di_L}}{i_C\dfrac{dt}{dv_C}}} = \sqrt{\frac{v_L}{i_C}\cdot\frac{dv_C}{di_L}} = \sqrt{\Omega\cdot\Omega} = \sqrt{\Omega^2} = \Omega$$



(a)

(b)

FIG. 16.25
Coax cable: (a) electrical equivalent (lossless line); (b) characteristic impedance.

The most common coax cables have characteristic impedances of either 50 Ω or 75 Ω, as shown in Fig. 16.25(b). In actuality they may be 53.5-Ω and 73.5-Ω lines, respectively, but they are usually grouped in the category of 50- or 75-Ω lines. The 75-Ω line is typically used for applications such as cable TV and RF equipment, while the 50-Ω line is typically used for test equipment, ham radio stations, and medical equipment. Two of the most common coax cables are listed in Table 16.1 with specific information about their characteristics.

TABLE 16.1
Characteristics of two frequently used coax cables.

	RG-59U 75 Ω (actually 73.5 Ω)	**RG-58U 50 Ω (actually 53.5 Ω)**
Core wire:	20 AWG, 40% aluminum	20 AWG, 95% tinned
Resistance:	44.5 Ω/1000 ft	10 Ω/1000 ft
Coating:	Duofoil, 100% shield coverage	Polyethylene
PVC jacket:	0.237-in. outside diameter	0.193-in. outside diameter
Capacitance:	16.2 pF/ft	28.5 pF/ft
Losses:	1 MHz, 0.8 dB/100 ft	1 MHz, 0.3 dB/100 ft
	10 MHz, 1 dB/100 ft	10 MHz, 1.1 dB/100 ft
	50 MHz, 1.8 dB/100 ft	50 MHz, 2.5 dB/100 ft
	100 MHz, 2.5 dB/100 ft	100 MHz, 3.8 dB/100 ft
	1 GHz, 7.9 dB/100 ft	1 GHz, 14.5 dB/100 ft

In reality, a transmission line will not be infinite in length as required for the definition of characteristic impedance. The result is that a 20-ft length of 75-Ω cable will not have an input impedance of 20 Ω but rather one that is determined by the load applied to the cable. However, if the transmission line is terminated by a resistance of 75 Ω, the characteristic impedance of 75 Ω will appear at the source. In other words, terminating a coax cable by its characteristic impedance will make it appear as an infinite line to the source. When the applied load equals the characteristic impedance of the line, the line is said to be *matched.* An applied load equal to the characteristic impedance also results in maximum power transfer to the load as established by the maximum power theorem. Any loading other than the characteristic impedance will result in a "reflection of power" back to the source. Matching the load to the line is therefore a major concern when using coax cables. For instance, take the folded-dipole antenna referred to as a *yagi* that was a common sight on roof tops before cable came along. The twin line cable running from the antenna to the TV had a characteristic impedance of 300 Ω. Today, most TVs have an input impedance of 75 Ω, and thus such antennas would have to be connected to the TV with a *matching transformer* (called a *Balun transformer*) that would make the 75-Ω load look like 300 Ω to the antenna for maximum power transfer, as shown in Fig. 16.26. In today's world, TVs are referred to as *cable ready* if they have a coax connection and an input impedance of 75 Ω to match the cable system.

One of the mistakes frequently made when installing a coax system is to hook up a splitter and fail to terminate all the output terminals. In Fig. 16.27(a), a three-way splitter is connected to two TVs with the third terminal left open for any possible future additions. The open third terminal will cause a mismatch on the incoming line, and less power

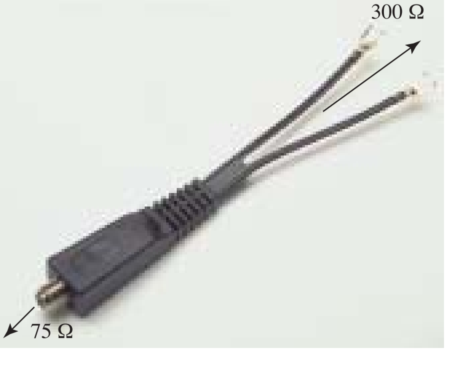
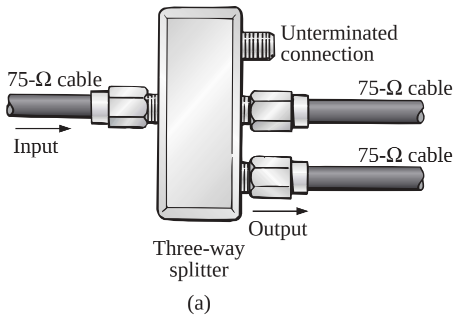


FIG. 16.26
Balun matching transformer.



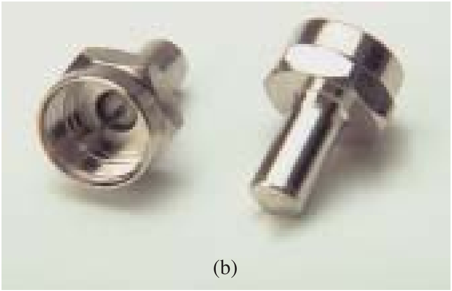


FIG. 16.27
Signal splitting: (a) three-way splitter; (b) F-type 75-Ω coax terminator.

will get to the connected TVs. This situation is corrected by terminating the unused terminal with a commercially available connector as shown in Fig. 16.27(b), which simply has a 75-Ω resistor inside. It is also important to realize that each time you split the signal, you lose power to each of the TVs connected to the system. In fact, you lose 3 dB for each split as shown in Fig. 16.28(a). Splitting the signal in two will result in a loss of 3 dB for each TV, while splitting it three ways will result in a 6-dB loss for each TV. The concept of decibels will be covered in Chapter 24, but be aware for the moment that a 3-dB drop represents a drop in power of one-half—certainly a significant amount. A TV can still respond pretty well with a drop of 3 dB or 6 dB, but anything approaching a 12-dB drop will probably result in a poor image and should be avoided. Whenever using a splitter, it is always best to connect an amplifier before the splitter as shown in Fig. 16.28(b). In essence, the amplifier compensates for the loss introduced by splitters and also (if well designed) will permit leaving a terminal open without disturbing the resulting signal power flow. In other words, a good

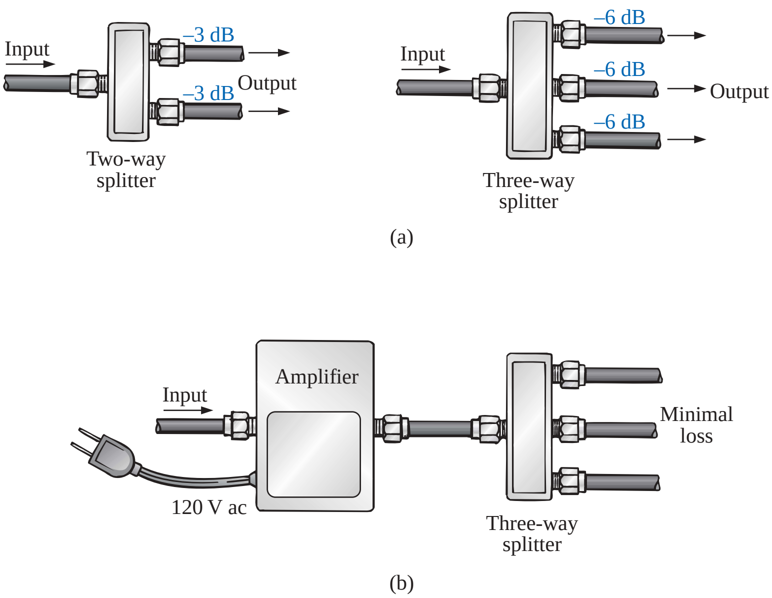


FIG. 16.28
Coax splitting losses: (a) dB losses introduced by two-way and three-way splitters; (b) using an amplifier.

amplifier knows how to compensate for a terminal that is improperly terminated.

Table 16.1 reveals that there is a measurable loss in power (dB) for every 100 ft of cable. For each cable, about 3 dB are lost for every 100 ft at 100 MHz, primarily because of the resistance of the center conductor (44.5 Ω/1000 ft for the 75-Ω line and 10 Ω/1000 ft for the 50-Ω line). This is one reason why it is not recommended to first split the signal and apply the amplifier at the location of the TV. In Fig. 16.29(a), the signal-to-noise (unwanted signals) ratio is quite high, and the reception will be quite good. However, as shown in Fig. 16.29(b), if the signal is sent down a 100-ft cable to a room distant from the source, there will be a drop in signal, and even if the noise component does not increase, the signal-to-noise ratio at the TV will be much higher. If an amplifier is connected at this point, it will amplify both the signal and the unwanted noise, and the poorer signal-to-noise ratio will be fed to the TV, resulting in a poorer reception. In general, therefore, amplifiers should be applied where the signal-to-noise ratio is the highest.

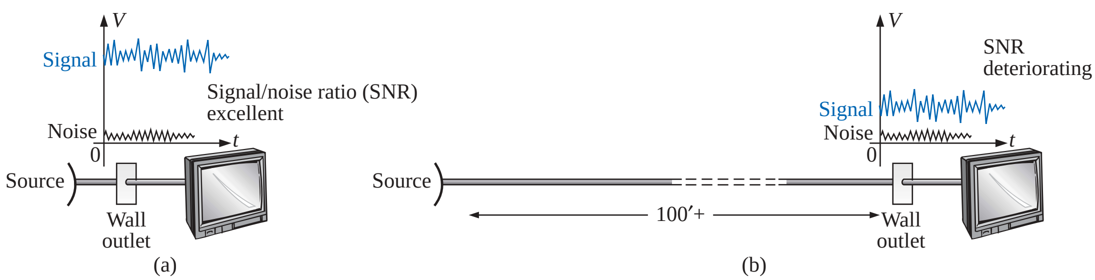


FIG. 16.29
Signal-to-noise ratios: (a) negligible line loss; (b) measurable line loss.

The discussion of coax cables and their proper use could go on for a number of pages. Priorities, however, require that any further investigation be left to the reader. Simply be aware that the matching process is an important one and that coax cables are not ideal systems and do have an internal resistance that can affect transmission—especially over long distances.

16.5 COMPUTER ANALYSIS

PSpice

ac Bridge Network We will be using Example 16.4 to demonstrate the power of the **VPRINT** option in the **SPECIAL** library. It permits a direct determination of the magnitude and angle of any voltage in an ac network. Similarly the **IPRINT** option does the same for ac currents. In Example 16.4, the ac voltages across R_1 and R_2 were first determined, and then Kirchhoff's voltage law was applied to determine the voltage between the two known points. Since PSpice is designed primarily to determine the voltage at a point with respect to ground, the network of Fig. 16.7 is entered as shown in Fig. 16.30 to permit a direct calculation of the voltages across R_1 and R_2.

The source and network elements are entered using a procedure that has been demonstrated several times in previous chapters, although for

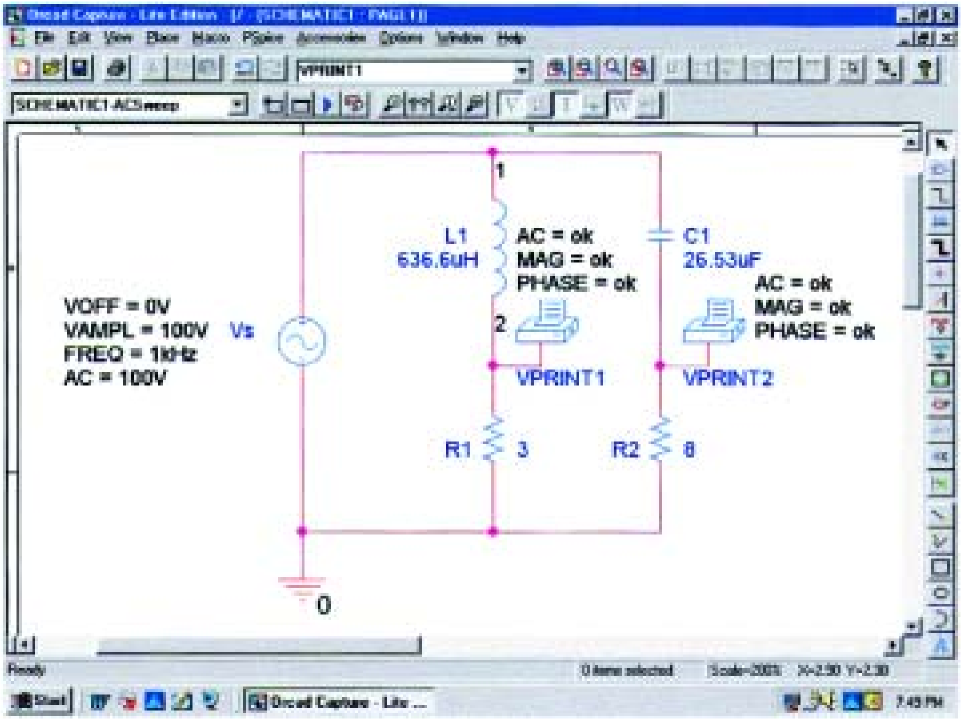


FIG. 16.30

Determining the voltage across R_1 and R_2 using the ***VPRINT*** *option of a PSpice analysis.*

the **AC Sweep** analysis to be performed in this example, the source must carry an **AC** level also. Fortunately, it is the same as **VAMPL** as shown in Fig. 16.30. It is introduced into the source description by double-clicking on the source symbol to obtain the **Property Editor** dialog box. The **AC** column is selected and the 100 V entered in the box below. Then **Display** is selected and **Name and Value** chosen. Click **OK** followed by **Apply,** and you can exit the dialog box. The result is **AC** = 100 V added to the source description on the diagram and in the system. Using the reactance values of Fig. 16.7, the values for L and C were determined using a frequency of 1 kHz. The voltage across R_1 and R_2 can be determined using the **Trace** command in the same manner as described in the previous chapter or by using the **VPRINT** option. Both approaches will be discussed in this section because they have application to any ac network.

The **VPRINT** option is under the **SPECIAL** library in the **Place Part** dialog box. Once selected, the printer symbol will appear on the screen next to the cursor, and it can be placed near the point of interest. Once the printer symbol is in place, a double-click on it will result in the **Property Editor** dialog box. Scrolling from left to right, type the word **ok** under **AC, MAG,** and **PHASE.** When each is active, the **Display** key should be selected and the option **Name and Value** chosen followed by **OK.** When all the entries have been made, choose **Apply** and exit the dialog box. The result appears Fig. 16.30 for the two applications of the **VPRINT** option. If you prefer, **VPRINT1** and **VPRINT2** can be added to distinguish between the two when you review the output data. This is accomplished by returning to the **Property Editor** dialog box for each by double-clicking on the printer symbol of each and selecting **Value** and then **Display** followed by **Value Only.** We are now ready for the simulation.

The simulation is initiated by selecting the **New Simulation Profile** icon and entering **ACSweep** as the **Name.** Then select **Create** to bring up the **Simulation Settings** dialog box. This time, we want to analyze the network at 1 kHz but are not interested in plots against time. Thus, the **AC Sweep/Noise** option will be selected under **Analysis type** in the **Analysis** section. An **AC Sweep Type** region will then appear in the dialog asking for the **Start Frequency.** Since we are interested in the response at only one frequency, the **Start** and **End Frequency** will be the same: 1 kHz. Since we need only one point of analysis, the **Points/Decade** will be 1. Click **OK,** and the **Run PSpice** icon can be selected. The **SCHEMATIC1** screen will appear, and the voltage across R_1 can be determined by selecting **Trace** followed by **Add Trace** and then **V(R1:1).** The result is the bottom display of Fig. 16.31 with only one plot point at 1 kHz. Since we fixed the frequency of interest at 1 kHz, this is the only frequency with a response. The magnitude of the voltage across R_1 is 60 V to match the longhand solution of Example 16.4. The phase angle associated with the voltage can be determined by the sequence **Plot-Add Plot to Window-Trace-Add Trace-P( )** from the **Functions or Macros** list and then **V(R1:1)** to obtain **P(V(R1:1))** in the **Trace Expression** box. Click **OK,** and the resulting plot shows that the phase angle is near just less than −50° which is certainly a close match with the −53.13° obtained in Example 16.4.

The above process made no use of the new **VPRINT** option just introduced. We will now see what this option provides. When the **SCHEMATIC1** window appears after the simulation, the window should be exited using the **X**, and **PSpice** should be selected on the top

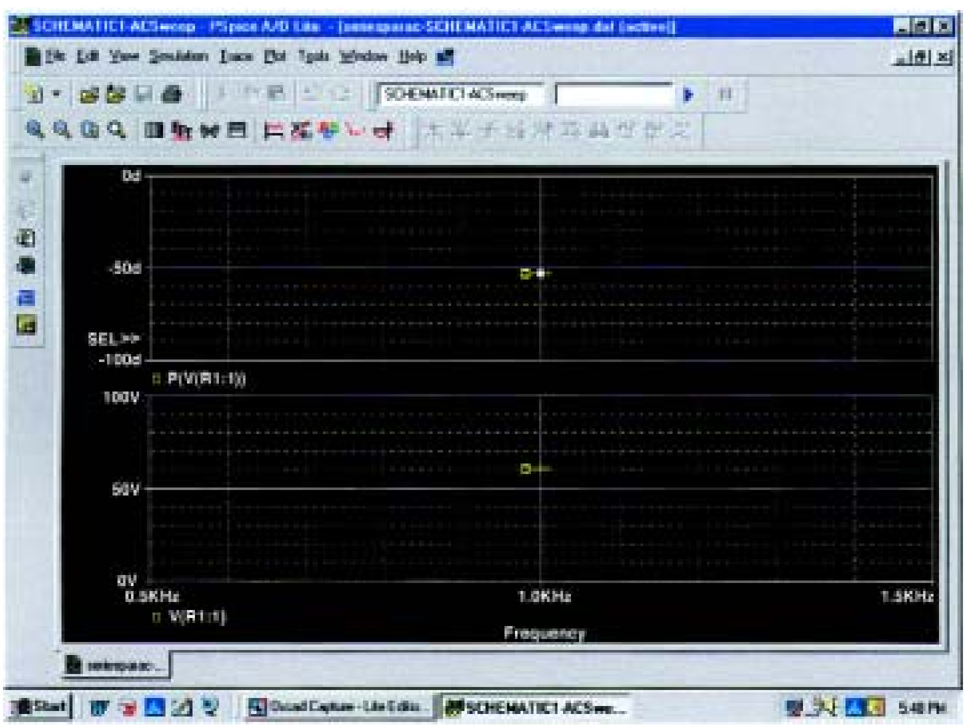


FIG. 16.31
The resulting magnitude and phase angle for the voltage V_{R_1} of Fig. 16.30.

menu bar of the resulting screen. A list will appear of which **View Output File** is an option. Selecting this option will result in a long list of data about the construction of the network and the results obtained from the simulation. In Fig. 16.32 the portion of the output file listing the resulting magnitude and phase angle for the voltages defined by **VPRINT1** and **VPRINT2** is provided. Note that the voltage across R_1 defined by **VPRINT1** is 60 V at an angle of 53.13°. The voltage across R_2 as defined by **VPRINT2** is 80 V at an angle of 36.87°. Both are exact matches of the solutions of Example 16.4. In the future, therefore, if the **VPRINT** option is used, the results will appear in the output file.

```

 ** Profile: "SCHEMATIC1-ACSweep"  [ C:\PSpice\seriesparac-SCHEMATIC1-ACSweep.s
im ]


 ****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ        VM(N00809)  VP(N00809)


   1.000E+03   6.000E+01  -5.313E+01

**** 06/30/01 17:45:20 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-ACSweep"  [ C:\PSpice\seriesparac-SCHEMATIC1-ACSweep.s
im ]


 ****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ        VM(N00717)  VP(N00717)


   1.000E+03   8.000E+01   3.687E+01

```

FIG. 16.32

*The **VPRINT1** (V_{R_1}) and **VPRINT2** (V_{R_2}) response for the network of Fig. 16.30.*

Now we will determine the voltage across the two branches from point *a* to point *b*. Return to **SCHEMATIC1,** and select **Trace** followed by **Add Trace** to obtain the list of **Simulation Output Variables.** Then, by applying Kirchhoff's voltage law around the closed loop, we find that the desired voltage is **V(R1:1)-V(R2:1)** which when followed by **OK** will result in the plot point in the screen in the bottom of Fig. 16.33. Note that it is exactly 100 V as obtained in the longhand solution. The phase angle can then be determined through **Plot-Add Plot to Window-Trace-Add Trace** and creating the expression **P(V(R1:1)-V(R2:1)).** Remember that the expression can be generated using the lists of **Output variables** and **Functions**, but it can

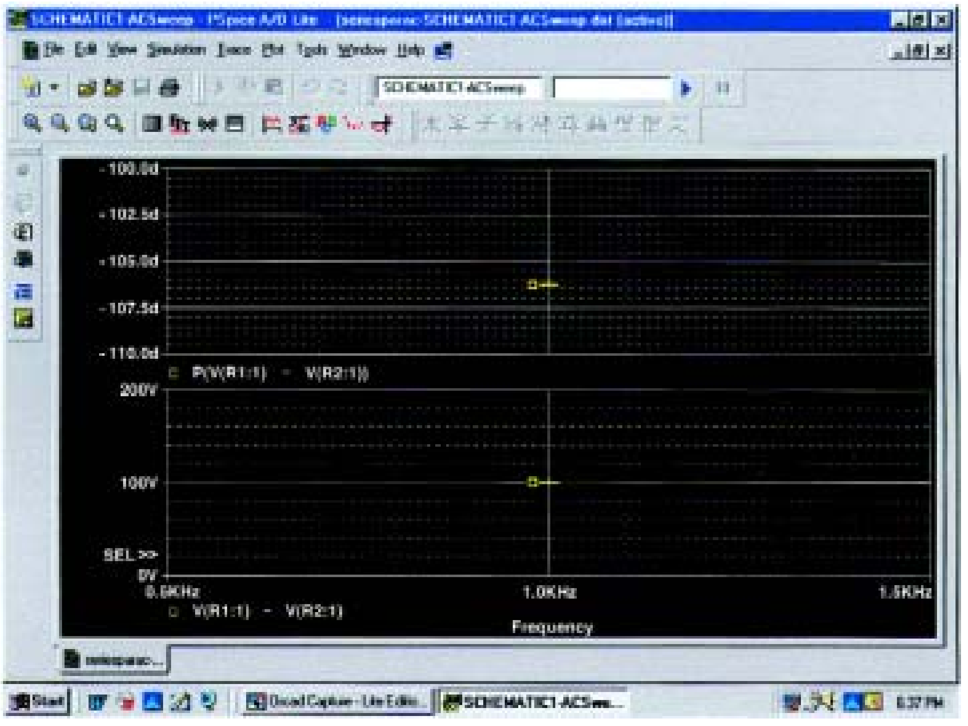


FIG. 16.33

The PSpice reponse for the voltage between the two points above resistors R_1 and R_2.

also be simply typed in from the keyboard. However, always be sure that there are as many left parentheses as there are right. Click **OK,** and a solution near −105° appears. A better reading can be obtained by using **Plot-Axis Settings-Y Axis-User Defined** and changing the scale to −100° to −110°. The result is the top screen of Fig. 16.33 with an angle closer to −106.5° or 73.5° which is very close to the theoretical solution of 73.74°.

Finally, the last way to find the desired bridge voltage is to remove the **VPRINT2** option and place the ground at that point as shown in Fig. 16.34. Now the voltage generated from a point above R_1 to ground will be the desired voltage. Repeating a full simulation will then result in the plot of Fig. 16.35 with the the same results as Fig. 16.33. Note, however, that even though the two figures look the same, the quantities listed in the bottom left of each plot are different.

Electronics Workbench

Electronics Workbench will now be used to determine the voltage across the last element of the ladder network of Fig. 16.36. The mathematical content of this chapter would certainly suggest that this analysis would be a lengthy exercise in complex algebra, with one mistake (a single sign or an incorrect angle) enough to invalidate the results. However, it will take only a few minutes to "draw" the network on the screen and only a few seconds to generate the results—results you can usually assume are correct if all the parameters were entered correctly. The results are certainly an excellent check against a longhand solution.

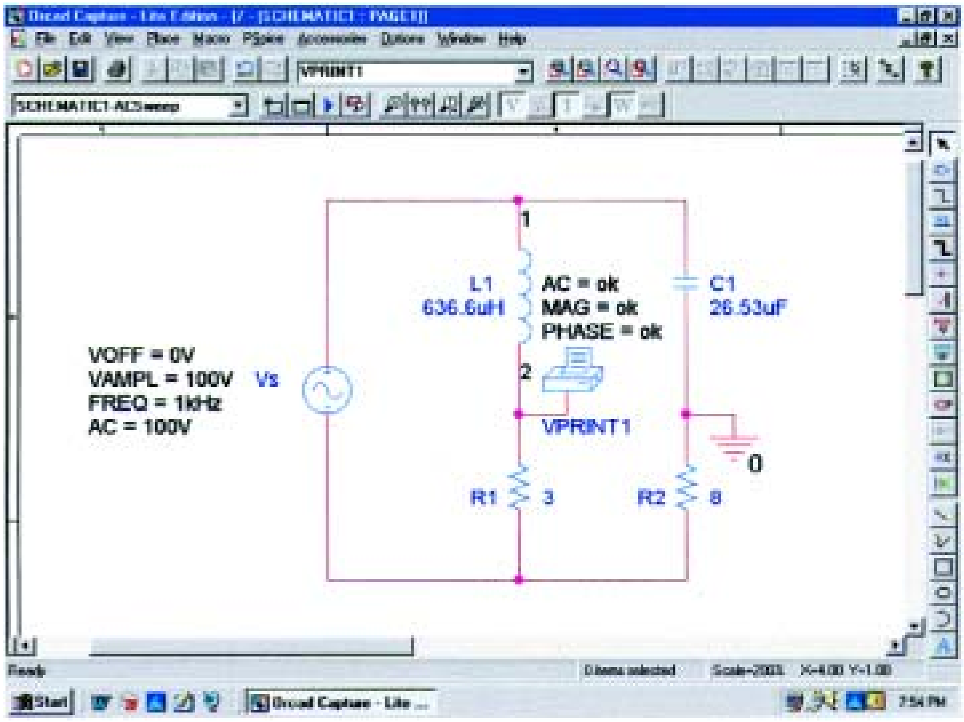


FIG. 16.34

Determining the voltage between the two points above resistors R_1 and R_2 by moving the ground connection of Fig. 16.30 to the position of ***VPRINT2****.*

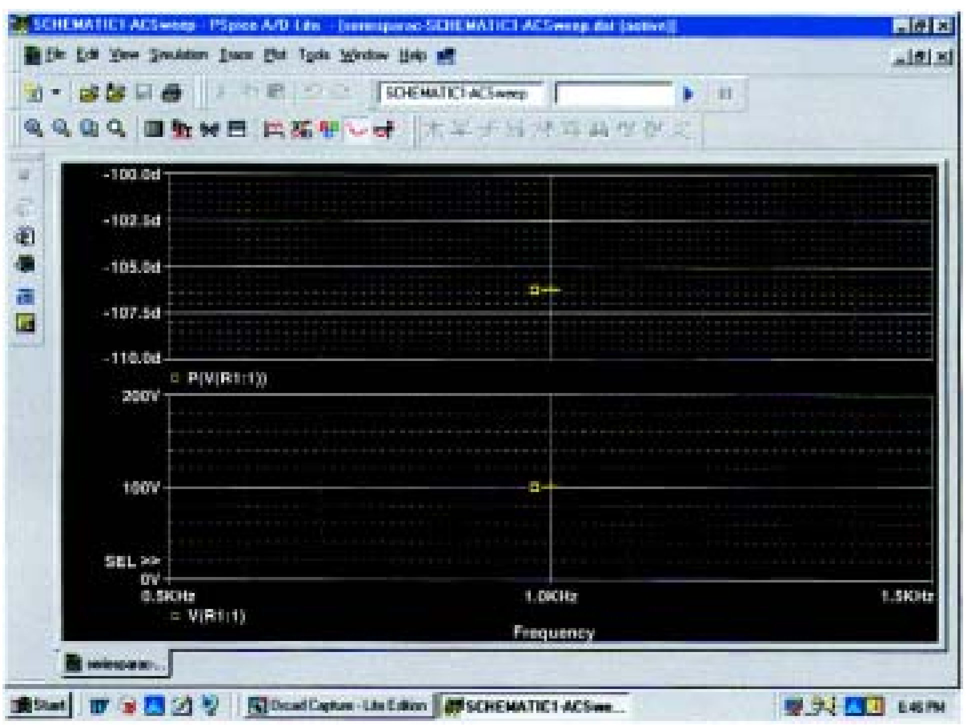


FIG. 16.35

PSpice response to the simulation of the network of Fig. 16.34.

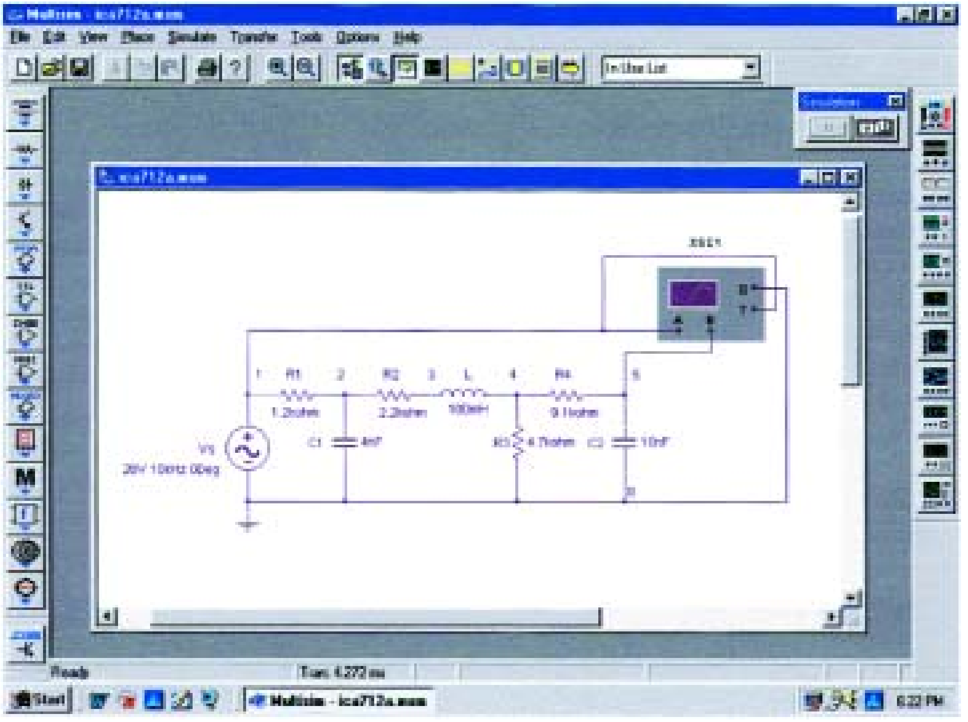


FIG. 16.36
Using the oscilloscope of Electronics Workbench to determine the voltage across the capacitor C_2.

Our first approach will be to use an oscilloscope to measure the amplitude and phase angle of the output voltage as shown in Fig. 16.36. Note that five nodes are defined, with node 5 the desired voltage. The oscilloscope settings include a **Time base** of 20 μs/div. since the period of the 10-kHz signal is 100 μs. Channel **A** was set on 10 V/div. so that the full 20 V of the applied signal will have a peak value encompassing two divisions. Note that **Channel A** in Fig. 16.36 is connected directly to the source **Vs** and to the **Trigger** input for synchronization. Expecting the output voltage to have a smaller amplitude resulted in a vertical sensitivity of 1 V/div. for **Channel B.** The analysis was initiated by placing the **Simulation** switch in the **1** position. It is important to realize that

when simulation is initiated, it will take time for networks with reactive elements to settle down and for the response to reach its steady-state condition. It is therefore wise to let a system run for a while after simulation before selecting **Single** ***on the oscilloscope to obtain a steady waveform for analysis.***

The resulting plots of Fig. 16.37 clearly show that the applied voltage has an amplitude of 20 V and a period of 100 μs (5 div. at 20 μs/div). The cursors sit ready for use at the left and right edges of the screen. Clicking on the small red arrow (with number 1) at the top of the oscilloscope screen will permit you to drag it to any location on the horizontal axis. As you move the cursor, the magnitude of each waveform will appear in the **T1** box below. By comparing positive slopes through the origin, you should see that the applied voltage is leading the output voltage by an angle that is more than 90°. Setting the cursor at the

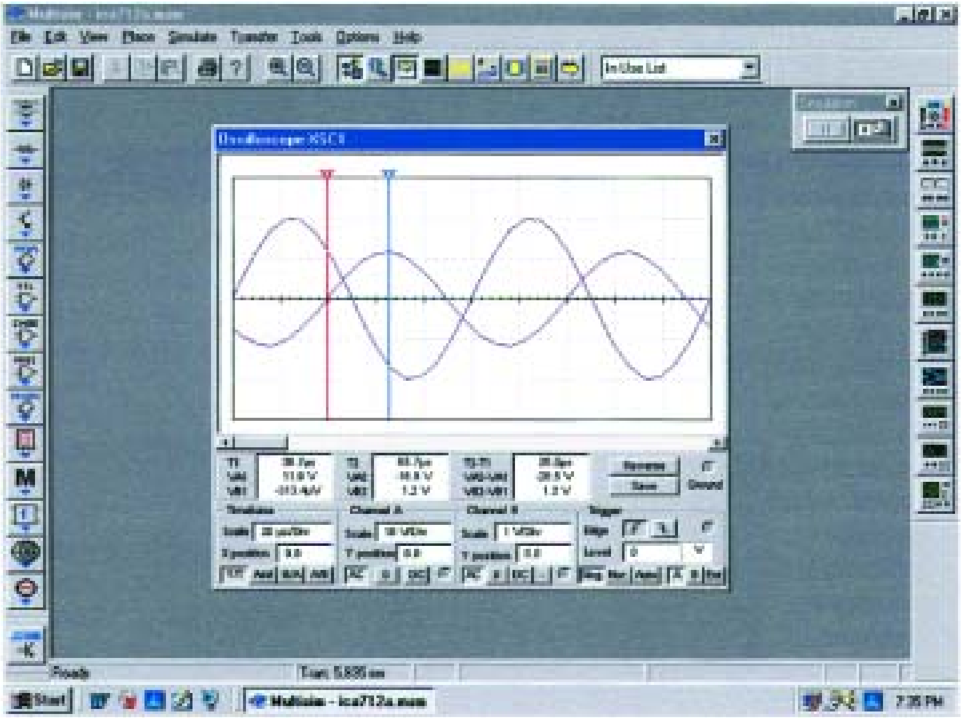


FIG. 16.37

Using Electronics Workbench to display the applied voltage and voltage across the capacitor C_2 for the network of Fig. 16.36.

point where the output voltage on channel B passes through the origin with a positive slope, we find that we cannot achieve exactly 0 V; but $-313.4\ \mu V = -0.313$ mV **(VB1)** is certainly very close at 39.7 μs **(T1).**

Knowing that the applied voltage passed through the origin at 0 μs permits the following claculation for the phase angle:

$$\frac{39.7\ \mu s}{100\ \mu s} = \frac{\theta}{360^\circ}$$

$$\theta = 142.92^\circ$$

with the result that the output voltage has an angle of -142.92° associated with it. The second cursor is found at the right edge of the screen and has a blue color. Selecting it and moving it to the peak value of the output voltage results in **VB2** = 1.2 V at 65.7 μs **(T2).** The result of all the above is

$$\mathbf{V}_{C_2} = 1.2\ \text{V} \angle -142.92^\circ.$$

Our second approach will be to use the **AC Analysis** option under the **Simulate** heading. First, realize that when we were using the oscilloscope as we did above, there was no need to pass through the sequence of dialog boxes to choose the desired analysis. All that was necessary was to simulate using either the switch or the **PSpice-Run** sequence—the oscilloscope was there to measure the output voltage. Remember that the source defined the magnitude of the applied voltage, the frequency, and the phase shift. This time we will use the sequence

Simulate-Analyses-AC Analysis to obtain the **AC Analysis** dialog box in which the **Start** and **Stop frequencies** will be 10 kHz and the **Selected variable for analysis** will be node 5 only. Selecting **Simulate** will then result in a magnitude-phase plot with no apparent indicators at 10 kHz. However, this is easily corrected by first selecting one of the plots by clicking on the **Voltage** label at the left of the plot. Then select the **Show/Hide Grid, Show/Hide Legend,** and **Show/Hide Cursors** keys to obtain the cursors, legend, and **AC Analysis** dialog box. Hook on the red cursor and move it to 10 kHz. At that location, and that location only, **x1** will appear as 10 kHz in the dialog box, and **y1** will be 1.1946 as shown in Fig. 16.38. In other words, the cursor has defined the magnitude of the voltage across the output capacitor as 1.1946 V or approximately 1.2 V as obtained above. If you then select the **Phase** curve and repeat the procedure, you will find that at 10 kHz **(x1)** the angle is $-142.15°$ **(y1)** which is very close to the $-142.92°$ obtained above.

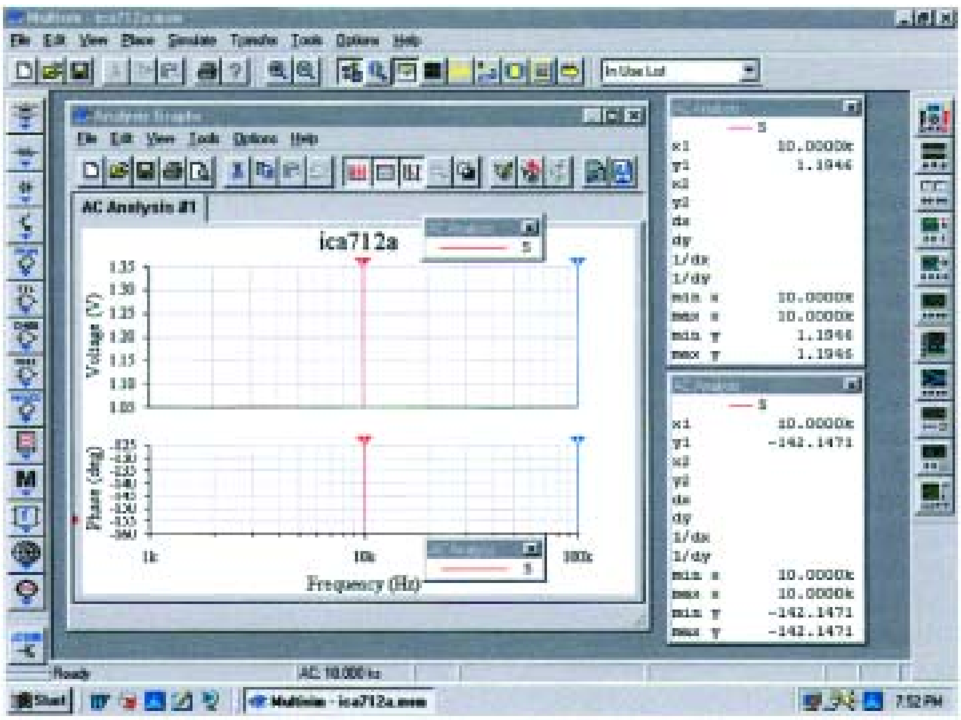


FIG. 16.38

*Using the **AC Analysis** option under Electronics Workbench to determine the magnitude and phase angle for the voltage V_{C_2} for the network of Fig. 16.36.*

In total, therefore, we have two methods to obtain an ac voltage in a network—one by instrumentation and the other through the computer methods. Both are valid, although, as expected, the computer approach has a higher level of accuracy.

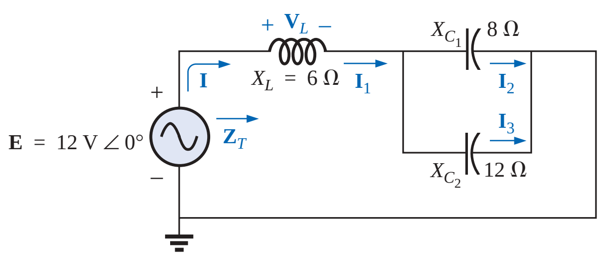


FIG. 16.39
Problems 1 and 19.

PROBLEMS

SECTION 16.2 Illustrative Examples

1. For the series-parallel network of Fig. 16.39:
- **a.** Calculate $\mathbf{Z}_T$.
- **b.** Determine **I**.
- **c.** Determine $\mathbf{I}_1$.
- **d.** Find $\mathbf{I}_2$ and $\mathbf{I}_3$.
- **e.** Find $\mathbf{V}_L$.

2. For the network of Fig. 16.40:
- **a.** Find the total impedance $\mathbf{Z}_T$.
- **b.** Determine the current $\mathbf{I}_s$.
- **c.** Calculate $\mathbf{I}_C$ using the current divider rule.
- **d.** Calculate $\mathbf{V}_L$ using the voltage divider rule.

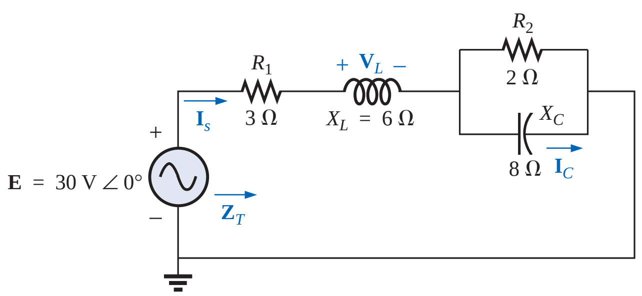


FIG. 16.40
Problems 2 and 15.

3. For the network of Fig. 16.41:
- **a.** Find the total impedance $\mathbf{Z}_T$ and the total admittance $\mathbf{Y}_T$.
- **b.** Find the current $\mathbf{I}_s$.
- **c.** Calculate $\mathbf{I}_2$ using the current divider rule.
- **d.** Calculate $\mathbf{V}_C$.
- **e.** Calculate the average power delivered to the network.

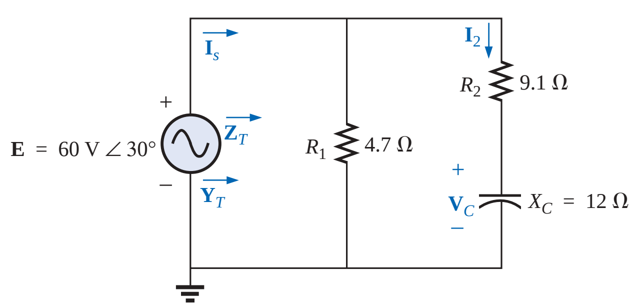


FIG. 16.41
Problems 3 and 20.

4. For the network of Fig. 16.42:
   a. Find the total impedance $\mathbf{Z}_T$.
   b. Calculate the voltage $\mathbf{V}_2$ and the current $\mathbf{I}_L$.
   c. Find the power factor of the network.

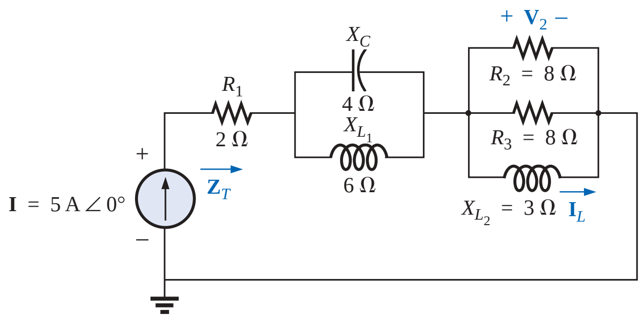


FIG. 16.42
Problem 4.

5. For the network of Fig. 16.43:
   a. Find the current **I.**
   b. Find the voltage $\mathbf{V}_C$.
   c. Find the average power delivered to the network.

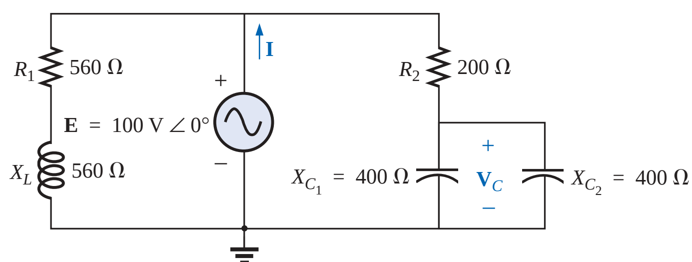


FIG. 16.43
Problems 5 and 21.

***6.** For the network of Fig. 16.44:
   a. Find the current $\mathbf{I}_1$.
   b. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
   c. Find the voltage $\mathbf{V}_{ab}$.

***7.** For the network of Fig. 16.45:
   a. Find the current $\mathbf{I}_1$.
   b. Find the voltage $\mathbf{V}_1$.
   c. Calculate the average power delivered to the network.

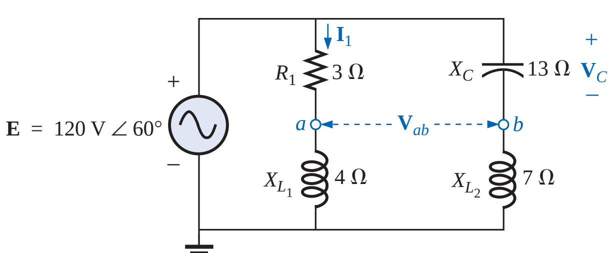


FIG. 16.44
Problem 6.

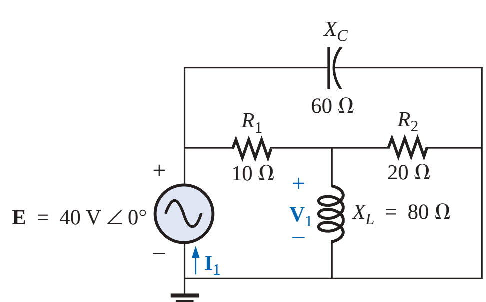


FIG. 16.45
Problems 7 and 16.

8. For the network of Fig. 16.46:
- **a.** Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.
- **b.** Find the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
- **c.** Verify Kirchhoff's current law by showing that $\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$.
- **d.** Find the power factor of the network, and indicate whether it is leading or lagging.

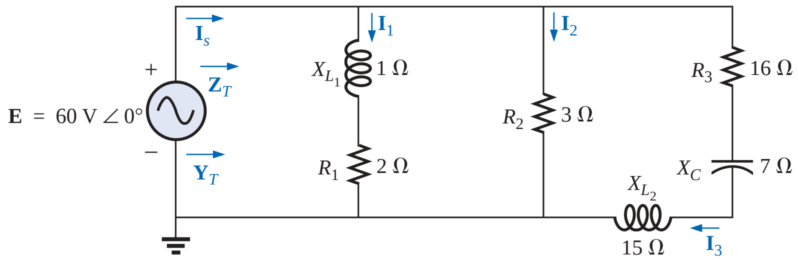


FIG. 16.46
Problem 8.

***9.** For the network of Fig. 16.47:
- **a.** Find the total admittance $\mathbf{Y}_T$.
- **b.** Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.
- **c.** Find the current $\mathbf{I}_3$.

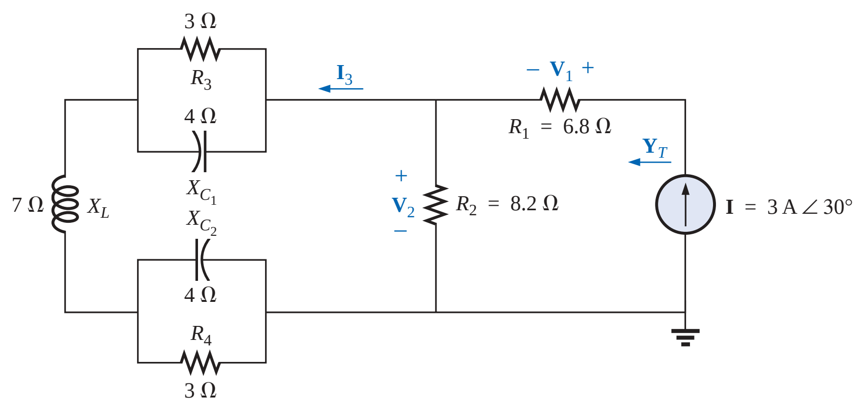


FIG. 16.47
Problem 9.

***10.** For the network of Fig. 16.48:
- **a.** Find the total impedance $\mathbf{Z}_T$ and the admittance $\mathbf{Y}_T$.
- **b.** Find the source current $\mathbf{I}_s$ in phasor form.
- **c.** Find the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in phasor form.
- **d.** Find the voltages $\mathbf{V}_1$ and $\mathbf{V}_{ab}$ in phasor form.
- **e.** Find the average power delivered to the network.
- **f.** Find the power factor of the network, and indicate whether it is leading or lagging.

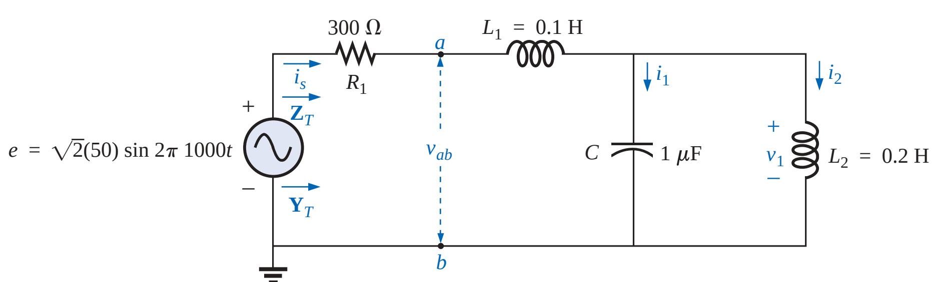


FIG. 16.48
Problem 10.

***11.** Find the current **I** for the network of Fig. 16.49.

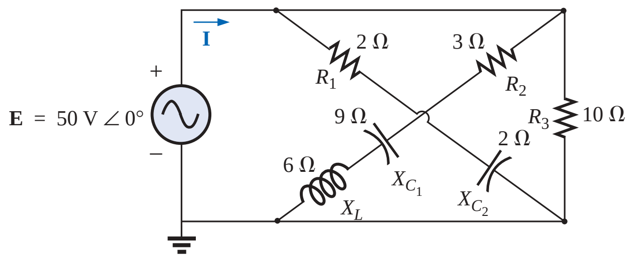


FIG. 16.49
Problems 11 and 17.

SECTION 16.3 Ladder Networks

12. Find the current $\mathbf{I}_5$ for the network of Fig. 16.50. Note the effect of one reactive element on the resulting calculations.

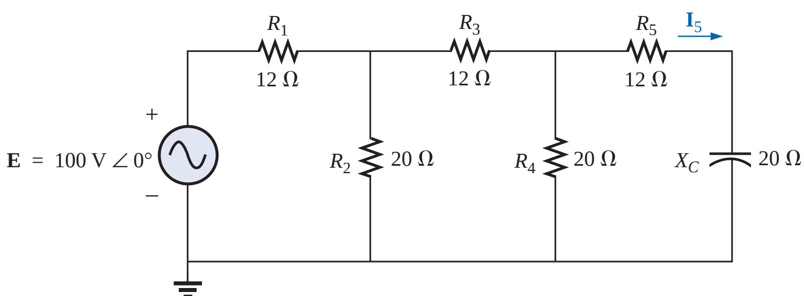


FIG. 16.50
Problem 12.

13. Find the average power delivered to R_4 in Fig. 16.51.

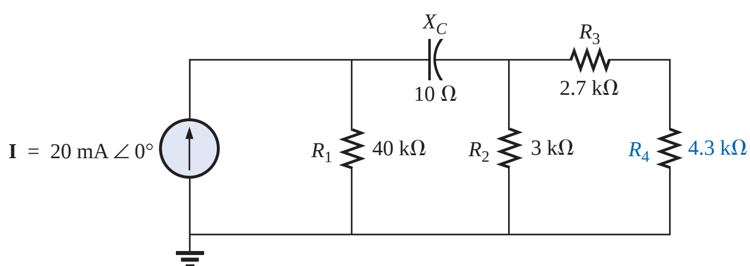


FIG. 16.51
Problem 13.

14. Find the current $\mathbf{I}_1$ for the network of Fig. 16.52.

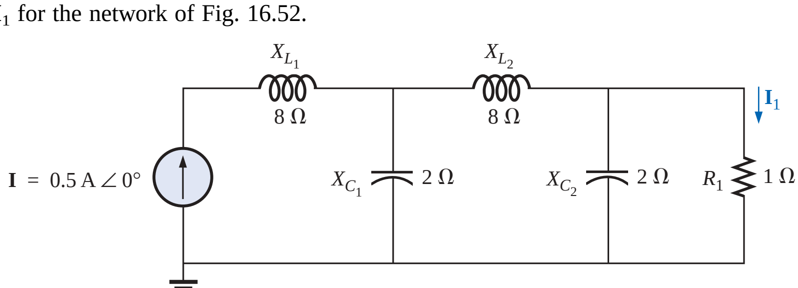


FIG. 16.52
Problems 14 and 18.

SECTION 16.5 Computer Analysis

PSpice or Electronics Workbench

For Problems 15 through 18, use a frequency of 1 kHz to determine the inductive and capacitive levels required for the input files. In each case write the required input file.

***15.** Repeat Problem 2 using PSpice or EWB.

***16.** Repeat Problem 7, parts (a) and (b), using PSpice or EWB.

***17.** Repeat Problem 11 using PSpice or EWB.

***18.** Repeat Problem 14 using PSpice or EWB.

Programming Language (C++, QBASIC, Pascal, etc.)

19. Write a program to provide a general solution to Problem 1; that is, given the reactance of each element, generate a solution for parts (a) through (e).

20. Given the network of Fig. 16.41, write a program to generate a solution for parts (a) and (b) of Problem 2. Use the values given.

21. Generate a program to obtain a general solution for the network of Fig. 16.43 for the questions asked in parts (a) through (c) of Problem 2. That is, given the resistance and reactance of the elements, determine the requested current, voltage, and power.

GLOSSARY

Ladder network A repetitive combination of series and parallel branches that has the appearance of a ladder.

Series-parallel ac network A combination of series and parallel branches in the same network configuration. Each branch may contain any number of elements whose impedance is dependent on the applied frequency.