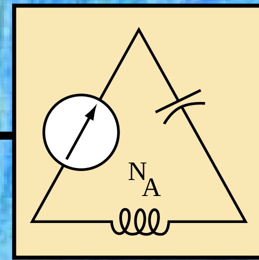
17



Methods of Analysis and Selected Topics (ac)

17.1 INTRODUCTION

For networks with two or more sources that are not in series or parallel, the methods described in the last two chapters cannot be applied. Rather, methods such as mesh analysis or nodal analysis must be employed. Since these methods were discussed in detail for dc circuits in Chapter 8, this chapter will consider the variations required to apply these methods to ac circuits. Dependent sources will also be introduced for both mesh and nodal analysis.

The branch-current method will not be discussed again because it falls within the framework of mesh analysis. In addition to the methods mentioned above, the bridge network and Δ-Y, Y-Δ conversions will also be discussed for ac circuits.

Before we examine these topics, however, we must consider the subject of independent and controlled sources.

17.2 INDEPENDENT VERSUS DEPENDENT (CONTROLLED) SOURCES

In the previous chapters, each source appearing in the analysis of dc or ac networks was an **independent source,** such as E and I (or $\mathbf{E}$ and $\mathbf{I}$) in Fig. 17.1.

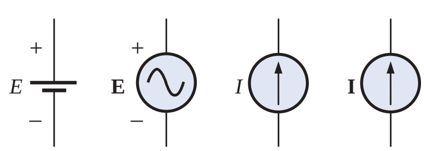


FIG. 17.1
Independent sources.

The term* independent *specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.

A* dependent *or* controlled source *is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.

Currently two symbols are used for controlled sources. One simply uses the independent symbol with an indication of the controlling element, as shown in Fig. 17.2. In Fig. 17.2(a), the magnitude and phase of the voltage are controlled by a voltage **V** elsewhere in the system, with the magnitude further controlled by the constant k_1. In Fig.

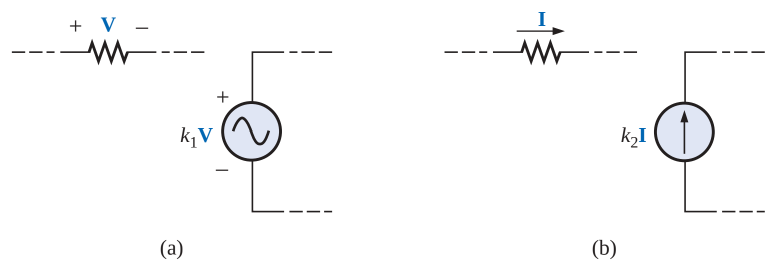


FIG. 17.2
Controlled or dependent sources.

17.2(b), the magnitude and phase of the current source are controlled by a current **I** elsewhere in the system, with the magnitude further controlled by the constant k_2. To distinguish between the dependent and independent sources, the notation of Fig. 17.3 was introduced. In recent years many respected publications on circuit analysis have accepted the notation of Fig. 17.3, although a number of excellent publications in the area of electronics continue to use the symbol of Fig. 17.2, especially in the circuit modeling for a variety of electronic devices such as the transistor and FET. This text will employ the symbols of Fig. 17.3.

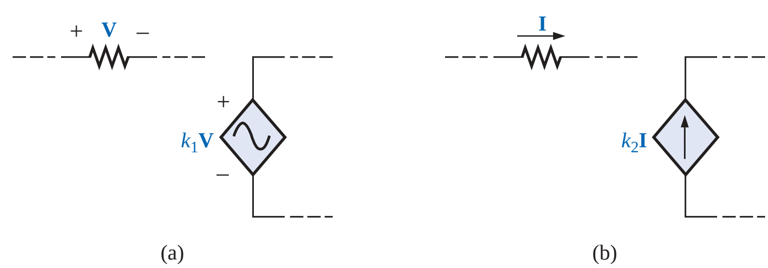


FIG. 17.3
Special notation for controlled or dependent sources.

Possible combinations for controlled sources are indicated in Fig. 17.4. Note that the magnitude of current sources or voltage sources can be controlled by a voltage and a current, respectively. Unlike with the independent source, isolation such that **V** or **I** = 0 in Fig. 17.4(a) will result in the short-circuit or open-circuit equivalent as indicated in Fig. 17.4(b). Note that the type of representation under these conditions is controlled by whether it is a current source or a voltage source, not by the controlling agent (**V** or **I**).

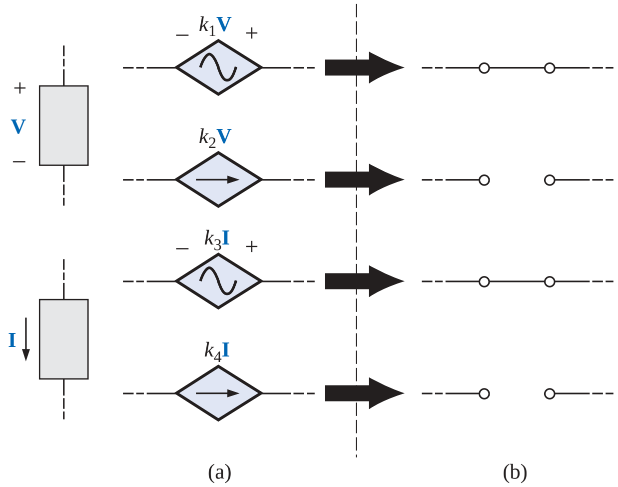


FIG. 17.4
Conditions of V = 0 V and I = 0 A for a controlled source.

17.3 SOURCE CONVERSIONS

When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source, or a voltage source to a current source. This **source conversion** can be accomplished in much the same manner as for dc circuits, except now we shall be dealing with phasors and impedances instead of just real numbers and resistors.

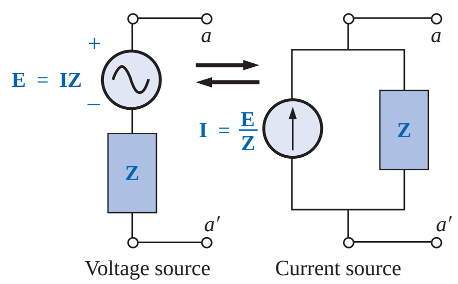


FIG. 17.5
Source conversion.

Independent Sources

In general, the format for converting one type of independent source to another is as shown in Fig. 17.5.

EXAMPLE 17.1 Convert the voltage source of Fig. 17.6(a) to a current source.

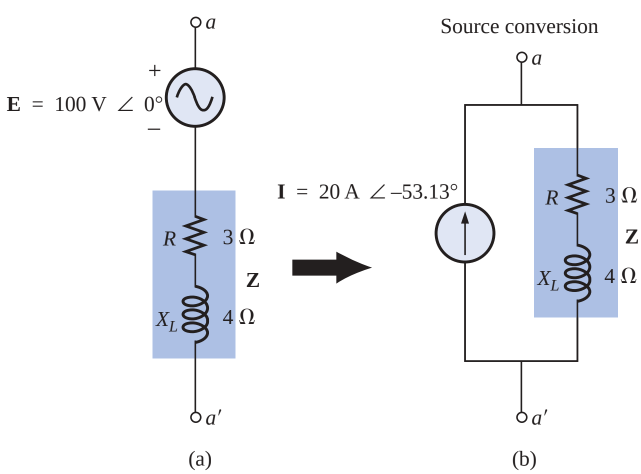


FIG. 17.6
Example 17.1.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ}$$
$$= \mathbf{20\text{ A}\angle -53.13^\circ} \qquad \text{[Fig. 17.6(b)]}$$

EXAMPLE 17.2 Convert the current source of Fig. 17.7(a) to a voltage source.

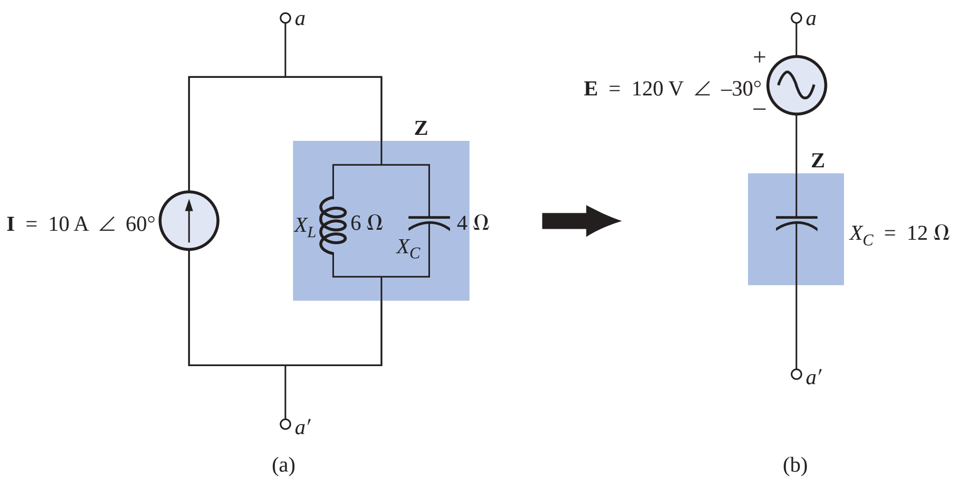


FIG. 17.7
Example 17.2.

Solution:

$$\mathbf{Z} = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C\angle -90^\circ)(X_L\angle 90^\circ)}{-j\,X_C + j\,X_L}$$
$$= \frac{(4\ \Omega\angle -90^\circ)(6\ \Omega\angle 90^\circ)}{-j\,4\ \Omega + j\,6\ \Omega} = \frac{24\ \Omega\angle 0^\circ}{2\angle 90^\circ}$$
$$= \mathbf{12\ \Omega\angle -90^\circ} \qquad \text{[Fig. 17.7(b)]}$$
$$\mathbf{E} = \mathbf{IZ} = (10\text{ A}\angle 60^\circ)(12\ \Omega\angle -90^\circ)$$
$$= \mathbf{120\text{ V}\angle -30^\circ} \qquad \text{[Fig. 17.7(b)]}$$

Dependent Sources

For dependent sources, the direct conversion of Fig. 17.5 can be applied if the controlling variable (**V** or **I** in Fig. 17.4) is not determined by a portion of the network to which the conversion is to be applied. For example, in Figs. 17.8 and 17.9, **V** and **I**, respectively, are controlled by an external portion of the network. Conversions of the other kind, where **V** and **I** are controlled by a portion of the network to be converted, will be considered in Sections 18.3 and 18.4.

EXAMPLE 17.3 Convert the voltage source of Fig. 17.8(a) to a current source.

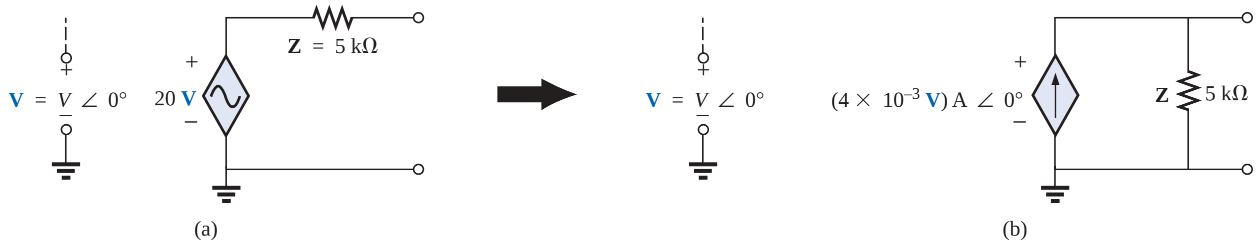


FIG. 17.8
Source conversion with a voltage-controlled voltage source.

Solution:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{(20\mathbf{V})\ \mathrm{V}\ \angle 0^\circ}{5\ \mathrm{k}\Omega\ \angle 0^\circ}$$
$$= \mathbf{(4 \times 10^{-3}\ V)\ A\ \angle 0^\circ} \qquad \text{[Fig. 17.8(b)]}$$

EXAMPLE 17.4 Convert the current source of Fig. 17.9(a) to a voltage source.

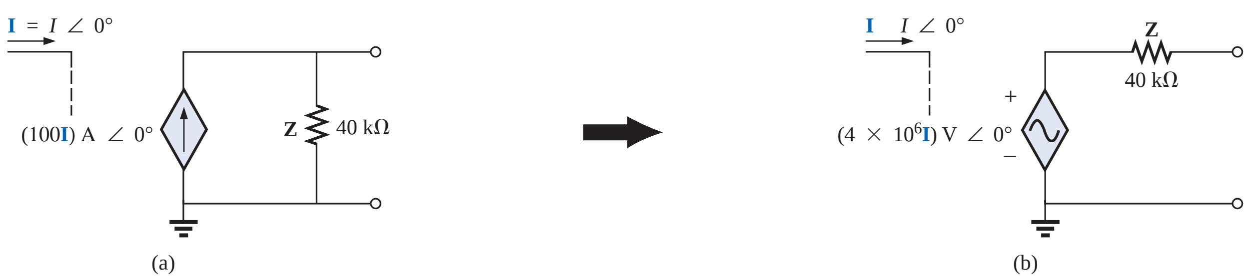


FIG. 17.9
Source conversion with a current-controlled current source.

Solution:

$$\mathbf{E} = \mathbf{IZ} = [(100\mathbf{I})\ \mathrm{A}\ \angle 0^\circ][40\ \mathrm{k}\Omega\ \angle 0^\circ]$$
$$= \mathbf{(4 \times 10^{6} I)\ V\ \angle 0^\circ} \qquad \text{[Fig. 17.9(b)]}$$

17.4 MESH ANALYSIS

General Approach

Independent Voltage Sources Before examining the application of the method to ac networks, the student should first review the appropriate sections on **mesh analysis** in Chapter 8 since the content of this section will be limited to the general conclusions of Chapter 8.

The general approach to mesh analysis for independent sources includes the same sequence of steps appearing in Chapter 8. In fact, throughout this section the only change from the dc coverage will be to substitute impedance for resistance and admittance for conductance in the general procedure.

1. ***Assign a distinct current in the clockwise direction to each independent closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. However, it eliminates the need to have to choose a direction for each application. Any direction can be chosen for each loop current with no loss in accuracy as long as the remaining steps are followed properly.***
2. ***Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.***
3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the format approach to follow.***
   a. ***If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents passing through in the opposite direction.***
   b. ***The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***
4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

The technique is applied as above for all networks with independent sources or for networks with *dependent sources where the controlling variable is not a part of the network under investigation.* If the controlling variable is part of the network being examined, a method to be described shortly must be applied.

EXAMPLE 17.5 Using the general approach to mesh analysis, find the current $\mathbf{I}_1$ in Fig. 17.10.

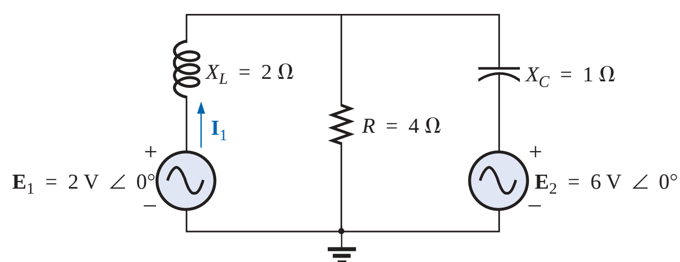


FIG. 17.10
Example 17.5.

Solution: When applying these methods to ac circuits, it is good practice to represent the resistors and reactances (or combinations thereof) by subscripted impedances. When the total solution is found in terms of these subscripted impedances, the numerical values can be substituted to find the unknown quantities.

The network is redrawn in Fig. 17.11 with subscripted impedances:

$$\mathbf{Z}_1 = +j\,X_L = +j\,2\ \Omega \qquad \mathbf{E}_1 = 2\ \text{V}\ \angle 0°$$
$$\mathbf{Z}_2 = R = 4\ \Omega \qquad \mathbf{E}_2 = 6\ \text{V}\ \angle 0°$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,1\ \Omega$$

Steps 1 and 2 are as indicated in Fig. 17.11.

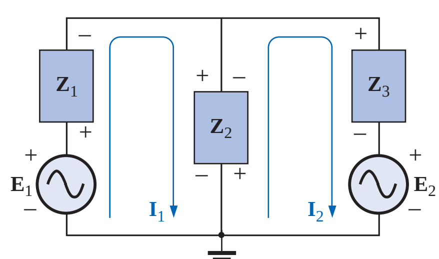


FIG. 17.11
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.10.

Step 3:

$$+\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

or

$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2\mathbf{Z}_2 = 0$$
$$-\mathbf{I}_2\mathbf{Z}_2 + \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

so that

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

which are rewritten as

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 \qquad\qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

Step 4: Using determinants, we obtain

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$
$$= \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2}$$
$$= \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_1 = \frac{(2\text{ V} - 6\text{ V})(4\ \Omega) + (2\text{ V})(-j\,1\ \Omega)}{(+j\,2\ \Omega)(4\ \Omega) + (+j\,2\ \Omega)(-j\,2\ \Omega) + (4\ \Omega)(-j\,2\ \Omega)}$$
$$= \frac{-16 - j\,2}{j\,8 - j^2 2 - j\,4} = \frac{-16 - j\,2}{2 + j\,4} = \frac{16.12\text{ A} \angle -172.87^\circ}{4.47 \angle 63.43^\circ}$$
$$= \mathbf{3.61\ A \angle -236.30^\circ} \quad \text{or} \quad \mathbf{3.61\ A \angle 123.70^\circ}$$

Dependent Voltage Sources For dependent voltage sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: Treat each dependent source like an independent source when Kirchhoff's voltage law is applied to each independent loop. However, once the equation is written, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen mesh currents.
3. Step 4 is as before.

EXAMPLE 17.6 Write the mesh currents for the network of Fig. 17.12 having a dependent voltage source.

Solution:

Steps 1 and 2 are defined on Fig. 17.12.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$

$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{V}_x - \mathbf{I}_2 R_3 = 0$

Then substitute $\mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$

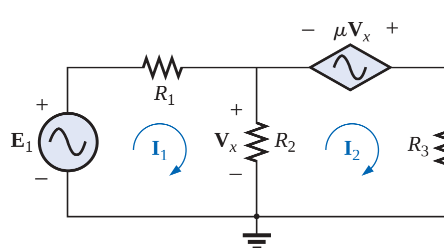


FIG. 17.12
Applying mesh analysis to a network with a voltage-controlled voltage source.

The result is two equations and two unknowns.

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I} - \mathbf{I}_2) = 0$$
$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2 R_3 = 0$$

Independent Current Sources For independent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each current source as an open circuit (recall the *supermesh* designation of Chapter 8), and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited simply to the mesh currents.
3. Step 4 is as before.

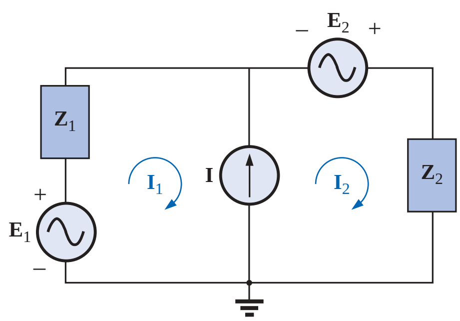


FIG. 17.13
Applying mesh analysis to a network with an independent current source.

EXAMPLE 17.7 Write the mesh currents for the network of Fig. 17.13 having an independent current source.

Solution:

Steps 1 and 2 are defined on Fig. 17.13.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 + \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2 = 0$ (only remaining independent path)

with $\mathbf{I}_1 + \mathbf{I} = \mathbf{I}_2$

The result is two equations and two unknowns.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent current sources, except now the dependent sources have to be defined in terms of the chosen mesh currents to ensure that the final equations have only mesh currents as the unknown quantities.
3. Step 4 is as before.

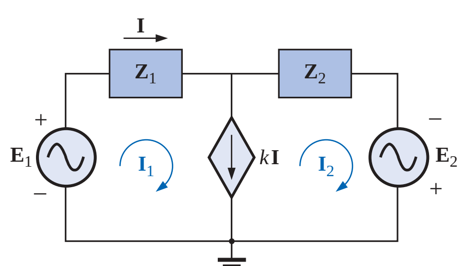


FIG. 17.14
Applying mesh analysis to a network with a current-controlled current source.

EXAMPLE 17.8 Write the mesh currents for the network of Fig. 17.14 having a dependent current source.

Solution:

Steps 1 and 2 are defined on Fig. 17.14.

Step 3: $\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0$

and $k\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$

Now $\mathbf{I} = \mathbf{I}_1$ so that $k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$ or $\mathbf{I}_2 = \mathbf{I}_1(1 - k)$

The result is two equations and two unknowns.

Format Approach

The format approach was introduced in Section 8.9. The steps for applying this method are repeated here with changes for its use in ac circuits:

1. ***Assign a loop current to each independent closed loop (as in the previous section) in a clockwise direction.***
2. ***The number of required equations is equal to the number of chosen independent closed loops. Column 1 of each equation is formed by simply summing the impedance values of those impedances through which the loop current of interest passes and multiplying the result by that loop current.***
3. ***We must now consider the mutual terms that are always subtracted from the terms in the first column. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. Each mutual term is the product of the mutual impedance and the other loop current passing through the same element.***
4. ***The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. Negative signs are assigned to those potentials for which the reverse is true.***
5. ***Solve the resulting simultaneous equations for the desired loop currents.***

The technique is applied as above for all networks with independent sources or for networks with dependent sources where the controlling variable is not a part of the network under investigation. If the controlling variable is part of the network being examined, additional care must be taken when applying the above steps.

EXAMPLE 17.9 Using the format approach to mesh analysis, find the current $\mathbf{I}_2$ in Fig. 17.15.

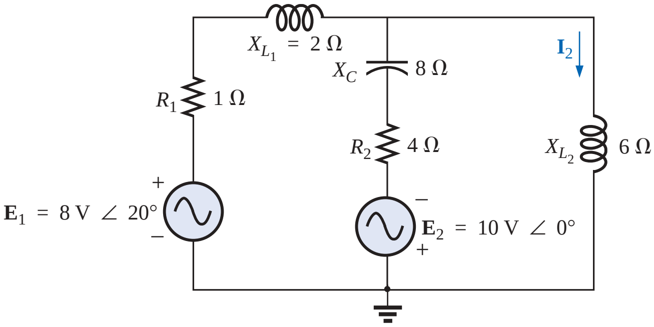


FIG. 17.15
Example 17.9.

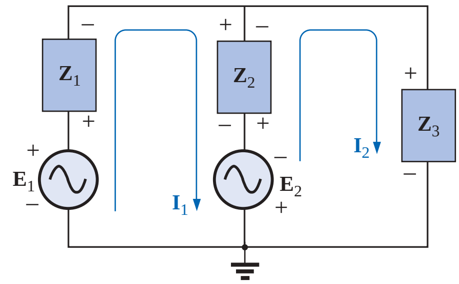


FIG. 17.16
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.16.

Solution 1: The network is redrawn in Fig. 17.16:

$$\mathbf{Z}_1 = R_1 + j\,X_{L_1} = 1\ \Omega + j\,2\ \Omega \qquad \mathbf{E}_1 = 8\text{ V} \angle 20°$$
$$\mathbf{Z}_2 = R_2 - j\,X_C = 4\ \Omega - j\,8\ \Omega \qquad \mathbf{E}_2 = 10\text{ V} \angle 0°$$
$$\mathbf{Z}_3 = +j\,X_{L_2} = +j\,6\ \Omega$$

Note the reduction in complexity of the problem with the substitution of the subscripted impedances.

Step 1 is as indicated in Fig. 17.16.

Steps 2 to 4:

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 + \mathbf{E}_2$$
$$\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 = -\mathbf{E}_2$$

which are rewritten as

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 \qquad\qquad &= \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{I}_1\mathbf{Z}_2 \qquad\qquad + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

Step 5: Using determinants, we have

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & \mathbf{E}_1 + \mathbf{E}_2 \\ -\mathbf{Z}_2 & -\mathbf{E}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}$$
$$= \frac{-(\mathbf{Z}_1 + \mathbf{Z}_2)\mathbf{E}_2 + \mathbf{Z}_2(\mathbf{E}_1 + \mathbf{E}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{Z}_2^2}$$
$$= \frac{\mathbf{Z}_2\mathbf{E}_1 - \mathbf{Z}_1\mathbf{E}_2}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

Substituting numerical values yields

$$\mathbf{I}_2 = \frac{(4\ \Omega - j\,8\ \Omega)(8\ \text{V}\angle 20^\circ) - (1\ \Omega + j\,2\ \Omega)(10\ \text{V}\angle 0^\circ)}{(1\ \Omega + j\,2\ \Omega)(4\ \Omega - j\,8\ \Omega) + (1\ \Omega + j\,2\ \Omega)(+j\,6\ \Omega) + (4\ \Omega - j\,8\ \Omega)(+j\,6\ \Omega)}$$
$$= \frac{(4 - j\,8)(7.52 + j\,2.74) - (10 + j\,20)}{20 + (j\,6 - 12) + (j\,24 + 48)}$$
$$= \frac{(52.0 - j\,49.20) - (10 + j\,20)}{56 + j\,30} = \frac{42.0 - j\,69.20}{56 + j\,30} = \frac{80.95\ \text{A}\angle -58.74^\circ}{63.53\angle 28.18^\circ}$$
$$= \mathbf{1.27\ A\ \angle -86.92^\circ}$$

Calculator The calculator (TI-86 or equivalent) can be an effective tool in performing the long, laborious calculations involved with the final equation appearing above. However, you must be very careful to use the correct number of brackets and to define by brackets the order of the arithmetic operations.

```
((4,−8)*8(∠20)−(1,2)*(10∠0))/((1,2)*(4,−8)+(1,2)*(0,6)+(4,−8)*(0,6)) ENTER
(67.854E−3,−1.272E0)
Ans ▶ Pol
(1.274E0∠−86.956E0)
```

CALC. 17.1

Mathcad Solution: This example provides an excellent opportunity to demonstrate the power of Mathcad. First the impedances and parameters are defined for the equations to follow as shown in Fig. 17.17. Then the **guess** values of the mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are entered. The label **Given** must then be entered followed by the equations for the net-

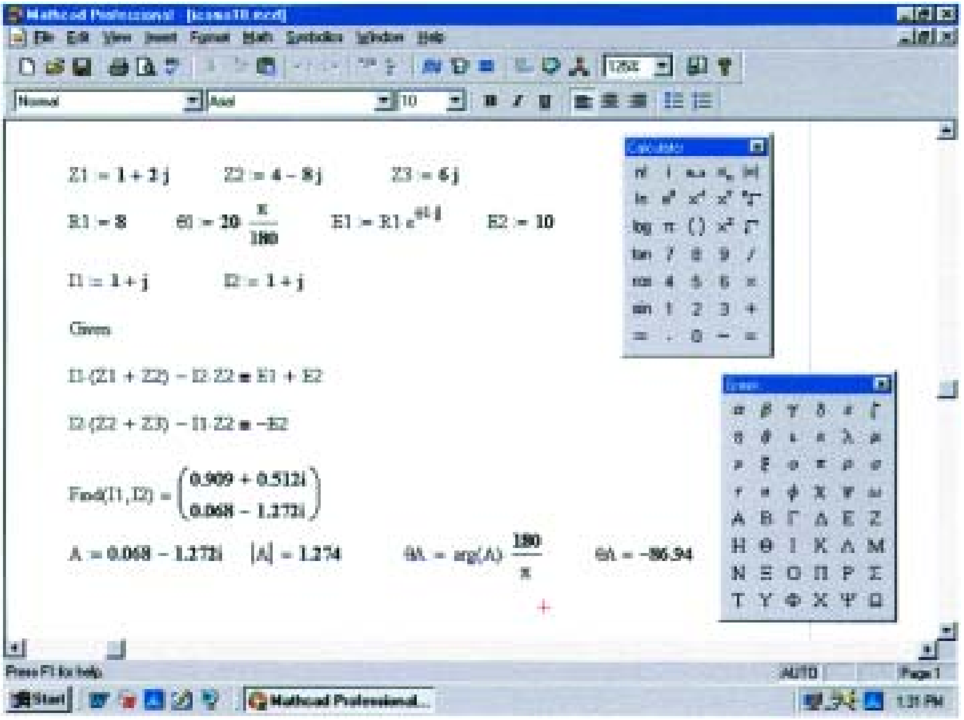


FIG. 17.17
Using Mathcad to verify the results of Example 17.9.

work. Note that in this example, we are not continuing with the analysis until the matrix is defined—we are working directly from the network equations. Once the equations have been properly entered, **Find(I1,I2)** is entered. Then selecting the equal sign will result in the single-column matrix with the results in rectangular form. Conversion to polar form requires defining a variable **A** and then calling for the magnitude and angle using the definitions entered earlier in the listing and both the **Calculator** and **Greek** toolbars. The result for $\mathbf{I}_2$ is 1.274 A $\angle -86.94°$ which is an excellent match with the theoretical solution.

EXAMPLE 17.10 Write the mesh equations for the network of Fig. 17.18. Do not solve.

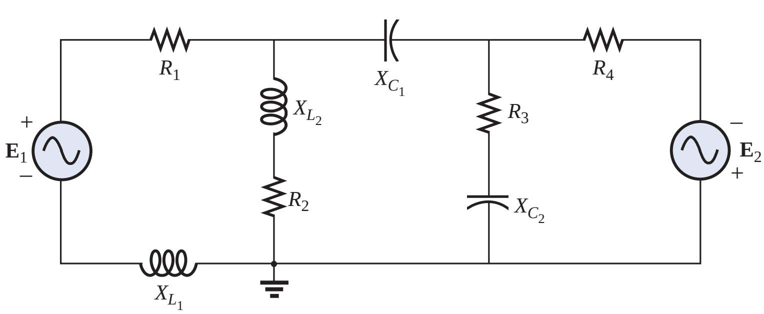


FIG. 17.18
Example 17.10.

Solution: The network is redrawn in Fig. 17.19. Again note the reduced complexity and increased clarity provided by the use of subscripted impedances:

$$\begin{aligned} \mathbf{Z}_1 &= R_1 + j X_{L_1} \qquad & \mathbf{Z}_4 &= R_3 - j X_{C_2} \\ \mathbf{Z}_2 &= R_2 + j X_{L_2} & \mathbf{Z}_5 &= R_4 \\ \mathbf{Z}_3 &= j X_{C_1} \end{aligned}$$

and

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 \\ \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 &= 0 \\ \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) - \mathbf{I}_2\mathbf{Z}_4 &= \mathbf{E}_2 \end{aligned}$$

or

$$\begin{array}{llll} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) & - \mathbf{I}_2(\mathbf{Z}_2) & + 0 & = \mathbf{E}_1 \\ \mathbf{I}_1\mathbf{Z}_2 & - \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) & + \mathbf{I}_3(\mathbf{Z}_4) & = 0 \\ 0 & - \mathbf{I}_2(\mathbf{Z}_4) & + \mathbf{I}_3(\mathbf{Z}_4 + \mathbf{Z}_5) & = \mathbf{E}_2 \end{array}$$

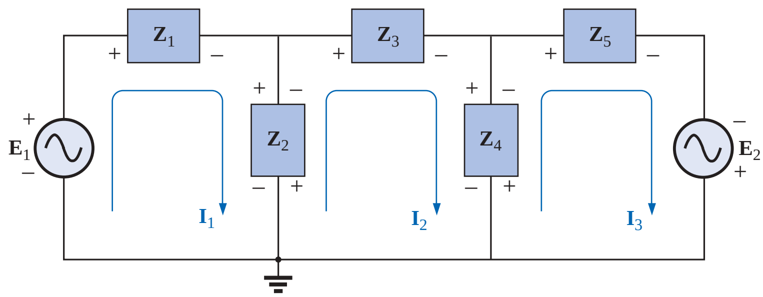


FIG. 17.19
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.18.

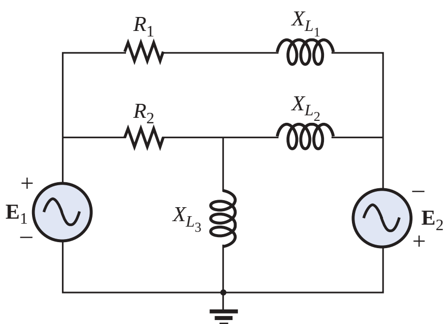


FIG. 17.20
Example 17.11

EXAMPLE 17.11 Using the format approach, write the mesh equations for the network of Fig. 17.20.

Solution: The network is redrawn as shown in Fig. 17.21, where

$$\begin{aligned} \mathbf{Z}_1 &= R_1 + j X_{L_1} \qquad & \mathbf{Z}_3 &= j X_{L_2} \\ \mathbf{Z}_2 &= R_2 & \mathbf{Z}_4 &= j X_{L_3} \end{aligned}$$

and

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_4 &= \mathbf{E}_1 \\ \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{I}_1\mathbf{Z}_2 - \mathbf{I}_3\mathbf{Z}_3 &= 0 \\ \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{I}_1\mathbf{Z}_4 &= \mathbf{E}_2 \end{aligned}$$

or

$$\begin{array}{llll} \mathbf{I}_1(\mathbf{Z}_2 + \mathbf{Z}_4) & - \mathbf{I}_2\mathbf{Z}_2 & - \mathbf{I}_3\mathbf{Z}_4 & = \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 & + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3) & - \mathbf{I}_3\mathbf{Z}_3 & = 0 \\ -\mathbf{I}_1\mathbf{Z}_4 & - \mathbf{I}_2\mathbf{Z}_3 & + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4) & = \mathbf{E}_2 \end{array}$$

Note the symmetry *about* the diagonal axis; that is, note the location of $-\mathbf{Z}_2$, $-\mathbf{Z}_4$, and $-\mathbf{Z}_3$ off the diagonal.

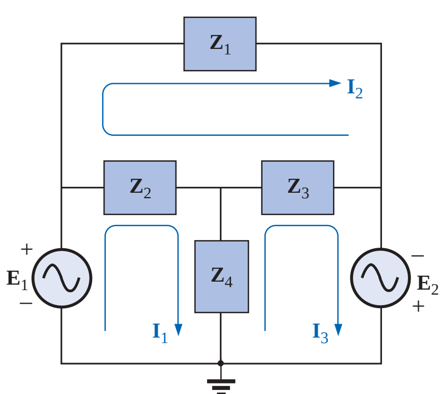


FIG. 17.21
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.20.

17.5 NODAL ANALYSIS

General Approach

Independent Sources Before examining the application of the method to ac networks, a review of the appropriate sections on **nodal**

analysis in Chapter 8 is suggested since the content of this section will be limited to the general conclusions of Chapter 8.

The fundamental steps are the following:

1. ***Determine the number of nodes within the network.***
2. ***Pick a reference node and label each remaining node with a subscripted value of voltage:*** $\mathbf{V}_1$, $\mathbf{V}_2$, ***and so on.***
3. ***Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.***
4. ***Solve the resulting equations for the nodal voltages.***

A few examples will refresh your memory about the content of Chapter 8 and the general approach to a nodal-analysis solution.

EXAMPLE 17.12 Determine the voltage across the inductor for the network of Fig. 17.22.

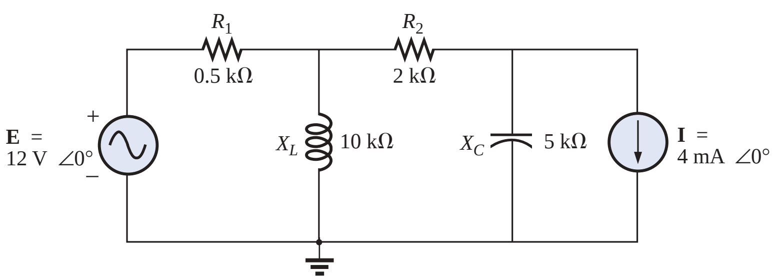


FIG. 17.22
Example 17.12.

Solution 1:

Steps 1 and 2 are as indicated in Fig. 17.23.

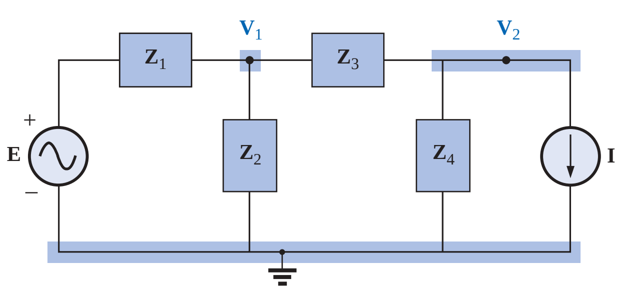


FIG. 17.23
Assigning the nodal voltages and subscripted impedances to the network of Fig. 17.22.

Step 3: Note Fig. 17.24 for the application of Kirchhoff's current law to node $\mathbf{V}_1$:

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

$$0 = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

$$\frac{\mathbf{V}_1 - \mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} = 0$$

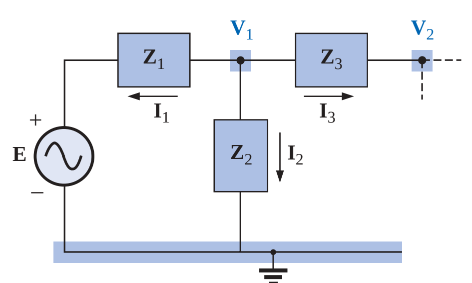


FIG. 17.24
Applying Kirchhoff's current law to the node $\mathbf{V}_1$ *of Fig. 17.23.*

Rearranging terms:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}_1}{\mathbf{Z}_1} \qquad \textbf{(17.1)}$$

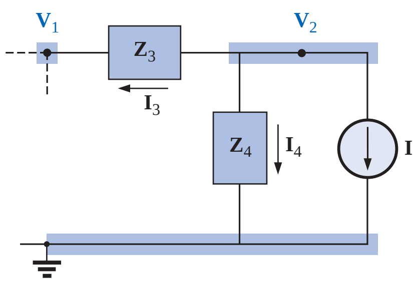


FIG. 17.25
Applying Kirchhoff's current law to the node $\mathbf{V}_2$ of Fig. 17.23.

Note Fig. 17.25 for the application of Kirchhoff's current law to node $\mathbf{V}_2$.

$$0=\mathbf{I}_3+\mathbf{I}_4+\mathbf{I}$$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{\mathbf{Z}_3}+\frac{\mathbf{V}_2}{\mathbf{Z}_4}+\mathbf{I}=0$$

Rearranging terms:

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]=-\mathbf{I} \qquad \textbf{(17.2)}$$

Grouping equations:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}}{\mathbf{Z}_1}$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]=\mathbf{I}$$

$$\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}=\frac{1}{0.5\text{ k}\Omega}+\frac{1}{j\,10\text{ k}\Omega}+\frac{1}{2\text{ k}\Omega}=2.5\text{ mS }\angle-2.29^\circ$$

$$\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}=\frac{1}{2\text{ k}\Omega}+\frac{1}{-j\,5\text{ k}\Omega}=0.539\text{ mS }\angle 21.80^\circ$$

and

$$\mathbf{V}_1[2.5\text{ mS }\angle-2.29^\circ]-\mathbf{V}_2[0.5\text{ mS }\angle 0^\circ]=24\text{ mA }\angle 0^\circ$$
$$\mathbf{V}_1[0.5\text{ mS }\angle 0^\circ]-\mathbf{V}_2[0.539\text{ mS }\angle 21.80^\circ]=4\text{ mA }\angle 0^\circ$$

with

$$\mathbf{V}_1=\frac{\begin{vmatrix}24\text{ mA }\angle 0^\circ & -0.5\text{ mS }\angle 0^\circ\\ 4\text{ mA }\angle 0^\circ & -0.539\text{ mS }\angle 21.80^\circ\end{vmatrix}}{\begin{vmatrix}2.5\text{ mS }\angle-2.29^\circ & -0.5\text{ mS }\angle 0^\circ\\ 0.5\text{ mS }\angle 0^\circ & -0.539\text{ mS }\angle 21.80^\circ\end{vmatrix}}$$

$$=\frac{(24\text{ mA }\angle 0^\circ)(-0.539\text{ mS }\angle 21.80^\circ)+(0.5\text{ mS }\angle 0^\circ)(4\text{ mA }\angle 0^\circ)}{(2.5\text{ mS }\angle-2.29^\circ)(-0.539\text{ mS }\angle 21.80^\circ)+(0.5\text{ mS }\angle 0^\circ)(0.5\text{ mS }\angle 0^\circ)}$$

$$=\frac{-12.94\times 10^{-6}\text{ V }\angle 21.80^\circ+2\times 10^{-6}\text{ V }\angle 0^\circ}{-1.348\times 10^{-6}\angle 19.51^\circ+0.25\times 10^{-6}\angle 0^\circ}$$

$$=\frac{-(12.01+j\,4.81)\times 10^{-6}\text{ V}+2\times 10^{-6}\text{ V}}{-(1.271+j\,0.45)\times 10^{-6}+0.25\times 10^{-6}}$$

$$=\frac{-10.01\text{ V}-j\,4.81\text{ V}}{-1.021-j\,0.45}=\frac{11.106\text{ V }\angle-154.33^\circ}{1.116\angle-156.21^\circ}$$

$$\mathbf{V}_1=\mathbf{9.95\text{ V }\angle 1.88^\circ}$$

Mathcad Solution: The length and the complexity of the above mathematical development strongly suggest the use of an alternative approach such as Mathcad. The printout of Fig. 17.26 first defines the letters **k** and **m** to specific numerical values so that the power-of-ten format did not have to be included in the equations. Thus, the results are cleaner and easier to review. When entering the equations, remember that the *j* is entered as 1*j* **without** the multiplication sign between the

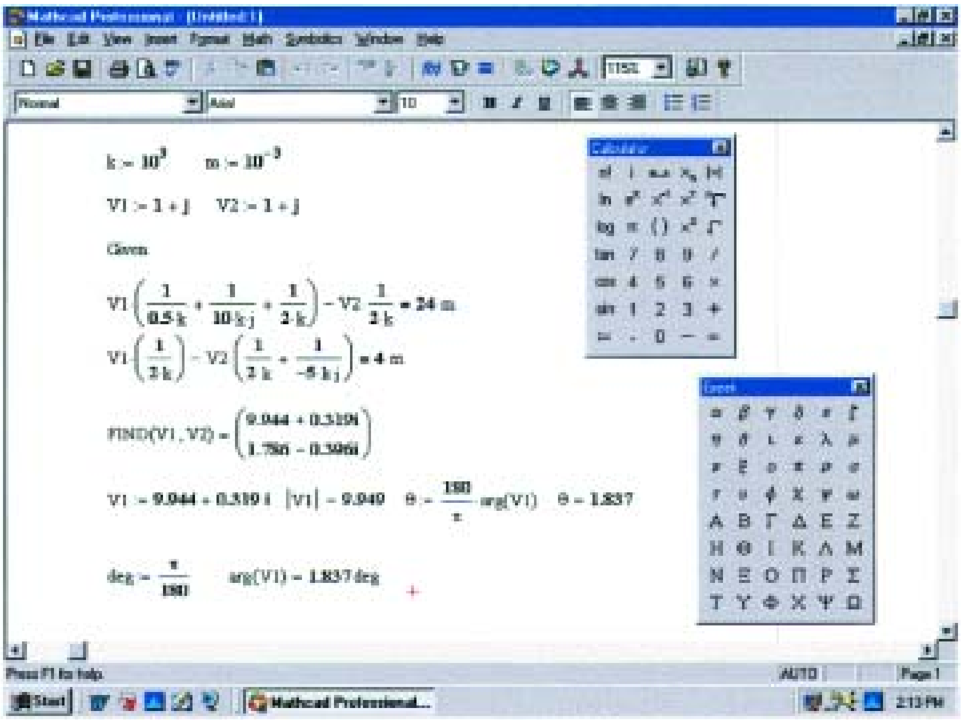


FIG. 17.26
Using Mathcad to verify the results of Example 17.12.

1 and the *j*. A multiplication sign between the two will define the *j* as another variable. Also be sure that the multiplication process is inserted between the nodal variables and the brackets. If an error signal continues to surface, it is often best to simply reenter the entire listing—errors are often not easy to spot simply by looking at the resulting equations.

Finally the results are obtained and converted to polar form for comparison with the theoretical solution. The solution of 9.949 A $\angle 1.837°$ is a very close confirmation of the longhand solution.

Before leaving this example, let's look at another method for obtaining the polar form of the solution. The method appears in the bottom of Fig. 17.26. First **deg** is defined as shown, and then **arg** is picked up from the **Insert-*f(x)*-Insert Function-arg** sequence. Next **V1** is entered; the result will be in radian form but with a small black rectangle in the place where the units normally appear. Click on that black rectangle, and the bracket will appear and **deg** can be typed. When the equal sign is selected, the angle in degrees will appear.

Dependent Current Sources For dependent current sources, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each dependent current source like an independent source when Kirchhoff's current law is applied to each defined node. However, once the equations are established, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen nodal voltages.
3. Step 4 is as before.

EXAMPLE 17.13 Write the nodal equations for the network of Fig. 17.27 having a dependent current source.

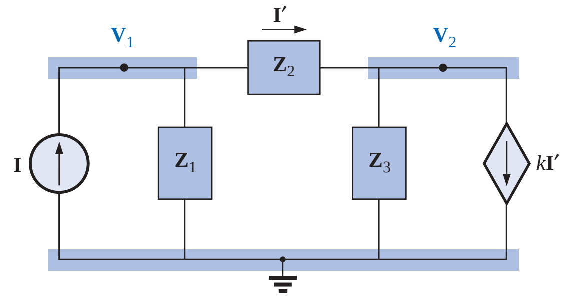


FIG. 17.27
Applying nodal analysis to a network with a current-controlled current source.

Solution:

Steps 1 and 2 are as defined in Fig. 17.27.

Step 3: At node $\mathbf{V}_1$,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} - \mathbf{I} = 0$$

and
$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}$$

At node $\mathbf{V}_2$,

$$\mathbf{I}_2 + \mathbf{I}_3 + k\mathbf{I}' = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\left[\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}\right] = 0$$

and
$$\mathbf{V}_1\left[\frac{1-k}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1-k}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] = 0$$

resulting in two equations and two unknowns.

Independent Voltage Sources between Assigned Nodes For independent voltage sources between assigned nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent sources.
2. Step 3 is modified as follows: Treat each source between defined nodes as a short circuit (recall the *supernode* classification of Chapter 8), and write the nodal equations for each remaining independent node. Then relate the chosen nodal voltages to the independent voltage source to ensure that the unknowns of the final equations are limited solely to the nodal voltages.
3. Step 4 is as before.

EXAMPLE 17.14 Write the nodal equations for the network of Fig. 17.28 having an independent source between two assigned nodes.

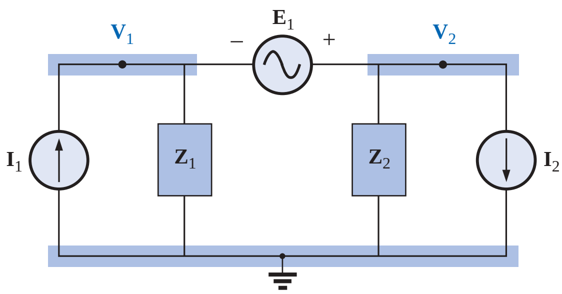


FIG. 17.28
Applying nodal analysis to a network with an independent voltage source between defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.28.

Step 3: Replacing the independent source **E** with a short-circuit equivalent results in a supernode that will generate the following equation when Kirchhoff's current law is applied to node $\mathbf{V}_1$:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2}{\mathbf{Z}_2} + \mathbf{I}_2$$

with $$\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{E}$$

and we have two equations and two unknowns.

Dependent Voltage Sources between Defined Nodes For dependent voltage sources between defined nodes, the procedure is modified as follows:

1. Steps 1 and 2 are the same as those applied for independent voltage sources.
2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent voltage sources, except now the dependent sources have to be defined in terms of the chosen nodal voltages to ensure that the final equations have only nodal voltages as their unknown quantities.
3. Step 4 is as before.

EXAMPLE 17.15 Write the nodal equations for the network of Fig. 17.29 having a dependent voltage source between two defined nodes.

Solution:

Steps 1 and 2 are defined in Fig. 17.29.

Step 3: Replacing the dependent source $\mu\mathbf{V}_x$ with a short-circuit equivalent will result in the following equation when Kirchhoff's current law is applied at node $\mathbf{V}_1$:

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{\mathbf{Z}_2} - \mathbf{I} = 0$$

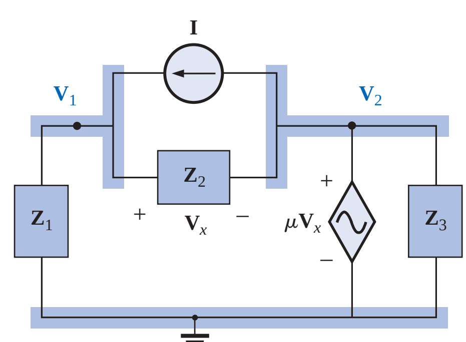


FIG. 17.29
Applying nodal analysis to a network with a voltage-controlled voltage source.

and $$\mathbf{V}_2 = \mu\mathbf{V}_x = \mu[\mathbf{V}_1 - \mathbf{V}_2]$$

or $$\mathbf{V}_2 = \frac{\mu}{1+\mu}\mathbf{V}_1$$

resulting in two equations and two unknowns. Note that because the impedance $\mathbf{Z}_3$ is in parallel with a voltage source, it does not appear in the analysis. It will, however, affect the current through the dependent voltage source.

Format Approach

A close examination of Eqs. (17.1) and (17.2) in Example 17.12 will reveal that they are the same equations that would have been obtained using the format approach introduced in Chapter 8. Recall that the approach required that the voltage source first be converted to a current source, but the writing of the equations was quite direct and minimized any chances of an error due to a lost sign or missing term.

The sequence of steps required to apply the format approach is the following:

1. ***Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining independent nodes of the network.***
2. ***The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the admittances tied to the node of interest and multiplying the result by that subscripted nodal voltage.***
3. ***The mutual terms are always subtracted from the terms of the first column. It is possible to have more than one mutual term if the nodal voltage of interest has an element in common with more than one other nodal voltage. Each mutual term is the product of the mutual admittance and the other nodal voltage tied to that admittance.***
4. ***The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node, and a negative sign if it draws current from the node.***
5. ***Solve resulting simultaneous equations for the desired nodal voltages. The comments offered for mesh analysis regarding independent and dependent sources apply here also.***

EXAMPLE 17.16 Using the format approach to nodal analysis, find the voltage across the 4-Ω resistor in Fig. 17.30.

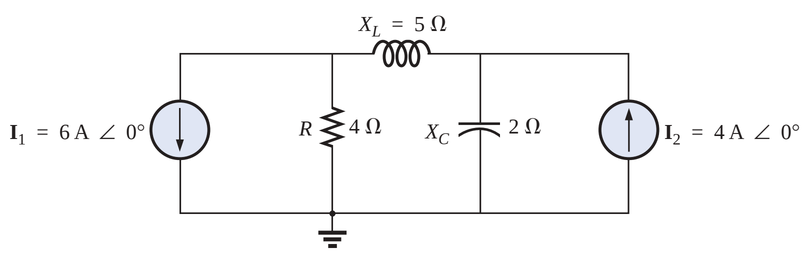


FIG. 17.30
Example 17.16.

Solution 1: Choosing nodes (Fig. 17.31) and writing the nodal equations, we have

$$\mathbf{Z}_1 = R = 4\,\Omega \qquad \mathbf{Z}_2 = j\,X_L = j\,5\,\Omega \qquad \mathbf{Z}_3 = -j\,X_C = -j\,2\,\Omega$$

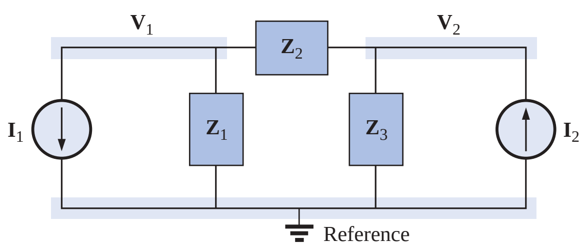


FIG. 17.31
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.30.

$$\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) = -\mathbf{I}_1$$
$$\mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{V}_1(\mathbf{Y}_2) = +\mathbf{I}_2$$

or

$$\begin{aligned} \mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) \qquad &= -\mathbf{I}_1 \\ -\mathbf{V}_1(\mathbf{Y}_2) \qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_2) &= +\mathbf{I}_2 \end{aligned}$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3}$$

Using determinants yields

$$\mathbf{V}_1 = \frac{\begin{vmatrix} -\mathbf{I}_1 & -\mathbf{Y}_2 \\ +\mathbf{I}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}}{\begin{vmatrix} \mathbf{Y}_1 + \mathbf{Y}_2 & -\mathbf{Y}_2 \\ -\mathbf{Y}_2 & \mathbf{Y}_3 + \mathbf{Y}_2 \end{vmatrix}} = \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{(\mathbf{Y}_1 + \mathbf{Y}_2)(\mathbf{Y}_3 + \mathbf{Y}_2) - \mathbf{Y}_2^2} = \frac{-(\mathbf{Y}_3 + \mathbf{Y}_2)\mathbf{I}_1 + \mathbf{I}_2\mathbf{Y}_2}{\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_2\mathbf{Y}_3 + \mathbf{Y}_1\mathbf{Y}_2}$$

Substituting numerical values, we have

$$\mathbf{V}_1 = \frac{-[(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)]6\text{ A }\angle 0° + 4\text{ A }\angle 0°(1/j\,5\,\Omega)}{(1/4\,\Omega)(1/-j\,2\,\Omega) + (1/j\,5\,\Omega)(1/-j\,2\,\Omega) + (1/4\,\Omega)(1/j\,5\,\Omega)}$$

$$= \frac{-(+j\,0.5 - j\,0.2)6\,\angle 0° + 4\,\angle 0°(-j\,0.2)}{(1/-j\,8) + (1/10) + (1/j\,20)}$$

$$= \frac{(-0.3\,\angle 90°)(6\,\angle 0°) + (4\,\angle 0°)(0.2\,\angle -90°)}{j\,0.125 + 0.1 - j\,0.05}$$

$$= \frac{-1.8\,\angle 90° + 0.8\,\angle -90°}{0.1 + j\,0.075}$$

$$= \frac{2.6\text{ V }\angle -90°}{0.125\,\angle 36.87°}$$

$$\mathbf{V}_1 = \mathbf{20.80\ V\angle -126.87°}$$

Mathcad Solution: For this example we will use the matrix format to find the nodal voltage $\mathbf{V}_1$. First the various parameters of the network are defined including the factor **deg** so that the phase angle will be displayed in degrees. Next the numerator is defined by **n,** and the **Matrix**

icon is selected from the **Matrix** toolbar. Within the **Insert Matrix** dialog box, the **Rows** and **Columns** are set as 2 followed by an **OK** to place the 2 × 2 matrix on the screen. The parameters are than entered as shown in Fig. 17.32 using a left click of the mouse to select the parameter to be entered. Once the numerator is set, the process is repeated to define the numerator. Finally the equation for **V1** is defined, and the result in rectangular form will appear when the equal sign is selected. The magnitude and the angle are then found in polar form as described in earlier sections of this chapter. The results are again a clear confirmation of the theoretical result.

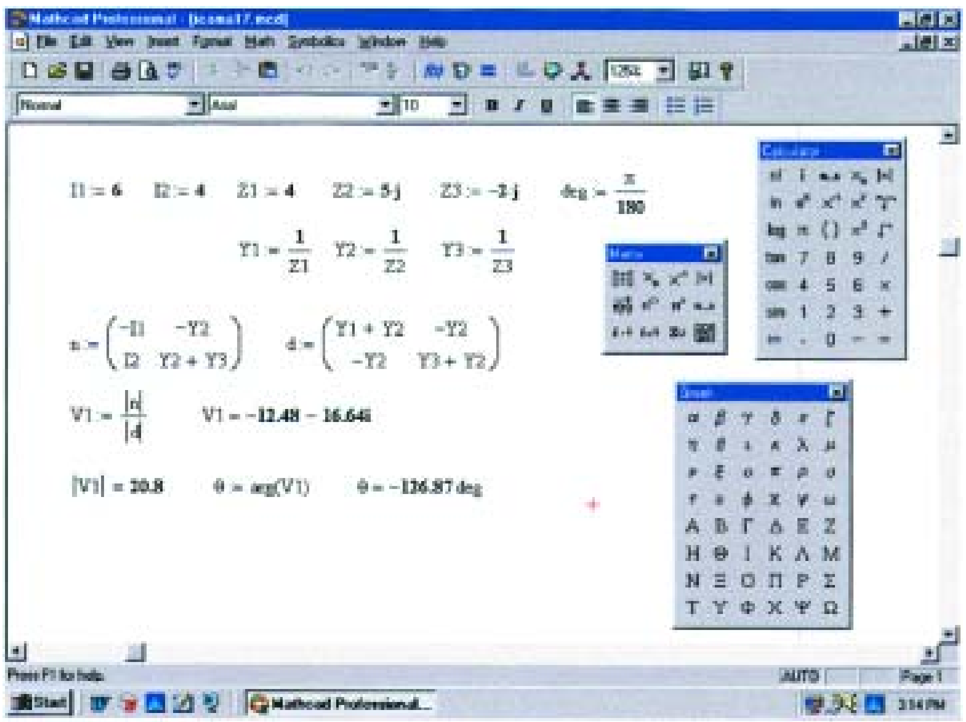


FIG. 17.32
Using Mathcad to verify the results of Example 17.16.

EXAMPLE 17.17 Using the format approach, write the nodal equations for the network of Fig. 17.33.

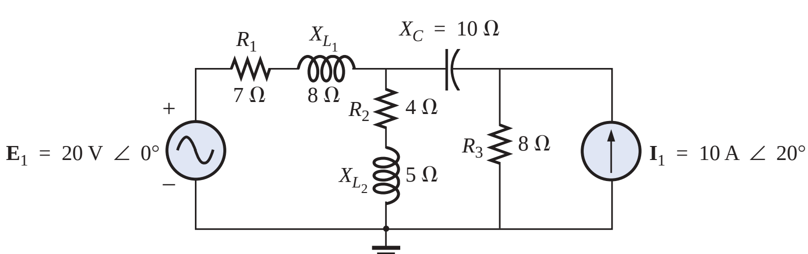


FIG. 17.33
Example 17.17.

Solution: The circuit is redrawn in Fig. 17.34, where

$$\mathbf{Z}_1 = R_1 + j X_{L_1} = 7\ \Omega + j\,8\ \Omega \qquad \mathbf{E}_1 = 20\text{ V}\angle 0°$$
$$\mathbf{Z}_2 = R_2 + j X_{L_2} = 4\ \Omega + j\,5\ \Omega \qquad \mathbf{I}_1 = 10\text{ A}\angle 20°$$
$$\mathbf{Z}_3 = -j X_C = -j\,10\ \Omega$$
$$\mathbf{Z}_4 = R_3 = 8\ \Omega$$

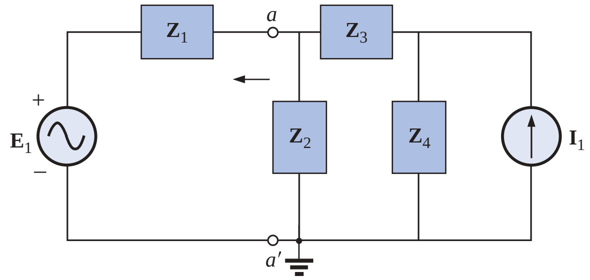


FIG. 17.34
Assigning the subscripted impedances for the network of Fig. 17.33.

Converting the voltage source to a current source and choosing nodes, we obtain Fig. 17.35. Note the “neat” appearance of the network using the subscripted impedances. Working directly with Fig. 17.33 would be more difficult and could produce errors.

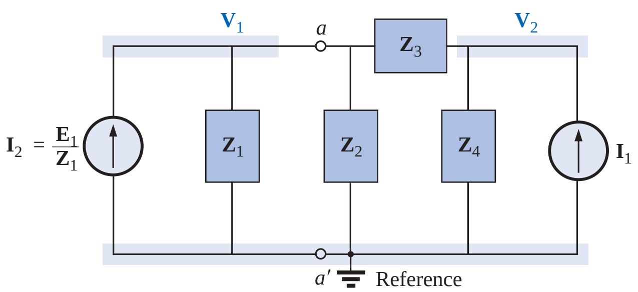


FIG. 17.35
Converting the voltage source of Fig. 17.34 to a current source and defining the nodal voltages.

Write the nodal equations:

$$\begin{aligned}\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) &= +\mathbf{I}_2\\ \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_3) &= +\mathbf{I}_1\end{aligned}$$

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} \qquad \mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4}$$

which are rewritten as

$$\begin{aligned}\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3) - \mathbf{V}_2(\mathbf{Y}_3) \qquad\qquad &= +\mathbf{I}_2\\ -\mathbf{V}_1(\mathbf{Y}_3) \qquad\qquad + \mathbf{V}_2(\mathbf{Y}_3 + \mathbf{Y}_4) &= +\mathbf{I}_1\end{aligned}$$

EXAMPLE 17.18 Write the nodal equations for the network of Fig. 17.36. Do not solve.

Solution: Choose nodes (Fig. 17.37):

$$\mathbf{Z}_1 = R_1 \qquad \mathbf{Z}_2 = j X_{L_1} \qquad \mathbf{Z}_3 = R_2 - j X_{C_2}$$
$$\mathbf{Z}_4 = -j X_{C_1} \qquad \mathbf{Z}_5 = R_3 \qquad \mathbf{Z}_6 = j X_{L_2}$$

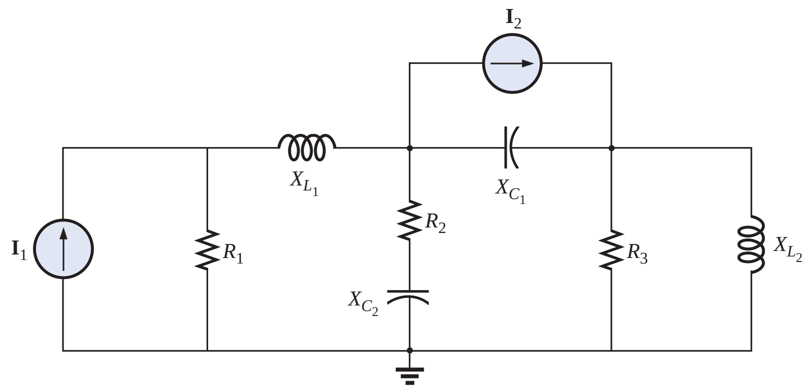


FIG. 17.36
Example 17.18.

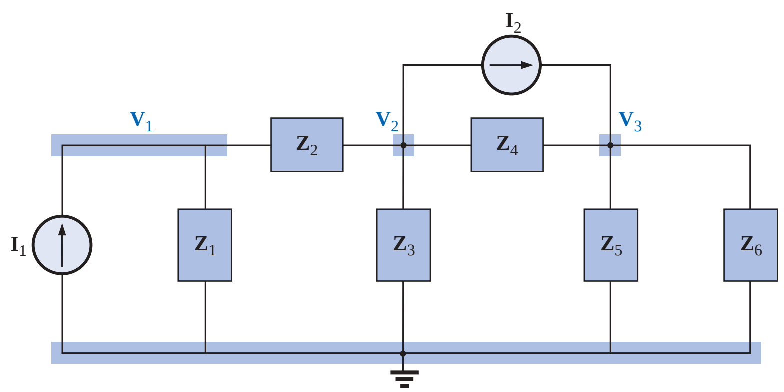


FIG. 17.37
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.36.

and write the nodal equations:

$$\begin{aligned}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) - \mathbf{V}_2(\mathbf{Y}_2) &= +\mathbf{I}_1 \\
\mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) - \mathbf{V}_1(\mathbf{Y}_2) - \mathbf{V}_3(\mathbf{Y}_4) &= -\mathbf{I}_2 \\
\mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) - \mathbf{V}_2(\mathbf{Y}_4) &= +\mathbf{I}_2
\end{aligned}$$

which are rewritten as

$$\begin{array}{llll}
\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) & - \mathbf{V}_2(\mathbf{Y}_2) & + 0 & = +\mathbf{I}_1 \\
-\mathbf{V}_1(\mathbf{Y}_2) & + \mathbf{V}_2(\mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4) & - \mathbf{V}_3(\mathbf{Y}_4) & = -\mathbf{I}_2 \\
0 & - \mathbf{V}_2(\mathbf{Y}_4) & + \mathbf{V}_3(\mathbf{Y}_4 + \mathbf{Y}_5 + \mathbf{Y}_6) & = +\mathbf{I}_2
\end{array}$$

$$\mathbf{Y}_1 = \frac{1}{R_1} \qquad \mathbf{Y}_2 = \frac{1}{j X_{L_1}} \qquad \mathbf{Y}_3 = \frac{1}{R_2 - j X_{C_2}}$$

$$\mathbf{Y}_4 = \frac{1}{-j X_{C_1}} \qquad \mathbf{Y}_5 = \frac{1}{R_3} \qquad \mathbf{Y}_6 = \frac{1}{j X_{L_2}}$$

Note the symmetry about the diagonal for this example and those preceding it in this section.

EXAMPLE 17.19 Apply nodal analysis to the network of Fig. 17.38. Determine the voltage $\mathbf{V}_L$.

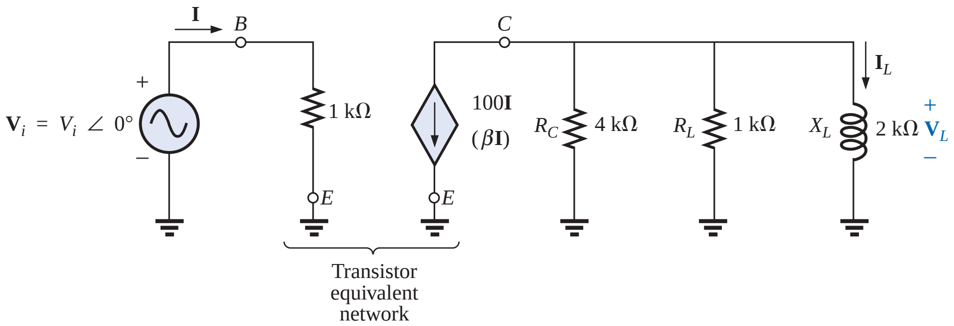


FIG. 17.38
Example 17.19.

Solution: In this case there is no need for a source conversion. The network is redrawn in Fig. 17.39 with the chosen nodal voltage and subscripted impedances.

Apply the format approach:

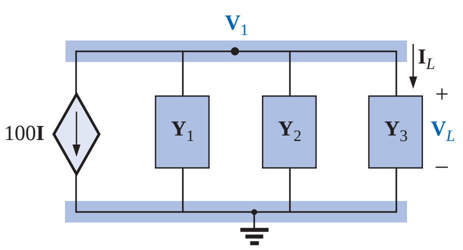


FIG. 17.39
Assigning the nodal voltage and subscripted impedances for the network of Fig. 17.38.

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{4\text{ k}\Omega} = 0.25\text{ mS}\angle 0° = G_1\angle 0°$$

$$\mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{1\text{ k}\Omega} = 1\text{ mS}\angle 0° = G_2\angle 0°$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{2\text{ k}\Omega\angle 90°} = 0.5\text{ mS}\angle -90°$$

$$= -j\,0.5\text{ mS} = -j\,B_L$$

$$\mathbf{V}_1:\quad (\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3)\mathbf{V}_1 = -100\mathbf{I}$$

and
$$\mathbf{V}_1 = \frac{-100\mathbf{I}}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3}$$

$$= \frac{-100\mathbf{I}}{0.25\text{ mS} + 1\text{ mS} - j\,0.5\text{ mS}}$$

$$= \frac{-100 \times 10^3\mathbf{I}}{1.25 - j\,0.5} = \frac{-100 \times 10^3\mathbf{I}}{1.3463\angle -21.80°}$$

$$= -74.28 \times 10^3\mathbf{I}\angle 21.80°$$

$$= -74.28 \times 10^3\left(\frac{\mathbf{V}_i}{1\text{ k}\Omega}\right)\angle 21.80°$$

$$\mathbf{V}_1 = \mathbf{V}_L = \mathbf{-(74.28V_i)\ V\angle 21.80°}$$

17.6 BRIDGE NETWORKS (ac)

The basic bridge configuration was discussed in some detail in Section 8.11 for dc networks. We now continue to examine **bridge networks** by considering those that have reactive components and a sinusoidal ac voltage or current applied.

We will first analyze various familiar forms of the bridge network using mesh analysis and nodal analysis (the format approach). The balance conditions will be investigated throughout the section.

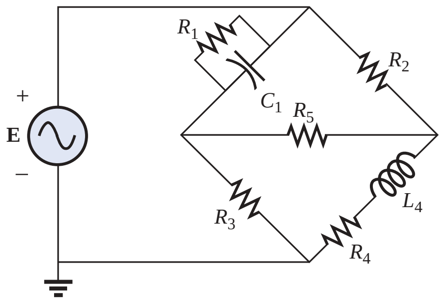


FIG. 17.40
Maxwell bridge.

Apply **mesh analysis** to the network of Fig. 17.40. The network is redrawn in Fig. 17.41, where

$$\mathbf{Z}_1 = \frac{1}{\mathbf{Y}_1} = \frac{1}{G_1 + jB_C} = \frac{G_1}{G_1^2 + B_C^2} - j\frac{B_C}{G_1^2 + B_C^2}$$

$$\mathbf{Z}_2 = R_2 \qquad \mathbf{Z}_3 = R_3 \qquad \mathbf{Z}_4 = R_4 + jX_L \qquad \mathbf{Z}_5 = R_5$$

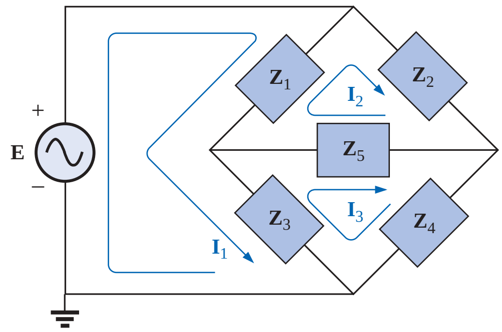


FIG. 17.41
Assigning the mesh currents and subscripted impedances for the network of Fig. 17.40.

Applying the format approach:

$$\begin{aligned}
(\mathbf{Z}_1 + \mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_1)\mathbf{I}_2 - (\mathbf{Z}_3)\mathbf{I}_3 &= \mathbf{E} \\
(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5)\mathbf{I}_2 - (\mathbf{Z}_1)\mathbf{I}_1 - (\mathbf{Z}_5)\mathbf{I}_3 &= 0 \\
(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5)\mathbf{I}_3 - (\mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_5)\mathbf{I}_2 &= 0
\end{aligned}$$

which are rewritten as

$$\begin{array}{llll}
\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_3) & - \mathbf{I}_2\mathbf{Z}_1 & - \mathbf{I}_3\mathbf{Z}_3 & = \mathbf{E} \\
-\mathbf{I}_1\mathbf{Z}_1 & + \mathbf{I}_2(\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) & - \mathbf{I}_3\mathbf{Z}_5 & = 0 \\
-\mathbf{I}_1\mathbf{Z}_3 & - \mathbf{I}_2\mathbf{Z}_5 & + \mathbf{I}_3(\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) & = 0
\end{array}$$

Note the symmetry about the diagonal of the above equations. For balance, $\mathbf{I}_{\mathbf{Z}_5} = 0$ A, and

$$\mathbf{I}_{\mathbf{Z}_5} = \mathbf{I}_2 - \mathbf{I}_3 = 0$$

From the above equations,

$$\mathbf{I}_2 = \frac{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & \mathbf{E} & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & 0 & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & 0 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_3 & -\mathbf{Z}_1 & -\mathbf{Z}_3 \\ -\mathbf{Z}_1 & (\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_5) & -\mathbf{Z}_5 \\ -\mathbf{Z}_3 & -\mathbf{Z}_5 & (\mathbf{Z}_3 + \mathbf{Z}_4 + \mathbf{Z}_5) \end{vmatrix}}$$

$$= \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_4 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

where Δ signifies the determinant of the denominator (or coefficients). Similarly,

$$\mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_5 + \mathbf{Z}_3\mathbf{Z}_5)}{\Delta}$$

and

$$\mathbf{I}_{\mathbf{Z}_5} = \mathbf{I}_2 - \mathbf{I}_3 = \frac{\mathbf{E}(\mathbf{Z}_1\mathbf{Z}_4 - \mathbf{Z}_3\mathbf{Z}_2)}{\Delta}$$

For $\mathbf{I}_{\mathbf{Z}_5} = 0$, the following must be satisfied (for a finite Δ not equal to zero):

$$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{I}_{\mathbf{Z}_5} = 0 \qquad \textbf{(17.3)}$$

This condition will be analyzed in greater depth later in this section.

Applying **nodal analysis** to the network of Fig. 17.42 will result in the configuration of Fig. 17.43, where

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1} = \frac{1}{R_1 - j\,X_C} \qquad \mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{R_2}$$

$$\mathbf{Y}_3 = \frac{1}{\mathbf{Z}_3} = \frac{1}{R_3} \qquad \mathbf{Y}_4 = \frac{1}{\mathbf{Z}_4} = \frac{1}{R_4 + j\,X_L} \qquad \mathbf{Y}_5 = \frac{1}{R_5}$$

and

$$\begin{aligned}(\mathbf{Y}_1 + \mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_1)\mathbf{V}_2 - (\mathbf{Y}_2)\mathbf{V}_3 &= \mathbf{I}\\ (\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5)\mathbf{V}_2 - (\mathbf{Y}_1)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_3 &= 0\\ (\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\mathbf{V}_3 - (\mathbf{Y}_2)\mathbf{V}_1 - (\mathbf{Y}_5)\mathbf{V}_2 &= 0\end{aligned}$$

which are rewritten as

$$\begin{array}{rrrl}\mathbf{V}_1(\mathbf{Y}_1 + \mathbf{Y}_2) & - \mathbf{V}_2\mathbf{Y}_1 & - \mathbf{V}_3\mathbf{Y}_2 & = \mathbf{I}\\ -\mathbf{V}_1\mathbf{Y}_1 & + \mathbf{V}_2(\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5) & - \mathbf{V}_3\mathbf{Y}_5 & = 0\\ -\mathbf{V}_1\mathbf{Y}_2 & - \mathbf{V}_2\mathbf{Y}_5 & + \mathbf{V}_3(\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5) & = 0\end{array}$$

Again, note the symmetry about the diagonal axis. For balance, $\mathbf{V}_{\mathbf{Z}_5} = 0$ V, and

$$\mathbf{V}_{\mathbf{Z}_5} = \mathbf{V}_2 - \mathbf{V}_3 = 0$$

From the above equations,

$$\mathbf{V}_2 = \frac{\begin{vmatrix}\mathbf{Y}_1 + \mathbf{Y}_2 & \mathbf{I} & -\mathbf{Y}_2\\ -\mathbf{Y}_1 & 0 & -\mathbf{Y}_5\\ -\mathbf{Y}_2 & 0 & (\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5)\end{vmatrix}}{\begin{vmatrix}\mathbf{Y}_1 + \mathbf{Y}_2 & -\mathbf{Y}_1 & -\mathbf{Y}_2\\ -\mathbf{Y}_1 & (\mathbf{Y}_1 + \mathbf{Y}_3 + \mathbf{Y}_5) & -\mathbf{Y}_5\\ -\mathbf{Y}_2 & -\mathbf{Y}_5 & (\mathbf{Y}_2 + \mathbf{Y}_4 + \mathbf{Y}_5\end{vmatrix}}$$

$$= \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_1\mathbf{Y}_4 + \mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_3\mathbf{Y}_5)}{\Delta}$$

Similarly,

$$\mathbf{V}_3 = \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_3 + \mathbf{Y}_3\mathbf{Y}_2 + \mathbf{Y}_1\mathbf{Y}_5 + \mathbf{Y}_3\mathbf{Y}_5)}{\Delta}$$

Note the similarities between the above equations and those obtained for mesh analysis. Then

$$\mathbf{V}_{\mathbf{Z}_5} = \mathbf{V}_2 - \mathbf{V}_3 = \frac{\mathbf{I}(\mathbf{Y}_1\mathbf{Y}_4 - \mathbf{Y}_3\mathbf{Y}_2)}{\Delta}$$

For $\mathbf{V}_{\mathbf{Z}_5} = 0$, the following must be satisfied for a finite Δ not equal to zero:

$$\boxed{\mathbf{Y}_1\mathbf{Y}_4 = \mathbf{Y}_3\mathbf{Y}_2} \qquad \mathbf{V}_{\mathbf{Z}_5} = 0 \qquad \textbf{(17.4)}$$

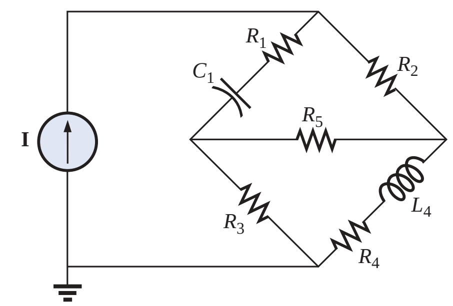


FIG. 17.42
Hay bridge.

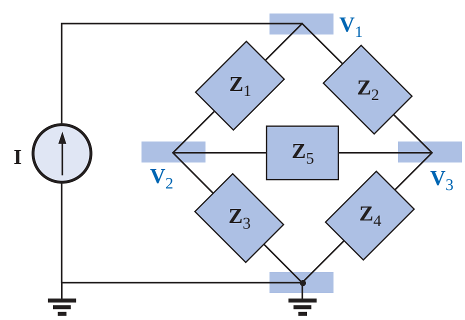


FIG. 17.43
Assigning the nodal voltages and subscripted impedances for the network of Fig. 17.42.

However, substituting $\mathbf{Y}_1 = 1/\mathbf{Z}_1$, $\mathbf{Y}_2 = 1/\mathbf{Z}_2$, $\mathbf{Y}_3 = 1/\mathbf{Z}_3$, and $\mathbf{Y}_4 = 1/\mathbf{Z}_4$, we have

$$\frac{1}{\mathbf{Z}_1\mathbf{Z}_4} = \frac{1}{\mathbf{Z}_3\mathbf{Z}_2}$$

or $$\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_3\mathbf{Z}_2} \qquad \mathbf{V}_{\mathbf{Z}_5} = 0$$

corresponding with Eq. (17.3) obtained earlier.

Let us now investigate the balance criteria in more detail by considering the network of Fig. 17.44, where it is specified that **I** and **V** = 0.

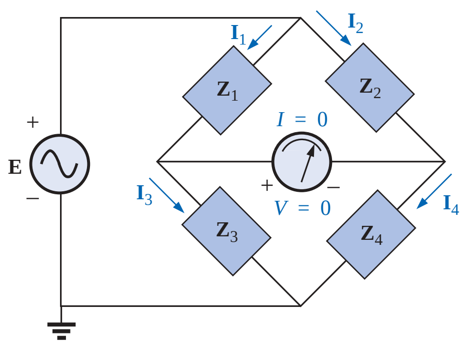


FIG. 17.44
Investigating the balance criteria for an ac bridge configuration.

Since **I** = 0,

$$\boxed{\mathbf{I}_1 = \mathbf{I}_3} \tag{17.5a}$$

and $$\boxed{\mathbf{I}_2 = \mathbf{I}_4} \tag{17.5b}$$

In addition, for **V** = 0,

$$\boxed{\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2} \tag{17.5c}$$

and $$\boxed{\mathbf{I}_3\mathbf{Z}_3 = \mathbf{I}_4\mathbf{Z}_4} \tag{17.5d}$$

Substituting the preceding current relations into Eq. (17.5d), we have

$$\mathbf{I}_1\mathbf{Z}_3 = \mathbf{I}_2\mathbf{Z}_4$$

and $$\mathbf{I}_2 = \frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1$$

Substituting this relationship for $\mathbf{I}_2$ into Eq. (17.5c) yields

$$\mathbf{I}_1\mathbf{Z}_1 = \left(\frac{\mathbf{Z}_3}{\mathbf{Z}_4}\mathbf{I}_1\right)\mathbf{Z}_2$$

and $$\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$$

as obtained earlier. Rearranging, we have

$$\boxed{\frac{\mathbf{Z}_1}{\mathbf{Z}_3} = \frac{\mathbf{Z}_2}{\mathbf{Z}_4}} \tag{17.6}$$

corresponding with Eq. (8.4) for dc resistive networks.

For the network of Fig. 17.42, which is referred to as a **Hay bridge** when $\mathbf{Z}_5$ is replaced by a sensitive galvanometer,

$$\mathbf{Z}_1 = R_1 - j\,X_C$$
$$\mathbf{Z}_2 = R_2$$
$$\mathbf{Z}_3 = R_3$$
$$\mathbf{Z}_4 = R_4 + j\,X_L$$

This particular network is used for measuring the resistance and inductance of coils in which the resistance is a small fraction of the reactance X_L.

Substitute into Eq. (17.6) in the following form:

$$\mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_4\mathbf{Z}_1$$

$$R_2R_3 = (R_4 + jX_L)(R_1 - jX_C)$$

or $$R_2R_3 = R_1R_4 + j(R_1X_L - R_4X_C) + X_CX_L$$

so that

$$R_2R_3 + j\,0 = (R_1R_4 + X_CX_L) + j(R_1X_L - R_4X_C)$$

For the equations to be equal, *the real and imaginary parts must be equal*. Therefore, for a balanced Hay bridge,

$$\boxed{R_2R_3 = R_1R_4 + X_CX_L} \tag{17.7a}$$

and $$\boxed{0 = R_1X_L - R_4X_C} \tag{17.7b}$$

or substituting $$X_L = \omega L \quad \text{and} \quad X_C = \frac{1}{\omega C}$$

we have $$X_CX_L = \left(\frac{1}{\omega C}\right)(\omega L) = \frac{L}{C}$$

and $$R_2R_3 = R_1R_4 + \frac{L}{C}$$

with $$R_1\omega L = \frac{R_4}{\omega C}$$

Solving for R_4 in the last equation yields

$$R_4 = \omega^2 LCR_1$$

and substituting into the previous equation, we have

$$R_2R_3 = R_1(\omega^2 LCR_1) + \frac{L}{C}$$

Multiply through by C and factor:

$$CR_2R_3 = L(\omega^2C^2R_1^2 + 1)$$

and $$\boxed{L = \frac{CR_2R_3}{1 + \omega^2C^2R_1^2}} \tag{17.8a}$$

With additional algebra this yields:

$$\boxed{R_4 = \frac{\omega^2C^2R_1R_2R_3}{1 + \omega^2C^2R_1^2}} \tag{17.8b}$$

Equations (17.7) and (17.8) are the balance conditions for the Hay bridge. Note that each is frequency dependent. For different frequencies, the resistive and capacitive elements must vary for a particular coil to achieve balance. For a coil placed in the Hay bridge as shown in Fig. 17.42, the resistance and inductance of the coil can be determined by Eqs. (17.8a) and (17.8b) when balance is achieved.

The bridge of Fig. 17.40 is referred to as a **Maxwell bridge** when $\mathbf{Z}_5$ is replaced by a sensitive galvanometer. This setup is used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Application of Eq. (17.6) in the form:

$$\mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_4\mathbf{Z}_1$$

and substituting

$$\mathbf{Z}_1 = R_1 \angle 0° \parallel X_{C_1} \angle -90° = \frac{(R_1 \angle 0°)(X_{C_1} \angle -90°)}{R_1 - j\,X_{C_1}}$$

$$= \frac{R_1 X_{C_1} \angle -90°}{R_1 - j\,X_{C_1}} = \frac{-j\,R_1 X_{C_1}}{R_1 - j\,X_{C_1}}$$

$$\mathbf{Z}_2 = R_2$$

$$\mathbf{Z}_3 = R_3$$

and $\mathbf{Z}_4 = R_4 + j\,X_{L_4}$

we have $$(R_2)(R_3) = (R_4 + j\,X_{L_4})\left(\frac{-j\,R_1 X_{C_1}}{R_1 - j\,X_{C_1}}\right)$$

$$R_2 R_3 = \frac{-j\,R_1 R_4 X_{C_1} + R_1 X_{C_1} X_{L_4}}{R_1 - j\,X_{C_1}}$$

or $$(R_2 R_3)(R_1 - j\,X_{C_1}) = R_1 X_{C_1} X_{L_4} - j\,R_1 R_4 X_{C_1}$$

and $$R_1 R_2 R_3 - j\,R_2 R_3 X_{C_1} = R_1 X_{C_1} X_{L_4} - j\,R_1 R_4 X_{C_1}$$

so that for balance

$$\not{R}_1 R_2 R_3 = \not{R}_1 X_{C_1} X_{L_4}$$

$$R_2 R_3 = \left(\frac{1}{\not{2}\not{\pi} f C_1}\right)(\not{2}\not{\pi} f L_4)$$

and $$\boxed{L_4 = C_1 R_2 R_3} \qquad \textbf{(17.9)}$$

and $$R_2 R_3 \not{X}_{C_1} = R_1 R_4 \not{X}_{C_1}$$

so that $$\boxed{R_4 = \frac{R_2 R_3}{R_1}} \qquad \textbf{(17.10)}$$

Note the absence of frequency in Eqs. (17.9) and (17.10).

One remaining popular bridge is the **capacitance comparison bridge** of Fig. 17.45. An unknown capacitance and its associated resistance can be determined using this bridge. Application of Eq. (17.6) will yield the following results:

$$\boxed{C_4 = C_3 \frac{R_1}{R_2}} \qquad \textbf{(17.11)}$$

$$\boxed{R_4 = \frac{R_2 R_3}{R_1}} \qquad \textbf{(17.12)}$$

The derivation of these equations will appear as a problem at the end of the chapter.

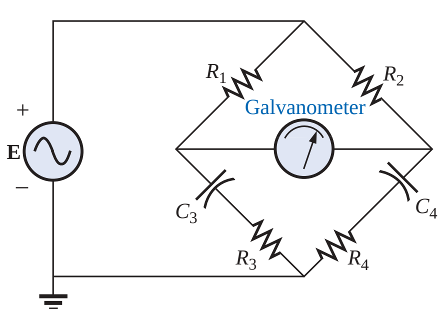


FIG. 17.45
Capacitance comparison bridge.

17.7 Δ-Y, Y-Δ CONVERSIONS

The Δ-Y, Y-Δ (or π-T, T-π as defined in Section 8.12) conversions for ac circuits will not be derived here since the development corresponds exactly with that for dc circuits. Taking the **Δ-Y configuration** shown in Fig. 17.46, we find the general equations for the impedances of the Y in terms of those for the Δ:

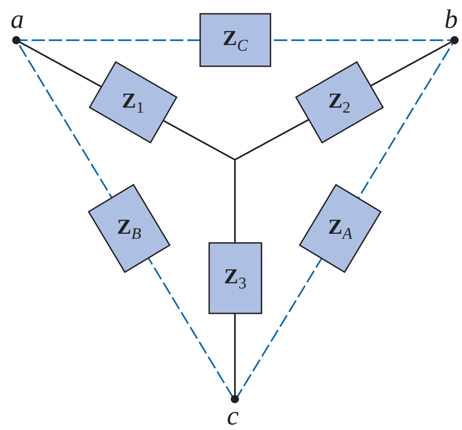


FIG. 17.46
Δ-Y configuration.

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} \tag{17.13}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} \tag{17.14}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} \tag{17.15}$$

For the impedances of the Δ in terms of those for the Y, the equations are

$$\mathbf{Z}_B = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2} \tag{17.16}$$

$$\mathbf{Z}_A = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} \tag{17.17}$$

$$\mathbf{Z}_C = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_3} \tag{17.18}$$

Note that each impedance of the Y is equal to the product of the impedances in the two closest branches of the Δ, divided by the sum of the impedances in the Δ.

Further, the value of each impedance of the Δ is equal to the sum of the possible product combinations of the impedances of the Y, divided by the impedances of the Y farthest from the impedance to be determined.

Drawn in different forms (Fig. 17.47), they are also referred to as the T and π configurations.

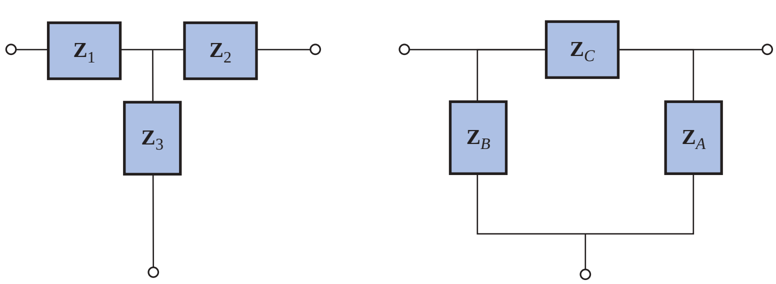


FIG. 17.47
The T and π configurations.

In the study of dc networks, we found that if all of the resistors of the Δ or Y were the same, the conversion from one to the other could be accomplished using the equation

$$R_\Delta = 3R_Y \quad \text{or} \quad R_Y = \frac{R_\Delta}{3}$$

For ac networks,

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad \text{or} \quad \mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} \tag{17.19}$$

Be careful when using this simplified form. It is not sufficient for all the impedances of the Δ or Y to be of the same magnitude: *The angle associated with each must also be the same.*

EXAMPLE 17.20 Find the total impedance $\mathbf{Z}_T$ of the network of Fig. 17.48.

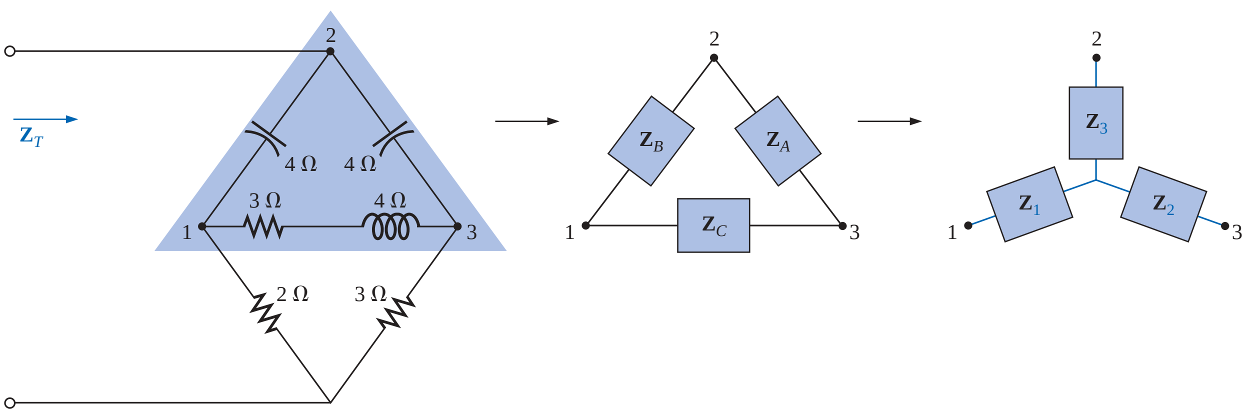


FIG. 17.48
Converting the upper Δ of a bridge configuration to a Y.

Solution:

$$\mathbf{Z}_B = -j\,4 \qquad \mathbf{Z}_A = -j\,4 \qquad \mathbf{Z}_C = 3 + j\,4$$

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_B\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(3\,\Omega + j\,4\,\Omega)}{(-j\,4\,\Omega) + (-j\,4\,\Omega) + (3\,\Omega + j\,4\,\Omega)}$$

$$= \frac{(4\,\angle -90^\circ)(5\,\angle 53.13^\circ)}{3 - j\,4} = \frac{20\,\angle -36.87^\circ}{5\,\angle -53.13^\circ}$$

$$= 4\,\Omega\,\angle 16.13^\circ = 3.84\,\Omega + j\,1.11\,\Omega$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_A\mathbf{Z}_C}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(3\,\Omega + j\,4\,\Omega)}{5\,\Omega\,\angle -53.13^\circ}$$

$$= 4\,\Omega\,\angle 16.13^\circ = 3.84\,\Omega + j\,1.11\,\Omega$$

Recall from the study of dc circuits that if two branches of the Y or Δ were the same, the corresponding Δ or Y, respectively, would also have two similar branches. In this example, $\mathbf{Z}_A = \mathbf{Z}_B$. Therefore, $\mathbf{Z}_1 = \mathbf{Z}_2$, and

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C} = \frac{(-j\,4\,\Omega)(-j\,4\,\Omega)}{5\,\Omega\,\angle -53.13°}$$
$$= \frac{16\,\Omega\,\angle -180°}{5\,\angle -53.13°} = 3.2\,\Omega\,\angle -126.87° = -1.92\,\Omega - j\,2.56\,\Omega$$

Replace the Δ by the Y (Fig. 17.49):

$$\mathbf{Z}_1 = 3.84\,\Omega + j\,1.11\,\Omega \qquad \mathbf{Z}_2 = 3.84\,\Omega + j\,1.11\,\Omega$$
$$\mathbf{Z}_3 = -1.92\,\Omega - j\,2.56\,\Omega \qquad \mathbf{Z}_4 = 2\,\Omega$$
$$\mathbf{Z}_5 = 3\,\Omega$$

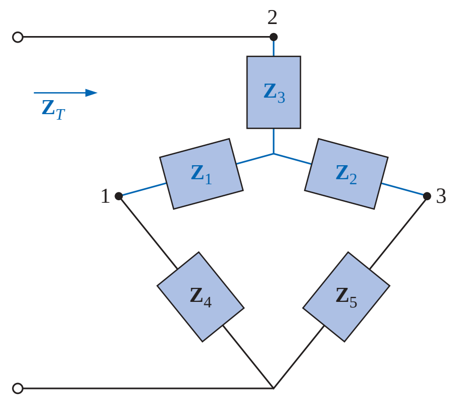


FIG. 17.49
The network of Fig. 17.48 following the substitution of the Y configuration.

Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_4$ are in series:

$$\mathbf{Z}_{T_1} = \mathbf{Z}_1 + \mathbf{Z}_4 = 3.84\,\Omega + j\,1.11\,\Omega + 2\,\Omega = 5.84\,\Omega + j\,1.11\,\Omega$$
$$= 5.94\,\Omega\,\angle 10.76°$$

Impedances $\mathbf{Z}_2$ and $\mathbf{Z}_5$ are in series:

$$\mathbf{Z}_{T_2} = \mathbf{Z}_2 + \mathbf{Z}_5 = 3.84\,\Omega + j\,1.11\,\Omega + 3\,\Omega = 6.84\,\Omega + j\,1.11\,\Omega$$
$$= 6.93\,\Omega\,\angle 9.22°$$

Impedances $\mathbf{Z}_{T_1}$ and $\mathbf{Z}_{T_2}$ are in parallel:

$$\mathbf{Z}_{T_3} = \frac{\mathbf{Z}_{T_1}\mathbf{Z}_{T_2}}{\mathbf{Z}_{T_1} + \mathbf{Z}_{T_2}} = \frac{(5.94\,\Omega\,\angle 10.76°)(6.93\,\Omega\,\angle 9.22°)}{5.84\,\Omega + j\,1.11\,\Omega + 6.84\,\Omega + j\,1.11\,\Omega}$$
$$= \frac{41.16\,\Omega\,\angle 19.98°}{12.68 + j\,2.22} = \frac{41.16\,\Omega\,\angle 19.98°}{12.87\,\angle 9.93°} = 3.198\,\Omega\,\angle 10.05°$$
$$= 3.15\,\Omega + j\,0.56\,\Omega$$

Impedances $\mathbf{Z}_3$ and $\mathbf{Z}_{T_3}$ are in series. Therefore,

$$\mathbf{Z}_T = \mathbf{Z}_3 + \mathbf{Z}_{T_3} = -1.92\,\Omega - j\,2.56\,\Omega + 3.15\,\Omega + j\,0.56\,\Omega$$
$$= 1.23\,\Omega - j\,2.0\,\Omega = \mathbf{2.35\,\Omega\,\angle -58.41°}$$

EXAMPLE 17.21 Using both the Δ-Y and Y-Δ transformations, find the total impedance $\mathbf{Z}_T$ for the network of Fig. 17.50.

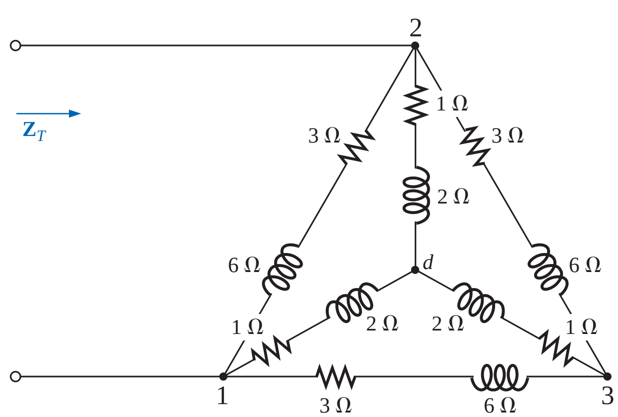


FIG. 17.50
Example 17.21.

Solution: *Using the Δ-Y transformation,* we obtain Fig. 17.51. In this case, since both systems are balanced (same impedance in each branch), the center point *d′* of the transformed Δ will be the same as point *d* of the original Y:

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = \frac{3\ \Omega + j\,6\ \Omega}{3} = 1\ \Omega + j\,2\ \Omega$$

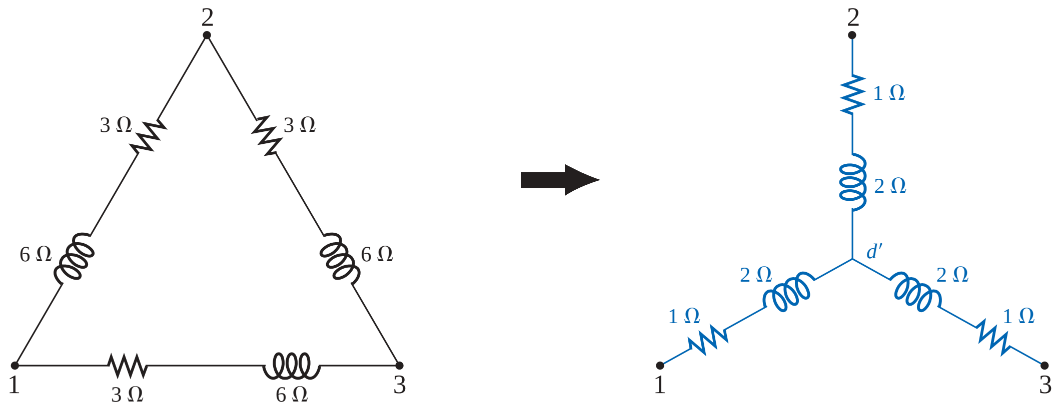


FIG. 17.51
Converting a Δ configuration to a Y configuration.

and (Fig. 17.52)

$$\mathbf{Z}_T = 2\left(\frac{1\ \Omega + j\,2\ \Omega}{2}\right) = \mathbf{1\ \Omega + j\,2\ \Omega}$$

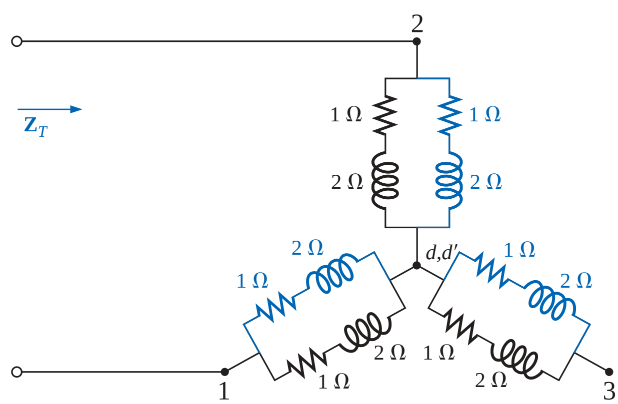


FIG. 17.52
Substituting the Y configuration of Fig. 17.51 into the network of Fig. 17.50.

Using the Y-Δ transformation (Fig. 17.53), we obtain

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 3(1\ \Omega + j\,2\ \Omega) = 3\ \Omega + j\,6\ \Omega$$

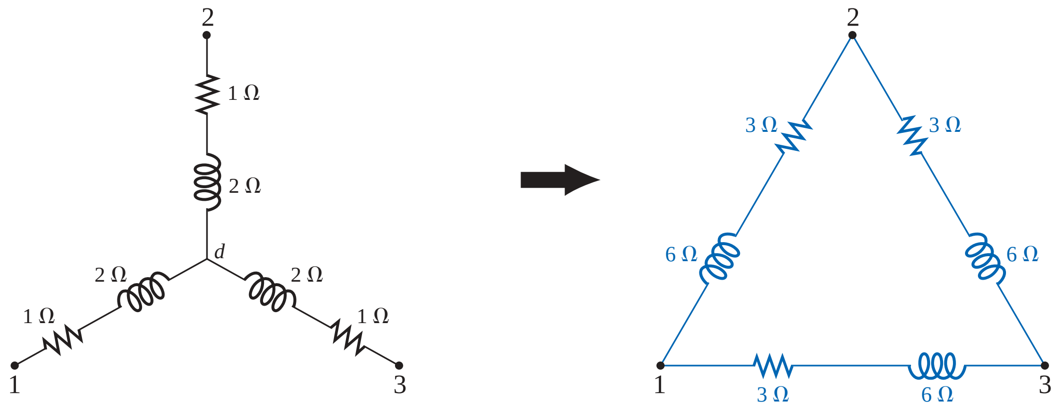


FIG. 17.53
Converting the Y configuration of Fig. 17.50 to a Δ.

Each resulting parallel combination in Fig. 17.54 will have the following impedance:

$$\mathbf{Z}' = \frac{3\ \Omega + j\,6\ \Omega}{2} = 1.5\ \Omega + j\,3\ \Omega$$

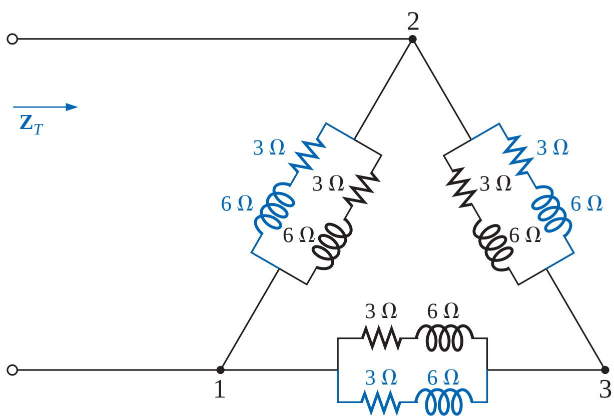


FIG. 17.54
Substituting the Δ configuration of Fig. 17.53 into the network of Fig. 17.50.

and
$$\mathbf{Z}_T = \frac{\mathbf{Z}'(2\mathbf{Z}')}{\mathbf{Z}' + 2\mathbf{Z}'} = \frac{2(\mathbf{Z}')^2}{3\mathbf{Z}'} = \frac{2\mathbf{Z}'}{3}$$
$$= \frac{2(1.5\ \Omega + j\,3\ \Omega)}{3} = \mathbf{1\ \Omega + \mathit{j}\,2\ \Omega}$$

which compares with the above result.

17.8 COMPUTER ANALYSIS

PSpice

Nodal Analysis The first application of PSpice will be to determine the nodal voltages for the network of Example 17.16 and compare solutions. The network will appear as shown in Fig. 17.55 using elements

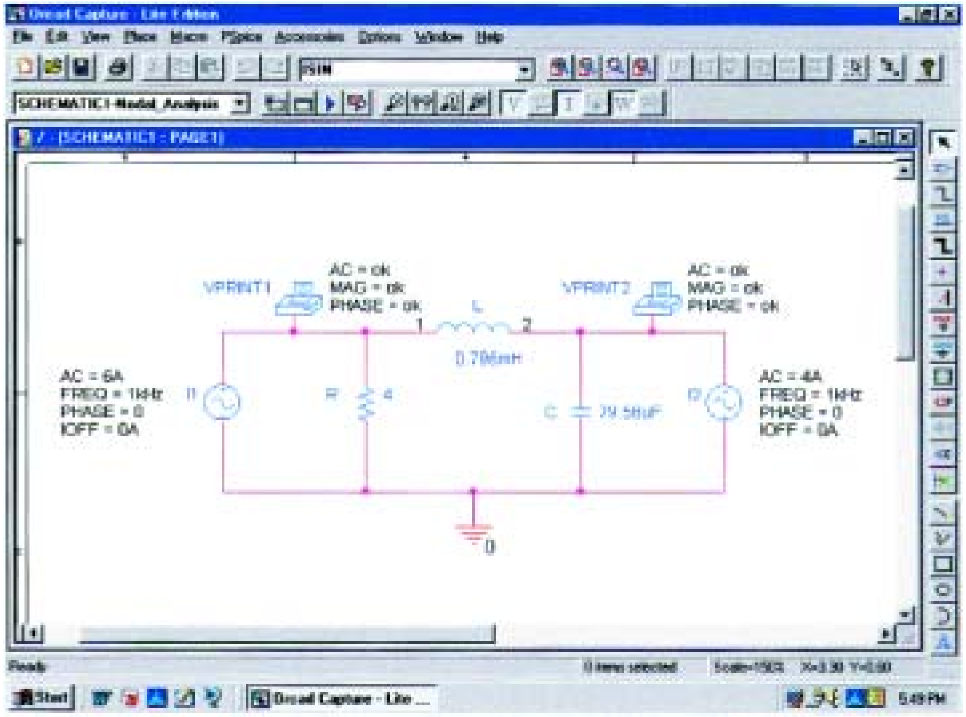


FIG. 17.55

Using PSpice to verify the results of Example 17.16.

that were determined from the reactance level at a frequency of 1 kHz. There is no need to continually use 1 kHz. Any frequency will do, but remember to use the chosen frequency to find the network components and when setting up the simulation.

For the current sources, **ISIN** was chosen so that the phase angle could be specified (even though it is 0°), although the symbol does not have the arrow used in the text material. The direction must be recognized as pointing from the + to − sign of the source. That requires that the sources $\mathbf{I}_1$ and $\mathbf{I}_2$ be set as shown in Fig. 17.55. The source $\mathbf{I}_2$ is reversed by using the **Mirror Vertically** option obtained by right-clicking the source symbol on the screen. Setting up the **ISIN** source is the same as that employed with the **VSIN** source. It can be found under the **SOURCE** library, and its attributes are the same as for the **VSIN** source. For each source, **IOFF** is set to 0 A, and the amplitude is the peak value of the source current. The frequency will be the same for each source. Then **VPRINT1** is selected from the **SPECIAL** library and placed to generate the desired nodal voltages. Finally the remaining elements are added to the network as shown in Fig. 17.55. For each source the symbol is double-clicked to generate the **Property Editor** dialog box. **AC** is set at the 6-A level for the $\mathbf{I}_1$ source and at 4 A for the $\mathbf{I}_2$ source, followed by **Display** and **Name and Value** for each. It will appear as shown in Fig. 17.55. A double-click on each **VPRINT1** option will also provide the **Property Editor,** so **OK** can be added under **AC, MAG,** and **PHASE.** For each quantity, **Display** is selected followed by **Name and Value** and **OK.** Then **Value** is selected and **VPRINT1** is displayed as **Value** only. Selecting **Apply** and leaving the dialog box will result in the listing next to each source in Fig. 17.55. For **VPRINT2** the listing on **Value** must first be changed from **VPRINT1** to **VPRINT2** before selecting **Display** and **Apply.**

Now the **New Simulation Profile** icon is selected and **ACNodal** entered as the **Name** followed by **Create.** In the **Simulation Settings** dialog box, **AC Sweep** is selected, and the **Start Frequency** and **End Frequency** are set at 1 kHz with 1 for the **Points/Decade.** Click **OK,** and select the **Run PSpice** icon; a **SCHEMATIC1** screen will result. Exiting **(X)** will bring us back to the **Orcad Capture** window. Selecting **PSpice** followed by **View Output File** will result in the display of Fig. 17.56, providing exactly the same results as obtained in Example 17.16 with $\mathbf{V}_1 = 20.8\text{ V} \angle -126.9°$. The other nodal voltage is $8.617\text{ V} \angle -15.09°$.

```
 ** Profile: "SCHEMATIC1-Nodal_Analysis"  [ C:\PSpice\nodal_analysis-SCHEMATIC1-N
odal_Analysis.sim ]

 ****     AC ANALYSIS                          TEMPERATURE =   27.000 DEG C

******************************************************************************

  FREQ        VM(N01310)  VP(N01310)

   1.000E+03   2.080E+01  -1.269E+02

**** 07/16/01 17:40:22 ************** PSpice Lite (Mar 2000) *****************

 ** Profile: "SCHEMATIC1-Nodal_Analysis"  [ C:\PSpice\nodal_analysis-SCHEMATIC1-N
odal_Analysis.sim ]

 ****     AC ANALYSIS                          TEMPERATURE =   27.000 DEG C

******************************************************************************

  FREQ        VM(N01383)  VP(N01383)

   1.000E+03   8.617E+00  -1.509E+01
```

FIG. 17.56

Output file for the nodal voltages for the network of Fig. 17.55.

Current-Controlled Current Source (CCCS) Our interest will now turn to controlled sources in the PSpice environment. Controlled sources are not particularly difficult to apply once a few important elements of their use are understood. The network of Fig. 17.14 has a current-controlled current source in the center leg of the configuration. The magnitude of the current source is *k* times the current through resistor R_1, where *k* can be greater or less than 1. The resulting schematic, appearing in Fig. 17.57, seems quite complex in the area of the controlled source, but once the role of each component is understood, it will not be that difficult to understand. First, since it is the only new element in the schematic, let us concentrate on the controlled source. Current-controlled current sources (CCCS) are called up under the **ANALOG** library as **F** and appear as shown in the center of Fig. 17.57.

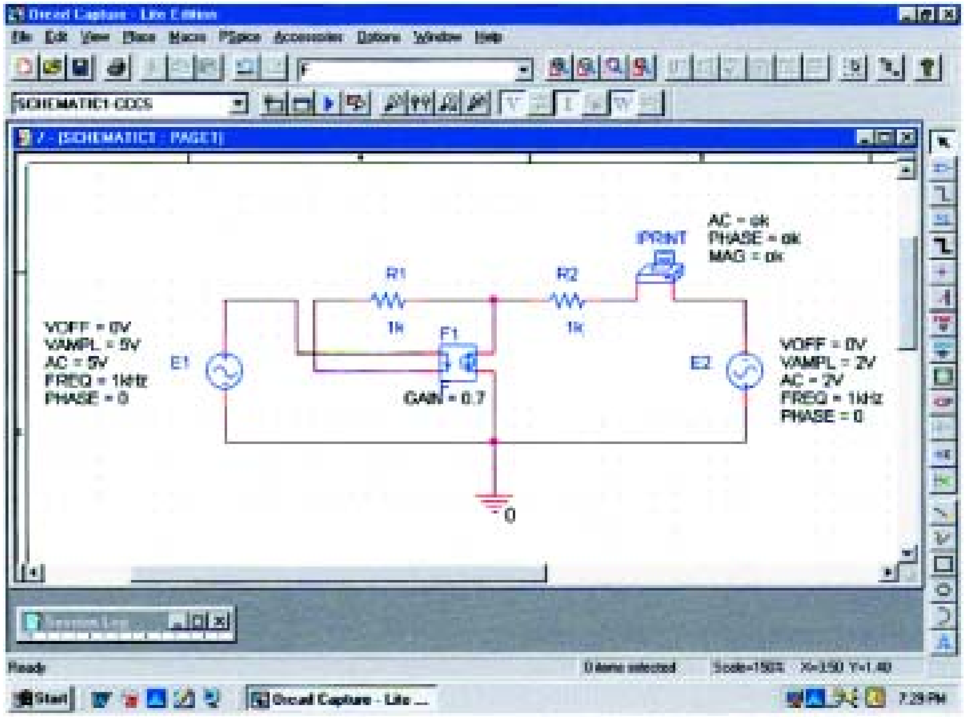


FIG. 17.57
Using PSpice to verify the results of Example 17.8.

Take special note of the direction of the current in each part of the symbol. In particular, note that the sensing current of **F** has the same direction as the defining controlling current in Fig. 17.14. In addition, note that the controlled current source also has the same direction as the source in Fig. 17.14. If we double-click on the CCCS symbol, the **Property Editor** dialog box will appear with the **GAIN** (*k* as described above) set at 1. In this example the gain must be set at **0.7,** so click on the region below the **GAIN** label and enter **0.7.** Then select **Display** followed by **Name and Value-OK.** Exit the **Property Editor,** and **GAIN = 0.7** will appear with the CCCS as shown in Fig. 17.57.

The other new component in this schematic is **IPRINT;** it can be found in the **SPECIAL** library. It is used to tell the program to list the current in the branch of interest in the output file. If you fail to tell the program which output data you would like, it will simply run through the simulation and list specific features of the network but will not provide any voltages or currents. In this case the current $\mathbf{I}_2$ through the resistor R_2 is desired. Double-clicking on the **IPRINT** component will result in the **Property Editor** dialog box with a number of elements that need to be defined—much like that for **VPRINT.** First enter **OK** beneath **AC** and follow with **Display-Name and Value-OK.** Repeat for **MAG** and **PHASE,** and then select **Apply** before leaving the dialog box. The **OK** is designed simply to tell the software program that these are the quantities that it is “ok” to generate and provide. The purpose of the **Apply** at the end of each visit to the **Property Editor** dialog box is to “apply” the changes made to the network under investigation. When you exit the **Property Editor,** the three chosen parameters will appear on the schematic with the **OK** directive. You may find that the labels will appear all over the **IPRINT** symbol. No problem—just click on each, and move to a more convenient location.

The remaining components of the network should be fairly familiar, but don't forget to **Mirror Vertically** the voltage source **E2.** In addition, do not forget to call up the **Property Editor** for each source and set the level of **AC, FREQ, VAMPL,** and **VOFF** and be sure that the **PHASE** is set on the default value of 0°. The value appears with each parameter in Fig. 17.57 for each source. Always be sure to select **Apply** before leaving the **Property Editor.** After placing all the components on the screen, you must connect them with a **Place wire** selection. Normally, this is pretty straightforward. However, with controlled sources there is often the need to cross over wires **without** making a connection. In general, when you're placing a wire over another wire and you don't want a connection to be made, click a spot on one side of the wire to be crossed to create the temporary red square. Then cross the wire, and make another click to establish another red square. If the connection is done properly, the crossed wire should not show a connection point (a small red dot). In this example the top of the controlling current was connected first from the **E1** source. Then a wire was connected from the lower end of the sensing current to the point where a 90° turn up the page was to be made. The wire was clicked in place at this point before crossing the original wire and clicked again before making the right turn to resistor R_1. You will not find a small red dot where the wires cross.

Now for the simulation. In the **Simulation Settings** dialog box, select **AC Sweep/Noise** with a **Start** and **End Frequency** of 1 kHz. There will be 1 **Point/Decade.** Click **OK,** and select the **Run Spice** key; a **SCHEMATIC1** will result that should be exited to obtain the **Orcad Capture** screen. Select **PSpice** followed by **View Output File,** and scroll down until you read **AC ANALYSIS** such as appearing in Fig. 17.58. The magnitude of the desired current is 1.615 mA with a phase angle of 0°, a perfect match with the theoretical analysis to follow. One would expect a phase angle of 0° since the network is composed solely of resistive elements.

The equations obtained earlier using the supermesh approach were

$$\mathbf{E} - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0 \quad \text{or} \quad \mathbf{I}_1\mathbf{Z}_1 + \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1 + \mathbf{E}_2$$

```

** Profile: "SCHEMATIC1-CCCS"  [ C:\PSpice\cccs-SCHEMATIC1-CCCS.sim ]


****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C


******************************************************************************



 FREQ        IM(V_PRINT1)IP(V_PRINT1)


  1.000E+03   1.615E-03   0.000E+00

```

FIG. 17.58

The output file for the mesh current $\mathbf{I}_2$ *of Fig. 17.14.*

and $$k\mathbf{I} = k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$$

resulting in $$\mathbf{I}_1 = \frac{\mathbf{I}_2}{1-k} = \frac{\mathbf{I}_2}{1-0.7} = \frac{\mathbf{I}_2}{0.3} = 3.333\mathbf{I}_2$$

so that $$\mathbf{I}_1(1\text{ k}\Omega) + \mathbf{I}_2(1\text{ k}\Omega) = 7\text{ V} \qquad \text{(from above)}$$

becomes $$(3.333\mathbf{I}_2)1\text{ k}\Omega + \mathbf{I}_2(1\text{ k}\Omega) = 7\text{ V}$$

or $$(4.333\text{ k}\Omega)\mathbf{I}_2 = 7\text{ V}$$

and $$\mathbf{I}_2 = \frac{7\text{ V}}{4.333\text{ k}\Omega} = \mathbf{1.615\text{ mA}\angle 0°}$$

confirming the computer solution.

PROBLEMS

SECTION 17.2 Independent versus Dependent (Controlled) Sources

1. Discuss, in your own words, the difference between a controlled and an independent source.

SECTION 17.3 Source Conversions

2. Convert the voltage sources of Fig. 17.59 to current sources.

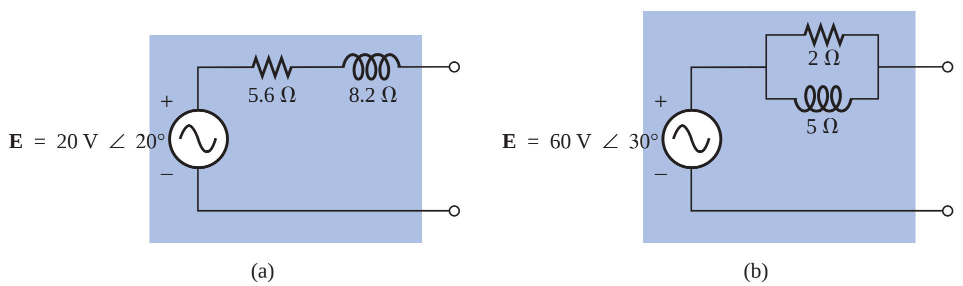


FIG. 17.59
Problem 2.

3. Convert the current sources of Fig. 17.60 to voltage sources.

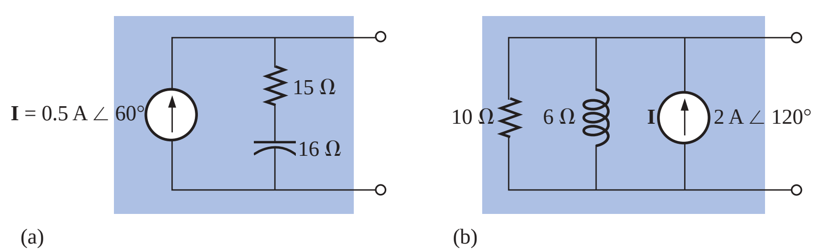


FIG. 17.60
Problem 3.

4. Convert the voltage source of Fig. 17.61(a) to a current source and the current source of Fig. 17.61(b) to a voltage source.

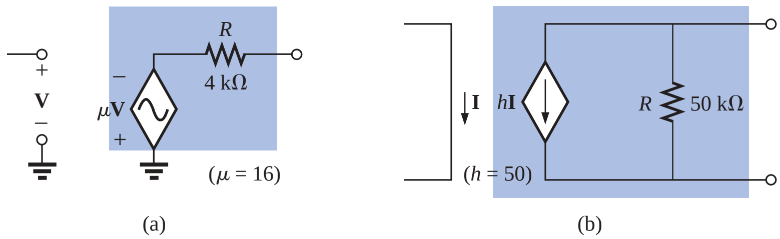


FIG. 17.61
Problem 4.

SECTION 17.4 Mesh Analysis

5. Write the mesh equations for the networks of Fig. 17.62. Determine the current through the resistor R.

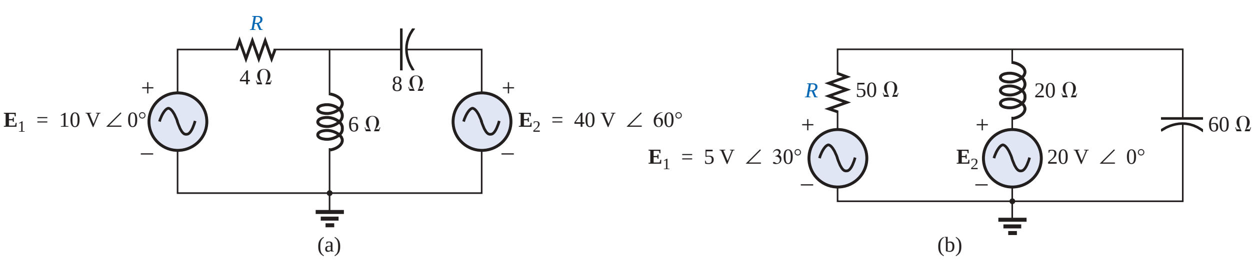


FIG. 17.62
Problems 5 and 34.

6. Write the mesh equations for the networks of Fig. 17.63. Determine the current through the resistor R_1.

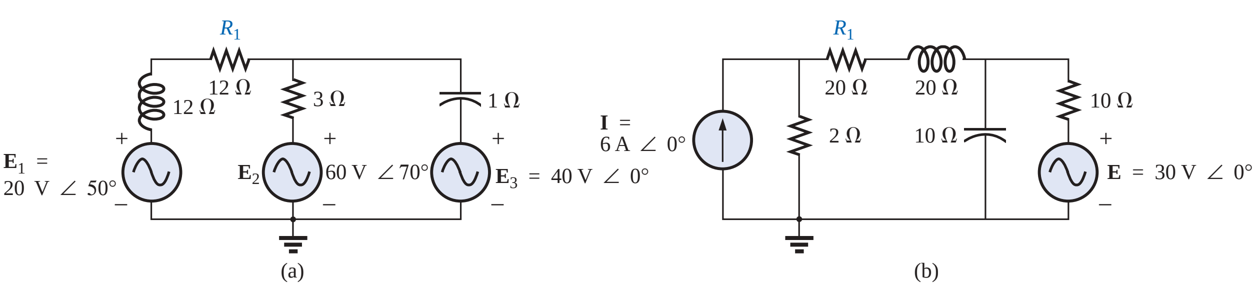


FIG. 17.63
Problems 6 and 16.

***7.** Write the mesh equations for the networks of Fig. 17.64. Determine the current through the resistor R_1.

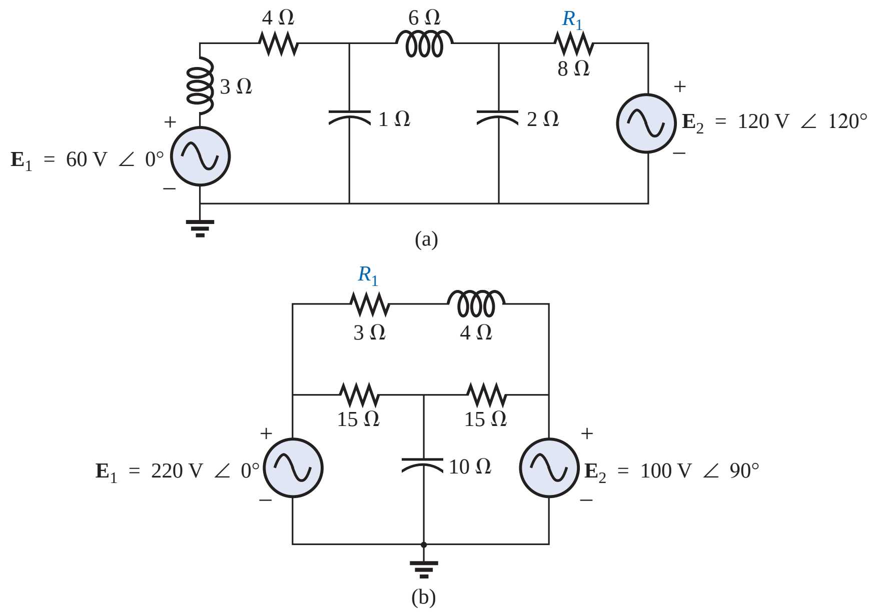


FIG. 17.64
Problems 7, 17, and 35.

***8.** Write the mesh equations for the networks of Fig. 17.65. Determine the current through the resistor R_1.

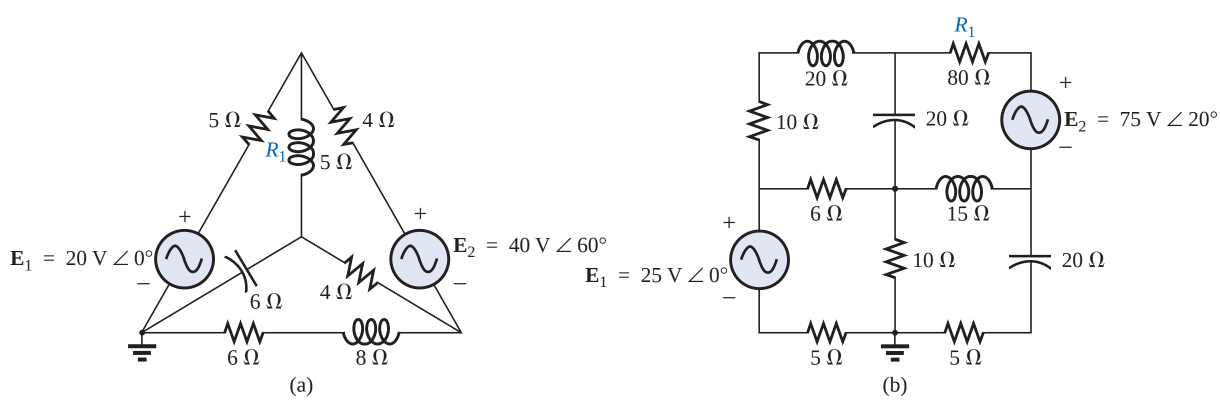


FIG. 17.65
Problems 8, 18, and 19.

9. Using mesh analysis, determine the current $\mathbf{I}_L$ (in terms of **V**) for the network of Fig. 17.66.

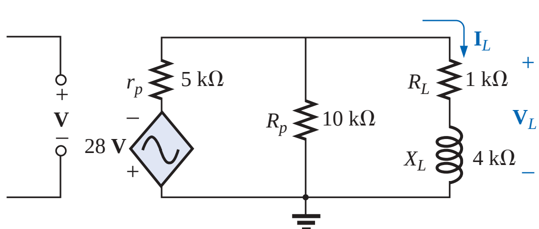


FIG. 17.66
Problem 9.

***10.** Using mesh analysis, determine the current $\mathbf{I}_L$ (in terms of **I**) for the network of Fig. 17.67.

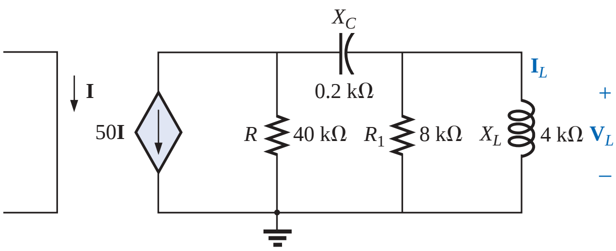


FIG. 17.67
Problem 10.

***11.** Write the mesh equations for the network of Fig. 17.68, and determine the current through the 1-kΩ and 2-kΩ resistors.

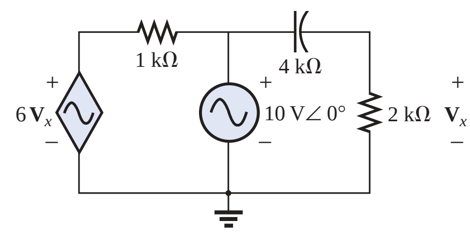


FIG. 17.68
Problems 11 and 36.

***12.** Write the mesh equations for the network of Fig. 17.69, and determine the current through the 10-kΩ resistor.

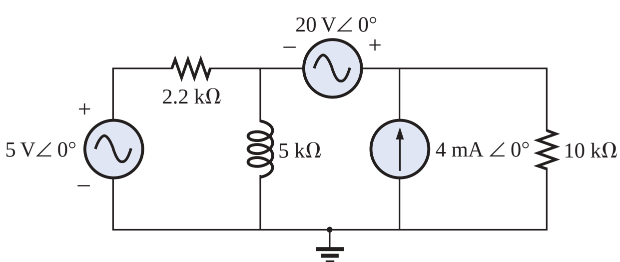


FIG. 17.69
Problems 12 and 37.

***13.** Write the mesh equations for the network of Fig. 17.70, and determine the current through the inductive element.

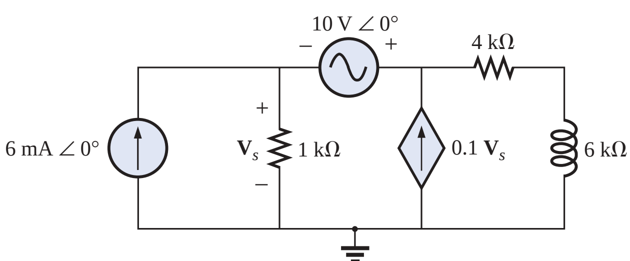


FIG. 17.70
Problems 13 and 38.

SECTION 17.5 Nodal Analysis

14. Determine the nodal voltages for the networks of Fig. 17.71.

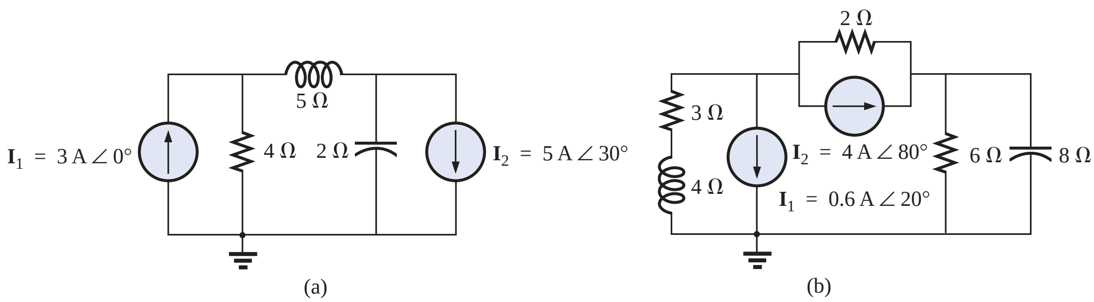


FIG. 17.71
Problems 14 and 39.

15. Determine the nodal voltages for the networks of Fig. 17.72.

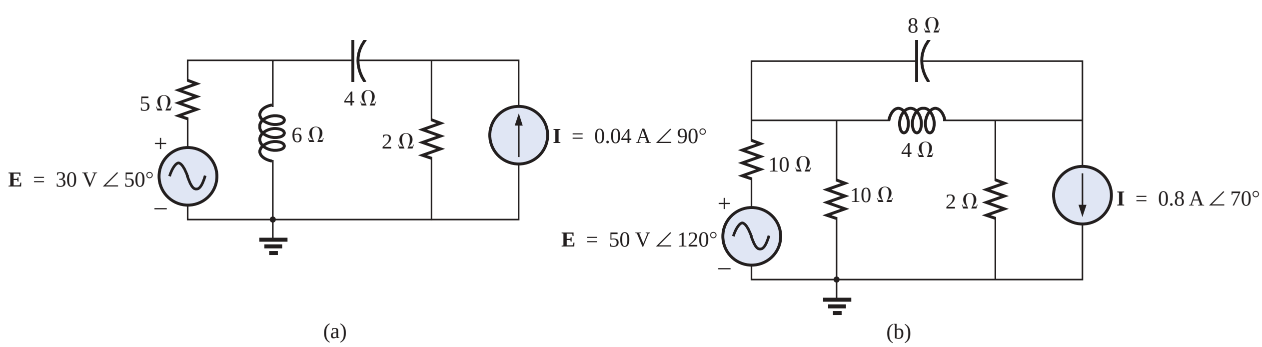


FIG. 17.72
Problem 15.

16. Determine the nodal voltages for the network of Fig. 17.63(b).

17. Determine the nodal voltages for the network of Fig. 17.64(b).

***18.** Determine the nodal voltages for the network of Fig. 17.65(a).

***19.** Determine the nodal voltages for the network of Fig. 17.65(b).

***20.** Determine the nodal voltages for the networks of Fig. 17.73.

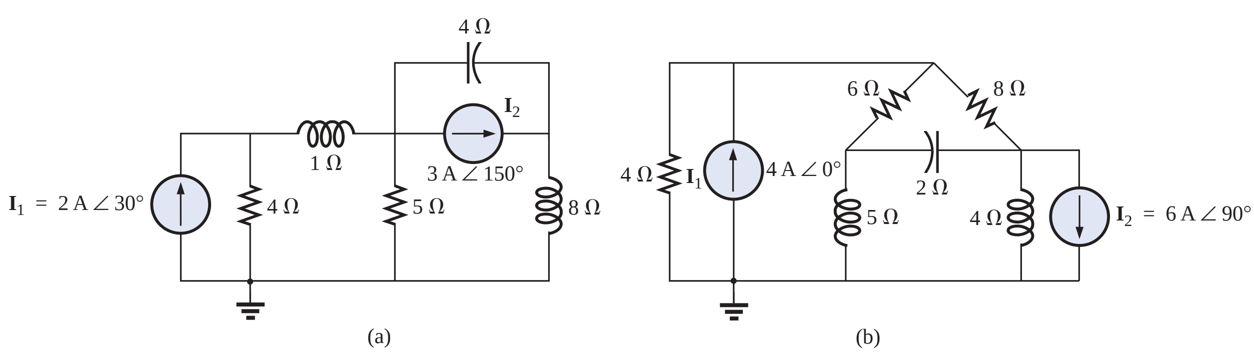


FIG. 17.73
Problem 20.

***21.** Write the nodal equations for the network of Fig. 17.74, and find the voltage across the 1-kΩ resistor.

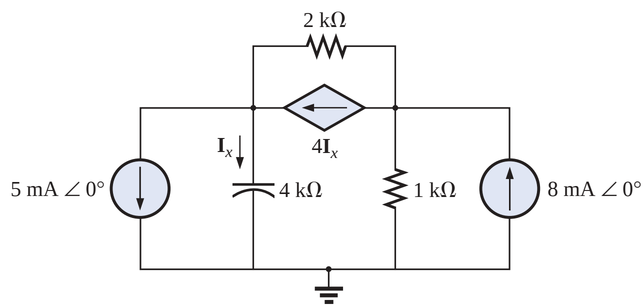


FIG. 17.74
Problems 21 and 40.

***22.** Write the nodal equations for the network of Fig. 17.75, and find the voltage across the capacitive element.

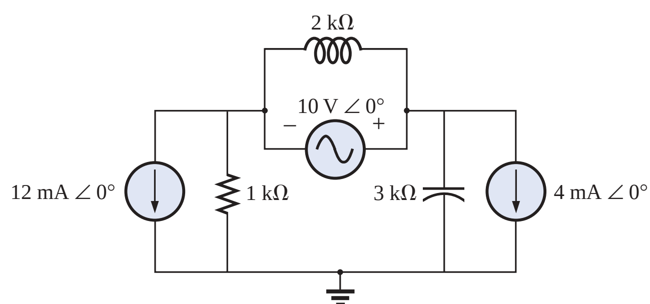


FIG. 17.75
Problems 22 and 41.

***23.** Write the nodal equations for the network of Fig. 17.76, and find the voltage across the 2-kΩ resistor.

***24.** Write the nodal equations for the network of Fig. 17.77, and find the voltage across the 2-kΩ resistor.

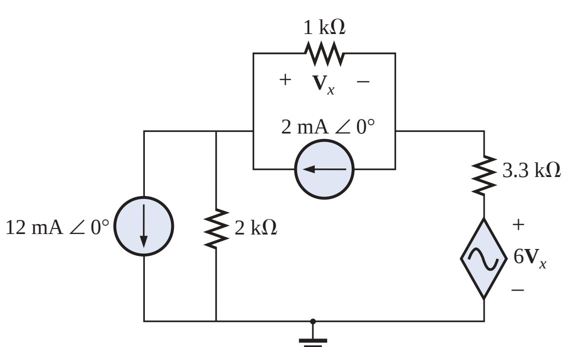


FIG. 17.76
Problems 23 and 42.

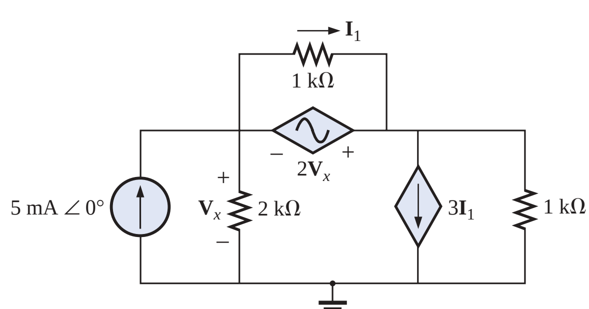


FIG. 17.77
Problems 24 and 43.

***25.** For the network of Fig. 17.78, determine the voltage $\mathbf{V}_L$ in terms of the voltage $\mathbf{E}_i$.

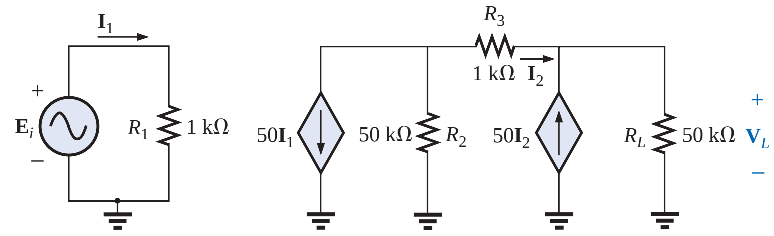


FIG. 17.78
Problem 25.

SECTION 17.6 Bridge Networks (ac)

26. For the bridge network of Fig. 17.79:
- **a.** Is the bridge balanced?
- **b.** Using mesh analysis, determine the current through the capacitive reactance.
- **c.** Using nodal analysis, determine the voltage across the capacitive reactance.

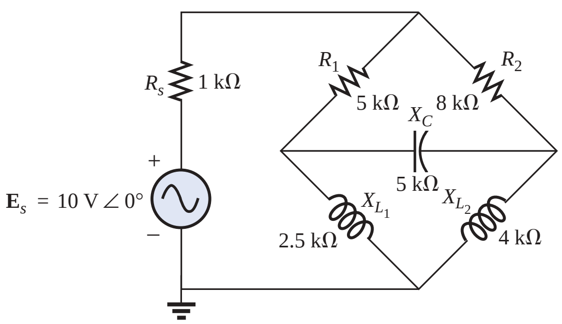


FIG. 17.79
Problem 26.

27. For the bridge network of Fig. 17.80:
- **a.** Is the bridge balanced?
- **b.** Using mesh analysis, determine the current through the capacitive reactance.
- **c.** Using nodal analysis, determine the voltage across the capacitive reactance.

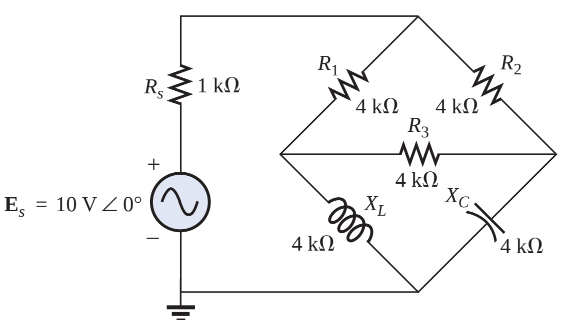


FIG. 17.80
Problem 27.

28. The Hay bridge of Fig. 17.81 is balanced. Using Eq. (17.3), determine the unknown inductance L_x and resistance R_x.

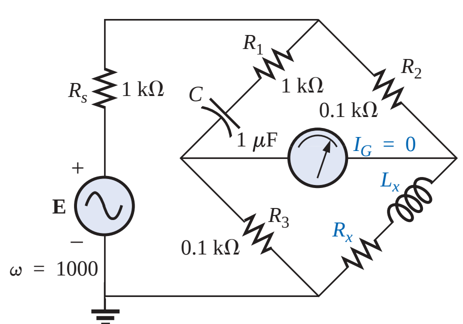


FIG. 17.81
Problem 28.

29. Determine whether the Maxwell bridge of Fig. 17.82 is balanced ($\omega = 1000$ rad/s).

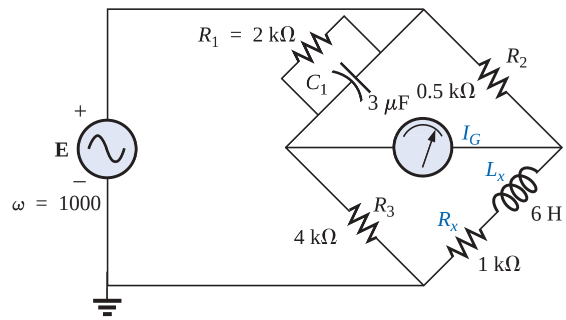


FIG. 17.82
Problem 29.

30. Derive the balance equations (17.11) and (17.12) for the capacitance comparison bridge.

31. Determine the balance equations for the inductance bridge of Fig. 17.83.

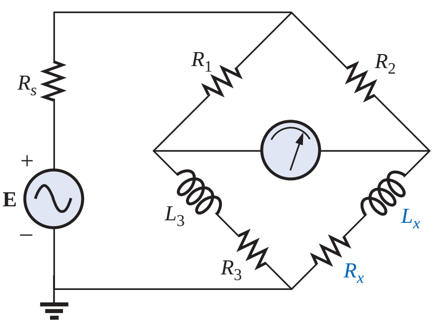


FIG. 17.83
Problem 31.

SECTION 17.7 Δ-Y, Y-Δ Conversions

32. Using the Δ-Y or Y-Δ conversion, determine the current **I** for the networks of Fig. 17.84.

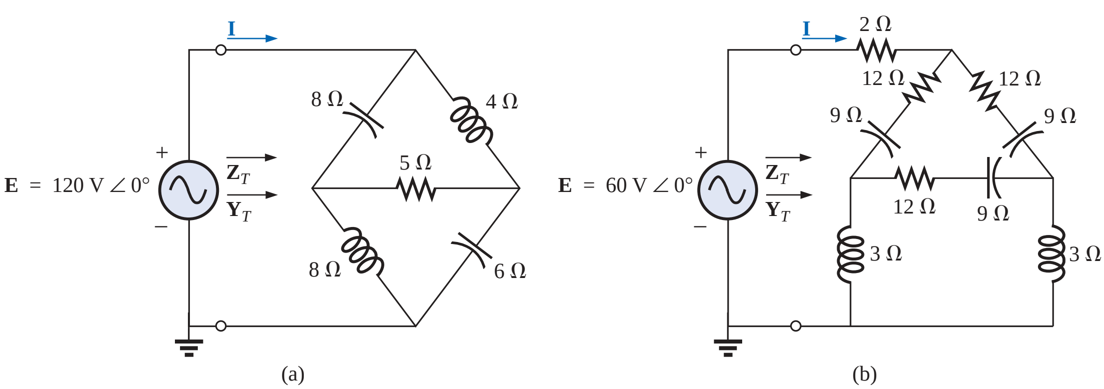


FIG. 17.84
Problem 32.

33. Using the Δ-Y or Y-Δ conversion, determine the current **I** for the networks of Fig. 17.85. (**E** = 100 V ∠0° in each case.)

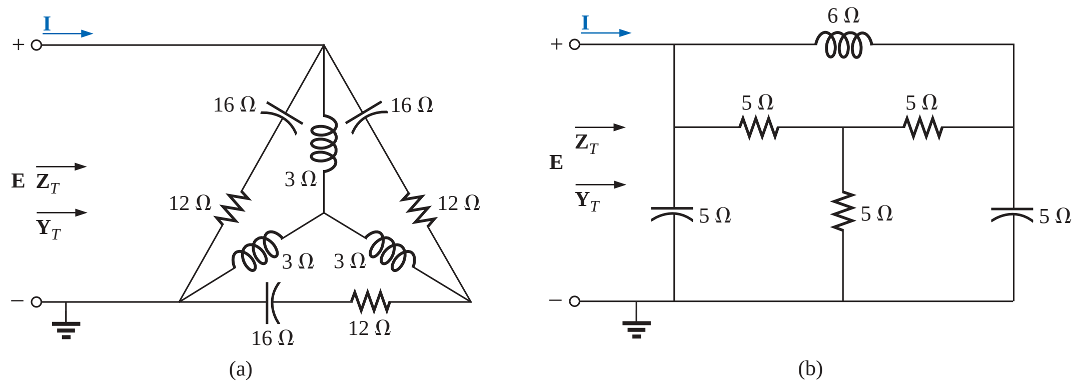


FIG. 17.85
Problem 33.

SECTION 17.8 Computer Analysis

PSpice or Electronics Workbench

34. Determine the mesh currents for the network of Fig. 17.62(a).

35. Determine the mesh currents for the network of Fig. 17.64(a).

***36.** Determine the mesh currents for the network of Fig. 17.68.

***37.** Determine the mesh currents for the network of Fig. 17.69.

***38.** Determine the mesh currents for the network of Fig. 17.70.

39. Determine the nodal voltages for the network of Fig. 17.71(b).

***40.** Determine the nodal voltages for the network of Fig. 17.74.

***41.** Determine the nodal voltages for the network of Fig. 17.75.

***42.** Determine the nodal voltages for the network of Fig. 17.76.

***43.** Determine the nodal voltages for the network of Fig. 17.77.

Programming Language (C++, QBASIC, Pascal, etc.)

44. Write a computer program that will provide a general solution for the network of Fig. 17.10. That is, given the reactance of each element and the parameters of the source voltages, generate a solution in phasor form for both mesh currents.

45. Repeat Problem 35 for the nodal voltages of Fig. 17.30.

46. Given a bridge composed of series impedances in each branch, write a program to test the balance condition as defined by Eq. (17.6).

GLOSSARY

Bridge network A network configuration having the appearance of a diamond in which no two branches are in series or parallel.

Capacitance comparison bridge A bridge configuration having a galvanometer in the bridge arm that is used to determine an unknown capacitance and associated resistance.

Delta (Δ) configuration A network configuration having the appearance of the capital Greek letter delta.

Dependent (controlled) source A source whose magnitude and/or phase angle is determined (controlled) by a current or voltage of the system in which it appears.

Hay bridge A bridge configuration used for measuring the resistance and inductance of coils in those cases where the resistance is a small fraction of the reactance of the coil.

Independent source A source whose magnitude is independent of the network to which it is applied. It displays its terminal characteristics even if completely isolated.

Maxwell bridge A bridge configuration used for inductance measurements when the resistance of the coil is large enough not to require a Hay bridge.

Mesh analysis A method through which the loop (or mesh) currents of a network can be determined. The branch currents of the network can then be determined directly from the loop currents.

Nodal analysis A method through which the nodal voltages of a network can be determined. The voltage across each element can then be determined through application of Kirchhoff's voltage law.

Source conversion The changing of a voltage source to a current source, or vice versa, which will result in the same terminal behavior of the source. In other words, the external network is unaware of the change in sources.

Wye (Y) configuration A network configuration having the appearance of the capital letter Y.