18

Network Theorems (ac)

18.1 INTRODUCTION

This chapter will parallel Chapter 9, which dealt with network theorems as applied to dc networks. It would be time well spent to review each theorem in Chapter 9 before beginning this chapter because many of the comments offered there will not be repeated.

Due to the need for developing confidence in the application of the various theorems to networks with controlled (dependent) sources, some sections have been divided into two parts: independent sources and dependent sources.

Theorems to be considered in detail include the superposition theorem, Thévenin's and Norton's theorems, and the maximum power theorem. The substitution and reciprocity theorems and Millman's theorem are not discussed in detail here because a review of Chapter 9 will enable you to apply them to sinusoidal ac networks with little difficulty.

18.2 SUPERPOSITION THEOREM

You will recall from Chapter 9 that the **superposition theorem** eliminated the need for solving simultaneous linear equations by considering the effects of each source independently. To consider the effects of each source, we had to remove the remaining sources. This was accomplished by setting voltage sources to zero (short-circuit representation) and current sources to zero (open-circuit representation). The current through, or voltage across, a portion of the network produced by each source was then added algebraically to find the total solution for the current or voltage.

The only variation in applying this method to ac networks with independent sources is that we will now be working with impedances and phasors instead of just resistors and real numbers.

The superposition theorem is not applicable to power effects in ac networks since we are still dealing with a nonlinear relationship. It can be applied to networks with sources of different frequencies only if the total response for *each* frequency is found independently and the results are expanded in a nonsinusoidal expression, as appearing in Chapter 25.

One of the most frequent applications of the superposition theorem is to electronic systems in which the dc and ac analyses are treated separately and the total solution is the sum of the two. It is an important application of the theorem because the impact of the reactive elements changes dramatically in response to the two types of independent sources. In addition, the dc analysis of an electronic system can often define important parameters for the ac analysis. Example 18.4 will demonstrate the impact of the applied source on the general configuration of the network.

We will first consider networks with only independent sources to provide a close association with the analysis of Chapter 9.

Independent Sources

EXAMPLE 18.1 Using the superposition theorem, find the current **I** through the 4-Ω reactance (X_{L_2}) of Fig. 18.1.

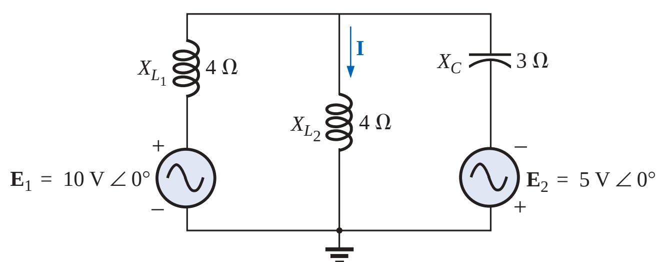

FIG. 18.1
Example 18.1.

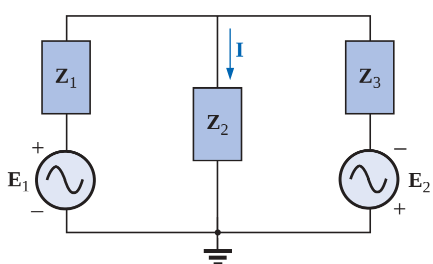

FIG. 18.2
Assigning the subscripted impedances to the network of Fig. 18.1.

Solution: For the redrawn circuit (Fig. 18.2),

$$\mathbf{Z}_1 = +j\,X_{L_1} = j\,4\,\Omega$$
$$\mathbf{Z}_2 = +j\,X_{L_2} = j\,4\,\Omega$$
$$\mathbf{Z}_3 = -j\,X_C = -j\,3\,\Omega$$

Considering the effects of the voltage source $\mathbf{E}_1$ (Fig. 18.3), we have

$$\mathbf{Z}_{2\|3} = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(j\,4\,\Omega)(-j\,3\,\Omega)}{j\,4\,\Omega - j\,3\,\Omega} = \frac{12\,\Omega}{j} = -j\,12\,\Omega$$
$$= 12\,\Omega\,\angle -90°$$

$$I_{s_1} = \frac{\mathbf{E}_1}{\mathbf{Z}_{2\|3} + \mathbf{Z}_1} = \frac{10\text{ V}\,\angle 0°}{-j\,12\,\Omega + j\,4\,\Omega} = \frac{10\text{ V}\,\angle 0°}{8\,\Omega\,\angle -90°}$$
$$= 1.25\text{ A}\,\angle 90°$$

and

$$\mathbf{I}' = \frac{\mathbf{Z}_3\mathbf{I}_{s_1}}{\mathbf{Z}_2 + \mathbf{Z}_3} \qquad \text{(current divider rule)}$$
$$= \frac{(-j\,3\,\Omega)(j\,1.25\text{ A})}{j\,4\,\Omega - j\,3\,\Omega} = \frac{3.75\text{ A}}{j\,1} = 3.75\text{ A}\,\angle -90°$$

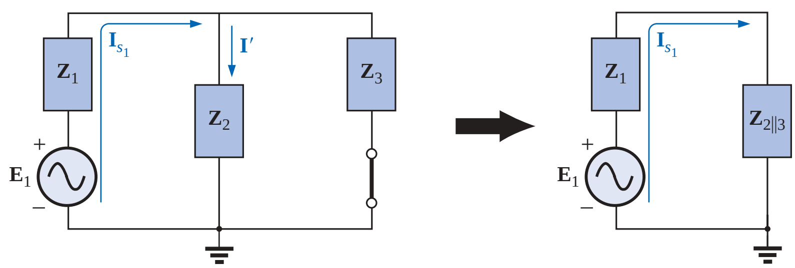

FIG. 18.3
Determining the effect of the voltage source $\mathbf{E}_1$ on the current $\mathbf{I}$ of the network of Fig. 18.1.

Considering the effects of the voltage source $\mathbf{E}_2$ (Fig. 18.4), we have

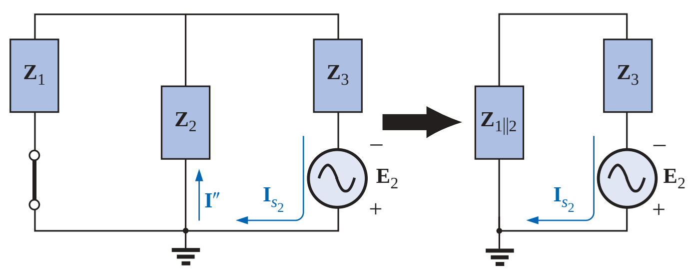

FIG. 18.4
Determining the effect of the voltage source $\mathbf{E}_2$ on the current $\mathbf{I}$ of the network of Fig. 18.1.

$$\mathbf{Z}_{1\|2} = \frac{\mathbf{Z}_1}{N} = \frac{j\,4\,\Omega}{2} = j\,2\,\Omega$$

$$\mathbf{I}_{s_2} = \frac{\mathbf{E}_2}{\mathbf{Z}_{1\|2} + \mathbf{Z}_3} = \frac{5\text{ V}\angle 0°}{j\,2\,\Omega - j\,3\,\Omega} = \frac{5\text{ V}\angle 0°}{1\,\Omega\angle -90°} = 5\text{ A}\angle 90°$$

and
$$\mathbf{I}'' = \frac{\mathbf{I}_{s_2}}{2} = 2.5\text{ A}\angle 90°$$

The resultant current through the 4-Ω reactance X_{L_2} (Fig. 18.5) is

$$\begin{aligned}\mathbf{I} &= \mathbf{I}' - \mathbf{I}''\\ &= 3.75\text{ A}\angle -90° - 2.50\text{ A}\angle 90° = -j\,3.75\text{ A} - j\,2.50\text{ A}\\ &= -j\,6.25\text{ A}\\ \mathbf{I} &= \mathbf{6.25\text{ A}\angle -90°}\end{aligned}$$

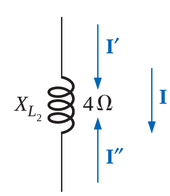

FIG. 18.5
Determining the resultant current for the network of Fig. 18.1.

EXAMPLE 18.2 Using superposition, find the current $\mathbf{I}$ through the 6-Ω resistor of Fig. 18.6.

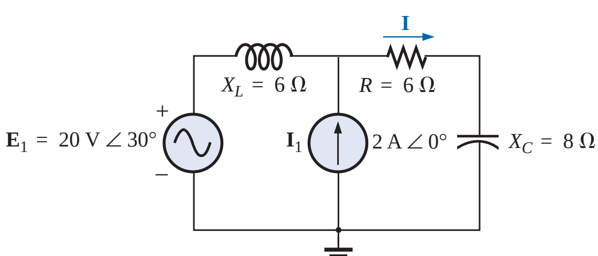

FIG. 18.6
Example 18.2.

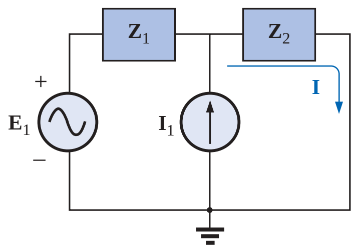

FIG. 18.7
Assigning the subscripted impedances to the network of Fig. 18.6.

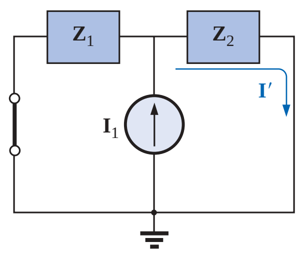

FIG. 18.8
Determining the effect of the current source $\mathbf{I}_1$ *on the current* **I** *of the network of Fig. 18.6.*

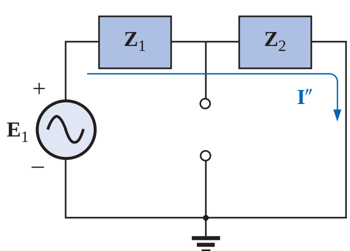

FIG. 18.9
Determining the effect of the voltage source $\mathbf{E}_1$ *on the current* **I** *of the network of Fig. 18.6.*

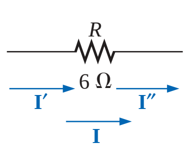

FIG. 18.10
Determining the resultant current **I** *for the network of Fig. 18.6.*

Solution: For the redrawn circuit (Fig. 18.7),

$$\mathbf{Z}_1 = j\,6\;\Omega \qquad \mathbf{Z}_2 = 6 - j\,8\;\Omega$$

Consider the effects of the current source (Fig. 18.8). Applying the current divider rule, we have

$$\begin{aligned}\mathbf{I}' &= \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\;\Omega)(2\text{ A})}{j\,6\;\Omega + 6\;\Omega - j\,8\;\Omega} = \frac{j\,12\text{ A}}{6 - j\,2}\\ &= \frac{12\text{ A}\;\angle 90^\circ}{6.32\;\angle -18.43^\circ}\\ \mathbf{I}' &= 1.9\text{ A}\;\angle 108.43^\circ\end{aligned}$$

Consider the effects of the voltage source (Fig. 18.9). Applying Ohm's law gives us

$$\begin{aligned}\mathbf{I}'' &= \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\text{ V}\;\angle 30^\circ}{6.32\;\Omega\;\angle -18.43^\circ}\\ &= 3.16\text{ A}\;\angle 48.43^\circ\end{aligned}$$

The total current through the 6-Ω resistor (Fig. 18.10) is

$$\begin{aligned}\mathbf{I} &= \mathbf{I}' + \mathbf{I}''\\ &= 1.9\text{ A}\;\angle 108.43^\circ + 3.16\text{ A}\;\angle 48.43^\circ\\ &= (-0.60\text{ A} + j\,1.80\text{ A}) + (2.10\text{ A} + j\,2.36\text{ A})\\ &= 1.50\text{ A} + j\,4.16\text{ A}\\ \mathbf{I} &= \mathbf{4.42\text{ A}\;\angle 70.2^\circ}\end{aligned}$$

EXAMPLE 18.3 Using superposition, find the voltage across the 6-Ω resistor in Fig. 18.6. Check the results against $\mathbf{V}_{6\Omega} = \mathbf{I}(6\;\Omega)$, where **I** is the current found through the 6-Ω resistor in Example 18.2.

Solution: For the current source,

$$\mathbf{V}'_{6\Omega} = \mathbf{I}'(6\;\Omega) = (1.9\text{ A}\;\angle 108.43^\circ)(6\;\Omega) = 11.4\text{ V}\;\angle 108.43^\circ$$

For the voltage source,

$$\mathbf{V}''_{6\Omega} = \mathbf{I}''(6) = (3.16\text{ A}\;\angle 48.43^\circ)(6\;\Omega) = 18.96\text{ V}\;\angle 48.43^\circ$$

The total voltage across the 6-Ω resistor (Fig. 18.11) is

$$\begin{aligned}\mathbf{V}_{6\Omega} &= \mathbf{V}'_{6\Omega} + \mathbf{V}''_{6\Omega}\\ &= 11.4\text{ V}\;\angle 108.43^\circ + 18.96\text{ V}\;\angle 48.43^\circ\\ &= (-3.60\text{ V} + j\,10.82\text{ V}) + (12.58\text{ V} + j\,14.18\text{ V})\\ &= 8.98\text{ V} + j\,25.0\text{ V}\\ \mathbf{V}_{6\Omega} &= \mathbf{26.5\text{ V}\;\angle 70.2^\circ}\end{aligned}$$

Checking the result, we have

$$\begin{aligned}\mathbf{V}_{6\Omega} &= \mathbf{I}(6\;\Omega) = (4.42\text{ A}\;\angle 70.2^\circ)(6\;\Omega)\\ &= \mathbf{26.5\text{ V}\;\angle 70.2^\circ}\quad\text{(checks)}\end{aligned}$$

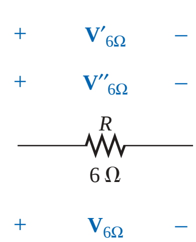

FIG. 18.11
Determining the resultant voltage $\mathbf{V}_{6\Omega}$ *for the network of Fig. 18.6.*

EXAMPLE 18.4 For the network of Fig. 18.12, determine the sinusoidal expression for the voltage v_3 using superposition.

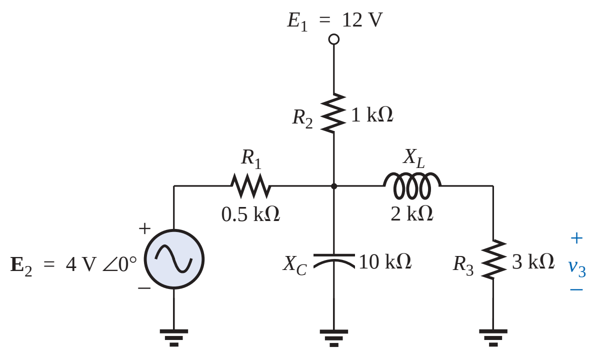

FIG. 18.12
Example 18.4.

Solution: For the dc source, recall that for dc analysis, in the steady state the capacitor can be replaced by an open-circuit equivalent, and the inductor by a short-circuit equivalent. The result is the network of Fig. 18.13.

The resistors R_1 and R_3 are then in parallel, and the voltage V_3 can be determined using the voltage divider rule:

$$R' = R_1 \parallel R_3 = 0.5\text{ k}\Omega \parallel 3\text{ k}\Omega = 0.429\text{ k}\Omega$$

and
$$V_3 = \frac{R'E_1}{R' + R_2}$$
$$= \frac{(0.429\text{ k}\Omega)(12\text{ V})}{0.429\text{ k}\Omega + 1\text{ k}\Omega} = \frac{5.148\text{ V}}{1.429}$$
$$V_3 \cong \mathbf{3.6\ V}$$

For ac analysis, the dc source is set to zero and the network is redrawn, as shown in Fig. 18.14.

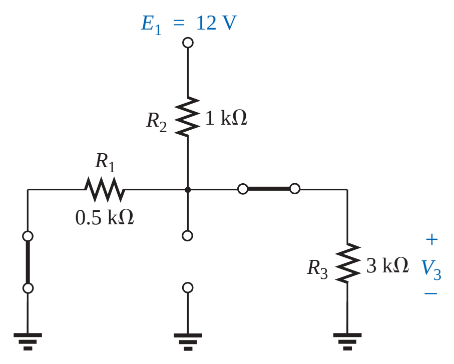

FIG. 18.13
Determining the effect of the dc voltage source E_1 on the voltage v_3 of the network of Fig. 18.12.

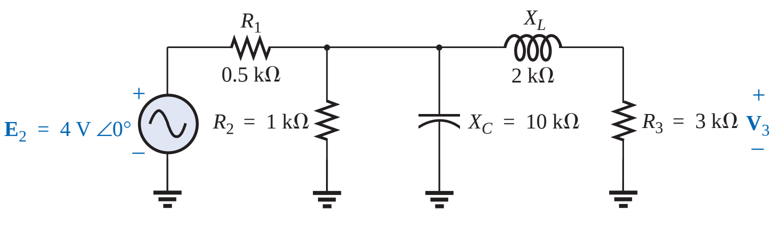

FIG. 18.14
Redrawing the network of Fig. 18.12 to determine the effect of the ac voltage source $\mathbf{E}_2$.

The block impedances are then defined as in Fig. 18.15, and series-parallel techniques are applied as follows:

$$\mathbf{Z}_1 = 0.5\text{ k}\Omega \angle 0°$$
$$\mathbf{Z}_2 = (R_2 \angle 0° \parallel (X_C \angle -90°)$$
$$= \frac{(1\text{ k}\Omega \angle 0°)(10\text{ k}\Omega \angle -90°)}{1\text{ k}\Omega - j\,10\text{ k}\Omega} = \frac{10\text{ k}\Omega \angle -90°}{10.05 \angle -84.29°}$$
$$= 0.995\text{ k}\Omega \angle -5.71°$$

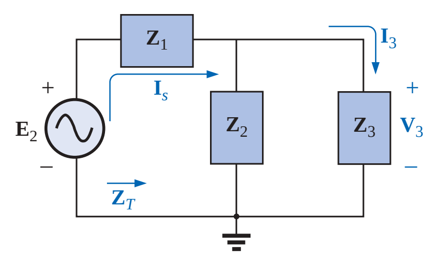

FIG. 18.15
Assigning the subscripted impedances to the network of Fig. 18.14.

$$\mathbf{Z}_3 = R_3 + j\,X_L = 3\text{ k}\Omega + j\,2\text{ k}\Omega = 3.61\text{ k}\Omega\ \angle 33.69^\circ$$

and

$$\begin{aligned}\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3\\ &= 0.5\text{ k}\Omega + (0.995\text{ k}\Omega\ \angle -5.71^\circ) \parallel (3.61\text{ k}\Omega\ \angle 33.69^\circ)\\ &= 1.312\text{ k}\Omega\ \angle 1.57^\circ\end{aligned}$$

Calculator Performing the above on the TI-86 calculator gives the following result:

```
(0.5,0)+((0.995∠−5.71)*(3.61∠33.69))/((0.995∠−5.71)+(3.61∠33.69)) (Enter)
   (1.311E0,35.373E−3)
Ans ► Pol
   (1.312E0∠1.545E0)
```

CALC. 18.1

$$\mathbf{I}_s = \frac{\mathbf{E}_2}{\mathbf{Z}_T} = \frac{4\text{ V}\ \angle 0^\circ}{1.312\text{ k}\Omega\ \angle 1.57^\circ} = 3.05\text{ mA}\ \angle -1.57^\circ$$

Current divider rule:

$$\begin{aligned}\mathbf{I}_3 &= \frac{\mathbf{Z}_2\mathbf{I}_s}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{(0.995\text{ k}\Omega\ \angle -5.71^\circ)(3.05\text{ mA}\ \angle -1.57^\circ)}{0.995\text{ k}\Omega\ \angle -5.71^\circ + 3.61\text{ k}\Omega\ \angle 33.69^\circ}\\ &= 0.686\text{ mA}\ \angle -32.74^\circ\end{aligned}$$

with

$$\begin{aligned}\mathbf{V}_3 &= (I_3\ \angle \theta)(R_3\ \angle 0^\circ)\\ &= (0.686\text{ mA}\ \angle -32.74^\circ)(3\text{ k}\Omega\ \angle 0^\circ)\\ &= \mathbf{2.06\text{ V}\ \angle -32.74^\circ}\end{aligned}$$

The total solution:

$$\begin{aligned}v_3 &= v_3\text{ (dc)} + v_3\text{ (ac)}\\ &= 3.6\text{ V} + 2.06\text{ V}\ \angle -32.74^\circ\\ v_3 &= \mathbf{3.6 + 2.91\sin(\omega t - 32.74^\circ)}\end{aligned}$$

The result is a sinusoidal voltage having a peak value of 2.91 V riding on an average value of 3.6 V, as shown in Fig. 18.16.

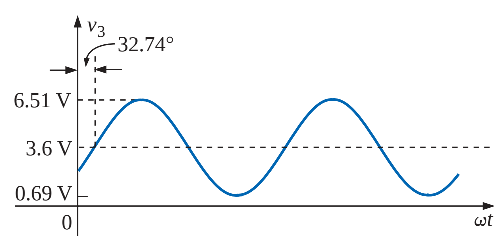

FIG. 18.16
The resultant voltage v_3 for the network of Fig. 18.12.

Dependent Sources

For dependent sources in which *the controlling variable is not determined by the network to which the superposition theorem is to be applied*, the application of the theorem is basically the same as for independent sources. The solution obtained will simply be in terms of the controlling variables.

EXAMPLE 18.5 Using the superposition theorem, determine the current $\mathbf{I}_2$ for the network of Fig. 18.17. The quantities μ and h are constants.

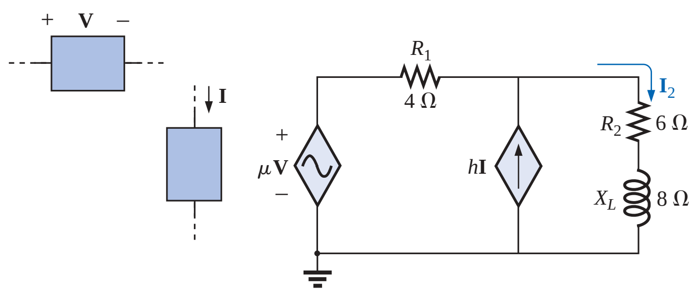

FIG. 18.17
Example 18.5.

Solution: With a portion of the system redrawn (Fig. 18.18),

$$\mathbf{Z}_1 = R_1 = 4\ \Omega \qquad \mathbf{Z}_2 = R_2 + j\,X_L = 6 + j\,8\ \Omega$$

For the voltage source (Fig. 18.19),

$$\mathbf{I}' = \frac{\mu\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mu\mathbf{V}}{4\ \Omega + 6\ \Omega + j\,8\ \Omega} = \frac{\mu\mathbf{V}}{10\ \Omega + j\,8\ \Omega}$$
$$= \frac{\mu\mathbf{V}}{12.8\ \Omega\ \angle 38.66^\circ} = 0.078\ \mu\mathbf{V}/\Omega\ \angle -38.66^\circ$$

For the current source (Fig. 18.20),

$$\mathbf{I}'' = \frac{\mathbf{Z}_1(h\mathbf{I})}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(4\ \Omega)(h\mathbf{I})}{12.8\ \Omega\ \angle 38.66^\circ} = 4(0.078)h\mathbf{I}\ \angle -38.66^\circ$$
$$= 0.312h\mathbf{I}\ \angle -38.66^\circ$$

The current $\mathbf{I}_2$ is

$$\mathbf{I}_2 = \mathbf{I}' + \mathbf{I}''$$
$$= 0.078\ \mu\mathbf{V}/\Omega\ \angle -38.66^\circ + 0.312h\mathbf{I}\ \angle -38.66^\circ$$

For $\mathbf{V} = 10\ \text{V}\ \angle 0^\circ$, $\mathbf{I} = 20\ \text{mA}\ \angle 0^\circ$, $\mu = 20$, and $h = 100$,

$$\mathbf{I}_2 = 0.078(20)(10\ \text{V}\ \angle 0^\circ)/\Omega\ \angle -38.66^\circ$$
$$+ 0.312(100)(20\ \text{mA}\ \angle 0^\circ)\ \angle -38.66^\circ$$
$$= 15.60\ \text{A}\ \angle -38.66^\circ + 0.62\ \text{A}\ \angle -38.66^\circ$$
$$\mathbf{I}_2 = \mathbf{16.22\ A\ \angle -38.66^\circ}$$

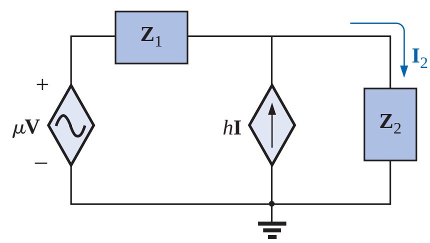

FIG. 18.18
Assigning the subscripted impedances to the network of Fig. 18.17.

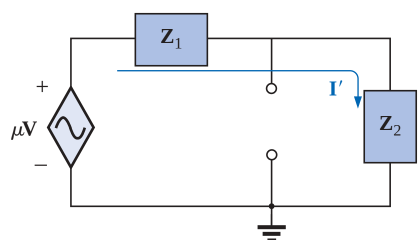

FIG. 18.19
Determining the effect of the voltage-controlled voltage source on the current $\mathbf{I}_2$ for the network of Fig. 18.17.

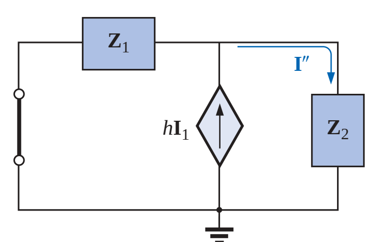

FIG. 18.20
Determining the effect of the current-controlled current source on the current $\mathbf{I}_2$ for the network of Fig. 18.17.

For dependent sources in which *the controlling variable is determined by the network to which the theorem is to be applied*, the dependent source cannot be set to zero unless the controlling variable is also zero. For networks containing dependent sources such as indicated in Example 18.5 and dependent sources of the type just introduced above, the superposition theorem is applied for each independent source and each dependent source not having a controlling variable in the portions of the network under investigation. It must be reemphasized that dependent sources are not sources of energy in the sense that, if all independent sources are removed from a system, all currents and voltages must be zero.

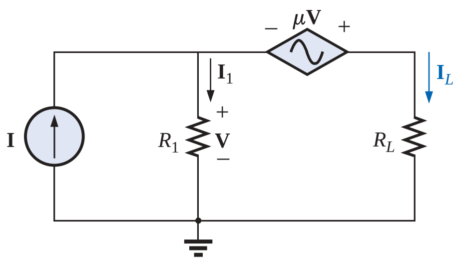

FIG. 18.21
Example 18.6.

EXAMPLE 18.6 Determine the current $\mathbf{I}_L$ through the resistor R_L of Fig. 18.21.

Solution: Note that the controlling variable $\mathbf{V}$ is determined by the network to be analyzed. From the above discussions, it is understood that the dependent source cannot be set to zero unless $\mathbf{V}$ is zero. If we set $\mathbf{I}$ to zero, the network lacks a source of voltage, and $\mathbf{V} = 0$ with $\mu\mathbf{V} = 0$. The resulting $\mathbf{I}_L$ under this condition is zero. Obviously, therefore, the network must be analyzed as it appears in Fig. 18.21, with the result that neither source can be eliminated, as is normally done using the superposition theorem.

Applying Kirchhoff's voltage law, we have

$$\mathbf{V}_L = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

and
$$\mathbf{I}_L = \frac{\mathbf{V}_L}{R_L} = \frac{(1 + \mu)\mathbf{V}}{R_L}$$

The result, however, must be found in terms of $\mathbf{I}$ since $\mathbf{V}$ and $\mu\mathbf{V}$ are only dependent variables.

Applying Kirchhoff's current law gives us

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_L = \frac{\mathbf{V}}{R_1} + \frac{(1 + \mu)\mathbf{V}}{R_L}$$

and
$$\mathbf{I} = \mathbf{V}\left(\frac{1}{R_1} + \frac{1 + \mu}{R_L}\right)$$

or
$$\mathbf{V} = \frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}$$

Substituting into the above yields

$$\mathbf{I}_L = \frac{(1 + \mu)\mathbf{V}}{R_L} = \frac{(1 + \mu)}{R_L}\left(\frac{\mathbf{I}}{(1/R_1) + [(1 + \mu)/R_L]}\right)$$

Therefore,
$$\boldsymbol{\mathbf{I}_L = \frac{(1 + \mu)R_1\mathbf{I}}{R_L + (1 + \mu)R_1}}$$

18.3 THÉVENIN'S THEOREM

Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term *impedance* instead of *resistance;* that is,

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series, as shown in Fig. 18.22.

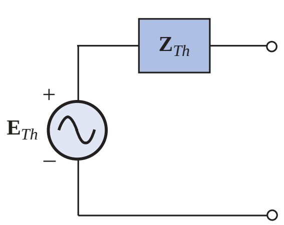

FIG. 18.22
Thévenin equivalent circuit for ac networks.

Since the reactances of a circuit are frequency dependent, the Thévenin circuit found for a particular network is applicable only at *one* frequency.

The steps required to apply this method to dc circuits are repeated here with changes for sinusoidal ac circuits. As before, the only change is the replacement of the term *resistance* with *impedance.* Again, dependent and independent sources will be treated separately.

Example 18.9, the last example of the independent source section, will include a network with dc and ac sources to establish the groundwork for possible use in the electronics area.

Independent Sources

1. ***Remove that portion of the network across which the Thévenin equivalent circuit is to be found.***
2. ***Mark (○, ●, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_{Th}$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{E}_{Th}$ by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.***
5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.***

EXAMPLE 18.7 Find the Thévenin equivalent circuit for the network external to resistor R in Fig. 18.23.

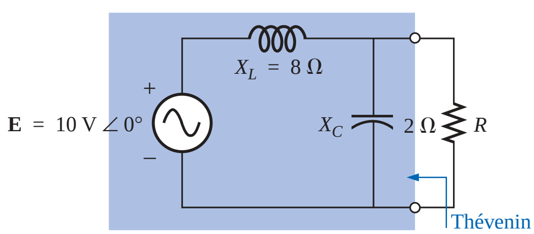

FIG. 18.23
Example 18.7.

Solution:

Steps 1 and 2 (Fig. 18.24):

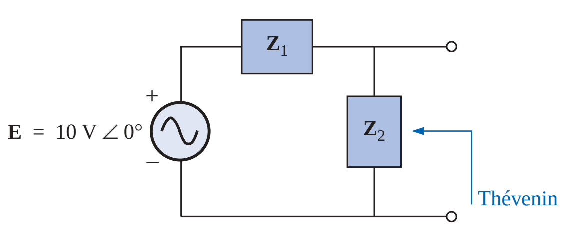

FIG. 18.24
Assigning the subscripted impedances to the network of Fig. 18.23.

$$\mathbf{Z}_1 = j\,X_L = j\,8\,\Omega \qquad \mathbf{Z}_2 = -j\,X_C = -j\,2\,\Omega$$

Step 3 (Fig. 18.25):

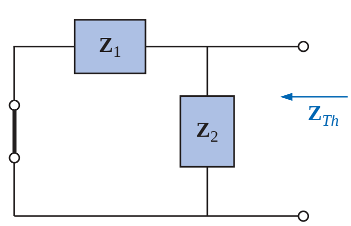

FIG. 18.25
Determining the Thévenin impedance for the network of Fig. 18.23.

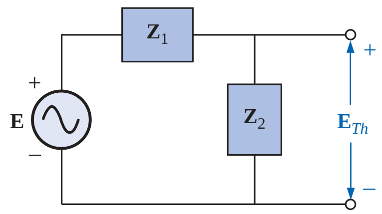

FIG. 18.26
Determining the open-circuit Thévenin voltage for the network of Fig. 18.23.

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2 16\,\Omega}{j\,6} = \frac{16\,\Omega}{6\angle 90^\circ}$$
$$= \mathbf{2.67\,\Omega\angle -90^\circ}$$

Step 4 (Fig. 18.26):

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$
$$= \frac{(-j\,2\,\Omega)(10\text{ V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\text{ V}}{j\,6} = \mathbf{3.33\text{ V}\angle -180^\circ}$$

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.27.

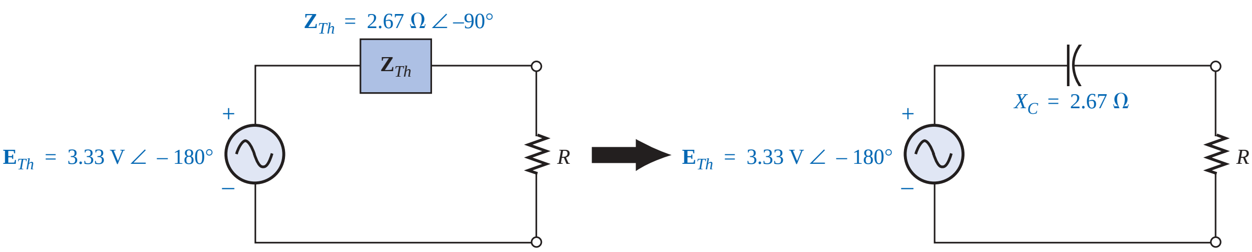

FIG. 18.27
The Thévenin equivalent circuit for the network of Fig. 18.23.

EXAMPLE 18.8 Find the Thévenin equivalent circuit for the network external to branch *a-a′* in Fig. 18.28.

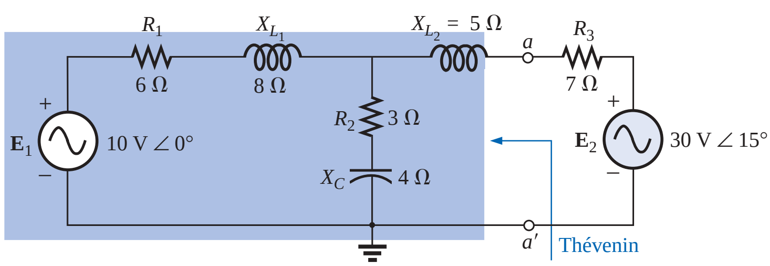

FIG. 18.28
Example 18.8.

Solution:

Steps 1 and 2 (Fig. 18.29): Note the reduced complexity with subscripted impedances:

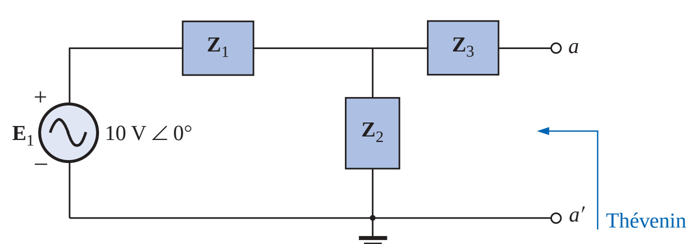

FIG. 18.29
Assigning the subscripted impedances to the network of Fig. 18.28.

$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 6\ \Omega + j\,8\ \Omega$$
$$\mathbf{Z}_2 = R_2 - jX_C = 3\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_3 = +jX_{L_2} = j\,5\ \Omega$$

Step 3 (Fig. 18.30):

$$\mathbf{Z}_{Th} = \mathbf{Z}_3 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = j\,5\ \Omega + \frac{(10\ \Omega\ \angle 53.13^\circ)(5\ \Omega\ \angle -53.13^\circ)}{(6\ \Omega + j\,8\ \Omega) + (3\ \Omega - j\,4\ \Omega)}$$
$$= j\,5 + \frac{50\ \angle 0^\circ}{9 + j\,4} = j\,5 + \frac{50\ \angle 0^\circ}{9.85\ \angle 23.96^\circ}$$
$$= j\,5 + 5.08\ \angle -23.96^\circ = j\,5 + 4.64 - j\,2.06$$
$$\mathbf{Z}_{Th} = \mathbf{4.64\ \Omega + \mathit{j}\,2.94\ \Omega = 5.49\ \Omega\ \angle 32.36^\circ}$$

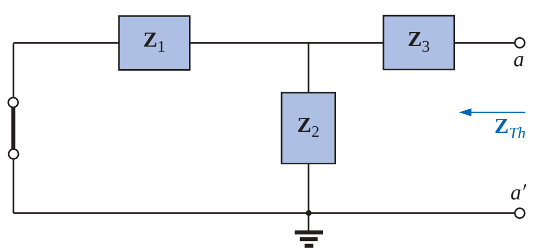

FIG. 18.30
Determining the Thévenin impedance for the network of Fig. 18.28.

Step 4 (Fig. 18.31): Since *a-a′* is an open circuit, $\mathbf{I}_{\mathbf{Z}_3} = 0$. Then $\mathbf{E}_{Th}$ is the voltage drop across $\mathbf{Z}_2$:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$
$$= \frac{(5\ \Omega\ \angle -53.13^\circ)(10\ \text{V}\ \angle 0^\circ)}{9.85\ \Omega\ \angle 23.96^\circ}$$
$$\mathbf{E}_{Th} = \frac{50\ \text{V}\ \angle -53.13^\circ}{9.85\ \angle 23.96^\circ} = \mathbf{5.08\ V\ \angle -77.09^\circ}$$

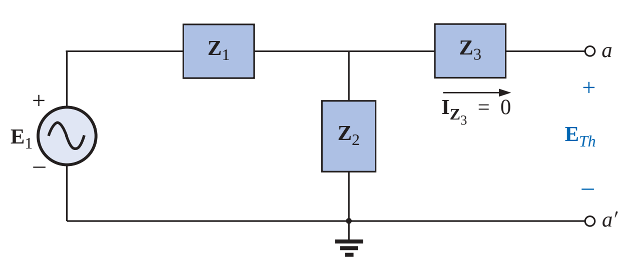

FIG. 18.31
Determining the open-circuit Thévenin voltage for the network of Fig. 18.28.

Step 5: The Thévenin equivalent circuit is shown in Fig. 18.32.

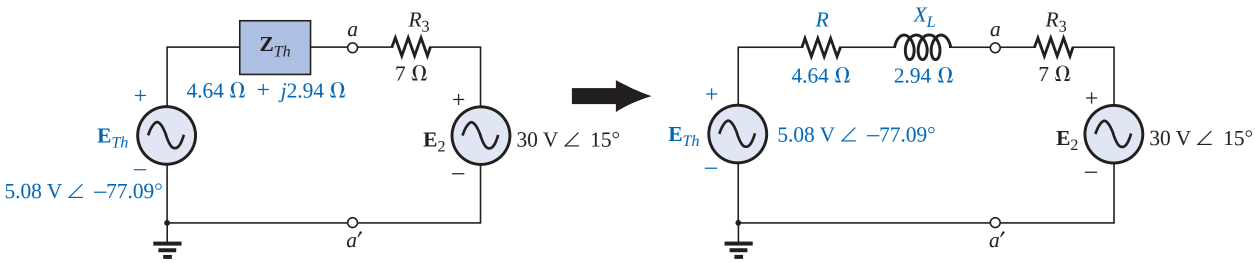

FIG. 18.32
The Thévenin equivalent circuit for the network of Fig. 18.28.

The next example demonstrates how superposition is applied to electronic circuits to permit *a separation of the dc and ac analyses.* The fact that the controlling variable in this analysis is not in the portion of the network connected directly to the terminals of interest permits an analysis of the network in the same manner as applied above for independent sources.

EXAMPLE 18.9 Determine the Thévenin equivalent circuit for the transistor network external to the resistor R_L in the network of Fig. 18.33. Then determine $\mathbf{V}_L$.

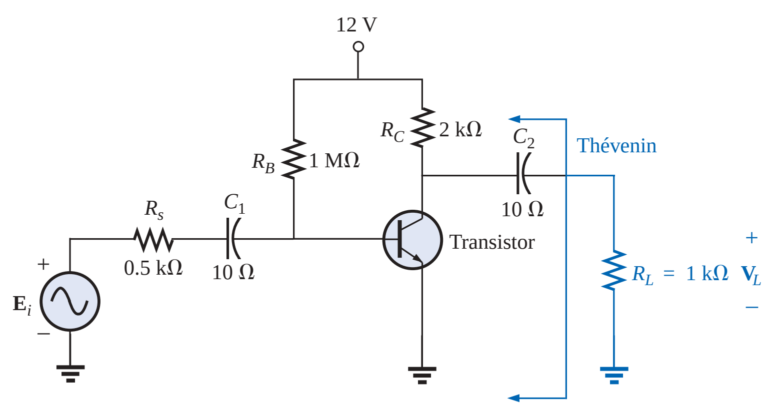

FIG. 18.33
Example 18.9.

Solution: Applying superposition.

dc Conditions Substituting the open-circuit equivalent for the coupling capacitor C_2 will isolate the dc source and the resulting currents from the load resistor. The result is that for dc conditions, $V_L = 0$ V. Although the output dc voltage is zero, the application of the dc voltage is important to the basic operation of the transistor in a number of important ways, one of which is to determine the parameters of the "equivalent circuit" to appear in the ac analysis to follow.

ac Conditions For the ac analysis, an equivalent circuit is substituted for the transistor, as established by the dc conditions above, that will behave like the actual transistor. A great deal more will be said about equivalent circuits and the operations performed to obtain the network of Fig. 18.34, but for now let us limit our attention to the manner in which the Thévenin equivalent circuit is obtained. Note in Fig. 18.34 that the equivalent circuit includes a resistor of 2.3 kΩ and a controlled current source whose magnitude is determined by the product of a factor of 100 and the current I_1 in another part of the network.

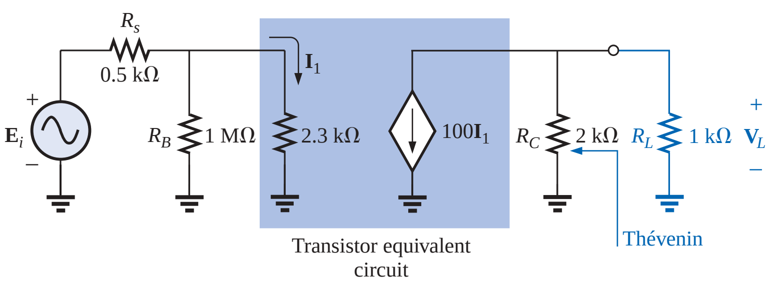

FIG. 18.34
The ac equivalent network for the transistor amplifier of Fig. 18.33.

Note in Fig. 18.34 the absence of the coupling capacitors for the ac analysis. In general, coupling capacitors are designed to be open circuits for dc analysis and short circuits for ac analysis. The short-circuit equivalent is valid because the other impedances in series with the coupling capacitors are so much larger in magnitude that the effect of the coupling capacitors can be ignored. Both R_B and R_C are now tied to ground because the dc source was set to zero volts (superposition) and replaced by a short-circuit equivalent to ground.

For the analysis to follow, the effect of the resistor R_B will be ignored since it is so much larger than the parallel 2.3-kΩ resistor.

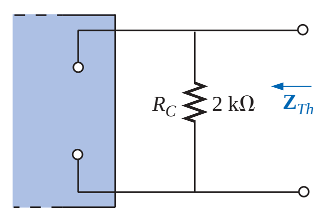

FIG. 18.35
Determining the Thévenin impedance for the network of Fig. 18.34.

$\mathbf{Z}_{Th}$ When $\mathbf{E}_i$ is set to zero volts, the current $\mathbf{I}_1$ will be zero amperes, and the controlled source $100\mathbf{I}_1$ will be zero amperes also. The result is an open-circuit equivalent for the source, as appearing in Fig. 18.35.

It is fairly obvious from Fig. 18.35 that

$$\mathbf{Z}_{Th} = \mathbf{2\ k\Omega}$$

$\mathbf{E}_{Th}$ For $\mathbf{E}_{Th}$, the current $\mathbf{I}_1$ of Fig. 18.34 will be

$$\mathbf{I}_1 = \frac{\mathbf{E}_i}{R_s + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{0.5\ \text{k}\Omega + 2.3\ \text{k}\Omega} = \frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}$$

and
$$100\mathbf{I}_1 = (100)\left(\frac{\mathbf{E}_i}{2.8\ \text{k}\Omega}\right) = 35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i$$

Referring to Fig. 18.36, we find that

$$\begin{aligned}\mathbf{E}_{Th} &= -(100\mathbf{I}_1)R_C\\ &= -(35.71 \times 10^{-3}/\Omega\ \mathbf{E}_i)(2 \times 10^3\ \Omega)\\ \mathbf{E}_{Th} &= \mathbf{-71.42E}_i\end{aligned}$$

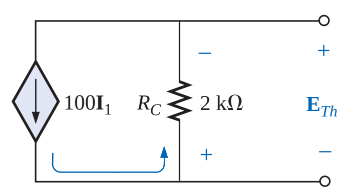

FIG. 18.36
Determining the Thévenin voltage for the network of Fig. 18.34.

The Thévenin equivalent circuit appears in Fig. 18.37 with the original load R_L.

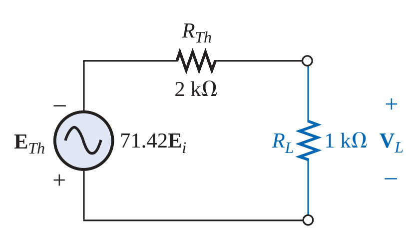

FIG. 18.37
The Thévenin equivalent circuit for the network of Fig. 18.34.

Output Voltage V_L

$$\mathbf{V}_L = \frac{-R_L\mathbf{E}_{Th}}{R_L + \mathbf{Z}_{Th}} = \frac{-(1\ \text{k}\Omega)(71.42\mathbf{E}_i)}{1\ \text{k}\Omega + 2\ \text{k}\Omega}$$

and
$$\mathbf{V}_L = \mathbf{-23.81E}_i$$

revealing that the output voltage is 23.81 times the applied voltage with a phase shift of 180° due to the minus sign.

Dependent Sources

For dependent sources with a *controlling variable not in the network under investigation,* the procedure indicated above can be applied. However, for dependent sources of the other type, where the *controlling variable is part of the network to which the theorem is to be applied,* another approach must be employed. The necessity for a different approach will be demonstrated in an example to follow. The method is *not limited to dependent sources* of the latter type. It can also be applied to any dc or sinusoidal ac network. However, for networks of independent sources, the method of application employed in Chapter 9 and presented in the first portion of this section is generally more direct, with the usual savings in time and errors.

The new approach to Thévenin's theorem can best be introduced at this stage in the development by considering the Thévenin equivalent circuit of Fig. 18.38(a). As indicated in Fig. 18.38(b), the open-circuit terminal voltage ($\mathbf{E}_{oc}$) of the Thévenin equivalent circuit is the Thévenin equivalent voltage; that is,

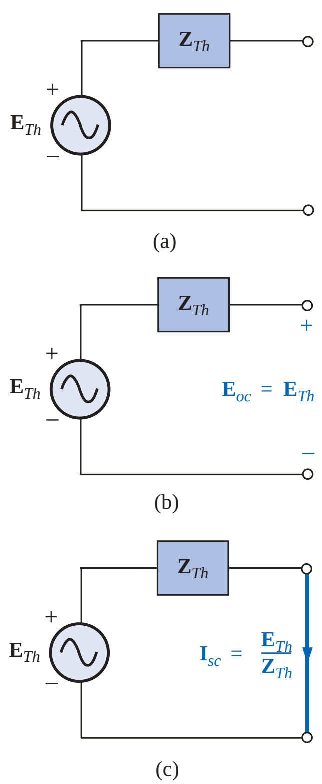

FIG. 18.38
Defining an alternative approach for determining the Thévenin impedance.

$$\boxed{\mathbf{E}_{oc} = \mathbf{E}_{Th}} \qquad \textbf{(18.1)}$$

If the external terminals are short circuited as in Fig. 18.38(c), the resulting short-circuit current is determined by

$$\boxed{\mathbf{I}_{sc} = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}}} \qquad \textbf{(18.2)}$$

or, rearranged,

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}}$$

and
$$\boxed{\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}} \qquad \textbf{(18.3)}$$

Equations (18.1) and (18.3) indicate that for any linear bilateral dc or ac network with or without dependent sources of any type, if the open-circuit terminal voltage of a portion of a network can be determined along with the short-circuit current between the same two terminals, the Thévenin equivalent circuit is effectively known. A few examples will make the method quite clear. The advantage of the method, which was stressed earlier in this section for independent sources, should now be more obvious. The current $\mathbf{I}_{sc}$, which is necessary to find $\mathbf{Z}_{Th}$, is in general more difficult to obtain since all of the sources are present.

There is a third approach to the Thévenin equivalent circuit that is also useful from a practical viewpoint. The Thévenin voltage is found as in the two previous methods. However, the Thévenin impedance is obtained by applying a source of voltage to the terminals of interest and determining the source current as indicated in Fig. 18.39. For this method, the source voltage of the original network is set to zero. The Thévenin impedance is then determined by the following equation:

$$\boxed{\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g}} \tag{18.4}$$

Note that for each technique, $\mathbf{E}_{Th} = \mathbf{E}_{oc}$, but the Thévenin impedance is found in different ways.

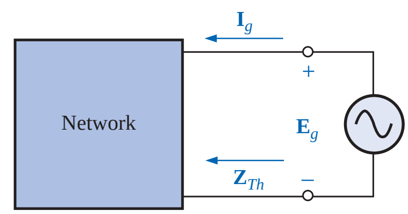

FIG. 18.39
Determining $\mathbf{Z}_{Th}$ using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

EXAMPLE 18.10 Using each of the three techniques described in this section, determine the Thévenin equivalent circuit for the network of Fig. 18.40.

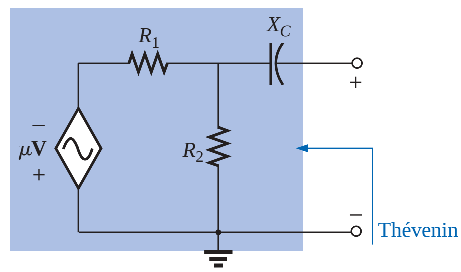

FIG. 18.40
Example 18.10.

Solution: Since for each approach the Thévenin voltage is found in exactly the same manner, it will be determined first. From Fig. 18.40, where $\mathbf{I}_{X_C} = 0$,

Due to the polarity for **V** and defined terminal polarities

$$\mathbf{V}_{R_1} = \mathbf{E}_{Th} = \mathbf{E}_{oc} = \downarrow -\frac{R_2(\mu\mathbf{V})}{R_1 + R_2} = -\frac{\boldsymbol{\mu R_2}\mathbf{V}}{\boldsymbol{R_1 + R_2}}$$

The following three methods for determining the Thévenin impedance appear in the order in which they were introduced in this section.

Method 1: See Fig. 18.41.

$$\mathbf{Z}_{Th} = \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

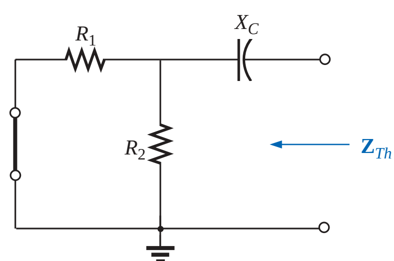

FIG. 18.41
Determining the Thévenin impedance for the network of Fig. 18.40.

Method 2: See Fig. 18.42. Converting the voltage source to a current source (Fig. 18.43), we have (current divider rule)

$$\mathbf{I}_{sc} = \frac{-(R_1 \parallel R_2)\dfrac{\mu\mathbf{V}}{R_1}}{(R_1 \parallel R_2) - j\,X_C} = \frac{-\dfrac{R_1R_2}{R_1 + R_2}\left(\dfrac{\mu\mathbf{V}}{R_1}\right)}{(R_1 \parallel R_2) - j\,X_C}$$

$$= \frac{\dfrac{-\mu R_2\mathbf{V}}{R_1 + R_2}}{(R_1 \parallel R_2) - j\,X_C}$$

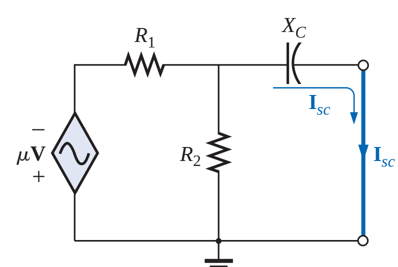

FIG. 18.42
Determining the short-circuit current for the network of Fig. 18.40.

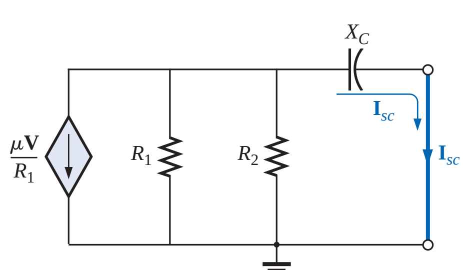

FIG. 18.43
Converting the voltage source of Fig. 18.42 to a current source.

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-\mu R_2 \mathbf{V}}{R_1 + R_2}}{\dfrac{\dfrac{-\mu R_2 \mathbf{V}}{R_1 + R_2}}{(R_1 \parallel R_2) - j X_C}} = \frac{1}{\dfrac{1}{(R_1 \parallel R_2) - j X_C}}$$

$$= \boldsymbol{R_1 \parallel R_2 - j X_C}$$

Method 3: See Fig. 18.44.

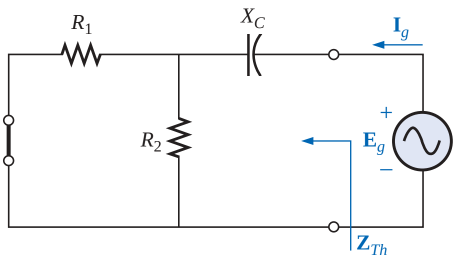

FIG. 18.44
Determining the Thévenin impedance for the network of Fig. 18.40 using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

$$\mathbf{I}_g = \frac{\mathbf{E}_g}{(R_1 \parallel R_2) - j X_C}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{R_1 \parallel R_2 - j X_C}$$

In each case, the Thévenin impedance is the same. The resulting Thévenin equivalent circuit is shown in Fig. 18.45.

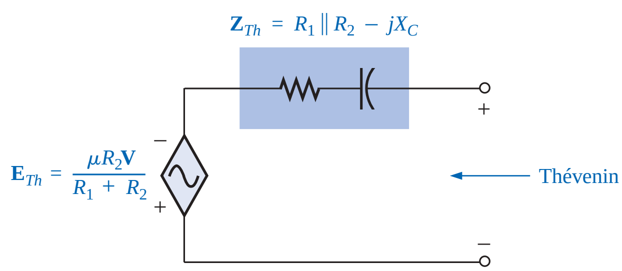

FIG. 18.45
The Thévenin equivalent circuit for the network of Fig. 18.40.

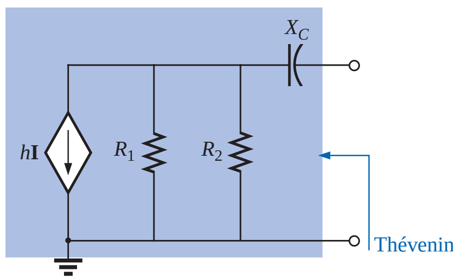

FIG. 18.46
Example 18.11.

EXAMPLE 18.11 Repeat Example 18.10 for the network of Fig. 18.46.

Solution: From Fig. 18.46, $\mathbf{E}_{Th}$ is

$$\mathbf{E}_{Th} = \mathbf{E}_{oc} = -h\mathbf{I}(R_1 \parallel R_2) = -\frac{\boldsymbol{hR_1R_2\mathbf{I}}}{\boldsymbol{R_1 + R_2}}$$

Method 1: See Fig. 18.47.

$$\mathbf{Z}_{Th} = \boldsymbol{R_1 \parallel R_2 - j X_C}$$

Note the similarity between this solution and that obtained for the previous example.

Method 2: See Fig. 18.48.

$$\mathbf{I}_{sc} = \frac{-(R_1 \parallel R_2)h\mathbf{I}}{(R_1 \parallel R_2) - j X_C}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{-h\mathbf{I}(R_1 \parallel R_2)}{\dfrac{-(R_1 \parallel R_2)h\mathbf{I}}{(R_1 \parallel R_2) - j X_C}} = \boldsymbol{R_1 \parallel R_2 - j X_C}$$

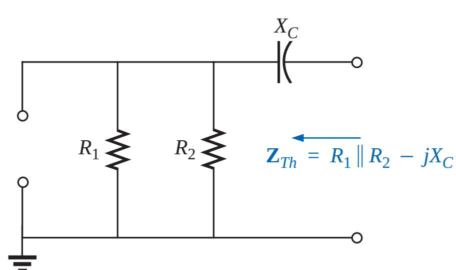

FIG. 18.47
Determining the Thévenin impedance for the network of Fig. 18.46.

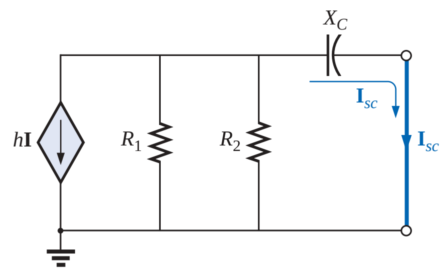

FIG. 18.48
Determining the short-circuit current for the network of Fig. 18.46.

Method 3: See Fig. 18.49.

$$\mathbf{I}_g = \frac{\mathbf{E}_g}{(R_1 \parallel R_2) - j\,X_C}$$

and
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \boldsymbol{R_1 \parallel R_2 - j\,X_C}$$

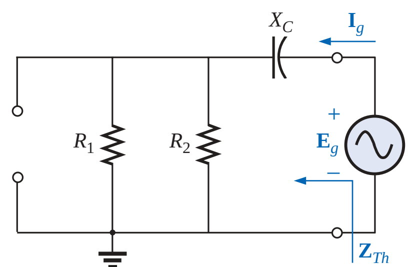

FIG. 18.49
Determining the Thévenin impedance using the approach $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

The following example has a dependent source that will not permit the use of the method described at the beginning of this section for independent sources. All three methods will be applied, however, so that the results can be compared.

EXAMPLE 18.12 For the network of Fig. 18.50 (introduced in Example 18.6), determine the Thévenin equivalent circuit between the indicated terminals using each method described in this section. Compare your results.

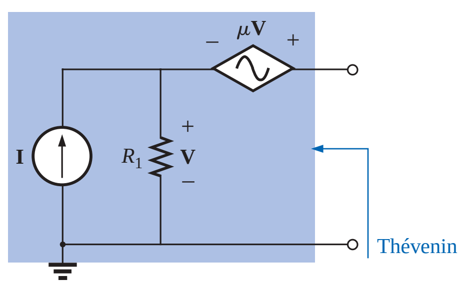

FIG. 18.50
Example 18.12.

Solution: First, using Kirchhoff's voltage law, $\mathbf{E}_{Th}$ (which is the same for each method) is written

$$\mathbf{E}_{Th} = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

However,
$$\mathbf{V} = \mathbf{I}R_1$$

so
$$\mathbf{E}_{Th} = \boldsymbol{(1 + \mu)\mathbf{I}R_1}$$

$\mathbf{Z}_{Th}$

Method 1: See Fig. 18.51. Since $\mathbf{I} = 0$, $\mathbf{V}$ and $\mu\mathbf{V} = 0$, and

$$\cancel{Z_{Th} = R_1} \qquad \text{(incorrect)}$$

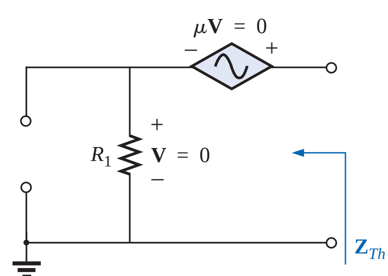

FIG. 18.51
Determining $\mathbf{Z}_{Th}$ incorrectly.

Method 2: See Fig. 18.52. Kirchhoff's voltage law around the indicated loop gives us

$$\mathbf{V} + \mu\mathbf{V} = 0$$

and
$$\mathbf{V}(1 + \mu) = 0$$

Since μ is a positive constant, the above equation can be satisfied only when $\mathbf{V} = 0$. Substitution of this result into Fig. 18.52 will yield the configuration of Fig. 18.53, and

$$\mathbf{I}_{sc} = \mathbf{I}$$

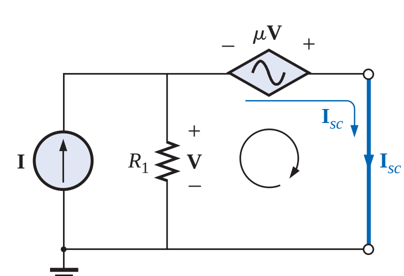

FIG. 18.52
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.50.

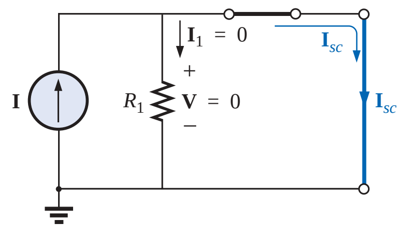

FIG. 18.53
Substituting $\mathbf{V} = 0$ *into the network of Fig. 18.52.*

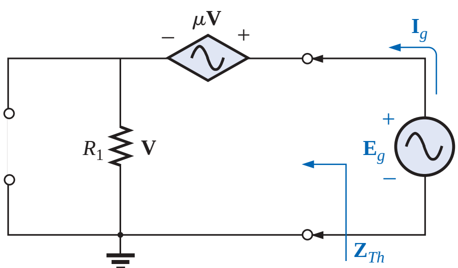

FIG. 18.54
Determining $\mathbf{Z}_{Th}$ *using the approach* $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

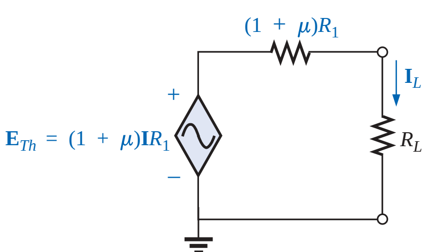

FIG. 18.55
The Thévenin equivalent circuit for the network of Fig. 18.50.

with

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{(1+\mu)\mathbf{I}R_1}{\mathbf{I}} = \mathbf{(1 + \mu)}\boldsymbol{R_1} \qquad \text{(correct)}$$

Method 3: See Fig. 18.54.

$$\mathbf{E}_g = \mathbf{V} + \mu\mathbf{V} = (1 + \mu)\mathbf{V}$$

or

$$\mathbf{V} = \frac{\mathbf{E}_g}{1 + \mu}$$

and

$$\mathbf{I}_g = \frac{\mathbf{V}}{R_1} = \frac{\mathbf{E}_g}{(1 + \mu)R_1}$$

and

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \mathbf{(1 + \mu)}\boldsymbol{R_1} \qquad \text{(correct)}$$

The Thévenin equivalent circuit appears in Fig. 18.55, and

$$\mathbf{I}_L = \frac{\mathbf{(1 + \mu)}\boldsymbol{R_1}\mathbf{I}}{\boldsymbol{R_L} + \mathbf{(1 + \mu)}\boldsymbol{R_1}}$$

which compares with the result of Example 18.6.

The network of Fig. 18.56 is the basic configuration of the transistor equivalent circuit applied most frequently today (although most texts in electronics will use the circle rather than the diamond outline for the source). Obviously, it is necessary to know its characteristics and to be adept in its use. Note that there are both a controlled voltage and a controlled current source, each controlled by variables in the configuration.

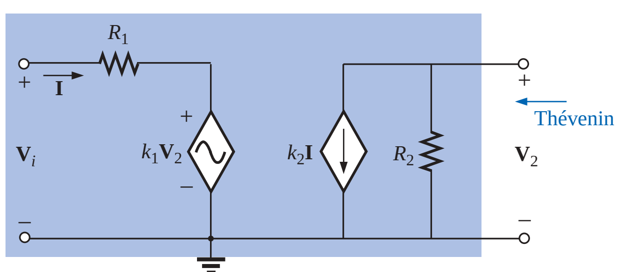

FIG. 18.56
Example 18.13: Transistor equivalent network.

EXAMPLE 18.13 Determine the Thévenin equivalent circuit for the indicated terminals of the network of Fig. 18.56.

Solution: Apply the second method introduced in this section.

E_{Th}

$$\mathbf{E}_{oc} = \mathbf{V}_2$$

$$\mathbf{I} = \frac{\mathbf{V}_i - k_1\mathbf{V}_2}{R_1} = \frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}$$

and

$$\mathbf{E}_{oc} = -k_2\mathbf{I}R_2 = -k_2R_2\left(\frac{\mathbf{V}_i - k_1\mathbf{E}_{oc}}{R_1}\right)$$

$$= \frac{-k_2R_2\mathbf{V}_i}{R_1} + \frac{k_1k_2R_2\mathbf{E}_{oc}}{R_1}$$

or
$$\mathbf{E}_{oc}\left(1 - \frac{k_1k_2R_2}{R_1}\right) = \frac{-k_2R_2\mathbf{V}_i}{R_1}$$

and
$$\mathbf{E}_{oc}\left(\frac{R_1 - k_1k_2R_2}{\cancel{R_1}}\right) = \frac{-k_2R_2\mathbf{V}_i}{\cancel{R_1}}$$

so
$$\boxed{\mathbf{E}_{oc} = \frac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2} = \mathbf{E}_{Th}} \qquad \textbf{(18.5)}$$

$\mathbf{I}_{sc}$ For the network of Fig. 18.57, where

$$\mathbf{V}_2 = 0 \qquad k_1\mathbf{V}_2 = 0 \qquad \mathbf{I} = \frac{\mathbf{V}_i}{R_1}$$

and
$$\mathbf{I}_{sc} = -k_2\mathbf{I} = \frac{-k_2\mathbf{V}_i}{R_1}$$

so
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{-k_2\mathbf{V}_i}{R_1}} = \frac{R_1R_2}{R_1 - k_1k_2R_2}$$

and
$$\boxed{\mathbf{Z}_{Th} = \frac{R_2}{1 - \dfrac{k_1k_2R_2}{R_1}}} \qquad \textbf{(18.6)}$$

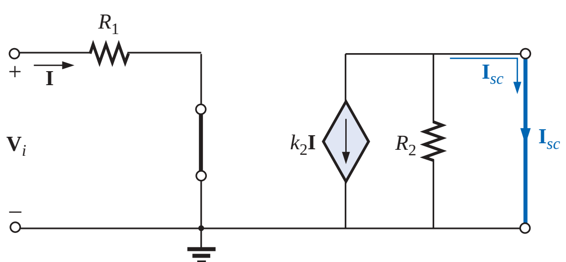

FIG. 18.57
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.56.

Frequently, the approximation $k_1 \cong 0$ is applied. Then the Thévenin voltage and impedance are

$$\boxed{\mathbf{E}_{Th} = \frac{-k_2R_2\mathbf{V}_i}{R_1}} \qquad k_1 = 0 \qquad \textbf{(18.7)}$$

$$\boxed{\mathbf{Z}_{Th} = R_2} \qquad k_1 = 0 \qquad \textbf{(18.8)}$$

Apply $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$ to the network of Fig. 18.58, where

$$\mathbf{I} = \frac{-k_1\mathbf{V}_2}{R_1}$$

But
$$\mathbf{V}_2 = \mathbf{E}_g$$

so
$$\mathbf{I} = \frac{-k_1\mathbf{E}_g}{R_1}$$

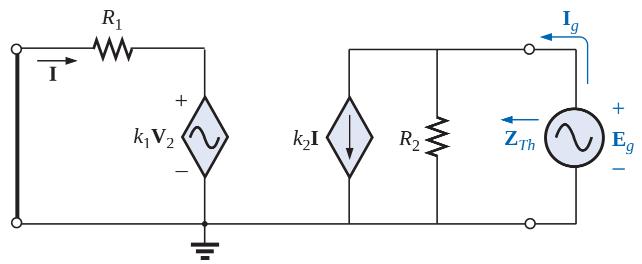

FIG. 18.58
Determining $\mathbf{Z}_{Th}$ *using the procedure* $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$.

Applying Kirchhoff's current law, we have

$$\mathbf{I}_g = k_2\mathbf{I} + \frac{\mathbf{E}_g}{R_2} = k_2\left(-\frac{k_1\mathbf{E}_g}{R_1}\right) + \frac{\mathbf{E}_g}{R_2}$$

$$= \mathbf{E}_g\left(\frac{1}{R_2} - \frac{k_1k_2}{R_1}\right)$$

and
$$\frac{\mathbf{I}_g}{\mathbf{E}_g} = \frac{R_1 - k_1k_2R_2}{R_1R_2}$$

or
$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \frac{\boldsymbol{R_1R_2}}{\boldsymbol{R_1 - k_1k_2R_2}}$$

as obtained above.

The last two methods presented in this section were applied only to networks in which the magnitudes of the controlled sources were dependent on a variable within the network for which the Thévenin equivalent circuit was to be obtained. Understand that both of these methods can also be applied to any dc or sinusoidal ac network containing only independent sources or dependent sources of the other kind.

18.4 NORTON'S THEOREM

The three methods described for Thévenin's theorem will each be altered to permit their use with **Norton's theorem.** Since the Thévenin and Norton impedances are the same for a particular network, certain portions of the discussion will be quite similar to those encountered in the previous section. We will first consider independent sources and the approach developed in Chapter 9, followed by dependent sources and the new techniques developed for Thévenin's theorem.

You will recall from Chapter 9 that Norton's theorem allows us to replace any two-terminal linear bilateral ac network with an equivalent circuit consisting of a current source and an impedance, as in Fig. 18.59.

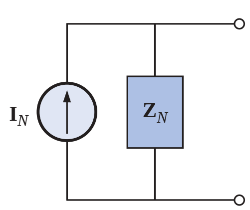

FIG. 18.59
The Norton equivalent circuit for ac networks.

The Norton equivalent circuit, like the Thévenin equivalent circuit, is applicable at only one frequency since the reactances are frequency dependent.

Independent Sources

The procedure outlined below to find the Norton equivalent of a sinusoidal ac network is changed (from that in Chapter 9) in only one respect: the replacement of the term *resistance* with the term *impedance.*

1. ***Remove that portion of the network across which the Norton equivalent circuit is to be found.***
2. ***Mark (○, ●, and so on) the terminals of the remaining two-terminal network.***
3. ***Calculate $\mathbf{Z}_N$ by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.***
4. ***Calculate $\mathbf{I}_N$ by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.***
5. ***Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.***

The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in Fig. 18.60. The source transformation is applicable for any Thévenin or Norton equivalent circuit determined from a network with any combination of independent or dependent sources.

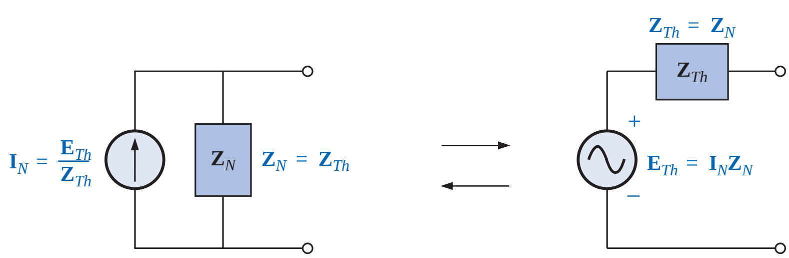

FIG. 18.60
Conversion between the Thévenin and Norton equivalent circuits.

EXAMPLE 18.14 Determine the Norton equivalent circuit for the network external to the 6-Ω resistor of Fig. 18.61.

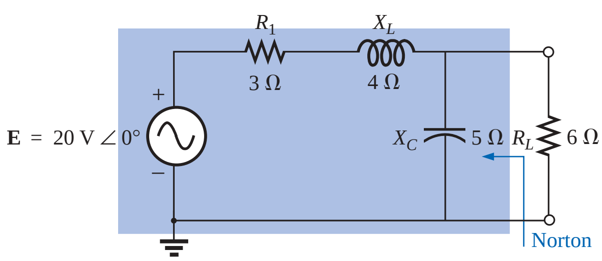

FIG. 18.61
Example 18.14.

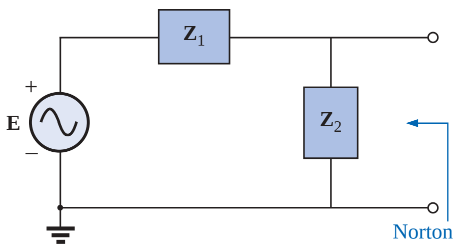

FIG. 18.62
Assigning the subscripted impedances to the network of Fig. 18.61.

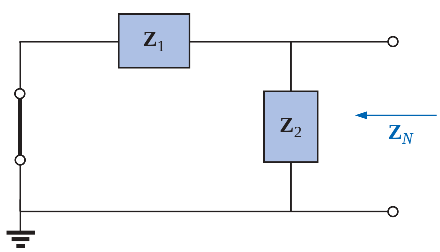

FIG. 18.63
Determining the Norton impedance for the network of Fig. 18.61.

Solution:

Steps 1 and 2 (Fig. 18.62):

$$\mathbf{Z}_1 = R_1 + j\,X_L = 3\,\Omega + j\,4\,\Omega = 5\,\Omega\,\angle 53.13°$$
$$\mathbf{Z}_2 = -j\,X_C = -j\,5\,\Omega$$

Step 3 (Fig. 18.63):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\,\Omega\,\angle 53.13°)(5\,\Omega\,\angle -90°)}{3\,\Omega + j\,4\,\Omega - j\,5\,\Omega} = \frac{25\,\Omega\,\angle -36.87°}{3 - j\,1}$$
$$= \frac{25\,\Omega\,\angle -36.87°}{3.16\,\angle -18.43°} = 7.91\,\Omega\,\angle -18.44° = \mathbf{7.50\,\Omega - j\,2.50\,\Omega}$$

Step 4 (Fig. 18.64):

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{20\,\text{V}\,\angle 0°}{5\,\Omega\,\angle 53.13°} = \mathbf{4\,A\,\angle -53.13°}$$

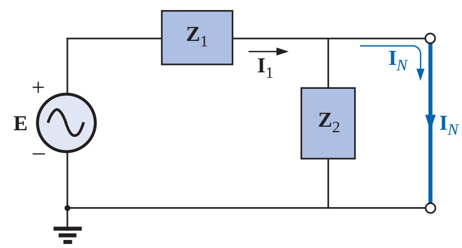

FIG. 18.64
Determining $\mathbf{I}_N$ for the network of Fig. 18.61.

Step 5: The Norton equivalent circuit is shown in Fig. 18.65.

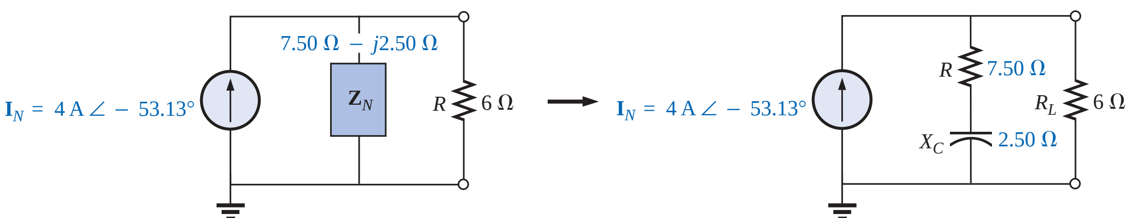

FIG. 18.65
The Norton equivalent circuit for the network of Fig. 18.61.

EXAMPLE 18.15 Find the Norton equivalent circuit for the network external to the 7-Ω capacitive reactance in Fig. 18.66.

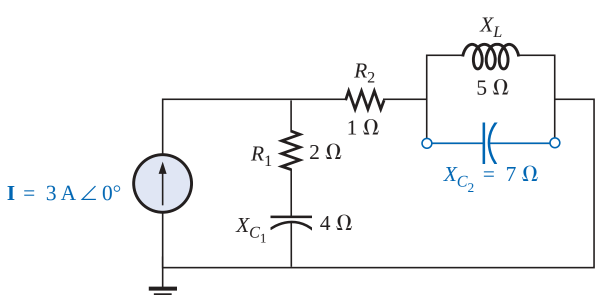

FIG. 18.66
Example 18.15.

Solution:

Steps 1 and 2 (Fig. 18.67):

$$\mathbf{Z}_1 = R_1 - j\,X_{C_1} = 2\,\Omega - j\,4\,\Omega$$
$$\mathbf{Z}_2 = R_2 = 1\,\Omega$$
$$\mathbf{Z}_3 = +j\,X_L = j\,5\,\Omega$$

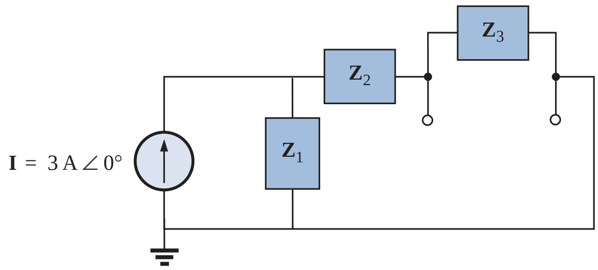

FIG. 18.67
Assigning the subscripted impedances to the network of Fig. 18.66.

Step 3 (Fig. 18.68):

$$\mathbf{Z}_N = \frac{\mathbf{Z}_3(\mathbf{Z}_1 + \mathbf{Z}_2)}{\mathbf{Z}_3 + (\mathbf{Z}_1 + \mathbf{Z}_2)}$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = 2\,\Omega - j\,4\,\Omega + 1\,\Omega = 3\,\Omega - j\,4\,\Omega = 5\,\Omega\,\angle -53.13^\circ$$

$$\mathbf{Z}_N = \frac{(5\,\Omega\,\angle 90^\circ)(5\,\Omega\,\angle -53.13^\circ)}{j\,5\,\Omega + 3\,\Omega - j\,4\,\Omega} = \frac{25\,\Omega\,\angle 36.87^\circ}{3 + j\,1}$$

$$= \frac{25\,\Omega\,\angle 36.87^\circ}{3.16\,\angle +18.43^\circ}$$

$$\mathbf{Z}_N = 7.91\,\Omega\,\angle 18.44^\circ = \mathbf{7.50\,\Omega + j\,2.50\,\Omega}$$

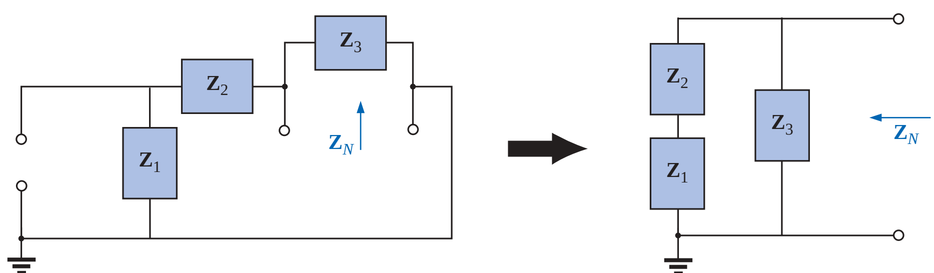

FIG. 18.68
Finding the Norton impedance for the network of Fig. 18.66.

Calculator Performing the above on the TI-86 calculator, we obtain the following:

```
((0,5)*((2,−4)+(1,0)))/((0,5)+((2,−4)+(1,0)))
  (7.500E0,2.500E0)
Ans ► Pol
  (7.906E0∠18.435E0)
```

CALC. 18.2

Step 4 (Fig. 18.69):

$$\mathbf{I}_N = \mathbf{I}_1 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(current divider rule)}$$

$$= \frac{(2\,\Omega - j\,4\,\Omega)(3\text{ A})}{3\,\Omega - j\,4\,\Omega} = \frac{6\text{ A} - j\,12\text{ A}}{5\,\angle -53.13^\circ} = \frac{13.4\text{ A}\,\angle -63.43^\circ}{5\,\angle -53.13^\circ}$$

$$\mathbf{I}_N = \mathbf{2.68\ A\,\angle -10.3^\circ}$$

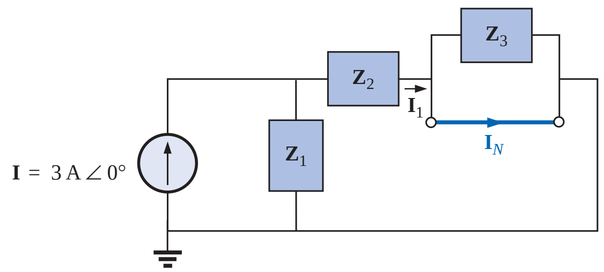

FIG. 18.69
Determining $\mathbf{I}_N$ for the network of Fig. 18.66.

Step 5: The Norton equivalent circuit is shown in Fig. 18.70.

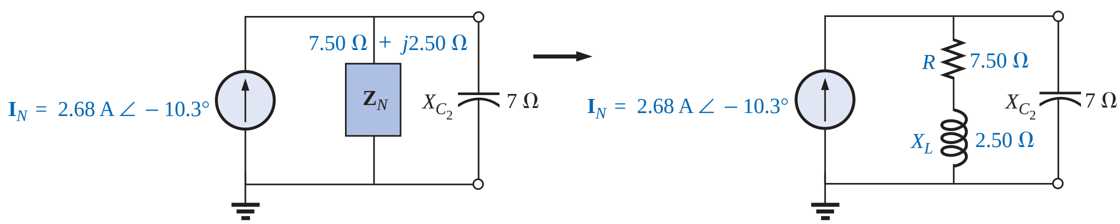

FIG. 18.70
The Norton equivalent circuit for the network of Fig. 18.66.

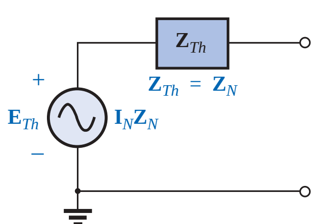

FIG. 18.71
Determining the Thévenin equivalent circuit for the Norton equivalent of Fig. 18.70.

EXAMPLE 18.16 Find the Thévenin equivalent circuit for the network external to the 7-Ω capacitive reactance in Fig. 18.66.

Solution: Using the conversion between sources (Fig. 18.71), we obtain

$$\mathbf{Z}_{Th} = \mathbf{Z}_N = \mathbf{7.50\ \Omega + j\,2.50\ \Omega}$$

$$\mathbf{E}_{Th} = \mathbf{I}_N\mathbf{Z}_N = (2.68\text{ A}\,\angle -10.3^\circ)(7.91\,\Omega\,\angle 18.44^\circ)$$

$$= \mathbf{21.2\ V\,\angle 8.14^\circ}$$

The Thévenin equivalent circuit is shown in Fig. 18.72.

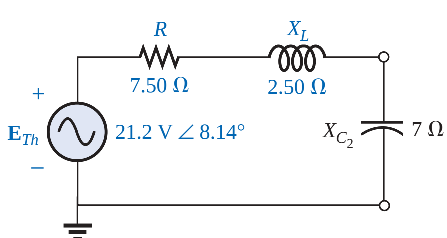

FIG. 18.72
The Thévenin equivalent circuit for the network of Fig. 18.66.

Dependent Sources

As stated for Thévenin's theorem, *dependent sources in which the controlling variable is not determined by the network* for which the Norton equivalent circuit is to be found do not alter the procedure outlined above.

For dependent sources of the other kind, one of the following procedures must be applied. Both of these procedures can also be applied to networks with any combination of independent sources and dependent sources not controlled by the network under investigation.

The Norton equivalent circuit appears in Fig. 18.73(a). In Fig. 18.73(b), we find that

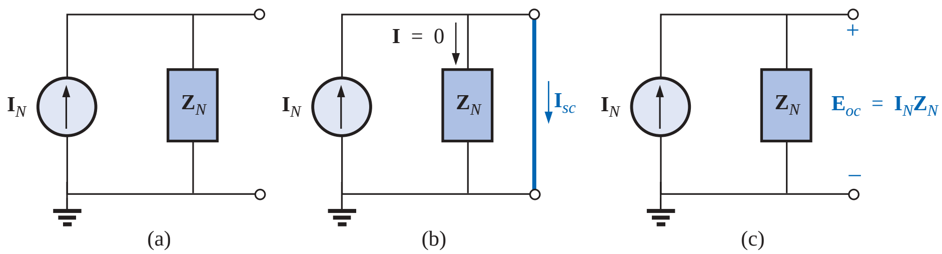

FIG. 18.73
Defining an alternative approach for determining $\mathbf{Z}_N$.

$$\boxed{\mathbf{I}_{sc} = \mathbf{I}_N} \tag{18.9}$$

and in Fig. 18.73(c) that

$$\mathbf{E}_{oc} = \mathbf{I}_N \mathbf{Z}_N$$

Or, rearranging, we have

$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_N}$$

and

$$\boxed{\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}}} \tag{18.10}$$

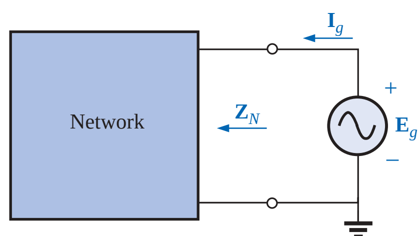

FIG. 18.74
Determining the Norton impedance using the approach $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{I}_g$.

The Norton impedance can also be determined by applying a source of voltage $\mathbf{E}_g$ to the terminals of interest and finding the resulting $\mathbf{I}_g$, as shown in Fig. 18.74. All independent sources and dependent sources not controlled by a variable in the network of interest are set to zero, and

$$\boxed{\mathbf{Z}_N = \frac{\mathbf{E}_g}{\mathbf{I}_g}} \tag{18.11}$$

For this latter approach, the Norton current is still determined by the short-circuit current.

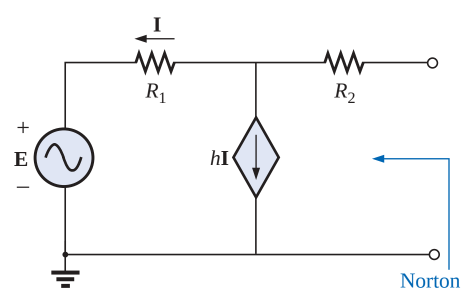

FIG. 18.75
Example 18.17.

EXAMPLE 18.17 Using each method described for dependent sources, find the Norton equivalent circuit for the network of Fig. 18.75.

Solution:

$\mathbf{I}_N$ For each method, $\mathbf{I}_N$ is determined in the same manner. From Fig. 18.76, using Kirchhoff's current law, we have

$$0 = \mathbf{I} + h\mathbf{I} + \mathbf{I}_{sc}$$

or

$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I}$$

Applying Kirchhoff's voltage law gives us

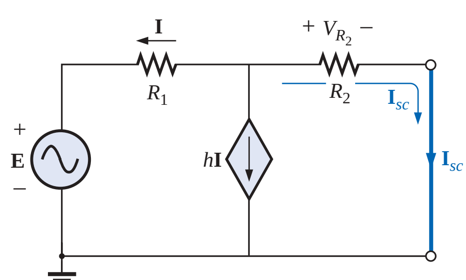

FIG. 18.76
Determining $\mathbf{I}_{sc}$ for the network of Fig. 18.75.

$$\mathbf{E} + \mathbf{I}R_1 - \mathbf{I}_{sc}R_2 = 0$$

and
$$\mathbf{I}R_1 = \mathbf{I}_{sc}R_2 - \mathbf{E}$$

or
$$\mathbf{I} = \frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}$$

so
$$\mathbf{I}_{sc} = -(1 + h)\mathbf{I} = -(1 + h)\left(\frac{\mathbf{I}_{sc}R_2 - \mathbf{E}}{R_1}\right)$$

or
$$R_1\mathbf{I}_{sc} = -(1 + h)\mathbf{I}_{sc}R_2 + (1 + h)\mathbf{E}$$
$$\mathbf{I}_{sc}[R_1 + (1 + h)R_2] = (1 + h)\mathbf{E}$$
$$\mathbf{I}_{sc} = \boldsymbol{\frac{(1 + h)\mathbf{E}}{R_1 + (1 + h)R_2}} = \mathbf{I}_N$$

$\mathbf{Z}_N$

Method 1: $\mathbf{E}_{oc}$ is determined from the network of Fig. 18.77. By Kirchhoff's current law,

$$0 = \mathbf{I} + h\mathbf{I} \quad \text{or} \quad \mathbf{I}(h + 1) = 0$$

For h, a positive constant $\mathbf{I}$ must equal zero to satisfy the above. Therefore,

$$\mathbf{I} = 0 \quad \text{and} \quad h\mathbf{I} = 0$$

and
$$\mathbf{E}_{oc} = \mathbf{E}$$

with
$$\mathbf{Z}_N = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}}{\dfrac{(1 + h)\mathbf{E}}{R_1 + (1 + h)R_2}} = \boldsymbol{\frac{R_1 + (1 + h)R_2}{(1 + h)}}$$

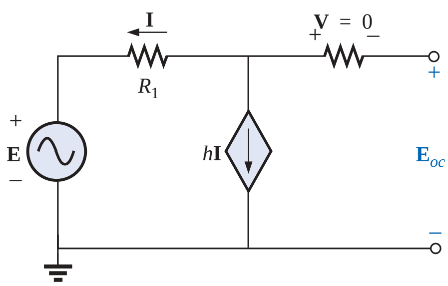

FIG. 18.77
Determining $\mathbf{E}_{oc}$ for the network of Fig. 18.75.

Method 2: Note Fig. 18.78. By Kirchhoff's current law,

$$\mathbf{I}_g = \mathbf{I} + h\mathbf{I} = (1 + h)\mathbf{I}$$

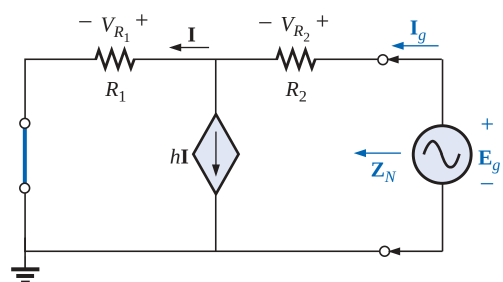

FIG. 18.78
Determining the Norton impedance using the approach $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{E}_g$.

By Kirchhoff's voltage law,

$$\mathbf{E}_g - \mathbf{I}_gR_2 - \mathbf{I}R_1 = 0$$

or
$$\mathbf{I} = \frac{\mathbf{E}_g - \mathbf{I}_gR_2}{R_1}$$

Substituting, we have

$$\mathbf{I}_g = (1 + h)\mathbf{I} = (1 + h)\left(\frac{\mathbf{E}_g - \mathbf{I}_gR_2}{R_1}\right)$$

and
$$\mathbf{I}_gR_1 = (1 + h)\mathbf{E}_g - (1 + h)\mathbf{I}_gR_2$$

so $$\mathbf{E}_g(1 + h) = \mathbf{I}_g[R_1 + (1 + h)R_2]$$

or $$\mathbf{Z}_N = \frac{\mathbf{E}_g}{\mathbf{I}_g} = \frac{\boldsymbol{R_1 + (1 + h)R_2}}{\boldsymbol{1 + h}}$$

which agrees with the above.

EXAMPLE 18.18 Find the Norton equivalent circuit for the network configuration of Fig. 18.56.

Solution: By source conversion,

$$\mathbf{I}_N = \frac{\mathbf{E}_{Th}}{\mathbf{Z}_{Th}} = \frac{\dfrac{-k_2R_2\mathbf{V}_i}{R_1 - k_1k_2R_2}}{\dfrac{R_1R_2}{R_1 - k_1k_2R_2}}$$

and $$\boxed{\mathbf{I}_N = \frac{-\boldsymbol{k_2}\mathbf{V}_i}{\boldsymbol{R_1}}} \qquad \textbf{(18.12)}$$

which is $\mathbf{I}_{sc}$ as determined in Example 18.13, and

$$\boxed{\mathbf{Z}_N = \mathbf{Z}_{Th} = \frac{\boldsymbol{R_2}}{1 - \dfrac{\boldsymbol{k_1k_2R_2}}{\boldsymbol{R_1}}}} \qquad \textbf{(18.13)}$$

For $k_1 \cong 0$, we have

$$\boxed{\mathbf{I}_N = \frac{-\boldsymbol{k_2}\mathbf{V}_i}{\boldsymbol{R_1}}} \qquad k_1 = 0 \qquad \textbf{(18.14)}$$

$$\boxed{\mathbf{Z}_N = \boldsymbol{R_2}} \qquad k_1 = 0 \qquad \textbf{(18.15)}$$

18.5 MAXIMUM POWER TRANSFER THEOREM

When applied to ac circuits, the **maximum power transfer theorem** states that

maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.

That is, for Fig. 18.79, for maximum power transfer to the load,

$$\boxed{Z_L = Z_{Th} \quad \text{and} \quad \theta_L = -\theta_{Th_Z}} \qquad \textbf{(18.16)}$$

or, in rectangular form,

$$\boxed{R_L = R_{Th} \quad \text{and} \quad \pm j\, X_{\text{load}} = \mp j\, X_{Th}} \qquad \textbf{(18.17)}$$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive, as indicated in Fig. 18.80:

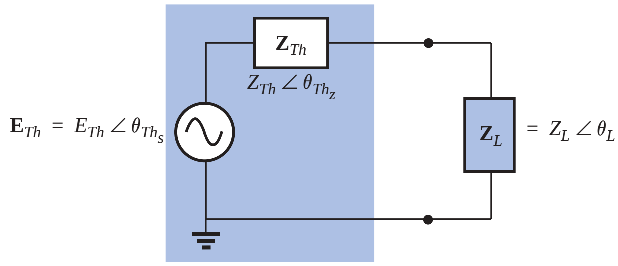

FIG. 18.79
Defining the conditions for maximum power transfer to a load.

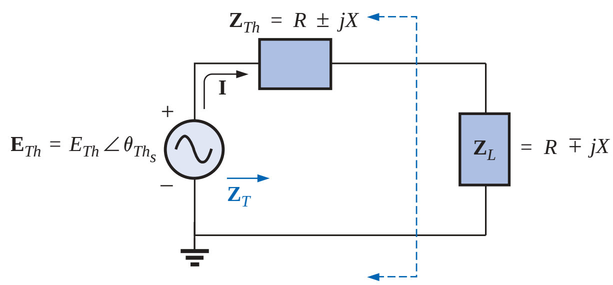

FIG. 18.80
Conditions for maximum power transfer to $\mathbf{Z}_L$.

$$\mathbf{Z}_T = (R \pm j\,X) + (R \mp j\,X)$$

and
$$\boxed{\mathbf{Z}_T = 2R} \qquad \textbf{(18.18)}$$

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1; that is,

$$\boxed{F_p = 1} \qquad \text{(maximum power transfer)} \qquad \textbf{(18.19)}$$

The magnitude of the current **I** of Fig. 18.80 is

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2R}$$

The maximum power to the load is

$$P_{\max} = I^2 R = \left(\frac{E_{Th}}{2R}\right)^2 R$$

and
$$\boxed{P_{\max} = \frac{E_{Th}^2}{4R}} \qquad \textbf{(18.20)}$$

EXAMPLE 18.19 Find the load impedance in Fig. 18.81 for maximum power to the load, and find the maximum power.

Solution: Determine $\mathbf{Z}_{Th}$ [Fig. 18.82(a)]:

$$\mathbf{Z}_1 = R - j\,X_C = 6\,\Omega - j\,8\,\Omega = 10\,\Omega \angle -53.13^\circ$$
$$\mathbf{Z}_2 = +j\,X_L = j\,8\,\Omega$$

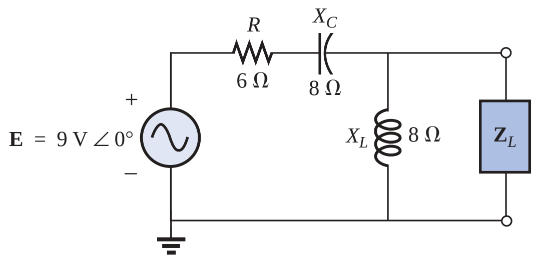

FIG. 18.81
Example 18.19.

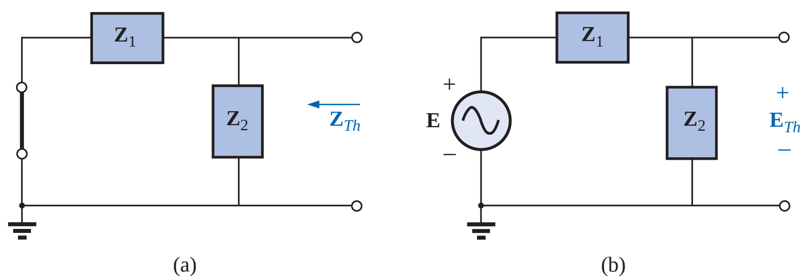

FIG. 18.82
Determining (a) $\mathbf{Z}_{Th}$ and (b) $\mathbf{E}_{Th}$ for the network external to the load in Fig. 18.81.

$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(10\ \Omega\ \angle -53.13^\circ)(8\ \Omega\ \angle 90^\circ)}{6\ \Omega - j\,8\ \Omega + j\,8\ \Omega} = \frac{80\ \Omega\ \angle 36.87^\circ}{6\ \angle 0^\circ}$$
$$= 13.33\ \Omega\ \angle 36.87^\circ = 10.66\ \Omega + j\,8\ \Omega$$

and $\qquad \mathbf{Z}_L = 13.3\ \Omega\ \angle -36.87^\circ = \mathbf{10.66\ \Omega - \mathit{j}\,8\ \Omega}$

To find the maximum power, we must first find $\mathbf{E}_{Th}$ [Fig. 18.82(b)], as follows:

$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_1} \qquad \text{(voltage divider rule)}$$
$$= \frac{(8\ \Omega\ \angle 90^\circ)(9\ \text{V}\ \angle 0^\circ)}{j\,8\ \Omega + 6\ \Omega - j\,8\ \Omega} = \frac{72\ \text{V}\ \angle 90^\circ}{6\ \angle 0^\circ} = 12\ \text{V}\ \angle 90^\circ$$

Then $\qquad P_{\max} = \dfrac{E_{Th}^2}{4R} = \dfrac{(12\ \text{V})^2}{4(10.66\ \Omega)} = \dfrac{144}{42.64} = \mathbf{3.38\ W}$

EXAMPLE 18.20 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

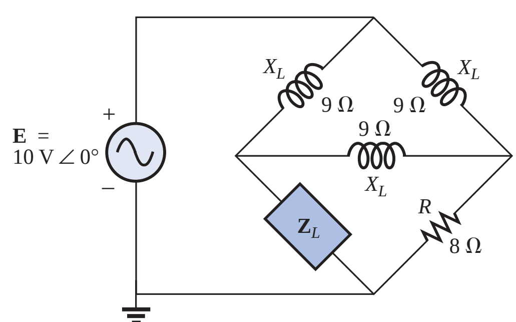

FIG. 18.83
Example 18.20.

Solution: First we must find $\mathbf{Z}_{Th}$ (Fig. 18.84).

$$\mathbf{Z}_1 = +j\,X_L = j\,9\ \Omega \qquad \mathbf{Z}_2 = R = 8\ \Omega$$

Converting from a Δ to a Y (Fig. 18.85), we have

$$\mathbf{Z}'_1 = \frac{\mathbf{Z}_1}{3} = j\,3\ \Omega \qquad \mathbf{Z}_2 = 8\ \Omega$$

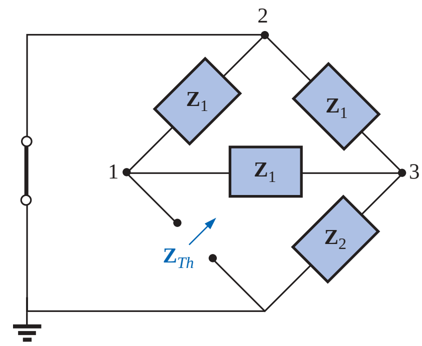

FIG. 18.84
Defining the subscripted impedances for the network of Fig. 18.83.

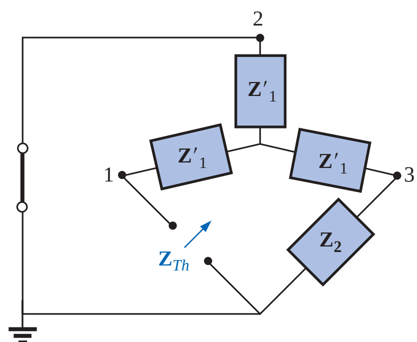

FIG. 18.85
Substituting the Y equivalent for the upper Δ configuration of Fig. 18.84.

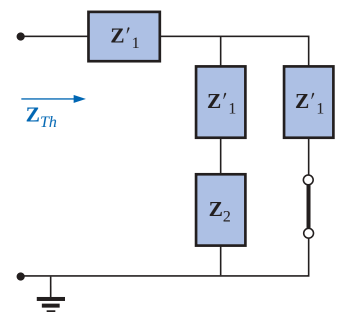

FIG. 18.86
Determining $\mathbf{Z}_{Th}$ for the network of Fig. 18.83.

The redrawn circuit (Fig. 18.86) shows

$$\begin{aligned}
\mathbf{Z}_{Th} &= \mathbf{Z}'_1 + \frac{\mathbf{Z}'_1(\mathbf{Z}'_1 + \mathbf{Z}_2)}{\mathbf{Z}'_1 + (\mathbf{Z}'_1 + \mathbf{Z}_2)} \\
&= j\,3\,\Omega + \frac{3\,\Omega\,\angle 90^\circ(j\,3\,\Omega + 8\,\Omega)}{j\,6\,\Omega + 8\,\Omega} \\
&= j\,3 + \frac{(3\,\angle 90^\circ)(8.54\,\angle 20.56^\circ)}{10\,\angle 36.87^\circ} \\
&= j\,3 + \frac{25.62\,\angle 110.56^\circ}{10\,\angle 36.87^\circ} = j\,3 + 2.56\,\angle 73.69^\circ \\
&= j\,3 + 0.72 + j\,2.46 \\
\mathbf{Z}_{Th} &= 0.72\,\Omega + j\,5.46\,\Omega
\end{aligned}$$

and $$\mathbf{Z}_L = \mathbf{0.72\,\Omega - \mathit{j}\,5.46\,\Omega}$$

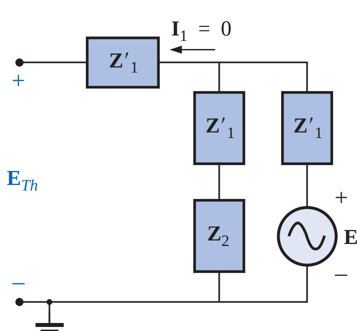

FIG. 18.87
Finding the Thévenin voltage for the network of Fig. 18.83.

For $\mathbf{E}_{Th}$, use the modified circuit of Fig. 18.87 with the voltage source replaced in its original position. Since $I_1 = 0$, $\mathbf{E}_{Th}$ is the voltage across the series impedance of $\mathbf{Z}'_1$ and $\mathbf{Z}_2$. Using the voltage divider rule gives us

$$\begin{aligned}
\mathbf{E}_{Th} &= \frac{(\mathbf{Z}'_1 + \mathbf{Z}_2)\mathbf{E}}{\mathbf{Z}'_1 + \mathbf{Z}_2 + \mathbf{Z}'_1} = \frac{(j\,3\,\Omega + 8\,\Omega)(10\text{ V}\,\angle 0^\circ)}{8\,\Omega + j\,6\,\Omega} \\
&= \frac{(8.54\,\angle 20.56^\circ)(10\text{ V}\,\angle 0^\circ)}{10\,\angle 36.87^\circ} \\
\mathbf{E}_{Th} &= 8.54\text{ V}\,\angle -16.31^\circ
\end{aligned}$$

and $$P_{max} = \frac{E_{Th}^2}{4R} = \frac{(8.54\text{ V})^2}{4(0.72\ \Omega)} = \frac{72.93}{2.88}\text{ W}$$
$$= \mathbf{25.32\ W}$$

If the load resistance is adjustable but the magnitude of the load reactance cannot be set equal to the magnitude of the Thévenin reactance, then the maximum power *that can be delivered* to the load will occur when the load reactance is made as close to the Thévenin reactance as possible and the load resistance is set to the following value:

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{load})^2}} \qquad \textbf{(18.21)}$$

where each reactance carries a positive sign if inductive and a negative sign if capacitive.

The power delivered will be determined by

$$\boxed{P = E_{Th}^2/4R_{av}} \qquad \textbf{(18.22)}$$

where $$\boxed{R_{av} = \frac{R_{Th} + R_L}{2}} \qquad \textbf{(18.23)}$$

The derivation of the above equations is given in Appendix G of the text. The following example demonstrates the use of the above.

EXAMPLE 18.21 For the network of Fig. 18.88:

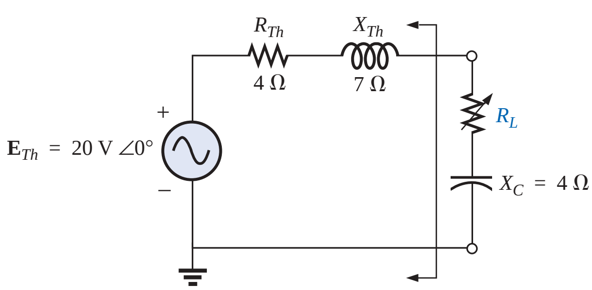

FIG. 18.88
Example 18.21.

a. Determine the value of R_L for maximum power to the load if the load reactance is fixed at 4 Ω.
b. Find the power delivered to the load under the conditions of part (a).
c. Find the maximum power to the load if the load reactance is made adjustable to any value, and compare the result to part (b) above.

Solutions:

a. Eq. (18.21): $$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_{load})^2}$$
$$= \sqrt{(4\ \Omega)^2 + (7\ \Omega - 4\ \Omega)^2}$$

$$= \sqrt{16 + 9} = \sqrt{25}$$
$$R_L = \mathbf{5\ \Omega}$$

b. Eq. (18.23): $R_{av} = \dfrac{R_{Th} + R_L}{2} = \dfrac{4\ \Omega + 5\ \Omega}{2}$
$$= \mathbf{4.5\ \Omega}$$

Eq. (18.22): $P = \dfrac{E_{Th}^2}{4R_{av}}$
$$= \frac{(20\text{ V})^2}{4(4.5\ \Omega)} = \frac{400}{18}\text{ W}$$
$$\cong \mathbf{22.22\ W}$$

c. For $\mathbf{Z}_L = 4\ \Omega - j\,7\ \Omega$,
$$P_{max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(20\text{ V})^2}{4(4\ \Omega)}$$
$$= \mathbf{25\ W}$$

exceeding the result of part (b) by 2.78 W.

18.6 SUBSTITUTION, RECIPROCITY, AND MILLMAN'S THEOREMS

As indicated in the introduction to this chapter, the **substitution** and **reciprocity theorems** and **Millman's theorem** will not be considered here in detail. A careful review of Chapter 9 will enable you to apply these theorems to sinusoidal ac networks with little difficulty. A number of problems in the use of these theorems appear in the problems section at the end of the chapter.

18.7 APPLICATIONS

Soldering Gun

Soldering and welding are two operations that are best performed by the application of heat that is unaffected by the thermal characteristics of the materials involved. In other words, the heat applied should not be sensitive to the changing parameters of the welding materials, the metals involved, or the welding conditions. The arc (a heavy current) established in the welding process should remain fixed in magnitude to ensure an even weld. This is best accomplished by ensuring a fixed current through the system even though the load characteristics may change—that is, by ensuring a constant current supply of sufficient amperage to establish the required arc for the welding equipment or even heating of the soldering iron tip. A further requirement for the soldering process is that the heat developed be sufficient to raise the solder to its melting point of about 800°F.

The soldering gun of Fig. 18.89(a) employs a unique approach to establishing a fixed current through the soldering tip. The soldering tip is actually part of a secondary winding of a transformer (Chapter 21) having only one turn as its secondary as shown in Fig. 18.89(b). Because of the heavy currents that will be established in this single-turn secondary, it is quite large in size to ensure that it can handle the current and to minimize its resistance level. The primary of the transformer

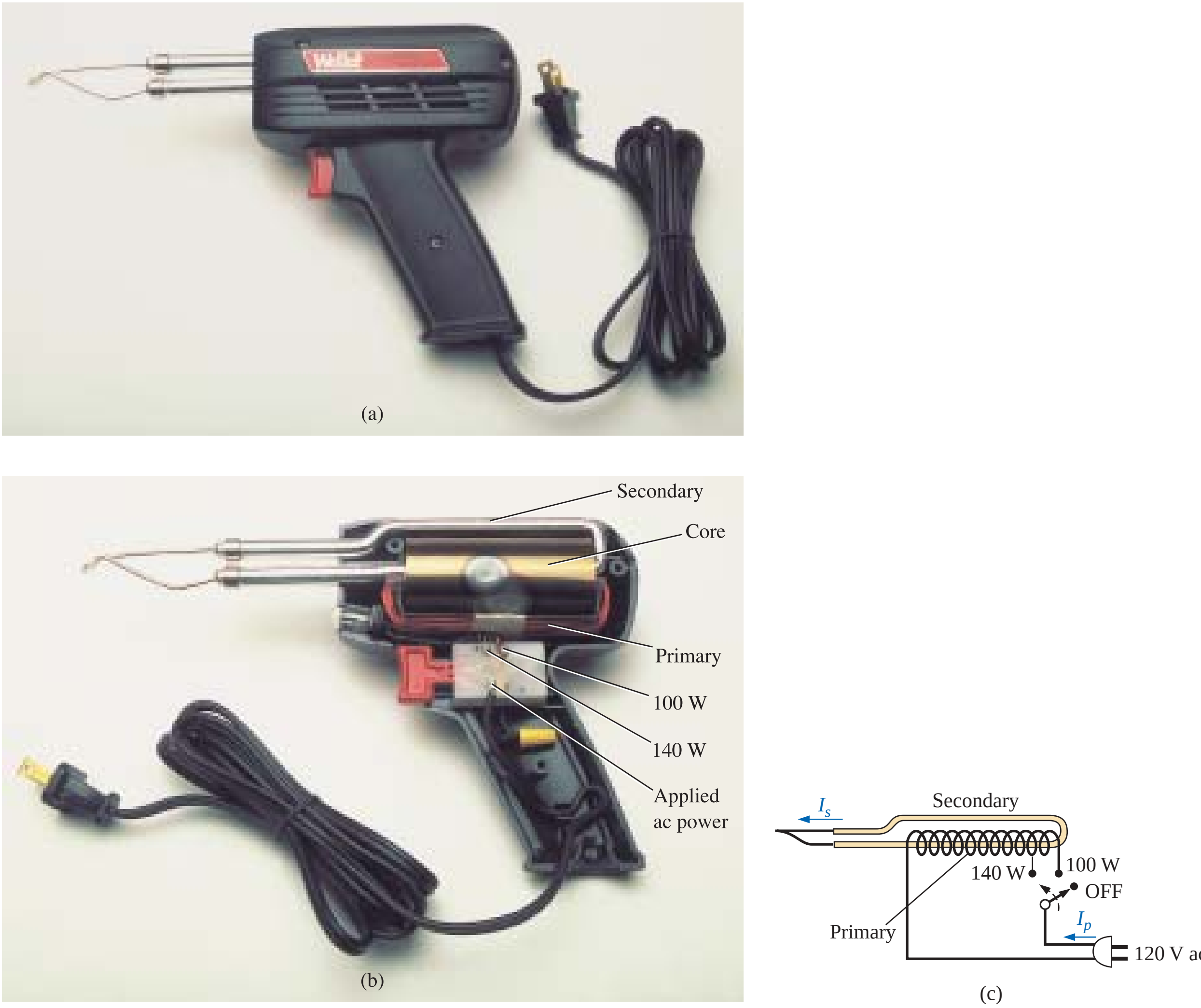

FIG. 18.89
Soldering gun: (a) appearance; (b) internal construction; (c) turns ratio control.

has many turns of thinner wire to establish the turns ratio necessary to establish the required current in the secondary. The Universal® unit of Fig. 18.89 is rated 140 W/100 W, indicating that it has two levels of power controlled by the trigger. As you pull the trigger, the first setting will be at 140 W, and a fully depressed trigger will provide 100 W of power. The inductance of the primary is 285 mH at the 140-W setting and 380 mH at the 100-W setting, indicating that the switch controls how many windings of the primary will be part of the transformer action for each wattage rating, as shown in Fig. 18.89(c). Since inductance is a direct function of the number of turns, the 140-W setting has fewer turns than the 100-W setting. The dc resistance of the primary was found to be about 11.2 Ω for the 140-W setting and 12.8 Ω for the 100-W setting, which makes sense also since more turns will require a longer wire and the resistance should increase accordingly.

Under rated operating conditions, the primary current for each setting can be determined using Ohm's law in the following manner:

For 140 W,

$$I_p = \frac{P}{V_p} = \frac{140\text{ W}}{120\text{ V}} = \mathbf{1.17\ A}$$

For 100 W,

$$I_p = \frac{P}{V_p} = \frac{100\text{ W}}{120\text{ V}} = \mathbf{0.83\ A}$$

As expected, the current demand is more for the 140-W setting than for the 100-W setting. Using the measured values of input inductance and resistance for the 140-W setting, the equivalent circuit of Fig. 18.90(a) will result. Using the applied 60 Hz to determine the reactance of the coil and then determining the total impedance seen by the primary will result in the following for the source current:

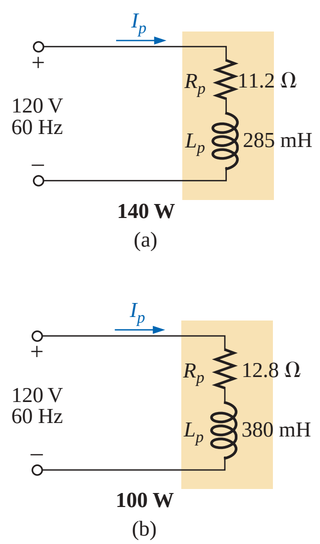

FIG. 18.90
Equivalent circuits for the soldering iron of Fig. 18.89(a): (a) at 140-W setting; (b) at 100-W setting.

$$X_L = 2\pi f L = 2\pi(60\text{ Hz})(285\text{ mH}) = 107.44\ \Omega$$

and $$\mathbf{Z}_T = R + j\,X_L = 11.2\ \Omega + j\,107.44\ \Omega = 108.02\ \Omega\ \angle 84.05°$$

so that $$|I_p| = \left|\frac{E}{Z_T}\right| = \frac{120\text{ V}}{108.02\ \Omega} = \mathbf{1.11\ A}$$

which is a close match with the rated level.

For the 100-W level of Fig. 18.90(b), the following analysis would result:

$$X_L = 2\pi f L = 2\pi(60\text{ Hz})(380\text{ mH}) = 143.26\ \Omega$$

and $$\mathbf{Z}_T = R + j\,X_L = 12.8\ \Omega + j\,143.26\ \Omega = 143.83\ \Omega\ \angle 84.89°$$

so that $$|I_p| = \left|\frac{E}{Z_T}\right| = \frac{120\text{ V}}{143.83\ \Omega} = \mathbf{0.83\ A}$$

which is a match to hundredths place with the value calculated from rated conditions.

Removing the tip and measuring the primary and secondary voltages resulted in 120 V/0.38 V for the 140-W setting and 120 V/0.31 V for the 100-W setting, respectively. Since the voltages of a transformer are directly related to the turns ratio, the number of turns in the primary (N_p) to that of the secondary (N_s) can be estimated by the following for each setting:

For 140 W,

$$\frac{N_p}{N_s} = \frac{120\text{ V}}{0.38\text{ V}} \cong \mathbf{316}$$

For 100 W,

$$\frac{N_p}{N_s} = \frac{120\text{ V}}{0.31\text{ V}} \cong \mathbf{387}$$

Looking at the photograph of Fig. 18.89(b), it would certainly appear that there are 300 or more turns in the primary winding.

The currents of a transformer are related by the turns ratio in the following manner, permitting a calculation of the secondary currents for each setting:

For 140 W,

$$I_s = \frac{N_p}{N_s} I_p = 316(1.17\text{ A}) \cong \mathbf{370\ A}$$

For 100 W,

$$I_s = \frac{N_p}{N_s} I_p = 387(0.83\ \text{A}) \cong \mathbf{321\ A}$$

Quite clearly, the secondary current is much higher for the 140-W setting. The resulting current levels are probably higher than you might have expected, but keep in mind that the above analysis does not include the effect of the reflected impedance from the secondary to the primary that will reduce the primary current level (to be discussed in Chapter 21). In addition, as the soldering tip heats up, its resistance increases, further reducing the resulting current levels. Using an Amp-Clamp®, the current in the secondary was found to exceed 300 A when the power was first applied and the soldering tip was cold. However, as the tip heated up because of the high current levels, the current through the primary dropped to about 215 A for the 140-W setting and to 180 A for the 100-W setting. These high currents are part of the reason that the lifetime of most soldering tips on soldering guns is about 20 hours. Eventually, the tip will simply begin to melt. Using these levels of current and the given power rating, the resistance of the secondary can be approximated as follows:

For 140 W,

$$R = \frac{P}{I^2} = \frac{140\ \text{W}}{(215\ \text{A})^2} \cong \mathbf{3\ m\Omega}$$

For 100 W,

$$R = \frac{P}{I^2} = \frac{100\ \text{W}}{(180\ \text{A})^2} \cong \mathbf{3\ m\Omega}$$

which is as low as expected when you consider the cross-sectional area of the secondary and the fact that the tip is a short section of low-resistance, tin-plated copper.

One of the obvious advantages of the soldering gun versus the iron is that the iron is off when you release the trigger, thus reducing energy costs and extending the life of the tip. Applying dc current rather than ac to develop a constant current would be impractical because the high current demand would require a series of large batteries in parallel.

The above investigation was particularly interesting because of the manner in which the constant current characteristic was established, the levels of current established, and the excellent manner in which some of the theory introduced in the text was verified.

Electronic Systems

One of the blessings in the analysis of electronic systems is that the superposition theorem can be applied so that the dc analysis and ac analysis can be performed separately. The analysis of the dc system will affect the ac response, but the analysis of each is a distinct, separate process. Even though electronic systems have not been investigated in this text, a number of important points can be made in the description to follow that support some of the theory presented in this and recent chapters, so inclusion of this description is totally valid at this point. Consider the network of Fig. 18.91 with a transistor power amplifier, an 8-Ω speaker as the load, and a source with an internal resistance of 800 Ω. Note that each component of the design was isolated by a color box

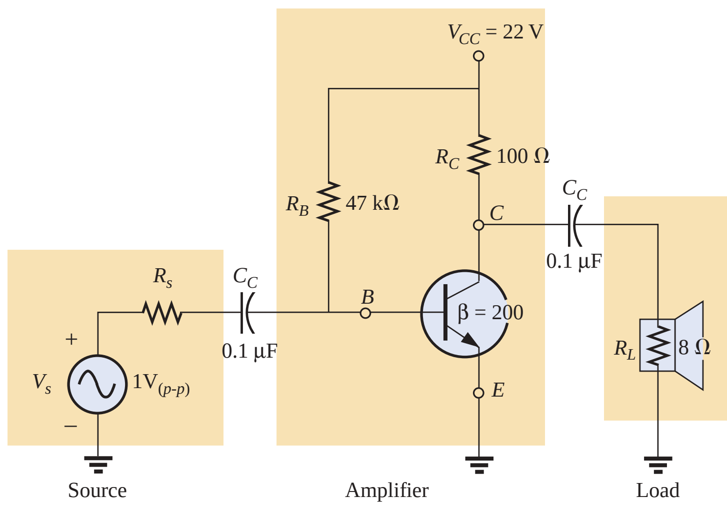

FIG. 18.91
Transistor amplifier.

to emphasize the fact that each component must be carefully weighed in any good design.

As mentioned above, the analysis can be separated into a dc and an ac component. For the dc analysis the two capacitors can be replaced by an open-circuit equivalent (Chapter 10), resulting in an isolation of the amplifier network as shown in Fig. 18.92. Given the fact that V_{BE} will be about 0.7 V dc for any operating transistor, the base current I_B can be found as follows using Kirchhoff's voltage law:

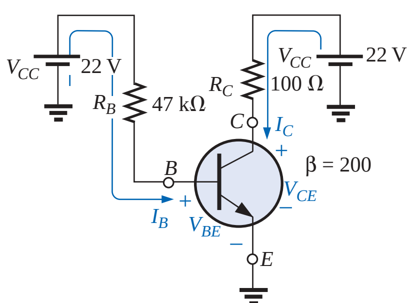

FIG. 18.92
dc equivalent of the transistor network of Fig. 18.91.

$$I_B = \frac{V_{R_B}}{R_B} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{22\text{ V} - 0.7\text{ V}}{47\text{ k}\Omega} = \mathbf{453.2\ \mu A}$$

For transistors, the collector current I_C is related to the base current by $I_C = \beta I_B$, and

$$I_C = \beta I_B = (200)(453.2\ \mu\text{A}) = \mathbf{90.64\ mA}$$

Finally, through Kirchhoff's voltage law, the collector voltage (also the collector-to-emitter voltage since the emitter is grounded) can be determined as follows:

$$V_C = V_{CE} = V_{CC} - I_C R_C = 22\text{ V} - (90.64\text{ mA})(100\ \Omega) = \mathbf{12.94\ V}$$

For the dc analysis, therefore,

$$I_B = \mathbf{453.2\ \mu A} \qquad I_C = \mathbf{90.64\ mA} \qquad V_{CE} = \mathbf{12.94\ V}$$

which will define a point of dc operation for the transistor. This is an important aspect of electronic design since the dc operating point will have an effect on the ac gain of the network.

Now, using superposition, we can analyze the network from an ac viewpoint by setting all dc sources to zero (replaced by ground connections) and replacing both capacitors by short circuits as shown in Fig. 18.93. Substituting the short-circuit equivalent for the capacitors is valid because at 10 kHz (the midrange for human hearing response), the reactance of the capacitor is determined by $X_C = 1/2\pi fC = 15.92\ \Omega$ which can be ignored when compared to the series resistors at the source and load. In other words, the capacitor has played the important role of isolating the amplifier for the dc response and completing the network for the ac response.

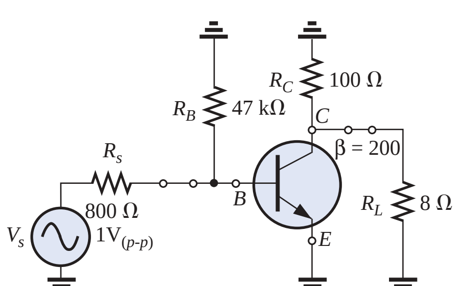

FIG. 18.93
ac equivalent of the transistor network of Fig. 18.91.

Redrawing the network as shown in Fig. 18.94(a) will permit an ac investigation of its reponse. The transistor has now been replaced by an equivalent network that will represent the behavior of the device. This process will be covered in detail in your basic electronics courses. This transistor configuration has an input impedance of 200 Ω and a current source whose magnitude is sensitive to the base current in the input circuit and to the amplifying factor for this transistor of 200. The 47-kΩ resistor in parallel with the 200-Ω input impedance of the transistor can be ignored, so the input current I_i and base current I_b are determined by

$$I_i \cong I_b = \frac{V_s}{R_s + R_i} = \frac{1\text{ V }(p\text{-}p)}{800\ \Omega + 200\ \Omega} = \frac{1\text{ V }(p\text{-}p)}{1\text{ k}\Omega} = 1\text{ mA }(p\text{-}p)$$

The collector current I_C is then

$$I_C = \beta I_b = (200)(1\text{ mA }(p\text{-}p)) = 200\text{ mA }(p\text{-}p)$$

and the current to the speaker is determined by the current divider rule as follows:

$$I_L = \frac{100\ \Omega(I_C)}{100\ \Omega + 8\ \Omega} = 0.926 I_C = 0.926(200\text{ mA }(p\text{-}p))$$
$$= 185.2\text{ mA }(p\text{-}p)$$

with the voltage across the speaker being

$$V_L = -I_L R_L = -(185.2\text{ mA }(p\text{-}p))(8\ \Omega) = -1.48\text{ V}$$

The power to the speaker is then determined as follows:

$$P_{\text{speaker}} = V_{L_{\text{rms}}} \cdot I_{L_{\text{rms}}} = \frac{(V_{L(p\text{-}p)})(I_{L(p\text{-}p)})}{8} = \frac{(1.48\text{ V})(185.2\text{ mA }(p\text{-}p))}{8}$$
$$= \mathbf{34.26\ mW}$$

which is relatively low. It would initially appear that the above was a good design for distribution of power to the speaker because a majority

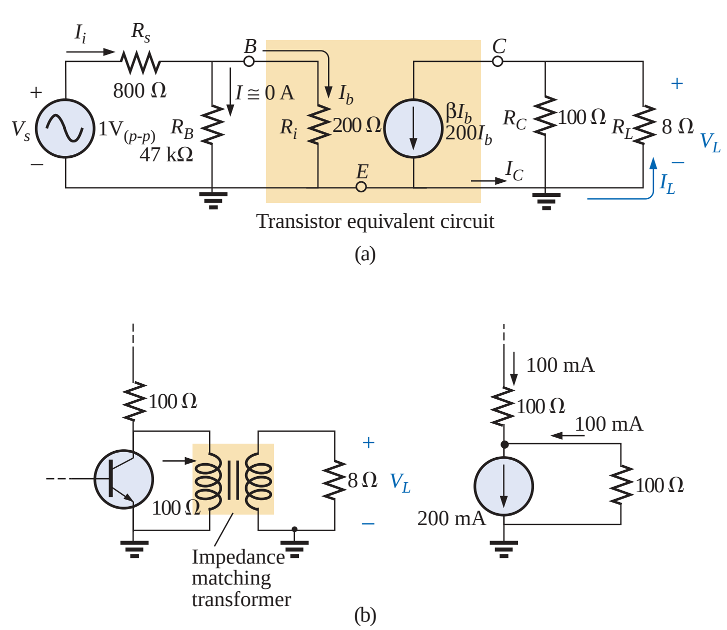

FIG. 18.94
(a) Network of Fig. 18.93 following the substitution of the transistor equivalent network; (b) effect of the matching transformer.

of the collector current went to the speaker. However, one must always keep in mind that power is the product of voltage and current. A high current with a very low voltage will result in a lower power level. In this case, the voltage level is too low. However, if we introduce a matching transformer that makes the 8-Ω resistive load "look like" 100 Ω as shown in Fig. 18.94(b), establishing maximum power conditions, the current to the load will drop to half of the previous amount because current splits through equal resistors. But the voltage across the load will increase to

$$V_L = I_L R_L = (100 \text{ mA } (p\text{-}p))(100\ \Omega) = 10 \text{ V } (p\text{-}p)$$

and the power level to

$$P_{\text{speaker}} = \frac{(V_{L(p\text{-}p)})(I_{L(p\text{-}p)})}{8} = \frac{(10 \text{ V})(100 \text{ mA})}{8} = \mathbf{125\ mW}$$

which is 3.6 times the gain without the matching transformer.

For the 100-Ω load, the dc conditions are unaffected due to the isolation of the capacitor C_C, and the voltage at the collector is 12.94 V as shown in Fig. 18.95(a). For the ac response with a 100-Ω load, the output voltage as determined above will be 10 V peak-to-peak (5 V peak) as shown in Fig. 18.95(b). Note the out-of-phase relationship with the input due to the opposite polarity of V_L. The full response at the collector terminal of the transistor can then be drawn by superimposing the ac response on the dc response as shown in Fig. 18.95(c) (another application of the superposition theorem). In other words, the dc level simply shifts the ac waveform up or down and does not disturb its shape. The peak-to-peak value remains the same, and the phase relationship is unaltered. The total waveform at the load will include only the ac response of Fig. 18.95(b) since the dc component has been blocked out by the capacitor.

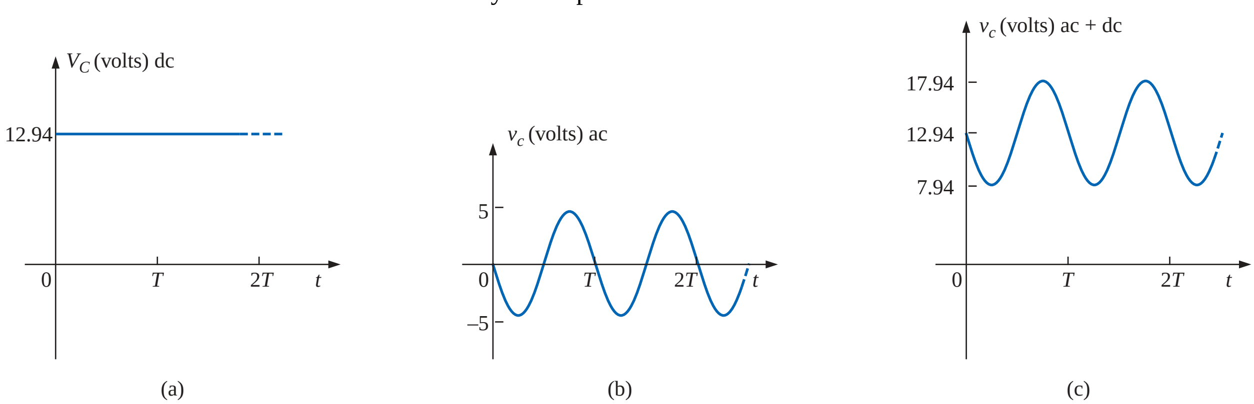

FIG. 18.95
Collector voltage for the network of Fig. 18.91: (a) dc; (b) ac; (c) dc and ac.

The voltage at the source will appear as shown in Fig. 18.96(a), while the voltage at the base of the transistor will appear as shown in Fig. 18.96(b) because of the presence of the dc component.

A number of important concepts were presented in the above example, with some probably leaving a question or two because of your lack of experience with transistors. However, if nothing else is evident from the above example, it should be the power of the superposition theorem to permit an isolation of the dc and ac responses and the ability to combine both if the total response is desired.

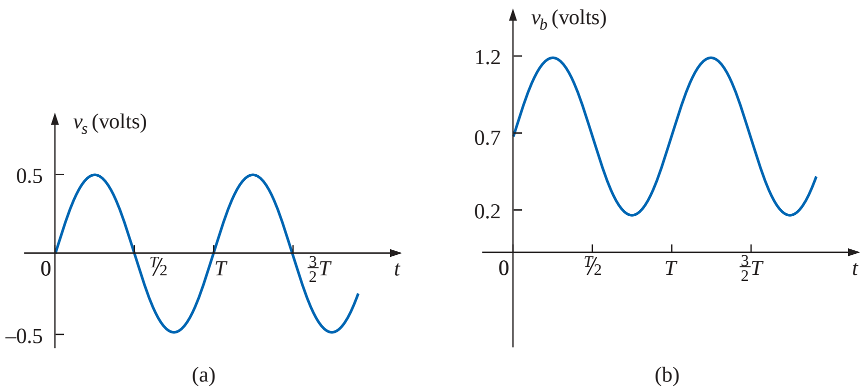

FIG. 18.96
Applied signal: (a) at the source; (b) at the base of the transistor.

18.8 COMPUTER ANALYSIS

PSpice

Superposition The analysis will begin with the network of Fig. 18.12 from Example 18.4 because it has both an ac and a dc source. You will find that it is not necessary to specify an analysis for each, even though one is essentially an ac sweep and the other is a bias point calculation. When **AC Sweep** is selected, the program will automatically perform the bias calculations and display the results in the output file.

The resulting schematic appears in Fig. 18.97 with **VSIN** and **VDC** as the **SOURCE** voltages. The placement of all the *R-L-C* elements and

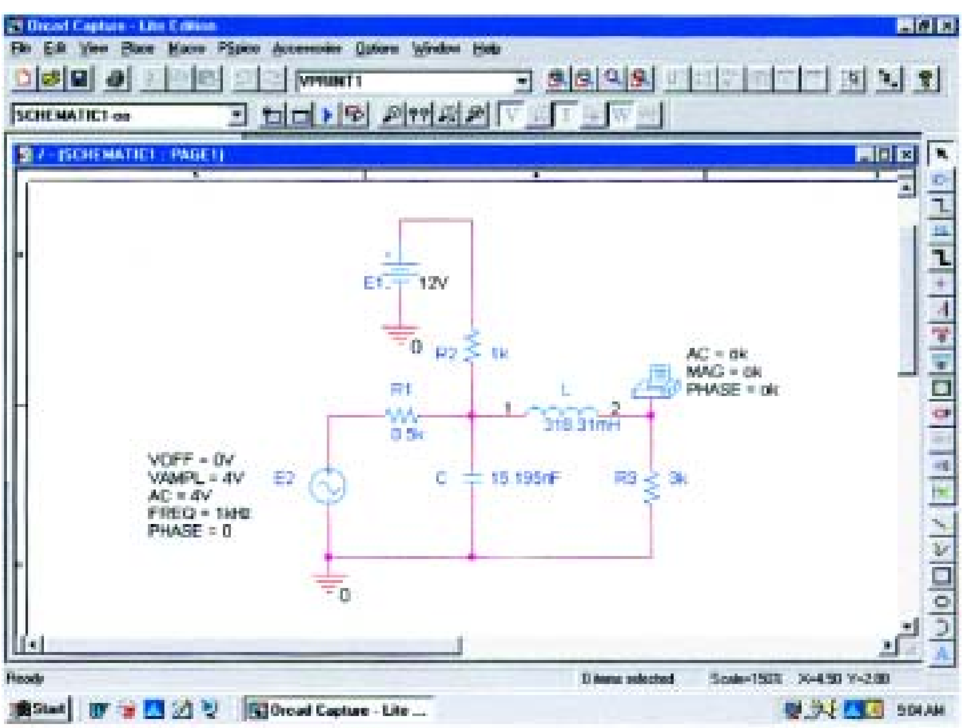

FIG. 18.97
Using PSpice to apply superposition to the network of Fig. 18.12.

the dc source should be quite straightforward at this point. For the ac source, be sure to double-click on the source symbol to obtain the **Property Editor** dialog box. Then set **AC** to 4 V, **FREQ** to 1 kHz, **PHASE** to 0°, **VAMPL** to 4 V, and **VOFF** to 0 V. In each case choose **Name and Value** under the **Display** heading so that we have a review of the parameters on the screen. Also, be sure to **Apply** before exiting the dialog box. Obtain the **VPRINT1** option from the **SPECIAL** library, place it as shown, and then double-click to obtain its **Property Editor**. The parameters **AC, MAG,** and **PHASE** must then recieve the **OK** listing, and **Name and Value** must be applied to each under **Display** before you choose **Apply** and **OK**. The network is then ready for simulation.

After you have selected the **New Simulation Profile** icon, the **New Simulation** dialog box will appear in which **SuperpositionAC** is entered as the **Name**. Following the selection of **Create**, the **Simulation Settings** dialog box will appear in which **AC Sweep/Noise** is selected. The **Start** and **End Frequencies** are both set at 1 kHz, and 1 is entered for the **Points/Decade** request. Click **OK**, and then select the **Run PSpice** key; the **SCHEMATIC1** screen will result with an axis extending from 0.5 kHz to 1.5 kHz. Through the sequence **Trace-Add Trace-V(R3:1)-OK**, the plot point appearing in the bottom of Fig. 18.98 will result. Its value is slightly above the 2-V level and could be read as 2.05 V which compares very nicely with the hand-calculated solution of 2.06 V. The phase angle can be obtained from **Plot-Add Plot to Window-Trace-Add Trace-P(V(R3:1))** to obtain a phase angle close to −33°. Additional accuracy can be added to the phase plot through the sequence **Plot-Axis Settings-Y Axis-User Defined −40d to −30d-OK**, resulting in the −32.5° reading of Fig. 18.98— again, very close to the hand cal-

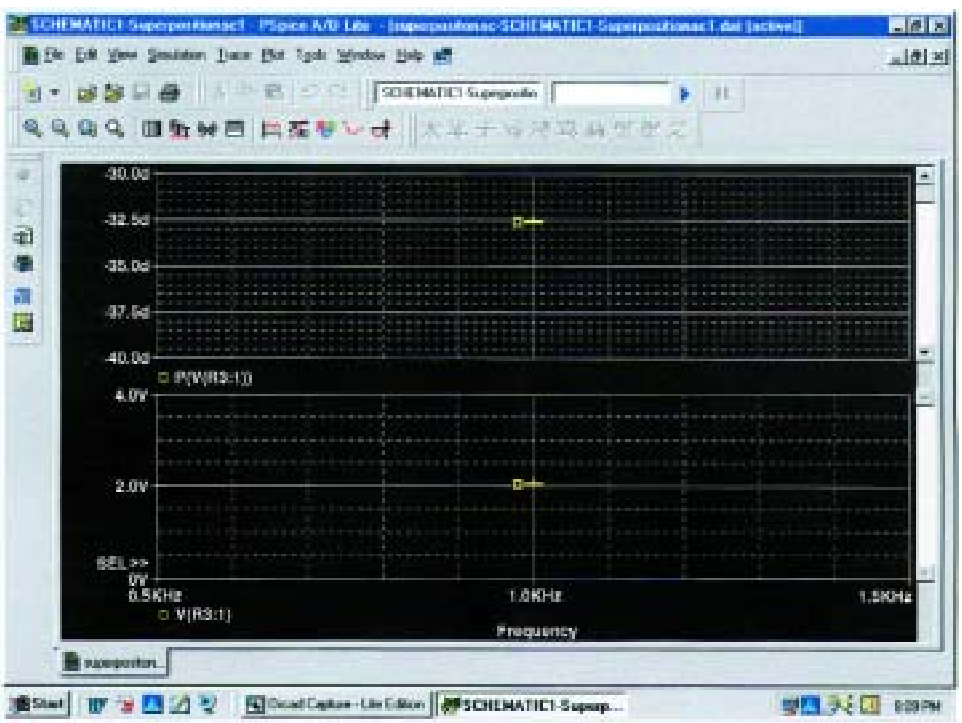

FIG. 18.98
The output results from the simulation of the network of Fig. 18.97.

culation of −32.74° of Example 18.4. Now this solution is fine for the ac signal, but it tells us nothing about the dc component.

By exiting the **SCHEMATIC1** screen, we obtain the **Orcad Capture** window on which **PSpice** can be selected followed by **View Output File**. The result is the printout of Fig. 18.99 which has both the dc and the ac solutions. The **SMALL SIGNAL BIAS SOLUTION** includes the nodes of the network and their dc levels. The node numbers are defined under the netlist starting on line 30. In particular, note the dc level of 3.6 V at node **N00676** which is at the top of resistor R_3 in Fig. 18.97. Also note that the dc level of both ends of the inductor is the same value because of the substitution of a short-circuit equivalent for the inductor for dc analysis. The ac solution appears under the **AC ANALYSIS** heading as 2.06 V at −32.66°, which again is a great verification of the results of Example 18.4.

```
49:
50:  ** Profile: "SCHEMATIC1-Superpositionac1"  [ C:\PSpice\superpositonac-SCHEMATIC
    1-Superpositionac1.sim ]
51:
52:
53:  ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
54:
55:
56: ******************************************************************************
57:
58:
59:
60:  NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
61:
62:
63: (N00530)    0.0000 (N00596)    3.6000 (N00623)   12.0000 (N00676)    3.6000
64:
65:
66:
67:
68:     VOLTAGE SOURCE CURRENTS
69:     NAME          CURRENT
70:
71:     V_E1         -8.400E-03
72:     V_E2          7.200E-03
73:
74:     TOTAL POWER DISSIPATION   1.01E-01  WATTS
75:
76:
77: **** 07/01/01 20:07:27 ************** PSpice Lite (Mar 2000) *****************
78:
79:  ** Profile: "SCHEMATIC1-Superpositionac1"  [ C:\PSpice\superpositonac-SCHEMATIC
    1-Superpositionac1.sim ]
80:
81:
82:  ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C
83:
84:
85: ******************************************************************************
86:
87:
88:
89:   FREQ        VM(N00676)  VP(N00676)
90:
91:
92:    1.000E+03   2.060E+00  -3.266E+01
93:
```

FIG. 18.99

*The output file for the dc **(SMALL SIGNAL BIAS SOLUTION)** and **AC ANALYSIS** for the network of Fig. 18.97.*

Finally, if a plot of the voltage across resistor R_3 is desired, we must return to the **New Simulation Profile** and enter a new **Name** such as **SuperpositionAC1** followed by **Create fill** in the **Simulation Profile** dialog box. This time, however, we will choose the **Time Domain(Transient)** option so that we can obtain a plot against time. The fact that the source has a defined frequency of 1 kHz will tell the program which frequency to apply. The **Run to time** will be 5 ms, resulting in a five-cycle display of the 1-kHz signal. The **Start saving data after** will remain at 0 s, and the **Maximum step size** will be 5 ms/1000 = 5 μs. Click **OK**, and select the **Run PSpice** icon; the **SCHEMATIC1** screen will result again. This time **Trace-Add Trace-V(R3:1)-OK** will result in the plot of Fig. 18.100 which clearly shows a dc level of 3.6 V. Setting a cursor at $t = 0$ s (**A1**) will result in 3.6 V in the **Probe Cursor** display box. Placing the other cursor at the peak value at 2.34 ms **(A2)** will result in a peak value of about 5.66 V. The difference between the peak and the dc level provides the peak value of the ac signal and is listed as 2.06 V in the same **Probe Cursor** display box. A variety of options have now been introduced to find a particular voltage or current in a network with both dc and ac sources. It is certainly satisfying that they all verify our theoretical solution.

Thévenin's Theorem The next application will parallel the methods employed to determine the Thévenin equivalent circuit for dc circuits. The network of Fig. 18.28 will appear as shown in Fig. 18.101 when the open-circuit Thévenin voltage is to be determined. The open circuit is simulated by using a resistor of 1 T (1 million MΩ). The resistor is necessary to establish a connection between the right side of inductor L_2 and ground—nodes cannot be left floating for Orcad simulations.

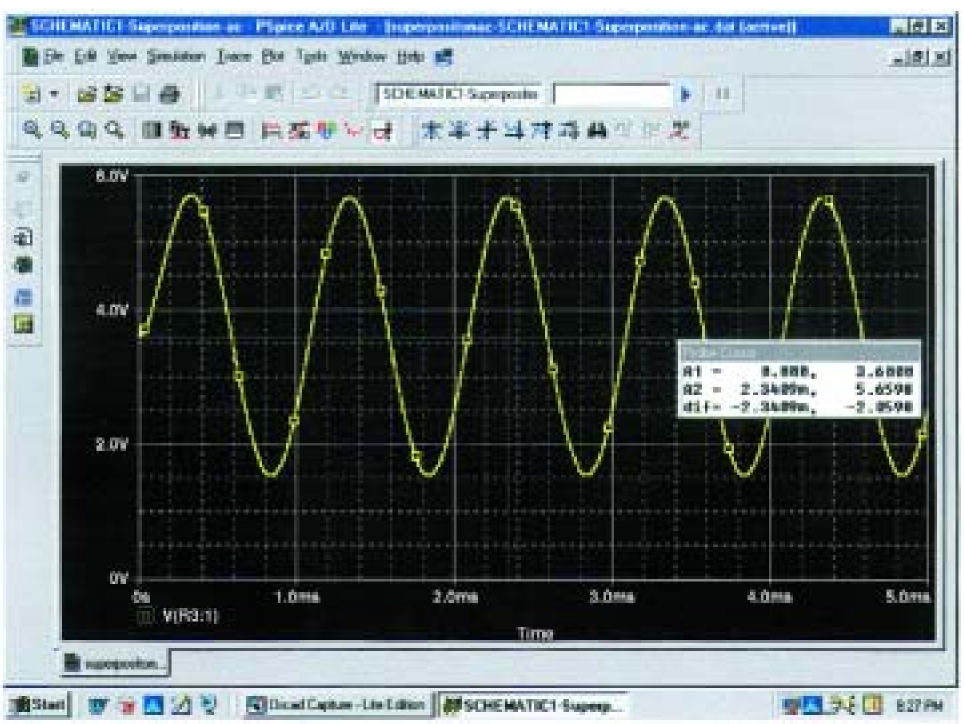

FIG. 18.100
Using PSpice to display the voltage across R_3 for the network of Fig. 18.97.

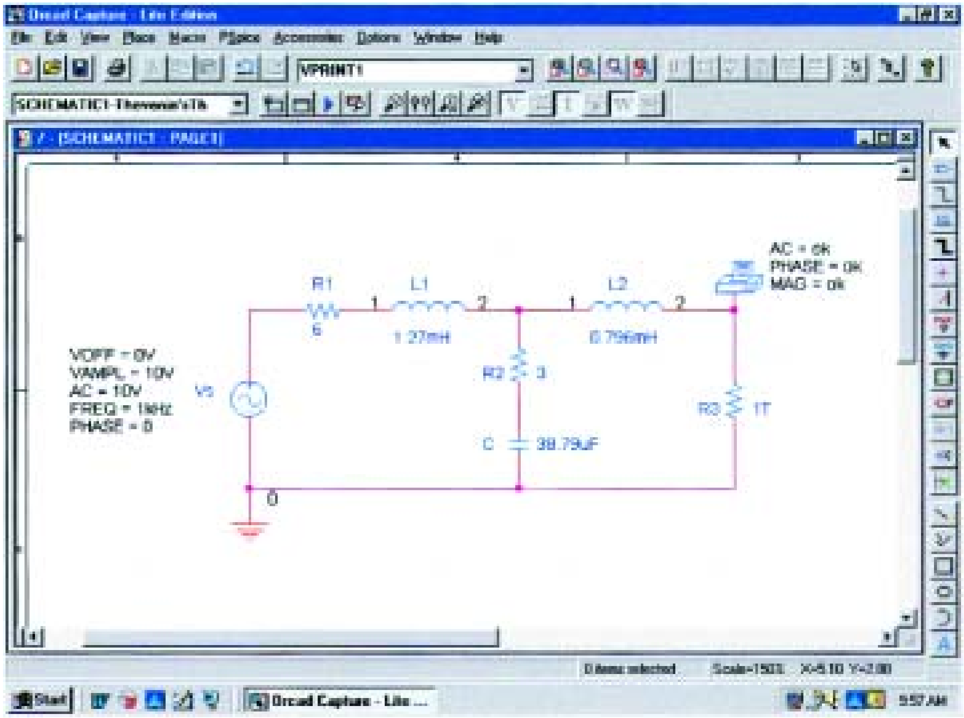

FIG. 18.101
Using PSpice to determine the open-circuit Thévenin voltage.

Since the magnitude and the angle of the voltage are required, **VPRINT1** is introduced as shown in Fig 18.101. The simulation was an **AC Sweep** simulation at 1 kHz, and when the **Orcad Capture** window was obtained, the results appearing in Fig. 18.102 were taken from the listing resulting from the **PSpice-View Output File**. The magnitude of the Thévenin voltage is 5.187 V to compare with the 5.08 V of Example 18.8, while the phase angle is −77.13° to compare with the −77.09° of the same example—excellent results.

```

 ** Profile: "SCHEMATIC1-Thevenin'sTh"  [ C:\PSpice\thevenin'sth-SCHEMATIC1-Thevenin'sTh.si
m ]


 ****     AC ANALYSIS                         TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ        VM(N00433)  VP(N00433)


   1.000E+03   5.187E+00  -7.713E+01

```

FIG. 18.102
The output file for the open-circuit Thévenin voltage for the network of Fig. 18.101.

Next, the short-circuit current will be determined using **IPRINT** as shown in Fig. 18.103, to permit a determination of the Thévenin impedance. The resistance R_{coil} of 1 $\mu\Omega$ had to be introduced because inductors cannot be treated as ideal elements when using PSpice; they must all show some series internal resistance. Note that the short-circuit current will pass directly through the printer symbol for **IPRINT.** Incidentally, there is no need to exit the **SCHEMATIC1** developed above to determine the Thévenin voltage. Simply delete **VPRINT** and **R3**, and insert **IPRINT.** Then run a new simulation to obtain the results of Fig. 18.104. The magnitude of the short-circuit current is 0.936 A at an angle of $-108°$. The Thévenin impedance is then defined by

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} = \frac{5.187\text{ V} \angle -77.13°}{0.936\text{ A} \angle -108.0°} = 5.54\ \Omega \angle 30.87°$$

which is an excellent match with 5.49 Ω $\angle 32.36°$ obtained in Example 18.8.

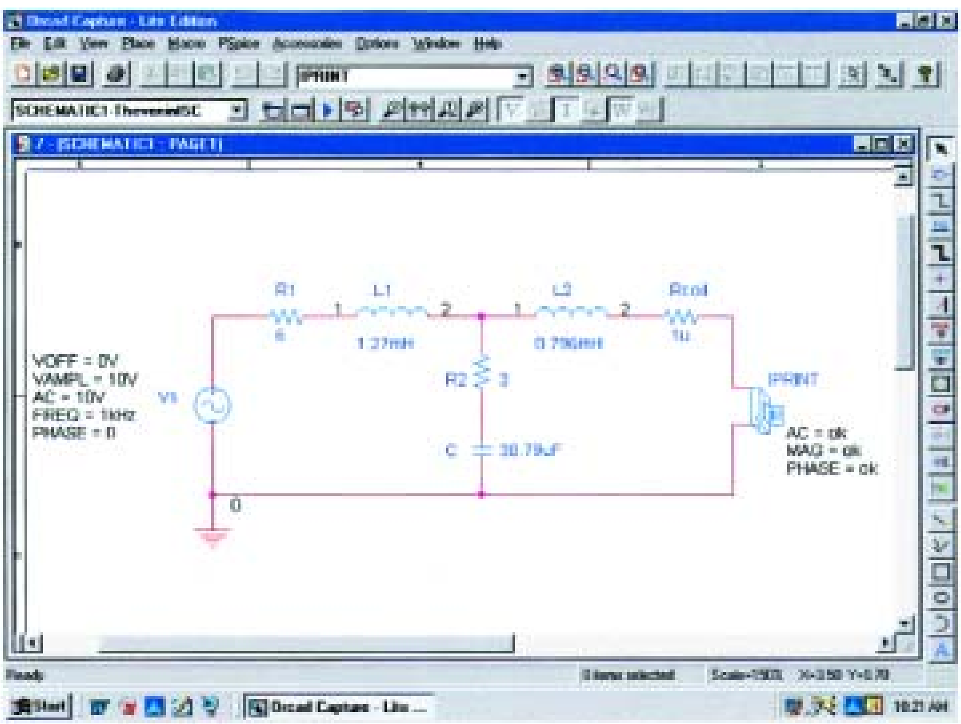

FIG. 18.103
Using PSpice to determine the short-circuit current.

VCVS The last application of this section will be to verify the results of Example 18.12 and to gain some practice using controlled (dependent) sources. The network of Fig. 18.50, with its voltage-controlled voltage source (VCVS), will have the schematic appearance of Fig. 18.105. The VCVS appears as **E** in the **ANALOG** library, with the voltage **E1** as the controlling voltage and **E** as the controlled voltage. In the **Property Editor** dialog box, the **GAIN** must be changed to 20 while

```
 ** Profile: "SCHEMATIC1-TheveninISC"  [ C:\PSpice\thevenin'sth-SCHEMATIC1-Thev
eninISC.sim ]


 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C

******************************************************************************



  FREQ        IM(V_PRINT2)IP(V_PRINT2)


   1.000E+03   9.361E-01  -1.086E+02

```

FIG. 18.104

The output file for the short-circuit current for the network of Fig. 18.103.

the rest of the columns can be left as is. After **Display-Name and Value, Apply** can be selected and the dialog box exited to result in **GAIN = 20** near the controlled source. Take particular note of the second ground inserted near **E** to avoid a long wire to ground that might overlap other elements. For this exercise the current source **ISRC** will be used because it has an arrow in its symbol, and frequency is not

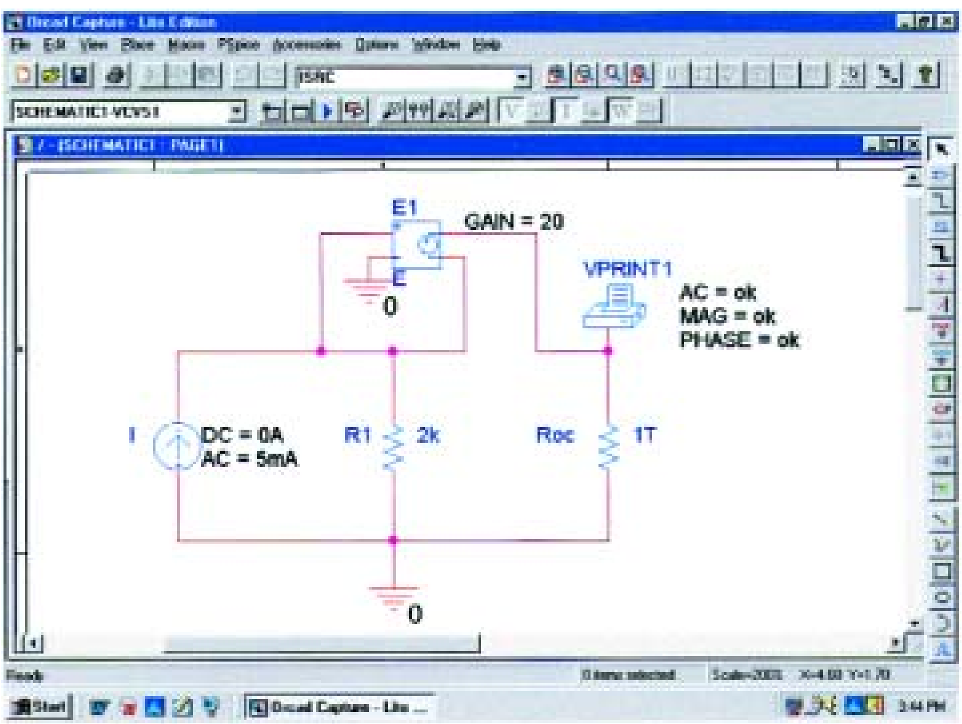

FIG. 18.105

Using PSpice to determine the open-circuit Thévenin voltage for the network of Fig. 18.50.

important for this analysis since there are only resistive elements present. In the **Property Editor** dialog box, the **AC** level is set to 5 mA, and the **DC** level to 0 A; both were displayed using **Display-Name and value. VPRINT1** is set up as in past exercises. The resistor **Roc** (open circuit) was given a very large value so that it would appear as an open circuit to the rest of the network. **VPRINT1** will provide the open circuit Thévenin voltage between the points of interest. Running the simulation in the **AC Sweep** mode at 1 kHz will result in the output file appearing in Fig. 18.106, revealing that the Thévenin voltage is 210 V $\angle 0°$. Substituting the numerical values of this example into the equation obtained in Example 18.12 confirms the result:

$$\mathbf{E}_{Th} = (1 + \mu)\mathbf{I}R_1 = (1 + 20)\,(5\text{ mA}\angle 0°)(2\text{ k}\Omega)$$
$$= \mathbf{210\ V\angle 0°}$$

```

** Profile: "SCHEMATIC1-VCVS1"  [ C:\PSpice\vcvs-SCHEMATIC1-VCVS1.sim ]


****     AC ANALYSIS                        TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ         VM(N01658)  VP(N01658)


   1.000E+03    2.100E+02    0.000E+00

```

FIG. 18.106
The output file for the open-circuit Thévenin voltage for the network of Fig. 18.105.

Next, the short-circuit current must be determined using the **IPRINT** option. Note in Fig. 18.107 that the only difference between this network and that of Fig. 18.106 is the replacement of **Roc** with **IPRINT** and the removal of **VPRINT1**. There is therefore no need to completely "redraw" the network. Just make the changes and run a new simulation. The result of the new simulation as shown in Fig. 18.108 is a current of 5 mA at an angle of 0°.

The ratio of the two measured quantities will result in the Thévenin impedance:

$$\mathbf{Z}_{Th} = \frac{\mathbf{E}_{oc}}{\mathbf{I}_{sc}} = \frac{\mathbf{E}_{Th}}{\mathbf{I}_{sc}} = \frac{210\text{ V}\angle 0°}{5\text{ mA}\angle 0°} = \mathbf{42\ k\Omega}$$

which also matches the longhand solution of Example 18.12:

$$\mathbf{Z}_{Th} = (1 + \mu)R_1 = (1 + 20)2\text{ k}\Omega = (21)2\text{ k}\Omega = \mathbf{42\ k\Omega}$$

The analysis of the full transistor equivalent network of Fig. 18.56 with two controlled sources can be found in the PSpice section of Chapter 26.

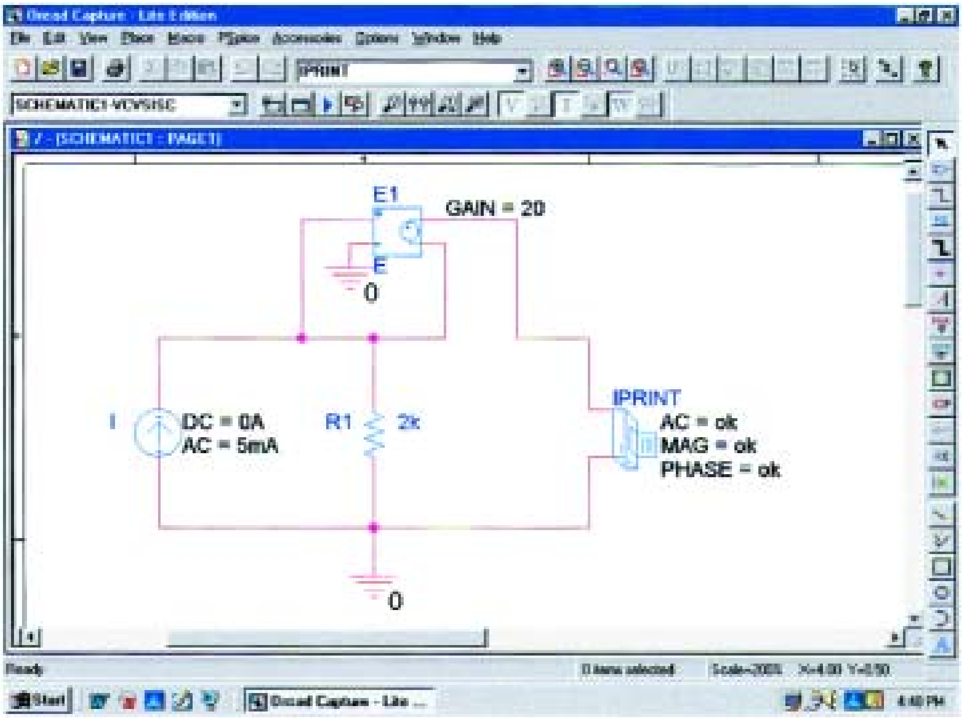

FIG. 18.107
Using PSpice to determine the short-circuit current for the network of Fig. 18.50.

```

 ** Profile: "SCHEMATIC1-VCVSISC"  [ C:\PSpice\vcvs-SCHEMATIC1-VCVSISC.sim ]


 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ        IM(V_PRINT3)IP(V_PRINT3)


   1.000E+03   5.000E-03   0.000E+00

```

FIG. 18.108
The output file for the short-circuit current for the network of Fig. 18.107.

PROBLEMS

SECTION 18.2 Superposition Theorem

1. Using superposition, determine the current through the inductance X_L for each network of Fig. 18.109.

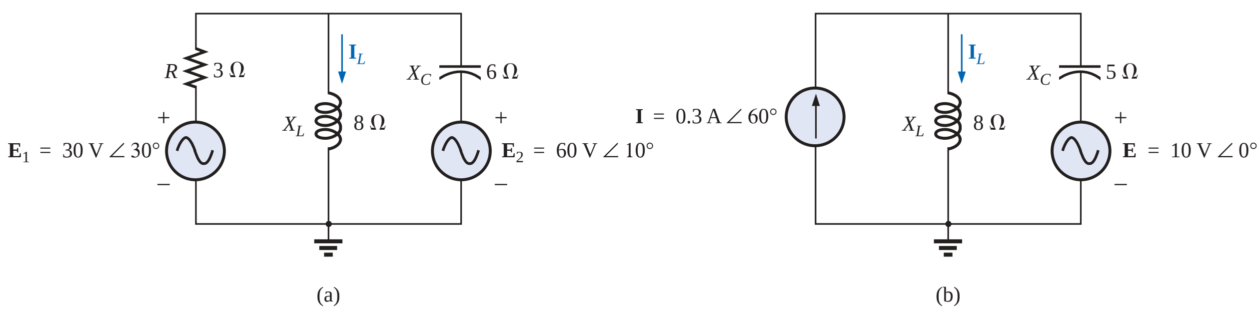

FIG. 18.109
Problem 1.

***2.** Using superposition, determine the current $\mathbf{I}_L$ for each network of Fig. 18.110.

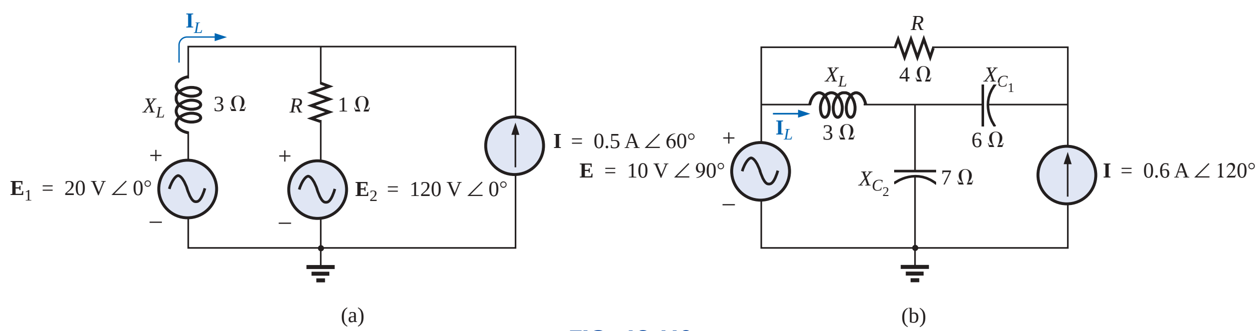

FIG. 18.110
Problem 2.

***3.** Using superposition, find the sinusoidal expression for the current i for the network of Fig. 18.111.

4. Using superposition, find the sinusoidal expression for the voltage v_C for the network of Fig. 18.112.

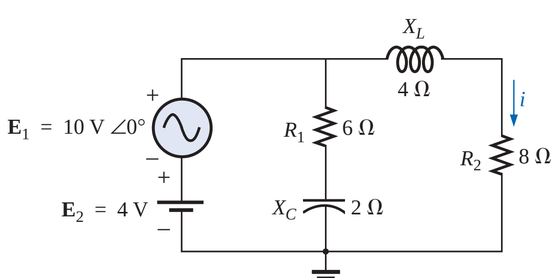

FIG. 18.111
Problems 3, 15, 30, and 42.

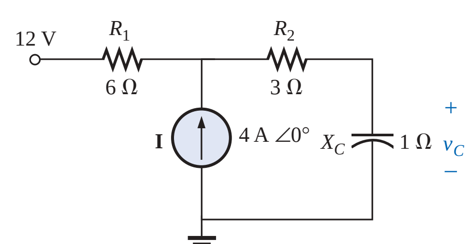

FIG. 18.112
Problems 4, 16, 31, and 43.

***5.** Using superposition, find the current **I** for the network of Fig. 18.113.

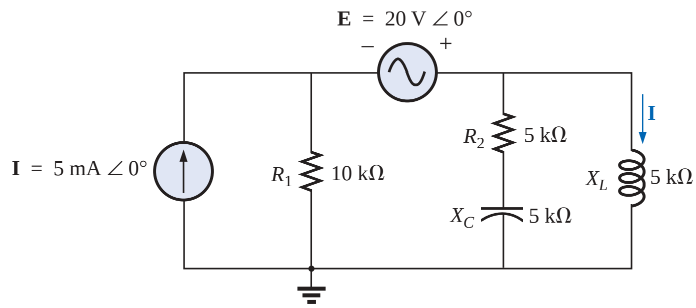

FIG. 18.113
Problems 5, 17, 32, and 44.

6. Using superposition, determine the current $\mathbf{I}_L$ ($h = 100$) for the network of Fig. 18.114.

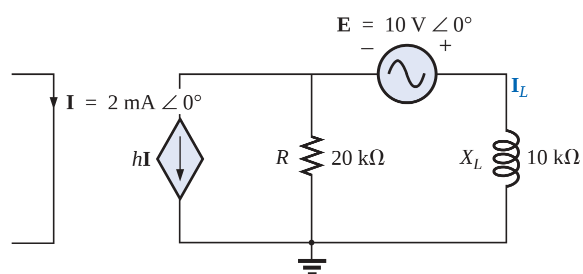

FIG. 18.114
Problems 6 and 20.

7. Using superposition, for the network of Fig. 18.115, determine the voltage $\mathbf{V}_L$ ($\mu = 20$).

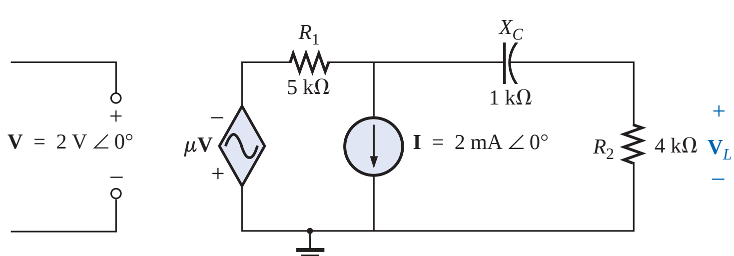

FIG. 18.115
Problems 7, 21, and 35.

***8.** Using superposition, determine the current $\mathbf{I}_L$ for the network of Fig. 18.116 ($\mu = 20$; $h = 100$).

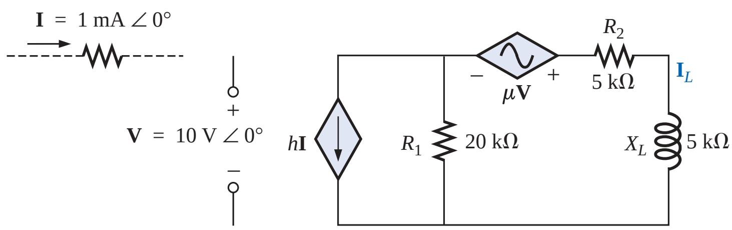

FIG. 18.116
Problems 8, 22, and 36.

***9.** Determine $\mathbf{V}_L$ for the network of Fig. 18.117 ($h = 50$).

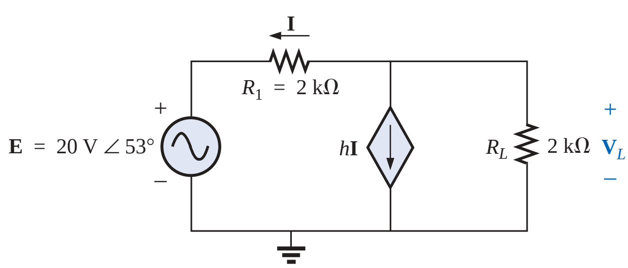

FIG. 18.117
Problems 9 and 23.

***10.** Calculate the current $\mathbf{I}$ for the network of Fig. 18.118.

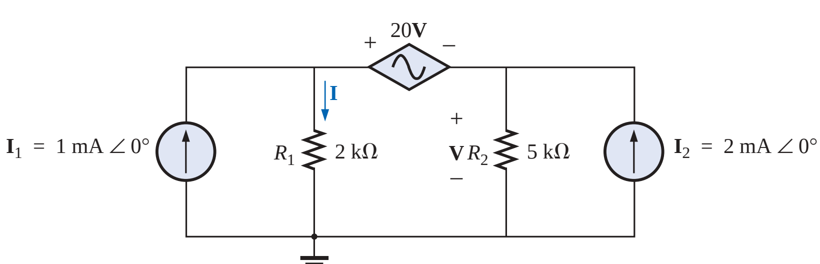

FIG. 18.118
Problems 10, 24, and 38.

11. Find the voltage $\mathbf{V}_s$ for the network of Fig. 18.119.

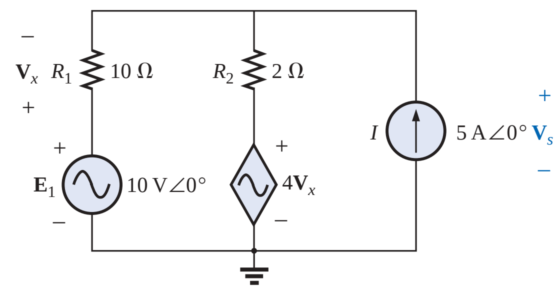

FIG. 18.119
Problem 11.

SECTION 18.3 Thévenin's Theorem

12. Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.120 external to the elements between points *a* and *b*.

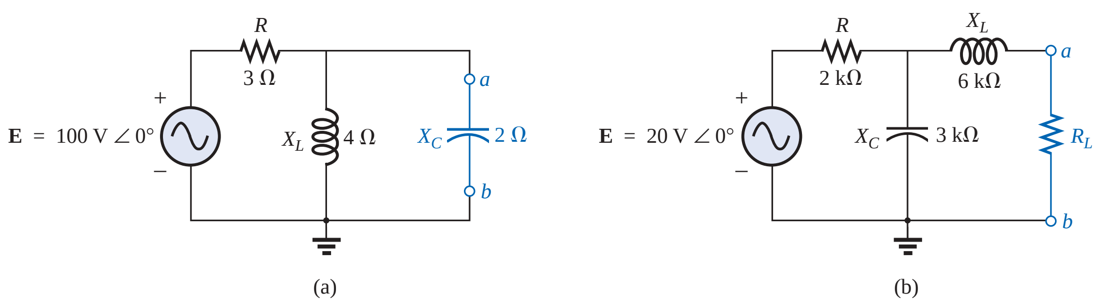

FIG. 18.120
Problems 12 and 26.

***13.** Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.121 external to the elements between points *a* and *b*.

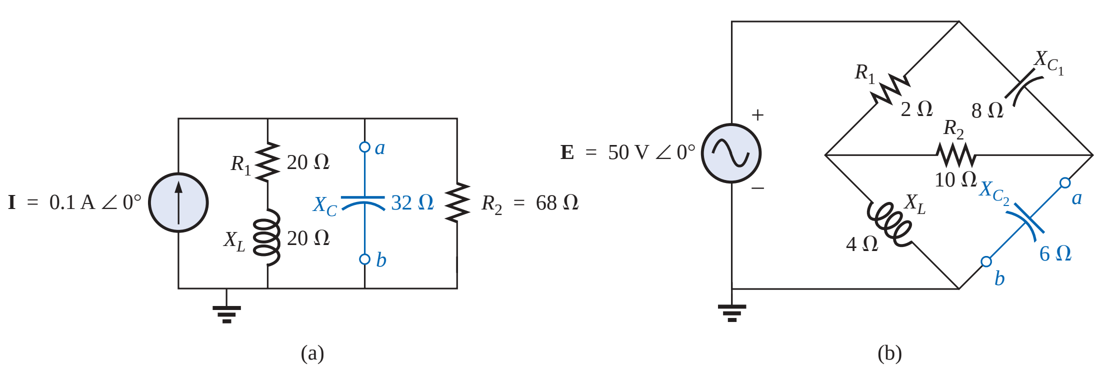

FIG. 18.121
Problems 13 and 27.

***14.** Find the Thévenin equivalent circuit for the portions of the networks of Fig. 18.122 external to the elements between points *a* and *b*.

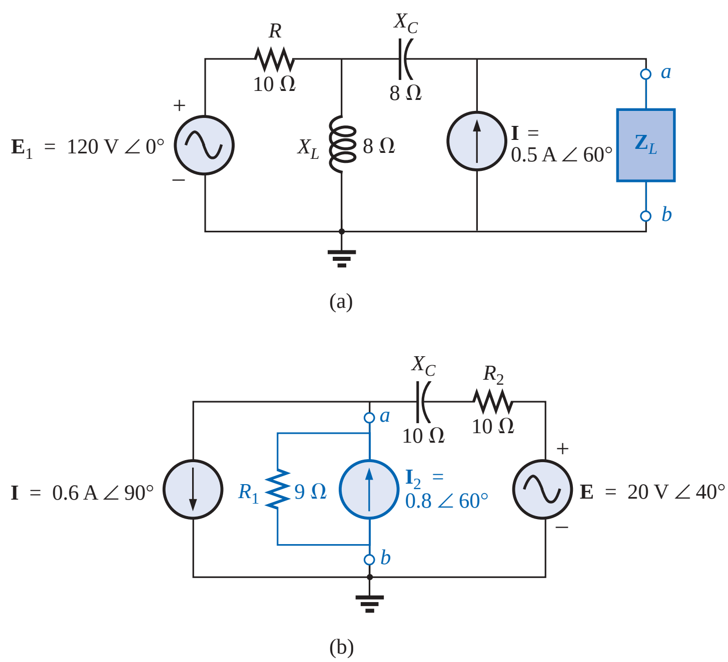

FIG. 18.122
Problems 14 and 28.

***15. a.** Find the Thévenin equivalent circuit for the network external to the resistor R_2 in Fig. 18.111.

b. Using the results of part (a), determine the current *i* of the same figure.

16. a. Find the Thévenin equivalent circuit for the network external to the capacitor of Fig. 18.112.

b. Using the results of part (a), determine the voltage $\mathbf{V}_C$ for the same figure.

***17. a.** Find the Thévenin equivalent circuit for the network external to the inductor of Fig. 18.113.

b. Using the results of part (a), determine the current **I** of the same figure.

18. Determine the Thévenin equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.123 (in terms of **V**).

19. Determine the Thévenin equivalent circuit for the network external to the 4-kΩ inductive reactance of Fig. 18.124 (in terms of **I**).

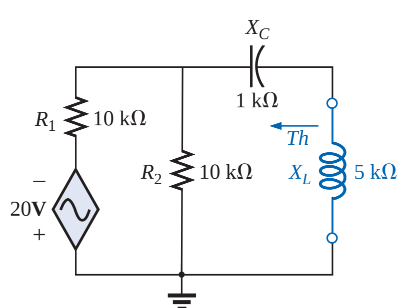

FIG. 18.123
Problems 18 and 33.

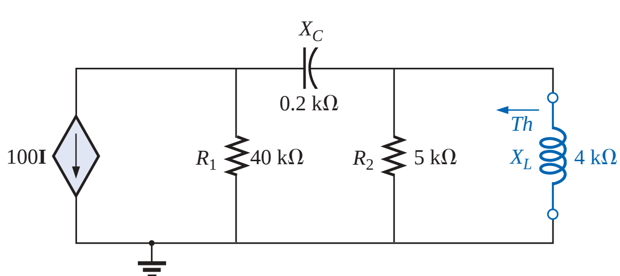

FIG. 18.124
Problems 19 and 34.

20. Find the Thévenin equivalent circuit for the network external to the 10-kΩ inductive reactance of Fig. 18.114.

21. Determine the Thévenin equivalent circuit for the network external to the 4-kΩ resistor of Fig. 18.115.

***22.** Find the Thévenin equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.116.

***23.** Determine the Thévenin equivalent circuit for the network external to the 2-kΩ resistor of Fig. 18.117.

***24.** Find the Thévenin equivalent circuit for the network external to the resistor R_1 of Fig. 18.118.

***25.** Find the Thévenin equivalent circuit for the network to the left of terminals *a*-*a*′ of Fig. 18.125.

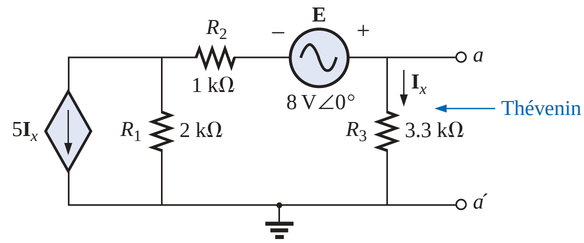

FIG. 18.125
Problem 25.

SECTION 18.4 Norton's Theorem

26. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks of Fig. 18.120.

27. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks of Fig. 18.121.

28. Find the Norton equivalent circuit for the network external to the elements between *a* and *b* for the networks of Fig. 18.122.

***29.** Find the Norton equivalent circuit for the portions of the networks of Fig. 18.126 external to the elements between points *a* and *b*.

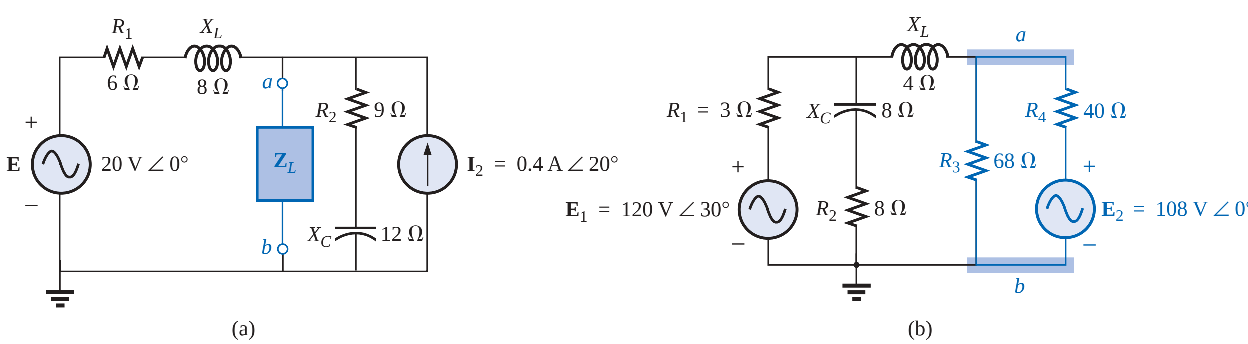

FIG. 18.126
Problem 29.

***30. a.** Find the Norton equivalent circuit for the network external to the resistor R_2 in Fig. 18.111.

b. Using the results of part (a), determine the current **I** of the same figure.

***31. a.** Find the Norton equivalent circuit for the network external to the capacitor of Fig. 18.112.

b. Using the results of part (a), determine the voltage $\mathbf{V}_C$ for the same figure.

***32. a.** Find the Norton equivalent circuit for the network external to the inductor of Fig. 18.113.

b. Using the results of part (a), determine the current **I** of the same figure.

33. Determine the Norton equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.123.

34. Determine the Norton equivalent circuit for the network external to the 4-kΩ inductive reactance of Fig. 18.124.

35. Find the Norton equivalent circuit for the network external to the 4-kΩ resistor of Fig. 18.115.

***36.** Find the Norton equivalent circuit for the network external to the 5-kΩ inductive reactance of Fig. 18.116.

***37.** For the network of Fig. 18.127, find the Norton equivalent circuit for the network external to the 2-kΩ resistor.

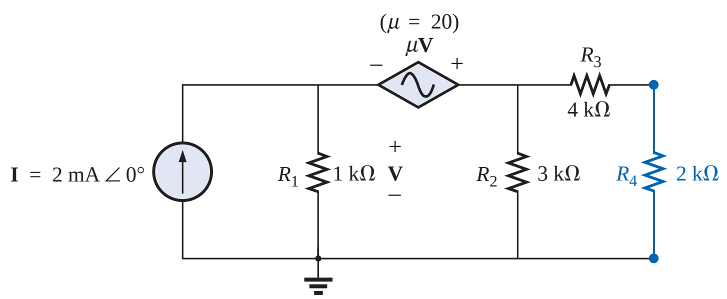

FIG. 18.127
Problem 37.

***38.** Find the Norton equivalent circuit for the network external to the $\mathbf{I}_1$ current source of Fig. 18.118.

SECTION 18.5 Maximum Power Transfer Theorem

39. Find the load impedance $\mathbf{Z}_L$ for the networks of Fig. 18.128 for maximum power to the load, and find the maximum power to the load.

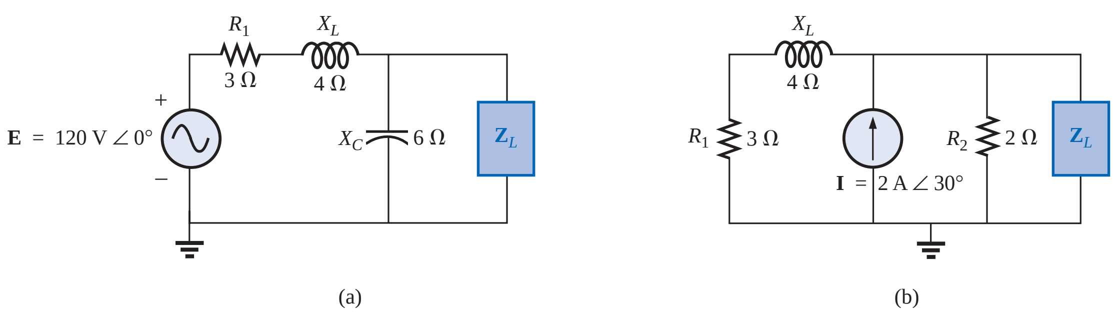

FIG. 18.128
Problem 39.

***40.** Find the load impedance $\mathbf{Z}_L$ for the networks of Fig. 18.129 for maximum power to the load, and find the maximum power to the load.

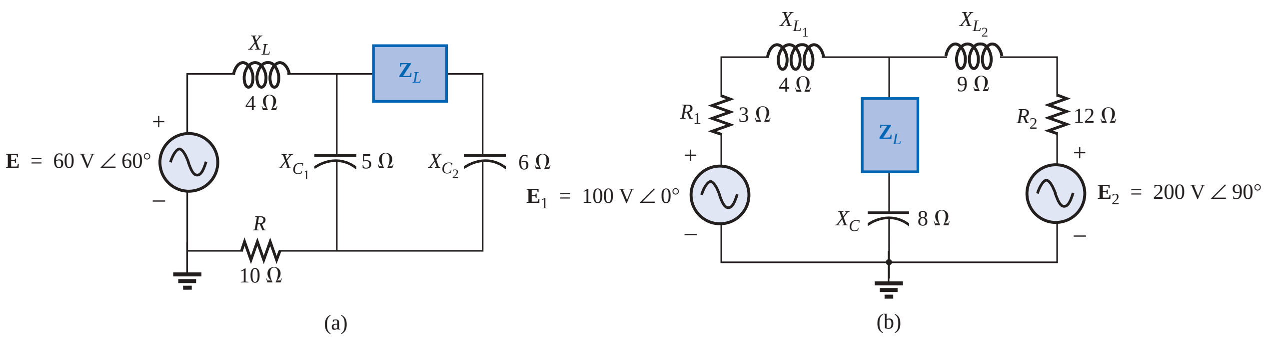

FIG. 18.129
Problem 40.

41. Find the load impedance R_L for the network of Fig. 18.130 for maximum power to the load, and find the maximum power to the load.

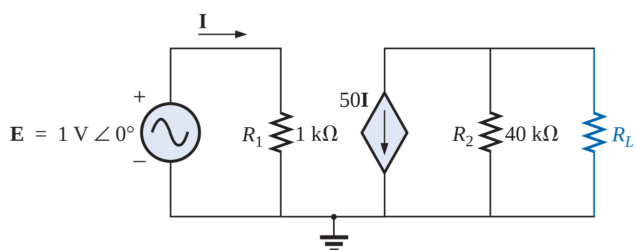

FIG. 18.130
Problem 41.

***42. a.** Determine the load impedance to replace the resistor R_2 of Fig. 18.111 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

***43. a.** Determine the load impedance to replace the capacitor X_C of Fig. 18.112 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

***44. a.** Determine the load impedance to replace the inductor X_L of Fig. 18.113 to ensure maximum power to the load.

b. Using the results of part (a), determine the maximum power to the load.

45. a. For the network of Fig. 18.131, determine the value of R_L that will result in maximum power to the load.

b. Using the results of part (a), determine the maximum power delivered.

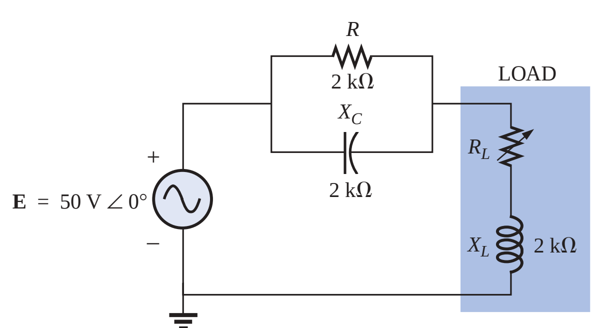

FIG. 18.131
Problem 45.

***46. a.** For the network of Fig. 18.132, determine the level of capacitance that will ensure maximum power to the load if the range of capacitance is limited to 1 nF to 5 nF.

b. Using the results of part (a), determine the value of R_L that will ensure maximum power to the load.

c. Using the results of parts (a) and (b), determine the maximum power to the load.

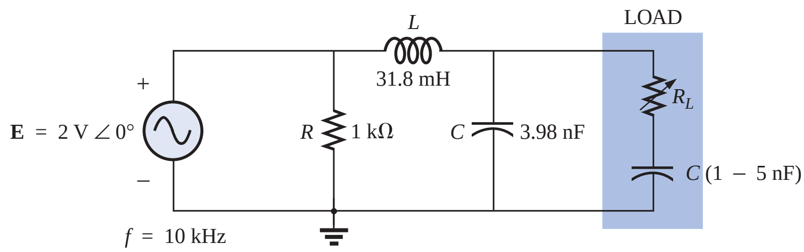

FIG. 18.132
Problem 46.

SECTION 18.6 Substitution, Reciprocity, and Millman's Theorems

47. For the network of Fig. 18.133, determine two equivalent branches through the substitution theorem for the branch *a-b*.

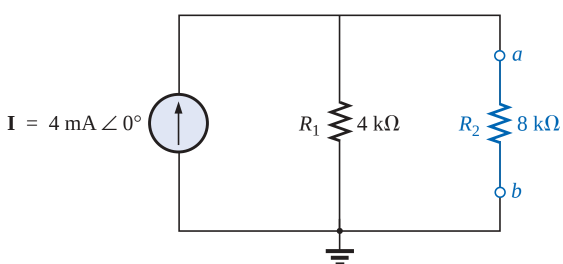

FIG. 18.133
Problem 47.

48. a. For the network of Fig. 18.134(a), find the current **I**.

b. Repeat part (a) for the network of Fig. 18.134(b).

c. Do the results of parts (a) and (b) compare?

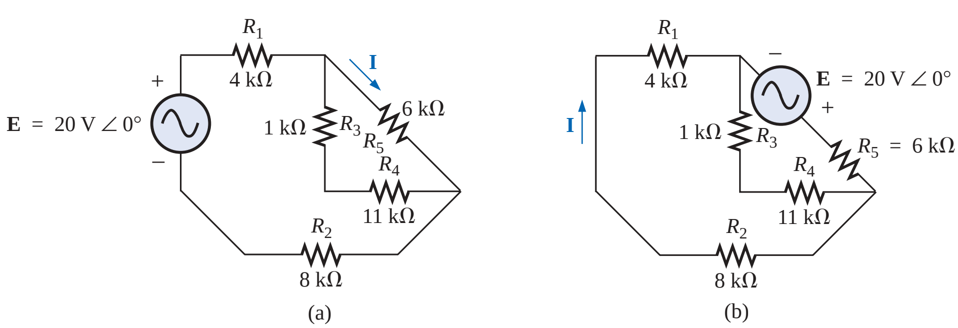

FIG. 18.134
Problem 48.

49. Using Millman's theorem, determine the current through the 4-kΩ capacitive reactance of Fig. 18.135.

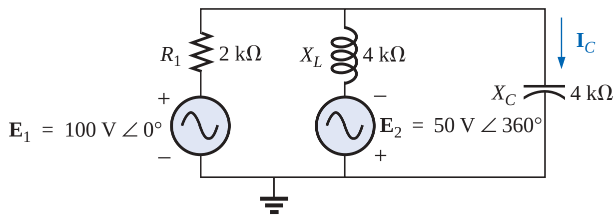

FIG. 18.135
Problem 49.

SECTION 18.8 Computer Analysis

PSpice or Electronics Workbench

50. Apply superposition to the network of Fig. 18.6. That is, determine the current **I** due to each source, and then find the resultant current.

***51.** Determine the current $\mathbf{I}_L$ for the network of Fig. 18.21 using schematics.

***52.** Using schematics, determine $\mathbf{V}_2$ for the network of Fig. 18.56 if $\mathbf{V}_i = 1\ \text{V}\ \angle 0°$, $R_1 = 0.5\ \text{k}\Omega$, $k_1 = 3 \times 10^{-4}$, $k_2 = 50$, and $R_2 = 20\ \text{k}\Omega$.

***53.** Find the Norton equivalent circuit for the network of Fig. 18.75 using schematics.

***54.** Using schematics, plot the power to the *R-C* load of Fig. 18.88 for values of R_L from 1 Ω to 10 Ω.

Programming Language (C++, QBASIC, Pascal, etc.)

55. Given the network of Fig. 18.1, write a program to determine a general solution for the current **I** using superposition. That is, given the reactance of the same network elements, determine **I** for voltage sources of any magnitude but the same angle.

56. Given the network of Fig. 18.23, write a program to determine the Thévenin voltage and impedance for any level of reactance for each element and any magnitude of voltage for the voltage source. The angle of the voltage source should remain at zero degrees.

57. Given the configuration of Fig. 18.136, demonstrate that maximum power is delivered to the load when $X_C = X_L$ by tabulating the power to the load for X_C varying from 0.1 kΩ to 2 kΩ in increments of 0.1 kΩ.

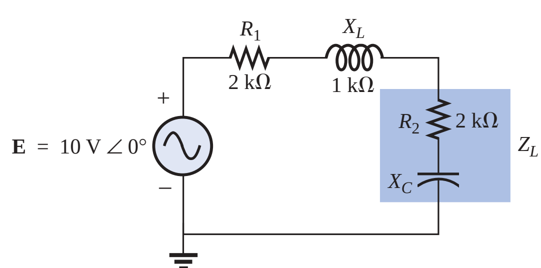

FIG. 18.136
Problem 57.

GLOSSARY

Maximum power transfer theorem A theorem used to determine the load impedance necessary to ensure maximum power to the load.

Millman's theorem A method employing voltage-to-current source conversions that will permit the determination of unknown variables in a multiloop network.

Norton's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single current source and parallel impedance. The resulting configuration can then be employed to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Reciprocity theorem A theorem stating that for single-source networks, the magnitude of the current in any branch of a network, due to a single voltage source anywhere else in the network, will equal the magnitude of the current through the branch in which the source was originally located if the source is placed in the branch in which the current was originally measured.

Substitution theorem A theorem stating that if the voltage across and current through any branch of an ac bilateral network are known, the branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

Superposition theorem A method of network analysis that permits considering the effects of each source independently. The resulting current and/or voltage is the phasor sum of the currents and/or voltages developed by each source independently.

Thévenin's theorem A theorem that permits the reduction of any two-terminal linear ac network to one having a single voltage source and series impedance. The resulting configuration can then be employed to determine a particular current or voltage in the original network or to examine the effects of a specific portion of the network on a particular variable.

Voltage-controlled voltage source (VCVS) A voltage source whose parameters are controlled by a voltage elsewhere in the system.