

Resonance

20.1 INTRODUCTION

This chapter will introduce the very important *resonant (or tuned) circuit*, which is fundamental to the operation of a wide variety of electrical and electronic systems in use today. The resonant circuit is a combination of R , L , and C elements having a frequency response characteristic similar to the one appearing in Fig. 20.1. Note in the figure that the response is a

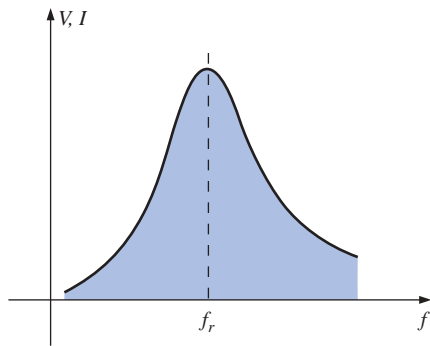


FIG. 20.1
Resonance curve.

maximum for the frequency f_r , decreasing to the right and left of this frequency. In other words, for a particular range of frequencies the response will be near or equal to the maximum. The frequencies to the far left or right have very low voltage or current levels and, for all practical purposes, have little effect on the system's response. The radio or television receiver has a response curve for each broadcast station of the type indicated in Fig. 20.1. When the receiver is set (or tuned) to a particular station, it is set on or near the frequency f_r of Fig. 20.1. Stations transmitting at frequencies to the far right or left of this resonant frequency are not carried through with significant power to affect the program of interest. The tuning process (setting the dial to f_r) as described above is the reason for



the terminology *tuned circuit*. When the response is at or near the maximum, the circuit is said to be in a state of **resonance**.

The concept of resonance is not limited to electrical or electronic systems. If mechanical impulses are applied to a mechanical system at the proper frequency, the system will enter a state of resonance in which sustained vibrations of very large amplitude will develop. The frequency at which this occurs is called the *natural frequency* of the system. The classic example of this effect was the Tacoma Narrows Bridge built in 1940 over Puget Sound in Washington State. Four months after the bridge, with its suspended span of 2800 ft, was completed, a 42-mi/h pulsating gale set the bridge into oscillations at its natural frequency. The amplitude of the oscillations increased to the point where the main span broke up and fell into the water below. It has since been replaced by the new Tacoma Narrows Bridge, completed in 1950.

The resonant electrical circuit *must* have both inductance and capacitance. In addition, resistance will always be present due either to the lack of ideal elements or to the control offered on the shape of the resonance curve. When resonance occurs due to the application of the proper frequency (f_r), the energy absorbed by one reactive element is the same as that released by another reactive element within the system. In other words, energy pulsates from one reactive element to the other. Therefore, once an ideal (pure C , L) system has reached a state of resonance, it requires no further reactive power since it is self-sustaining. In a practical circuit, there is some resistance associated with the reactive elements that will result in the eventual “damping” of the oscillations between reactive elements.

There are two types of resonant circuits: *series* and *parallel*. Each will be considered in some detail in this chapter.

SERIES RESONANCE

20.2 SERIES RESONANT CIRCUIT

A resonant circuit (series or parallel) must have an inductive and a capacitive element. A resistive element will always be present due to the internal resistance of the source (R_s), the internal resistance of the inductor (R_l), and any added resistance to control the shape of the response curve (R_{design}). The basic configuration for the series resonant circuit appears in Fig. 20.2(a) with the resistive elements listed above. The “cleaner” appearance of Fig. 20.2(b) is a result of combining the series resistive elements into one total value. That is,

$$R = R_s + R_l + R_d \quad (20.1)$$

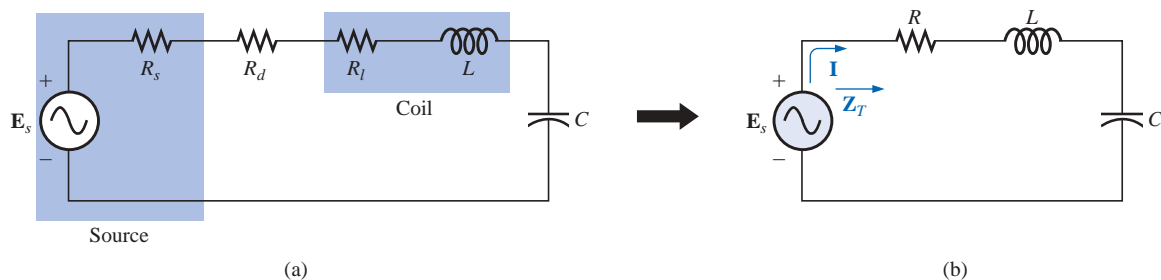


FIG. 20.2

Series resonant circuit.



The total impedance of this network at any frequency is determined by

$$\mathbf{Z}_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

The resonant conditions described in the introduction will occur when

$$X_L = X_C \tag{20.2}$$

removing the reactive component from the total impedance equation. The total impedance at resonance is then simply

$$\mathbf{Z}_{T_s} = R \tag{20.3}$$

representing the minimum value of \mathbf{Z}_T at any frequency. The subscript s will be employed to indicate series resonant conditions.

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance [Eq. (20.2)]:

$$X_L = X_C$$

Substituting yields

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

and

$$\omega_s = \frac{1}{\sqrt{LC}} \tag{20.4}$$

or

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad \begin{array}{l} f = \text{hertz (Hz)} \\ L = \text{henries (H)} \\ C = \text{farads (F)} \end{array} \tag{20.5}$$

The current through the circuit at resonance is

$$\mathbf{I} = \frac{E \angle 0^\circ}{R \angle 0^\circ} = \frac{E}{R} \angle 0^\circ$$

which you will note is the maximum current for the circuit of Fig. 20.2 for an applied voltage \mathbf{E} since \mathbf{Z}_T is a minimum value. Consider also that *the input voltage and current are in phase at resonance*.

Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but 180° out of phase at resonance:

$$\left. \begin{array}{l} \mathbf{V}_L = (I \angle 0^\circ)(X_L \angle 90^\circ) = IX_L \angle 90^\circ \\ \mathbf{V}_C = (I \angle 0^\circ)(X_C \angle -90^\circ) = IX_C \angle -90^\circ \end{array} \right\} \begin{array}{l} 180^\circ \\ \text{out of} \\ \text{phase} \end{array}$$

and, since $X_L = X_C$, the magnitude of V_L equals V_C at resonance; that is,

$$V_{L_s} = V_{C_s} \tag{20.6}$$

Figure 20.3, a phasor diagram of the voltages and current, clearly indicates that the voltage across the resistor at resonance is the input voltage, and \mathbf{E} , and \mathbf{I} , and \mathbf{V}_R are in phase at resonance.

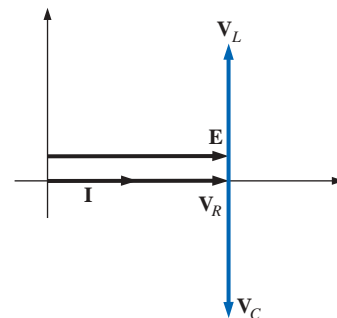


FIG. 20.3
Phasor diagram for the series resonant circuit at resonance.

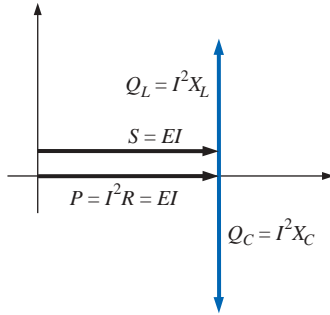


FIG. 20.4
Power triangle for the series resonant circuit at resonance.

The average power to the resistor at resonance is equal to I^2R , and the reactive power to the capacitor and inductor are I^2X_C and I^2X_L , respectively.

The power triangle at resonance (Fig. 20.4) shows that the total apparent power is equal to the average power dissipated by the resistor since $Q_L = Q_C$. The power factor of the circuit at resonance is

$$F_p = \cos \theta = \frac{P}{S}$$

and

$$F_{p_s} = 1 \tag{20.7}$$

Plotting the power curves of each element on the same set of axes (Fig. 20.5), we note that, even though the total reactive power at any instant is equal to zero (note that $t = t'$), energy is still being absorbed and released by the inductor and capacitor at resonance.

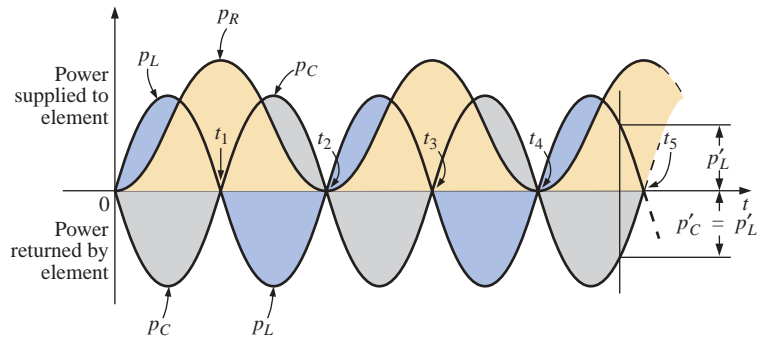


FIG. 20.5
Power curves at resonance for the series resonant circuit.

A closer examination reveals that the energy absorbed by the inductor from time 0 to t_1 is the same as the energy released by the capacitor from 0 to t_1 . The reverse occurs from t_1 to t_2 , and so on. Therefore, the total apparent power continues to be equal to the average power, even though the inductor and capacitor are absorbing and releasing energy. This condition occurs only at resonance. The slightest change in frequency introduces a reactive component into the power triangle, which will increase the apparent power of the system above the average power dissipation, and resonance will no longer exist.

20.3 THE QUALITY FACTOR (Q)

The **quality factor** Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

$$Q_s = \frac{\text{reactive power}}{\text{average power}} \tag{20.8}$$

The quality factor is also an indication of how much energy is placed in storage (continual transfer from one reactive element to the other) compared to that dissipated. The lower the level of dissipation for the same

reactive power, the larger the Q_s factor and the more concentrated and intense the region of resonance.

Substituting for an inductive reactance in Eq. (20.8) at resonance gives us

$$Q_s = \frac{I^2 X_L}{I^2 R}$$

and

$$Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R} \quad (20.9)$$

If the resistance R is just the resistance of the coil (R_l), we can speak of the Q of the coil, where

$$Q_{\text{coil}} = Q_l = \frac{X_L}{R_l} \quad R = R_l \quad (20.10)$$

Since the quality factor of a coil is typically the information provided by manufacturers of inductors, it is often given the symbol Q without an associated subscript. It would appear from Eq. (20.10) that Q_l will increase linearly with frequency since $X_L = 2\pi fL$. That is, if the frequency doubles, then Q_l will also increase by a factor of 2. This is approximately true for the low range to the midrange of frequencies such as shown for the coils of Fig. 20.6. Unfortunately, however, as the frequency increases, the effective resistance of the coil will also increase, due primarily to skin effect phenomena, and the resulting Q_l will decrease. In addition, the capacitive effects between the windings will increase, further reducing the Q_l of the coil. For this reason, Q_l must be specified for a particular frequency or frequency range. For wide frequency applications, a plot of Q_l versus frequency is often provided. The maximum Q_l for most commercially available coils is less than 200, with most having a maximum near 100. Note in Fig. 20.6 that for coils of the same type, Q_l drops off more quickly for higher levels of inductance.

If we substitute

$$\omega_s = 2\pi f_s$$

and then

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

into Eq. (20.9), we have

$$\begin{aligned} Q_s &= \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R} \left(\frac{1}{2\pi\sqrt{LC}} \right) L \\ &= \frac{L}{R} \left(\frac{1}{\sqrt{LC}} \right) = \left(\frac{\sqrt{L}}{\sqrt{L}} \right) \frac{L}{R\sqrt{LC}} \end{aligned}$$

and

$$Q_s = \frac{1}{R} \sqrt{\frac{L}{C}} \quad (20.11)$$

providing Q_s in terms of the circuit parameters.

For series resonant circuits used in communication systems, Q_s is usually greater than 1. By applying the voltage divider rule to the circuit of Fig. 20.2, we obtain

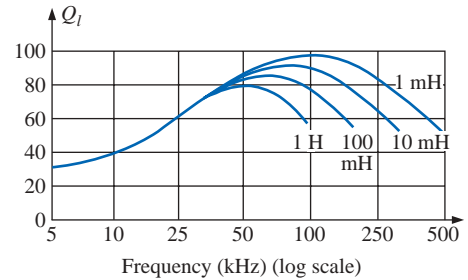


FIG. 20.6

Q_l versus frequency for a series of inductors of similar construction.



$$V_L = \frac{X_L E}{Z_T} = \frac{X_L E}{R} \quad (\text{at resonance})$$

and $V_{L_s} = Q_s E$ (20.12)

or
$$V_C = \frac{X_C E}{Z_T} = \frac{X_C E}{R}$$

and $V_{C_s} = Q_s E$ (20.13)

Since Q_s is usually greater than 1, the voltage across the capacitor or inductor of a series resonant circuit can be significantly greater than the input voltage. In fact, in many cases the Q_s is so high that careful design and handling (including adequate insulation) are mandatory with respect to the voltage across the capacitor and inductor.

In the circuit of Fig. 20.7, for example, which is in the state of resonance,

$$Q_s = \frac{X_L}{R} = \frac{480 \Omega}{6 \Omega} = 80$$

and $V_L = V_C = Q_s E = (80)(10 \text{ V}) = \mathbf{800 \text{ V}}$

which is certainly a potential of significant magnitude.

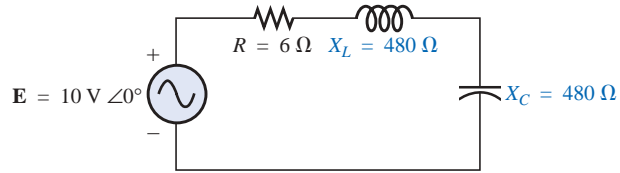


FIG. 20.7
High-Q series resonant circuit.

20.4 Z_T VERSUS FREQUENCY

The total impedance of the series R - L - C circuit of Fig. 20.2 at any frequency is determined by

$$\mathbf{Z}_T = R + jX_L - jX_C \quad \text{or} \quad \mathbf{Z}_T = R + j(X_L - X_C)$$

The magnitude of the impedance \mathbf{Z}_T versus frequency is determined by

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

The total-impedance-versus-frequency curve for the series resonant circuit of Fig. 20.2 can be found by applying the impedance-versus-frequency curve for each element of the equation just derived, written in the following form:

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2} \quad (20.14)$$

where $Z_T(f)$ “means” the total impedance as a *function* of frequency. For the frequency range of interest, we will assume that the resistance R does not change with frequency, resulting in the plot of Fig. 20.8. The curve for the inductance, as determined by the reactance equation, is a

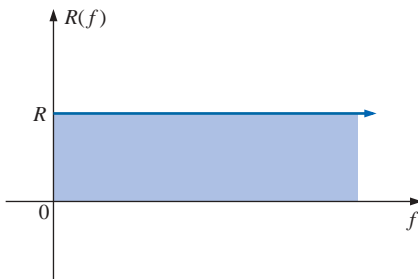


FIG. 20.8
Resistance versus frequency.

straight line intersecting the origin with a slope equal to the inductance of the coil. The mathematical expression for any straight line in a two-dimensional plane is given by

$$y = mx + b$$

Thus, for the coil,

$$X_L = 2\pi fL + 0 = (2\pi L)(f) + 0$$

$$\begin{matrix} \downarrow & & \downarrow & \downarrow & \downarrow \\ y = & & m \cdot x & + & b \end{matrix}$$

(where $2\pi L$ is the slope), producing the results shown in Fig. 20.9.

For the capacitor,

$$X_C = \frac{1}{2\pi fC} \quad \text{or} \quad X_C f = \frac{1}{2\pi C}$$

which becomes $yx = k$, the equation for a hyperbola, where

$$\begin{aligned} y \text{ (variable)} &= X_C \\ x \text{ (variable)} &= f \\ k \text{ (constant)} &= \frac{1}{2\pi C} \end{aligned}$$

The hyperbolic curve for $X_C(f)$ is plotted in Fig. 20.10. In particular, note its very large magnitude at low frequencies and its rapid drop-off as the frequency increases.

If we place Figs. 20.9 and 20.10 on the same set of axes, we obtain the curves of Fig. 20.11. The condition of resonance is now clearly defined by the point of intersection, where $X_L = X_C$. For frequencies less than f_s , it is also quite clear that the network is primarily capacitive ($X_C > X_L$). For frequencies above the resonant condition, $X_L > X_C$, and the network is inductive.

Applying

$$\begin{aligned} Z_T(f) &= \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2} \\ &= \sqrt{[R(f)]^2 + [X(f)]^2} \end{aligned}$$

to the curves of Fig. 20.11, where $X(f) = X_L(f) - X_C(f)$, we obtain the curve for $Z_T(f)$ as shown in Fig. 20.12. The minimum impedance occurs at the resonant frequency and is equal to the resistance R . Note that the curve is not symmetrical about the resonant frequency (especially at higher values of Z_T).

The phase angle associated with the total impedance is

$$\theta = \tan^{-1} \frac{(X_L - X_C)}{R} \tag{20.15}$$

For the $\tan^{-1} x$ function (resulting when $X_L > X_C$), the larger x is, the larger the angle θ (closer to 90°). However, for regions where $X_C > X_L$, one must also be aware that

$$\tan^{-1}(-x) = -\tan^{-1} x \tag{20.16}$$

At low frequencies, $X_C > X_L$, and θ will approach -90° (capacitive), as shown in Fig. 20.13, whereas at high frequencies, $X_L > X_C$, and θ will approach 90° . In general, therefore, for a series resonant circuit:

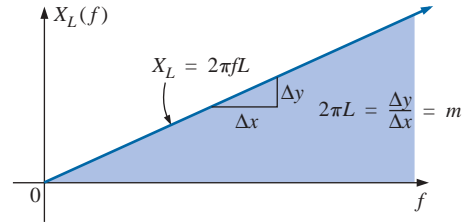


FIG. 20.9
Inductive reactance versus frequency.

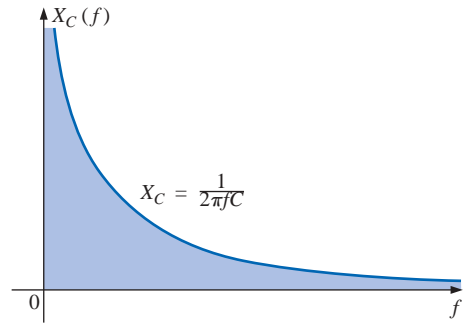


FIG. 20.10
Capacitive reactance versus frequency.

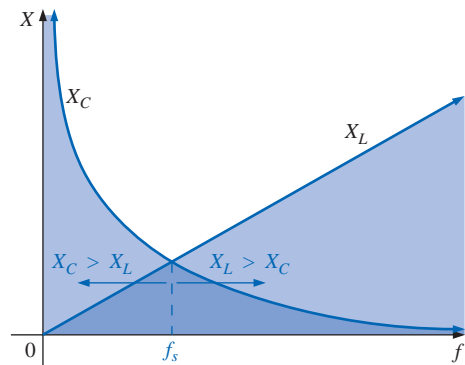


FIG. 20.11
Placing the frequency response of the inductive and capacitive reactance of a series R-L-C circuit on the same set of axes.

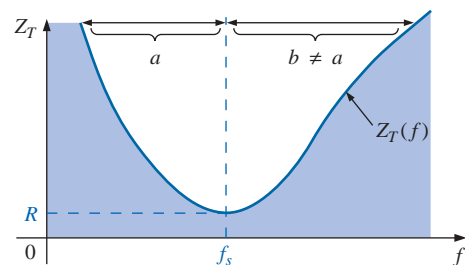


FIG. 20.12
 Z_T versus frequency for the series resonant circuit.

$f < f_s$:	network capacitive; I leads E
$f > f_s$:	network inductive; E leads I
$f = f_s$:	network resistive; E and I are in phase

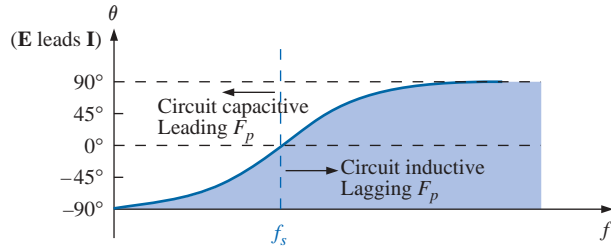


FIG. 20.13

Phase plot for the series resonant circuit.

20.5 SELECTIVITY

If we now plot the magnitude of the current $I = E/Z_T$ versus frequency for a fixed applied voltage E , we obtain the curve shown in Fig. 20.14, which rises from zero to a maximum value of E/R (where Z_T is a minimum) and then drops toward zero (as Z_T increases) at a slower rate than it rose to its peak value. The curve is actually the inverse of the impedance-versus-frequency curve. Since the Z_T curve is not absolutely symmetrical about the resonant frequency, the curve of the current versus frequency has the same property.

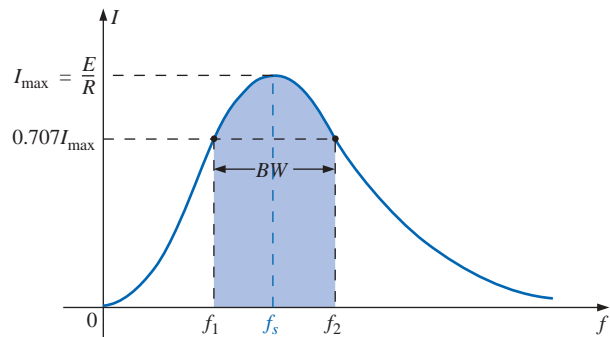


FIG. 20.14

I versus frequency for the series resonant circuit.

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of the maximum current are called the **band frequencies, cutoff frequencies, or half-power frequencies**. They are indicated by f_1 and f_2 in Fig. 20.14. The range of frequencies between the two is referred to as the **bandwidth** (abbreviated BW) of the resonant circuit.

Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency; that is,

$$P_{\text{HPF}} = \frac{1}{2} P_{\text{max}} \tag{20.17}$$

The above condition is derived using the fact that

$$P_{\max} = I_{\max}^2 R$$

and
$$P_{\text{HPF}} = I^2 R = (0.707 I_{\max})^2 R = (0.5)(I_{\max}^2 R) = \frac{1}{2} P_{\max}$$

Since the resonant circuit is adjusted to select a band of frequencies, the curve of Fig. 20.14 is called the **selectivity curve**. The term is derived from the fact that one must be *selective* in choosing the frequency to ensure that it is in the bandwidth. The smaller the bandwidth, the higher the selectivity. The shape of the curve, as shown in Fig. 20.15, depends on each element of the series R - L - C circuit. If the resistance is made smaller with a fixed inductance and capacitance, the bandwidth decreases and the selectivity increases. Similarly, if the ratio L/C increases with fixed resistance, the bandwidth again decreases with an increase in selectivity.

In terms of Q_s , if R is larger for the same X_L , then Q_s is less, as determined by the equation $Q_s = \omega_s L/R$.

A small Q_s , therefore, is associated with a resonant curve having a large bandwidth and a small selectivity, while a large Q_s indicates the opposite.

For circuits where $Q_s \geq 10$, a widely accepted approximation is that the resonant frequency bisects the bandwidth and that the resonant curve is symmetrical about the resonant frequency.

These conditions are shown in Fig. 20.16, indicating that the cutoff frequencies are then equidistant from the resonant frequency.

For any Q_s , the preceding is not true. The cutoff frequencies f_1 and f_2 can be found for the general case (any Q_s) by first employing the fact that a drop in current to 0.707 of its resonant value corresponds to an increase in impedance equal to $1/0.707 = \sqrt{2}$ times the resonant value, which is R .

Substituting $\sqrt{2}R$ into the equation for the magnitude of Z_T , we find that

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

becomes

$$\sqrt{2}R = \sqrt{R^2 + (X_L - X_C)^2}$$

or, squaring both sides, that

$$2R^2 = R^2 + (X_L - X_C)^2$$

and

$$R^2 = (X_L - X_C)^2$$

Taking the square root of both sides gives us

$$R = X_L - X_C \quad \text{or} \quad R = X_L + X_C = 0$$

Let us first consider the case where $X_L > X_C$, which relates to f_2 or ω_2 . Substituting $\omega_2 L$ for X_L and $1/\omega_2 C$ for X_C and bringing both quantities to the left of the equal sign, we have

$$R - \omega_2 L + \frac{1}{\omega_2 C} = 0 \quad \text{or} \quad R\omega_2 - \omega_2^2 L + \frac{1}{C} = 0$$

which can be written

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

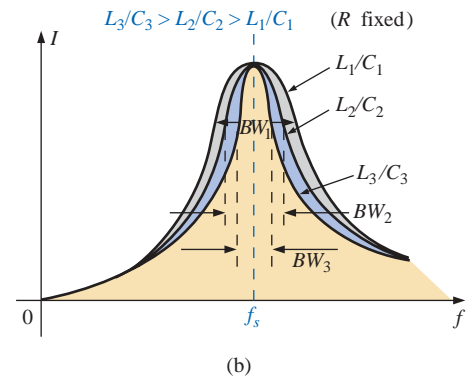
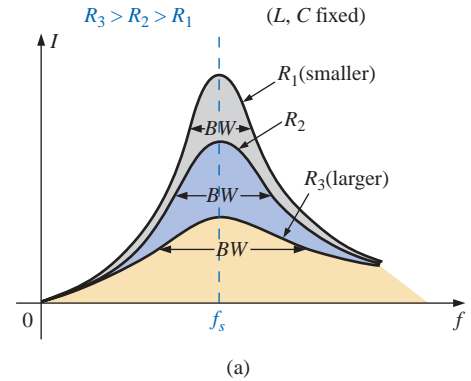


FIG. 20.15

Effect of R , L , and C on the selectivity curve for the series resonant circuit.

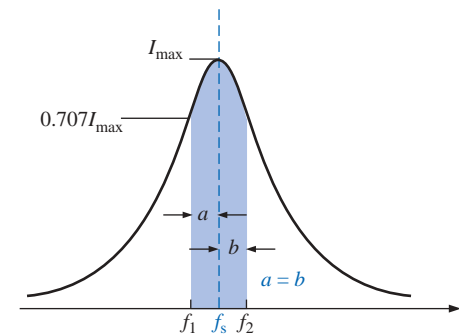


FIG. 20.16

Approximate series resonance curve for $Q_s \geq 10$.



Solving the quadratic, we have

$$\omega_2 = \frac{-(-R/L) \pm \sqrt{[-(R/L)]^2 - [-(4/LC)]}}{2}$$

and

$$\omega_2 = +\frac{R}{2L} \pm \frac{1}{2} \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

with

$$f_2 = \frac{1}{2\pi} \left[\frac{R}{2L} + \frac{1}{2} \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right] \quad (\text{Hz}) \quad (20.18)$$

The negative sign in front of the second factor was dropped because $(1/2)\sqrt{(R/L)^2 + 4/LC}$ is always greater than $R/(2L)$. If it were not dropped, there would be a negative solution for the radian frequency ω .

If we repeat the same procedure for $X_C > X_L$, which relates to ω_1 or f_1 such that $Z_T = \sqrt{R^2 + (X_C - X_L)^2}$, the solution f_1 becomes

$$f_1 = \frac{1}{2\pi} \left[-\frac{R}{2L} + \frac{1}{2} \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right] \quad (\text{Hz}) \quad (20.19)$$

The bandwidth (BW) is

$$BW = f_2 - f_1 = \text{Eq. (20.18)} - \text{Eq. (20.19)}$$

and

$$BW = f_2 - f_1 = \frac{R}{2\pi L} \quad (20.20)$$

Substituting $R/L = \omega_s/Q_s$ from $Q_s = \omega_s L/R$ and $1/2\pi = f_s/\omega_s$ from $\omega_s = 2\pi f_s$ gives us

$$BW = \frac{R}{2\pi L} = \left(\frac{1}{2\pi}\right) \left(\frac{R}{L}\right) = \left(\frac{f_s}{\omega_s}\right) \left(\frac{\omega_s}{Q_s}\right)$$

or

$$BW = \frac{f_s}{Q_s} \quad (20.21)$$

which is a very convenient form since it relates the bandwidth to the Q_s of the circuit. As mentioned earlier, Equation (20.21) verifies that the larger the Q_s , the smaller the bandwidth, and vice versa.

Written in a slightly different form, Equation (20.21) becomes

$$\frac{f_2 - f_1}{f_s} = \frac{1}{Q_s} \quad (20.22)$$

The ratio $(f_2 - f_1)/f_s$ is sometimes called the *fractional bandwidth*, providing an indication of the width of the bandwidth compared to the resonant frequency.

It can also be shown through mathematical manipulations of the pertinent equations that the resonant frequency is related to the geometric mean of the band frequencies; that is,

$$f_s = \sqrt{f_1 f_2} \quad (20.23)$$

20.6 $V_R, V_L, \text{ AND } V_C$

Plotting the magnitude (effective value) of the voltages V_R , V_L , and V_C and the current I versus frequency for the series resonant circuit on the same set of axes, we obtain the curves shown in Fig. 20.17. Note that the V_R curve has the same shape as the I curve and a peak value equal to the magnitude of the input voltage E . The V_C curve builds up slowly at first from a value equal to the input voltage since the reactance of the capacitor is infinite (open circuit) at zero frequency and the reactance of the inductor is zero (short circuit) at this frequency. As the frequency increases, $1/\omega C$ of the equation

$$V_C = IX_C = (I) \left(\frac{1}{\omega C} \right)$$

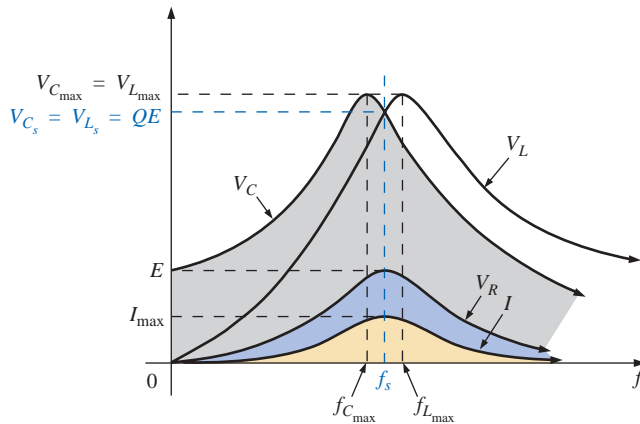


FIG. 20.17

$V_R, V_L, V_C,$ and I versus frequency for a series resonant circuit.

becomes smaller, but I increases at a rate faster than that at which $1/\omega C$ drops. Therefore, V_C rises and will continue to rise due to the quickly rising current, until the frequency nears resonance. As it approaches the resonant condition, the rate of change of I decreases. When this occurs, the factor $1/\omega C$, which decreased as the frequency rose, will overcome the rate of change of I , and V_C will start to drop. The peak value will occur at a frequency just before resonance. After resonance, both V_C and I drop in magnitude, and V_C approaches zero.

The higher the Q_s of the circuit, the closer $f_{C_{\max}}$ will be to f_s , and the closer $V_{C_{\max}}$ will be to $Q_s E$. For circuits with $Q_s \geq 10$, $f_{C_{\max}} \cong f_s$, and $V_{C_{\max}} \cong Q_s E$.

The curve for V_L increases steadily from zero to the resonant frequency since both quantities ωL and I of the equation $V_L = IX_L = (I)(\omega L)$ increase over this frequency range. At resonance, I has reached its maximum value, but ωL is still rising. Therefore, V_L will reach its maximum value after resonance. After reaching its peak value, the voltage V_L will drop toward E since the drop in I will overcome the rise in ωL . It approaches E because X_L will eventually be infinite, and X_C will be zero.

As Q_s of the circuit increases, the frequency $f_{L_{\max}}$ drops toward f_s , and $V_{L_{\max}}$ approaches $Q_s E$. For circuits with $Q_s \geq 10$, $f_{L_{\max}} \cong f_s$, and $V_{L_{\max}} \cong Q_s E$.

The V_L curve has a greater magnitude than the V_C curve for any frequency above resonance, and the V_C curve has a greater magnitude than the V_L curve for any frequency below resonance. This again verifies that the series R - L - C circuit is predominantly capacitive from zero to the resonant frequency and predominantly inductive for any frequency above resonance.

For the condition $Q_s \geq 10$, the curves of Fig. 20.17 will appear as shown in Fig. 20.18. Note that they each peak (on an approximate basis) at the resonant frequency and have a similar shape.

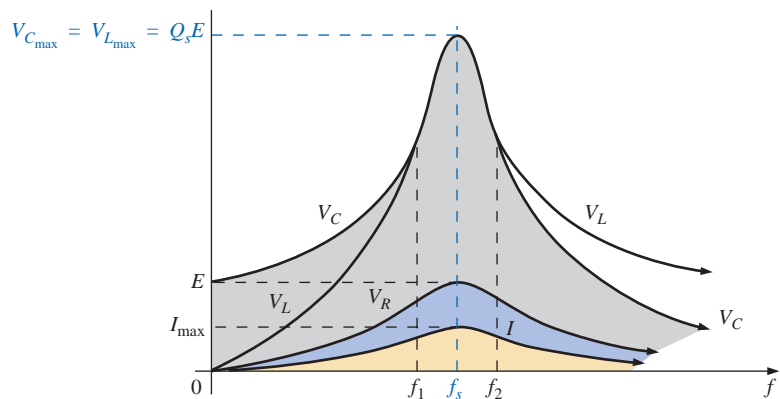


FIG. 20.18

V_R , V_L , V_C , and I for a series resonant circuit where $Q_s \geq 10$.

In review,

1. V_C and V_L are at their maximum values at or near resonance (depending on Q_s).
2. At very low frequencies, V_C is very close to the source voltage and V_L is very close to zero volts, whereas at very high frequencies, V_L approaches the source voltage and V_C approaches zero volts.
3. Both V_R and I peak at the resonant frequency and have the same shape.

20.7 EXAMPLES (SERIES RESONANCE)

EXAMPLE 20.1

- a. For the series resonant circuit of Fig. 20.19, find I , V_R , V_L , and V_C at resonance.
- b. What is the Q_s of the circuit?
- c. If the resonant frequency is 5000 Hz, find the bandwidth.
- d. What is the power dissipated in the circuit at the half-power frequencies?

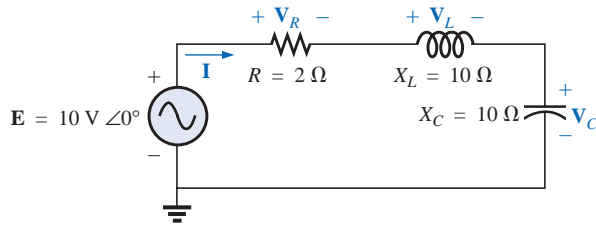


FIG. 20.19
Example 20.1.

Solutions:

- a. $Z_{T_s} = R = 2 \Omega$
- $$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_{T_s}} = \frac{10 \text{ V } \angle 0^\circ}{2 \Omega \angle 0^\circ} = 5 \text{ A } \angle 0^\circ$$
- $$\mathbf{V}_R = \mathbf{E} = 10 \text{ V } \angle 0^\circ$$
- $$\mathbf{V}_L = (I \angle 0^\circ)(X_L \angle 90^\circ) = (5 \text{ A } \angle 0^\circ)(10 \Omega \angle 90^\circ) = 50 \text{ V } \angle 90^\circ$$
- $$\mathbf{V}_C = (I \angle 0^\circ)(X_C \angle -90^\circ) = (5 \text{ A } \angle 0^\circ)(10 \Omega \angle -90^\circ) = 50 \text{ V } \angle -90^\circ$$
- b. $Q_s = \frac{X_L}{R} = \frac{10 \Omega}{2 \Omega} = 5$
- c. $BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{5000 \text{ Hz}}{5} = 1000 \text{ Hz}$
- d. $P_{\text{HPF}} = \frac{1}{2}P_{\text{max}} = \frac{1}{2}I_{\text{max}}^2 R = \left(\frac{1}{2}\right)(5 \text{ A})^2(2 \Omega) = 25 \text{ W}$

EXAMPLE 20.2 The bandwidth of a series resonant circuit is 400 Hz.

- If the resonant frequency is 4000 Hz, what is the value of Q_s ?
- If $R = 10 \Omega$, what is the value of X_L at resonance?
- Find the inductance L and capacitance C of the circuit.

Solutions:

- a. $BW = \frac{f_s}{Q_s}$ or $Q_s = \frac{f_s}{BW} = \frac{4000 \text{ Hz}}{400 \text{ Hz}} = 10$
- b. $Q_s = \frac{X_L}{R}$ or $X_L = Q_s R = (10)(10 \Omega) = 100 \Omega$
- c. $X_L = 2\pi f_s L$ or $L = \frac{X_L}{2\pi f_s} = \frac{100 \Omega}{2\pi(4000 \text{ Hz})} = 3.98 \text{ mH}$
- $$X_C = \frac{1}{2\pi f_s C} \quad \text{or} \quad C = \frac{1}{2\pi f_s X_C} = \frac{1}{2\pi(4000 \text{ Hz})(100 \Omega)}$$
- $$= 0.398 \mu\text{F}$$

EXAMPLE 20.3 A series R - L - C circuit has a series resonant frequency of 12,000 Hz.

- If $R = 5 \Omega$, and if X_L at resonance is 300 Ω , find the bandwidth.
- Find the cutoff frequencies.

Solutions:

- a. $Q_s = \frac{X_L}{R} = \frac{300 \Omega}{5 \Omega} = 60$
- $$BW = \frac{f_s}{Q_s} = \frac{12,000 \text{ Hz}}{60} = 200 \text{ Hz}$$

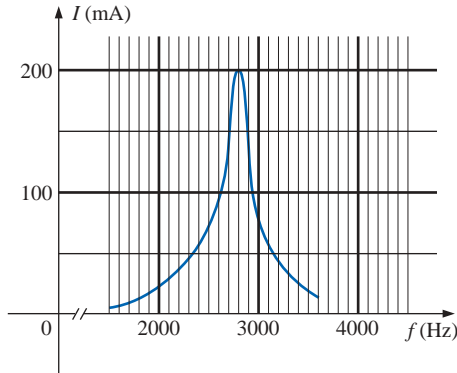


FIG. 20.20
Example 20.4.

b. Since $Q_s \geq 10$, the bandwidth is bisected by f_s . Therefore,

$$f_2 = f_s + \frac{BW}{2} = 12,000 \text{ Hz} + 100 \text{ Hz} = \mathbf{12,100 \text{ Hz}}$$

$$\text{and } f_1 = 12,000 \text{ Hz} - 100 \text{ Hz} = \mathbf{11,900 \text{ Hz}}$$

EXAMPLE 20.4

- Determine the Q_s and bandwidth for the response curve of Fig. 20.20.
- For $C = 101.5 \text{ nF}$, determine L and R for the series resonant circuit.
- Determine the applied voltage.

Solutions:

a. The resonant frequency is 2800 Hz. At 0.707 times the peak value,

$$BW = \mathbf{200 \text{ Hz}}$$

and

$$Q_s = \frac{f_s}{BW} = \frac{2800 \text{ Hz}}{200 \text{ Hz}} = \mathbf{14}$$

$$\text{b. } f_s = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad L = \frac{1}{4\pi^2 f_s^2 C}$$

$$= \frac{1}{4\pi^2 (2.8 \times 10^3 \text{ Hz})^2 (101.5 \times 10^{-9} \text{ F})}$$

$$= \mathbf{31.832 \text{ mH}}$$

$$Q_s = \frac{X_L}{R} \quad \text{or} \quad R = \frac{X_L}{Q_s} = \frac{2\pi(2800 \text{ Hz})(31.832 \times 10^{-3} \text{ H})}{14}$$

$$= \mathbf{40 \Omega}$$

$$\text{c. } I_{\max} = \frac{E}{R} \quad \text{or} \quad E = I_{\max} R$$

$$= (200 \text{ mA})(40 \Omega) = \mathbf{8 \text{ V}}$$

EXAMPLE 20.5 A series R - L - C circuit is designed to resonant at $\omega_s = 10^5 \text{ rad/s}$, have a bandwidth of $0.15\omega_s$, and draw 16 W from a 120-V source at resonance.

- Determine the value of R .
- Find the bandwidth in hertz.
- Find the nameplate values of L and C .
- Determine the Q_s of the circuit.
- Determine the fractional bandwidth.

Solutions:

$$\text{a. } P = \frac{E^2}{R} \quad \text{and} \quad R = \frac{E^2}{P} = \frac{(120 \text{ V})^2}{16 \text{ W}} = \mathbf{900 \Omega}$$

$$\text{b. } f_s = \frac{\omega_s}{2\pi} = \frac{10^5 \text{ rad/s}}{2\pi} = 15,915.49 \text{ Hz}$$

$$BW = 0.15f_s = 0.15(15,915.49 \text{ Hz}) = \mathbf{2387.32 \text{ Hz}}$$

c. Eq. (20.20):

$$BW = \frac{R}{2\pi L} \quad \text{and} \quad L = \frac{R}{2\pi BW} = \frac{900 \Omega}{2\pi(2387.32 \text{ Hz})} = \mathbf{60 \text{ mH}}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad C = \frac{1}{4\pi^2 f_s^2 L} = \frac{1}{4\pi^2 (15,915.49 \text{ Hz})^2 (60 \times 10^{-3} \text{ H})}$$

$$= \mathbf{1.67 \text{ nF}}$$



$$d. Q_s = \frac{X_L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi(15,915.49 \text{ Hz})(60 \text{ mH})}{900 \Omega} = \mathbf{6.67}$$

$$e. \frac{f_2 - f_1}{f_s} = \frac{BW}{f_s} = \frac{1}{Q_s} = \frac{1}{6.67} = \mathbf{0.15}$$

PARALLEL RESONANCE

20.8 PARALLEL RESONANT CIRCUIT

The basic format of the series resonant circuit is a series R - L - C combination in series with an applied voltage source. The parallel resonant circuit has the basic configuration of Fig. 20.21, a parallel R - L - C combination in parallel with an applied current source.

For the series circuit, the impedance was a minimum at resonance, producing a significant current that resulted in a high output voltage for \mathbf{V}_C and \mathbf{V}_L . For the parallel resonant circuit, the impedance is relatively high at resonance, producing a significant voltage for \mathbf{V}_C and \mathbf{V}_L through the Ohm's law relationship ($\mathbf{V}_C = \mathbf{I}\mathbf{Z}_T$). For the network of Fig. 20.21, resonance will occur when $X_L = X_C$, and the resonant frequency will have the same format obtained for series resonance.

If the practical equivalent of Fig. 20.21 had the format of Fig. 20.22, the analysis would be as direct and lucid as that experienced for series resonance. However, in the practical world, the internal resistance of the coil must be placed in series with the inductor, as shown in Fig. 20.22. The resistance R_l can no longer be included in a simple series or parallel combination with the source resistance and any other resistance added for design purposes. Even though R_l is usually relatively small in magnitude compared with other resistance and reactance levels of the network, it does have an important impact on the parallel resonant condition, as will be demonstrated in the sections to follow. In other words, the network of Fig. 20.21 is an ideal situation that can be assumed only for specific network conditions.

Our first effort will be to find a parallel network equivalent (at the terminals) for the series R - L branch of Fig. 20.22 using the technique introduced in Section 15.10. That is,

$$\mathbf{Z}_{R-L} = R_l + jX_L$$

$$\begin{aligned} \text{and } \mathbf{Y}_{R-L} &= \frac{1}{\mathbf{Z}_{R-L}} = \frac{1}{R_l + jX_L} = \frac{R_l}{R_l^2 + X_L^2} - j \frac{X_L}{R_l^2 + X_L^2} \\ &= \frac{1}{\frac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j \left(\frac{R_l^2 + X_L^2}{X_L} \right)} = \frac{1}{R_p} + \frac{1}{jX_{Lp}} \end{aligned}$$

with

$$R_p = \frac{R_l^2 + X_L^2}{R_l} \quad (20.24)$$

and

$$X_{Lp} = \frac{R_l^2 + X_L^2}{X_L} \quad (20.25)$$

as shown in Fig. 20.23.

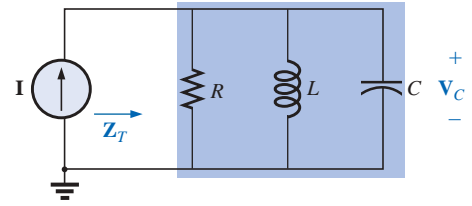


FIG. 20.21

Ideal parallel resonant network.

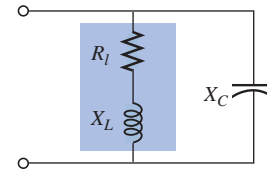


FIG. 20.22

Practical parallel L-C network.

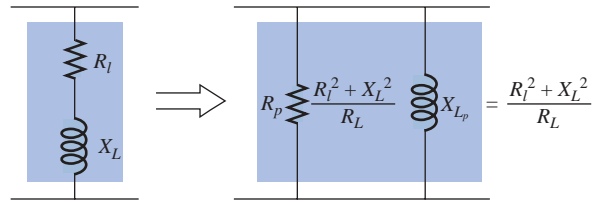


FIG. 20.23

Equivalent parallel network for a series R-L combination.

Redrawing the network of Fig. 20.22 with the equivalent of Fig. 20.23 and a practical current source having an internal resistance R_s will result in the network of Fig. 20.24.

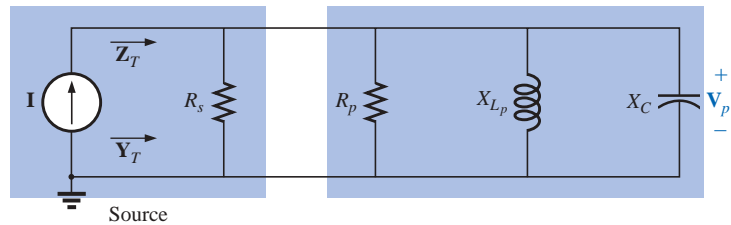


FIG. 20.24

Substituting the equivalent parallel network for the series R-L combination of Fig. 20.22.

If we define the parallel combination of R_s and R_p by the notation

$$R = R_s \parallel R_p \tag{20.26}$$

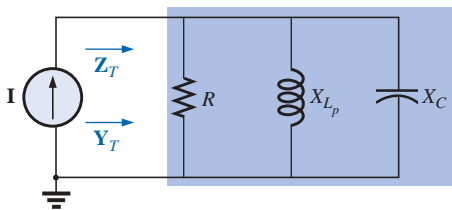


FIG. 20.25

Substituting $R = R_s \parallel R_p$ for the network of Fig. 20.24.

the network of Fig. 20.25 will result. It has the same format as the ideal configuration of Fig. 20.21.

We are now at a point where we can define the resonance conditions for the practical parallel resonant configuration. Recall that for series resonance, the resonant frequency was the frequency at which the impedance was a minimum, the current a maximum, and the input impedance purely resistive, and the network had a unity power factor. For parallel networks, since the resistance R_p in our equivalent model is frequency dependent, the frequency at which maximum V_C is obtained is not the same as required for the unity-power-factor characteristic. Since both conditions are often used to define the resonant state, the frequency at which each occurs will be designated by different subscripts.

Unity Power Factor, f_p

For the network of Fig. 20.25,

$$\begin{aligned} \mathbf{Y}_T &= \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{R} + \frac{1}{jX_{L_p}} + \frac{1}{-jX_C} \\ &= \frac{1}{R} - j\left(\frac{1}{X_{L_p}}\right) + j\left(\frac{1}{X_C}\right) \end{aligned}$$



and

$$\mathbf{Y}_T = \frac{1}{R} + j \left(\frac{1}{X_C} - \frac{1}{X_{L_p}} \right) \quad (20.27)$$

For unity power factor, the reactive component must be zero as defined by

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

Therefore,

$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

and

$$X_{L_p} = X_C \quad (20.28)$$

Substituting for X_{L_p} yields

$$\frac{R_i^2 + X_L^2}{X_L} = X_C \quad (20.29)$$

The resonant frequency, f_p , can now be determined from Eq. (20.29) as follows:

$$R_i^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C} \right) \omega L = \frac{L}{C}$$

or

$$X_L^2 = \frac{L}{C} - R_i^2$$

with

$$2\pi f_p L = \sqrt{\frac{L}{C} - R_i^2}$$

and

$$f_p = \frac{1}{2\pi L} \sqrt{\frac{L}{C} - R_i^2}$$

Multiplying the top and bottom of the factor within the square-root sign by C/L produces

$$f_p = \frac{1}{2\pi L} \sqrt{\frac{1 - R_i^2(C/L)}{C/L}} = \frac{1}{2\pi L \sqrt{C/L}} \sqrt{1 - \frac{R_i^2 C}{L}}$$

and

$$f_p = \frac{1}{2\pi \sqrt{LC}} \sqrt{1 - \frac{R_i^2 C}{L}} \quad (20.30)$$

or

$$f_p = f_s \sqrt{1 - \frac{R_i^2 C}{L}} \quad (20.31)$$

where f_p is the resonant frequency of a parallel resonant circuit (for $F_p = 1$) and f_s is the resonant frequency as determined by $X_L = X_C$ for series resonance. Note that unlike a series resonant circuit, the resonant

frequency f_p is a function of resistance (in this case R_l). Note also, however, the absence of the source resistance R_s in Eqs. (20.30) and (20.31). Since the factor $\sqrt{1 - (R_l^2 C/L)}$ is less than 1, f_p is less than f_s . Recognize also that as the magnitude of R_l approaches zero, f_p rapidly approaches f_s .

Maximum Impedance, f_m

At $f = f_p$ the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of R_p . The frequency at which maximum impedance will occur is defined by f_m and is slightly more than f_p , as demonstrated in Fig. 20.26. The frequency f_m is determined by differentiating (calculus) the general equation for Z_T with respect to frequency and then determining the frequency at which the resulting equation is equal to zero. The algebra is quite extensive and cumbersome and will not be included here. The resulting equation, however, is the following:

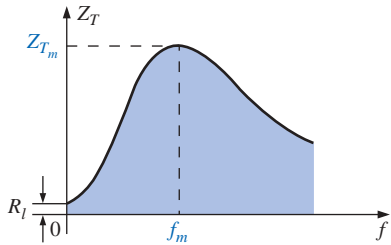


FIG. 20.26
 Z_T versus frequency for the parallel resonant circuit.

$$f_m = f_s \sqrt{1 - \frac{1}{4} \left(\frac{R_l^2 C}{L} \right)} \tag{20.32}$$

Note the similarities with Eq. (20.31). Since the square-root factor of Eq. (20.32) is always more than the similar factor of Eq. (20.31), f_m is always closer to f_s and more than f_p . In general,

$$f_s > f_m > f_p \tag{20.33}$$

Once f_m is determined, the network of Fig. 20.25 can be used to determine the magnitude and phase angle of the total impedance at the resonance condition simply by substituting $f = f_m$ and performing the required calculations. That is,

$$Z_{T_m} = R \parallel X_{L_p} \parallel X_C \quad f = f_m \tag{20.34}$$

20.9 SELECTIVITY CURVE FOR PARALLEL RESONANT CIRCUITS

The Z_T -versus-frequency curve of Fig. 20.26 clearly reveals that a parallel resonant circuit exhibits maximum impedance at resonance (f_m), unlike the series resonant circuit, which experiences minimum resistance levels at resonance. Note also that Z_T is approximately R_l at $f = 0$ Hz since $Z_T = R_s \parallel R_l \cong R_l$.

Since the current I of the current source is constant for any value of Z_T or frequency, the voltage across the parallel circuit will have the same shape as the total impedance Z_T , as shown in Fig. 20.27.

For the parallel circuit, the resonance curve of interest is that of the voltage V_C across the capacitor. The reason for this interest in V_C derives from electronic considerations that often place the capacitor at the input to another stage of a network.

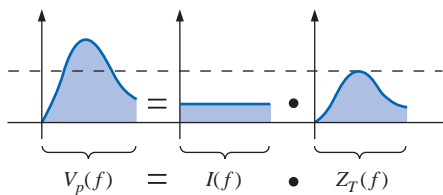


FIG. 20.27
Defining the shape of the $V_p(f)$ curve.

Since the voltage across parallel elements is the same,

$$V_C = V_p = IZ_T \quad (20.35)$$

The resonant value of V_C is therefore determined by the value of Z_{Tm} and the magnitude of the current source I .

The quality factor of the parallel resonant circuit continues to be determined by the ratio of the reactive power to the real power. That is,

$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

where $R = R_s \parallel R_p$, and V_p is the voltage across the parallel branches. The result is

$$Q_p = \frac{R}{X_{L_p}} = \frac{R_s \parallel R_p}{X_{L_p}} \quad (20.36a)$$

or since $X_{L_p} = X_C$ at resonance,

$$Q_p = \frac{R_s \parallel R_p}{X_C} \quad (20.36b)$$

For the ideal current source ($R_s = \infty \Omega$) or when R_s is sufficiently large compared to R_p , we can make the following approximation:

$$R = R_s \parallel R_p \cong R_p$$

and

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = \frac{(R_l^2 + X_L^2)/R_l}{(R_l^2 + X_L^2)/X_L}$$

so that

$$Q_p = \frac{X_L}{R_l} = Q_l \quad (20.37) \quad R_s \gg R_p$$

which is simply the quality factor Q_l of the coil.

In general, the bandwidth is still related to the resonant frequency and the quality factor by

$$BW = f_2 - f_1 = \frac{f_r}{Q_p} \quad (20.38)$$

The cutoff frequencies f_1 and f_2 can be determined using the equivalent network of Fig. 20.25 and the unity power condition for resonance. The half-power frequencies are defined by the condition that the output voltage is 0.707 times the maximum value. However, for parallel resonance with a current source driving the network, the frequency response for the driving point impedance is the same as that for the output voltage. This similarity permits defining each cutoff frequency as the frequency at which the input impedance is 0.707 times its maximum value. Since the maximum value is the equivalent resistance R of Fig. 20.25, the cutoff frequencies will be associated with an impedance equal to $0.707R$ or $(1/\sqrt{2})R$.



Setting the input impedance for the network of Fig. 20.25 equal to this value will result in the following relationship:

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = 0.707R$$

which can be written as

$$\mathbf{Z} = \frac{1}{\frac{1}{R}\left[1 + jR\left(\omega C - \frac{1}{\omega L}\right)\right]} = \frac{R}{\sqrt{2}}$$

or

$$\frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{R}{\sqrt{2}}$$

and finally

$$\frac{1}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} = \frac{1}{\sqrt{2}}$$

The only way the equality can be satisfied is if the magnitude of the imaginary term on the bottom left is equal to 1 because the magnitude of $1 + j1$ must be equal to $\sqrt{2}$.

The following relationship, therefore, defines the cutoff frequencies for the system:

$$R\left(\omega C - \frac{1}{\omega L}\right) = 1$$

Substituting $\omega = 2\pi f$ and rearranging will result in the following quadratic equation:

$$f^2 - \frac{f}{2\pi RC} - \frac{1}{4\pi^2 LC} = 0$$

having the form $af^2 + bf + c = 0$

with $a = 1$ $b = -\frac{1}{2\pi RC}$ and $c = -\frac{1}{4\pi^2 LC}$

Substituting into the equation:

$$f = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

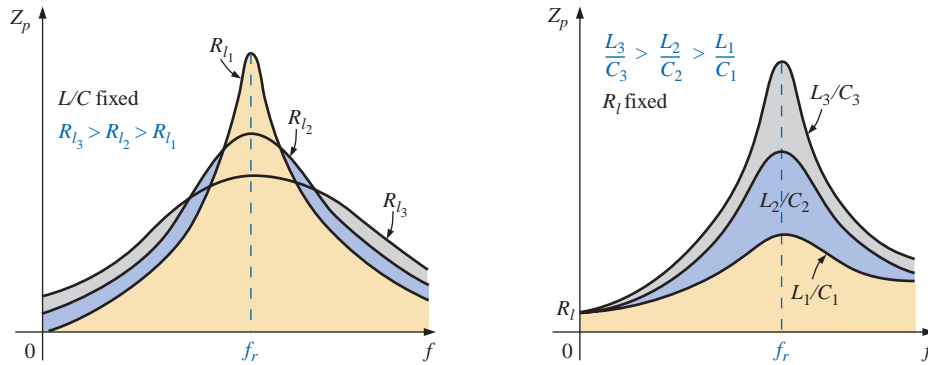
will result in the following after a series of careful mathematical manipulations:

$$f_1 = \frac{1}{4\pi C} \left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right] \quad (20.39a)$$

$$f_2 = \frac{1}{4\pi C} \left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right] \quad (20.39b)$$

Since the term in the brackets of Eq. (20.39a) will always be negative, simply associate f_1 with the magnitude of the result.

The effect of R , L , and C on the shape of the parallel resonance curve, as shown in Fig. 20.28 for the input impedance, is quite similar

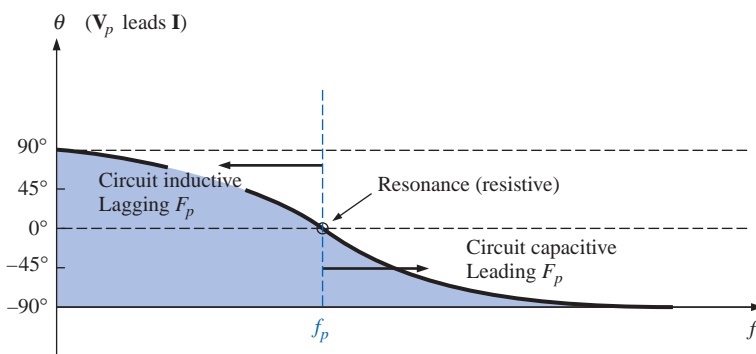

FIG. 20.28

Effect of R , L , and C on the parallel resonance curve.

to their effect on the series resonance curve. Whether or not R_l is zero, the parallel resonant circuit will frequently appear in a network schematic as shown in Fig. 20.28.

At resonance, an increase in R_l or a decrease in the ratio L/C will result in a decrease in the resonant impedance, with a corresponding increase in the current. The bandwidth of the resonance curves is given by Eq. (20.38). For increasing R_l or decreasing L (or L/C for constant C), the bandwidth will increase as shown in Fig. 20.28.

At low frequencies, the capacitive reactance is quite high, and the inductive reactance is low. Since the elements are in parallel, the total impedance at low frequencies will therefore be inductive. At high frequencies, the reverse is true, and the network is capacitive. At resonance (f_p), the network appears resistive. These facts lead to the phase plot of Fig. 20.29. Note that it is the inverse of that appearing for the series resonant circuit because at low frequencies the series resonant circuit was capacitive and at high frequencies it was inductive.


FIG. 20.29

Phase plot for the parallel resonant circuit.

20.10 EFFECT OF $Q_l \geq 10$

The content of the previous section may suggest that the analysis of parallel resonant circuits is significantly more complex than encountered for series resonant circuits. Fortunately, however, this is not the case since, for the majority of parallel resonant circuits, the quality factor of the coil Q_l is sufficiently large to permit a number of approximations that simplify the required analysis.



Inductive Reactance, X_{L_p}

If we expand X_{L_p} as

$$X_{L_p} = \frac{R_i^2 + X_L^2}{X_L} = \frac{R_i^2(X_L)}{X_L(X_L)} + X_L = \frac{X_L}{Q_i^2} + X_L$$

then, for $Q_i \geq 10$, $X_L/Q_i^2 \cong 0$ compared to X_L , and

$$\boxed{X_{L_p} \cong X_L} \quad Q_i \geq 10 \quad (20.40)$$

and since resonance is defined by $X_{L_p} = X_C$, the resulting condition for resonance is reduced to:

$$\boxed{X_L \cong X_C} \quad Q_i \geq 10 \quad (20.41)$$

Resonant Frequency, f_p (Unity Power Factor)

We can rewrite the factor $R_i^2 C/L$ of Eq. (20.31) as

$$\frac{R_i^2 C}{L} = \frac{1}{L} = \frac{1}{(\omega) L} = \frac{1}{\omega L} = \frac{1}{X_L X_C} \frac{R_i^2}{R_i^2}$$

and substitute Eq. (20.41) ($X_L \cong X_C$):

$$\frac{1}{X_L X_C} = \frac{1}{X_L^2} = \frac{1}{R_i^2}$$

Equation (20.31) then becomes

$$\boxed{f_p = f_s \sqrt{1 - \frac{1}{Q_i^2}}} \quad Q_i \geq 10 \quad (20.42)$$

clearly revealing that as Q_i increases, f_p becomes closer and closer to f_s .

For $Q_i \geq 10$,

$$1 - \frac{1}{Q_i^2} \cong 1$$

and

$$\boxed{f_p \cong f_s = \frac{1}{2\pi\sqrt{LC}}} \quad Q_i \geq 10 \quad (20.43)$$

Resonant Frequency, f_m (Maximum V_C)

Using the equivalency $R_i^2 C/L = 1/Q_i^2$ derived for Eq. (20.42), Equation (20.32) will take on the following form:

$$\boxed{f_m \cong f_s \sqrt{1 - \frac{1}{4} \left(\frac{1}{Q_i^2} \right)}} \quad Q_i \geq 10 \quad (20.44)$$

The fact that the negative term under the square root will always be less than that appearing in the equation for f_p reveals that f_m will always be closer to f_s than f_p .

For $Q_l \geq 10$ the negative term becomes very small and can be dropped from consideration, leaving:

$$\boxed{f_m \cong f_s = \frac{1}{2\pi\sqrt{LC}}} \quad Q_l \geq 10 \quad (20.45)$$

In total, therefore, for $Q_l \geq 10$,

$$\boxed{f_p \cong f_m \cong f_s} \quad Q_l \geq 10 \quad (20.46)$$

R_p

$$\begin{aligned} R_p &= \frac{R_l^2 + X_L^2}{R_l} = R_l + \frac{X_L^2}{R_l} \left(\frac{R_l}{R_l} \right) = R_l + \frac{X_L^2}{R_l^2} R_l \\ &= R_l + Q_l^2 R_l = (1 + Q_l^2) R_l \end{aligned}$$

For $Q_l \geq 10$, $1 + Q_l^2 \cong Q_l^2$, and

$$\boxed{R_p \cong Q_l^2 R_l} \quad Q_l \geq 10 \quad (20.47)$$

Applying the approximations just derived to the network of Fig. 20.24 will result in the approximate equivalent network for $Q_l \geq 10$ of Fig. 20.30, which is certainly a lot “cleaner” in general appearance.

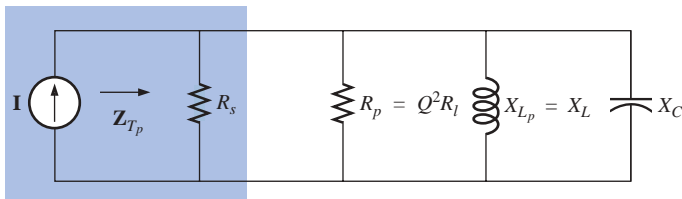


FIG. 20.30

Approximate equivalent circuit for $Q_l \geq 10$.

Substituting $Q_l = \frac{X_L}{R_l}$ into Eq. (20.47),

$$R_p \cong Q_l^2 R_l = \left(\frac{X_L}{R_l} \right)^2 R_l = \frac{X_L^2}{R_l} = \frac{X_L X_C}{R_l} = \frac{2\pi f L}{R_l (2\pi f C)}$$

and

$$\boxed{R_p \cong \frac{L}{R_l C}} \quad Q_l \geq 10 \quad (20.48)$$

Z_{T_p}

The total impedance at resonance is now defined by

$$\boxed{Z_{T_p} \cong R_s \parallel R_p = R_s \parallel Q_l^2 R_l} \quad Q_l \geq 10 \quad (20.49)$$



For an ideal current source ($R_s = \infty \Omega$), or if $R_s \gg R_p$, the equation reduces to

$$Z_{T_p} \cong Q_l^2 R_l \quad Q_l \geq 10, R_s \gg R_p \quad (20.50)$$

Q_p

The quality factor is now defined by

$$Q_p = \frac{R}{X_{L_p}} \cong \frac{R_s \parallel Q_l^2 R_l}{X_L} \quad (20.51)$$

Quite obviously, therefore, R_s does have an impact on the quality factor of the network and the shape of the resonant curves.

If an ideal current source ($R_s = \infty \Omega$) is employed, or if $R_s \gg R_p$,

$$Q_p \cong \frac{R_s \parallel Q_l^2 R_l}{X_L} = \frac{Q_l^2 R_l}{X_L} = \frac{Q_l^2}{X_L/R_l} = \frac{Q_l^2}{Q_l}$$

and

$$Q_p \cong Q_l \quad Q_l \geq 10, R_s \gg R_p \quad (20.52)$$

BW

The bandwidth defined by f_p is

$$BW = f_2 - f_1 = \frac{f_p}{Q_p} \quad (20.53)$$

By substituting Q_p from above and performing a few algebraic manipulations, we can show that

$$BW = f_2 - f_1 \cong \frac{1}{2\pi} \left[\frac{R_l}{L} + \frac{1}{R_s C} \right] \quad (20.54)$$

clearly revealing the impact of R_s on the resulting bandwidth. Of course, if $R_s = \infty \Omega$ (ideal current source):

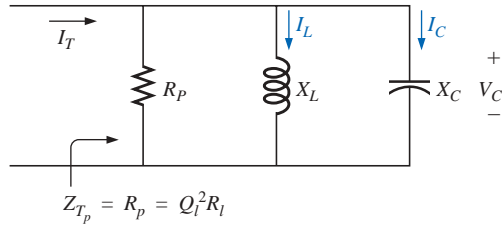
$$BW = f_2 - f_1 \cong \frac{R_l}{2\pi L} \quad R_s = \infty \Omega \quad (20.55)$$

I_L and I_C

A portion of Fig. 20.30 is reproduced in Fig. 20.31, with I_T defined as shown.

As indicated, Z_{T_p} at resonance is $Q_l^2 R_l$. The voltage across the parallel network is, therefore,

$$V_C = V_L = V_R = I_T Z_{T_p} = I_T Q_l^2 R_l$$


FIG. 20.31

Establishing the relationship between I_C and I_L and the current I_T .

The magnitude of the current I_C can then be determined using Ohm's law, as follows:

$$I_C = \frac{V_C}{X_C} = \frac{I_T Q_l^2 R_l}{X_C}$$

Substituting $X_C = X_L$ when $Q_l \geq 10$,

$$I_C = \frac{I_T Q_l^2 R_l}{X_L} = I_T \frac{Q_l^2}{\frac{X_L}{R_l}} = I_T \frac{Q_l^2}{Q_l}$$

and

$$I_C \cong Q_l I_T \quad Q_l \geq 10 \quad (20.56)$$

revealing that the capacitive current is Q_l times the magnitude of the current entering the parallel resonant circuit. For large Q_l , the current I_C can be significant.

A similar derivation results in

$$I_L \cong Q_l I_T \quad Q_l \geq 10 \quad (20.57)$$

Conclusions

The equations resulting from the application of the condition $Q_l \geq 10$ are obviously a great deal easier to apply than those obtained earlier. It is, therefore, a condition that should be checked early in an analysis to determine which approach must be applied. Although the condition $Q_l \geq 10$ was applied throughout, many of the equations are still good approximations for $Q_l < 10$. For instance, if $Q_l = 5$, $X_{L_p} = (X_L/Q_l^2) + X_L = (X_L/25) + X_L = 1.04X_L$, which is very close to X_L . In fact, for $Q_l = 2$, $X_{L_p} = (X_L/4) + X_L = 1.25X_L$, which agreeably is not X_L , but it is only 25% off. In general, be aware that the approximate equations can be applied with good accuracy with $Q_l < 10$. The smaller the level of Q_l , however, the less valid the approximation. The approximate equations are certainly valid for a range of values of $Q_l < 10$ if a rough approximation to the actual response is desired rather than one accurate to the hundredths place.

20.11 SUMMARY TABLE

In an effort to limit any confusion resulting from the introduction of f_p and f_m and an approximate approach dependent on Q_l , summary Table 20.1 was developed. One can always use the equations for any Q_l , but



TABLE 20.1
Parallel resonant circuit ($f_s = 1/(2\pi\sqrt{LC})$).

	Any Q_l	$Q_l \geq 10$	$Q_l \geq 10, R_s \gg Q_l^2 R_l$
f_p	$f_s \sqrt{1 - \frac{R_l^2 C}{L}}$	f_s	f_s
f_m	$f_s \sqrt{1 - \frac{1}{4} \left[\frac{R_l^2 C}{L} \right]}$	f_s	f_s
Z_{T_p}	$R_s \parallel R_p = R_s \parallel \left(\frac{R_l^2 + X_L^2}{R_l} \right)$	$R_s \parallel Q_l^2 R_l$	$Q_l^2 R_l$
Z_{T_m}	$R_s \parallel \mathbf{Z}_{R-L} \parallel \mathbf{Z}_C$	$R_s \parallel Q_l^2 R_l$	$Q_l^2 R_l$
Q_p	$\frac{Z_{T_p}}{X_{L_p}} = \frac{Z_{T_p}}{X_C}$	$\frac{Z_{T_p}}{X_L} = \frac{Z_{T_p}}{X_C}$	Q_l
BW	$\frac{f_p}{Q_p}$ or $\frac{f_m}{Q_p}$	$\frac{f_p}{Q_p} = \frac{f_s}{Q_p}$	$\frac{f_p}{Q_l} = \frac{f_s}{Q_l}$
I_L, I_C	Network analysis	$I_L = I_C = Q_l I_T$	$I_L = I_C = Q_l I_T$

a proficiency in applying the approximate equations defined by Q_l will pay dividends in the long run.

For the future, the analysis of a parallel resonant network might proceed as follows:

1. Determine f_s to obtain some idea of the resonant frequency. Recall that for most situations, f_s , f_m , and f_p will be relatively close to each other.
2. Calculate an approximate Q_l using f_s from above, and compare it to the condition $Q_l \geq 10$. If the condition is satisfied, the approximate approach should be the chosen path unless a high degree of accuracy is required.
3. If Q_l is less than 10, the approximate approach can be applied, but it must be understood that the smaller the level of Q_l , the less accurate the solution. However, considering the typical variations from nameplate values for many of our components and that a resonant frequency to the tenths place is seldom required, the use of the approximate approach for many practical situations is usually quite valid.

20.12 EXAMPLES (PARALLEL RESONANCE)

EXAMPLE 20.6 Given the parallel network of Fig. 20.32 composed of “ideal” elements:

- a. Determine the resonant frequency f_p .
- b. Find the total impedance at resonance.
- c. Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system.

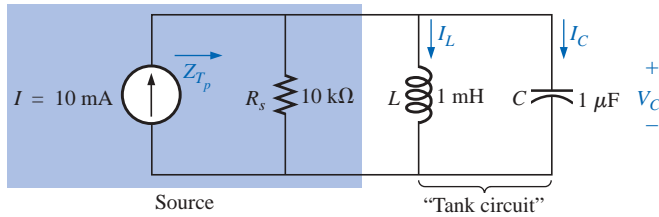


FIG. 20.32
Example 20.6.

- d. Find the voltage V_C at resonance.
e. Determine the currents I_L and I_C at resonance.

Solutions:

- a. The fact that R_l is zero ohms results in a very high $Q_l (= X_L/R_l)$, permitting the use of the following equation for f_p :

$$f_p = f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1 \text{ mH})(1 \mu\text{F})}}$$

$$= \mathbf{5.03 \text{ kHz}}$$

- b. For the parallel reactive elements:

$$\mathbf{Z_L} \parallel \mathbf{Z_C} = \frac{(X_L \angle 90^\circ)(X_C \angle -90^\circ)}{+j(X_L - X_C)}$$

but $X_L = X_C$ at resonance, resulting in a zero in the denominator of the equation and a very high impedance that can be approximated by an open circuit. Therefore,

$$\mathbf{Z_{Tp}} = R_s \parallel \mathbf{Z_L} \parallel \mathbf{Z_C} = R_s = \mathbf{10 \text{ k}\Omega}$$

c. $Q_p = \frac{R_s}{X_{L_p}} = \frac{R_s}{2\pi f_p L} = \frac{10 \text{ k}\Omega}{2\pi(5.03 \text{ kHz})(1 \text{ mH})} = \mathbf{316.41}$

$$BW = \frac{f_p}{Q_p} = \frac{5.03 \text{ kHz}}{316.41} = \mathbf{15.90 \text{ Hz}}$$

Eq. (20.39a):

$$f_1 = \frac{1}{4\pi C} \left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$= \frac{1}{4\pi(1 \mu\text{F})} \left[\frac{1}{10 \text{ k}\Omega} - \sqrt{\frac{1}{(10 \text{ k}\Omega)^2} + \frac{4(1 \mu\text{F})}{1 \text{ mH}}} \right]$$

$$= \mathbf{5.025 \text{ kHz}}$$

Eq. (20.39b):

$$f_2 = \frac{1}{4\pi C} \left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$= \mathbf{5.041 \text{ kHz}}$$

d. $V_C = IZ_{Tp} = (10 \text{ mA})(10 \text{ k}\Omega) = \mathbf{100 \text{ V}}$

e. $I_L = \frac{V_L}{X_L} = \frac{V_C}{2\pi f_p L} = \frac{100 \text{ V}}{2\pi(5.03 \text{ kHz})(1 \text{ mH})} = \frac{100 \text{ V}}{31.6 \Omega} = \mathbf{3.16 \text{ A}}$

$$I_C = \frac{V_C}{X_C} = \frac{100 \text{ V}}{31.6 \Omega} = \mathbf{3.16 \text{ A}} (= Q_p I)$$

Example 20.6 demonstrates the impact of R_s on the calculations associated with parallel resonance. The source impedance is the only factor to limit the input impedance and the level of V_C .

EXAMPLE 20.7 For the parallel resonant circuit of Fig. 20.33 with $R_s = \infty \Omega$:

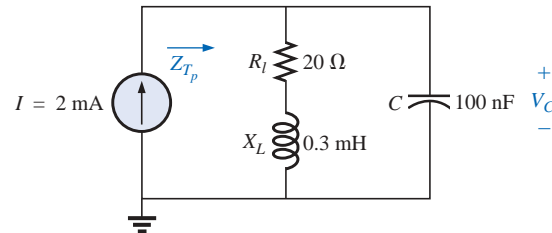


FIG. 20.33

Example 20.7.

- Determine f_s , f_m , and f_p , and compare their levels.
- Calculate the maximum impedance and the magnitude of the voltage V_C at f_m .
- Determine the quality factor Q_p .
- Calculate the bandwidth.
- Compare the above results with those obtained using the equations associated with $Q_l \geq 10$.

Solutions:

$$\text{a. } f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3 \text{ mH})(100 \text{ nF})}} = \mathbf{29.06 \text{ kHz}}$$

$$\begin{aligned} f_m &= f_s \sqrt{1 - \frac{1}{4} \left[\frac{R_l^2 C}{L} \right]} \\ &= (29.06 \text{ kHz}) \sqrt{1 - \frac{1}{4} \left[\frac{(20 \Omega)^2 (100 \text{ nF})}{0.3 \text{ mH}} \right]} \\ &= \mathbf{28.58 \text{ kHz}} \end{aligned}$$

$$\begin{aligned} f_p &= f_s \sqrt{1 - \frac{R_l^2 C}{L}} = (29.06 \text{ kHz}) \sqrt{1 - \left[\frac{(20 \Omega)^2 (100 \text{ nF})}{0.3 \text{ mH}} \right]} \\ &= \mathbf{27.06 \text{ kHz}} \end{aligned}$$

Both f_m and f_p are less than f_s , as predicted. In addition, f_m is closer to f_s than f_p , as forecast. f_m is about 0.5 kHz less than f_s , whereas f_p is about 2 kHz less. The differences among f_s , f_m , and f_p suggest a low- Q network.

- b.** $Z_{T_m} = (R_l + jX_L) \parallel -jX_C$ at $f = f_m$

$$X_L = 2\pi f_m L = 2\pi(28.58 \text{ kHz})(0.3 \text{ mH}) = 53.87 \Omega$$

$$X_C = \frac{1}{2\pi f_m C} = \frac{1}{2\pi(28.58 \text{ kHz})(100 \text{ nF})} = 55.69 \Omega$$

$$R_l + jX_L = 20 \Omega + j53.87 \Omega = 57.46 \Omega \angle 69.63^\circ$$

$$\begin{aligned} Z_{T_m} &= \frac{(57.46 \Omega \angle 69.63^\circ)(55.69 \Omega \angle -90^\circ)}{20 \Omega + j53.87 \Omega - j55.69 \Omega} \\ &= \mathbf{159.34 \Omega \angle -15.17^\circ} \end{aligned}$$

$$V_{C_{\max}} = IZ_{T_m} = (2 \text{ mA})(159.34 \Omega) = \mathbf{318.68 \text{ mV}}$$

c. $R_s = \infty \Omega$; therefore,

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l} \\ = \frac{2\pi(27.06 \text{ kHz})(0.3 \text{ mH})}{20 \Omega} = \frac{51 \Omega}{20 \Omega} = \mathbf{2.55}$$

The low Q confirms our conclusion of part (a). The differences among f_s , f_m , and f_p will be significantly less for higher- Q networks.

d. $BW = \frac{f_p}{Q_p} = \frac{27.06 \text{ kHz}}{2.55} = \mathbf{10.61 \text{ kHz}}$

e. For $Q_l \geq 10$, $f_m = f_p = f_s = \mathbf{29.06 \text{ kHz}}$

$$Q_p = Q_l = \frac{2\pi f_s L}{R_l} = \frac{2\pi(29.06 \text{ kHz})(0.3 \text{ mH})}{20 \Omega} = \mathbf{2.74} \\ \text{(versus 2.55 above)}$$

$$Z_{T_p} = Q_l^2 R_l = (2.74)^2 \cdot 20 \Omega = \mathbf{150.15 \Omega \angle 0^\circ} \\ \text{(versus } 159.34 \Omega \angle -15.17^\circ \text{ above)}$$

$$V_{C_{\max}} = I Z_{T_p} = (2 \text{ mA})(150.15 \Omega) = \mathbf{300.3 \text{ mV}} \\ \text{(versus 318.68 mV above)}$$

$$BW = \frac{f_p}{Q_p} = \frac{29.06 \text{ kHz}}{2.74} = \mathbf{10.61 \text{ kHz}} \\ \text{(versus 10.61 kHz above)}$$

The results reveal that, even for a relatively low Q system, the approximate solutions are still in the ballpark compared to those obtained using the full equations. The primary difference is between f_s and f_p (about 7%), with the difference between f_s and f_m at less than 2%. For the future, using f_s to determine Q_l will certainly provide a measure of Q_l that can be used to determine whether the approximate approach is appropriate.

EXAMPLE 20.8 For the network of Fig. 20.34 with f_p provided:

- Determine Q_l .
- Determine R_p .
- Calculate Z_{T_p} .
- Find C at resonance.
- Find Q_p .
- Calculate the BW and cutoff frequencies.

Solutions:

a. $Q_l = \frac{X_L}{R_l} = \frac{2\pi f_p L}{R_l} = \frac{2\pi(0.04 \text{ MHz})(1 \text{ mH})}{10 \Omega} = \mathbf{25.12}$

b. $Q_l \geq 10$. Therefore,

$$R_p \cong Q_l^2 R_l = (25.12)^2 (10 \Omega) = \mathbf{6.31 \text{ k}\Omega}$$

c. $Z_{T_p} = R_s \parallel R_p = 40 \text{ k}\Omega \parallel 6.31 \text{ k}\Omega = \mathbf{5.45 \text{ k}\Omega}$

d. $Q_l \geq 10$. Therefore,

$$f_p \cong \frac{1}{2\pi\sqrt{LC}}$$

and $C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (0.04 \text{ MHz})^2 (1 \text{ mH})} = \mathbf{15.83 \text{ nF}}$

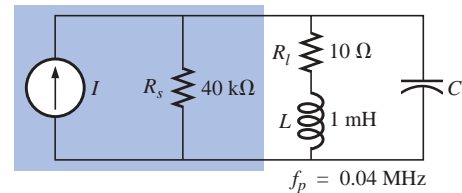


FIG. 20.34
Example 20.8.

e. $Q_l \geq 10$. Therefore,

$$Q_p = \frac{Z_{T_p}}{X_L} = \frac{R_s \parallel Q_l^2 R_l}{2\pi f_p L} = \frac{5.45 \text{ k}\Omega}{2\pi(0.04 \text{ MHz})(1 \text{ mH})} = \mathbf{21.68}$$

f. $BW = \frac{f_p}{Q_p} = \frac{0.04 \text{ MHz}}{21.68} = \mathbf{1.85 \text{ kHz}}$

$$f_1 = \frac{1}{4\pi C} \left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$= \frac{1}{4\pi(15.9 \text{ mF})} \left[\frac{1}{5.45 \text{ k}\Omega} - \sqrt{\frac{1}{(5.45 \text{ k}\Omega)^2} + \frac{4(15.9 \text{ mF})}{1 \text{ mH}}} \right]$$

$$= 5.005 \times 10^6 [183.486 \times 10^{-6} - 7.977 \times 10^{-3}]$$

$$= 5.005 \times 10^6 [-7.794 \times 10^{-3}]$$

$$= \mathbf{39.009 \text{ kHz}} \quad (\text{ignoring the negative sign})$$

$$f_2 = \frac{1}{4\pi C} \left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$= 5.005 \times 10^6 [183.486 \times 10^{-6} + 7.977 \times 10^{-3}]$$

$$= 5.005 \times 10^6 [8.160 \times 10^{-3}]$$

$$= \mathbf{40.843 \text{ kHz}}$$

Note that $f_2 - f_1 = 40.843 \text{ kHz} - 39.009 \text{ kHz} = 1.834 \text{ kHz}$, confirming our solution for the bandwidth above. Note also that the bandwidth is not symmetrical about the resonant frequency, with 991 Hz below and 843 Hz above.

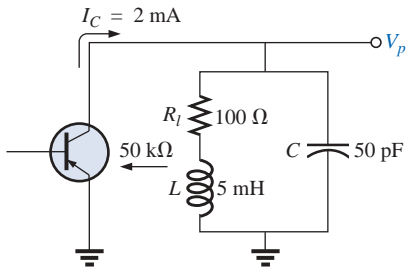


FIG. 20.35
Example 20.9.

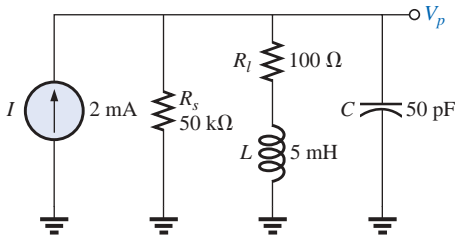


FIG. 20.36
Equivalent network for the transistor configuration of Fig. 20.35.

EXAMPLE 20.9 The equivalent network for the transistor configuration of Fig. 20.35 is provided in Fig. 20.36.

- Find f_p .
- Determine Q_p .
- Calculate the BW .
- Determine V_p at resonance.
- Sketch the curve of V_C versus frequency.

Solutions:

a. $f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \text{ mH})(50 \text{ pF})}} = 318.31 \text{ kHz}$

$$X_L = 2\pi f_s L = 2\pi(318.31 \text{ kHz})(5 \text{ mH}) = 10 \text{ k}\Omega$$

$$Q_l = \frac{X_L}{R_l} = \frac{10 \text{ k}\Omega}{100 \Omega} = 100 > 10$$

Therefore, $f_p = f_s = \mathbf{318.31 \text{ kHz}}$. Using Eq. (20.31) would result in $\cong 318.5 \text{ kHz}$.

b. $Q_p = \frac{R_s \parallel R_p}{X_L}$

$$R_p = Q_l^2 R_l = (100)^2 100 \Omega = 1 \text{ M}\Omega$$

$$Q_p = \frac{50 \text{ k}\Omega \parallel 1 \text{ M}\Omega}{10 \text{ k}\Omega} = \frac{47.62 \text{ k}\Omega}{10 \text{ k}\Omega} = \mathbf{4.76}$$

Note the drop in Q from $Q_l = 100$ to $Q_p = 4.76$ due to R_s .

c. $BW = \frac{f_p}{Q_p} = \frac{318.31 \text{ kHz}}{4.76} = \mathbf{66.87 \text{ kHz}}$

On the other hand,

$$BW = \frac{1}{2\pi} \left(\frac{R_l}{L} + \frac{1}{R_s C} \right) = \frac{1}{2\pi} \left[\frac{100 \Omega}{5 \text{ mH}} + \frac{1}{(50 \text{ k}\Omega)(50 \text{ pF})} \right]$$

$$= \mathbf{66.85 \text{ kHz}}$$

compares very favorably with the above solution.

- d. $V_p = IZ_{T_p} = (2 \text{ mA})(R_s \parallel R_p) = (2 \text{ mA})(47.62 \text{ k}\Omega) = \mathbf{95.24 \text{ V}}$
 e. See Fig. 20.37.

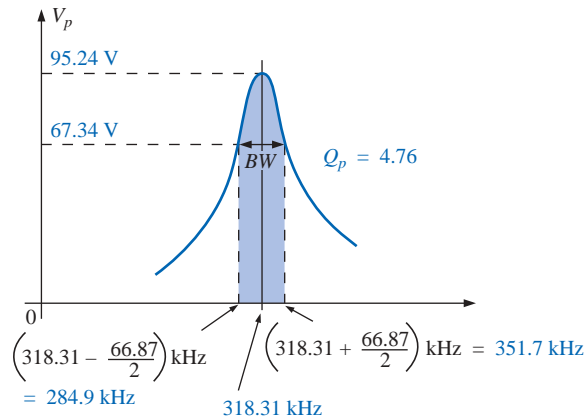


FIG. 20.37

Resonance curve for the network of Fig. 20.36.

EXAMPLE 20.10 Repeat Example 20.9, but ignore the effects of R_s , and compare results.

Solutions:

- a. f_p is the same, **318.31 kHz**.
 b. For $R_s = \infty \Omega$,

$$Q_p = Q_l = \mathbf{100} \quad (\text{versus } 4.76)$$

c. $BW = \frac{f_p}{Q_p} = \frac{318.31 \text{ kHz}}{100} = \mathbf{3.183 \text{ kHz}}$ (versus 66.87 kHz)

d. $Z_{T_p} = R_p = 1 \text{ M}\Omega$ (versus 47.62 k Ω)
 $V_p = IZ_{T_p} = (2 \text{ mA})(1 \text{ M}\Omega) = \mathbf{2000 \text{ V}}$ (versus 95.24 V)

The results obtained clearly reveal that the source resistance can have a significant impact on the response characteristics of a parallel resonant circuit.

EXAMPLE 20.11 Design a parallel resonant circuit to have the response curve of Fig. 20.38 using a 1-mH, 10- Ω inductor and a current source with an internal resistance of 40 k Ω .

Solution:

$$BW = \frac{f_p}{Q_p}$$

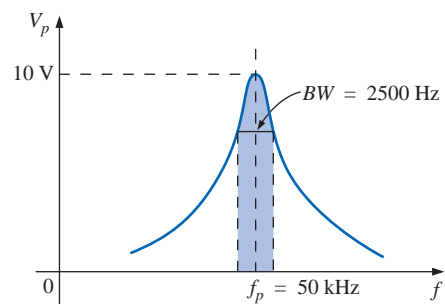


FIG. 20.38
 Example 20.11.



Therefore,

$$Q_p = \frac{f_p}{BW} = \frac{50,000 \text{ Hz}}{2500 \text{ Hz}} = 20$$

$$X_L = 2\pi f_p L = 2\pi(50 \text{ kHz})(1 \text{ mH}) = 314 \Omega$$

and $Q_l = \frac{X_L}{R_l} = \frac{314 \Omega}{10 \Omega} = 31.4$

$$R_p = Q_l^2 R = (31.4)^2(10 \Omega) = 9859.6 \Omega$$

$$Q_p = \frac{R}{X_L} = \frac{R_s \parallel 9859.6 \Omega}{314 \Omega} = 20 \quad (\text{from above})$$

so that $\frac{(R_s)(9859.6)}{R_s + 9859.6} = 6280$

resulting in $R_s = 17.298 \text{ k}\Omega$

However, the source resistance was given as 40 kΩ. We must therefore add a parallel resistor (R') that will reduce the 40 kΩ to approximately 17.298 kΩ; that is,

$$\frac{(40 \text{ k}\Omega)(R')}{40 \text{ k}\Omega + R'} = 17.298 \text{ k}\Omega$$

Solving for R' :

$$R' = 30.481 \text{ k}\Omega$$

The closest commercial value is **30 kΩ**. At resonance, $X_L = X_C$, and

$$X_C = \frac{1}{2\pi f_p C}$$

$$C = \frac{1}{2\pi f_p X_C} = \frac{1}{2\pi(50 \text{ kHz})(314 \Omega)}$$

and $C \cong 0.01 \mu\text{F}$ (commercially available)

$$\begin{aligned} Z_{T_p} &= R_s \parallel Q_l^2 R_l \\ &= 17.298 \text{ k}\Omega \parallel 9859.6 \Omega \\ &= 6.28 \text{ k}\Omega \end{aligned}$$

with $V_p = IZ_{T_p}$

and $I = \frac{V_p}{Z_{T_p}} = \frac{10 \text{ V}}{6.28 \text{ k}\Omega} \cong 1.6 \text{ mA}$

The network appears in Fig. 20.39.

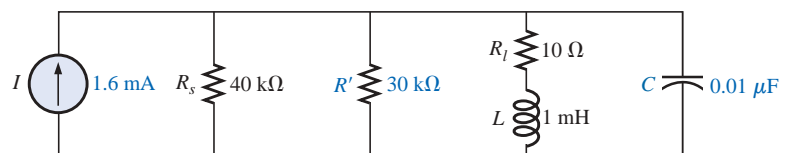


FIG. 20.39

Network designed to meet the criteria of Fig. 20.38.



20.13 APPLICATIONS

Stray Resonance

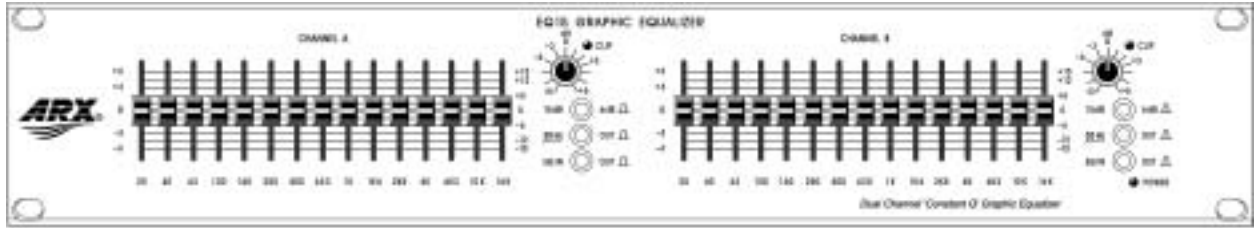
Stray resonance, like stray capacitance and inductance and unexpected resistance levels, can occur in totally unexpected situations and can severely affect the operation of a system. All that is required to produce stray resonance would be, for example, a level of capacitance introduced by parallel wires or copper leads on a printed circuit board, or simply two parallel conductive surfaces with residual charge and inductance levels associated with any conductor or components such as tape recorder heads, transformers, and so on, that provide the elements necessary for a resonance effect. In fact, this resonance effect is a very common effect in the everyday cassette tape recorder. The play/record head is a coil that can act like an inductor and an antenna. Combine this factor with the stray capacitance and real capacitance in the network to form the tuning network, and the tape recorder with the addition of a semiconductor diode can respond like an AM radio. As you plot the frequency response of any transformer, you will normally find a region where the response has a peaking effect (look ahead at Fig. 25.23). This peaking is due solely to the inductance of the coils of the transformer and the stray capacitance between the wires.

In general, any time you see an unexpected peaking in the frequency response of an element or a system, it is normally caused by a resonance condition. If the response has a detrimental effect on the overall operation of the system, a redesign may be in order, or a filter can be added that will block the frequencies that result in the resonance condition. Of course, when you add a filter composed of inductors and/or capacitors, you must be careful that you don't add another unexpected resonance condition. It is a problem that can be properly weighed only by constructing the system and exposing it to the full range of tests.

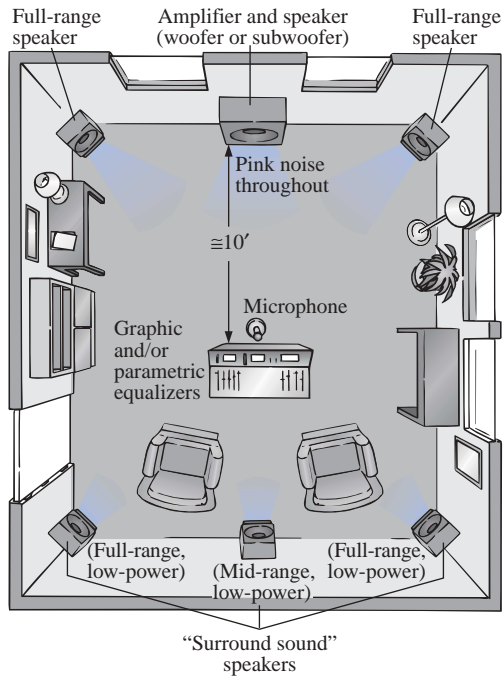
Graphic and Parametric Equalizers

We have all noticed at one time or another that the music we hear in a concert hall doesn't quite sound the same when we play it at home on our entertainment center. Even after we check the specifications of the speakers and amplifiers and find that both are nearly perfect (and the most expensive we can afford), the sound is still not what it should be. In general, we are experiencing the effects of the local environmental characteristics on the sound waves. Some typical problems are hard walls or floors (stone, cement) that will make high frequencies sound louder. Curtains and rugs, on the other hand, will absorb high frequencies. The shape of the room and the placement of the speakers and furniture will also affect the sound that reaches our ears. Another criterion is the echo or reflection of sound that will occur in the room. Concert halls are designed very carefully with their vaulted ceilings and curved walls to allow a certain amount of echo. Even the temperature and humidity characteristics of the surrounding air will affect the quality of the sound. It is certainly impossible, in most cases, to redesign your listening area to match a concert hall, but with the proper use of electronic systems you can develop a response that will have all the qualities that you can expect from a home entertainment center.

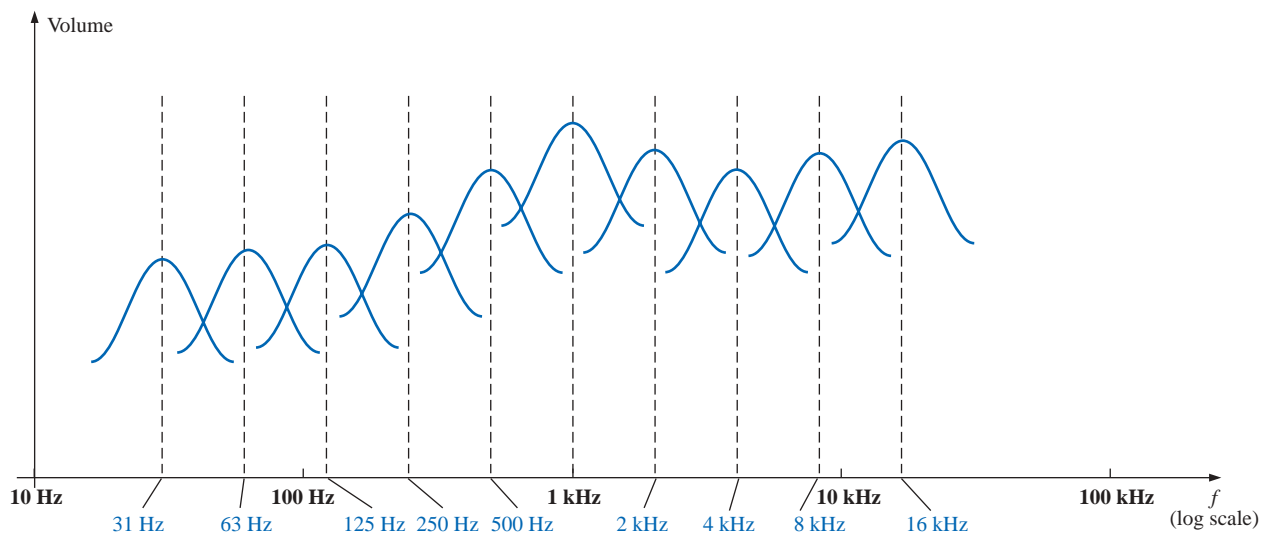
For a quality system a number of steps can be taken: *characterization and digital delay (surround sound)* and *proper speaker and ampli-*



(a)



(b)



(c)

FIG. 20.40

(a) Dual-channel 15-band “Constant Q ” graphic equalizer (Courtesy of ARX Systems.); (b) setup; (c) frequency response.

fier selection and placement. Characterization is a process whereby a thorough sound absorption check of the room is performed and the frequency response determined. A *graphic equalizer* such as appearing in Fig. 20.40(a) is then used to make the response “flat” for the full range of frequencies. In other words, the room is made to appear as though all the frequencies receive equal amplification in the listening area. For instance, if the room is fully carpeted with full draping curtains, there will be a lot of high-frequency absorption, requiring that the high frequencies have additional amplification to match the sound levels of the mid and low frequencies. To *characterize* the typical rectangular-shaped room, a setup such as shown in Fig. 20.40(b) may be used. The amplifier and speakers are placed in the center of one wall, with additional speakers in the corners of the room facing the reception area. A mike is then placed in the reception area about 10 ft from the amplifier and centered between the two other speakers. A *pink noise* will then be sent out from a spectrum analyzer (often an integral part of the graphic equalizer) to the amplifier and speakers. Pink noise is actually a square-wave signal whose amplitude and frequency can be controlled. A square-wave signal was chosen because a Fourier breakdown of a square-wave signal will result in a broad range of frequencies for the system to check. You will find in Chapter 24 that a square wave can be constructed of an infinite series of sine waves of different frequencies. Once the proper volume of pink noise is established, the spectrum analyzer can be used to set the response of each slide band to establish the desired flat response. The center frequencies for the slides of the graphic equalizer of Fig. 20.40(a) are provided in Fig. 20.40(c), along with the frequency response for a number of adjoining frequencies evenly spaced on a logarithmic scale. Note that each center frequency is actually the resonant frequency for that slide. The design is such that each slide can control the volume associated with that frequency, but the bandwidth and frequency response stay fairly constant. A good spectrum analyzer will have each slide set against a decibel (dB) scale (decibels will be discussed in detail in Chapter 23). The decibel scale simply establishes a scale for the comparison of audio levels. At a normal listening level, usually a change of about 3 dB is necessary for the audio change to be detectable by the human ear. At low levels of sound, a 2-dB change may be detectable, but at loud sounds probably a 4-dB change would be necessary for the change to be noticed. These are not strict laws but simply rules of thumb commonly used by audio technicians. For the room in question, the mix of settings may be as shown in Fig. 20.40(c). Once set, the slides are not touched again. A flat response has been established for the room for the full audio range so that every sound or type of music is covered.

A *parametric equalizer* such as appearing in Fig. 20.41 is similar to a *graphic equalizer*, but instead of separate controls for the individual frequency ranges, it uses three basic controls over three or four broader frequency ranges. The typical controls—the *gain*, *center frequency*, and

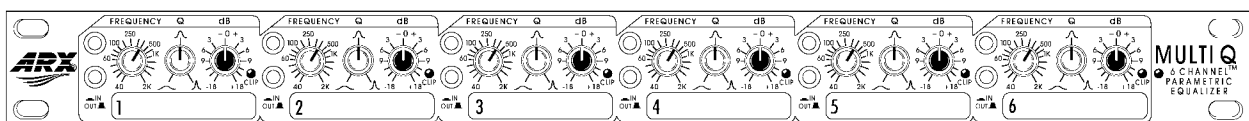


FIG. 20.41

Six-channel parametric equalizer. (Courtesy of ARX Systems.)



bandwidth—are typically available for the *low*-, *mid*-, and *high-frequency* ranges. Each is fundamentally an independent control; that is, a change in one can be made without affecting the other two. For the parametric equalizer of Fig. 20.41, each of the six channels has a frequency control switch which, in conjunction with the $f \times 10$ switch, will give a range of center frequencies from 40 Hz through 16 kHz. It has controls for BW (“ Q ”) from 3 octaves to $\frac{1}{20}$ octave, and ± 18 dB cut and boost. Some like to refer to the parametric equalizer as a *sophisticated tone control* and will actually use them to enrich the sound after the flat response has been established by the graphic equalizer. The effect achieved with a standard tone control knob is sometimes referred to as “boring” compared to the effect established by a good parametric equalizer, primarily because the former can control only the volume and not the bandwidth or center frequency. In general, graphic equalizers establish the important *flat response* while parametric equalizers are adjusted to provide the *type* and *quality* of sound you like to hear. You can “notch out” the frequencies that bother you and remove tape “hiss” and the “sharpness” often associated with CDs.

One characteristic of concert halls that is more difficult to fake is the fullness of sound that concert halls are able to provide. In the concert hall you have the direct sound from the instruments and the reflection of sound off the walls and the vaulted ceilings which were all carefully designed expressly for this purpose. Any reflection results in a delay in the sound waves reaching the ear, creating the fullness effect. Through digital delay, speakers can be placed to the back and side of a listener to establish the surround sound effect. In general, the delay speakers are much lower in wattage, with 20-W speakers typically used with a 100-W system. The echo response is one reason that people often like to play their stereos louder than they should for normal hearing. By playing the stereo louder, they create more echo and reflection off the walls, bringing into play some of the fullness heard at concert halls.

It is probably safe to say that any system composed of quality components, a graphic and parametric equalizer, and surround sound will have all the components necessary to have a quality reproduction of the concert hall effect.

20.14 COMPUTER ANALYSIS

PSpice

Series Resonance This chapter provides an excellent opportunity to demonstrate what computer software programs can do for us. Imagine having to plot a detailed resonance curve with all the calculations required for each frequency. At every frequency, the reactance of the inductive and capacitive elements changes, and the phasor operations would have to be repeated—a long and arduous task. However, with PSpice, taking a few moments to enter the circuit and establish the desired simulation will result in a detailed plot in a few seconds that can have plot points every microsecond!

For the first time, the horizontal axis will be in the frequency domain rather than in the time domain as in all the previous plots. For the series resonant circuit of Fig. 20.42, the magnitude of the source was chosen to produce a maximum current of $I = 400 \text{ mV}/40 \Omega = 10 \text{ mA}$ at resonance, and the reactive elements will establish a resonant frequency of

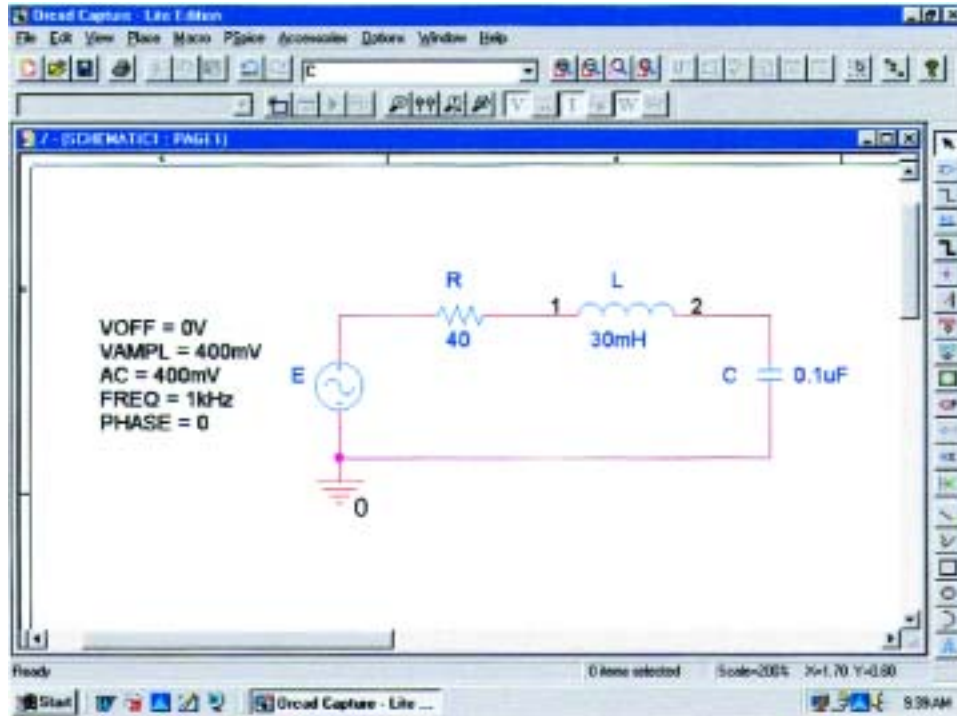


FIG. 20.42

Series resonant circuit to be analyzed using PSpice.

$$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(30 \text{ mH})(0.1\mu\text{F})}} \cong 2.91 \text{ kHz}$$

The quality factor is

$$Q_l = \frac{X_L}{R_l} = \frac{546.64 \Omega}{40 \Omega} \cong 13.7$$

which is relatively high and should give us a nice sharp response.

The bandwidth is

$$BW = \frac{f_s}{Q_l} = \frac{2.91 \text{ kHz}}{13.7} \cong 212 \text{ Hz}$$

which will be verified using our cursor options.

For the ac source, **VSIN** was chosen again. All the parameters were set by double-clicking on the source symbol and entering the values in the **Property Editor** dialog box. For each, **Name and Value** was selected under **Display** followed by **Apply** before leaving the dialog box.

In the **Simulation Settings** dialog box, **AC Sweep/Noise** was selected, and the **Start Frequency** was set at 1 kHz, the **End Frequency** at 10 kHz, and the **Points/Decade** at 10,000. The **Logarithmic scale** and **Decade** settings remain at their default values. The 10,000 for **Points/Decade** was chosen to ensure a number of data points near the peak value. When the **SCHEMATIC1** screen of Fig. 20.43 appears, **Trace-Add Trace-I(R)-OK** will result in a logarithmic plot that peaks just to the left of 3 kHz. The spacing between grid lines on the **X-axis** should be increased, so **Plot-Axis Settings-X Grid-unable Automatic Spacing-Log-0.1-OK** is implemented. Next, select the **Toggle cursor**

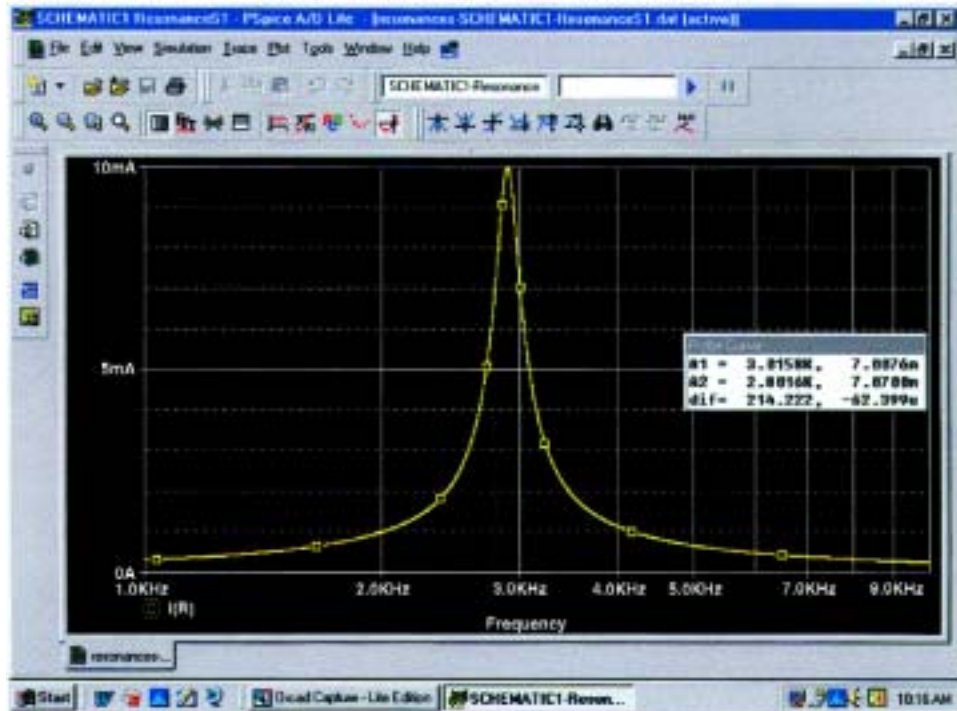


FIG. 20.43

Resonance curve for the current of the circuit of Fig. 20.42.

icon, and with a right click of the mouse move the right cursor as close to 7.07 mA as possible (0.707 of the peak value to define the bandwidth) to obtain **A1** with a frequency of 3.02 kHz at a level of 7.01 mA—the best we can do with the 10,000 data points per decade. Now do a left click, and place the left cursor as close to the same level as possible. The result is 2.8 kHz at a level of 7.07 mA for **A2**. The cursors were set in the order above to obtain a positive answer for the difference of the two as appearing in the third line of the **Probe Cursor** box. The resulting 214.22 Hz is an excellent match with the calculated value of 212 Hz.

Parallel Resonance Let us now investigate the parallel resonant circuit of Fig. 20.33 and compare the results with the longhand solution. The network appears in Fig. 20.44 using **ISRC** as the ac source voltage. Under the **Property Editor** heading, the following values were set: **DC** = 0 A, **AC** = 2 mA, and **TRAN** = 0. Under **Display, Do Not Display** was selected for both **DC** and **TRAN** since they do not play a part in our analysis. In the **Simulation Settings** dialog box, **AC Sweep/Noise** was selected, and the **Start Frequency** was selected as 10 kHz since we know that it will resonate near 30 kHz. The **End Frequency** was chosen as 100 kHz for a first run to see the results. The **Points/Decade** was set at 10,000 to ensure a good number of data points for the peaking region. After simulation, **Trace-Add Trace-V(C:1)-OK** resulted in the plot of Fig. 20.45 with a resonant frequency near 30 kHz. The selected range appears to be a good one, but the initial plot needed more grid lines on the *x*-axis, so **Plot-Axis Settings-X-Grid-unenable Automatic-Spacing-Log-0.1-OK** was used to obtain a grid line at 10-kHz intervals. Next the **Toggle cursor** pad was selected and a left-click

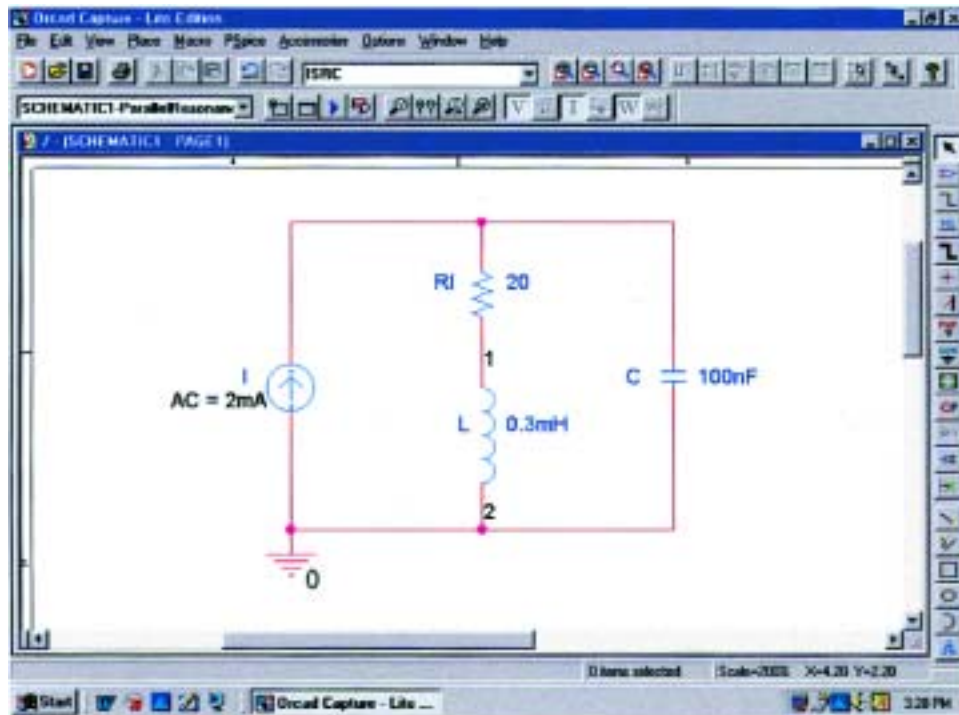


FIG. 20.44

Parallel resonant network to be analyzed using PSpice.

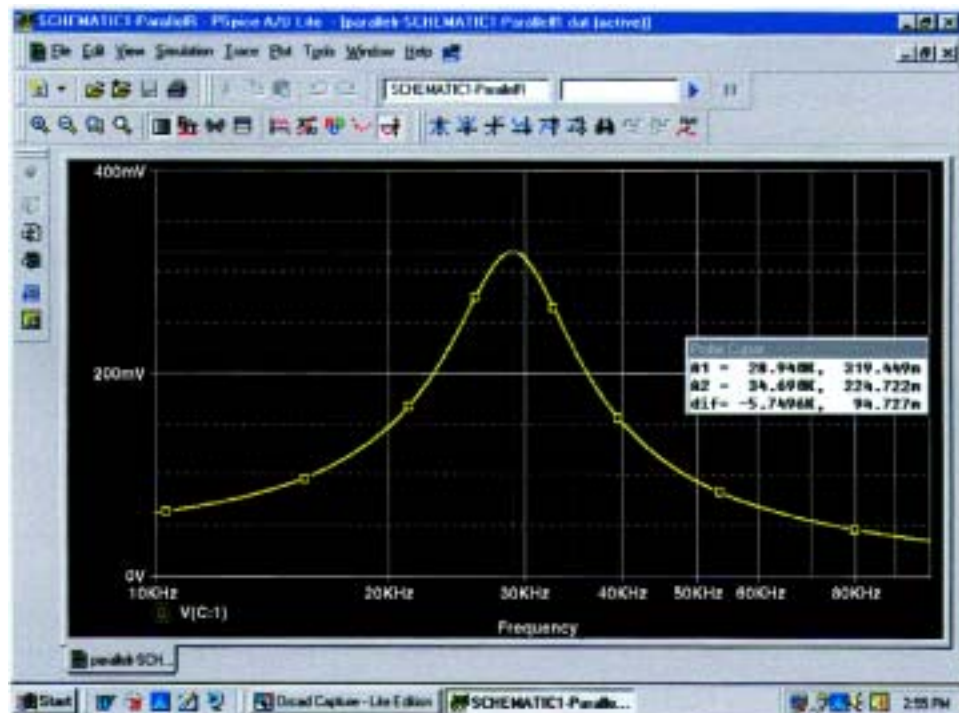


FIG. 20.45

Resonance curve for the voltage across the capacitor of Fig. 20.44.

cursor established on the screen. The **Cursor Peak** pad was then chosen to find the peak value of the curve. The result was $A1 = 319.45$ mV at 28.94 kHz which is a very close match with the calculated value of 318.68 mV at 28.57 kHz for the maximum value of V_C . The bandwidth is defined at a level of $0.707(319.45 \text{ mV}) = 225.85$ mV. Using the right-click cursor, we find that the closest we can come is 224.72 mV for the 10,000 points of data per decade. The resulting frequency is 34.69 kHz as shown in the **Probe Cursor** box of Fig. 20.45.

We can now use the left-click cursor to find the same level to the left of the peak value so that we can determine the bandwidth. The closest that the left-click cursor can come to 225.85 mV is 224.96 mV at a frequency of 23.97 kHz. The bandwidth will then appear as 10.72 kHz in the **Probe Cursor** box, comparing very well with the longhand solution of 10.68 kHz in Example 20.7.

It would now be interesting to look at the phase angle of the voltage across the parallel network to find the frequency when the network appears resistive and the phase angle is 0° . First use **Trace-Delete All Traces**, and call up **P(V(C:1))** followed by **OK**. The result is the plot of Fig. 20.46, revealing that the phase angle is close to -90° at very high frequencies as the capacitive element with its decreasing reactance takes over the characteristics of the parallel network. At 10 kHz the inductive element has a lower reactance than the capacitive element, and the network has a positive phase angle. Using the cursor option, we can move the left click along the horizontal axis until the phase angle is at its minimum value. As shown in Fig. 20.46, the smallest angle available with the determined data points is $49.86 \text{ mdegrees} = 0.05^\circ$ which is certainly very close to 0° . The corresponding frequency is 27.046 kHz which is essentially an exact match with the longhand solution of

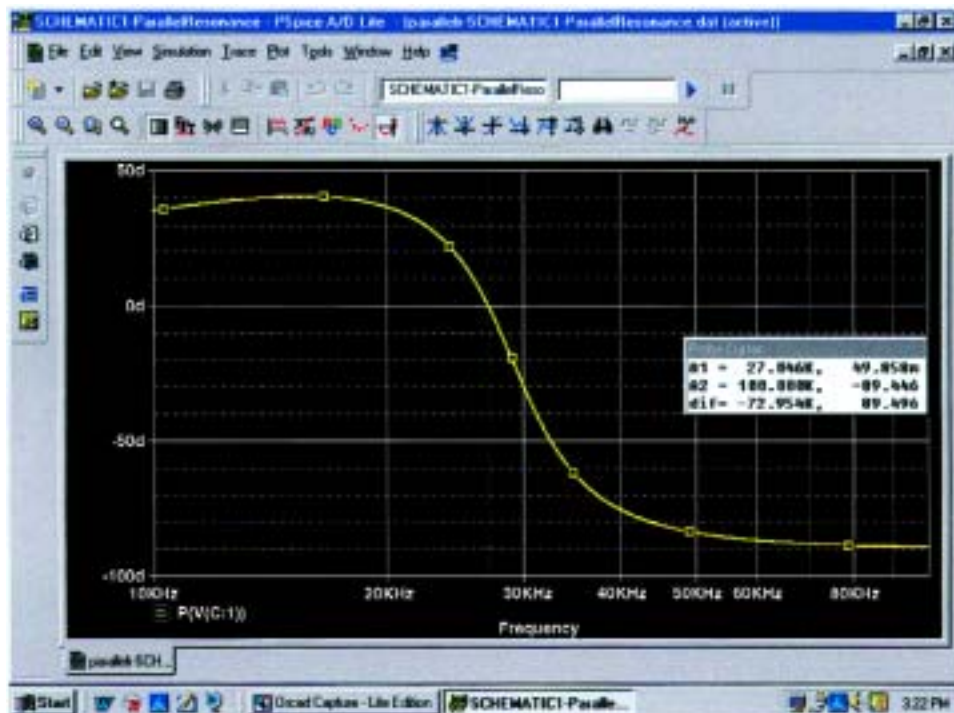


FIG. 20.46

Phase plot for the voltage v_C for the parallel resonant network of Fig. 20.44.

27.051 kHz. Clearly, therefore, the frequency at which the phase angle is zero and the total impedance appears resistive is less than the frequency at which the output voltage is a maximum.

Electronics Workbench

The results of Example 20.9 will now be confirmed using Electronics Workbench. The network of Fig. 20.36 will appear as shown in Fig. 20.47 after all the elements have been placed as described in earlier chapters. In particular, note that the frequency assigned to the 2-mA ac current source is 100 kHz. Since we have some idea that the resonant frequency is a few hundred kilohertz, it seemed appropriate that the starting frequency for the plot begin at 100 kHz and extend to 1 MHz. Also, be sure that the **AC Magnitude** is set to 2 mA in the **Analysis Setup** within the **AC Current** dialog box.

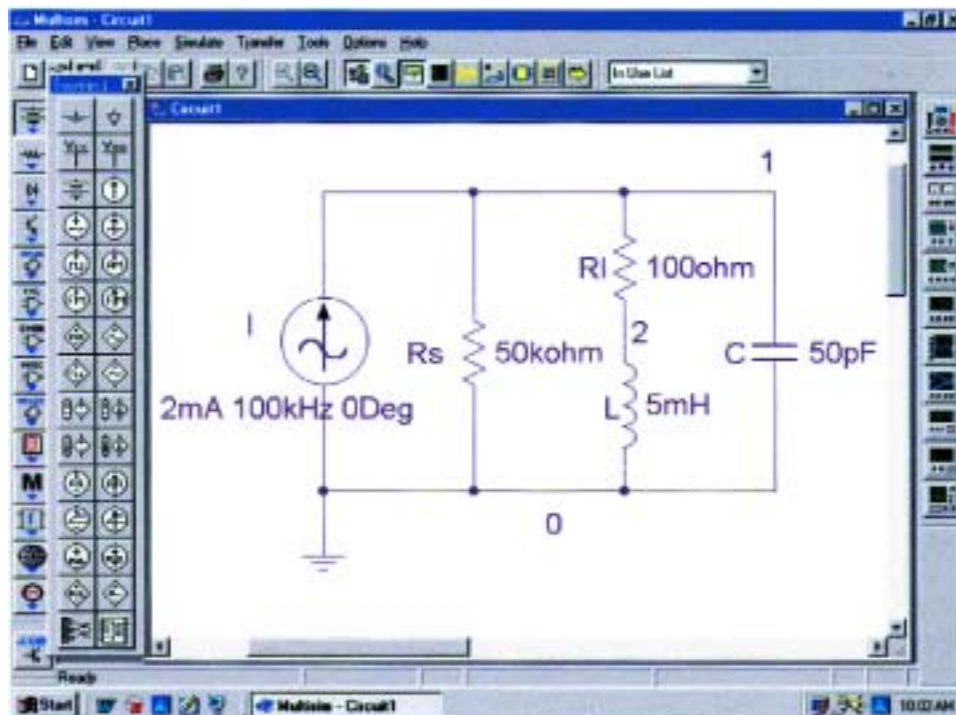


FIG. 20.47

Using Electronics Workbench to confirm the results of Example 20.9.

For simulation, the sequence **Simulate-Analyses-AC Analysis** is first selected to obtain the **AC Analysis** dialog box. The **Start frequency** is set at 100 kHz, and the **Stop frequency** at 1 MHz; **Sweep type** is **Decade**; **Number of points per decade** is 1000; and the **Vertical scale** is **Linear**. Under **Output variables**, node number 1 is selected as a **Variable for analysis** followed by **Simulate** to run the program. The results are the magnitude and phase plots of Fig. 20.48. Starting with the **Voltage** plot, the **Show/Hide Grid** key, **Show/Hide Legend** key, and **Show/Hide Cursors** key are selected. You will immediately note under the **AC Analysis** cursor box that the maximum value is 95.24 V and the minimum value is 6.94 V. By moving the cursor until we reach 95.24 V (**y1**), the resonant frequency can be found. As shown

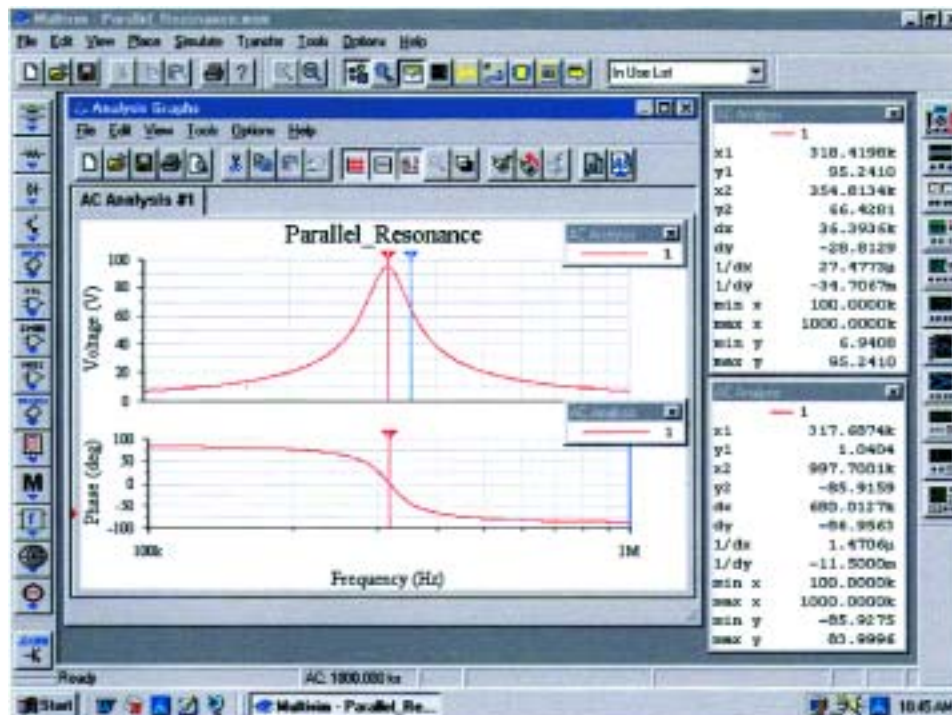


FIG. 20.48

Magnitude and phase plots for the voltage v_C of the network of Fig. 20.47.

in the top cursor dialog box of Fig. 20.48, this is achieved at 318.42 kHz (**x1**). The other (blue) cursor can be used to define the high cutoff frequency for the bandwidth by first calculating the 0.707 level of the output voltage. The result is $0.707(95.24 \text{ V}) = 67.33 \text{ V}$. The closest we can come to this level with the cursor is 66.43 V (**y2**) which defines a frequency of 354.81 kHz (**x2**). If we now use the red cursor to find the corresponding level below the resonant frequency, we will find a level of 67.49 V (**y1**) at 287.08 kHz (**x1**). The resulting bandwidth is therefore $354.81 \text{ kHz} - 287.08 \text{ kHz} = 67.73 \text{ kHz}$.

It would now be interesting to determine the resonant frequency if we define resonance as that frequency that results in a phase angle of 0° for the output voltage. By repeating the process described above for the phase plot, we can set the red cursor as close to 0° as possible. The result is 1.04° (**y1**) at 317.69 kHz (**x1**), clearly revealing that the resonant frequency defined by the phase angle is less than that defined by the peak voltage. However, with a Q_I of about 100, the difference of 0.73 kHz (less than 1 kHz) is not significant. Also note that when the second cursor was set on approximately 1 MHz (997.7 kHz), the phase angle of -85.91° is very close to that of a pure capacitor. The shorting effect of a capacitor at high frequencies has taken over the characteristics of the parallel resonant circuit.

Again, the computer solution was a very close match with the long-hand solution of Example 20.9 with a perfect match of 95.24 V for the peak value and only a small difference in bandwidth with 66.87 kHz in Example 20.9 and 67.73 kHz here. For the high cutoff frequency, the computer generated a result of 354.8 kHz, while the theoretical solution was 351.7 kHz. For the low cutoff frequency, the computer responded with 287.08 kHz compared to a theoretical solution of 284 kHz.



PROBLEMS

SECTIONS 20.2 THROUGH 20.7 Series Resonance

- Find the resonant ω_s and f_s for the series circuit with the following parameters:
 - $R = 10 \Omega$, $L = 1 \text{ H}$, $C = 16 \mu\text{F}$
 - $R = 300 \Omega$, $L = 0.5 \text{ H}$, $C = 0.16 \mu\text{F}$
 - $R = 20 \Omega$, $L = 0.28 \text{ mH}$, $C = 7.46 \mu\text{F}$
- For the series circuit of Fig. 20.49:
 - Find the value of X_C for resonance.
 - Determine the total impedance of the circuit at resonance.
 - Find the magnitude of the current I .
 - Calculate the voltages V_R , V_L , and V_C at resonance. How are V_L and V_C related? How does V_R compare to the applied voltage E ?
 - What is the quality factor of the circuit? Is it a high- or low- Q circuit?
 - What is the power dissipated by the circuit at resonance?
- For the series circuit of Fig. 20.50:
 - Find the value of X_L for resonance.
 - Determine the magnitude of the current I at resonance.
 - Find the voltages V_R , V_L , and V_C at resonance, and compare their magnitudes.
 - Determine the quality factor of the circuit. Is it a high- or low- Q circuit?
 - If the resonant frequency is 5 kHz, determine the value of L and C .
 - Find the bandwidth of the response if the resonant frequency is 5 kHz.
 - What are the low and high cutoff frequencies?
- For the circuit of Fig. 20.51:
 - Find the value of L in millihenries if the resonant frequency is 1800 Hz.
 - Calculate X_L and X_C . How do they compare?
 - Find the magnitude of the current I_{rms} at resonance.
 - Find the power dissipated by the circuit at resonance.
 - What is the apparent power delivered to the system at resonance?
 - What is the power factor of the circuit at resonance?
 - Calculate the Q of the circuit and the resulting bandwidth.
 - Find the cutoff frequencies, and calculate the power dissipated by the circuit at these frequencies.
- Find the bandwidth of a series resonant circuit having a resonant frequency of 6000 Hz and a Q_s of 15.
 - Find the cutoff frequencies.
 - If the resistance of the circuit at resonance is 3Ω , what are the values of X_L and X_C in ohms?
 - What is the power dissipated at the half-power frequencies if the maximum current flowing through the circuit is 0.5 A?
- A series circuit has a resonant frequency of 10 kHz. The resistance of the circuit is 5Ω , and X_C at resonance is 200Ω .
 - Find the bandwidth.

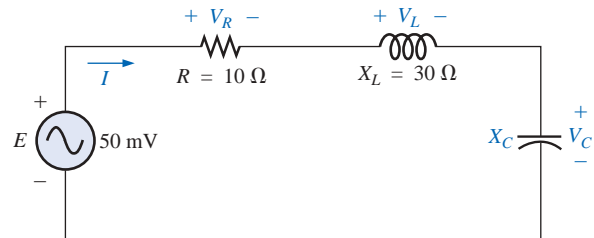


FIG. 20.49
Problem 2.

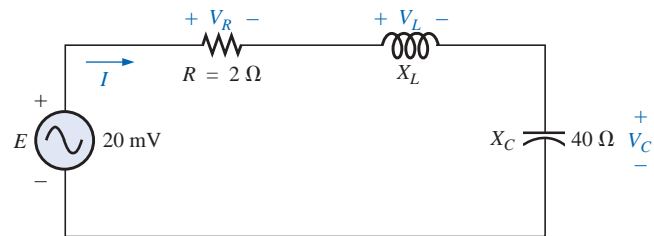


FIG. 20.50
Problem 3.

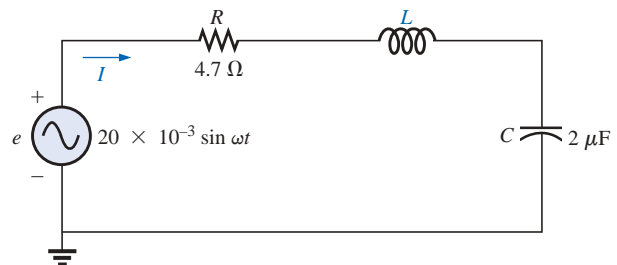


FIG. 20.51
Problem 4.



- b. Find the cutoff frequencies.
 - c. Find Q_s .
 - d. If the input voltage is $30\text{ V} \angle 0^\circ$, find the voltage across the coil and capacitor in phasor form.
 - e. Find the power dissipated at resonance.
7. a. The bandwidth of a series resonant circuit is 200 Hz. If the resonant frequency is 2000 Hz, what is the value of Q_s for the circuit?
 - b. If $R = 2\ \Omega$, what is the value of X_L at resonance?
 - c. Find the value of L and C at resonance.
 - d. Find the cutoff frequencies.
 8. The cutoff frequencies of a series resonant circuit are 5400 Hz and 6000 Hz.
 - a. Find the bandwidth of the circuit.
 - b. If Q_s is 9.5, find the resonant frequency of the circuit.
 - c. If the resistance of the circuit is $2\ \Omega$, find the value of X_L and X_C at resonance.
 - d. Find the value of L and C at resonance.
 - *9. Design a series resonant circuit with an input voltage of $5\text{ V} \angle 0^\circ$ to have the following specifications:
 - a. A peak current of 500 mA at resonance
 - b. A bandwidth of 120 Hz
 - c. A resonant frequency of 8400 Hz
 Find the value of L and C and the cutoff frequencies.
 - *10. Design a series resonant circuit to have a bandwidth of 400 Hz using a coil with a Q_l of 20 and a resistance of $2\ \Omega$. Find the values of L and C and the cutoff frequencies.
 - *11. A series resonant circuit is to resonate at $\omega_s = 2\pi \times 10^6$ rad/s and draw 20 W from a 120-V source at resonance. If the fractional bandwidth is 0.16:
 - a. Determine the resonant frequency in hertz.
 - b. Calculate the bandwidth in hertz.
 - c. Determine the values of R , L , and C .
 - d. Find the resistance of the coil if $Q_l = 80$.
 - *12. A series resonant circuit will resonate at a frequency of 1 MHz with a fractional bandwidth of 0.2. If the quality factor of the coil at resonance is 12.5 and its inductance is $100\ \mu\text{H}$, determine the following:
 - a. The resistance of the coil.
 - b. The additional resistance required to establish the indicated fractional bandwidth.
 - c. The required value of capacitance.

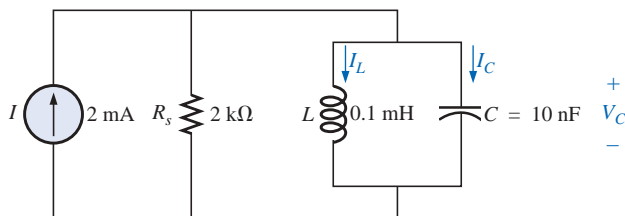


FIG. 20.52
Problem 13.

SECTIONS 20.8 THROUGH 20.12 Parallel Resonance

13. For the “ideal” parallel resonant circuit of Fig. 20.52:
 - a. Determine the resonant frequency (f_p).
 - b. Find the voltage V_C at resonance.
 - c. Determine the currents I_L and I_C at resonance.
 - d. Find Q_p .

14. For the parallel resonant network of Fig. 20.53:
- Calculate f_s .
 - Determine Q_l using $f = f_s$. Can the approximate approach be applied?
 - Determine f_p and f_m .
 - Calculate X_L and X_C using f_p . How do they compare?
 - Find the total impedance at resonance (f_p).
 - Calculate V_C at resonance (f_p).
 - Determine Q_p and the BW using f_p .
 - Calculate I_L and I_C at f_p .

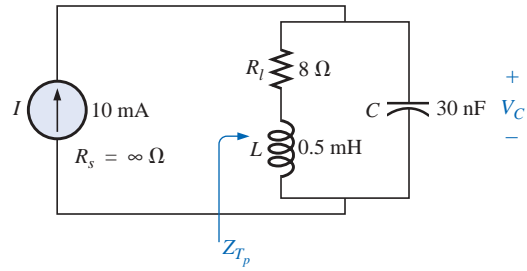


FIG. 20.53
Problem 14.

15. Repeat Problem 14 for the network of Fig. 20.54.

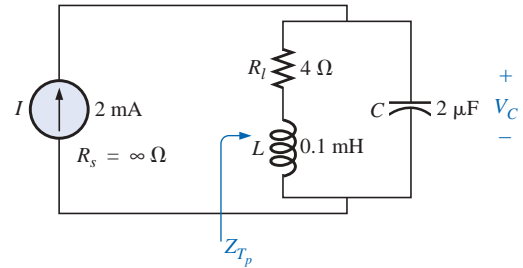


FIG. 20.54
Problem 15.

16. For the network of Fig. 20.55:
- Find the value of X_C at resonance (f_p).
 - Find the total impedance Z_{T_p} at resonance (f_p).
 - Find the currents I_L and I_C at resonance (f_p).
 - If the resonant frequency is 20,000 Hz, find the value of L and C at resonance.
 - Find Q_p and the BW .

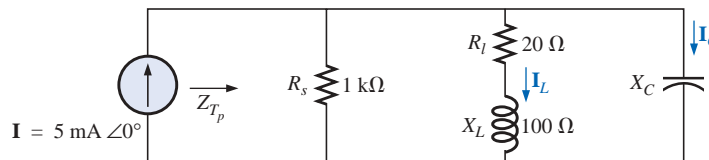


FIG. 20.55
Problem 16.

17. Repeat Problem 16 for the network of Fig. 20.56.

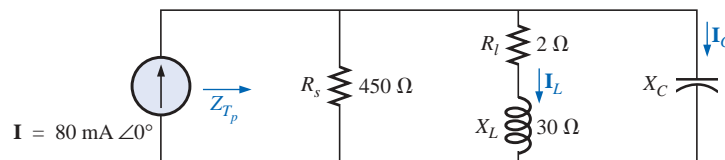


FIG. 20.56
Problem 17.



18. For the network of Fig. 20.57:

- Find the resonant frequencies f_s , f_p , and f_m . What do the results suggest about the Q_p of the network?
- Find the values of X_L and X_C at resonance (f_p). How do they compare?
- Find the impedance Z_{T_p} at resonance (f_p).
- Calculate Q_p and the BW .
- Find the magnitude of currents I_L and I_C at resonance (f_p).
- Calculate the voltage V_C at resonance (f_p).

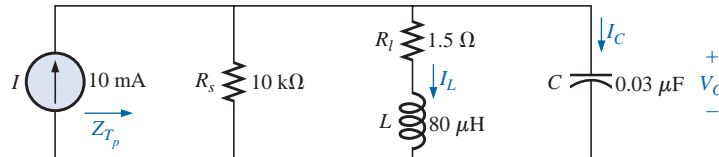


FIG. 20.57
Problem 18.

*19. Repeat Problem 18 for the network of Fig. 20.58.

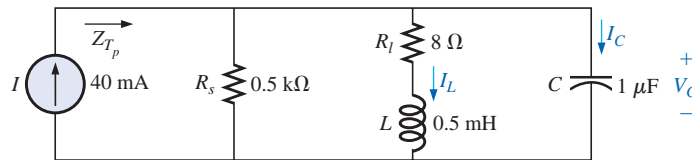


FIG. 20.58
Problems 19 and 29.

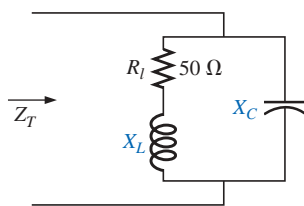


FIG. 20.59
Problem 20.

20. It is desired that the impedance Z_T of the high- Q circuit of Fig. 20.59 be $50\text{ k}\Omega \angle 0^\circ$ at resonance (f_p).

- Find the value of X_L .
- Compute X_C .
- Find the resonant frequency (f_p) if $L = 16\text{ mH}$.
- Find the value of C .

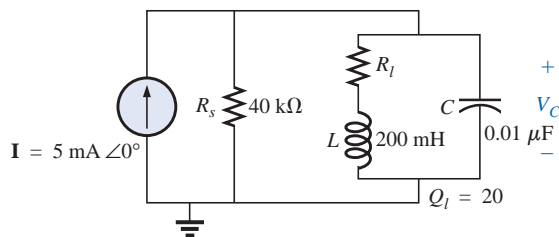


FIG. 20.60
Problem 21.

21. For the network of Fig. 20.60:

- Find f_p .
- Calculate the magnitude of V_C at resonance (f_p).
- Determine the power absorbed at resonance.
- Find the BW .

*22. For the network of Fig. 20.61:

- Find the value of X_L for resonance.
- Find Q .
- Find the resonant frequency (f_p) if the bandwidth is 1 kHz.
- Find the maximum value of the voltage V_C .
- Sketch the curve of V_C versus frequency. Indicate its peak value, resonant frequency, and band frequencies.

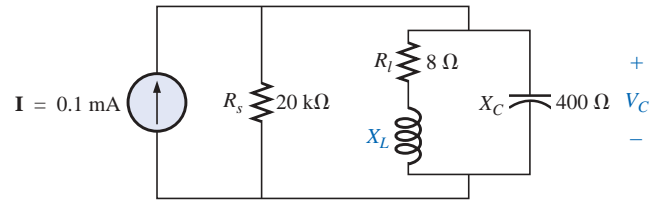


FIG. 20.61
Problem 22.

*23. Repeat Problem 22 for the network of Fig. 20.62.

*24. For the network of Fig. 20.63:

- Find f_s , f_p , and f_m .
- Determine Q_l and Q_p at f_p after a source conversion is performed.
- Find the input impedance Z_{T_p} .
- Find the magnitude of the voltage V_C .
- Calculate the bandwidth using f_p .
- Determine the magnitude of the currents I_C and I_L .

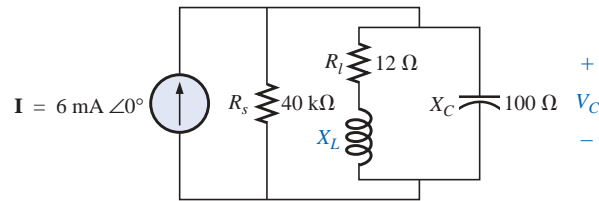


FIG. 20.62
Problem 23.

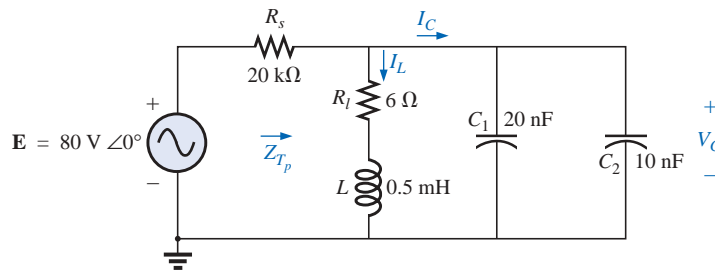


FIG. 20.63
Problem 24.

*25. For the network of Fig. 20.64, the following are specified:

$$\begin{aligned} f_p &= 20 \text{ kHz} \\ BW &= 1.8 \text{ kHz} \\ L &= 2 \text{ mH} \\ Q_l &= 80 \end{aligned}$$

Find R_s and C .

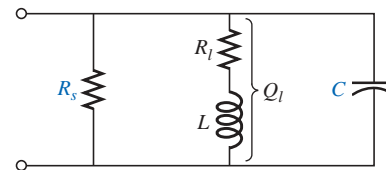


FIG. 20.64
Problem 25.

*26. Design the network of Fig. 20.65 to have the following characteristics:

- $BW = 500 \text{ Hz}$
- $Q_p = 30$
- $V_{C_{\max}} = 1.8 \text{ V}$

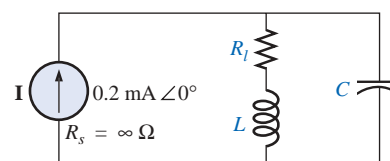


FIG. 20.65
Problem 26.

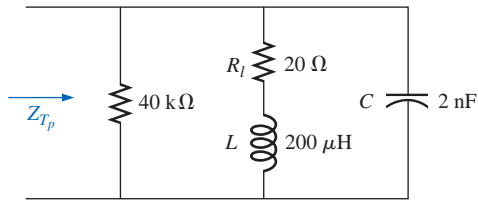


FIG. 20.66
Problem 27.

- *27. For the parallel resonant circuit of Fig. 20.66:
- Determine the resonant frequency.
 - Find the total impedance at resonance.
 - Find Q_p .
 - Calculate the BW .
 - Repeat parts (a) through (d) for $L = 20 \mu\text{H}$ and $C = 20 \text{nF}$.
 - Repeat parts (a) through (d) for $L = 0.4 \text{mH}$ and $C = 1 \text{nF}$.
 - For the network of Fig. 20.66 and the parameters of parts (e) and (f), determine the ratio L/C .
 - Do your results confirm the conclusions of Fig. 20.28 for changes in the L/C ratio?

SECTION 20.14 Computer Analysis

PSpice or Electronics Workbench

- Verify the results of Example 20.8. That is, show that the resonant frequency is in fact 40 kHz, the cutoff frequencies are as calculated, and the bandwidth is 1.85 kHz.
- Find f_p and f_m for the parallel resonant network of Fig. 20.58, and comment on the resulting bandwidth as it relates to the quality factor of the network.

Programming Language (C++, QBASIC, Pascal, etc.)

- Write a program to tabulate the impedance and current of the network of Fig. 20.2 versus frequency for a frequency range extending from $0.1f_s$ to $2f_s$ in increments of $0.1f_s$. For the first run, use the parameters defined by Example 20.1.
- Write a program to provide a general solution for the network of Fig. 20.36; that is, determine the parameters requested in parts (a) through (e) of Example 20.9.

GLOSSARY

Band (cutoff, half-power, corner) frequencies Frequencies that define the points on the resonance curve that are 0.707 of the peak current or voltage value. In addition, they define the frequencies at which the power transfer to the resonant circuit will be half the maximum power level.

Bandwidth (BW) The range of frequencies between the band, cutoff, or half-power frequencies.

Quality factor (Q) A ratio that provides an immediate indication of the sharpness of the peak of a resonance curve. The higher the Q , the sharper the peak and the more quickly it drops off to the right and left of the resonant frequency.

Resonance A condition established by the application of a particular frequency (the resonant frequency) to a series or parallel R - L - C network. The transfer of power to the system is a maximum, and, for frequencies above and below, the power transfer drops off to significantly lower levels.

Selectivity A characteristic of resonant networks directly related to the bandwidth of the resonant system. High selectivity is associated with small bandwidth (high Q 's), and low selectivity with larger bandwidths (low Q 's).