26

System Analysis: An Introduction

26.1 INTRODUCTION

The growing number of packaged systems in the electrical, electronic, and computer fields now requires that some form of system analysis appear in the syllabus of the introductory course. Although the content of this chapter will be a surface treatment at best, the material will introduce a number of important terms and techniques employed in the system analysis approach. The increasing use of packaged systems is quite understandable when we consider the advantages associated with such structures: reduced size, sophisticated and tested design, reduced construction time, reduced cost compared to discrete designs, and so forth. The use of any packaged system is limited solely to the proper utilization of the provided terminals of the system. Entry into the internal structure is not permitted, which also eliminates the possibility of repair to such systems.

System analysis includes the development of two-, three-, or multiport models of devices, systems, or structures. The emphasis in this chapter will be on the configuration most frequently subject to modeling techniques: the two-port system of Fig. 26.1.

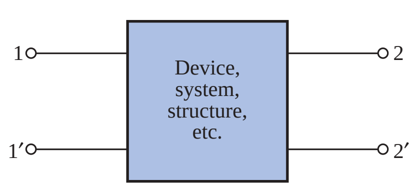

FIG. 26.1
Two-port system.

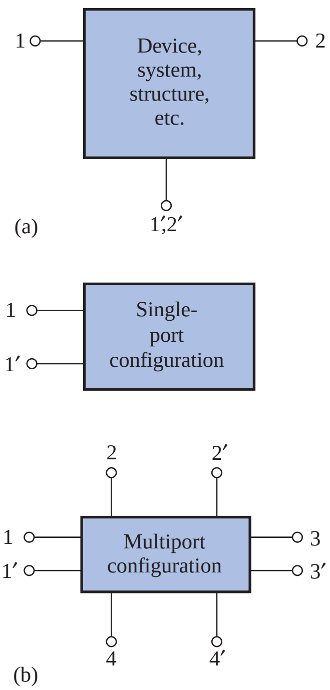

FIG. 26.2
(a) Two-port system; (b) single-port system and multiport system.

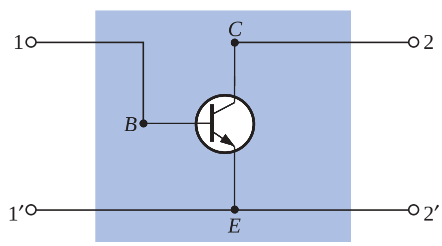

FIG. 26.3
Two-port transistor configuration.

Note that in Fig. 26.1 there are two ports of entry or interest, each having a pair of terminals. For some devices, the **two-port network** of Fig. 26.1 may appear as shown in Fig. 26.2(a). The block diagram of Fig. 26.2(a) simply indicates that terminals 1′ and 2′ are in common, which is a particular case of the general two-port network. A **single-port network** and a multiport network appear in Fig. 26.2(b). The former has been analyzed throughout the text, while the characteristics of the latter will be touched on in this chapter, with a more extensive coverage left for a more advanced course.

The latter part of this chapter introduces a set of equations (and, subsequently, networks) that will allow us to model the device or system appearing within the enclosed structure of Fig. 26.1. That is, we will be able to establish a network that will display the same terminal characteristics as those of the original system, device, and so on. In Fig. 26.3, for example, a transistor appears between the four external terminals. Through the analysis to follow, we will find a combination of network elements that will allow us to replace the transistor with a network that will behave very much like the original device for a specific set of operating conditions. Methods such as mesh and nodal analysis can then be applied to determine any unknown quantities. The models, when reduced to their simplest forms as determined by the operating conditions, can also provide very quick estimates of network behavior without a lengthy mathematical derivation. In other words, someone well-versed in the use of models can analyze the operation of large, complex systems in short order. The results may be only approximate in most cases, but this quick return for a minimum of effort is often worthwhile.

The analysis of this chapter is limited to linear (fixed-value) systems with bilateral elements. Three sets of parameters are developed for the two-port configuration, referred to as the **impedance (z), admittance (y),** and **hybrid (h) parameters.** Table 26.1 at the end of the chapter relates the three sets of parameters.

26.2 THE IMPEDANCE PARAMETERS Z_i AND Z_o

For the two-port system of Fig. 26.4, $\mathbf{Z}_i$ is the **input impedance** between terminals 1 and 1′, and $\mathbf{Z}_o$ is the **output impedance** between terminals 2 and 2′. For multiport networks an impedance level can be defined between any two (adjacent or not) terminals of the network.

The input impedance is defined by Ohm’s law in the following form:

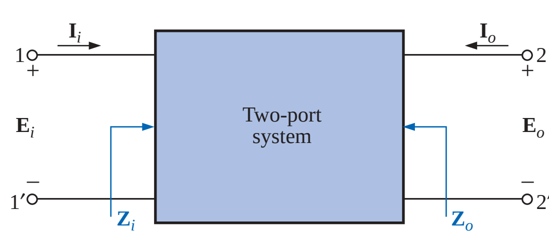

FIG. 26.4
Defining $\mathbf{Z}_i$ and $\mathbf{Z}_o$.

$$\mathbf{Z}_i = \frac{\mathbf{E}_i}{\mathbf{I}_i} \qquad \text{(ohms, } \Omega) \tag{26.1}$$

with $\mathbf{I}_i$ the current resulting from the application of a voltage $\mathbf{E}_i$.

The output impedance $\mathbf{Z}_o$ is defined by

$$\mathbf{Z}_o = \frac{\mathbf{E}_o}{\mathbf{I}_o}\bigg|_{\mathbf{E}_i = 0\text{ V}} \qquad \text{(ohms, } \Omega) \tag{26.2}$$

with $\mathbf{I}_o$ the current resulting from the application of a voltage $\mathbf{E}_o$ to the output terminals, with $\mathbf{E}_i$ set to zero.

Note that both $\mathbf{I}_i$ and $\mathbf{I}_o$ are defined as entering the package. This is common practice for a number of system analysis methods to avoid concern about the actual direction for each current and also to define $\mathbf{Z}_i$ and $\mathbf{Z}_o$ as positive quantities in Eqs. (26.1) and (26.2), respectively. If $\mathbf{I}_o$ were chosen to be leaving the system, $\mathbf{Z}_o$ as defined in Eq. (26.2) would have to have a negative sign.

An experimental setup for determining $\mathbf{Z}_i$ for any two input terminals is provided in Fig. 26.5. The sensing resistor R_s is chosen small enough not to disturb the basic operation of the system or to require too large a voltage $\mathbf{E}_g$ to establish the desired level of $\mathbf{E}_i$. Under operating conditions, the voltage across R_s is $\mathbf{E}_g - \mathbf{E}_i$, and the current through the sensing resistor is

$$\mathbf{I}_{R_s} = \frac{\mathbf{V}_{R_s}}{R_s} = \frac{\mathbf{E}_g - \mathbf{E}_i}{R_s}$$

But $$\mathbf{I}_i = \mathbf{I}_{R_s} \quad \text{and} \quad \mathbf{Z}_i = \frac{\mathbf{E}_i}{\mathbf{I}_i} = \frac{\mathbf{E}_i}{\mathbf{I}_{R_s}}$$

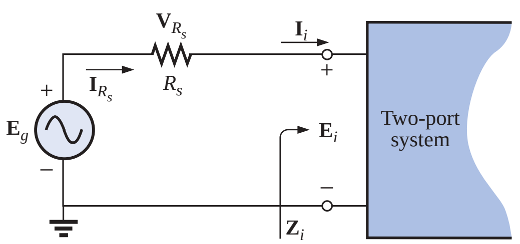

FIG. 26.5
Determining $\mathbf{Z}_i$.

The sole purpose of the sensing resistor, therefore, was to determine $\mathbf{I}_i$ using purely voltage measurements.

As we progress through this chapter, keep in mind that we cannot use an ohmmeter to measure $\mathbf{Z}_i$ or $\mathbf{Z}_o$ since we are dealing with ac systems whose impedance may be sensitive to the applied frequency. Ohmmeters can be used to measure resistance in a dc or an ac network, but recall that ohmmeters are employed only on a de-energized network, and their internal source is a dc battery.

The output impedance $\mathbf{Z}_o$ can be determined experimentally using the setup of Fig. 26.6. Note that a sensing resistor is introduced again, with $\mathbf{E}_g$ being an applied voltage to establish typical operating conditions. In addition, note that the input signal must be set to zero, as defined by Eq.

(26.2). The voltage across the sensing resistor is $\mathbf{E}_g - \mathbf{E}_o$, and the current through the sensing resistor is

$$\mathbf{I}_{R_s} = \frac{\mathbf{V}_{R_s}}{R_s} = \frac{\mathbf{E}_g - \mathbf{E}_o}{R_s}$$

but
$$\mathbf{I}_o = \mathbf{I}_{R_s} \quad \text{and} \quad \mathbf{Z}_o = \frac{\mathbf{E}_o}{\mathbf{I}_o} = \frac{\mathbf{E}_o}{\mathbf{I}_{R_s}}$$

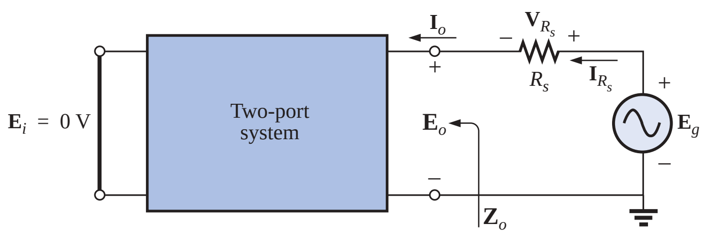

FIG. 26.6
Determining $\mathbf{Z}_o$*.*

For the majority of situations, $\mathbf{Z}_i$ and $\mathbf{Z}_o$ will be purely resistive, resulting in an angle of zero degrees for each impedance. The result is that either a DMM or a scope can be used to find the required magnitude of the desired quantity. For instance, for both $\mathbf{Z}_i$ and $\mathbf{Z}_o$, $\mathbf{V}_{R_s}$ can be measured directly with the DMM, as can the required levels of $\mathbf{E}_g$, $\mathbf{E}_i$, or $\mathbf{E}_o$. The current for each case can then be determined using Ohm's law, and the impedance level can be determined using either Eq. (26.1) or Eq. (26.2).

If we use an oscilloscope, we must be more sensitive to the common ground requirement. For instance, in Fig. 26.4, $\mathbf{E}_g$ and $\mathbf{E}_i$ can be measured with the oscilloscope since they have a common ground. Trying to measure $\mathbf{V}_{R_s}$ directly with the ground of the oscilloscope at the top input terminal of $\mathbf{E}_i$ would result in a shorting effect across the input terminals of the system due to the common ground between the supply and oscilloscope. If the input impedance of the system is "shorted out," the current $\mathbf{I}_i$ can rise to dangerous levels because the only resistance in the input circuit is the relatively small sensing resistor R_s. If we use the DMM to avoid concern about the grounding situation, we must be sure that the meter is designed to operate properly at the frequency of interest. Many commercial units are limited to a few kilohertz.

If the input impedance has an angle other than zero degrees (purely resistive), a DMM cannot be used to find the reactive component at the input terminals. The magnitude of the total impedance will be correct if measured as described above, but the angle from which the resistive and reactive components can be determined will not be provided. If an oscilloscope is used, the network must be hooked up as shown in Fig. 26.7. Both the voltage $\mathbf{E}_g$ and $\mathbf{V}_{R_s}$ can be displayed on the oscilloscope at the same time, and the phase angle between $\mathbf{E}_g$ and $\mathbf{V}_{R_s}$ can be determined. Since $\mathbf{V}_{R_s}$ and $\mathbf{I}_i$ are in phase, the angle determined will also be the angle between $\mathbf{E}_g$ and $\mathbf{I}_i$. The angle we are looking for is between $\mathbf{E}_i$ and $\mathbf{I}_i$, not between $\mathbf{E}_g$ and $\mathbf{I}_i$, but since R_s is usually chosen small enough, we can assume that the voltage drop across R_s is so small compared to $\mathbf{E}_g$ that $\mathbf{E}_i \cong \mathbf{E}_g$. Substituting the peak, peak-to-peak, or rms values from the oscilloscope measurements, along with the angle just determined, will permit a determination of the magnitude and angle for

$\mathbf{Z}_i$, from which the resistive and reactive components can be determined using a few basic geometric relationships. The reactive nature (inductive or capacitive) of the input impedance can be determined when the angle between $\mathbf{E}_i$ and $\mathbf{I}_i$ is computed. For a dual-trace oscilloscope, if $\mathbf{E}_g$ leads $\mathbf{V}_{R_s}$ ($\mathbf{E}_i$ leads $\mathbf{I}_i$), the network is inductive; if the reverse is true, the network is capacitive.

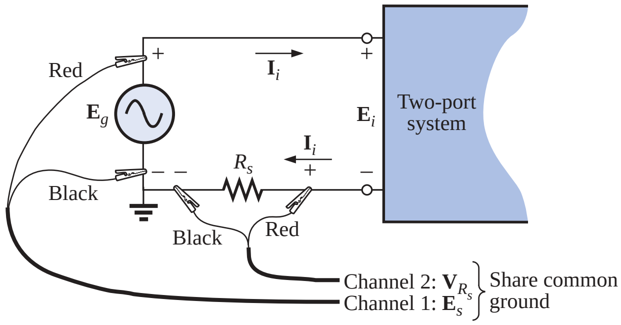

FIG. 26.7
Determining $\mathbf{Z}_i$ using an oscilloscope.

To determine the angle associated with $\mathbf{Z}_o$, the sensing resistor must again be moved to the bottom to form a common ground with the supply $\mathbf{E}_g$. Then, using the approximation $\mathbf{E}_g \cong \mathbf{E}_o$, the magnitude and angle of $\mathbf{Z}_o$ can be determined.

EXAMPLE 26.1 Given the DMM measurements appearing in Fig. 26.8, determine the input impedance $\mathbf{Z}_i$ for the system if the input impedance is known to be purely resistive.

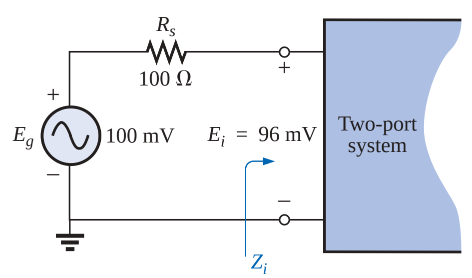

FIG. 26.8
Example 26.1.

Solution:

$$V_{R_s} = E_g - E_i = 100\text{ mV} - 96\text{ mV} = 4\text{ mV}$$

$$I_i = I_{R_s} = \frac{V_{R_s}}{R_s} = \frac{4\text{ mV}}{100\ \Omega} = 40\ \mu\text{A}$$

$$Z_i = R_i = \frac{E_i}{I_i} = \frac{96\text{ mV}}{40\ \mu\text{A}} = \mathbf{2.4\ k\Omega}$$

EXAMPLE 26.2 Using the provided DMM measurements of Fig. 26.9, determine the output impedance Z_o for the system if the output impedance is known to be purely resistive.

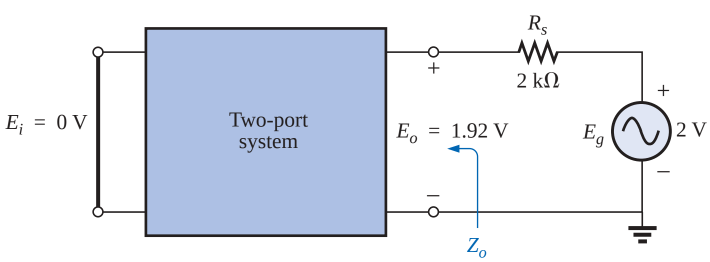

FIG. 26.9
Example 26.2.

Solution:

$$V_{R_s} = E_g - E_o = 2\text{ V} - 1.92\text{ V} = 0.08\text{ V} = 80\text{ mV}$$

$$I_o = I_{R_s} = \frac{V_{R_s}}{R_s} = \frac{80\text{ mV}}{2\text{ k}\Omega} = 40\ \mu\text{A}$$

$$Z_o = \frac{E_o}{I_o} = \frac{1.92\text{ V}}{40\ \mu\text{A}} = \mathbf{48\ k\Omega}$$

EXAMPLE 26.3 The input characteristics for the system of Fig. 26.10(a) are unknown. Using the oscilloscope measurements of Fig. 26.10(b), determine the input impedance for the system. If a reactive component exists, determine its magnitude and whether it is inductive or capacitive.

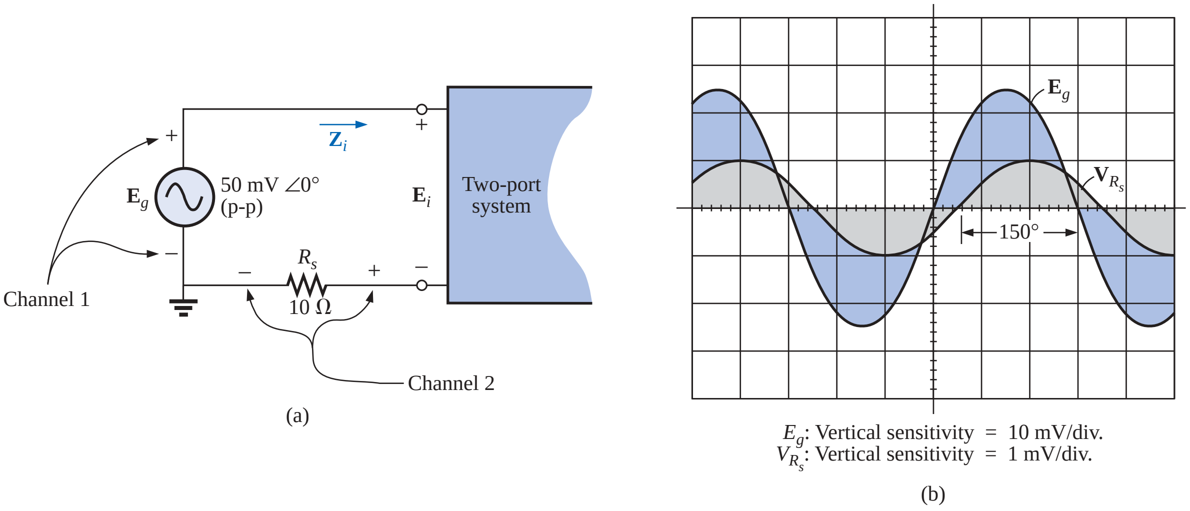

FIG. 26.10
Example 26.3.

Solution: The magnitude of $\mathbf{Z}_i$:

$$I_{i(p\text{-}p)} = I_{R_{s(p\text{-}p)}} = \frac{V_{R_{s(p\text{-}p)}}}{R_s} = \frac{2\text{ mV}}{10\ \Omega} = 200\ \mu\text{A}$$

$$Z_i = \frac{E_i}{I_i} \cong \frac{E_g}{I_i} = \frac{50\text{ mV}}{200\ \mu\text{A}} = 250\ \Omega$$

The angle of $\mathbf{Z}_i$: The phase angle between $\mathbf{E}_g$ and $\mathbf{V}_{R_s}$ (or $\mathbf{I}_{R_s} = \mathbf{I}_i$) is

$$180° - 150° = 30°$$

with $\mathbf{E}_g$ leading $\mathbf{I}_i$, so the system is inductive. Therefore,

$$\mathbf{Z}_i = 250\ \Omega\ \angle 30°$$
$$= \mathbf{216.51\ \Omega + \mathit{j}\,125\ \Omega} = R + j\,X_L$$

26.3 THE VOLTAGE GAINS $\mathbf{A}_{v_{NL}}$, $\mathbf{A}_v$, AND $\mathbf{A}_{v_T}$

The voltage gain for the two-port system of Fig. 26.11 is defined by

$$\mathbf{A}_{v_{NL}} = \frac{\mathbf{E}_o}{\mathbf{E}_i} \tag{26.3}$$

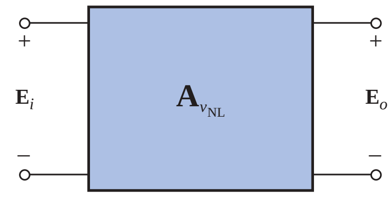

FIG. 26.11
Defining the no-load gain $\mathbf{A}_{v_{NL}}$.

The capital letter **A** in the notation was chosen from the term *a*mplification factor, with the subscript *v* selected to specify that voltage levels are involved. The subscript *NL* reveals that the ratio was determined under *no-load* conditions; that is, a load was not applied to the output terminals when the gain was determined. The no-load voltage gain is the gain typically provided with packaged systems since the applied load is a function of the application.

The magnitude of the ratio can be determined using a DMM or an oscilloscope. The oscilloscope, however, must be used to determine the phase shift between the two voltages.

In Fig. 26.12 a load has been introduced to establish a loaded gain that will be denoted simply as $\mathbf{A}_v$ and defined by

$$\mathbf{A}_v = \left.\frac{\mathbf{E}_o}{\mathbf{E}_i}\right|_{\text{with } R_L} \tag{26.4}$$

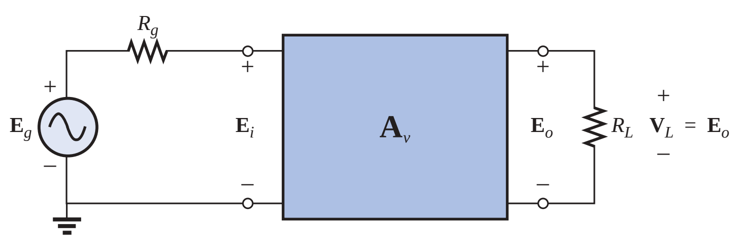

FIG. 26.12
Defining the loaded voltage gain $\mathbf{A}_v$ *(and* $\mathbf{A}_{v_T}$*).*

For all two-port systems the loaded gain $\mathbf{A}_v$ ***will always be less than the no-load gain.***

In other words, the application of a load will always reduce the gain below the no-load level.

A third voltage gain can be defined using Fig. 26.12 since it has an applied voltage source with an associated internal resistance—a situation often encountered in electronic systems. The total voltage gain of the system is represented by $\mathbf{A}_{v_T}$ and is defined by

$$\mathbf{A}_{v_T} = \frac{\mathbf{E}_o}{\mathbf{E}_g} \tag{26.5}$$

It is the voltage gain from the source $\mathbf{E}_g$ to the output terminals $\mathbf{E}_o$. Due to loss of signal voltage across the source resistance,

the voltage gain $\mathbf{A}_{v_T}$ ***is always less than the loaded voltage gain*** $\mathbf{A}_v$ ***or unloaded gain*** $\mathbf{A}_{v_{NL}}$***.***

If we expand Eq. (26.5) as follows:

$$\mathbf{A}_{v_T} = \frac{\mathbf{E}_o}{\mathbf{E}_g} = \frac{\mathbf{E}_o}{\mathbf{E}_g}(1) = \frac{\mathbf{E}_o}{\mathbf{E}_g}\left(\frac{\mathbf{E}_i}{\mathbf{E}_i}\right) = \frac{\mathbf{E}_o}{\mathbf{E}_i}\cdot\frac{\mathbf{E}_i}{\mathbf{E}_g}$$

then

$$\mathbf{A}_{v_T} = \mathbf{A}_v \frac{\mathbf{E}_i}{\mathbf{E}_g} \qquad \text{(if loaded)}$$

or

$$\mathbf{A}_{v_T} = \mathbf{A}_{v_{NL}} \frac{\mathbf{E}_i}{\mathbf{E}_g} \qquad \text{(if unloaded)}$$

The relationship between $\mathbf{E}_i$ and $\mathbf{E}_g$ can be determined from Fig. 26.12 if we recognize that $\mathbf{E}_i$ is across the input impedance $\mathbf{Z}_i$ and thus apply the voltage divider rule as follows:

$$\mathbf{E}_i = \frac{\mathbf{Z}_i(\mathbf{E}_g)}{\mathbf{Z}_i + R_g}$$

or

$$\frac{\mathbf{E}_i}{\mathbf{E}_g} = \frac{\mathbf{Z}_i}{\mathbf{Z}_i + R_g}$$

Substituting into the above relationships will result in

$$\mathbf{A}_{v_T} = \mathbf{A}_v \frac{\mathbf{Z}_i}{\mathbf{Z}_i + R_g} \qquad \text{(if loaded)} \qquad \textbf{(26.6)}$$

$$\mathbf{A}_{v_T} = \mathbf{A}_{v_{NL}} \frac{\mathbf{Z}_i}{\mathbf{Z}_i + R_g} \qquad \text{(if unloaded)} \qquad \textbf{(26.7)}$$

A two-port equivalent model for an unloaded system based on the definitions of $\mathbf{Z}_i$, $\mathbf{Z}_o$, and $\mathbf{A}_{v_{NL}}$ is provided in Fig. 26.13. Both $\mathbf{Z}_i$ and $\mathbf{Z}_o$ appear as resistive values since this is typically the case for most electronic amplifiers. However, both $\mathbf{Z}_i$ and $\mathbf{Z}_o$ can have reactive components and not invalidate the equivalency of the model.

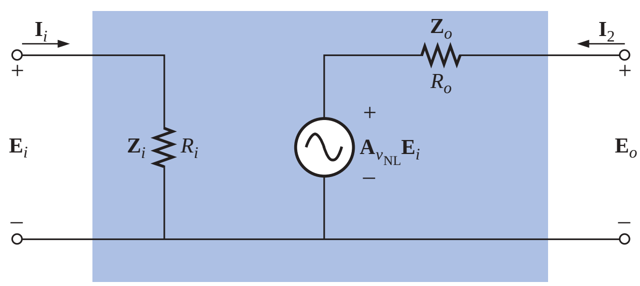

FIG. 26.13
Equivalent model for two-port amplifier.

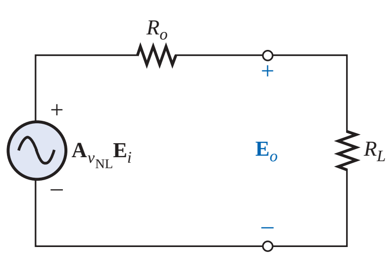

FIG. 26.14
Applying a load to the output of Fig. 26.13.

The input impedance is defined by $\mathbf{Z}_i = \mathbf{E}_i/\mathbf{I}_i$ and the voltage $\mathbf{E}_o = \mathbf{A}_{v_{NL}}\mathbf{E}_i$ in the absence of a load, resulting in $\mathbf{A}_{v_{NL}} = \mathbf{E}_o/\mathbf{E}_i$ as defined. The output impedance is defined with $\mathbf{E}_i$ set to zero volts, resulting in $\mathbf{A}_{v_{NL}}\mathbf{E}_i = 0$ V, which permits the use of a short-circuit equivalent for the controlled source. The result is $\mathbf{Z}_o = \mathbf{E}_o/\mathbf{I}_o$, as defined, and the parameters and structure of the equivalent model are validated.

If a load is applied as in Fig. 26.14, an application of the voltage divider rule will result in

$$\mathbf{E}_o = \frac{R_L(\mathbf{A}_{v_{NL}}\mathbf{E}_i)}{R_L + R_o}$$

and
$$\mathbf{A}_v = \frac{\mathbf{E}_o}{\mathbf{E}_i} = A_{v_{NL}}\frac{R_L}{R_L + R_o} \qquad (26.8)$$

For any value of R_L or R_o, the ratio $R_L/(R_L + R_o)$ must be less than 1, mandating that $\mathbf{A}_v$ is always less than $\mathbf{A}_{v_{NL}}$ as stated earlier. Further,

for a fixed output impedance (R_o), the larger the load resistance (R_L), the closer the loaded gain to the no-load level.

An experimental procedure for determining R_o can be developed if we solve Eq. (26.8) for the output impedance R_o:

$$\mathbf{A}_v = \frac{R_L}{R_L + R_o}\mathbf{A}_{v_{NL}}$$

or
$$\mathbf{A}_v(R_L + R_o) = R_L\mathbf{A}_{v_{NL}}$$
$$\mathbf{A}_vR_L + \mathbf{A}_vR_o = R_L\mathbf{A}_{v_{NL}}$$

and
$$\mathbf{A}_vR_o = R_L\mathbf{A}_{v_{NL}} - \mathbf{A}_vR_L$$

with
$$R_o = \frac{R_L(\mathbf{A}_{v_{NL}} - \mathbf{A}_v)}{\mathbf{A}_v}$$

or
$$R_o = R_L\left(\frac{\mathbf{A}_{v_{NL}}}{\mathbf{A}_v} - 1\right) \qquad (26.9)$$

Equation (26.9) reveals that the output impedance R_o of an amplifier can be determined by first measuring the voltage gain $\mathbf{E}_o/\mathbf{E}_i$ without a load in place to find $\mathbf{A}_{v_{NL}}$ and then measuring the gain with a load R_L to find $\mathbf{A}_v$. Substitution of $\mathbf{A}_{v_{NL}}$, $\mathbf{A}_v$, and R_L into Eq. (26.9) will then provide the value for R_o.

EXAMPLE 26.4 For the system of Fig. 26.15(a) employed in the loaded amplifier of Fig. 26.15(b):

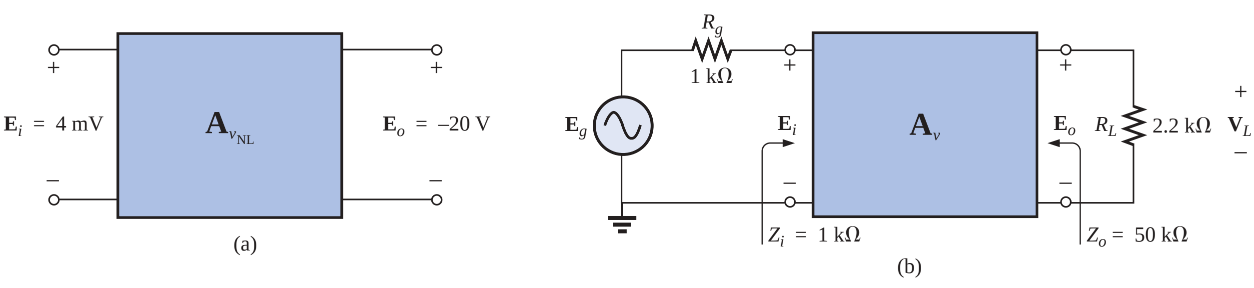

FIG. 26.15
Example 26.4.

a. Determine the no-load voltage gain $\mathbf{A}_{v_{NL}}$.
b. Find the loaded voltage gain $\mathbf{A}_v$.
c. Calculate the loaded voltage gain $\mathbf{A}_{v_T}$.
d. Determine R_o from Eq. (26.9), and compare it to the specified value of Fig. 26.15.

Solutions:

a. $\mathbf{A}_{v_{NL}} = \dfrac{\mathbf{E}_o}{\mathbf{E}_i} = \dfrac{-20\text{ V}}{4\text{ mV}} = \mathbf{-5000}$

b. $\mathbf{A}_v = \mathbf{A}_{v_{NL}} \dfrac{R_L}{R_L + R_o} = (-5000)\left(\dfrac{2.2\text{ k}\Omega}{2.2\text{ k}\Omega + 50\text{ k}\Omega}\right)$

$= (-5000)(0.0421) = \mathbf{-210.73}$

c. $\mathbf{A}_{v_T} = \mathbf{A}_v \dfrac{\mathbf{Z}_i}{\mathbf{Z}_i + R_g} = (-210.73)\left(\dfrac{1\text{ k}\Omega}{1\text{ k}\Omega + 1\text{ k}\Omega}\right)$

$= (-210.73)\left(\dfrac{1}{2}\right) = \mathbf{-105.36}$

d. $R_o = R_L\left(\dfrac{\mathbf{A}_{v_{NL}}}{\mathbf{A}_v} - 1\right) = 2.2\text{ k}\Omega\left(\dfrac{-5000}{-210.73} - 1\right)$

$= 2.2\text{ k}\Omega(23.727 - 1) = 2.2\text{ k}\Omega(22.727)$

$= \mathbf{50\text{ k}\Omega}$ as specified

26.4 THE CURRENT GAINS A_i AND A_{i_T}, AND THE POWER GAIN A_G

The current gain of two-port systems is typically calculated from voltage levels. A no-load gain is not defined for current gain since the absence of R_L requires that $\mathbf{I}_o = \mathbf{E}_o/R_L = 0$ A and $\mathbf{A}_i = \mathbf{I}_o/\mathbf{I}_i = 0$.

For the system of Fig. 26.16, however, a load has been applied, and

$$\mathbf{I}_o = -\frac{\mathbf{E}_o}{R_L}$$

with

$$\mathbf{I}_i = \frac{\mathbf{E}_i}{\mathbf{Z}_i}$$

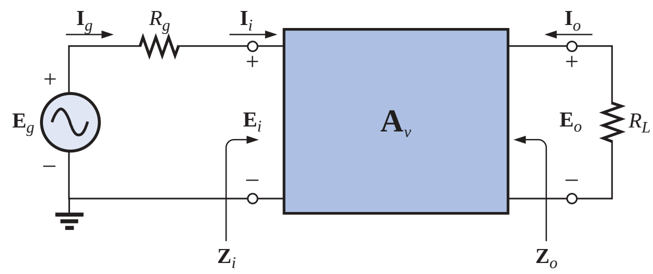

FIG. 26.16
Defining $\mathbf{A}_i$ *and* $\mathbf{A}_{i_T}$.

Note the need for a minus sign when $\mathbf{I}_o$ is defined, because the defined polarity of $\mathbf{E}_o$ would establish the opposite direction for $\mathbf{I}_o$ through R_L.

The loaded current gain is

$$\mathbf{A}_i = \frac{\mathbf{I}_o}{\mathbf{I}_i} = \frac{-\mathbf{E}_o/R_L}{\mathbf{E}_i/\mathbf{Z}_i} = -\frac{\mathbf{E}_o}{\mathbf{E}_i}\left(\frac{\mathbf{Z}_i}{R_L}\right)$$

and
$$\mathbf{A}_i = -\mathbf{A}_v \frac{\mathbf{Z}_i}{R_L} \tag{26.10}$$

In general, therefore, the loaded current gain can be obtained directly from the loaded voltage gain and the ratio of impedance levels, $\mathbf{Z}_i$ over R_L.

If the ratio $\mathbf{A}_{i_T} = \mathbf{I}_o/\mathbf{I}_g$ were required, we would proceed as follows:

$$\mathbf{I}_o = -\frac{\mathbf{E}_o}{R_L}$$

with
$$\mathbf{I}_i = \frac{\mathbf{E}_g}{R_g + \mathbf{Z}_i}$$

and
$$\mathbf{A}_{i_T} = \frac{\mathbf{I}_o}{\mathbf{I}_g} = \frac{-\mathbf{E}_o/R_L}{\mathbf{E}_g/(R_g + \mathbf{Z}_i)} = -\left(\frac{\mathbf{E}_o}{\mathbf{E}_g}\right)\left(\frac{R_g + \mathbf{Z}_i}{R_L}\right)$$

or
$$\mathbf{A}_{i_T} = \frac{\mathbf{I}_o}{\mathbf{I}_g} = -\mathbf{A}_{v_T}\left(\frac{R_g + \mathbf{Z}_i}{R_L}\right) \tag{26.11}$$

The result obtained with Eq. (26.10) or (26.11) will be the same since $\mathbf{I}_g = \mathbf{I}_i$, but the option of which gain is available or which you choose to use is now available.

Returning to Fig. 26.13 (repeated in Fig. 26.17), an equation for the current gain can be determined in terms of the no-load voltage gain.

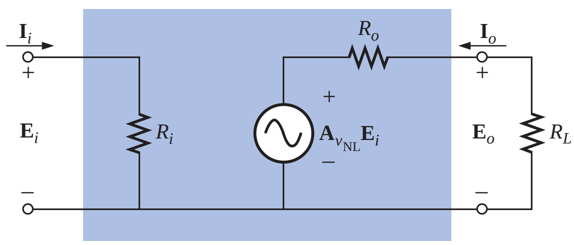

FIG. 26.17
Developing an equation for $\mathbf{A}_i$ *in terms of* $\mathbf{A}_{v_{NL}}$.

Through Ohm's law:

$$\mathbf{I}_o = -\frac{\mathbf{A}_{v_{NL}}\mathbf{E}_i}{R_L + R_o}$$

but
$$\mathbf{E}_i = \mathbf{I}_i R_i$$

and
$$\mathbf{I}_o = -\frac{\mathbf{A}_{v_{NL}}(\mathbf{I}_i R_i)}{R_L + R_o}$$

so that
$$\mathbf{A}_i = \frac{\mathbf{I}_o}{\mathbf{I}_i} = -\mathbf{A}_{v_{NL}}\frac{R_i}{R_L + R_o} \tag{26.12}$$

The result is an equation for the loaded current gain of an amplifier in terms of the nameplate no-load voltage gain and the resistive elements of the network.

Recall an earlier conclusion that the larger the value of R_L, the larger the loaded voltage gain. For current levels, Equation (26.12) reveals that

the larger the level of R_L, the less the current gain of a loaded amplifier.

In the design of an amplifier, therefore, one must balance the desired voltage gain with the current gain and the resulting ac output power level.

For the system of Fig. 26.17, the power delivered to the load is determined by E_o^2/R_L, whereas the power delivered at the input terminals is E_i^2/R_i. The power gain is therefore defined by

$$A_G = \frac{P_o}{P_i} = \frac{E_o^2/R_L}{E_i^2/R_i} = \frac{E_o^2}{E_i^2}\frac{R_i}{R_L} = \left(\frac{E_o}{E_i}\right)^2\frac{R_i}{R_L}$$

and

$$A_G = A_v^2\frac{R_i}{R_L} \qquad \textbf{(26.13)}$$

Expanding the conclusion,

$$A_G = (A_v)\left(A_v\frac{R_i}{R_L}\right) = (A_v)(-A_i)$$

so

$$A_G = -A_vA_i \qquad \textbf{(26.14)}$$

Don't be concerned about the minus sign. A_v or A_i will be negative to ensure that the power gain is positive, as obtained from Eq. (26.13).

If we substitute $A_v = -A_iR_L/R_i$ [from Eq. (26.10)] into Eq. (26.14), we will find

$$A_G = -A_vA_i = -\left(\frac{-A_iR_L}{R_i}\right)A_i$$

or

$$A_G = A_i^2\frac{R_L}{R_i} \qquad \textbf{(26.15)}$$

which has a format similar to that of Eq. (26.13), but now A_G is given in terms of the current gain of the system.

The last power gain to be defined is the following:

$$A_{G_T} = \frac{P_L}{P_g} = \frac{E_o^2/R_L}{E_gI_g} = \frac{E_o^2/R_L}{E_g^2/(R_g + R_i)} = \left(\frac{E_o}{E_g}\right)^2\left(\frac{R_g + R_i}{R_L}\right)$$

or

$$A_{G_T} = A_{v_T}^2\left(\frac{R_g + R_i}{R_L}\right) \qquad \textbf{(26.16)}$$

Expanding:

$$A_{G_T} = A_{v_T}\left(A_{v_T}\frac{R_g + R_i}{R_L}\right)$$

and

$$A_{G_T} = -A_{v_T}A_{i_T} \qquad \textbf{(26.17)}$$

EXAMPLE 26.5 Given the system of Fig. 26.18 with its nameplate data:

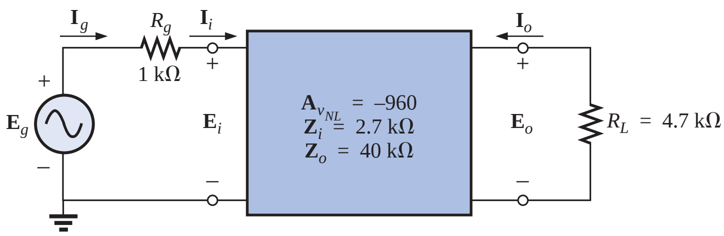

FIG. 26.18
Example 26.5.

a. Determine $\mathbf{A}_v$.
b. Calculate $\mathbf{A}_i$.
c. Increase R_L to double its current value, and note the effect on $\mathbf{A}_v$ and $\mathbf{A}_i$.
d. Find $\mathbf{A}_{i_T}$.
e. Calculate A_G.
f. Determine A_i from Eq. (26.1), and compare it to the value obtained in part (b).

Solutions:

a. $$\mathbf{A}_v = \mathbf{A}_{v_{NL}}\frac{R_L}{R_L + R_o} = (-960)\left(\frac{4.7\text{ k}\Omega}{4.7\text{ k}\Omega + 40\text{ k}\Omega}\right) = \mathbf{-100.94}$$

b. $$\mathbf{A}_i = -\mathbf{A}_{v_{NL}}\frac{R_i}{R_L + R_o} = -(-960)\left(\frac{2.7\text{ k}\Omega}{4.7\text{ k}\Omega + 40\text{ k}\Omega}\right) = \mathbf{57.99}$$

c. $R_L = 2(4.7\text{ k}\Omega) = 9.4\text{ k}\Omega$

$$\mathbf{A}_v = \mathbf{A}_{v_{NL}}\left(\frac{R_L}{R_L + R_o}\right) = (-960)\left(\frac{9.4\text{ k}\Omega}{9.4\text{ k}\Omega + 40\text{ k}\Omega}\right)$$

$= \mathbf{-182.67}$ versus -100.94, which is a significant increase

$$\mathbf{A}_i = -\mathbf{A}_{v_{NL}}\left(\frac{R_i}{R_L + R_o}\right) = -(-960)\left(\frac{2.7\text{ k}\Omega}{40\text{ k}\Omega + 9.4\text{ k}\Omega}\right)$$

$= \mathbf{52.47}$ versus 57.99

which is a drop in level but not as significant as the change in $\mathbf{A}_v$.

d. $\mathbf{A}_{i_T} = \mathbf{A}_i = \mathbf{57.99}$ as obtained in part (b)

However, $$\mathbf{A}_{i_T} = -\mathbf{A}_{v_T}\left(\frac{R_g + R_i}{R_L}\right)$$

$$= -\left[\mathbf{A}_v\frac{R_i}{(R_i + R_g)}\right]\left[\frac{(R_g + R_i)}{R_L}\right]$$

$$= -\mathbf{A}_v\frac{R_i}{R_L} = -(-100.94)\left(\frac{2.7\text{ k}\Omega}{4.7\text{ k}\Omega}\right)$$

$= \mathbf{57.99}$ as well

e. $$A_G = A_v^2\frac{R_i}{R_L} = (100.94)^2\left(\frac{2.7\text{ k}\Omega}{4.7\text{ k}\Omega}\right) = \mathbf{5853.19}$$

f. $A_G = -A_v A_i$

$$\text{or } A_i = \frac{A_G}{A_v} = -\frac{(5853.19)}{(-100.94)}$$

$$= \mathbf{57.99} \quad \text{as found in part (b)}$$

26.5 CASCADED SYSTEMS

When considering cascaded systems, as in Fig. 26.19, the most important fact to remember is that

the equations for cascaded systems employ the loaded voltage and current gains for each stage and not the nameplate unloaded levels.

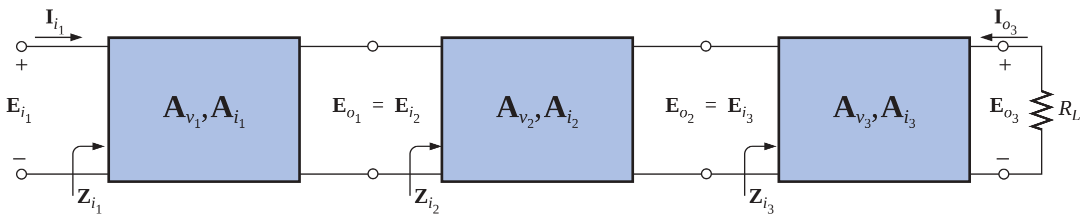

FIG. 26.19
Cascaded system.

Too often the labeled no-load gains are employed, resulting in enormous overall gains and unreasonably high expectations for the system. In addition, bear in mind that the input impedance of stage 3 may affect the input impedance of stage 2 and, therefore, the load on stage 1.

In general, therefore, the equations for cascaded systems initially appear to offer a high level of simplicity to the analysis. Simply be aware, however, that each term of the overall equations must be carefully evaluated before using the equation.

The total voltage gain for the system of Fig. 26.19 is

$$\mathbf{A}_{v_T} = \mathbf{A}_{v_1} \cdot \mathbf{A}_{v_2} \cdot \mathbf{A}_{v_3} \tag{26.18}$$

where, as noted above, the amplification factor of each stage is determined under loaded conditions.

The total current gain for the system of Fig. 26.19 is

$$\mathbf{A}_{i_T} = \mathbf{A}_{i_1} \cdot \mathbf{A}_{i_2} \cdot \mathbf{A}_{i_3} \tag{26.19}$$

where, again, the gain of each stage is determined under loaded (connected) conditions.

The current gain between any two stages can also be determined using an equation developed earlier in the chapter. For cascaded systems, the equation has the following general format:

$$\mathbf{A}_i = \mathbf{A}_v \frac{Z_i}{R_L} \tag{26.20}$$

where $\mathbf{A}_v$ is the loaded voltage gain corresponding to the desired loaded current gain. That is, if the gain is from the first to the third stages, then the voltage gain substituted is also from the first to third stages. The input impedance $\mathbf{Z}_i$ is for the first stage of interest, and R_L is the loading on the last stage of interest.

For example, for the three-stage amplifier of Fig. 26.19,

$$\mathbf{A}_{i_T} = \mathbf{A}_{v_T} \frac{Z_{i_1}}{R_L}$$

whereas for the first two stages

$$\mathbf{A}'_i = \mathbf{A}'_v \frac{Z_{i_1}}{Z_{i_3}}$$

where $$\mathbf{A}'_i = \frac{\mathbf{I}_{o_2}}{\mathbf{I}_{i_1}} \quad \text{and} \quad \mathbf{A}'_v = \frac{\mathbf{E}_{o_2}}{\mathbf{E}_{i_1}}$$

The total power gain is determined by

$$A_{G_T} = A_{v_T} A_{i_T} \tag{26.21}$$

whereas the gain between specific stages is simply the product of the voltage and current gains for each section. For example, for the first two stages of Fig. 26.19,

$$A'_G = A'_{v_2} \cdot A'_{i_2}$$

where $$A'_{v_2} = A_{v_1} \cdot A_{v_2} \quad \text{and} \quad A'_{i_2} = A_{i_1} \cdot A_{i_2}$$

EXAMPLE 26.6 For the cascaded system of Fig. 26.20, with its nameplate no-load parameters:

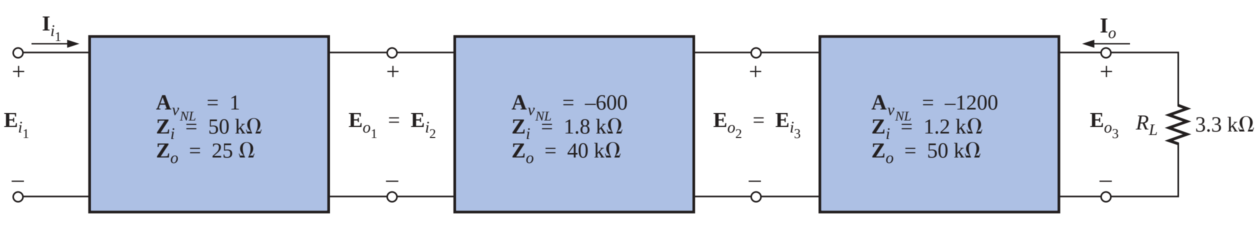

FIG. 26.20
Example 26.6.

a. Determine the load voltage and current gain for each stage, and redraw the system of Fig. 26.20 with the loaded parameters.
b. Calculate the total voltage and current gain.
c. Find the total power gain of the system using Eq. (26.21).
d. Calculate the voltage and current gain for the first two stages using Eqs. (26.18) and (26.19).
e. Determine the current gain for the first two stages using Eq. (26.20), and compare your answer with the result of part (d).
f. Calculate the power gain for the first two stages using Eq. (26.21).
g. Determine the power gain for the first two stages using Eq. (26.13). Compare this answer with the result of part (f).
h. Calculate the incorrect voltage gain for the entire system using Eq. (26.18) and the no-load nameplate level for each stage. Compare this answer to the result of part (b).

Solutions:

a. $$\mathbf{A}_{v_1} = \mathbf{A}_{v_{NL1}}\frac{R_L}{R_L + R_o} = \mathbf{A}_{v_{NL1}}\frac{Z_{i_2}}{Z_{i_2} + R_{o_1}} = (1)\frac{1.8\text{ k}\Omega}{1.8\text{ k}\Omega + 25\ \Omega}$$
$$= \mathbf{0.986}$$

$$\mathbf{A}_{v_2} = \mathbf{A}_{v_{NL2}}\frac{\mathbf{Z}_{i_3}}{Z_{i_3} + R_{o_2}} = (-600)\frac{1.2\text{ k}\Omega}{1.2\text{ k}\Omega + 40\text{ k}\Omega} = \mathbf{-17.476}$$

$$\mathbf{A}_{v_3} = \mathbf{A}_{v_{NL3}}\frac{R_L}{R_L + R_{o_3}} = (-1200)\frac{3.3\text{ k}\Omega}{3.3\text{ k}\Omega + 50\text{ k}\Omega} = \mathbf{-74.296}$$

$$\mathbf{A}_{i_1} = -\mathbf{A}_{v_{NL}}\frac{R_i}{R_L + R_o} = -\mathbf{A}_{v_{NL1}}\frac{Z_{i_1}}{Z_{i_2} + R_{o_1}} = -(1)\frac{50\text{ k}\Omega}{1.8\text{ k}\Omega + 25\ \Omega}$$
$$= \mathbf{-27.397}$$

$$\mathbf{A}_{i_2} = -\mathbf{A}_{v_{NL2}}\frac{Z_{i_2}}{Z_{i_3} + R_{o_2}} = -(-600)\frac{1.8\text{ k}\Omega}{1.2\text{ k}\Omega + 40\text{ k}\Omega} = \mathbf{26.214}$$

$$\mathbf{A}_{i_3} = -\mathbf{A}_{v_{NL3}}\frac{Z_{i_3}}{R_L + R_{o_3}} = -(-1200)\frac{1.2\text{ k}\Omega}{3.3\text{ k}\Omega + 50\text{ k}\Omega} = \mathbf{27.017}$$

Note Fig. 26.21.

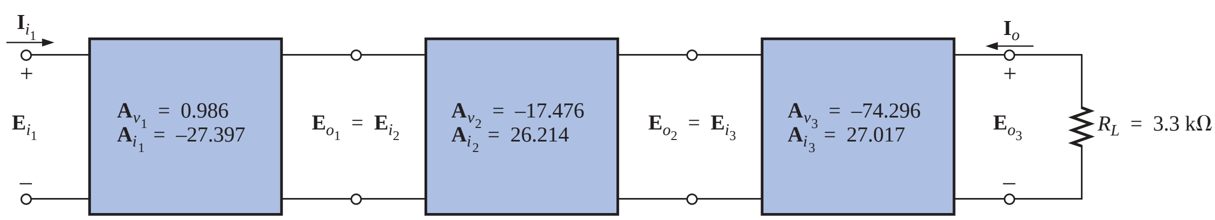

FIG. 26.21
Solution to Example 26.6.

b. $$\mathbf{A}_{v_T} = \frac{\mathbf{E}_{o_3}}{\mathbf{E}_{i_1}} = \mathbf{A}_{v_1}\cdot\mathbf{A}_{v_2}\cdot\mathbf{A}_{v_3} = (0.986)(-17.476)(-74.296)$$
$$= \mathbf{1280.22}$$

$$\mathbf{A}_{i_T} = \frac{\mathbf{I}_{o_3}}{\mathbf{I}_{i_1}} = \mathbf{A}_{i_1}\cdot\mathbf{A}_{i_2}\cdot\mathbf{A}_{i_3} = (-27.397)(26.214)(27.017)$$
$$= \mathbf{-19{,}403.20}$$

c. $A_{G_T} = -A_{v_T}\cdot A_{i_T} = -(1280.22)(-19{,}403.20) = \mathbf{24.84\times 10^6}$

d. $A'_{v_2} = \mathbf{A}_{v_1}\cdot\mathbf{A}_{v_2} = (0.986)(-17.476) = \mathbf{-17.231}$

$A'_{i_2} = \mathbf{A}_{i_1}\cdot\mathbf{A}_{i_2} = (-27.397)(26.214) = \mathbf{-718.185}$

e. $$\mathbf{A'}_{i_2} = \mathbf{A}_v\frac{\mathbf{Z}_i}{R_L} = \mathbf{A'}_{v_2}\frac{\mathbf{Z}_{i_1}}{\mathbf{Z}_{i_3}} = (-17.231)\frac{50\text{ k}\Omega}{1.2\text{ k}\Omega}$$
$$= \mathbf{-717.958} \qquad \text{versus } -718.185$$

with the difference due to the level of accuracy carried through the calculations.

f. $A'_{G_2} = A'_{v_2}\cdot A'_{i_2} = (-17.231)(-718.185) = \mathbf{12{,}375.05}$

g. $$A'_{G_2} = A_v^2\frac{R_i}{R_L} = (A'_{v_2})^2\frac{R_{i_1}}{Z_{i_3}} = (-17.231)^2\frac{50\text{ k}\Omega}{1.2\text{ k}\Omega} = \mathbf{12{,}371.14}$$

h. $\mathbf{A}_{v_T} = \mathbf{A}_{v_1} \cdot \mathbf{A}_{v_2} \cdot \mathbf{A}_{v_3} = (1)(-600)(-1200) = 7.2 \times 10^5$
$720{,}000 : 1280.22 = 562.40 : 1$
which is certainly a significant difference in results.

26.6 IMPEDANCE (z) PARAMETERS

For the two-port configuration of Fig. 26.22, four variables are specified. For most situations, if any two are specified, the remaining two variables can be determined. The four variables can be related by the following equations:

$$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \tag{26.22a}$$

$$\mathbf{E}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \tag{26.22b}$$

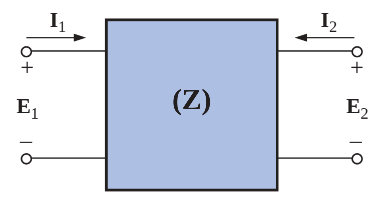

FIG. 26.22
Two-port impedance parameter configuration.

The **impedance parameters** $\mathbf{z}_{11}$, $\mathbf{z}_{12}$, and $\mathbf{z}_{22}$ are measured in ohms.

To model the system, each impedance parameter must be determined by setting a particular variable to zero.

$\mathbf{z}_{11}$

For $\mathbf{z}_{11}$, if $\mathbf{I}_2$ is set to zero, as shown in Fig. 26.23, Equation (26.22a) becomes

$$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}(0)$$

and

$$\mathbf{z}_{11} = \left.\frac{\mathbf{E}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2 = 0} \quad \text{(ohms, } \Omega) \tag{26.23}$$

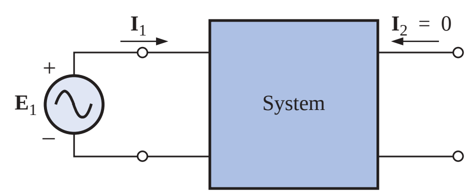

FIG. 26.23
Determining $\mathbf{z}_{11}$.

Equation (26.23) reveals that with $\mathbf{I}_2$ set to zero, the impedance parameter is determined by the resulting ratio of $\mathbf{E}_1$ to $\mathbf{I}_1$. Since $\mathbf{E}_1$ and $\mathbf{I}_1$ are both input quantities, with $\mathbf{I}_2$ set to zero, the parameter $\mathbf{z}_{11}$ is formally referred to in the following manner:

$\mathbf{z}_{11}$ = open-circuit, input-impedance parameter

$\mathbf{z}_{12}$

For $\mathbf{z}_{12}$, $\mathbf{I}_1$ is set to zero, and Equation (26.22a) results in

$$\mathbf{z}_{12} = \left.\frac{\mathbf{E}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_2 = 0} \quad \text{(ohms, } \Omega) \tag{26.24}$$

For most systems where input and output quantities are to be compared, the ratio of interest is usually that of the output quantity divided by the input quantity. In this case, the *reverse* is true, resulting in the following:

$\mathbf{z}_{12}$ = open-circuit, reverse-transfer impedance parameter

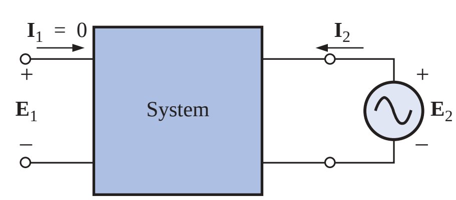

FIG. 26.24
Determining $\mathbf{z}_{12}$.

The term *transfer* is included to indicate that $\mathbf{z}_{12}$ will relate an input and output quantity (for the condition $\mathbf{I}_1 = 0$). The network configuration for determining $\mathbf{z}_{12}$ is shown in Fig. 26.24.

For an applied source $\mathbf{E}_2$, the ratio $\mathbf{E}_1/\mathbf{I}_2$ will determine $\mathbf{z}_{12}$ with $\mathbf{I}_1$ set to zero.

$\mathbf{z}_{21}$

To determine $\mathbf{z}_{21}$, set $\mathbf{I}_2$ to zero and find the ratio $\mathbf{E}_2/\mathbf{I}_1$ as determined by Eq. (26.22b). That is,

$$\mathbf{z}_{21} = \frac{\mathbf{E}_2}{\mathbf{I}_1}\bigg|_{\mathbf{I}_2 = 0} \quad \text{(ohms, } \Omega) \qquad \textbf{(26.25)}$$

In this case, input and output quantities are again the determining variables, requiring the term *transfer* in the nomenclature. However, the ratio is that of an output to an input quantity, so the descriptive term *forward* is applied, and

$\mathbf{z}_{21}$ = open-circuit, forward-transfer impedance parameter

The determining network is shown in Fig. 26.25. For an applied voltage $\mathbf{E}_1$, it is determined by the ratio $\mathbf{E}_2/\mathbf{I}_1$ with $\mathbf{I}_2$ set to zero.

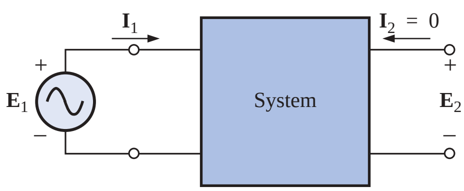

FIG. 26.25
Determining $\mathbf{z}_{21}$.

$\mathbf{z}_{22}$

The remaining parameter, $\mathbf{z}_{22}$, is determined by

$$\mathbf{z}_{22} = \frac{\mathbf{E}_2}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1 = 0} \quad \text{(ohms, } \Omega) \qquad \textbf{(26.26)}$$

as derived from Eq. (26.22b) with $\mathbf{I}_1$ set to zero. Since it is the ratio of the output voltage to the output current with $\mathbf{I}_1$ set to zero, it has the terminology

$\mathbf{z}_{22}$ = open-circuit, output-impedance parameter

The required network is shown in Fig. 26.26. For an applied voltage $\mathbf{E}_2$, it is determined by the resulting ratio $\mathbf{E}_2/\mathbf{I}_2$ with $\mathbf{I}_1 = 0$.

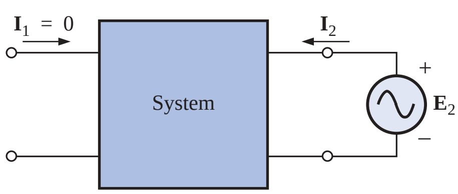

FIG. 26.26
Determining $\mathbf{z}_{22}$.

EXAMPLE 26.7 Determine the impedance (**z**) parameters for the T network of Fig. 26.27.

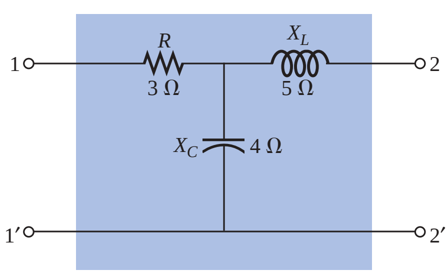

FIG. 26.27
T configuration.

Solution: For $\mathbf{z}_{11}$, the network will appear as shown in Fig. 26.28, with $\mathbf{Z}_1 = 3\ \Omega \angle 0°$, $\mathbf{Z}_2 = 5\ \Omega \angle 90°$, and $\mathbf{Z}_3 = 4\ \Omega \angle -90°$:

$$\mathbf{I}_1 = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_3}$$

Thus

$$\mathbf{z}_{11} = \frac{\mathbf{E}_1}{\mathbf{I}_1}\bigg|_{\mathbf{I}_2 = 0}$$

and

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{Z}_3 \qquad \textbf{(26.27)}$$

For $\mathbf{z}_{12}$, the network will appear as shown in Fig. 26.29, and

$$\mathbf{E}_1 = \mathbf{I}_2\mathbf{Z}_3$$

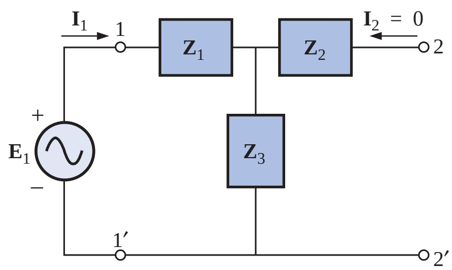

FIG. 26.28
Determining $\mathbf{z}_{11}$.

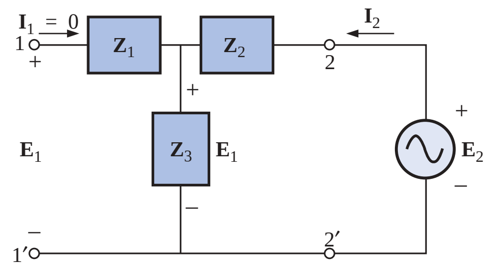

FIG. 26.29
Determining $\mathbf{z}_{12}$.

Thus
$$\mathbf{z}_{12} = \left.\frac{\mathbf{E}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1 = 0} = \frac{\mathbf{I}_2\mathbf{Z}_3}{\mathbf{I}_2}$$

and
$$\boxed{\mathbf{z}_{12} = \mathbf{Z}_3} \qquad \textbf{(26.28)}$$

For $\mathbf{z}_{21}$, the required network appears in Fig. 26.30, and

$$\mathbf{E}_2 = \mathbf{I}_1\mathbf{Z}_3$$

Thus,
$$\mathbf{z}_{21} = \left.\frac{\mathbf{E}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2 = 0} = \frac{\mathbf{I}_1\mathbf{Z}_3}{\mathbf{I}_1}$$

and
$$\boxed{\mathbf{z}_{21} = \mathbf{Z}_3} \qquad \textbf{(26.29)}$$

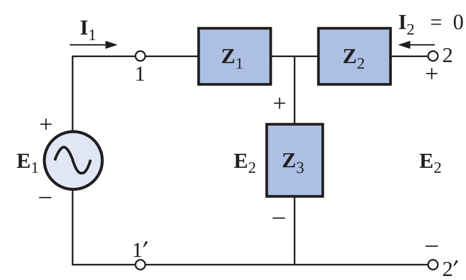

FIG. 26.30
Determining $\mathbf{z}_{21}$.

For $\mathbf{z}_{22}$, the determining configuration is shown in Fig. 26.31, and

$$\mathbf{I}_2 = \frac{\mathbf{E}_2}{\mathbf{Z}_2 + \mathbf{Z}_3}$$

Thus
$$\mathbf{z}_{22} = \left.\frac{\mathbf{E}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1 = 0} = \frac{\mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3)}{\mathbf{I}_2}$$

and
$$\boxed{\mathbf{z}_{22} = \mathbf{Z}_2 + \mathbf{Z}_3} \qquad \textbf{(26.30)}$$

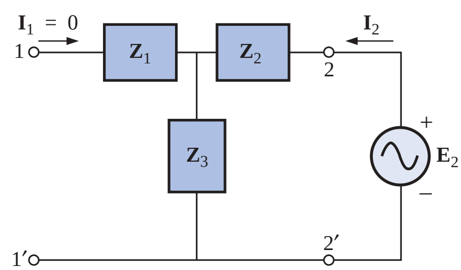

FIG. 26.31
Determining $\mathbf{z}_{22}$.

Note that for the T configuration, $\mathbf{z}_{12} = \mathbf{z}_{21}$. For $\mathbf{Z}_1 = 3\ \Omega\ \angle 0°$, $\mathbf{Z}_2 = 5\ \Omega\ \angle 90°$, and $\mathbf{Z}_3 = 4\ \Omega\ \angle -90°$, we have

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{3\ \Omega - \mathit{j}\,4\ \Omega}$$
$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_3 = 4\ \Omega\ \angle -90° = \mathbf{-\mathit{j}\,4\ \Omega}$$
$$\mathbf{z}_{22} = \mathbf{Z}_2 + \mathbf{Z}_3 = 5\ \Omega\ \angle 90° + 4\ \Omega\ \angle -90° = 1\ \Omega\ \angle 90° = \mathbf{\mathit{j}\,1\ \Omega}$$

For a set of impedance parameters, the terminal (external) behavior of the device or network within the configuration of Fig. 26.22 is determined. An *equivalent circuit* for the system can be developed using the impedance parameters and Eqs. (26.22a) and (26.22b). Two possibilities for the impedance parameters appear in Fig. 26.32.

Applying Kirchhoff's voltage law to the input and output loops of the network of Fig. 26.32(a) results in

$$\mathbf{E}_1 - \mathbf{z}_{11}\mathbf{I}_1 - \mathbf{z}_{12}\mathbf{I}_2 = 0$$

and
$$\mathbf{E}_2 - \mathbf{z}_{22}\mathbf{I}_2 - \mathbf{z}_{21}\mathbf{I}_1 = 0$$

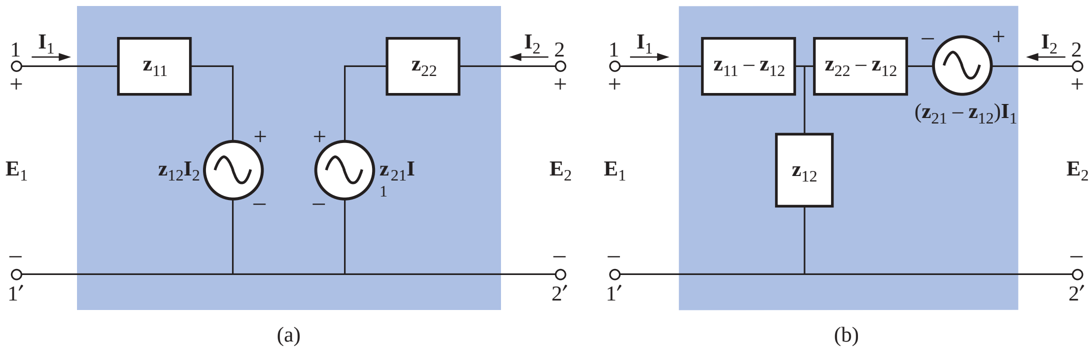

FIG. 26.32
*Two possible two-port, **z**-parameter equivalent networks.*

which, when rearranged, become

$$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad \mathbf{E}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

matching Eqs. (26.22a) and (26.22b).

For the network of Fig. 26.32(b),

$$\mathbf{E}_1 - \mathbf{I}_1(\mathbf{z}_{11} - \mathbf{z}_{12}) - \mathbf{z}_{12}(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

and
$$\mathbf{E}_2 - \mathbf{I}_1(\mathbf{z}_{21} - \mathbf{z}_{12}) - \mathbf{I}_2(\mathbf{z}_{22} - \mathbf{z}_{12}) - \mathbf{z}_{12}(\mathbf{I}_1 + \mathbf{I}_2) = 0$$

which, when rearranged, are

$$\mathbf{E}_1 = \mathbf{I}_1(\mathbf{z}_{11} - \mathbf{z}_{12} + \mathbf{z}_{12}) + \mathbf{I}_2\mathbf{z}_{12}$$
$$\mathbf{E}_2 = \mathbf{I}_1(\mathbf{z}_{21} - \mathbf{z}_{12} + \mathbf{z}_{12}) + \mathbf{I}_2(\mathbf{z}_{22} - \mathbf{z}_{12} + \mathbf{z}_{12})$$

and
$$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{E}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

Note in each network the necessity for a current-controlled voltage source, that is, a voltage source the magnitude of which is determined by a particular current of the network.

The usefulness of the impedance parameters and the resulting equivalent networks can best be described by considering the system of Fig. 26.33(a), which contains a device (or system) for which the impedance

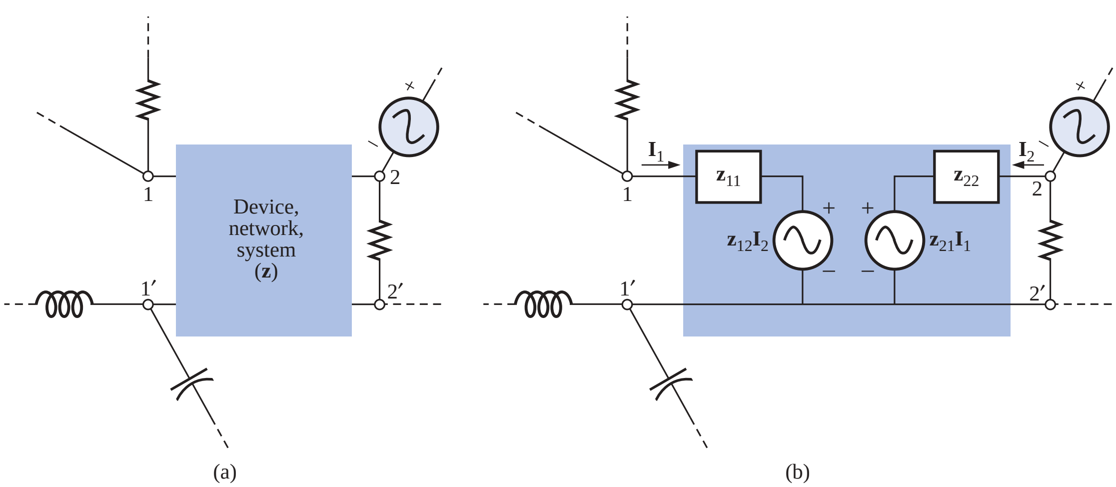

FIG. 26.33
*Substitution of the **z**-parameter equivalent network into a complex system.*

parameters have been determined. As shown in Fig. 26.33(b), the equivalent network for the device (or system) can then be substituted, and methods such as mesh analysis, nodal analysis, and so on, can be employed to determine required unknown quantities. The device itself can then be replaced with an equivalent circuit and the desired solutions obtained more directly and with less effort than is required using only the characteristics of the device.

EXAMPLE 26.8 Draw the equivalent circuit in the form shown in Fig. 26.32(b) using the impedance parameters determined in Example 26.7.

Solution: The circuit appears in Fig. 26.34.

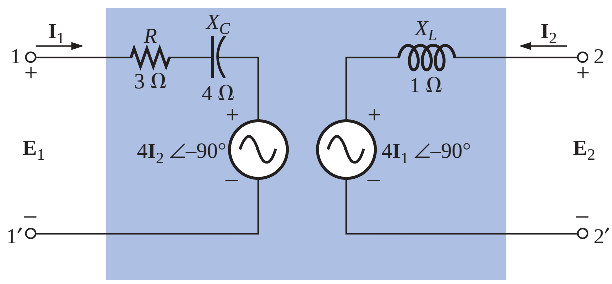

FIG. 26.34
Example 26.8.

26.7 ADMITTANCE (y) PARAMETERS

The equations relating the four terminal variables of Fig. 26.22 can also be written in the following form:

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{E}_1 + \mathbf{y}_{12}\mathbf{E}_2 \qquad \textbf{(26.31a)}$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{E}_1 + \mathbf{y}_{22}\mathbf{E}_2 \qquad \textbf{(26.31b)}$$

Note that in this case each term of each equation has the units of current, compared to voltage for each term of Eqs. (26.22a) and (26.22b). In addition, the unit of each coefficient is siemens, compared with the ohm for the impedance parameters.

The impedance parameters were determined by setting a particular current to zero through an open-circuit condition. For the **admittance (y) parameters** of Eqs. (26.31a) and (26.31b), a voltage is set to zero through a short-circuit condition.

The terminology applied to each of the admittance parameters follows directly from the descriptive terms applied to each of the impedance parameters. The equations for each are determined directly from Eqs. (26.31a) and (26.31b) by setting a particular voltage to zero.

$\mathbf{y}_{11}$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{E}_1}\bigg|_{\mathbf{E}_2 = 0} \quad \text{(siemens, S)} \tag{26.32}$$

$\mathbf{y}_{11}$ = short-circuit, input-admittance parameter

The determining network appears in Fig. 26.35.

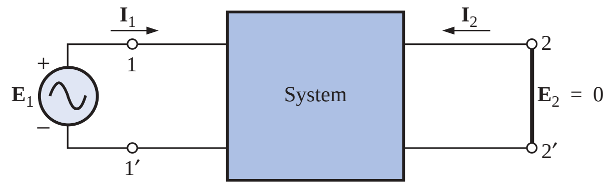

FIG. 26.35
$\mathbf{y}_{11}$ *determination.*

$\mathbf{y}_{12}$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{E}_2}\bigg|_{\mathbf{E}_1 = 0} \quad \text{(siemens, S)} \tag{26.33}$$

$\mathbf{y}_{12}$ = short-circuit, reverse-transfer admittance parameter

The network for determining $\mathbf{y}_{12}$ appears in Fig. 26.36.

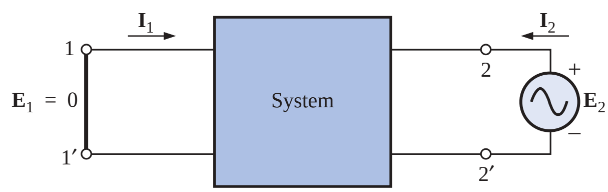

FIG. 26.36
$\mathbf{y}_{12}$ *determination.*

$\mathbf{y}_{21}$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{E}_1}\bigg|_{\mathbf{E}_2 = 0} \quad \text{(siemens, S)} \tag{26.34}$$

$\mathbf{y}_{21}$ = short-circuit, forward-transfer admittance parameter

The network for determining $\mathbf{y}_{21}$ appears in Fig. 26.37.

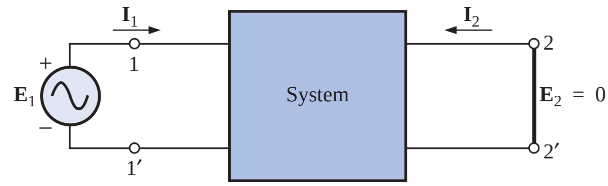

FIG. 26.37
$\mathbf{y}_{21}$ *determination.*

$\mathbf{y}_{22}$

$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{E}_2}\right|_{\mathbf{E}_1 = 0} \quad \text{(siemens, S)} \qquad \textbf{(26.35)}$$

$\mathbf{y}_{22}$ = short-circuit, output-admittance parameter

The required network appears in Fig. 26.38.

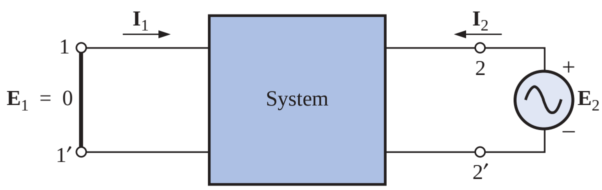

FIG. 26.38
$\mathbf{y}_{22}$ determination.

EXAMPLE 26.9 Determine the admittance parameters for the π network of Fig. 26.39.

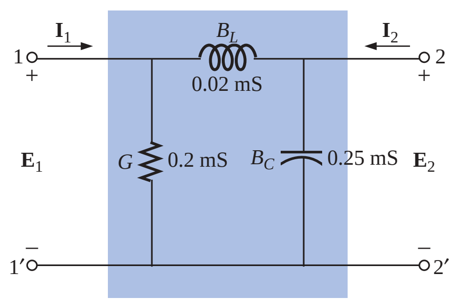

FIG. 26.39
π network.

Solution: The network for $\mathbf{y}_{11}$ will appear as shown in Fig. 26.40, with

$$\mathbf{Y}_1 = 0.2 \text{ mS} \angle 0° \qquad \mathbf{Y}_2 = 0.02 \text{ mS} \angle -90° \qquad \mathbf{Y}_3 = 0.25 \text{ mS} \angle 90°$$

We use
$$\mathbf{I}_1 = \mathbf{E}_1\mathbf{Y}_T = \mathbf{E}_1(\mathbf{Y}_1 + \mathbf{Y}_2)$$

with
$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{E}_1}\right|_{\mathbf{E}_2 = 0}$$

and
$$\mathbf{y}_{11} = \mathbf{Y}_1 + \mathbf{Y}_2 \qquad \textbf{(26.36)}$$

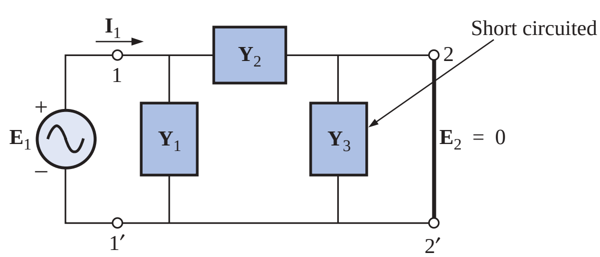

FIG. 26.40
Determining $\mathbf{y}_{11}$.

The determining network for $\mathbf{y}_{12}$ appears in Fig. 26.41. $\mathbf{Y}_1$ is short circuited; so $\mathbf{I}_{\mathbf{Y}_2} = \mathbf{I}_1$, and

$$\mathbf{I}_{\mathbf{Y}_2} = \mathbf{I}_1 = -\mathbf{E}_2\mathbf{Y}_2$$

The minus sign results because the defined direction of $\mathbf{I}_1$ in Fig. 26.41 is opposite to the actual flow direction due to the applied source $\mathbf{E}_2$; that is,

$$\mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{E}_2}\right|_{\mathbf{E}_1 = 0}$$

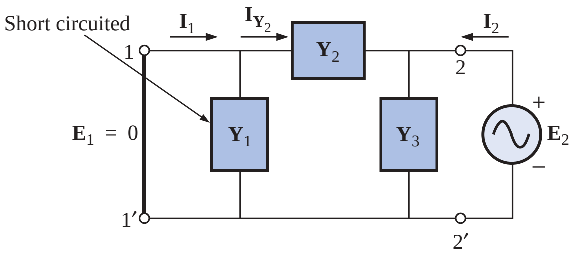

FIG. 26.41
Determining $\mathbf{y}_{12}$.

and

$$\mathbf{y}_{12} = -\mathbf{Y}_2 \tag{26.37}$$

The network employed for $\mathbf{y}_{21}$ appears in Fig. 26.42. In this case, $\mathbf{Y}_3$ is short circuited, resulting in

$$\mathbf{I}_{\mathbf{Y}_2} = \mathbf{I}_2 \quad \text{and} \quad \mathbf{I}_{\mathbf{Y}_2} = \mathbf{I}_2 = -\mathbf{E}_1\mathbf{Y}_2$$

with

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{E}_1}\bigg|_{\mathbf{E}_2 = 0}$$

and

$$\mathbf{y}_{21} = -\mathbf{Y}_2 \tag{26.38}$$

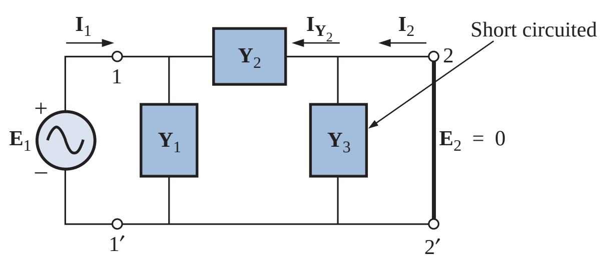

FIG. 26.42
Determining $\mathbf{y}_{21}$.

Note that for the π configuration, $\mathbf{y}_{12} = \mathbf{y}_{21}$, which was expected since the impedance parameters for the T network were such that $\mathbf{z}_{12} = \mathbf{z}_{21}$. A T network can be converted directly to a π network using the Y-Δ transformation.

The determining network for $\mathbf{y}_{22}$ appears in Fig. 26.43, and

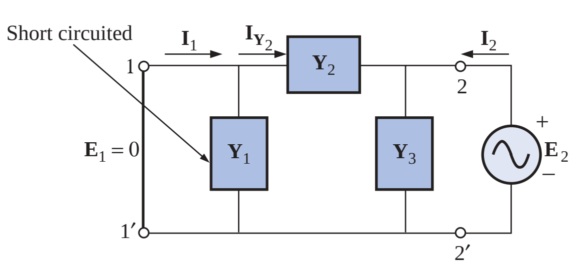

FIG. 26.43
Determining $\mathbf{y}_{22}$.

$$\mathbf{Y}_T = \mathbf{Y}_2 + \mathbf{Y}_3 \quad \text{and} \quad \mathbf{I}_2 = \mathbf{E}_2(\mathbf{Y}_2 + \mathbf{Y}_3)$$

Thus
$$\mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{E}_2}\right|_{\mathbf{E}_1 = 0}$$

and
$$\mathbf{y}_{22} = \mathbf{Y}_2 + \mathbf{Y}_3 \qquad \textbf{(26.39)}$$

Substituting values, we have

$$\mathbf{Y}_1 = 0.2\text{ mS } \angle 0°$$
$$\mathbf{Y}_2 = 0.02\text{ mS } \angle -90°$$
$$\mathbf{Y}_3 = 0.25\text{ mS } \angle 90°$$
$$\mathbf{y}_{11} = \mathbf{Y}_1 + \mathbf{Y}_2$$
$$= \mathbf{0.2\text{ mS} - \mathit{j}\,0.02\text{ mS } (\mathit{L})}$$
$$\mathbf{y}_{12} = \mathbf{y}_{21} = -\mathbf{Y}_2 = -(-j\,0.02\text{ mS})$$
$$= \mathbf{\mathit{j}\,0.02\text{ mS } (\mathit{C})}$$
$$\mathbf{y}_{22} = \mathbf{Y}_2 + \mathbf{Y}_3 = -j\,0.02\text{ mS} + j\,0.25\text{ mS}$$
$$= \mathbf{\mathit{j}\,0.23\text{ mS } (\mathit{C})}$$

Note the similarities between the results for $\mathbf{y}_{11}$ and $\mathbf{y}_{22}$ for the π network compared with $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ for the T network.

Two networks satisfying the terminal relationships of Eqs. (26.31a) and (26.31b) are shown in Fig. 26.44. Note the use of parallel branches

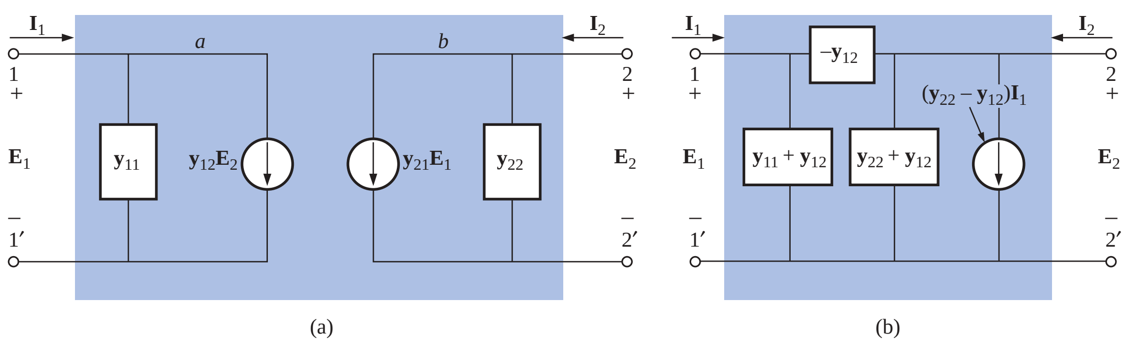

FIG. 26.44
Two possible two-port, **y***-parameter equivalent networks.*

since each term of Eqs. (26.31a) and (26.31b) has the units of current, and the most direct route to the equivalent circuit is an application of Kirchhoff's current law in reverse. That is, find the network that satisfies Kirchhoff's current law relationship. For the impedance parameters, each term had the units of volts, so Kirchhoff's voltage law was applied in reverse to determine the series combination of elements in the equivalent circuit of Fig. 26.44(a).

Applying Kirchhoff's current law to the network of Fig. 26.44(a), we have

$$\text{Node } a\text{:}\quad \overbrace{\mathbf{I}_1}^{\text{Entering}} = \overbrace{\mathbf{y}_{11}\mathbf{E}_1 + \mathbf{y}_{12}\mathbf{E}_2}^{\text{Leaving}}$$

$$\text{Node } b\text{:}\quad \mathbf{I}_2 = \mathbf{y}_{22}\mathbf{E}_2 + \mathbf{y}_{21}\mathbf{E}_1$$

which, when rearranged, are Eqs. (26.31a) and (26.31b).

For the results of Example 26.9, the network of Fig. 26.45 will result if the equivalent network of Fig. 26.44(a) is employed.

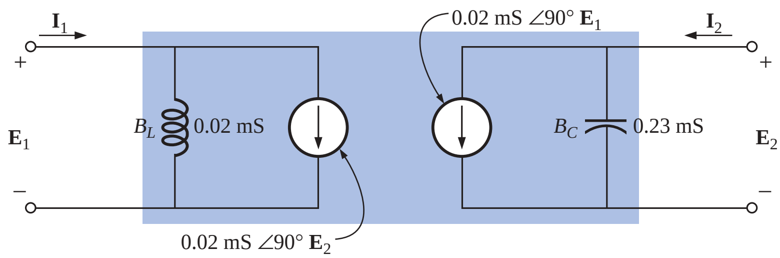

FIG. 26.45
Equivalent network for the results of Example 26.9.

26.8 HYBRID (h) PARAMETERS

The **hybrid (h) parameters** are employed extensively in the analysis of transistor networks. The term *hybrid* is derived from the fact that the parameters have a mixture of units (a hybrid set) rather than a single unit of measurement such as ohms or siemens, used for the **z** and **y** parameters, respectively. The defining hybrid equations have a mixture of current *and* voltage variables on one side, as follows:

$$\boxed{\mathbf{E}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{E}_2} \qquad \textbf{(26.40a)}$$

$$\boxed{\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{E}_2} \qquad \textbf{(26.40b)}$$

To determine the hybrid parameters, it will be necessary to establish both the short-circuit and the open-circuit condition, depending on the parameter desired.

$\mathbf{h}_{11}$

$$\boxed{\mathbf{h}_{11} = \frac{\mathbf{E}_1}{\mathbf{I}_1}}\Bigg|_{\mathbf{E}_2 = 0} \qquad \text{(ohms, } \Omega) \qquad \textbf{(26.41)}$$

$\mathbf{h}_{11}$ = ***short-circuit, input-impedance parameter***

The determining network is shown in Fig. 26.46.

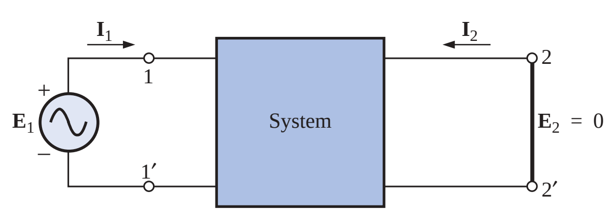

FIG. 26.46
$\mathbf{h}_{11}$ *determination.*

$\mathbf{h}_{12}$

$$\mathbf{h}_{12} = \left.\frac{\mathbf{E}_1}{\mathbf{E}_2}\right|_{\mathbf{I}_1 = 0} \quad \text{(dimensionless)} \tag{26.42}$$

$\mathbf{h}_{12}$ = open-circuit, reverse-transfer voltage ratio parameter

The network employed in determining $\mathbf{h}_{12}$ is shown in Fig. 26.47.

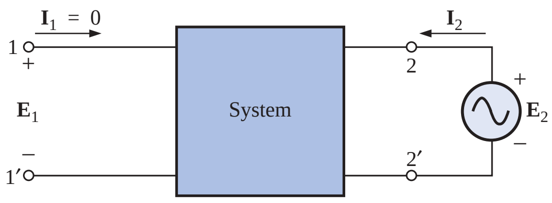

FIG. 26.47
$\mathbf{h}_{12}$ *determination.*

$\mathbf{h}_{21}$

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{E}_2 = 0} \quad \text{(dimensionless)} \tag{26.43}$$

$\mathbf{h}_{21}$ = short-circuit, forward-transfer current ratio parameter

The determining network appears in Fig. 26.48.

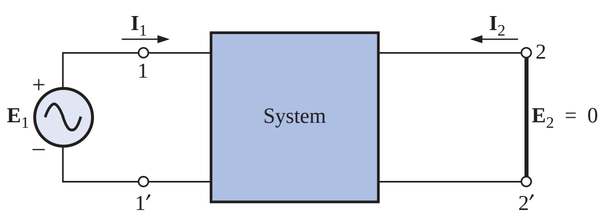

FIG. 26.48
$\mathbf{h}_{21}$ *determination.*

$\mathbf{h}_{22}$

$$\mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{E}_2}\right|_{\mathbf{I}_1 = 0} \quad \text{(siemens, S)} \tag{26.44}$$

$\mathbf{h}_{22}$ = open-circuit, output admittance parameter

The network employed to determine $\mathbf{h}_{22}$ is shown in Fig. 26.49.

The subscript notation for the hybrid parameters is reduced to the following for most applications. The letter chosen is that letter appearing in boldface in the preceding description of each parameter:

$$\mathbf{h}_{11} = \mathbf{h}_i \qquad \mathbf{h}_{12} = \mathbf{h}_r \qquad \mathbf{h}_{21} = \mathbf{h}_f \qquad \mathbf{h}_{22} = \mathbf{h}_o$$

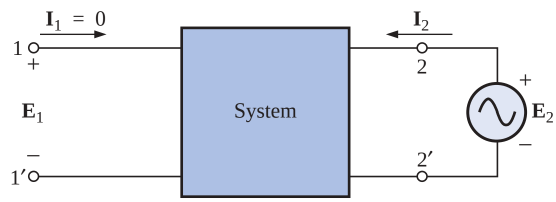

FIG. 26.49
$\mathbf{h}_{22}$ *determination.*

The hybrid equivalent circuit appears in Fig. 26.50. Since the unit of measurement for each term of Eq. (26.40a) is the volt, Kirchhoff's voltage law was applied in reverse to obtain the series input circuit indicated. The unit of measurement of each term of Eq. (26.40b) has the units of current, resulting in the parallel elements of the output circuit as obtained by applying Kirchhoff's current law in reverse.

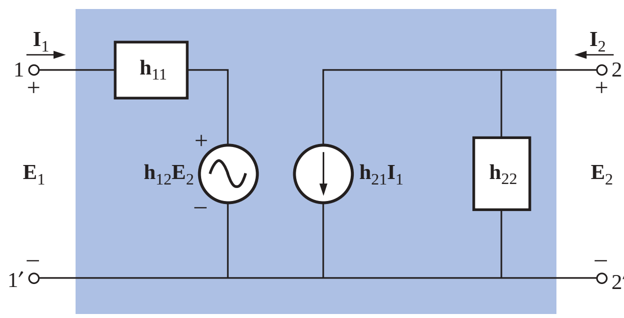

FIG. 26.50
Two-port, hybrid-parameter equivalent network.

Note that the input circuit has a voltage-controlled voltage source whose controlling voltage is the output terminal voltage, while the output circuit has a current-controlled current source whose controlling current is the current of the input circuit.

EXAMPLE 26.10 For the hybrid equivalent circuit of Fig. 26.51:

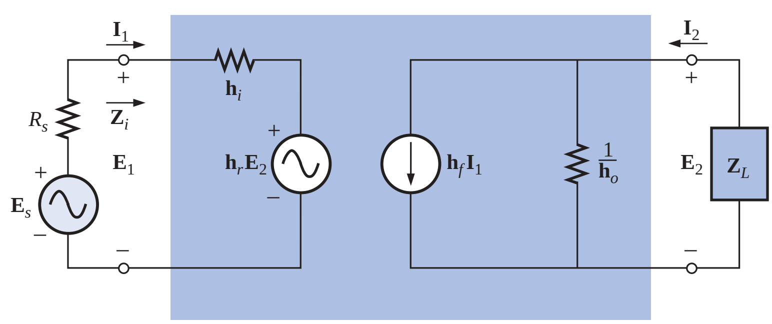

FIG. 26.51
Example 26.10.

a. Determine the current ratio (gain) $\mathbf{A}_i = \mathbf{I}_2/\mathbf{I}_1$.
b. Determine the voltage ratio (gain) $\mathbf{A}_v = \mathbf{E}_2/\mathbf{E}_1$.

Solutions:

a. Using the current divider rule, we have

$$\mathbf{I}_2 = \frac{(1/\mathbf{h}_o)\mathbf{h}_f\mathbf{I}_1}{(1/\mathbf{h}_o) + \mathbf{Z}_L} = \frac{\mathbf{h}_f\mathbf{I}_1}{1 + \mathbf{h}_o\mathbf{Z}_L}$$

and

$$\mathbf{A}_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{h}_f}{1 + \mathbf{h}_o\mathbf{Z}_L} \qquad \textbf{(26.45)}$$

b. Applying Kirchhoff's voltage law to the input circuit gives us

$$\mathbf{E}_1 - \mathbf{h}_i\mathbf{I}_1 - \mathbf{h}_r\mathbf{E}_2 = 0 \quad \text{and} \quad \mathbf{I}_1 = \frac{\mathbf{E}_1 - \mathbf{h}_r\mathbf{E}_2}{\mathbf{h}_i}$$

Apply Kirchhoff's current law to the output circuit:

$$\mathbf{I}_2 = \mathbf{h}_f\mathbf{I}_1 + \mathbf{h}_o\mathbf{E}_2$$

However,

$$\mathbf{I}_2 = -\frac{\mathbf{E}_2}{\mathbf{Z}_L}$$

so

$$-\frac{\mathbf{E}_2}{\mathbf{Z}_L} = \mathbf{h}_f\mathbf{I}_1 + \mathbf{h}_o\mathbf{E}_2$$

Substituting for $\mathbf{I}_1$ gives us

$$-\frac{\mathbf{E}_2}{\mathbf{Z}_L} = \mathbf{h}_f\left(\frac{\mathbf{E}_1 - \mathbf{h}_r\mathbf{E}_2}{\mathbf{h}_i}\right) + \mathbf{h}_o\mathbf{E}_2$$

or

$$\mathbf{h}_i\mathbf{E}_2 = -\mathbf{h}_f\mathbf{Z}_L\mathbf{E}_1 + \mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L\mathbf{E}_2 - \mathbf{h}_i\mathbf{h}_o\mathbf{Z}_L\mathbf{E}_2$$

and

$$\mathbf{E}_2(\mathbf{h}_i - \mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L + \mathbf{h}_i\mathbf{h}_o\mathbf{Z}_L) = -\mathbf{h}_f\mathbf{Z}_L\mathbf{E}_1$$

with the result that

$$\mathbf{A}_v = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{-\mathbf{h}_f\mathbf{Z}_L}{\mathbf{h}_i(1 + \mathbf{h}_o\mathbf{Z}_L) - \mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L} \qquad \textbf{(26.46)}$$

EXAMPLE 26.11 For a particular transistor, $\mathbf{h}_i = 1\ \text{k}\Omega$, $\mathbf{h}_r = 4 \times 10^{-4}$, $\mathbf{h}_f = 50$, and $\mathbf{h}_o = 25\ \mu\text{s}$. Determine the current and the voltage gain if $\mathbf{Z}_L$ is a 2-kΩ resistive load.

Solution:

$$\mathbf{A}_i = \frac{\mathbf{h}_f}{1 + \mathbf{h}_o\mathbf{Z}_L} = \frac{50}{1 + (25\ \mu\text{S})(2\ \text{k}\Omega)}$$

$$= \frac{50}{1 + (50 \times 10^{-3})} = \frac{50}{1.050} = \mathbf{47.62}$$

$$\mathbf{A}_v = \frac{-\mathbf{h}_f\mathbf{Z}_L}{\mathbf{h}_i(1 + \mathbf{h}_o\mathbf{Z}_L) - \mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L}$$

$$= \frac{-(50)(2\ \text{k}\Omega)}{(1\ \text{k}\Omega)(1.050) - (4 \times 10^{-4})(50)(2\ \text{k}\Omega)}$$

$$= \frac{-100 \times 10^3}{(1.050 \times 10^3) - (0.04 \times 10^3)} = -\frac{100}{1.01} = \mathbf{-99}$$

The minus sign simply indicates a phase shift of 180° between $\mathbf{E}_2$ and $\mathbf{E}_1$ for the defined polarities in Fig. 26.51.

26.9 INPUT AND OUTPUT IMPEDANCES

The input and output impedances will now be determined for the hybrid equivalent circuit and a **z**-parameter equivalent circuit. The input impedance can always be determined by the ratio of the input voltage to the input current with or without a load applied. The output impedance is always determined with the source voltage or current set to zero. We found in the previous section that for the hybrid equivalent circuit of Fig. 26.51,

$$\mathbf{E}_1 = \mathbf{h}_i\mathbf{I}_1 + \mathbf{h}_r\mathbf{E}_2$$

$$\mathbf{E}_2 = -\mathbf{I}_2\mathbf{Z}_L$$

and

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{h}_f}{1 + \mathbf{h}_o\mathbf{Z}_L}$$

By substituting for $\mathbf{I}_2$ in the second equation (using the relationship of the last equation), we have

$$\mathbf{E}_2 = -\left(\frac{\mathbf{h}_f\mathbf{I}_1}{1 + \mathbf{h}_o\mathbf{Z}_L}\right)\mathbf{Z}_L$$

so the first equation becomes

$$\mathbf{E}_1 = \mathbf{h}_i\mathbf{I}_1 + \mathbf{h}_r\left(-\frac{\mathbf{h}_f\mathbf{I}_1\mathbf{Z}_L}{1 + \mathbf{h}_o\mathbf{Z}_L}\right)$$

and

$$\mathbf{E}_1 = \mathbf{I}_1\left(\mathbf{h}_i - \frac{\mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L}{1 + \mathbf{h}_o\mathbf{Z}_L}\right)$$

Thus,

$$\mathbf{Z}_i = \frac{\mathbf{E}_1}{\mathbf{I}_1} = \mathbf{h}_i - \frac{\mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L}{1 + \mathbf{h}_o\mathbf{Z}_L} \qquad \textbf{(26.47)}$$

For the output impedance, we will set the source voltage to zero but preserve its internal resistance R_s as shown in Fig. 26.52.

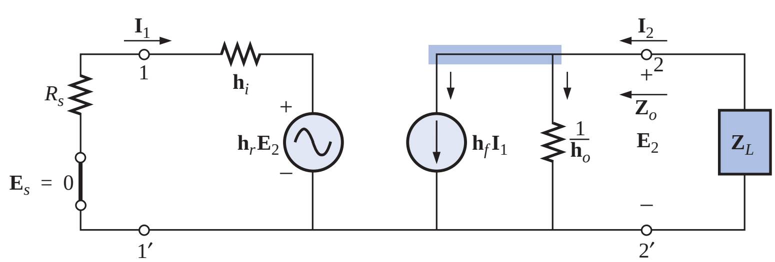

FIG. 26.52
Determining $\mathbf{Z}_o$ for the hybrid equivalent network.

Since

$$\mathbf{E}_s = 0$$

then

$$\mathbf{I}_1 = -\frac{\mathbf{h}_r\mathbf{E}_2}{\mathbf{h}_i + R_s}$$

From the output circuit,

$$\mathbf{I}_2 = \mathbf{h}_f\mathbf{I}_1 + \mathbf{h}_o\mathbf{E}_2$$

or

$$\mathbf{I}_2 = \mathbf{h}_f\left(-\frac{\mathbf{h}_r\mathbf{E}_2}{\mathbf{h}_i + R_s}\right) + \mathbf{h}_o\mathbf{E}_2$$

and $$\mathbf{I}_2 = \left(-\frac{\mathbf{h}_r\mathbf{h}_f}{\mathbf{h}_i + R_s} + \mathbf{h}_o\right)\mathbf{E}_2$$

Thus, $$\mathbf{Z}_o = \frac{\mathbf{E}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{h}_o - \dfrac{\mathbf{h}_r\mathbf{h}_f}{\mathbf{h}_i + R_s}} \tag{26.48}$$

EXAMPLE 26.12 Determine $\mathbf{Z}_i$ and $\mathbf{Z}_o$ for the transistor having the parameters of Example 26.11 if $R_s = 1\ \text{k}\Omega$.

Solution:

$$\mathbf{Z}_i = \mathbf{h}_i - \frac{\mathbf{h}_r\mathbf{h}_f\mathbf{Z}_L}{1 + \mathbf{h}_o\mathbf{Z}_L} = 1\ \text{k}\Omega - \frac{0.04\ \text{k}\Omega}{1.050}$$
$$= 1 \times 10^3 - 0.0381 \times 10^3 = \mathbf{961.9\ \Omega}$$
$$\mathbf{Z}_o = \frac{1}{\mathbf{h}_o - \dfrac{\mathbf{h}_r\mathbf{h}_f}{\mathbf{h}_i + R_s}} = \frac{1}{25\ \mu\text{S} - \dfrac{(4 \times 10^{-4})(50)}{1\ \text{k}\Omega + 1\ \text{k}\Omega}}$$
$$= \frac{1}{25 \times 10^{-6} - 10 \times 10^{-6}} = \frac{1}{15 \times 10^{-6}}$$
$$= \mathbf{66.67\ k\Omega}$$

For the **z**-parameter equivalent circuit of Fig. 26.53,

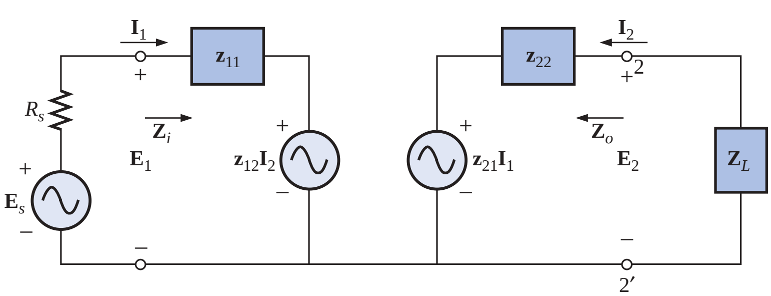

FIG. 26.53
*Determining $\mathbf{Z}_i$ for the **z**-parameter equivalent network.*

$$\mathbf{I}_2 = -\frac{\mathbf{z}_{21}\mathbf{I}_1}{\mathbf{z}_{22} + \mathbf{Z}_L}$$

and $$\mathbf{I}_1 = \frac{\mathbf{E}_1 - \mathbf{z}_{12}\mathbf{I}_2}{\mathbf{z}_{11}}$$

or $$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\left(-\frac{\mathbf{z}_{21}\mathbf{I}_1}{\mathbf{z}_{22} + \mathbf{Z}_L}\right)$$

and $$\mathbf{Z}_i = \frac{\mathbf{E}_1}{\mathbf{I}_1} = \mathbf{z}_{11} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22} + \mathbf{Z}_L} \tag{26.49}$$

For the output impedance, $\mathbf{E}_s = 0$, and

$$\mathbf{I}_1 = -\frac{\mathbf{z}_{12}\mathbf{I}_2}{R_s + \mathbf{z}_{11}} \quad \text{and} \quad \mathbf{I}_2 = \frac{\mathbf{E}_2 - \mathbf{z}_{21}\mathbf{I}_1}{\mathbf{z}_{22}}$$

or
$$\mathbf{E}_2 = \mathbf{z}_{22}\mathbf{I}_2 + \mathbf{z}_{21}\mathbf{I}_1 = \mathbf{z}_{22}\mathbf{I}_2 + \mathbf{z}_{21}\left(-\frac{\mathbf{z}_{12}\mathbf{I}_2}{R_s + \mathbf{z}_{11}}\right)$$

and
$$\mathbf{E}_2 = \mathbf{z}_{22}\mathbf{I}_2 - \frac{\mathbf{z}_{12}\mathbf{z}_{21}\mathbf{I}_2}{R_s + \mathbf{z}_{11}}$$

Thus,
$$\mathbf{Z}_o = \frac{\mathbf{E}_2}{\mathbf{I}_2} = \mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{R_s + \mathbf{z}_{11}} \tag{26.50}$$

26.10 CONVERSION BETWEEN PARAMETERS

The equations relating the **z** and **y** parameters can be determined directly from Eqs. (26.22) and (26.31). For Eqs. (26.31a) and (26.31b),

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{E}_1 + \mathbf{y}_{12}\mathbf{E}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{E}_1 + \mathbf{y}_{22}\mathbf{E}_2$$

The use of determinants will result in

$$\mathbf{E}_1 = \frac{\begin{vmatrix} \mathbf{I}_1 & \mathbf{y}_{12} \\ \mathbf{I}_2 & \mathbf{y}_{22} \end{vmatrix}}{\begin{vmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{vmatrix}} = \frac{\mathbf{y}_{22}\mathbf{I}_1 - \mathbf{y}_{12}\mathbf{I}_2}{\mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}}$$

Substituting the notation

$$\Delta_{\mathbf{y}} = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$$

we have
$$\mathbf{E}_1 = \frac{\mathbf{y}_{22}}{\Delta_{\mathbf{y}}}\mathbf{I}_1 - \frac{\mathbf{y}_{12}}{\Delta_{\mathbf{y}}}\mathbf{I}_2$$

which, when related to Eq. (26.22a),

$$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

indicates that

$$\mathbf{z}_{11} = \frac{\mathbf{y}_{22}}{\Delta_{\mathbf{y}}} \quad \text{and} \quad \mathbf{z}_{12} = -\frac{\mathbf{y}_{12}}{\Delta_{\mathbf{y}}}$$

and, similarly,

$$\mathbf{z}_{21} = -\frac{\mathbf{y}_{21}}{\Delta_{\mathbf{y}}} \quad \text{and} \quad \mathbf{z}_{22} = \frac{\mathbf{y}_{11}}{\Delta_{\mathbf{y}}}$$

For the conversion of **z** parameters to the admittance domain, determinants are applied to Eqs. (26.22a) and (26.22b). The impedance parameters can be found in terms of the hybrid parameters by first forming the determinant for $\mathbf{I}_1$ from the hybrid equations:

$$\mathbf{E}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{E}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{E}_2$$

That is,

$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & \mathbf{h}_{12} \\ \mathbf{I}_2 & \mathbf{h}_{22} \end{vmatrix}}{\begin{vmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{vmatrix}} = \frac{\mathbf{h}_{22}}{\Delta_{\mathbf{h}}}\mathbf{E}_1 - \frac{\mathbf{h}_{12}}{\Delta_{\mathbf{h}}}\mathbf{I}_2$$

and $$\frac{\mathbf{h}_{22}}{\Delta_{\mathbf{h}}}\mathbf{E}_1 = \mathbf{I}_1 + \frac{\mathbf{h}_{12}}{\Delta_{\mathbf{h}}}\mathbf{I}_2$$

or $$\mathbf{E}_1 = \frac{\Delta_{\mathbf{h}}\mathbf{I}_1}{\mathbf{h}_{22}} + \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}\mathbf{I}_2$$

which, when related to the impedance-parameter equation,

$$\mathbf{E}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

indicates that

$$\mathbf{z}_{11} = \frac{\Delta_{\mathbf{h}}}{\mathbf{h}_{22}} \quad \text{and} \quad \mathbf{z}_{12} = \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$$

The remaining conversions are left as an exercise. A complete table of conversions appears in Table 26.1.

TABLE 26.1

Conversions between **z**, **y**, *and* **h** *parameters.*

To ↓ \ From →	z		y		h	
z	$\mathbf{z}_{11}$	$\mathbf{z}_{12}$	$\frac{\mathbf{y}_{22}}{\Delta_{\mathbf{y}}}$	$\frac{-\mathbf{y}_{12}}{\Delta_{\mathbf{y}}}$	$\frac{\Delta_{\mathbf{h}}}{\mathbf{h}_{22}}$	$\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$
	$\mathbf{z}_{21}$	$\mathbf{z}_{22}$	$\frac{-\mathbf{y}_{21}}{\Delta_{\mathbf{y}}}$	$\frac{\mathbf{y}_{11}}{\Delta_{\mathbf{y}}}$	$\frac{-\mathbf{h}_{21}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{h}_{22}}$
y	$\frac{\mathbf{z}_{22}}{\Delta_{\mathbf{z}}}$	$\frac{-\mathbf{z}_{12}}{\Delta_{\mathbf{z}}}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\frac{1}{\mathbf{h}_{11}}$	$\frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}}$
	$\frac{-\mathbf{z}_{21}}{\Delta_{\mathbf{z}}}$	$\frac{\mathbf{z}_{11}}{\Delta_{\mathbf{z}}}$	$\mathbf{y}_{21}$	$\mathbf{y}_{22}$	$\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$	$\frac{\Delta_{\mathbf{h}}}{\mathbf{h}_{11}}$
h	$\frac{\Delta_{\mathbf{z}}}{\mathbf{z}_{22}}$	$\frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{y}_{11}}$	$\frac{-\mathbf{y}_{12}}{\mathbf{y}_{11}}$	$\mathbf{h}_{11}$	$\mathbf{h}_{12}$
	$\frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}}$	$\frac{1}{\mathbf{z}_{22}}$	$\frac{\mathbf{y}_{21}}{\mathbf{y}_{11}}$	$\frac{\Delta_{\mathbf{y}}}{\mathbf{y}_{11}}$	$\mathbf{h}_{21}$	$\mathbf{h}_{22}$

26.11 COMPUTER ANALYSIS

PSpice

Hybrid Equivalent Network—Voltage Gain The computer analysis of this section will be limited to a practice session in the use of controlled sources. The system to be analyzed will be the hybrid equivalent network of Fig. 26.54. Both the voltage gain and the output impedance will be determined using schematics.

Using Eq. (26.46), the magnitude and the phase of the output voltage are determined in the following manner:

$$\mathbf{A}_v = \frac{-\mathbf{h}_f R_L}{\mathbf{h}_i(1 + \mathbf{h}_o R_L) - \mathbf{h}_r\mathbf{h}_f R_L}$$

$$= \frac{-(50)(2\text{ k}\Omega)}{(1\text{ k}\Omega)(1 + (25 \times 10^{-6}\text{ S})(2\text{ k}\Omega)) - (4 \times 10^{-4})(50)(2\text{ k}\Omega)}$$

$$= \frac{-100 \times 10^3}{(1\text{ k}\Omega)(1 + 50 \times 10^{-3}) - 40} = \frac{-100 \times 10^3}{1050 - 40} = -99.01$$

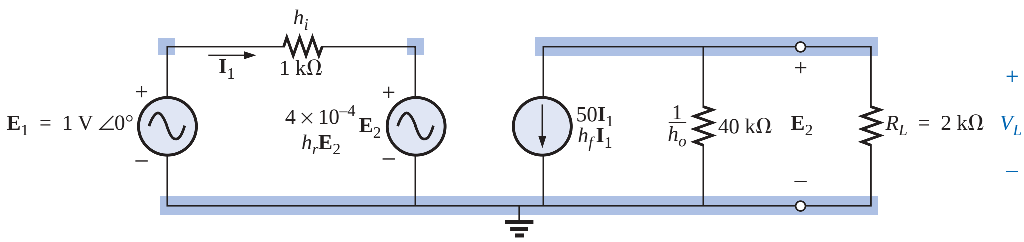

FIG. 26.54
Hybrid equivalent model to be investigated under loaded conditions using PSpice.

and
$$\mathbf{A}_v = \frac{\mathbf{E}_2}{\mathbf{E}_1} = \frac{\mathbf{V}_L}{\mathbf{E}_1}$$

so that
$$\mathbf{V}_L = \mathbf{A}_v\mathbf{E}_1 = (-99.01)(1\text{ V}\angle 0°)$$
$$= \mathbf{99.01\text{ V}\angle 180°}$$

The schematic representation has been established as shown in Fig. 26.55. Note that both a **CCCS** and a **VCVS** must be used along with the ac source **VSIN.** Most of the construction and setting up of the various components through the **Property Editor** dialog box is quite straightforward. However, you must be very careful when setting up the connections for the controlling variables. When you cross a line, be absolutely sure that a small circular dot does not appear where you cross the line; otherwise, a connection is being made. Simply click the wire in place before crossing the line, and then click the wire construction again after crossing the line.

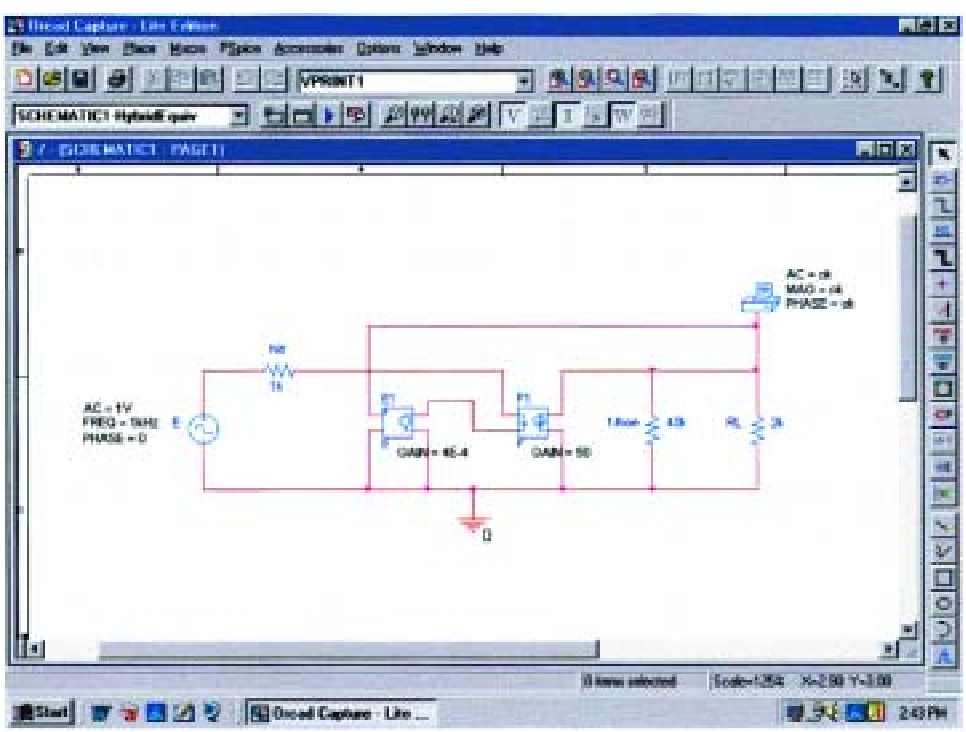

FIG. 26.55
Using PSpice to analyze the network of Fig. 26.54.

The **Simulation Settings** were **AC Sweep, Start** and **End Frequencies** at 1 kHz, and 1 data **Point/Decade.** Select **PSpice-View Output File,** click **OK,** and run the simulation. The **AC ANALYSIS** listing of Fig. 26.56 results. There is an exact match between the theoretical solution provided above and the computer analysis.

Hybrid Equivalent Network—Output Impedance For the output impedance, the applied source **VSIN** is set to 0 V by replacing it with a direct 0-Ω connection. Then a 1-A current source is applied as shown in Fig. 26.57. The **ISRC** current source was selected because it

```

 ** Profile: "SCHEMATIC1-HybridEquiv"  [ C:\PSpice\hybridequivalent-SCHEMATIC1-HybridEquiv.s
im ]


 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C


******************************************************************************



  FREQ        VM(N00869)  VP(N00869)


   1.000E+03   9.901E+01   1.800E+02

```

FIG. 26.56
Output file for the voltage across the load resistor of Fig. 26.55.

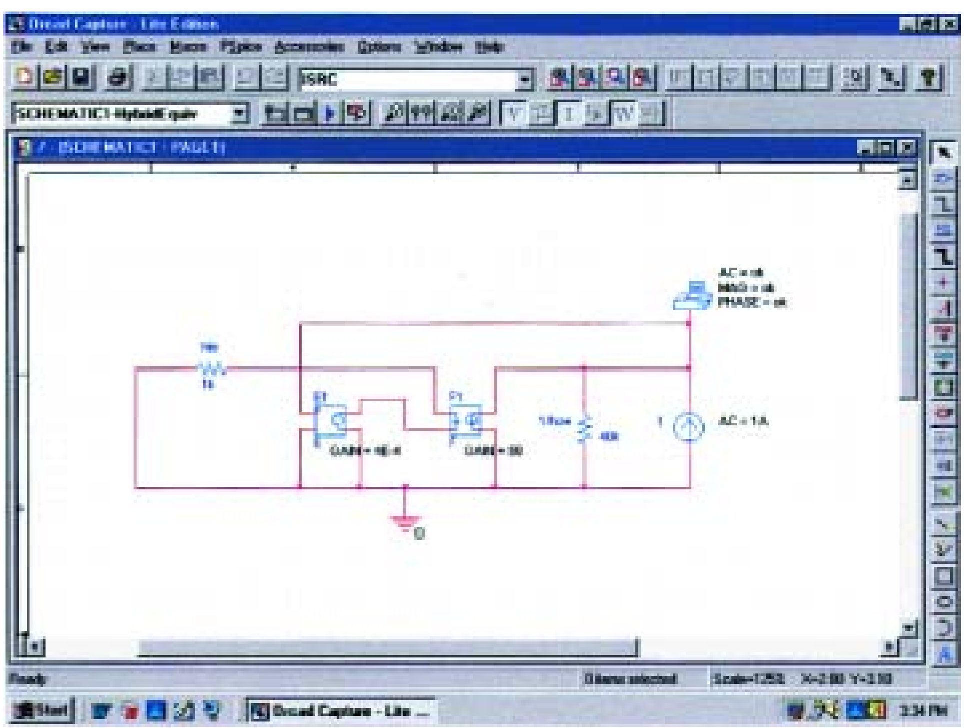

FIG. 26.57
Modification of the schematic of Fig. 26.55 to determine the output impedance of the network.

has the arrow in the symbol. Everything else in the network remains the same, so there is no need to rebuild the entire network. Simply make the changes and run the simulation. Even the simulation does not have to be changed since the chosen parameters will remain the same. The current source was given a magnitude of 1 A so that the magnitude of the **VPRINT1** voltage would also be the magnitude of the output impedance. The results of Fig. 26.58 indicate an output impedance of 200 kΩ. The following theoretical analysis reveals that the output impedance is indeed 200 kΩ:

$$\mathbf{Z}_o = \frac{1}{\mathbf{h}_o - \dfrac{\mathbf{h}_r\mathbf{h}_f}{\mathbf{h}_i + R_s}} = \frac{1}{25 \times 10^{-6}\,\text{S} - \dfrac{(4 \times 10^{-4})(50)}{1\,\text{k}\Omega + 0}}$$

$$= \frac{1}{25 \times 10^{-6}\,\text{S} - 20 \times 10^{-6}\,\text{S}} = \frac{1}{5 \times 10^{-6}\,\text{S}} = \mathbf{200\,k\Omega}$$

```
 ** Profile: "SCHEMATIC1-HybridEquiv"  [ C:\PSpice\hybridequivalent-schematic1-hybridequiv.s
im ]
 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C
******************************************************************************
  FREQ        VM(N00869)   VP(N00869)
   1.000E+03   2.000E+05   0.000E+00
```

FIG. 26.58
Output file for the voltage across the 1-A current source (and output impedance) of the network of Fig. 26.57.

PROBLEMS

SECTION 26.2 The Impedance Parameters Z_i and Z_o

1. Given the indicated voltage levels of Fig. 26.59, determine the magnitude of the input impedance $\mathbf{Z}_i$.

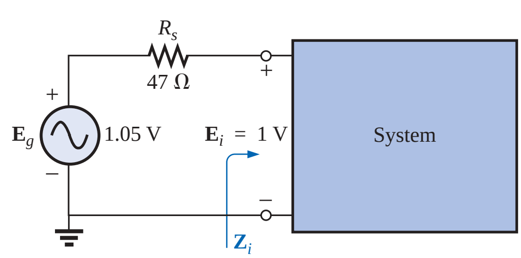

FIG. 26.59
Problem 1.

2. For a system with

$$\mathbf{E}_i = 120\ \text{V}\ \angle 0^\circ \quad \text{and} \quad \text{I}_i = 6.2\ \text{A}\ \angle -10.8^\circ$$

determine the input impedance in rectangular form. At a frequency of 60 Hz, determine the nameplate values of the parameters.

3. For the multiport system of Fig. 26.60:
   a. Determine the magnitude of $\mathbf{I}_{i_1}$ if $\mathbf{E}_{i_1} = 20$ mV.
   b. Find $\mathbf{Z}_{i_2}$ using the information provided.
   c. Calculate the magnitude of $\mathbf{E}_{i_3}$.

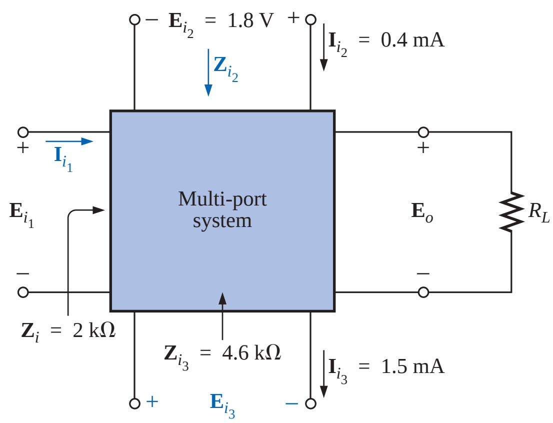

FIG. 26.60
Problem 3.

4. Given the indicated voltage levels of Fig. 26.61, determine $\mathbf{Z}_o$.

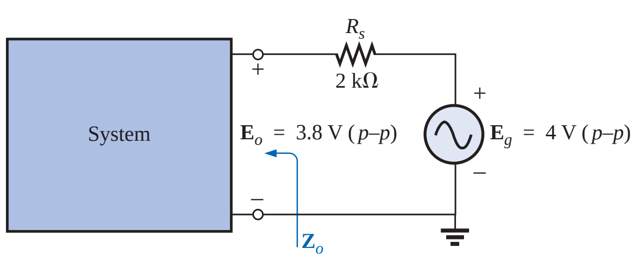

FIG. 26.61
Problems 4 through 6.

5. For the configuration of Fig. 26.61, determine $\mathbf{Z}_o$ if $e_g = 2 \sin 377t$ and $v_R = 40 \times 10^{-3} \sin 377t$, with $R_s = 0.91$ kΩ.

6. Determine $\mathbf{Z}_o$ for the system of Fig. 26.61 if $E_g = 1.8$ V **(**p-p) and $E_o = 0.6$ V rms.

7. Determine the output impedance for the system of Fig. 26.62 given the indicated scope measurements.

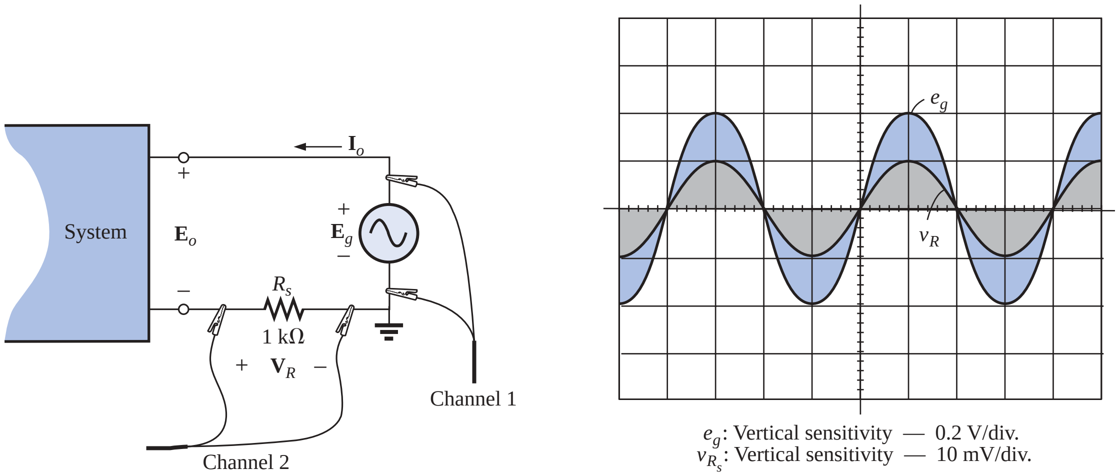

FIG. 26.62
Problem 7.

SECTION 26.3 The Voltage Gains $A_{v_{NL}}$, A_v, and A_{v_T}

8. Given the system of Fig. 26.63, determine the no-load voltage gain $\mathbf{A}_{v_{NL}}$.

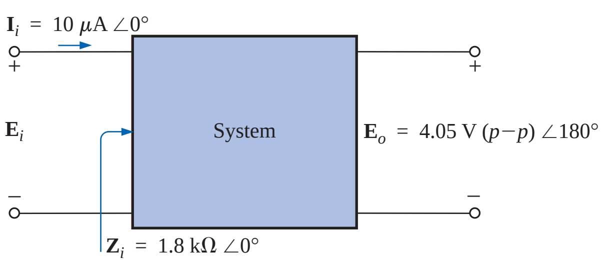

FIG. 26.63
Problem 8.

9. For the system of Fig. 26.64:
   a. Determine $\mathbf{A}_v = \mathbf{E}_o/\mathbf{E}_i$.
   b. Find $\mathbf{A}_{v_T} = \mathbf{E}_o/\mathbf{E}_g$.

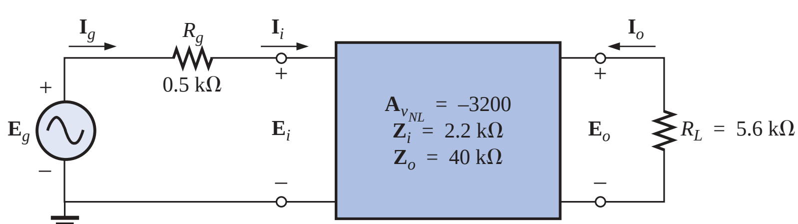

FIG. 26.64
Problems 9, 12, and 13.

10. For the system of Fig. 26.65(a), the no-load output voltage is −1440 mV, with 1.2 mV applied at the input terminals. In Fig. 26.65(b), a 4.7-kΩ load is applied to the same system, and the output voltage drops to −192 mV, with the same applied input signal. What is the output impedance of the system?

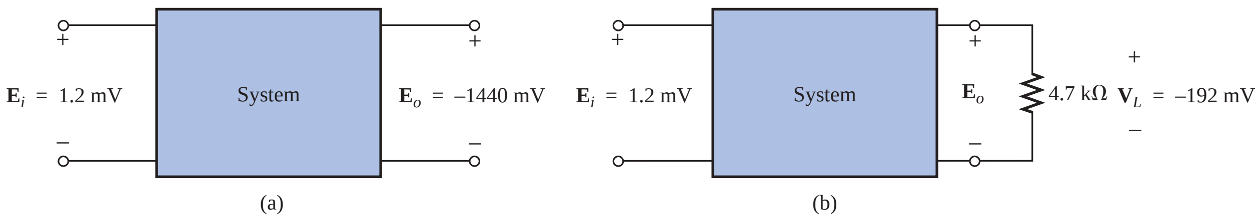

FIG. 26.65
Problem 10.

***11.** For the system of Fig. 26.66, if

$\mathbf{A}_v = -160 \qquad \mathbf{I}_o = 4\text{ mA} \angle 0° \qquad \mathbf{E}_g = 70\text{ mV} \angle 0°$

a. Determine the no-load voltage gain.
b. Find the magnitude of $\mathbf{E}_i$.
c. Determine $\mathbf{Z}_i$.

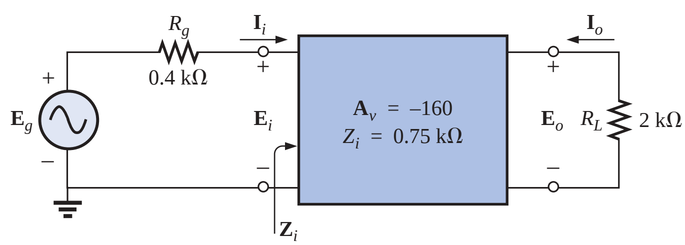

FIG. 26.66
Problems 11 and 14.

SECTION 26.4 The Current Gains A_i and A_{i_T}, and the Power Gain A_G

12. For the system of Fig. 26.64:
a. Determine $\mathbf{A}_i = \mathbf{I}_o/\mathbf{I}_i$.
b. Find $\mathbf{A}_{i_T} = \mathbf{I}_o/\mathbf{I}_g$.
c. Compare the results of parts (a) and (b), and explain why the results compare as they do.

13. For the system of Fig. 26.64:
a. Determine A_G using Eq. (26.13), and compare the value with the result obtained using Eq. (26.14).
b. Find A_{G_T} using Eq. (26.16), and compare the value to the result obtained using Eq. (26.17).

14. For the system of Fig. 26.66:
a. Determine the magnitude of $\mathbf{A}_i = \mathbf{I}_o/\mathbf{I}_i$.
b. Find the power gain $A_{G_T} = P_L/P_g$.

SECTION 26.5 Cascaded Systems

15. For the two-stage system of Fig. 26.67:

a. Determine the total voltage gain $\mathbf{A}_{v_T} = \mathbf{V}_L/\mathbf{E}_i$.

b. Find the total current gain $\mathbf{A}_{i_T} = \mathbf{I}_o/\mathbf{I}_i$.

c. Find the current gain of each stage $\mathbf{A}_{i_1}$ and $\mathbf{A}_{i_2}$.

d. Determine the total current gain using the results of part (c), and compare it to the result obtained in part (b).

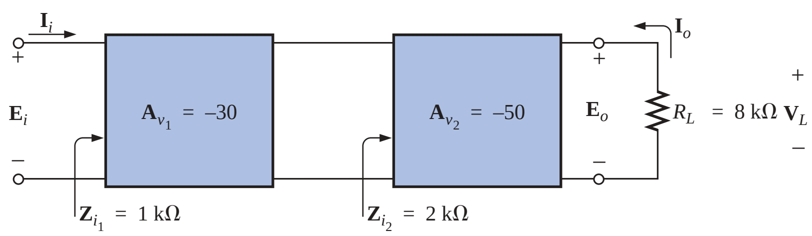

FIG. 26.67
Problem 15.

***16.** For the system of Fig. 26.68:

a. Determine $\mathbf{A}_{v_2}$ if $\mathbf{A}_{v_T} = -6912$.

b. Determine $\mathbf{Z}_{i_2}$ using the information provided.

c. Find $\mathbf{A}_{i_3}$ and $\mathbf{A}_{i_T}$ using the information provided in Fig. 26.68.

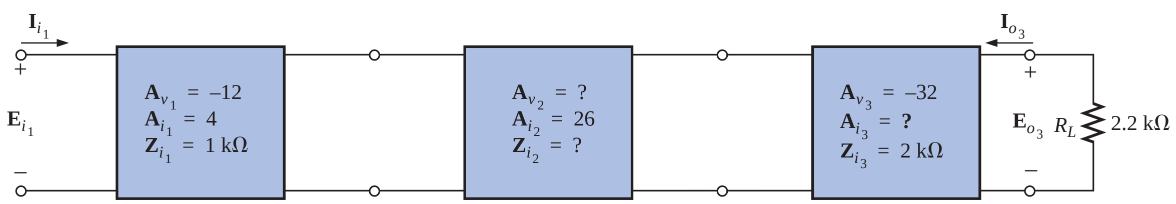

FIG. 26.68
Problem 16.

SECTION 26.6 Impedance (z) Parameters

17. a. Determine the impedance (**z**) parameters for the π network of Fig. 26.69.

b. Sketch the **z**-parameter equivalent circuit (using either form of Fig. 26.32).

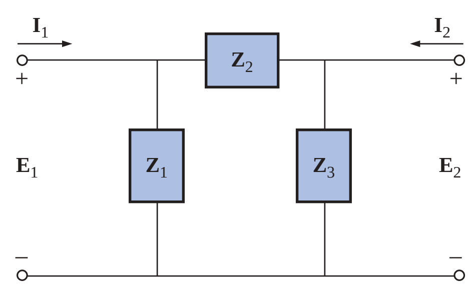

FIG. 26.69
Problems 17 and 21.

18. a. Determine the impedance (**z**) parameters for the network of Fig. 26.70.

b. Sketch the **z**-parameter equivalent circuit (using either form of Fig. 26.32).

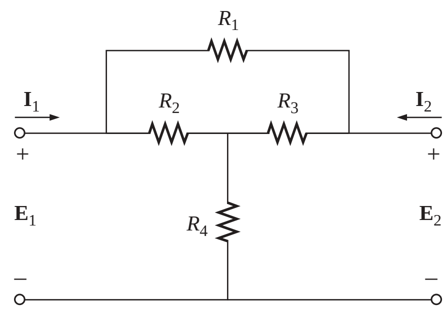

FIG. 26.70
Problems 18 and 22.

SECTION 26.7 Admittance (y) Parameters

19. a. Determine the admittance (**y**) parameters for the T network of Fig. 26.71.

b. Sketch the **y**-parameter equivalent circuit (using either form of Fig. 26.44).

20. a. Determine the admittance (**y**) parameters for the network of Fig. 26.72.

b. Sketch the **y**-parameter equivalent circuit (using either form of Fig. 26.44).

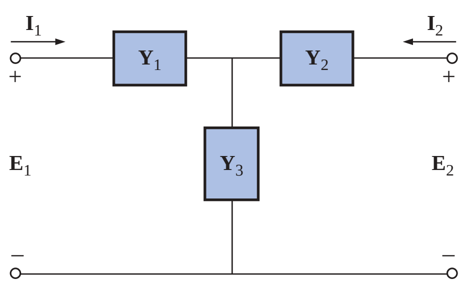

FIG. 26.71
Problems 19 and 23.

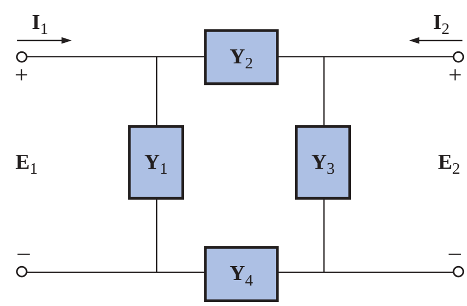

FIG. 26.72
Problems 20 and 24.

SECTION 26.8 Hybrid (h) Parameters

21. a. Determine the **h** parameters for the network of Fig. 26.69.

b. Sketch the hybrid equivalent circuit.

22. a. Determine the **h** parameters for the network of Fig. 26.70.

b. Sketch the hybrid equivalent circuit.

23. a. Determine the **h** parameters for the network of Fig. 26.71.

b. Sketch the hybrid equivalent circuit.

24. a. Determine the **h** parameters for the network of Fig. 26.72.

b. Sketch the hybrid equivalent circuit.

25. For the hybrid equivalent circuit of Fig. 26.73:

a. Determine the current gain $A_i = \mathbf{I}_2/\mathbf{I}_1$.

b. Determine the voltage gain $A_v = \mathbf{E}_2/\mathbf{E}_1$.

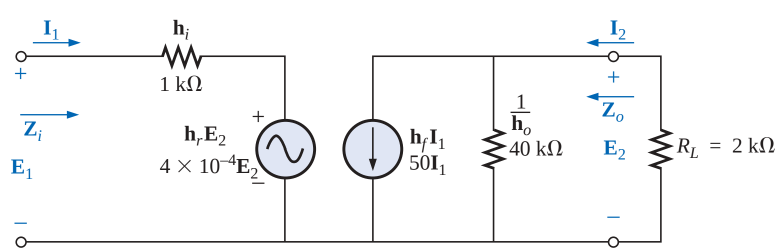

FIG. 26.73
Problems 25 and 26.

SECTION 26.9 Input and Output Impedances

26. For the hybrid equivalent circuit of Fig. 26.73:
a. Determine the input impedance.
b. Determine the output impedance.

27. Determine the input and output impedances for the **z**-parameter equivalent circuit of Fig. 26.74.

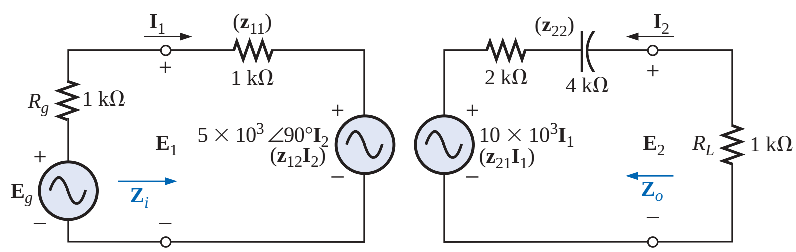

FIG. 26.74
Problems 27, 32, and 34.

28. Determine the expression for the input and output impedance of the **y**-parameter equivalent circuit.

SECTION 26.10 Conversion between Parameters

29. Determine the **h** parameters for the following **z** parameters:

$$\mathbf{z}_{11} = 4\ \text{k}\Omega$$
$$\mathbf{z}_{12} = 2\ \text{k}\Omega$$
$$\mathbf{z}_{21} = 3\ \text{k}\Omega$$
$$\mathbf{z}_{22} = 4\ \text{k}\Omega$$

30. a. Determine the **z** parameters for the following **h** parameters:

$$\mathbf{h}_{11} = 1\ \text{k}\Omega$$
$$\mathbf{h}_{12} = 2 \times 10^{-4}$$
$$\mathbf{h}_{21} = 100$$
$$\mathbf{h}_{22} = 20 \times 10^{-6}\ \text{S}$$

b. Determine the **y** parameters for the hybrid parameters indicated in part (a).

SECTION 26.11 Computer Analysis

PSpice or Electronics Workbench

31. For $\mathbf{E}_1 = 4\ \text{V} \angle 30°$, determine $\mathbf{E}_2$ across a 2-kΩ resistive load between 2 and 2′ for the network of Fig. 26.34.

32. For $\mathbf{E}_g = 2\ \text{V} \angle 0°$, determine $\mathbf{E}_2$ for the network of Fig. 26.74.

33. Determine $\mathbf{Z}_i$ for the network of Fig. 26.34 with a 2-kΩ resistive load from 2 to 2′.

34. Determine $\mathbf{Z}_i$ for the network of Fig. 26.74.

GLOSSARY

Admittance (y) parameters A set of parameters, having the units of siemens, that can be used to establish a two-port equivalent network for a system.

Hybrid (h) parameters A set of mixed parameters (ohms, siemens, some unitless) that can be used to establish a two-port equivalent network for a system.

Impedance (z) parameters A set of parameters, having the units of ohms, that can be used to establish a two-port equivalent network for a system.

Input impedance The impedance appearing at the input terminals of a system.

Output impedance The impedance appearing at the output terminals of a system with the energizing source set to zero.

Single-port network A network having a single set of access terminals.

Two-port network A network having two pairs of access terminals.