Appendix G

MAXIMUM POWER TRANSFER CONDITIONS

Derivation of maximum power transfer conditions for the situation where the resistive component of the load is adjustable but the load reactance is set in magnitude.*

For the circuit of Fig. G.1, the power delivered to the load is determined by

$$P = \frac{V_{R_L}^2}{R_L}$$

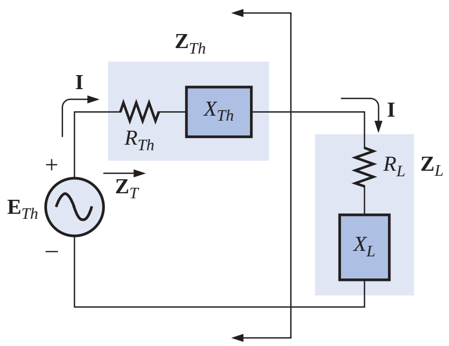

FIG. G.1

Applying the voltage divider rule:

$$\mathbf{V}_{R_L} = \frac{R_L\mathbf{E}_{Th}}{R_L + R_{Th} + X_{Th}\angle 90° + X_L\angle 90°}$$

The magnitude of $\mathbf{V}_{R_L}$ is determined by

$$V_{R_L} = \frac{R_L E_{Th}}{\sqrt{(R_L + R_{Th})^2 + (X_{Th} + X_L)^2}}$$

and

$$V_{R_L}^2 = \frac{R_L^2 E_{Th}^2}{(R_L + R_{Th})^2 + (X_{Th} + X_L)^2}$$

with

$$P = \frac{V_{R_L}^2}{R_L} = \frac{R_L E_{Th}^2}{(R_L + R_{Th})^2 + (X_{Th} + X_L)^2}$$

Using differentiation (calculus), maximum power will be transferred when $dP/dR_L = 0$. The result of the preceding operation is that

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2} \qquad \text{[Eq. (18.21)]}$$

The magnitude of the total impedance of the circuit is

$$Z_T = \sqrt{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

Substituting this equation for R_L and applying a few algebraic maneuvers will result in

$$Z_T = 2R_L(R_L + R_{Th})$$

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*With sincerest thanks for the input of Professor Harry J. Franz of the Beaver Campus of Pennsylvania State University.

and the power to the load R_L will be

$$P = I^2R_L = \frac{E_{Th}^2}{Z_T^2} R_L = \frac{E_{Th}^2 R_L}{2R_L(R_L + R_{Th})}$$

$$= \frac{E_{Th}^2}{4\left(\frac{R_L + R_{Th}}{2}\right)}$$

$$= \frac{E_{Th}^2}{4R_{av}}$$

with $$R_{av} = \frac{R_L + R_{Th}}{2}$$