PROBLEM SOLUTIONS CHAPTER 10

SOLUTION 10.1 Using KCL, we can write

$$C\frac{dv_C}{dt}+\frac{v_C}{R}=i_{in}(t)$$

Dividing by C:

$$\frac{dv_C}{dt}+\frac{1}{RC}v_C=\frac{i_{in}(t)}{C}$$

We know that $i_{in}(t)=20\sin(400t)$ mA, which can be represented by a complex exponential, $i_{in}(t)=\text{Re}\left[20e^{j400t}e^{-j\pi/2}\right]$ mA. For convenience we will simply let $i_{in}(t)=20e^{j400t}e^{-j\pi/2}$ mA, knowing that we must take the real part to complete our solution. The output voltage will also be reparesented as a complex exponential:

$$v_C(t)=V_m e^{j(400t+\phi)}=V_m e^{j400t}e^{j\phi}$$

Substituting this expression into the differential equation and canceling e^{j400t}:

$$j400V_m e^{j\phi}+\frac{V_m}{RC}e^{j\phi}=\frac{20\times10^{-3}e^{-j\pi/2}}{C}$$

Thus

$$V_m e^{j\phi}\left(\frac{1}{RC}+j400\right)=\frac{20\times10^{-3}}{C} \qquad V_m e^{j\phi}=\frac{-j4000}{1000+j400}=3.714 \quad -111.8^{o}$$

where the values for R = 100 Ω and C = 5 mF were substituted in. Thus,

$$V_m=3.714$$

$$\phi=0-\tan^{-1}\left(\frac{400}{1000}\right)=-111.8^\circ$$

Taking into account a 90° phase shift we obtain

$$v_C(t)=3.714\cos(400t-111.8^\circ)=3.714\sin(400t-21.8^\circ)\ \text{V}$$

and

$$i_{out}(t)=18.57\sin(400t-21.8^\circ)\ \text{mA}$$

SOLUTION 10.2 From KCL and component definitions:

$$i_{in}(t) - \frac{v_L}{25} - i_L = 0 \qquad \frac{0.1}{25}\frac{di_L}{dt} + i_L = i_{in}(t) \qquad \frac{di_L}{dt} + 250 i_L = 250 i_{in}(t)$$

We represent the input signal by the complex exponential: $i_{in}(t) = 0.2e^{j250t}$ A and the unknown current can be represented as $i_L(t) = I_L e^{j(250t+\phi)}$.

Substituting this into the differential equation and canceling e^{j250t}:

$$j250 I_L e^{j\phi} + 250 I_L e^{j\phi} = 50$$

Thus

$$I_L e^{j\phi}(j250 + 250) = 50 \qquad I_L e^{j\phi} = \frac{50}{250 + j250} = 0.14142 \quad -45^o$$

and

$$I_L = 0.141,\ \phi = -45° \qquad i_L(t) = 0.141\cos(250t - 45°) \text{ A}$$

SOLUTION 10.3. Construct differential equation by KVL and device definitions:

$$v_{in}(t) - 0.5\frac{di_L}{dt} - 200 i_L = 0 \qquad \frac{di_L}{dt} + 400 i_L = 2 v_{in}(t)$$

We represent $v_{in}(t)$ as the complex exponential function, $v_{in}(t) = 20e^{j400t}$ V. The current in the inductor has the form: $i_L = I_L e^{j(400t+\phi)}$. Substituting into the differential equation and canceling e^{j400t}:

$$j400 I_L e^{j\phi} + 400 I_L e^{j\phi} = 40$$

Thus

$$I_L e^{j\phi}(j\omega + 400) = 40 \qquad I_L e^{j\phi} = \frac{40}{400 + j400} = 0.070711 \quad -45^o$$

and

$$I_L = 0.0707,\ \phi = -45°, \qquad i_L(t) = 70.7\cos(400t - 45°) \text{ mA}$$

Hence,

$$v_{out}(t) = 14.14\cos(400t - 45°) \text{ V}$$

SOLUTION 10.4. Construct differential equation using KVL and device definitions:

$$v_{in}(t) - v_C - C\frac{dv_C}{dt}R = 0 \qquad RC\frac{dv_C}{dt} + v_C = v_{in}(t)$$

The output voltage is defined as:

$$v_{out}(t) = v_{in}(t) - v_C(t)$$

This means that finding v_C is enough to be able to obtain the output voltage. The input voltage is represented by the complex exponential:

$$v_{in}(t) = 20e^{j250t}e^{-j\pi/2} \text{ V}$$

and $v_C(t) = V_m e^{j(250t+\phi)}$. Substituting into the differential equation, dividing by e^{j250t}, and rearranging:

$$j250RCV_Ce^{j\phi} + V_Ce^{j\phi} = -j20 \qquad V_Ce^{j\phi}(j250RC + 1) = -j20$$

$$V_Ce^{j\phi} = \frac{-j20}{1+j} = 14.142 \quad -135^o$$

Now

$$V_{out}e^{j\theta} = -j20 - V_Ce^{j\phi} \qquad V_{out}e^{j\theta} = 10 - j10 = 14.142 \quad -45^o$$

Thus, in the time-domain,

$$v_{out}(t) = 14.142\cos(250t - 45°) \text{ V}$$

SOLUTION 10.5. The circuit is identical to that of problem 10.1. Thus,

$$RC\frac{dv_s}{dt} + v_s = Ri_s(t) \qquad Ri_s(t) - RC\frac{dv_s}{dt} = v_s$$

Moreover, the complex exponential solution is given by

$$v_s(t) = V_m e^{j\phi} = 223.6e^{-j63.43^o} \text{ V}$$

Hence

$$R - jRC223.6e^{-j63.43^o} = 223.6e^{-j63.43^o} = 100 - j200$$

i.e.,

$$R - jRC(100 - j200) = R - 200RC - j100RC = 100 - j200$$

Thus $RC = 2$ s, $R = 100 + 200RC = 500\ \Omega$, and C = 4 mF.

SOLUTION 10.6. First use KVL

$$v_{in} - v_L - v_c - v_{out} = 0 \qquad v_{in} - L\frac{di_L}{dt} - \frac{1}{C}\int_{-\infty}^{t} i_L dt - Ri_L = 0$$

Differentiating this equation, rearranging, and dividing by L,

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{1}{L}\frac{dv_{in}}{dt}$$

We represent the input signal with $\omega = 10^4$ as $v_{in}(t) = 100e^{j\omega t}e^{-j\pi/2}$ V and $i_L(t) = I_L e^{j\omega t}e^{j\phi}$ A. Substituting these two expressions into the differential equation and dividing out $e^{j\omega t}$:

$$-\omega^2 I_L e^{j\phi} + j\frac{R\omega}{L}I_L e^{j\phi} + \frac{1}{LC}I_L e^{j\phi} = \frac{-j100\times j\omega}{L} \qquad I_L e^{j\phi}\left(\frac{1}{LC} - \omega^2 + j\frac{R\omega}{L}\right) = 10^8$$

$$I_L e^{j\phi} = \frac{10^8}{-75\times 10^6 + j10^8} = -0.48 - j0.64 = 0.8 \quad -126.87^o$$

Solving for the magnitude and angle (by hand or using MATLAB):

$$i_L(t) = 0.8\cos(10{,}000t - 126.87°) = 0.8\sin(10{,}000t - 36.87°)\ \text{A}$$

and

$$v_{out}(t) = 80\sin(10{,}000t - 36.87°)\ \text{V}$$

SOLUTION 10.7. Using standard reference directions, from KCL and component definitions,

$$i_{in} = i_R + i_L + i_C = \frac{v_{out}}{R} + i_L(0) + \frac{1}{L}\int_0^t v_{out}(\tau)\,d\tau + C\frac{dv_{out}}{dt}$$

Taking a second derivative and dividing by C yields

$$\frac{d^2 v_{out}}{dt^2} + \frac{1}{RC}\frac{dv_{out}}{dt} + \frac{1}{LC}v_{out} = \frac{1}{C}\frac{di_{in}}{dt}$$

We now let $\omega = 2500$ rad/s and represent $i_{in}(t)$ by the real part of the complex exponential $0.02e^{j\omega t}$.

Further we represent $v_{out}(t)$ as the real part of the complex exponential $V_m e^{j(\omega t+\varphi)} = V_m e^{j\omega t} e^{j\varphi}$.
Substituting these expressions into the differential equation and taking the indicated derivatives yields

$$V_m (j\omega)^2 e^{j\omega t} e^{j\varphi} + \frac{V_m}{RC} j\omega e^{j\omega t} e^{j\varphi} + \frac{1}{LC} V_m e^{j\omega t} e^{j\varphi} = \frac{0.02}{C} j\omega e^{j\omega t}$$

Observe that $e^{j\omega t}$ cancels out on both sides of this equation producing

$$V_m e^{j\varphi} \left(\frac{1}{LC} - \omega^2 + \frac{j\omega}{RC} \right) = \frac{j0.02\omega}{C}$$

Hence

$$V_m e^{j\varphi} = \frac{j0.02\omega}{C\left(\frac{1}{LC} - \omega^2 + \frac{j\omega}{RC} \right)} = 1.28 + j0.96$$

This was obtained using MATLAB as follows:

```
»w = 2500;
»L = 40e-3;
»C = 1e-6;
»R = 100;
»Vout = j*0.02*w/(1/L - C*w^2 +j*w/R)
Vout =  1.2800e+00 + 9.6000e-01i

»magVout = abs(Vout)
magVout =   1.6000e+00
»angVout = angle(Vout)*180/pi
angVout =   3.6870e+01
```

Therefore

$$v_{out}(t) = 1.6\cos(2500t + 36.87^{o})\ \text{V}$$

SOLUTION 10.8. **(a)** From KCL: $3\angle 15° - 5\angle 45° - I = 0 \qquad I = 3\angle 15° - 5\angle 45°$. In MATLAB,

```
»Ibar = 3*exp(j*15*pi/180) - 5*exp(j*45*pi/180)
```

Ibar =-6.3776e-01 - 2.7591e+00i

»abs(Ibar)

ans =2.8318e+00

»angle(Ibar)*180/pi

ans =-1.0302e+02

Therefore,

$$i(t) = \text{Re}\{Ibar\} = 2.83\cos(\omega t - 103°) \text{ A}$$

(b) From KCL,

»Ibar = (1 + 2*j) - (-2 + j*6)

Ibar =

3.0000e+00 - 4.0000e+00i

»abs(Ibar)

ans = 5

»angle(Ibar)*180/pi

ans =-5.3130e+01

Therefore $i(t) = 5\cos(50\pi t - 53°)$ A.

SOLUTION 10.9. We define a Gaussian surface encompassing the three bottom nodes. Thus, KCL dictates that the sum of 4 currents be zero:

»Ibar = -(-2-j*8) + (3 +j*12) + 10

Ibar =

1.5000e+01 + 2.0000e+01i

»abs(Ibar)

ans =

25

»angle(Ibar)*180/pi

ans =

5.3130e+01

Therefore $i(t) = 25\cos(1000t + 53.13°)$ A.

SOLUTION 10.10. First represent the time-domain functions as phasors:

$\mathbf{V_1} = 2\angle 0°$, $\mathbf{V_2} = 2\sqrt{2}\angle -45°$

Then, by KVL

```
»V1 = 2; V2 = 2*sqrt(2)*exp(-j*pi/4);
»VL = V1 - V2
VL =
0 + 2.0000e+00i
```

Therefore, $\mathbf{V_L} = 2\angle 90°$ V and $v_L(t) = 2\cos(\omega t + 90°) = -2\sin(\omega t)$ V

SOLUTION 10.11. Apply KVL by simply following the loop defined by the independent voltage sources:

$$\mathbf{V_x} = 4j - 2j - 1 - 1 + (1 - j) - (1 + j) = -2$$

SOLUTION 10.12. First note that $\mathbf{I_{out}} = \frac{\mathbf{V_R}}{R}$. $\mathbf{V_R} = -j10 + (5 - j5) - 10 \qquad \mathbf{V_R} = -5 - j15$ V.

Thus, $\mathbf{I_{out}} = \frac{-5 - j15}{5} = -1 - j3 = 3.16\angle -108.4^{\circ} \qquad i_{out}(t) = 3.16\cos(500\pi t - 108.4°)$ A.

SOLUTION 10.13. Using a Gaussian surface,

$$\mathbf{I_y} = -(2 + j3) - (1 + j2) + (1 - j5) = -2 - j10 = 10.2\angle -101.3^{o} \text{ A.}$$

Therefore, $i(t) = 10.2\cos(2000\pi t - 101.3°)$ A. Now applying KVL,

$$V_x = (2 + j2) + (2 + j3) - (1 - 4j) = 3 + 9j$$

```
»Vx=2+2*j+2+j*3-1+4*j
Vx =
3.0000e+00 + 9.0000e+00i
»abs(Vx)
ans =
9.4868e+00
»angle(Vx)*180/pi
```

ans =
7.1565e+01

Thus, $v_x(t) = 9.487\cos(2000\pi t + 71.6°)$ V.

SOLUTION 10.14. **(a)** At $\omega = 1000\pi$ rad/s,

$$Y_C(j\omega) = j\omega C = j4.7 \times 10^{-3} \qquad C = 1.496\ \mu\text{F}$$

So, at $\omega = 50\pi$, $Y_C(j\omega) = j\omega C = j2.3499 \times 10^{-4}\ S \qquad Z_C(j\omega) = -j4255\ \Omega$.

(b) At $\omega = 1000\pi$ rad/s, $Z_L = j\omega L = j18.85 \qquad L = 6$ mH. Since impedance is proportional to frequency, multiplying the frequency by 20 means the impedance is multiplied by 20. Thus, at $\omega = 20 \times (10^3\pi)$ rad/s,

$$Z_L = j18.85 \times 20 = j377 \qquad Y_L = -j2.652\ \text{mS}$$

SOLUTION 10.15. The input voltagephasor as $2 \quad 0°$. By inspection:

$$\mathbf{I_1} = j\omega C\ \ \mathbf{V_C} = j10 \times 0.1 \times 2 = j2$$

From the definition of a dependent V-source and an inductor:

$$\mathbf{I_2} = \frac{5\mathbf{I_1}}{j\omega L} = \frac{j10}{j2} = 5$$

Finally,

$$\mathbf{V_{out}} = 5 \times 2(\mathbf{I_1} + \mathbf{I_2}) = 10(5 + j2) = 50 + j20 = 53.85 \quad 21.8°$$

$$v_{out}(t) = 53.85\cos(10t + 21.5°)\ \text{V}$$

SOLUTION 10.16. Using KCL and the definition of a resistor:

$$V_{10\Omega} = 10(6 \quad 0° - 3 \quad 90°) = 10(6 - 3j) = 60 - j30\ \text{V}$$

Thus,

$$Z(j\omega_0) = \frac{60 - j30}{j3} = -10 - j20\ \Omega$$

And, the combination of this $Z(j\omega_0)$ with the 10 Ω resistance, at this frequency, is 10 – j5.

```
»Z=-10-j*20;  R = 10;
»ZZ = R*Z/(R+Z)
ZZ =1.0000e+01 - 5.0000e+00i
»C = 1/(-5*j*j*2000*pi)
C =3.1831e-05
```

This is equivalent to a 10 Ω resistor in series with a 31.83 μF capacitor.

SOLUTION 10.17.

```
»w = 2*pi*60;
»VL = 3 +12*exp(-j*30*pi/180) + 6 -12*exp(j*30*pi/180)
VL =9.0000e+00 - 1.2000e+01i
»ZL = j*w/60
ZL =0 + 6.2832e+00i
»IL = VL/ZL
IL =-1.9099e+00 - 1.4324e+00i
»abs(IL)
ans =2.3873e+00
»angle(IL)*180/pi
ans =-1.4313e+02
```

Therefore, $i_L(t) = 2.387\cos(120\pi t - 143.1^o)$ A.

Solution 10.18 (a)

First represent the inputs with their phasors: $\mathbf{I_{s1}} = 10\angle 30° = 8.66 + j5$ and $\mathbf{V_{s2}} = 100\angle 0°$.

The admittance of the RC combination is: $Y_1 = \frac{1}{R} + j\omega C = 0.1 + j0.1$. Using superposition and noting that a 0-volt V-source is a short circuit and a 0-amp current source is an open circuit,

$$\mathbf{I_x} = Y_1\mathbf{V_{s2}} - \mathbf{I_{s1}} = 1.3397 + j5 = 5.1764\angle 75^o \text{ A}$$

Therefore,

$$i_x(t) = 5.176\cos(100t + 75°)$$

(b) Let i_{x1} be the contribution to i_x generated by the current source and i_{x2} the contribution generated by the voltage source. Then $i_{x1}(t) = -10\cos(50t + 30^o)$ A, and $i_{x2}(t) = -14.142\cos(100t + 45^o)$ A since

```
»Vs2 = 100;
»w = 100;
»R = 10; C = 1e-3;
»Y1 = 1/R + j*w*C
Y1 =1.0000e-01 + 1.0000e-01i
»Ix2 = Y1*Vs2
Ix2 =1.0000e+01 + 1.0000e+01i
»abs(Ix2)
ans =1.4142e+01
»angle(Ix2)*180/pi
ans =45
```

Therefore $i_x(t) = -10\cos(50t + 30^o) - 14.142\cos(100t + 45^o)$ A.

Solution 10.19 First of all, write out the given values: $\omega = 200$ rad/s, $\mathbf{I_1} = 0.5 \quad 90° = 0.5j$ A, and $\mathbf{V_{s2}} = 4 \quad 0°$. From KVL $\mathbf{V_{s1}} = 3\mathbf{I_1} + j\omega L\mathbf{I_1} + \dfrac{\mathbf{I_1}}{j\omega C} + \mathbf{V_{s2}}$ which leads to:

```
»w = 200;  I1 = 0.5j;  Vs2 = 4;
»Vs1 = 3*I1 + j*w*0.04*I1 + I1/(j*w*1e-3) +Vs2
Vs1 =2.5000e+00 + 1.5000e+00i
»abs(Vs1)
ans =
2.9155e+00
»angle(Vs1)*180/pi
ans =
3.0964e+01
```

Therefore $v_{s1}(t) = 2.9155\cos(200t + 30.964^o)$ V.

Solution 10.20 The phasor for the input can be written as $\omega = 1000$ rad/s and $\mathbf{V_{in}} = 10 \quad 60° = 5 + j8.66$ V. The currents can be obtained easily by applying Ohm's law for phasors:

$\mathbf{I_R} = \frac{10\angle 60}{500} = 0.02\angle 60^\circ$ A, $\mathbf{I_L} = \frac{10\angle 60}{j1000\times 0.25} = 0.04\angle -30^\circ$ A, and

$\mathbf{I_C} = 10\angle 60\times\left(j1000\times 2\times 10^{-6}\right) = 0.02\angle 150^\circ$ A. Thus

$$\mathbf{I_{in}} = 0.02\angle 60^\circ + 0.04\angle -30^\circ + 0.02\angle 150^\circ = 0.0283\angle 15^\circ \text{ A}$$

and

$$i_{in}(t) = 0.0283\cos(1000t + 15^\circ) \text{ A}$$

using the following MATLAB code:

```
»Vin = 10*exp(j*60*pi/180)
Vin =5.0000e+00 + 8.6603e+00i
»R = 500;  L = 0.25; C = 2e-6;
»w = 1e3;
»IR = Vin/500
IR =1.0000e-02 + 1.7321e-02i
»IL = Vin/(j*w*L)
IL =3.4641e-02 - 2.0000e-02i
»IC = j*w*C*Vin
IC =-1.7321e-02 + 1.0000e-02i
»Iin = IR + IL + IC
Iin =2.7321e-02 + 7.3205e-03i
»abs(Iin)
ans =2.8284e-02
»angle(Iin)*180/pi
ans =1.5000e+01
```

Solution 10.21 In **MATLAB**

```
»Iin = -100*j*1e-3;  R = 100;
»L = 0.04; C = 2e-6; w = 2500;
»VR = R*Iin
VR =0 - 1.0000e+01i
»VL = j*w*L*Iin
VL =10
»VC = Iin/(j*w*C)
VC =-20
»Vin = VR + VL + VC
```

Vin =-1.0000e+01 - 1.0000e+01i
»abs(Vin)
ans =1.4142e+01
»angle(Vin)*180/pi
ans =-135

Therefore, $v_{in}(t) = 14.14\cos(2500t - 135°)$ V.

Solution 10.22 (a) Here, $i_1(t) = 0.6\cos(200t)$ A and $v_{out}(t) = 20\sin(200t) = 20\cos(200t - 90°)$ V. For $\omega = 200$ rad/s, the phasors are by inspection: $\mathbf{I_1} = 0.6\ \angle 0°$ A, $\mathbf{V_{out}} = 20\ \angle -90°$ V .

(b) Write down the resistor, inductor, and capacitor current phasors, given $\mathbf{V_{out}}$:

$$\mathbf{I_R} = \frac{20\ \angle -90}{1/0.03} = 0.6\ \angle -90° = -j0.6$$

$$\mathbf{I_L} = \frac{20\ \angle -90}{j200 \times 0.1} = 1\ \angle -180° = -1$$

$$\mathbf{I_C} = 20\ \angle -90 \times \left(j200 \times 0.4 \times 10^{-3}\right) = 1.6\ \angle 0°$$

Now, by KCL

$$\mathbf{I_2} = \mathbf{I_R} + \mathbf{I_L} + \mathbf{I_C} - \mathbf{I_1} = -j0.6$$

where we have substituted the above values of branch currents. The time-domain function is:

$$i_2(t) = 0.6\cos(200t - 90°) = 0.6\sin(200t) \text{ A}$$

Solution 10.23 (a) $\omega = 400$ rad/s and $\mathbf{V_{in}} = 20\ \angle 0°$ V. We can easily use the voltage divider formula for phasors and substitute values to obtain:

$$\mathbf{V_{out}} = \frac{200}{200 + j\omega L}\mathbf{V_{in}} = \frac{20}{1 + j} = 14.14\ \angle -45°$$

in which case $v_{out}(t) = 14.14\cos(400t - 45°)$ V

(b) $\omega = 250$ rad/s and $\mathbf{V_{in}} = 20\ \angle -90° = -j20$ V. Again, we can easily use voltage division:

$$\mathbf{V_{out}} = \frac{400}{400 + \dfrac{1}{j\omega C}}\mathbf{V_{in}} = \frac{20\ \angle -90°}{1 - j} = 14.14\ \angle -45° \text{ V}$$

Thus, in the time-domain,

$$v_{out}(t) = 14.142\cos(250t - 45°) \text{ V}$$

Solution 10.24 (a) $\omega = 10{,}000$ rad/s and $\mathbf{V_{in}} = 100\angle -90° = -j100$ V. Apply the voltage divider formula:

$$\mathbf{V_{out}} = \frac{100}{100 + j\omega L + 1/j\omega C}\mathbf{V_{in}} = \frac{100}{100 + j100 - j0.25}\mathbf{V_{in}} = 80\angle -126.87°$$

The steady-state response is thus,

$$v_{out}(t) = 80\sin(10{,}000t - 36.87°) \text{ V}$$

The phasor method provides for a much easier way of obtaining the steady-state response.

(b) Here, $\omega = 2500$ rad/s and $\mathbf{I_{in}} = 0.02\angle 0°$ A. Now, apply current division:

$$\mathbf{I_R} = \frac{0.01}{0.01 + j\omega C + 1/j\omega L}\mathbf{I_{in}} = \frac{0.01}{0.01 + j0.0025 - j0.01}\mathbf{I_{in}} = 0.016\angle 36.86°$$

By Ohm's law:

$$\mathbf{V_{out}} = 100\mathbf{I_R} = 1.6\angle 36.87° \text{ V}$$

Therefore

$$v_{out}(t) = 1.6\cos(2500t + 36.87^{o}) \text{ V}$$

Solution 10.25 (a) It is easier to find the admittance first:

$$Y_{in}(j100) = \frac{1}{j\omega L} + j\omega C = -j0.2 + j12.5 = j12.3$$

$$Z_{in}(j100) = -j0.0813$$

(b) $Y_{in}(j100) = \dfrac{1}{25j} = \dfrac{1}{j100 \times 0.05} + j100C$. Hence, in MATLAB,

```
»C = (1/(25*j) - 1/(j*w*L) )/(j*100)
C =   1.6000e-03
```

Solution 10.26 (a)

$$Z_{in}(j\omega) = \frac{1}{j\omega C} + \frac{R \times j\omega L}{R + j\omega L} = \frac{-j}{\omega C} + \frac{j\omega RL}{R + j\omega L} = 9.975\angle -0.0072°$$

```
»R = 10; L = 0.1; C = 1e-3; w = 2e3;
»Zin = -j/(w*C) + j*w*R*L/(R + j*w*L)
Zin =  9.9751e+00 - 1.2469e-03i
```

```
»abs(Zin)
ans =   9.9751e+00
»angle(Zin)*180/pi
ans =  -7.1620e-03
```

(b) As the frequency increases, the capacitor becomes a short circuit and the inductor becomes an open circuit. Thus, the impedance approaches R. Analytically,

$$\lim_{\omega\to\infty} Z_{in}(j\omega) = \lim_{\omega\to\infty}\frac{1}{j\omega C} + \lim_{\omega\to\infty}\frac{R}{\frac{R}{j\omega L}+1} = R$$

(c) $Z_{in}(j\omega) = \frac{-j}{\omega C} + \frac{j\omega RL(R - j\omega L)}{R^2+\omega^2 L^2} = j\left(\frac{\omega LR^2}{R^2+\omega^2L^2} - \frac{1}{\omega C}\right) + \frac{R\omega^2 L^2}{R^2+\omega^2L^2}$. It follows that w must satisfy

$\omega^2 LR^2 C = R^2 + \omega^2 L^2$ or equivalently $\omega^2 = \frac{R^2}{LR^2C - L^2} = \frac{1}{LC\left(1 - \frac{L}{CR^2}\right)}$. For the given component

values $\left(1 - \frac{L}{CR^2}\right) = 0$; hence there is no finite value of frequency for which the impedance is real.

SOLUTION 10.27. We note that the input admittance is given by:

$$Y_{in}(j\omega) = \frac{1}{100 + j\omega 0.1} + j\omega\times 10^{-6} = \frac{100 - j\omega 0.1}{10^4 + 0.01\omega^2} + j\omega\times 10^{-6}$$
$$= \frac{100}{10^4 + 0.01\omega^2} + j\omega\times 10^{-6} - j\frac{\omega 0.1}{10^4+0.01\omega^2}$$

Thus, ω must satisfy $10^5 = 10^4 + 0.01\omega^2$ or equivalently, $\omega = \sqrt{10^7 - 10^6} = 3000$ rad/s.

SOLUTION 10.28. (a) As usual we will use MATLAB.

```
»R1 = 20; R2 = 10;
»L = 0.04; C = 0.6e-3;
»w = 250;
```

```
»Yin = 1/R1 + 1/(R2 + j*w*L) + j*w*C
Yin =  1.0000e-01 + 1.0000e-01i
```

Hence

$$Y_{in}(j250) = 0.1 + j0.1 \text{ S}$$

(b) For this part we observe that

$$Y_{in}(j250) = \frac{1}{R_1} + \frac{1}{R_2 + j\omega L} + j\omega C = 0.1 - 0.05j + j250C$$

For this to be real, the imaginary part must be zero, i.e.,

$$C = \frac{0.05}{250} = \frac{1}{5000} = 0.2 \text{ mF}$$

Solution 10.29 (a) We can derive an expression for the input impedance by noting that it is the series combination of the resistance and the inductor/capacitor pair connected in parallel. Thus,

$$Z_{in}(j\omega) = R + \frac{\frac{1}{j\omega C} \times j\omega L}{\frac{1}{j\omega C} + j\omega L} = R - j\frac{\frac{L}{C}}{\omega L - \frac{1}{\omega C}}$$

Equating the real and imaginary parts of the given impedance, $R = 4 \ \Omega$ and

$$\frac{-L}{\omega LC - 1/\omega} = \frac{-L}{2L - 0.25} = 2 \qquad L = 0.1 \text{ H}$$

(b) At zero inductance, the above reactance is zero. Also, at L = 0.125, the denominator of the above reactance is zero, which means that the reactance is infinite.

Solution 10.30 (a) First derive an expression for the input impedance as a function of frequency:

$$Z_{in}(j\omega) = 5 + j\omega L + \frac{1}{j\omega C} = 5 + j\left(\omega L - \frac{1}{\omega C}\right)$$

We want the imaginary part to be equal to zero. Thus,

$$\omega L - \frac{1}{\omega C} = 0 \qquad \omega^2 = \frac{1}{LC} \qquad \omega = 2500 \text{ rad/s}$$

where we have substituted the values of L and C.

The magnitude of the impedance is minimum when the imaginary part is zero, and Z_{in}(j2500) = 5.

(b) Derive an expression for the admittance:

$$Y_{in}(\omega) = \frac{1}{5} + j\omega C + \frac{1}{j\omega L} = \frac{1}{5} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Again, the imaginary part is equal to zero when $\omega = \frac{1}{\sqrt{LC}}$ or $\omega = 2500$, at which point the admittance is 0.2.

Solution 10.31 The input admittance is $Y_{in}(\omega) = \frac{1}{R} + j\omega C$ = 0.008+0.004. Equating this to the given admittance at $\omega = 500$ yields R = 125 Ω, ωC = 0.004 or C = 8 μF.

Solution 10.32 Derive an expression for the admittance at $\omega = 1000$:

$$Y_{in}(j\omega) = \frac{1}{j\omega L} + \frac{0.25 \times j\omega C}{0.25 + j\omega C} = -j0.5 + \frac{j0.125}{0.25 + j0.5} = 0.2 - j0.4 \text{ S}$$

Note that this is equivalent to a 0.2 S conductance (i.e. 5 Ω resistance) in parallel with a 2.5 mH inductance (at the given frequency!). Now, the impedance is:

$$Z_{in} = \frac{1}{0.2 - j0.4} = 1 + j2 \ \Omega$$

This is equivalent to a resistance of 1 Ω in series with a 2 mH inductance (at the given frequency).

Solution 10.33 The current is zero when the input impedance of the parallel combination of inductor and capacitor is infinite. The latter is given by:

$$Z_{LC}(j\omega) = \frac{\frac{1}{j\omega C} \times j\omega L}{\frac{1}{j\omega C} + j\omega L} = -j\frac{\frac{L}{C}}{\omega L - \frac{1}{\omega C}}$$

The magnitude of this is infinite when $\omega_r L - \frac{1}{\omega C} = 0$ $\quad \omega_r^2 = \frac{1}{LC}$ $\quad \omega_r = 15{,}811$ rad/s. Observe that

$$\mathbf{I_s} = \frac{\mathbf{V_s}}{R + Z_{LC}(j\omega_r)} = 0$$

Hence $i_s(t) = 0$ at $\omega_r = 15{,}811$ rad/s. At this frequency, the voltage across the LC tank is equal to the input voltage (since there is no drop across the resistor).

Solution 10.34 (a) By inspection:

$$Z_{in} = \frac{2 \times j\omega 0.4}{2 + j\omega 0.4} = \frac{j\omega 0.8}{2 + j\omega 0.4}$$

$$Y_{in} = \frac{1}{R} + \frac{1}{j\omega L} = 0.5 - j\frac{2.5}{\omega}$$

(b) Use the current divider formula, and substitute the given frequency, to obtain:

$$\mathbf{I_L} = \frac{R}{R + j\omega L}\sqrt{2} = \frac{2\sqrt{2}}{2 + j2} = 1 \quad -45^\circ$$

and $i_L(t) = \cos(5t - 45^o)$ A.

Solution 10.35 (a) Using voltage division, $\mathbf{V_{out}} = \dfrac{R}{R - j/\omega C}\mathbf{V_{in}} = \dfrac{j\omega RC}{1 + j\omega RC}\mathbf{V_{in}}$ which

$$\mathbf{V_{out}} = 90^o - \phi_{in} - \tan^{-1}(\omega RC) \text{ and } |\mathbf{V_{out}}| = \frac{\omega RC|\mathbf{V_{in}}|}{\sqrt{1 + \omega^2 R^2 C^2}}$$

(b) For this part, we need to make sure that $\tan^{-1}(\omega RC) = 45^\circ \qquad \omega RC = 1 \qquad \omega = 1/RC$.

(c) At this frequency,

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\frac{1}{RC}RC}{\sqrt{1 + \frac{1}{R^2C^2}R^2C^2}} = \frac{1}{\sqrt{2}}$$

Solution 10.36 Here, $\omega = 1/RC$ and $\dfrac{\mathbf{V_C}}{\mathbf{V_{in}}} = \dfrac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \dfrac{1}{1 + j\omega RC} = \dfrac{1}{1 + j}$. Therefore

$$\mathbf{V_C} = \frac{1}{1 + j}\mathbf{V_{in}} = 0.707 V_m \quad -45^\circ$$

The time-domain function is

$$v_C(t) = 0.707 V_m \cos\left(\frac{1}{RC}t - 45^\circ\right) \text{ V}$$

Solution 10.37 (a) The magnitude of the capacitor voltage is 10/14.14 = 0.707 times the magnitude of the input signal. We just showed in the above problem that

$$\frac{\mathbf{V_C}}{\mathbf{V_{in}}} = \frac{1}{1 + j10RC} \qquad \left|\frac{\mathbf{V_C}}{\mathbf{V_{in}}}\right| = \frac{1}{\sqrt{1 + 100R^2C^2}}$$

And we also showed that the ratio is 0.707 when the frequency is 1/RC. So, C = 1/(10R) = 0.01 F.

(b) Again, from the results of the previous problem, the angle is –45 degrees.

Solution 10.38 $\omega = 1000$ rad/s and $\mathbf{I_{in}} = 2\angle 45°$ A. The equivalent admittance is

$$Y_{eq}(j\omega) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = 0.25 + j(0.25 - 0.25) = 0.25 \text{ S}$$

In MATLAB

```
»Yeq = 0.25;
»Iin = 2*exp(j*pi/4)
Iin =
  1.4142e+00 + 1.4142e+00i
»Vout = Iin/Yeq
Vout =
  5.6569e+00 + 5.6569e+00i
»abs(Vout)
ans =     8
»angle(Vout)*180/pi
ans =   4.5000e+01

»%  Using Current Division
»IL = (1/(j*1000*4e-3)/Yeq)*Iin
IL =
  1.4142e+00 - 1.4142e+00i
»abs(IL)
ans =     2
»angle(IL)*180/pi
ans =   -45
```

Therefore, $v_{out}(t) = 8\cos(1000t + 45^o)$ V and $i_L(t) = 2\cos(1000t - 45^o)$ A.

(b) If $\omega = 618$ rad/s, then

$$Y_{eq}(\omega) = \frac{1}{4} - j0.405 + j0.1545 = 0.25 - j0.25 = 0.3536 \quad -45°$$

and

```
»Vout = Iin/Yeq
Vout =
 -3.4779e-04 + 5.6565e+00i
»abs(Vout)
ans =
   5.6565e+00
»angle(Vout)*180/pi
ans =
   9.0004e+01
»IL = Iin*(1/(j*618*4e-3))/Yeq
IL =
  2.2882e+00 + 1.4069e-04i
»abs(IL)
ans =
   2.2882e+00
»angle(IL)*180/pi
ans =
   3.5228e-03
```

Therefore, $v_{out}(t) = 5.657\cos(1000t + 90^o) = -5.657\sin(1000t)$ V and $i_L(t) = 2.288\cos(1000t)$ A.

Solution 10.39 Write the input phasor: $\omega = 1000$ rad/s and $\mathbf{I_{in}} = 0.01\sqrt{2} \quad 60°$ A.

```
»w =1000;
»Iin = 0.01*sqrt(2)*exp(j*60*pi/180)
Iin =
  7.0711e-03 + 1.2247e-02i
»Yeq = 1/500 +1/(j*w*0.25) +j*w*2e-6
Yeq =
  2.0000e-03 - 2.0000e-03i
»Vin = Iin/Yeq
```

```
Vin =
 -1.2941e+00 + 4.8296e+00i
»abs(Vin)
ans =
     5
»angle(Vin)*180/pi
ans =
   1.0500e+02
»IR  = Iin*(1/500)/Yeq
IR =
 -2.5882e-03 + 9.6593e-03i
»abs(IR)
ans =
   1.0000e-02
»angle(IR)*180/pi
ans =
   1.0500e+02
»IL = Iin*(1/(j*w*0.25))/Yeq
IL =
  1.9319e-02 + 5.1764e-03i
»abs(IL)
ans =
   2.0000e-02
»angle(IL)*180/pi
ans =
   1.5000e+01
»IC = Iin*j*w*2e-6/Yeq
IC =
 -9.6593e-03 - 2.5882e-03i
»abs(IC)
ans =
   1.0000e-02
»angle(IC)*180/pi
ans =
  -1.6500e+02
```

Therefore, $v_{in}(t) = 5\cos(1000t - 105°)$ V, $i_C(t) = 0.01\cos(1000t - 165°)$ A, $i_L(t) = 0.02\cos(1000t + 15°)$ A, and $i_R(t) = 0.01\cos(1000t + 105°)$ A.

SOLUTION 10.40. $\omega = 4$. Using voltage division

$$\mathbf{V_1} = \frac{R}{R + \dfrac{1}{j\omega C} + j\omega L}\mathbf{V_{in}} = \frac{2}{2 - 4j + 2j}(-8j) = 5.657 \angle -45^o$$

Converting back to time:

$$v_{out}(t) = 5.657\cos(4t - 45°) \text{ V}$$

SOLUTION 10.41. $\omega = 25$ rad/s, $\mathbf{V_s} = 10 \angle 0°$ V. By voltage division,

$$\mathbf{V_C} = \frac{-j/0.02 \times 25}{10 + j0.08 \times 25 - j/0.02 \times 25} 10 \angle 0° = -2j = 2 \angle -90°$$

Thus, $v_C(t) = 2\cos(25t - 90°)$ V.

SOLUTION 10.42. $\dfrac{|\mathbf{V_{out}}|}{|\mathbf{V_{in}}|} = \left|\dfrac{1}{3 + j8 + 1/j4C}\right| = 0.2 \qquad 25 = \left|3 + j\left(8 - \dfrac{1}{4C}\right)\right|^2$.

Thus $\left(8 - \dfrac{1}{4C}\right)^2 = 25 - 9 \qquad C = \dfrac{1}{16} = 0.0625$ F.

SOLUTION 10.43. Here, $\omega = 3.33 \times 10^3$ rad/s and $\mathbf{V_{in}} = 50 \angle 0°$ V. Using phasors,

$$\mathbf{V_R} = \frac{400}{400 - \dfrac{j}{3.33 \times 10^3 \times 10^{-6}}} \times 50 = 40 \angle 36.897° \text{ V}$$

and

$$\mathbf{V_{out}} = \frac{100\mathbf{V_R}/j\omega C}{100 + \dfrac{1}{j\omega C}} = \frac{100\mathbf{V_R}}{j100\omega C + 1} = 3985 \angle 32.1° \text{ V}$$

Hence, $v_{out}(t) = 3985\cos(3.33\times 10^3 t + 32.1°)$ V.

SOLUTION 10.44. Here, $\omega = 10^4$ rad/s, $\mathbf{I_{in}} = 0.01\angle 0°$ A.

$$Z_1 = \frac{100\times j\omega 0.1}{100 + j\omega 0.1} = 1 + j1000 \qquad \mathbf{V_L} = Z_L\mathbf{I_{in}} = 0.01\times Z_L = 0.01 + j10$$

Now, in MATLAB

```
»w=1e4;  R = 100;L = 0.1;  C = 0.1e-6;
»Z1 = R*j*w*L/(R+j*w*L)
Z1 =  9.9010e+01 + 9.9010e+00i
»Iin = 0.01;
»VL = Z1*Iin
VL =9.9010e-01 + 9.9010e-02i
»Z2 = 1/(1/R + j*w*C)
Z2 =  9.9010e+01 - 9.9010e+00i
»VC = Z2*VL
VC =  9.9010e+01 + 1.7764e-15i
»abs(VC)
ans =   9.9010e+01
»angle(VC)*180/pi
ans =   1.0280e-15
```

Thus, $v_C(t) = 99\cos(10000t)$ V.

SOLUTION 10.45. Here $\omega = 40$ rad/s and $\mathbf{V_{in}} = 120\angle 0°$. This problem is best done in MATLAB using parallel impedance computation, voltage division, and Ohm's law for phasors:

```
»R1 = 500; R2 = 80;
»C = 0.1e-3; L =  2;
»w = 40; Vin = 120;
»Z1 = R1/(j*w*C)/(R1 + 1/(j*w*C))
Z1 =  1.0000e+02 - 2.0000e+02i
»Z2 = R2*j*w*L/(R2 + j*w*L)
Z2 =  4.0000e+01 + 4.0000e+01i
»  Use voltage division
»VC = Z1*Vin/(Z1+Z2)
VC =  1.2212e+02 - 3.1858e+01i
»abs(VC)
ans =   1.2621e+02
»angle(VC)*180/pi
ans =  -1.4621e+01
```

```
» Use voltage division and Ohm's law for inductors
»VL = Z2*Vin/(Z1+Z2)
VL =-2.1239e+00 + 3.1858e+01i
»IL = VL/(j*w*L)
IL =3.9823e-01 + 2.6549e-02i
»abs(IL)
ans =  3.9911e-01
»angle(IL)*180/pi
ans =  3.8141e+00
```

Therefore, $v_C(t) = 126.21\cos(40t - 14.621^o)$ V and $i_L(t) = 0.399\cos(40t + 3.814)$ A.

SOLUTION 10.46. Here $\omega = 40$ rad/s and $\mathbf{I_{in}} = 0.120 \angle 0°$ A. This problem is best solved using MATLAB.

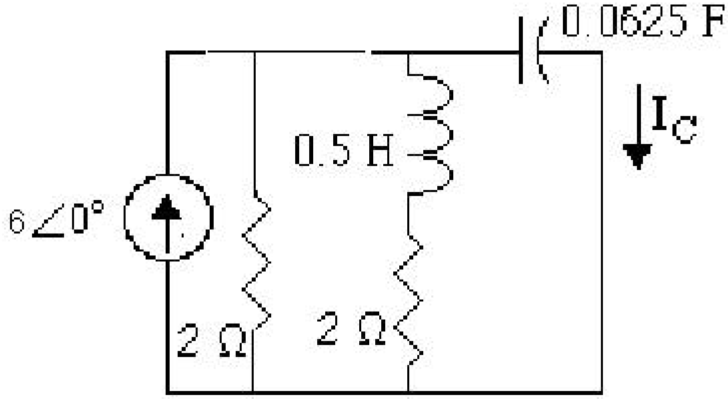
```
»w = 40; Iin = 0.12;R = 5;
»C = 0.004; L = 0.1;
»Y1 = 1/R + j*w*C
Y1 =  2.0000e-01 + 1.6000e-01i

»Y2 = 1/R + 1/(j*w*L)
Y2 =  2.0000e-01 - 2.5000e-01i

»% USING CURRENT DIVISION
»IL = Iin*Y2/(Y1 + Y2)
IL =  7.3171e-02 - 5.8537e-02i
»abs(IL)
ans =   9.3704e-02
»angle(IL)*180/pi
ans =  -3.8660e+01

»% AGAIN USING CURRENT DIVISION
»IC = Iin*Y1/(Y1 + Y2)
IC =  4.6829e-02 + 5.8537e-02i

»% USING OHM'S LAW FOR CAPACITORS
»VC = IC/(j*w*C)
VC =  3.6585e-01 - 2.9268e-01i
»abs(VC)
ans =   4.6852e-01
»angle(VC)*180/pi
ans =  -3.8660e+01
```

SOLUTION 10.47 Here, $\omega = 25$ rad/s and $\mathbf{I_S} = 2 \angle 0°$ A. Now perform a source transformation. The combination of current source in parallel with resistor is changed into a voltage source in series with the same resistor. The voltage source value is: $\mathbf{V_S} = 1 \times \mathbf{I_S} = 2 \angle 0°$ V. Apply Ohm's law to obtain:

$$\mathbf{I_1} = \frac{2\angle 0°}{2 + 1/j25 \times 0.02} = 0.5 + j0.5 = 0.707 \angle 45°$$

Hence, $i_1(t) = 0.707\cos(25t + 45^o)$ A.

SOLUTION 10.48 Apply a source transformation to obtain:



The impedance of the inductor branch is $Z_L(j4) = 2 + j4 \times 0.5 = 2 + j2\ \Omega$ and

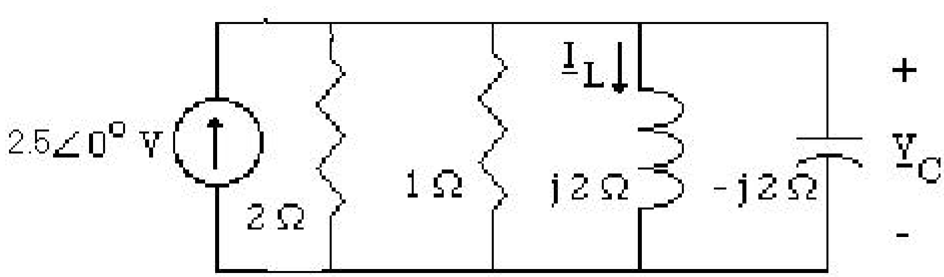
$Y_L(j4) = \dfrac{1}{Z_L(j4)} = 0.25 - j0.25$ S. Now, the total admittance seen by the source:

$$Y_{eq}(j4) = \frac{1}{2} + j0.25 + Y_L(j4) = 0.75\ \text{S}$$

Thus, by inspection, $\mathbf{V_C} = \dfrac{6\quad 0^\circ}{0.75} = 8\quad 0^\circ$ V and $\mathbf{I_C} = 0.25\quad 90^\circ \times 8\quad 0^\circ = 2\quad 90^\circ$ A. Therefore

$i_C(t) = 2\cos(4t + 90^o) = -2\sin(4t)$ A.

SOLUTION 10.49. Apply a source transformation to obtain:



Then, by inspection, $Y_{eq}(j500) = \dfrac{1}{2} + \dfrac{1}{1} + \dfrac{1}{j2} - \dfrac{1}{j2} = 1.5$ S. Thus,

$$\mathbf{V_C} = \frac{2.5\angle 0°}{1.5} = 1.667\angle 0° \text{ V} = \mathbf{V_L} \qquad \mathbf{I_L} = 1.667/j2 = 0.833\angle -90° \text{ A}$$

In the time-domain:

$$v_C(t) = 1.667\cos(500t) \text{ V and } i_L(t) = 0.833\cos(500t - 90°) = 0.833\sin(500t) \text{ A.}$$