PROBLEM SOLUTIONS CHAPTER 10

SOLUTION 10.50. The input voltage phasor is $\omega = 2000$ rad/s and $\mathbf{V_S} = 20\angle 0°$ V. Now, do a source transformation on the phasor circuit:

where

$$\mathbf{I_S} = \frac{\mathbf{V_S}}{Z_L} = \frac{20\angle 0°}{j2000 \times 10 \times 10^{-3}} = 1\angle -90° = -j \text{ A}$$

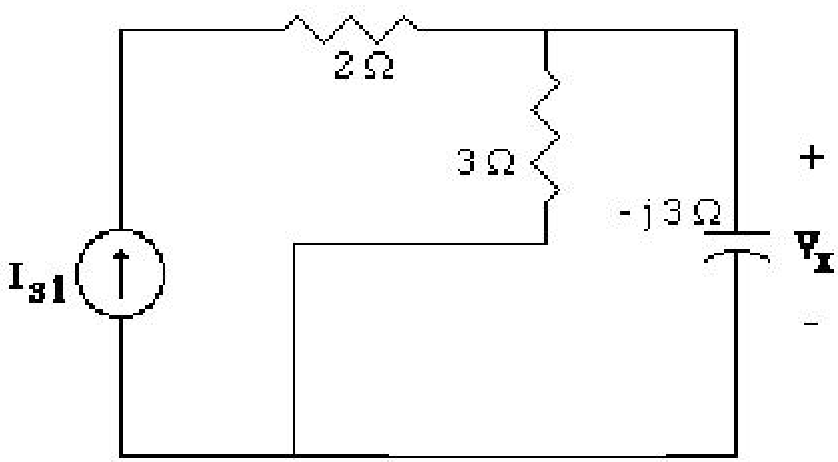
Now,

$$Y_{eq} = \frac{1}{20} + \frac{1}{j20} + j2000 \times 50 \times 10^{-6} = 0.0707\angle 45° \text{ S}$$

and

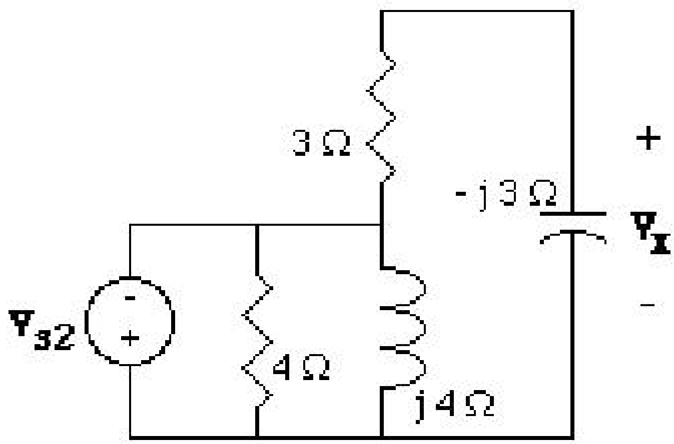
$$\mathbf{V_C} = \frac{\mathbf{I_S}}{Y_{eq}} = \frac{1\angle -90°}{0.0707\angle 45°} = 14.14\angle -135° \text{ V}$$

SOLUTION 10.51. Use superposition. First, find response to current source using circuit below:



$$\mathbf{V_{x\_1}} = \mathbf{I_{s1}} Z_{RC} = I_{s1}\frac{3 \times (-j3)}{3 - j3} = 2\angle 0° \times 2.121\angle -45° = 4.242\angle -45°$$

Now, find the response due to the voltage source using the following circuit:



The voltage across the inductor is the same as the input source, and this voltage divides between the series combination of capacitor and resistor:

$$\mathbf{V_{x\_2}} = \frac{j3}{3-j3}\mathbf{V_{s2}} = \frac{j3}{3-j3}3\angle 90^{o} = 2.121\angle -135^{o}\text{ V}$$

Combining the two contributions implies that:

$$\mathbf{V_x} = 4.242\angle -45^{o} + 2.121\angle -135^{o} = 4.74\angle -71.6^{o}\text{ V}$$

SOLUTION 10.52. **(a)** As stated, $\mathbf{V}_L = a\mathbf{V}_{s1} + b\mathbf{I}_{s2}$. To find a, set $\mathbf{I}_{s2} = 0$ and use voltage division:

$$\mathbf{V_L} = -\frac{j30}{j30+j30}\mathbf{V_{s1}} = -0.5\mathbf{V_{s1}} = -a\mathbf{V_{s1}}$$

To find b, set $\mathbf{V}_{s1} = 0$ and use parallel impedance and Ohm's law:

$$\mathbf{V_L} = (j30//j30)\mathbf{I_{s2}} = j15\mathbf{I_{s2}} = b\mathbf{I_{s2}}$$

Hence

$$\mathbf{V}_L = -0.5\mathbf{V}_{s1} + j15\mathbf{I}_{s2}$$

(b) For this part, $\mathbf{V_{s1}} = 10$ and $\mathbf{I_{s2}} = 0.5\angle -90^{o}$. Hence from the formula,

$$\mathbf{V_L} = -0.5V_{s1} + j15\mathbf{I_{s2}} = -0.5\times 10 + j15\times 0.5(-j) = -5 + 7.5 = 2.5\text{ V}$$

Therefore $v_L(t) = 2.5\cos(100\pi t)$ V.

SOLUTION 10.53. In this problem, we can make use of the linearity property for phasors. Specifically, from the given information, we can write

$$\frac{\mathbf{V_1}}{\mathbf{I_{in}}} = \frac{20 \quad 45^\circ}{10 \quad 0^\circ} = 2 \quad 45^\circ = a \text{ and } \frac{\mathbf{V_1}}{\mathbf{V_{in}}} = \frac{5 \quad 90^\circ}{10 \quad 45^\circ} = 0.5 \quad 45^\circ = b$$

Hence,

$$\mathbf{V_1} = a\mathbf{I_{in}} + b\mathbf{V_{in}}$$

Substituting the new values of input current and voltage, we obtain:

$$\mathbf{V_1} = 10 \quad 0^\circ + 10 \quad 45^\circ = 18.48 \quad 22.5^\circ \text{ V}$$

SOLUTION 10.54. **(a)** For $\mathbf{V_{out}}$ to be zero, we want

$$\frac{R_C}{R_C - \dfrac{j}{\omega C}} = \frac{j\omega L}{R_L + j\omega L} \qquad R_C R_L = \frac{L}{C}$$

(b) Substituting $R_C C = 2$ s and $R_L = 3\ \Omega$ into the above expression gives: $L = R_C C R_L = 6$ H.

(c) The bridge circuit can be represented by an impedance $Z_{bridge}(j\omega)$. The voltage that appears across the bridge, say V_{bridge}, is obtained by voltage division. Hence, by the voltage substitution theorem, the problem may be solved as in part (a) with this new source voltage V_{bridge} appearing across $Z_{bridge}(j\omega)$.

SOLUTION 10.55 The input phasor is: $\omega = 1000$ rad/s and $\mathbf{I_{in}} = 2 \quad 45^\circ$ A, assuming peak value. First compute

$$Y_{th}(j10^3) = 0.25 - \frac{j}{1000 \times 4 \times 10^{-3}} + j1000 \times 0.25 \times 10^{-3} = 0.25\text{S} \qquad Z_{th}(j10^3) = 4\ \Omega$$

Then,

$$\mathbf{V_{oc}} = \mathbf{I_{in}} \times 4 = 8 \quad 45^\circ \text{ V}$$

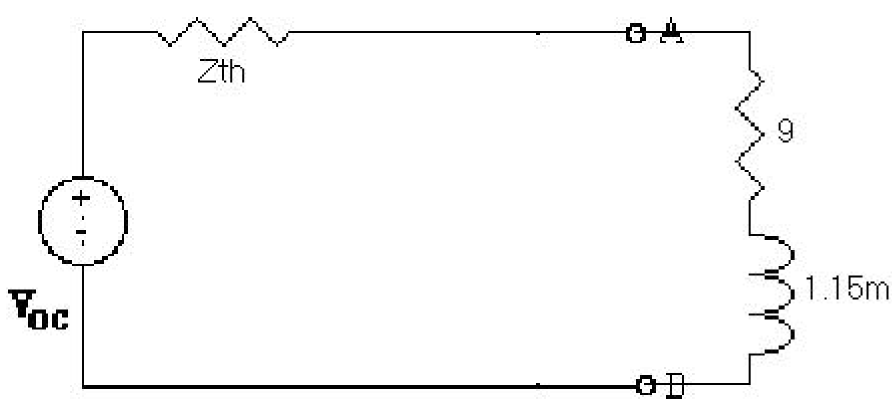
in which case $v_{oc}(t) = 8\cos(10^3 t + 45^o)$ V. The final equivalent is a voltage source (having value $\mathbf{V_{oc}}$) in series with a resistance of 4 Ω.

SOLUTION 10.56. **(a)** First note that the frequency is given in Hz, so, $\omega = 1281.77$ rad/s and $\mathbf{I_{in}} = 10\angle 0°$ A. Then, in MATLAB,

```
»R = 0.25; L = 1.17e-3;  C = 520e-6;
»w = 2*pi*204;
»Yin = j*w*C +1/(R + j*w*L)
Yin =
  1.0815e-01 + 1.7737e-02i
»Zin = 1/Yin
Zin =
  9.0039e+00 - 1.4766e+00i
»abs(Zin)
ans =
   9.1242e+00
»angle(Zin)*180/pi
ans =
  -9.3134e+00
```

Therefore $Z_{th} = 9.1242\angle -9.313°$ Ω. Finally, $\mathbf{V_{oc}} = Z_{th}\mathbf{I_{in}} = 91.2\angle -9.313°$ V.

(b) Now, the circuit looks like the following:



Simple voltage division can yield:

$$\mathbf{V_L} = \frac{Z_L}{Z_L + Z_{TH}}\mathbf{V_{oc}} = 46.219\angle 0^o \qquad v_L(t) = 46.2\cos(1281.77t) \text{ V}$$

SOLUTION 10.57. For this problem we short the V-source and compute Z_{th} and use voltage division to find $\mathbf{V_{oc}}$. Specifically

$$Z_{th} = \frac{1}{j\omega C + \frac{1}{0.1 + j\omega L}} = 10^3 - j10 \ \Omega$$

and

$$\mathbf{V_{oc}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + 0.1 + j\omega L} \times 2 = -200j \ \text{V}$$

where

```
»w=1000;
»L = 0.01;
»C = 0.1e-3;
»Zth = 1/(j*w*C + 1/(0.1 + j*w*L))
Zth =  1.0000e+03 - 1.0000e+01i

»Voc = 2*(1/(j*w*C))/(0.1 + j*w*L + 1/(j*w*C))
Voc =          0 - 2.0000e+02i
```

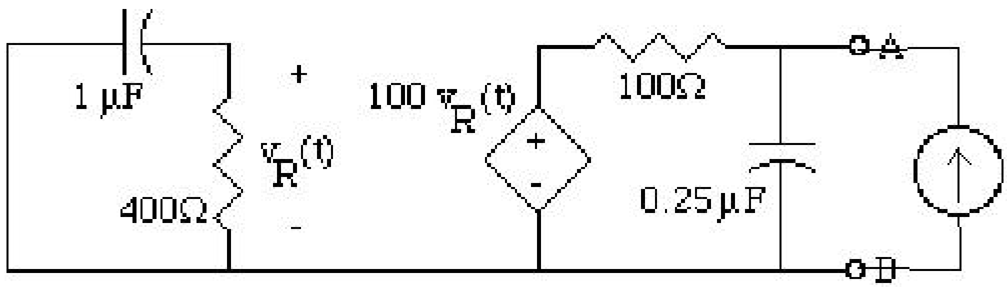
To compute the load voltage, define

$$Z_{load} = 10^3 + j10^{-2}\omega = 10^3 + j10 \ \Omega$$

Again using voltage division and MATLAB we have

```
»Zload = 1e3 + j*10;
»Vload = Voc*Zload/(Zload + Zth)
Vload =
  1.0000e+00 - 1.0000e+02i
»magVload = abs(Vload)
magVload =
   1.0000e+02  (volts)
```

SOLUTION 10.58. Here, $\omega = 3.33\times10^3$ rad/s and $\mathbf{V_{in}} = 50 \quad 0°$ V. Here we note that we already found $\mathbf{V_{oc}}$ in Problem 10.43. Thus, $\mathbf{V_{oc}} = 3985 \quad 32.1°$ V. In order to find Z_{th}, we introduce a fictitious 1 A current source at the A and B terminals:



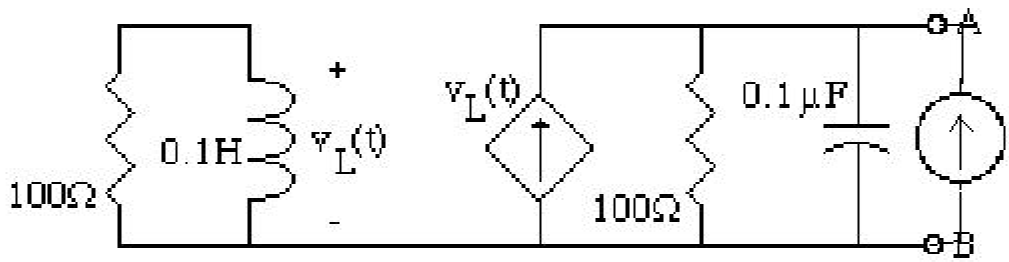
Now, we note that the $\mathbf{V_R}$ voltage phasor is zero. Thus, the dependent source has zero volts across it. This way, the temporary current source sees the parallel combination of a resistor and a capacitor:

$$\mathbf{V_{AB}} = \frac{100\times\dfrac{-j}{3.33\times10^3\times0.25\times10^{-6}}}{100-j1200} = 99.65 \quad -4.77° \text{ V}$$

Thus, $Z_{th} = 99.65 \quad -4.77°\ \Omega$. The Thevenin equivalent consists of $\mathbf{V_{oc}}$ in series with $Z_{th} = 99.65 \quad -4.77° = 99.3 - j8.2865\ \Omega$. Thus the Norton equivalent is the parallel combination of Z_{th} and the current source with value

$$\mathbf{I_{sc}} = \frac{\mathbf{V_{oc}}}{Z_{th}} = \frac{3985 \quad 32.1°}{99.65 \quad -4.77°} = 39.99 \quad 36.91^o \text{ A.}$$

SOLUTION 10.59. $\omega = 10{,}000$ rad/s and $\mathbf{V_{in}} = 10 \quad 0°$ V. Again, we have already found $\mathbf{V_{oc}}$ in Problem 10.44: $\mathbf{V_{oc}} = 99 \quad 0°$ V. Now, to compute the impedance, we introduce the temporary current source of 1 A:



Again, the inductor has no voltage across it. So, the dependent source generates no current. Hence, the independent source sees the parallel combination of a resistor and a capacitor:

$$\mathbf{V_{AB}} = \frac{100 \times (-j1000)}{100 - j1000} = 99.5 \angle -5.71^\circ \text{ V}$$

Thus $Z_{th}(j10^4) = 99.5 \angle -5.71^\circ\ \Omega$. So, the Thevenin equivalent is the series combination of the $\mathbf{V_{oc}}$ source and the above $Z_{th}(j10^4) = 99.5 \angle -5.71^\circ\ \Omega$.

SOLUTION 10.60. Inject a current source at terminals A and B. Then, write a KCL equation at node A:

$$\mathbf{I_R} = \mathbf{I_S} - \mathbf{I_1} \qquad \mathbf{V_R} = \mathbf{I_S} - \mathbf{I_1} \qquad \mathbf{V_C} = \mathbf{V_{AB}} = \mathbf{V_R} + \mathbf{I_1} = \mathbf{I_S} - \mathbf{I_1} + \mathbf{I_1} = \mathbf{I_S}$$

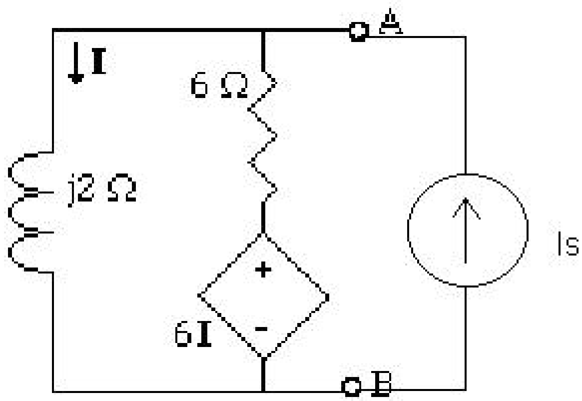
Since the voltage across the current source is equal to its current, the equivalent impedance across this current source is 1 Ω.

SOLUTION 10.61. Inject a current source as usual. Then, write Ohm's law for phasors for the equivalent series RLC circuit. Note that the controlling current for the dependent source is the input current:

$$\mathbf{V_{AB}} = j0.01 \times 200 \times \mathbf{I_S} - \frac{j}{200 \times 0.005}\mathbf{I_S} - 2\mathbf{I_S} = (2 + j)\mathbf{I_S}$$

Therefore, $Z_{th} = \dfrac{\mathbf{V_{AB}}}{\mathbf{I_s}} = 2 + j\ \Omega$.

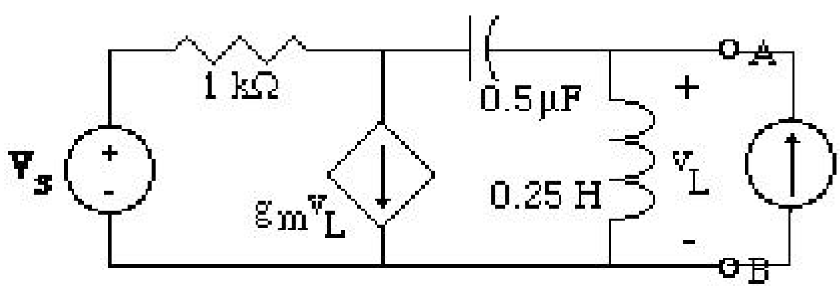
SOLUTION 10.62. Inject a current source $\mathbf{I_s}$:



Apply KCL: $\dfrac{j2\mathbf{I}-6\mathbf{I}}{6}=\mathbf{I_s}-\mathbf{I}$ $\qquad \mathbf{I}=\dfrac{6}{j2}\mathbf{I_s}$. Now, $\mathbf{V_{AB}}$ is the voltage across the inductor:

$$\mathbf{V_{AB}}=\frac{6\times j2}{j2}\mathbf{I_s}=6\mathbf{I_s} \qquad Z_{th}=6\ \Omega.$$

Solution 10.63. $\omega = 2000$ rad/s and $\mathbf{V_S} = 25\angle 0°$ V. Now, inject a current source, and express $\mathbf{V_{AB}}$ as a function of this current source and $\mathbf{V_{oc}}$:



Now, write two nodal equations at A and the top of the dependent current source:

$$\frac{\mathbf{V_1}-\mathbf{V_S}}{10^3}+g_m\mathbf{V_{AB}}+j\omega C\left(\mathbf{V_1}-\mathbf{V_{AB}}\right)=0$$

$$j\omega C\left(\mathbf{V_{AB}}-\mathbf{V_1}\right)+\frac{\mathbf{V_{AB}}}{j\omega L}=\mathbf{I_S}$$

Substituting values, these can be cast into the following matrix equation:

$$10^{-3}\begin{bmatrix} 1+j & 3-j \\ -j & -j \end{bmatrix}\begin{bmatrix} \mathbf{V_1} \\ \mathbf{V_{AB}} \end{bmatrix} = \begin{bmatrix} 10^{-3}\mathbf{V_S} \\ \mathbf{I_S} \end{bmatrix}$$

This can be solved in MATLAB to obtain:

$$\begin{bmatrix} \mathbf{V_1} \\ \mathbf{V_{AB}} \end{bmatrix} = 100 \times \begin{bmatrix} -2.5 - j2.5 & -5 + j10 \\ 2.5 + j2.5 & 5 \end{bmatrix}\begin{bmatrix} 10^{-3}\mathbf{V_S} \\ \mathbf{I_S} \end{bmatrix}$$

Thus

$$\mathbf{V_{AB}} = (0.25 + j0.25)\mathbf{V_s} + 500\mathbf{I_s} = 500\mathbf{I_s} + 6.25 + j6.25$$

Therefore $Z_{th} = 500\ \Omega$ and $\mathbf{V_{oc}} = 6.25 + j6.25 = 8.839\angle 45^o$ V. For the Norton equivalent we need

$$\mathbf{I_{sc}} = \frac{\mathbf{V_{oc}}}{Z_{th}} = \frac{8.839}{500}\angle 45^o = 0.01768\angle 45^o \text{ A.}$$

SOLUTION 10.64. Inject an upward current source, $\mathbf{I_{S2}}$, at $\mathbf{V_{AB}}$. Then, write the following two nodal equations at $\mathbf{V_x}$ and $\mathbf{V_A}$: let $R_1 = R_2 = R$, then

$$\frac{\mathbf{V_x}}{R} + j\omega C_1\mathbf{V_x} + j\omega C_2\left(\mathbf{V_x} - \mathbf{V_{AB}}\right) = \mathbf{I_S}$$

$$j\omega C_2\left(\mathbf{V_{AB}} - \mathbf{V_x}\right) + g_m\mathbf{V_x} + \frac{\mathbf{V_{AB}}}{R} = \mathbf{I_{S2}}$$

which after grouping terms becomes

$$\left(\frac{1}{R} + j\omega C_1 + j\omega C_2\right)\mathbf{V_x} - j\omega C_2\mathbf{V_{AB}} = \mathbf{I_S}$$

$$\left(-j\omega C_2 + g_m\right)\mathbf{V_x} + \left(j\omega C_2 + \frac{1}{R}\right)\mathbf{V_{AB}} = \mathbf{I_{S2}}$$

In MATLAB

```
»R = 100e3; C1 = 1e-9; C2 =  1e-10;
»gm = 0.1e-3; w =  1e3;
»Nodal = [1/R+j*w*C1+j*w*C2   -j*w*C2;-j*w*C2+gm  j*w*C2+1/R]
Nodal =
  1.0000e-05 + 1.1000e-06i          0 - 1.0000e-07i
```

1.0000e-04 - 1.0000e-07i 1.0000e-05 + 1.0000e-07i
»Nodalinv = inv(Nodal)
Nodalinv =
  9.5680e+04 - 2.0070e+04i 2.1024e+02 + 9.5470e+02i
 -9.5449e+05 + 2.1120e+05i 9.7783e+04 - 1.0523e+04i

Thus

$$\begin{bmatrix} \mathbf{V_x} \\ \mathbf{V_{AB}} \end{bmatrix} = 10^3 \times \begin{bmatrix} 95.68 - j20 & 0.21 + j0.955 \\ -954.5 + j211.2 & 97.78 - j10.52 \end{bmatrix} \begin{bmatrix} 40 \times 10^{-6} \\ \mathbf{I_{S2}} \end{bmatrix}$$

Therefore

$$\mathbf{V_{AB}} = -38.18 + j8.448 + (97.8 - j10.5) \times 10^3 I_{S2}$$

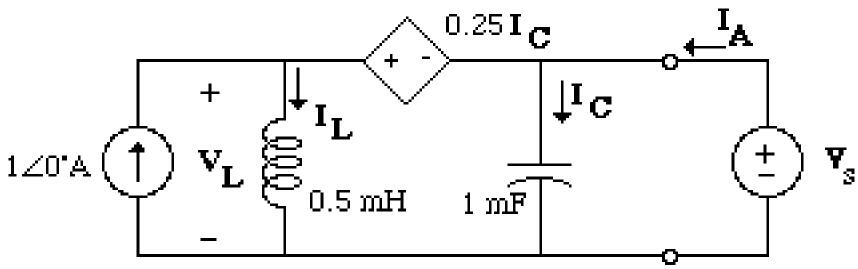
from which we identify

$$\mathbf{V_{oc}} = -38.18 + j8.448 = 39.1\angle 167.5^o \text{ V}$$

and $Z_{th} = (97.8 - j10.5) \times 10^3 = 98.35 \times 10^3 \angle -6.14^o\ \Omega$.

SOLUTION 10.65. We solve this problem by the method illustrated in example 6.3 where a fictitious source is applied and the repsonse is calculated. One can either apply a voltage source or a current source (see figure 6.10 a and b). Generally speaking, neither choice can be claimed as superior to the other. But for a specific circuit, one choice can lead to a much simpler solution than the other. To illustrate this point, we solve the problem with both choices below. Note that although the first method is much simpler than the second, it lacks the generality. If one more resistor were inserted into the circuit, the simplicity of solution of solution may disappear totally, whereas the second method will proceed with very few changes.

Method 1. For this solution we apply an arbitrary voltage source, labeled in phasor form as $\mathbf{V_s}$ as indicated in the circuit below.



For this circuit we will compute an equation of the form ofequation 6.11:

$$\mathbf{I_A} = \frac{1}{Z_{th}}\mathbf{V_s} - \mathbf{I_{sc}} \qquad (1)$$

By inspection of the circuit,

$$\mathbf{I_A} = \mathbf{I_C} + \mathbf{I_L} - 1 = j\omega C\mathbf{V_s} + \frac{\mathbf{V_L}}{j\omega L} - 1 = j2\mathbf{V_s} - j\mathbf{V_L} - 1 \qquad (2)$$

But

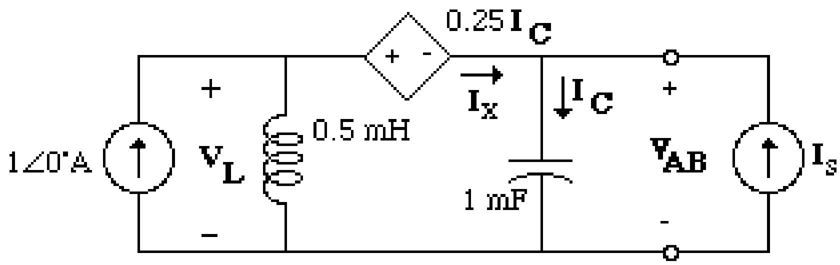
$$\mathbf{V_L} = 0.25\mathbf{I_C} + \mathbf{V_s} = 0.25\,j\omega C\mathbf{V_s} + \mathbf{V_s} = (0.5j + 1)\mathbf{V_s} \qquad (3)$$

Substituting (3) into (2) produces

$$\mathbf{I_A} = (0.5 + j)\mathbf{V_s} - 1 \qquad (4)$$

By comparing (4) with (1), we obtain the answers for the Norton equivlent circuit: $\mathbf{I_{sc}}$ = 1 A and Z_{th} =1/(0.5 + 0.4 – j0.8 Ω.

Method 2: For this method we apply an arbitrary current source, labeled in phasor form as $\mathbf{I_s}$ as indicated in the circuit below.



Notice that we have added a current label $\mathbf{I_x}$ as we plan to use modified nodal analysis method to obtain the desired answer. For this we will compute an equation of the form

$$\mathbf{V_{AB}} = Z_{th}\mathbf{I_s} + \mathbf{V_{oc}}$$

Because of the addition of $\mathbf{I_x}$, we can write the modified nodal equations more or less by inspection:

$$\begin{bmatrix} 1/j\omega L & 0 & 1 \\ 0 & j\omega C & -1 \\ 1 & -(1+j0.25\omega C) & 0 \end{bmatrix}\begin{bmatrix} \mathbf{V_L} \\ \mathbf{V_{AB}} \\ \mathbf{I_x} \end{bmatrix} = \begin{bmatrix} -j & 0 & 1 \\ 0 & 2j & -1 \\ 1 & -1-0.5j & 0 \end{bmatrix}\begin{bmatrix} \mathbf{V_L} \\ \mathbf{V_{AB}} \\ \mathbf{I_x} \end{bmatrix} = \begin{bmatrix} 1 \\ \mathbf{I_s} \\ \mathbf{0} \end{bmatrix}$$

where from MATLAB,

```
»w=2000;
»L = 0.5e-3;
»C = 1e-3;
»Y11 = 1/(j*w*L)
Y11 =          0 - 1.0000e+00i
»Y22 = j*w*C
Y22 =          0 + 2.0000e+00i
»Y32 = -(1+j*0.25*w*C)
Y32 = -1.0000e+00 - 5.0000e-01i
```

Solving the equations in MATLAB produces,

```
»A = [-j  0  1;0  2*j  -1; 1  -1-0.5*j  0];
»Ainv = inv(A)
Ainv =
  8.0000e-01 - 6.0000e-01i  8.0000e-01 - 6.0000e-01i  1.6000e+00 + 8.0000e-01i
  4.0000e-01 - 8.0000e-01i  4.0000e-01 - 8.0000e-01i  8.0000e-01 + 4.0000e-01i
  1.6000e+00 + 8.0000e-01i  6.0000e-01 + 8.0000e-01i -8.0000e-01 + 1.6000e+00i
```

Multiplying the second row of Ainv times the right-most vector of our equations produces

$$\mathbf{V_{AB}} = (0.4 - j0.8)\mathbf{I_s} + (0.4 - 0.8j)$$

This implies that Z_{th} = 0.4 – j0.8 Ω and $\mathbf{V_{oc}}$ = 0.4–j0.8 V. For the Norton equivalent we need

$$\mathbf{I_{sc}} = \frac{\mathbf{V_{oc}}}{Z_{th}} = 1 \text{ A}$$

SOLUTION 10.66. (a) $\omega = 1000$ rad/s and $\mathbf{V_S} = 20 \quad 45°$ V. By voltage division:

$$\mathbf{V_{out}} = \frac{1}{1+j} 20 \quad 45° = 14.14 \quad 0° \text{ V} \qquad \mathbf{I_L} = 14.14 \quad 0° \text{ A}$$

(b) At dc, $\mathbf{V_{out}} = \mathbf{V_{in}}$. We want the frequency at which $\mathbf{V_{out}} = 0.1V_{in}$. Thus, we want:

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1}{1 + j\omega \times 0.001}\right| = 0.1 \qquad \frac{1}{1+\omega^2 \times 10^{-6}} = \frac{1}{100} \qquad \omega = 9950 \text{ rad/s}$$

SOLUTION 10.67. $Z = 25 - j20\ \Omega$. By KCL:

$$\frac{\mathbf{V_Z} - 14\angle 0°}{j15} + \frac{\mathbf{V_Z}}{25 - j20} + \frac{\mathbf{V_Z} + 8\angle 90°}{-j20} = 0$$

This equation simply needs to be manipulated in order to obtain:

$$\mathbf{V_Z} = 41.35\angle -73.46°$$

SOLUTION 10.68. In this problem, we note that the impedance, jX, is in series with the parallel RLC circuit to the right. Thus, all we need to do is to find an expression for the equivalent impedance of the parallel RLC circuit:

$$Y_{RLC} = \frac{1}{30} - \frac{j}{20} + j0.025 \qquad Z_{RLC} = 19.2 + j14.4$$

Now, the total impedance seen by the source is 19.2 + j14.4 +jX. Therefore, for this to be real, the unknown reactance has to be –14.4 Ω. Also, the input current now is **I** = 96/19.2 = 5 A. Hence

$i(t) = 5\cos(10t)$ A.

SOLUTION 10.69. First, compute the current through the series RC section: $\mathbf{I_C} = \frac{1}{1-j}$ A.

Now, by KCL, we can write

$$0 = \mathbf{I_C} - \mathbf{I_x} + \frac{1 - 2\mathbf{I_x}}{j} = \frac{1}{1-j} - \mathbf{I_x} + \frac{1-2\mathbf{I_x}}{j} \qquad \mathbf{I_x} = 0.3 + j0.1 = 0.316\angle 18.44^o \text{ A}$$

SOLUTION 10.70. $\omega = 20$ rad/s and $Y_{in} = 0.05 + j0.0866$ S.

(a) Since $Y_{in} = \frac{1}{R} - \frac{j}{\omega L} + j\omega C$, equating the real parts of the above two expressions implies that R = 1/0.05 = 20 Ω.

(b) Similarly, equating the imaginary parts and substituting, we obtain:

$$j\left(\omega C - \frac{1}{\omega L}\right) = j0.0866 \qquad L = 3.73\ \text{H}$$

(c)

$$\mathbf{V_C} = \frac{\mathbf{I_{in}}}{Y_{in}} = \frac{20\angle 30°}{0.05 + j0.0866} = 200\angle -30°\ \text{V} \qquad v_C(t) = 200\cos\left(20t - 30^o\right)\ \text{V}$$

(d)

$$\mathbf{I_L} = \frac{200\angle 30°}{j20 \times 3.73} = 2.68\angle -120°\,\text{A} \qquad i_L(t) = 2.68\cos\left(20t - 120^o\right)\ \text{A}$$

(e) $\mathbf{V_{oc}}$ is just the voltage $\mathbf{V_C}$, which was obtained in (c), and $Z_{th} = \frac{1}{Y_{in}} = 10\angle -60°\ \Omega$.

(f) Z_{th} = 5 – j8.66 Ω. This is equivalent to a series combination of a 5 Ω resistance and a 5.77 mF capacitance at the given frequency $\omega = 20$ rad/s.

SOLUTION 10.71. (a) $\mathbf{V_s} = 2 + j0\text{V},\ \omega = 1000$ rad/s. By KCL

$$\frac{\mathbf{V_A}}{-j1.33} + \frac{\mathbf{V_A}}{2 + j2} + \frac{\mathbf{V_A} - \mathbf{V_s}}{4} = 0 \qquad \mathbf{V_A} = 0.5 - j0.5 = 0.707\angle -45°\ \text{V}$$

The time-domain expression is:

$$v_C(t) = v_A(t) = 0.707\cos\left(1000t - 45^o\right)\ \text{V}$$

(b) $\mathbf{I_L} = \frac{0.707\angle -45°}{2 + j2} = 0.25\angle -90°$ A implies that $i_L(t) = 0.25\cos\left(1000t - 90^o\right) = 0.25\sin(1000t)$ A.

(c) We already determined V_{oc} in part (a). Now, turn off the source to compute the equivalent impedance:

$Y_{th} = 0.75j + \frac{1}{2 + j2} + 0.25 = 0.5 + j0.5 \qquad Z_{th} = 1 - j\ \Omega$. This is the series connection of a 1 Ω resistor with a capacitor of value $C = \frac{1}{\omega} = 10^{-3}$ F. This completes the definition of the Thevenin equivalent.

SOLUTION 10.72. In MATLAB,

```
»R = 5; L = 1e-3; C = 20e-6;Vs1 = 5; Is2 = 0.5*j;
»G = 1/R; w = 10e3;
»YL = 1/(j*w*L)
YL =
        0 - 1.0000e-01i
```

```
»YC = j*w*C
YC =
        0 + 2.0000e-01i
```

Hence

$$G(\mathbf{V_C} - \mathbf{V_{s1}}) + Y_C \mathbf{V_C} + Y_L(\mathbf{V_C} - \mathbf{V_{s1}} - 2\mathbf{V_R}) = \mathbf{I_{s2}}$$

Substituting for V_R,

$$G(\mathbf{V_C} - \mathbf{V_{s1}}) + Y_C \mathbf{V_C} + Y_L(\mathbf{V_C} - \mathbf{V_{s1}} - 2(\mathbf{V_{s1}} - \mathbf{V_C})) = \mathbf{I_{s2}}$$

Therefore

$$\left(G + Y_C + 3Y_L\right)\mathbf{V_C} = \left(G + 3Y_L\right)\mathbf{V_{s1}} + \mathbf{I_{s2}}$$

Again, using MATLAB,

```
»a =  G + YC + 3*YL
a =
  2.0000e-01 - 1.0000e-01i
»b = (G+3*YL)*Vs1 + Is2
b =
  1.0000e+00 - 1.0000e+00i
»VC = b/a
VC =
  6.0000e+00 - 2.0000e+00i
»abs(VC)
ans =
   6.3246e+00
»angle(VC)*180/pi
ans =
  -1.8435e+01
```

Hence, $v_C(t) = 6.325\cos(10^4 t - 18.44^o)$ V.

SOLUTION 10.73. Denote by v_{C1}, the node voltage of the 2.5 mF capacitor . Note that at $\omega = 800$ rad/s, $\mathbf{V_s} = 20 \quad 0^o$ V. From this we write a set of nodal equations by inspection after observing the following from MATLAB:

```
»w = 800;
»C1 = 2.5e-3;
»L = 1.25e-3;
»Y1 = 0.5+j*w*C1-j/(L*w)
Y1 =  5.0000e-01 + 1.0000e+00i

»Yoff=j/(L*w)
Yoff =           0 + 1.0000e+00i

»Y2 = 0.25+j*w*3.75e-3 - j*w*1.25e-3
Y2 =  2.5000e-01 + 2.0000e+00i
```

This information leads to the following matrix nodal equation:

$$\begin{bmatrix} 0.5 + j & j \\ j & 0.25 + j2 \end{bmatrix} \begin{bmatrix} \mathbf{V_{C1}} \\ \mathbf{V_{out}} \end{bmatrix} = \begin{bmatrix} 0.5\mathbf{V_s} \\ 0.25\mathbf{V_s} \end{bmatrix}$$

To solve these equations we again use MATLAB:

```
»A = [Y1, Yoff;Yoff, Y2]
A =
  5.0000e-01 + 1.0000e+00i           0 + 1.0000e+00i
           0 + 1.0000e+00i  2.5000e-01 + 2.0000e+00i

»b = [0.5*20;0.25*20]
b =
    10
     5

»Vnodes = inv(A)*b
Vnodes =
  7.1141e+00 - 6.9799e+00i
 -3.6242e+00 + 5.3691e-01i

»magVnodes = abs(Vnodes)
magVnodes =
   9.9664e+00
```

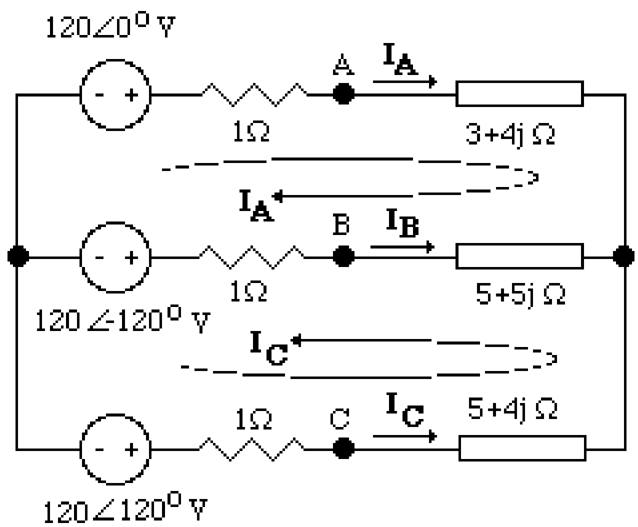
3.6637e+00

»angVnodes = angle(Vnodes)*180/pi
angVnodes =
-4.4454e+01
1.7157e+02

Therefore

$$v_{out}(t) = 3.664\cos(800t + 171.57^{o})\ \text{V}$$

SOLUTION 10.74. For this problem we use loop analysis with loops indicated in the figure below.



Since there are no controlled sources, we can write down the loop equations by inspection:

$$\begin{bmatrix} 10+9j & 6+5j \\ 6+5j & 12+9j \end{bmatrix}\begin{bmatrix} \mathbf{I_A} \\ \mathbf{I_C} \end{bmatrix} = \begin{bmatrix} 120 - 120\angle -120^{o} \\ 120\angle 120^{o} - 120\angle -120^{o} \end{bmatrix} = \begin{bmatrix} 180 + 103.92 \\ j207.85 \end{bmatrix}$$

The solution of this equation is done in MATLAB as follows:
»b1=120-120*exp(-j*2*pi/3)
b1 = 1.8000e+02 + 1.0392e+02i

»b2=120*exp(j*2*pi/3)-120*exp(-j*2*pi/3)
b2 = 0 + 2.0785e+02i

```
»A = [10+j*9, 6+j*5;6+j*5,12+j*9]
A =
  1.0000e+01 + 9.0000e+00i  6.0000e+00 + 5.0000e+00i
  6.0000e+00 + 5.0000e+00i  1.2000e+01 + 9.0000e+00i

»I=inv(A)*[b1;b2]
I =
  1.4472e+01 - 1.3469e+01i
  4.2926e-01 + 1.7703e+01i

»%  Please note that using the commands I=inv(A)*[b1,b2]'
»%  will lead to the wrong answer because a conjugate is
»%  inserted along with the transpose.

»magI = abs(I)
magI =
   1.9770e+01
   1.7708e+01

»angleI = angle(I)*180/pi
angleI =
  -4.2944e+01
   8.8611e+01

»IB = -I(1)-I(2)
IB =  1.8288e+00 + 1.7234e+01i

»magIB = abs(IB)
magIB =   1.7331e+01

»angleIB = angle(IB)*180/pi
angleIB =   8.3943e+01
```

Changing the sign on each source amounts to multiplying its value by "–1". This means that all magnitudes remain the same, but there is a 180^o phase shift for each current, i.e., add 180^o to each current angle.

SOLUTION P10.75. For this problem we have both a transient component to the response and a steady state component. The steady state component is computed in the usual way because the circuit is stable, i.e., the time constant is positive. Once the steady state part is computed, we use initial conditions to obtain the coefficient B in the response.

Part 1: Compute steady state response. For this we use MATLAB,

```
»R = 0.5;  L = 0.866;
»Vs = 10;
»w = 1;
»Zin = R + j*w*L
Zin =  5.0000e-01 + 8.6600e-01i
»IL = Vs/Zin
IL =  5.0002e+00 - 8.6604e+00i
»magIL = abs(IL)
magIL =   1.0000e+01
»angIL = angle(IL)*180/pi
angIL =  -5.9999e+01
```

Hence

$$i_L(t) = 10\cos(t - 60^o) + Be^{-0.577t} \text{ A}$$

Part 2: From the initial conditions we have

$$i_L(0) = 1 = \left[10\cos(t - 60^o) + Be^{-0.577t}\right]_{t=0} = 10\cos(-60^o) + B = 5 + B$$

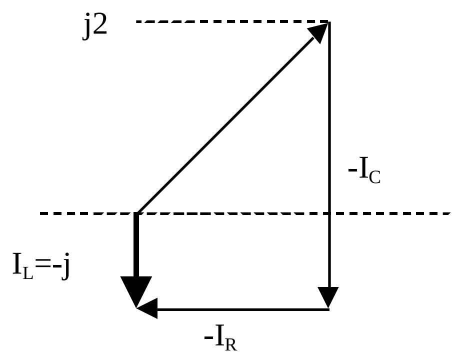
Hence B = –4. It follows that

$$i_L(t) = 10\cos(t - 60^o) - 4e^{-0.577t} \text{ A}$$

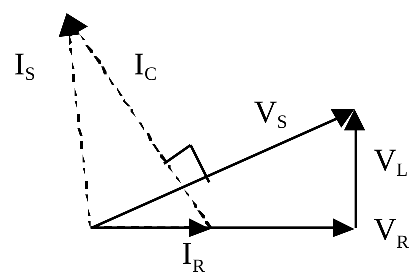
SOLUTION 10.76. KCL dictates that:

$$\mathbf{I_L} + \mathbf{I_R} + \mathbf{I_C} = \mathbf{I_S} \qquad \mathbf{I_L} = \mathbf{I_S} - \mathbf{I_R} - \mathbf{I_C}$$

We can perform this sum graphically as follows:

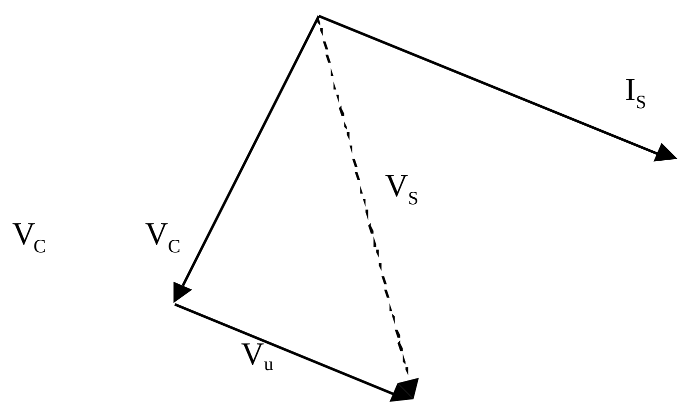


SOLUTION 10.77. (a) Note that $\mathbf{V_S} = \mathbf{V_R} + \mathbf{V_L}$, but that $\mathbf{V_L}$ leads $\mathbf{V_R}$ by 90 degrees. Similarly, $\mathbf{I_S} = \mathbf{I_R} + \mathbf{I_C}$, but $\mathbf{I_C}$ leads $\mathbf{V_S}$ by 90 degrees. Also note that the inductor current is also $\mathbf{I_R}$, and the capacitor voltage is $\mathbf{V_S}$.



(b) Using graph paper to construct the phasor diagram to scale, we find the difference between the phase angles of $\mathbf{I_s}$ and $\mathbf{V_s}$ is zero.

Solution 10.78. First note that $\mathbf{V_C}$, the capacitor voltage, will lag $\mathbf{I_S}$ by 90 degrees. Now, $\mathbf{V_C}$ plus the unknown element voltage should result in a vector that runs diagonally between the two vectors. From the following illustration, it follows that the unknown voltage should have the same phase as the input current:

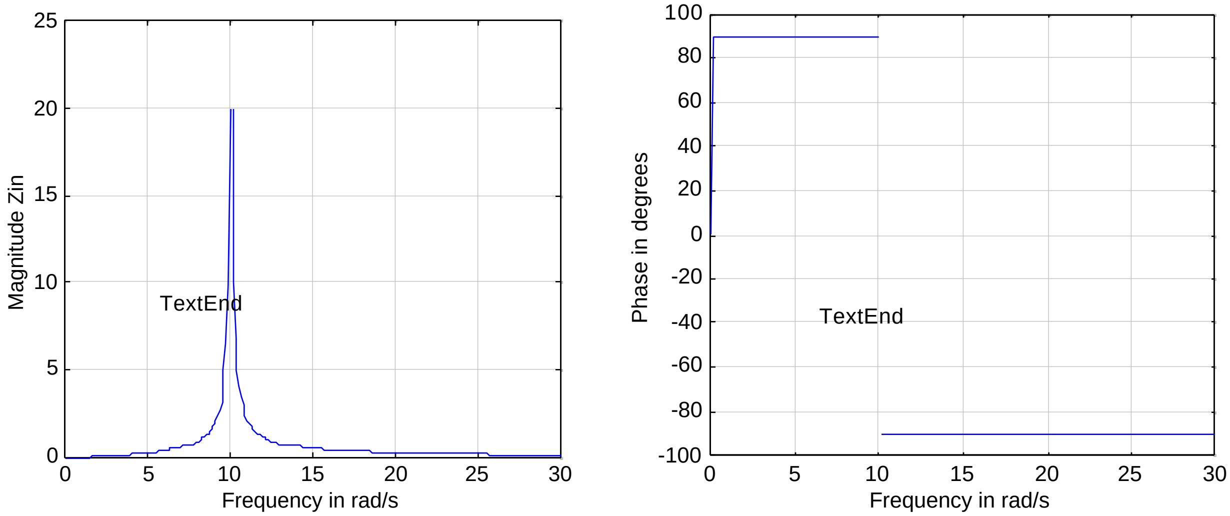


This means that the unknown element is a resistor. The 45o phase difference implies that $|\mathbf{V_C}| = |\mathbf{V_u}|$ or $\frac{I_s}{\omega C} = I_s R$. Therefore $R = \frac{1}{\omega C} = \frac{1}{10^3 \times 10^{-6}} = 1000\ \Omega$.

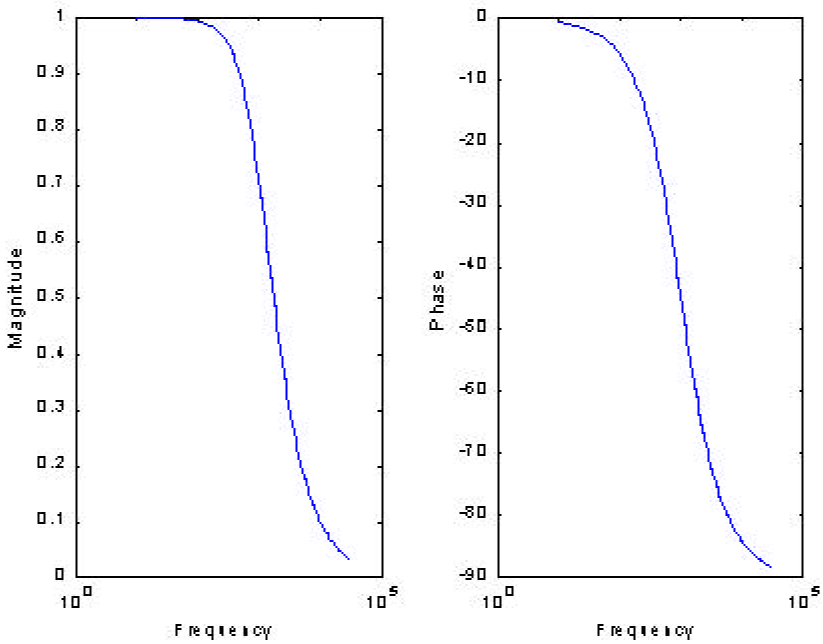
SOLUTION 10.79. The student can construct the phasor diagram using graph paper. The diagram is going to look like that in the problem statement, except that the proper lengths and angles will be used.

SOLUTION 10.80. As the frequency approaches infinity, the capacitor shorts and the inductor opens. So, the output voltage is zero. As the frequency approaches zero, the capacitor opens, but the inductor shorts, so the output is also zero. A plot of the complete response is shown below. (Note that the magnitude response at 10rad/s is infinite):

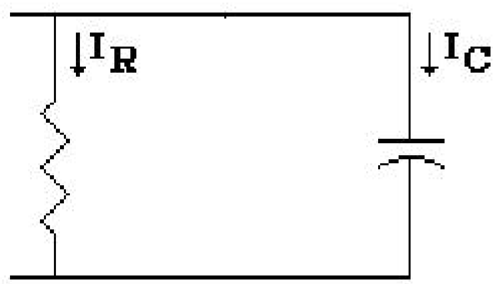
```
»L = 0.04; C = 0.25;
»w = 0: 30/300:30;
»%  Vout = Zin * Iin
»Zin = j*w*L ./(j*w*L*j.*w*C + 1);
»plot(w, abs(Zin))
»grid
»ylabel('Magnitude Zin')
»xlabel('Frequency in rad/s')
»plot(w,angle(Zin)*180/pi)
»grid
»xlabel('Phase in degrees')
»ylabel('Phase in degrees')
»xlabel('Frequency in rad/s')
```



SOLUTION 10.81. At infinite frequency, the resistor current is zero (because the inductor opens). So, the output voltage is zero. At DC, the inductor is short, and the output voltage is equal to the input voltage. The plot of the frequency response is shown below (a logarithmic x-axis is used):



SOLUTION 10.82. The circuit inside the black box is



At DC, the capacitor is an open circuit. Thus, the voltage across the resistor is 1mA×R. But we know that this voltage is 1 from the graph. This means that R = 1 kΩ. Now, in general for the above diagram:

$$\frac{\mathbf{V}}{\mathbf{I}} = \frac{R}{1 + j\omega RC}$$

The magnitude of this function is R/sqrt(2) when $\omega = 1/RC$. Substituting the frequency from the graph (1000 rad/s), we get C = 1 μF.

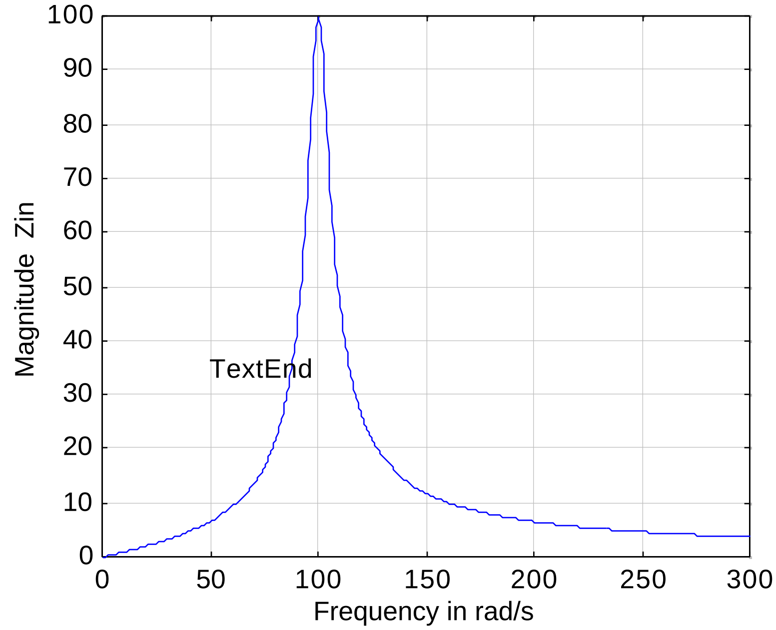
Solution 10.83. The admittance of the parallel RLC circuit is:

$$Y_{in} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$Z_{in} = \frac{1}{Y_{in}} = \frac{\mathbf{V_s}}{\mathbf{I_s}}$$

The function we want to find the frequency response for is nothing but the input admittance of the circuit. Using MATLAB, the following plot can be obtained:

```
»R = 100; L = 0.1; C = 1e-3;
»w = 0:0.5:300;
»w = 0.01:0.5:300;
»Yin = 1/R + 1. ./(j*w*L) + j*w*C;
»Zin = 1 ./Yin;
»plot(w,abs(Zin))
»grid
»xlabel('Frequency in rad/s')
»ylabel('Magnitude  Zin')
```

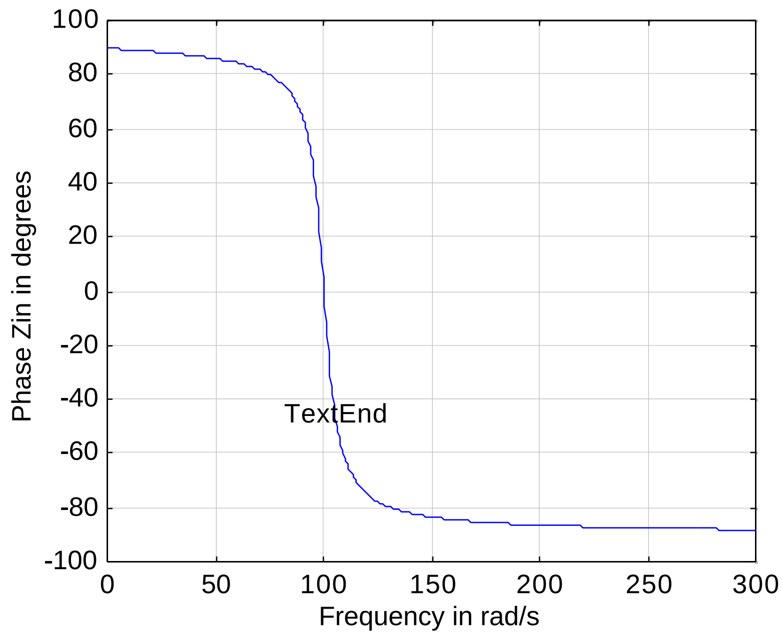


»plot(w,angle(Zin)*180/pi)

»grid

»xlabel('Frequency in rad/s')

»ylabel('Phase Zin in degrees')



SOLUTION 10.84. The circuit inside the box is a series RLC circuit. It cannot be a parallel RLC, because as per problem 83, the admittance of a parallel RLC does not approach zero as w approaches infinity. Thus,

$$\frac{\mathbf{I}}{\mathbf{V}} = Y_{in} = \frac{1}{R + \frac{1}{j\omega C} + j\omega L}$$

The resonance frequency is 50 rad/s and is determined by 1/sqrt(LC). Given L = 0.4 H, C = 1 mF.

To obtain R, we make use of the fact that, from the given graph at $\omega = 57$ rad/s, the current magnitude is approximately 0.2 times the peak magnitude. Therefore

$$\frac{1}{\sqrt{R^2 + \left(57L - \frac{1}{57C}\right)^2}} \quad \frac{0.2}{\sqrt{R^2 + \left(50L - \frac{1}{50C}\right)^2}} = \frac{0.2}{R}$$

Hence

$$R^2 + \left(57 \times 0.4 - \frac{1}{57 \times 10^{-3}}\right)^2 = \frac{R^2}{0.04} = 25R^2$$

$$R^2 + \left(57x0.4 - \frac{1}{57x0.001}\right)^2 \qquad \frac{R^2}{0.04} = 25R^2$$

From which $R = \frac{5.2561}{\sqrt{24}} = 1.0728\ \Omega$.

SOLUTION 10.85. Create three mesh currents in the three planar loops. All currents are clockwise: I_1 in the voltage source loop, I_2 in the top bridge loop, and I_3 in the bottom one. The three mesh equations are:

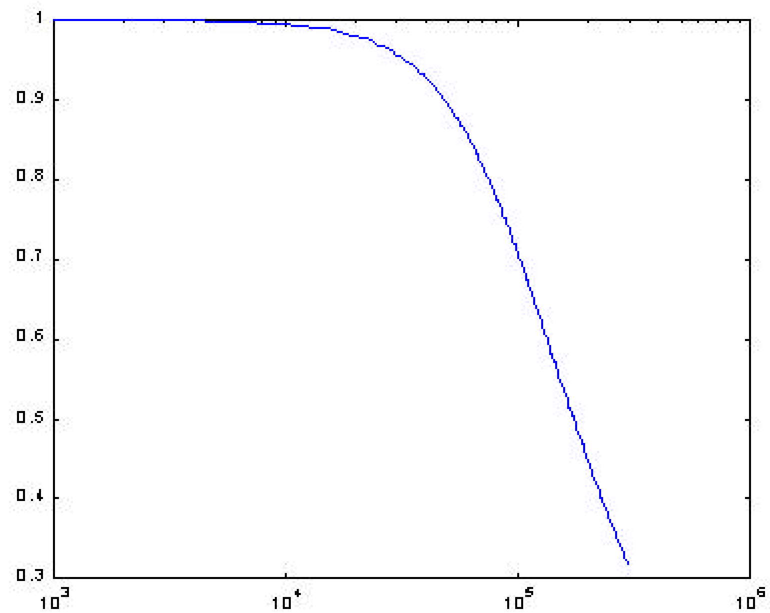
$$\mathbf{V} - \mathbf{I_1}R_1 - R_2(\mathbf{I_1} - \mathbf{I_2}) - R_3(\mathbf{I_1} - \mathbf{I_3}) = 0$$

$$R_2(\mathbf{I_2} - \mathbf{I_1}) + \frac{1}{j\omega C_1}\mathbf{I_2} + R_{meter}(\mathbf{I_2} - \mathbf{I_3}) = 0$$

$$R_3(\mathbf{I_3} - \mathbf{I_1}) + R_{meter}(\mathbf{I_3} - \mathbf{I_2}) + \frac{1}{j\omega C_2}\mathbf{I_3} = 0$$

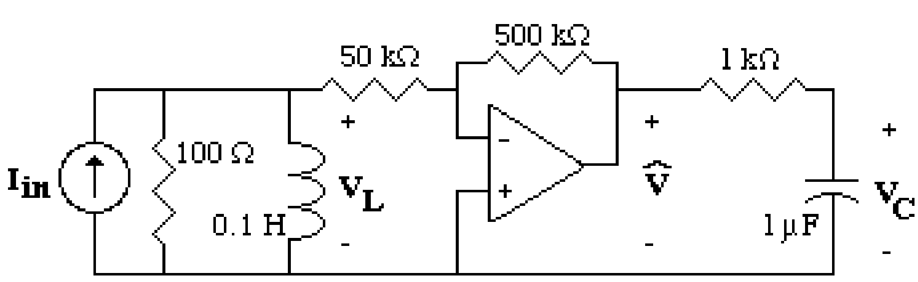
The plots of the magnitude and phase of $\mathbf{V_B} - \mathbf{V_C} = R_{meter}(\mathbf{I_3} - \mathbf{I_2})$ are shown in the text.

SOLUTION 10.86. We can see all ranges by plotting on a logarithmic scale:



Note that the output will decay when we start to reach the bandwidth of the op-amp. In other words, the inverting amplifier says that the output is –1 times the input (provided the op amp works properly). Once the op amp's gain starts dropping, the output voltage also decays with it.

SOLUTION 10.87. **Correction**: Change the 0.01 μF capacitor to 1 μF. **(a)** For this part consider the diagram below,



From the problem statement, $\omega = 320\pi$ rad/s, and $I_{in} = 0.01 \quad 0^o$ A. Observe that the 50 kΩ resistor input to the inverting op amp terminal is in parallel with the 100 Ω resistor because of the virtual ground at the op amp terminals. However, for all practical purposes, this has no effect on the 100 Ω resistor, hence from Ohm's law

$$\mathbf{V_L} = \frac{100 \times j0.1\omega}{100 + j0.1\omega}\mathbf{I_{in}} = (50.265 + j50) \times 0.01 = 0.50265 + j0.5$$

From the inverting op amp configuration,

$$\hat{\mathbf{V}} = \mathbf{-10V_L} = -5.0265 - j5$$

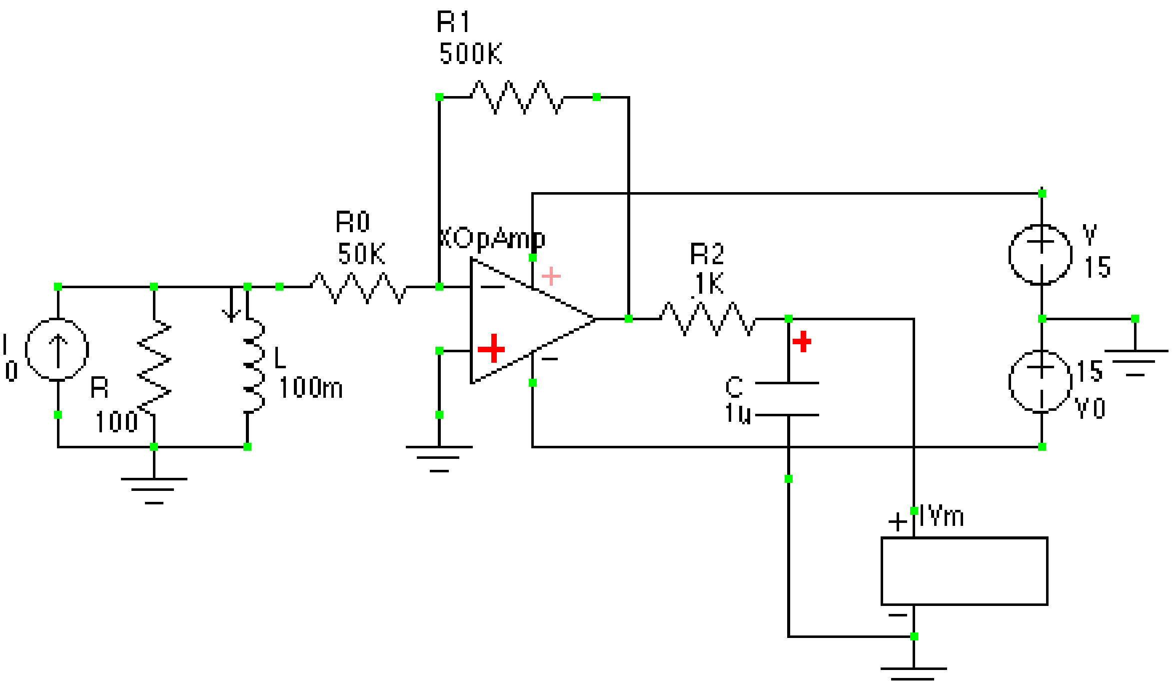
From voltage division,

$$\mathbf{V_C} = \frac{10^{-3}}{10^{-3} + j\omega 10^{-9}}\hat{\mathbf{V}} = (0.49735 - j0.5) \times (-5.0265 - j5) = 5\angle 179.7^o \text{ V}$$

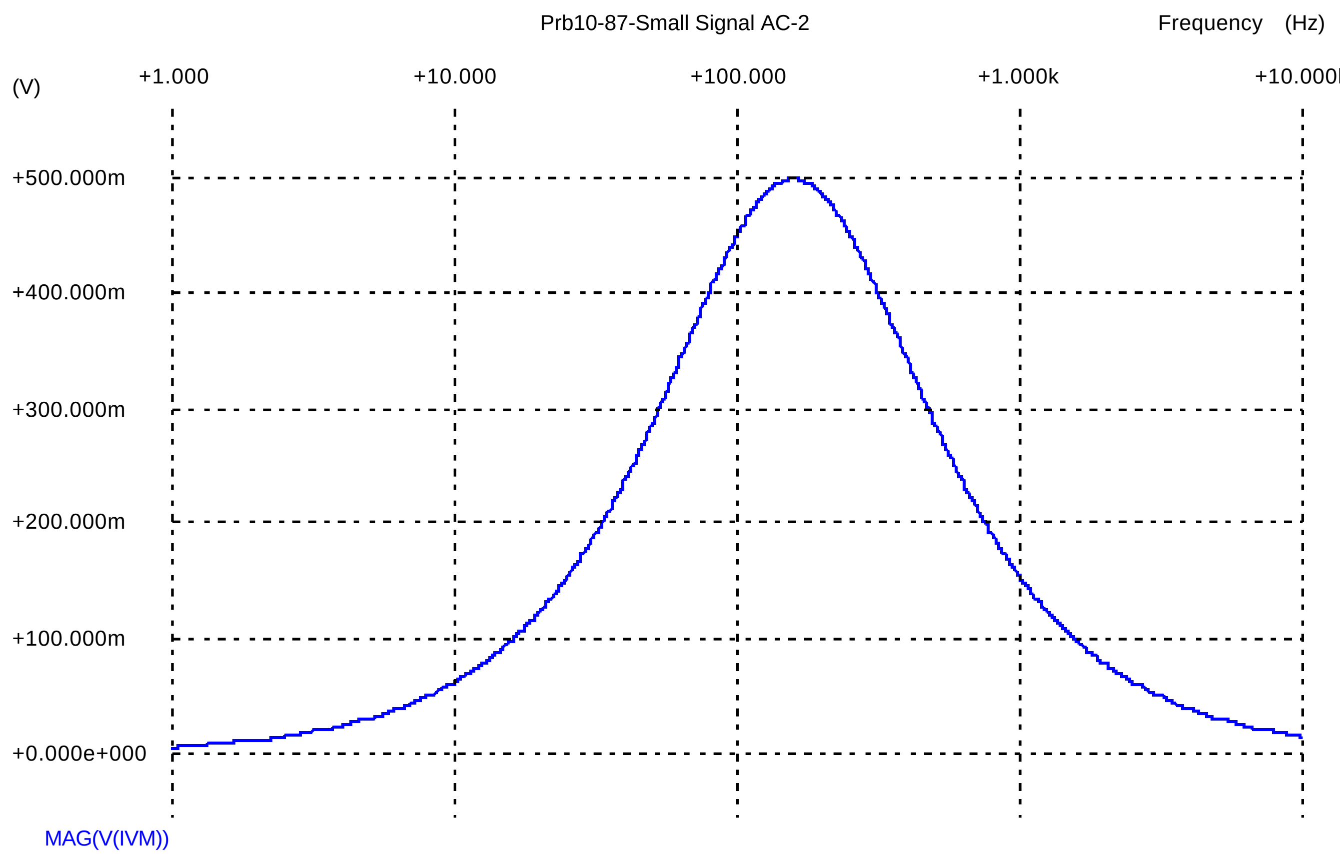
Therefore

$$v_C(t) = 5\cos(320\pi t + 179.7^o)\text{ V}$$

Parts (b) and (c). For the SPICE simulation we have the following circuit in B^2-SPICE:



which leads to the response below



The magnitude at 160 Hz is 0.499 for a 1 mA current input. Thus a 10 mA input current should lead to 4.99 V by linearity which approximates the 5 V computed analytically in part (a). Hence with a 15 V saturation limit, the input magnitude may increase by a factor of 3 to 30 mA.

SOLUTION 10.88. **(a)** $\omega = 400$ rad/s and $\mathbf{V_{in}} = 10^{-3} \quad -90°$ V. By the virtual short property:

$$\mathbf{I_{in}} = \frac{\mathbf{V_{in}}}{1/\,j\omega C} = j\omega C\mathbf{V_{in}}$$

All this current flows through the 1 MΩ resistor: $\mathbf{V_{in}} = -j\omega C10^{6}\,\mathbf{V_{in}} = 0.4 \quad -180°$. Thus,

$$v_{out}(t) = -\,0.4\cos(400t)\text{ V}$$

(b) $\omega = 200$ rad/s, $\mathbf{V_{in}} = 10^{-3} \quad 0°$ V, and $\mathbf{V_{out}} = 10^{-3} \quad -90°$ V. Again,

$$\mathbf{V_{out}} = -j10^{-3} = -j\omega C \times 4 \times 10^{5}\mathbf{V_{in}} = -j80 \times 10^{3}C \qquad C = 12.5\text{ nF}$$

SOLUTION 10.89. **(a)** $\omega = 800\text{rad/s}$ and $\mathbf{V_{in}} = 1\ \angle -90° \text{V}$ $\quad \mathbf{I}_{in} = \frac{\mathbf{V_{in}}}{10^6}$. No current flows into Op-Amp terminals:

$$\mathbf{V_{out}} = -\mathbf{I_{in}}\frac{1}{j\omega C} = \frac{j\times(-j)}{800} = 1.25\times10^{-3}\text{ V}$$

Thus, $v_{out}(t) = 1.25\cos(800t)$ mV.

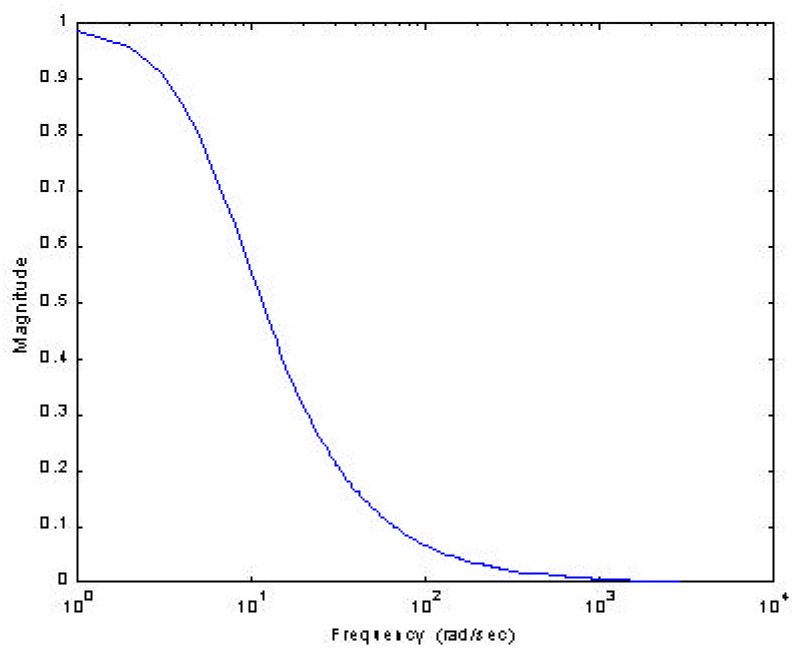
(b) Again, $\mathbf{I_{in}} = \frac{\mathbf{V_{in}}}{200\times10^3}$ A, and $\mathbf{V_{out}} = -\mathbf{I_{in}}\frac{1}{j\omega C}$ $\quad 10 = \frac{j\times(-j)}{\omega C 2\times10^5}$ $\quad C = 2.5\text{ nF}$.

Solution 10.90 (a) $\omega = 2\pi700\text{rad/s}$ and $\mathbf{V_{in}} = 1\ \angle 0°\text{ V}$ $\quad \mathbf{I_{in}} = \frac{1}{150\times10^3}$ A. Further,

$$\mathbf{V_{out}} = -\frac{150\times10^3\frac{-j}{\omega C}}{150\times10^3 - \frac{j}{\omega C}}\mathbf{I_{in}} = \frac{-1}{150\times10^3}\times\frac{150\times10^3\frac{-j}{\omega C}}{150\times10^3 - \frac{j}{\omega C}} = 0.0015\ \angle 90^o\text{ V}$$

Thus, $v_{out}(t) = 1.5\cos(2\pi700t + 90^o)$ mV.

(b)



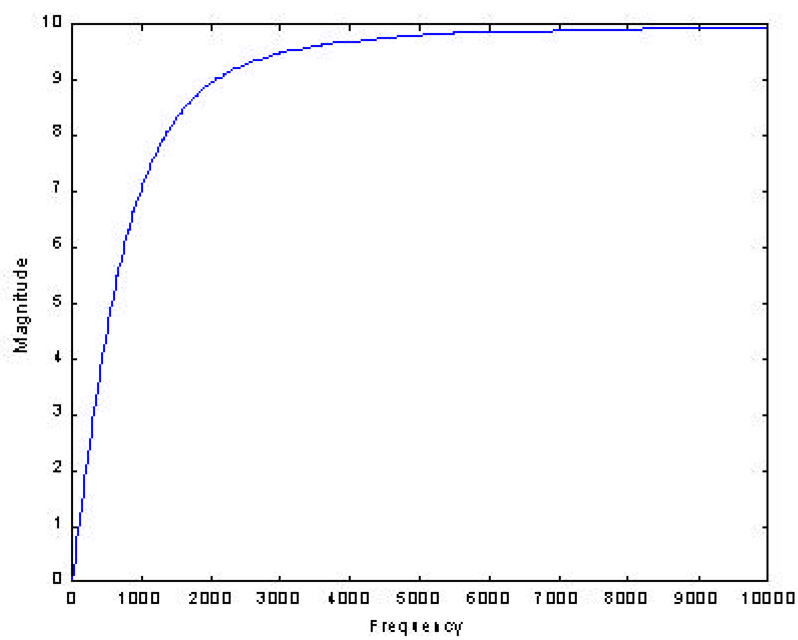
(c) The output lags the DC response by 45 degrees (note that at DC, the amplifier is inverting, or has a phase of –180 degrees). Now, the frequency response is really determined by the RC circuit in the feedback path of the op-amp. The first resistance at the input simply converts the input into a current that drives this RC circuit. It can be shown that a 45 degree phase shift occurs in an RC circuit when the frequency is $1/R_fC$ (directly from the results of an analysis on an RC circuit). So,

$$R_fC = 1/2000\pi \qquad C = 0.016 \text{ nF} \ \ (\text{We know } R_f = 10 \text{ M}\Omega.)$$

Also, at this frequency, the response is 0.707×DC response. The DC response is $-10^7/R$. So, the DC gain is 14.14. Thus, R = 7.07 kΩ.

Solution 10.91 (a) Use the virtual short property: $\mathbf{I_{in}} = \dfrac{\mathbf{V_{in}}}{10^3 + \frac{1}{j\omega C}}$. All of current flows through the feedback path: $\mathbf{V_{out}} = -10^4\mathbf{I_{in}} = \dfrac{10^4\,\mathbf{V_{in}}}{10^3 - \frac{j}{\omega C}}$ $\qquad \dfrac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = 7.07 \quad -135^o$.

(b)

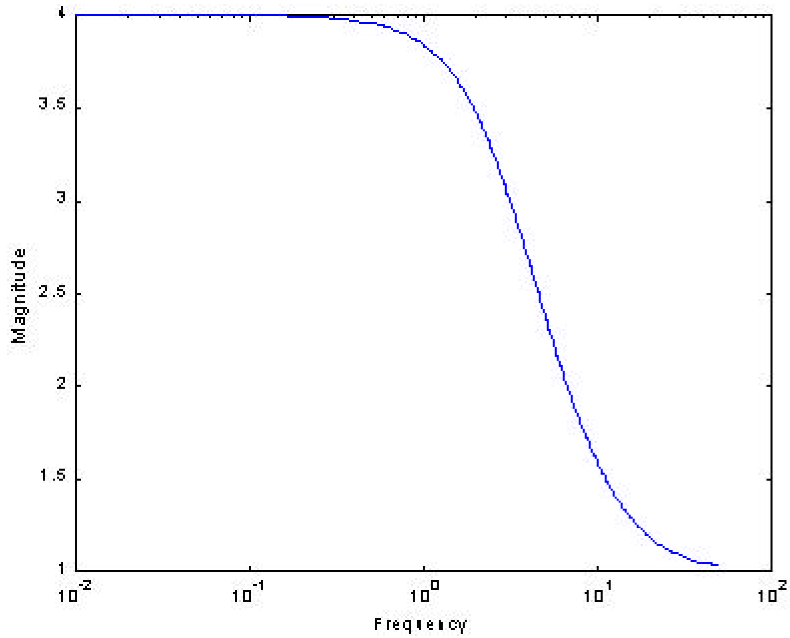


(c) The spice result looks the same at low frequencies. However, at high frequencies, the response falls back to zero as the op-amp non-ideal frequency response starts to affect the behavior of the circuit.

SOLUTION 10.92. (a) The negative terminal of the op-amp is at $\mathbf{V}_s$. This implies $\mathbf{I_R} = \dfrac{\mathbf{V_S}}{R}$. By KVL,

$$\mathbf{V_S} + Z_{RC}\mathbf{I_R} = \mathbf{V_o} \qquad \frac{\mathbf{V_o}}{\mathbf{V_S}} = 1 + \frac{1}{R}\frac{\frac{-j3R}{\omega C}}{3R - \frac{j}{\omega C}} = 1 + \frac{3}{j3\omega RC + 1}$$

The MATLAB plot for the given values is:



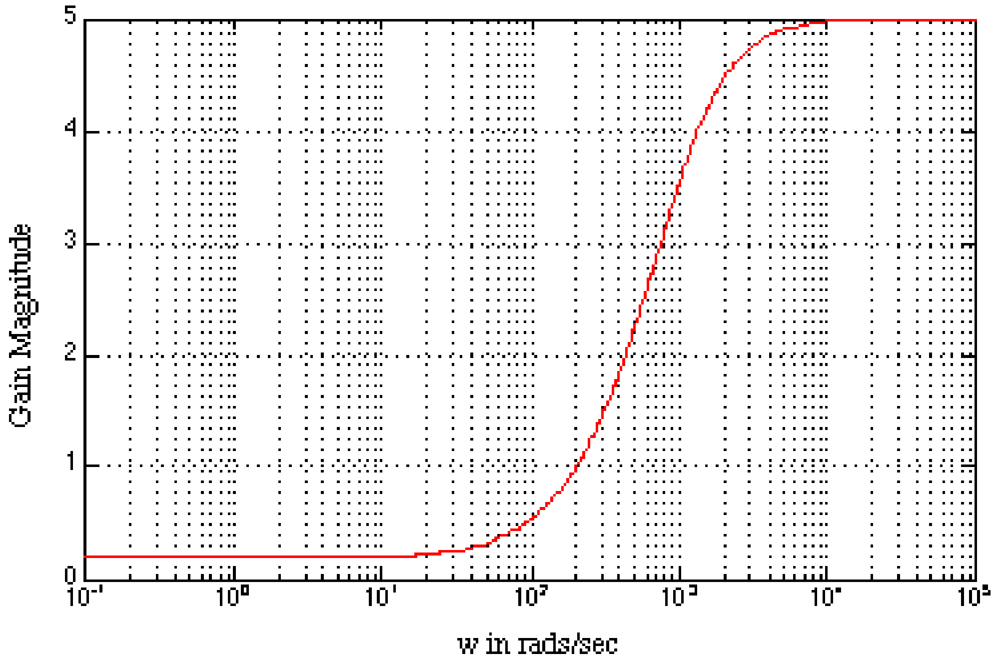
(c) The spice result looks pretty much the same, especially since the cut-off frequency of this circuit is much lower than the frequency at which the op-amp ceases to operate as an ideal op-amp.

SOLUTION 10.93. To compute the gain as a function of ω we observe that by the properties of an ideal op amp,

$$\text{Gain} = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = -\frac{2\times10^{-5} + j0.5\times10^{-6}\omega}{10^{-4} + j0.1\times10^{-6}\omega} = \frac{20 + j0.5\omega}{100 + j0.1\omega}$$

In MATLAB

```
»G1 = 1/50e3;
»G2 = 1/10e3;
»C1 = 0.5e-6;
»C2 = 0.1e-6;
»w = logspace(-1,5,1500);
»Y1 = G1 +j*w*C1;
»Y2 = G2 + j*w*C2;
»H = Y1 ./ Y2;
»semilogx(w,abs(H))
»grid
```



SOLUTION 10.94. **(a)** The equations are:

$$\frac{\mathbf{V_1} - \mathbf{V_{in}}}{1000} + \frac{\mathbf{V_1} - \mathbf{V_2}}{1000} + \frac{\mathbf{V_1} - \mathbf{V_2}}{1/\ jwC_f} = 0$$

$$\frac{\mathbf{V_2} - \mathbf{V_1}}{1000} + \frac{\mathbf{V_2}}{1/\ j\omega C_2} = 0$$

$$\mathbf{V_2} = \mathbf{V_{out}}$$

Substituting values and solving for $\mathbf{V_1}$ and $\mathbf{V_2}$,

$$\mathbf{V_1} = 0.8535 \quad -82^o \mathbf{V_{in}}$$

$$\mathbf{V_2} = 0.723 \quad -114.7^o \mathbf{V_{in}}$$

The second one is the relation that we are looking for.

(b) At 100 Hz,

$$\mathbf{V_1} = 1.00 \quad -3.68^o \mathbf{V_{in}}$$

$$\mathbf{V_2} = 1.00 \quad -7.3^o \mathbf{V_{in}}$$

At 3000 Hz,

$$\mathbf{V_1} = 0.15 \quad -102^o \mathbf{V_{in}}$$

$$\mathbf{V_2} = 0.0728 \quad -164^o \mathbf{V_{in}}$$

(c) The response is that of a low-pass filter, as predicted from the results of part (b) above.

SOLUTION 10.95. (a) The two nodal equations:

$$\frac{\mathbf{V_x} - \mathbf{V_{in}}}{200} + j\omega C_1 \mathbf{V_x} + j\omega C_2 \left(\mathbf{V_x} - \mathbf{V_{out}}\right) = 0$$

$$j\omega C_2 \left(\mathbf{0} - \mathbf{V_x}\right) - \frac{\mathbf{V_{out}}}{28 \times 10^3} = 0$$

In MATLAB,

```
»w =  2*pi*1.34e3;  C = 0.05e-6; R1 = 200; R2 = 28e3;
»A = [1/R1+j*w*C+j*w*C  -j*w*C;-j*w*C  -1/R2]
A =
  5.0000e-03 + 8.4195e-04i           0 - 4.2097e-04i
           0 - 4.2097e-04i -3.5714e-05
»b = [1/R1; 0];
»V = A\b
V =
  2.6664e-01 - 5.9266e+00i
 -6.9859e+01 - 3.1429e+00i
»Vout = V(2)
Vout =
 -6.9859e+01 - 3.1429e+00i
»abs(Vout)
ans =
   6.9929e+01
»angle(Vout)*180/pi
ans =
  -1.7742e+02
```

Hence, $\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = 69.93 \quad -177.4^{o}$.

When the capacitors are shorts, the output is shorted to the virtual ground input, at 0 V. Similarly, when they are opens, the virtual ground makes sure that v_{out} is zero, since there is no drop across the feedback resistor.

(b) The band-pass response can be computed using any SPICE program.

SOLUTION 10.96. First, note the input-output relationship:

$$\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = -\frac{Z_f}{R_1}$$

where Z_f is the impedance of the parallel RLC circuit. We have studied this circuit extensively earlier, and we have shown that at "resonance", the impedance of this circuit is going to be real and equal to the value of resistance, in this case R_2. That's exactly the requirement of this problem, since we want $\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = -\frac{Z_f}{R_1}$ to simply be equal to R_2/R_1. It remains to note that this resonance occurs at a frequency $\omega = 1/\text{sqrt}(LC)$.

SOLUTION 10.97. (a) First, analyze the feedback amplifier circuit. The output of this op-amp circuit is:

$$\mathbf{V_{op2}} = -\frac{1/\,j\omega C}{2\times 10^6}\mathbf{V_{out}}$$

Also, by voltage division, the voltage at the resistive voltage divider (+ terminal of first op-amp):

$$\mathbf{V_{RR}} = \frac{2\times 10^3}{2\times 10^5}\times -\frac{1}{j\omega C_2 2\times 10^6}\mathbf{V_{out}} = \frac{-0.01}{j\omega C_2 2\times 10^6}\mathbf{V_{out}}$$

Now, the first op-amp circuit is an inverting amplifier, but it's + terminal is at VRR now. Thus,

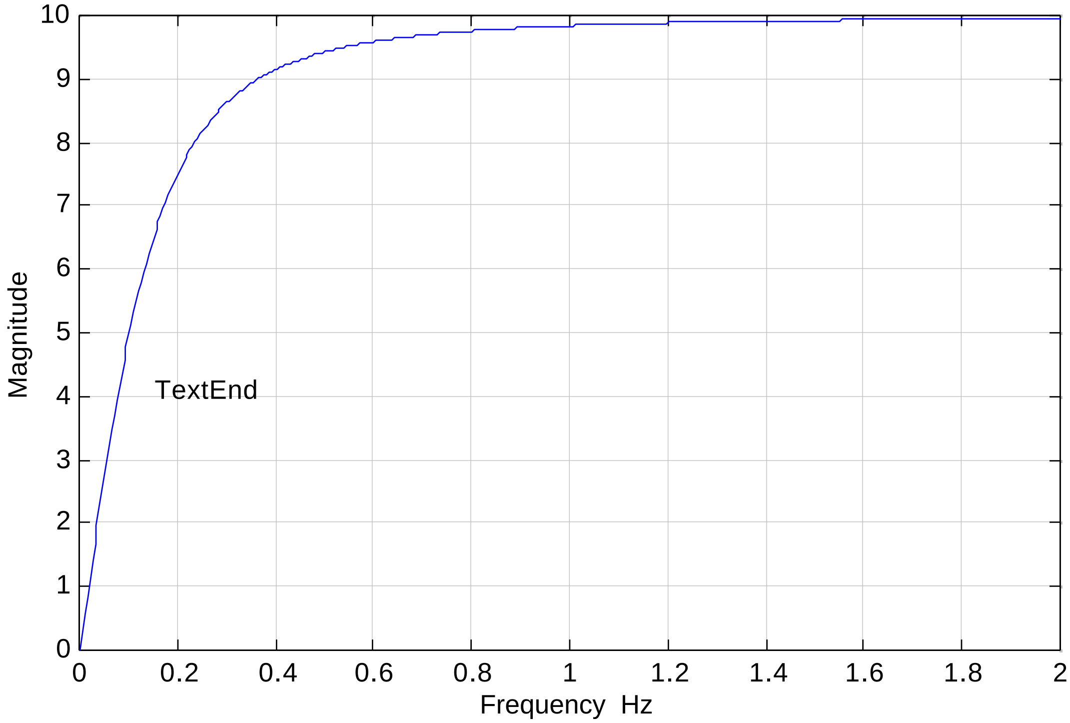
$$\mathbf{V_{out}} = -10\left(\mathbf{V_{in}} - \mathbf{V_{RR}}\right) + \mathbf{V_{RR}}$$

Substituting the above $\mathbf{V_{RR}}$ means that:

$$\mathbf{V_{out}} = \frac{-10}{1+\frac{1.1}{j\omega}}\mathbf{V_{in}}$$

Since the input voltage has unity magnitude and zero phase, the above expression gives the required magnitude and phase of the output voltage.

(b)



(c) As can be seen, the response to zero frequency (i.e. DC) is zero. Also, the circuit goes back very quickly (less than 2 Hz) to provide the required operation, which is to achieve a gain of 10.