CHAPTER 11. PROBLEM SOLUTIONS

SOLUTION 11.1. Using equation 11.3, $P_{av} = \frac{1}{2}\int_0^1 (e^t - 1)^2 R dt = \left[\frac{e^{2t}}{2} + t - 2e^t \right]_0^1 = 0.758 \text{ W}.$

SOLUTION 11.2.

(a) From 11.6, $P_{av} = \frac{V_m^2}{2R} = 50 \text{ mW}$ for a sinusoidal input.

(b) From 11.3,

$$P_{av} = \frac{R}{2}\left[\int_0^{\pi/20} (10\cos(10t))^2 dt + \int_{\pi/20}^{3\pi/20} (-10\cos(10t))^2 dt + \int_{3\pi/20}^{2\pi/10} (10\cos(10t))^2 dt \right] = \frac{I_m^2 R}{2} = 50 \text{ mW}$$

just as the previous case since the square of the absolute cos(10t) is the same as the square of cos(10t).

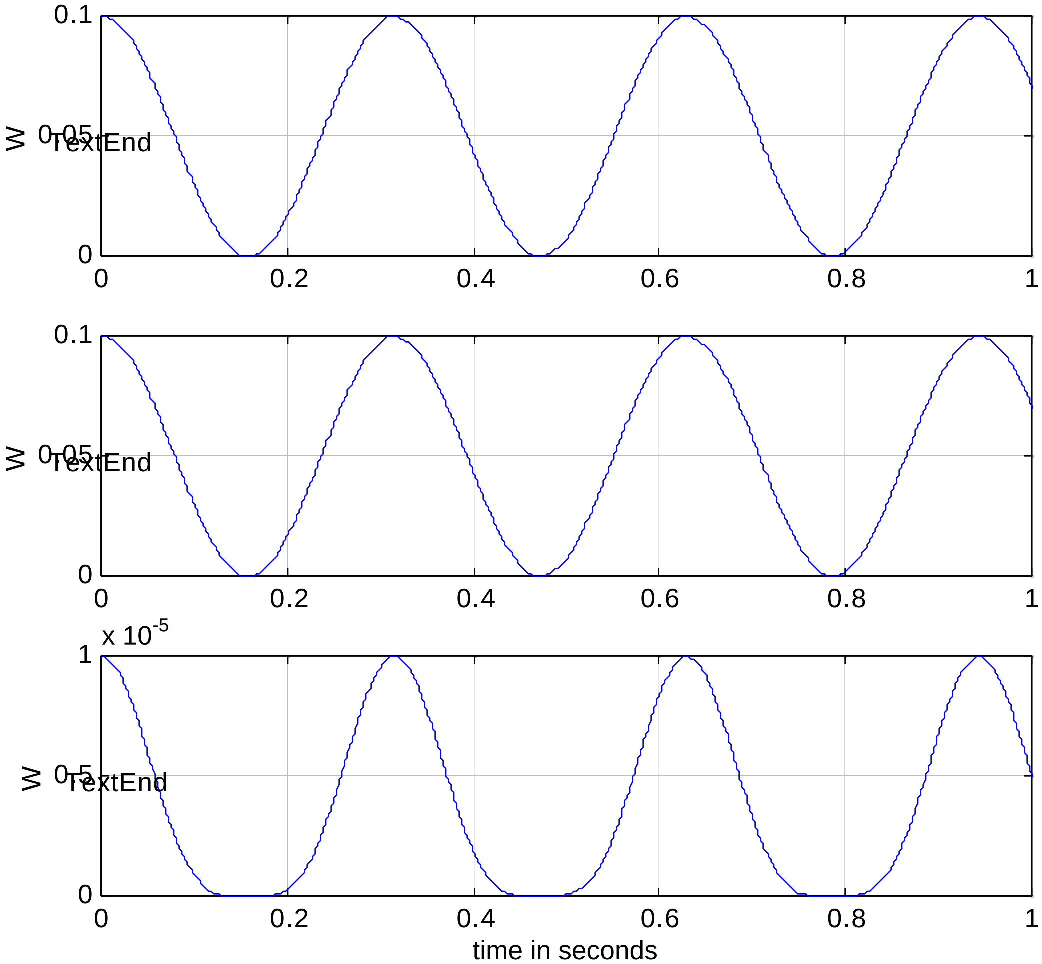
(c)

$$P_{av} = \frac{10R}{2\pi} \int_0^{2\pi/10} \left[0.01\cos^2(10t) \right]^2 dt = \frac{10^{-3}}{2\pi} R \int_0^{2\pi/10} \left[0.5\cos(20t) + 0.125\cos(40t) + 0.375 \right] dt$$

$$= \frac{1}{2\pi} \left[\frac{0.5}{20}\sin(20t) + \frac{0.125}{40}\sin(40t) + 0.375t \right]_0^{2\pi/10} = 37.5 \text{ mW}$$

(d)

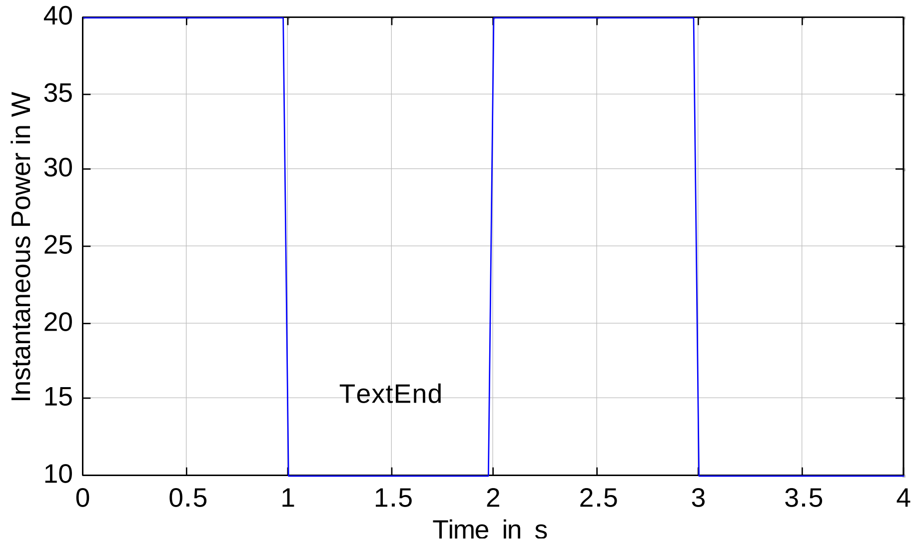
```
t=0:1/1000:1;
R=1e3;

pta= (0.01*cos(10.*t)).^2.*R;
ptb= (0.01*abs(cos(10.*t))).^2.*R;
ptc= (0.01*cos(10.*t)).^4.*R;

subplot(3,1,1);
plot(t,pta);
grid
ylabel('W');
subplot(3,1,2);
plot(t,ptb);
grid
ylabel('W');
subplot(3,1,3);
plot(t,ptc);
grid
ylabel('W');
xlabel('time in seconds');
```

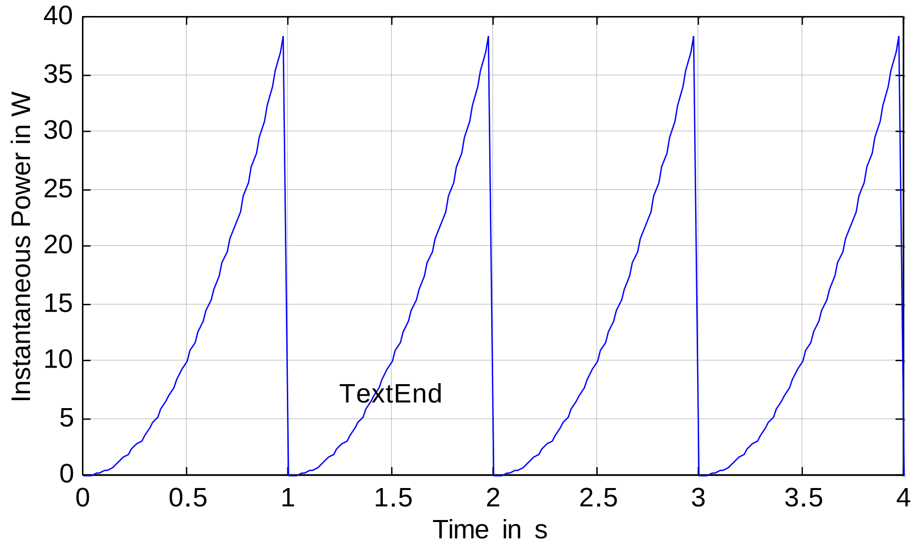


SOLUTION 11.3. (a) For figure a, the period is 2, and $P_{av} = \frac{1}{2}\int_0^2 \frac{v^2(t)}{R}dt = \frac{1}{2R}(400 + 100) = 25$ W.

In figure b, the period is 1, and $P_{av} = \int_0^1 \frac{(20t)^2}{R}dt = \frac{400}{R}\left[\frac{t^3}{3}\right]_0^1 = 13.3$ W.

(b)





SOLUTION 11.4. (a) For (a), looking the definition for the effective voltage, one sees graphically that the integral over one period, 2, of the squared waveform, is 500. Dividing by the period, and taking the square root, $V_{eff} = 15.81$ V. For fig. b, $V_{eff} = \sqrt{\int_0^1 (20t)^2\, dt} = \sqrt{400\frac{1}{3}} = 11.55$ V

(b) $P_{av} = I_{eff}^2 R = \left(\frac{V_{eff}}{10}\right)^2 8 = 20$ W.

(c) $P_{av} = \left(\frac{V_{eff}}{10}\right)^2 8 = 10.67$ W

SOLUTION 11.5. (a) This can be done graphically quite easily. The period of fig a, is 9s. The total area of one period of the squared waveform is 75. This yields $I_{eff} = \sqrt{\frac{75}{9}} = 2.89$ A. In fig. b, the area over one period is 25 which yields $I_{eff} = \sqrt{\frac{25}{3}} = 2.89$ A.

(b) Using current division, $P_{av} = \left(I_{eff}\,\frac{60}{90}\right)^2 30 = 111.36$ W.

(c) The same result as (b) is obtained since the effective current is the same.

SOLUTION 11.6. (a) $I_{eff} = \sqrt{\frac{1}{2}\int_0^1 \left(e^t - 1\right)^2 dt} = 0.615$ A.

(b) $P_{av} = I_{eff}^2 R = 0.758$ W.

SOLUTION 11.7. (a)

$$V_{eff}^2 = \frac{20}{2\pi}\int_0^{2\pi/20} \left(10 + 2\cos(20t)\right)^2 dt = \frac{20}{2\pi}\left[100t + 2t + \frac{2}{40}\sin(40t) + 2\sin(20t)\right]_0^{2\pi/20} = 102.01$$

Hence $V_{eff} = 10.1$ V.

(b)

$$V_{eff}^2 = \frac{1}{\pi}\int_0^{\pi} \left(10\cos(2t) + 5\cos(4t)\right)^2 dt = \frac{1}{\pi}\left[62.5t\right]_0^{\pi} = 62.568$$

Hence $V_{eff} = 7.91$ V.

(c)

Without going into detailed calculation, note the following fact about $v_3^2(t)$. Only the product terms that have the same frequency will produce a non-zero result when integrated. Thus the integral reduces to the following:

$$V_{eff}^2 = \frac{1}{\pi}\int_0^{\pi} \left(100\cos^2(2t) + 25\cos^2(4t) + 25\cos^2(4t - \pi/4) + 50\cos(4t)\cos(4t - \pi/4)\right) dt$$

Hence

$$V_{eff} = \sqrt{\left[50 + 12.5 + 12.5 + 25\cos(-\pi/4)\right]} = 9.627 \text{ V}$$

SOLUTION 11.8. The voltage is $\mathbf{V} = 50\angle 0$ V and the impedance $Z_{eq} = 100 - 100j = 141.42\angle -45^o$ Ω. Thus $\mathbf{I} = \frac{\mathbf{V}}{Z_{eq}} = 353.6\angle 45^o$ mA. Hence, $P_{av} = R\left|\mathbf{I_{eff}}\right|^2 = 100 \times \left(\frac{0.3536}{\sqrt{2}}\right)^2 = 6.2516$ W.

SOLUTION 11.9. **(a)** The equivalent load seen by the source, $Z_{eq} = \dfrac{1}{j\omega C + 1/R} = 2 \quad -36.87^o$. Thus

$\mathbf{V_L} = \mathbf{I}Z_{eq} = 10 \quad -36.87^o$ V and $v_L(t) = 10\cos(30t - 36.87^o)$ V.

(b) $P_{av} = \dfrac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \dfrac{10 \times 5}{2}\cos(-36.87^o) = 20$ W, and from 11.4

$$p_L(t) = 25\cos(-36.87^o) + 25\cos(60t - 36.87^o) \text{ W}$$

SOLUTION 11.10. (a) First find the impedance $Z_{eq} = 3 + j4 = 5 \quad 53.13^o$ Ω, then

$\mathbf{I_s} = \dfrac{\mathbf{V_s}}{Z_{eq}} = 10 \quad -143.13^o$ A. The magnitude is 10 A rms or 14.14 A peak-value.

(b) $P_{av} = 10 \;\; 50\cos(53.13^o) = 300$ W.

(c) $3|\mathbf{I_s}|^2 = 300$ W

SOLUTION 11.11. (a) By KVL, $\mathbf{V_s} = 10\mathbf{I_L} + j100\mathbf{I_L} + 9 \;\; 10\mathbf{I_L}$ and

$\mathbf{I_L} = \dfrac{\mathbf{V_s}}{100 + j100} = \dfrac{100 \quad 0}{141.42 \quad 45^o} = \dfrac{1}{\sqrt{2}} \quad -45^o$ A.

(b) $P_{avVs} = |\mathbf{V_s}||\mathbf{I_L}|\cos(45^o) = 50$ W and $P_{av9V1} = |9 \;\; 10\mathbf{I_L}||\mathbf{I_L}|\cos(180^o) = -45$ W.

SOLUTION 11.12. **(a)** By KCL the current through the resistor is 5**I**. So by KVL,

$\mathbf{V_s} = j1000\mathbf{I} - j500\mathbf{I} + 200 \;\; 5\mathbf{I}$ and hence

$$\mathbf{I} = \frac{\mathbf{V_s}}{1000 + j500} = 107.33 \quad -26.57^o \text{ A}$$

The power absorbed by the resistor is, $P_{av} = |200 \;\; 5\mathbf{I}||5\mathbf{I}| = 57.6$ W.

(b) $P_{avVs} = |\mathbf{V_s}||\mathbf{I}|\cos(26.57^o) = 11.52$ W and $P_{av4I} = |4\mathbf{I}||200 \;\; 5\mathbf{I}|\cos(0) = 46.08$ W.

SOLUTION 11.13. First find the input impedance, $Z_{eq} = 2 - j8 + 6 + j2 = 8 - j6$ Ω. Then calculate the current $\mathbf{I_L} = \dfrac{100}{8 - j6} = 8 + j6$ A. The complex power is then computed:

```
»ZL = 6 + j*2;
»IL = 8 + j*6;
»VL = ZL*IL
VL =
  3.6000e+01 + 5.2000e+01i
»SL = VL * conj(IL)
SL =
  6.0000e+02 + 2.0000e+02i
```

```
»abs(SL)
ans =
   6.3246e+02
```

Thus the apparent power is 790 VA, the average power 600 W, the reactive power 200 var, and the apparent power is 632.46 VA.

SOLUTION 11.14. (a) Using MATLAB

```
»Vseff = 100*exp(j*pi/6)
Vseff =  8.6603e+01 + 5.0000e+01i

»ZL = 350 +j*1*300;
»Zin = 50  +  ZL
Zin =  4.0000e+02 + 3.0000e+02i

»ILeff = Vseff/Zin
ILeff =  1.9856e-01 - 2.3923e-02i

»ILpk = sqrt(2)*abs(ILeff)
ILpk =   2.8284e-01

»ILang = angle(ILeff)*180/pi
ILang =  -6.8699e+00

»SL = ZL*ILeff*conj(ILeff)
SL =  1.4000e+01 + 1.2000e+01i
```

Therefore, $i_L(t) = \sqrt{2}(0.2)\cos(300t - 6.87^o) = 0.2828\cos(300t - 6.87^o)$ A. $\mathbf{S_L} = 14 + j12$ VA, and the average power is 14 W.

SOLUTION 11.15. First use voltage division and observe that $\mathbf{V_L} = \frac{1}{2}\mathbf{V_s} = 60 \angle 60°$ V. Now the complex power is $\mathbf{S_L} = \mathbf{V_L}\frac{\mathbf{V_L^*}}{(6 + j8)^*} = 360 \angle 53.13° = 216 + j288$ VA. Thus the average power is 216 W.

***SOLUTION 11.16. (a)** To find $\mathbf{V_2}$ we write a node equation. First we note that $Y_i = 1/Z_i$ is the corresponding admittance. Hence

$$Y_1(\mathbf{V_2} - \mathbf{V_a}) + Y_2\mathbf{V_2} + Y_3(\mathbf{V_2} - \mathbf{V_b}) = 0$$

Hence

$$\mathbf{V_2} = \frac{Y_1\mathbf{V_a} + Y_3\mathbf{V_b}}{Y_1 + Y_2 + Y_3} = 100 + j50 = 111.8 \angle 26.57^o \text{ V}$$

obtained using MATLAB as follows

```
»Z1 = 0.1+j*0.1; Z2 = 0.4+j*2.2;
»Z3 = 0.2 + j*0.2; Va = 104 + j*50; Vb = 106 + j*48;
»Y1 = 1/Z1
```

```
Y1 =  5.0000e+00 - 5.0000e+00i

»Y2 = 1/Z2
Y2 =  8.0000e-02 - 4.4000e-01i

»Y3 = 1/Z3
Y3 =  2.5000e+00 - 2.5000e+00i

»V2 = (Y1*Va + Y3*Vb)/(Y1 + Y2 + Y3)
V2 =  1.0000e+02 + 5.0000e+01i

»magV2 = abs(V2)
magV2 =   1.1180e+02
»angleV2 = angle(V2)*180/pi
angleV2 =   2.6565e+01
```

(b) Again working strictly in MATLAB we have the following complex powers of the loads and the two sources:

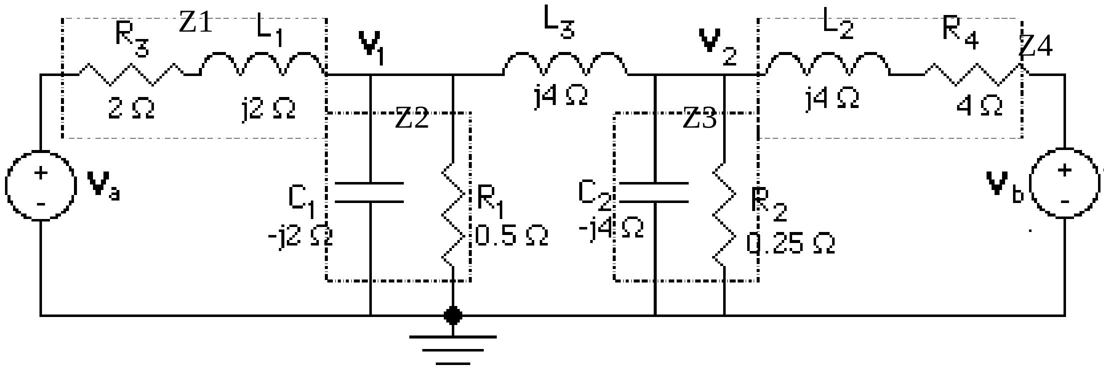
```
»Sz1 = (V2 - Va)*conj((V2-Va)*Y1)
Sz1 =
  8.0000e+01 + 8.0000e+01i

»Sz2 = V2*conj(V2*Y2)
Sz2 =
  1.0000e+03 + 5.5000e+03i

»Sz3 = (V2 - Vb)*conj((V2-Vb)*Y3)
Sz3 =
  1.0000e+02 + 1.0000e+02i

»Sva = Va*conj((Va - V2)*Y1)
Sva =
  1.0800e+03 + 3.0800e+03i

»Svb = Vb*conj((Vb - V2)*Y3)
Svb =
  1.0000e+02 + 2.6000e+03i
```

(c) To verify conservation of power observe that:

```
»TotSrsPwr = Sva + Svb
TotSrsPwr =
  1.1800e+03 + 5.6800e+03i

»TotLdPwer = Sz1 + Sz2 + Sz3
TotLdPwer =
  1.1800e+03 + 5.6800e+03i
```

which provides the desired verification.

SOLUTION 11.17. Use MATLAB and refer to the following figure:




```
%(a)
%Bundle the impedances as per the following figure and
%obtain the following.
Z1=2+2*j;
Y1=1/Z1;
Y2=2+0.5*j;
Z2=1/Y2;
ZL3=4*j;
YL3=1/ZL3;
Y3=4+0.25*j;
Z3=1/Y3;
Z4=4+4*j;
Y4=1/Z4;
V1=10+2*j;
V2=12+2*j;

%Write out KCL for node 1 and 2
%(Va-V1)*Y1=V1*Y2+(V1-V2)*YL3
%(Vb-V2)*Y4=V2*Y3+(V2-V1)*YL3
Va= (V1*Y2+(V1-V2)*YL3)/Y1+V1
Vb= (V2*Y3+(V2-V1)*YL3)/Y4+V2
Va = 29.0000 +59.0000i
Vb =  1.6000e+02+  2.3400e+02i
```

(b)

```
Sr3=( (Va-V1)*Y1*2)*conj((Va-V1)*Y1)
Sl1= ((Va-V1)*Y1*(2*j))*conj((Va-V1)*Y1)
Sc1= V1*conj(V1*0.5*j)
Sr1= V1*conj(V1/0.5)
Sl3= (V1-V2)*conj((V1-V2)*YL3)
Sc2= V2*conj(V2*0.25*j)
Sr2= V2*conj(V2/0.25)
Sl2= (V2-Vb)*Y4*4*j*conj((V2-Vb)*Y4)
Sr4= (V2-Vb)*Y4*4*conj((V2-Vb)*Y4)
SVA = Va * conj((Va-V1)*Y1)
SVB = Vb * conj((Vb-V2)*Y4)

Sr3 =   9.0250e+02

Sl1 =  0 + 9.0250e+02i

Sc1 =   0 - 5.2000e+01i
```

Sr1 = 208

Sl3 = 0 + 1.0000e+00i

Sc2 = 0 - 3.7000e+01i

Sr2 = 592

Sl2 = 0 + 9.4660e+03i

Sr4 = 9466

SVA = 1.1115e+03 + 8.4550e+02i

SVB = 1.0057e+04 + 9.4350e+03i

(c) Take the real part of each of the complex power found in (b). The only components with non-zero average power will be the resistors which have 208 W, 592 W, 902.5 W, and 9466 W average power respectively.

```
%(d)
Total_passive=Sr1+Sr2+Sr3+Sr4+Sl1+Sl2+Sl3+Sc1+Sc2
Total_active= Va*conj((Va-V1)*Y1)+Vb*conj((Vb-V2)*Y4)
Total_passive =
  1.1168e+04+  1.0280e+04i

Total_active  =  1.1168e+04+  1.0280e+04i
```

which verifies the conservation of power.

SOLUTION 11.18. (a) From conservation of energy, the complex power is the sum of the complex power absorbed by every circuit elements. Thus $\mathbf{S_s} = 1240 + j145$ VA, and the apparent power is 1248.4 VA. The average power is 1240 W.

(b) From $\mathbf{S_s} = \mathbf{V_s I_s^*}$, $I_S = \dfrac{|\mathbf{S_s}|}{|\mathbf{V_s}|} = 5.428$ A.

SOLUTION 11.19. (a) The complex power delivered by the source is the sum of the complex power consumed by the circuit elements. Thus $\mathbf{S_s} = 44 + j28$ kVA.

(b) $|\mathbf{I_s}| = \dfrac{|\mathbf{S_s}|}{|\mathbf{V_s}|} = 22.675$ A

(c) The total power delivered to the three groups of impedance following $\mathbf{V_1}$ is $\mathbf{S_1} = 41.5 + j22$. From the current obtained in (b) $|\mathbf{V_1}| = \dfrac{|\mathbf{S_1} + \mathbf{S_2} + \mathbf{S_4}|}{|\mathbf{I_s}|} = 2071.5$ V.

(d) From V_1, and the total power delivered to Z_4 and Z_2, $|\mathbf{I_2}| = \dfrac{|\mathbf{S_4} + \mathbf{S_2}|}{|\mathbf{V_1}|} = 12.385$ A. Finally

$$|\mathbf{V_2}| = \frac{|\mathbf{S_2}|}{|\mathbf{I_2}|} = 1805.5 \text{ V.}$$

SOLUTION 11.20. (a) $\mathbf{I_s} = \left(\dfrac{44 + j28}{2.3}\right)^* = 22.675 \angle -(32.47 - 0) = 22.675 \angle -32.47° \text{ A}.$

(b) $\mathbf{V_1} = \left[\dfrac{\mathbf{S_1} + \mathbf{S_2} + \mathbf{S_4}}{\mathbf{I_s^*}}\right] = 2071.5 \angle -4.54° \text{ V}$

(c) Similarly as before $\mathbf{I_2} = \left[\dfrac{\mathbf{S_1} + \mathbf{S_2}}{\mathbf{V_1}}\right]^* = 12.385 \angle -37.61° \text{ A}$, and $\mathbf{V_2} = Z_2\mathbf{I_2} = 1805.5 \angle -11.05° \text{ V}.$

SOLUTION 11.21. From equation 11.30, we find $Q = P\sqrt{\dfrac{1}{pf^2} - 1} = 455.61$ var; thus $\mathbf{S} = 1000 + j455.61 = 1099 \angle 24.5° \text{ VA}.$

SOLUTION 11.22. (a) $\mathbf{S_1} = 1000$ W, and $\mathbf{S_2} = 800 + j600$ VA. Thus the total power delivered by the source is $\mathbf{S_{tot}} = 1800 + j600 = 1897.37 \angle 18.43°$ VA, and $\mathbf{I_s} = \dfrac{\mathbf{S_{tot}^*}}{\mathbf{V_s^*}} = 15.81 \angle -18.43° = 15 - j5$ A rms.

(b) $\mathbf{V_1} = \dfrac{\mathbf{S_1}}{\mathbf{I_s^*}} = 63.25 \angle -18.43° \text{ V}.$

(c) $\mathbf{V_2} = \dfrac{\mathbf{S_2}}{\mathbf{I_s^*}} = 63.25 \angle 18.43° \text{ V}.$

SOLUTION 11.23. Using 11.25, find $|\mathbf{S_L}| = \dfrac{P_{ave}}{pf} = \dfrac{3000}{0.75} = 4000$ VA, and the load current $|\mathbf{I_L}| = \dfrac{|\mathbf{S_L}|}{|\mathbf{V_L}|} = \dfrac{4000}{120} = 33.33$ A. The power absorbed by the transmission line is then from $P_{line} = R|\mathbf{I_L}|^2 = 0.2 \times (33.33)^2 = 222.2$ W.

SOLUTION 11.24. The capacitor must absorb a reactive power of $Q^{new} - Q^{old} = -17.9$ kvar. Thus $jQ_C = -j17.9 = -j\omega C|V_s|^2$, and $C = \dfrac{-Q_C}{\omega|V_s|^2} = 0.897$ mF.

SOLUTION 11.25. From equation 11.30, $Q^{new} = 86.6\sqrt{\dfrac{1}{(0.94)^2} - 1} = 31.43$ VA. Thus the reactive power absorbed by the capacitor is –18.57 var. Hence $C = \dfrac{-Q_C}{\omega|V_s|^2} = 3.714$ μF.

SOLUTION 11.26. Device 1 has a complex power of $\mathbf{S_1} = P_1 + jQ_1 = 360 + j\,0$ VA. Recall equation 11.29, $pf = \frac{P_{ave}}{|\mathbf{S}|}$, and equation 11.30 $Q^{new} = P_{ave}\sqrt{\frac{1}{(pf)^2} - 1}$

where with a lagging pf, Q > 0, and with a leading pf for Q < 0. Using equation 11.30, we have for device 2:

$$Q_2 = 1440\sqrt{\frac{1}{(0.8)^2} - 1} = 1080 \text{ var}$$

$\mathbf{S_2}$ = 1440 + j 1080 VA.

$$\mathbf{S_{1,2}} = \mathbf{S_1} + \mathbf{S_2} = 1800 + j1080 \text{ VA}$$

As an aside we compute the magnitude of the current without the capacitor attached.

$$|\mathbf{I_s}| = \frac{|\mathbf{S_{1,2}}|}{120} = 17.493 \text{ A}$$

The capacitor is used to achieve a lower source current with the same average power. The first step is to find the desired QC. Here

$$\mathbf{S_{tot}} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_C} = 1800 + j1080 + jQ_C$$

Hence

$$\frac{|\mathbf{S_{tot}}|}{120} = \frac{|1800 + j1080 + jQ_C|}{120} = 15$$

In MATLAB we have:

```
»QC = sqrt((15*120)^2 - 1800^2) - 1080
QC =      -1080
```

From the formula on page 451 of the text,

$$Q_C = -\omega C |\mathbf{V_{source}}|^2$$

Hence C = 0.2 mF. Finally

$$pf = \frac{P_{ave}}{|\mathbf{S_1} + \mathbf{S_2} + \mathbf{S_C}|} = \frac{1800}{1800} = 1$$

SOLUTION 11.27. (a) From equation 11.30, $Q^{old} = 7\sqrt{\frac{1}{(0.65)^2} - 1} = 8.1839$ kvar. Therefore, the power absorbed without the capacitor bank is $\mathbf{S_{old}} = 7 + j8.1839$ kVA. When the bank is added we want $Q^{new} = 7\sqrt{\frac{1}{(0.8)^2} - 1} = 5.25$ kvar, and hence $\mathbf{S_{new}} = 7 + j5.25$ kVA. Thus the reactance that must be absorbed by the bank is –2.934 kvar, and $C_{eq} = \frac{-Q_C}{\omega |V_s|^2} = 0.13511$ mF.

(b) As was just determined 5.25 kVA.

(c) $|\mathbf{S_{old}}| = \dfrac{7}{0.65} = 10.77$ kVA. $|\mathbf{S_{new}}| = \dfrac{7}{0.8} = 8.75$ kVA. The kVA saving is: 10.77 – 8.75 = 2.02 kVA.

»Savings = 20*2.02*12
Savings =
  4.8480e+02

i.e., $484.80.

SOLUTION 11.28. (a) The apparent power is simply 94kW/0.78=120.51 kVA.

(b) $\mathbf{S_m} = 120.51\angle 38.74^o = 94 + j75.41$ kVA.

(c) 75.41 kvar.

(d) $\mathbf{I_{eff}} = \dfrac{\mathbf{S_m^*}}{\mathbf{V_{eff}^*}} = 523.96\angle -38.74$ A.

(e) By KVL, $\mathbf{V_s} = R_{line}\mathbf{I_{eff}} + \mathbf{V_{eff}} = 516.08 - j229.52$ V.

(f) $\mathbf{S_s} = \mathbf{V_s}\mathbf{I_{eff}^*} = 295.94\angle 14.76°$ kVA.

(g) The efficiency is $= \dfrac{120.51\cos(38.74)}{295.94\cos(14.76)} 100 = 32.96\%$. Note that the line resistance of 0.7 Ω is much to large for practical usage. This value is chosen for pedagogical reasons.

(h) With a power factor of 0.94, $\mathbf{S_m^{new}} = 94 + j34.12$ kVA. The average power of the motor must be kept the same. The reactance that must be provided by the capacitor is, $Q_{new} - Q_{old} = Q_C = -41.29$ kvar. The proper capacitor current will be $\mathbf{I_C} = \dfrac{j42.29k}{\mathbf{V_{eff}^*}}$, and $Z_C = \dfrac{1}{j\omega C} = \dfrac{\mathbf{V_{eff}}}{\mathbf{I_C}} = \dfrac{|\mathbf{V_{eff}}|^2}{j42.29\times 10^3}$. Solving for

$$C = \frac{42.29\times 10^3}{\omega |V_{eff}|^2} = 2.12 \text{ mF.}$$

(i) $\mathbf{I_{eff}^{new}} = \left(\dfrac{\mathbf{S_m^{new}}}{\mathbf{V_{eff}}}\right)^* = 434.79\angle -19.95°$ A

(j) By KVL, $\mathbf{V_s} = R_{line}\mathbf{I_{eff}^{new}} + \mathbf{V_{eff}} = 516.1 - j103.84 = 526.43\angle -11.38^o$ V.

(k) $\mathbf{S_s^{new}} = \mathbf{V_s^{new}}\left(\mathbf{I_{eff}^{new}}\right)^* = 228.89\angle 8.57° = 226.33 + j34.12$ kVA, and the efficiency is 41.5%.

SOLUTION 11.29. (a) The Thevenin equivalent seen at the output is, $Z_{th} = 5\Omega - j/(0.1\omega)$. For maximum power transfer, $Z_L = Z_{th}^* = 5 + j$. Note that $\mathbf{V_{OC}} = 50$ V rms, and that $\mathbf{I_{eff}} = \dfrac{\mathbf{V_{OC}}}{Z_{th} + Z_L} = 5$ A rms. Thus $\mathbf{S_L} = Z_L\mathbf{I_{eff}}\mathbf{I_{eff}^*} = 125 + j25$ VA, and the average power is 125 W.

SOLUTION 11.30. First find the following Thevenin equivalent,

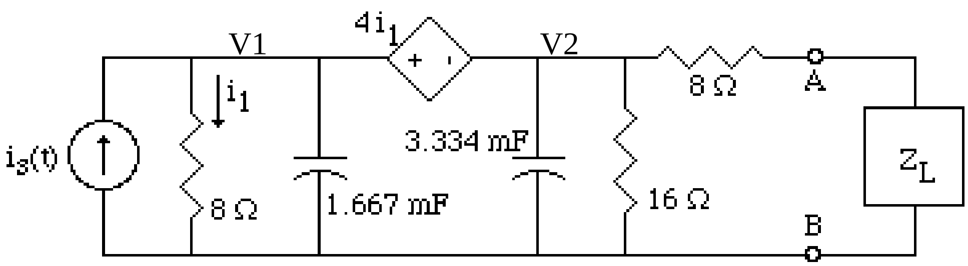
$Z_{th} = 2 + j4$ Ω, $\mathbf{V_{oc}} = \dfrac{10\sqrt{2}}{3}$ V.

By the maximum transfer property, $R_L = 2\Omega$, and $C = 1/(4\omega) = 0.25$ mF.

SOLUTION 11.31. (a) The thevenin equivalent left of the load is by KCL,
$\mathbf{I_{test}} + \mathbf{I_s} = \mathbf{V_R}(j0.001 + 0.001)$ and $\mathbf{V_{test}} = 3\mathbf{V_R} + j500\mathbf{I_{test}}$ V. Substituting for $\mathbf{V_R}$,
$\mathbf{V_{test}} = \mathbf{I_{test}}[1500 - j1000] + \mathbf{I_s}[1500 - j1500]$ V, and $Z_{th} = 1500 - j1000\ \Omega$ with
$\mathbf{V_{oc}} = \mathbf{I_s} \times 2121.3 \quad -45°$ V. The value of the load for maximum power transfer is then,
$Z_L = 1500 + j1000\ \Omega$.

(b) The complex power absorbed by the load is, $\mathbf{S_L} = Z_L \left| \dfrac{\mathbf{V_{oc}}}{Z_{th} + Z_L} \right|^2 = 750 + j500$ VA, and the average power 750 W.

SOLUTION 11.32. Consider



(a) Using nodal analysis get the following equations:

$$\mathbf{V_1} - \mathbf{V_2} = 4(\mathbf{V_1}/8)$$
$$\mathbf{I_s} + \mathbf{I_{test}} = \mathbf{V_1}(1/8 + j0.25) + \mathbf{V_2}(1/16 + j0.5)$$
$$\mathbf{V_{test}} = \mathbf{V_2} + 8\mathbf{I_{test}}$$

Using MATLAB we get the following expression,

$$\mathbf{V_{test}} = (8.2847\text{- j}0.9110)\mathbf{I_{test}} + (0.2847\text{- j}0.9110)\mathbf{I_s}$$

From this expression $\mathbf{V_{oc}} = 28.47 - j91.1$ V, and $Z_{th} = 8.2847\text{- j}0.9110\ \Omega$. The phasor equivalent of this circuit is a $\mathbf{V_{oc}}$ source in series with a 8.28 Ω resistor and a 7.32 mF capacitor.
(b) $Z_L = 8.2847\text{+ j}0.9110$, which is a 8.28 Ohm resistor in series with a 6.07mH inductor. Same configuration as before.

(c) $\mathbf{S_L} = Z_L \left| \dfrac{\mathbf{V_{oc}}}{Z_{th} + Z_L} \right|^2 = 275.2 + j30.26$ VA and the average power is 275.2 W.

SOLUTION 11.33. (a) The Norton equivalent may be found by inspection as $Z_{th} = 10 + j20\ \Omega$ and $\mathbf{I_{sc}} = 10$ A. Thus for maximum power transfer, $Z_L = 20 - j20\ \Omega$. This is a 10 Ω resistor and a 0.005 F capacitor in series. The maximum power is $\mathbf{S_L} = Z_L \left| \dfrac{\mathbf{I_{sc}} Z_{th}}{Z_{th} + Z_L} \right|^2 = 1250 - j2500$ VA. The maximum average power is 1250 W.
(b) If R is set to 20 Ω, the closest that can be achieved to maximum power transfer is $Z_L = 20 - j20$, or C equal to 0.005 F. With Z_L as above, by current division

$$\mathbf{I_{load}} = 10 \times \frac{10 + j20}{(10 + j20) + (20 - j20)} = 3.333 - j6.6667 = 7.45 \quad 63.434^o \text{ A}$$

The maximum average power then $20 \times |\mathbf{I_{load}}|^2 = 1111$ W.

(c) Using 11.38, $R_L = 31.62\ \Omega$. $P_{av} = R_L \left| \frac{\mathbf{I_{sc}} Z_{th}}{Z_{th} + Z_L} \right|^2 = 600.63$ W.

SOLUTION 11.34. (a) From Thevenin $Z_{th} = 19.2 - j14.4\ \Omega$, and from 11.38, set $R_L = 24\ \Omega$.
$\mathbf{V_{oc}} = \frac{\mathbf{V_s}(-j40)}{30 - j40} = 80 \quad -36.87°$. The maximum power is $P_{av} = \left| \frac{\mathbf{V_{oc}}}{Z_{th} + R_L} \right|^2 R_L = 74.07$ W.

(b) The voltage is $\mathbf{V} = \mathbf{V_{oc}} \frac{R_L}{Z_{th} + R_L}$, from this relationship, one sees that as the load resistance increase to infinity the output voltage goes to $\mathbf{V_{oc}}$, which is the maximum output voltage.

SOLUTION 11.35. Correction: the inductor symbol in the load should be a resistor. Since the source resistance is variable, example 6.21 serves as a reference suggesting that R = 0 is the answer. To see this consider that

$$P = 2|\mathbf{I_{load}}|^2 = \frac{2 \times 100}{(R+2)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{2 \times 100}{(R+2)^2 + (2-2)^2}$$

Hence, decreasing R produces increasing power and the maximum power is transferred when R = 0 with $P_{max} = 50$ W assuming that the source voltage is given in rms V.

SOLUTION 11.36. As per problem 35,

$$P = 10|\mathbf{I_{load}}|^2 = \frac{10 \times \left(50/\sqrt{2}\right)^2}{(R+10)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{12.5 \times 10^3}{(R+10)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Here, again R = 0 with C chosen to eliminate the reactive term maximizes power transfer. Hence

$$C = \frac{1}{\omega^2 L} = 0.01 \text{ F}$$

with $P_{max} = 125$ W.

SOLUTION 11.37. **(a)** By the maximum power transfer theorem, P_1 is maximized when Z_L is chosen as the conjugate of Z_{source}, i.e.

$$Z_L = 10 + j1000\ \Omega$$

(b) To find the appropriate values of L and C observe that

$$Z_L(j\omega) = j\omega L + \frac{1}{10^{-4} + j\omega C} = j10^7 L + \frac{10^{-4} - j\omega C}{10^{-8} + 10^{14}C^2}$$

$$= \frac{10^{-4}}{10^{-8} + 10^{14}C^2} + j\left(10^7 L - \frac{\omega C}{10^{-8} + 10^{14}C^2}\right) = 10 + j1000$$

Equating real parts leads to:

```
»%10^-4 = 10^-7 + 10^15*C^2
»
»C = sqrt(1e-4 - 1e-7)/sqrt(1e15)
C =   3.1607e-10
```

Thus C = 0.31607 nF.

Equating imaginary parts using the above value of C leads to:

```
»w = 1e7;
»L = (1e3 + w*C/(1e-8 + 1e14*C^2))/1e7
L =   1.3161e-04
```

Thus L = 0.1316 mH.

(c) In part (b), L and C are chosen to maximize P_1, the power delivered to Z_L. Since L and C consume no average power, this maximum power is transferred to the 10 kΩ fixed resistor with the computed values of L and C. Thus Z_L is the same as in part (b) or $Z_L = 10 + j1000\ \Omega$.

Since we know Z_L,

$$P_{max} = \frac{(0.1)^2}{4 \times 10} = 0.25 \text{ mW}$$

This is the average power consumed by the 10 kΩ resistor. Therefore

$$|\mathbf{V_2}|^2 = 0.25 \times 10^{-3} \times 10^4 = 2.5$$

It follows that $|\mathbf{V_2}| = 1.5811$ V. Power to the 10 kΩ fixed resistor is

$$P_{10k\Omega} = \frac{|\mathbf{V_2}|^2}{10^4}$$

Thus if $P_{10k\Omega}$ is maximized, then $|\mathbf{V_2}|$ is maximized.

SOLUTION 11.38. (a) From equation 11.4, if we substitute $\frac{2\pi}{T}$ ω, then the resulting instantaneous power will be $p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\frac{4\pi}{T}t + \theta_v + \theta_i)$ W where it is clearly seen that the fundamental period will now be halved. Note that by the same argument the fundamental frequency of the instantaneous power is double that of the voltage and current.

(b) As a sinusoid, the fundamental period is 2π/10, any integer multiple of this period will also be periodic.
(c) This is the same as (b) with an offset of 1 V added.

SOLUTION 11.39. First, $F_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_o+T} f^2(t)dt}$. Without going into detailed calculation, $f^2(t)$ will give a summation of two types of products, a product of each element with themselves, and products of each element with the other element. In the later case, we know that two cosines multiplied with one another and integrated over one period will yield zero if their angular frequency are different. As for the former case the integral will yields the result we are looking for. For example look at the first two terms,

$$F_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_o+T} f^2(t)dt} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \left(F_o^2 + 2F_1^2\cos^2(\omega_1 t + \theta_1) + ...\right)dt}$$

$$= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \left(F_o^2 + F_1^2 + F_1^2\cos(2\omega_1 t + 2\theta_1) + ...\right)dt}$$

$$= \sqrt{\frac{1}{T}\left[F_o^2 t + F_1^2 t + F_1^2\sin(2\omega_1 t + 2\theta_1) + ...\right]_0^T} = \sqrt{F_o^2 + F_1^2 ...}$$

SOLUTION 11.40. (a) We are given that $v_C(t) = V_m\sin(\omega t)$ V. Hence

$$i_C(t) = C\frac{dv_C}{dt} = \omega CV_m\cos(\omega t) \text{ A}$$

It follows that

$$p(t) = v_C(t)i_C(t) = \omega CV_m^2\sin(\omega t)\cos(\omega t) = 0.5\omega CV_m^2\sin(2\omega t) \text{ Watts}$$

Clearly, p(t) has a peak value of $0.5\omega CV_m^2$ and the integral of the sign over one period is zero implying that the average value of p(t) is zero.

(b) Here

$$W_C(t) = 0.5Cv_C^2(t) = 0.5CV_m^2\sin^2(\omega t) = 0.25CV_m^2(1-\cos(2\omega t)) \text{ J}$$

Here the peak value occurs when cos(2ωt) = –1 in which case the peak value is $0.5CV_m^2$. Further, the average value of cos(2ωt) over one period, T = π/ω, is zero whereas the average of a constant over the same period is simply the constant. Hence, $W_{C,ave} = 0.25CV_m^2$ J.

(c) From example 11.6,

$$Q_C = -I_{C,eff}V_{C,eff} = -\omega CV_{C,eff}^2 = -0.5\omega CV_m^2 = -2\omega\left(0.25CV_m^2\right) = -2\omega W_{C,ave}.$$

Therefore, $W_{C,ave} = -\frac{Q_C}{2\omega}$.

SOLUTION 11.41. (a) We are given that $i_L(t) = I_m \sin(\omega t)$ A. Hence

$$v_L(t) = L\frac{di_L}{dt} = \omega L I_m \cos(\omega t) \text{ V}$$

It follows that

$$p(t) = v_L(t) i_L(t) = \omega L I_m^2 \sin(\omega t)\cos(\omega t) = 0.5\omega L I_m^2 \sin(2\omega t) \text{ watts}$$

Clearly, p(t) has a peak value of $0.5\omega L I_m^2$ and the integral of the sign over one period is zero implying that the average value of p(t) is zero.

(b) Here

$$W_L(t) = 0.5 L i_L^2(t) = 0.5 L I_m^2 \sin^2(\omega t) = 0.25 L I_m^2 \big(1 - \cos(2\omega t)\big) \text{ J}$$

Here the peak value occurs when cos(2ωt) = –1 in which case the peak value is $0.5 L I_m^2$. Further, the average value of cos(2ωt) over one period, T = π/ω, is zero whereas the average of a constant over the same period is simply the constant. Hence, $W_{L,ave} = 0.25 L I_m^2$ J.

(c) As an extension to example 11.5,

$$Q_L = I_{L,eff} V_{L,eff} = \omega L I_{L,eff}^2 = 0.5\omega L I_m^2 = 2\omega\left(0.25 C I_m^2\right) = 2\omega W_{L,ave}.$$

Therefore, $W_{L,ave} = \dfrac{Q_L}{2\omega}$.

SOLUTION 11.42. (a) The complex power absorbed by the load is, $\mathbf{S_L} = \mathbf{VI}^* = Z\mathbf{II}^* = Z|\mathbf{I}|^2$. Now note that the average power is the real part of the complex power. Also note that a complex number multiplied by its complex conjugate will yield a real value. Therefore the real part of $Z|\mathbf{I}|^2$ is just the real part of Z, R, multiplied by $|\mathbf{I}|^2$, $P_{av} = R|\mathbf{I}|^2$. With the same reasoning, the reactance is the imaginary part of Z, X, multiplied by $|\mathbf{I}|^2$, $Q = X|\mathbf{I}|^2$.

(b) The complex power absorbed by the load is $\mathbf{S_L} = \mathbf{VI}^* = \mathbf{V}(Y\mathbf{V})^* = Y|\mathbf{V}|^2$. The same reasoning as in (a) holds thus the real part of the admittance times $|\mathbf{V}|^2$, yields $P_{av} = G|\mathbf{V}|^2$. Using the imaginary part, $Q = B|\mathbf{V}|^2$.

SOLUTION 11.43. (a) The equivalent resistance seen by the source is R_{eq} = 6 – j9 Ω. So the current delivered by the source is:

$$\mathbf{I_s} = \frac{V_s}{6 - j9} = \frac{-j110}{6 - j9} = 10.17 \quad -33.69° \text{ A}$$

or $i_L(t) = 10.17\sqrt{2}\cos(120\pi t - 33.69^o)$ A. Similarly

$$\mathbf{V_C} = -j15 \times \mathbf{I_s} = \frac{-15 \times 110}{6 - j9} = 152.54 \quad -123.69° \text{ V}$$

or $v_C(t) = 152.54\sqrt{2}\cos(120\pi t - 123.69^o)$ V. Further, $L = \frac{6}{\omega} = 15.9$ mH and $C = \frac{1}{15\omega} = 176.8$ μF. The instantaneous energy stored in the inductor is

$$W_L(t) = 0.5Li_L^2(t) = 1.646\cos^2(120\pi t - 33.69^o) \text{ J}$$

and the instantaneous energy stored in the capacitor is

$$W_C(t) = 0.5Cv_C^2(t) = 4.115\cos^2(120\pi t - 123.69^o) \text{ J}$$

The source voltage is zero when t = 0. Therefore

$$W_L(0) = 1.646\cos^2(-33.69^o) = 1.1395 \text{ J}$$

and

$$W_C(0) = 4.115\cos^2(-123.69^o) = 1.266 \text{ J}$$

(b) and (c) Observe that $i_L(t)$ and $v_C(t)$ are 90^o out of phase. When one is zero, the other has a peak value. Therefore when $W_C = 0$ implies $v_C(t_0) = 0$ for appropriate t_0; hence $W_L(t_0) = 1.646$ J. Similarly, when $W_L = 0$, say at t_0, then $W_C(t_0) = 4.115$ J.

SOLUTION 11.44. In order to solve this problem, we want to express the power in terms or R's and L's in both circuits. First, looking at the circuit with just the coil and the 110 V source: $\mathbf{I} = \mathbf{V}/Z_{coil}$, $Z_{coil} = R + j\omega L$, and

$$P_{coil} = |\mathbf{I}|^2 R = \frac{110^2}{R^2 + \omega^2 L^2} R = 300 \text{ watts} \qquad (*)$$

Next, looking at the circuit when a resistance is added in series with the coil, $\mathbf{I} = \mathbf{V}/(8 + Z_{coil})$, $Z_{coil} = R + j\omega L$,

$$P_{coil} = |I|^2 R = \frac{220^2}{(8+R)^2 + \omega^2 L^2} R = 300 \text{ watts} \qquad (**)$$

To find R, solve equation (*) for $R^2 + \omega^2 L^2$ and substitute into equation (**) to obtain

```
»R = 300*64/(220^2 - 300*16 -110^2)
R =   6.0952e-01
```

Substituting R into equation (*) yields $L = 13.05$ mH.

SOLUTION 11.45. The average power consumed by the 2.7 Ω resistor is 250 watts. This allows us to compute the magnitude of $|\mathbf{I}_{coil}|$. We know that $|\mathbf{V}_{coil}|$ is 150 Vrms. Thus we can compute the magnitude of the coil impedance and hence L as follows:

```
»magIcoil = sqrt(250/2.7)
magIcoil =   9.6225e+00

»magZcoil = 150/magIcoil
magZcoil =   1.5588e+01
```

```
»% magZcoil^2 = 2.7^2 + (w*L)^2
»
»w = 2*pi*60;
»L = sqrt( magZcoil^2 - 2.7^2)/w
L =
   4.0725e-02

»%  Magnitude of impedance seen by 220 V source is:
»
»magZin = 220/magIcoil
magZin =   2.2863e+01

»%  magZin^2 = (R + 2.7)^2 + (w*L)^2
»
»Realpart = sqrt(magZin^2 - (w*L)^2)
Realpart =   1.6941e+01

»R = Realpart - 2.7
R =   1.4241e+01
```