

CHAPTER 12. PROBLEM SOLUTIONS

SOLUTION P12.1. By conservation of energy, the instantaneous power consumed by each load when summed together is equivalent to the total power consumed by the three phase load. Thus writing out

$$\begin{aligned}
 P_{tot}(t) &= p_{AB}(t) + p_{BC}(t) + p_{CA}(t) = v_{AB}(t)i_{AB}(t) + v_{BC}(t)i_{BC}(t) + v_{CA}(t)i_{CA}(t) \\
 &= V_L\sqrt{2}\cos(\omega t + \theta_v) \frac{V_L\sqrt{2}}{|Z|}\cos(\omega t + \theta_i) + V_L\sqrt{2}\cos(\omega t + \theta_v - 120^\circ) \frac{V_L\sqrt{2}}{|Z|}\cos(\omega t + \theta_i - 120^\circ) \\
 &\quad + V_L\sqrt{2}\cos(\omega t + \theta_v + 120^\circ) \frac{V_L\sqrt{2}}{|Z|}\cos(\omega t + \theta_i + 120^\circ) \\
 &= \frac{V_L^2}{|Z|}(\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)) + \frac{V_L^2}{|Z|}(\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i + 120^\circ)) \\
 &\quad + \frac{V_L^2}{|Z|}(\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 120^\circ)) \\
 &= \frac{3V_L^2}{|Z|}\cos(\theta_v - \theta_i) = \frac{3V_L^2}{|Z|} \times pf
 \end{aligned}$$

SOLUTION P12.2. To justify the point of this problem we equate the following two equations:

(i) For the 3 phase system:

$$P'_{loss} = 3 \times |\mathbf{I}_L|^2 R' = 3 \times \frac{3}{9} \times \frac{P_L^2}{V_L^2} R' = \frac{P_L^2}{V_L^2} R'$$

(ii) For the single phase system:

$$P_{loss} = 2 \times (|\mathbf{I}_L|^2 R) = \frac{P_L^2}{V_L^2} 2R$$

It follows that $R' = 2R$. Since both systems have the same distance of transmission and the resistance of a wire is inversely proportional to the cross sectional area, the condition $R' = 2R$ implies that the cross section A' of each wire in the three-phase system need only be half of the area A of the wire in the single phase system. But there are two wires in the single phase system and three wires in the three-phase system. Therefore the ratio of the materials used is:

$$\frac{\text{material in 3-phase system}}{\text{material in 1-phase system}} = \frac{3A'}{2A} = \frac{3}{2} \times \frac{1}{2} = 75\%$$

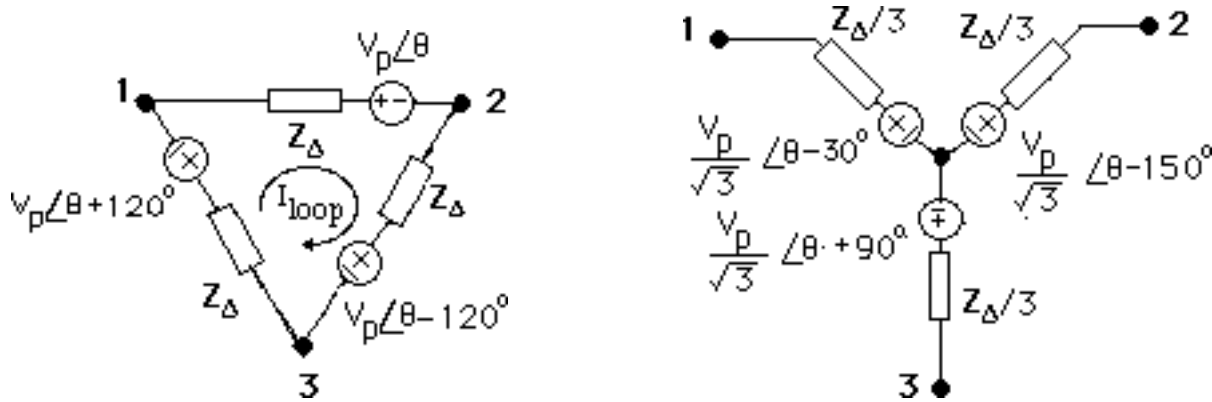
SOLUTION P12.3. For row 1 of table 12.1, the impedance in (a) seen between each pair of terminal is

$$Z_{ik} = Z \parallel 2Z = 2\frac{Z}{3}. \text{ In (b) the impedance seen between any two terminals is}$$

$$Z_{jk} = \frac{Z}{3} + \frac{Z}{3} = 2\frac{Z}{3}.$$

In row 2, the impedance between any two terminals is $Z_{jk} = Z_Y + Z_Y = 2Z_Y$ for (c), and $Z_{ik} = 3Z_Y \parallel 6Z_Y = 2Z_Y$ for (d).

SOLUTION P12.4. Consider the Δ -Y relationship of the figures below (row 3 of table 12.1):



Let us consider the terminal pair (1,2):

(i) For the Y-connected case, $Z_{th} = 2 \times \frac{Z}{3}$ and

$$V_{oc} = \frac{V_p}{\sqrt{3}} \angle(\theta - 30^\circ) - \frac{V_p}{\sqrt{3}} \angle(\theta - 150^\circ) = \frac{V_p}{\sqrt{3}} \angle(\theta - 30^\circ - 150^\circ) = V_p \angle\theta$$

(ii) For the D-connected case,

$$Z_{th} = Z \parallel (2Z) = \frac{2(Z)^2}{3Z} = \frac{2}{3}Z$$

Now note that there is no load connected to the Δ -configuration. Applying KVL to the indicated loop implies that:

$$0 = 3Z I_{loop} + V_p \angle\theta + V_p \angle(\theta - 120^\circ) + V_p \angle(\theta + 120^\circ) = 3Z I_{loop}$$

Hence $I_{loop} = 0$. Finally, $V_{oc} = V_{12} = Z I_{loop} + V_p \angle\theta = V_p \angle\theta$. Therefore, looking into terminals 1-2, both the Δ -configuration and the Y-configuration have the same Thevenin equivalent. For terminal pairs (1-3) and (2-3), the proof is virtually the same. Hence this establishes the equivalence in row 3 of table 12.1.

To establish the equivalence in row 4 of table 12.1, we do all the same computations with the slightly different labeling to obtain the same result, i.e., the circuits are equivalent.

SOLUTION 12.5. For (a) first note the following relationship $V_N = \frac{V_1}{Z_1} + \frac{V_2}{Z_2} (Z_1 \parallel Z_2 \parallel Z_3)$, which is obtained by KCL at the center node, N, with node 3 as the reference node.. Write out KCL at terminal 1,

$$I_1 = \frac{V_1}{Z_1} - \frac{V_1}{Z_1 Z_1} + \frac{V_2}{Z_2 Z_1} (Z_1 \parallel Z_2 \parallel Z_3) = V_1 \frac{Z_3 + Z_2}{Z_2 Z_3 + Z_1 Z_3 + Z_1 Z_2} - V_2 \frac{Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

. Do the

same for terminal 1 in (b), $I_1 = V_1 \frac{1}{Z_{31}} + \frac{1}{Z_{12}} - \frac{V_2}{Z_{12}}$. Now substitute the formulas in the problem

statement into the later equation, $I_1 = V_1 \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} - V_2 \frac{Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$, which is the same as the equation for (a).

Using the same method, the same result is obtained for node 2. So the fact that the substitution of the equivalence in (b) yields the same equation as in (a) proves that the equivalences are accurate.

SOLUTION 12.6. By Ohm's law $\mathbf{I}_A = \frac{\mathbf{V}_p}{Z} = \frac{200 \angle 0^\circ}{10 + j5} = 17.89 \angle -26.57^\circ$ A. By the same relationship $\mathbf{I}_B = 17.89 \angle -146.57^\circ$ A, and $\mathbf{I}_C = 17.89 \angle 93.43^\circ$ A. The neutral line current is the sum of the other three and is zero. The power of each phase is the same, as they have the same load, and current magnitude. Using the current at terminal A, the total power is, $P = 3R|\mathbf{I}_A|^2 = 9602$ W.

SOLUTION 12.7. For this balanced Y-Y connection, we follow the method of example 12.6. Using MATLAB:

```

»Z = 20 + j*10'
Z = 2.0000e+01 + 1.0000e+01i

»VAN = 200;
»IA = VAN/Z
IA = 8.0000e+00 - 4.0000e+00i

»IAmag = abs(IA)
IAmag = 8.9443e+00

»IAangle = angle(IA)*180/pi
IAangle = -2.6565e+01

»% Use phase inference to obtain:
»IBmag = IAmag
IBmag = 8.9443e+00

»IBangle = IAangle - 120
IBangle = -1.4657e+02

»ICmag = IAmag
ICmag = 8.9443e+00

»ICangle = IAangle + 120
ICangle = 9.3435e+01

»Ptotave = 3*real(Z)*IAmag^2
Ptotave = 4.8000e+03

```

SOLUTION 12.8. Using the same approach as in the example 12.6 we compute

$$|V_{AN}| = |V_{sA}| \times \left| \frac{Z}{Z + Z_g} \right| = 120 \times \left| \frac{4 + j3}{4 + j3 + 0.1 + j0.2} \right| = 116.94 \text{ V}$$

Thus the drop is: $\frac{120 - 116.94}{120} \times 100 = 2.55 \%$.

SOLUTION 12.9. A 5.5 % drop corresponds to $|V_{AN}| = 113.4 \text{ V}$. This means

$$\frac{|Z|}{|Z + \alpha Z_1 + Z_g|} = \frac{|V_{AN}|}{120} = \frac{0.945 \times 120}{120} = 0.945$$

From this equation,

$$|Z + \alpha Z_1 + Z_g| = \frac{|Z|}{0.945} = 5.291.$$

So $(4.05 + \alpha 0.1)^2 + (3.15 + \alpha 0.2)^2 = 27.99$. Solving the quadratic equation results in $\alpha = 0.7893$.

SOLUTION 12.10. Solving the single phase equivalent circuit for phase A,

$$\mathbf{I}_A = \frac{V_p - V_p \angle 120^\circ}{10 + j5} = 58.87 \angle -56.57^\circ$$

Using phase inference,

$$\begin{aligned} \mathbf{I}_B &= \mathbf{I}_A \times \angle -120^\circ = 58.87 \angle -176.57^\circ \\ \mathbf{I}_C &= \mathbf{I}_A \times \angle 120^\circ = 58.87 \angle 64.43^\circ \end{aligned}$$

Alternately, using much more work,

$$\begin{aligned} \mathbf{I}_B &= \frac{V_p \angle -120^\circ - V_p}{10 + j5} = 58.87 \angle -176.57^\circ \\ \mathbf{I}_C &= \frac{V_p \angle 120^\circ - V_p \angle -120^\circ}{10 + j5} = 58.87 \angle 64.43^\circ \end{aligned}$$

The total power, using the previously derived relationship, is

$$P = \sqrt{3} V_L I_L pf = \sqrt{3} \times 380 \times 58.87 \times pf = 34.655 \text{ kW}$$

where

$$pf = \cos \tan^{-1} \frac{5}{10} = 0.8944$$

SOLUTION 12.11. (a) By KVL and Ohm's law,

$$\mathbf{I}_{AB} = \frac{V_p}{30 + j15} = 6.57 \angle -26.57^\circ$$

By phase inference,

$$\mathbf{I}_{BC} = \mathbf{I}_{AB} \times -120^\circ = 6.57 \angle -146.57^\circ$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB} \times 120^\circ = 6.57 \angle 93.43^\circ$$

Alternately,

$$\mathbf{I}_{BC} = \frac{V_p \angle -120^\circ}{30 + j15} = 6.57 \angle -146.57^\circ$$

$$\mathbf{I}_{CA} = \frac{V_p \angle 120^\circ}{30 + j15} = 6.57 \angle 93.43^\circ$$

To compute the line currents we have in Amps:

$$\mathbf{I}_A = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 11.36 \angle -56.57^\circ$$

$$\mathbf{I}_B = \mathbf{I}_{BC} - \mathbf{I}_{AB} = 11.36 \angle -176.57^\circ$$

$$\mathbf{I}_C = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 11.36 \angle 63.43^\circ$$

(b) The average power is $P = 3 \times 30 |\mathbf{I}_{AB}|^2 = 3.88 \text{ kW}$, and the total reactive power is $Q = 3 \times 15 |\mathbf{I}_{AB}|^2 = 1.94 \text{ kvar}$.

SOLUTION 12.12. From the efficiency we know that $\frac{25 \text{ 746}}{P_{delivered}} = 0.85$, so the total power being delivered to the load is 21941 W. Using 12.4, $I_L = \frac{21941}{\sqrt{3} V_L pf} = 63.31 \text{ A}$.

SOLUTION 12.13. (a) Using the power efficiency relationship, $P_{tot} = \frac{300 \text{ 746}}{0.935} = 239.36 \text{ kW}$.

(b) Using 12.4, $I_L = \frac{239.36 \times 10^3}{\sqrt{3} V_L pf} = 68.28 \text{ A}$.

SOLUTION 12.14. The magnitude of the power delivered to each delta connected load is 79.787 kW. We can now perform the analysis on a single phase. Using equation 11.30, we obtain

$$Q_{old} = 79787 \sqrt{\frac{1}{0.88} - 1} = 43064 \text{ vars}$$

and

$$Q_{new} = 79787 \sqrt{\frac{1}{0.95} - 1} = 26225 \text{ vars}$$

Hence

$$\mathbf{S}_{old} = 79787 + j43064$$

$$\mathbf{S}_{new} = 79787 + j26225$$

So the reactive power to be supplied by the capacitor is $|Q_C| = 16839 = 43064 - 26225$. The capacitance

$$C = \frac{|Q_C|}{\omega |V_L|^2} = 8.44 \text{ uF.}$$

SOLUTION 12.15. Replacing the delta configuration of the source by its Y-equivalent, the following voltages (in V) are obtained,

$$\mathbf{V}_{AN} = \frac{V_p}{\sqrt{3}} \angle -30^\circ, \quad \mathbf{V}_{BN} = \frac{V_p}{\sqrt{3}} \angle -150^\circ, \quad \mathbf{V}_{CN} = \frac{V_p}{\sqrt{3}} \angle 90^\circ$$

From the single phase equivalent,

$$\mathbf{I}_A = \frac{\mathbf{V}_{AN}}{Z} = 19.63 \angle -56.57^\circ$$

From phase inference

$$\mathbf{I}_B = \mathbf{I}_A \times -120^\circ = 19.63 \angle -176.57^\circ$$

$$\mathbf{I}_C = \mathbf{I}_A \times 120^\circ = 19.63 \angle 63.43^\circ$$

The power factor of the load is

$$pf = \cos \tan^{-1} \frac{5}{10} = 0.8944$$

Hence, the total power in the balanced system is

$$P = \sqrt{3} V_L I_L pf = \sqrt{3} \times 380 \times 19.63 \times 0.8944 = 11.556 \text{ kW.}$$

SOLUTION 12.16. First replace the delta connected load by its Y-equivalent. Then analyzing a single phase and using phase inference, we obtain in amps,

$$\mathbf{I}_A = \frac{V_p}{(Z/3)} = 34.08 \angle -26.56^\circ$$

$$\mathbf{I}_B = \mathbf{I}_A \times -120^\circ = 34.08 \angle -146.56^\circ$$

$$\mathbf{I}_C = \mathbf{I}_A \times 120^\circ = 34.08 \angle 93.43^\circ$$

The power factor of the load is

$$pf = \cos \tan^{-1} \frac{5}{10} = 0.8944$$

The total power is $P = \sqrt{3}V_L I_L pf = 3 \times 220 \times 34.08 \times 0.8944 = 11.61 \text{ kW}$. The voltage across each load is $\sqrt{3} V_p = V_L = 219.97 \text{ V}$.

SOLUTION 12.17. (a) Referring to figure 12.2, the following relationships may be pointed out:

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} + \mathbf{V}_{NB} = \mathbf{V}_{AN} - \mathbf{V}_{BN}$$

$$\mathbf{V}_{BC} = \mathbf{V}_{BN} + \mathbf{V}_{NC} = \mathbf{V}_{BN} - \mathbf{V}_{CN}$$

$$\mathbf{V}_{CA} = \mathbf{V}_{CN} + \mathbf{V}_{NA} = \mathbf{V}_{CN} - \mathbf{V}_{AN}$$

(b) Perform a delta to Y-transformation and use Ohm's law and phase inference to obtain in amps:

$$\mathbf{I}_A = \frac{220 \angle -30^\circ}{\sqrt{3} Z} = 11.36 \angle -56.57^\circ$$

$$\mathbf{I}_B = 11.36 \angle -176.57^\circ$$

$$\mathbf{I}_C = 11.36 \angle 63.43^\circ$$

(c) First,

$$pf = \cos \tan^{-1} \frac{5}{10} = 0.8944$$

The average power, $P_{av} = \sqrt{3}V_L I_L pf = \sqrt{3} \times 220 \times 11.36 \times 0.8944 = 3.87 \text{ kW}$.

The reactive power may be found as follows, $Q = 3 \times 5 |\mathbf{I}_L|^2 = 1.936 \text{ kvar}$.

SOLUTION 12.18. Part (a) Apply Δ -Y transformation to the source.

Apply the transformation of row 3, Table 12.1 to the source. The result is the circuit shown in figure 12.19 of example 12.8. Therefore we may use many of the calculated values in example 12.9. In particular, the line currents are

$$\mathbf{I}_A = 19.49 \angle -68.9^\circ \text{ A}, \mathbf{I}_B = 19.49 \angle -188.9^\circ \text{ A}, \text{ and } \mathbf{I}_C = 19.49 \angle 51.09^\circ \text{ A}$$

Part (b). Compute line-to-neutral voltage. From figure 12.21 and by Ohm's law

$$\mathbf{V}_{AN} = \mathbf{I}_A Z_Y = 19.49 / -68.9^\circ \times 5 \angle 36.87^\circ = 97.45 \angle -32.03^\circ \text{ V}$$

By inference on phase angles

$$\mathbf{V}_{BN} = 97.45 \angle -152.03^\circ \text{ V and } \mathbf{V}_{CN} = 97.45 \angle 87.97^\circ \text{ V}$$

Part (c). Compute line voltages \mathbf{V}_{AB} , \mathbf{V}_{BC} , and \mathbf{V}_{CA} . From the voltage phasor diagram of figure 12.2.

$$\mathbf{V}_{AB} = \sqrt{3} \mathbf{V}_{AN} e^{j30^\circ} = 168.8 e^{-j2.03^\circ} \text{ V}$$

By inference on phase angles,

$$\mathbf{V}_{BC} = \mathbf{V}_{AB} e^{-j120^\circ} = 168.8 e^{-j122.03^\circ} \text{ V}$$

$$\mathbf{V}_{CA} = \mathbf{V}_{AB} e^{j120^\circ} = 168.8 e^{-j117.97^\circ} \text{ V}$$

Part (d). Compute the total power. From equation 12.4

$$P_{total} = \sqrt{3} \times V_L \times I_L \times pf = \sqrt{3} \times 168.8 \times 19.49 \times 0.8 = 4558 \text{ W}$$

SOLUTION 12.19. Perform a delta to Y transformation on the source. This yields

$\mathbf{V}_{IN} = \frac{V_p}{\sqrt{3}} \angle -30^\circ = \frac{380}{\sqrt{3}} \angle -30^\circ \text{ V}$. Perform a delta to Y-transformation on the load. This yields a new impedance $Z_{new} = \frac{Z}{3} = 4 + j3 \ \Omega$. Now analyzing a single phase and finding the others by inference yields, in amps,

$$\mathbf{I}_A = \frac{\mathbf{V}_{IN}}{Z_1 + Z_{new}} = 42.18 \angle -67.97^\circ$$

$$\mathbf{I}_B = 42.18 \angle -187.97^\circ$$

$$\mathbf{I}_C = 42.18 \angle 52.03^\circ$$

The average power delivered to the load is $P_{av} = 3 \times |\mathbf{I}_A|^2 \times 4 = 21.35 \text{ kW}$.

SOLUTION 12.20. Performing a delta-to-Y-transformation on the source and the load, the new source magnitudes are multiplied by $1/\sqrt{3}$, and the loads are divided by 3. Observe that $Z_g = 0.15 + j0.45 \ \Omega$, $Z_1 = 0.1 + j0.2 \ \Omega$, and $Z = 12 + j9 \ \Omega$. Using Ohm's law on phase one, and then inference on the phase angles for the other currents, yields in amps,

$$\mathbf{I}_A = \frac{V_p \angle -30^\circ}{\sqrt{3} \left(\frac{Z_g}{3} + Z_1 + \frac{Z}{3} \right)} = 41.14 \angle -68.91^\circ$$

$$\mathbf{I}_B = 41.14 \angle -188.91^\circ$$

$$\mathbf{I}_C = 41.14 \angle 51.09^\circ$$

The total power may be calculated as follows, $P_{av} = 3 \frac{12}{3} |I_L|^2 = 12 \times (41.14)^2 = 20.310 \text{ kW}$.

SOLUTION 12.21. Perform delta-to-Y-transformation on the delta load. Because of the property of the neutral to be ground in balance circuit, the two loads in the same phase combine in parallel. Thus the new impedance seen in one branch is

$$Z_{Y_{new}} = Z_Y \parallel \frac{Z}{3} = \frac{(5 + j5) \times (5 - j3)}{(5 + j5) + (5 - j3)} = 4.044 + j0.1923$$

Now looking at a single phase and inferring for the others,

$$\begin{aligned} \mathbf{I}_A &= \frac{V_p \angle -30^\circ}{\sqrt{3} Z_{Y_{new}}} = \frac{230 \angle -30^\circ}{\sqrt{3}(4.044 + j0.1923)} = 32.84 \angle -32.73^\circ \text{ A} \\ \mathbf{I}_B &= 32.84 \angle -152.73^\circ \text{ A} \\ \mathbf{I}_C &= 32.84 \angle 87.27^\circ \text{ A} \end{aligned}$$

The complex power is $\mathbf{S} = 3 Z_{Y_{new}} |\mathbf{I}_L|^2 = 13.07 + j0.615 \text{ kVA}$.

SOLUTION 12.22. For this problem we do not need to use any 3-phase knowledge if we are clever. We will provide a clever solution here. $\mathbf{V}_{DN} = \mathbf{V}_{DB} - \mathbf{V}_{NB}$. Using B as the reference we write a node equation at N to obtain:

$$0 = \frac{\mathbf{V}_{NB}}{Z_1} + \frac{\mathbf{V}_{NA}}{Z_1} + \frac{\mathbf{V}_{NC}}{Z_1} = \frac{\mathbf{V}_{NB}}{Z_1} + \frac{\mathbf{V}_{NB} - \mathbf{V}_{AB}}{Z_1} + \frac{\mathbf{V}_{NB} - \mathbf{V}_{CB}}{Z_1}$$

Therefore

$$3\mathbf{V}_{NB} = \mathbf{V}_{AB} + \mathbf{V}_{CB} = 115 - 115 \angle 120^\circ = 172.5 + j99.59$$

From voltage division

$$\mathbf{V}_{DB} = \frac{Z_3}{Z_3 + Z_2} \mathbf{V}_{AB} = \frac{2 + j4}{2 + j4 + 4 - j2} 115 = 57.5 + j57.5 \text{ V}$$

Now using MATLAB we have

```

»115-115*exp(-j*2*pi/3)
ans = 1.7250e+02 + 9.9593e+01i
»VNB =ans/3
VNB = 5.7500e+01 + 3.3198e+01i
»VDB = 115*(2+j*4)/(6+j*2)
VDB = 5.7500e+01 + 5.7500e+01i

»VDN = VDB - VNB
VDN = 7.1054e-15 + 2.4302e+01i

»magVDN = abs(VDN)
magVDN = 2.4302e+01

»angVDN = angle(VDN)*180/pi
angVDN = 9.0000e+01

```

SOLUTION P12.23. First we write the following two loop equations,

$$\mathbf{V}_{sA} - \mathbf{V}_{sA} - 120 = \mathbf{I}_A (Z_g + Z_1 + Z_A) + (\mathbf{I}_C + \mathbf{I}_A)(Z + Z_1 + Z_g)$$

$$\mathbf{V}_{sA} - 120 - \mathbf{V}_{sA} - 120 = (\mathbf{I}_A + 2\mathbf{I}_C)(Z_g + Z_1 + Z)$$

and solve in MATLAB:

(a)

```
%Problem 12.23a
```

```
Zg=0.05+0.15i;
```

```
Z=4+3i;
```

```
Z1=0.1+0.2i;
```

```
ZA=5+2i;
```

```
VA= 120;
```

```
VB= 120*exp(-120*i*pi/180);
```

```
VC= 120*exp(120*i*pi/180);
```

```
% Write mesh equations in matrix form and solve:
```

```
% i.e. [IA;IC] = A - 1*C
```

```
A=[ 2*(Z1+Zg)+ZA+Z Z+Z1+Zg; Zg+Z1+Z 2*(Zg+Z1+Z)];
```

```
Ainv= A^(-1);
```

```
C=[VA-VB;VC-VB];
```

```
B=Ainv*C;
```

```
IA= B(1);
```

```
IC= B(2);
```

```
IB= -IA-IC;
```

```
IA
```

```
abs(IA)
```

```
180*angle(IA)/pi
```

```
IB
```

```
abs(IB)
```

```
180*angle(IB)/pi
```

```
IC
```

```
abs(IC)
```

```
180*angle(IC)/pi
```

```
% And By Ohm's law,
```

```
VAN= IA*ZA
```

```
abs(VAN)
```

```
180*angle(VAN)/pi
```

```
VBN= IB*Z
```

```
abs(VBN)
```

```
180*angle(VBN)/pi
```

```
VCN= IC*Z
```

```
abs(VCN)
```

```
180*angle(VCN)/pi
```

```
IA = 21.76 -29.12° A
```

```
IB = 23.88 -155.60° A
```

```
IC = 20.64 82.39° A
```

$$V_{AN} = 117.20 \angle -7.32^\circ \text{ V}$$

$$V_{BN} = 119.4 \angle -118.73^\circ \text{ V}$$

$$V_{CN} = 103.2 \angle 119.26^\circ \text{ V}$$

(b)

% From phasor diagram

$$V_{AB} = V_{AN} - V_{BN}$$

$$\text{abs}(V_{AB})$$

$$180 * \text{angle}(V_{AB}) / \pi$$

$$V_{BC} = V_{BN} - V_{CN}$$

$$\text{abs}(V_{BC})$$

$$180 * \text{angle}(V_{BC}) / \pi$$

$$V_{CA} = V_{CN} - V_{AN}$$

$$\text{abs}(V_{CA})$$

$$180 * \text{angle}(V_{CA}) / \pi$$

$$V_{AB} = 195.5 \angle 27.34^\circ$$

$$V_{BC} = 194.9 \angle -92.04^\circ$$

$$V_{CA} = 196.99 \angle 147.8^\circ$$

(c) Calculating the average power delivered to each load using the general formula

$$P_{tot} = R|I|^2 = 5 \times (21.76)^2 + 4 \times (23.88)^2 + 4 \times (20.64)^2 = 6.353 \text{ kW}$$

SOLUTION P12.24. Doing this problem in MATLAB,

% Problem 12.24

$$Z_{Dg} = 0.15 + 0.45i;$$

$$Z_D = 12 + 9i;$$

$$Z_1 = 0.1 + 0.2i;$$

$$Z_{DA} = 15 + 6i;$$

% Apply the delta to Y transformation to the load as follows,

$$Z_A = (Z_D * Z_{DA}) / (2 * Z_D + Z_{DA});$$

$$Z_B = (Z_{DA} * Z_D) / (2 * Z_D + Z_{DA});$$

$$Z_C = (Z_D * Z_D) / (2 * Z_D + Z_{DA});$$

% Apply delta to Y trans. to the source,

$$V_1 = 180 / \sqrt{3} * \exp(-30 * i * \pi / 180);$$

$$V_2 = 180 / \sqrt{3} * \exp(-150 * i * \pi / 180);$$

$$V_3 = 180 / \sqrt{3} * \exp(90 * i * \pi / 180);$$

$$Z_g = Z_{Dg} / 3;$$

% Write nodal equations in matrix form and solve

% i.e. $[I_A; I_C] = A^{-1} * C$

$$A = [2 * (Z_1 + Z_g) + Z_A + Z_B \quad Z_B + Z_1 + Z_g; \quad Z_g + Z_1 + Z_B \quad 2 * (Z_g + Z_1) + Z_C + Z_B];$$

$$A_{inv} = A^{-1};$$

$$C = [V_1 - V_2; \quad V_3 - V_2];$$

$$B = A_{inv} * C;$$

$$I_A = B(1);$$

$$I_C = B(2);$$

$$I_B = -I_A - I_C;$$

$$I_A$$

$$\text{abs}(I_A)$$

```

180*angle(IA)/pi
IB
abs(IB)
180*angle(IB)/pi
IC
abs(IC)
180*angle(IC)/pi

```

$$IA = 17.4 \quad -63.1^\circ \text{ A}$$

$$IB = 20.1 \quad 178.9^\circ \text{ A}$$

$$IC = 19.5 \quad 51^\circ \text{ A}$$

(b)

```

% By Ohm's law,
VAN= IA*ZA;
VBN= IB*ZB;
VCN= IC*ZC;
%From phasor diagram
VAB=VAN - VBN
abs(VAB)
180*angle(VAB)/pi
VBC = VBN-VCN
abs(VBC)
180*angle(VBC)/pi
VCA =VCN - VAN
abs(VCA)
180*angle(VCA)/pi

```

$$VAB = 170.40 \quad -2.5^\circ \text{ V}$$

$$VBC = 168.58 \quad -122.4^\circ \text{ V}$$

$$VCA = 169.75 \quad 118.1^\circ \text{ V}$$

(c)

```

% by Ohm's law
IAB=VAB/ZDA
abs(IAB)
180*angle(IAB)/pi
IBC=VBC/ZD
abs(IBC)
180*angle(IBC)/pi
ICA=VCA/ZD
abs(ICA)
180*angle(ICA)/pi

```

$$IAB = 10.55 \quad -24.3^\circ \text{ A}$$

$$IBC = 11.24 \quad -159.3^\circ \text{ A}$$

$$ICA = 11.32 \quad 81.2^\circ \text{ A}$$

(d)

$$P_{tot} = R|\mathbf{I}|^2 = 15 \times (10.55)^2 + 12 \times (11.24)^2 + 12 \times (11.32)^2 = 4.721 \text{ kW}$$

SOLUTION P12.25. The average power that the load draws is $P = 440 \cdot 40 \cdot pf = 8.8 \text{ kW}$. From this we obtain the old and new complex powers:

$$\mathbf{S}_{\text{old}} = 8.8 + j15.242 \text{ kVA} \text{ and } \mathbf{S}_{\text{new}} = 8.8 + j15.242 + jQ_C \text{ kVA}$$

Here the reactance supplied by the capacitor is $Q_C = -\omega C |V_l|^2 = -7299 \text{ var}$. So the new power factor is

$$pf = \cos \tan^{-1} \frac{Q_{\text{new}}}{8800} = 0.742. \text{ From } |\mathbf{S}| = |\mathbf{V}_l| |\mathbf{I}_l| = 11.855 \text{ kVA, we solve for}$$

$$|\mathbf{I}_l| = |\mathbf{I}_A| = |\mathbf{I}_B| = 26.94 \text{ A.}$$

SOLUTION P12.26. Using Ohm's law and the power factor of the load,

$$\mathbf{I}_C = -440 \cdot \omega C \cdot 90^\circ = 16.59 \angle -90^\circ \text{ A, } \mathbf{I}_A = 40 \angle 60^\circ \text{ A, and } \mathbf{I}_B = -\mathbf{I}_C - \mathbf{I}_A = 26.94 \angle -137.9^\circ \text{ A.}$$

SOLUTION P12.27. By simply applying Ohm's law to each load and then KCL at node N, we get the following:

$$\mathbf{I}_A = \frac{\mathbf{V}_A}{50} = 4.4 \text{ A, } \mathbf{I}_B = \frac{\mathbf{V}_B}{50} = 4.4 \angle -120^\circ \text{ A, } \mathbf{I}_C = \frac{\mathbf{V}_C}{250} = 0.88 \angle 120^\circ \text{ A, and}$$

$$\mathbf{I}_N = \mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 3.52 \angle -60^\circ \text{ A.}$$

Finally,

$$P_A = 50 \times (4.4)^2 = 968 \text{ W, } P_B = 50 \times (4.4)^2 = 968 \text{ W, } P_C = 250 \times (0.88)^2 = 193.6 \text{ W.}$$

SOLUTION P12.28. Using MATLAB,

$$VA = 220$$

$$VB = 220 \cdot \exp(-120 \cdot \pi \cdot i / 180);$$

$$VC = 220 \cdot \exp(120 \cdot \pi \cdot i / 180);$$

% Use KVL to solve for the different currents

$$IA = (VA - VB)/50 + (VA - VC)/50$$

$$\text{abs}(IA)$$

$$180 \cdot \text{angle}(IA) / \pi$$

$$IB = (VB - VC)/100 + (VB - VA)/50$$

$$\text{abs}(IB)$$

$$180 \cdot \text{angle}(IB) / \pi$$

$$IC = (VC - VB)/100 + (VC - VA)/50$$

$$\text{abs}(IC)$$

$$180 \cdot \text{angle}(IC) / \pi$$

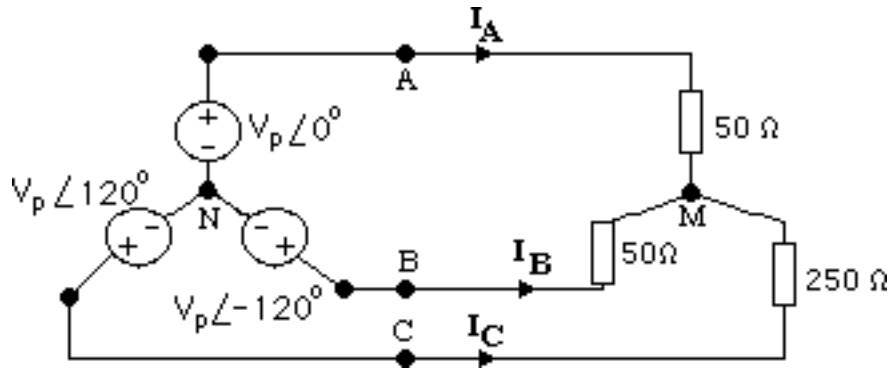
$$IA = 13.2 \text{ A}$$

$$IB = 10.1 \angle -130.9^\circ \text{ A}$$

$$I_C = 10.1 \angle 130.9^\circ \text{ A}$$

The total power delivered to each resistor may be calculated using the following relationship $P = \frac{|V_l|^2}{R}$, where $|V_l|^2 = 3|V_p|^2$ using Y to Delta source transformation. Thus $P_{AB} = 2904 \text{ W}$, $P_{BC} = 1452 \text{ W}$, and $P_{CA} = 2904 \text{ W}$.

SOLUTION P12.29. Consider the circuit of figure 12.29 with two additional node labeling:



Choose N as the reference node and apply KCL to node M. This yields the following node equation:

$$0.02(\mathbf{V}_M - 220) + 0.02 \mathbf{V}_M - 220e^{-j120^\circ} + 0.004 \mathbf{V}_M - 220e^{j120^\circ} = 0$$

Equivalently

$$2.2\mathbf{V}_M = 88 - j152.42 \text{ or } \mathbf{V}_M = 40 - j69.282 \text{ V}$$

The remaining calculations proceed in MATLAB as follows:

```

»% Compute node voltage VM
»X = +220 +220*exp(-j*2*pi/3) + 0.2*220*exp(j*2*pi/3)
X = 8.8000e+01 - 1.5242e+02i
»VM = X/2.2
VM = 4.0000e+01 - 6.9282e+01i

»% Compute line currents
»IA = 0.02*(220 - VM)
IA = 3.6000e+00 + 1.3856e+00i
»magIA = abs(IA)
magIA = 3.8575e+00
»angIA = angle(IA)*180/pi
angIA = 2.1052e+01
»IB = 0.02*(220*exp(-j*2*pi/3) - VM)
IB = -3.0000e+00 - 2.4249e+00i
»magIB = abs(IB)
magIB = 3.8575e+00
»angIB = angle(IB)*180/pi
angIB = -1.4105e+02
»IC = 0.004*(220*exp(j*2*pi/3) - VM)
IC = -6.0000e-01 + 1.0392e+00i
»magIC = abs(IC)

```

```
magIC = 1.2000e+00
»angIC = angle(IC)*180/pi
angIC = 1.2000e+02
```

```
»% Compute total power
»Ptot = 50*magIA^2 + 50*magIB^2 + 250*magIC^2
Ptot = 1.8480e+03
```

SOLUTION P12.30. (a) The equivalent resistance of a 40 ft size #14 wire is

$R_{eq1} = 40 \frac{2.575}{1000} = 0.103 \text{ } \Omega$. The voltage at the light and appliance is then

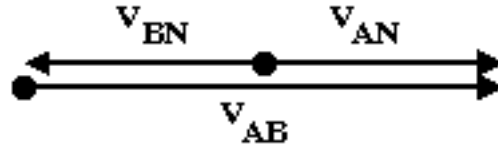
$V_{app} = 115 - 2 R_{eq1} 10 = 112.94 \text{ V}$, which is equivalent to a 1.8% drop, thus the wire gauge is appropriate.

(b) Repeating the previous calculations, $R_{eq2} = 50 \frac{1.619}{1000} = 0.08095 \text{ } \Omega$.

$V_{app} = 115 - 2 R_{eq2} 14 = 112.7334 \text{ V}$, which corresponds to 1.97%. So the wire gauge is appropriate.

(c) Under normal operating condition the current in the live wire should equal the current in the neutral wire; thus no current should be in the ground wire.

(d) Note the following relationship, $V_{AB} = V_{AN} + V_{NB} = V_{AN} - V_{BN}$. This means that $V_{AN} = 115 \text{ V}$ and $V_{BN} = 115 \text{ } 180^\circ \text{ V}$.

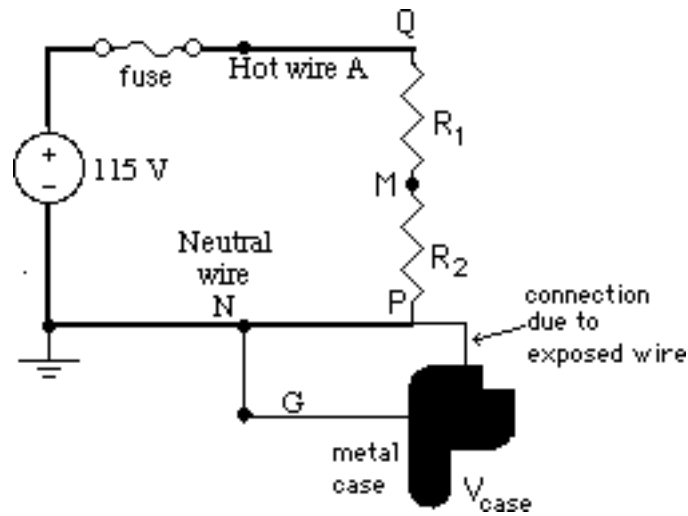


SOLUTION P12.31. Assuming a very large resistance for the person who touches the prong. For prong A, the voltage is approximately $V = 115 \text{ V}$. For prong N, the voltage is approximately $V = 0$. For the ground prong G, no current flows through the wire, so the voltage should be ground or zero.

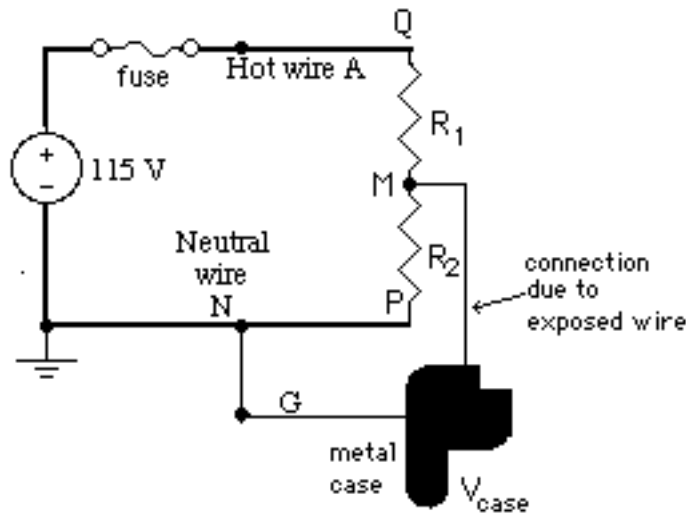
SOLUTION P12.32. For an approximate analysis, we use the circuit models below given the following assumptions:

- (i) The resistance from body to ground is very large (possibly due to rubber shoes);
- (ii) The resistances of all connecting wires are negligible.
- (iii) The motor winding is represented by R_1 in series with R_2 . Here we can further estimate that $R_1 + R_2 = 115/3 = 38.3 \text{ } \Omega$, and $R_1 = R_2 = 19.15 \text{ } \Omega$.
- (iv) The trigger switch of the drill has been depressed.

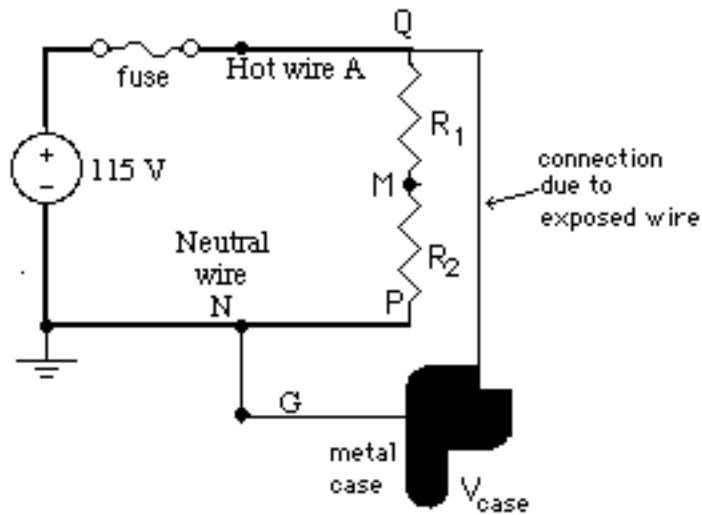
(a) The point P is connected to the metal case as shown below. It is obvious that $V_{case} = V_p = 0$.



(b) The point M is connected to the metal case as shown below. The current through the hot wire is $115/19.15 = 6$ A. If the fuse capacity is smaller than 6 A, then it will blow and $V_{case} = V_M = 0$. On the other hand, if the fuse capacity is greater than 6 A, then R_2 is nearly shorted and $V_{case} = V_M = 0$.



(c) The point Q is connected to the metal case as shown below.



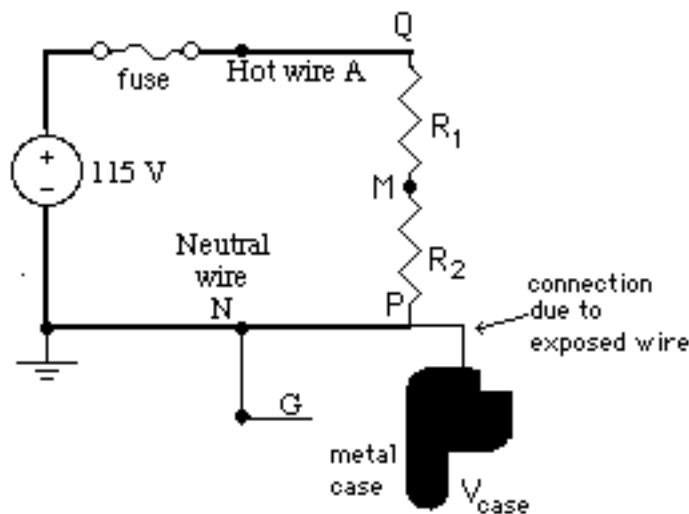
Both R_1 and R_2 are nearly shorted. The line current is so large that it will blow the fuse. Hence $V_{\text{case}} = V_Q = 0$.

CONCLUSION: In all three cases, the person touching the metal case of the defective appliance will experience zero or a very low voltage. The circuit is safe.

SOLUTION P12.33. For an approximate analysis, we use the circuit models below given the following assumptions:

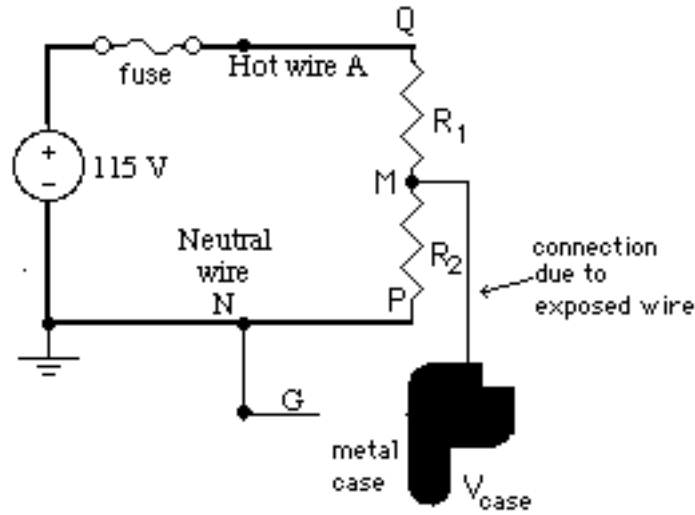
- (i) The resistance from body to ground is very large (possibly due to rubber shoes);
- (ii) The resistances of all connecting wires are negligible.
- (iii) The motor winding is represented by R_1 in series with R_2 . Here we can further estimate that. $R_1 + R_2 = 115/3 = 38.3 \ \Omega$, and $R_1 = R_2 = 19.15 \ \Omega$.
- (iv) The trigger switch of the drill has been depressed.

(a) The point P is connected to the metal case as shown below. It is obvious $V_{\text{case}} = V_P = 0$.

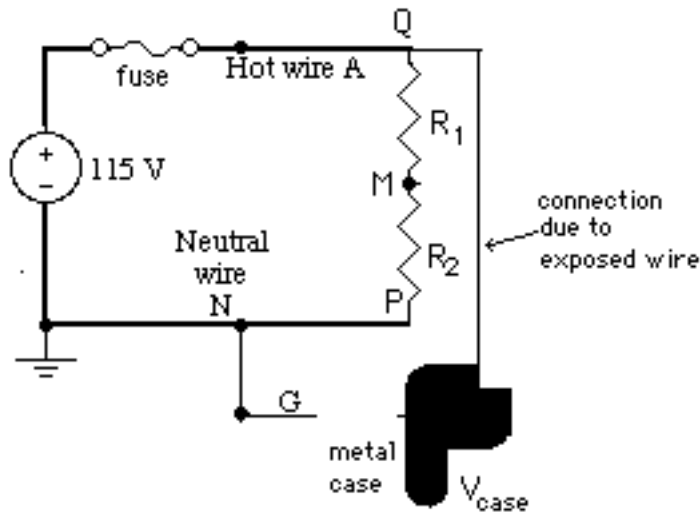


(b) The point M is connected to the metal case as shown below. Simple voltage divider action leads to

$$V_{\text{case}} = V_M = 0.5 \times 115 = 57.5 \text{ V}$$



(c) The point Q is connected to the metal case as shown below.



It is obvious that $V_{\text{case}} = V_Q = 115 \text{ V}$.

CONCLUSION: In all three cases, the fuse will not blow. For cases 2 and 3, the voltage appearing on the metal case may cause serious injury to the person.