

SOLUTION 16.31.

In order to compute the area beneath $v(t-\tau)h(\tau)$ seven regions will be considered: $t<0$, $0 \le t < 1$, $1 \le t < 2$, $2 \le t < 3$, $3 \le t < 4$, $4 \le t < 5$ and $5 \le t$.

Step 1: $t<0$. For t in this region $v(t-\tau)h(\tau)=0$ for all τ. Hence

$$y(t) = v(t) \quad h(t) = 0 \text{ for } t<0.$$

Step 2: $0 \le t < 1$. In this case $v(t-\tau)h(\tau) = v0 \times h0$ for $0 \le \tau \le t$ and is zero otherwise. Therefore the area beneath $v(t-\tau)h(\tau)$ equals

$$y(t) = v(t) \quad h(t) = v0 \times h0 \times t \text{ for } 0 \le t < 1.$$

Step 3: $1 \le t < 2$. For t in this region we have

$$v(t-\tau)h(\tau) = \begin{cases} v1 \times h0, & 0 \le \tau \le t-1 \\ v0 \times h0, & t-1 < \tau < 1 \\ v0 \times h1, & 1 \le \tau < t \\ 0, & \text{otherwise} \end{cases}$$

Therefore the area beneath $v(t-\tau)h(\tau)$ equals

$$y(t) = v(t) \quad h(t) = v1 \times h0 \times \left[(t-1)-0\right] + v0 \times h0 \times \left[1-(t-1)\right] + v0 \times h1 \times (t-1) =$$
$$= t \times (v1 \times h0 - v0 \times h0 + v0 \times h1) - v1 \times h0 + 2 \times v0 \times h0 - v0 \times h1, \text{ for } 1 \le t < 2$$

Step 4: $2 \le t < 3$. In this case

$$v(t-\tau)h(\tau) = \begin{cases} v1 \times h0, & t-2 < \tau < 1 \\ v1 \times h1, & 1 \le \tau \le t-1 \\ v0 \times h1, & t-1 < \tau < 2 \\ v0 \times h2, & 2 \le \tau < t \\ 0, & \text{otherwise} \end{cases}$$

Hence, for $2 \le t < 3$,

$$y(t) = v(t) \quad h(t) =$$
$$= v1 \times h0 \times \left[1-(t-2)\right] + v1 \times h1 \times \left[(t-1)-1\right] + v0 \times h1 \times \left[2-(t-1)\right] + v0 \times h2 \times (t-2) =$$
$$= t \times (-v1 \times h0 + v1 \times h1 - v0 \times h1 + v0 \times h2) + 3 \times v1 \times h0 - 2 \times v1 \times h1 + 3 \times v0 \times h1 - 2 \times v0 \times h2$$

Step 5: $3 \le t < 4$. In this case

$$v(t-\tau)h(\tau) = \begin{cases} v1 \times h1, & t-2 < \tau < 2 \\ v1 \times h2, & 2 \le \tau \le t-1 \\ v0 \times h2, & t-1 < \tau < 3 \\ 0, & \text{otherwise} \end{cases}$$

Hence, for $3 \le t < 4$,

$$y(t) = v(t) \quad h(t) =$$
$$= v1 \times h1 \times \left[2-(t-2)\right] + v1 \times h2 \times \left[(t-1)-2\right] + v0 \times h2 \times \left[3-(t-1)\right] =$$
$$= t \times (-v1 \times h1 + v1 \times h2 - v0 \times h2) + 4 \times v1 \times h1 - 3 \times v1 \times h2 + 4 \times v0 \times h2$$

Step 6: $4 \le t < 5$. In this case $v(t-\tau)h(\tau) = v1 \times h2$ for $t-2 < \tau < 3$ and is zero otherwise. Therefore

$$y(t) = v(t) \quad h(t) = v1 \times h2 \times \left[3-(t-2)\right] = v1 \times h2 \times (5-t) \text{ for } 4 \le t < 5.$$

Step 7: $5 \le t$. For t in this region $v(t-\tau)h(\tau) = 0$ for all τ. Hence

$$y(t) = v(t) \quad h(t) = 0 \text{ for } 5 \le t.$$

In sum,

$$y(t) = \begin{cases} v0 \times h0 \times t, & 0 \le t < 1 \\ t \times (v1 \times h0 - v0 \times h0 + v0 \times h1) - v1 \times h0 + 2 \times v0 \times h0 - v0 \times h1, & 1 \le t < 2 \\ t \times (-v1 \times h0 + v1 \times h1 - v0 \times h1 + v0 \times h2) + 3 \times v1 \times h0 - 2 \times v1 \times h1 + 3 \times v0 \times h1 - 2 \times v0 \times h2, & 2 \le t < 3 \\ t \times (-v1 \times h1 + v1 \times h2 - v0 \times h2) + 4 \times v1 \times h1 - 3 \times v1 \times h2 + 4 \times v0 \times h2, & 3 \le t < 4 \\ v1 \times h2 \times (5 - t), & 4 \le t < 5 \\ 0, & \text{otherwise} \end{cases}$$

Hence,

$$y_1 = y(1) = v0 \times h0 = 6$$
$$y_2 = y(2) = v0 \times h1 + v1 \times h0 = 8$$
$$y_3 = y(3) = v0 \times h2 + v1 \times h1 = -6$$
$$y_4 = y(4) = v1 \times h2 = 4$$

(b) Using the expressions of $p(x)$ and $q(x)$ it follows that

$$p(x) \times q(x) = x^3 \times (v0 \times h0) + x^2 \times (v0 \times h1 + v1 \times h0) + x \times (v0 \times h0 + v1 \times h1) + v1 \times h2$$

We observe that the coefficients of $p(x) \times q(x)$ are exactly y_1, y_2, y_3 and y_4, respectively. Therefore

$$r(x) = p(x) \times q(x).$$

SOLUTION 16.32.

(a) This part will be solved using the techniques of convolution algebra. Therefore we can write $f_3(t)$ as

$$f_3(t) = f_1^{(-1)}(t) \quad f_2^{(1)}(t)$$

Where the superscript (-1) means integration and the superscript (1) means differentiation. From figure P16.32 we observe that

$$f_1(t) = 4\left[u(t) - u(t-4)\right]$$

Hence

$$f_1^{(-1)}(t) = 4\left[tu(t) - (t-4)u(t-4)\right] =$$
$$= 4\left[r(t) - r(t-4)\right]$$

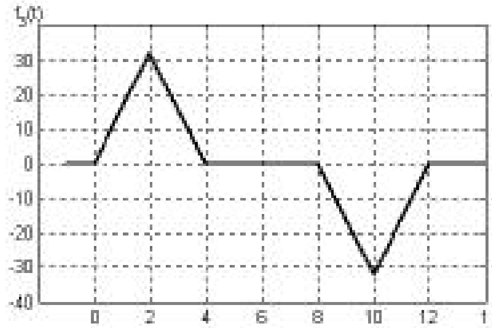
By inspection, from the same figure, we have

$$f_2^{(1)}(t) = 4\left[\delta(t) - 2\delta(t-2) + 2\delta(t-4) - 2\delta(t-6) + \delta(t-8)\right]$$

Using the sifting property of the delta function $f_3(t)$ can be computed as follows

$$f_3(t) = \left\{4\left[r(t) - r(t-4)\right]\right\} \quad \left\{4\left[\delta(t) - 2\delta(t-2) + 2\delta(t-4) - 2\delta(t-6) + \delta(t-8)\right]\right\} =$$
$$= 16\left[r(t) - 2r(t-2) + r(t-4) - r(t-8) + 2r(t-10) - r(t-12)\right]$$

A picture of $f_3(t)$ is sketched in the next figure.



(b) Using the techniques of problem 16.31 and considering the time step tstep = 2, the polynomials $p(x)$, $q(x)$ and $r(x)$ can be associated with the functions $f_1(t)$, $f_2(t)$ and $f_3(t)$, respectively, as below:

$$p(x) = 4x + 4$$
$$q(x) = 4x^3 - 4x^2 + 4x - 4$$
$$r(x) = 32x^4 - 32$$

We need to verify that the equality

$$p(x)\ \ q(x)\ \ \text{tstep} = r(x)$$

holds. The equality indeed holds because

$$p(x)\ \ q(x)\ \ \text{tstep} = 32(x+1)(x^3 - x^2 + x - 1) = 32(x^4 - 1) = r(x).$$

The results obtained in part (a) and part (b) coincide.

SOLUTION 16.33.

(a) Using the techniques of convolution algebra $f_3(t)$ can be written as

$$f_3(t) = f_1^{(-1)}(t) \quad f_2^{(1)}(t)$$

Where the superscript (-1) means integration and the superscript (1) means differentiation. From figure P16.33 we observe that

$$f_1(t) = 2\left[u(t+1) - u(t-4)\right]$$

Therefore

$$f_1^{(-1)}(t) = 4\left[(t+1)u(t+1) - (t-4)u(t-4)\right] =$$
$$= 4\left[r(t+1) - r(t-4)\right] = g(t)$$

By inspection, from the same figure, we have

$$f_2^{(1)}(t) = 4\delta(t) - 8\delta(t-2) + 6\delta(t-4) - 2\delta(t-6)$$

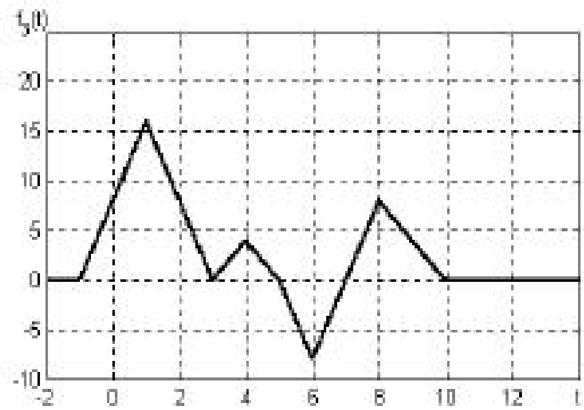
Using the sifting property of the delta function $f_3(t)$ can be computed as follows

$$f_3(t) = 4g(t) - 8g(t-2) + 6g(t-4) - 2g(t-6)$$

This is plotted in MATLAB as follows:

```
>> t = -2:0.01:14;
>> g = 2*(t+1).*u(t+1)-2*(t-4).*u(t-4);
>> g1 = 4*g;
>> g2 = -8*( 2*(t-1).*u(t-1)-2*(t-6).*u(t-6) );
>> g3 = 6*( 2*(t-3).*u(t-3)-2*(t-8).*u(t-8) );
>> g4 = -2*( 2*(t-5).*u(t-5)-2*(t-10).*u(t-10) );
>> f3 = g1+g2+g3+g4;
>> plot(t,f3);
>> grid;
```

A picture of $f_3(t)$ is sketched in the next figure.

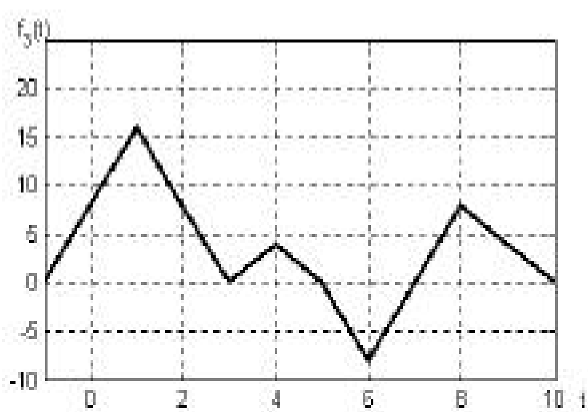


(b) To account for the fact that $f_1(t)$ is nonzero for negative t the following formula (see problem 16.22)

$$f_1(t) \quad f_2(t) = \left[f_1(t-1) \quad f_2(t) \right]_{t=t+1}$$

will be used to compute $f_3(t)$. Using a slightly modified version of the code of problem 16.31, we have

```
>> f1 = [2, 2, 2, 2, 2];
>> f2 = [4, 4, -4, -4, 2, 2];
>> T = 1;
>> tstep = T;
>> f3 = tstep*conv(f1,f2);
>> f3 = [0 f3 0];
>> t = -1:tstep:tstep*(length(f1)+length(f2))-1;
>> plot(t,f3)
>> grid
```



The results of parts (a) and (b) coincide.

SOLUTION 16.34. This problem is solved using the techniques of the convolution algebra with the graphical method left to the student.

$$f_3(t) = f_1(t) * f_2(t) = \left[f_1(t)\right]^{(-1)} * \left[f_2(t)\right]^{(1)}$$

where the superscript (-1) means integration and the superscript (1) means differentiation. By inspection,

$$\left[f_1(t)\right]^{(-1)} = 4tu(t) - 4(t-6)u(t-6) = g(t)$$

and

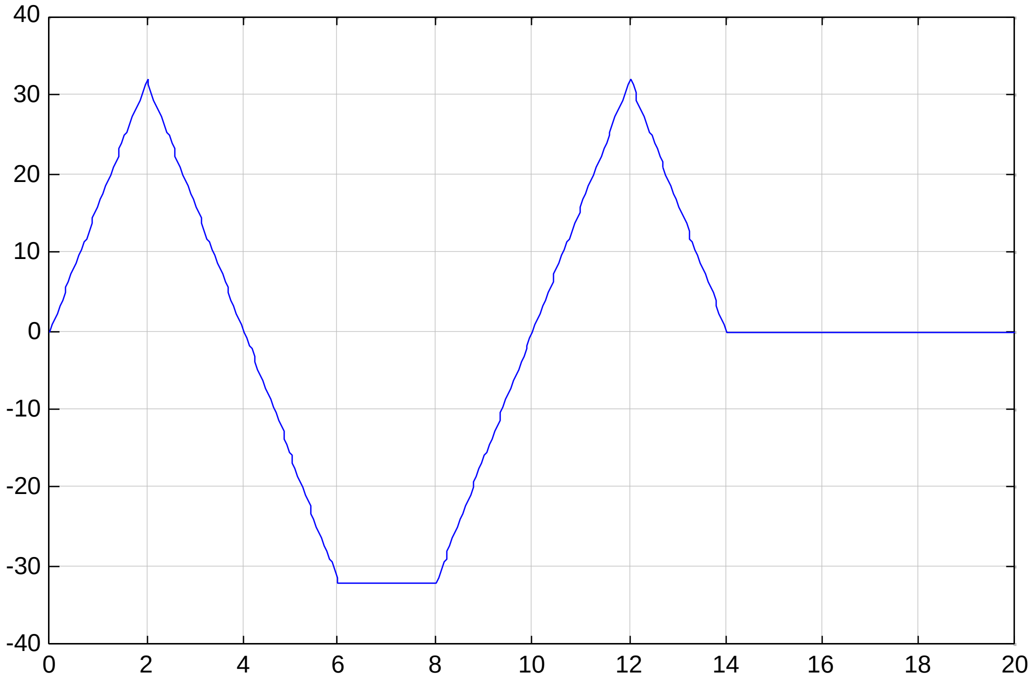
$$\left[f_2(t)\right]^{(1)} = 4\delta(t) - 8\delta(t-2) + 8\delta(t-6) - 4\delta(t-8)$$

Hence the response say y(t) satisfies

$$f_3(t) = 4g(t) - 8g(t-2) + 8g(t-6) - 4g(t-8)$$

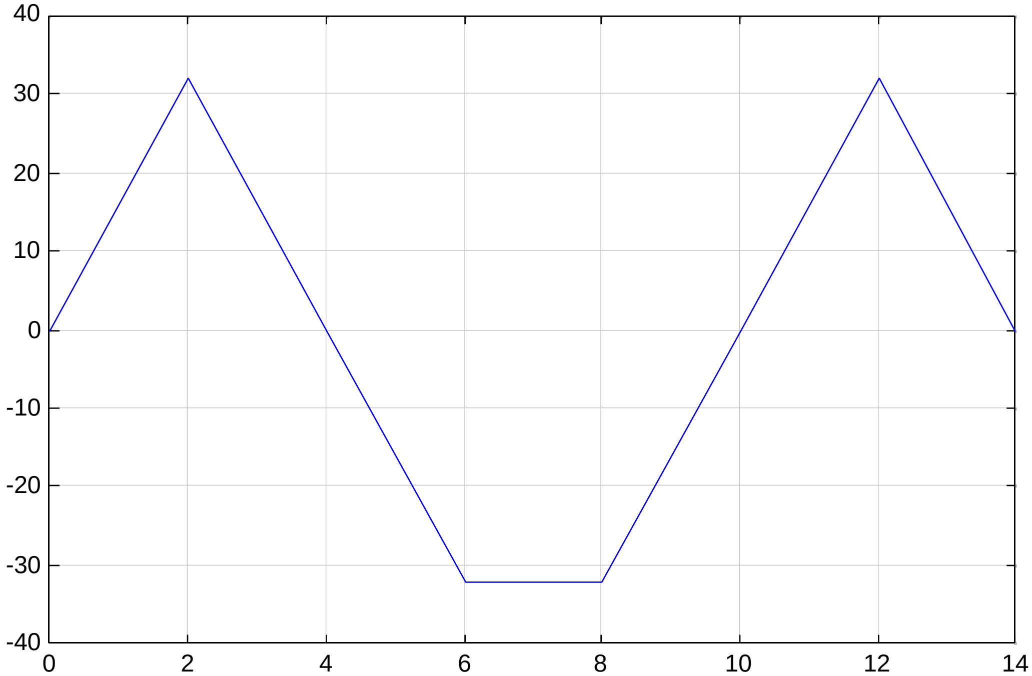
This is plotted in MATLAB as follows:

```
»t=0:.05:20;
»g = 4*t .* u(t) - 4*(t-6) .*u(t-6);
»g1=4*g;
»g2 = -8*(4*(t-2) .* u(t-2) - 4*(t-8) .*u(t-8));
»g3 = 8*(4*(t-6) .* u(t-6) - 4*(t-12) .*u(t-12));
»g4 = -4*(4*(t-8) .* u(t-8) - 4*(t-14) .*u(t-14));
»f3 = g1+g2+g3+g4;
»plot(t,f3)
»grid
```



(b) Using the code of problem 31, we have

```
»f1 = [4   4   4];
»f2 = [4   -4   -4   4];
»T = 2;
»tstep = T;
»f3 = [0    conv(f1,f2)*tstep      0];
»t = 0: tstep : tstep* (length(f1) + length(f2));
»plot(t,f3)
»grid
```



The results of parts (a) and (b) coincide.

SOLUTION 16.35.

(a) In order to compute the area beneath $f_1(t-\tau)\ \ f_1(\tau)$ four regions will be considered: $t<0$, $0 \le t < 1$, $1 \le t < 2$, and $2 \le t$.

Step 1: $t<0$. Here $f_1(t-\tau)\ \ f_1(\tau) = 0$ for all τ. Hence

$$f_3(t) = f_1(t)\ \ f_1(t) = 0 \text{ for } t<0.$$

Step 2: $0 \le t < 1$. In this case $f_1(t-\tau)\ \ f_1(\tau) = 1$ for $0 \le \tau \le t$ and is zero otherwise. Therefore the area beneath $f_1(t-\tau)\ \ f_1(\tau)$ equals

$$f_3(t) = t \text{ for } 0 \le t < 1.$$

Step 3: $1 \le t < 2$. In this case $f_1(t-\tau)\ \ f_1(\tau) = 1$ for $t-1 < \tau < 1$ and is zero otherwise. Therefore the area beneath $f_1(t-\tau)\ \ f_1(\tau)$ equals

$$f_3(t) = 1-(t-1) = 2-t \text{ for } 1 \le t < 2.$$

Step 4: $2 \le t$. Here $f_1(t-\tau)\ \ f_1(\tau) = 0$ for all τ. Hence

$$f_3(t) = f_1(t)\ \ f_1(t) = 0 \text{ for } 2 \le t.$$

In sum,

$$f_3(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

(b) In order to compute the area beneath $f_1(t-\tau)\ \ f_2(\tau)$ four regions will be considered: $t<0$, $0\le t<1$, $1\le t<2$, and $2\le t$.
Step 1: $t<0$. Here $f_1(t-\tau)\ \ f_2(\tau)=0$ for all τ. Hence

$$f_4(t)=f_1(t)\ \ f_2(t)=0 \text{ for } t<0.$$

Step 2: $0\le t<1$. In this case $f_1(t-\tau)\ \ f_2(\tau)=\tau$ for $0\le\tau\le t$ and is zero otherwise. Therefore the area beneath $f_1(t-\tau)\ \ f_2(\tau)$ equals

$$f_4(t)=0.5t^2 \text{ for } 0\le t<1.$$

Step 3: $1\le t<2$. For t in this region $f_1(t-\tau)\ \ f_2(\tau)=\tau$ for $t-1<\tau<1$ and is zero otherwise. Therefore the area beneath $f_1(t-\tau)\ \ f_2(\tau)$ equals

$$f_4(t)=0.5-0.5t^2 \text{ for } 1\le t<2.$$

Step 4: $2\le t$. Here $f_1(t-\tau)\ \ f_2(\tau)=0$ for all τ. Hence

$$f_4(t)=f_1(t)\ \ f_1(t)=0 \text{ for} 2\le t.$$

In sum,

$$f_4(t)=\begin{cases}0.5t^2, & 0\le t<1\\ 0.5-0.5t^2, & 1\le t<2\\ 0, & \text{otherwise}\end{cases}$$

(c) In order to compute the area beneath $f_2(t-\tau)\ \ f_2(\tau)$ four regions will be considered: $t<0$, $0\le t<1$, $1\le t<2$, and $2\le t$.

Step 1: $t<0$. Here $f_2(t-\tau)\ \ f_2(\tau)=0$ for all τ. Hence

$$f_5(t)=f_2(t)\ \ f_2(t)=0 \text{ for } t<0.$$

Step 2: $0\le t<1$. In this case $f_2(t-\tau)\ \ f_2(\tau)=(t-\tau)\tau$ for $0\le\tau\le t$ and is zero otherwise. Therefore the area beneath $f_2(t-\tau)\ \ f_2(\tau)$ equals

$$f_5(t)=\int_0^t (t-\tau)\tau d\tau=\left[-0.3333\tau^3+0.5t\tau^2\right]_0^t=0.1667t^3 \text{ for } 0\le t<1.$$

Step 3: $1\le t<2$. For t in this region $f_2(t-\tau)\ \ f_2(\tau)=(t-\tau)\tau$ for $t-1<\tau<1$ and is zero otherwise. Therefore the area beneath $f_2(t-\tau)\ \ f_2(\tau)$ equals

$$f_5(t)=\int_{t-1}^{1}(t-\tau)\tau d\tau=\left[-0.3333\tau^3+0.5t\tau^2\right]_{t-1}^{1}$$
$$=-0.1667t^3+t-0.6667 \text{ for } 1\le t<2.$$

Step 4: $2\le t$. Here $f_2(t-\tau)\ \ f_2(\tau)=0$ for all τ. Hence

$$f_5(t) = f_2(t) \quad f_2(t) = 0 \text{ for } 2 \le t.$$

In sum,

$$f_5(t) = \begin{cases} 0.1667t^3, & 0 \le t < 1 \\ -0.1667t^3 + t - 0.6667, & 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

SOLUTION 16.36.

In order to compute the area beneath $f_1(t-\tau) \quad f_2(\tau)$ five regions will be considered: $t < 0$, $0 \le t < 1$, $1 \le t < 2$, $2 \le t < 3$ and $3 \le t$.

Step 1: $t < 0$. Here $f_1(t-\tau) \quad f_2(\tau) = 0$ for all τ. Hence

$$f_3(t) = f_1(t) \quad f_2(t) = 0 \text{ for } t < 0.$$

Step 2: $0 \le t < 1$. In this case $f_1(t-\tau) \quad f_2(\tau) = \tau$ for $0 \le \tau \le t$ and is zero otherwise. Therefore the area beneath $f_1(t-\tau) \quad f_2(\tau)$ equals

$$f_3(t) = 0.5t^2 \text{ for } 0 \le t < 1.$$

Step 3: $1 \le t < 2$. For t in this region

$$f_1(t-\tau) \quad f_2(\tau) = \begin{cases} \tau, & t-1 < \tau < 1 \\ 2-\tau, & 1 \le \tau \le t \\ 0, & \text{otherwise} \end{cases}$$

Therefore the area beneath $f_1(t-\tau) \quad f_2(\tau)$ equals

$$f_3(t) = \left[0.5 - 0.5(t-1)^2\right] + \left[0.5 - 0.5(2-t)^2\right]$$
$$= -t^2 + 3t - 1.5 \text{ for } 1 \le t < 2.$$

Step 4: $2 \le t < 3$. For t in this region $f_1(t-\tau) \quad f_2(\tau) = 2-\tau$ for all $t-1 < \tau < 2$. Hence

$$f_3(t) = f_1(t) \quad f_2(t) = 0.5 - 0.5(2-t)^2 = -0.5t^2 + 2t - 1.5 \text{ for } 2 \le t < 3.$$

Step 5: $3 \le t$. Here $f_1(t-\tau) \quad f_2(\tau) = 0$ for all τ. Hence

$$f_3(t) = f_1(t) \quad f_2(t) = 0 \text{ for } 3 \le t.$$

In sum,

$$f_3(t) = \begin{cases} 0.5t^2, & 0 \le t < 1 \\ -t^2 + 3t - 1.5, & 1 \le t < 2 \\ -0.5t^2 + 2t - 1.5, & 2 \le t < 3 \\ 0, & \text{otherwise} \end{cases}$$

SOLUTION 16.37.

(a) In order to compute the area beneath $f_1(\tau)\ \ f_2(t-\tau)$ three regions will be considered: $t<0$, $0 \le t < 2$, and $2 \le t$.

Step 1: $t<0$. Here $f_1(\tau)\ \ f_2(t-\tau) = 0$ for all τ. Hence

$$f_3(t) = f_1(t)\ \ \ f_2(t) = 0 \text{ for } t<0.$$

Step 2: $0 \le t < 2$. In this case $f_1(\tau)\ \ f_2(t-\tau) = 8\tau$ for $0 \le \tau \le t$ and is zero otherwise. Therefore the area beneath $f_1(\tau)\ \ f_2(t-\tau)$ equals

$$f_3(t) = 0.5(t\ \ 8t) = 4t^2 \text{ for } 0 \le t < 2.$$

Step 3: $2 \le t$. For t in this region

$$f_1(\tau)\ \ f_2(t-\tau) = \begin{cases} 8\tau, & 0 \le \tau < 2 \\ 16, & 2 \le \tau < t \\ 0, & \text{otherwise} \end{cases}$$

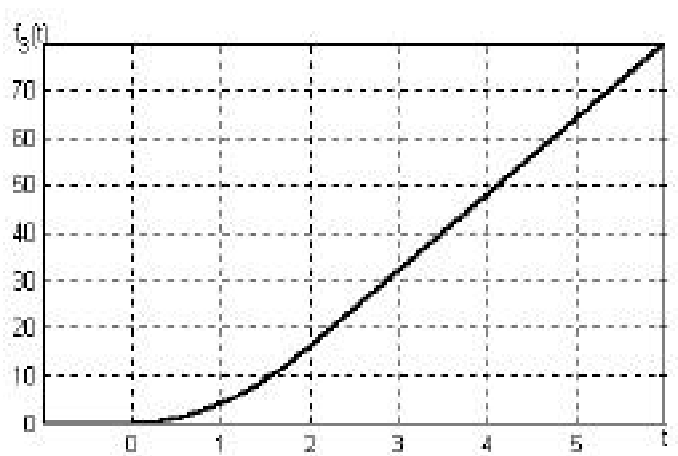
Therefore the area beneath $f_1(\tau)\ \ f_2(t-\tau)$ equals

$$f_3(t) = 16 + 16(t-2) = 16(t-1) \text{ for } 2 \le t.$$

In sum,

$$f_3(t) = \begin{cases} 4t^2, & 0 \le t < 2 \\ 16(t-1), & 2 \le t \\ 0, & \text{otherwise} \end{cases}$$

A picture of $f_3(t)$ is sketched in the next figure.



(b) From figure P16.37 we observe that

$$f_1(t) = 2tu(t) - 2(t-2)u(t-2)$$

From table 13.1 and the time shift property of the Laplace transform it follows that

$$F_1(s) = \frac{2}{s^2}\left(1 - e^{-2s}\right)$$

$$F_2(s) = \frac{4}{s}$$

By the convolution theorem

$$F_3(s) = F_1(s)F_2(s)$$

Therefore

$$F_3(s) = \frac{8}{s^3}\left(1 - e^{-2s}\right)$$

Taking the inverse Laplace transform yields

$$f_3(t) = 4t^2 u(t) - 4(t-2)^2 u(t-2)$$

The results of parts (a) and (b) coincide.

SOLUTION 16.38.

(a) In order to compute the area beneath $f_1(t-\tau)\ \ f_2(\tau)$ six regions will be considered: $t < 0$, $0 \le t < 2$, $2 \le t < 6$, $6 \le t < 8$, $8 \le t < 10$, and $10 \le t$.

Step 1: $t < 0$. Here $f_1(t-\tau)\ \ f_2(\tau) = 0$ for all τ. Hence

$$f_3(t) = f_1(t)\ \ \ f_2(t) = 0 \text{ for } t < 0.$$

Step 2: $0 \le t < 2$. In this case $f_1(t-\tau)\ \ f_2(\tau) = 8(t-\tau)$ for $0 \le \tau \le t$ and is zero otherwise. Therefore the area beneath $f_1(\tau)\ \ f_2(t-\tau)$ equals

$$f_3(t) = 0.5(t\ \ 8t) = 4t^2 \text{ for } 0 \le t < 2.$$

Step 3:$2 \le t < 6$. For t in this region

$$f_1(t-\tau)\ \ f_2(\tau) = \begin{cases} 16, & 0 \le \tau < t-2 \\ 8(t-\tau), & t-2 \le \tau < t \\ 0, & \text{otherwise} \end{cases}$$

Therefore the area beneath $f_1(t-\tau)\ \ f_2(\tau)$ equals

$$f_3(t) = 16 + 16(t-2) = 16(t-1) \text{ for } 2 \le t < 6.$$

Step 4: $6 \le t < 8$. For t in this region

$$f_1(t-\tau)\ \ f_2(\tau) = \begin{cases} 16, & 0 \le \tau < t-2 \\ 8(t-\tau), & t-2 \le \tau < 6 \\ -8(t-\tau), & 6 \le \tau < t \\ 0, & \text{otherwise} \end{cases}$$

Therefore, for $6 \le t < 8$, the area beneath $f_1(t-\tau)\ \ f_2(\tau)$ equals

$$f_3(t) = 16(t-2) + \left[16 - \frac{8(t-6)^2}{2}\right] - \frac{8(t-6)^2}{2}$$

$$= -8(t-6)^2 + 16t - 16 = -8t^2 + 112t - 304$$

Step 5: $8 \le t < 10$. For t in this region

$$f_1(t-\tau)\ f_2(\tau) = \begin{cases} 16, & 0 \le \tau < 6 \\ -16, & 6 \le \tau < t-2 \\ -8(t-\tau), & t-2 \le \tau < 8 \\ 0, & \text{otherwise} \end{cases}$$

Therefore, for $8 \le t < 10$, the area beneath $f_1(t-\tau)\ f_2(\tau)$ equals

$$f_3(t) = 96 - 16(t-8) - \left[16 - \frac{8(t-8)^2}{2}\right] = 4t^2 - 80t + 464$$

Step 6: $10 \le t$. For t in this region

$$f_1(t-\tau) f_2(\tau) = \begin{cases} 16, & 0 \le \tau < 6 \\ -16, & 6 \le \tau < 8 \\ 0, & \text{otherwise} \end{cases}$$

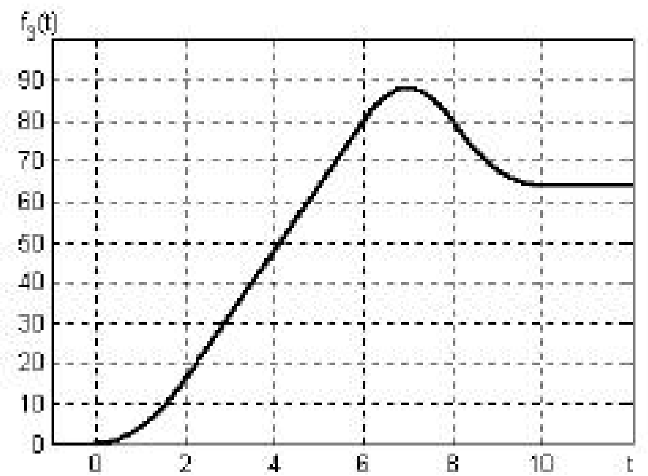
Therefore,

$$f_3(t) = 96 - 32 = 64 \text{ for } 10 \le t.$$

In sum,

$$f_3(t) = \begin{cases} 0, & t < 0 \\ 4t^2, & 0 \le t < 2 \\ 16t - 16, & 2 \le t < 6 \\ -8t^2 + 112t - 304, & 6 \le t < 8 \\ 4t^2 - 80t + 464, & 8 \le t < 10 \\ 64, & 10 \le t \end{cases}$$

A picture of $f_3(t)$ is sketched in the next figure.



(b) From figure P16.38 we observe that

$$f_2(t) = 4\left[u(t) - u(t-6)\right] - 4\left[u(t-6)u - (t-8)\right]$$
$$= 4u(t) - 8u(t-6) + 4u(t-8)$$

From table 13.1 and the time shift property of the Laplace transform it follows that

$$F_2(s) = \frac{4}{s} - \frac{8}{s}e^{-6s} + \frac{4}{s}e^{-8s}$$

From problem 16.37 we have that

$$F_1(s) = \frac{2}{s^2}\left(1 - e^{-2s}\right)$$

By the convolution theorem

$$F_3(s) = F_1(s)F_2(s)$$

Therefore

$$F_3(s) = \frac{8}{s^3} - \frac{8}{s^3}e^{-2s} - \frac{16}{s^3}e^{-6s} + \frac{24}{s^3}e^{-8s} - \frac{8}{s^3}e^{-10s}$$

Taking the inverse Laplace transform yields

$$f_3(t) = 4t^2u(t) - 4(t-2)^2u(t-2) - 8(t-6)^2u(t-6)$$
$$+12(t-8)^2u(t-8) - 4(t-10)^2u(t-10)$$

The results of parts (a) and (b) coincide.

SOLUTION 16.39.

$p(t) \quad q(t)$ will be computed using the techniques of convolution algebra. Therefore we can write

$$p(t) \quad q(t) = p^{(-1)}(t) \quad q^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation. By inspection

$$p(t) = (t+4)\left[u(t+4) - u(t)\right] + (-t+4)\left[u(t) - u(t-4)\right] + 4\left[u(t-4) - u(t-8)\right] =$$
$$= (t+4)u(t+4) + (-2t)u(t) + tu(t-4) - 4u(t-8)$$

Therefore,

$$p^{(-1)}(t) = 0.5(t+4)^2u(t+4) - t^2u(t) + (0.5t^2 - 8)u(t-4) - (4t - 32)u(t-8)$$

By inspection we also have

$$q^{(1)}(t) = 4\delta(t)$$

By the sifting property of the delta function it follows that

$$p(t) \quad q(t) = 4p^{(-1)}(t) =$$
$$= 2(t+4)^2u(t+4) - 4t^2u(t) + 2(t^2 - 16)u(t-4) - 16(t-8)u(t-8).$$

SOLUTION 16.40.

(a) First observe that

$$h(t) = 0.1u(t-0.1) + 0.2u(t-0.2) + 0.2u(t-0.3) + 0.2u(t-0.4) + 0.2u(t-0.5) +$$
$$+0.1u(t-0.6) - 0.1u(t-1) - 0.2u(t-1.3) - 0.2u(t-1.5) -$$
$$-0.2u(t-1.7) - 0.2u(t-2) - 0.1u(t-2.2)$$

Due to the fact that $h(t)$ is a linear combination of terms of the type $Ku(t-T)$, the convolution of $h(t)$ and $v_{in}(t)$ reduces to a linear combination of terms of the following type: $\left[K_1 u(t)\right] \quad \left[K_2 u(t-T)\right]$. Using the definition of the convolution, the previous convolution product is computed below

$$\left[K_1 u(t)\right] \quad \left[K_2 u(t-T)\right] = K_1 K_2 \int_{-\infty}^{\infty} u(\tau)u(t-\tau-T)d\tau =$$

$$= K_1 K_2 \int_0^{\infty} u(t-\tau-T)d\tau = \begin{cases} 0, & t < T \\ K_1 K_2 \int_0^{t-T} d\tau, & T \le t \end{cases}$$

$$= \begin{cases} 0, & t < T \\ K_1 K_2 (t-T), & T \le t \end{cases} = K_1 K_2 (t-T)u(t-T)$$

Therefore $v_{out}(t)$ is a linear combination of functions of type $K_1 K_2 (t-T)u(t-T)$,

$$v_{out}(t) = h(t) \quad v_{in}(t) =$$
$$= 10(t-0.1)u(t-0.1) + 20(t-0.2)u(t-0.2) + 20(t-0.3)u(t-0.3) + 20(t-0.4)u(t-0.4) +$$
$$+20(t-0.5)u(t-0.5) + 10(t-0.6)u(t-0.6) - 10(t-1)u(t-1) - 20(t-1.3)u(t-1.3) -$$
$$-20(t-1.5)u(t-1.5) - 20(t-1.7)u(t-1.7) - 20(t-2)u(t-2) - 10(t-2.2)u(t-2.2)$$

At $t = 0s$

$$v_{out}(0) = 0 \text{ V.}$$

At $t = 0.5s$

$$v_{out}(0.5) = 16 \text{ V.}$$

At $t = 1s$

$$v_{out}(1) = 65 \text{ V.}$$

At $t = 1.5s$

$$v_{out}(1.5) = 106 \text{ V.}$$

(b) In this case $v_{out}(t)$ will be computed using the techniques of the convolution algebra. Hence we have

$$v_{out}(t) = v_{in}(t) \quad h(t) = v_{in}^{(-1)}(t) \quad h^{(1)}(t)$$
$$= \left[50t^2 u(t)\right] \quad [0.1\delta(t-0.1) + 0.2\delta(t-0.2) + 0.2\delta(t-0.3) + 0.2\delta(t-0.4) +$$
$$+0.2\delta(t-0.5) + 0.1\delta(t-0.6) - 0.1\delta(t-1) - 0.2\delta(t-1.3) -$$
$$-0.2\delta(t-1.5) - 0.2\delta(t-1.7) - 0.2\delta(t-2) - 0.1\delta(t-2.2)]$$

Using the sifting property of delta function it follows that

$$v_{out}(t) = 5(t-0.1)^2 u(t-0.1) + 10(t-0.2)^2 u(t-0.2) + 10(t-0.3)^2 u(t-0.3) +$$
$$+10(t-0.4)^2 u(t-0.4) + 10(t-0.5)^2 u(t-0.5) + 5(t-0.6)^2 u(t-0.6) -$$
$$-5(t-1)^2 u(t-1) - 10(t-1.3)^2 u(t-1.3) - 10(t-1.5)^2 u(t-1.5) -$$
$$-10(t-1.7)^2 u(t-1.7) - 10(t-2)^2 u(t-2) - 5(t-2.2)^2 u(t-2.2) \text{ V.}$$

At $t = 1s$

$$v_{out}(1) = 22.25 \text{ V.}$$

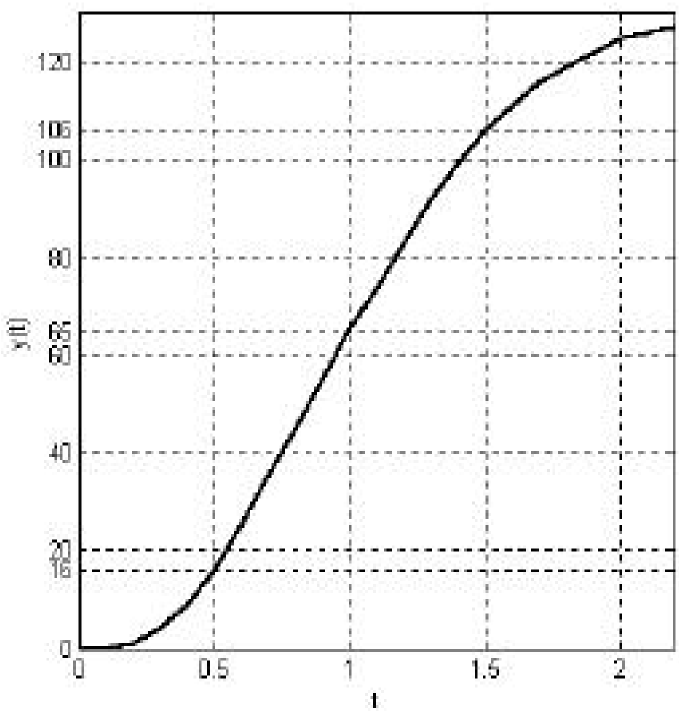
(c) From the expression of $v_{out}(t)$ obtained in part (a) we observe that $v_{out}(t)$ does not change after $t = 2.2s$. Therefore it is sufficient to compute $v_{out}(t)$ for $t \le 2.2s$. Hence $v_{in}(t)$ can be considered to be equal to

$$v_{in}(t) = 100[u(t) - u(t-2.2)] \text{ V.}$$

Using the code of problem 16.31 we have

```
>> vin = 100*ones(1,22);
>> h = [0, 0.1, 0.3, 0.5, 0.7, 0.9, 1, 1, 1, 1, 0.9, 0.9, 0.9, 0.7, 0.7, 0.5, 0.5, 0.3, 0.3, …
                                                                              …0.3, 0.1, 0.1];
>> T = 0.1;
>> tstep = T;
>> y = tstep*conv(vin,h);
>> y = [0 y 0];
>> t = 0:tstep:tstep*(length(h)+length(vin));
% After t = 2.2s vout(t) does not change
>> t = t(1:length(h)+1);
>> y = y(1:length(h)+1);
>> plot(t,y)
>> grid
```

A picture of $y(t)$ is sketched in the next figure.



Using the previous MATLAB code we the values of $y(t)$ at the specified instants of time are:
At $t = 0s$

$$y(0) = 0 \text{ V}$$

At $t = 0.5s$

$$y(0.5) = 16 \text{ V}$$

At $t = 1s$

$$y(1) = 65 \text{ V}$$

At $t = 1.5s$

$$y(1.5) = 106 \text{ V}$$

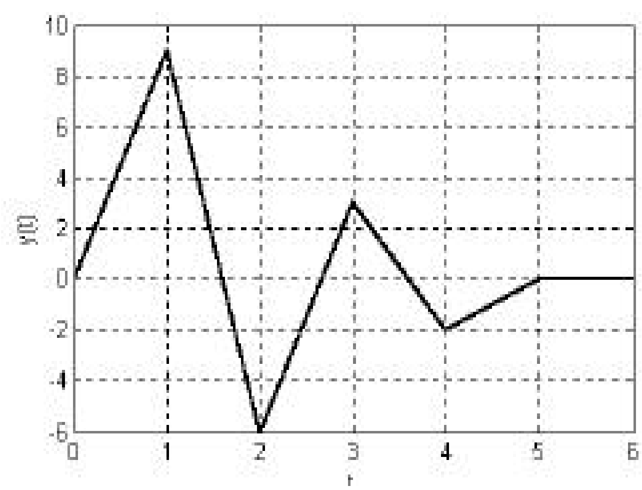
The results of parts (a) and (c) coincide.

SOLUTION 16.41.

Using the MATLAB code of problem 16.31 we have:

```
>> vin = [1];
>> h = [9, -6, 3, -2];
>> T = 1;
>> tstep = T;
>> y = tstep*conv(vin,h);
>> y = [0 y 0];
>> t = 0:tstep:tstep*(length(h)+length(vin));
>> plot(t,y)
>> grid
```

The breakpoints in $y(t)$ of the above figure are [9, -6, 3 –2] as expected because the polynomial associated with $v_{in}(t)$ is the constant 1 and the polynomial associated with $h(t)$ is the polynomial $9x^3 - 6x^2 + 3x - 2$, as it can be observed from figure P16.41.



SOLUTION 16.42.

(a) Let $v_{out,40}(t)$ denote the response that has been obtained in problem 16.40, part (a), to the input $100u(t)$.

The expression of $v_{out,40}(t)$ is (see problem 16.40, part (a)):

$$v_{out,40}(t) = h(t) \quad [100u(t)] =$$
$$= 10(t-0.1)u(t-0.1) + 20(t-0.2)u(t-0.2) + 20(t-0.3)u(t-0.3) + 20(t-0.4)u(t-0.4) +$$

$$+20(t-0.5)u(t-0.5)+10(t-0.6)u(t-0.6)-10(t-1)u(t-1)-20(t-1.3)u(t-1.3)-$$
$$-20(t-1.5)u(t-1.5)-20(t-1.7)u(t-1.7)-20(t-2)u(t-2)-10(t-2.2)u(t-2.2)$$

Using the distributive property of the convolution product and the time invariance property it follows that

$$v_{out}(t)=h(t)\ast v_{in}(t)=h(t)\ast\left[100u(t)-100u(t-0.2)\right]=$$
$$=h(t)\ast\left[100u(t)\right]-h(t)\ast\left[100u(t-0.2)\right]=$$
$$=v_{out,40}(t)-v_{out,40}(t-0.2)$$

Using the above expression of $v_{out,40}(t)$ we have:

$$v_{out,40}(0)=0\text{ V and }v_{out,40}(-0.2)=0\text{ V},$$
$$v_{out,40}(0.5)=16\text{ V and }v_{out,40}(0.3)=4\text{ V},$$
$$v_{out,40}(1)=65\text{ V and }v_{out,40}(0.8)=45\text{ V},$$
$$v_{out,40}(1.5)=106\text{ V and }v_{out,40}(1.3)=92\text{ V}.$$

At $t=0s$

$$v_{out}(0)=v_{out,40}(0)-v_{out,40}(-0.2)=0\text{ V}.$$

At $t=0.5s$

$$v_{out}(0.5)=v_{out,40}(0.5)-v_{out,40}(0.3)=12\text{ V}.$$

At $t=1s$

$$v_{out}(1)=v_{out,40}(1)-v_{out,40}(0.8)=20\text{ V}.$$

At $t=1.5s$

$$v_{out}(1.5)=v_{out,40}(1.5)-v_{out,40}(1.3)=14\text{ V}.$$

(b) In this case $v_{out}(t)$ will be computed using the techniques of convolution algebra.
We have

$$v_{out}(t)=v_{in}(t)\ast h(t)=v_{in}^{(-1)}(t)\ast h^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation.
From figure P16.42 observe that

$$v_{in}(t)=100t\left[u(t)-u(t-0.5)\right]+100(1-t)\left[u(t-0.5)-u(t-1)\right]=$$
$$=100tu(t)+100(1-2t)u(t-0.5)+100(t-1)u(t-1)$$

Therefore

$$v_{in}^{(-1)}(t)=50t^2u(t)+(-100t^2+100t-25)u(t-0.5)+(50t^2-100t+50)u(t-1)=g(t)$$

By the sifting property of the delta function we have

$$v_{out}(t)=v_{in}^{(-1)}(t)\ast h^{(1)}(t)=g(t)\ast h^{(1)}(t)$$
$$=g(t)\ast\left[0.1\delta(t-0.1)+0.2\delta(t-0.2)+0.2\delta(t-0.3)+0.2\delta(t-0.4)+\right.$$
$$+0.2\delta(t-0.5)+0.1\delta(t-0.6)-0.1\delta(t-1)-0.2\delta(t-1.3)-$$
$$\left.-0.2\delta(t-1.5)-0.2\delta(t-1.7)-0.2\delta(t-2)-0.1\delta(t-2.2)\right]=$$
$$=0.1g(t-0.1)+0.2g(t-0.2)+0.2g(t-0.3)+0.2g(t-0.4)+$$
$$+0.2g(t-0.5)+0.1g(t-0.6)-0.1g(t-1)-0.2g(t-1.3)-$$

$$-0.2g(t-1.5)-0.2g(t-1.7)-0.2g(t-2)-0.1g(t-2.2)$$

The values of $v_{out}(t)$ at the specified instants of time can be computed using MATLAB. The results are:

$$v_{out}(0)=0\,\text{V}$$
$$v_{out}(0.5)=2.2\,\text{V}$$
$$v_{out}(1)=17.85\,\text{V}$$
$$v_{out}(1.5)=23.3\,\text{V}.$$

SOLUTION 16.43.

Using the techniques of convolution algebra we have

$$v_{out}(t)=h(t)\quad v_{in}(t)=h^{(-1)}(t)\quad v_{in}^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation.
We have

$$h^{(-1)}(t)=2(1-e^{-2t})u(t)$$

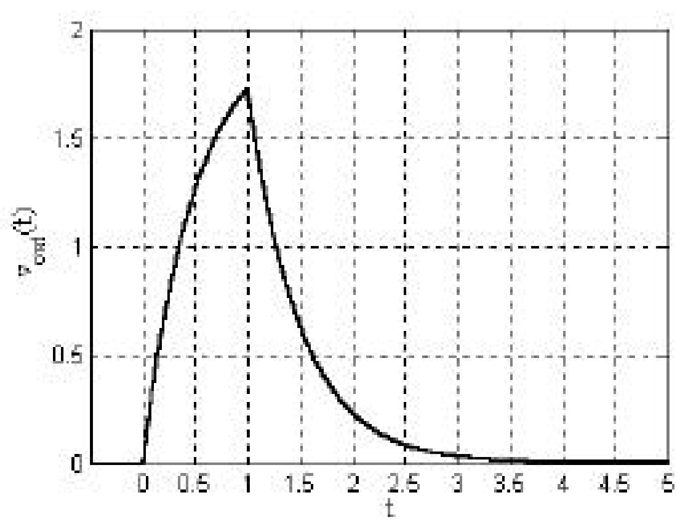
and

$$v_{in}^{(1)}(t)=\delta(t)-\delta(t-1)$$

Using the sifting property of the delta function it follows that

$$v_{out}(t)=\left[2(1-e^{-2t})u(t)\right]\quad[\delta(t)-\delta(t-1)]=$$
$$=2(1-e^{-2t})u(t)-2(1-e^{-2(t-1)})u(t-1)$$

A picture of $v_{out}(t)$ is sketched in the next figure.



SOLUTION 16.44.
(a) From table 13.1 it follows that the Laplace transform of $v_{in}(t)$ is

$$V_{in}(s)=\frac{1}{s}-\frac{1}{s+1}-\frac{1}{s+2}=\frac{-s^2+2}{s(s+1)(s+2)}$$

And the Laplace transform of $v_{out}(t)$ is

$$V_{out}(s) = \frac{1}{s} - \frac{2}{s+1} - \frac{1}{(s+1)^2} + \frac{1}{s+2} = \frac{-s^2+2}{s(s+1)^2(s+2)}$$

Therefore the transfer function of the circuit is

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s+1}$$

A simple RC circuit that represents this transfer function is a series RC circuit with $R = 1\Omega$ and $C = 1F$. $v_{out}(t)$ is represented by the capacitor voltage and $v_{in}(t)$ is the source voltage.

(b) The impulse response of the circuit is

$$h(t) = \mathcal{L}^{-1}\left[H(s)\right] = \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = e^{-t}u(t).$$

(c) Assuming zero initial conditions we have

$$v_{out}(t) = v_{in}(t) \quad h(t) = v_{in}^{(1)}(t) \quad h^{(-1)}(t) = \delta(t) \quad \left[(1-e^{-t})u(t)\right] = (1-e^{-t})u(t) \text{ V.}$$

(d) Using the techniques of convolution algebra the zero-state response can be computed as

$$v_{out}(t) = v_{in}(t) \quad h(t) = v_{in}^{(2)}(t) \quad h^{(-2)}(t)$$

where the superscript (2) means double differentiation and the superscript (-2) means double integration. First, from figure P16.44, observe that

$$v_{in}^{(1)}(t) = \left[u(t-1) - u(t-2)\right] + \left[u(t-3) - u(t-4)\right]$$

Therefore

$$v_{in}^{(2)}(t) = \delta(t-1) - \delta(t-2) + \delta(t-3) - \delta(t-4)$$

$h^{(-2)}(t)$ is computed as the integral of $h^{(-1)}(t)$.

$$h^{(-2)}(t) = \int_{-\infty}^{t} h^{(-1)}(\tau)d\tau = \int_{-\infty}^{t} (1-e^{-\tau})u(\tau)d\tau = \int_{0^-}^{t} (1-e^{-\tau})d\tau = (t+e^{-t}-1)u(t)$$

The zero-state response can now be computed

$$v_{out}(t) = v_{in}^{(2)}(t) \quad h^{(-2)}(t) =$$
$$= \left[\delta(t-1) - \delta(t-2) + \delta(t-3) - \delta(t-4)\right] \quad \left[(t+e^{-t}-1)u(t)\right]$$

By the sifting property of the delta function it follows that

$$v_{out}(t) = \left(t-2+e^{-(t-1)}\right)u(t-1) - \left(t-3+e^{-(t-2)}\right)u(t-2)$$
$$+\left(t-4+e^{-(t-3)}\right)u(t-3) - \left(t-5+e^{-(t-4)}\right)u(t-4) \text{ V.}$$

SOLUTION 16.45.
Using the techniques of the convolution algebra we have

$$y(t) = f(t) \quad g(t) = f^{(2)}(t) \quad g^{(-2)}(t)$$

where

$$g^{(-1)}(t) = \pi^2 \int_{0^-}^{t} \cos(\pi\tau)d\tau = \pi \sin(\pi t)u(t)$$

and

$$g^{(-2)}(t) = \pi^2 \int_{0^-}^{t} \sin(\pi\tau)d\tau = [1 - \cos(\pi t)]u(t)$$

Differentiating $f(t)$ twice leads to

$$f^{(2)}(t) = -\delta(t) + 2\delta(t-1) - 2\delta(t-3) + \delta(t-4)$$

Therefore

$$y(t) = f^{(2)}(t) \quad g^{(-2)}(t) =$$
$$= [-\delta(t) + 2\delta(t-1) - 2\delta(t-3) + \delta(t-4)] \quad \{[1-\cos(\pi t)]u(t)\}$$

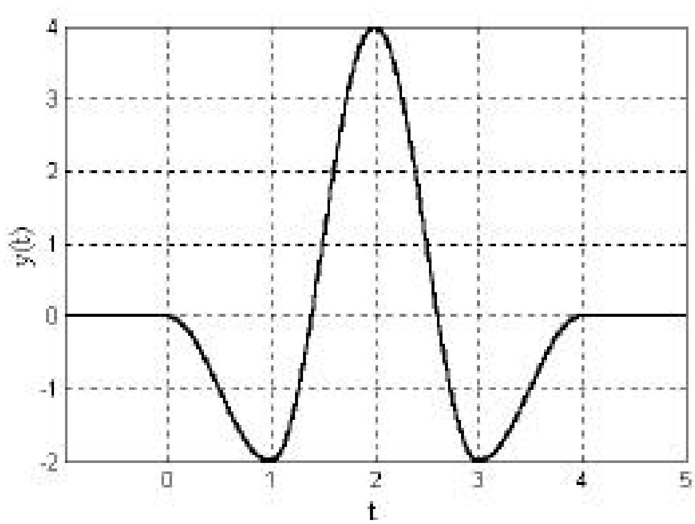
Using the sifting property of the delta function it follows that

$$y(t) = -[1-\cos(\pi t)]u(t) + 2\{1-\cos[\pi(t-1)]\}u(t-1)$$
$$-2\{1-\cos[\pi(t-3)]\}u(t-3) + \{1-\cos[\pi(t-4)]\}u(t-4)$$

Simplifying the expression of $y(t)$ yields

$$y(t) = -[1-\cos(\pi t)] \; [u(t) - u(t-4)] + 2 \; [1+\cos(\pi t)] \; [u(t-1) - u(t-3)].$$

A picture of $y(t)$ is sketched in the next figure.



SOLUTION 16.46.
(a) By the current division formula

$$I_C(s) = \frac{Cs}{Cs + \dfrac{1}{Ls}} \; I_{in}(s) = \frac{s^2}{s^2+1} \; I_{in}(s)$$

The transfer function of the circuit can be computed as below

$$H(s) = \frac{V_C(s)}{I_{in}(s)} = \frac{I_C(s) \;\; Z_C(s)}{I_{in}(s)} = \frac{s^2}{s^2+1} \; \frac{1}{s} = \frac{s}{s^2+1}$$

(b) The impulse response is computed as the inverse Laplace transform of the transfer function

$$h(t) = \mathrm{L}^{-1}\left[H(s)\right] = \cos(t)u(t).$$

(c) Assuming zero initial conditions it follows, by the impulse response theorem, that

$$v_{out}(t) = i_{in}(t) \quad h(t)$$

Using the techniques of the convolution algebra we have

$$v_{out}(t) = i_{in}^{(2)}(t) \quad h^{(-2)}(t)$$

By inspection

$$i_{in}^{(1)}(t) = \left[u(t) - u(t-2\pi)\right] + \left[u(t-4\pi) - u(t-6\pi)\right]$$

Therefore

$$i_{in}^{(2)}(t) = \delta(t) - \delta(t-2\pi) + \delta(t-4\pi) - \delta(t-6\pi)$$

And

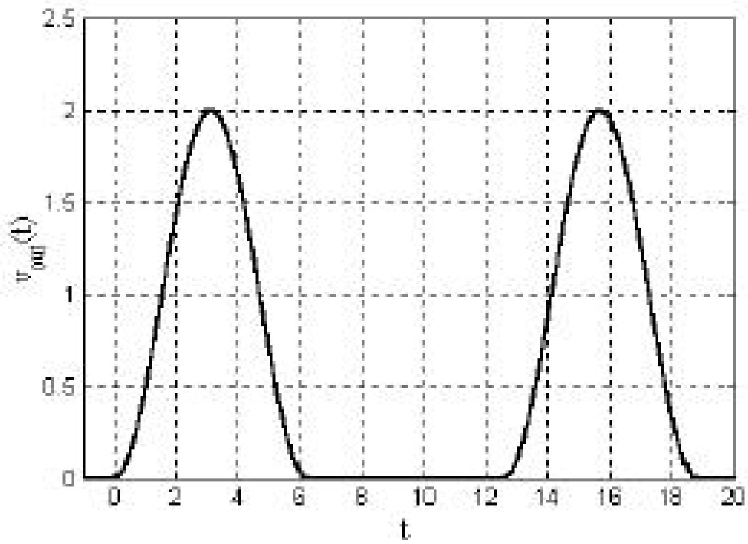
$$h^{(-1)}(t) = \int_{0^-}^{t} \cos(\tau)d\tau = \sin(t)u(t)$$

Hence

$$h^{(-2)}(t) = \int_{0^-}^{t} \sin(\tau)d\tau = \left[1-\cos(t)\right]u(t)$$

Using the sifting property of the delta function we have

$$\begin{aligned} v_{out}(t) &= \left[\delta(t) - \delta(t-2\pi) + \delta(t-4\pi) - \delta(t-6\pi)\right] \quad \left\{\left[1-\cos(t)\right]u(t)\right\} \\ &= \left[1-\cos(t)\right]u(t) - \left[1-\cos(t-2\pi)\right]u(t-2\pi) \\ &\quad + \left[1-\cos(t-4\pi)\right]u(t-4\pi) - \left[1-\cos(t-6\pi)\right]u(t-6\pi) \\ &= \left[1-\cos(t)\right] \quad \left[u(t) - u(t-2\pi) + u(t-4\pi) - u(t-6\pi)\right] \end{aligned}$$



V.

(d) A picture of $v_{out}(t)$ is sketched in the next figure.

SOLUTION 16.47.

(a) The step response, $v_{out}(t)$, is computed using the convolution algebra techniques. We have

$$v_{out}(t) = h(t) \quad v_{in}(t) = h^{(1)}(t) \quad v_{in}^{(-1)}(t)$$

From figure P16.47 observe that

$$h(t) = 2u(t) - u(t-1) - 2u(t-2) - u(t-3) + u(t-5) + 2u(t-6) + u(t-7) - 2u(t-8)$$

Differentiating we have

$$h^{(1)}(t) = 2\delta(t) - \delta(t-1) - 2\delta(t-2) - \delta(t-3) + \delta(t-5) + 2\delta(t-6) + \delta(t-7) - 2\delta(t-8)$$

Since

$$v_{in}(t) = u(t),$$

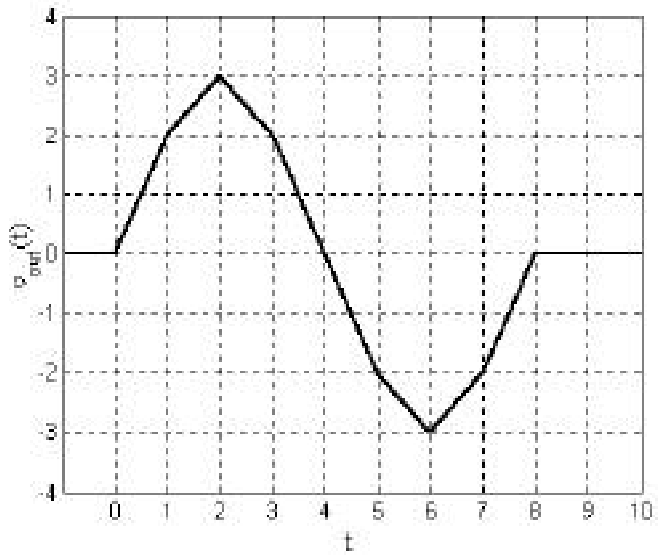
by integration it follows that

$$v_{in}^{(-1)}(t) = tu(t).$$

Using the sifting property of the delta function it follows that

$$\begin{aligned} v_{out}(t) &= h^{(1)}(t) \quad v_{in}^{(-1)}(t) \\ &= \left[2\delta(t) - \delta(t-1) - 2\delta(t-2) - \delta(t-3) + \delta(t-5) + 2\delta(t-6) + \delta(t-7) - 2\delta(t-8)\right] \quad \left[tu(t)\right] \\ &= 2tu(t) - (t-1)u(t-1) - 2(t-2)u(t-2) - (t-3)u(t-3) + \\ &\qquad + (t-5)u(t-5) + 2(t-6)u(t-6) + (t-7)u(t-7) - 2(t-8)u(t-8)\,\text{V}. \end{aligned}$$

A picture of the step response is sketched in the next figure.



(b) Using the convolution algebra techniques we have

$$v_{out}(t) = h(t) \quad v_{in}(t) = h^{(1)}(t) \quad v_{in}^{(-1)}(t)$$

For computing $v_{in}^{(-1)}(t)$ we have

$$v_{in}^{(-1)}(t) = \int_{-\infty}^{t} e^{\tau} u(-\tau) d\tau = \begin{cases} \int_{-\infty}^{t} e^{\tau} d\tau, & t < 0 \\ \int_{-\infty}^{0} e^{\tau} d\tau, & 0 \le t \end{cases} = \begin{cases} e^{t}, & t < 0 \\ 1, & 0 \le t \end{cases}$$

Using the sifting property of the delta function it follows that

$$v_{out}(t) = h^{(1)}(t) \quad v_{in}^{(-1)}(t) =$$

$$= \left[2\delta(t) - \delta(t-1) - 2\delta(t-2) - \delta(t-3) + \delta(t-5) + 2\delta(t-6) + \delta(t-7) - 2\delta(t-8)\right] \quad \left[v_{in}^{(-1)}(t)\right] =$$

$$= 2v_{in}^{(-1)}(t) - v_{in}^{(-1)}(t-1) - 2v_{in}^{(-1)}(t-2) - v_{in}^{(-1)}(t-3) +$$

$$+ v_{in}^{(-1)}(t-5) + 2v_{in}^{(-1)}(t-6) + v_{in}^{(-1)}(t-7) - 2v_{in}^{(-1)}(t-8) \text{ V.}$$

Using the expression of $v_{in}^{(-1)}(t)$ computed above it follows that

$$v_{out}(7.5) = 0.7869 \text{ V},$$
$$v_{out}(6.5) = 1.1603 \text{ V},$$
$$v_{out}(5.5) = 0.2720 \text{ V},$$
$$v_{out}(0.5) = 0.8848 \text{ V}.$$

SOLUTION 16.48.

First observe, from figure P16.48(a), that

$$h(t) = (1-t) \quad \left[u(t) - u(t-1)\right] =$$

$$= (1-t)u(t) - (1-t)u(t-1).$$

Using the convolution algebra techniques it follows that the step response $y(t)$ is

$$y(t) = v(t) \quad h(t) = v^{(1)}(t) \quad h^{(-1)}(t) =$$
$$= \delta(t) \quad h^{(-1)}(t) = h^{(-1)}(t)$$

where

$$h^{(-1)}(t) = \int_{-\infty}^{t} (1-\tau)u(\tau)d\tau - \int_{-\infty}^{t} (1-\tau)u(\tau-1)d\tau =$$
$$= \left[\tau - 0.5\tau^2\right]_0^t \; u(t) - \left[\tau - 0.5\tau^2\right]_1^t \; u(t-1) =$$
$$= (t - 0.5t^2)u(t) - (t - 0.5t^2 - 0.5)u(t-1).$$

Hence the step response is

$$y(t) = (t - 0.5t^2)u(t) - (t - 0.5t^2 - 0.5)u(t-1).$$

Let $y^v(t)$ denote the zero-state response to the input $v(t)$.
From figure P16.48(b) we observe that

$$v(t) = u(t) + u(t-1) - 2u(t-2).$$

Using the distributive property of the convolution it follows that

$$y^v(t) = h(t) \quad v(t) =$$
$$= h(t) \quad \left[u(t) + u(t-1) - 2u(t-2)\right] =$$
$$= h(t) \quad u(t) + h(t) \quad u(t-1) - 2 \; h(t) \quad u(t-2).$$

Due to the fact that

$$y(t) = h(t) \quad u(t)$$

by the linearity and time invariance properties it follows that

$$y^v(t) = y(t) + y(t-1) - 2y(t-2) =$$
$$= (t - 0.5t^2)u(t) + (t-1)u(t-1) +$$
$$+ (1.5t^2 - 8t + 12)u(t-2) + (0.5t^2 - 3t + 4)u(t-3).$$

SOLUTION 16.49.

(a) By the voltage division formula it follows that

$$V_{out}(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + Ls} \; V_{in}(s) = \frac{1}{LCs^2 + 1} \; V_{in}(s)$$

Therefore the transfer function is

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{LCs^2 + 1}$$

Taking the inverse Laplace transform yields

$$h(t) = \mathcal{L}^{-1}\left[\frac{1}{LCs^2 + 1}\right] = \frac{1}{\sqrt{LC}} \mathcal{L}^{-1}\left[\frac{\frac{1}{\sqrt{LC}}}{s^2 + \frac{1}{LC}}\right] = \frac{1}{\sqrt{LC}} \sin\left(\frac{1}{\sqrt{LC}} t\right) u(t).$$

(b) The step response is computed as the convolution of the impulse response and the step function.

$$v_{out}(t) = h(t) \quad u(t)$$

Using the techniques of the convolution algebra it follows that

$$v_{out}(t) = h^{(-1)}(t) \quad u^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation. Taking the integral of $h(t)$ we have

$$h^{(-1)}(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{LC}} \sin\left(\frac{1}{\sqrt{LC}}\tau\right) u(\tau)\, d\tau =$$

$$= u(t) \left[-\cos\left(\frac{1}{\sqrt{LC}}\tau\right)\right]_0^t = \left[1 - \cos\left(\frac{1}{\sqrt{LC}} t\right)\right] u(t)\,.$$

Therefore

$$v_{out}(t) = h^{(-1)}(t) \quad u^{(1)}(t) =$$

$$= \left\{ \left[1 - \cos\left(\frac{1}{\sqrt{LC}} t\right)\right] u(t) \right\} \quad \delta(t) =$$

$$= \left[1 - \cos\left(\frac{1}{\sqrt{LC}} t\right)\right] u(t) \text{ V.}$$

(c) We denote by $v_{out}^{T}(t)$ the output to the rectangular pulse in figure P16.49(b). Observe, from figure P16.49(b), that

$$v_{in}(t) = \frac{1}{T} \left[u(t) - u(t-T)\right] \text{ V.}$$

By linearity and time invariance it follows that

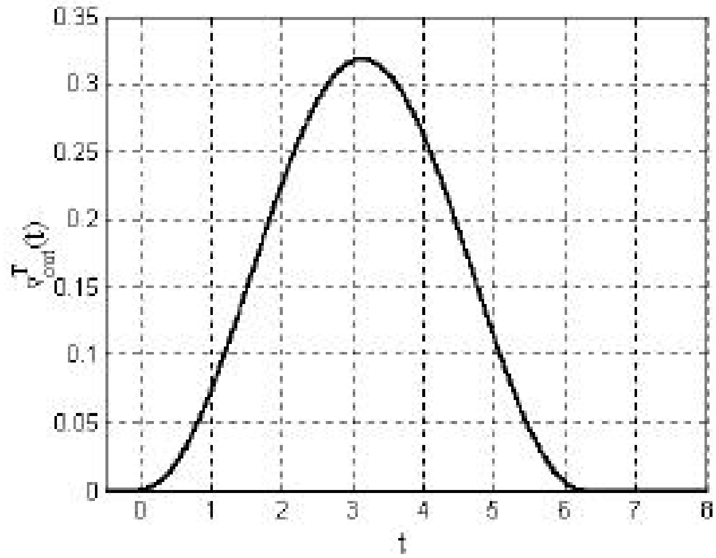
$$v_{out}^{T}(t) = \frac{1}{T} \left[v_{out}(t) - v_{out}(t-T)\right]$$

where $v_{out}(t)$ is the step response obtained in part (b). Therefore

$$v_{out}^{T}(t) = \frac{1}{2\pi\sqrt{LC}} \left[1 - \cos\left(\frac{1}{\sqrt{LC}} t\right)\right] u(t) - \frac{1}{2\pi\sqrt{LC}} \left[1 - \cos\left(\frac{1}{\sqrt{LC}} t - 2\pi\right)\right] u(t - 2\pi\sqrt{LC})$$

$$= \frac{1}{2\pi\sqrt{LC}} \left[1 - \cos\left(\frac{1}{\sqrt{LC}} t\right)\right] \left[u(t) - u(t - 2\pi\sqrt{LC})\right]$$

A picture of $v_{out}^{T}(t)$, for $L = 1H$ and $C = 1F$, is sketched in the next figure.



SOLUTION 16.50.
The impulse response of the configuration in the figure P16.50 is

$$h(t) = h_1(t) \quad \left[h_2(t) + h_3(t)\right] \quad h_4(t)$$

Due to the fact that $h_4(t) = 2\delta(t)$, the sifting property of the delta function can be applied and it follows that

$$h(t) = 2 \;\; h_1(t) \quad \left[h_2(t) + h_3(t)\right]$$

Using the techniques of the convolution algebra we can further write

$$h(t) = 2 \;\; h_1^{(1)}(t) \quad \left[h_2(t) + h_3(t)\right]^{(-1)} =$$
$$h(t) = 2 \;\; h_1^{(1)}(t) \quad \left[h_2^{(-1)}(t) + h_3^{(-1)}(t)\right]$$

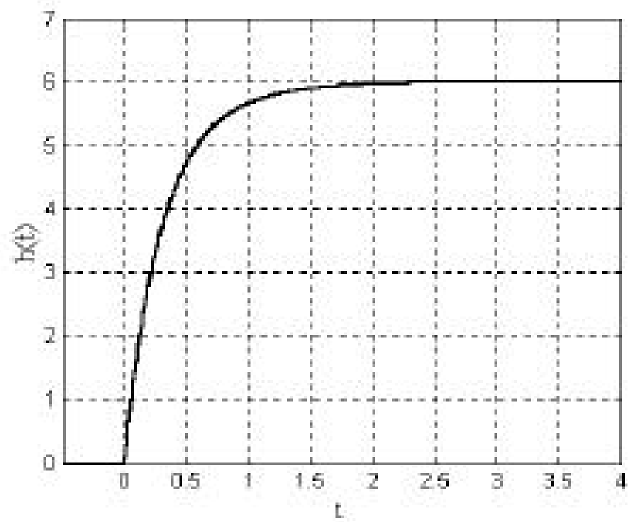
We have

$$h_1^{(1)}(t) = u^{(1)}(t) = \delta(t)$$
$$h_2^{(-1)}(t) = \int_{-\infty}^{t} 2e^{-2\tau} u(\tau) d\tau = u(t) \;\left[-e^{-2\tau}\right]_0^t = \left(1 - e^{-2t}\right)u(t)$$
$$h_3^{(-1)}(t) = \int_{-\infty}^{t} 8e^{-4\tau} u(\tau) d\tau = u(t) \;\left[-2e^{-4\tau}\right]_0^t = 2\left(1 - e^{-4t}\right)u(t)$$

Substituting the expressions of $h_1^{(-1)}(t)$, $h_2^{(-1)}(t)$ and $h_3^{(-1)}(t)$ in the expression of $h(t)$, and using the sifting property of the delta function we have

$$h(t) = 2\left(1 - e^{-2t}\right)u(t) + 4\left(1 - e^{-4t}\right)u(t).$$

A picture of $h(t)$ is sketched in the next figure.



SOLUTION 16.51.
Observe first that $h_2(t)$, $h_3(t)$ and $h_4(t)$ have the same expressions as in the problem 16.50. In problem 16.50 the following convolution has been computed

$$u(t) \quad \left[h_2(t)+h_3(t)\right] \quad h_4(t) = 2\left(1-e^{-2t}\right)u(t) + 4\left(1-e^{-4t}\right)u(t)$$

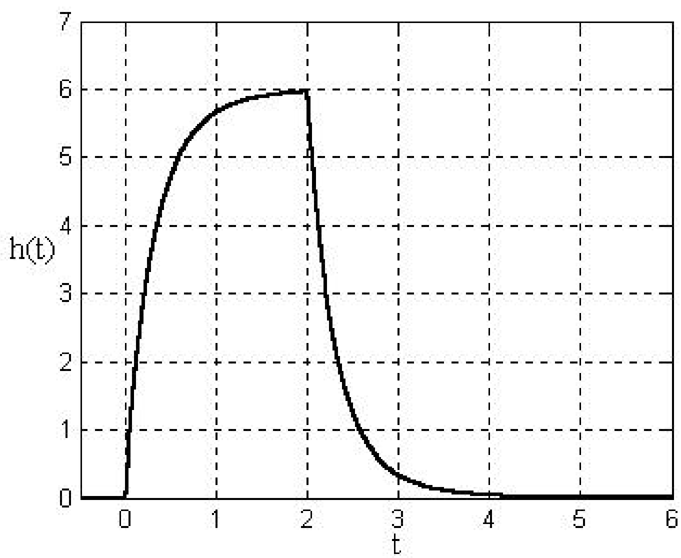
Using the time shift property of the convolution (see problem 16.22, part (a)) it follows that

$$u(t-2) \quad \left[h_2(t)+h_3(t)\right] \quad h_4(t) = 2\left(1-e^{-2(t-2)}\right)u(t-2) + 4\left(1-e^{-4(t-2)}\right)u(t-2)$$

Using the above expressions and the distributive property of the convolution it follows that the overall impulse response can be computed as below

$$\begin{aligned} h(t) &= h_1(t) \quad \left[h_2(t)+h_3(t)\right] \quad h_4(t) = \left[u(t)-u(t-2)\right] \quad \left[h_2(t)+h_3(t)\right] \quad h_4(t) \\ &= u(t) \quad \left[h_2(t)+h_3(t)\right] \quad h_4(t) - u(t-2) \quad \left[h_2(t)+h_3(t)\right] \quad h_4(t) = \\ &= 2\left(1-e^{-2t}\right)u(t) + 4\left(1-e^{-4t}\right)u(t) - 2\left(1-e^{-2(t-2)}\right)u(t-2) + 4\left(1-e^{-4(t-2)}\right)u(t-2). \end{aligned}$$

A picture of $h(t)$ is sketched in the next figure.



SOLUTION 16.52.
The overall impulse response of the configuration is

$$h(t) = h_1(t) \quad \left[h_2(t) + h_3(t)\right] \quad h_4(t)$$

Using the distributive property of the convolution we have

$$h(t) = h_1(t) \quad h_2(t) \quad h_4(t) + h_1(t) \quad h_3(t) \quad h_4(t)$$

Replacing the expressions for $h_2(t)$ and $h_3(t)$, and using the sifting property of the delta function it follows that

$$\begin{aligned} h(t) &= 2\; h_1(t) \quad \delta(t) \quad h_4(t) - 2\; h_1(t) \quad \delta(t-2) \quad h_4(t) \\ &= 2\; h_1(t) \quad h_4(t) - 2\left[h_1(t) \quad h_4(t)\right]_{t=t-2} \end{aligned}$$

Using the techniques of the convolution algebra it follows that

$$h_1(t) \quad h_4(t) = h_1^{(1)}(t) \quad h_4^{(-1)}(t)$$

where the superscript (1) means differentiation and the superscript (-1) means integration.
We have

$$h_1^{(1)}(t) = u^{(1)}(t) = \delta(t)$$

and

$$h_4^{(-1)}(t) = \int_{-\infty}^{t} 2e^{-\tau} u(\tau) d\tau = 2u(t)\int_0^t e^{-\tau}\, d\tau = 2u(t)\left[-e^{-\tau}\right]_0^t = 2(1-e^{-t})u(t)$$

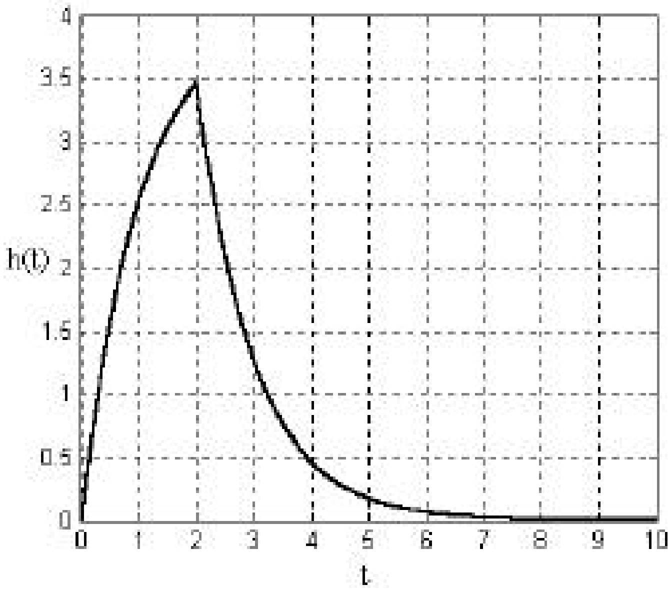
Therefore

$$h_1(t) \quad h_4(t) = \delta(t) \quad \left[2(1-e^{-t})u(t)\right] = 2(1-e^{-t})u(t)$$

Replacing the above expression into the expression of $h(t)$ it follows that

$$h(t) = 4(1-e^{-t})u(t) - 4\left[(1-e^{-t})u(t)\right]_{t=t-2} = 4(1-e^{-t})u(t) - 4\left[1-e^{-(t-2)}\right]u(t-2).$$

A picture of $h(t)$ is sketched in the next picture.



SOLUTION 16.53.

(a) The overall impulse response of the configuration in figure P16.53 is

$$h(t) = \left[h_1(t) + h_2(t)\right] \quad h_3(t)$$

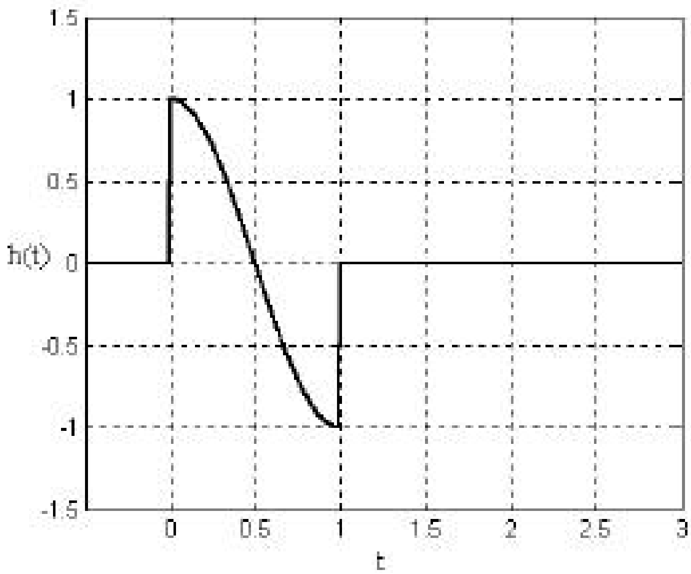
By the distributive property of convolution it follows that

$$h(t) = h_1(t) \quad h_3(t) + h_2(t) \quad h_3(t) = \delta(t) \quad \cos(\pi t)u(t) + \delta(t-1) \quad \cos(\pi t)u(t)$$

By the sifting property of the delta function it follows that

$$h(t) = \cos(\pi t)u(t) + \cos\left[\pi(t-1)\right]u(t-1).$$

A picture of $h(t)$ is sketched in the next figure.



(b) The response $y(t)$ is computed as

$$y(t) = h(t) \ast u(t) = \left\{\cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1)\right\} \ast u(t)$$

Using the distributive property and the time shift property of convolution(see problem 16.22, part (a)) we have

$$\begin{aligned} y(t) &= [\cos(\pi t)u(t)] \ast u(t) + \left\{\cos[\pi(t-1)]u(t-1)\right\} \ast u(t) = \\ &= [\cos(\pi t)u(t)] \ast u(t) + \left\{[\cos(\pi t)u(t)] \ast u(t)\right\}_{t=t-1} \end{aligned}$$

Using the techniques of convolution algebra, the convolution $\cos(\pi t)u(t) \ast u(t)$ is computed as

$$[\cos(\pi t)u(t)] \ast u(t) = [\cos(\pi t)u(t)]^{(-1)} \ast u^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation.
We have

$$[\cos(\pi t)u(t)]^{(-1)} = \int_{-\infty}^{t} \cos(\pi\tau)u(\tau)d\tau = u(t)\int_{0}^{t}\cos(\pi\tau)d\tau = u(t)\frac{1}{\pi}[\sin(\pi t)]_0^t = \frac{\sin(\pi t)}{\pi}u(t).$$

Therefore

$$[\cos(\pi t)u(t)] \ast u(t) = \left[\frac{\sin(\pi t)}{\pi}u(t)\right] \ast \delta(t) = \frac{\sin(\pi t)}{\pi}u(t)$$

Hence the step response is

$$y(t) = \frac{\sin(\pi t)}{\pi}u(t) + \frac{\sin[\pi(t-1)]}{\pi}u(t-1) = \frac{\sin(\pi t)}{\pi}[u(t) - u(t-1)].$$

SOLUTION 16.54.
(a) The overall impulse response of the configuration in figure P16.53 is

$$h(t) = \left[h_1(t) + h_2(t)\right] \quad h_3(t)$$

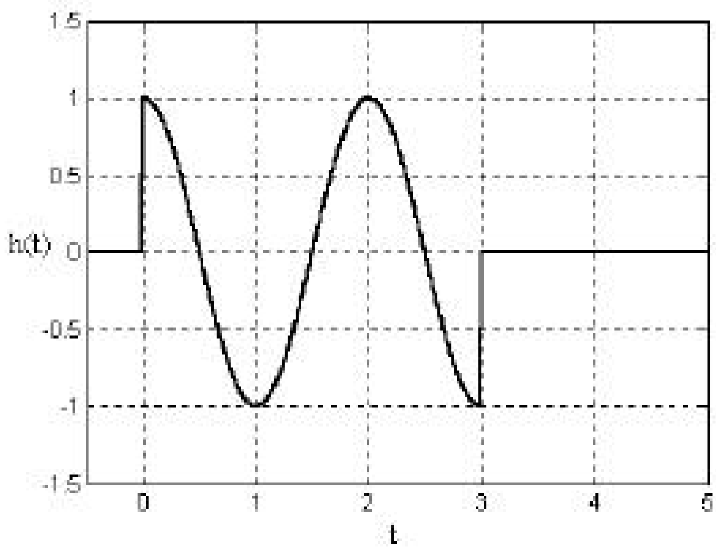
By the distributive property of convolution it follows that

$$h(t) = h_1(t) \quad h_3(t) + h_2(t) \quad h_3(t) = \delta(t) \quad \cos(\pi t)u(t) + \delta(t-3) \quad \cos(\pi t)u(t)$$

By the sifting property of the delta function it follows that

$$h(t) = \cos(\pi t)u(t) + \cos\left[\pi(t-3)\right]u(t-3).$$

A picture of $h(t)$ is sketched in the next figure.



(b) The response $y(t)$ is computed as

$$y(t) = h(t) \quad u(t) = \left\{\cos(\pi t)u(t) + \cos\left[\pi(t-3)\right]u(t-3)\right\} \quad u(t)$$

Using the distributive property and the time shift property of convolution(see problem 16.22, part (a) we have

$$\begin{aligned} y(t) &= \left[\cos(\pi t)u(t)\right] \quad u(t) + \left\{\cos\left[\pi(t-3)\right]u(t-3)\right\} \quad u(t) \\ &= \left[\cos(\pi t)u(t)\right] \quad u(t) + \left\{\left[\cos(\pi t)u(t)\right] \quad u(t)\right\}_{t=t-3} \end{aligned}$$

Using the techniques of convolution algebra, the convolution $\cos(\pi t)u(t) \quad u(t)$ is computed as

$$\left[\cos(\pi t)u(t)\right] \quad u(t) = \left[\cos(\pi t)u(t)\right]^{(-1)} \quad u^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation. We have

$$\left[\cos(\pi t)u(t)\right]^{(-1)} = \int_{-\infty}^{t} \cos(\pi\tau)u(\tau)d\tau = u(t)\int_{0}^{t} \cos(\pi\tau)d\tau = u(t)\frac{1}{\pi}\left[\sin(\pi t)\right]_0^t = \frac{\sin(\pi t)}{\pi}u(t).$$

Therefore

$$[\cos(\pi t)u(t)] \ast u(t) = \left[\frac{\sin(\pi t)}{\pi} u(t)\right] \ast \delta(t) = \frac{\sin(\pi t)}{\pi} u(t)$$

Hence the step response is

$$y(t) = \frac{\sin(\pi t)}{\pi} u(t) + \frac{\sin[\pi(t-3)]}{\pi} u(t-3) = \frac{\sin(\pi t)}{\pi}[u(t) - u(t-3)].$$

SOLUTION 16.55.
(a) The overall impulse response of the configuration in figure P16.55 is

$$h(t) = [h_1(t) + h_2(t) + h_3(t) + h_4(t)] \ast h_5(t)$$

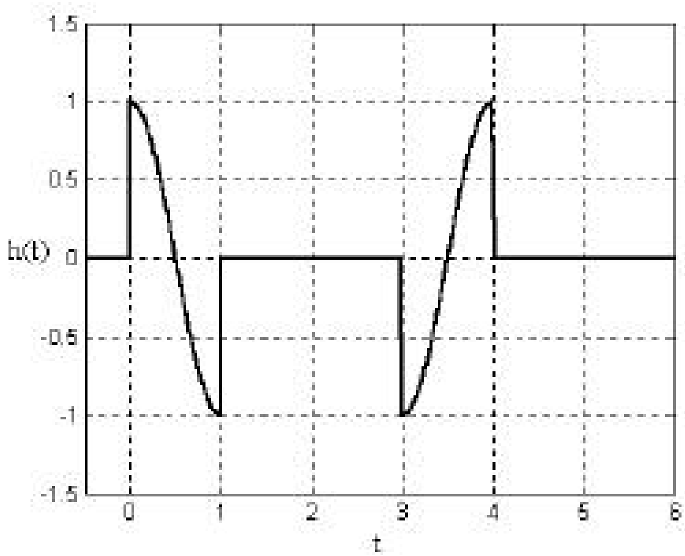
By the distributive property of convolution it follows that

$$\begin{aligned} h(t) &= h_1(t) \ast h_5(t) + h_2(t) \ast h_5(t) + h_3(t) \ast h_5(t) + h_4(t) \ast h_5(t) = \\ &= \delta(t) \ast \cos(\pi t)u(t) + \delta(t-1) \ast \cos(\pi t)u(t) - \\ &\quad -\delta(t-3) \ast \cos(\pi t)u(t) - \delta(t-4) \ast \cos(\pi t)u(t) \end{aligned}$$

By the sifting property of the delta function it follows that

$$\begin{aligned} h(t) &= \cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1) - \\ &\quad -\cos[\pi(t-3)]u(t-3) - \cos[\pi(t-4)]u(t-4). \end{aligned}$$

A picture of $h(t)$ is sketched in the next figure.



(b) The response $y(t)$ is computed as

$$\begin{aligned} y(t) = h(t) \ast u(t) = \{&\cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1) \\ &-\cos[\pi(t-3)]u(t-3) - \cos[\pi(t-4)]u(t-4)\} \ast u(t) \end{aligned}$$

Using the distributive property and the time shift property of convolution (see problem 16.22, part (a)) we have

$$\begin{aligned} y(t) &= \left[\cos(\pi t)u(t)\right] \ast u(t) + \left\{\cos[\pi(t-1)]u(t-1)\right\} \ast u(t) - \\ &\quad -\left\{\cos[\pi(t-3)]u(t-3)\right\} \ast u(t) - \left\{\cos[\pi(t-4)]u(t-4)\right\} \ast u(t) = \\ &= \left[\cos(\pi t)u(t)\right] \ast u(t) + \left\{\left[\cos(\pi t)u(t)\right] \ast u(t)\right\}_{t=t-1} - \\ &\quad -\left\{\left[\cos(\pi t)u(t)\right] \ast u(t)\right\}_{t=t-3} - \left\{\left[\cos(\pi t)u(t)\right] \ast u(t)\right\}_{t=t-4} \end{aligned}$$

Using the techniques of convolution algebra, the convolution $\cos(\pi t)u(t) \ast u(t)$ is computed as

$$\left[\cos(\pi t)u(t)\right] \ast u(t) = \left[\cos(\pi t)u(t)\right]^{(-1)} \ast u^{(1)}(t)$$

where the superscript (-1) means integration and the superscript (1) means differentiation. We have

$$\left[\cos(\pi t)u(t)\right]^{(-1)} = \int_{-\infty}^{t} \cos(\pi\tau)u(\tau)d\tau = u(t)\int_{0}^{t}\cos(\pi\tau)d\tau = u(t)\frac{1}{\pi}\left[\sin(\pi t)\right]_0^t = \frac{\sin(\pi t)}{\pi}u(t).$$

Therefore

$$\left[\cos(\pi t)u(t)\right] \ast u(t) = \left[\frac{\sin(\pi t)}{\pi}u(t)\right] \ast \delta(t) = \frac{\sin(\pi t)}{\pi}u(t)$$

Hence the step response is

$$\begin{aligned} y(t) &= \frac{\sin(\pi t)}{\pi}u(t) + \frac{\sin[\pi(t-1)]}{\pi}u(t-1) - \frac{\sin[\pi(t-3)]}{\pi}u(t-3) - \frac{\sin[\pi(t-4)]}{\pi}u(t-4) \\ &= \frac{\sin(\pi t)}{\pi}\left[u(t) - u(t-1) + u(t-3) - u(t-4)\right]. \end{aligned}$$

SOLUTION 16.56.
(a) By definition

$$h(t) \ast f(t) = \int_{-\infty}^{\infty} h(t-\tau)f(\tau)d\tau$$

By making a change of variable

$$\tau_1 = t - \tau$$

we have

$$h(t) \ast f(t) = \int_{-\infty}^{\infty} h(\tau_1)f(t-\tau_1)d\tau_1 = \int_{-\infty}^{\infty} f(t-\tau_1)h(\tau_1)d\tau_1$$

By definition

$$f(t) \ast h(t) = \int_{-\infty}^{\infty} f(t-\tau)h(\tau)d\tau$$

From the above expressions we observe that

$$h(t) \ast f(t) = f(t) \ast h(t)$$

because τ and τ_1 are only variables of integration.

(b) Using the definition of convolution we have

$$[h(t) \ast f(t)] \ast g(t) = \left[\int_{-\infty}^{\infty} h(\tau) f(t-\tau)d\tau\right] \ast g(t) = \int_{-\infty}^{\infty}\left\{\left[\int_{-\infty}^{\infty} h(\tau) f(t-\tau_1-\tau)d\tau\right] g(\tau_1)\right\}d\tau_1 =$$

$$= \int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}\left[h(\tau) f(t-\tau_1-\tau)g(\tau_1)\right]d\tau\right\}d\tau_1$$

Changing the order of integration we have

$$[h(t) \ast f(t)] \ast g(t) = \int_{-\infty}^{\infty}\left\{h(\tau)\left[\int_{-\infty}^{\infty} f(t-\tau_1-\tau)g(\tau_1)d\tau_1\right]\right\}d\tau$$

By the definition of the convolution we have

$$f(t) \ast g(t) = \int_{-\infty}^{\infty} f(t-\theta)g(\theta)d\theta$$

Therefore

$$[f(t) \ast g(t)]_{t=t-\tau} = \int_{-\infty}^{\infty} f(t-\theta-\tau)g(\theta)d\theta$$

Hence

$$[h(t) \ast f(t)] \ast g(t) = \int_{-\infty}^{\infty}\left\{h(\tau)\,[f(t) \ast g(t)]_{t=t-\tau}\right\}d\tau$$

By the definition of the convolution product we have

$$h(t) \ast [f(t) \ast g(t)] = \int_{-\infty}^{\infty}\left\{h(\tau)\,[f(t) \ast g(t)]_{t=t-\tau}\right\}d\tau$$

Therefore

$$[h(t) \ast f(t)] \ast g(t) = h(t) \ast [f(t) \ast g(t)].$$

Thus the associative property of convolution is proved.

SOLUTION 16.57.

We have

$$f(t)\, h(t) = \sum_{k=0}^{\infty} f(t)\delta(t-kT)$$

For some nonnegative k, the Laplace transform of $f(t)\delta(t-kT)$ is

$$\mathcal{L}[f(t)\delta(t-kT)] = f(kT)e^{-skT}$$

by the sifting property of the delta function. Therefore we have

$$\mathcal{L}\left[f(t)h(t)\right] = \mathcal{L}\left[\sum_{k=0}^{\infty} f(t)\delta(t-kT)\right] = \sum_{k=0}^{\infty} f(kT)e^{-skT}$$

Using the notation $z = e^{sT}$ we have

$$\mathcal{L}\left[f(t)h(t)\right] = \sum_{k=0}^{\infty} f(kT)z^{-k}.$$

SOLUTION 16.58.
(a) By the voltage division formula we have

$$V_{out}(s) = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} \; V_{in}(s)$$

Therefore the transfer function of the circuit is

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{1}{CRs+1} = \frac{1}{2s+1}.$$

Taking the inverse Laplace transform yields

$$h(t) = \mathcal{L}^{-1}\left[\frac{1}{2s+1}\right] = \mathcal{L}^{-1}\left[\frac{\frac{1}{2}}{s+\frac{1}{2}}\right] = 0.5e^{-0.5t}u(t).$$

(b) By the impulse response theorem it follows that

$$v_{out}(t) = h(t) \quad v_{in}(t) =$$
$$= \left[0.5e^{-0.5t}u(t)\right] \quad \left[\delta(t)+\delta(t-1)+\delta(t-2)+\ldots\right]$$

Using the sifting property of the delta function it follows that

$$v_{out}(t) = 0.5e^{-0.5t}u(t) + 0.5e^{-0.5(t-1)}u(t-1) + 0.5e^{-0.5(t-2)}u(t-2) + \ldots \text{ V}.$$

Therefore for $0 < t < 1$

$$v_{out}(t) = 0.5e^{-0.5t}\text{ V}$$

because only $u(t)$ is nonzero for $0 < t < 1$.

(c) From the expression of $v_{out}(t)$ obtained in the part (b) it follows that

$$v_{out}(t) = 0.5e^{-0.5t} + 0.5e^{-0.5(t-1)} \text{ V, for } 1 < t < 2$$

because only $u(t)$ and $u(t-1)$ are nonzero for $1 < t < 2$.

(d) For t in the interval $(4,5)$, only $u(t)$, $u(t-1)$, $u(t-2)$, $u(t-3)$ and $u(t-4)$ are nonzero. Therefore

$$v_{out}(t) = 0.5e^{-0.5t} + 0.5e^{-0.5(t-1)} + 0.5e^{-0.5(t-2)} + 0.5e^{-0.5(t-3)} + 0.5e^{-0.5(t-4)} \text{ V.}$$

The above expression can be written as

$$v_{out}(t) = 0.5e^{-0.5(t-4)}\left[1 + \left(e^{-0.5}\right) + \left(e^{-0.5}\right)^2 + \left(e^{-0.5}\right)^3 + \left(e^{-0.5}\right)^4\right] \text{ V.}$$

(e) Using the sum formula for geometric series with $\lambda = e^{-0.5}$ we have

$$1 + \left(e^{-0.5}\right) + \left(e^{-0.5}\right)^2 + \left(e^{-0.5}\right)^3 + \left(e^{-0.5}\right)^4 = \frac{1-\left(e^{-0.5}\right)^5}{1-e^{-0.5}}$$

$$v_{out}(t) = 0.5e^{-0.5(t-4)} \; \frac{1-\left(e^{-0.5}\right)^5}{1-e^{-0.5}} \text{ V.}$$

Therefore

$$v_{out}(t) = 8.6189 \;\; e^{-0.5t} \text{ V.}$$

(f) Using the expression of $v_{out}(t)$ obtained in part (b) it follows that, for $n < t < n+1$,

$$v_{out}(t) = 0.5e^{-0.5t} + 0.5e^{-0.5(t-1)} + 0.5e^{-0.5(t-2)} + \ldots + 0.5e^{-0.5(t-n)} =$$
$$= 0.5e^{-0.5(t-n)}\left[1 + \left(e^{-0.5}\right) + \left(e^{-0.5}\right)^2 + \ldots + \left(e^{-0.5}\right)^n\right] \text{ V.}$$

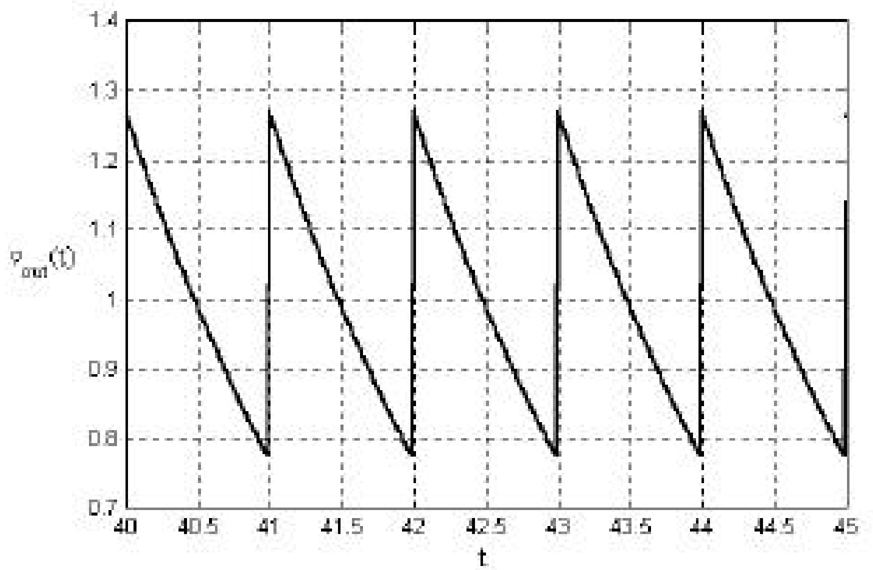
Using the sum formula for geometric series we have

$$v_{out}(t) = 0.5e^{-0.5(t-n)} \; \frac{1-\left(e^{-0.5}\right)^{n+1}}{1-e^{-0.5}} \text{ V.}$$

(g) For large n, $\left(e^{-0.5}\right)^{n+1} \quad 0$. Therefore, for large n we have

$$v_{out}(t) \quad e^{-0.5(t-n)} \; \frac{0.5}{1-e^{-0.5}} \text{ V for } n < t < n+1.$$

A picture of $v_{out}(t)$, for large n, is sketched in the next figure.



SOLUTION 16.59.
By the impulse response theorem we have

$$v_{out}(t) = h(t) \quad v_{in}(t) = \left[0.5e^{-0.5t}u(t)\right] \quad \left[\delta(t) + \delta(t+1) + \delta(t+2) + \ldots\right] \text{ V.}$$

Using the sifting property of the delta function it follows that

$$v_{out}(t) = 0.5e^{-0.5t}u(t) + 0.5e^{-0.5(t+1)}u(t+1) + 0.5e^{-0.5(t+2)}u(t+2) + \ldots \text{ V.}$$

For $0 < t$ we have

$$v_{out}(t) = 0.5e^{-0.5t} + 0.5e^{-0.5(t+1)} + 0.5e^{-0.5(t+2)} + \ldots =$$
$$= 0.5e^{-0.5t}\left[1 + \left(e^{-0.5}\right) + \left(e^{-0.5}\right)^2 + \ldots\right] \text{ V.}$$

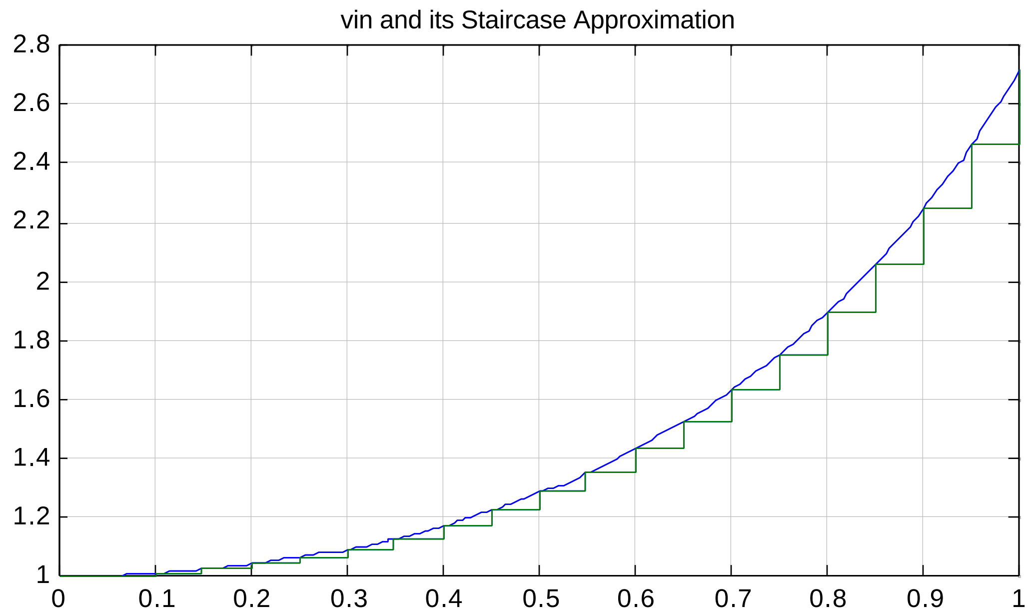
Using the sum formula for geometric series(for $n = \infty$) we have

$$v_{out}(t) = 0.5e^{-0.5t} \quad \frac{1}{1 - e^{-0.5}} \text{ V for } 0 < t.$$

Simplifying the expression of $v_{out}(t)$ it follows that

$$v_{out}(t) = 1.2707 \quad e^{-0.5t} \text{ V for } 0 < t.$$

SOLUTION 16.60. First we plot for reader convenience $v_{in}(t)$ and its staircase approximation.

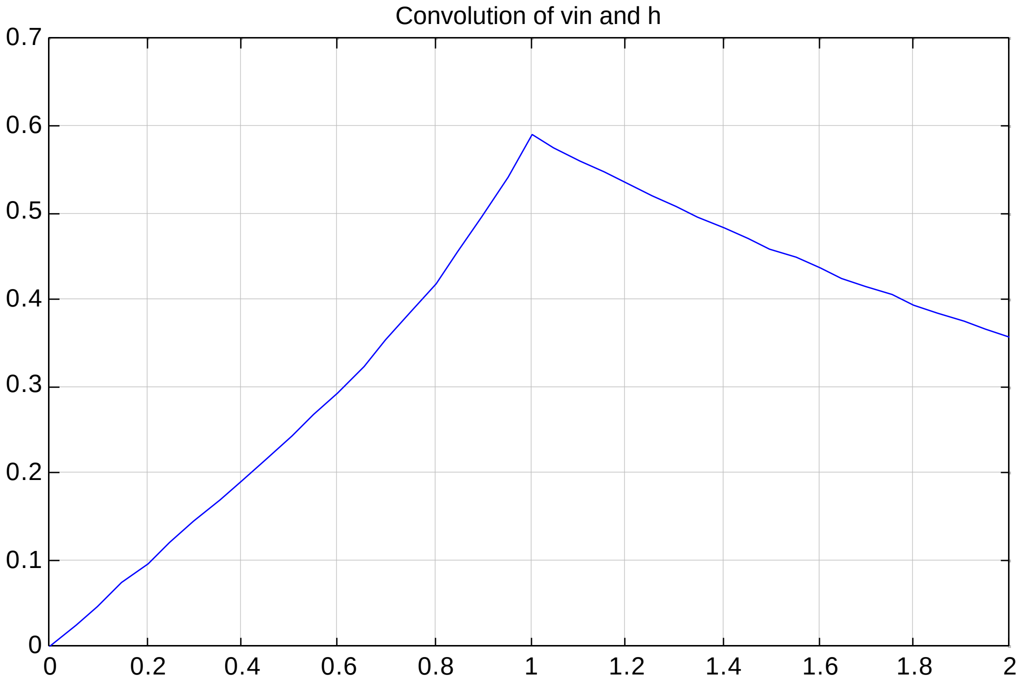


The transfer function of the circuit of figure 16.58 is $H(s) = \frac{1}{2s+1}$ in which case

$h(t) = 0.5e^{-0.5t}u(t)$. Because we only want the output for 0 ≤ t ≤ 2, we only need h(t) for 0 ≤ t ≤ 2 s.

Hence we need to generate staircase approximations to both $v_{in}(t)$ and h(t) as follows:

```
t = 0:0.05:2;
vin = exp(t .^2) .* (u(t) - u(t - 1));
h = 0.5*exp( - 0.5*t) .* (u(t) - u(t - 2));
T = 0.05;
tstep = T;
y = [0 conv(vin,h)*tstep 0];
t = 0:tstep:tstep*(length(vin)+length(h));
% For plotting through time 2 s we set
t=t(1:41);
y = y(1:41);
plot(t,y)
grid
```


Convolution of vin and h
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2