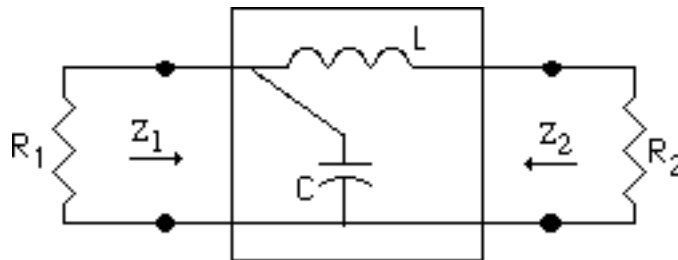


CHAPTER 17 PROBLEM SOLUTIONS

SOLUTION PROBLEM 17.11. Case 1: suppose $R_1 > R_2$. From example 17.3, page 696, if L and C are connected as indicated in part (a), then Z_1 can be made real and larger than R_2 . This means we can solve the problem at least for Z_1 . Specifically, consider the figure below



From example 17.3, at a specified frequency, r , for which Z_1 is real, then L , C , and R_2 must satisfy,

$$r = \sqrt{\frac{1}{LC} - \frac{R_2^2}{L^2}} \quad (1)$$

Further from example 17.3, at r ,

$$Z_1(jr) = \frac{L}{R_2 C}$$

We require that $Z_1(jr) = R_1$ in which case

$$R_1 R_2 = \frac{L}{C} \quad (2)$$

It is necessary to solve equations (1) and (2) simultaneously for L and C . From (2), $L = R_1 R_2 C$. Substituting into the square of (1) yields

$$r^2 = \frac{1}{R_1 R_2 C^2} - \frac{R_2^2}{R_1^2 R_2^2 C^2}$$

Hence

$$C = \frac{1}{r R_1} \sqrt{\frac{R_1 - R_2}{R_2}}$$

It follows that

$$L = R_1 R_2 \frac{1}{r R_1} \sqrt{\frac{R_1 - R_2}{R_2}} = \frac{1}{r} \sqrt{R_2 (R_1 - R_2)}$$

Observe that since $R_1 > R_2$, both C and L are real, i.e., exist. Please note that this connection would not result in real values of C and L had $R_1 < R_2$. If we can now show that $Z_2 = R_2$, then parts (a) and (b) are valid for this case.

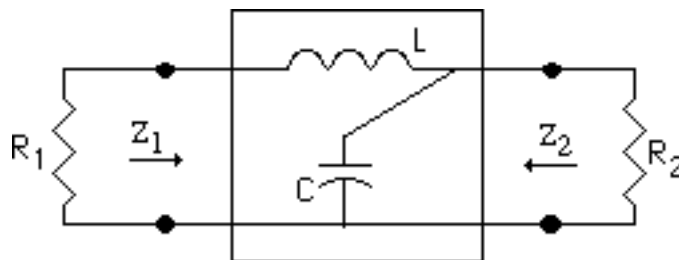
By direct computation

$$\begin{aligned} Z_2(j\omega) &= j\omega L + \frac{1}{j\omega C + \frac{1}{R_1}} = j\omega L + \frac{R_1}{j\omega C R_1 + 1} = j\sqrt{R_2(R_1 - R_2)} + \frac{R_1}{j\sqrt{\frac{R_1 - R_2}{R_2}} + 1} \\ &= j\sqrt{R_2(R_1 - R_2)} + \frac{R_1 - jR_1\sqrt{\frac{R_1 - R_2}{R_2}}}{1 + \frac{R_1 - R_2}{R_2}} = j\sqrt{R_2(R_1 - R_2)} + R_2 - jR_2\sqrt{\frac{R_1 - R_2}{R_2}} = R_2 \end{aligned}$$

Thus, (a) and (b) are true for the case $R_1 > R_2$.

We can also arrive at the conclusion that $Z_2 = R_2$ using maximum power transfer concepts. Since Z_1 is constructed so that $Z_1 = R_1$, we have set up the conditions for maximum power transfer of a V-source in series with R_1 to the "load" Z_1 . Since the LC coupling network is lossless, whatever average power is received by the network to the right of R_1 , will be dissipated by R_2 . Therefore maximum power is transferred to the load R_2 . Looking back from R_2 , it must be that R_2 sees a Thevenin resistance $Z_2 = R_2$ since it is known that there is a non-zero R_1 .

Case 2, $R_1 < R_2$. Now consider the configuration



Interchanging the subscripts of 1 and 2 in case 1 produces the derivation for this case.

Combining cases 1 and 2 using the text notation produces

$$C = \frac{1}{R_{\text{large}}} \sqrt{\frac{R_{\text{large}} - R_{\text{small}}}{R_{\text{small}}}} \quad L = \frac{1}{\sqrt{R_{\text{small}}(R_{\text{large}} - R_{\text{small}})}}$$

SOLUTION PROBLEM 17.16.

(a)

»C = 0.1e-6;

»R = 510;

»wr = 2*pi*1.56e3;

»%From equation 17.4 in example 17.3 which analyzes

»%this particular circuit,

»%L must satisfy

»% $wr^2 L^2 - L/C + R^2 = 0$.

»% Therefore

»

»L = roots([wr^2 -1/C R^2])

L =

5.3133e-02

5.0952e-02

(b) From HW problem 17.9,

$$\left(\frac{r}{L} \right)^2 = \frac{1}{LC} - \frac{1}{R^2 C^2}$$

Hence, in MATLAB,

»Linv = C*(wr^2 + 1/(R^2*C^2))

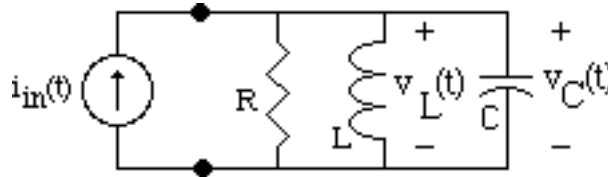
Linv =

4.8054e+01

»L = 1/Linv

L = 2.0810e-02

SOLUTION PROBLEM 17.22. (Correction: change R_L in figure P17.22 to R .) We solve this problem using phasors rather than the Laplace transform approach as it is simpler. Consider



As such, since by definition $\omega_0 = 1/\sqrt{LC}$, the resonant frequency,

$$\mathbf{V}_C = \mathbf{V}_L = Z_{in}(j\omega_0)\mathbf{I}_{in} = R\mathbf{I}_{in} = RI_m$$

Hence

$$v_C(t) = v_L(t) = RI_m \cos(\omega_0 t) \quad \text{and} \quad i_L(t) = \frac{RI_m}{\omega_0 L} \sin(\omega_0 t).$$

(a) By direct computation

$$w_C(t) = \frac{1}{2} C v_C^2(t) = \frac{1}{2} C R^2 I_m^2 \cos^2(\omega_0 t)$$

(b) By direct computation

$$w_L(t) = \frac{1}{2} L i_L^2(t) = \frac{R^2 I_m^2}{2 \omega_0^2 L} \sin^2(\omega_0 t)$$

(c) Also by direct computation, since $\omega_0^2 = 1/LC$,

$$\begin{aligned} w_C(t) + w_L(t) &= \frac{1}{2} C R^2 I_m^2 \cos^2(\omega_0 t) + \frac{R^2 I_m^2}{2 \omega_0^2 L} \sin^2(\omega_0 t) \\ &= \frac{1}{2} R^2 I_m^2 C \cos^2(\omega_0 t) + \frac{1}{2 \omega_0^2 L} \sin^2(\omega_0 t) = \frac{1}{2} R^2 I_m^2 C \end{aligned}$$

(d) The energy dissipated in the resistance in one period is

$$w_R(0, T) = R \int_0^T i_R^2(t) dt = R \int_0^T i_{in}^2(t) dt = R I_m^2 \int_0^T \cos^2(\omega_0 t) dt$$

$$= \frac{RI_m^2}{2} \int_0^T dt + \frac{RI_m^2}{2} \int_0^T \cos(2\omega t) dt = \frac{RI_m^2}{2} T = \frac{RI_m^2}{2}$$

(e) Finally

$$2 \frac{\text{maximum energy stored}}{\text{total energy lost per period}} = 2 \frac{0.5 R^2 I_m^2 C}{\frac{RI_m^2}{2}} = \omega RC = Q$$

by equation 17.13.

SOLUTION PROBLEM 17.27.

$$(a) H_1(s) = \frac{I_L}{V_s} = \frac{1}{Z_{in}(s)} = \frac{1}{R + Ls + \frac{1}{Cs}} = \frac{\frac{1}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$H_2(s) = H_1(s) \times \frac{1}{Cs} = \frac{1}{V_s} I_L \times \frac{1}{Cs} = \frac{V_C}{V_s} = \frac{\frac{s}{L} \times \frac{1}{Cs}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

(b) $H_1(s)$ is precisely of the form of equation 17.18 with a single zero at the origin. It follows that $\omega_p = \frac{1}{LC}$ and $\omega_{2p} = \frac{R}{L}$. Hence, from equation 17.19, $\omega_m = \omega_p = \frac{1}{\sqrt{LC}}$, $H_1(j\omega_m) = \frac{1}{R}$,

$$B = \omega_{2p} = \frac{R}{L}, \text{ and } Q_{cir} = Q_p = \frac{\omega_p}{\omega_{2p}} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}.$$

(c) With $s = j\omega$,

$$|H_2(j\omega)|^2 = \frac{\frac{1}{LC}}{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\omega\right)^2} = \frac{1}{\left(1 - LC\omega^2\right)^2 + (RC\omega)^2}$$

Instead of maximizing $|H_2(j\omega)|^2$ we minimize its reciprocal, through differentiation. Let

$$f(C) = \frac{1}{|H_2(j\omega)|^2} = (1 - LC^2)^2 + (RC)^2$$

Then

$$f'(C) = 2C(R)^2 + 2(1 - LC^2)(-2L) = 0$$

implies that

$$C = \frac{L}{R^2(L)^2} = \frac{1}{L^2} \frac{1}{\frac{R^2}{L^2} + 1} = \frac{1}{L^2} \frac{1}{\frac{1}{Q_{coil}^2} + 1}$$

If the coil has high Q, then

$$C \approx \frac{1}{L^2}$$

in which case

$$|H_2(j\omega)| = \frac{|V_C|}{|V_S|} = \frac{L}{R} = Q_{coil}$$

Therefore

$$|V_C|_{\max} = Q_{coil}|V_S|$$

SOLUTION PROBLEM 17.31. Here

$$|H(j\omega)|^2 = \left| K \frac{a + j\omega}{\left(\frac{\omega}{p} - \frac{\omega^2}{2}\right) + j2\frac{\omega}{p}} \right|^2 = K^2 \frac{a^2 + \omega^2}{\left(\frac{\omega}{p} - \frac{\omega^2}{2}\right)^2 + 4\frac{\omega^2}{p^2}} = K^2 f(\omega^2)$$

(a) Differentiating with respect to ω^2 and setting the derivative equal to zero yields

$$0 = \frac{df(\omega^2)}{d\omega^2} = \frac{d}{d\omega^2} \frac{a^2 + \omega^2}{\left(\frac{\omega}{p} - \frac{\omega^2}{2}\right)^2 + 4\frac{\omega^2}{p^2}}$$

$$= \frac{1}{\left(\frac{2}{p} - 2\right)^2 + 4\frac{2}{p}^2} - \frac{\left(a^2 + 2\right)\left(-2\left(\frac{2}{p} - 2\right) + 4\frac{2}{p}\right)}{\left[\left(\frac{2}{p} - 2\right)^2 + 4\frac{2}{p}^2\right]^2}$$

Given that the denominator of the first term is non-zero, this is equivalent to

$$\begin{aligned} 0 &= \left(\frac{2}{p} - 2\right)^2 + 4\frac{2}{p}^2 - \left(a^2 + 2\right)\left(-2\left(\frac{2}{p} - 2\right) + 4\frac{2}{p}\right) \\ &= \left(\frac{2}{p}\right)^2 + \frac{4}{p} - 2\frac{2}{p}^2 + 4\frac{2}{p}^2 + 2\left(a^2 + 2\right)\left(\frac{2}{p} - 2\right) - 4\left(a^2 + 2\right)\frac{2}{p} \\ &= \left(\frac{2}{p}\right)^2 + \frac{4}{p} - 2\frac{2}{p}^2 + 4\frac{2}{p}^2 + 2a^2\frac{2}{p} - 2a^2^2 + 2\frac{2}{p}^2 - 2\left(\frac{2}{p}\right)^2 - 4a^2\frac{2}{p} - 4\frac{2}{p}^2 \\ &= -\left(\frac{2}{p}\right)^2 - 2a^2^2 + \frac{4}{p} + 2a^2\frac{2}{p} - 4a^2\frac{2}{p} \end{aligned}$$

Hence

$$\left(\frac{2}{p}\right)^2 + 2a^2^2 + a^4 = \frac{4}{p} + 2a^2\frac{2}{p} - 4a^2\frac{2}{p} + a^4$$

where we have added a^4 to produce perfect squares, i.e.,

$$\left(\frac{2}{p} + a^2\right)^2 = \left(\frac{2}{p} + a^2\right)^2 - \left(2a\frac{2}{p}\right)^2$$

This implies that

$$2 = -a^2 \pm \sqrt{\left(\frac{2}{p} + a^2\right)^2 - \left(2a\frac{2}{p}\right)^2}$$

Thus, to achieve a real positive solution we obtain,

$$m = \sqrt{-a^2 + \sqrt{\left(\frac{2}{p} + a^2\right)^2 - \left(2a\frac{2}{p}\right)^2}}$$

(b) Here, as in part (a), the arithmetic is simpler if we deal with $1/H(s)$ rather than $H(s)$.

Specifically

$$\frac{1}{H(s)} = \frac{s^2 + 2\frac{2}{p}s + \frac{2}{p}}{K(s+a)} = \frac{s^2 + 2\frac{2}{p}(s+a) - 2\frac{2}{p}a + \frac{2}{p}}{K(s+a)} = \frac{2}{K} + \frac{s^2 - 2\frac{2}{p}a + \frac{2}{p}}{K(s+a)}$$

The problem asks for a non-zero value of ω . For zero phase shift of $H(j\omega)$, $\text{Im}[H(j\omega)] = 0$

$\text{Im}[H(j\omega)] > 0$. This necessarily requires that the imaginary part of the above expression at $s = j\omega$ must be zero. Thus

$$\begin{aligned} \text{Im} \frac{1}{H(j\omega)} &= \text{Im} \frac{-\omega^2 - 2\omega p a + \frac{2}{p}}{K(j\omega + a)} = \text{Im} \frac{\left(-\omega^2 - 2\omega p a + \frac{2}{p}\right)(a - j\omega)}{K(\omega^2 + a^2)} \\ &= \frac{\left(-\omega^2 - 2\omega p a + \frac{2}{p}\right)(-\omega)}{K(\omega^2 + a^2)} = 0 \end{aligned}$$

The solutions to this are dc, i.e., $\omega = 0$, and

$$\text{(for zero phase shift)} \quad \omega = \sqrt{\frac{2}{p} - 2\omega p a}$$

Note that such a frequency may not exist if the quantity under the radical is negative.

SOLUTION PROBLEM 17.40. (a) Compute the transfer function:

$$\begin{aligned} H(s) &= \frac{V_{out}}{I_{in}} = \frac{1}{Y_{in}(s)} = \frac{1}{\frac{1}{Ls + R_s} + Cs + \frac{1}{R_p}} = \frac{1}{C} \times \frac{s + \frac{R_s}{L}}{s^2 + \frac{R_s}{L} + \frac{1}{R_p C} s + \frac{R_s}{R_p} + 1} \frac{1}{LC} \\ &= 8 \times 10^5 \frac{s + 1.333 \times 10^3}{s^2 + 2000s + 1.0006 \times 10^6} \end{aligned}$$

where $\frac{2}{0} = 1.6 \times 10^9$ and $\frac{2}{p} = 1.0006 \times 10^6$, $\frac{2}{0} = 1.60089 \times 10^9$ and $p = 4.001 \times 10^4$ rad/sec.

(b) $Q_p = \frac{p}{2} = 20$ which is high Q_p . Further $a = \frac{R_s}{L} = 1.333 \times 10^3 \ll p = 40.011109 \times 10^3$.

(c) Equation 17.28,

$$m = \sqrt{-a^2 + \sqrt{\left(\frac{2}{p} + a^2\right)^2 - (2a/p)^2}}$$

gives the exact value of m . In MATLAB,

```
»wmsqrd = -a^2 + sqrt((wp^2+a^2)^2 - (twosig*a)^2)
wmsqrd = 1.600886670362673e+09
```

```
»wm = sqrt(wmsqrd)
wm = 4.001108184444246e+04
```

Clearly, m approximates p . Finally,

$$\left| \frac{H(j\omega_m)}{H(j\omega_p)} \right| = 8 \times 10^5 \left| \frac{j\omega_p + 1.333 \times 10^3}{-j\omega_p + j2000} \right| = \frac{8 \times 10^5}{2000} \left| 1 - j \frac{1.333 \times 10^3}{\omega_p} \right|$$

400

(d) From equation 17.30,

$$\begin{aligned} \omega_{1m} - 0.5B &= 39.011 \times 10^3 \text{ rad/sec, and} \\ \omega_{2m} + 0.5B &= 41.011 \times 10^3 \text{ rad/sec.} \end{aligned}$$

(e) From equation 17.30,

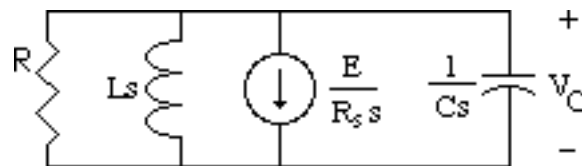
$$Q_{cir} = Q_p = \frac{p}{2} = \frac{\sqrt{\frac{R_s}{R_p} + 1} \frac{1}{LC}}{\frac{R_s}{L} + \frac{1}{R_p C}} = \frac{0}{\frac{R_s}{L} + \frac{1}{R_p C}}$$

$$\frac{1}{Q_{cir}} = \frac{\frac{R_s}{L} + \frac{1}{R_p C}}{0} = \frac{R_s}{L} + \frac{1}{R_p C} = \frac{1}{Q_{coil}} + \frac{1}{Q_{cap}} = \frac{Q_{coil} + Q_{cap}}{Q_{coil} Q_{cap}}$$

Hence,

$$Q_{cir} = \frac{Q_{coil}Q_{cap}}{Q_{coil} + Q_{cap}}$$

SOLUTION PROBLEM 17.65. Here we consider the equivalent circuit valid for $t > 0$. Note that $i_L(0^-) = i_L(0^+) = E/Rs$ and $v_C(0^-) = v_C(0^+) = 0$. Hence



It follows that

$$V_C(s) = \frac{1}{\frac{1}{R} + \frac{1}{Ls} + Cs} \times \frac{E}{R_s s} = \frac{1}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} \times \frac{E}{CR_s}$$

Complex roots occur when

$$\frac{1}{RC}^2 - \frac{4}{LC} = \frac{1}{RC}^2 - 4 \frac{2}{0} < 0$$

Equivalently,

$$0RC > \frac{1}{2}$$

as was to be shown. Further, since $v_C(0^+) = 0$, the general form of the capacitor voltage for constant excitation is (as per chapter 10)

$$v_C(t) = e^{-t} (A \cos(\omega_d t) + B \sin(\omega_d t)) = B e^{-t} \sin(\omega_d t)$$

From the characteristic equation, the complex roots are

$$- \pm j \omega_d = \frac{-1}{2RC} \pm j \sqrt{\frac{1}{LC} - \frac{1}{2RC}^2} = \frac{-1}{2RC} \pm j \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

Here

$$a = \frac{0}{2Q} \quad d = \sqrt{1 - \frac{1}{4Q^2}}$$

Letting $B = V_m$, for the appropriate value of V_m , we obtain the desired result.

(b) If Q is large, $d \approx 1$. The $V_m e^{-at}$ will drop to $1/e$ of its peak value in $t = \frac{1}{a} = \frac{2Q}{0} = \frac{2Q}{d}$

seconds. The period of oscillation of the damped sinusoid is $\frac{2}{d}$. Therefore the number of cycles

contained in this interval is

$$\frac{\frac{2Q}{d}}{\frac{2}{d}} = \frac{Q}{1}$$