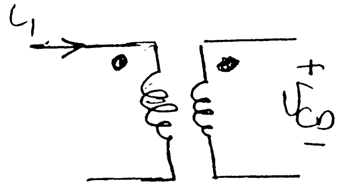
Chapter 18/Problem 1

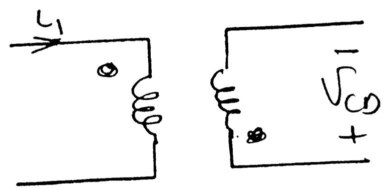
Because $i_1(t) = 2t\,u(t)$ with positive current at point-A, then, because V_{CD} is $20\,u(t)$, the dots are placed at point-A and -C

Fig P:19.1



$$M = M_{21} = \frac{|v_2(t)|}{\left|\frac{di_1}{dt}\right|} = \frac{20}{2} = 10H$$

When V_{CD} is $-20\,u(t)$, then the dots on coil-2 are reversed to appear at point-A and -D



$$M = M_{21} = \frac{|v_2(t)|}{\left|\frac{di_1}{dt}\right|} = \frac{20}{2} = 10H$$

Chapter 18/Problem 2

The waveform can be broken into components

$i_1 = -2t$ A; $v_2(t) = 4$ V $\quad 0 < t < \frac{1}{2}$

$i_1 = 2t$ A; $v_2(t) = -4$ V $\quad \frac{1}{2} < t < \frac{3}{2}$

$i_1 = -2t$ A; $v_2(t) = 4$ V $\quad \frac{3}{2} < t < \frac{3}{2}$

Fig P18.2a

Fig P18.2b

Fig P18.2c

(a) The opposite polarity of $i_1(t)$ and $v_2(t)$ indicated here shows that the dots should be placed at terminals -A and -D (or terminals -B and -C)

$$v_2(t) = M\frac{di_1}{dt}$$

$$4 = M(-2)$$

$$M = \frac{4}{-2} = -2H$$

(b) If $i(t) = 2(1-e^{-100t})u(t)$ A

$$v_2(t) = -M\frac{di_1}{dt}$$

$$= -M\frac{d}{dt}\left[2(1-e^{-100t})\right]$$

$$= -2(200e^{-100t})$$

$$= -400e^{-100t}u(t) \text{ V}$$

Chapter 18/Problem 3

(a)

(i) For the dots at points-A and-C

$$v_2(t) = M\frac{di_1}{dt}$$

(ii) For the dots at points-A and-D

$$v_2(t) = -M\frac{di_1}{dt}$$

(iii) For the dots at points-B and-C

$$v_2(t) = -M\frac{di_1}{dt}$$

(iv) For the dot at point-B and-D

$$v_2(t) = M\frac{di_1}{dt}$$

(b) The foregoing development constitutes the sought after verification

$\boxed{\text{Fig P18.3}}$

Chapter 18/Problem 4

Fig P18.4a

Fig P18.4b

(a)

For Fig P18.4a

$$v_x = 2i_a + 4\frac{di_a}{dt} + 2\frac{di_b}{dt}$$

$$v_y = 3\frac{di_b}{dt} + 2\frac{di_a}{dt}$$

For Fig P18.4b

$$v_x = R_1 i_a + L_1\frac{di_a}{dt} - M\frac{di_b}{dt}$$

$$v_y = L_2\frac{di_b}{dt} - M\frac{di_a}{dt}$$

(b) For Fig P18.4a

$$V_x(s) = (4s+2)I_a(s) + 2sI_b(s) + 4i_a(0^+) + 2i_b(0^+)$$

$$V_y(s) = 3sI_b(s) + 2sI_a(s) + 3i_b(0^+) + 2I_a(0^+)$$

For Fig P18.4b

$$V_x(s) = (L_1 s + R_1)I_a(s) - MsI_b(s) + L_1 i_a(0^+) - Mi_b(0^+)$$

$$V_y(s) = L_2 s I_b(s) - MsI_a(s) + L_2 i_b(0^+) - Mi_a(0^+)$$

Chapter 18 / Problem 5

Fig P18.5a

Fig P18.5b

(a) For Fig P18.5a

$$v_x = 2i_a + 4\frac{di_a}{dt} - 2\frac{di_b}{dt}$$

$$v_y = 3\frac{di_b}{dt} - 2\frac{di_a}{dt}$$

For Fig P18.5b

$$v_x = R_1 i_a + L_1\frac{di_a}{dt} + M\frac{di_b}{dt}$$

$$v_y = L_2\frac{di_b}{dt} + M\frac{di_a}{dt}$$

(b) For Fig P18.5a

$$V_x(s) = 2I_a(s) + 4sI_a(s) - 2sI_b(s) + 4i_a(0^-) - 2i_b(0^-)$$

$$V_y(s) = 3sI_b(s) - 2sI_a(s) + 3i_b(0^-) - 2i_a(0^-)$$

For Fig P18.5b

$$V_x(s) = (L_1 s + R_1)I_a(s) + MsI_b(s) + L_1 i_a(0^-) + Mi_b(0^-)$$

$$V_y(s) = L_2 sI_b(s) + MsI_a(s) + L_2 i_b(0^-) + Mi_a(0^-)$$

Chapter 18/Problem 6

$\boxed{\text{Fig } P18.6}$

Write by inspection

$$i_a - i_b + 2\left(\frac{di_a}{dt} - \frac{di_b}{dt}\right) - \frac{di_c}{dt} = v_{in}(t)$$

$$2\left(\frac{di_b}{dt} - \frac{di_a}{dt}\right) + \frac{di_c}{dt} + i_b - i_c + i_b = 0$$

$$i_c - i_a + i_c - i_b + 2\frac{di_c}{dt} + \frac{di_b}{dt} - \frac{di_a}{dt} = 0$$

or

$$i_a + 2\frac{di_a}{dt} - i_b - 2\frac{di_b}{dt} - \frac{di_c}{dt} = v_{in}(t)$$

$$-2\frac{di_a}{dt} + 2i_b + 2\frac{di_b}{dt} - i_c + \frac{di_c}{dt} = 0$$

$$-i - \frac{di_a}{dt} - i_b + \frac{di_b}{dt} + 2i_c + 2\frac{di_c}{dt} = 0$$

Chapter 18/Problem 7

Fig P18.7a

Fig P18.7b

For Fig P18.7a

$$v = 3\frac{di}{dt} + 2.5\frac{di}{dt} + 5\frac{di}{dt} + 2.5\frac{di}{dt}$$

With

$$v = L_{eq}\frac{di}{dt}$$

observe that

$$L_{eq} = 3 + 2.5 + 5 + 2.5 = 13\,H$$

For Fig P18.7b

$$v = 3\frac{di}{dt} - 2.5\frac{di}{dt} + 5\frac{di}{dt} - 2.5\frac{di}{dt}$$

with

$$v = L_{eq}\frac{di}{dt}$$

then

$$L_{eq} = 3 - 2.5 + 5 - 2.5 = 3\,H$$

Chapter 18/Problem 8

Fig P18.8a

Fig P18.8b

For Fig P18.8a

$$v_{in}(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$0 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

From the second

$$\frac{di_2}{dt} = -\frac{M}{L_2} \frac{di_1}{dt}$$

and with this in the first

$$v_{in}(t) = L_{eq} \frac{di_1}{dt} = \left(L_1 - \frac{M^2}{L_2}\right) \frac{di_1}{dt}$$

$$L_{eq} = L_1 - \frac{M^2}{L_2}$$

For Fig P18.8b

$$v_{in}(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$0 = (L_2 + L_3) \frac{di_2}{dt} + M \frac{di_1}{dt}$$

From the second

$$\frac{di_2}{dt} = -\frac{M}{L_2 + L_3} \frac{di_1}{dt}$$

and with this in the first

$$v_{in}(t) = L_{eq} \frac{di_1}{dt} = \left(L_1 - \frac{M^2}{L_2 + L_3}\right) \frac{di_1}{dt}$$

and

$$L_{eq} = L_1 - \frac{M^2}{L_2 + L_3}$$

These results are independent of the dot positions.

Chapter 18/Problem 9

$$V_{IN}(s) = (La + L_1) s I_1(s) + MsI_2(s)$$

$$0 = (L_2 + L_5) s I_2(s) + MsI_1(s)$$

fig P18.9

From the second

$$I_2(s) = -\frac{M}{L_2 + L_5} I_1(s)$$

and with this in the first

$$V_{IN}(s) = Z_{IN}(s) I_1(s) = \left(La + L_1 - \frac{M^2}{L_2 + L_5}\right) s\, I_1(s)$$

This makes

$$Z_{IN}(s) = \left(La + L_1 - \frac{M^2}{L_2 + L_6}\right) s$$

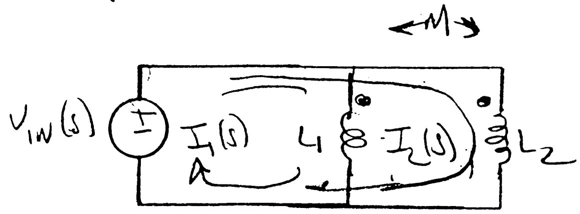
Chapter 18/Problem 10 ①

Treat these as problems in loop analysis

For Fig P18.10a

Fig P18.10a

Fig P18.10b



Using V_{IN}, I_1 and I_2 for $V_{IN}(s)$, $I_1(s)$ and $I_2(s)$ leads to

$$V_{IN} = L_1 s I_1 + M s I_2$$

$$V_{IN} = M s I_1 + L_2 s I_2$$

In matrix form

$$s\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{IN} \\ V_{IN} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix}}{s(L_1L_2 - M^2)}\begin{bmatrix} V_{IN} \\ V_{IN} \end{bmatrix}$$

$$I_1 = \frac{(L_2 - M)V_{IN}}{s(L_1L_2 - M^2)}$$

$$I_2 = \frac{(L_1 - M)V_{IN}}{s(L_1L_2 - M^2)}$$

Then by KCL

$$I = I_1 + I_2 = \frac{L_1 + L_2 - 2M}{L_1L_2 - M^2}\,\frac{V_{IN}}{s}$$

and with $V_{IN} = L_{eq}\, s I$

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

18/10 Cont'd

(b) If the dots are reversed, then

$$V_{IN} = L_1 s I_1 - M s I_2$$
$$V_{IN} = -M s I_1 + L_2 s I_2$$

$$s \begin{bmatrix} L_1 & -M \\ -M & L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{IN} \\ V_{IN} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} L_2 & M \\ M & L_1 \end{bmatrix}}{s(L_1 L_2 - M^2)} \begin{bmatrix} V_{IN} \\ V_{IN} \end{bmatrix}$$

$$I_1 = \frac{L_2 + M}{L_1 L_2 - M^2} \frac{V_{IN}}{s}$$

$$I_2 = \frac{L_1 + M}{L_1 L_2 - M^2} \frac{V_{IN}}{s}$$

$$I_1 + I_2 = \frac{L_1 + L_2 + 2M}{L_1 L_2 - M^2} \frac{V_{IN}}{s}$$

which makes

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 L_2 + 2M}$$

Chapter 18/Problem 11

Fig P18.11a

Fig P18.11b

(a) For fig 18.11a, with

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

$$Z_{IN}(s) = \frac{R L_{eq} s}{R + L_{eq} s}$$

This is the answer given but

$$Z_{IN}(s) = \frac{\dfrac{s(L_1L_2 - M^2)R}{L_1 + L_2 - 2M}}{R + \dfrac{s(L_1L_2 - M^2)}{L_1 + L_2 - 2M}}$$

$$Z_{IN}(s) = \frac{R(L_1L_2 - M^2)s}{R(L_1 + L_2 - 2M) + (L_1L_2 - M^2)s}$$

(b) For fig 18.11b with

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

$$Z_{IN}(s) = \frac{R L_{eq} s}{R + L_{eq} s}$$

$$= \frac{\dfrac{s(L_1L_2 - M^2)R}{L_1 + L_2 + 2M}}{R + \dfrac{s(L_1L_2 - M^2)}{L_1 + L_2 + 2M}}$$

$$= \frac{R(L_1 + L_2 - M^2)s}{R(L_1 + L_2 + 2M) + (L_1L_2 - M^2)s}$$

18/12

Please be careful

Somehow, this one
has a peculiar aroma
associated with it

Chapter 18/Problem 12

The mesh equations in the s-domain with V_{IN}, I_a, I_b and I_c used for $V_{IN}(s)$, $I_a(s)$, $I_b(s)$ and $I_c(s)$ are

$\boxed{\text{Fig P18.6}}$

$$(2s+1)I_a - (2s+1)I_b - sI_c = V_{IN}$$
$$-2sI_a + (2s+2)I_b + (s-1)I_c = 0$$
$$-(s+1)I_a + (s-1)I_b + (2s+2)I_c = 0$$

Solve for I_a

$$I_a = \frac{\begin{vmatrix} V_{IN} & -(2s+1) & -s \\ 0 & 2s+2 & s+1 \\ 0 & s-1 & 2s+2 \end{vmatrix}}{\begin{vmatrix} 2s+1 & -(2s+1) & -s \\ -2s & 2s+2 & s+1 \\ -(s+1) & s-1 & 2s+2 \end{vmatrix}} = \frac{(3s^2+8s+5)V_{IN}}{21s^3+29s^2+22s+8}$$

Then

$$Z_{IN}(s) = \frac{V_{IN}(s)}{I_a(s)} = \frac{21s^3+29s^2+22s+8}{3s^2+8s+5}$$

Chapter 18/Problem 15

In the aiding configuration

$$L_{eq} = L_1 + L_2 + 2M = 3.7$$

In the opposing configuration

$$L_{eq} = L_1 + L_2 - 2M = 0.25$$

Adding and subtracting these gives

$$2(L_1 + L_2) = 6.2$$

$$L_1 + L_2 = 3.1$$

And

$$4M = 1.2$$

$$M = 0.3H \leftarrow$$

The parallel combination Leq with dotted terminals connected together is given by part (a) of Problem 10

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{121}{250}$$

$$= \frac{L_1 L_2 - 0.09}{3.1 - 0.6} = \frac{121}{250}$$

$$L_1 L_2 = 1.30$$

Then with

$$L_2 = 3.10 - L_1$$

$$L_1(3.10 - L_1) = 1.30$$

$$-L_1^2 + 3.10L_1 - 1.30 = 0$$

$$L_1^2 - 3.10L_1 + 1.30 = 0$$

$$(L_1 - 2.60)(L_2 - 0.50) = 0$$

$$L_1 = 2.60 \leftarrow$$

$$L_2 = 0.50 \leftarrow$$

Note $L_1 > L_2$

Chapter 18/Problem 16

Fig P18.16

The open circuit at the right hand terminals means that the 1H inductor make NO contribution to the current through the 0.2H inductor.

Thus, by voltage division

$$V_2(s) = \frac{0.2s}{0.2s+200} V_{IN}(s)$$

With $V_{IN}(t) = 10u(t)$ V

$$V_{IN}(s) = \frac{0.2s}{0.2s+200} \frac{10}{s} = \frac{2}{s+1000}$$

and

$$v_{IN}(t) = 2e^{-1000t} u(t) \text{ V}$$

Chapter 18/Problem 17

Fig P18.17

(a) The open circuit at the right hand terminals means that the 5H inductor makes no contribution to the current through the 0.2H inductor.

Thus, by voltage division

$$V_2(s) = \frac{0.2s}{0.2s+200} V_{IN}(s)$$

With $v_{IN}(t) = 12u(t)$

$$V_2(s) = \frac{0.2s}{0.2s+200} \cdot \frac{12}{s} = \frac{2.4}{s+1000}$$

and $$v_2(t) = 2.4e^{-1000t}u(t) \text{ V}$$

The time constant is $T = 1$ ms

(b) If $v_{IN}(t) = -12u(-t)$, linearity gives

$$v_2(t) = -2.4e^{+1000t}u(-t) \text{ V}$$

(c) If $v_{IN}(t) = -6u(-t) + 6u(t)$ V, linearity and superposition gives

$$v_2(t) = -1.2e^{+1000t}u(-t) + 1.2e^{-1000t}u(t) \text{ V}$$

Chapter 18/Problem 18

With the open circuit at the right hand terminals, the 2H inductor at the right makes no mutual inductive contribution to the current through the 1H inductor. Because of this, it doesn't matter where the dot on the 1-H inductor is placed

Fig P18.18

(a) and (b) will have the same zero-state response.

By voltage division

$$V_{out}(s) = \frac{s}{s+1} V_{IN}(s)$$

with $v_{IN}(t) = 10e^{-2t}u(t)\,V$

$$V_{out}(s) = \frac{10s}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$$K_1 = \left.\frac{10s}{s+2}\right|_{s=-1} = \frac{-10}{1} = -10$$

$$K_2 = \left.\frac{10s}{s+1}\right|_{s=-2} = \frac{-10}{-1} = 10$$

$$V_{out}(s) = 10\left[\frac{1}{s+2} - \frac{1}{s+1}\right]$$

$$v_{out}(t) = 10\left(e^{-2t} - e^{-t}\right)u(t)\,V$$

Chapter 18/Problem 19

Fig P18.19

With the open circuit at the left terminals, the 0.1H inductor at the left make no contribution to the current through the 0.2H inductor. Thus, it doesn't matter where the dot is placed on the 0.1H inductor.

(a) and (b) will have the same zero-state response

$$i_{IN}(t) = 10\,u(t)\ \text{mA}$$

$$Y(s) = Cs + \frac{1}{Ls} = \frac{LCs^2+1}{Ls} = \frac{C\left(s^2+\frac{1}{LC}\right)}{s}$$

and with $L=0.2H$ and $C=0.1F$

$$Z(s) = \frac{10s}{s^2+50}$$

and

$$V_{out}(s) = Z(s)\,I_{IN}(s)$$

$$I_{IN}(s) = \frac{10}{s}$$

$$V_{out}(s) = \frac{100}{s^2+50} = \frac{100}{\sqrt{50}}\,\frac{\sqrt{50}}{s^2+50}$$

$$V_{out}(s) = 10\sqrt{2}\sin(7.071t)\,u(t)\ \text{mA}$$

Chapter 18/Problem 20

Fig P18.20

Because the left hand end of the circuit has an open circuit, the 0.1H inductor makes no contribution on $V_{out}(t)$.

For both part (a) and part (b)

$$Y(s) = \frac{8}{5s} + \frac{s}{10} + 1 = \frac{5s^2+50s+80}{50s}$$

and

$$Z(s) = \frac{10s}{s^2+10s+16}\ \Omega$$

(a) With $i_{IN}(t) = 10u(t)$ mA

$$I_{IN}(s) = \frac{10}{s}$$

$$V_{out}(s) = Z(s)\,I_{IN}(s) = \frac{100}{s^2+10s+16} = \frac{100}{(s+2)(s+8)} = \frac{K_1}{s+2} + \frac{K_2}{s+8}$$

$$K_1 = \left.\frac{100}{s+8}\right|_{s=-2} = \frac{100}{6} = \frac{50}{3}$$

$$K_2 = \left.\frac{100}{s+2}\right|_{s=-8} = \frac{100}{-6} = -\frac{50}{3}$$

$$V_{out}(s) = \frac{50}{3}\left(e^{-2t} - e^{-8t}\right)u(t)\ \text{mV}$$

(c) With $i_{IN}(t) = 10e^{-4t}u(t)$ mA

$$I_{IN}(s) = \frac{10}{s+4}$$

$$V_{out}(s) = \frac{100s}{(s+2)(s+4)(s+8)} = \frac{K_1}{s+2} + \frac{K_2}{s+4} + \frac{K_3}{s+8}$$

18/20 Cont'd

$$K_1 = \frac{100s}{(s+4)(s+8)}\Bigg|_{s=-2} = \frac{-200}{2(6)} = -\frac{50}{3}$$

$$K_2 = \frac{100s}{(s+2)(s+8)}\Bigg|_{s=-4} = \frac{-400}{(-2)(4)} = \frac{150}{3}$$

$$K_3 = \frac{100s}{(s+2)(s+4)}\Bigg|_{s=-8} = \frac{-800}{(-6)(-4)} = -\frac{100}{3}$$

$$V_{out}(s) = \frac{50}{3}\left(3e^{-4t} - e^{-2t} - 2e^{-8t}\right) u(t) \quad mV$$

Chapter 18/Problem 21

(a) With the dot in position-A

$L_{eq} = L_1 + L_2 - 2M$

$= 2 + 2 - 2 = 2H$

Fig P18.21

$Z(s) = 8 + 2s$

$Z(j\omega) = 8 + j2\omega$

$Z(j2) = 8 + j4$

With

$V_s = 4\cos 2t\,V, \quad \hat{V}_s = 4\angle 0^\circ$

$\hat{I} = \frac{\hat{V}_s}{8+j4} = \frac{4\angle 0^\circ}{8.94\angle 26.57^\circ} = 1.789\angle -26.51^\circ\ A$

$i(t) = 1.789\cos(2t - 26.51^\circ)\ A$

and

$p_R(t) = Ri(t)^2 = 8(1.789)^2\cos^2(2t)$

$p_R(t) = 25.6\cos^2(2t)\ W$

(b) With the dot in position-B

$L_{eq} = L_1 + L_2 + 2M = 2 + 2 + 2 = 6H$

$Z(s) = 8 + 6s$

$Z(j\omega) = 8 + j6\omega$

$Z(j2) = 8 + j12$

$\hat{I} = \frac{4\angle 0^\circ}{14.42\angle 56.31^\circ} = 0.277\angle -56.31^\circ$

$i(t) = 0.277\cos(2t - 56.31^\circ)\ A$

and

$p_R(t) = Ri(t)^2 = 8(0.277)^2\cos^2(2t)$

$p_R(t) = 0.615\cos^2(2t)\ W$

Chapter 18/Problem 22

Fig P18.22

(a) The dots are opposing

$$L_{eq} = L_1 + L_2 - 2M$$
$$= 3 + 2 - 2(2) = 1\,H$$

(b) The step response with all IC's equal to zero

$$Z(s) = s + 2 + \frac{1}{s} = \frac{s^2 + 2s + 1}{s}$$

For $v_{IN}(t) = u(t)$

$$V_{IN}(s) = \frac{1}{s}$$

$$I_{out}(s) = \frac{V_{IN}(s)}{Z(s)} = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

$$i_{out}(t) = te^{-t}u(t)\,A$$

(c) This is a simple RLC series circuit with

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1}} = 1\ \text{rad/s}$$

(d)

$$Q = \frac{\omega_0 L}{R} = \frac{1(1)}{2} = \frac{1}{2}\ \text{rad/s}$$

$$BW = \frac{\omega_0}{Q} = \frac{1}{1/2} = 2\ \text{rad/s}$$

Check Q

$$Q = \frac{1}{RC\omega_0} = \frac{1}{2(1)(1)} = \frac{1}{2}\ ✓$$