CHAPTER 18 PROBLEM SOLUTIONS

SOLUTION PROBLEM 18.34. There is a correction to this problem: set M = 3 H.

(a) The stored energy at t = 0 is:

$$W(0) = 0.5L_1 i_1^2(0) + 0.5L_2 i_2^2(0) + M i_1(0) i_2(0) = 8 \text{ J}$$

```
»L1 = 10; L2 = 2; M = 3; i10 = 1; i20 = -3;
»W0 = 0.5*L1*i10^2 + 0.5*L2*i20^2 + M*i10*i20
W0 =     5
```

(b) Writing two differential mesh equations we obtain

$$L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} + R_1 i_1 = 10\frac{di_1}{dt} + 3\frac{di_2}{dt} + i_1 = 0$$

and

$$L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} + R_2 i_2 = 2\frac{di_2}{dt} + 3\frac{di_1}{dt} + i_2 = 0$$

Taking the Laplace transform of these equations yields

$$10sI_1 - 10i_1(0) + 3sI_2 - 3i_2(0) + I_1 = (10s+1)I_1 + 3sI_2 - 10i_1(0) - 3i_2(0) = 0$$

and

$$2sI_2 - 2i_2(0) + 3sI_1 - 3i_1(0) + I_2 = (2s+1)I_2 + 3sI_1 - 3i_1(0) - 2i_2(0) = 0$$

Putting these equations in matrix form yields

$$\begin{bmatrix} (10s+1) & 3s \\ 3s & (2s+1) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10i_1(0) + 3i_2(0) \\ 3i_1(0) + 2i_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Solving yields

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} (10s+1) & 3s \\ 3s & (2s+1) \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \frac{1}{11s^2 + 12s + 1}\begin{bmatrix} 2s+1 & -3s \\ -3s & 10s+1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$= \frac{1}{(11s+1)(s+1)}\begin{bmatrix} 11s+1 \\ -33s-3 \end{bmatrix} = \frac{1}{s+1}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Therefore, by inspection,

$$i_1(t) = e^{-t}u(t) \text{ A} \quad \text{and} \quad i_2(t) = -3e^{-t}u(t) \text{ A}$$

Remark: normally, i1(t) and i2(t) would have two exponential terms present. Because of the special choice of initial conditions, a pole cancelled out.

(c) From equation 18.24 with the lower limit changed to zero and the upper limit changed to ∞, we have

$$W(0,\infty) = \int_0^\infty \left(v_1 i_1 + v_2 i_2\right) dt = 0.5 L_1 i_1^2(\infty) + 0.5 L_2 i_2^2(\infty) + M i_1(\infty) i_2(\infty)$$

$$-0.5 L_1 i_1^2(0) + 0.5 L_2 i_2^2(0) + M i_1(0) i_2(0)$$

From part (b) all currents at t = ∞ are zero, hence

$$W(0,\infty) = -0.5 L_1 i_1^2(0) + 0.5 L_2 i_2^2(0) + M i_1(0) i_2(0) = -5 \text{ J}$$

The result of part (a) indicates that the initial store energy is 5 J. The result of part (c) indicates that the energy returned to the circuit is also 5 J, i.e., the total energy accumulated in the inductors over [0,∞) is –5 J. Hence 5 J is dissipated in the resistors.

Remark: the interested student might computer the integral $\int_0^\infty \left(R_1 i_1^2(t) + R_2 i_2^2(t)\right) dt$, the actual energy dissipated in the resistors over [0,∞), and show that this is 5 J.

SOLUTION PROBLEM 18.35.

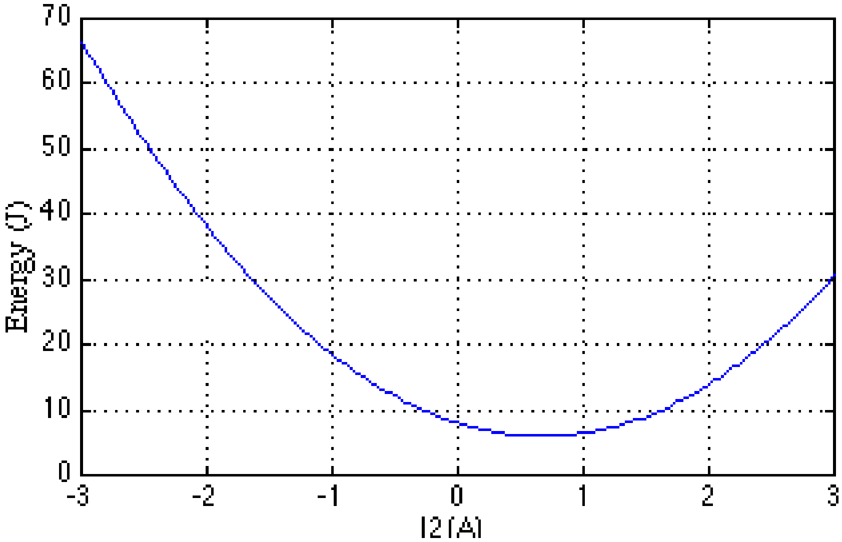
(a)

```
»L1 = 4; L2 = 9; M = 3;
»I1 = 2; I2 = -3;
»W = 0.5*L1*I1^2 + 0.5*L2*I2^2 - M*I1*I2
W =
  6.6500e+01
```

(b)

```
»K = 0.5*L1*I1^2
K =     8
»%  Minimize (over I2) K + 0.5*9*I2^2 - 3*2*I2
»%  Take Derivative and set to zero; then solve for I2.
»%  Derivative is: 9*I2 – 6 = 0
»%  The result is I2 = 2/3 A.
»I2 = 2/3;
»Wmin = 0.5*L1*I1^2 + 0.5*L2*I2^2 - M*I1*I2
Wmin =     6
```

(c)



(d)
»L1 = 4; L2 = 9; M = 3;
»k = M/sqrt(L1*L2)
k = 5.0e-01

SOLUTION PROBLEM 18.37.

»k = 0.5;
»L1 = 9; L2 = 4;L3 = 1;
»M = 0.5*sqrt(L1*L2)
M = 3
»Lcpld = L1 + L2 + 2*M
Lcpld = 19
»Leq = Lcpld + L3
Leq = 20
»Imax = 2;
»Wmax = 0.5*Leq*Imax^2
Wmax = 40 J

SOLUTION PROBLEM 18.41.

»RL = 100; Rs = 300e3; R = 10e3;
»m = 20; n = 5;

(a)
»Z2 = RL*m^2
Z2 = 40000
»Req1 = Z2*R/(Z2+R)
Req1 = 8000
»Z1 = Req1*n^2
Z1 = 200000

(b)
```
»%Gv1 = v1/vin
»%v1 = [Z1/(Rs + Z1)]vin
»Gv1 = Z1/(Rs+Z1)
Gv1 =   4.0000e-01

»%  Gv2 = v2/vin
»%  Gv2 = v2/v1 * v1/vin  =  (1/n) * G1
»Gv2 = G1/n
Gv2 =   8.0000e-02

»%  Gv3 = v3/vin
»Gv3 = -Gv2/m
Gv3 =  -4.0000e-03
```

(c)
```
»% Gi2 = i2/iin
»Gi2 = n*R/(R+Z2)
Gi2 =     1

»%  Gi3 = i3/iin = i3/i2  *  i2/iin  = –m*Gi2
»Gi3 = -m*Gi2
Gi3 =   -20
```

SOLUTION PROBLEM 18.55. **(a)** The parameters in the circuit of figure P18.55b are given by equations in figure 18.22b. Specifically, since $k = M\big/\sqrt{L_1 L_2} = 0.16\big/\sqrt{3.5 \times 0.008} = 0.95618$

```
»M = 0.16; L1 = 3.5; L2 = 0.008;
»k=M/sqrt(L1*L2)
k =
  9.5618e-01
»La = (1 - k^2)*L1
La =
  3.0000e-01
»Lb = k^2 * L1
Lb =
  3.2000e+00
»N =M/L2
N =
   20
```

(b)
```
»R = 500;
»w = 2*pi*60;
»Vseff = 110;
»Zin = R + j*La*w + j*Lb*w
Zin =
 5.0000e+02 + 1.3195e+03i
»Iseff = Vseff/Zin
Iseff =
 2.7624e-02 - 7.2899e-02i
»Pave = R*abs(Iseff)^2
```

```
Pave =
   3.0387e+00
```

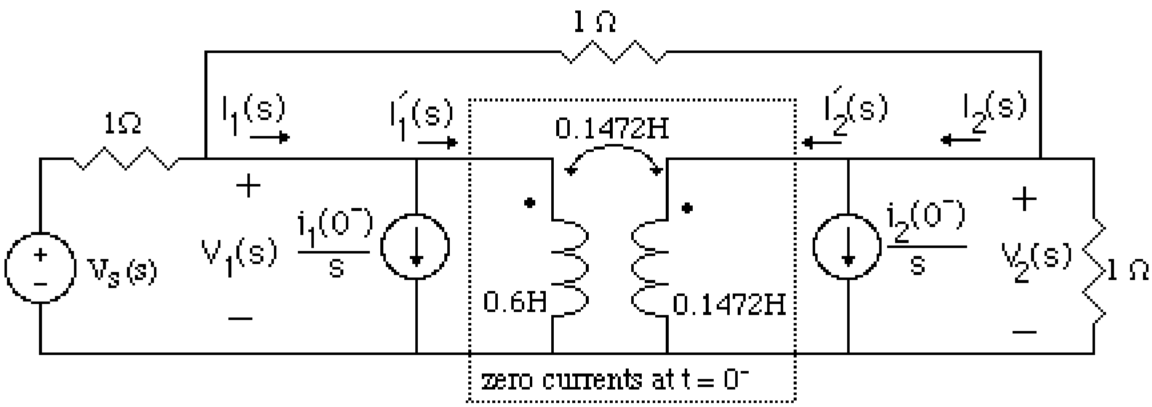
(c)

```
»Zin2 = R + j*La*w
Zin2 =
  5.0000e+02 + 1.1310e+02i
»Iseff2 = Vseff/Zin2
Iseff2 =
  2.0929e-01 - 4.7341e-02i
»Is2mag = abs(Iseff2)
Is2mag =
   2.1458e-01

» %  The current in the secondary is (in A):

»Isecmag = Ismag*N
Isecmag =
   4.2916e+00
```

(d)

```
»%  Our first step is to compute the reflected impedance:
»Zrefl = 100*N^2
Zrefl =
       40000

»%  We now compute the impedance of the parallel combination
»%  of Lb and Zrefl denoted Zpar
»Zpar = 1/(1/Zrefl + 1/(j*w*Lb))
Zpar =
  3.6350e+01 + 1.2053e+03i

»%  We now compute the input impedance:

»Zin = R+j*w*La + Zpar
Zin =
  5.3635e+02 + 1.3184e+03i

»%  Now we compute the voltage across the primary of the
»%  ideal transformer, by voltage division:

»Vpar = Vseff*Zpar/Zin
Vpar =
  8.7342e+01 + 3.2500e+01i

»%  Now we compute the voltage across the load:
»Vload = Vpar/N
Vload =
  4.3671e+00 + 1.6250e+00i
»Vloadmag = abs(Vload)
Vloadmag =
   4.6596e+00
»Iloadmag = Vloadmag/100
Iloadmag =
   4.6596e-02
```

SOLUTION PROBLEM 18.65. (a) The equivalent circuit accounting for initial conditions is given below:



(b) From the definition of coupled inductors

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.6s & 0.1472s \\ 0.1472s & 0.1472s \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}$$

(c) Hence

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} 0.6s & 0.1472s \\ 0.1472s & 0.1472s \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{15}{s} \begin{bmatrix} 0.1472 & -0.1472 \\ -0.1472 & 0.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(d) Writing nodal equations we obtain,

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{s} \begin{bmatrix} i_1(0^-) \\ i_2(0^-) \end{bmatrix} + \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}$$

(e) Now we substitute our result of part (c):

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \frac{1}{s} \begin{bmatrix} -2 \\ -2 \end{bmatrix} + \frac{1}{s} \begin{bmatrix} 2.208 & -2.208 \\ -2.208 & 9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

which simplifies to

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} - \frac{1}{s} \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \frac{1}{s} \begin{bmatrix} 2s + 2.208 & -(s + 2.208) \\ -(s + 2.208) & 2s + 9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or equivalently

$$\begin{bmatrix} 2s + 2.208 & -(s + 2.208) \\ -(s + 2.208) & 2s + 9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} sV_s + 2 \\ 2 \end{bmatrix}$$

(f) Solving these equations we obtain:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2s+2.208 & -(s+2.208) \\ -(s+2.208) & 2s+9 \end{bmatrix}^{-1} \begin{bmatrix} sV_s+2 \\ 2 \end{bmatrix}$$

$$= \frac{1}{3s^2+18s+15} \begin{bmatrix} 2s+9 & (s+2.208) \\ (s+2.208) & 2s+2.208 \end{bmatrix} \begin{bmatrix} sV_s+2 \\ 2 \end{bmatrix}$$

(g) If $v_s(t) = 10u(t)$ V, then

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{3s^2+18s+15} \begin{bmatrix} 2s+9 & (s+2.208) \\ (s+2.208) & 2s+2.208 \end{bmatrix} \begin{bmatrix} 12 \\ 2 \end{bmatrix} = \frac{1}{3s^2+18s+15} \begin{bmatrix} 26s+112.42 \\ 16s+30.912 \end{bmatrix}$$

From MATLAB

```
»[r,p,k]=residue([16 30.912], [3 18 15])
r =
   4.0907e+00
   1.2427e+00
p =
    -5
    -1
k =
     []
```

Therefore,

$$v_2(t) = \left(4.0907e^{-5t} + 1.2427e^{-t}\right)u(t) \text{ V}$$

SOLUTION PROBLEM 18.67. The solution to this problem is based upon the following: (i) $L_{eq} = L_1 + L_2 + 2M$ for series aiding connection (see example 18.4) and (ii) $k = M/\sqrt{L_1L_2}$ (a definition), and (iii) k = 1 (an assumption).

(a) Given $L_1 = L_2 = L$ and k = 1, $L_{eq} = L_1 + L_2 + 2M = L + L + 2k\sqrt{L^2} = 4L$. Hence, when the number of turns is doubled, the inductance is quadrupled.

(b) For this part, let us first consider L_2 which has 2N turns. We can view L_2 as two coils of N turns each connected in series aiding with coupling coefficient k = 1. Hence, according to part (a), the inductance of L_2 is four times that of L_1 which only has N turns. Hence,

$$L_{eq} = L_1 + L_2 + 2M = L + 4L + 2k\sqrt{4L^2} = 9L$$

Observe that the coil has 3N turns yielding an inductance of $9L = 3^2L$.

(c) Suppose coil 1 and coil 2 consist of one turn each. Here the total number of turns is 2N where N = 1 turn. Suppose further that $L_1 = L$. From part (a), $L_{eq} = 4L = (2)^2 L$. Now suppose coil one consists of one turn and coil 2 consists of M turns. We assume here as an induction hypothesis that

$$L_{eq} = (M+1)^2 L$$

We must show that if coil 2 has (M+1)N turns then,

$$L_{eq} = (M+2)^2 L$$

Our first step is to compute the equivalent inductance of coil 2. However, coil 2 consists of a single turn coupled to an M-turn coil, which by the induction hypothesis means that

$$L_2 = (M+1)^2 L$$

Thus coil 1 in a series aiding connection with L2 leads to

$$L_{eq} = L_1 + L_2 + 2M = L + (M+1)^2 L + 2\sqrt{(M+1)^2 L^2} = L + (M+1)^2 L + 2(M+1)L$$

$$= L\left[(M+1)^2 + 2(M+1) + 1\right] = L\left[(M+1)+1\right]^2 = (M+2)^2 L$$

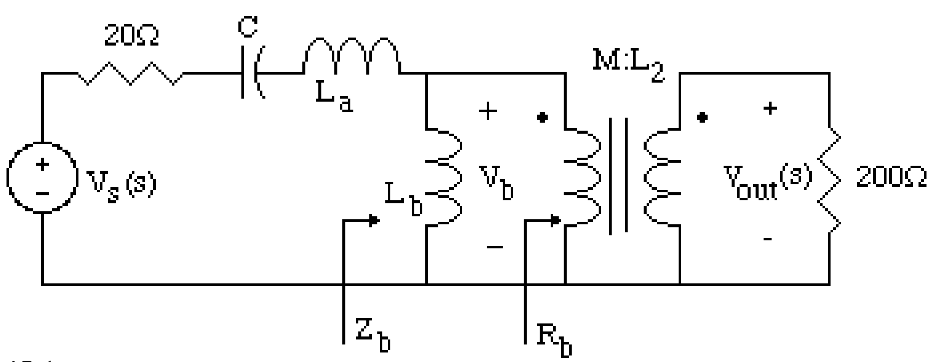
Given this relationship, if coil 1 consists of N1 turns, and one turn has an inductance L, then $L_1 = (N_1)^2 L$. Similarly, $L_2 = (N_2)^2 L$, and $M = k\sqrt{L_1 L_2} = \sqrt{L_1 L_2} = \sqrt{(N_1)^2 (N_2)^2 L^2} = N_1 N_2 L$. It immediately follows that

$$L_1 : L_2 : M = N_1{}^2 : N_2{}^2 : (N_1 N_2)$$

SOLUTION PROBLEM 18.70. (a) $k = M / \sqrt{L_1 L_2}$

»M = 1.5; L1 = 1.5; L2 = 6;
»k = M/sqrt(L1*L2)
k = 5.0000e-01

(b) For this part and the remaining parts consider the following equivalent circuit where the coupled coils have been replaced by the model of figure 18.22(b).



»La = (1 - k^2)*L1
La =
 1.1250e+00
»Lb = k^2 * L1
Lb =
 3.7500e-01

»% The turns ratio is M:L2, i.e.,
»1.5/6
ans = 2.5000e-01
»% Therefore the turns ratio is 1:4. It follows that

»Rb = 200*(1/4)^2
Rb =
 1.2500e+01

To compute Z(s) we have,

$$Z(s) = 20 + \frac{1}{Cs} + L_a s + \frac{R_b L_b s}{L_b s + R_b} = 1.125s + 20 + \frac{12.5s}{s + \frac{100}{3}} + \frac{1}{Cs}$$

Hence,

$$Z(j\omega) = j1.125\omega + 20 + \frac{j12.5\omega}{j\omega + \frac{100}{3}} - \frac{j}{C\omega} = 20 + \frac{12.5\omega^2}{\omega^2 + \frac{10^4}{9}} + j\left(1.125\omega - \frac{1}{C\omega} + \frac{416.67\omega}{\omega^2 + \frac{10^4}{9}}\right)$$

(c) For this part we need to make the imaginary part of Z(jw) real. To this end:

»K1 = 12.5*100/3
K1 = 4.1667e+02
»w = 1333;
»K2 = 1.125*w + K1*w/(w^2 + 1e4/9)
K2 = 1.4999e+03
»C = 1/(K2*w)
C = 5.0015e-07

Hence, we take C = 5 μF.

(d) At resonance, we have

$$Z(j\omega_r) = Z(j1333) = 20 + \frac{12.5\omega_r^2}{\omega_r^2 + \frac{10^4}{9}} = 32.5 \ \Omega$$

and

$$Z_b(j\omega_r) = \frac{j12.5\omega_r}{j\omega_r + \frac{100}{3}} = \frac{j16{,}662}{33.33 + j1333} = 12.492 \ + j0.31238$$

By voltage division

$$\frac{V_{out}(j\omega_r)}{V_s(j\omega_r)} = \frac{V_{out}}{V_b} \times \frac{V_b}{V_s} = \frac{4}{1} \times \frac{Z_b(j\omega_r)}{Z(j\omega_r)} = 1.5379 \ + j0.038456$$

»Zb = j*12.5*w/(j*w + 100/3)
Zb =
 1.2492e+01 + 3.1238e-01i

»Zwr=20 + 12.5*w^2/(w^2 + 1e4/9)
Zwr =

3.2492e+01

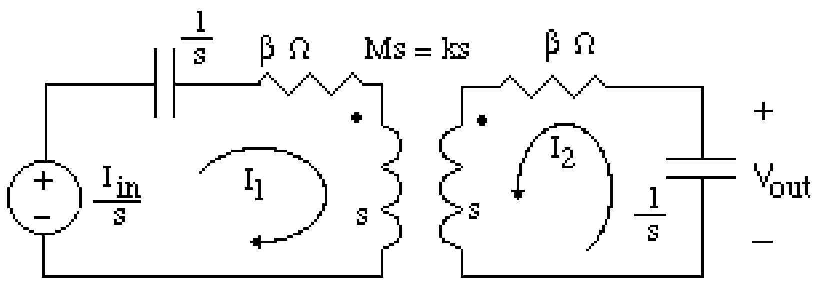
»Gv = 4*Zb/Zwr
Gv =
1.5379e+00 + 3.8456e-02i

»MagGv = abs(Gv)
MagGv =
1.5384e+00

»AngGv = angle(Gv)*180/pi
AngGv =
1.4325e+00

SOLUTION PROBLEM 18.71.

(a) Following the hint we apply a source transformation to obtain



Writing two mesh equations we obtain the following matrix form of the mesh equations:

$$\begin{bmatrix} s+\beta+1/s & ks \\ ks & s+\beta+1/s \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{in}/s \\ 0 \end{bmatrix}$$

Solving for I_2 yields

$$I_2 = \frac{\det\begin{bmatrix} s+\beta+1/s & I_{in}/s \\ ks & 0 \end{bmatrix}}{\det\begin{bmatrix} s+\beta+1/s & ks \\ ks & s+\beta+1/s \end{bmatrix}} = \frac{-kI_{in}}{(s+\beta+1/s)^2-(ks)^2}$$

To find H(s) we have

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{-I_2/s}{I_{in}} = \frac{k/s}{(s+\beta+1/s)^2-(ks)^2} = \frac{k/s}{(s+ks+\beta+1/s)(s-ks+\beta+1/s)}$$

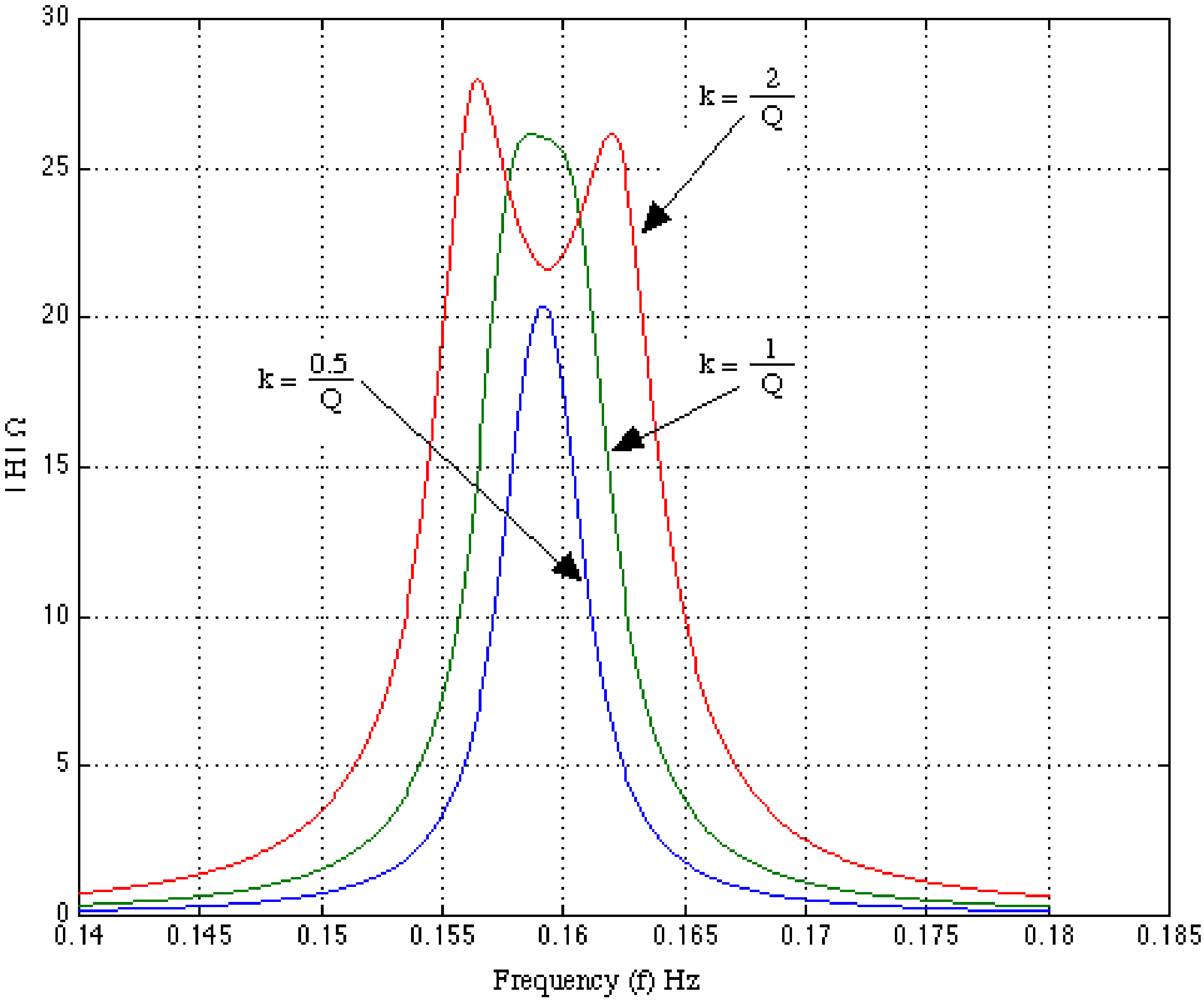
$$= \frac{ks}{\left[(1+k)s^2+\beta s+1\right]\left[(1-k)s^2+\beta s+1\right]}$$

(b) R = 0.02 Ω, b = R = 0.02 Ω, Q = 50, and k = 0.01, 0.02, and 0.04.

```
»beta = 0.02; k1 = 0.01; k2 = 0.02; k3 = 0.04;
»p11 = roots([(1+k1)  beta 1]);
»p12 = roots([(1-k1)  beta 1]);
»p1 = [p11;p12]
p1 =
 -9.9010e-03 + 9.9499e-01i
 -9.9010e-03 - 9.9499e-01i
 -1.0101e-02 + 1.0050e+00i
 -1.0101e-02 - 1.0050e+00i

»p21 = roots([(1+k2)  beta 1]);
»p22 = roots([(1-k2)  beta 1]);
»p2 = [p21;p22]
p2 =
 -9.8039e-03 + 9.9010e-01i
 -9.8039e-03 - 9.9010e-01i
 -1.0204e-02 + 1.0101e+00i
 -1.0204e-02 - 1.0101e+00i

»p31 = roots([(1+k3)  beta 1]);
»p32 = roots([(1-k3)  beta 1]);
»p3 = [p31;p32]
p3 =
 -9.6154e-03 + 9.8053e-01i
 -9.6154e-03 - 9.8053e-01i
 -1.0417e-02 + 1.0206e+00i
 -1.0417e-02 - 1.0206e+00i
```

(c)

```
»f = 0.14:.0001:.18;
»n1 = [k1/(1-k1)^2   0];
n2 = [k2/(1-k2)^2  0];
n3 = [k3/(1-k3)^2  0];
»h1 = freqs(n1, poly(p1), 2*pi*f);
»h2 = freqs(n2, poly(p2), 2*pi*f);
»h3 = freqs(n3, poly(p3), 2*pi*f);
»plot(f, abs(h1), f, abs(h2), f, abs(h3))
»grid
```



(d) An inspection of [f, abs(h2)] (i.e., a tabulation of the values) in part (c) indicates that f_{peak} = 0.15865 Hz and H_{peak} = 26.148 Ω. The frequency scale factor Kf is defined according to:

```
»Kf = 455e3/fpeak
Kf =   2.8679e+06
```

Further,

```
»Km = Kf*2.35e-3
Km =   6.7397e+03

»Lnew = Km*1/Kf
Lnew =
  2.3500e-03
»Cnew = 1/(Km*Kf)
Cnew =
  5.1736e-11
»Rnew = Km*0.02
Rnew =
  1.3479e+02
```

The 3 dB down value of h2 is Hpeak/sqrt(2). Hence

```
»Hmax = max(abs(h2))
Hmax =
   2.6148e+01
»H3db = Hmax/sqrt(2)
H3db =
   1.8490e+01
```

Again, inspecting the tabulated values indicates that the 3 dB frequencies are: f_1 = 0.1569 Hz and f_2 = 0.1614 Hz. Finally

```
»Bf = f2 - f1
Bf = 4.5000e-03 Hz
»Bfnew = Kf*Bf
Bfnew = 1.2906e+04 Hz
```

(e) For this part we redo part (a) with R, L, and C as literals.

Writing two mesh equations we obtain the following matrix form of the mesh equations:

$$\begin{bmatrix} Ls + R + 1/Cs & kLs \\ kLs & Ls + R + 1/Cs \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{in}/Cs \\ 0 \end{bmatrix}$$

Solving for I_2 yields

$$I_2 = \frac{\det\begin{bmatrix} Ls + R + 1/Cs & I_{in}/Cs \\ kLs & 0 \end{bmatrix}}{\det\begin{bmatrix} Ls + R + 1/Cs & kLs \\ kLs & Ls + R + 1/Cs \end{bmatrix}} = \frac{-kL/C}{(Ls + R + 1/Cs)^2 - (kLs)^2} I_{in}$$

To find H(s) we have

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{-I_2/Cs}{I_{in}} = \frac{kL/C^2 s}{(Ls + R + 1/Cs)^2 - (kLs)^2} = \frac{kL/C^2 s}{(Ls + kLs + R + 1/Cs)(Ls - kLs + R + 1/Cs)}$$

$$= \frac{kLs}{\left(LC(1+k)s^2 + CRs + 1\right)\left(LC(1-k)s^2 + CRs + 1\right)} = \frac{ks/LC^2}{\left((1+k)s^2 + \frac{R}{L}s + \frac{1}{LC}\right)\left((1-k)s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$= \frac{k\omega_0^2 s/C}{\left((1+k)s^2 + \frac{\omega_0}{Q}s + \omega_0^2\right)\left((1-k)s^2 + \frac{\omega_0}{Q}s + \omega_0^2\right)}$$

Evaluating this expression at $s = j\omega_0$, yields

$$H(j\omega_0) = \frac{jk\omega_0^3 / C}{\left(-(1+k)\omega_0^2 + \frac{j\omega_0^2}{Q} + \omega_0^2\right)\left(-(1-k)\omega_0^2 + \frac{j\omega_0^2}{Q} + \omega_0^2\right)}$$

$$= \frac{-jk / C\omega_0}{\left(k - \frac{j}{Q}\right)\left(k + \frac{j1}{Q}\right)} = \frac{-jk / C\omega_0}{k^2 + \frac{1}{Q^2}}$$

Therefore

$$|H(j\omega_0)| = \frac{1}{C\omega_0} \times \frac{k}{k^2 + \frac{1}{Q^2}}$$

(f) To solve this part we differentiate and set equal to zero as follows:

$$\frac{d|H(j\omega_0)|}{dk} = \frac{1}{C\omega_0} \times \frac{d}{dk}\left(\frac{k}{k^2 + \frac{1}{Q^2}}\right) = \frac{1}{C\omega_0} \times \left[\frac{1}{k^2 + \frac{1}{Q^2}} - \frac{2k^2}{\left(k^2 + \frac{1}{Q^2}\right)^2}\right] = 0$$

It follows that

$$\frac{2k^2}{k^2 + \frac{1}{Q^2}} = 1$$

Hence

$$k^2 = \frac{1}{Q^2}$$

or k = 1/Q. With this value of k,

$$|H(j\omega_0)|_{max} = \frac{1}{C\omega_0} \times \frac{1/Q}{\frac{2}{Q^2}} = \frac{Q}{2C\omega_0}$$

At ω_0, the magnitude of the transfer function increases with increasing k, reaching a peak at k = 1/Q and then decreases with a further increase in k as born out in the plots of part (c).