CHAPTER 19 PROBLEM SOLUTIONS

SOLUTION PROBLEM 19.40. (a) Let Z_1 = R and Z_2 = 1/Cs or Y_2 = Cs. From problem 38,

$$h = \begin{bmatrix} Z_1 & 1 \\ -1 & Y_2 \end{bmatrix} = \begin{bmatrix} R & 1 \\ -1 & Cs \end{bmatrix}$$

(b) This part is a cascade of an ideal transformer and part (a). Label the voltage and current at the port 1 of N_1 as $\hat{V}_1$ and $\hat{I}_1$. From the properties of the ideal transformer, $V_1 = -b\hat{V}_1$ and $I_1 = -\hat{I}_1 / b$. Hence

$$\begin{bmatrix} \hat{V}_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} R & 1 \\ -1 & Cs \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} -V_1/b \\ I_2 \end{bmatrix} = \begin{bmatrix} R & 1 \\ -1 & Cs \end{bmatrix} \begin{bmatrix} -bI_1 \\ V_2 \end{bmatrix}$$

Therefore

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} b^2R & -b \\ b & Cs \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From table 19.1, if $h_{22} = Cs \neq 0$, then the z-parameters exist and if $h_{11} = b^2R \neq 0$, the y-parameters exist, i.e., if C ≠ 0 and R ≠ 0 (assuming reasonably that b ≠ 0) respectively.

SOLUTION PROBLEM 19.41. For this solution we apply the definition of h-parameters: by inspection

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{1}{2+2s}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \left.\frac{2V_1 - 2sV_1}{I_1}\right|_{V_2=0} = \left.\frac{(2-2s)V_1}{(2+2s)V_1}\right|_{V_2=0} = \frac{1-s}{s+1}$$

When $I_1 = 0$, then $I_2 = 2V_1 + 2V_1 = 4V_1$ and $V_2 = 0.5(2V_1) + \frac{1}{2s}(2V_1) = \frac{s+1}{s}V_1$.

Therefore, $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{s}{s+1}$ and $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \left.\frac{4V_1}{V_2}\right|_{I_1=0} = \frac{4s}{s+1}$.

SOLUTION PROBLEM 19.42. (a) In MATLAB

```
»h11 = 250; h12 = 0.025; h21 = 12.5; h22 = 2.25e-3;
»Zs = 1e3; ZL = 500;
»YL = 1/ZL
YL = 2.0000e-03
»Zin = h11 - h12*h21/(h22 + (1/ZL))
Zin =   1.7647e+02
»Yout = h22 - h12*h21/(h11 + Zs)
Yout =   2.0000e-03
»Zout = 1/Yout
Zout =   500
```

(b)

```
»%  Gv1 = V1/Vs
»Gv1 = Zin/(Zin + Zs)
Gv1 =   1.5000e-01
»%  Gv2 = V2/V1
»Gv2 =  -h21/(Zin*(h22 + YL))
Gv2 =  -1.6667e+01
»Gv = Gv1*Gv2
Gv =  -2.5000e+00
```

(c) Given the above, the Thevenin equivalent seen by the capacitor is $\mathbf{V_{oc}} = -2.5\mathbf{V_{in}}$ and $R_{th} = 500\ \Omega$.

In MATLAB

```
»Zth = ZL*Zout/(ZL + Zout)
Zth =   250
»Vin = 10;
»Voc = -2.5*Vin;
»w = 400;
```

```
»Zc = 1/(j*w*10e-6)
Zc =          0 - 2.5000e+02i
»Vc = Voc*Zc/(Zth + Zc)
Vc = -1.2500e+01 + 1.2500e+01i»V2mag = abs(Vc)
»V2mag = abs(Vc)
V2mag =
  1.7678e+01
»V2ang = angle(Vc)*180/pi
V2ang =
  135
```

From above,

$$v_2(t) = 17.678\sqrt{2}\cos(400t + 135^o)\ \text{V}$$

Therefore

```
»Pave = V2mag^2/500
Pave =
  6.2500e-01
```

SOLUTION PROBLEM 19.43.

(a) Using the h-parameters of stage 2

$$Z_{in2} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L} = 1000 + \frac{0.966 \times 51}{0.0008 + 1/64} = 4000\ \Omega$$

The load for stage 1 is the parallel combination of Z_{in2} and the 3 kΩ resistance. However, because $h_{12} = 0$, the input impedance is unaffected by the load, and hence for stage 1,

$$Z_{in1} = h_{11} = 2000\ \Omega$$

(b) For stage 1, because $h_{12} = 0$, the output impedance is unaffected by the source impedance. Thus,

$$Y_{out1} = h_{22} = 0.05 \times 10^{-3}\ \text{S},$$

$$Z_{out1} = 1/Y_{out1} = 20 \times 10^{3}\ \Omega$$

For stage 2, the source impedance is the parallel combination of Z_{out1} and the 3 kΩ resistance. Thus

$$Z_{s2} = \frac{20000\times 3000}{20000 + 3000} = 2608.7\ \Omega$$

and

$$Y_{out2} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_{s2}} = 0.0008 + \frac{0.966\times 51}{1000 + 2608.7} = 0.0145\ \mathrm{S}$$

$$Z_{out2} = 1/Y_{out2} = 69.19\ \Omega$$

(c)

$$\frac{V_1}{V_s} = \frac{Z_{in1}}{Z_{in1} + Z_s} = \frac{2000}{2000 + 2000} = 0.5$$

The load of stage 1 is the parallel combination of Z_{in2} and Z_m. Thus

$$Y_{L1} = Y_{in2} + 1/3000 = 5.834\times 10^{-4}\ \mathrm{S}$$

Hence

$$Z_{L1} = 1/Y_{L1} = 1714\ \Omega$$

and

$$\left(\frac{V_2}{V_1}\right)_{\text{stage 1}} = \frac{1}{Z_{in1}}\times\frac{-h_{21}}{h_{22} + Y_{L1}} = \frac{1}{2000}\times\frac{-50}{(0.05 + 0.5834)\times 10^{-3}} = -39.46$$

For stage 2, the load is 64 Ω. Hence

$$\left(\frac{V_2}{V_1}\right)_{\text{stage 2}} = \frac{1}{Z_{in2}}\times\frac{-h_{21}}{h_{22} + Y_{L2}} = \frac{1}{4000}\times\frac{51}{(0.8 + 1000/64)\times 10^{-3}} = 0.7762$$

Finally, the overall voltage gain is the product of the three gains calculated above

$$\frac{V_{out}}{V_s} = 0.5\times(-39.46)\times(0.7762) = -15.32$$

(d) The input circuit consists of a series connection of V_S, Z_S, C and Z_{in1}. The remainder of the circuit is resistive and has no effect on the frequency response. The magnitude response is of the high pass type with

$$f_{3dB} = \frac{1}{2\pi(R_s + R_{in1})C} = \frac{1}{2\pi(2000 + 2000)10^{-6}} = 39.789\ \mathrm{Hz}$$

SOLUTION PROBLEM 19.44. To meet the required matching, we must have

$$Z_{out} = Z_{out2} = Z_L = 64 = \frac{1}{Y_{out2}}$$

This requires that

$$Y_{out2} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_{s2}} = 0.0008 + \frac{0.966 \times 51}{1000 + Z_{s2}} = \frac{1}{64}$$

Solving for Z_{s2}, we obtain $Z_{s2} = 2323.2\ \Omega$. Now for stage 2, Z_{s2} is the parallel combination of $Z_{out1} = 20\ \text{k}\Omega$ and Z_m:

$$Z_{s2} = \frac{20000 \times Z_m}{20000 + Z_m} = 2323.2\ \ \Omega$$

from which $Z_m = 2628.5\ \Omega$. With this new value of Z_m, we repeat the calculations of problem 19.43 to obtain $Z_{in} = 2000\ \Omega$ and $V_{out}/V_s = -14\ .26$. Details follow.

Using the h parameters of stage 2

$$Z_{in2} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L} = 1000 + \frac{0.966 \times 51}{0.0008 + 1/64} = 4000\ \ \Omega$$

For stage 1, because $h_{12} = 0$, the input impedance is not affected by the load and

$Z_{in1} = h_{11} = 2000\ \Omega$. The voltage gains of the various stages are:

$$\frac{V_1}{V_s} = \frac{Z_{in1}}{Z_{in1} + Z_s} = \frac{2000}{2000 + 2000} = 0.5$$

The load of stage 1 is the parallel combination of Z_{in2} and Z_m. Thus

$$Y_{L1} = Y_{in2} + 1/2628.5 = 6.3044 \times 10^{-4}\ \text{S}$$

Hence

$$\left(\frac{V_2}{V_1}\right)_{\text{stage 1}} = \frac{1}{Z_{in1}} \times \frac{-h_{21}}{h_{22} + Y_{L1}} = \frac{1}{2000} \times \frac{-50}{(0.05 + 0.63044) \times 10^{-3}} = -36.74$$

For stage 2, the load is 64 Ω. Hence

$$\left(\frac{V_2}{V_1}\right)_{\text{stage 2}} = \frac{1}{Z_{in2}} \times \frac{-h_{21}}{h_{22} + Y_{L2}} = \frac{1}{4000} \times \frac{51}{(0.8 + 1000/64) \times 10^{-3}} = 0.7762$$

Finally, the overall voltage gain is the product of the three gains calculated above

$$\frac{V_{out}}{V_s} = 0.5 \times (-36.74) \times (0.7762) = -14.26$$

SOLUTION PROBLEM 19.45.

(a) Using the h-parameters of stage 2

$$Z_{in2} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L} = 500 + \frac{0.966 \times 51}{0.0016 + 1/32} = 2000 \ \Omega$$

The load for stage 1 is the parallel combination of Z_{in2} and the 1.5 kΩ resistance. However, because $h_{12} = 0$, the input impedance is unaffected by the load, and $Z_{in1} = h_{11} = 1000 \ \Omega$.

(b) For stage 1, because $h_{12} = 0$, the output impedance is unaffected by the source impedance, and

$$Y_{out1} = h_{22} = 0.1 \times 10^{-3} \text{ S},$$

$$Z_{out1} = 1/Y_{out1} = 10^4 \ \Omega$$

For stage 2, the source impedance is the parallel combination of Z_{out1} and the 1.5 kΩ resistor

$$Z_{s2} = \frac{10^4 \times 1.5 \times 10^3}{10^4 + 1.5 \times 10^3} = 1304.3 \ \Omega$$

and

$$Y_{out2} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_{s2}} = 0.0016 + \frac{0.966 \times 51}{500 + 1304.3} = 0.0289 \text{ S}$$

$$Z_{out2} = 1/Y_{out2} = 34.6 \ \Omega$$

(c)

$$\frac{V_1}{V_s} = \frac{Z_{in1}}{Z_{in1} + Z_s} = \frac{1000}{1000 + 1000} = 0.5$$

The load of stage 1 is the parallel combination of Z_{in2} and Z_m. Thus

$$Y_{L1} = Y_{in2} + 1/1500 = 1.1667 \times 10^{-3} \text{ S}$$

Hence

$$Z_{L1} = 1/ Y_{L1} = 857.1 \ \Omega$$

$$\left(\frac{V_2}{V_1}\right)_{\text{stage 1}} = \frac{1}{Z_{in1}} \times \frac{-h_{21}}{h_{22} + Y_{L1}} = \frac{1}{1000} \times \frac{-50}{(0.1 + 1.1667) \times 10^{-3}} = -39.47$$

For stage 2, the load is 32 Ω. Hence

$$\left(\frac{V_2}{V_1}\right)_{\text{stage 2}} = \frac{1}{Z_{in2}} \times \frac{-h_{21}}{h_{22} + Y_{L2}} = \frac{1}{2000} \times \frac{51}{0.0016 + 1/32} = 0.7762$$

Finally, the overall voltage gain is the product of the three gains calculated above

$$\frac{V_{out}}{V_s} = 0.5 \times (-39.47) \times (0.7762) = -15.32$$

(d) The input circuit consists of a series connection of V_S, Z_S, C and Z_{in1}. The remainder of the circuit is resistive and has no effect on the frequency response. The magnitude response is of the highpass type with

$$f_{3dB} = \frac{1}{2\pi(R_s + R_{in1})C} = \frac{1}{2\pi(1000+1000)10^{-6}} = 79.58 \text{ Hz}$$

SOLUTION PROBLEM 19.46. (a) Since the currents through Y_L and h_{22} are the same, $h_{22} = Y_L$.

(b) From current division, $I_2 = \dfrac{Y_L}{Y_L + h_{22}} h_{21} I_1 \qquad \dfrac{I_2}{I_1} = \dfrac{Y_L h_{21}}{Y_L + h_{22}}$.

(c) $150 = \dfrac{I_2}{I_1} = \dfrac{Y_L h_{21}}{Y_L + h_{22}} = 0.5h_{21} \qquad h_{21} = 300$.

(d) $h_{12} = \left.\dfrac{V_1}{V_2}\right]_{I_1=0} = \dfrac{-1}{2} = -0.5$.

(e) $\dfrac{I_1}{I_s} = \dfrac{Z_s}{Z_s + Z_{in}} = \dfrac{9 \times 10^3}{9 \times 10^3 + Z_{in}} = 0.9 \qquad Z_{in} = 1000\ \Omega$. Given this quantity,

$$h_{11} = Z_{in} + \frac{h_{12}h_{21}}{h_{22} + Y_L} = 1000 - \frac{150}{0.25} = 400\ \Omega.$$

SOLUTION PROBLEM 19.47. Recall that

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(a) From this expression and specification 1, $h_{12} = \left.\frac{V_1}{V_2}\right]_{I_1=0} = 0$.

(b) From the formula for Y_{out} (equation 19.50), specification 2, and the result of part (a), we have

$$Y_{out} = \frac{1}{Z_{out}} = \frac{1}{800} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_s} = h_{22}$$

Hence, $h_{22} = \frac{1}{800} = 1.25$ mS.

For maximum power transfer from amplifier to the load,

$$Z_{out} = 800 = b^2 Z_L = 8b^2$$

Therefore, b = 10.

(c) and **(d)** Observe that

$$Z_{in} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L/b^2} = h_{11}$$

From specification 3 and voltage division,

$$\frac{V_1}{V_s} = \frac{24}{25} = \frac{Z_{in}}{Z_{in} + 40}$$

Equivalently,

$$\frac{V_s}{V_1} = \frac{25}{24} = 1 + \frac{40}{Z_{in}}$$

Hence $h_{11} = Z_{in} = 40 \times 24 = 960\ \Omega$.

(e) From equation 19.51,

$$G_{v2} = \frac{V_2}{V_1} = -100 = \frac{-h_{21}}{Z_{in}\left(h_{22} + Y_L/b^2\right)} = \frac{-h_{21}}{960\left(1.25\times10^{-3} + 1.25\times10^{-3}\right)} = \frac{-h_{21}}{2.4}$$

Hence, $h_{21} = 240$.

(f) The power delivered to the load is $P_L = \frac{V_2^2}{800}$ and the power delivered to the amplifier is $P_{amp} = \frac{V_1^2}{Z_{in}} = \frac{V_1^2}{960}$. Therefore the power gain is

$$\frac{P_L}{P_{amp}} = \frac{96}{80}\left(\frac{V_2}{V_1}\right)^2 = 1.2\times 10^4$$

SOLUTION PROBLEM 19.48. Recall that

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(a) From this expression and specification 1, $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 0.01$.

(b) For maximum power transfer from amplifier to the load,

$$Z_{out} = 800 = b^2 Z_L = 8b^2$$

Therefore, b = 10.

Now we find Z_{in}. From specification 3 and voltage division,

$$\frac{V_1}{V_s} = \frac{24}{25} = \frac{Z_{in}}{Z_{in}+40} \qquad (1)$$

Equivalently,

$$\frac{V_s}{V_1} = \frac{25}{24} = 1 + \frac{40}{Z_{in}} \qquad Z_{in} = 40\times 24 = 960\ \Omega$$

Using the formula for Z_{in} we have the following equation

$$Z_{in} = 960 = h_{11} - \frac{0.01 h_{21}}{h_{22} + 1.25\times 10^{-3}} \qquad h_{11} = 960 + 0.01\frac{h_{21}}{h_{22}+1.25\times 10^{-3}} \qquad (2)$$

But from the given specs,

$$\frac{V_2}{V_1} = -100 = \frac{-h_{21}}{Z_{in}\left(h_{22} + Y_L/b^2\right)} = \frac{-h_{21}}{960\left(h_{22} + 1.25\times 10^{-3}\right)}$$

which implies that

$$\frac{h_{21}}{\left(h_{22} + 1.25\times 10^{-3}\right)} = 960\times 10^2 \qquad (3)$$

Substituting (3) into (2) allows us to solve for h_{11}:

$$h_{11} = 960 + 0.01\times 960\times 10^2 = 1920\ \Omega$$

Now let us rewrite equation (3) as:

$$h_{21} - 960\times 10^2 h_{22} = 960\times 10^2 \times 1.25\times 10^{-3} = 120 \qquad (4)$$

Also

$$Y_{out} = 1.25\times 10^{-3} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_s} = h_{22} - \frac{0.01 h_{21}}{1960}$$

Equivalently,

$$1960\times 1.25\times 10^{-3} = 2.45 = 1960 h_{22} - 0.01 h_{21} \qquad (5)$$

Solving equations (4) and (5) simultaneously in MATLAB yields

```
»A = [1  -960e2;-0.01  1960]
A =
   1.0000e+00  -9.6000e+04
  -1.0000e-02   1.9600e+03
»b = [120; 2.45]
b =
   1.2000e+02
   2.4500e+00
»x = A\b
x =
```

```
   4.7040e+02
   3.6500e-03
»h21 = x(1); h22 = x(2);
h = [1920  0.01;h21  h22]
h =
   1.9200e+03   1.0000e-02
   4.7040e+02   3.6500e-03
```

We can verify these results as well as compute the overall amplifier gain using the following m-file:

```
% two-port analysis in terms of h-parameters
function [zin, zout] =twoport(h, zL, zs)
['twoport analysis using h-parameters']
h11= h(1,1); h12=h(1,2); h21=h(2,1); h22=h(2,2);
zin = h11 - h12*h21/(h22+ 1/zL)

yout= h22 - h12*h21/(h11+zs);
zout= 1/yout
v1tovs= zin/(zin+zs)
v2tov1= -h21/(zin*(h22+1/zL))
v2tovs= v1tovs*v2tov1
```

```
»twoporth(h,ZL,Zs)
ans =
twoport analysis using h-parameters
zin =
   960
zout =
   8.0000e+02
v1tovs =
   9.6000e-01
v2tov1 =
  -100
v2tovs =
   -96
```

Hence the overall voltage gain is V_L/V_s = –96/10 = –96 because of the transformer.

Finally to compute power gains,

```
»Vs = 1;  Vin = 24/25;
»VL = -9.6;
»Pin = Vin^2/960
Pin =
  9.6000e-04
»Pload = VL^2/8
Pload =
  1.1520e+01
»Pgain = Pload/Pin
Pgain =
      12000
```

SOLUTION PROBLEM 19.49.

(a) $$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-C_\mu s V_\pi + g_m V_\pi}{\left(\frac{1}{R_\pi} + \left(C_\pi + C_\mu\right)s\right)V_\pi} = \frac{-C_\mu s + g_m}{\left(C_\pi + C_\mu\right)s + \frac{1}{R_\pi}}$$

(b) $$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{\left(R_x + \frac{1}{\frac{1}{R_\pi} + \left(C_\pi + C_\mu\right)s}\right)I_1}{I_1} = R_x + \frac{1}{\frac{1}{R_\pi} + \left(C_\pi + C_\mu\right)s}$$

(c) Under the condition that $I_1 = 0$, $V_1 = V\pi$. Using voltage division from V_2 to V_π:

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{V_\pi}{V_2} = \frac{\left(\dfrac{1}{\dfrac{1}{R_\pi} + C_\pi s}\right)}{\dfrac{1}{C_\mu s} + \dfrac{1}{\dfrac{1}{R_\pi} + C_\pi s}} = \frac{C_\mu s}{\dfrac{1}{R_\pi} + C_\pi s + C_\mu s} = \frac{C_\mu s}{\left(C_\pi + C_\mu\right)s + \dfrac{1}{R_\pi}}$$

(d) $$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{g_m V_\pi + \left(\dfrac{1}{R_\pi} + C_\pi s\right)V_\pi}{\dfrac{\left(C_\pi + C_\mu\right)s + \dfrac{1}{R_\pi}}{C_\mu s}V_\pi} = \frac{C_\mu s\left(C_\pi s + g_m + \dfrac{1}{R_\pi}\right)}{\left(C_\pi + C_\mu\right)s + \dfrac{1}{R_\pi}}$$

SOLUTION PROBLEM 19.50: (a) Recall t-parameter relationship:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For the given network, $V_2 = Z_L(-I_2)$ and

$$V_1 = t_{11}V_2 + t_{12}(-I_2) = t_{11}Z_L(-I_2) + t_{12}(-I_2) = (t_{11}Z_L + t_{12})(-I_2)$$

Further,

$$I_1 = t_{21}V_2 + t_{22}(-I_2) = (t_{21}Z_L + t_{22})(-I_2)$$

Hence,

$$Z_{in} = \frac{V_1}{I_1} = \frac{(t_{11}Z_L + t_{12})(-I_2)}{(t_{21}Z_L + t_{22})(-I_2)} = \frac{t_{11}Z_L + t_{12}}{t_{21}Z_L + t_{22}}$$

(b) For the output impedance relationship, from the t-parameter relationships

$$V_1 = Z_s(-I_1) = t_{11}V_2 + t_{12}(-I_2) = -Z_s(t_{21}V_2 + t_{22}(-I_2))$$

Grouping V2 and I2 terms together on separate sides of the equation implies

$$(t_{21}Z_s + t_{11})V_2 = (t_{22}Z_s + t_{12})I_2$$

Thus

$$Z_{out} = \frac{V_2}{I_2} = \frac{t_{22}Z_s + t_{12}}{t_{21}Z_s + t_{11}}$$

SOLUTION PROBLEM 19.51:

45. From the z-parameter relationships

$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad V_1 - z_{11}I_1 = z_{12}I_2 = -z_{12}(-I_2)$$

and

$$V_2 = z_{21}I_1 + z_{22}I_2 \qquad z_{21}I_1 = V_2 + z_{22}(-I_2)$$

These two equations in matrix form are:

$$\begin{bmatrix} 1 & -z_{11} \\ 0 & z_{21} \end{bmatrix}\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & -z_{12} \\ 1 & z_{22} \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Solving for $\begin{bmatrix} V_1 & I_1 \end{bmatrix}^T$ yields

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \frac{1}{z_{21}}\begin{bmatrix} z_{21} & z_{11} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -z_{12} \\ 1 & z_{22} \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \frac{1}{z_{21}}\begin{bmatrix} z_{11} & \Delta z \\ 1 & z_{22} \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

SOLUTION PROBLEM 19.52: For figure 19.52a, by inspection

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Therefore T is as indicated in the problem.

For figure 19.52b, by inspection

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Therefore T is as indicated in the problem.

SOLUTION PROBLEM 19.53: Here we use the results of problem 19.52:

(a) For figure (a)

$$T_{new} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 + Z_1Y_2 & Z_1 \\ Y_2 & 1 \end{bmatrix}$$

(b) For figure (b)

$$T_{new} = \begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix}\begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ Y_2 & 1 + Z_1 Y_2 \end{bmatrix}$$

SOLUTION PROBLEM 19.54: By the properties of an ideal transformer,

$$V_1 = nV_2 \quad I_1 = \frac{-1}{n} I_2 = \left(\frac{1}{n}\right)(-I_2)$$

Therefore

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

with the t-parameters given by the 2x2 matrix.

SOLUTION PROBLEM 19.55: This problem uses the results of the previous two problems.

(a)

$$T = \begin{bmatrix} 1 + Z_1 Y_2 & Z_1 \\ Y_2 & 1 \end{bmatrix}\begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} = \begin{bmatrix} n\left(1 + Z_1 Y_2\right) & Z_1/n \\ nY_2 & 1/n \end{bmatrix}$$

(b)

$$T = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}\begin{bmatrix} 1 + Z_1 Y_2 & Z_1 \\ Y_2 & 1 \end{bmatrix} = \begin{bmatrix} n\left(1 + Z_1 Y_2\right) & nZ_1 \\ Y_2/n & 1/n \end{bmatrix}$$

SOLUTION PROBLEM 19.56:

(a)

$$T = \begin{bmatrix} 0.25 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 20 \end{bmatrix}$$

(b)

$$T = \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 2 + 8/R & 5 + 4/R \end{bmatrix}$$

(c)

$$T = \begin{bmatrix} 1 + Z_1 Y_2 & Z_1 \\ Y_2 & 1 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 + 0.25s & 0.5s \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 + 3s & 4 + 3.5s \\ 6 & 7 \end{bmatrix}$$

SOLUTION PROBLEM 19.57: For each 2-port of the form of figure P19.53a, we have that the t-parameters are given by

$$T = \begin{bmatrix} 1 + Z_1 Y_2 & Z_1 \\ Y_2 & 1 \end{bmatrix}$$

The given network consists of three such sections in cascade whose t-parameters are respectiely,

$$T_1 = \begin{bmatrix} 1 + s^2 & s \\ s & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 1 + 1 \times 0.5s & 1 \\ 0.5s & 1 \end{bmatrix} = \begin{bmatrix} 1 + 0.5s & 1 \\ 0.5s & 1 \end{bmatrix}, \text{ and}$$

$$T_3 = \begin{bmatrix} 1 + 2s \times \frac{1}{4}s & 2s \\ \frac{1}{4}s & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}s^2 & 2s \\ \frac{1}{4}s & 1 \end{bmatrix}$$

Observe that

$$T_1 T_2 = \begin{bmatrix} \frac{1}{2}s^3 + \frac{3}{2}s^2 + \frac{1}{2}s + 1 & s^2 + s + 1 \\ \frac{1}{2}s^2 + \frac{3}{2}s & s + 1 \end{bmatrix}$$

and the overall t-parameters are

$$T = T_1 T_2 T_3 = \begin{bmatrix} \frac{1}{4}s^5 + \frac{3}{4}s^4 + s^3 + \frac{9}{4}s^2 + \frac{3}{4}s + 1 & s^4 + 3s^3 + 2s^2 + 3s + 1 \\ \frac{1}{4}s^4 + \frac{3}{4}s^3 + \frac{3}{4}s^2 + \frac{7}{4}s & s^3 + 3s^2 + s + 1 \end{bmatrix}$$

SOLUTION PROBLEM 19.58: This poblem is done primarily in MATLAB.

Part (a)

```
% The following  code solves part (a) of the problem.
% Parameter Specification
t11= 0.895+j*0.022;
t22= t11;
t12= 40 + j*180;

% t21= (t11*t22 -1)/t12;
%  The above formula follows because it  is  a reciprocal network.
%  The actual value is  specified.
t21= -2.6175e-05+j*1.1023e-03;
t=[t11 t12; t21 t22]

% Part (a) calculations
vr= 115200
ir = 361

vs= t11*vr +t12*ir
magvs=abs(vs)
angvs= angle(vs)*180/pi
```

```
is=t21*vr+t22*ir
magis= abs(is)
angis= angle(is)*180/pi
pscomp=vs*conj(is)
ps=real(pscomp)
pr=real(vr*conj(ir))
eff= pr/ps
pf= ps/abs(pscomp)
ploss= ps- pr
```

The MATLAB output is as follows:

```
T =
  8.9500e-01 + 2.2000e-02i  4.0000e+01 + 1.8000e+02i
 -2.6175e-05 + 1.1023e-03i  8.9500e-01 + 2.2000e-02i
vr = 115200
ir =   361

vs =  1.1754e+05 + 6.7514e+04i
magvs =   1.3555e+05
angvs =   2.9872e+01

is =  3.2008e+02 + 1.3493e+02i
magis =   3.4736e+02
angis =   2.2857e+01

pscomp =  4.6733e+07 + 5.7501e+06i
ps =   4.6733e+07
pr =    41587200

eff =   8.8989e-01
pf =   9.9252e-01
ploss =   5.1458e+06
```

Part (b)

```
% The following  code solves part (b) of the problem.

zL=500;
zin= (t11*zL + t12)/( t21*zL + t22)
yin = 1/zin
vsnew=134000;
iin= yin*vsnew
psnew= vsnew^2*real(yin)
m= inv(t)
v2=m(1,1)*vsnew +m(1,2) *iin
magv2=abs(v2)
iload= m(2,1) *vsnew + m(2,2)* iin
magild = abs(iload)
%  Check value of rload
rload=abs(v2)/abs(iload)
```

The MATLAB output for part (b) is:

```
zin =  4.8759e+02 - 1.0031e+02i
yin =  1.9676e-03 + 4.0478e-04i
iin =  2.6366e+02 + 5.4241e+01i
psnew =   3.5331e+07

% m is the inverse of T-matrix.

m =
  8.9500e-01 + 2.2000e-02i -4.0000e+01 - 1.8000e+02i
  2.6175e-05 - 1.1023e-03i  8.9500e-01 + 2.2000e-02i

v2 =  1.1915e+05 − 4.6681e+04i
magv2 =   1.2796e+05
iload =  2.3829e+02 - 9.3362e+01i
magild =   2.5593e+02
rload =   5.0000e+02
```

SOLUTION PROBLEM 19.59. From the given information, the circuit is linear and reciprocal. (a) Here $i_2(t)$ is the integral of $i_1(t)$. Therefore, the new $v_1(t)$ is the integral of the old $v_2(t)$. The result for $t \ge 0$ is:

$$v_1(t) = 3.005 - 3e^{-t} + e^{-t}[0.00865\sin(500t) - 0.005\cos(500t)]$$

$$= 3.005 - 3e^{-t} + e^{-t}\left[-2\ \ 10^{-5}\cos(500t - \pi/6) + 0.01\sin(500t - \pi/6)\right]$$

(b) From the problem statement

$$z_{21}(s) = \frac{3}{s+1} + 5\frac{\cos\left(\frac{\pi}{6}\right)(s+1) + 250}{(s+1)^2 + (500)^2}$$

From reciprocity, $z_{12}(s) = z_{21}(s)$. For steady state analysis, we use phasors to obtain

$$\mathrm{V}_1 = z_{12}(j500)\mathbf{I_2} = \frac{3}{1+j500} + 5\frac{\cos\left(\frac{\pi}{6}\right)(1+j500) + 250}{(1+j500)^2 + (500)^2}$$

$$v_{1ss}(t) = 2.505\cos(500t - 30.15^\circ)\ \mathrm{V}$$

SOLUTION PROBLEM 19.60. Writing loop equations we have:

(i) For the left loop,

$$V_1 - aV_2 - 3(I_1 + I_3) = 0 \qquad V_1 - aV_2 = 3I_1 + 3I_3$$

(ii) For the right loop,

$$V_2 + bI_1 - (I_2 - I_3) = 0 \qquad V_2 = -bI_1 + I_2 - I_3$$

(iii) For the middle loop,

$$bI_1 + (I_3 - I_2) + I_3 + 3(I_1 + I_3) = 0 \qquad 0 = (b+3)I_1 - I_2 + 5I_3$$

Writing the first two equations in matrix form yields

$$\begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ -b & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

whose solution is

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ -b & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 3-ab & a & 3-a \\ -b & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Hence

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3-ab & a \\ -b & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} 3-a \\ -1 \end{bmatrix} I_3$$

From the third equation

$$I_3 = \begin{bmatrix} -0.2b - 0.6 & 0.2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left(\begin{bmatrix} 3-ab & a \\ -b & 1 \end{bmatrix} + \begin{bmatrix} 3-a \\ -1 \end{bmatrix} \begin{bmatrix} -0.2b - 0.6 & 0.2 \end{bmatrix} \right) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -0.6b - 0.8ab + 0.6a + 1.2 & 0.6 + 0.8a \\ -0.8b + 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For reciprocity,

$$z_{12} = z_{21} \qquad a = -b$$

SOLUTION PROBLEM 19.61. (a) and (b) together: Observe that

$$V_1 - 0.5V_1 - I_1 = V_2 \qquad I_1 = 0.5V_1 - V_2 \qquad V_1 = 2I_1 + 2V_2$$

and

$$I_1 + I_2 = 0.5(V_2 - aI_1) \qquad I_2 = -(0.5a + 1)I_1 + 0.5V_2$$

The h-parameters are

$$H = \begin{bmatrix} 2 & 2 \\ -(0.5a + 1) & 0.5 \end{bmatrix}$$

Reciprocity requires that

$$h_{12} = -h_{21} \qquad 2 = 0.5a + 1 \qquad a = 2$$

Thus

$$H = \begin{bmatrix} 2 & 2 \\ -2 & 0.5 \end{bmatrix}$$

SOLUTION PROBLEM 19.62. There are 2 corrections in the problem statement concerning the second set of expressions:

(1) $v_1(t) = 2e^{-t} - 1.5e^{-1.5t}$ V

(2) $i_2(t) = 0.5\, e^{-1.5t}$ A

For both parts, recall, the y-parameters:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(a) Part-1: From the first set of given data ($V_2 = 0$)

$$I_1 = \frac{1}{s},\ I_2 = \frac{0.5}{s+1},\ V_1 = \frac{0.5}{s+1} + \frac{0.5}{(s+1)^2} = \frac{0.5(s+2)}{(s+1)^2}$$

Hence,

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{2(s+1)^2}{s(s+2)},\ y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{s+1}{s+2}$$

(a) Part-2: From the second set of given data ($Z_L = 1\ \Omega$, $I_1 = 1/s$, etc.), we have

$$I_2 = \frac{0.5}{s+1.5},\ V_1 = \frac{2}{s+1} - \frac{1.5}{s+1.5} = \frac{0.5(s+3)}{(s+1)(s+1.5)}$$

For a terminated 2-port,

$$I_2 = \frac{-V_2}{Z_L} = -V_2 = \frac{y_{21}}{y_{22}+Y_L} V_1 = \frac{y_{21}}{y_{22}+1} V_1$$

Therefore

$$y_{22} = y_{21}\frac{V_1}{I_2} - 1 = \frac{s+1}{s+2} \times \frac{0.5(s+3)}{(s+1)(s+1.5)} \times \frac{s+1.5}{0.5} - 1 = \frac{(s+3)}{(s+2)} - 1 = \frac{1}{s+2}$$

Also, $I_1 = y_{11}V_1 + y_{12}V_2$ $\qquad y_{12} = \dfrac{I_1 - y_{11}V_1}{V_2} = \dfrac{I_1 - y_{11}V_1}{-I_2}$. Hence

$$y_{12} = -\frac{s+1.5}{0.5} \times \left(\frac{1}{s} - \frac{2(s+1)^2}{s(s+2)} \frac{0.5(s+3)}{(s+1)(s+1.5)} \right) = -\frac{(2s+3)(s+2)}{s(s+2)} + \frac{2(s+1)(s+3)}{s(s+2)}$$

$$= \frac{1}{(s+2)}$$

In conclusion

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{2(s+1)^2}{s(s+2)} & \dfrac{1}{(s+2)} \\ \dfrac{s+1}{s+2} & \dfrac{1}{(s+2)} \end{bmatrix} = \frac{1}{(s+2)} \begin{bmatrix} \dfrac{2(s+1)^2}{s} & 1 \\ s+1 & 1 \end{bmatrix}$$

(b) This is a straightforward application of the conversion table 19.1.

SOLUTION PROBLEM 19.63. (a) Consider figure (a). Write two mesh equations:

$$V_1 = 16I_1 + \frac{2}{s}(I_1 + I_2) + 4I_1 = \left(20 + \frac{2}{s}\right)I_1 + \frac{2}{s}I_2$$

$$V_2 = sI_2 + \frac{2}{s}(I_1 + I_2) + 4I_1 = \left(4 + \frac{2}{s}\right)I_1 + \left(s + \frac{2}{s}\right)I_2$$

Therefore

$$Z = \frac{1}{s}\begin{bmatrix} 20s+2 & 2 \\ 4s+2 & s^2+2 \end{bmatrix}$$

Taking the inverse yields the y-parameters

$$Y = \frac{1}{\Delta Z}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix} = \frac{1}{20s^2 + 2s + 32}\begin{bmatrix} s^2+2 & -2 \\ -(4s+2) & 20s+2 \end{bmatrix}$$

where

$$\Delta Z = \frac{(20s+2)(s^2+2) - 2(4s+2)}{s} = \frac{20s^3 + 2s^2 + 32s}{s} = 20s^2 + 2s + 32$$

Finally, the h-parameters are given as

$$H = \begin{bmatrix} 1/y_{11} & z_{12}/z_{22} \\ -z_{21}/z_{22} & 1/z_{22} \end{bmatrix} = \frac{1}{s^2+2}\begin{bmatrix} 20s^2 + 2s + 32 & 2 \\ -(4s+2) & s \end{bmatrix}$$

(b) Now consider figure (b). z_{11} is V_1 when $I_2 = 0$. But if $I_2 = 0$, then because of the ideal transformer $I_1 = 0$, meaning that the ratio is not defined. Hence the z-parameters do not exist.

To find the h-parameters, observe that because of the ideal transformer, $I_2 = -0.5I_1$, $I_1 = -2I_2$, and $V_{pri} = 0.5V_{\sec}$. Writing a mesh equation at the right mesh first we obtain

$$V_2 = RI_2 + V_{\sec} + (I_1 + I_2)R = V_{\sec}$$

Now writing a mesh equation on the left we have

$$V_1 = RI_1 + V_{pri} + (I_1 + I_2)R = 1.5RI_1 + V_{pri} = 1.5RI_1 + 0.5V_2$$

Therefore

$$H = \begin{bmatrix} 1.5R & 0.5 \\ -0.5 & 0 \end{bmatrix}$$

To obtain the y-parameters we use table 19.1:

$$Y = \frac{1}{h_{11}} \begin{bmatrix} 1 & -h_{12} \\ h_{21} & \Delta h \end{bmatrix} = \frac{1}{6R} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

Note: the det[Y] = 0 implying again that the z-parameters do not exist.

(c) For this network we consider it as a cascade (left to right) of an ideal transformer, the middle network of a transformer and an inductor across the top, and finally another ideal transformer. The t-parameters of these two ports are respectively:

$$T_1 = \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix},\ T_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix},\ T_3 = \begin{bmatrix} 1/b & 0 \\ 0 & b \end{bmatrix}$$

To find T_2, we replace the mutually coupled inductors by the pi-equivalent circuit of figure 18.25c where $L_1 = 4$ H, $L_2 = 9$ H, $M = k\sqrt{L_1 L_2} = 3$ H, and $\Delta = 27$. Thus $L_{left} = \dfrac{\Delta}{L_2 - M} = 4.5$ H, $L_{right} = \dfrac{\Delta}{L_1 - M} = 27$ H, and $L_{top} = \dfrac{\Delta}{M} = 9$ H. Notice that the external 9 H inductor is in parallel with L_{top} leading to $L_{par} = 4.5$ H. The y-matrix of this new pi-network is by inspection:

$$Y_{mid} = \frac{1}{s} \begin{bmatrix} 2/4.5 & -1/4.5 \\ -1/4.5 & 1/27 + 1/4.5 \end{bmatrix} = \frac{1}{27s} \begin{bmatrix} 12 & -6 \\ -6 & 7 \end{bmatrix}$$

To compute the t-parameters we have

$$T_2 = \begin{bmatrix} \dfrac{-y_{22}}{y_{21}} & \dfrac{-1}{y_{21}} \\ \dfrac{-\Delta y}{y_{21}} & \dfrac{-y_{11}}{y_{21}} \end{bmatrix} = \begin{bmatrix} 7/6 & 4.5s \\ 8/27s & 2 \end{bmatrix}$$

Therefore

$$T = T_1 T_2 T_3 = \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} \begin{bmatrix} 7/6 & 4.5s \\ 8/27s & 2 \end{bmatrix} \begin{bmatrix} 1/b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 7a/6b & 4.5abs \\ 8/27abs & 2b/a \end{bmatrix}$$

From table 19.1, we obtain

$$H=\begin{bmatrix} a^2 2.25s & 0.5a/b \\ -0.5a/b & 4\big/27b^2 \end{bmatrix},\quad Z=s\begin{bmatrix} \dfrac{63a^2}{16} & \dfrac{27ab}{8} \\ \dfrac{27ab}{8} & \dfrac{27b^2}{4} \end{bmatrix},\quad Y=\frac{1}{27a^2b^2s}\begin{bmatrix} 12b^2 & -6ab \\ -6ab & 7a^2 \end{bmatrix}$$

SOLUTION PROBLEM 19.64.

(a) The defining equation for the g-parameters is:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{12} \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Because of the load impedance Z$_L$, we have $V_2=-Z_L I_2$. Hence substituting for V$_2$ in the second equation yields

$$V_2=g_{21}V_1+g_{22}I_2=-Z_L I_2 \qquad I_2=\frac{-g_{21}V_1}{g_{22}+Z_L}$$

Substituting this equation into $I_1=g_{11}V_1+g_{12}I_2$ we obtain

$$I_1=g_{11}V_1-g_{12}\left(\frac{g_{21}V_1}{g_{22}+Z_L}\right)=\left(g_{11}-\frac{g_{12}g_{21}}{g_{22}+Z_L}\right)V_1$$

Therefore,

$$Y_{in}=g_{11}-\frac{g_{12}g_{21}}{g_{22}+Z_L}$$

(b) Because of the source impedance Z$_s$, we have $V_1=-Z_s I_1$ or $I_1=-Y_s V_1$. Hence substituting for V$_1$ in the first equation yields

$$I_1=g_{11}V_1+g_{12}I_2=-Y_sV_1 \qquad V_1=\frac{-g_{12}I_2}{g_{11}+Y_s}$$

Substituting this equation into $V_2=g_{21}V_1+g_{22}I_2$ we obtain

$$V_2=g_{21}\left(\frac{-g_{12}I_2}{g_{11}+Y_s}\right)+g_{22}I_2=\left(g_{22}-\frac{g_{12}g_{21}}{g_{11}+Y_s}\right)I_2 \qquad Z_{out}=g_{22}-\frac{g_{12}g_{21}}{g_{11}+Y_s}$$

(c)

$$G_1=\frac{V_1}{V_s}=\frac{Z_{in}}{Z_s+Z_{in}}=\frac{Y_s}{Y_s+Y_{in}}=\frac{Y_s}{Y_s+g_{11}-\dfrac{g_{12}g_{21}}{g_{22}+Z_L}}=\frac{Y_s\left(g_{22}+Z_L\right)}{\left(g_{22}+Z_L\right)\left(g_{11}+Y_s\right)-g_{12}g_{21}}$$

(d) Refer to figure P19.64, where $V_2=\dfrac{Z_L}{g_{22}+Z_L}g_{21}V_1 \qquad G_2=\dfrac{V_2}{V_1}=\dfrac{g_{21}Z_L}{g_{22}+Z_L}$.

(e) $G_v = G_1G_2 = \dfrac{Y_s\left(g_{22}+Z_L\right)}{\left(g_{22}+Z_L\right)\left(g_{11}+Y_s\right)-g_{12}g_{21}} \times \dfrac{g_{21}Z_L}{g_{22}+Z_L} = \dfrac{g_{21}Y_sZ_L}{\left(g_{22}+Z_L\right)\left(g_{11}+Y_s\right)-g_{12}g_{21}}$

SOLUTION PROBLEM 19.65. Recall

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

For the given circuit

$$V_2 = \frac{1}{4}(I_2 - 4V_2) + V_1 \qquad V_2 = \frac{1}{2}V_1 + \frac{1}{8}I_2$$

Also

$$V_2 = V_1 - \frac{1}{4}\left(I_1 - sV_1\right)$$

This implies that

$$I_1 = (4+s)V_1 - 4V_2 = (4+s)V_1 - 4\left(\frac{1}{2}V_1 + \frac{1}{8}I_2\right) = (2+s)V_1 - \frac{1}{2}I_2$$

Thus the g-parameter matrix is:

$$G = \begin{bmatrix} 2+s & -\dfrac{1}{2} \\ \dfrac{1}{2} & \dfrac{1}{8} \end{bmatrix}$$

SOLUTION PROBLEM 19.66. We first convert the y-parameters to t-parameters using table 19.1:

$$Y_{N1} \to T_{N1} = \begin{bmatrix} \dfrac{-y_{22}}{y_{21}} & \dfrac{-1}{y_{21}} \\ \dfrac{-\Delta y}{y_{21}} & \dfrac{-y_{11}}{y_{21}} \end{bmatrix} = \begin{bmatrix} -0.1 & -0.5 \\ -3.82 & -25.1 \end{bmatrix}$$

where $\Delta y = 50.2 \times 0.2 - 2 \times 1.2 = 7.64$. Now we convert the h-parameters to t-parameters using table 19.1:

$$H_{N2} \to T_{N2} = \begin{bmatrix} \dfrac{-\Delta h}{h_{21}} & \dfrac{-h_{11}}{h_{21}} \\ \dfrac{-h_{21}}{h_{21}} & \dfrac{-1}{h_{21}} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 56 & 13 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 11.2 & 2.6 \\ 0.4 & 0.2 \end{bmatrix}$$

where $\Delta h = 13 \times 2 + 6 \times 5 = 56$. To obtain the cascaded t-parameters we compute

$$T_{cas} = T_{N1}T_{N2} = \frac{1}{5}\begin{bmatrix} -6.6 & -1.8 \\ -264.12 & -74.76 \end{bmatrix} = \begin{bmatrix} -1.32 & -0.36 \\ -52.824 & -14.952 \end{bmatrix}$$

Thus,

$$Y_{in} = \frac{1}{Z_{in}} = \frac{t_{21}Z_L + t_{22}}{t_{11}Z_L + t_{12}} = \frac{\frac{603}{5}}{\frac{15}{5}} = 40.2\ \text{S}$$

To obtain the voltage gain, we first convert the t-parameters back to y-parameters (table 19.1) and then use the derived voltage gain formula:

$$T_{cas} = \begin{bmatrix} -1.32 & -0.36 \\ -52.824 & -14.952 \end{bmatrix} \rightarrow Y_{cas} = \begin{bmatrix} 41.533 & 2 \\ 2.7778 & 3.6667 \end{bmatrix} \text{S}$$

Hence,

$$G_v = \frac{V_L}{V_1} = \frac{-y_{21}}{y_{22} + Y_L} = \frac{-2.7778}{3.6667 + 0.5} = -0.66667$$

Alternately, one could consider the load as a 2-port, compute its t-parameters, construct the overall t-parameters as a cascade of three networks, and then use $G_v = \frac{1}{t_{11}}$.

SOLUTION PROBLEM 19.67.

(a) Since only one C or L, weh have in general: $H(s) = \frac{as+b}{cs+d}$. Since $H(\infty) = 0 = \frac{a}{c}$, we have that a = 0. Therefore,

$$H(s) = \frac{\frac{b}{c}}{s + \frac{d}{c}} = \frac{K}{s + \omega_c}$$

(b) To prove that $\omega_c = \frac{1}{R_{th}C}$ or $\frac{R_{th}}{L}$ where R_{th} is the Thevenin resistance seen by the energy storage element, we refer the reader to problem 19.69 which provides a general derivation with H(∞) arbitrary; hence this problem is the special case of H(∞) = 0.

(c) Here $R_{th} = 2000//(50 + 1000//200)) = 195.49\ \Omega$. Hence

$$\omega_c = \frac{1}{R_{th}C} = 25.58 \times 10^6\ \text{rad/s}$$

SOLUTION PROBLEM 19.68.

(a) Because there is only one energy storage element, it follows that the most general form of the transfer function is: $H(s)=\dfrac{as+b}{cs+d}$. Since $H(0)=0=\dfrac{b}{d}$, we have b = 0. Further, since the transfer function is high pass, c ≠ 0, and

$$H(s)=\frac{as}{cs+d}=\frac{\left(\frac{a}{c}\right)s}{s+\left(\frac{d}{c}\right)}=\frac{Ks}{s+\omega_c}$$

(b) To prove that $\omega_c=\dfrac{1}{R_{th}C}$ or $\dfrac{R_{th}}{L}$ where R_{th} is the Thevenin resistance seen by the energy storage element, we refer the reader to problem 19.69 which provides a general derivation with H(0) arbitrary; hence this problem is the special case of H(0) = 0.

(c) Let us apply the result of example 19.1 to that part of the circuit to the right of the 1 kΩ resistor and call the associated resistance Z_{in}. Here $Z_1 = \infty$, $Z_2 = 2$ kΩ, $Z_L = 100$ Ω, and beta = 50. Hence

$$Z_{in}=(beta+1)Z_L=5.1\text{ k}\Omega$$

Thus

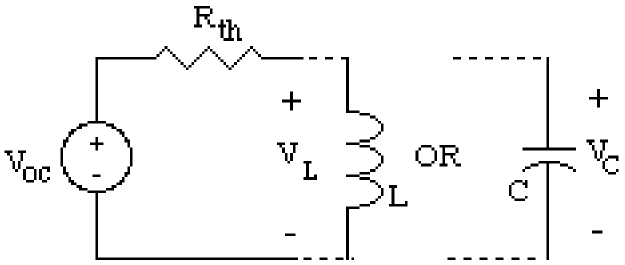
$$R_{th}=200+1000\,//(2000+5100)=1076.5\ \Omega$$

Hence,

$$\omega_c=\frac{1}{R_{th}C}=\frac{1}{1076.54\times 2\times 10^{-6}}=464.45\ \text{rad/s}$$

SOLUTION PROBLEM 19.69. In this problem we assume (i) a single input single output system and that linear circuit seen by the energy storage element has a Thevenin equivalent or a Norton equivalent. For simplicity we will presume the existence of a Thevenin equivalent.

(a) The Thevenin equivalent seen by the dynamic element L or C consists of $Z_{th}(s)$ in series with $V_{oc}(s)$. Since the remainder network seen by L or C is non-dynamic (resistive, resistive with dependent sources and ideal op amps, etc), we have $Z_{th}(s) = R_{th}$ and $V_{oc}(s) = K_O \times \text{Input}(s)$, R_{th} and K_O being real constants.



By voltage division,

$$V_L = \frac{Ls}{Ls + R_{th}} V_{oc} = \frac{K_0 s}{s + \frac{R_{th}}{L}} \times Input(s) \qquad (1)$$

and

$$V_C = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R_{th}} V_{oc} = \frac{\frac{K_0}{R_{th}C}}{s + \frac{1}{R_{th}C}} \times Input(s) \qquad (2)$$

After $V_L(s)$ or $V_C(s)$ has been determined, we can find the Laplace transform of any other output (voltage or current) using the voltage source substitution theorem (chapter 6) and linearity (chapter 5):

$$Ouput(s) = K_1 \times Input(s) + K_2\big(V_L(s) \quad or \quad V_C(s)\big) \qquad (3)$$

For the case of $V_L(s)$,

$$Ouput(s) = \left(K_1 + K_2 \frac{K_0 s}{s + \frac{R_{th}}{L}} \right) \times Input(s) \qquad (4)$$

and for the case of $V_C(s)$,

$$Ouput(s) = \left(K_1 + K_2 \frac{\frac{K_0}{R_{th}C}}{s + \frac{1}{R_{th}C}} \right) \times Input(s) \qquad (5)$$

For either case, the transfer function H(s) has the form

$$H(s) = \frac{Ouput(s)}{Input(s)} = \frac{K_3 s + K_4}{s + \omega_c} \qquad (6)$$

where $\omega_c = \dfrac{R_{th}}{L}$ *or* $\dfrac{1}{R_{th}C}$ with K_3 and K_4 real constants.

It remains to give K_3 and K_4 some physical interpretations. In (6), let s = ∞, we have

$$H(\infty) = \left.\frac{K_3 s + K_4}{s + \omega_c}\right]_{s=\infty} = K_3$$

On the other hand, letting s = 0 in (6) produces

$$H(0) = \left.\frac{K_3 s + K_4}{s + \omega_c}\right]_{s=0} = \frac{K_4}{\omega_c}$$

Substituting K_3 and K_4 into (6), we obtain the desired result

$$H(s) = \frac{Ouput(s)}{Input(s)} = \frac{H(\infty)s + \omega_c H(0)}{s + \omega_c} \qquad (7)$$

(b) When s = ∞, the impedance of C is

$$Z_C(\infty) = \left.\frac{1}{Cs}\right]_{s=\infty} = 0$$

and the impedance of L is

$$Z_L(\infty) = Ls\big]_{s=\infty} = \infty$$

Therefore in calculating H(∞), we may replace C by a short circuit and L by an open circuit. On the other hand, when s = 0, the impedance of C is

$$Z_C(0) = \left.\frac{1}{Cs}\right]_{s=0} = \infty$$

and the iimpedance of L is

$$Z_L(0) = Ls\big]_{s=0} = 0$$

Therefore in calculating H(0), we may replace C by an open circuit and L by a short circuit.

(c) For figure P19.69a, by inspection

$R_{th} = 3//(2 + 4) = 3//6 = 2\ \Omega$
$\omega_c = 1/(R_{th}\,C) = 1/(2 \times 0.5) = 1$
$H(\infty) = 4/(2 + 4) = 2/3$
$H(0) = 4/(2 + 3 + 4) = 4/9$

Therefore

$$H(s) = \frac{H(\infty)s + \omega_c H(0)}{s+\omega_c} = \frac{\frac{2}{3}s + \frac{4}{9}}{s+1} = \frac{\frac{2}{3}\left(s + \frac{2}{3}\right)}{s+1}$$

For figure P19.69b

$R_{th} = 3//(2 + 4) = 3//6 = 2\ \Omega$
$\omega_c = R_{th}/ L = 2/2 =1$
$H(\infty) = 4/(2 + 4) = 2/3$
$H(0) = 4/(2 + 3 + 4) = 4/9$

Therefore

$$H(s) = \frac{H(\infty)s + \omega_c H(0)}{s+\omega_c} = \frac{\frac{4}{9}s + \frac{2}{3}}{s+1} = \frac{\frac{4}{9}(s+1.5)}{s+1}$$

SOLUTION PROBLEM 19.70. According to problem 19.68, the transfer function of the circuit is

$$H(s) = \frac{Ks}{s+\omega_c}$$

where $\omega_c = \dfrac{1}{R_{th}C}$ and R_{th} is the Thevenin resistance seen by the storage element C. To find Rth we make use of figure 19.4 and the associated formula. The details are in the MATLAB code below:

```
»R1 = 200*1e3/1200
R1 =
  1.6667e+02
»Z1 = R1+2e3
Z1 =
  2.1667e+03
»Z3 = 100;
```

```
»beta = -50;
»Zout = Z1/(1+beta)
Zout =
  -4.4218e+01
»Rth = 2000 + Zout
Rth =
   1.9558e+03
»wc = 1/(Rth*2e-6)
wc =
   2.5565e+02
```

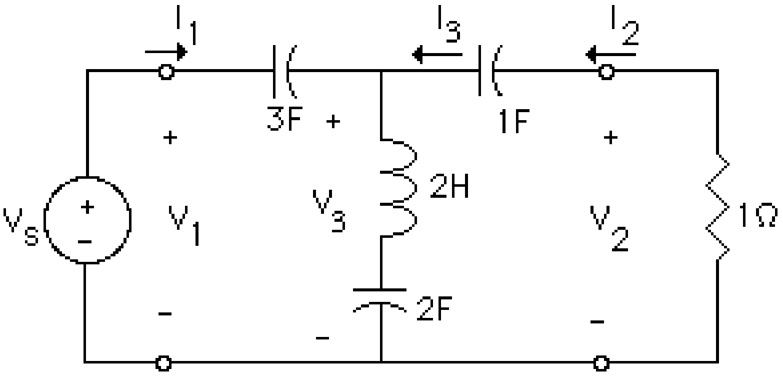
Hence

$$\omega_c = \frac{1}{1955.8 \times 2 \times 10^{-6}} = 255.65 \text{ rad/s}$$

SOLUTION PROBLEM 19.71. (a) Except for the terminating resistor, let the other element branches of the circuit be given by

$$Z_1 = \frac{1}{3s_1},\ Y_2 = \frac{1}{2s + \frac{1}{2s}} = \frac{2s}{4s^2} + 1, \text{ and } Z_3 = \frac{1}{s}$$

Consider the circuit



Here from problem 19.53a and from 19.52 a we obtain

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{\frac{2}{3}}{4s^2+1} & \dfrac{1}{3s} \\ \dfrac{2s}{4s^2+1} & 1 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1}{s} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

which implies that

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \frac{1}{4s^2+1} \begin{bmatrix} 4s^2 + 5/3 & \dfrac{\left(16s^2+6\right)}{3s} \\ 2s & 4s^2+3 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

(b)

$$G_V(s) = \frac{-y_{21}}{y_{22} + y_L}$$

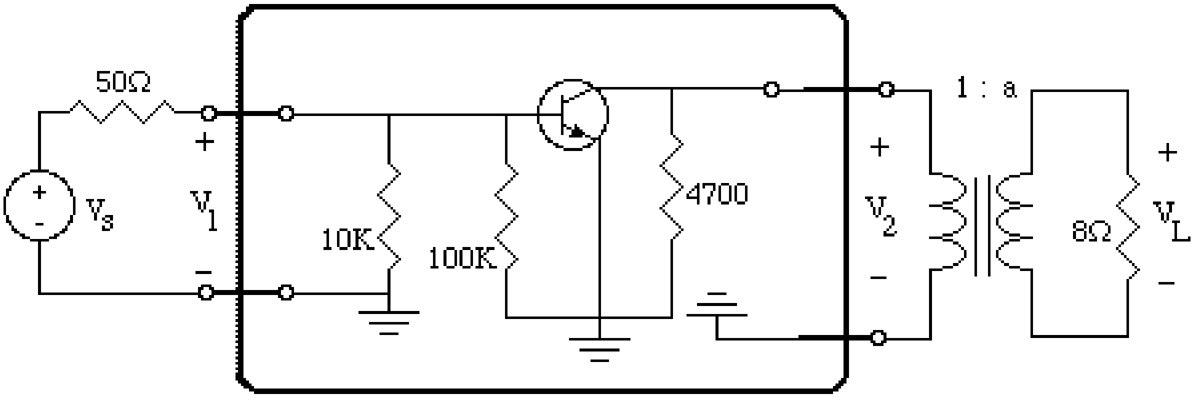
(c)

$$y_{21} = -\frac{1}{t_{12}} = -\frac{3s(4s^2+1)}{16s^2+6}, \; y_{22} = \frac{t_{11}}{t_{12}} = \frac{3s\left(4s^2 + \frac{5}{3}\right)}{16s^2+6} = \frac{s(12s^2+5)}{16s^2+6}, \text{ and } Y_L = 1$$

(d) Hence,

$$G_V(s) = \frac{-y_{21}}{y_{22} + y_L} = \frac{\dfrac{3s(4s^2+1)}{16s^2+6}}{\dfrac{s(12s^2+s)}{16s^2+6} + 1} = \frac{3s(4s^2+1)}{12s^3 + 16s^2 + 5s + 6}$$

SOLUTION PROBLEM 19.72. For part (a), treating each capacitor as a short circuit yields the equivalent circuit below.



```
»h11T = 4.2e3;  h12T = 0;  h21T = 150; h22T = 0.1e-3;
»hT = [h11T, h12T;h21T  h22T];
»yT = htoy(hT)
yT =
   2.3810e-04            0
   3.5714e-02   1.0000e-04
```

By inspection, the y11 parameter of the overall two-port consists of the sum of y11T plus the conductances of the two front end resistors. Also, the y22 parameter of the overall two port is y22T plus the conductance of the 4.7k Ω resistor. Hence,

```
»y = yT + [1/1e4+1/1e5  0;0  1/4700]
y =
   3.4810e-04            0
   3.5714e-02   3.1277e-04
```

Hence, the overall h-parameters are:

```
»h = ytoh(y)
h =
   2.8728e+03            0
   1.0260e+02   3.1277e-04
```

(b)

```
»yout = h(2,2) - (h(1,2)*h(2,1)/(h(1,1)+50))
yout =
```

```
   3.1277e-04
»Zout = 1/yout
Zout =
   3.1973e+03
»a = sqrt(8/Zout)
a =
   5.0021e-02
```

(c) To compute the gain we first need Z_{in}. From equation 19.49, since $h_{12} = 0$, $Z_{in} = h_{11}$.

```
» Zin = h(1,1)
Zin =
   2.8728e+03
```

From equation 19.51,

```
»Gv2 = -h(2,1)/(Zin*(h(2,2)+h(2,2)))
Gv2 =
  -5.7094e+01
```

From equation 19.52,

```
»Gv1 = Zin/(Zin + 50)
Gv1 =
   9.8289e-01
```

To compute V_L/V_s we use:

```
»Gv = Gv1*Gv2*a
Gv =
  -2.8071e+00
```

To compute V_L/V_1 we use:

```
»Gvv = Gv2*a
Gvv =
```

```
  -2.8559e+00
```

(d) For these calculations, we assume Vs is normalized to 1 V. Since we are computing gains, we may do this without loss of generality.

```
»Vs = 1;
»Is = 1/(50 + Zin)
Is =
   3.4214e-04
```

Now we compute the normalized power delivered by the voltage source.

```
»Psnorm = Vs*Is
Psnorm =
   3.4214e-04
```

Next we compute the normalized power absorbed by the load.

```
»VL = Gv*1
VL =
  -2.8071e+00

»PLnorm = VL^2/8
PLnorm =
   9.8496e-01
```

Next, the power gain from source to load is:

```
»GpLs = PLnorm/Psnorm
GpLs =
   2.8788e+03
```

Further, we compute the power gain from input to the two port to the load as follows:

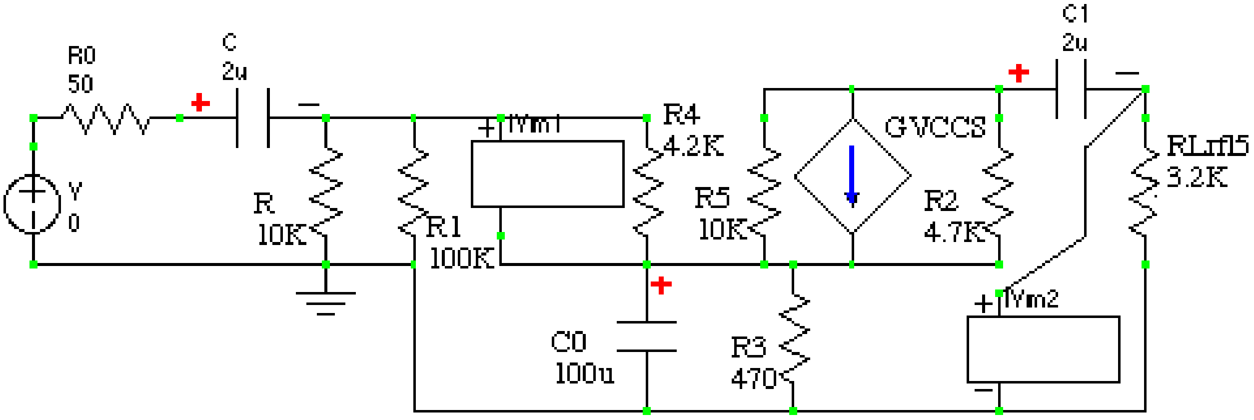
```
»V1 = Vs*Zin/(Zin + 50)
V1 =
```

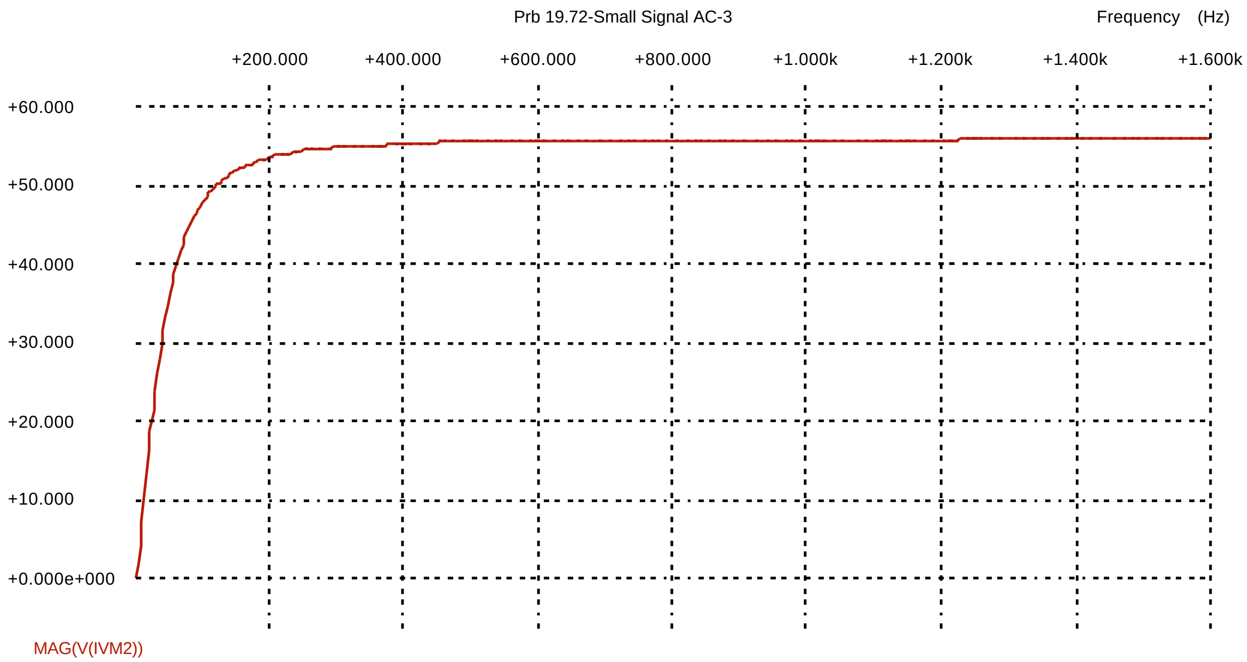
```
   9.8289e-01

»P1 = V1*Is
P1 =
   3.3629e-04

»GpL1 = PLnorm/P1
GpL1 =
   2.9289e+03
```

(e) SPICE Simulation Because the frequency response is flat for freqency above 800 Hz, we only plotted up to 1.6k Hz. The circuit diagram reflects the load back to the primary of the ideal transformer. In general, this is not possible. Hence for a SPICE simulation, it is necessary to use one of the models given in Figure 18.15 consisting of two controlled sources. For this example, this is not necessary. Note however that the actual output voltage is 0.05 times the values on the graph given below. This simulation assumes a 1 V source voltage and the parameter of GVCCS is 0.035714. Notice that in this problem





(f) For this part, we change 100 μF to 10 μF. The resulting plot shows degradation of the low end frequency response.