PROBLEM SOLUTIONS CHAPTER 1.

Solution 1.1. (a) Charge on one electron: -1.6019×10^{-19} C. This means that charge on 10^{13} electrons is: -1.6019×10^{-6} C. Net charge on sphere is: 1.6019×10^{-6} C (POSITIVE).

Solution 1.2. (a) 1 atom ≡ -4.646×10^{-18} C. By proportionality, $64g \equiv NA$ atoms.
$3.1g \equiv ?$ atoms $\quad 3.1g \equiv \frac{3.1NA}{64}$ atoms.

$$\text{Total Charge } = -4.646\times10^{-18}\,\frac{C}{atom}\times\frac{3.1\times6.023\times10^{23}}{64}\,atoms = -1.3554\times10^{5}\,C$$

(b) Total charge per atom is -4.646×10^{-18} C. Total charge per electron is -1.6019×10^{-19} C. Therefore, there are 29 electrons per atom of copper.

(c) 0.91 A ≡ 0.91 C/s. $i = \frac{\Delta Q}{\Delta t} \qquad \Delta t = \frac{\Delta Q}{i} = \frac{1.36\times10^{5}}{0.91} = 1.49\times10^{5}\,\text{sec}$.

(d) We know there are $\frac{3.1NA}{64} = 2.9174\times10^{22}$ atoms in the penny. Removing 1 electron from $0.05\times\frac{3.1NA}{64}$ atoms means removing $0.05\times\frac{3.1NA}{64}$ electrons. Therefore,

$$\text{Net charge } = 0.05\times\frac{3.1NA}{64}\times1.6019\times10^{-19} = 234C$$

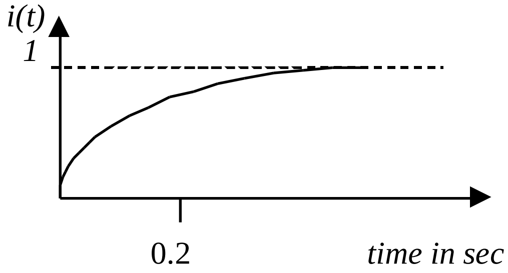
Solution 1.3 (a) $7.573\times10^{17}\times\left(-1.6019\times10^{-19}\right) = -0.1213C$

(b) $\text{Current} = \frac{0.1213}{10^{-3}} = 121.3\text{A}$ flowing from right to left.

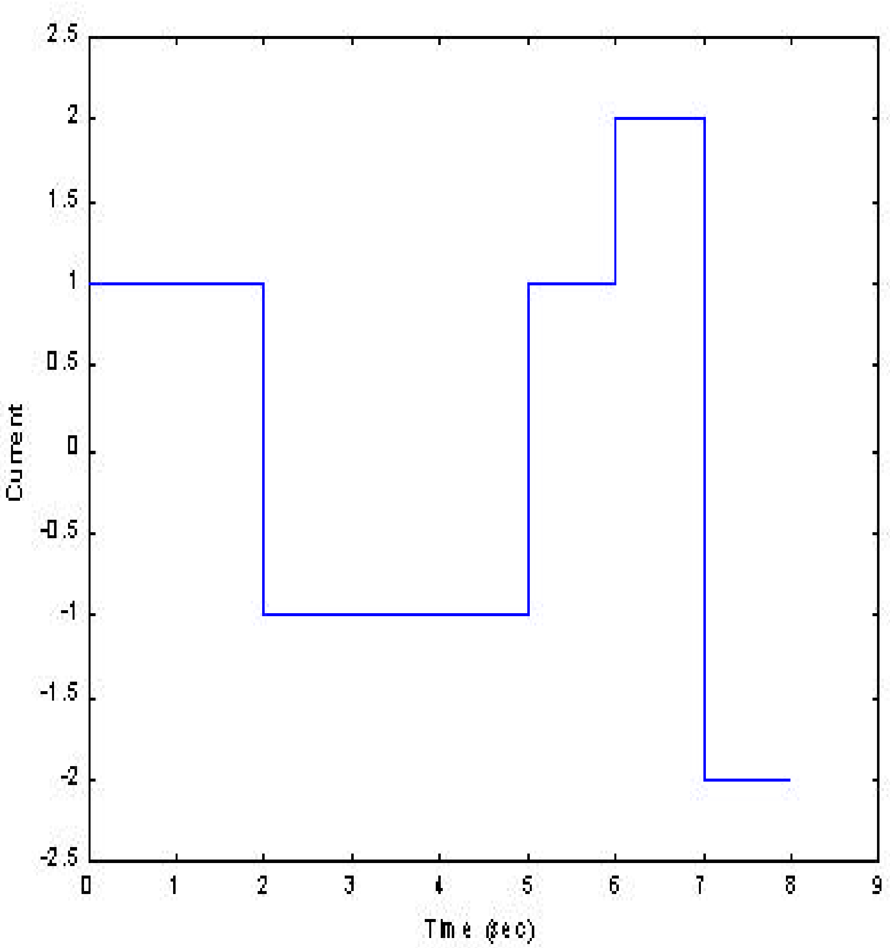
(c) Again, use proportionality:

$$10\,A = \frac{x\times1.6019\times10^{-19}}{60\text{sec}} \qquad x = \frac{10\times60}{1.6019\times10^{-19}} = 3.75\times10^{21}$$

(d) $i(t) = \frac{dq}{dt} = 1 - e^{-5t}$ A. This is an exponential evolution with an initial value of 0, a final value of 1, and a time-constant of 1/5 (signal reaches ~63% of it's final value in one time-constant).



(e) Current is the slope of the charge waveform. Therefore, by inspection:



Solution 1.4 **(a)** $6.023\times10^{23}\times(-1.6019\times10^{-19}) = -9.65\times10^{4}$ C.

(b) Current flows from right to left (opposite electrons), and:

$$I = \frac{9.65\times10^{4}}{10^{-3}} = 9.65\times10^{7}\ A$$

(c) Using proportionality:

$$5A = \frac{x \times 1.6019 \times 10^{-19}}{60\text{sec}} \qquad x = \frac{5 \times 60}{1.6019 \times 10^{-19}} = 1.87 \times 10^{21}$$

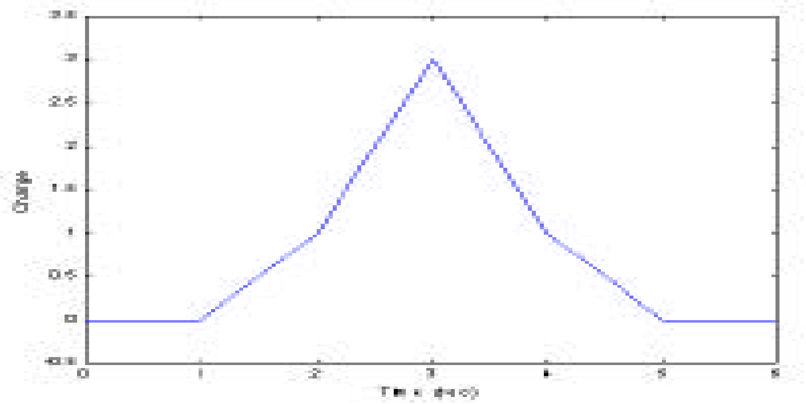
(d) $i(t) = \frac{dq}{dt} = 1 + 0.5\pi\cos(\pi t)$ $\qquad$ $i(1\text{sec}) = 1 - 1.57 = -0.57A$. Current flows from left to right.

Solution 1.5 **(a)** $i(t) = 1 - 4e^{-2t} + 3e^{-3t}$ t ≥ 0. Then

$$q(t) = \int_0^t i(t)dt = \int_0^t \left(1 - 4e^{-2\tau} + 3e^{-3\tau}\right)d\tau = \tau\Big]_{-\infty}^{t} - 4\int_0^t e^{-2\tau}d\tau + 3\int_0^t e^{-3\tau}d\tau$$

$$= t - 4\left[-0.5e^{-2\tau}\right]_0^t + 3\left[-0.333e^{-3\tau}\right]_0^t = t + 2e^{-2t} - e^{-3t} - 1$$

(b) By inspection:



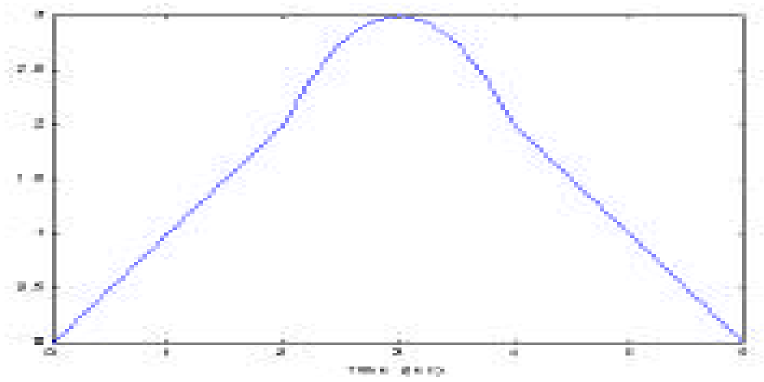
(c) $q(t) = 120\cos(120\pi t)$. Hence

$$i(t) = \frac{dq}{dt} = -120\pi \times 120\sin(120\pi t) = -14400\pi\sin(120\pi t) \text{ A}$$

Solution 1.6. **(a)** $i(t) = 1 - \cos(\pi t)$ A. Hence

$$q(t) = \int_0^t i(\tau)d\tau = \int_0^t (1 - \cos(\pi\tau))d\tau = t - \frac{1}{\pi}\sin(\pi\tau)\Big]_0^t = t - \frac{1}{\pi}\sin(\pi t) \text{ C}$$

(b) Charge is integral of current. Graphically, the charge at time t is the area under the current curve up to time t: (note the quadratic nature between 2 and 4 seconds)



Solution 1.7
Again, Q is the running area under the current curve. Between 0 and 3 seconds, current decreases linearly until zero. So, Q_{tot} = 7.5 C. From 0 to 6: Q_{tot} = 7.5 + Q_{3_6} = 7.5 -1/1×0.5 + -1/1×0.5 + -1×1 = 5.5 C, where the curve from 3 to 6 was divided into two triangular sections and one rectangular one.

Solution 1.8 Charge is the area under the current curve. Thus, Q = 0.1*4 – 0.1*2 = 0.2 C.

Solution 1.9 Calculate the change in energy for the electron: $\Delta E = Q\,\Delta V = 3.218\times10^{-15}$.

Equate this to kinetic energy:

$$3.218\times10^{-15} = \frac{1}{2}mv^2 \qquad v = 8.4\times10^{7}\,m/s$$

where the mass of an electron, 9.1066×10^{-31} has been substituted.

Solution 1.10 P = VI. Hence I = P/V = $2\times10^{3}/120$ = 16.6667 A

PROBLEM Solution 1.11 (a) It is necessary to integrate the i(t) curve to obtain q(t). We do this interval by interval:

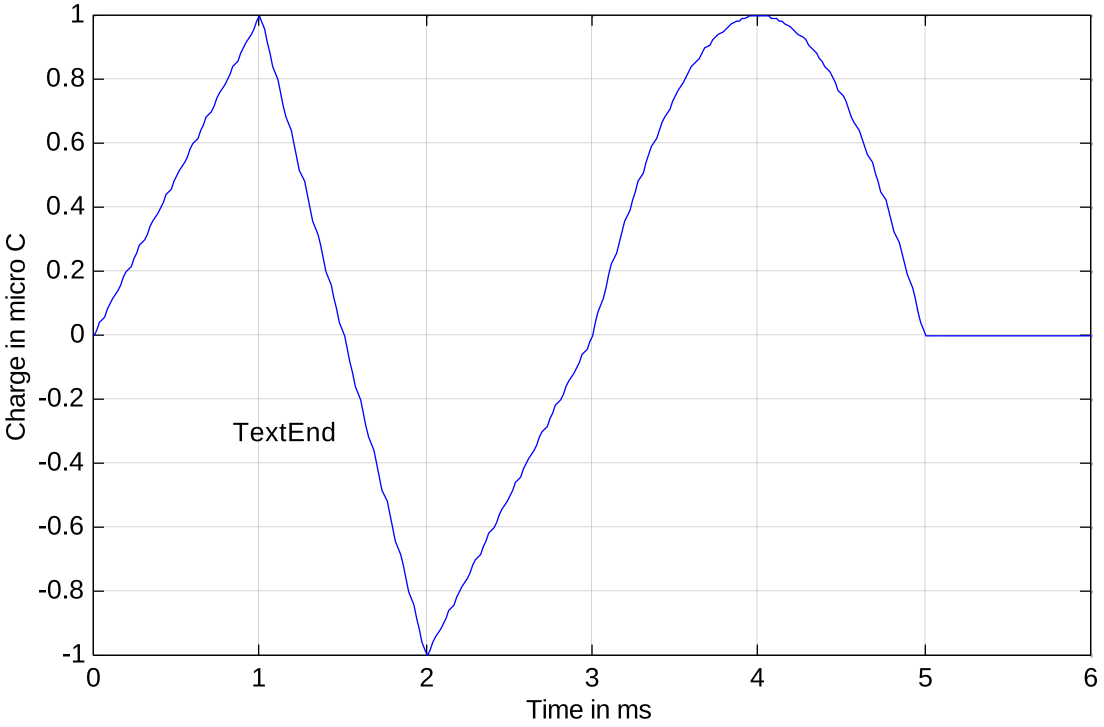
(i) $0 \le t \le 1$ ms, $q(t) = 0 + \int_0^t d\tau = t$ μC

(ii) 1 ms $\le t \le 2$ ms, $q(t) = 1 - 2\int_1^t d\tau = 3 - 2t$ μC

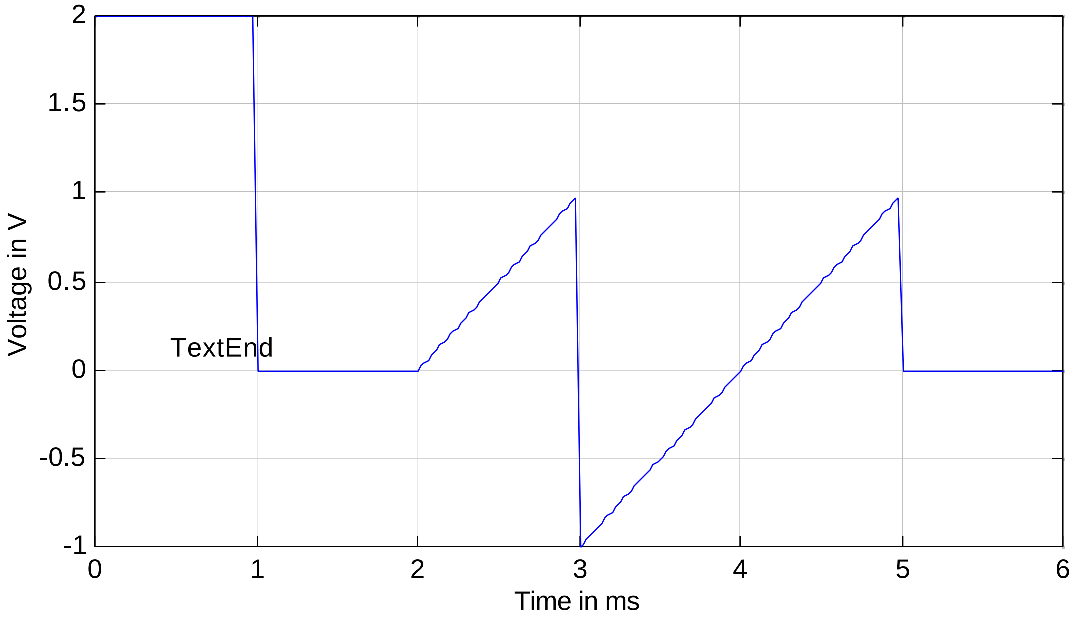
(iii) 2 ms $\le t \le 3$ ms, $q(t) = -1 + \int_2^t d\tau = -3 + t$ μC

(iv) $3 \text{ ms} \le t \le 5 \text{ ms}, \quad q(t) = 0 + \int_3^t (8 - 2\tau)\, d\tau = 8t - t^2 - 15 \ \mu\text{C}$

(v) $5 \text{ ms} \le t, \quad q(t) = 0 \ \mu\text{C}$



(b) Voltage is the ratio of the power and current curve. In this case, the division can be done graphically by inspection. Note that the ratio of a quadratic function and a linear function is a linear function:



Solution 1.12 **(a)** $V_A = P/I = 20/4 = 5$ V

(b) $P_B = VI = 2 \times 7 = 14$ W

(c) $V_C = P/I = -3W/3A = -1V$

(d) $I_D = P/V = -27W/3V = -9A$

(e) $I_E = P/V = 2/1 = 2A$

(f) $P_F = VI = -4 \times 5 = -20W$

In all of the above, note that the direction of the current flow relative to the polarity of the voltage across a device determines whether power is delivered or absorbed. Power is absorbed when current flows from the positive terminal of the device to the negative one.

Solution 1.13 **(a)** By inspection: Circuit Element (CE) 1 absorbs –5W, and CE 2 absorbs 6W.
(b) Compute power *absorbed* by all elements including independent sources:

I_{3A}:	-15
CE1:	-5
V_{3V}:	-12
CE2:	+6
V_{5V}:	10
I_{2A}:	16

Sum: 0 (Verifies conservation of power.)

Solution 1.14 (a) Compute power absorbed:

I_{5A}: -85
CE1: 98
V_{3V}: 33
CE2: 16
V_{7V}: -42
I_{2A}: -20

Sum: 0

(b) Add all terms:

I-source: $P_{absorbed} = -3\left(1-e^{-t}\right) = -3+3e^{-t}$ watts

V-source: $P_{absorbed} = -2\left(3e^{-t}-1\right) = -6e^{-t}+2$ watts

CE1: $P_{absorbed} = 3e^{-t}\times 3\left(1-e^{-t}\right) = 9e^{-t}-9e^{-2t}$ watts

CE2: $P_{absorbed} = \left(3e^{-t}-1\right)\left(3e^{-t}-1\right) = 9e^{-2t}-6e^{-t}+1$ watts

Simple algebraic manipulation of the the sum of all the above terms reveals that the result is zero.

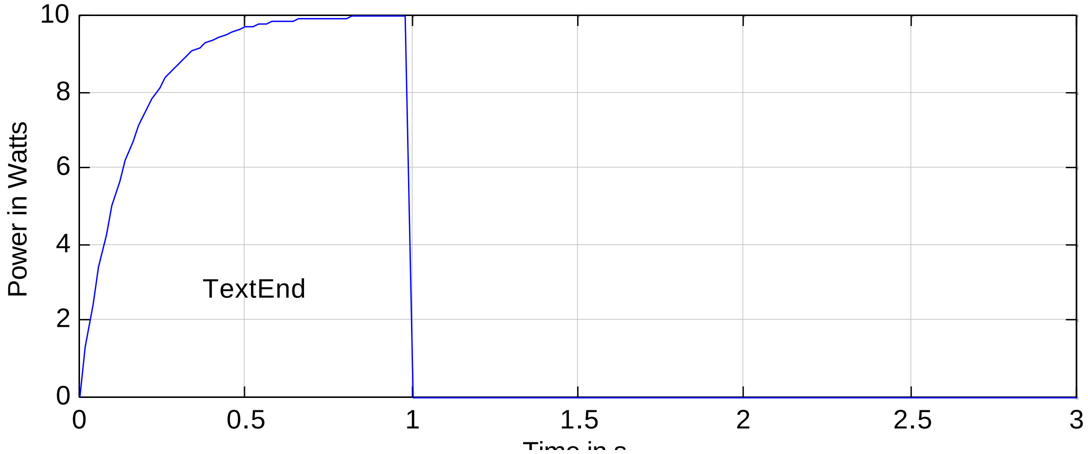
Solution 1.15 (a) When $I_L = 1$, $P = V_L I_L = (16-4)\times1 = 12$ W. When $I_L = 2$, $P = V_L I_L = (16-16)\times1 = 0$.

(b) $P = (16-4I_L^2)I_L$. Differentiate this w.r.t. I_L and set to zero: $16 – 12I_L^2 = 0$. Therefore, $I_L = 1.155$A.

Solution 1.16 (a) When $I_L = 2$, $P = (16-4)\times2 = 24$W. When $I_L = 3$, $P = (16 - 9)\times3 = 21$ W.

(b) Maximum occurs in the interval from 0 to 4: $P = (16 - I_L^2)\, I_L$
Differentiate w.r.t. I_L and set to zero: $16 – 3I_L^2 = 0$.
Therefore, $I_L = 2.31$ A.

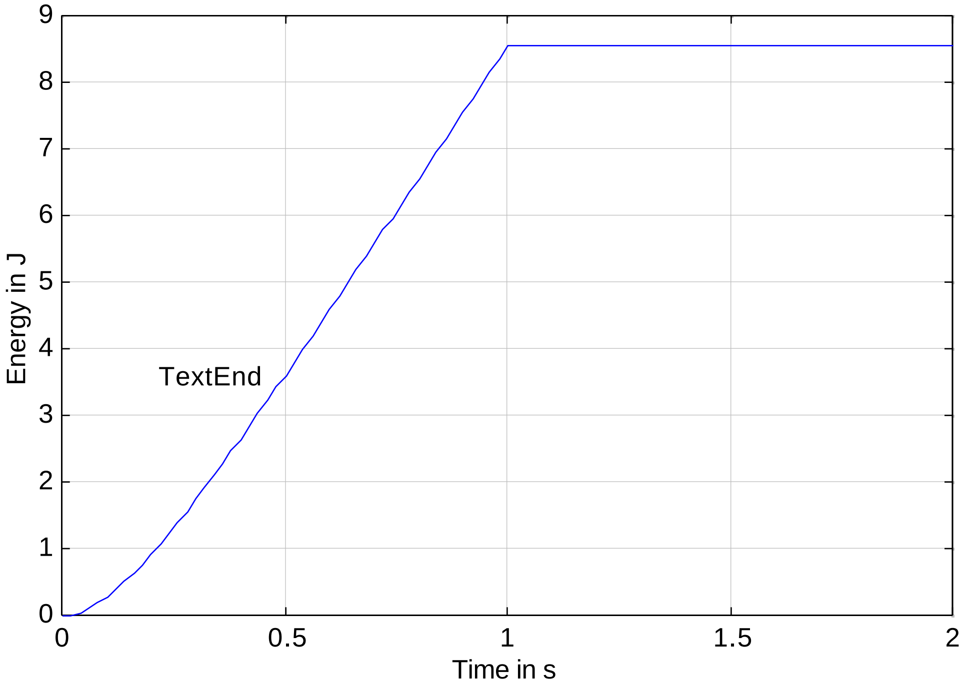
Solution 1.17 (a) Power is the product of the current and voltage. We can compute the product graphically:



(b)

$$W(t) = \int_0^t p(t)dt = \int_0^t \left(10 - 10e^{-7\tau}\right)d\tau = 10\tau\Big]_0^t - \left[-\tfrac{10}{7}\, e^{-7\tau}\right]_0^t = 10t + \frac{10}{7}e^{-7t} - \frac{10}{7}$$

This can be used as an aid to plot the work function:



Solution 1.18 (a) Since, $i(t) = 115 - 23t$ mA ,

$$q(7) = \int_0^7 i(\tau)\,d\tau = \left. 115t - \frac{23t^2}{2} \right]_0^7 \times 10^{-3} = 0.2415 \text{ C}$$

(b) Energy is the integral of power:

$$E = \int_0^7 p(t)dt = \int_0^7 v(t)\times i(t)dt = 25\int_0^7 i(t)dt$$

$$= 25 \times 0.2415 = 6.0375 \text{ C}$$

Solution 1.19 (a) Δt = 100°F, Rate of temp. increase is 2.5 Wh/°F per gallon:

Energy = 2.5Wh/°F/gallon×100 °F×30gallons = 7500 Wh = 2.7×10^7J.

(b) Heater generates P = 120×10 = 1200 W. We want 7500 Wh. Therefore, the total number of hours needed is 7500Wh/1200W = 6.25 h.

Solution 1.20 First compute the change in temperature required, in °F:

Δt = 80-25 = 55°C = 55×9/5 °F= 99°F

Next, compute the energy spent every hour, which means on 40 gallons of water:

E = 2.5 Wh/°F/gallon×99°F×40gallons = 9900 Wh

Since the heater is not 100 % efficient, we spend more energy than is actually needed to heat the water:

E_spent = 9900 Wh/0.9 = 11000 Wh

So, far, this was the energy spent every hour. Over six hours, the total energy spent is:

E_{6h} = 11000×6 = 66,000 Wh

Finally, the total energy spent per month is E_m = 66,000×30 = 1980 kWh

and the bill is 1980 kWh×0.14$/kWh = $277.2

Solution 1.21

Energy = 120 W × 6 h = 720 Wh = 0.72 kWh

Therefore, cost per day = 0.72 kWh × 8 = 5.76 cents, and cost per month is 5.76×31 = $1.785.

Solution 1.22

We need to compute the difference between the inner diameter of the tube and the outer one in order to get the cross-sectional area:

$$area = \pi R_{out}^2 - \pi R_{in}^2 = \pi 0.003^2 - \pi 0.0018^2 = 1.81 \times 10^{-3}\, m^2$$

Then, $R = 1.7\times10^{-5}\times(12/1.81) = 11.3$ mΩ.

Solution 1.23 L = 20 m, W = 0.015m, H = 0.001 m. Thus, A = W×H, and $R = 5.1\times\rho_{copper}\times L/A$

= 0.116 Ω.

Solution 1.24. **(a)** 500 ft, 20 gauge wire: 10.35 Ω/1000 ft from table 1.3. This implies that

R = 5.175 Ω.

(b) 55 ft, 20 gauge, nickel wire:

$$R = 5.1 \times \frac{10.35}{1000} \times 55 = 2.9\ \Omega$$

(c) $R_{tot} = 2.9 + 5.175 = 8.08$ Ω.

Solution 1.25. $R(T) = R(20)\left[1 + \alpha(T - 20)\right]$. Substituting at T = –10 yields:

$$21 = R(20)\left[1 + 0.0039(-30)\right] \quad \text{or} \quad R(20) = 23.78\ \Omega$$

Evaluating at T = +10 yields,

$$R(10) = 23.78 + 23.78 \times 0.0039 \times (-10) \quad \text{or} \quad R(10) = 22.85\Omega$$

Solution 1.26. For tungsten, we know that α = 0.0045. Therefore:

$$\begin{aligned} R(150) &= R(20)\left[1 + \alpha(T - 20)\right] = 200\left[1 + 0.0045(150 - 20)\right] \\ &= 317\Omega \end{aligned}$$

Rate of change of resistance is (317-200)/(150-20) = 0.9 Ω/°C.

Solution 1.27. Plug numbers directly into the same formula as problem 1.26:

$$0.0022 = 0.002 + 0.002\times0.0039(T-20)$$

Rearrange to obtain: $T = 45.64^{o}$ C.

Solution 1.28. (a) Power in a wire: $P = I^2R$. Rearranging, we can express the current as

$$I = \sqrt{P/R}\,.$$

Substitute given P and R to obtain $I = 0.707$ mA.

(b) Use the same formula for current obtained above to get 50 A.

Solution 1.29. Use formula for power: $P = V^2/R$. Rearranging, $R = V^2/P = 96\ \Omega$.

Solution 1.30 (a) $I = V/R = 12$ A, out of the positive terminal of the battery.

(b) Up through the resistor.

(c) Absorbed power by resistor: $P = V^2/R = 14.4$ W. Same power is delivered by source.

(d) From table 1.2 and 1.3, 1000 feet of 18 AWG aluminum wire has resistance:

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»R1000ft = 1.6*6.51
R1000ft =   1.0416e+01
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By proportionality, $1000 \times 0.1 = L \times 10.416$. Hence,

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»L =  100/10.416
L =   9.6006e+00 meters.
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Solution 1.31 (a) $V = 10$ V.

(b) $P = V^2/R$, which means that $R = V^2/P = 100/25 = 4\ \Omega$.

(c) $I = V/R = 10/4 = 2.5$ A. Current flow is downwards through resistor.

(d) Up through resistor.

(e) $P = V^2/R_{10} = 100/10 = 10$ W. Hence, $I_{10} = V/R_{10} = 1$ A. Without applying material from a future chapter, a legitimate way to obtain I_{source} is to apply conservation of power first and then compute I_{source} from the power formula. Hence, $P_{source} = 10 + 25 = 35$ watts. Using material from a later chapter, in particular KCL, we may conclude that, $I_{src} = 2.5 + 1 = 3.5$ A. Thus, $P_{source} = VI_{source} = 10\times3.5 = 35$ W. This approach indicates that power is conserved.

Solution 1.32 **(a)** From 0 to 1 s, $i(t) = 10^{-3}t$. Thus, $i^2R = 10^{-6}t^2R$ is the power absorbed during this interval. Integrating this expression for the power from 0 to 1 s gives us the total energy used:

$$\left.\frac{10^{-6}t^3R}{3}\right|_0^1 = 5000\frac{10^{-6}}{3} = 0.001667 \text{ J.}$$

Finally, we need to multiply this by 2 to account for the interval from 1 to 2 seconds. Thus, the total energy spent is 3.33 mJ.

(b) The same charge that got transported in one direction during the interval from 0 to 1 is being transported back in the interval from 1 to 2 (by symmetry). Therefore, total charge transfer is zero.

Solution 1.33. **(a)** 60 W + 120 W = 180 W.

(b) P = IV ⇨ I = P/V = 180/12 = 15 A.

(c) P = Energy/Time ⇨ Time = 1.2 MJ/180 W = 6.67×10^4 sec = 1.85 h.

Solution 1.34. $P = I^2R$. Therefore, 325 = 25×(5+4+2R). Solving for R, yields R = 2 Ω.

Solution 1.35. **(a)** Use definition of power and substitute given power:

$$V_2 = \sqrt{P \times R} = \sqrt{98 \times 2} = 14 \text{ V}$$

Similarly, $I_3 = \sqrt{\frac{P}{R}} = \sqrt{\frac{12}{3}} = 2$ A, $V_4 = \sqrt{P \times R} = \sqrt{16 \times 4} = 8$ V, $I_5 = \sqrt{\frac{768.8}{5}} = 12.4$ A, and $V_6 = \sqrt{486 \times 6} = 54$ V.

(b) $P_{tot} = P_{dissipated}$ = 98 + 12 + 16 + 768.8 + 486 = 1380.8 W.

(c) $V_{in} = V_2 + V_6 = 68$ V. $I_{in} + I_3 = I_5 + I_4$ and $I_4 = \frac{V_4}{4}$. Thus, $I_{in} = I_5 + I_4 - I_3 = 12.4$ A.

Solution 1.36. **(a)** Sources are the top, right-most, and bottom left. The reason is that current flows out of the positive terminal of the device.

(b) The 32/16 element is a 2 Ω resistor. The 54.5/18.167 element is a 3Ω resistor. The 13/2.167 element is a 6 Ω resistor. The 93/2.833 element is a 32.827 Ω resistor. The 24/5 element is a 4.8 Ω resistor.

Solution 1.37. Power: $12 = I_x^2R$, which means that $R = 12/I_x^2$. Now, analyze the loop: $16 = I_x(R+4)$.

Substitute the value of R into this expression: $16 = I_x\frac{12}{I_x^2} + 4I_x$. Hence: $I_x^2 - 4I_x + 3 = 0$.

This equation has two solutions: one is at $I_x = 1$ A or R = 12 Ω. The other is at $I_x = 3$ A or R = 4/3 Ω.

Solution 1.38. (a) Conservation of power:

$$16I_x = 4I_x^2 + 12 + 10 - 6$$

Hence

$$0 = I_x^2 - 4I_x + 4 = (I_x - 2)^2$$

Thus, $I_x = 2$ A.

(b) $32I_x = 4I_x^2 + 28$ $\quad I_x^2 - 8I_x + 7 = (I_x - 7)(I_x - 1) = 0$. Hence, $I_x = 7A$ or $I_x = 1A$.

Solution 1.39. (a)

(i) AA: I = 36/12 = 3A

BB: I = 24/12 = 2A

CC: I = 14.4/12 = 1.2A

(ii) Sum = 6.2A

(iii) P = VI = 6.2×12 = 74.4W. This is equal to the sum of the powers absorbed by the bulbs.

(iv) R = V/I

AA: R = 12/3 = 4Ω

BB: R = 12/2 = 6Ω

CC: R = 12/1.2 = 10Ω

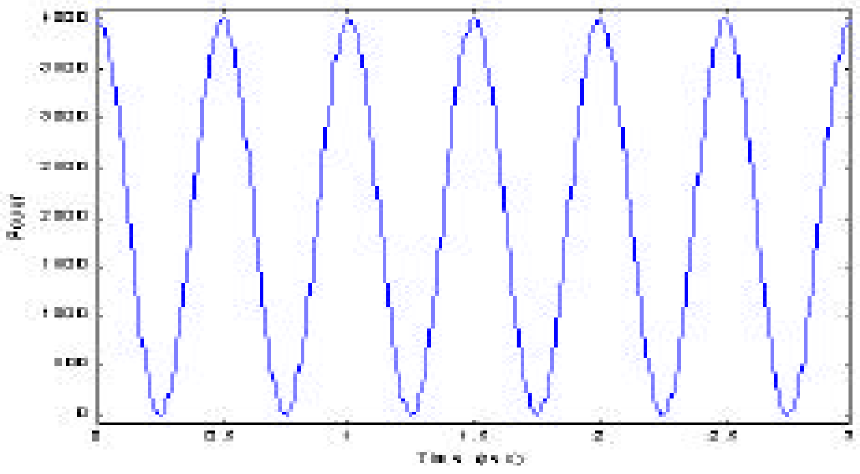
(b) Each AA bulb draws 3 A. Thus, up to five bulbs can be connected without blowing the fuse (5×3=15).

So, 6 or more would blow the fuse.

(c) Similar analysis suggests that 13 or more bulbs would blow the fuse. Intuitively, the bulbs draw less current, so more of them can be used.

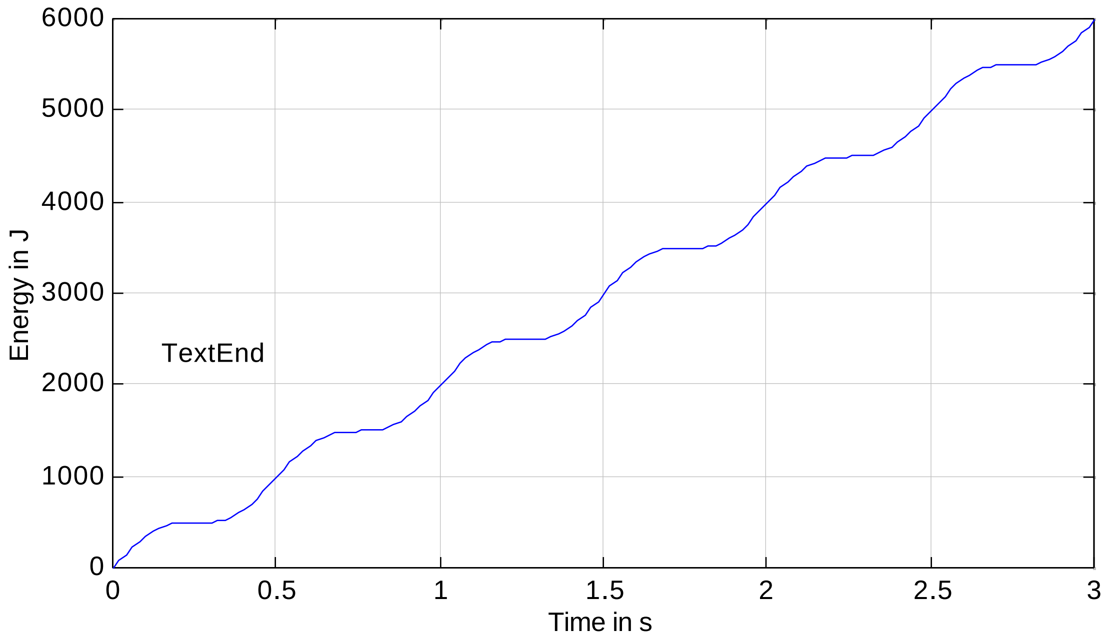
Solution 1.40. (a)

$$p(t) = i^2(t)R = [20\cos(2\pi t)]^2 \times 10 = 4000\frac{1+\cos(4\pi t)}{2} = 2000(1+\cos(4\pi t)) \text{ W}$$



(b)

$$W(t) = \int_0^t p(t)dt = 2000t + 2000\int_0^t \cos(4\pi t)dt = 2000t + \frac{2000\sin(4\pi t)}{4\pi} \text{ J}$$



Solution. 1.41. When the switch is closed, a constant current of 5/10000 = 0.5 mA flows through the circuit. When the switch is open, no current flows. So, 50% of the time, a 0.5 mA current flows, and the other 50% no current flows. The average current is therefore 0.25 mA.

Solution. 1.42 When the switch is at A, the current is 5/5000 = 1 mA. When the switch is at B, the current is 5/10000 = 0.5 mA. Now, the switch is at position A 20% of the time (1ms out of a 5ms period, after which the events repeat). So, the average current is 0.2*1 mA + 0.8*0.5 mA = 0.6 mA.

Solution. 1.43 The current in the load resistor is 2 A. So, the power is $2^2 \times R_L = 8$ W.

Solution 1.44. $V_{in} = I_{in}R_1 \Rightarrow I_{out} = \mu V_{in}/R_2 = \mu I_{in}R_1/R_2$.

Solution 1.45 (a) $I_1 = V_{in}/R_1$. Hence, $V_{out} = \alpha V_{in}R_2/R_1$.

(b)

$$\frac{V_{out}}{V_{in}} = \frac{\alpha R_2}{R_1} = \frac{100 \times 10}{R_1} = 5 \qquad R_1 = 200\ \Omega$$

(c)

$$\text{Power} - \text{gain} = \frac{\frac{\alpha^2 V_{in}^2}{R_1^2} R_2}{\frac{V_{in}^2}{R_1}} = \frac{\alpha^2 R_2}{R_1} = 500$$

Solution 1.46 (a) $V_1 = 200$ mA × 5 Ω = 1 V implies $V_2 = 0.8 \times 8 = 6.4$ V. Hence

$V_{out} = 5 \times 6.4 = 32$ V and $I_{out} = 32/64 = 0.5$ A.

(b) Current Gain = 0.5/0.2 = 2.5.

(c) Power values for the 5, 8, and 64 Ω resistors are, respectively, $P_5 = 0.2$ W, $P_8 = 5.12$ W, $P_{64} = 16$ W.

Solution 1.47 (a) $I_1 = 5\text{A} \Rightarrow I_2 = 3 \times 5/3 = 5\text{ A} \Rightarrow I_{out} = 25$ A, $V_{out} = 50$ V.

(b) Voltage Gain = 5.

(c) $P_{in} = 5 \times 5 \times 2 = 50$ W, $P_1 = 5 \times 5 \times 3 = 75$W, $P_2 = 25 \times 50 = 1250$W.

Solution 1.48 $I_1 = V_{in}/10 = 0.1$ A, $V_R = 10 \times (V_{in}/10) \times R = R$; $V_{out} = 5R \times 10 = 50R = 50RV_{in}$

$V_{out}/V_{in} = 50R$. If we want V_{out}/V_{in} to be 150, R has to be 3 Ω.