SOLUTIONS PROBLEMS CHAPTER 20

USEFUL MATLAB M-FILES FOR USE IN THE SOLUTION TO PROBLEMS IN THIS CHAPTER.

Program 1: converts y-parameters to t-parameters

```
% convert y parameters to t parameters
function [t,t11,t12,t21,t22] = ytot(y)
y11=y(1,1);
y12=y(1,2);
y21=y(2,1);
y22=y(2,2);
deltay= y11*y22-y12*y21;
t11=-y22/y21;
t12 = -1/y21;
t21= -deltay/y21;
t22= -y11/y21;
t= [ t11 t12; t21 t22];
```

Program 2: Computes Zin, Zout, and gains using two port h-parameters.

```
»% two-port analysis in terms of h-parameters
»function [zin, zout] =twoport(h, zL, zs)
»['twoport analysis using h-parameters']
»h11= h(1,1); h12=h(1,2); h21=h(2,1); h22=h(2,2);
»zin = h11 - h12*h21/(h22+ 1/zL)
»yout= h22 - h12*h21/(h11+zs);
»zout= 1/yout
»v1tovs= zin/(zin+zs)
»v2tov1= -h21/(zin*(h22+1/zL))
»v2tovs= v1tovs*v2tov1
```

Program 3: Computes Zin, Zout, and gains using two port y-parameters.

```
% two-port analysis in terms of y-parameters
function [zin, zout] =twoporty(y, zL, zs)
['twoport analysis using y-parameters']
y11= y(1,1); y12=y(1,2); y21=y(2,1); y22=y(2,2);
yin = y11 - y12*y21/(y22+ 1/zL)
zin= 1/yin
yout= y22 - y12*y21/(y11+1/zs)
zout= 1/yout
v1tovs= zin/(zin+zs)
v2tov1= -y21/(y22+1/zL)
v2tovs= v1tovs*v2tov1
```

Program 4: Computes Zin, Zout, and gains using two port t-parameters.

```
% two-port analysis in terms of t-parameters
function [zin, zout] =twoportt(t, zL, zs)
```

```
['analysis of terminated twoport using t-parameters']
t11= t(1,1); t12=t(1,2); t21=t(2,1); t22=t(2,2);
zin= (t11*zL + t12)/(t21*zL + t22)
zout= (t22*zs + t12)/(t21*zs + t11)
v2tov1= zL/(t11*zL + t12)
v1tovs= zin/(zin+zs)
v2tovs= v2tov1*v1tovs
```

Program 5: converts z-parameters to t-parameters

```
%converting z to t paramters (same sormulas as
%converting t to z parameters)

function [t,t11,t12,t21,t22] = ztot(z)
z11=z(1,1); z12=z(1,2); z21=z(2,1); z22=z(2,2);
deltaz= z11*z22 - z12*z21;
t11= z11/z21;
t12= deltaz/z21;
t21= 1/z21;
t22= z22/z21;
t= [ t11 t12; t21 t22];
```

SOLUTION 20.1.

For network a, the z-parameters are by inspection

$$Z_a = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

Similarly, for network b, the z-parameters are the same:

$$Z_b = \begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

The interconnection of networks a and b conforms to figure 20.2b, which is a series interconnection. Hence, the new overall z-parameters are

$$Z_{new} = Z_a + Z_b = 2\begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$$

SOLUTION 20.2. For networks a and b, the y-parameters are by inspection

$$Y_a = Y_b = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \text{S}$$

Hence, their z-parameters are the inverse of the y-parameter matrix:

$$Z_a = Z_b = \frac{1}{8}\begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \Omega$$

The interconnection of networks a and b conforms to figure 20.2b, which is a series interconnection. Hence, the new overall z-parameters are

$$Z_{new} = Z_a + Z_b = \frac{1}{4}\begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \Omega$$

SOLUTION 20.3. For network a consisting of the single inductor,

$$Z_a = \begin{bmatrix} s & s \\ s & s \end{bmatrix} \Omega$$

For network b, we have

$$Z_b = \left[Y_b\right]^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 1 & 2 \end{bmatrix} \Omega$$

The interconnection of networks a and b conforms to figure 20.2b, which is a series interconnection. Hence, the new overall z-parameters are

$$Z_{new} = Z_a + Z_b = \begin{bmatrix} s+0.5 & s+0.5 \\ s+1 & s+2 \end{bmatrix} \Omega$$

SOLUTION 20.4. For network a consisting of the single capacitor,

$$Z_a = \frac{1}{s}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Omega$$

The interconnection of networks a and b conforms to figure 20.2b, which is a series interconnection. Hence, the new overall z-parameters are

$$Z_{new} = Z_a + Z_b = \frac{1}{s}\begin{bmatrix} 1+0.5s & 1+0.5s \\ 1+s & 1+2s \end{bmatrix} \Omega$$

SOLUTION 20.5. For network a, the y-parameters are by inspection

$$Y_a = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \text{S}$$

Hence, their z-parameters are the inverse of the y-parameter matrix:

$$Z_a = \frac{1}{5}\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \Omega$$

The interconnection of networks a and b conforms to figure 20.2b, which is a series interconnection. Hence, the new overall z-parameters are

$$Z_{new} = Z_a + Z_b = \begin{bmatrix} 1.6 & 0.9 \\ 0.4 & -0.4 \end{bmatrix} \Omega$$

SOLUTION 20.6. For networks Na and Nb, the y-parameters are:

$$Y_{Na} = Y_{Nb} = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.7 \end{bmatrix} \text{S}$$

Hence, their z-parameters are the inverse of the y-parameter matrix:

$$Z_{Na} = Z_{Nb} = \frac{20}{9}\begin{bmatrix} 0.7 & 0.2 \\ 0.2 & 0.7 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 14 & 4 \\ 4 & 14 \end{bmatrix} \Omega$$

The network Na* has the same z-parameters as Na and continues to act as a two when series interconnected to another 2-port because of the transformer. Hence, the interconnection of networks Na* and Nb forms a valid series interconnection in which cas

$$Z_{new} = Z_{Na*} + Z_{Nb} = \frac{2}{9}\begin{bmatrix} 14 & 4 \\ 4 & 14 \end{bmatrix} \Omega$$

SOLUTION 20.7. For network Nb, the y-parameters are the same as in problem 20.6, i.e.,

$$Y_{Nb} = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.7 \end{bmatrix} \text{S} \quad \text{and} \quad Z_{Nb} = \frac{1}{9}\begin{bmatrix} 14 & 4 \\ 4 & 14 \end{bmatrix} \Omega$$

For network Na, consider the purely resistive part without the transformer. The y-parameters of this part are half the y-parameters of Nb, i.e.,

$$Y_R = \begin{bmatrix} 0.35 & -0.1 \\ -0.1 & 0.35 \end{bmatrix} S$$

In MATLAB we use the m-file which converts y-parameters to t-parameters:

```
»y = [0.35  -0.1;-0.1  0.35];
»t = ytot(y)
t =
   3.5000e+00   1.0000e+01
   1.1250e+00   3.5000e+00
```

From figure 19.34, the transformer has to parameters with a = 2 given by

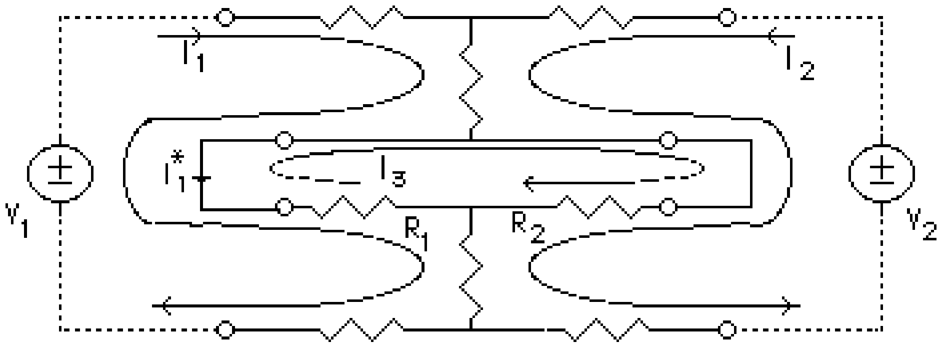
$$t_{trans} = \begin{bmatrix} 1/a & 0 \\ 0 & a \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

```
»ttrans = [0.5  0;0  2];
»tNa = ttrans*t
tNa =
   1.7500e+00   5.0000e+00
   2.2500e+00   7.0000e+00
»
»zNa = ttoz(tNa)
zNa =
   7.7778e-01   4.4444e-01
   4.4444e-01   3.1111e+00
»
»zNb = [14  4;4  14]/9;
»znew = zNa + zNb
znew =
   2.3333e+00   8.8889e-01
   8.8889e-01   4.6667e+00
»
```

SOLUTION 20.8. This problem is identical numerically to problem 20.6. Here however the isolation transformer is on the right hand side which makes no difference to the interconnection. Therefore,

$$Z_{new} = Z_{Na*} + Z_{Nb} = \frac{2}{9}\begin{bmatrix} 14 & 4 \\ 4 & 14 \end{bmatrix} \Omega$$

SOLUTION 20.9. Figure 20.4 is



Using the values in figure 20.3, we obtain the following mesh equation matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4+R_1 & 2 & -R_1 \\ 2 & 4+R_2 & R_2 \\ -R_1 & R_2 & R_1+R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

For I3 to be zero, the third equation implies that $R_1I_1 = R_2I_2$. Therefore,

$$V_1 = \left(4+R_1\right)I_1 + 2\frac{R_1}{R_2}I_1 = \left(4 + R_1 + \frac{2R_1}{R_2}\right)I_1$$

Similarly

$$V_2 = 2I_1 + \left(4+R_2\right)\frac{R_1}{R_2}I_1 + = \left(2 + \frac{(4+R_2)R_1}{R_2}\right)I_1$$

Therefore

$$\frac{V_1}{V_2} = \frac{\left(4 + R_1 + \frac{2R_1}{R_2}\right)}{\left(2 + \frac{(4+R_2)R_1}{R_2}\right)} = \frac{\left(4R_2 + R_1R_2 + 2R_1\right)}{\left(2R_2 + R_1R_2 + 4R_1\right)} = \frac{42}{48} = \frac{7}{8}$$

as was to be shown.

Now suppose, $\frac{V_1}{V_2} = \frac{7}{8}$ or equivalently $V_1 = \frac{7}{8}V_2$. With specific values

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix} = V_2\begin{bmatrix} 7/8 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 2 & -6 \\ 2 & 7 & 3 \\ -6 & 3 & 9 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

In MATLAB

```
»Z = [10 2 -6;2 7 3;-6 3 9];
»b = [7/8   1  0]';
»I = inv(Z)*b
I =
   6.2500e-02
   1.2500e-01
```

2.7756e-17

Thus

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = V_2 \begin{bmatrix} 0.0625 \\ 0.124 \\ 0 \end{bmatrix}$$

SOLUTION 20.10

(a) Write two loop equations assuming I_1 and I_2 are the usual port currents. Here

$$V_1 = Z_1 I_1 + V_{13} + Z_3(I_1 + I_2)$$

and

$$V_2 = Z_2 I_2 + V_{23} + Z_3(I_1 + I_2)$$

which in matrix form are

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix}$$

However,

$$\begin{bmatrix} V_{13} \\ V_{23} \end{bmatrix} = Z_A \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Therefore

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + Z_A \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left(\begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} + Z_A \right) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(b) The procedure of part (a) is repeated to produce the same result.

REMARK: this problem states that the two networks are equivalent two ports. Thus the configurations can be used interchangeably.

SOLUTION 20.11. Because of the isolation transformers, the overall Z-parameters are the sum of the three component Z-parameters. For N_a,

$$Z_a = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega$$

For N_b,

$$Z_b = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix} \Omega$$

For N_c,

$$Z_c = Z_a = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega$$

Therefore

$$Z = Z_a + Z_b + Z_c = \begin{bmatrix} 12 & 3 \\ 3 & 9 \end{bmatrix} \Omega$$

SOLUTION 20.12. (a) Given the Z-parameters of N, then

$$Y_N = Z_N^{-1} = \begin{bmatrix} 7 & 2 \\ 10 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -2 \\ -10 & 7 \end{bmatrix} \text{S}$$

The 1-F capacitor considered as a two port has y-parameters

$$Y_C = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix} \text{S}$$

Therefore, the parallel connection of the two ports has y-parameters

$$Y = Y_N + Y_C = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -10 & 7 \end{bmatrix} = \begin{bmatrix} s+3 & -(s+2) \\ -(s+10) & s+7 \end{bmatrix} \text{S}$$

(b) For this part, the same reasoning applies with s replaced by 1/s.

SOLUTION 20.13. The isolation transformer allows for the valid parallel connection of N_a^* and N_b in the sense that the overall y-parameters are the sum of the individual y-parameters. Further, because the ideal transformer is 1:1 with the standard dot locations, the y-parameters of N_a^* are those of N_a. Further, the y-parameters of N_b are the same as those of N_a as the circuits are simple vertical flips of each other. Therefore

$$Y_a^* = Y_a = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.7 \end{bmatrix} = Y_b$$

Hence, the overall y-parameters are:

$$Y = 2 \times \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.7 \end{bmatrix} = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \text{S}$$

True-False: FALSE because the connection does not conform to figure 20.2a.

SOLUTION 20.14. From the previous example, the y-parameters of N_b are

$$Y_b = \begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 0.7 \end{bmatrix} \text{S}$$

The resistance values of the resistive part of N_a are twice those of N_b. Hence, the y-parameters of the resistive part are half those of N_b, i.e.,

$$\begin{bmatrix} I_1' \\ I_2 \end{bmatrix} = Y_{a,R} \begin{bmatrix} V_1' \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.35 & -0.1 \\ -0.1 & 0.35 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2 \end{bmatrix} \text{S}$$

We obtain the y-parameters of N_a by considering the effect of the transformer on these y-parameters. Observe that

$$I_1 = 2I_1' \text{ and } V_1 = 0.5V_1'$$

Hence

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.2 \\ -0.1 & 0.35 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.2 \\ -0.1 & 0.35 \end{bmatrix} \begin{bmatrix} 2V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.4 & -0.2 \\ -0.2 & 0.35 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

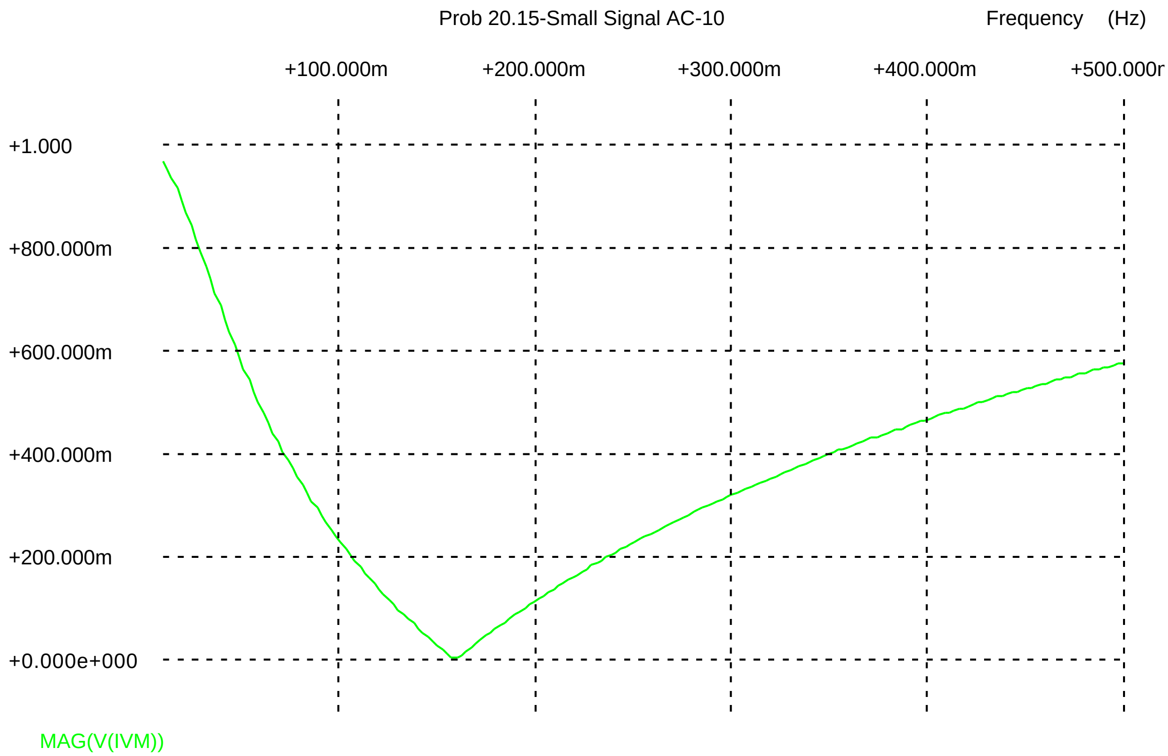
Thus

$$Y_a = \begin{bmatrix} 1.4 & -0.2 \\ -0.2 & 0.35 \end{bmatrix} \text{S}$$

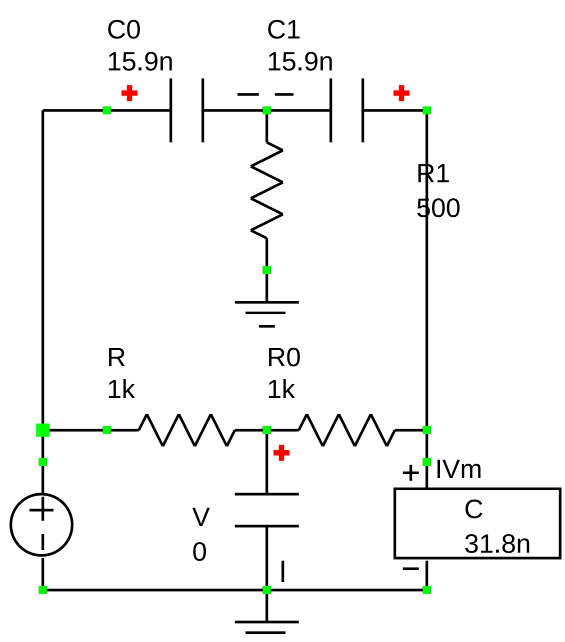
It follows that

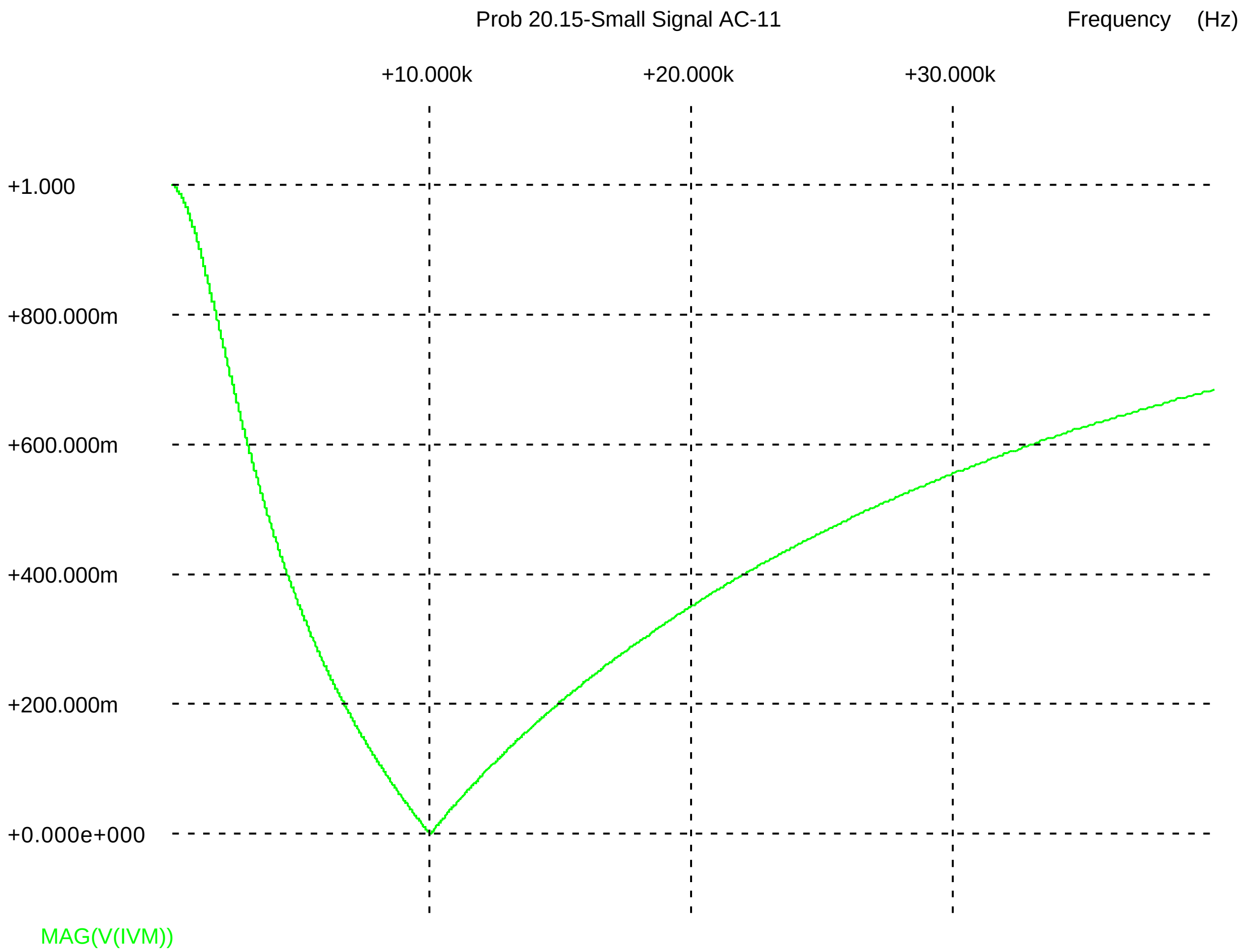
$$Y = Y_a + Y_b = \begin{bmatrix} 2.1 & -0.4 \\ -0.4 & 1.05 \end{bmatrix} \text{S}$$

SOLUTION 20.15. (a)


Prob 20.15-Small Signal AC-10
Frequency (Hz)
+100.000m
+200.000m
+300.000m
+400.000m
+500.000r
+1.000
+800.000m
+600.000m
+400.000m
+200.000m
+0.000e+000
MAG(V(IVM))

(b)


C0
15.9n
C1
15.9n
R1
500
R
1k
R0
1k
IVm
C
31.8n
V
0
I



SOLUTION 20.16. (a) For network N_a, the y-parameters by inspection are:

$$Y_a = \begin{bmatrix} G_1 + j\omega C_1 & 0 \\ g_m & G_0 \end{bmatrix} = \begin{bmatrix} 2 + j10.21 & 0 \\ 95 & 0.07143 \end{bmatrix} \text{mS}$$

For network N_b, the y-parameters by inspection are:

$$Y_b = \left(G_f + j\omega C_2\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \left(0.8333 + j1.021\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{mS}$$

Therefore

$$Y = Y_a + Y_b = \begin{bmatrix} 2.8333 + j11.23 & -0.8333 - j1.021 \\ 94.167 - j1.021 & 0.9048 + j1.021 \end{bmatrix} \text{mS}$$

(b), (c), and (d). Here we use the MATLAB m-file for two port analysis in terms of y-parameters:

```
»zs = 50;  zL = 50;
»twoporty(Y, zL, zs)
ans =
twoport analysis using y-parameters
yin =
  6.8501e-03 + 1.5594e-02i
zin =
  2.3614e+01 - 5.3756e+01i
yout =
  5.3617e-03 + 3.0023e-03i
zout =
  1.4199e+02 - 7.9506e+01i
v1tovs =
  5.5701e-01 - 3.2349e-01i
v2tov1 =
 -4.4915e+00 + 2.6821e-01i
v2tovs =
 -2.4150e+00 + 1.6023e+00i
»
```

SOLUTION 20.17. The t-parameters of the LR circuit follow from problem 19.53 with Z_1 = Ls = s Ω and Z_2 = 0.5 Ω:

$$T_{LR} = \begin{bmatrix} 1+2s & s \\ 2 & 1 \end{bmatrix}$$

Therefore

$$T_{cascade} = T * T_{LR} = \begin{bmatrix} 5+2s & 2+s \\ 5+2s & 2+s \end{bmatrix}$$

SOLUTION 20.18. This problem can be solved in many ways. Here we emphasize the cascade nature of the two ports.

```
»%  The y-parameters of Nb are:
»Yb = [8  2;20  6];
»%  The z-parameters of Na are:
»Za = [0.75  -0.25;-2.5  1];
»%  The t-parameters of Na are:
»Ta = ztot(Za)
Ta =
  -3.0000e-01  -5.0000e-02
  -4.0000e-01  -4.0000e-01
»%  The t-parameters of Nb are:
»Tb = ytot(Yb)
Tb =
```

```
  -3.0000e-01  -5.0000e-02
  -4.0000e-01  -4.0000e-01
»% The t-parameters of the cascaded two port are:
»
»Tab = Ta*Tb
Tab =
   1.1000e-01   3.5000e-02
   2.8000e-01   1.8000e-01
»%  Doing a t-parameter analysis we obtain:
»twoportt(Tab,0.25,0.5)
ans =
analysis of terminated twoport using t-parameters
zin =
   2.5000e-01
zout =
   5.0000e-01
v2tov1 =
     4
v1tovs =
   3.3333e-01
v2tovs =
   1.3333e+00
ans =
   2.5000e-01
»
```

SOLUTION 20.19. Use the h-parameter analysis m-file.

```
»H1 = [1e3  0.001;50  6e-5];  H2 = [1e3  0.99;-5  8e-4];
»zL = 100; zs = 2e3; zm = 10e3;
»%  For part (a), the common collector stage:
»twoporth(H2,zL,0)
ans =
twoport analysis using h-parameters
zin =
   1.4583e+03
zout =
   1.7391e+02
v1tovs =
     1
v2tov1 =
   3.1746e-01
v2tovs =
   3.1746e-01
```

Remark: in the above, v2 is Vout and v1 is vm; zout has no signifigance.

```
»%  We now compute the load on the first stage.
»zin2 = 1.4583e+03;
»zLm = zm*zin2/(zin2 + zm)
zLm =
   1.2727e+03
»%  zLm is the load impedance on stage 1.

»%  For part (a), the common emitter stage:
»twoporth(H1,zLm,zs)
ans =
twoport analysis using h-parameters
zin =
   9.4088e+02
zout =
   2.3077e+04
v1tovs =
   3.1993e-01
v2tov1 =
  -6.2835e+01
v2tovs =
  -2.0103e+01
```

Remark: in the above, v2 is vm and v1 is Vin; zout is the output impedance of stage 1.

Conclusion: the input impedance to stage 1 is 940.88 Ω and the input impedance to stage 2 is 1.4583 kΩ.

(b) From the above output and remark, Vm/Vin = -62.835. Further Vout/Vm = 0.31746.

(c) From above Vm/Vs = -20.103. Therefore Vout/Vs = Vout/Vm * Vm/Vs = -6.38.

(d)

```
»zout1 =  2.3077e+04;
»zs2 = zm*zout1/(zm + zout1)
zs2 =
   6.9768e+03
»twoporth(H2,zL,zs2)
ans =
twoport analysis using h-parameters
zin =
   1.4583e+03
zout =
   7.0395e+02
v1tovs =
```

1.7289e-01
v2tov1 =
3.1746e-01
v2tovs =
5.4885e-02

Conclusion: Zout of amplifier is 704 Ω.

SOLUTION 20.20

(a) Using MATLAB

```
»na = 1.1514;
»nb  =  3.4012;
»Zlprime = nb^2*75 + j*1042.94
ZLprime =
  8.6761e+02 + 1.0429e+03i

»Zsprime = 75/na^2 + j*30
Zsprime =
  5.6573e+01 + 3.0000e+01i
```

(b) Since the h-parameters of the transistors are given, we can again use MATLAB and the m-file twoporth defined earlier. Hence:

```
»h = [ 60-j*50   0.01; -j*2   0.0005+j*0.0004];
»[Zin, Zout] = twoporth(h,ZLprime, Zsprime)
Zin =
  5.6569e+01 - 3.0000e+01i
Zout =
  8.6763e+02 - 1.0429e+03i
```

(c) Observe that Zin = 5.6569e+01 - 3.0000e+01i and Zsprime = 5.6573e+01 + 3.0000e+01i, which are clearly conjugates of each other. Further, Zout = 8.6763e+02 - 1.0429e+03i and ZLprime = 8.6761e+02 + 1.0429e+03i, which are also conjugates of each other. Hence maximum power is transferred into and out of the transistor.

(d) For this part, we change all cascaded two ports to t-parameters. Specifically,

```
t0 = [1   75;0  1];
t1 = [na  0; 0  1/na];
t2 = [ 1 j*30; 0 1];
t3 = htot(h);
t4 = [ 1  j*1042.9; 0  1];
t5 = [nb  0; 0  1/nb];
t6 = [1  0;1/75  1];
t = t0*t1*t2*t3*t4*t5*t6
```

```
t =
  2.2176e-01 - 2.5959e-01i  8.3160e+00 - 9.7351e+00i
  1.4785e-03 - 1.7307e-03i  6.6577e-02 - 7.4415e-02i
»gain = 1/t(1,1)
gain =  1.9024e+00 + 2.2270e+00i

»gainmag = abs(gain)
gainmag =   2.9289e+00

»gainangle = angle(gain)*180/pi
gainangle =   4.9494e+01
```

In this case, Vout/Vs = gain = 1.9024e+00 + 2.2270e+00i = 2.9289 49.494^o

SOLUTION 20.21.

(a)

```
»Y2N = [1  0;20.1  0]*1e-3;
»Y10k = [1 -1;-1  1]*1e-4;
»Yshade = Y2N + Y10k
Yshade =
   1.1000e-03  -1.0000e-04
   2.0000e-02   1.0000e-04
»
```

(b) This is a series connection of two 2-ports. Hence we first convert the answer of part (a) to z-parameters.

```
»Y2N = [1  0;20.1  0]*1e-3;
»Y10k = [1 -1;-1  1]*1e-4;
»Yshade = Y2N + Y10k
Yshade =
   1.1000e-03  -1.0000e-04
   2.0000e-02   1.0000e-04
»Zshade = inv(Yshade)
Zshade =
   4.7393e+01   4.7393e+01
  -9.4787e+03   5.2133e+02
»Z1k = [1  1;1  1]*1e3;
»Zdashed = Zshade + Z1k
Zdashed =
   1.0474e+03   1.0474e+03
  -8.4787e+03   1.5213e+03
»
```

(c)

```
»twoportz(Zdashed,1e12,1e3)
```

```
ans =
twoport analysis using z-parameters
zin =
   1.0474e+03
zout =
   5.8588e+03
v1tovs =
   5.1157e-01
v2tov1 =
  -8.0950e+00
v2tovs =
  -4.1412e+00
```

Conclusion: Vout/Vs = -4.1412.

SOLUTION 20.22.

```
»Z = [3  1;5   2]*1e3;
»Y = inv(Z)
Y =
   2.0000e-03  -1.0000e-03
  -5.0000e-03   3.0000e-03
»%  Consider the parallel connection of Y with the 1 kΩ resistor
»Y1 = Y + [1 -1;-1 1]*1e-3
Y1 =
   3.0000e-03  -2.0000e-03
  -6.0000e-03   4.0000e-03
»
»%  Now consider Y/Z in parallel with 1 kΩ connected between B and C
»Y2 = Y + [0  0;0  1]*1e-3
Y2 =
   2.0000e-03  -1.0000e-03
  -5.0000e-03   4.0000e-03
»
»%  This  combination is in series with another 1 kΩ resistor
»%  Hence  we need to compute Z2 first
»Z2 = inv(Y2)
Z2 =
   1.3333e+03   3.3333e+02
   1.6667e+03   6.6667e+02
»%  Now we compute the series combo of Z2 and the 1 kΩ resistor
»Z3 = Z2 + [1  1;1  1]*1e3
Z3 =
   2.3333e+03   1.3333e+03
   2.6667e+03   1.6667e+03
»
```

```
»
»%  We now convert Y1 and Z3 to t-parameters and then multiply
»%  together to obtain the overall t-parameters
»
»T1 = ytot(Y1)
T1 =
   6.6667e-01   1.6667e+02
            0   5.0000e-01
»T3 = ztot(Z3)
T3 =
   8.7500e-01   1.2500e+02
   3.7500e-04   6.2500e-01
»T = T1*T3
T =
   6.4583e-01   1.8750e+02
   1.8750e-04   3.1250e-01
»Y = ttoy(T)
Y =
   1.6667e-03  -8.8889e-04
  -5.3333e-03   3.4444e-03
»
```

SOLUTION 20.23. Refer to figure 20.13 for all problems.

(a) By inspection,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 & -4 & -2 \\ -4 & 4+5s & -5s \\ -2 & -5s & 2+5s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

The 3x3 coefficient matrix is the desired $Y_{ind}(s)$.

(b) Writing the nodal equation by inspection yields

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} 0.5 & 0 & -0.25 & -0.25 \\ 0 & 1 & -0.5 & -0.5 \\ -0.25 & -0.5 & 1.25 & -0.5 \\ \hline -0.25 & -0.5 & -0.5 & 1.25 \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \hline V_4 \end{bmatrix}$$

Using MATLAB,

```
»W11=[0.5, 0 -0.25;0 1 -0.5;-0.25 -0.5 1.25]
W11 =
   5.0000e-01            0  -2.5000e-01
            0   1.0000e+00  -5.0000e-01
  -2.5000e-01  -5.0000e-01   1.2500e+00
»W12 = [-0.25 -0.5 -0.5]'
```

```
W12 =
  -2.5000e-01
  -5.0000e-01
  -5.0000e-01
»W21 = [-0.25 -0.5 -0.5]
W21 =
  -2.5000e-01  -5.0000e-01  -5.0000e-01
»W22 = 1.25
W22 =
   1.2500e+00
»Yind = W11 - W12*inv(W22)*W21
Yind =
   4.5000e-01  -1.0000e-01  -3.5000e-01
  -1.0000e-01   8.0000e-01  -7.0000e-01
  -3.5000e-01  -7.0000e-01   1.0500e+00
»
```

(c) Again, writing the nodal equation by inspection yields

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} 5 & 0 & 0 & -5 \\ 0 & 6 & -7 & 1 \\ 0 & -4 & 8 & -4 \\ \hline -5 & -2 & -1 & 8 \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \hline V_4 \end{bmatrix}$$

Again, using MATLAB,

```
»W11 = [5 0 0;0 6 -7;0 -4 8]
W11 =
     5     0     0
     0     6    -7
     0    -4     8
»W12 = [-5 1 -4]'
W12 =
    -5
     1
    -4
»W21 = [-5 -2 -1]
W21 =
    -5    -2    -1
»W22 = 8;
»Yind = W11 - W12*inv(W22)*W21
Yind =
   1.8750e+00  -1.2500e+00  -6.2500e-01
   6.2500e-01   6.2500e+00  -6.8750e+00
  -2.5000e+00  -5.0000e+00   7.5000e+00
»
```

(d) This part is similar to part (a) as it does not require the method of matrix partitioning. By inspection,

$$\begin{bmatrix} I_G \\ I_D \\ I_S \end{bmatrix} = \begin{bmatrix} (C_{GD}+C_{GS})s & -C_{GD}s & -C_{GS}s \\ -C_{GD}s+g_m & C_{GD}s & -g_m \\ -C_{GS}s-g_m & 0 & C_{GS}s+g_m \end{bmatrix} \begin{bmatrix} V_G \\ V_D \\ V_S \end{bmatrix}$$

The 3x3 coefficient matrix is the desired $Y_{ind}(s)$.

SOLUTION 20.24. (a) With regard to the given information, the associated indefinite admittance matrix is the coefficient matrix in the following nodal equation given reference to figure 20.13:

$$\begin{bmatrix} I_G \\ I_D \\ I_S \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ g_m & 0 & -g_m \\ -g_m & 0 & g_m \end{bmatrix} \begin{bmatrix} V_G \\ V_D \\ V_S \end{bmatrix}$$

We use property 5 to compute the remaining answers:

(a)-(a): Y_{GD} is as given.

(a)-(b): $Y_{SD} = \begin{bmatrix} g_m & 0 \\ -g_m & 0 \end{bmatrix}$ S

(a)-(c): $Y_{GS} = \begin{bmatrix} 0 & 0 \\ -g_m & g_m \end{bmatrix}$ S

(a)-(d): $Y_{DG} = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix}$ S

(a)-(e): $Y_{DS} = \begin{bmatrix} 0 & -g_m \\ 0 & g_m \end{bmatrix}$ S

(a)-(e): $Y_{SG} = \begin{bmatrix} g_m & -g_m \\ 0 & 0 \end{bmatrix}$ S

(b) Transmission from port-1 to port-2 occurs when the 2-1 entry of the 2-port y-matrix is nonzero. Hence, the following all have the desired transmission: Y_{GD}, Y_{SD}, Y_{GS}.

SOLUTION 20.25. Using the zero-sum properties of the rows and columns, we have by inspection:

$$Y_{ind} = \begin{bmatrix} 40 & 2 & -42 \\ ? & ? & -50 \\ ? & -22 & 92 \end{bmatrix} = \begin{bmatrix} 40 & 2 & -42 \\ 30 & 20 & -50 \\ -70 & -22 & 92 \end{bmatrix} \text{S}$$

SOLUTION 20.26. (a) $Y_{ind} = \begin{bmatrix} 8 & ? & ? \\ -2 & 9 & -7 \\ -6 & ? & ? \end{bmatrix} = \begin{bmatrix} 8 & -2 & -6 \\ -2 & 9 & -7 \\ -6 & -7 & 13 \end{bmatrix}$ S, where the second equality arises because a purely

resistive network has a symmetric indefinite admittance matrix.
(b) Construct a common ground 2-port with input terminal C, common terminal A, and output terminal B:

$$Y_{CB} = \begin{bmatrix} 13 & -7 \\ -7 & 9 \end{bmatrix} \text{ S}$$

Therefore, since the output terminal B is shorted, $Y_L = \infty$, i.e.,

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = 13 \text{ S}$$

Hence

$$Z_{in} = \frac{1}{13}\ \Omega$$

(c) Construct a common ground 2-port with input terminal A, common terminal B, and output terminal C:

$$Y_{AC} = \begin{bmatrix} 8 & -6 \\ -6 & 13 \end{bmatrix} \text{ S}$$

Hence, the required voltage gain is

$$G_V = \frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + Y_L} = \frac{-y_{21}}{y_{22}} = \frac{6}{13}$$

Therefore, $V_2 = \frac{6}{13}$ V.

SOLUTION 20.27.
(a) Using the zero-sum property of the rows and columns of an indefinite admittance matrix we can write down by inspection (in mS)

$$\begin{bmatrix} I_B \\ I_E \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -100 & 100.1 & -0.1 \\ 99 & 99.1 & 0.1 \end{bmatrix} \begin{bmatrix} V_B \\ V_E \\ V_C \end{bmatrix}$$

(b) The y-parameters (also in mS) of the common emitter configuration are easily computed as

$$\begin{bmatrix} I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 99 & 0.1 \end{bmatrix} \begin{bmatrix} V_{BE} \\ V_{CE} \end{bmatrix}$$

Hence in MATLAB

```
»Y = [1  0;99 0.1];
»Z = inv(Y)
Z =
   1.0000e+00            0
  -9.9000e+02   1.0000e+01
```

where Z is in kΩ. It follows that $V_{CE} = -990 I_B + 10 I_C$ where I_B and I_C are in mA and V_{CE} is in volts.

SOLUTION 20.28. Expanding the given y-parameter matrix into an indefinite admittance matrix yields

$$\begin{bmatrix} I_G \\ I_D \\ I_S \end{bmatrix} = Y_{ind} \begin{bmatrix} V_G \\ V_D \\ V_S \end{bmatrix} = \begin{bmatrix} 0.2 + j2.5 & -0.01 - j0.65 & -0.19 - j1.85 \\ 3.1 - j0.65 & 0.05 + j0.8 & -3.15 - j0.15 \\ -3.3 - j1.85 & -0.04 - j0.15 & 3.34 + j2 \end{bmatrix} \begin{bmatrix} V_G \\ V_D \\ V_S \end{bmatrix}$$

By inspection, with G as the common terminal, S as the input terminal, and D as the output terminal, we obtain,

$$Y_{new} = \begin{bmatrix} 3.34 + j2 & -0.04 - j0.15 \\ -3.15 - j0.15 & 0.05 + j0.8 \end{bmatrix} \text{mS}$$

SOLUTION 20.29. Here we use the zero-sum properties of the columns and rows to complete the indefinite admittance matrix:

$$Y_{ind} = \begin{bmatrix} -s & ? & ? \\ s-1 & 2 & ? \\ 1 & 2s-1 & -2s \end{bmatrix} = \begin{bmatrix} -s & -2s-1 & 3s+1 \\ s-1 & 2 & -s-1 \\ 1 & 2s-1 & -2s \end{bmatrix} \text{S}$$

In figure (b), the top 2-port has $y_{top} = \begin{bmatrix} -s & -2s-1 \\ s-1 & 2 \end{bmatrix}$ S and the bottom 2-port has y-parameters $y_{bot} = \begin{bmatrix} -2s & 2s-1 \\ -s-1 & 2 \end{bmatrix}$ S. Since these 2-ports are connected in parallel, the overall 2-port y-parameters are

$$y = y_{top} + y_{bot} = \begin{bmatrix} -s & -2s-1 \\ s-1 & 2 \end{bmatrix} + \begin{bmatrix} -2s & 2s-1 \\ -s-1 & 2 \end{bmatrix} = \begin{bmatrix} -3s & -2 \\ -2 & 4 \end{bmatrix} \text{ S}$$

SOLUTION 20.30. From problem 28,

$$\begin{bmatrix} I_G \\ I_D \\ I_S \end{bmatrix} = Y_{ind} \begin{bmatrix} V_G \\ V_D \\ V_S \end{bmatrix} = \begin{bmatrix} 0.2 + j2.5 & -0.01 - j0.65 & -0.19 - j1.85 \\ 3.1 - j0.65 & 0.05 + j0.8 & -3.15 - j0.15 \\ -3.3 - j1.85 & -0.04 - j0.15 & 3.34 + j2 \end{bmatrix} \begin{bmatrix} V_G \\ V_D \\ V_S \end{bmatrix}$$

Hence

$$y_{GS} = \begin{bmatrix} 0.2 + j2.5 & -0.19 - j1.85 \\ -3.3 - j1.85 & 3.34 + j2 \end{bmatrix} \text{ S}$$

SOLUTION 20.31. (a) Here, by inspection we can compute the indefinite admittance matrix as the coefficient matrix of the following nodal equations:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = Y_{ind} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 + 2 & -Y_2 & -Y_1 - 2 \\ Y_2 - 2 & -Y_2 & 2 \\ -Y_1 - 2Y_2 & 2Y_2 & Y_1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

(b)

$$y_{AB} = \begin{bmatrix} Y_1 + Y_2 + 2 & -Y_2 \\ Y_2 - 2 & -Y_2 \end{bmatrix}$$

(c)

$$y_{AC} = \begin{bmatrix} Y_1 + Y_2 + 2 & -Y_1 - 2 \\ -Y_1 - 2Y_2 & Y_1 \end{bmatrix}$$

SOLUTION 20.32. (a) Here, by inspection we can compute the indefinite admittance matrix as the coefficient matrix of the following nodal equations:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = Y_{ind} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 & -Y_1 \\ -Y_2 + g_m & Y_2 + Y_3 & -Y_3 - g_m \\ -Y_1 - g_m & -Y_3 & Y_1 + Y_3 + g_m \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

(b)

$$y_{AB} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 + g_m & Y_2 + Y_3 \end{bmatrix}$$

(c)

$$y_{AC} = \begin{bmatrix} Y_1 + Y_2 & -Y_1 \\ -Y_1 - g_m & Y_1 + Y_3 + g_m \end{bmatrix}$$

SOLUTION 20.33. (a) Here the nodal equation matrix is:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ \hline 0 \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ \hline V_D \end{bmatrix} = \left[\begin{array}{ccc|c} 8 & 0 & 0 & -8 \\ 0 & 2+2s & -2s-10 & 8 \\ 0 & -2s & 16+2s & -16 \\ \hline -8 & -2 & -6 & 16 \end{array}\right] \begin{bmatrix} V_A \\ V_B \\ V_C \\ \hline V_D \end{bmatrix}$$

where V_D is the internal node voltage. Using the method of matrix partitioning,

$$Y_{ind} = W_{11} - W_{12} W_{22}^{-1} W_{21} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2+2s & -2s-10 \\ 0 & -2s & 16+2s \end{bmatrix} - \frac{1}{16} \begin{bmatrix} -8 \\ 8 \\ -16 \end{bmatrix} \begin{bmatrix} -8 & -2 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2+2s & -2s-10 \\ 0 & -2s & 16+2s \end{bmatrix} - \begin{bmatrix} -0.5 \\ 0.5 \\ -1 \end{bmatrix} \begin{bmatrix} -8 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 4 & -1 & -3 \\ 4 & 3+2s & -2s-7 \\ -8 & -2s-2 & 10+2s \end{bmatrix} \text{S}$$

(b) When C is grounded,

$$y_{AB} = \begin{bmatrix} 4 & -1 \\ 4 & 3+2s \end{bmatrix} \text{S} \quad \text{and} \quad z_{AB} = \frac{1}{8s+16} \begin{bmatrix} 3+2s & 1 \\ -4 & 4 \end{bmatrix} \Omega$$

(c) When B is grounded

$$y_{AC} = \begin{bmatrix} 4 & -3 \\ -8 & 10+2s \end{bmatrix} \text{S}$$

SOLUTION 20.34. (a) Writing the usual node equations we have,

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ \hline 0 \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ \hline V_D \end{bmatrix} = \left[\begin{array}{ccc|c} s+0.5 & -s & 0 & -0.5 \\ -s & s+0.5 & 0 & -0.5 \\ 0 & 0 & 0.5s & -0.5s \\ \hline -0.5 & -0.5 & -0.5s & 0.5s+1 \end{array}\right] \begin{bmatrix} V_A \\ V_B \\ V_C \\ \hline V_D \end{bmatrix}$$

Using the method of matrix partitioning,

$$Y_{ind} = W_{11} - W_{12}W_{22}^{-1}W_{21} = \begin{bmatrix} s+0.5 & -s & 0 \\ -s & s+0.5 & 0 \\ 0 & 0 & 0.5s \end{bmatrix} - \frac{0.5}{s+2}\begin{bmatrix} 1 \\ 1 \\ s \end{bmatrix}\begin{bmatrix} 1 & 1 & s \end{bmatrix}$$

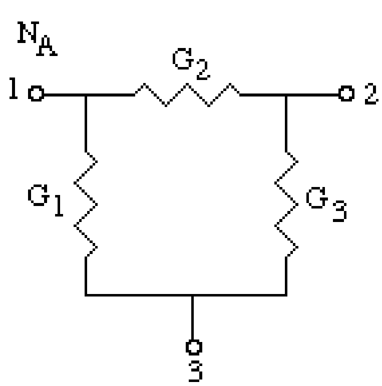
$$= \begin{bmatrix} s+0.5 & -s & 0 \\ -s & s+0.5 & 0 \\ 0 & 0 & 0.5s \end{bmatrix} - \frac{0.5}{s+2}\begin{bmatrix} 1 & 1 & s \\ 1 & 1 & s \\ s & s & s^2 \end{bmatrix}$$

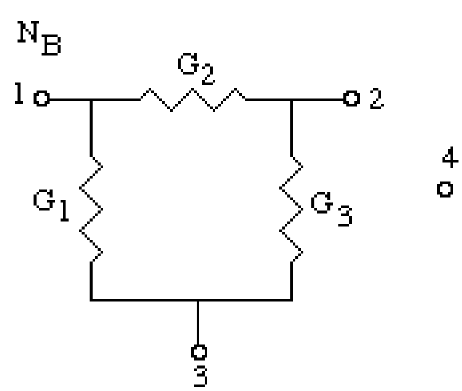
$$= \frac{1}{s+2}\begin{bmatrix} s^2+2.5s+0.5 & -s^2-2s-0.5 & -0.5s \\ -s^2-2s-0.5 & s^2+2.5s+0.5 & -0.5s \\ -0.5s & -0.5s & s \end{bmatrix} \text{S}$$

(b) Here

$$y_{AB} = \frac{1}{s+2}\begin{bmatrix} s^2+2.5s+0.5 & -s^2-2s-0.5 \\ -s^2-2s-0.5 & s^2+2.5s+0.5 \end{bmatrix} \text{S}$$

SOLUTION 20.35. (a) Rule 1: Consider the two networks N_A (3 external nodes) and N_B (4 external nodes) given below:



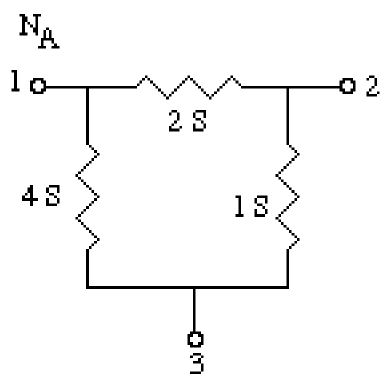


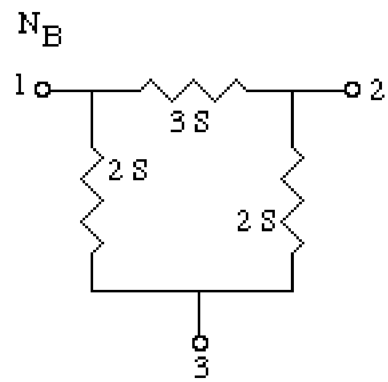
The two indefinite admittance matrices are

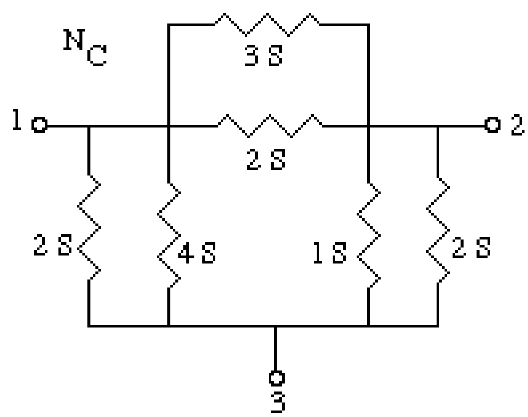
$$Y_{indN_A} = \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & G_2+G_3 & -G_3 \\ -G_1 & -G_3 & G_1+G_3 \end{bmatrix} \text{ and } Y_{indN_B} = \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 & 0 \\ -G_2 & G_2+G_3 & -G_3 & 0 \\ -G_1 & -G_3 & G_1+G_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Observe that Y_{indN_B} can be obtained from Y_{indN_A} by adding a column of zeros and a row of zeros to form a 4x4 matrix.

(b) Rule 2: Consider two networks N_A and N_B and a third network N_C which combines elements of N_A and N_B as given below:







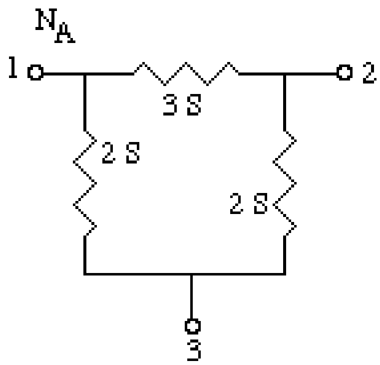
The corresponding indefinite admittance matrices are:

$$Y_{indN_A} = \begin{bmatrix} 6 & -2 & -4 \\ -2 & 3 & -1 \\ -4 & -1 & 5 \end{bmatrix}, \; Y_{indN_B} = \begin{bmatrix} 5 & -3 & -2 \\ -3 & 5 & -2 \\ -2 & -2 & 4 \end{bmatrix}, \text{ and } Y_{indN_C} = \begin{bmatrix} 11 & -5 & -6 \\ -5 & 8 & -3 \\ -6 & -3 & 9 \end{bmatrix}.$$

Clearly,

$$Y_{indN_C} = Y_{indN_A} + Y_{indN_B}$$

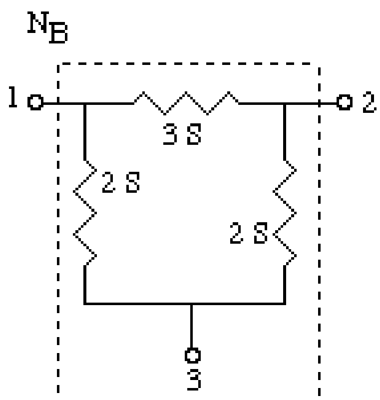
(c) Rule 3: Consider the 3-terminal network



with indefinite admittance matrix

$$Y_{indN_A} = \begin{bmatrix} 5 & -3 & -2 \\ -3 & 5 & -2 \\ -2 & -2 & 4 \end{bmatrix} \text{S}$$

If we move node 3 inside to form a 2-terminal network and labeled as N_B,



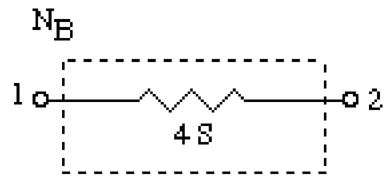
then from nodal analysis we have

$$\begin{bmatrix} I_1 \\ I_2 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cc|c} 5 & -3 & -2 \\ -3 & 5 & -2 \\ \hline -2 & -2 & 4 \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ \hline V_3 \end{bmatrix}$$

Using the method of matrix partitioning,

$$Y_{indN_B} = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} -2 \\ -2 \end{bmatrix} \begin{bmatrix} -2 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \text{S}$$

This computation is the one given by the formula in the problem. To see that this is correct, we observe that the internal simplification of NB leads to the following:



SOLUTION 20.36.

Part (a)

```
»Yinda= [1/2   -1/4   0   -1/4   0; -1/4   1/2   0   0   -1/4; ...
0   0   0   0   0; -1/4   0   0   1/2   -1/4; 0   -1/4   0   -1/4   1/2]

Yinda =

   5.0000e-01  -2.5000e-01           0   -2.5000e-01             0
  -2.5000e-01   5.0000e-01           0             0   -2.5000e-01
           0             0           0             0             0
  -2.5000e-01            0           0    5.0000e-01   -2.5000e-01
           0   -2.5000e-01           0   -2.5000e-01    5.0000e-01



»Yindb= [0   0   0   0   0; 0   0   0   0   0; 0   0   3/4   -1/2   -1/4; ...
0   0   -1/2   5/8   -1/8; 0   0   -1/4   -1/8   3/8]
Yindb =

           0            0            0            0            0
           0            0            0            0            0
           0            0   7.5000e-01  -5.0000e-01  -2.5000e-01
           0            0  -5.0000e-01   6.2500e-01  -1.2500e-01
           0            0  -2.5000e-01  -1.2500e-01   3.7500e-01
```

Part (b)

```
»Yind = Yinda + Yindb
Yind =

   5.0000e-01   -2.5000e-01            0   -2.5000e-01             0
  -2.5000e-01    5.0000e-01            0             0   -2.5000e-01
           0             0   7.5000e-01   -5.0000e-01   -2.5000e-01
  -2.5000e-01            0  -5.0000e-01    1.1250e+00   -3.7500e-01
           0   -2.5000e-01  -2.5000e-01   -3.7500e-01    8.7500e-01

% To suppress nodes 4 and 5 we use the partitioned matrix formula as follows:

»W11= [Yind(1:3, 1:3)]
W11 =
```

```
   5.0000e-01  -2.5000e-01            0
  -2.5000e-01   5.0000e-01            0
            0            0   7.5000e-01

»W12=[Yind(1:3, 4:5)]
W12 =
  -2.5000e-01            0
            0  -2.5000e-01
  -5.0000e-01  -2.5000e-01

»W21= [Yind(4:5, 1:3)]
W21 =
  -2.5000e-01            0  -5.0000e-01
            0  -2.5000e-01  -2.5000e-01

»W22= [Yind(4:5, 4:5)]
W22 =
   1.1250e+00  -3.7500e-01
  -3.7500e-01   8.7500e-01

»Yind123 = W11 - W12*inv(W22)*W21
Yind123 =
   4.3519e-01  -2.7778e-01  -1.5741e-01
  -2.7778e-01   4.1667e-01  -1.3889e-01
  -1.5741e-01  -1.3889e-01   2.9630e-01
```

(d) For the required Y-matrix we delete row and column 3 to obtain

```
»Ysc = Yind123(1:2,1:2)
Ysc =
   4.3519e-01  -2.7778e-01
  -2.7778e-01   4.1667e-01

»Zoc = inv(Ysc)
Zoc =
   4.0000e+00   2.6667e+00
   2.6667e+00   4.1778e+00
```

SOLUTION 20.37. Since complex roots must occur in conjugate pairs, we will only check $j\omega_0$.

$$0 = p(j\omega_0) = -ja_3\omega_0^3 - a_2\omega_0^2 + ja_1\omega_0 + a_0 = a_0 - a_2\omega_0^2 + j\left(a_1\omega_0 - a_3\omega_0^3\right)$$

Both real and imaginary parts must be zero, i.e.,

$$a_0 - a_2\omega_0^2 = 0 \text{ and } a_1\omega_0 - a_3\omega_0^3 = \omega_0\left(a_1 - a_3\omega_0^2\right) = 0$$

From the first equation, $a_2\omega_0^2 = a_0$. The second equation above must be true for arbitrary ω_0 which implies that $a_1 = a_3\omega_0^2$. Equivalently $a_1a_2 = a_3a_2\omega_0^2 = a_3a_0$. Conclusion: this condition leads to imaginary complex roots.

(b) Given the above condition, what are the resulting imaginary roots of the polynomial? Since the polynomial is cubic, we can assume $a_3 \neq 0$. In this case,

$$0 = p(s) = s^3 + \frac{a_2}{a_3}s^2 + \frac{a_1}{a_3}s + \frac{a_0}{a_3} = s\left(s^2 + \frac{a_1}{a_3}\right) + \frac{a_2}{a_3}s^2 + \frac{a_1a_2}{a_3^2} = s\left(s^2 + \frac{a_1}{a_3}\right) + \frac{a_2}{a_3}\left(s^2 + \frac{a_1}{a_3}\right)$$

$$= \left(s^2 + \frac{a_1}{a_3}\right)\left(s + \frac{a_2}{a_3}\right)$$

Therefore, the roots are: $s = \pm j\sqrt{\frac{a_1}{a_3}}, -\frac{a_2}{a_3}$.

SOLUTION 20.38.

(a) The four 2-port equations arising from the interconnection are:

$$V_1 = V_{1a} - V_{1b} \quad I_1 = I_{1a} = -I_{1b} \quad V_2 = V_{2a} = V_{2b} \quad I_2 = I_{2a} + I_{2b}$$

Thus

$$V_1 = V_{1a} - V_{1b} = \left(h_{11a}I_{1a} + h_{12a}V_{2a}\right) - \left(h_{11b}I_{1b} + h_{12b}V_{2b}\right) = \left(h_{11a} + h_{11b}\right)I_1 + \left(h_{12a} - h_{12b}\right)V_2$$

and

$$I_2 = I_{2a} + I_{2b} = \left(h_{21a}I_{1a} + h_{22a}V_{2a}\right) + \left(h_{21b}I_{1b} + h_{22b}V_{2b}\right) = \left(h_{21a} - h_{21b}\right)I_1 + \left(h_{22a} + h_{22b}\right)V_2$$

This proves that the series-parallel connection has the required h-parameters.

(b) The four 2-port equations arising from the interconnection are:

$$V_2 = V_{2a} - V_{2b} \quad I_2 = I_{2a} = -I_{2b} \quad V_1 = V_{1a} = V_{1b} \quad I_1 = I_{1a} + I_{1b}$$

Thus,

$$I_1 = I_{1a} + I_{1b} = \left(g_{11a}V_{1a} + g_{12a}I_{2a}\right) + \left(g_{11b}V_{1b} + g_{12b}I_{2b}\right) = \left(g_{11a} + g_{11b}\right)V_1 + \left(g_{12a} - g_{12b}\right)I_2$$

and

$$V_2 = V_{2a} - V_{2b} = \left(g_{21a}V_{1a} + g_{22a}I_{2a}\right) - \left(g_{21b}V_{1b} + g_{22b}I_{2b}\right) = \left(g_{21a} - g_{21b}\right)V_1 + \left(g_{22a} + g_{22b}\right)I_2$$

This proves that the parallel-series connection has the required g-parameters.

SOLUTION 20.39.

(a) Refer here to N_a in figure P20.39b. With reference to figure 19.28b, h_{11} = 11 kΩ, h_{12} = 0, h_{21} = 95.9, and $h_{22} = 1/10^5 = 10^{-5}$ S. Similarly, by inspection with reference to equation 19.33, h_{11} = 90||10 = 9 kΩ, h_{12} = 0.1 (reverse voltage division), h_{21} = –0.1, and $h_{22} = 10^{-5}$ S.

(b) By problem 20.38, part (a),

$$h_{11} = h_{11a} + h_{11b} = 20\text{ k}\Omega \qquad h_{12} = h_{12a} - h_{12b} = -0.1$$

$$h_{21} = h_{21a} - h_{21b} = 96 \qquad h_{22} = h_{22a} + h_{22b} = 2\times10^{-5}\text{ S}$$

(c) Recall from chapter 19 that $Z_{in} = \frac{V_1}{I_1} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + Y_L}$ and $Y_{out} = \frac{I_2}{V_2} = h_{22} - \frac{h_{12}h_{21}}{h_{11} + Z_s}$ in which case Z_{out} is the reciprocal. Using our MATLAB script, we have

```
h = [20e3  -0.1;  96   0.02e-3 ];
zL= 1e8;
zs= 5e3;
»twoporth(h,zL,zs)
ans =
twoport analysis using h-parameters
zin =
   4.9976e+05
zout =
   2.4752e+03
v1tovs =
   9.9009e-01
v2tov1 =
  -9.5998e+00
v2tovs =
  -9.5047e+00
```

REMARK: We have used the following m-file code for "twoporth":

```
»% two-port analysis in terms of h-parameters
»function [zin, zout] =twoport(h, zL, zs)
»['twoport analysis using h-parameters']
»h11= h(1,1); h12=h(1,2); h21=h(2,1); h22=h(2,2);
»zin = h11 - h12*h21/(h22+ 1/zL)
»yout= h22 - h12*h21/(h11+zs);
```

```
»zout= 1/yout
»v1tovs= zin/(zin+zs)
»v2tov1= -h21/(zin*(h22+1/zL))
»v2tovs= v1tovs*v2tov1
```

SOLUTION 20.40.

(a) The y-parameters for N_a are:

$$y_{AB} = \begin{bmatrix} 7 & 4 \\ 4 & 7 \end{bmatrix}^{-1} = \frac{1}{33}\begin{bmatrix} 7 & -4 \\ -4 & 7 \end{bmatrix} \text{S}$$

Hence the indefinite admittance matrix for N_a is:

$$Y_{indN_a} = \frac{1}{33}\begin{bmatrix} 7 & -4 & -3 \\ -4 & 7 & -3 \\ -3 & -3 & 6 \end{bmatrix} \text{S}$$

Let us consider the associate 2-port with port A grounded and B as the new port 1 input. The problem is then solved by computing the input impedance with port 2 open circuited. Hence, the new y-parameters are:

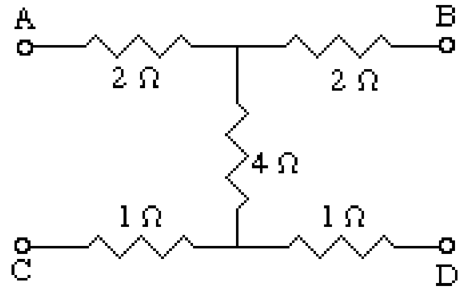
$$y_{BC} = \frac{1}{33}\begin{bmatrix} 7 & -3 \\ -3 & 6 \end{bmatrix} \text{S}$$

Thus $Y_{in} = y_{11} - \dfrac{y_{12}y_{21}}{y_{22}} = \dfrac{1}{33}\left(7 - \dfrac{9}{6}\right) = \dfrac{5.5}{33} = \dfrac{1}{6}$ S. Hence, $Z_{in} = 6\ \Omega$ is the unique reading.

(b) The answer is not unique as demonstrated in part (c).

(c) The following two networks have the given Z-parameters, but the meter reading for N_1 is 4 Ω but for N_2 it is 2 Ω.

N_1



N_2