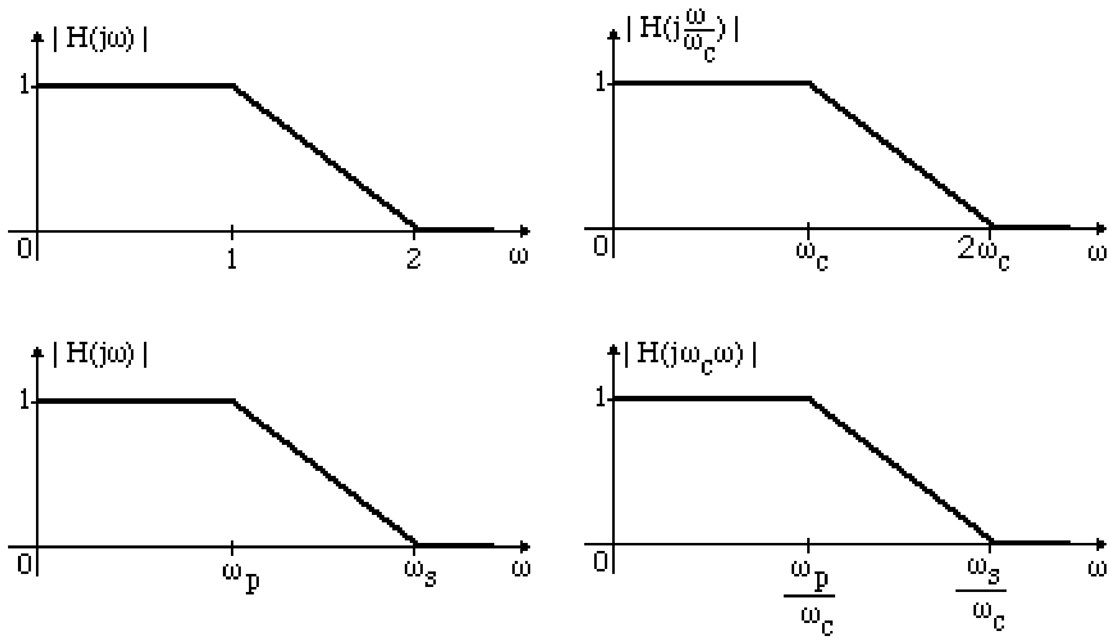
SOLUTIONS CHAPTER 21 PROBLEMS

SOLUTION TO 21.1.

(a) Low pass

(b) High pass

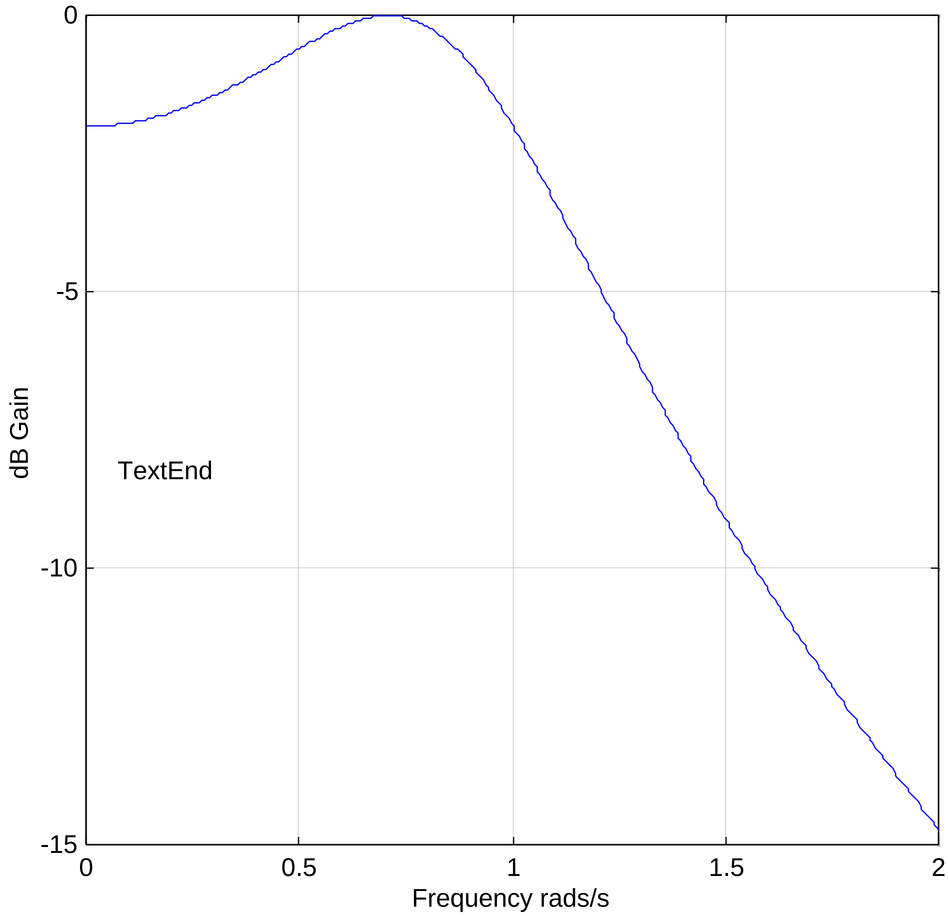
SOLUTION TO 21.2.



SOLUTION TO 21.3.

(a)

```
»n = 0.65378;
»d = [1  0.80381643 0.82306043];
»w = 0:0.005:2;
»h = freqs(n,d,w);
»plot(w, 20*log10(abs(h)))
»grid
»xlabel('Frequency rads/s')
»ylabel('dB Gain')
»
```



(b) »poles = roots(d)

```
poles =
 -4.0191e-01 + 8.1335e-01i
 -4.0191e-01 - 8.1335e-01i
```

(c)

```
»%  Poles of new transfer function
»wp = 2*pi*750'
wp =
   4.7124e+03
»wp = 2*pi*750;
»polesnew = poles*wp
polesnew =
 -1.8939e+03 + 3.8328e+03i
 -1.8939e+03 - 3.8328e+03i
»%  All zeros remain at infinity.
```

Further

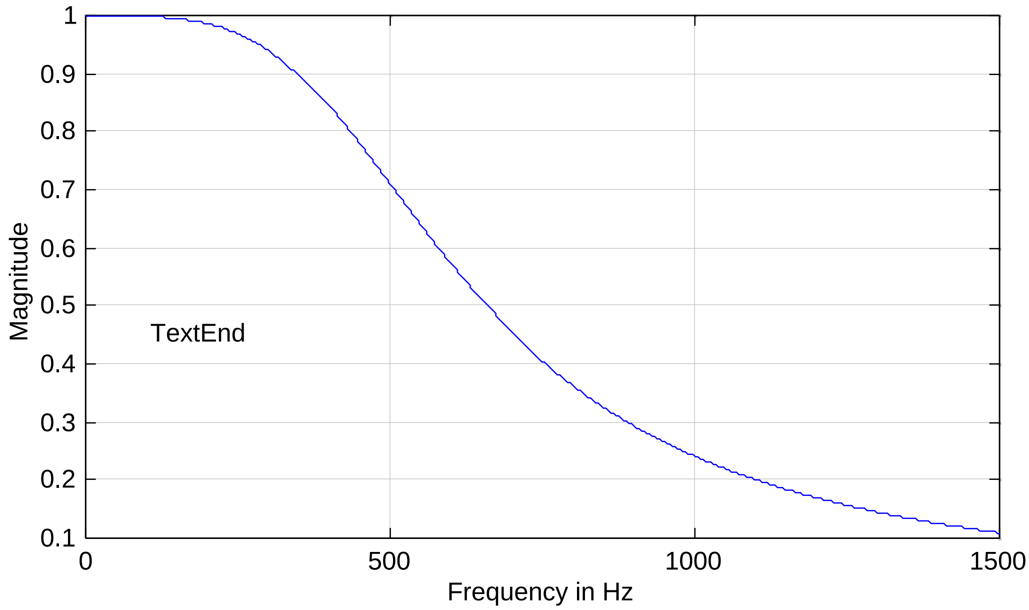
$$H(s) = H_{NLP}(s/\omega_p) = \frac{(\omega_p)^2}{s^2 + 0.80381643\omega_p s + 0.82306043(\omega_p)^2} = \frac{2.2207 \times 10^7}{s^2 + 3.7879 \times 10^3 s + 1.8277 \times 10^7}$$

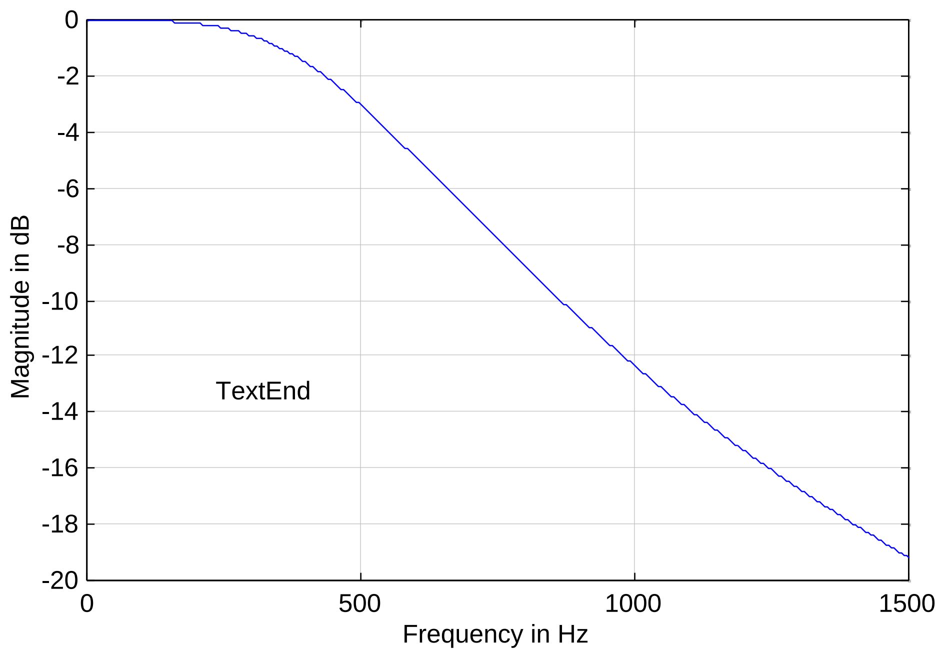
SOLUTION TO 21.4. (a) The 2nd order normalized LP transfer function is $H_{NLP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$. This must be frequency scaled by $K_f = 1000\pi$. Hence,

$$H(s) = H_{NLP}(s/K_f) = \frac{(K_f)^2}{s^2 + K_f\sqrt{2}s + (K_f)^2} = \frac{9.8696 \times 10^6}{s^2 + 4.4429 \times 10^3 s + 9.8696 \times 10^6}$$

(b) Using MATLAB,

```
»n = (1000*pi)^2;
»d = [1  sqrt(2)*pi*1e3  (1000*pi)^2];
»w = 0:1:2*pi*1500;
»h = freqs(n,d,w);
»plot(w/(2*pi),abs(h))
»grid
»xlabel('Frequency in Hz')
»ylabel('Magnitude')
»plot(w/(2*pi),20*log10(abs(h)))
»grid
»xlabel('Frequency in Hz')
»ylabel('Magnitude in dB')
```





(c)

```
»n
n =   9.8696e+06
»d
d =   1.0000e+00   4.4429e+03   9.8696e+06
»w = j*2000*pi;
»mag = abs(n/(w^2 + d(2)*w + d(3)))
mag =   2.4254e-01
```

SOLUTION TO 21.5. (a) ε_{max} is that value of e that places the magnitude response curve through A_{max} at $\omega = \omega_p$. Therefore

$$A_{max} = 10\log_{10}\left|H(j\omega_p)\right|^2 = 10\log_{10}\left(1+\varepsilon_{max}^2\left(\frac{\omega_p}{\omega_p}\right)^{2n}\right) = 10\log_{10}\left(1+\varepsilon_{max}^2\right)$$

Therefore $\varepsilon_{max}^2 = 10^{0.1A_{max}} - 1$ which upon a square root yields the final answer.

(b) Similarly, ε_{min} puts the magnitude response curve through the A_{min} spec. Hence

$$A_{min} = 10\log_{10}\left|H(j\omega_s)\right|^2 = 10\log_{10}\left(1+\varepsilon_{min}^2\left(\frac{\omega_s}{\omega_p}\right)^{2n}\right)$$

Therefore

$$\varepsilon_{min}^2 = \frac{10^{0.1A_{min}} - 1}{\left(\dfrac{\omega_s}{\omega_p}\right)^{2n}}$$

which is equivalent to the required formula.

SOLUTION TO 21.6. The relationship of ε and ω_c is given by the formula: $\omega_c = \dfrac{\omega_p}{(\varepsilon)^{1/n}}$. Further, ε_{max} in putting the magnitude response curve through the A_{max} spec produces $\omega_{c\,min}$, and ε_{min} in putting the magnitude response curve through the A_{min} spec produces $\omega_{c\,max}$. Hence, from the solution to problem 5,

$$\omega_{c\,min} = \frac{\omega_p}{(\varepsilon_{max})^{1/n}} = \frac{\omega_p}{\sqrt[2n]{10^{0.1A_{max}} - 1}} \le \omega \le \omega_{c\,max} = \frac{\omega_p}{(\varepsilon_{min})^{1/n}} = \frac{\omega_s}{\sqrt[2n]{10^{0.1A_{min}} - 1}}$$

SOLUTION TO 21.7. (a) From above material, the second order Butterworth NLP transfer function is

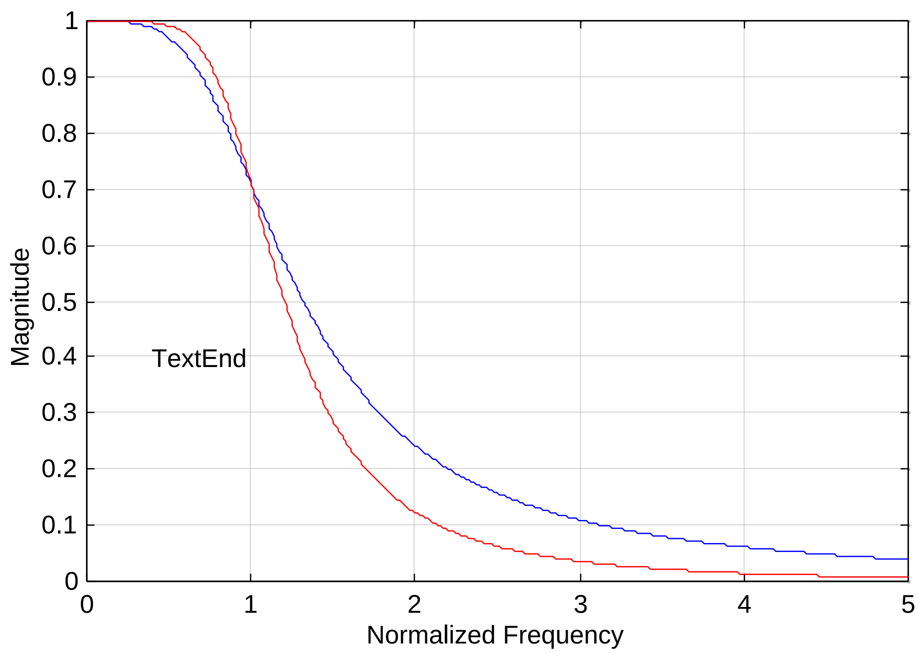
$$H_{NLP2}(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

and from tables, the third order is

$$H_{NLP3}(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

(b)

```
n1 = 1;  d1 = [1  sqrt(2)  1];
n2 = 1;  d2 = [1  2  2  1];
w = 0:.01:5;
h1 = freqs(n1,d1,w);
h2 = freqs(n2,d2,w);
plot(w,abs(h1))
grid
xlabel('Normalized Frequency')
ylabel('Magnitude')
hold
plot(w,abs(h2),'r')
hold off
```



Notice how the 3rd order filter has a sharper transition to zero.

(c) »% The simplest way to obtain the step response is as follows:

```
»
»syms s t
»StepResp1 = ilaplace(1/(s^3 + sqrt(2)*s^2 + s))
StepResp1 =
1-exp(-1/2*2^(1/2)*t)*cos(1/2*2^(1/2)*t)-exp(-1/2*2^(1/2)*t)*sin(1/2*2^(1/2)*t)
»StepResp2 = ilaplace(1/(s^4 + 2*s^3 + 2*s^2 + s))
```

```
StepResp2 =
1-exp(-t)-2/3*exp(-1/2*t)*3^(1/2)*sin(1/2*3^(1/2)*t)
»
```

Thus the step response of the second order Butterworth normalized LP filter is:

$$v(t) = u(t) - e^{-0.70711t}\left[\cos(0.70711t) - \sin(0.70711t)\right]u(t)$$

and that of the third order Butterworth normalized LP filter is:

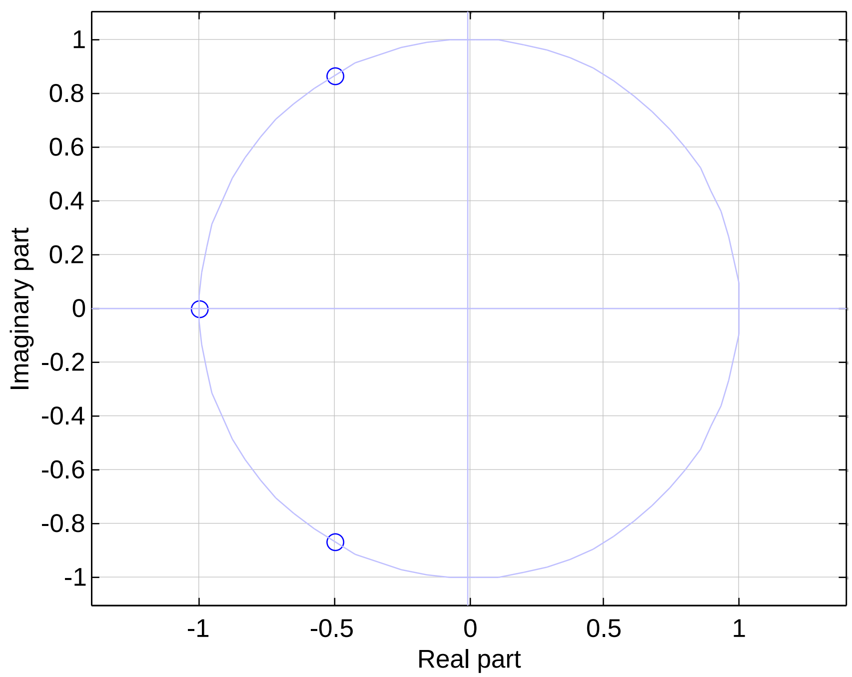
$$v(t) = \left(1 - e^{-t}\right)u(t) - 1.1547e^{-0.5t}\sin(0.86603t)u(t)$$

SOLUTION TO 21.8.

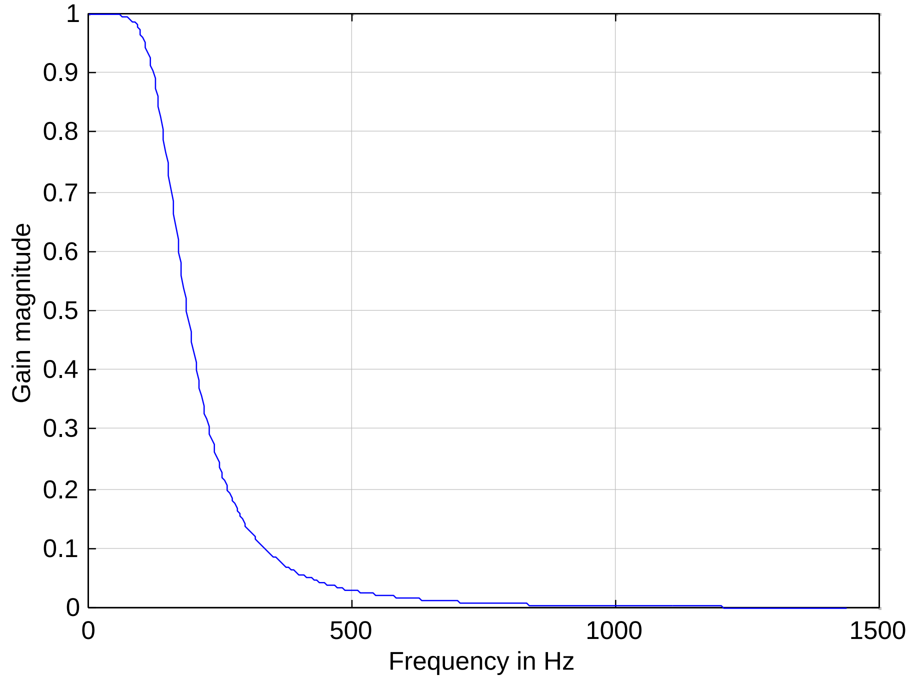
```
fp = 100; fs = 1200;Amax = 0.3; Amin = 35;
n = buttord(fp,fs,Amax,Amin,'s')
emax = sqrt(10^(0.1*Amax) - 1)
emin = sqrt(10^(0.1*Amin) - 1)/(fs/fp)^n
fcmin = fp/((10^(0.1*Amax)-1)^(1/(2*n)))
fcmax = fs/((10^(0.1*Amin)-1)^(1/(2*n)))
wcmin = 2*pi*fcmin
wcmax = 2*pi*fcmax
wc = wcmin;
fc = fcmin;
[z,p,k] = buttap(n)
% Numerators are each 1.  Denominators are the polynomials
d1 = poly(p(1:2))
d2 = poly(p(3))
zplane(p)
grid
pause
znew = z*wc
pnew = p*wc
knew = k*wc^n
f = 0:fc/50:1.2*fs;
h = freqs(knew*poly(znew),poly(pnew),2*pi*f);
plot(f,abs(h))
grid
xlabel('Frequency in Hz')
ylabel('Gain magnitude')
pause
plot(f,20*log10(abs(h)))
xlabel('Frequency in Hz')
ylabel('Gain in dB')
```

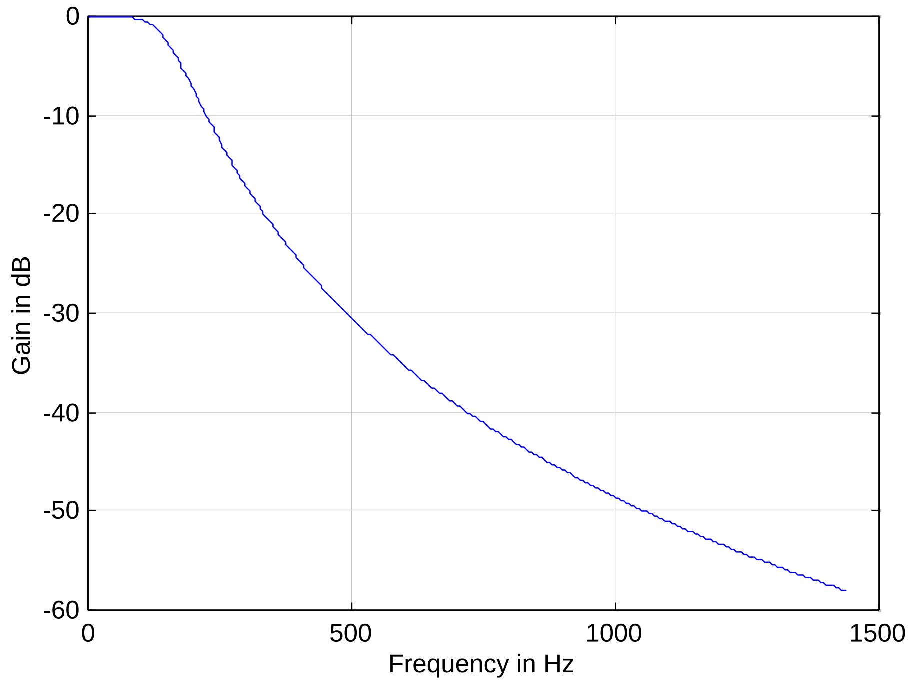
grid

n =   3
emax =   2.6743e-01
emin =   3.2538e-02
fcmin =   1.5521e+02
fcmax =   3.1324e+02
wcmin =   9.7524e+02
wcmax =   1.9681e+03
z =   []
p =
 -5.0000e-01 + 8.6603e-01i
 -5.0000e-01 - 8.6603e-01i
 -1.0000e+00
k =   1
% Numerators are each 1. Denominators are the polynomials
d1 =   1.0000e+00   1.0000e+00   1.0000e+00
d2 =   1   1



znew =   []
pnew =
 -4.8762e+02 + 8.4458e+02i
 -4.8762e+02 - 8.4458e+02i
 -9.7524e+02
knew =   9.2753e+08

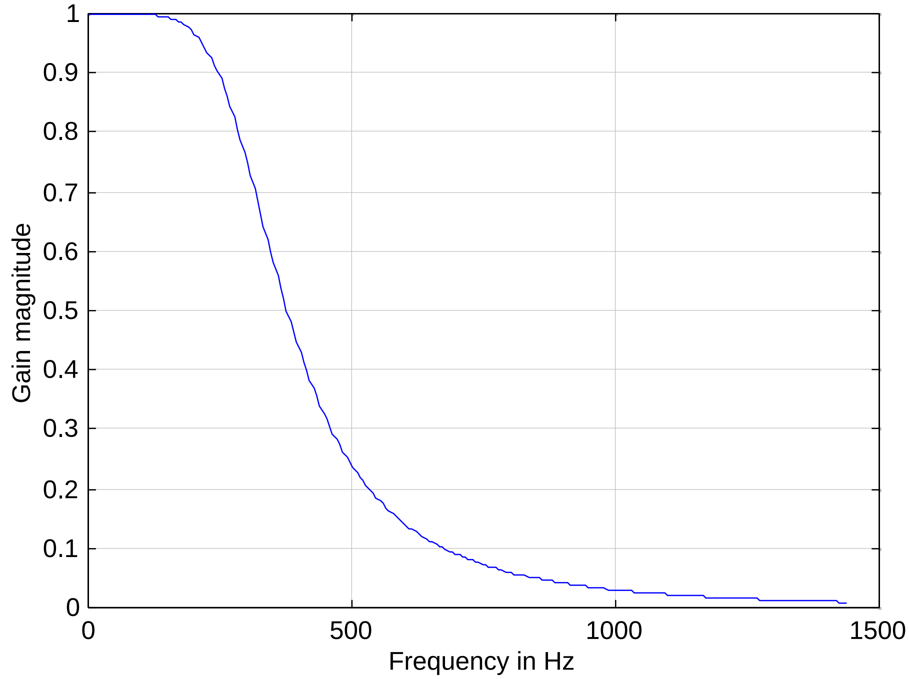


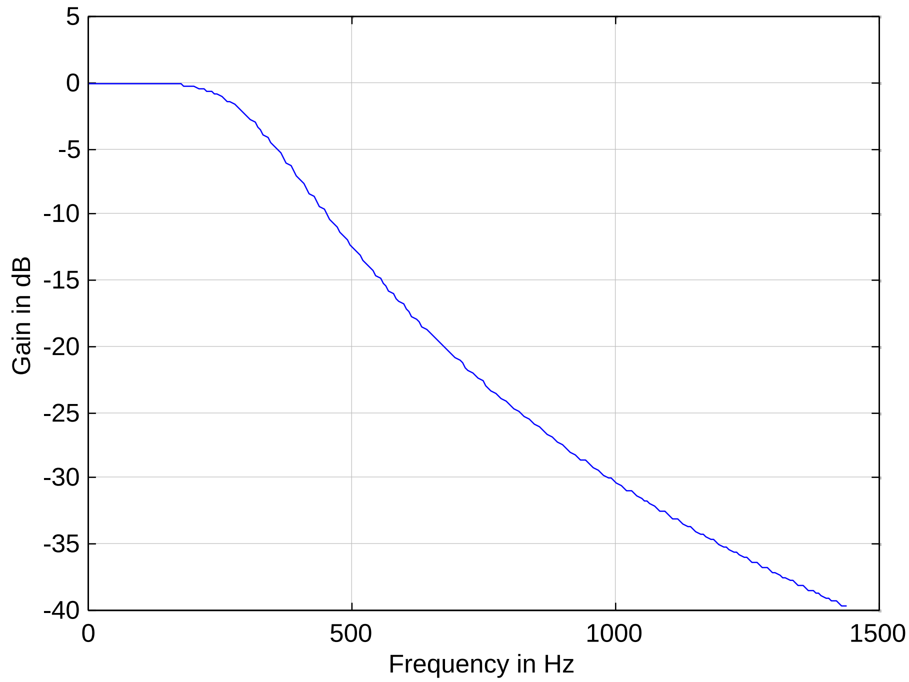


SOLUTION TO 21.9.

```
fp = 100; fs = 1200;Amax = 0.3; Amin = 35;
n = buttord(fp,fs,Amax,Amin,'s');
emax = sqrt(10^(0.1*Amax) - 1);
emin = sqrt(10^(0.1*Amin) - 1)/(fs/fp)^n;
fcmin = fp/((10^(0.1*Amax)-1)^(1/(2*n)));
fcmax = fs/((10^(0.1*Amin)-1)^(1/(2*n)));
wcmin = 2*pi*fcmin;
wcmax = 2*pi*fcmax;
[z,p,k] = buttap(n);
wc = wcmax;
fc = fcmax;
znew = z*wc
pnew = p*wc
knew = k*wc^n
f = 0:fc/50:1.2*fs;
h = freqs(knew*poly(znew),poly(pnew),2*pi*f);
plot(f,abs(h))
grid
xlabel('Frequency in Hz')
ylabel('Gain magnitude')
pause
plot(f,20*log10(abs(h)))
xlabel('Frequency in Hz')
ylabel('Gain in dB')
grid
```

```
znew =     []
pnew =
 -9.8406e+02 + 1.7044e+03i
 -9.8406e+02 - 1.7044e+03i
 -1.9681e+03
knew =   7.6235e+09
```



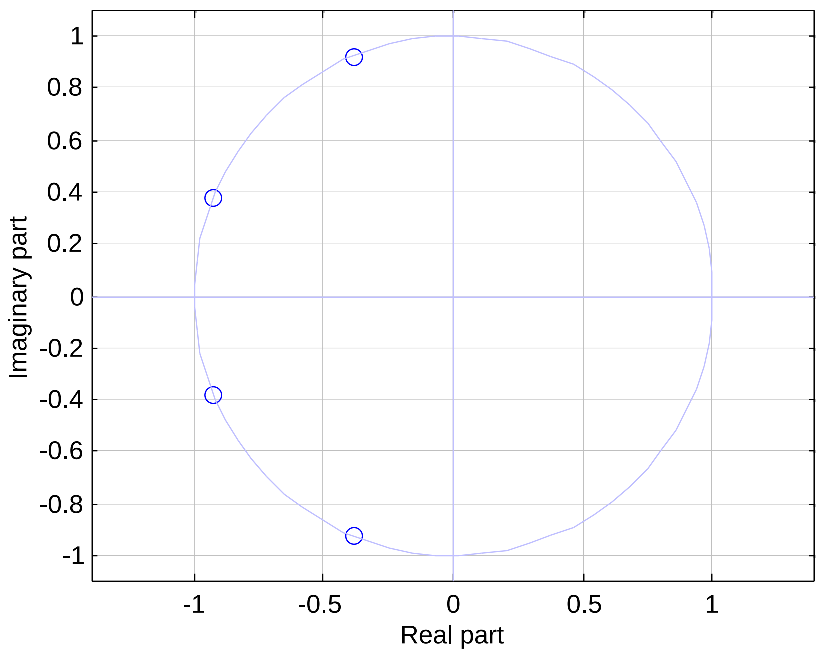


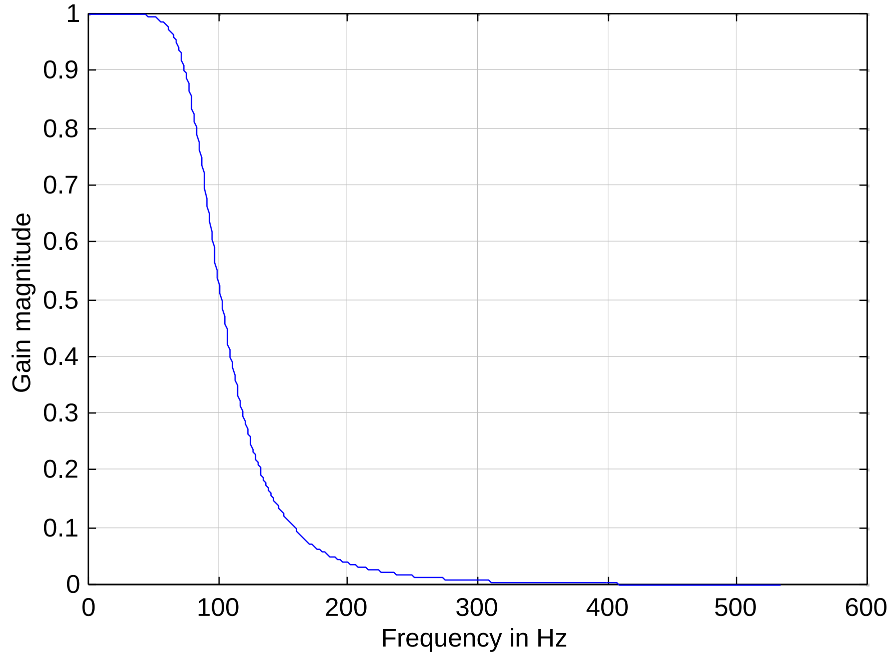
SOLUTION TO 21.10.

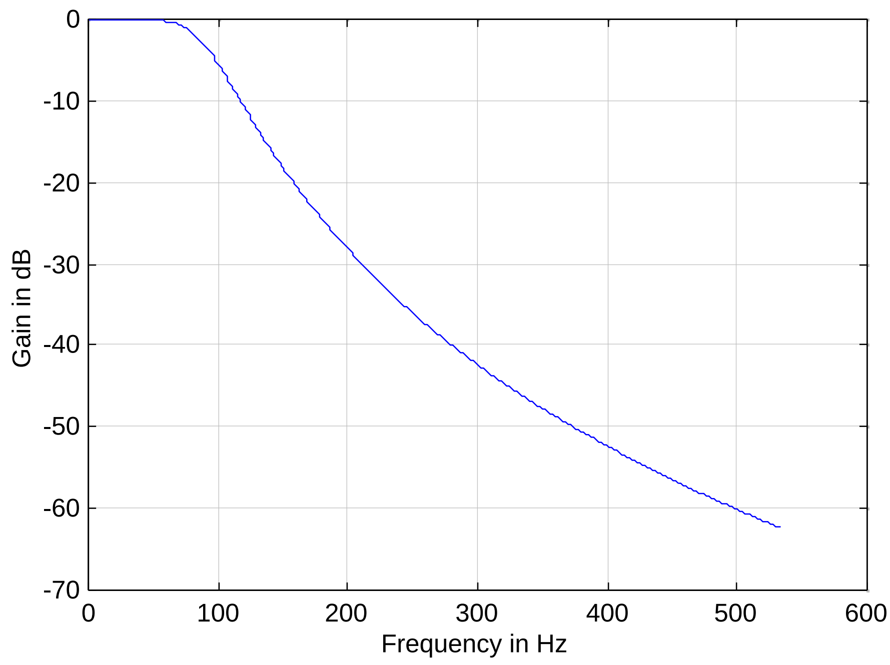
```
fp = 75; fs = 450;Amax = 1; Amin = 45;
n = buttord(fp,fs,Amax,Amin,'s')
emax = sqrt(10^(0.1*Amax) - 1)
emin = sqrt(10^(0.1*Amin) - 1)/(fs/fp)^n
fcmin = fp/((10^(0.1*Amax)-1)^(1/(2*n)))
fcmax = fs/((10^(0.1*Amin)-1)^(1/(2*n)))
wcmin = 2*pi*fcmin
wcmax = 2*pi*fcmax
[z,p,k] = buttap(n)
d1 = poly(p(1:2))
d2 = poly(p(3:4))
zplane(p)
grid
pause
wc = wcmin;
fc = fcmin;
znew = z*wc
pnew = p*wc
knew = k*wc^n
W = 0:0.01:fs/fp;
h = freqs(k*poly(z),poly(p),W);
plot(W*wc/(2*pi),abs(h))
grid
xlabel('Frequency in Hz')
ylabel('Gain magnitude')
pause
plot(W*wc/(2*pi),20*log10(abs(h)))
xlabel('Frequency in Hz')
ylabel('Gain in dB')
grid
```

```
n =     4
emax =   5.0885e-01
emin =   1.3721e-01
fcmin =   8.8800e+01
fcmax =   1.2323e+02
wcmin =   5.5795e+02
wcmax =   7.7427e+02
z =     []
p =
 -3.8268e-01 + 9.2388e-01i
 -3.8268e-01 - 9.2388e-01i
 -9.2388e-01 + 3.8268e-01i
 -9.2388e-01 - 3.8268e-01i
k =     1
```

d1 =   1.0000e+00   7.6537e-01   1.0000e+00
d2 =   1.0000e+00   1.8478e+00   1.0000e+00
znew =   []
pnew =
 -2.1352e+02 + 5.1548e+02i
 -2.1352e+02 - 5.1548e+02i
 -5.1548e+02 + 2.1352e+02i
 -5.1548e+02 - 2.1352e+02i
knew =   9.6912e+10





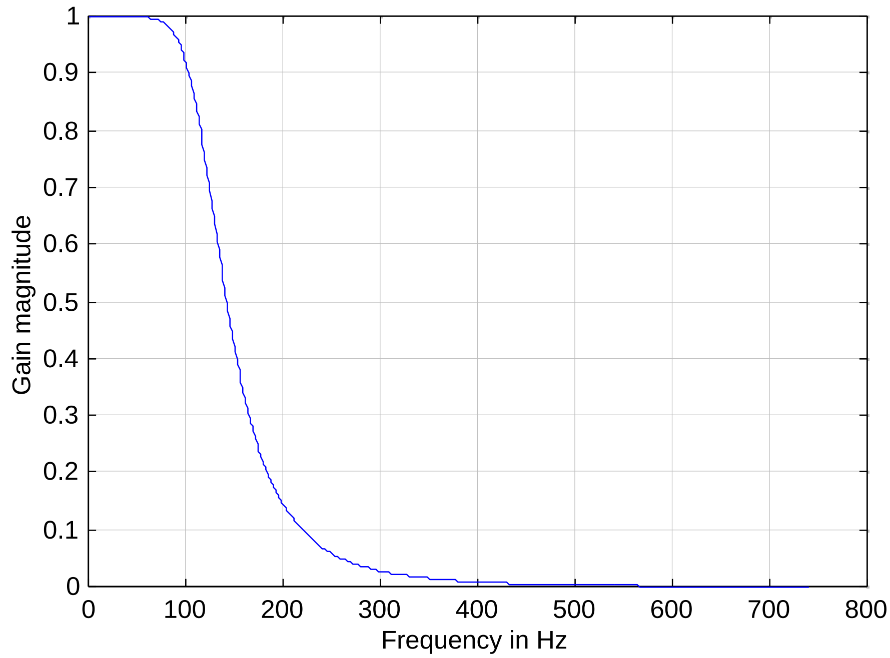


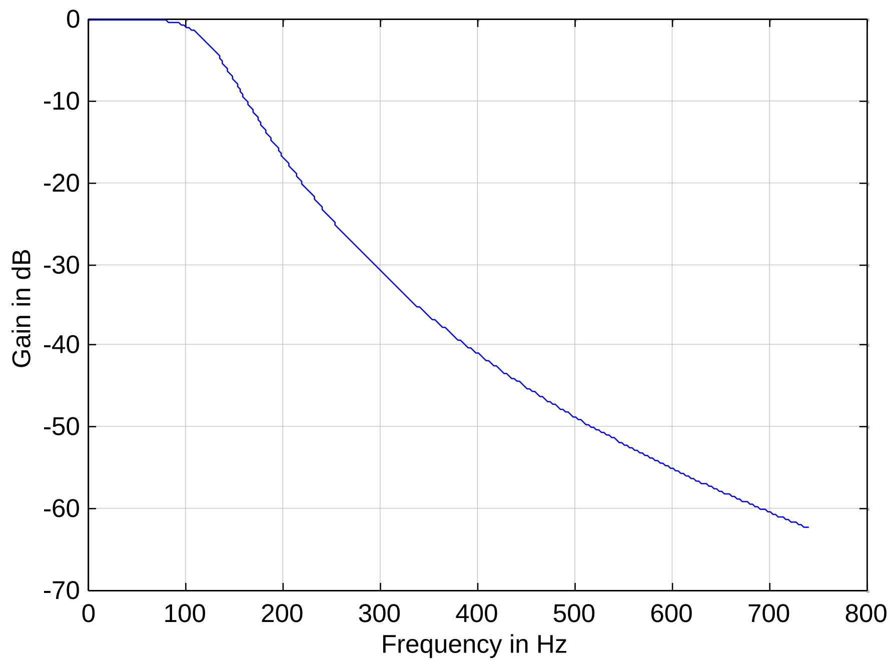
SOLUTION TO 21.11.

fp = 75; fs = 450;Amax = 1; Amin = 45;

```
n = buttord(fp,fs,Amax,Amin,'s');
%  The order mfile may not be available in the student edition.
emax = sqrt(10^(0.1*Amax) - 1);
emin = sqrt(10^(0.1*Amin) - 1)/(fs/fp)^n;
fcmin = fp/((10^(0.1*Amax)-1)^(1/(2*n)));
fcmax = fs/((10^(0.1*Amin)-1)^(1/(2*n)));
wcmin = 2*pi*fcmin;
wcmax = 2*pi*fcmax;
[z,p,k] = buttap(n);
wc = wcmax;
fc = fcmax
znew = z*wc
pnew = p*wc
knew = k*wc^n
W = 0:0.01:fs/fp;
h = freqs(k*poly(z),poly(p),W);
plot(W*wc/(2*pi),abs(h))
grid
xlabel('Frequency in Hz')
ylabel('Gain magnitude')
pause
plot(W*wc/(2*pi),20*log10(abs(h)))
xlabel('Frequency in Hz')
ylabel('Gain in dB')
grid

fc =   1.2323e+02
znew =     []
pnew =
 -2.9630e+02 + 7.1533e+02i
 -2.9630e+02 - 7.1533e+02i
 -7.1533e+02 + 2.9630e+02i
 -7.1533e+02 - 2.9630e+02i
knew =   3.5940e+11
```





SOLUTION TO 21.12. Here we require that

$$s^3 + 2s^2 + 2s + 1 = s^3 + \frac{C_1 + C_2}{C_1 C_2} s^2 + \frac{C_1 + C_2 + L}{LC_1 C_2} s + \frac{2}{LC_1 C_2}$$

Thus

$$LC_1C_2 = 2 \qquad C_1 + C_2 + L = 4 \text{ and } 2 = \frac{C_1 + C_2}{C_1 C_2} \times \frac{L}{L} = \frac{LC_1 + LC_2}{2}$$

in which case, $LC_1 + LC_2 = 4$. Therefore, $L(C_1 + C_2 + L) = 4L = 4 + L^2$. Equivalently

$$L^2 - 4L + 4 = (L-2)(L-2) = 0$$

Hence L = 2 H is the only solution. Thus $C_1C_2 = 1 \qquad C_1 = \frac{1}{C_2} \qquad \frac{1}{C_2} + C_2 + 2 = 4$ or equivalently $C_2^2 - 2C_2 + 1 = (C_2 - 1)(C_2 - 1) = 0$ which implies that $C_1 = C_2 = 1$ F is the only solution, as was to be shown.

SOLUTION TO 21.13. (a) By voltage division

$$H(s) = \frac{\frac{1}{Cs+G}}{Ls + \frac{1}{Cs+G}} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

(b) With R = 1 Ω,

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

requires that C = 1/$\sqrt{2}$ F and L = $\sqrt{2}$ H.

(c)
```
»fc = 1000;
»wc = 2*pi*fc
wc =   6.2832e+03
»Kf = wc;
»Km = 1000;
»C = 1/sqrt(2);
»L = sqrt(2);
»Cnew = C/(Kf*Km)
Cnew =   1.1254e-07
»Lnew = L*Km/Kf
Lnew =   2.2508e-01
```

(d)

```
»Km = C/(wc*1e-6)
Km =   1.1254e+02
»Kf = wc;
»Rnew = Km
Rnew =   1.1254e+02
»Lnew = L*Km/wc
Lnew =
   2.5330e-02
```

SOLUTION TO 21.14. (a) By voltage division

$$H(s) = \frac{\frac{1}{Cs}}{Ls + R_s + \frac{1}{Cs}} = \frac{\frac{1}{LC}}{s^2 + \frac{R_s}{L}s + \frac{1}{LC}}$$

(b) With R = 1 Ω,

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R_s}{L}s + \frac{1}{LC}} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

requires that L = 1/$\sqrt{2}$ F and C = $\sqrt{2}$ H.

(c)

```
»L = 1/sqrt(2); C = 1/L;
»Km = 10;
»Kf = 2*pi*500;
»Rnew = 10;
»Cnew = C/(Km*Kf)
Cnew =   4.5016e-05
»Lnew = Km*L/Kf
Lnew =   2.2508e-03
```

(d)

```
»Km = C/(1e-6*Kf)
Km =   4.5016e+02
»Lnew = L*Km/Kf
Lnew =   1.0132e-01
»Cnew = C/(Km*Kf)
Cnew =   1.0000e-06
```

SOLUTION TO 21.15.

(a) Since 1 = 2/(LC), L = 2/C. Since (1/L + 1/C) = (C/2 + 1/C) = sqrt(2), we have that C is a root of the quadratic $0.5C^2$ – sqrt(2)C + 1 = 0. Hence

```
»v = [0.5  -sqrt(2)  1];
»r = roots(v)
r =
   1.4142e+00
   1.4142e+00
»C = r(1)
C =   1.4142e+00
»L = 2/C
L =   1.4142e+00
```

(b)

```
»Km = 1e3;
»Kf = 2*pi*3500;
»Cnew = C/(Km*Kf)
Cnew =   6.4308e-08
»Lnew = L*Km/Kf
Lnew =   6.4308e-02
```

(c)

```
»Km = C/(Kf*10e-9)
Km =   6.4308e+03
»Cnew = C/(Km*Kf)
Cnew =   1.0000e-08
»Lnew = L*Km/Kf
Lnew =   4.1356e-01
»Rs = Km
Rs =   6.4308e+03
»RL = Rs
RL =   6.4308e+03
```

SOLUTION TO 21.16.

(a) Let G = R_L. Then by voltage division

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\dfrac{1}{Cs+G}}{Ls + R_s + \dfrac{1}{Cs+G}} = \frac{\dfrac{1}{LC}}{s^2 + \left(\dfrac{1}{CR_L} + \dfrac{R_s}{L}\right)s + \dfrac{1 + R_s / R_L}{LC}}$$

(b) Since 1 = 1.25/(LC), L = 1.25/C. Since (R_s/L + 1/R_LC) = (2C/1.25 + 1/8C) = sqrt(2), we have that C is a root of the quadratic (16/1.25)C^2 – 8sqrt(2)C + 1 = 0. Hence

```
»C = roots([16/1.25   -8*sqrt(2)   1])
C =
   7.8427e-01
   9.9615e-02
»L = 1.25 ./C
L =
   1.5938e+00
   1.2548e+01
```

(c)

```
»Km = 1e3;
»Kf = 2*pi*5e3
Kf =   3.1416e+04
»Cnew = C/(Km*Kf)
Cnew =
   2.4964e-08
   3.1708e-09
»Lnew = L*Km/Kf
Lnew =
   5.0734e-02
   3.9942e-01
```

SOLUTION TO 21.17.

(a) Define G = 1/R_s, execute two source transformations, and apply voltage division to obtain

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{\frac{1}{Cs+G} + Ls + 1} \times \frac{G}{Cs+G} = \frac{\frac{1}{R_s LC}}{s^2 + \left(\frac{1}{R_s C} + \frac{1}{L}\right)s + \frac{1+1/R_s}{LC}}$$

(b) Since L = 1.5/C, the values of C are the roots of the quadratic, (2/1.5)C^2 – 2sqrt(2)C + 1 = 0. Hence

```
»C = roots([(2/1.5)   -2*sqrt(2)   1])
C =
   1.6730e+00
   4.4829e-01
»L = 1.5 ./C
L =
   8.9658e-01
   3.3461e+00
»Km = 2e3;
»Kf = 2*pi*5e3;
»Cnew = C/(Km*Kf)
```

Cnew =
   2.6627e-08
   7.1347e-09
»Lnew = Km*L/Kf
Lnew =
   5.7078e-02
   2.1302e-01
»Rsnew = 2*Km
Rsnew =   4000

SOLUTION TO 21.18.

(a)

$$|H(j\omega)| = \frac{|K|}{\left| j\left(\frac{\omega}{\omega_p}\right) + 1 \right|^2} = \frac{|K|}{\left(\frac{\omega}{\omega_p}\right)^2 + 1}$$

The 3 dB down frequency, ω_c, occurs when

$$\frac{1}{\left(\frac{\omega_c}{\omega_p}\right)^2 + 1} = \frac{1}{\sqrt{2}}$$

Equivalently

$$\omega_c = \omega_p \sqrt{\sqrt{2} - 1} = 0.64359\omega_p = 6.4359 \times 10^4 \text{ rad/sec.}$$

(b) $h(t) = K\omega_p^2 t e^{-\omega_p t} u(t)$ since

$$H(s) = \frac{K}{\left(\frac{s}{\omega_p} + 1\right)^2} = \frac{K\omega_p^2}{\left(s + \omega_p\right)^2}.$$

Further,

$$H(s)\frac{1}{s} = \frac{K\omega_p^2}{s\left(s + \omega_p\right)^2} = \frac{K}{s} - \frac{K}{s + \omega_p} - \frac{K\omega_p}{\left(s + \omega_p\right)^2}$$

Hence, the step response is

$$K\left(1 - e^{-\omega_p t} - \omega_p t e^{-\omega_p t}\right)u(t)$$

SOLUTION TO 21.19.

(a) Using voltage division,

$$H(s) = \frac{V_C}{V_{in}} \times \frac{V_{out}}{V_C} = \frac{\dfrac{1}{Cs + \dfrac{1}{L_2 s + 1}}}{1 + L_1 s + \dfrac{1}{Cs + \dfrac{1}{L_2 s + 1}}} \times \frac{1}{L_2 s + 1}$$

$$= \frac{\dfrac{1}{L_1 L_2 C}}{s^3 + \left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right)s^2 + \dfrac{L_1 + L_2 + C}{L_1 L_2 C} s + \dfrac{2}{L_1 L_2 C}}$$

(b) Matching coefficients in

$$\frac{\dfrac{1}{L_1 L_2 C}}{s^3 + \left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right)s^2 + \dfrac{L_1 + L_2 + C}{L_1 L_2 C} s + \dfrac{2}{L_1 L_2 C}} = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

yields $C = \dfrac{2}{L_1 L_2}$ and $\dfrac{L_1 + L_2 + C}{L_1 L_2 C} = \dfrac{L_1 + L_2 + \dfrac{2}{L_1 L_2}}{2} = 2$; equivalently, $L_1 + L_2 + \dfrac{2}{L_1 L_2} = 4$. Further, $\left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right) = \dfrac{L_1 + L_2}{L_1 L_2} = 2$ implies that $L_1 + L_2 = 2L_1 L_2$ implies $2L_1 L_2 + \dfrac{2}{L_1 L_2} = 4$. This requires that $L_1 L_2 = 1$ and from earlier equations that $L_1 + L_2 = 2L_1 L_2 = 2$ which forces $L_1 = L_2 = 1$ H and $C = 2$ F. The idea is to match the denominator coefficients and thus the dc gain is 0.5 instead of the desired 1. A transformer or some amplifier device is needed to increase the gain to 1.

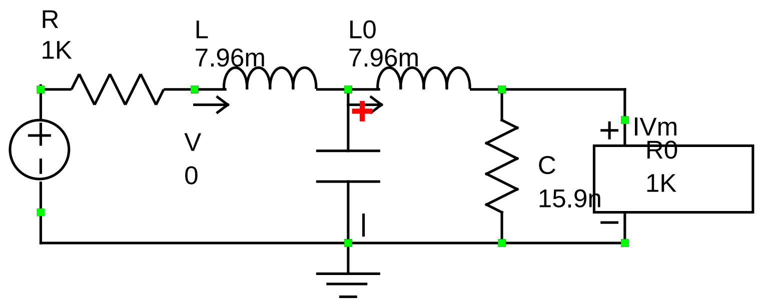
(c) Using MATLAB:

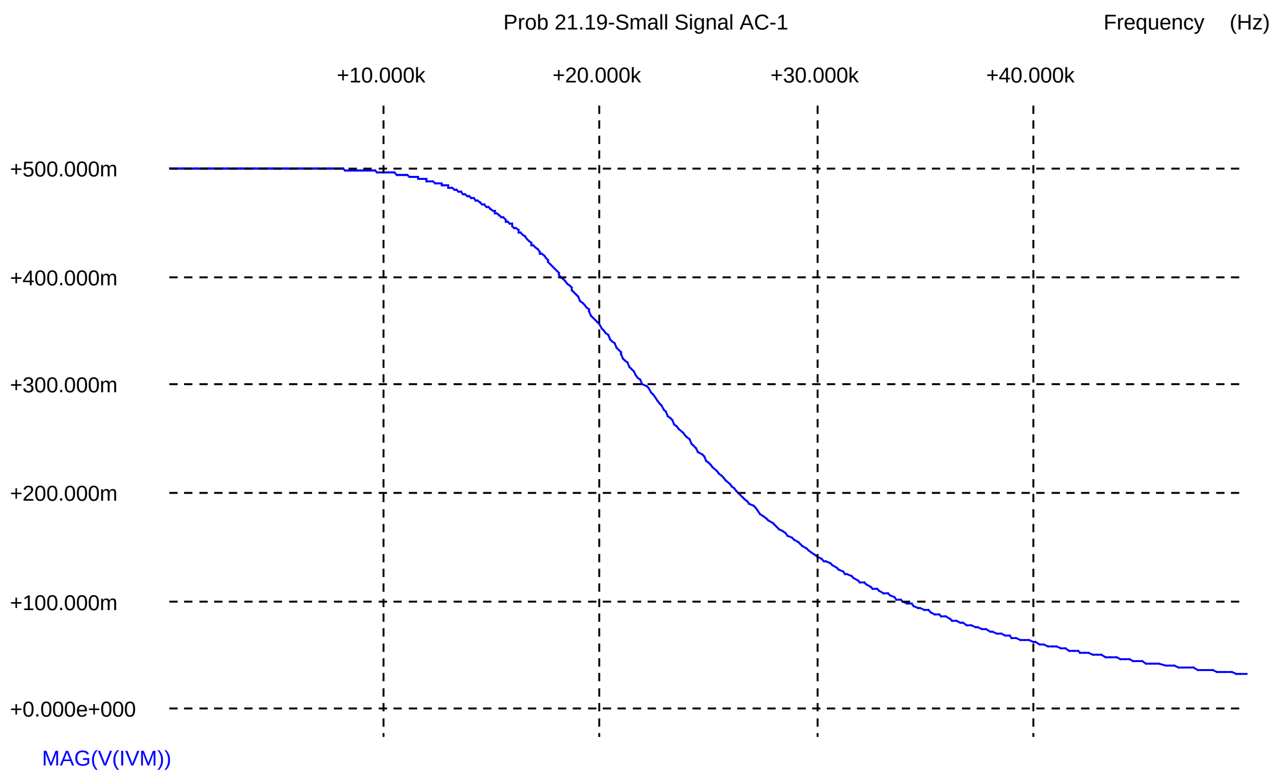
```
»Km = 1000;
»Kf = 2*pi*20e3;
»Lnew = Km/Kf
Lnew =
  7.9577e-03
»Cnew = 2/(Km*Kf)
Cnew =
  1.5915e-08
```

»

Hence, $L_{1new} = L_{2new} = 7.96$ mH and $C = 15.9$ nF.

(d) SPICE simulation





SOLUTION TO 21.20. (a) From figure P21.19a

$$\frac{V_{in} - V_1}{R_s} = I_1, \qquad V_2 = V_{out}$$

in which case

$$V_1 = z_{11} I_1 + z_{12} \times 0 = V_{in} - R_s I_1$$

Also

$$V_2 = z_{21} I_1 + z_{22} \times 0 = V_{out}$$

This implies that $V_{in} = (z_{11} + R_s) I_1$ and $V_{out} = z_{21} I_1$. Finally we conclude that

$$\frac{V_{out}}{V_{in}} = \frac{z_{21}}{z_{11} + R_s}$$

(b) Now from figure P21.20b, we have $V_1 = V_{in}$ and $V_2 = V_{out} = -R_L I_2$. This implies that

$$I_2 = -\frac{V_{out}}{R_L} = y_{21} V_1 + y_{22} V_2$$

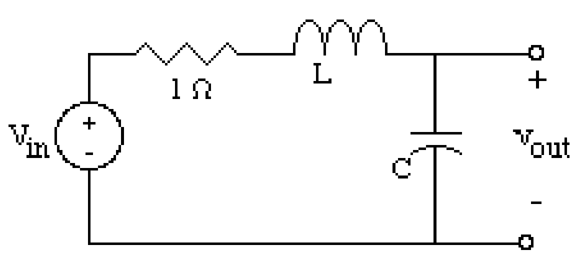
Thus

$$\frac{V_{out}}{V_{in}} = \frac{-y_{21}}{y_{22} + \frac{1}{R_L}} = \frac{-y_{21}}{y_{22} + G_L}$$

Consider here that

$$\frac{V_{out}}{V_{in}} = \frac{z_{21}}{z_{11} + 1} = \frac{1}{s^2 + \sqrt{2}s + 1} = \frac{\frac{1}{s}}{\left(s + \frac{1}{s} + \sqrt{2}\right)} = \frac{\frac{1}{\sqrt{2}s}}{\left(\frac{s}{\sqrt{2}} + \frac{1}{\sqrt{2}s}\right)}$$

Hence $z_{21} = \frac{1}{\sqrt{2}s}$ and $z_{11} = \left(\frac{s}{\sqrt{2}} + \frac{1}{\sqrt{2}s}\right)$. This leads to the circuit



with $L = \frac{1}{\sqrt{2}}$ H and $C = \sqrt{2}$ F.

(c) Similarly,

$$\frac{-y_{21}}{y_{22}+1} = \frac{\frac{1}{\sqrt{2}s}}{\left(\frac{s}{\sqrt{2}} + \frac{1}{\sqrt{2}s}\right)+1}$$

implies $y_{21} = -\frac{1}{\sqrt{2}s}$ and $y_{22} = \frac{s}{\sqrt{2}s} + \frac{1}{\sqrt{2}s}$.

This yields the same circuit as above with

$$L = \sqrt{2} \text{ H and } C = \frac{1}{\sqrt{2}} \text{ F.}$$

(d) Here $K_m = 1000$ and $K_f = 5000 rad / s$

(d-i) For (b),

$$L = \frac{1}{\sqrt{2}} \times \frac{10^3}{5 \times 10^3} = \frac{1}{5\sqrt{2}} = 0.1414 \text{ H}$$

and

$$C = \sqrt{2} \times \frac{1}{5\ \ 10^6} = 0.2828 \ \mu\text{F}$$

(d-ii) For (c)

$$L = 0.2828 \text{ H and } C = 0.1414 \ \mu\text{F.}$$

SOLUTION TO 21.21.

(a) From earlier developments

$$H(s) = -\frac{Y_{in}}{Y_{out}} = -\frac{\frac{1}{R_1}}{Cs + \frac{1}{R_2}}$$

(b) Let C = 1 F, and $R_1 = R_2 = 1\ \Omega$.

(c) »Kf = 2*pi*3500

Kf =   2.1991e+04
»Km = 1/(Kf*1e-9)
Km =   4.5473e+04

In the final design, $R_1 = R_2 = 45.5$ kΩ.

SOLUTION TO 21.22.

(a) From problem 21 and voltage division,

$$H(s) = -\frac{\frac{1}{R_1}}{Cs + \frac{1}{R_2}} \times \frac{\frac{1}{C_2 s}}{R_3 + \frac{1}{C_2 s}} = -\frac{\frac{1}{C_1 R_1}}{s + \frac{1}{C_1 R_2}} \times \frac{\frac{1}{R_3 C_2}}{s + \frac{1}{R_3 C_2}}$$

(b) By inspection, let $C_1 = 0.1$ F, $R_1 = 1$ Ω, $R_2 = 10$ Ω, $R_3 = 10$ Ω, and $C_2 = 0.1$ F, in which case

$$H(s) = -\frac{10}{s+1} \times \frac{1}{s+1} = \frac{-10}{(s+1)^2}$$

(c)
»Kf = 1e5;
»Km = 0.1/(Kf*1e-9)
Km =   1000

Hence, in the final design

$C_1 = 1$ nF, $R_1 = 1$ kΩ, $R_2 = 10$ kΩ, $R_3 = 10$ kΩ, and $C_2 = 1$ nF.

(d) Cascade the circuit of figure P21.22 with another op amp section. For the first part of the design, again set $\omega_p = 1$ and use the same values as in part (c). The extra op amp section has the same values as the first section. As such, final values are the same as in part (c).

SOLUTION TO 21.23. The 2nd order NLP Butterworth transfer function is: $H_{NLP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$. The design parameters and steps are detailed in the excel spread sheet below. An additional design called design C is also listed. For input attenuation, the resistor R_1 is replaced by the voltage divider R_3-R_4 combination.

w0^2	w0/Q	Num	w0	Q	KNLP	KMA	Kf=wp	KmR	KMB	KMC
1	1.41	1.00	1.0000	0.7071	1.00	22507.86	6283.20	10000.00	22507.86	15915.46

KMS
19492.37

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0	1	2Q	1/(2Q)	1	1	KNLP/K	1/alpha	1/(1-alpha)
Design B	1	1	2	1	1/Q	1	Q	KNLP/K	1/alpha	1/(1-alpha)
Design C	1	1-1/Q	3-1/Q	1	1	1	1	KNLP/K	1/alpha	1/(1-alpha)
Saraga	RA	RA/3	3-Apr	rt(3)Q	1	1/Q	1/rt(3)	KNLP/K	1/alpha	1/(1-alpha)

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0.0000	1.0000	1.4142	0.7071	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	1.4142	1.0000	0.7071	0.5000	2.0000	2.0000
Design C	1.00	-0.4142	1.5858	1.0000	1.0000	1.0000	1.0000	0.6306	1.5858	2.7071
Saraga	3.00	1.0000	1.3333	1.2247	1.0000	1.4142	0.5774	0.7500	1.3333	5.6569
w0 scale										
Design A	∞	0.0000	1.0000	1.4142	0.7071	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	1.4142	1.0000	0.7071	0.5000	2.0000	2.0000
Design C	1.00	0.1140	2.1140	1.0000	1.0000	1.0000	1.0000	0.6306	1.5858	2.7071
Saraga	3.00	1.0000	1.3333	1.2247	1.0000	1.4142	0.5774	0.7500	1.3333	5.6569
wp scale										
Design A	∞	0.000E+00	1.000E+00	2.251E-04	1.125E-04	1.000E+00	1.000E+00	1.000E+00	1.000E+00	#DIV/0!
Design B	1.00	1.000E+00	2.000E+00	1.592E-04	2.251E-04	1.000E+00	7.071E-01	5.000E-01	2.000E+00	2.000E+00
Design C	1.00	1.140E-01	2.114E+00	1.592E-04	1.592E-04	1.000E+00	1.000E+00	6.306E-01	1.586E+00	2.707E+00
Saraga	3.00	1.000E+00	1.333E+00	1.949E-04	1.592E-04	1.414E+00	5.774E-01	7.500E-01	1.333E+00	5.657E+00
Km scale										
Design A	∞	0.000E+00	1.000E+00	1.000E-08	5.000E-09	2.251E+04	2.251E+04	1.000E+00	2.251E+04	#DIV/0!
Design B	10000	1.000E+04	2.000E+00	7.0711E-09	1.000E-08	2.251E+04	1.592E+04	5.000E-01	4.502E+04	4.502E+04
Design C	10000	1.140E+03	2.114E+00	1.000E-08	1.000E-08	1.592E+04	1.592E+04	6.306E-01	2.524E+04	4.308E+04
Saraga	30000	1.000E+04	1.333E+00	1.000E-08	8.165E-09	2.757E+04	1.125E+04	7.500E-01	2.599E+04	1.103E+05

SOLUTION TO 21.24 AND 21.25. In problem 21.8, the transfer function information was computed in MATLAB as:

% Numerators are each 1. Denominators are the polynomials
d1 = 1.0000e+00 1.0000e+00 1.0000e+00
d2 = 1 1

Further we know from MATLAB that
fcmin = 1.5521e+02

The Saraga design and Design A for d1, the second order section of each filter, are given by the excel spread sheet below, as well as two alternate designs labeled B and C.

w0^2	w0/Q	Num	w0	Q	KNLP	KMA	Kf=wp	KmR	KMB	KMC
1	1.00	1.00	1.0000	1.0000	1.000	41016.58	975.22	10000.00	20508.29	20508.29

KMS
35521.40

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0	1	2Q	1/(2Q)	1	1	KNLP/K	1/alpha	1/(1-alpha)
Design B	1	1	2	1	1/Q	1	Q	KNLP/K	1/alpha	1/(1-alpha)
Design C	1	1-1/Q	3-1/Q	1	1	1	1	KNLP/K	1/alpha	1/(1-alpha)
Saraga	RA	RA/3	3-Apr	rt(3)Q	1	1/Q	1/rt(3)	KNLP/K	1/alpha	1/(1-alpha)

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0.0000	1.0000	2.0000	0.5000	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	1.0000	1.0000	1.0000	0.5000	2.0000	2.0000
Design C	1.00	0.0000	2.0000	1.0000	1.0000	1.0000	1.0000	0.5000	2.0000	2.0000
Saraga	3.00	1.0000	1.3333	1.7321	1.0000	1.0000	0.5774	0.7500	1.3333	4.0000
w0 scale										
Design A	∞	0.0000	1.0000	2.0000	0.5000	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	1.0000	1.0000	1.0000	0.5000	2.0000	2.0000
Design C	1.00	0.1140	2.1140	1.0000	1.0000	1.0000	1.0000	0.5000	2.0000	2.0000
Saraga	3.00	1.0000	1.3333	1.7321	1.0000	1.0000	0.5774	0.7500	1.3333	4.0000
wp scale										
Design A	∞	0.000E+00	1.000E+00	2.051E-03	5.127E-04	1.000E+00	1.000E+00	1.000E+00	1.000E+00	#DIV/0!
Design B	1.00	1.000E+00	2.000E+00	1.025E-03	1.025E-03	1.000E+00	1.000E+00	5.000E-01	2.000E+00	2.000E+00
Design C	1.00	1.140E-01	2.114E+00	1.025E-03	1.025E-03	1.000E+00	1.000E+00	5.000E-01	2.000E+00	2.000E+00
Saraga	3.00	1.000E+00	1.333E+00	1.776E-03	1.025E-03	1.000E+00	5.774E-01	7.500E-01	1.333E+00	4.000E+00
Km scale										
Design A	∞	0.000E+00	1.000E+00	5.000E-08	1.250E-08	4.102E+04	4.102E+04	1.000E+00	4.102E+04	#DIV/0!
Design B	10000	1.000E+04	2.000E+00	5.000E-08	5.000E-08	2.051E+04	2.051E+04	5.000E-01	4.102E+04	4.102E+04
Design C	10000	1.140E+03	2.114E+00	5.000E-08	5.000E-08	2.051E+04	2.051E+04	5.000E-01	4.102E+04	4.102E+04
Saraga	30000	1.000E+04	1.333E+00	5.000E-08	2.887E-08	3.552E+04	2.051E+04	7.500E-01	4.736E+04	1.421E+05

The first order (leaky integrator) section is common to both problems. This section consists of an input resistor (conductance) R_1 (G_1) connected to the inverting terminal with a parallel R_2-C combination feeding back from the output. The transfer function is: $H(s) = \dfrac{G_1}{Cs + G_2}$. For the normalized design we set $G_1 = G_2 = 1$ S ($R_1 = R_2 = 1\ \Omega$) and C = 1 F. This design can be scaled independently of the S&K 2^{nd} order section. Hence we set C_{new} = 50 nF. Thus K_m = 20,508.29. Hence $R_1 = R_2$ = 20,508.29 Ω.

SOLUTION TO 21.26 AND 21.27. The relevent data from the solution of problem 21.10 is:

k = 1
d1 = 1.0000e+00 7.6537e-01 1.0000e+00
d2 = 1.0000e+00 1.8478e+00 1.0000e+00
fcmin = 8.8800e+01
wcmin = 5.5795e+02

In providing the designs, we set forth all the possible S&K designs using two excel spreadsheets, one for each second order section.

The designs for denominator d1 with numerator equal to 1 are:

w0^2	w0/Q	Num	w0	Q	KNLP	KMA	Kf=wp	KmR	KMB	KMC
1	0.7654	1.00000	1.0000	1.3066	1.0000	46834.37	557.95	10000.00	17922.81	17922.81

KMS
40559.76

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0	1	2Q	1/(2Q)	1	1	KNLP/K	1/alpha	1/(1-alpha)
Design B	1	1	2	1	1/Q	1	Q	KNLP/K	1/alpha	1/(1-alpha)
Design C	1	1-1/Q	3-1/Q	1	1	1	1	KNLP/K	1/alpha	1/(1-alpha)
Saraga	RA	RA/3	3-Apr	rt(3)Q	1	1/Q	1/rt(3)	KNLP/K	1/alpha	1/(1-alpha)

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0.0000	1.0000	2.6131	0.3827	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	0.7654	1.0000	1.3066	0.5000	2.0000	2.0000
Design C	1.00	0.2346	2.2346	1.0000	1.0000	1.0000	1.0000	0.4475	2.2346	1.8100
Saraga	3.00	1.0000	1.3333	2.2630	1.0000	0.7654	0.5774	0.7500	1.3333	3.0615
w0 scale										
Design A	∞	0.0000	1.0000	2.6131	0.3827	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	0.7654	1.0000	1.3066	0.5000	2.0000	2.0000
Design C	1.00	0.1140	2.1140	1.0000	1.0000	1.0000	1.0000	0.4475	2.2346	1.8100
Saraga	3.00	1.0000	1.3333	2.2630	1.0000	0.7654	0.5774	0.7500	1.3333	3.0615
wp scale										
Design A	∞	0.000E+00	1.000E+00	4.683E-03	6.859E-04	1.000E+00	1.000E+00	1.000E+00	1.000E+00	#DIV/0!
Design B	1.00	1.000E+00	2.000E+00	1.792E-03	1.372E-03	1.000E+00	1.307E+00	5.000E-01	2.000E+00	2.000E+00
Design C	1.00	1.140E-01	2.114E+00	1.792E-03	1.792E-03	1.000E+00	1.000E+00	4.475E-01	2.235E+00	1.810E+00
Saraga	3.00	1.000E+00	1.333E+00	4.056E-03	1.792E-03	7.654E-01	5.774E-01	7.500E-01	1.333E+00	3.061E+00
Km scale										
Design A	∞	0.000E+00	1.000E+00	1.000E-07	1.464E-08	4.683E+04	4.683E+04	1.000E+00	4.683E+04	#DIV/0!
Design B	10000	1.000E+04	2.000E+00	1.000E-07	7.654E-08	1.792E+04	2.342E+04	5.000E-01	3.585E+04	3.585E+04
Design C	10000	1.140E+03	2.114E+00	1.000E-07	1.000E-07	1.792E+04	1.792E+04	4.475E-01	4.005E+04	3.244E+04
Saraga	30000	1.000E+04	1.333E+00	1.000E-07	4.419E-08	3.104E+04	2.342E+04	7.500E-01	5.408E+04	1.242E+05

The designs for denominator d2 with numerator equal to 1 are:

w0^2	w0/Q	Num	w0	Q	KNLP	KMA	Kf=wp	KmR	KMB	KMC
1	1.8478	1.00	1.00	0.54	1.00	19399.08	557.95	10000.00	33117.77	17922.81

KMS
17922.81

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0	1	2Q	1/(2Q)	1	1	KNLP/K	1/alpha	1/(1-alpha)
Design B	1	1	2	1	1/Q	1	Q	KNLP/K	1/alpha	1/(1-alpha)
Design C	1	1-1/Q	3-1/Q	1	1	1	1	KNLP/K	1/alpha	1/(1-alpha)
Saraga	RA	RA/3	3-Apr	rt(3)Q	1	1/Q	1/rt(3)	KNLP/K	1/alpha	1/(1-alpha)

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0.0000	1.0000	1.0824	0.9239	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	1.8478	1.0000	0.5412	0.5000	2.0000	2.0000
Design C	1.00	-0.8478	1.1522	1.0000	1.0000	1.0000	1.0000	0.8679	1.1522	7.5703
Saraga	3.00	1.0000	1.3333	0.9374	1.0000	1.8478	0.5774	0.7500	1.3333	7.3912
w0 scale										
Design A	∞	0.0000	1.0000	1.0824	0.9239	1.0000	1.0000	1.0000	1.0000	#DIV/0!
Design B	1.00	1.0000	2.0000	1.0000	1.8478	1.0000	0.5412	0.5000	2.0000	2.0000
Design C	1.00	0.1140	2.1140	1.0000	1.0000	1.0000	1.0000	0.8679	1.1522	7.5703
Saraga	3.00	1.0000	1.3333	0.9374	1.0000	1.8478	0.5774	0.7500	1.3333	7.3912
wp scale										
Design A	∞	0.000E+00	1.000E+00	1.940E-03	1.656E-03	1.000E+00	1.000E+00	1.000E+00	1.000E+00	#DIV/0!
Design B	1.00	1.000E+00	2.000E+00	1.792E-03	3.312E-03	1.000E+00	5.412E-01	5.000E-01	2.000E+00	2.000E+00
Design C	1.00	1.140E-01	2.114E+00	1.792E-03	1.792E-03	1.000E+00	1.000E+00	8.679E-01	1.152E+00	7.570E+00
Saraga	3.00	1.000E+00	1.333E+00	1.680E-03	1.792E-03	1.848E+00	5.774E-01	7.500E-01	1.333E+00	7.391E+00
Km scale										
Design A	∞	0.000E+00	1.000E+00	1.000E-07	8.536E-08	1.940E+04	1.940E+04	1.000E+00	1.940E+04	#DIV/0!
Design B	10000	1.000E+04	2.000E+00	5.412E-08	1.000E-07	3.312E+04	1.792E+04	5.000E-01	6.624E+04	6.624E+04
Design C	10000	1.140E+03	2.114E+00	1.000E-07	1.000E-07	1.792E+04	1.792E+04	8.679E-01	2.065E+04	1.357E+05
Saraga	30000	1.000E+04	1.333E+00	9.374E-08	1.000E-07	3.312E+04	1.035E+04	7.500E-01	2.390E+04	1.325E+05

SOLUTION TO 21.28. For this problem we use the excel spread sheet given below. First we observe that

$$H(s) = \frac{\hat{K}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad \rightarrow \quad H(\omega_0 s) = \frac{\hat{K}}{(\omega_0 s)^2 + \frac{\omega_0}{Q}(\omega_0 s) + \omega_0^2} = \frac{\hat{K}/\omega_0^2 \ (= K_{NLP})}{s^2 + \frac{1}{Q}s + 1}$$

Thus after this type of frequency scaling, the new transfer function is:

$$H_{new}(s) = \frac{0.7943}{s^2 + \frac{1}{1.1286}s + 1}$$

The dc gain is of course 0.7943 and the modification of the circuit to achieve the correct dc gain is given in the spread sheet below via R_3 and R_4 which constitute a voltage divider that replaces R_1.

w0^2	w0/Q	Num	w0	Q	KNLP	KMA	Kf=wp	KmR	KMB	KMC
0.82306	0.8038	0.65378	0.9072	1.1286	0.7943	1131.42	43982.40	10000.00	501.23	501.23

KMS
979.83

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0	1	2Q	1/(2Q)	1	1	KNLP/K	1/alpha	1/(1-alpha)
Design B	1	1	2	1	1/Q	1	Q	KNLP/K	1/alpha	1/(1-alpha)
Design C	1	1-1/Q	3-1/Q	1	1	1	1	KNLP/K	1/alpha	1/(1-alpha)
Saraga	RA	RA/3	3-Apr	rt(3)Q	1	1/Q	1/rt(3)	KNLP/K	1/alpha	1/(1-alpha)

	RA	RB	K	C1	C2	R1	R2	alpha	R3	R4
Design A	∞	0.0000	1.0000	2.2573	0.4430	1.0000	1.0000	0.7943	1.2589	4.8621
Design B	1.00	1.0000	2.0000	1.0000	0.8860	1.0000	1.1286	0.3972	2.5179	1.6588
Design C	1.00	0.1140	2.1140	1.0000	1.0000	1.0000	1.0000	0.3758	2.6613	1.6019
Saraga	3.00	1.0000	1.3333	1.9549	1.0000	0.8860	0.5774	0.5957	1.6786	2.1917
w0 scale										
Design A	∞	0.0000	1.0000	2.4881	0.4883	1.0000	1.0000	0.7943	1.2589	4.8621
Design B	1.00	1.0000	2.0000	1.1023	0.9766	1.0000	1.1286	0.3972	2.5179	1.6588
Design C	1.00	0.1140	2.1140	1.1023	1.1023	1.0000	1.0000	0.3758	2.6613	1.6019
Saraga	3.00	1.0000	1.3333	2.1548	1.1023	0.8860	0.5774	0.5957	1.6786	2.1917
wp scale										
Design A	∞	0.000E+00	1.000E+00	5.657E-05	1.110E-05	1.000E+00	1.000E+00	7.943E-01	1.259E+00	4.862E+00
Design B	1.00	1.000E+00	2.000E+00	2.506E-05	2.220E-05	1.000E+00	1.129E+00	3.972E-01	2.518E+00	1.659E+00
Design C	1.00	1.140E-01	2.114E+00	2.506E-05	2.506E-05	1.000E+00	1.000E+00	3.758E-01	2.661E+00	1.602E+00
Saraga	3.00	1.000E+00	1.333E+00	4.899E-05	2.506E-05	8.860E-01	5.774E-01	5.957E-01	1.679E+00	2.192E+00
Km scale										
Design A	∞	0.000E+00	1.000E+00	5.000E-08	9.813E-09	1.131E+03	1.131E+03	7.943E-01	1.424E+03	5.501E+03
Design B	10000	1.000E+04	2.000E+00	5.000E-08	4.430E-08	5.012E+02	5.657E+02	3.972E-01	1.262E+03	8.315E+02
Design C	10000	1.140E+03	2.114E+00	5.000E-08	5.000E-08	5.012E+02	5.012E+02	3.758E-01	1.334E+03	8.029E+02
Saraga	30000	1.000E+04	1.333E+00	5.000E-08	2.558E-08	8.682E+02	5.657E+02	5.957E-01	1.645E+03	2.148E+03

SOLUTION TO 21.29. (a) $H_{HP}(s) = H_{NLP}\left(\frac{\omega_c}{s}\right) = \frac{1}{\left(\frac{\omega_c}{s}\right)^2 + \sqrt{2}\left(\frac{\omega_c}{s}\right) + 1}$. At $s = j\omega_p$,

$$\left|H_{HP}(j\omega_p)\right| = \left|H_{NLP}\left(\frac{\omega_c}{j\omega_p}\right)\right| = \left|\frac{1}{\left(\frac{\omega_c}{j\omega_p}\right)^2 + \sqrt{2}\left(\frac{\omega_c}{j\omega_p}\right) + 1}\right| = \left|\frac{1}{-\left(\frac{5.5}{7}\right)^2 - j\sqrt{2}\left(\frac{5.5}{7}\right) + 1}\right|$$

Thus in MATLAB,

```
»Magfp = 1/abs(1 - (5.5/7)^2 -j*sqrt(2)*(5.5/7))
Magfp =   8.5091e-01
»Attenfp = -20*log10(Magfp)
```

```
Attenfp =   1.4023e+00
»Magfs = 1/abs(1 - (5.5/1)^2 -j*sqrt(2)*(5.5/1))
Magfs =   3.3040e-02
»Attenfs =  -20*log10(Magfs)
Attenfs =   2.9619e+01
```

Thus the attenuation at f_p is 1.4023 dB and that at f_s is 29.619 dB.

(b) From problem 21.15, the transfer function is

$$H_{cir}(s) = \frac{1/LC}{s^2 + \left(\frac{1}{C} + \frac{1}{L}\right)s + \frac{2}{LC}}$$

and the values of L and C realizing the 2nd order Butterworth NLP transfer function can be computed according to

$$\frac{1}{C} + \frac{1}{L} = \sqrt{2},\ \frac{2}{LC} = 1 \qquad \frac{1}{C} + \frac{C}{2} = \sqrt{2} \qquad C^2 - 2\sqrt{2}C + 2 = 0$$

$$(C - \sqrt{2})^2 = 0 \qquad C = \sqrt{2}\ \text{ F} \qquad L = \sqrt{2}\ \text{ H}$$

(c) Here K_m = 1000. $Ls \rightarrow \frac{L\omega_C}{s} \rightarrow \frac{1}{\frac{1}{LK_m\omega_C}s}$ and $\frac{1}{Cs} \rightarrow \frac{s}{C\omega_C} \rightarrow \frac{K_m}{C\omega_C}s$. Thus in MATLAB,

```
»wc = 2*pi*5.5e3
wc =   3.4558e+04
»Km = 1000;
»C = sqrt(2);  L = sqrt(2);
»Lhp = Km/(C*wc)
Lhp =   2.0462e-02
»Chp = 1/(Km*wc*L)
Chp =   2.0462e-08
```

Therefore, the resistors take on values of 1 kΩ, the inductor is changed to a capacitor of value of C_{hp} = 20.46 nF and the capacitor is changed to an inductor of value L_{hp} = 20.46 mH.

SOLUTION TO 21.30. (a)

$$\Omega = \frac{\omega_p}{\omega},\ A_{\max} = 2 \text{ dB},\ \varepsilon = \varepsilon_{\max} = \sqrt{10^{0.1\times 2} - 1} = 0.76478,\ \Omega_c = \frac{1}{\sqrt[6]{10^{0.1\times 2} - 1}} = 1.0935$$

```
»Wc=1/(10^0.2-1)^(1/6)
Wc =   1.0935e+00
```

(b)

```
»wchp = 2*pi*5e3/Wc
wchp =   2.8730e+04
»fchp = wchp/(2*pi)
fchp =   4.5725e+03
```

Thus $\omega_{chp} = \dfrac{\omega_p}{\Omega_c} = 28.73$ krad/s.

(c) (i) $NLP \rightarrow HP$ transformation: scale by w_{chp}.

$$C_{hp} = \frac{1}{L_{lp}\omega_{chp}} = 17.404 \mu\text{F},\; L_{hp} = \frac{1}{C_{lp}\omega_{chp}} = 34.807\ \mu\text{H}$$

(ii) Magnitude scale to obtain proper value of capacitors.

```
»Clp = 1;
»Llp = 2;
»Chp = 1/(Llp*wchp)
Chp = 1.7404e-05
»Lhp = 1/(Clp*wchp)
Lhp =   3.4807e-05
»Cnewhp = 100e-9;
»Km = Chp/Cnewhp
Km =   1.7404e+02
»Lnewhp = Km*Lhp
Lnewhp =   6.0578e-03
```

$$K_m = 174.02,\; C_{hp,new} = \frac{C_{hp}}{K_m} = 100\ n\text{F},\; L_{hp,new} = K_m L_{hp} = 6.06\ m\text{H}$$

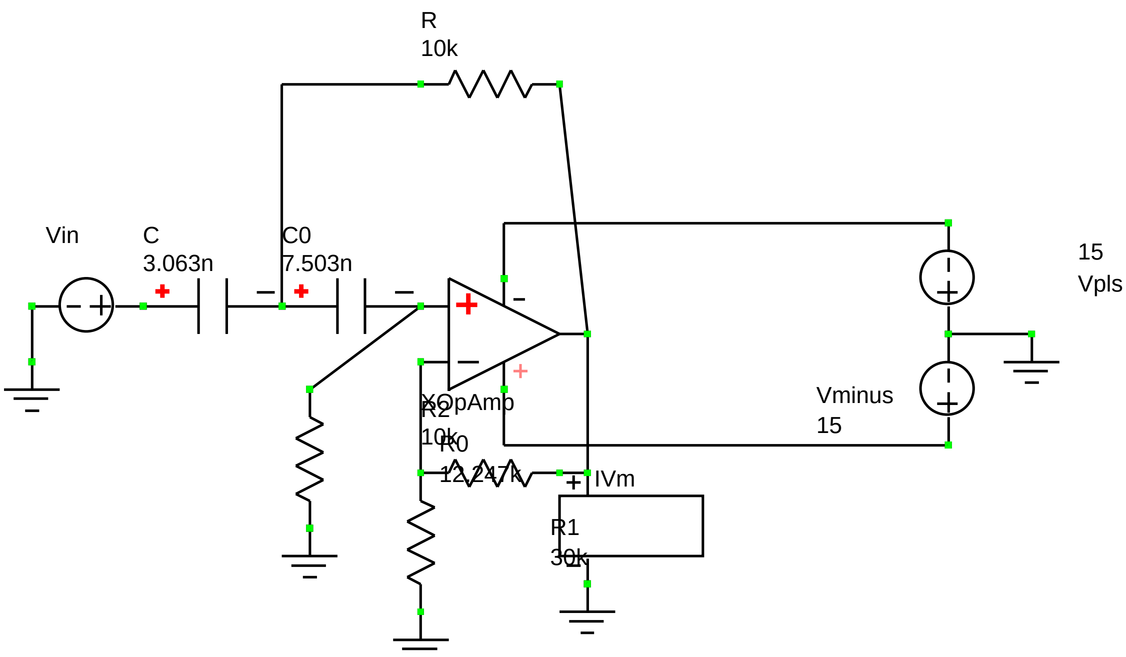
SOLUTION TO 21.31.

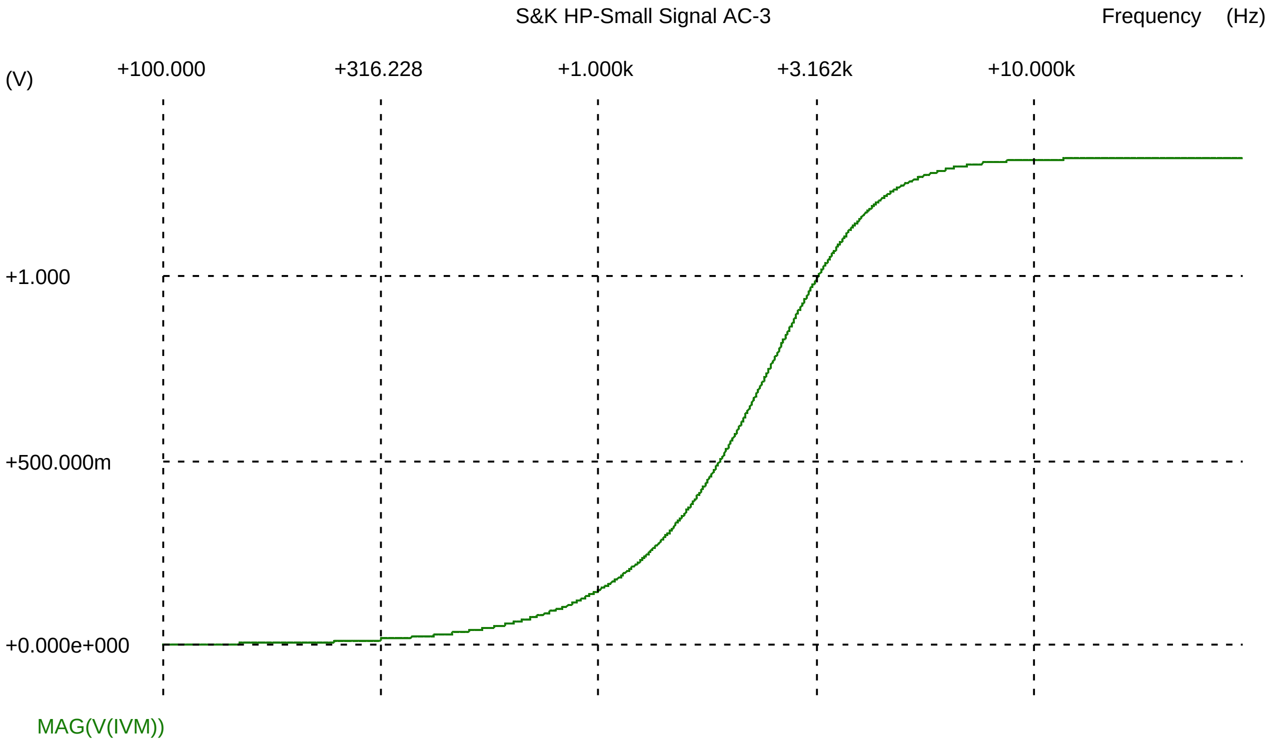
The 2nd order NLP Butterworth transfer function is: $H_{NLP}(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$. Using the transformation s to 1/s, we obtain the NHP Butterworth transfer function:

$$H_{NHP}(s) = H_{NLP}\left(\frac{1}{s}\right) = \frac{s^2}{s^2 + \sqrt{2}s + 1} = K_\infty \frac{s^2}{s^2 + d(1)\, s + d(2)}$$

INPUT:	**d(1)**	**d(2)**	**K**	**K∞**	**Kf**	**Km**	**KmR**
	1.414213562	1	1.33334	1	18849.6	12247.44871	30000
NHP Params	**w0**	**Q**	**alpha**				
	1	0.70710678	0.74999625				
NHP Crt Pars	**C1 = Q**	**C2 = rt(3)**	**R1 = 1**	**R2 =1/(rt(3)Q)**	**R**	**R/3**	
	0.707106781	1.73205081	1	0.816496581	1	0.333333333	
HP Crt Params	**C1new=C1/(Km*Kf)**	**C2new**	**R1new=R1*Km**	**R2new=R2*Km**	**Rnew=R*KmR**	**Rnew/3**	
	3.06293E-09	7.5026E-09	12247.44871	10000	30000	10000	

A plot of the design without input attenuation is shown below. Notice that as predicted the gain is 4/3.





Input attenuation requires that we replace C1 with a series combination of capacitors in which C1 = C3 + C4 and (1/C3)/(1/C4 + 1/C3) = alpha. Here then, C1 = C3 + C4 and alpha = C4/(C3 + C4) = C4/C1. Thus C4 = alpha*C1 and C3 = (1 – alpha)*C1. Thus

Input Attenuation	**C3 = (1 - alpha)C1**	**C4 = alpha*C1**
	7.65744E-10	2.29719E-09

SOLUTION TO 21.32. The fourth order Butterworth NLP transfer function can be obtained from tables or from MATLAB as follows:

```
»[z,p,k] = buttap(4)
z =
    []
p =
 -3.8268e-01 + 9.2388e-01i
 -3.8268e-01 - 9.2388e-01i
 -9.2388e-01 + 3.8268e-01i
 -9.2388e-01 - 3.8268e-01i
k =
    1
»%  Second Order Sections
```

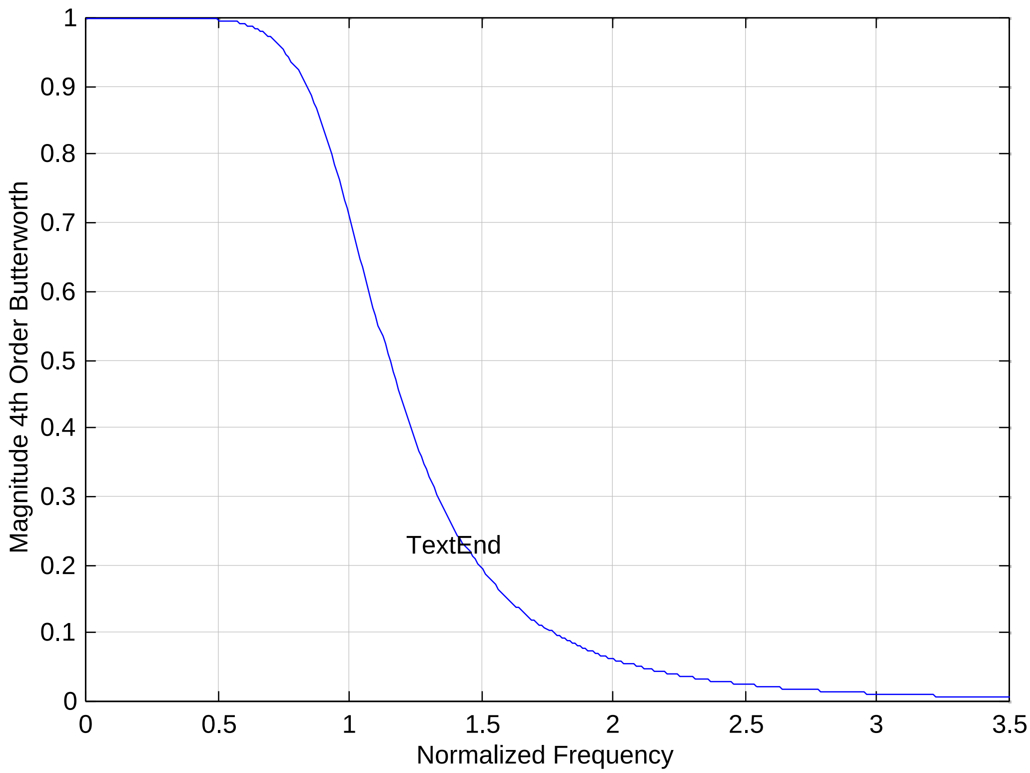
```
»n1 = 1;
»d1 = poly([p(1),conj(p(1))])
d1 =
   1.0000e+00   7.6537e-01   1.0000e+00
»n2 = 1;
»d2 = poly([p(3),conj(p(3))])
d2 =
   1.0000e+00   1.8478e+00   1.0000e+00
»
```

Thus,

$$H_{NLP}(s) = \frac{1}{s^2 + 0.76537s + 1} \times \frac{1}{s^2 + 1.8478s + 1}$$

having frequency response

```
»w = 0:0.01:3.5;
»h = freqs(k*poly(z),poly(p),w);
»plot(w,abs(h))
»grid
»xlabel('Normalized Frequency')
»ylabel('Magnitude 4th Order Butterworth')
```



The Saraga design parameters are given in the following Excel tables:

INPUT:	d(1)	d(2)	KNLP/NHP∞	w0LP/HP	Q	
NLP H(s)	0.76537	1	1	1	1.306557613	
NHP H(s)	0.76537	1	1	1	1.306557613	
K	**Kf**	**Km**	**KmR**	**alpha**		
1.33334	18849.6	22630.24168	60000	0.74999625		
NHP Crt Pars	**C1 = Q**	**C2 = rt(3)**	**R1 = 1**	**R2 =1/(rt(3)Q)**	**R**	**R/3**
	1.306557613	1.732050808	1	0.441886576	1	0.333333333
Scale by w0HP	1.306557613	1.732050808	1	0.441886576	1	0.333333333
HP Crt Params	**C1new=C1/(Km*Kf)**	**C2new**	**R1new=R1*Km**	**R2new=R2*Km**	**Rnew=R*KmR**	**Rnew/3**
	3.06293E-09	4.0604E-09	22630.24168	10000	60000	20000
Input Attenuation	**C3 = (1 - alpha)C1**	**C4 = alpha*C1**				
	7.65744E-10	2.29719E-09				

INPUT:	d(1)	d(2)	KNLP/NHP∞	w0LP/HP	Q	
NLP H(s)	1.8478	1	1	1	0.541184111	
NHP H(s)	1.8478	1	1	1	0.541184111	
K	**Kf**	**Km**	**KmR**	**alpha**		
1.33334	18849.6	10000	30000	0.74999625		
NHP Crt Pars	**C1 = Q**	**C2 = rt(3)**	**R1 = 1**	**R2 =1/(rt(3)Q)**	**R**	**R/3**
	0.541184111	1.732050808	1	1.066827827	1	0.333333333
Scale by w0HP	0.541184111	1.732050808	1	1.066827827	1	0.333333333
HP Crt Params	**C1new=C1/(Km*Kf)**	**C2new**	**R1new=R1*Km**	**R2new=R2*Km**	**Rnew=R*KmR**	**Rnew/3**
	2.87106E-09	9.18879E-09	10000	10668.27827	30000	10000
Input Attenuation	**C3 = (1 - alpha)C1**	**C4 = alpha*C1**				
	7.17777E-10	2.15329E-09				

SOLUTION TO 21.33. Using MATLAB,

```
»fp = 5e3; fs = 1.5e3;
»wp = 2*pi*fp; ws = 2*pi*fs;
»Amax = 3; Amin = 40;
»n = buttord(wp,ws,Amax,Amin,'s')
n =4

»[z,p,k] = buttap(n)
z =[]
p =
-3.8268e-01 + 9.2388e-01i
 -3.8268e-01 - 9.2388e-01i
 -9.2388e-01 + 3.8268e-01i
```

-9.2388e-01 - 3.8268e-01i
k =   1

»d1 = real(poly([p(1),p(2)]))
d1 =
1.0000e+00   7.6537e-01   1.0000e+00
»d2 = real(poly([p(3),p(4)]))
d2 =
1.0000e+00   1.8478e+00   1.0000e+00

In general,

$$H_{NHP}(s) = H_{NLP}\left(\frac{1}{s}\right) = \frac{1}{\left(\frac{1}{s}\right)^2 + \frac{\omega_{0LP}}{Q}\left(\frac{1}{s}\right) + (\omega_{0LP})^2} = \frac{1}{(\omega_{0LP})^2} \times \frac{s^2}{s^2 + \frac{(1/\omega_{0LP})}{Q}s + (1/\omega_{0LP})^2}$$

$$= \frac{(\omega_{0HP})^2 s^2}{s^2 + \frac{\omega_{0HP}}{Q}s + (\omega_{0HP})^2}$$

The S&K Saraga design for d1 is given by the following excel spreadsheet:

INPUT:	**d(1)**	**d(2)**	**KNLP/NHP∞**	**w0LP/HP**	**Q**	
NLP H(s)	0.76537	1	1	1	1.306557613	
NHP H(s)	0.76537	1	1	1	1.306557613	
K	**Kf**	**Km**	**KmR**	**alpha**		
1.33334	31416	45260.48336	60000	0.74999625		
NHP Crt Pars	**C1 = Q**	**C2 = rt(3)**	**R1 = 1**	**R2 =1/(rt(3)Q)**	**R**	**R/3**
	1.306557613	1.732050808	1	0.441886576	1	0.333333333
Scale by w0HP	1.306557613	1.732050808	1	0.441886576	1	0.333333333
HP Crt Params	**C1new=C1/(Km*Kf)**	**C2new**	**R1new=R1*Km**	**R2new=R2*Km**	**Rnew=R*KmR**	**Rnew/3**
	9.18879E-10	1.21812E-09	45260.48336	20000	60000	20000
Input Attenuation	**C3 = (1 - alpha)C1**	**C4 = alpha*C1**				
	2.29723E-10	6.89156E-10				

The S&K Saraga design for d2 is given by the following excel spreadsheet:

INPUT:	d(1)	d(2)	KNLP/NHP∞	w0LP/HP	Q	
NLP H(s)	1.8478	1	1	1	0.541184111	
NHP H(s)	1.8478	1	1	1	0.541184111	
K	Kf	Km	KmR	alpha		
1.33334	31416	20000	60000	0.74999625		
NHP Crt Pars	C1 = Q	C2 = rt(3)	R1 = 1	R2 =1/(rt(3)Q)	R	R/3
	0.541184111	1.732050808	1	1.066827827	1	0.333333333
Scale by w0HP	0.541184111	1.732050808	1	1.066827827	1	0.333333333
HP Crt Params	C1new=C1/(Km*Kf)	C2new	R1new=R1*Km	R2new=R2*Km	Rnew=R*KmR	Rnew/3
	8.61319E-10	2.75664E-09	20000	21336.55655	60000	20000
Input Attenuation	C3 = (1 - alpha)C1	C4 = alpha*C1				
	2.15333E-10	6.45986E-10				

This completes the design.

SOLUTION TO 21.34. For the woofer,

$$H(s) = \frac{8}{Ls+8} = \frac{\frac{8}{L}}{s+\frac{8}{L}}$$

Thus, $\frac{8}{L} = 2000 \times 2\pi \qquad L = 636\ \mu\text{H}.$

For the tweeter,

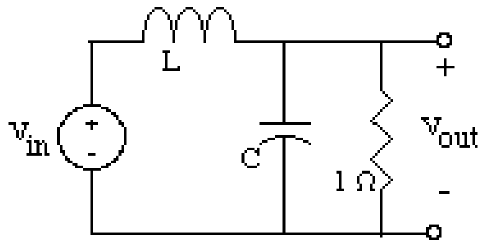
$$H(s) = \frac{8}{\frac{1}{Cs}+8} = \frac{8Cs}{8Cs+1} = \frac{s}{s+\frac{1}{8C}}$$

Thus, $\frac{1}{8C} = 2000 \times 2\pi \qquad C = 9.95\ \mu\text{F}.$

SOLUTION TO 21.35. For the woofer,

$$H_{NLP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

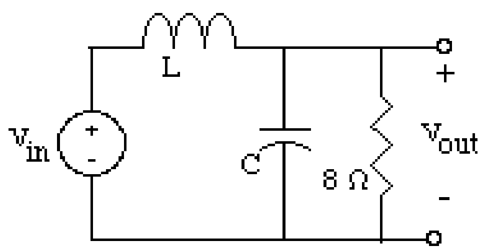
The transfer function of the following circuit



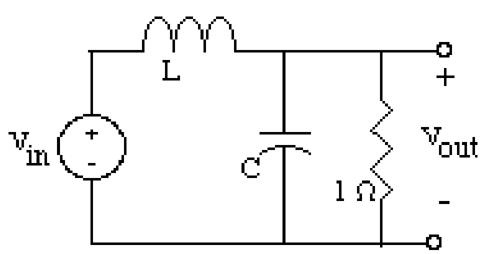
is

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{C}s + \frac{1}{LC}}$$

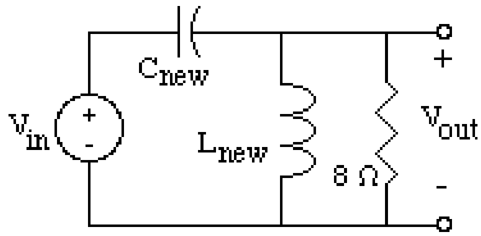
Thus $\sqrt{2} = \frac{1}{C}$ $\quad C = 0.70711$ F and since $\frac{1}{LC} = 1$, $L = \sqrt{2} = 1.4142$ H. Frequency scaling the element values by $K_f = 4000\pi$ and magnitude scaling by $K_m = 8$ yields $C = \frac{0.70711}{K_m K_f} = 7.0337$ μF and $L = \frac{1.4142 K_m}{K_f} = 0.90032$ mH:



For the tweeter we first realize the NLP Butterworth transfer function as above to obtain as above



with Thus $C = 0.70711$ F and $L = \sqrt{2} = 1.4142$ H. We now apply the frequency transformation $s \rightarrow \frac{\omega_C}{s}$ to each element (capacitors become inductors and inductors become capacitors according to figure 21.24) and we obtain the HP circuit topology



where C_{new} = 7.0337 μF and L_{new} = 0.90032 mH.

SOLUTION TO 21.36. Consider figure (a). Let the current entering the RC network from Z_1 be dentoted by I_{fa}. Let the voltage from this point to ground be denoted V_{fa}. Then

$$V_{out,a} = V_{fa} + I_{fa}Z_1 = H_a(s)V_{in}$$

For figure (b) with a similar denotation of voltage and current, we have

$$V_{out,b} = V_{fb} + \left(I_{fb} + \frac{V_{fb}}{\frac{Z_1}{k-1}} \right)\frac{Z_1}{k} = V_{fb} + I_{fb}\frac{Z_1}{k} + \frac{k-1}{k}V_{fb} = \frac{1}{k}\left((2k-1)V_{fb} + \frac{I_{fb}}{Z_1} \right)$$

If

$$(2k-1)V_{fb} \quad V_{fa} \text{ and } I_{fb} \quad I_{fa} \qquad\qquad (**)$$

then

$$V_{out,b} \quad \frac{1}{k}H_a(s)V_{in} = \frac{V_{out,a}}{k}$$

For gain enhancement, k < 1. However, for the (**) to be valid, we require that $\frac{Z_1}{k-1}$ be large relative to what it sees in the RC network. Hence, in general, k must be close to 1. Thus only small gain enhancements are possible. For such a potentially sensitive approach to gain enhancement, it might be better simply to add another op amp stage as op amps are comparatively inexpensive.