CHAPTER 22 PROBLEM SOLUTIONS

SOLUTION TO PROBLEM 22.1

(a) For figure P22.1a, $T_0 = 2$ and $\omega_0 = \pi$. Let $t_0 = -1$ in equation 22.5b. Then $f(t) = \delta(t)$ and

$$c_n = 0.5\int_{-1}^{1}\delta(t)e^{-jn\pi t}dt = 0.5 \quad \text{for all n}$$

From equation 22.6, $a_n = 1$ and $b_n = 0$ for all n. Finally from equation 22.2

$$f(t) = 0.5 + \sum_{n=1}^{\infty}\cos(n\pi t)$$

(b) For figure P22.1b, $T_0 = 2$ and $\omega_0 = \pi$. Let $t_0 = 0$ in equation 22.5b. Then $f(t) = -\delta(t-1)$ and

$$c_n = -0.5\int_{-1}^{1}\delta(t-1)e^{-jn\pi t}dt = -0.5e^{-jn\pi}$$

From equation 22.6, $b_n = 0$ for all n, and

$a_n = -1$ for n even

$a_n = 1$ for n odd

Finally from equation 22.2

$$f(t) = -0.5 + \cos(\pi t) - \cos(2\pi t) + \cos(3\pi t) - \cos(4\pi t) + \ldots$$

SOLUTION TO PROBLEM 22.2

(a) $T_0 = 1$ and $\omega_0 = 2\pi$. Let $t_0 = 0$ in equation 22.5b. Then

$$f(t) = e^{-(\ell n(2))t}$$

and from equation 22.5b

$$c_n = \int_0^1 e^{-(\ell n(2))t}e^{-j2n\pi t}dt = \int_0^1 e^{-(\ell n(2)+j2n\pi)t}dt = \frac{-1}{(\ell n(2)+j2n\pi)}\left[e^{-(\ell n(2)+j2n\pi)} - 1\right]$$

$$= \frac{0.5}{(\ell n(2)+j2n\pi)}$$

(b) Using the above result for c_n

$c_0 = \dfrac{0.5}{\ln 2} = 0.7213,$

$c_1 = \dfrac{0.5}{\ln 2 + j2\pi} = 0.7213e^{-j1.4609}$

$c_2 = \dfrac{0.5}{\ln 2 + j4\pi} = 0.397e^{-j1.516}$

From equation 22.6

$d_0 = c_0 = 0.7213$

$d_1 = 2|c_1| = 0.158$,

$\theta_1 = -1.461 \times 180/\pi = -83.7^o$

$d_2 = 2|c_2| = 0.0795$

$\theta_2 = -1.516 \times 180/\pi = -86.84^o$

Thus, f(t) in the form of equation 22.3 is

$f(t) = 0.7213 + 0.158\cos(2\pi t - 83.7^o) + 0.0795os(2\pi t - 86.84^o)$

SOLUTION TO PROBLEM 22.3.

(a) $T_0 = 1$ and $\omega_0 = 2\pi$. Let $t_0 = 0$ in equation 22.5b. Then

$$f(t) = e^{-(\ell n(2))t}\left[u(t) - u(t-0.5)\right]$$

and from equation 22.5b

$$c_n = \int_0^{0.5} e^{-(\ell n(2))t} e^{-j2n\pi t}dt = \int_0^{0.5} e^{-(\ell n(2)+j2n\pi)t}dt = \frac{-1}{(\ell n(2)+j2n\pi)}\left[e^{-0.5(\ell n(2)+j2n\pi)} - 1\right]$$

$$= \frac{1 - \frac{(-1)^n}{\sqrt{2}}}{(\ell n(2)+j2n\pi)}$$

(b) Using the above result for c_n, and MATLAB to evaluate the numerical result,

```
»n= 0;
»c0= (1- (-1)^n/sqrt(2))/(log(2) + j*2*n*pi
c0
   4.2256e-01
»n=1;
»c1= (1- (-1)^n/sqrt(2))/(log(2) + j*2*n*pi)
c1 =
  2.9612e-02-  2.6843e-01i
»abs(c1)
ans =
  2.7006e-01
»degreec1=angle(c1)*180/pi
degreec1 =
 -8.3705e+01
»n=2;
 »c2= (1- (-1)^n/sqrt(2))/(log(2) + j*2*n*pi)
c2 =
 1.2817e-03-  2.3237e-02i
```

```
»abs(c2)
ans =
   2.3272e-02
degreec2= angle(c2)*180/pi
degreec2 =
-8.6843e+01
```

From equation 22.6 and equation 22.3

$$f(t) = 0.4226 + 0.54\cos(2\pi t - 83.7^o) + 0.04654os(2\pi t - 86.84^o)$$

SOLUTION PROBLEM 22.4. (a) $f(t) = \cos(4t)\sin(2t) = 0.5[\sin(6t) - \sin(2t)]$.
The fundamental angular frequency of f(t) is ω_0= 2 rad/s. The given f(t) can be expressed as
$f(t) = -0.5 \sin(\omega_0 t) + 0.5\sin(3\omega_0 t)$. Observe that b_1= -0.5, b_3= 0.5 and all other a_i and b_i are zero.
From equation 22.4 , d_1= 0.5 $\underline{/-90^o}$ and d_3 = 0.5 $\underline{/90^o}$. From equations 22.6a and 22.6b.
c_1 = 0.25j and c_3 = -0.25j. All other c_n are zero for n positive
(b) $f(t) = \sin^2(4t)\cos^2(8t)= 0.5[1 - \cos(8t)]x0.5[1+ \cos(16t)]$

$$= 0.25 [1 - \cos(8t) + \cos(16t) - \cos(8t)\cos(16t)]$$

$$= 0.25 - 0.375 \cos(8t) + 0.25\cos(16t) - 0.125\cos(24t)$$

The fundamental angular frequency of f(t) is ω_0= 8 rad/s. The given f(t) can be expressed as
$f(t) = 0.25 -0.375 \cos(\omega_0 t) + 0.25\cos(2\omega_0 t) - 0.125sicos(3\omega_0 t)$. Observe that a0= 0.5, a_1=-0.375, a_2= 0.25 , a_3= -0.125 and all other a_i and b_i are zero.

Fom equation 22.4,

d_0= 0.25, d_1= 0.375 $\underline{/180^o}$, d_2 = 0.25 $\underline{/0^o}$, and d_3 = 0.125 $\underline{/180^o}$,

From equations 22.6a and 22.6b.

c_0 = 0.25, c_1 = -0.375 and c_2 = 0.25, c3 = -0.125. All other c_n are zero for n positive.

(c)

$$f(t) = [2 + 1.5 \sin(500t)- 2\cos(2000t)]\cos(10^6 t)]$$
$$=2\cos(10^6 t) + 0.75\sin(1000500t) + 0.75\sin(999500t)$$
$$- \cos(1002000t) - \cos(998000t)$$

The fundamental angular frequency of f(t) is ω_0= 500 rad/s. The given f(t) can be expressed as
$f(t) =2\cos(2000\omega_0 t) + 0.75\sin(2001\omega_0 t) + 0.75\sin(1999\omega_0 t) - \cos(2004\omega_0 t) - \cos(1996\omega_0 t)$
Observe that: a_{1996} = -1, b_{1999}= 0.75, a_{2000}= 2, b_{2001}= 0.75 , a_{2004}= -1, and all other a_i and b_i are zero. From equation 22.4 , d_{1996} = 1 $\underline{/180^o}$, d_{1999} = 0.75 $\underline{/-90^o}$, d_{2000} = 2 $\underline{/0^o}$,
d_{2001} = 0.75$\underline{/-90^o}$, d_{2004} = 1 $\underline{/180^o}$, and all other d_i are zero.
From equations 22.6a and 22.6b.
c_{1996} = -0.5 , c_{1999} = -j0.375, c_{2000} = 1 ,

c_{2001} = -j0.375^o, dc_{2004} =-0.5 , and all other c_n are zero for n positive..

SOLUTION PROBLEM 22.5

By inspesction, the derivative of f(t) is

$$f'(t) = \frac{A}{T} - \sum_{n=-\infty}^{\infty} A\delta(t - nT) = \frac{A}{T} - f_\delta(t)$$

where $f_\delta(t)$ is shown in figure 22/7, with its Fourier series given by equation 22.20b, i.e.

$$f_\delta(t) = \frac{A}{T} + \frac{2A}{T}\sum_{n=1}^{\infty}\cos(n\omega_0 t)$$

Therefore

$$f'(t) = -\frac{2A}{T}\sum_{n=1}^{\infty}\cos(n\omega_0 t)$$

The dc component is the average value of f(t) and is given by 0.5A. Other terms in the Fourier series of f(t) are obtained by integrating the cosine terms in the above expression. The result is

$$f(t) = 0.5A - \frac{A}{n\pi}\sum_{n=1}^{\infty}\sin(n\omega_0 t)$$

SOLUTION TO PROBLEM 22.6

Denote by $f_5(t)$ the waveform of figure P22.5, with A= 0.5 and T =1. Then by inspection

$$f(t) = f_{prob5}(-t) + 0.5$$

Substituting the result of problem 22.5 into the above equation, we have

$$f(t) = 0.75 + \frac{0.5}{n\pi}\sum_{n=1}^{\infty}\sin(n\omega_0 t)$$

SOLUTION PROBLEM 22.7

Consider the square wave g(t) shown in figure 22.4 with its Fourier series given by equation 22.13. By inspection, the derivative of f(t) is

$$f'(t) = -\frac{4}{T}g(t)$$ $$f'(t) = -\frac{4}{T}g(t)$$

Substituting equation 22.13 into the above expression, we have

$$f'(t) = -\frac{8A}{\pi T}\sum_{n=1,odd}^{\infty}\frac{\sin(n\omega_0 t)}{n}$$

The dc component is the average value of f(t) and is given by 0.5A. Other terms in the Fourier series of f(t) are obtained by integrating the sine terms in the above expression. The result is

$$f(t) = 0.5A + \frac{4A}{\pi^2}\sum_{n=1,odd}^{\infty}\frac{\cos(n\omega_0 t)}{n^2}$$

SOLUTION PROBLEM 22.8

The method used below is simpler than that suggested in the hint.

We first sketch the waveform of f'(t) and observe that it may be expressed as the sum of two periodic rectangu pulse trains:

$$f'(t) = \frac{1}{\alpha T}[\ f_p(t + 0.5\alpha T) + f_p(t - 0.5\alpha T)]$$

where $f_p(t)$ is sketched in figure 22.8. Using equation 22.23, we have

$$f'(t) = \frac{1}{\alpha T}\{\ [\alpha A + \sum_{n=1}^{\infty}\frac{2A\sin(n\alpha\pi)}{n\pi}\cos(n\omega_0(t+0.5\alpha T))]$$

$$-[\alpha A + \sum_{n=1}^{\infty}\frac{2A\sin(n\alpha\pi)}{n\pi}\cos(n\omega_0(t-0.5\alpha T))]\}$$

$$= \frac{1}{\alpha T}\{\sum_{n=1}^{\infty}\frac{2A\sin(n\alpha\pi)}{n\pi}[\cos(n\omega_0(t+0.5\alpha T))-\cos(n\omega_0(t-0.5\alpha T)]$$ \

$$= \frac{1}{\alpha T}\left[\sum_{n=1}^{\infty} \frac{-4A\sin^2(n\alpha\pi)}{n\pi} \sin(n\omega_0 t)\right]$$

The dc component is the average value of f(t) and is given by αA. Other terms in the Fourier series of f(t) are obtained by integrating the sine terms in the above expression. The result is

$$f(t) = \alpha A + \frac{2A}{\alpha\pi^2}\left\{\sum_{n=1}^{\infty} \left[\frac{\sin(n\alpha\pi)}{n}\right]^2 \cos(n\omega_0 t)\right\}$$

SOLUTION TO PROBLEM 22.9

We first sketch the waveform of f'(t) and observe that it may be expressed as the sum of a periodic rectangular pulse train and a periodic impulse train:

$$f'(t) = \frac{1}{\alpha T} f_p(t + 0.5\alpha T) - f_\delta(t)$$

where $f_p(t)$ is sketched in figure 22.8, and $f_\delta(t)$ in figure 22.7.

Using equations 22.23 and 22.20b, we have

$$f'(t) = \frac{1}{\alpha T}\left[\alpha A + \sum_{n=1}^{\infty} \frac{2A\sin(n\alpha\pi)}{n\pi} \cos(n\omega_0(t+0.5\alpha T))\right]$$

$$-\left[\frac{A}{T} + \frac{2A}{T}\sum_{n=1}^{\infty} \cos(n\omega_0 t)\right]$$

$$= \frac{2A}{T}\left\{\sum_{n=1}^{\infty}\left[\frac{\sin(n\alpha\pi)}{n\alpha\pi}\cos(n\omega_0(t+0.5\alpha T)) - \cos(n\omega_0 t)\right]\right\}$$

The dc component is the average value of f(t) and is given by $0.5\alpha A$. Other terms in the Fourier series of f(t) are obtained by integrating the sine terms in the above expression. The result is

$$f(t) = 0.5\alpha A + \frac{A}{n\pi}\left\{\sum_{n=1}^{\infty}\left[\frac{\sin(n\alpha\pi)}{n\alpha\pi}\sin(n\omega_0(t+0.5\alpha T)) - \sin(n\omega_0 t)\right]\right\}$$

It remains to rewrite the expression in the form of equation 22.2.
To this end, let $b = \sin(n\alpha\pi)/(n\alpha\pi)$ and re-write the terms within [ ] as follows:

$$b\sin(n\alpha\pi)\cos(n\omega_0 t) + \{b\cos(n\alpha\pi) - 1\}\sin(n\omega_0 t)$$

Hence, f(t) in the form of equation 22.2 has the coefficients, for n=1,2...

$$a_n = \frac{A}{\alpha\pi^2 n^2}\sin^2(n\alpha\pi)$$

$$b_n = \frac{A}{\alpha\pi^2 n^2}\{\sin(n\alpha\pi)\cos(n\alpha\pi) - n\alpha\pi\}$$

$$d_n = \sqrt{a_n^2 + b_n^2} = \frac{A}{\alpha\pi^2 n^2}\sqrt{\sin^4(n\alpha\pi) + \{\sin(n\alpha\pi)\cos(n\alpha\pi) - n\alpha\pi\}^2}$$

$$= \frac{A}{\alpha\pi^2 n^2}\sqrt{\sin^2(n\alpha\pi) + (n\alpha\pi)[n\alpha\pi - \sin(2n\alpha\pi)]}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

The result is item 6 of table 22.4

SOLUTION PROBLEM 22.10.

Observe that the present f(t) can be derived from that of problem 22.9 by (a) replacing t by -t; and (b) replacing α by β. Thus the Fourier series for f(t) is:

$$f(t) = 0.5\beta A + \sum_{n=1}^{\infty}[\, a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\,]$$

where

$$a_n = \frac{A}{\beta\pi^2 n^2}\sin^2(n\beta\pi)$$

$$b_n = \frac{-A}{\beta\pi^2 n^2}\{\sin(n\beta\pi)\cos(n\beta\pi) - n\beta\pi\}$$

$$d_n = \sqrt{a_n^2 + b_n^2} = \frac{A}{\beta\pi^2 n^2}\sqrt{\sin^4(n\beta\pi) + \{\sin(n\beta\pi)\cos(n\beta\pi) - n\beta\pi\}^2}$$

$$= \frac{A}{\beta\pi^2 n^2}\sqrt{\sin^2(n\beta\pi) + (n\beta\pi)[n\beta\pi - \sin(2n\beta\pi)]}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

SOLUTION PROBLEM 22.11. Following the hint, we have the second derivative of f(t) given by

$$f''(t) = \frac{1}{\alpha(1-\alpha)T} f_\delta(t+\alpha T) - \frac{1}{\alpha(1-\alpha)T} f_\delta(t)$$

where fd(t) is given in example 22.5. Notice that we have focused on the part of the waveform over [–αT, (1–α)T]. By making use of equation 22.20b, we obtain

$$f''(t) = \frac{2A}{\alpha(1-\alpha)T^2}\sum_{n=1}^{\infty}\left[\cos(n\omega_0 t + 2n\alpha\pi) - \cos(n\omega_0 t)\right]$$

$$= \frac{-4A}{\alpha(1-\alpha)T^2}\sum_{n=1}^{\infty}\left[\sin(n\omega_0 t + n\alpha\pi)\sin(n\alpha\pi)\right]$$

Therefore,

$$f(t) = \left[f(t)\right]_{ave} + \frac{4A}{\alpha(1-\alpha)T^2}\sum_{n=1}^{\infty}\left[\frac{\sin(n\alpha\pi)}{(n\omega_0)^2}\sin(n\omega_0 t + n\alpha\pi)\right]$$

$$= \frac{A}{2} + \sum_{n=1}^{\infty}\left[\frac{A\sin(n\alpha\pi)}{n^2\pi^2\alpha(1-\alpha)}\cos\left(n\omega_0 t + (n\alpha - 0.5)\pi\right)\right]$$

Letting T = 1 and α = 0.25 we obtain,

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty}\left[\frac{16A\sin\left(n\frac{\pi}{4}\right)}{3n^2\pi^2}\cos\left(n2\pi t + (0.25n - 0.5)\pi\right)\right]$$

Therefore, $d_0 = 0.5A$, and

$$d_n = \frac{16A\sin\left(n\frac{\pi}{4}\right)}{3n^2\pi^2}.$$

It follows that d_1 = 0.38211A and d_2 = 0.13509A.

SOLUTION TO PROBLEM 22.12.

Denote by $f_p(t, \alpha)$ the period rectangular waveform of figure 22.5, with A=1.

Then we can express the present f(t), with T= 4, as the sum of 3 terms:

$$f(t) = 3f_p(t-0.125T, 0.25) + 4\, f_p(t - 0.5T, 0.5) -2$$

From equation 22.14b, and equation 22.12c, for n= 1,2,...

$$c_n = \frac{3}{\pi n}\sin(0.25n\pi)e^{-jn\omega_0 x0.125T} + \frac{4}{\pi n}\sin(0.5n\pi)e^{-jn\omega_0 x0.5T}$$

$$= \frac{3}{\pi n}\sin(0.25n\pi)e^{-j0.25n\pi} + \frac{4}{\pi n}\sin(0.5n\pi)e^{-jn\pi}$$

The numerical values of the first few Fourier series coefficients are:

$$c_0 = \text{averge value of } f(t) = \frac{1}{4}(1 + 4 - 2) = 0.75$$
$$c_1 = \frac{3}{\pi}\sin(0.25\pi)e^{-j0.25\pi} + \frac{4}{\pi}\sin(0.5\pi)e^{-j\pi} = -0.7958 - j\,0.4775$$
$$c_2 = \frac{3}{2\pi}\sin(0.5\pi)e^{-j0.5\pi} + \frac{4}{2\pi}\sin(\pi)e^{-j2\pi} = -j0.4775$$

Solution Problem 22.13

(a) For sinusoidal steady analysis, the transfer function is

$$H(j\omega) = \frac{Y_L}{Y_L + Y_C + Y_R} = \frac{1}{1 + Z_L Y_C + Z_L Y_R} = \frac{1}{(1 - \omega^2 LC) + j\frac{\omega L}{R}} = \frac{1}{(1 - 4\times 10^{-5}\omega^2) + j\times 10^{--3}\omega}$$

The transfer function evaluated at various input frequencies are listed below.

$H(0) = 1$

$H(j377) = 0.2128\ \underline{/-175.4}^o$

$H(j3x377) = 0.0199\underline{/-178.7}^o$

$H(j5x377) = 0.0071\underline{/-179.2}^o$

Using equation 15.7 and superposition, we obtain the steady state output voltage (in V):

$$v_{out}(t) = 200 + 200\times\sqrt{2}\times 0.2128\cos(377t - 175.4^o) + 60\times\sqrt{2}\times 0.0199\cos(3\times 377t + 30^o - 178.7^o)$$
$$+ 80\times\sqrt{2}\times 0.0071\cos(5\times 377t + 50^o - 17.2^o)$$

$$= 200 + \sqrt{2}\times 42.55\cos(377t - 175.4^o) + \sqrt{2}\times 1.196\cos(3\times 377t - 148.7^o)$$
$$+ \sqrt{2}\times 0.5668\cos(5\times 377t - 129.2^o)$$

(b) From equation derived in P11.39,

$$V_{out,eff} = \sqrt{200^2 + 42.55^2 + 1.196^2 + 0.5668^2} = 204.48 \text{ V}$$

The average power absorbed by the 10 kΩ resistor is

$$P_{av} = \frac{204.48^2}{10^4} = 4.1812 \text{ W}$$

Solution Problem 22.14

One correction in the problem statement: in the angle expression, 5000 should be 10000.

(a) Using the identity cos(x)cos(y)= 0.5cos(x+y) + 0.5cos(x-y), we have

$$v_{in}(t) = 0.1\cos(998{,}000t) + 0.2\cos(999{,}000t) + 2\cos(1{,}000{,}000t)$$
$$+ 0.2\cos(1{,}001{,}000t) + 0.1\cos(1{,}002{,}000t) \quad \text{V}$$

(b) The transfer function has a constant magnitude of 10, and a phase shift proportional to the deviation from ω_c. at $\omega = \omega_c + 2\omega_m$, the phase shift is -9 degrees. From these facts, we can write directly

$$v_{out}(t) = \cos(998{,}000t + 9^o) + 2\cos(999{,}000t + 4.5^o) + 20\cos(1{,}000{,}000t)$$
$$+ 2\cos(1{,}001{,}000 - 4.5^o) + \cos(1{,}002{,}000t - 9^o) \quad \text{V}$$

(c) Using the identity

$$\cos(x)\cos(y) = 2\cos(\frac{x+y}{2})\cos(\frac{x+y}{2})$$

we can group the terms in $v_{out}(t)$ and re-write it as

$$v_{out}(t) = 2\cos(2\omega_m t - 9^o)\cos(\omega_c t) + 4\cos(\omega_m t - 4.5^o)\cos(\omega_c t) + 20\cos(\omega_c t)$$
$$= [20 + 4\cos(\omega_m t - 4.5^o) + 2\cos(2\omega_m - 9^o)]\cos(\omega_c t) = g(t)\cos(\omega_c t)$$

Thus

$$g(t) = 20 + 4\cos(\omega_m t - 4.5^o) + 2\cos(2\omega_m t - 9^0)$$

With t_d = 78.54 μs, then $\omega_m t_d = 1000 \times 78.54\text{x}10^{-6} = 0.07854$ rad, or 4.5 degrees. We have

$$10 f(t - t_d) = 20[1 + 0.2\cos(\omega_m(t - td) + 0.1\cos(2\omega_m(t - td)]$$

$$= 20[1 + 0.2\cos(\omega_m t - 4.5^o) + 0.1\cos(2\omega_m t - 9^o)] = g(t)$$

SOLUTION PROBLEM 22.15. (a) This proof is a special case of the general proof given in the solution to Problem 22.16. See the solution to problem 22.16, below.

FOR THE REMAINING PARTS, WE USE THE FOLLOWING MATLAB CODE:

```
% chapter 22, problem 15.
%part (b).
Vmax= 30*pi;
Vmin=0;
T=4;R = 1; C = 1;
voutmin= Vmin+(Vmax - Vmin)/(1+ exp(0.5*T/(R*C)))
voutmax = Vmax - (Vmax -Vmin)/(1+ exp(0.5*T/(R*C)))
t1= 0: 0.05: 2;
vseg1= Vmax+(voutmin- Vmax)*exp((-t1/(R*C)));
t2= 2:0.05: 4;
vseg2= Vmin+(voutmax- Vmin)*exp(-(t2-T/2)/(R*C));
t= [ t1 t2];
v= [vseg1 vseg2];
plot(t,v)
grid
xlabel('t in seconds')
ylabel('Steady State Output Voltage in V')
```

```
%part (c)
error1= 100*(12.235- voutmin)/voutmin
error2= 100*(82.013 -  voutmax)/voutmax
```

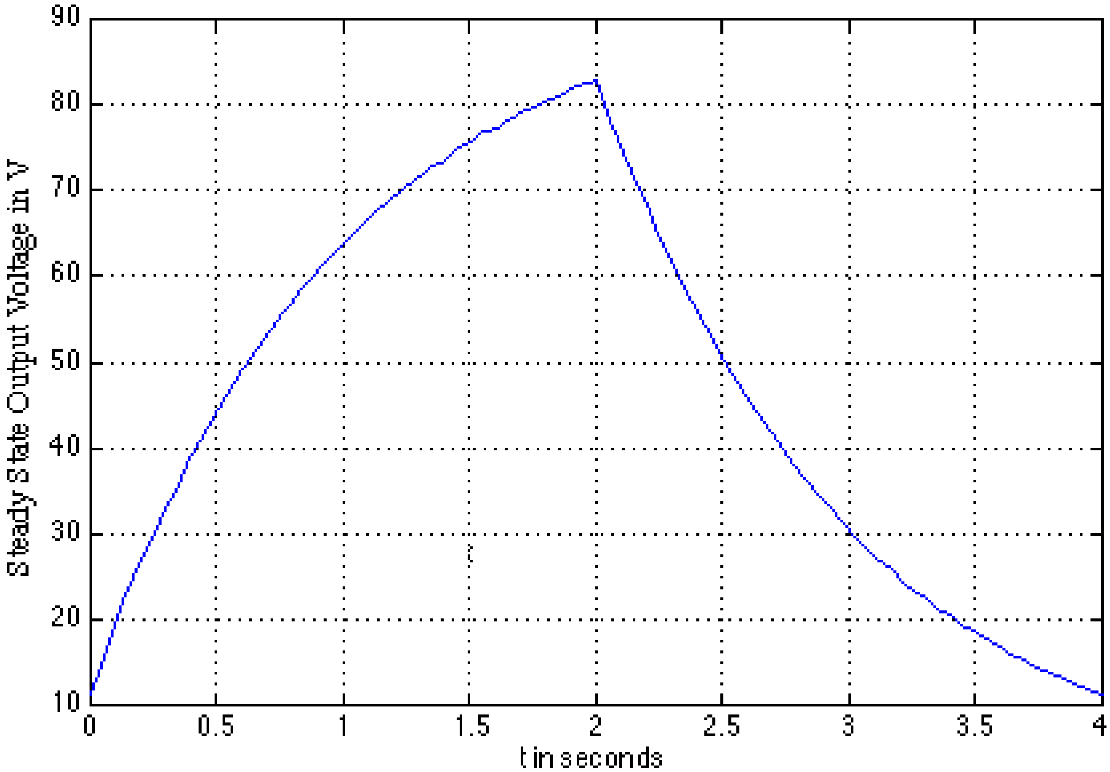
TO OBTAIN

(b)

```
voutmin =
  1.1235e+01
voutmax =
  8.3013e+01
```

(c)

```
error1 =
  8.9045e+00
error2 =
 -1.2048e+00
```



SOLUTION PROBLEM 22.16. CORRECTION: In the problem statement, $\frac{s}{\tau}$ should be τs.

For simplicity, let us consider the case when the transfer function is a voltage ratio, i.e., $H(s) = V_{out}/V_{in}$.

If a constant input $v_{in}(t) = V_{con}$ is applied to the stable network, then the $v_{out}(\infty) = KV_{con}$, independent of the initial conditions. This is because the zero-input response for a stable network approaches zero as t approaches infinite. To see this observe that the zero-state response is given by

$$v_{out}(t) = L^{-1}\left\{\frac{K/\tau}{s+1/\tau} \times \frac{V_{con}}{s}\right\} = L^{-1}\left\{\frac{KV_{con}}{s} - \frac{KV_{con}}{s+1/\tau}\right\} = KV_{con}\left(1-e^{-t/\tau}\right)u(t)$$

from which $v_{out}(\infty) = KV_{con}$ as asserted.

For the remainder of our proof we make use of the fact that in steady state, $v_{out}(t+T) = v_{out}(t)$ with t = 0 in our case. Specifically, after the first order network has reached steady state, the $v_{out}(t)$ waveform will be periodic as shown in figure P22.15c, where the time reference has been chosen so that $v_{out,min}$ occurs at t = 0.

Recall equation 8.19

$$x(t) = x(\infty) + \left[x(t_0^+) - x(\infty)\right]e^{-(t-t_0)/\tau}$$

Applying this equation to the interval [0, T/2], we have

$$b_0 = KV_{\max} + \left(a_0 - KV_{\max}\right)e^{-0.5T/\tau} \equiv KV_{\max} + \left(a_0 - KV_{\max}\right)\alpha \qquad (1)$$

Note that $\alpha = e^{-0.5T/\tau}$. Similarly, applying equation 8.19 to the interval [T/2, T] leads to

$$a_0 = KV_{\min} + \left(b_0 - KV_{mn}\right)\alpha \qquad (2)$$

Equations (1) and (2) can be written as a single matrix equation

$$\begin{bmatrix} -\alpha & 1 \\ 1 & -\alpha \end{bmatrix}\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} (1-\alpha)KV_{\max} \\ (1-\alpha)KV_{\min} \end{bmatrix} \qquad (3)$$

Solving equation (3) by matrix inverse (or Cramer's rule) results in

$$\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \frac{-1}{\alpha^2-1}\begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}\begin{bmatrix} (1-\alpha)KV_{\max} \\ (1-\alpha)KV_{\min} \end{bmatrix} = \frac{K}{\alpha+1}\begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}\begin{bmatrix} V_{\max} \\ V_{\min} \end{bmatrix} = \frac{K}{\alpha+1}\begin{bmatrix} \alpha V_{\max} + V_{\min} \\ V_{\max} + \alpha V_{\min} \end{bmatrix}$$

$$\left[v_{out}(t)\right]_{\min} = a_0 = K\frac{\alpha V_{\max} + V_{\min}}{\alpha+1} = K\frac{\alpha V_{\max} + V_{\min} + \alpha V_{\min} - \alpha V_{\min}}{\alpha+1}$$

$$= KV_{\min} + \alpha\frac{V_{\max} - V_{\min}}{\alpha+1} = KV_{\min} + K\frac{V_{\max} - V_{\min}}{1+1/\alpha} = KV_{\min} + K\frac{V_{\max} - V_{\min}}{1+e^{0.5T/\tau}}$$

and

$$\left[v_{out}(t)\right]_{\max} = b_0 = K\frac{V_{\max} + \alpha V_{\min}}{\alpha + 1} = K\frac{V_{\max} + \alpha V_{\min} + \alpha V_{\max} - \alpha V_{\max}}{\alpha + 1}$$

$$= KV_{\max} - K\frac{V_{\max} - V_{\min}}{1 + 1/\alpha} = KV_{\max} - K\frac{V_{\max} - V_{\min}}{1 + e^{0.5T/\tau}}$$

This complete completes the derivation of the desired formulas.

SOLUTION TO PROBLEM 22.17

There is one correction in the problem statement. 1-kHz should be 5.1mHz.

(a) The transfer function is, according to equation 4.3, and using the given element values R_1 = 10 kΩ, R_f = 50 kΩ and C = 20 mF:

$$H(s) = \frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_1} = -\frac{Y_1}{Y_f} = -\frac{\frac{1}{R_1}}{Cs + \frac{1}{R_f}} = -\frac{\frac{R_f}{R_1}}{1 + R_f Cs} = \frac{-5}{1 + 1000s}$$

(b) Here we have We have K= -5, t = 1000 s, Vmax = 1 V, Vmin = -1 V and T = 1/f = 1/0.0051= 196.15 s. In using the equations derived in problem 22.16, we note that the subscripts min and max should be switched in this case because K is negative. The following MATLAB codes perform the needed numerical calculations.

```
f= 5.1e-3;
Cf=20e-3;
Rf=50e3;
Rs= 10e3;
T=1/f;
K= -Rf/Rs;
tau=Rf*Cf;
['part (b)']
Vmax=1;
Vmin=-1;
voutmax= K*( Vmin + (Vmax-Vmin)/(1+exp(0.5*T/tau)))
voutmin= K*( Vmax - (Vmax-Vmin)/(1+exp(0.5*T/tau)))
```

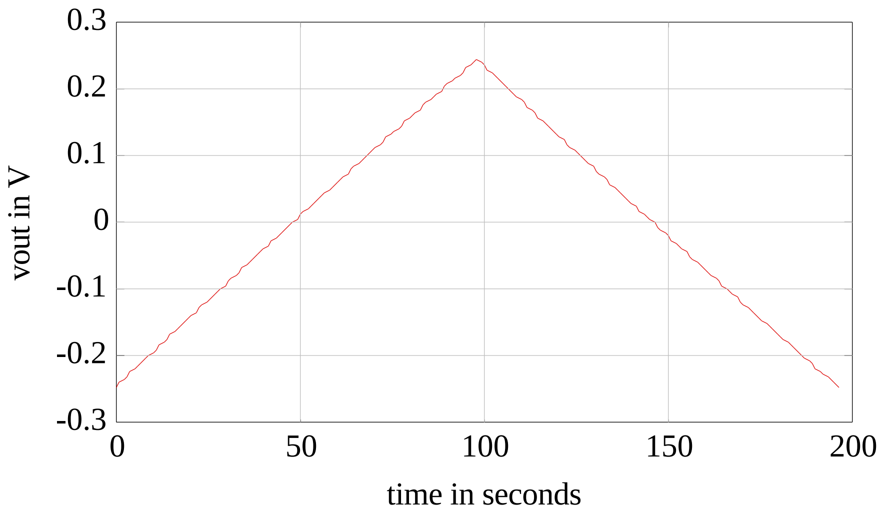
answers from MATLAB are:

```
voutmax =
    0.2449
voutmin =
   -0.2449
```

(c)

```
['part (c)']
```

```
t1= 0:0.005*T:0.5*T;
v1= 5 + (-0.245 -5).*exp(-(t1/tau)) ;
t2= 0.5*T:0.005*T:T;
v2= -5 + (0.245 +5).*exp(-(t2-0.5*T)/tau) ;
t=[t1,t2];
v=[v1,v2];
plot (t,v)
xlabel('time in seconds')
ylabel('vout in V')
grid
```



SOLUTION PROBLEM 22.18.

$$v_i(t) = \cos(\omega t) = \cos(6000t)$$

$$v_o(t) = 10v_i + v_i^2 + v_i^3$$
$$= 10\cos(\omega t) + \cos^2(\omega t) + \cos^3(\omega t)$$
$$= 10\cos(\omega t) + 0.5 + 0.5\cos(2\omega t) + 0.75\cos(\omega t) + 0.25\cos(3\omega t)$$
$$= 0.5 + 10.75\cos(\omega t) + 0.5\cos(2\omega t) \;\; 0.25\cos(3\omega t)$$

The effective values of various components of the output are:

dc	fundamental	2nd harmonic	3rd harmonic	total harmonic
0.5	$10.75/\sqrt{2}$	$0.5/\sqrt{2}$	$0.25/\sqrt{2}$	$0.559/\sqrt{2}$

The total harmonic distortion is

$$\frac{0.559}{10.75} \times 100\% = 5.2\%$$

and the average power at the fundamental frequency is

$$0.5 \times \frac{10.75^2}{100} = 0.5778 \text{ W}$$

SOLUTION TO PROBLEM 22.19.

$$v_o(t) = V_0 + V_1 \cos(\omega t) + V_2 \cos(2\omega t) + V_3 \cos(3\omega t)$$

We use equation 22.30 and the values of $v_o(t)$ read from the oscilloscope to compute V_k. The total harmonic distortion is given by

$$\text{H.D.} = \frac{\sqrt{V_2^2 + V_3^2}}{|V_1|} \times 100\ \%$$

MATLAb codes:

```
vo0= 10; vo60= 5.2; vo120= -4.6; vo180= -9.6;
V0= (vo0+2*vo60+2*vo120 +vo180)/6
V1= (vo0+ vo60 - vo120 -vo180)/3
V2= (vo0 - vo60 - vo120 +vo180)/3
V3= (vo0-2*vo60+2*vo120 -vo180)/6
% total harmonic distortion
HD= 100*sqrt( V2^2 +V3^2)/V1
```

The following answers are obtained frm MATLAB output.

V0 = 0.2667 V

V1 = 9.8000 V

V2 = -0.0667

V3 = 0 V

HD = 0.6803 (percent)

SOLUTION PROBLEM 22.20.

We shall follow the solution given in example 22.12, and only indicate the needed changes below. There are t corrections in example 22.12: (1) $20\cos(\omega\alpha T)$ should be $20\cos(0.5\omega\alpha T)$ and (2) $1.842\cos(\omega t)$ should be 1.8 $\cos(3\omega t)$.

The new input is

$$v_i(t) = 0.9 \cos(6000t)$$

The positive peak of the output sine wave is clipped for αT.

$$0.9 \times 20\cos(0.5\omega\alpha T) = 15 \rightarrow \alpha\pi = \cos^{-1}(15/18) \rightarrow \alpha = 0.1864$$

Equation 22.35 becomes

$$v_0(t) = -18\cos(\omega t) + 2a_1\cos(\omega t) + 2a_3\cos(3\omega t) + 2a_5\cos(5\omega t) + \ldots$$

Equations 2.36 becomes

$$v_{pp}(t) = [18\cos(\omega t) - 15] f_p(t)$$

Using A=1, and $\alpha = 0.1864$n item #2 of table 22.4, equation 22.37 becomes

$$f_p(t) = 0.1864 + 0.316\cos(\omega t) + 0.293\cos(2\omega t) + 0.208\cos(3\omega t)$$

$$+ 0.114\cos(4\omega t) + 0.0269\cos(5\omega t) - 0.0386\cos(6\omega t) + \ldots$$

and equation 22.38 becomes

$$v_{pp}(t) = 0.7164\cos(\omega t) + 0.5376\cos(3\omega t) + 0.2748\cos(5\omega t) +$$

$$\text{higher order odd harmonics} + \text{even harmonics}$$

Equation 22.39 becomes

$$v_0(t) = -16.57\cos(\omega t) + 1.076\cos(3\omega t) + 0.55\cos(5\omega t)$$

$$+ \text{higher order odd harmonics.}$$

from which the harmonic distortions are:

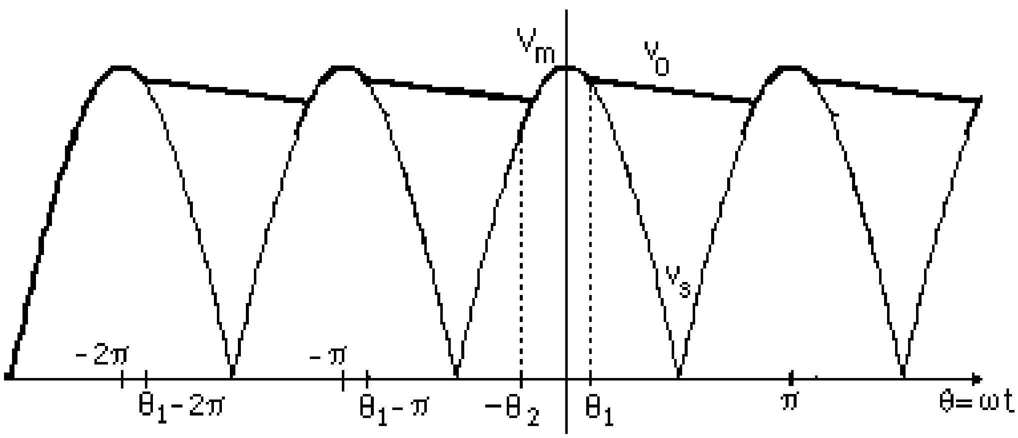
third order 1.076/16.57 = 6.595%

fifth order 0.55/16.57 = 3.319%

SOLUTION PROBLEM 22.21. Corrections: (1) $\cos(\theta_1)e^{\frac{-\theta_2-\theta_1+\pi}{\omega RC}} = \cos(\theta_2)$ should be $\cos(\theta_1)e^{-\left(\frac{\theta_2-\theta_1+\pi}{\omega RC}\right)} = \cos(\theta_2)$ and (2) on page 943, equation 22.46 , θ_2 should be $-\theta_2$, i.e., equation 22.46 should be $\cos(\theta_1)e^{-\left(\frac{\theta_2-\theta_1+\pi}{\omega RC}\right)}$.

The proof is similar to that given on page 943 for the half-wave rectifier case, except for some minor changes described below.

For the case of a full-wave rectifier, the output voltage waveform is a modification of figure 22.17 as shown below.



The exponential decay of $v_o(t)$ starts with the value $V_m\cos(\theta_1)$ at $\theta = \theta_1 - \pi$ (instead of $\theta_1 - 2\pi$, as in the half-wave rectifier case). Therefore, in equation 22.45, change T to T/2 and 2π to π. In other words

$$v_0(t) = \left(V_m\cos(\theta_1)\right)e^{-\left(\frac{t-\left(\frac{\theta_1}{\omega}-\frac{T}{2}\right)}{RC}\right)} = \left(V_m\cos(\theta_1)\right)e^{-\frac{\omega t-(\theta_1-\pi)}{\omega RC}}$$

In equation 22.46, make correction (2) above, and change 2π to π, i.e.,

$$\cos(\theta_1)e^{-\left(\frac{\theta_2-\theta_1+\pi}{\omega RC}\right)} = \cos(\theta_2)$$

The desired proof is complete.

SOLUTION PROBLEM P22.22

Refer to figure 22.17. Assuming $\theta_1 = 0$, we have

$$v_0(t) = V_m \exp\left(\frac{-t}{RC}\right) = V_m\left\{1 - \left(\frac{t}{RC}\right) + \left(\frac{t}{RC}\right)^2 - \left(\frac{t}{RC}\right)^3 + \dots\right\}$$

Assuming $\theta_2 = 0$, we compute the average value of $v_{o(t)}$ over the time interval [0, T]. For the case RC >> T, we can approximate $v_o(t)$ over this interval by keeping only the first two terms of the infinite series. Thus

$$v_0(t) \quad V_m\left[1 - \frac{t}{RC}\right]$$

which indicates that the plot of v_o vs. t over the interval [0 T] is approximately a straight line. Therefore the average of the $v_o(t)$ over [0 T] is equal to $v_o(T/2)$. Thus, for the case RC >> T, or equivalently $\omega RC >> 2\pi$,

$$V_{dc} \quad v_o\left(\frac{T}{2}\right) = V_m\left[1 - \frac{T}{2RC}\right] = V_m\left[1 - \frac{\pi}{\omega RC}\right]$$

SOLUTION PROBLEM 22.23

Given values are: C = 20e-6 F, R =100kΩ, V_m = 20 V; f = 60 Hz. From equation 22.48b

$$V_{dc} \quad \left(1 - \frac{\pi}{\omega RC}\right)V_m = \left(1 - \frac{1}{2fRC}\right)V_m = \left(1 - \frac{1}{2\times 60\times 10^5\times 20\times 10^{-6}}\right)\times 20 = 19.916 \text{ V}$$

To calculate the ripple factor, we first calculate θ_2 from equation 22.47,

and then use the result in equation 22.50.

$$\theta_2 = \cos^{-1}\left(\frac{2\pi}{RC}\right) = 0.1289 \text{ rad}$$

ripple factor $\frac{1 - \cos(\theta_2)}{\sqrt{3}\,[\,1 + \cos(\theta_2)]}$ =0.2406%

For the diode average and peak currents, use equations 22.49a and 22.52

$$I_{dc} = \frac{V_{dc}}{R} = \frac{19.917}{100{,}000} = 0.199\ \times 10^{-3}\ \text{A}$$

$$i_{d,peak} \quad V_m \omega C \sin(\theta_2) = 19.4 \times 10^{-3}\ \text{A}$$

SOLUTION PROBLEM 22.24

Equations 22.44 - 22.52 are derived for a half-wave rectifier. For a full-wave rectifier, some of these equations be modified slightly as given below. The difference arises from changing T to T/2.

The new equation for θ_2 is derived in problem 22.21, and repeated below.

The given values are: C = 20e-6 F, R =100kΩ, V_m = 20 V; f = 60 Hz.

A modification of equation 22.48b gives

$$V_{dc} = (1 - \frac{\pi}{2\omega RC})V_m = (1 - \frac{T}{4RC})V_m = (1 - \frac{1}{4 \times 60 \times 10^5 \times 20 \times 10^{-6}}) \times 20 = 19.96\ \ \text{V}$$

To calculate the ripple factor, we first calculate θ_2 from equation derived in problem 22.21, and then use the res in equation 22.50.

$$\theta_2 \quad \cos^{-1}(e^{-\frac{\pi}{\omega RC}}) = 5.23 \text{ degrees}$$

ripple factor $\frac{1 - \cos(\theta_2)}{\sqrt{3}\,[\,1 + \cos(\theta_2)]}$ =0.12%

For the diode average and peak currents, use equations 22.49a and 22.52

$$I_{dc} = \frac{V_{dc}}{R} = \frac{19.96}{100{,}000} = 0.1996\ \times 10^{-3}\ \text{A}$$

$$i_{d,peak} \quad V_m \omega C \sin(\theta_2) = 13.75 \times 10^{-3}\ \text{A}$$

–

SOLUTION PROBLEM 22.25. CORRECTIONS: (1) 195 Ω should be 1950 Ω. (2) 100 μF should be 10 μF.

Since this problem only requires an estimate of the answer, we can use reasonable approximations to sim the solution. Let $H(s) = V_o(s)/I(s)$ be the transfer function for the linear circuit to the right of the diodes. The

$$H(s) = \frac{V_{in}(s)}{I(s)} \times \frac{V_o(s)}{V_{in}(s)} = Z_{in}(s)\frac{Z_{par}(s)}{R + Z_{par}(s)}$$

where $Z_{par}(s)$ is the impedance of the parallel C-R_L.

The first step is to find the magnitude $|H(j\omega)|$. As long as $1/R_L << \omega C$ and $1/(\omega C) << R$, the following approximations are valid:

$$|Z_{par}(j\omega)| = \left|\frac{1}{j\omega C + \frac{1}{R_L}}\right| \quad \frac{1}{\omega C}$$

which means that the parallel impedance is essentially that of the capacitor, and

$$\left|\frac{Z_{par}(j\omega)}{R + Z_{par}(j\omega)}\right| \quad \left|\frac{Z_{par}(j\omega)}{R}\right| \quad \frac{1}{\omega RC}$$

and again since $1/(\omega C) << R$,

$$|Z_{in}(j\omega)| \quad \frac{1}{\omega C}$$

Using these approximations,

$$|H(j\omega)| = |Z_{in}(j\omega)| \times \left|\frac{Z_{par}(j\omega)}{R + Z_{par}(j\omega)}\right| \quad \frac{1}{\omega C} \times \frac{1}{\omega RC} = \frac{R}{(\omega RC)^2}$$

It is given that $I_{dc} = 0.01$ A. From the short pulse property, the input current i(t) consists of very short pulses 120 Hz and all ac components of i(t) have peak magnitudes approximately equal to twice the average dc value. Hence the peak magnitudes are 0.02 A. Therefore the magnitude of 120-Hz component of the output voltage

$$|H(j2\pi \times 120)| \times 0.02 \quad \frac{R}{(\omega RC)^2} \times 0.02 = \frac{1950}{\left(2\pi \times 120 \times 1950 \times 10 \times 10^{-6}\right)^2} \times 0.02 = 0.18042 \text{ V}$$

Hence, the effective value is $0.18042/\sqrt{2} = 0.12757$ V and the ripple factor is

$$\frac{0.12757}{30} \times 100 = 0.42524\,\%$$

SOLUTION PROBLEM 22.26

Since this problem only requires an estimate of the answer, we can use reasonable approximations to simply the solution. Let $H(s) = V_o(s)/I(s)$ be the transfer function for the linear circuit to the right of the diodes. Then

$$\mathrm{H(s)} = \frac{V_o}{I} = \frac{V_{in}}{I} \times \frac{V_o}{V_{in}}$$

Under the condition $1/(\omega C) << R$ and $1/(\omega C) << R_L$, we have

$$Z_{in}(s) \quad Z_C(s) = \frac{1}{Cs}$$

and

$$\frac{V_o}{V_{in}} \quad \left(\frac{1}{\omega RC}\right)^2$$

Using these approximations,

$$|H(j\omega)| \quad |Z_{in}|\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\omega C}\left(\frac{1}{\omega RC}\right)^2 = R\left(\frac{1}{\omega RC}\right)^3$$

For a full-wave rectifier, the fundamental frequency is 120 Hz, and $\omega = 240\pi$ rad/s. It is given that I_{dc} = 0.01 A. From the short pulse property, the input current i(t) consists of very short pulses at 120 Hz and all ac components of i(t) have peak magnitudes approximately equal to twice the average dc value. Hence the peak magnitudes are 0.02 A. Therefore the peak magnitude of 120-Hz component of the output voltage is

$$|H(j120\pi)| \times 0.02 \quad = R\left(\frac{1}{\omega RC}\right)^3 \times 0.02 = 975 \times \left(\frac{1}{120\pi \times 975 \times 16 \times 10^{-6}}\right)^3 \times 0.02 = 0.012 \text{ V}$$

Hence, the effective value is $0.012/\sqrt{2} = 0.0848$ V and the ripple factor is

$$\frac{0.0848}{30} \times 100\% = 0.2827\ \%$$

SOLUTION PROBLEM 22.27

Using H(jω) of figure P22.27 and the given formula for c_n, we have

$$c_n = \frac{1}{\omega_s}\int_{-0.5\,\omega_s}^{0.5\omega_s} H(j\omega)\, e^{-jnT_0\omega} d\omega = \frac{1}{\omega_s}\int_{-0.25\,\omega_s}^{0.25\omega_s} e^{-jnT_0\omega} d\omega = \frac{1}{-jnT_0\omega_s}\left[\, e^{-jnT_0\omega}\right]_{-0.25\,\omega_s}^{0.25\omega_s}$$

$$= \frac{1}{n\pi} \sin\left(n\frac{\pi}{2}\right), \quad n = 1,2,...$$

For n = 0, we have $c_0 = d_0 = a_0/2$ = [Average value of H(jω)]= 0.5 .

For n = 1,2,..., from equation 22.6

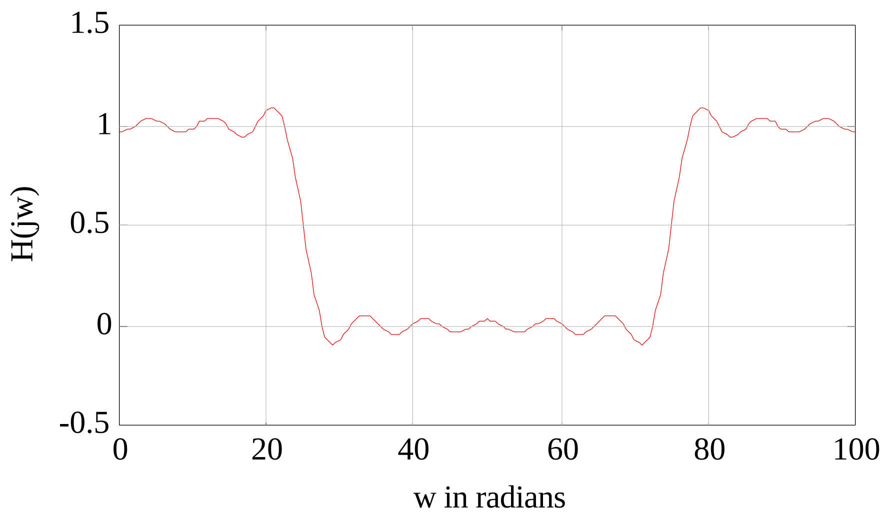
$$b_n = 0,$$

$$d_n = a_n = \frac{2}{n\pi} \sin\left(n\frac{\pi}{2}\right),$$

The Fourier series representation of H(jω) is

$$H(j\omega) = 0.5 + \frac{2}{\pi}\left[\cos(T_0\omega) - -\frac{1}{3}\cos(3T_0\omega) + -\frac{1}{3}\cos(5T_0\omega) - -\frac{1}{7}\cos(7T_0\omega + ...\right]$$

A plot of H(jω) vs. ωcurve using the first 11 terms (n=0,1,...11) of the Fourier series is given below together with the MATLAB codes.




```
%chapter 2, problem 27.
ws= 100;
T0=2*pi/ws;
w= 0: ws/200:ws;
d0w = 0.5;
d1w = (2/pi)*cos(T0*w);
d3w= -(2/pi/3)*cos(3*T0*w);
d5w= (2/pi/5)*cos(5*T0*w);
d7w= -(2/pi/7)*cos(7*T0*w);
d9w= +(2/pi/9)*cos(9*T0*w);
d11w= -(2/pi/10)*cos(11*T0*w);
H =d0w +d1w + d3w +d5w +d7w + d9w +d11w;
plot(w,H)
xlabel(' w in radians')
ylabel('H(jw)')
grid
```