PROBLEM SOLUTIONS CHAPTER 3.

Solution 3.1. Select the bottom node as the reference node, and write a node equation at the positive terminal of the V_1 resistor:

$$\frac{V_1 - V_0}{3R} + \frac{V_1}{6R} + \frac{V_1 - 4V_0}{6R} = 0$$

$$2V_1 - 2V_0 + V_1 + V_1 - 4V_0 = 0$$

$$4V_1 = 6V_0$$

$$V_1 = 1.5V_0$$

Solution 3.2 Write a node equation at the top node:

$$0.6 - \frac{V_x}{100} - \frac{2V_x}{100} - \frac{V_x}{50} = 0$$

$$-V_x - 2V_x - 2V_x = -60$$

$$-5V_x = -60$$

$$V_x = 12V$$

Solution 3.3

$$0.6 - \frac{V_x}{100} + \frac{25V_x}{100} - \frac{V_x}{50} - \frac{V_x - 0.2V_x}{40} = 0$$

$$-\frac{3V_x}{100} + \frac{25V_x}{100} - \frac{8V_x}{400} = -0.6$$

$$V_x = -\frac{0.6 \times 400}{80}$$

$$V_x = -3V$$

Solution 3.4 (a)

It is evident from the figure that V_c = 20. We need to write two equations in V_a and V_b and put them in matrix form. In this case, we can write the matrix equation by inspection. Note that the resistors are identified by conductance values.

$$\begin{bmatrix} 15m & -5m \\ -5m & 35m \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

(b) Solve the matrix equation by inverting the left-most matrix:

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \frac{1}{525\mu - 25\mu}\begin{bmatrix} 35m & 5m \\ 5m & 15m \end{bmatrix}\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$
$$= \frac{1}{0.5}\begin{bmatrix} 35 & 5 \\ 5 & 15 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$
$$= \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

(c) $V_x = V_{ab} = V_a - V_b = 20V$, V_{da} = -V_a = -40, V_{db} = -V_b = -20.

(d) P_i = 0.5×40 = 20W, P_v = 20×(20-20) = 0. P_{diss} = 40×40×10m + 20×20×5m + 20×20×5m = 20 W.

Power delivered equals dissipated power.

Solution 3.6 Write two nodal equations:

$$\frac{V_{s1} - V_1}{3000} = I_{s3} + \frac{V_1 - V_2}{6000}$$
$$\frac{V_2}{30000} = \frac{V_{s2} - V_2}{12000} + \frac{V_1 - V_2}{6000}$$

Rewrite equations as:

$$2V_{s1} - 2V_1 = 6000 I_{s3} + V_1 - V_2$$
$$2V_2 = 5V_{s2} - 5V_2 + 10V_1 - 10V_2$$

Cast into a matrix equation

$$\begin{bmatrix} -3 & 1 \\ -10 & 17 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6000 I_{s3} - 2V_{s1} \\ 5V_{s2} \end{bmatrix}$$

Solving the matrix equation yields:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 181.46 \\ 124.39 \end{bmatrix}$$

Power absorbed by the 6k resistor is $(V_1\text{-}V_2)^2/R = 0.5429W$.

Similarly, P_{s1} = (V_{s1}-V_1)/3000×V_{s1} = 4.7W, $P_{s2} = (V_{s2} - V_2)/12000 \times V_{s2} = -0.32W$

$$P_{s3} = I_{s3} \times (V_{s2} - V_1) = -1.21W$$

Solution 3.7 (a) Again, the matrix equation can be written by inspection:

$$\begin{bmatrix} G_1 + G_2 + G_4 & -G_4 \\ -G_4 & G_3 + G_4 + G_s \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 50G_1 \\ 50G_3 \end{bmatrix}$$

(b) Substituting the values of conductances and inverting the above matrix equation yields:

$V_B = 34.0132\,\text{V}$

$V_C = 33.6842\,\text{V}$

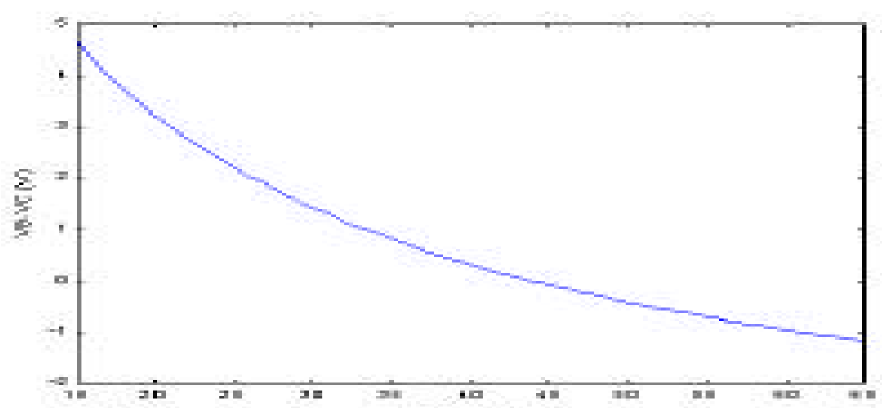
(c) Power delivered is $80.7566W$. Using the Principle of Conservation of Power:

$$P_{del} = P_1 + P_2 + P_3 + P_4 + P_5$$

or,

$$P_{del} = 50 \times \left[\frac{V_A - V_B}{20} + \frac{V_A - V_C}{20} \right] = 80.7566W$$

(d) In this part, we take the above matrix equation and solve it for each value of G_s. If we do this, we can get a feel for the behavior of V_B and V_C w.r.t. changes in G_s. The following plot is the voltage difference between the two nodes as a function of G_s, and hence as a function of temperature.



As can be seen, in this figure, the voltage difference between B and C does not change linearly with R_s. Since this resistance itself changes linearly with temperature, this means that V_B-V_C does not change proportionally to temperature.

Solution 3.8 The answer is:

$$\begin{bmatrix} G_1+G_2 & -G_1 & -G_2 & 0 \\ -G_1 & G_1+G_3+G_4+G_7 & -G_4 & -G_7 \\ -G_2 & -G_4 & G_4+G_5+G_6+G_2 & -G_6 \\ 0 & -G_7 & -G_6 & G_6+G_8+G_7 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -I_{s1} \\ 0 \\ 0 \\ I_{s2} \end{bmatrix}$$

Solution 3.9 Write the matrix equation by inspection:

$$\begin{bmatrix} 4/100 & -1/100 & -1/100 \\ -1/100 & 4/100 & -1/100 \\ -1/100 & -1/100 & 4/100 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_{s1}}{100} \\ \frac{V_{s1}}{100} \\ I_{s2} \end{bmatrix}$$

Solving the equation in MATLAB, we get: $V_1 = 7$, $V_2 = 7$, $V_3 = 11$, and $P = 0.6$.

$$P = V_{s1} \times \left(\frac{V_{S1} - V_1}{100} + \frac{V_{S1} - V_2}{100} \right) = 10 \times \left(\frac{3}{10} + \frac{3}{10} \right) = 0.6W$$

Solution 3.10 (a)

Nodal equation for A:

$$\frac{V_A - V_{s1}}{10} + \frac{V_A - V_B}{10} + \frac{V_A - V_C}{10} = 0$$

(b) At node B:

$$\frac{V_B}{10} + \frac{V_B - V_A}{10} - I_{s2} = 0$$

(c) At node C:

$$I_{s2} - I_{s3} + \frac{V_C - V_A}{10} = 0$$

(d) Manipulate algebraically to cast as the following matrix equation:

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_{s1} \\ 10I_{s2} \\ 10(I_{s3} - I_{s2}) \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -13\frac{1}{3} \\ 13\frac{1}{3} \\ -43\frac{1}{3} \end{bmatrix}$$

(e) P = (-10-13.333)/10×(-10) = 23.33W.

Solution 3.12

We are required to write the equations in matrix form. First, write a node equation at V_A and V_{out}:

$$V_A - 5 + \frac{V_A}{5} + \frac{V_A - V_{out}}{10} = 0$$
$$\frac{V_{out} - V_A}{10} - 7.5V_A + \frac{V_{out}}{10} = 0$$

Now group the coefficients for V_A and V_{out}, and write the matrix equation:

$$\begin{bmatrix} 1 + 1/5 + 1/10 & -1/10 \\ -1/10 + 7.5 & 2/10 \end{bmatrix} \begin{bmatrix} V_A \\ V_{out} \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} V_A \\ V_{out} \end{bmatrix} = \begin{bmatrix} 1 \\ -37 \end{bmatrix}$$

where the matrix inversion was performed in MATLAB. The ratio of the output voltage to the input voltage is -37/5.

Solution 3.13 (a) Nodes A and B are already labeled:

$$(V_A - 9)0.1 + (V_A - 0.5V_B)0.2 + (V_A - V_B)0.3 = 0$$
$$(V_B - V_A)0.3 + (V_B - 9)0.5 + 0.4V_B = 0$$

This can be rearranged Into:

$$(0.1 + 0.2 + 0.3)V_A - 0.3V_B - (0.2 \times 0.5)V_B = 0.9$$
$$-0.3V_A + (0.3 + 0.5 + 0.4)V_B = 4.5$$

The matrix equation can now be easily obtained:

$$\begin{bmatrix} 0.6 & -(0.5)(0.2) - 0.3 \\ -0.3 & 1.2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0.9 \\ 4.5 \end{bmatrix}$$
$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 4.8 \\ 4.95 \end{bmatrix}$$

(b) $I_{in} = -(V_A - V_s)0.1 - (V_B - V_s)0.5 = 2.445A$

(c) $P_s = V_S I_{in}$ = 22.005W, $P_{dep} = 0.5 \times V_{out} \times (0.5V_{out} - V_A) \times 0.2 = -1.151W$.

(d) $P = V^2/R = (4.95)\times(4.95)\times 0.4 = 9.801W$.

Solution 3.14 **(a)** We write node equations at V_A and V_B:

$$-I_{s1} + \frac{V_A}{20k} + gm_1 V_A + \frac{V_A - V_B}{10k} = 0$$
$$\frac{V_B - V_A}{10k} - gm_2\left(V_A - V_B\right) + \frac{V_B}{2.5k} + I_{s2} - gm_1 V_A = 0$$

Rearranging, we have:

$$\left(\frac{1}{20k} + gm_1 + \frac{1}{10k}\right)V_A - \frac{V_B}{10k} = I_{s1}$$
$$\left(\frac{-1}{10k} - gm_2 - gm_1\right)V_A + \left(\frac{1}{10k} + gm_2 + \frac{1}{2.5k}\right)V_B = -I_{s2}$$

(b)

$$\begin{bmatrix} 1/20k + gm_1 + 1/10k & -1/10k \\ -1/10k - gm_2 - gm_1 & 1/10k + gm_2 + 1/2.5k \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} I_{s1} \\ -I_{s2} \end{bmatrix}$$

(c) The above matrix is inserted into MATLAB, with all the values substituted, to obtain:

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 9.722 \\ 5.972 \end{bmatrix}$$

(d) $V_o = V_A - V_B$ = 3.75V

(e) $P_1 = V_A I_{s1}$ = 0.0292W, $P_{gm1} = -V_o gm_1 V_A$ = -0.008W, $P_{gm2} = V_B gm_2 V_o$ = 0.0112W, $P_2 = -V_B I_{s2}$ = -0.0119W.

Solution 3.15 I_1 = 0.4. Write nodal equations at A and B

$$\frac{V_A}{100} + \frac{V_A - V_B}{20} + 0.03\left(V_A - V_B\right) = 0.4$$
$$\frac{V_B - V_A}{20} + \frac{V_B}{40} + \frac{V_B - 80\,{}^{V_B}\!/_{40}}{40} = 0$$

Rearranging and casting into matrix form:

$$\begin{bmatrix} 1/100+1/20+0.03 & -1/20-0.03 \\ -1/20 & 1/20+1/40-1/40 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 40 \\ 40 \end{bmatrix}$$

It is obvious then, that V_x, the voltage between A and B, is zero.

Solution 3.16

1) $V_A = 3000 i_x = 3000 \times \frac{V_A - V_B}{9000} = \frac{V_A - V_B}{3}$

Equation at node B:

$\frac{V_B - V_A}{9000} + \frac{V_B}{6000} + \frac{V_B - V_D}{18000} = 0$ is equivalent to:

2) $-2V_A + 6V_B - V_D = 0$

Equation at node D:

$\frac{V_D - V_B}{18000} + \frac{V_D}{9000} + I_S = 0$ which can be rewritten as:

3) $3V_D - V_B = -360$ Solving the system formed by equations (1), (2) and (3) we obtain:

$V_A = 9V$, $V_B = -18V$, $V_D = -126V$.

Solution 3.17 (a) Choose E as the reference node

$V_A = 2i_x$

At node B

$6 = (V_B - V_C)/3 + (V_B - V_A)/2$

Or $6 = 5/6V_B - 1/3V_C - V_A/2$

At node C, $V_C = 2i_y$

At node D, VD = -12V

iy = (VD-VA)/2 = VD/2 –ix

From here on, the solution involves algebraic manipulations to solve the system of equations. MATLAB or hand analysis can be performed to obtain:

V_A = 48V, V_B = 12V, V_C = -60V.

(b) P_{6A} = 6(12 = 72W

I_{12V} = 30-8 = 22A (P12V = 264W.

$P_{2i_x} = 2i_x \times (-i_z - i_Y) = 2 \times 24 \times (+18 + 30) = 2304W$

$P_{2i_y} = 2i_y \times (-i_x + \frac{v_c - v_D}{6}) = 2 \times (-30) \times (-24 - 8) = 1920W$

(c) $P_{3\Omega} = i_x \times (V_B - V_C) = 1728W$ = 1728W

$P_{6\Omega}$ = 6×8×8 = 384W

$P_{2\Omega y} = 2 \times i_y \times i_y$ = 1800W

$P_{2\Omega} = (V_B - 2i_x)(V_B - 2i_x)/2$ = 648W.

(c) V_D = -12V, $V_A = 2i_x$, $V_C = 2i_z$.

Substitute the above V_A and V_C into the node equation for node B:

$i_z = (V_B - 2i_x)/2 = V_B/2 - i_x$ and

$i_x = (V_B - 2i_z)/3 = V_B/3 - 2/3 i_z$

Substitute i_z into i_x to obtain: $i_x = 0$, $V_A = 0$. Then, V_B can be deduced to be 12.

Finally, V_C = 12V.

Now, compute the powers.

$P_{6A} = 72W$, $P_{3\Omega} = 0$, $P_{6\Omega} = (V_C - V_D)^2/6 = 96W$, $P_{2\Omega y} = 72W$, $P_{2\Omega z} = 72W$.

$P_{12V} = 12 \times (\frac{V_A - V_D}{2} + \frac{V_C - V_D}{6}) = 12 \times (6 + 4) = 120W$,

$P_{2ix} = 0W$, $P_{2iy} = 12 \times 4 = 48W$

Solution 3.18 The three node equations at A, C, and D are:

$$-0.8 - 0.3 = 0.015V_A + 0.02V_A - 0.02V_C$$
$$V_c = 440$$
$$-0.8 + 2.5 = -0.005V_D + 0.025V_C - 0.025V_D$$

As can be seen, these really reduce to only two equations in two unknowns. These can be solved rather easily either by hand or by MATLAB to obtain: V_A = 220V, $V_D = 310V$. Note that all of these voltages are already referenced to node B (i.e. $V_A = V_{AB}$, etc).

Solution 3.19 **(a)** Supernode is BC (50 V source).

(b) Only one node equation needs to be written:

$$\frac{V_B}{90} - \frac{V_A}{90} + \frac{V_B}{10} + \frac{V_C}{10} - \frac{V_A}{10} + \frac{V_C}{90} = 0$$

with the constraint that $V_C - V_B = 50$.

(c) The constraint equation can be substituted into the B node equation to obtain

V_B = 125V. Thus, V_C = 175V, and $i_x = (V_C-V_A)/10$ = -12.5A.

(d) $(V_A-V_B)/90$ = 1.94A ➔ P_{300} = 300×(1.94+12.5) = 4332W.

$V_C/90$ = 1.94 ➔ P_{50} = -528W.

$$P_{50} = 50 \times (i_x + \frac{V_C}{90}) = -528W$$

Solution 3.20 (a) $V_B = V_A - 440$ and $V_C = V_A - 460$.

(b) Supernode is one including A, B, and C.

(c) $(V_C - 40)0.15 + 0.05V_B + 0.25V_A - 25 + 0.2(V_A - 40) = 0$

This can easily be rearranged to get V_A = 200V.

(d) power $P_s = V_A \times I = 200 \times 25 = 5000W = 5KW$

Solution 3.22 (a) $V_C = V_{s2}$ = 6V

(b) $I_x = 0.01V_A$

(c) Supernode at A,B, encompassing the controlled source. So, we have one equation:

$$I_{s1} = 0.01V_A + 0.0125V_B + 0.1V_B - 0.1V_C$$

(d) Substitute the constraint equations, $V_A - V_B = 20I_x = 0.2V_A$, (equivalently: $V_A = \frac{V_B}{0.8}$) into the above equation: $V_A = \frac{V_B}{0.8}$

$$I_{s1} = 0.01\frac{V_B}{0.8} + 0.0125V_B + 0.1V_B - 0.1V_C$$

$$V_B = 6.4V$$

$$V_A = 8V$$

(e) I_x = 0.08A.

(f) $P_{0.0125\Omega} = V_B^2 / R = 0.512W$

(g) $P = I_{s_1} \times V_A = 0.2 \times 8 = 1.6W$

Solution 3.23 (a) $V_C = V_{s2} = 50V$.

(b) $i_x = V_A/100$.

(c) Supernode A,B:

$$I_{s1} = 0.01V_A + 0.05V_B + 0.05V_B - 0.05V_{s2} + 0.09V_A - 0.09V_{s2}$$
$$V_A = V_B + 300i_x = V_B + 3V_A$$

(d) Solving the above two equations yields: $V_A = -90V$, $V_B = 180V$.

(e) $i_x = \frac{V_A}{100} = \frac{-90}{100} = -0.9A$

(f) $(V_B-V_C)\ (V_B-V_C)/R = 845W$.

(g) $P_{S_1} = I_{S_1} \times V_A = 2 \times (-90) = -180W$

Solution 3.24 (a) $V_B - V_C = 3V_x = 3V_B;\ V_C = -2V_B$

(b) Supernode at B and C, encompassing controlled source.

(c)

$$I_{s2} = (\frac{V_B}{10} - \frac{V_A}{10}) + \frac{V_B}{10} + \frac{V_C}{10} + (\frac{V_C}{10} - \frac{V_A}{10});\ V_C = -2V_B;\ 10I_{s_2} = -2(V_B + V_A)$$

equivalently: $2V_A + 2V_B = -10$

(d)

$$(0.1V_A - 0.1V_{s1}) + (0.1V_A - 0.1V_B) + (0.1V_A - 0.1V_C) = 0;\ 0.3V_A + 0.1V_B = 0.1V_{S_1};$$ equivalently:

$3V_A - V_B = -10$

(e) Again, any method can be used to simplify and solve the system of two equations. The solution is:

$V_A = -2.5V$, $V_B = -2.5V$.

Solution 3.25 (a) $V_A = V_{s1} = 16V$.

(b) Supernode at C and D, encompassing controlled voltage source.

(c) $I_{s2} = (0.75mV_D - 0.75mV_B) + (1mV_C - 1mV_A)$

(d) $V_C = 4V_B + V_D$

(e)

$$0.75mV_D + (0.75mV_B - 0.75mV_D) + (0.25mV_B - 0.25mV_A) = 0$$
or $1mV_B - 0.25mV_A = 0$

(f) We now have three equations in V_B, V_C, and V_D. These can be solved using any method. By inspection, we can immediately deduce V_B from V_A using the last equation: $V_B = 4V$.

The remaining two equations can be solved to obtain: $V_C = 20V$ and $V_D = 4V$.

Solution 3.26 (a) The supernode is the combination of A, C, and the controlled voltage source.

(b) Write node equations starting at the supernode:

$$(G_2V_A - G_2V_{in}) + (G_3V_A) + (G_4V_A - G_4V_B) + (G_6V_C) + (G_5V_C - G_5V_B) = 0$$
$$(G_2 + G_3 + G_4)V_A + (-G_4 - G_5)V_B + (G_6 + G_5)V_C = G_2V_{in}$$

and

$$(2G_6V_C) + (G_4V_B - G_4V_A) + (G_5V_B - G_5V_C) + (G_1V_B - G_1V_{in}) = 0$$
$$(-G_4)V_A + (G_1 + G_4 + G_5)V_B + (2G_6 - G_5)V_C = G_1V_{in}$$

and

$$V_A - V_C = 3V_x,\ V_A - V_C = 3(V_{in} - V_A),\ 4V_A - V_C = 3V_{in}$$

In matrix form:

$$\begin{bmatrix} 0.8 & -0.5 & 0.2 \\ -0.4 & 0.6 & 0.1 \\ 4 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 180 \end{bmatrix}$$

(c) The above system of equations can be solved to obtain: $V_A = 38.75V$, $V_B = 40V$, $V_C = -25V$.

(d) $I_{in} = (G_7V_{in}) + (G_2V_{in} - G_2V_A) + (G_1V_{in} - G_1V_B) = 5A$.

➔ $R_{eq} = 12\Omega$ and P = 300W.

$$P = V_{in} \times I_{in} = 60 \times 5 = 300W$$

(e) $I_{out} = V_CG_6 = -2.5A$ ➔ P = 62.5W.

Solution 3.27 (a) Supernode is A,B encompassing controlled voltage source.

(b)

$(V_A - V_{s1}) + 0.4V_B + (0.2V_B - 0.2V_C) = 0$

➔ $V_A + 0.6V_B - 0.2V_C = V_{s1}$

(c) $V_A - V_B = I_B = 0.4\ V_B$ ➔ $V_A = 1.4V_B$.

(d) $I_{s2} = 0.2V_B + (0.2V_C - 0.2V_B) = 0.2V_C$.

(e)

In matrix form:

$$\begin{bmatrix} 1 & 0.6 & -0.2 \\ 1 & -1.4 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 2 \end{bmatrix}$$

The solution is: $V_C = 10V, V_A = 7V, V_B = 5V$.

(f) $i = (V_A - V_{s1})1S = -1$ ➔ $P_{ccvs} = (V_A - V_B) \times (V_A - V_{s_1}) \times 1S = -2W$

$P_{vccs} = (V_{s1} - V_C)(0.2V_B) = -2W$.

Solution 3.28 (a) Supernode at A,C, CCVS.

(b) Node equation at supernode:

$$I_s + 0.25mV_A = G_1V_A + (G_2V_A - G_2V_B) + (G_5V_C) + (G_4V_C - G_4V_B)$$

➔ $$I_s = (G_1 + G_2 - 0.25m)V_A + (-G_2 - G_4)V_B + (G_4+G_5)V_C$$

Constraint:

$$V_A - V_C = 10^4 i_x = 10^4 G_3 V_B$$

➔ $$0 = V_A - 10^4 G_3 V_B - V_C$$

At node B:

$$G_3V_B + (G_2V_B - G_2V_A) + (G_4V_B - G_4V_C) = 0$$

➔ $$-G_2V_A + (G_3+G_2+G_4) - G_4V_C = 0$$

➔ (c) Matrix equation:

$$\begin{bmatrix} G_1 + G_2 - 0.25m & -G_2 - G_4 & G_4 + G_5 \\ 1 & -10^4 G_3 & -1 \\ -G_2 & G_3 + G_2 + G_4 & -G_4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \\ 0 \end{bmatrix}; \begin{bmatrix} 0 & -1 & 1 \\ 1000 & -1000 & -1000 \\ -0.2 & 1.1 & -0.8 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

(d) Substitute the values of conductances and solve the above matrix equation in MATLAB to obtain:

$V_A = -38V$, $V_B = -20V$, $V_C = -18V$.

(e) $P = V \times I = (10^4 G_3 V_B) \times \left[-I_s + V_A G_1 + G_2(V_A - V_B)\right] = (-20) \times \left[-2m - 1.9m - 3.6m\right] = 0.15W$

Solution 3.29

Loop equation: $V_{in} = 2kI_1 + 500(I_1 + 20m)$

➔ $V_{in} = 2500I_1 + 10$ ➔ $I_1 = 20mA$.

$P_{vin} = 20m \times 60 = 1.2W$.

$P_I = 20m \times (500I_1 + 500 \times 20m) = 0.4W$.

$P_{2k} = I_1 \times I_1 \times R = 0.8W$.

$P_{500} = (I_1 + 20m)^2 R = 0.8W$.

total power absorbed by resistors: $P_R = 0.8 + 0.8 = 1.6$

total power delivered by sources: $P_s = 1.2 + 0.4\ W = 1.6W$

Conservation of power is verified.

Solution 3.30

Loop equation: $100(I_1 - 0.5) + 200I_1 + 500 \times (I_1 + 20m) = 0$

➔ $I_1 = 0.05$A.

$P_{0.5A} = I \times V_{100\Omega}$; $V_{100\Omega} = 100 \times (0.5 - 0.05)$; where $V_{100\Omega}$ is the voltage on the 100Ω resistor.

$P_{0.5A} = 0.5(0.5\times100 - 0.05\times100) = 22.5$W.

$P_{20m} = 20m(I_1+20m)500 = 0.7$W.

Solution 3.31

Loop equation: $3.3 = 50I_1 + (50m + I_1)100 + (I_1 - 30m)40 + (I_1 - 50m)60$

➔ $I_1 = 0.01$A.

The power delivered by the independent voltage source:

$P = I_1\times3.3 = 0.033$W.

Solution 3.32

Loop equation: $50 = 300I_1 + (I_1 - 0.4I_1)500$

$50 = (300 + 500 - 200)I_1$ ➔ $I_1 = 0.0833$A.

Power absorbed by the 500Ω resistor.

$P_{500} = (I_1 - 0.4I_1)^2 500 = 1.25$W.

Solution 3.33

Loop equation: $1000(I_1 - Is) + 4000I_1 + 5000(I_1 - g_m V_x) = 0 \rightarrow 10000I_1 - 2V_x = 50$

and

$V_x = 1000(I_s - I_1) \rightarrow 1000I_1 + V_X = 50$.

Solve the above two equations in I_1 and V_x to obtain: $I_1 = 12.5$mA, $V_x = 37.5$V.

Thus, $R_{eq} = V_x/I_s = 750$_,

$P = I_{vccs} \times V_{vccs} = g_m \times V_x \times 5000 \times (g_m V_x - I_1)$=0.1875W

Solution 3.34

Loop equation: $V_{in} = 2I_{in} + 14I_{in} - 10V_1$

$V_1 = 2I_{in}$

After replacing V_1 in the loop equation we obtain:

➔ $V_{in} = -4I_{in}$ ➔

$$R_{1eg} = \frac{V_{in}}{I_{in}} = -4\Omega$$

Solution 3.35

Loop equation: $V_s = 500I_1 + 100(I_1 + 0.5) + 400(I_1 - 0.001V_x) + 100(I_1 + 0.005V_y)$

$V_x = 500I_1, V_y = 400I_1 - 400\times 0.001V_x = 400I_1 - 200I_1 = 200I_1$

After replacing V_x and V_y in the loop equation we obtain:

$V_s - 50 = 1000I_1$ ➔ $I_1 = 0.1A$

$V_y = 200I_1 = 20V$ ➔ $P_{400ohm} = V_y^2 / 400\Omega = 1W$

$R_{eq} = V_s/I_1 = 150/0.1 = 1500\Omega$.

Solution 3.36

Select clockwise loop current I_1 in the left loop. Select anti clockwise loop current I_2 in the right loop.

The two mesh equations are:

$12 = I_1 + 10(I_1 + I_2)$

and $10(I_2 + I_1) + 2I_2 + 12 = 0$

The two simultaneous equations can be solved easily to obtain: $I_1 = 0.75A$, $I_2 = +0.375A$.

$P_{10ohm} = (I_1 + I_2)^2/10 = 0.127W$.

Battery 1 supplies more current. $(I_1 > I_2)$

Solution 3.37

(a) The equation for the left loop is:

$$660 = I_1R + 1.296(I_1 + I_2) + 590 + I_1R$$

The equation for the right loop is:

$$660 = (0.3 - R)\ I_2 + 1.296\ (I_1 + I_2) + 590 + (0.3 - R)\ I_2$$

Simplifying the two equations:

$$70 = 1.596I_1 + 1.296I_2$$
$$70 = 1.296I_1 + 1.596I_2$$

The solution of these two equations is: $I_1 = I_2 = 24.2$A.

(b) $I_1 + I_2 = 48.4$, voltage across locomotive = 590 + 48.4×1.296 ➔ power = 31592W.

(c) Because the locomotive is 1/3 distance from either station it follows that

R = 1/3×0.3 = 0.1Ω. The two equations become:

$$70 = I_1(2R + 1.296) + 1.296I_2$$
$$70 = 1.296I_1 + (1.296 + 0.6 - 2R)I_2$$

The solution of these two equations is: $I_1 = 32.64$A, $I_2 = 16.32$A.

Current in locomotive motor $I_1 + I_2 = 48.96$A.

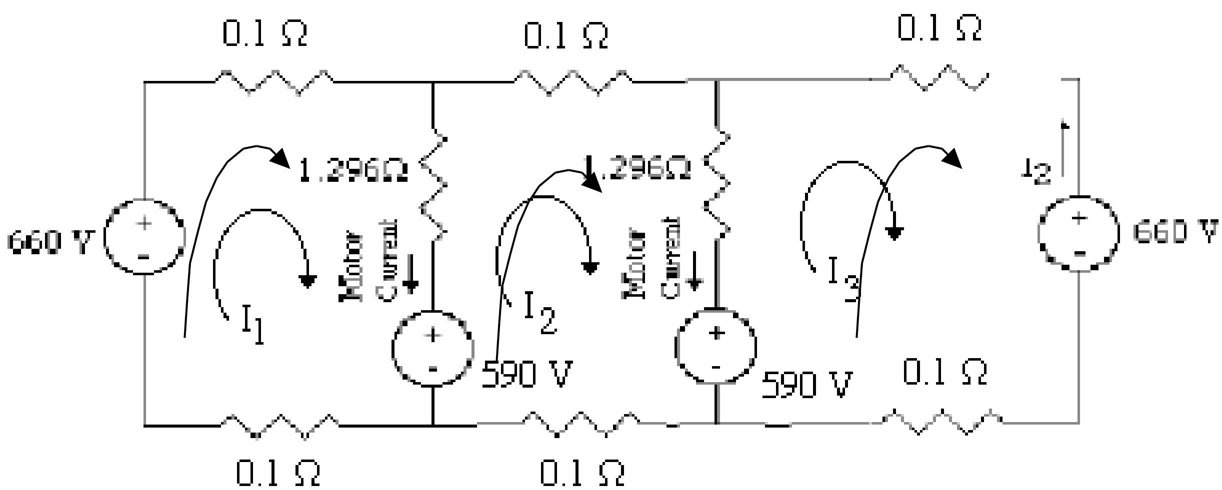
Voltage across locomotive 590 + $(I_1 + I_2) \times 1.296$

It follows that:

➔ $P = (I_1 + I_2)(590 + 49 \times 1.296) = 31993$W.

Solution 3.38

(a)



(b) The three loop equations are:

$$660 - 590 = 0.1I_1 + 1.296\,(I_1 - I_2) + 0.1I_1$$
$$0 = 1.296\,(I_2 - I_1) + 0.2I_2 + 1.296\,(I_2 - I_3)$$
$$-70 = 1.296(I_3 - I_2) + 0.2I_3$$

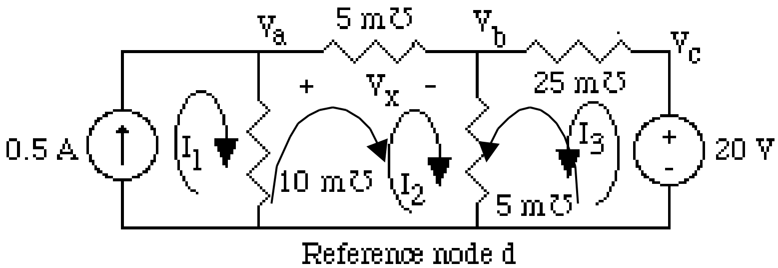
These three equations can be solved using any method to obtain:

I_1 = 46.8A, I_2 = 0, I_3 = – 46.8A.

(c) Motor currents are 46.8A each.

(d) Ps = VI = 660×46.8 = 30.9kW. Each source supplies 30.9kW.

Solution 3.39 (a) Define three meshes with three mesh currents. The first, I_1, is a clockwise current around the first mesh. The second, I_2, is a clockwise current around the middle loop of the circuit (through the 10mS, 5ms, and 5ms conductances). The third, I_3, is a *counterclockwise* current through the right-most loop containing the voltage source.



*current names shown above.

(b) $I_1 = 0.5A$

$$\left(\frac{I_2}{10m} - \frac{I_1}{10m}\right) + \frac{I_2}{5m} + \left(\frac{I_2}{5m} + \frac{I_3}{5m}\right) = 0.$$

$$\left(\frac{I_3}{25m} + \frac{I_3}{5m}\right) + \frac{I_2}{5m} = 20$$

These are two equations in two unknown currents. After grouping the terms, it can be verified that:

I_2 = 0.1A, I_3 = 0.

(c) V_x = 20V

V_{ad} = (0.5 – I_2)/10m = 40V

V_{bd} = 20V

(d) $P_{0.5}$ = $V_a \times 0.5$ = 40×0.5 = 20W

P_{20V} = 0W

$P_{resistors}$ = 2× $I_2{}^2$/5m + (0.5 – I_2)^2/10m = 20W

The conservation of power is verified.

Solution 3.40 (a) We can either write down the equations or evaluate the matrix by inspection:

$$90 \times (I_2 - 4.8m) + 10kI_2 + 50 = 0$$

$$90kI_3 + 10 \times (I_3 - 4.8m) = 50$$

OR

$$\begin{bmatrix} 100k & 0 \\ 0 & 100k \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 90k \times 4.8m - 50 \\ 10k \times 4.8m + 50 \end{bmatrix}$$

(b) The solution of the above equation is: I_2 = 3.82mA, I_3 = 0.98mA.

(c) Current source: P = 4.8m×[(4.8m – I_2)90k + (4.8m – I_3)10k] = 0.61W.

Voltage source: P = 50(I_3 – I_2) = – 142mW.

Solution 3.41 **(a)** By inspection:

$$\begin{bmatrix} 112k & -90k & -10k \\ -90k & 100k & 0 \\ -10k & 0 & 100k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 180 \\ -60 \\ 60 \end{bmatrix}$$

(b) Using MATLAB:

I_1 = 4.4mA, I_2 = 3.36mA, I_3 = 1.04mA

$\rightarrow i_x = -I_2 = -3.36mA$

(c) P_{180} = 180×4.4m = 0.792W, P_{60} = 60($I_3 - I_2$) = -0.139W.

Solution 3.42 The matrix equation is:

$$\begin{bmatrix} 8 & -6 & 3 \\ -6 & 8 & -2 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 0 \\ 6 \end{bmatrix}$$

whose solution is: I_1 = 4A, I_2 = 3A, I_3 = $0A$

v = $(I_1 + I_3 + I_2) \times 2$ = 2V

Solution 3.43 (a) First, note that two mesh currents are needed. Two clockwise currents are defined: I_1 in the middle loop, and I_2 in the right-most loop:

Middle loop equation:

$100I_1 - 100I_{s1} + 20I_x + 80I_1 - 80I_2 = 0$, where $I_x = I_{s1} - I_1$

and

Right-most loop equation:

$$80I_2 - 80I_1 + 10I_2 + V_{s2} = 0$$

These can easily be cast into the following matrix equation:

$$\begin{bmatrix} 160 & -80 \\ -80 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 16 \\ -6 \end{bmatrix}$$

(b) The solution of this equation is: $I_1 = 0.12A$, $I_2 = 0.04A$.

(c) $V_A = 100(I_{s1} - I_1) = 8V$ and $V_B = 80x(I_1 - I_2) = 6.4V$.

(d) $P_{S_1} = I_{s1}V_A = 1.6W$.

(e) $P_{0.0125S} = (I_1 - I_2)^2/0.0125 = 0.512W$.

Solution 3.44 (a) Create two clockwise mesh currents in the top loop (I_2) and the bottom-left loop (I_1). The bottom-right loop has an independent current source. Writing the loop equations:

$$V_{s1} = 200\,(I_1 - I_2) + 200\,(I_1 + I_{s2})$$

$$200\,(I_2 - I_1) + 100I_2 + 300I_x + 200\,(I_2 + I_{s2}) = 0, \text{ where } I_x = I_1 - I_2$$

(b) Solving, we get: $I_1 = -0.1A$, $I_2 = -0.7A$, $I_x = 0.6A$.

(c) $V_B = (I_1 + I_{s2})200 = 130V$.

(d) $P_{vs1} = I_1V_{s1} = -25W$, $P_{is2} = (V_B + (I_{s2} + I_2)200)I_{s2} = 105W$, $P_{300ix} = (-I_2)(300I_x) = 126W$.

Solution 3.45 (a) Create two clockwise mesh currents in the top loop (I_1) and the middle loop (I_2) (all resistor loop):

Top loop equation:

$$0.5v_x = 500\,(I_1 - I_2,) + 500I_1 \text{ where } v_x = -500I_1$$

and

Middle loop equation:

$$600\,(I_2 - I_{s1}) + 500\,(I_2 - I_1) + 900\,(I_2 + I_{s2}) = 0$$

(b) Solving, we get: $I_1 = 0.015A$, $I_2 = 0.0375A$, $v_x = -7.5V$.

(c) $P_{is1} = I_{s1}[0.5v_x + (I_{s1} - I_2)600] = 109.7W$

$P_{0.5vx} = 0.5v_x(I_1 - I_{s1}) = 1.63W$

$P_{is2} = (I_2 + I_{s2})\times 900\times I_{s2} = 53.2W$

Solution 3.46 Write the mesh equations in terms of R's and then substitute the values from the matrix:

Mesh 1 equation:

$$v_1 = R_1\,(i_1 - i_3) + R_2\,(i_1 - i_2) - 25 i_2$$

From this equation, and the first row of the matrix equation, we can deduce that

$$R_1 = 5\ \Omega$$

and

$$R_2 + 25 = 40 \qquad R_2 = 15\,\Omega.$$

Similarly:

Mesh 3 equation: $R_1(i_3 - i_1) + i_3 R_4 + R_3(i_3 - i_2) = 0$

From which we can deduce:

$$R_3 = 25\ \Omega \qquad \text{and} \qquad R_4 = 5\ \Omega.$$

Solution 3.47

Modified loop 1 equation:

$$V_{s1} = 3MI_1 + v + 2MI_1$$

Constraint equation:

$$-I_1 + I_2 = I_{s3}$$

Modified loop 2 equation:

$$v = 2MI_2 + V_{s2} + 8MI_2$$

Or in matrix form:

$$\begin{bmatrix} 5M & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -10M & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ v \end{bmatrix} = \begin{bmatrix} V_{s1} \\ I_{s3} \\ V_{s2} \end{bmatrix}$$

Solving: $I_1 = -1.1\ \mu$ A, $I_2 = -0.95\ \mu$ A.

The power $P_{s3} = I_{s3} \times v = 7.58\,\mu$ W

Solution 3.50 $I_2 = 2\text{A}$, $I_3 = -7\text{A}$

Loop 1 equation:

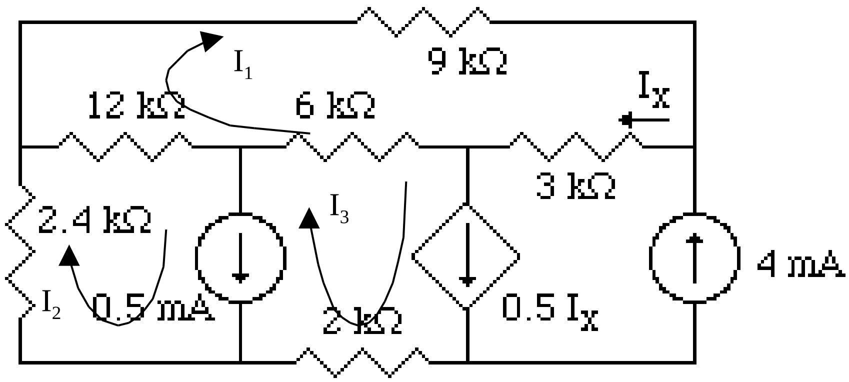
$$V_s = 3I_1 + 3(I_1 - I_2) + 6(I_1 + I_3 - I_2) + 2v_y + 2(I_1 + I_3) = 14I_1 + 2v_y - 74 \qquad 14I_1 + 2V_Y = 88$$

$$v_y = 3(I_1 - I_2) = 3I_1 - 6$$

Solving the above system, we obtain: I_1 = 5A ➔ v_y = 9, v can be found from the loop 3 equation $v + (I_1 + I_3 - I_2).6 + 2v_y + 2(I_3 + I_1) = 0$. Solving, we obtain $v = 10V$

Finally, $P_{vs} = V_s \times I_1$ = 70W.

Solution 3.51



Mesh 1 equation:

$9kI_1 + 3k(I_1 - I_4) + 6k(I_1 - I_3) + 12k(I_1 - I_2) = 0$

where we have used the fact that $I_x = I_1 - I_4$ (and I_4 = – 4mA)

Mesh 3 equation:

$6k(I_3 - I_1) - v_2 + 2kI_3 + v = 0$

Mesh 2 equation:

$2.4kI_2 + 12k(I_2 - I_1) - v = 0$

Constraint equations:

$I_2 - I_3 = 0.5mA$

$I_3 - I_4 = 0.5I_x = 0.5I_1 - -0.51I_4 = 0.51I_1 + 2mA$; $0.51I_1 - I_3 = -2mA$

The above five equations need to be put into matrix form:

$$\begin{bmatrix} 30k & -12k & -6k & 0 & 0 \\ -6k & 0 & 8k & 1 & -1 \\ -12k & 14.4k & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0.5 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ v \\ v_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 0 \\ 0.5m \\ 2m \end{bmatrix}$$

The solution is:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ v \\ v_2 \end{bmatrix} = \begin{bmatrix} -0.002A \\ -0.0025A \\ -0.003A \\ -12V \\ -24V \end{bmatrix}$$

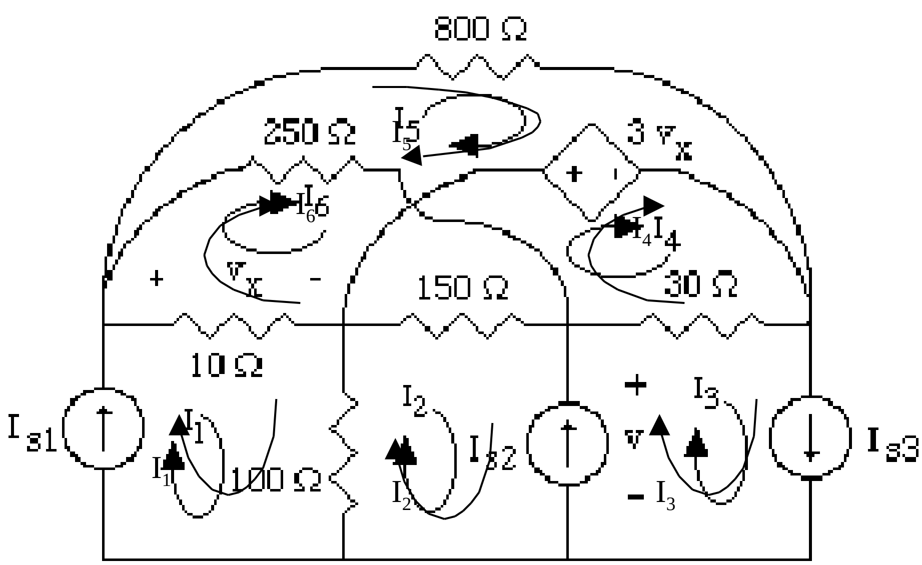
$I_x = I_1 - I_4 = 2mA$

Power delivered by 0.5mA current source: $P_1 = 0.5mxv = -6mW$

Power delivered by the dependent current source: $P_2 = 0.5I_x \; v_2 = 1m \times (-24) = -24mW$

Power delivered by 4mA current source: $P_3 = 4m \times (3k \; I_x - V_2) = 120mW$

Solution 3.52



Write the following equations:

Mesh 2 equation:

$$100\,(I_2 - I_1) + 150\,(I_2 - I_4 - I_6) + v = 0$$

Mesh 4 equation:

$$3v_x + 30\,(I_4 - I_3) + 150\,(I_4 - I_2 + I_6) = 0$$

Mesh 5 equation:

$$800I_5 - 3v_x + 10(I_5 - I_1 + I_6) = 0$$

Mesh 6 equation:

$$250I_6 + 150(I_6 + I_4 - I_2) + 10(I_6 + I_5 - I_1) = 0$$

Constraint equation:

$$I_{s_2} = -I_2 + I_3$$

Substituting the values of $I_1 = I_{s_1} = 1.15$ and $I_3 = I_{s3} = 0.95$, and noting that

$$v_x = (I_1 - I_6 - I_5)10$$

we can write:

$$\begin{bmatrix} 250 & -150 & 0 & -150 & 1 \\ -150 & 180 & -30 & 120 & 0 \\ 0 & 0 & 840 & 40 & 0 \\ -150 & 150 & 10 & 410 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_2 \\ I_4 \\ I_5 \\ I_6 \\ v \end{bmatrix} = \begin{bmatrix} 100I_1 \\ 30I_3 - 30I_1 \\ 40I_1 \\ 10I_1 \\ I_3 - I_{S2} \end{bmatrix}$$

$$\begin{bmatrix} I_2 \\ I_4 \\ I_5 \\ I_6 \\ v \end{bmatrix} = \begin{bmatrix} 0.65 \\ 0.45 \\ 0.05 \\ 0.1 \\ 35 \end{bmatrix}$$

Finally, v_x = 10V and v is as given above.

Solution 3.53 (a)

Replace the voltage source by current sources:

At node 1

$$i_{s1} = (V_2 - V_1)/12k$$

At node 2

$$(V_2 - V_1)/12k + (V_2 - V_3)/90k + (V_2 - V_4)/10k = 0$$

At node 3

$$i_{s2} = (V_3 - V_2)/90k + V_3/10k$$

At node 4

$$i_{s2} + (V_4 - V_2)/10k + V_4/90k = 0$$

$$V_1 = 180$$

$$V_4 - V_3 = 60$$

(b) In matrix form:

$$\begin{bmatrix} -1/12k & 1/12k & 0 & 0 & -1 & 0 \\ -1/12k & 1/12k+1/90k+1/10k & -1/90k & -1/10k & 0 & 0 \\ 0 & -1/90k & 1/90k+1/10k & 0 & 0 & -1 \\ 0 & -1/10k & 0 & 1/90k+1/10k & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ i_{s1} \\ i_{s2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 180 \\ 60 \end{bmatrix}$$

The solution from MATLAB is

180.0000

127.2000

33.6000

93.6000

-0.0044

0.00232

(c) Power delivered by S_1 is:

$$P_{s1} = V_{s1} \times (-I_{s1}) = 0.792W$$

Power delivered by S_2 is:

$$P_{s2} = V_{s2} \times (-I_{s2}) = -0.139W$$

Solution 3.54 (a) Replace the 100 ohm resistor, the controlled voltage source, and v_{s2} by current source. Then write the node equations:

$$I_{s1} = i_x + I_1 + 0.09\,(V_1 - V_3)$$

$$I_1 = V_2/20 + 0.05\,(V_2 - V_3)$$

$$-I_2 = 0.05\,(V_3 - V_2) + 0.09\,(V_3 - V_1)$$

$$V_3 = V_{s2}$$

$$V_1 - V_2 = 300 i_x = 300 V_1/100$$

In matrix form:

$$\begin{bmatrix} 0.1 & 0 & -0.09 & 1 & 0 \\ 0 & 0.1 & -0.05 & -1 & 0 \\ -0.09 & -0.05 & 0.14 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 50 \\ 0 \end{bmatrix}$$

(b) Using MATLAB to solve the above system the solution is:

V_1 = -90.0000V

V_2 = 180.0000V

V_3 = 50.0000V

I_1 = 15.5000A

I_2 = -6.1000A

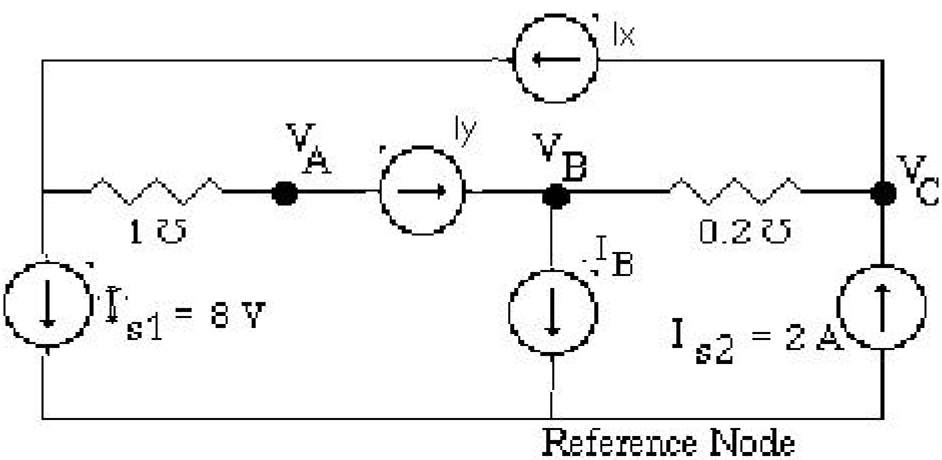
(c) Power delivered by the current source is

$$P_{s1} = I_{s1} \times V_1 = -180W$$

Power delivered by the voltage source is:

$$P_{s2} = V_{s2} \times (-I_2) = 305W$$

Solution 3.55 Modify the circuit so that it looks like the following:



The modified node equations are:

$$I_{s1} = -I_y + I_x$$

$$I_b = I_y + 0.2\,(V_C - V_B)$$

$$I_{s2} = 0.2\,(V_C - V_B) + I_x$$

The equations describing the constitutive relationships of elements in the original network are:

$$V_A - V_B = I_b$$

$$I_x = 0.2V_b$$

$$I_b = 0.4V_b$$

$$I_y = 8 - V_A$$

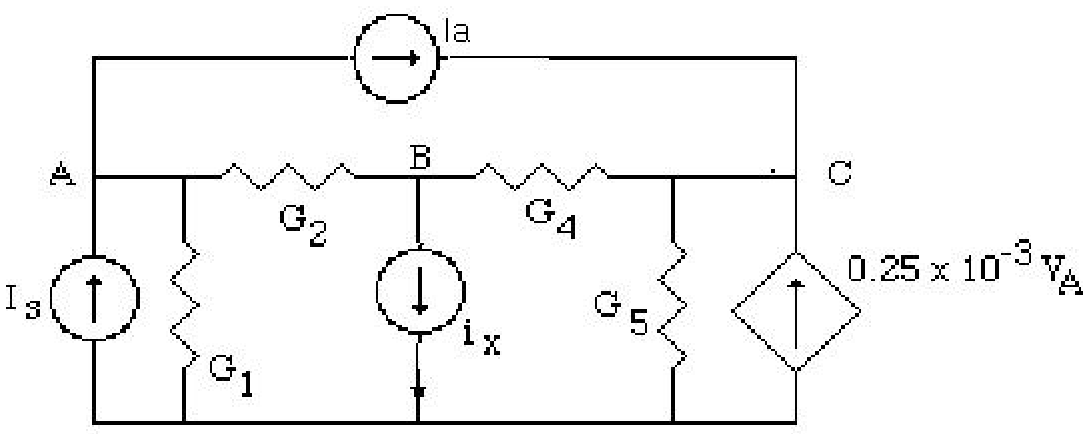
These can be cast into a matrix equation and solved easily to obtain the same result as previously arrived at.

In matrix form:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & -0.2 & 0.2 & 0 & 1 & -1 & 0 \\ 0 & -0.2 & 0.2 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0.2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ I_x \\ I_y \\ I_b \\ I_{s1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \\ V_C \\ I_x \\ I_y \\ I_b \\ I_{s1} \end{bmatrix} = \begin{bmatrix} 7V \\ 5V \\ 10V \\ 1A \\ 1A \\ 2A \\ 0A \end{bmatrix}$$

Solution 3.56 Modify the circuit as follows:



The modified node equations are:

At node A: $I_s = G_1V_A + G_2(V_A - V_B) + I_a$

At node B: $G_2(V_A - V_B) = I_x + G_4(V_B - V_C)$

At node C: $0.25\text{m}V_A + I_a = G_4(V_C - V_B) + G_5V_C$

The equations describing the constitutive relationships of elements in the original network are:

$I_x = G_3V_B$

$V_A - V_C = 10^4I_x$

These can be cast into a matrix equation that can be solved in MATLAB.
In Matrix form:

$$\begin{bmatrix} 0.25m & -0.2m & 0 & 1 & 0 \\ -0.2m & 1m & -0.8m & 0 & 1 \\ -0.25m & -0.8m & 1m & -1 & 0 \\ 0 & -0.1m & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & -10^4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ I_a \\ I_x \end{bmatrix} = \begin{bmatrix} 2m \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution is:

$$\begin{bmatrix} V_A \\ V_B \\ V_C \\ I_a \\ I_x \end{bmatrix} = \begin{bmatrix} -38V \\ -20V \\ -18V \\ 0.0075A \\ -0.002A \end{bmatrix}$$

We observe that we have obtained the same results as in problem 3.28.

Solution 3.57 Replace dependent source by i_{35} (from 3 to 5). Also, replace voltage source by i_{10} (from 1 to 0). Now, write the modified noted equations. The reference node is $O{:}V_O = 0V$:

At node 1: $i_{10} = (V_6 - V_1) + (V_2 - V_1)$

At node 2: $2 = (V_2 - V_1) + (V_2 - V_3)$

At node 3: $i_{35} = (V_4 - V_3) + (V_2 - V_3)$

At node 4: $2 = 2 + (V_4 - V_3) + V_4$

At node 5: $i_{35} = (V_5 - V_6) + V_5$

At node 6: $2 = (V_5 - V_6) - V_6$

Constraints:

$V_3 - V_5 = 15v_x = 15V_4$

$V_1 = 5$

The following matrix equation is obtained:

$$\begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -15 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ i10 \\ i35 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 5 \end{bmatrix}$$

The solution of this equation is obtained from MATLAB:

$V_1 = 5.0000\text{V}$

$V_2 = 3.3571\text{V}$

$V_3 = 0.2857\text{V}$

$V_4 = -0.1429\text{V}$

$V_5 = 1.8571\text{V}$

$V_6 = -0.0714\text{V}$

$i_{10} = -6.7143\text{A}$

$i_{35} = 3.7857\text{A}$

The power delivered by the dependent voltage source connected between nodes 3 and 5:

$$P_{35} = 15v_x(-i_{35}) = 15 \times V_4 \times (-i_{35}) = 8.115W$$

The power delivered by the current source connected between nodes 2 and 4:

$$P_{24} = (2A) \times (V_2 - V_4) = 7W$$

The power delivered by the current source connected between nodes 4 and 6:

$$P_{46} = (2A) \times (V_4 - V_6) = -0.143W$$

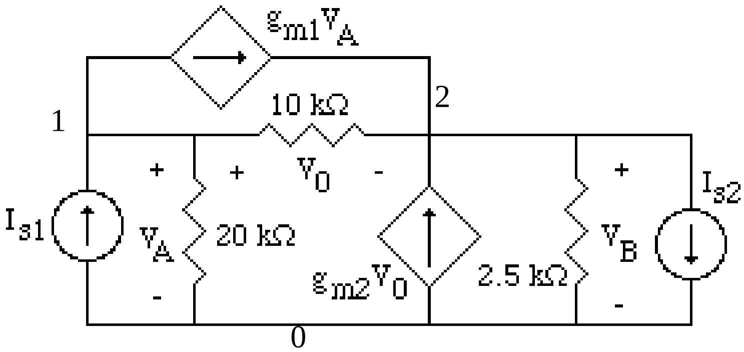
The power delivered by the voltage source connected between nodes 1 and 0:

$$P_{10} = 5V \times (I_{12} + I_{16}) = 5x\left[(V_1 - V_2) + (V_1 - V_6)\right] = 33.57W$$

Solution 3.59 Using the appropriate element stamps for each element of the circuit, we obtain the following system:

$$\begin{bmatrix} 0.15+0.2 & -0.15 & -0.2 & 0 \\ -0.15 & 0.15+0.05 & 0 & -1 \\ -0.2 & 0 & 0.25+0.2 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ I_x \end{bmatrix} = \begin{bmatrix} -8-3 \\ 3 \\ 25 \\ 440 \end{bmatrix}$$

Solution 3.60



$$\begin{bmatrix} 1/20k + 1/10k + g_{m1} & -1/10k \\ -1/10k - g_{m1} - g_{m2} & g_{m2} + 1/10k + 1/2.5k \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{s1} \\ -I_{s2} \end{bmatrix}$$

The solution is the same as that of problem 3.14.

Solution 3.62 (a) Because $R_T(T)$ can be approximated by a straight line between $(250\Omega, 0^\circ C)$ and $(80\Omega, 50^\circ C)$ it follows that:

$$R_T(T) = -3.4T + 250$$

(b) For $T = 25^\circ C$, $R_T = 165\,\Omega$

(c) The voltage across the $R_T + R_L$ series combination can be obtained from voltage division:

$$V_{T,L} = \frac{R_T + R_L}{R_T + 2R_L + R}\, 12 = 4.7857V$$

This is the same as the voltage across Rx because the meter is at zero deflection. Thus,

$\dfrac{R_x}{R_x + R}\, 12 = 4.7857.$ It follows that $R_x = 165.84\Omega.$

(d) We first denote the nodes:

A - the node common to R, R_x and the voltmeter;

B - the node common to R, R and the voltage source;

C - the node common to R_L, R_T and R_L;

D - the node common to R_x, R_L and the voltage source.

The reference node is $D: V_D = 0$. It follows that $V_B = 12V$.

We also have: $v_{out} = V_A - V_C$

The node equations are:

At node A: $\frac{V_A - V_C}{R_m} + \frac{V_A - 12}{R} + \frac{V_A}{R_x} = 0$

Equivalently: $V_A(RR_x + R_mR_x + R_mR) - V_C \; RR_x = 12R_mR_x$

$$V_A \times 4199.86 - V_C \times 41.46 = 19900.8 \qquad (1)$$

At node C: $\frac{V_C - 12}{R + R_L} + \frac{V_C}{R_T + R_L} + \frac{V_C - V_A}{R_m} = 0$

Equivalently: $(V_C - 12) \times 0.004 + \frac{V_C}{R_T + 2.5} + (V_C - V_A) \; 10^{-4} = 0$

$$V_C(0.004 \; R_T + 0.01 + 1 + 0.00025 + R_T \times 10^{-4}) -$$

$$-V_A(R_T \times 10^{-4} + 0.00025) = 0.48 \times (R_T + 2.5)$$

The last equation can be rewritten as:

$$V_C(0.0041R_T + 1.01025) - V_A(R_T \times 10^{-4} + 0.00025) = 0.48(R_T + 2.5) \qquad (2)$$

From (1) and (2), we obtain:

$$(0.0041R_T + 1.01025) \times \frac{-19900.8 + V_A \times 4199.86}{41.46} -$$

$$- V_A \times (R_T \times 10^{-4} + 0.00025) = 0.48(R_T + 2.5)$$

Equivalently: $V_A \times (R_T \times 0.415 + 102.337) = 2.448 \times R_T + 486.12$

It follows that $V_A = \frac{2.448 \times R_T + 486.12}{0.415 \times R_T + 102.337}$

From the equation at node A:

$$v_{out} = V_A - V_C = -R_m \times \left(\frac{V_A - 12}{R} + \frac{V_A}{R_x} \right) = \frac{2.448 \times R_T + 486.12}{0.415 \times R_T + 102.337} \times (-100.3) + 480$$

At $T = 0^\circ C$: $R_T = 250\Omega$. It follows that $v_{out} = -54.4415$V

At $T = 50^\circ c$: $R_T = 80\Omega$. It follows that $v_{out} =$ 80V

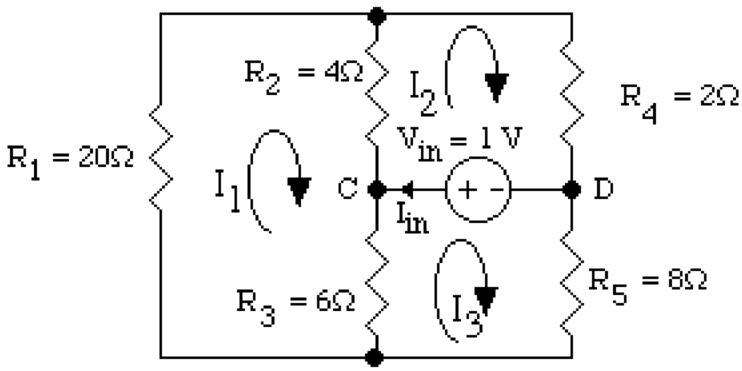
(e): The formula has been derived at part d):

T	R_T	v_{out}
$0^\circ C$	250Ω	$-54.4415V$
$5^\circ C$	233Ω	$-52.4136V$
$10^\circ C$	216Ω	$-50.2368V$
$15^\circ C$	199Ω	$-47.8938V$
$20^\circ C$	182Ω	$-45.3650V$
$25^\circ C$	165Ω	$-42.6273V$

$30^{\circ}C$	148Ω	$-39.6537V$

Solution 3.63

Place a source V_{in} between C and D, and calculate the current drawn from the source as below:



Loop 1 equation:

$$I_1R_1 + (I_1 - I_2)R_2 + (I_1 - I_3)R_3 = 0$$

Equivalently:

$$I_1(R_1 + R_2 + R_3) - I_2R_2 - I_3R_3 = 0$$

Loop 2 equation:

$$(I_2 - I_1)R_2 + I_2R_4 - 1 = 0$$

Equivalently:

$$-I_1R_2 + I_2(R_2 + R_4) = 1$$

Loop 3 equation:

$$1 + I_3R_5 + (I_3 - I_1)R_3 = 0$$

Equivalently:

$$-R_3I_1 + I_3(R_3 + R_5) = -1$$

We obtain the following system of equations:

$$\begin{cases} 30I_1 - 4I_2 - 6I_3 = 0 \\ -4I_1 + 6I_2 = 1 \\ -6I_1 + 14I_3 = -1 \end{cases} \qquad I_1 = 0.0096A,\ I_2 = 0.1731A,\ I_3 = -0.0673A;$$

$$I_{in} = I_2 - I_3 = 0.2404A$$

$$R_{eg,CD} = \frac{V_{in}}{I_{in}} = \frac{1}{0.2404} = 4.16\Omega$$

Solution 3.64

The node equation at node A is:

$$V_A G_1 + (V_A - V_B)G_2 + (V_A - V_C)G_3 = 0$$

Equivalently:

$$(G_1 + G_2 + G_3)V_A - V_B G_2 - V_C G_3 = 0$$

The supernode is identified by a Gaussian surface enclosing the controlled voltage source. The supernode equation is:

$$G_2(-V_A + V_B) - 6 + G_4 V_C + G_3(V_C - V_A) = 0$$

Equivalently, we have:

$$-V_A(G_2 + G_3) + G_2 V_B + V_C(G_3 + G_4) = 6$$

One way of obtaining the solution to the problem is:

We multiply the above two equations by 30.

$$-30(G_2 + G_3)V_A + 30G_2 V_B + 30(G_3 + G_4)V_C = 180$$

and

$$30(G_1 + G_2 + G_3)V_A - 30G_2 V_B - 30G_3 V_C = 0$$

By equating the coefficient of the above two equations with the coefficients of the first and second given equations, we obtain:

$$30G_2 = 30 \qquad G_2 = 0.1S$$

$$30G_3 = 2 \qquad G_3 = 0.067S$$

$$30(G_1 + G_2 + G_3) = 11 \qquad G_1 = 0.2S$$

$$30(G_3 + G_4) = 32 \qquad G_4 = 0.87S$$

β can be obtained as follows:

$$V_C - V_B = \beta V_X = \beta(V_C - V_A)$$

Equivalently:

$$\beta V_A - V_B + (1-\beta)V_C = 0$$

By comparing with the third given equation $\quad \beta = 3$.

Solution of 3.66

(a)

At node A: $\quad (V_A-V_C)/2 + (V_A-V_B)/2 + (V_A-V_D)/2 = 14$

At node B: $\quad (V_B-V_A)/2 + (V_B-V_C)/2 + (V_B-V_D)/2 = 7$

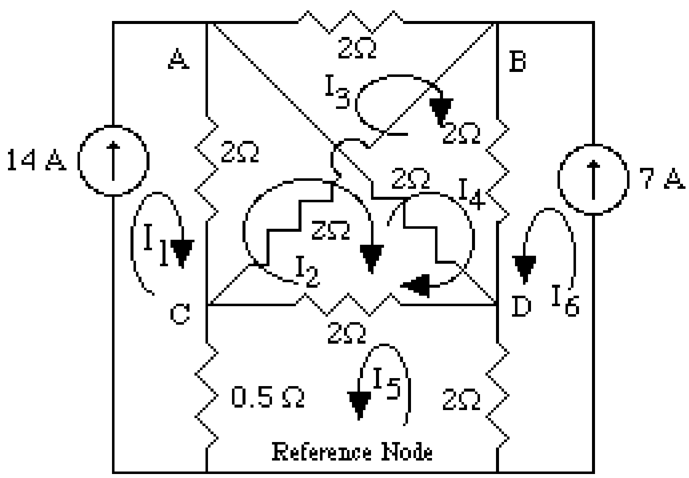
At node C: $\quad (V_C-V_A)/2 + (V_C-V_B)/2 + (V_C-V_D)/2 + 2V_C = 0$

At node D: $\quad (V_D-V_A)/2 + (V_D-V_B)/2 + (V_D-V_C)/2 + 0.5V_D = 0$

These can be solved in MATLAB to obtain:

22.0000
18.5000
7.5000
12.0000

(b) Mesh analysis would result in the same voltages



The loops and their current loops are:

$$\begin{cases} A, C, 14A: I_1 \\ A, C, D: I_2 \\ A, B, D: I_3 \\ B, C, D: I_4 \\ C, D, \text{reference node}: I_5 \\ B, D, 7A: I_6 \end{cases}$$

$$I_1 = 14A, I_6 = 7A$$

Loop ACD equation: $2(I_2 - I_1) + 2(I_2 - I_3) + 2(I_2 + I_4 + I_5) = 0$

Loop BCD equation: $2(I_3 + I_6 + I_4) + 2(I_2 + I_4 + I_5) + 2I_4 = 0$

Loop ABD equation: $2(I_3 + I_4 + I_6) + 2(I_3 - I_2) + 2I_3 = 0$

Loop CD_{ref} node equation: $2(I_5 + I_2 + I_4) + 2(I_5 - I_6) + 0.5(I_5 + I_1) = 0$

In matrix form:

$$\begin{bmatrix} 6 & -2 & 2 & 2 \\ 2 & 2 & 6 & 2 \\ -2 & 6 & 2 & 0 \\ 2 & 0 & 2 & 4.5 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 28 \\ -14 \\ -14 \\ 7 \end{bmatrix}, \quad \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 6.75 \\ 1.75 \\ -5.5 \\ 1 \end{bmatrix}$$

$$V_A = 2(I_1 - I_2) + 0.5(I_1 + I_5) = 22V$$

$$V_B = 2(I_4 + I_3 + I_6) + 2(I_6 - I_5) = 18.5V$$

$$V_C = 0.5(I_1 + I_5) = 7.5V$$

$$V_D = 2(I_6 - I_5) = 12V$$

(c) Mesh analysis requires more work.

(d) The removal of the top resistor will result in more node equations than loop equations. The addition of a resistor between node A and reference node will result in more loop equations than node equations.