PROBLEM SOLUTIONS CHAPTER 6

SOLUTION 6.1. **(a)** V_{oc} is found by removing R_L and doing voltage division.

$$V_{OC} = 63V \frac{(600 \,||\, 700)}{(600 \,||\, 700) + 300} \times \frac{600}{600 + 100} = 28V$$

R_{TH} is found by setting the source to zero and by calculating the equivalent resistance seen looking back between the A and B terminal.

$$R_{TH} = [(300 \,||\, 600) + 100] \,||\, 600 = 200\Omega$$

(b) Using

$$P = R_L \quad I_L$$

the power for each resistance may be found by substituting the appropriate R_L in the following equation.

$$P = \left(\frac{V_{OC}}{R_{TH} + R_L} \right)^2 R_L$$

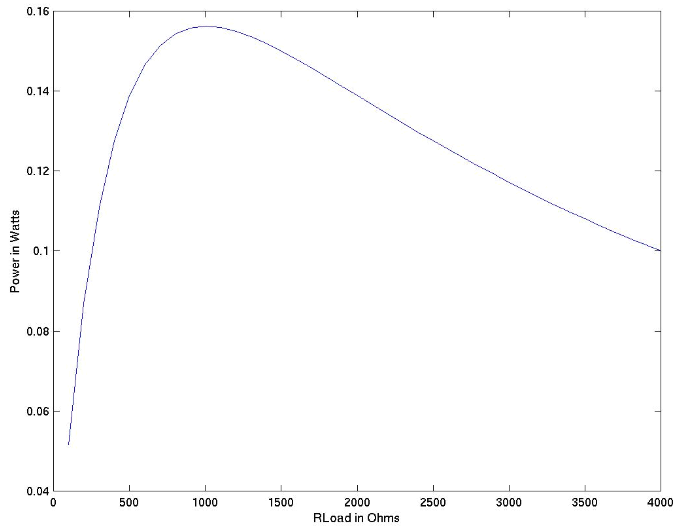
For 50 Ω, 200 Ω, and 800 Ω , the power obtained is 627.2 mW, 980.0 mW, and 627.2 mW respectively. The use of Thevenin equivalent does reduce the effort in obtaining the answer.

SOLUTION 6.2. **(a)** To find R_{TH}, open circuit the current source and short-circuit the voltage source. The resulting resistance seen from terminal A-B is 1 kΩ. Using superposition, the contribution of the current and voltage source at the open circuit output may be summed as 30 V (2 k/4 k) + 10 mA (2 k/4 k) (2 kΩ). V_{OC} is then 25 V and $I_{SC} = V_{OC}/R_{TH}$ is 25 mA.

(b) Following is a plot of

$$P = \left(\frac{V_{OC}}{R_{TH} + R_L} \right)^2 R_L$$

for R_L from 100 Ω to 4 k Ω.



SOLUTION 6.3. **(a)** Turning off the two sources

$$R_{TH} = (60+60)||40 = 30\,\Omega,$$

and using superposition

$$V_{OC} = 6V\left(\frac{40}{40+60+60}\right) + 0.1A\left(\frac{60}{40+60+60}\right)40 = 3\ \text{V}.$$

I_{SC} is the obtained as 100 mA.

(b) Using

$$P = \left(\frac{V_{OC}}{R_{TH}+R_L}\right)^2 R_L$$

a load of 90 Ω will absorb 56.25 mW.

(c) It absorbs 75 mW; hence the 30 Ω resistor absorbs more power.

SOLUTION 6.4. As both resistor divider ration are the same (3/6), the voltage at A and B is the same resulting in a V_{OC} of 0 V.

$$R_{TH} = (3K||6K) + (9K||18K) = 8\ \text{k}\Omega$$

The relation V_{OC}/I_{SC} cannot be used in this situation.

SOLUTION 6.5. Using superposition

$$V_{OC} = 20\left(\frac{20\times10^3}{20\times10^3+5\times10^3}\right) - 10\left(\frac{5\times10^3}{20\times10^3+5\times10^3}\right) + 20\quad 20\sin(50t) = 14 + 400\sin(50t)\ \text{V},$$

and

$$R_{TH} = (20k\,||\,5k) + 20k = 24 \text{ k}\Omega.$$

I_{SC} can then be found using the V_{OC}/R_{TH} relationship as 0.58+20sin(50t) mA.

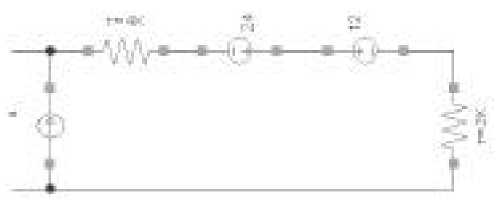
SOLUTION 6.6. Once again using superposition

$$V_{OC} = 72V\left(\frac{6K}{6K+8K+4K}\right) + 54mA\left(\frac{4K}{4K+8K+5K}\right) 6K + 54mA \;\; 2K = 204V ,$$

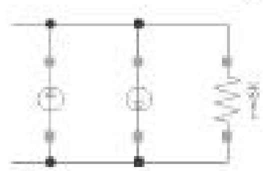
and

$$R_{TH} = [(4K+8K)\,||\,6K] + 4K + 2K = 10K\Omega$$

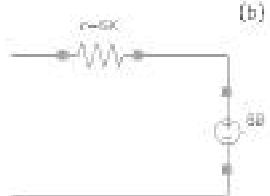
SOLUTION 6.7. Using source transformation, (a) is obtained from the original circuit. Then combining in series the resistors and voltage sources, and retransforming them (b) is obtained. Finally adding the two currents and transforming back the circuit to its Thevenin form (c) is obtained.



(a)



(b)



(c)

From (c),

$V_{oc} = 60V$

$R_{TH} = 6k\Omega$

$I_L = 60/(6k+6k) = 5mA$

$P = I_L^2 R_L = 0.15W$

SOLUTION 6.8. First, each source in series with 2R, can be replaced by an up going current source of $V_x / 2R$ in parallel with 2R. Then starting from the left, the two 2R in parallel are combined and then retransformed to a voltage source of $V_o / 2$ in series with 2R once added to the series resistance. Repeating the previous steps,

$V_o / 2 + 2R ? \quad V_o / 4\, R \| 2R \| 2R \| V_1 / 2\, R$

$V_o / 4\, R + V_1 / 2\, R \|\, R ? \quad V_o / 4 + V_1 / 2 + R + R$

$V_o / 4 + V_1 / 2 + 2R ? \quad V_o / 8\, R + V_1 / 4\, R \| 2R \| 2R \| V_2 / 2\, R$

$V_o / 8\, R + V_1 / 4\, R + V_2 / 2\, R \|\, R ? \quad V_o / 8 + V_1 / 4 + V_2 / 2 + R + R$

$V_o / 8 + V_1 / 4 + V_2 / 2 + 2R ? \quad V_o / 16\, R + V_1 / 8\, R + V_2 / 4\, R \| 2R \| 2R \| V_3 / 2\, R$

$V_o / 16\, R + V_1 / 8\, R + V_2 / 4\, R + V_3 / 2\, R \|\, R ? \quad V_o / 16 + V_1 / 8 + V_2 / 4 + V_3 / 2 + R + R$

Thus $V_{oc} = V_o / 16 + V_1 / 8 + V_2 / 4 + V_3 / 2$ and $R_{TH} = 2R$.

SOLUTION 6.9. First we find R_{TH}

$$R_{TH} = 2K + [(6K \| 3K) + (15K \| 10K)] \| 24K = 8K\Omega$$

Use nodal analysis to solve for V_{OC}. At node a

$1mA = 1/\,6K(V_a - V_c) + 1/\,3K(V_a - V_d) + 1/\,24K(V_a)$ or

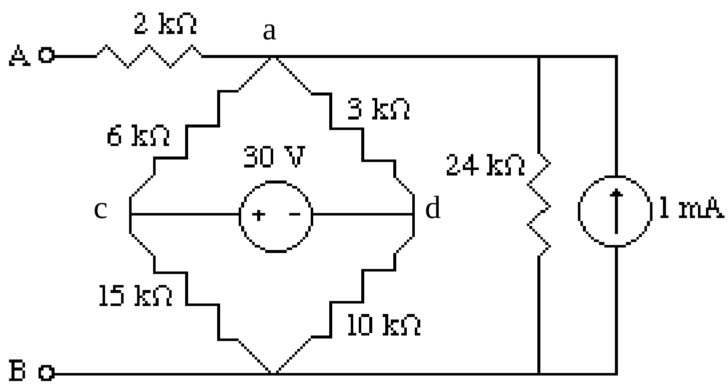
$(1/\,6K + 1/\,3K + 1/\,24K)V_a - 1/\,6K(V_c) - 1/\,3K(V_d) = 1m$

doing the same for the supernode, and the equation inside the supernode yields the following two equations:

$(1/\,6K + 1/\,3K)V_a - (1/\,6K + 1/15K)V_c - (1/\,3K + 1/10K)V_d = 0$, and

$V_c - V_d = 30$

using these three equations, it is now possible to solve for the three unknowns V_A, V_C, and V_D. Using MATLAB they are respectively 4.5V, 22.8750V, and -7.1250V. V_{OC} being V_A, 4.5V.



SOLUTION 6.10.

$R_{TH} = [(18K \| 9K) + 66K] \| 36K = 24K$

By superposition, noting that all resistances are in k-ohms,

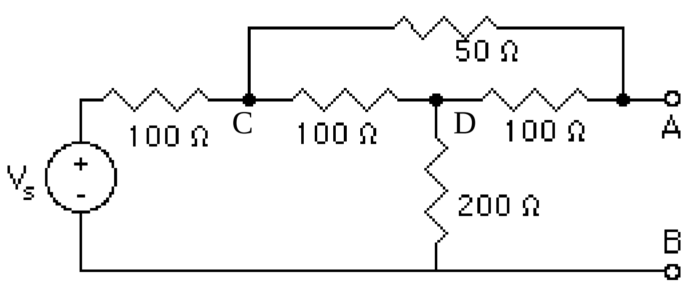
$$V_{OC} = \frac{66+(9||18)}{66+(9||18)+36}\ 36\ \ 2.5 + \frac{(66+36)||9}{[(66+36)||9]+18}\ \frac{36}{36+66}\ \ 18 - \frac{36}{36+(9||18)+66} 30 = 52\ \text{V}$$

When a 2 kΩ is connected, the current I_L becomes $V_{OC}/(R_{TH}+2\text{k}) = 2\text{mA}$; thus the power absorbed is 8 mW.

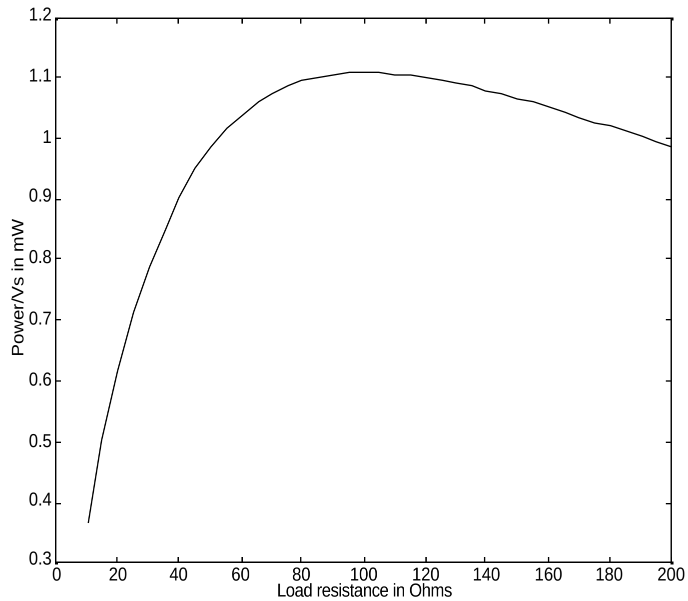
SOLUTION 6.11. (a) Introduce a test voltage source at the output, and write the nodal equations in matrix form:

$$\begin{bmatrix} (1/100+1/50) & -1/50 & -1/100 \\ -1/100 & -1/100 & (1/100+1/200+1/100) \\ -1/50 & (1/50+1/100+1/100) & -1/100 \end{bmatrix} \begin{bmatrix} V_{test} \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} i_{test} \\ 0 \\ V_s/100 \end{bmatrix}$$

Solving we obtain, $V_{test} = 100 i_{test} + 2V_s/3$. From eq. 6.10 $R_{TH} = 100\ \Omega$, $v_{oc} = 2V_s/3$.



(b) To obtained the power the following equation is used, $P = \left(\frac{v_{oc}}{R_{TH}+R_L}\right)^2 R_L$.



SOLUTION 6.12.

V_{OC} = 0, as no independent sources are present. Writing the following nodal equation where v_x is the voltage across both ports,

$i_x = ((v_x - i_z) - \alpha i_x) + (v_x - i_z)/\ 2$, R_{TH} can be found as $v_x / i_x = \dfrac{2.5+\alpha}{1.5}$,

SOLUTION 6.13.

Defining v_i and i_i as the voltage across and current into the input ports, writing the nodal equation at the input: $i_i + g_m v_x = 1/\ 200K(v_i - v_x)$. We can also get the following equation $v_x = 200K(g_m v_x + i_i)$. Using the previous two equations we can solve for $R_{TH} = v_i / i_i = -200K\left(1 + \dfrac{200K\ \ g_m}{1+200K\ \ g_m} + \dfrac{1}{1+200K\ \ g_m}\right)$.

This yields a g_m of 10 μS.

SOLUTION 6.14.

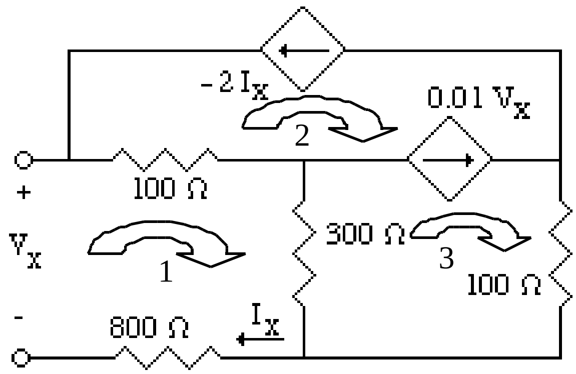
I_{SC} is null as no independent source are present. To find R_{TH} v_i and i_i are defined as the voltage across and current into the input ports. Writing the nodal equation we get:

$i_i = V_x / 1.8K + (1/\ 200)(V_x - 3/4V_x)$, and $v_i = V_x - 300\ \ i_i$. Solving $R_{TH} = v_i / i_i = -600\Omega$.

SOLUTION 6.15.

First, write out the equation around loop 1:

$V_x = i_1\ 1200 - i_2\ 100 - i_3\ 300$. Then substituting the following relationships, $i_1 = I_x$, $i_2 = 2I_x$, and $i_3 = 0.01V_x + i_2 = 0.01V_x + 2I_x$, and solving for $V_x / I_x = R_{TH} = 100\Omega$. $V_{OC} = 0$ as no independent sources are present.

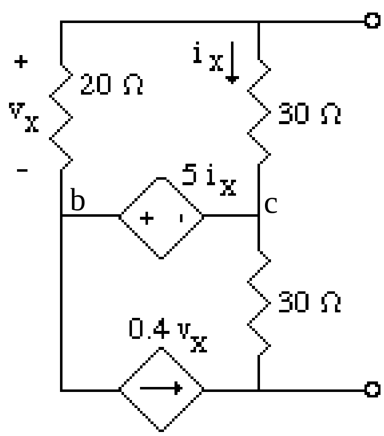


SOLUTION 6.16.

Introduce a test source at the output terminals, and write out the nodal equations in matrix for the top node, and the supernode comprised of the current controlled voltage source (ccvs). v_b is the node left of ccvs, and v_c the node to the right.

$$\begin{bmatrix} 1/30 + 1/20 & -1/20 & -1/30 \\ -1/20 - 1/30 + 0.4 & 1/20 - 0.4 & 2/30 \\ 1/6 & -1 & 5/6 \end{bmatrix} \begin{bmatrix} v_{test} \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} i_{test} \\ 0 \\ 0 \end{bmatrix}$$

Solving, we obtain $v_{test} = -90 i_{test}$, thus $R_{TH} = -90\ \Omega$.



SOLUTION 6.17.

(a) Turn off independent source. Introduce a test source and write loop equation: $v_x = 6i_x - 4i_1 + 10i_x$. Note that $i_1 = i_x$. Now solve for $v_x / i_x = R_{TH} = 12\Omega$.

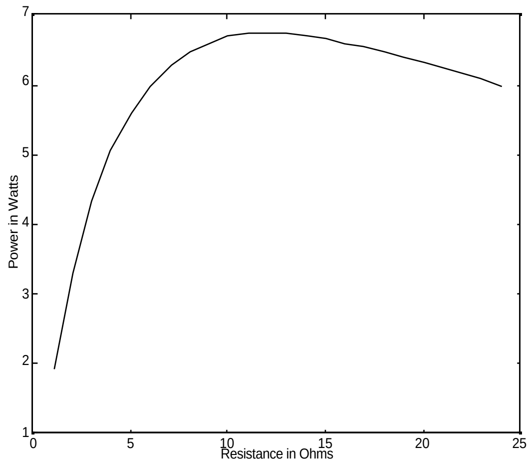
Short the input and write the loop and nodal equations:

$3A = i_1 + I_{SC}$

$10i_1 = 4i_1 + 6I_{SC}$

Solving yields $I_{SC} = 1.5A$, and $V_{OC} = I_{SC}R_{TH} = 18V$.

(b) In MATLAB the following plot is generated: $P = \left(\frac{V_{OC}}{R_{TH} + R_L}\right)^2 R_L$ for $1 \le R_L \le 24\Omega$.



Maximum power is absorbed by 12 Ω load.

SOLUTION 6.18.

To find thevenin resistance, introduce a test source and write the following equations: $v_s = 20i_1 + 40i_1 + 40i_1 = 100i_1$, and $i_s = v_s/100 + i_1$. Solving for $v_s/i_s = R_{TH} = 50\Omega$.

Next, use the following nodal equation; $0.2A = i_1 + I_{SC}$, and loop equation $20i_1 + 40i_1 = 40I_{SC}$. Solving using these two equations yields $I_{SC} = 0.12A$, and consequently $V_{OC} = R_{TH}I_{SC} = 6V$.

SOLUTION 6.19.

(a) Introduce a test source, v_s, and get the following two equations: $v_s - 15i_s = -V_x$, and $kV_x + V_x/3 + V_x/5 + i_s = 0$. Solving yields $v_s/i_s = R_{TH} = 15 + \left(\frac{1}{k + 8/15}\right)$, or 95/6 Ω for k = 2/3.

Next, write the following nodal equation $kV_x + V_x/3 + (V_x - 1)/5 = 0$ and observe that $V_{OC} = 1V - V_x$.

Thus solving yields $V_{OC} = 1 - \left(\frac{1/5}{k + \frac{8}{15}}\right) = 5/6V$ for k = 2/3, and $I_{SC} = V_{OC}/R_{TH} = 1/19A$.

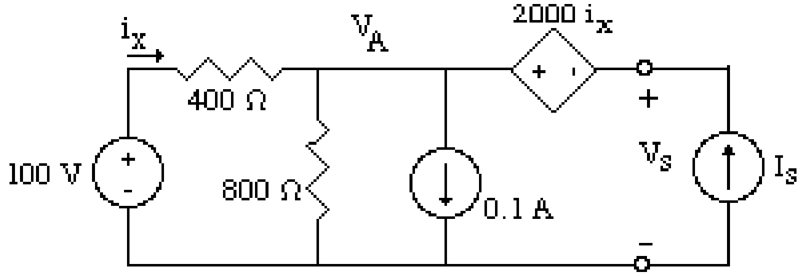
(b) Solving the previously obtained equation for k when $V_{OC} = 0$, yields k = -1/3, and consequently $R_{TH} = 20\ \Omega$.

SOLUTION 6.20.

Introduce a test source, v_t, and get the following equations: $i_t + i_x = i_x \quad \frac{V_s - 300 i_x}{300} \qquad V_s = 300 i_t + 300 i_x$, and $v_t = V_s - 300 i_x + 100 i_t$. Solving yields $v_t / i_t = R_{TH} = 400\Omega$.

Observe how no current flows through the 300 resistor in parallel with the dependent source. Thus V_{OC} will always be 0 V and is independent of V_S.

SOLUTION 6.21. **(a)** For this part, consider the modified circuit below.



Step 1: Applying KCL to node A, we have

$$\frac{1}{400}\left(V_A - 100\right) + \frac{V_A}{800} + 0.1 - I_S = 0$$

Multiplying through by 800 yields

$$3V_A = 800 I_S + 120$$

Step 2. Computing V_s, we have

$$V_S = V_A - 2000 i_x = V_A - \frac{2000}{400}\left(100 - V_A\right) = 6V_A - 500$$

Hence

$$V_S = 6\left(\frac{800}{3} I_S + 40\right) - 500 = 1600 I_S - 260 = R_{th} I_S + V_{oc}$$

Therefore $R_{th} = 1.6\ \text{k}\Omega$ and $V_{oc} = -260\ \text{V}$.

(b) By linearity, $V_{oc} = -130\ \text{V}$.

SOLUTION 6.22.

Introduce a test source, v_t,, and get the following two equations: $v_t = (30m - V_1/100)400 - V_1 = 12 - 5V_1$, and $i_t = 0.06V_1 + 30m - V_1/100 = 30m + 0.05V_1$. Solving obtain $v_t = -100i_t + 15$. Thus $R_{TH} = -100\Omega$, and $I_{SC} = V_{OC}/R_{TH} = -150mA$.

SOLUTION 6.23. Insert I_{test} as per text. Hence

$$\left(\frac{1}{40} + \frac{1}{120}\right)V_C - \frac{16}{40} = I_{test}$$

Solving, we obtain

$$V_C = 30I_{test} + 12$$

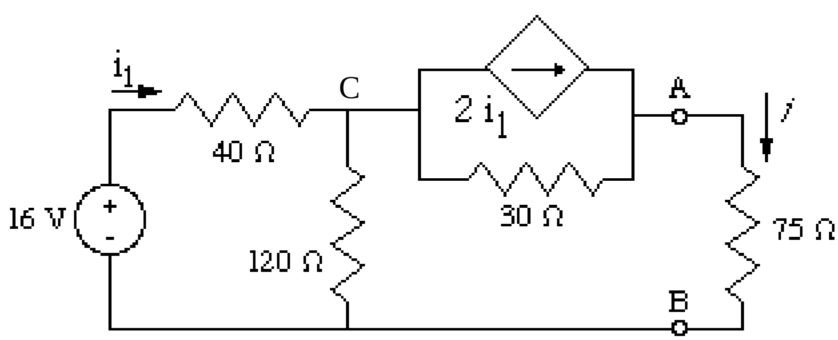
By KVL,

$$V_{test} = V_C + 30 \times \left(2i_1 + I_{test}\right) = V_C + \frac{60}{40}\left(16 - V_C\right) + 30I_{test} = -0.5V_C + 30I_{test} + 24$$

Substituting for V_C yields

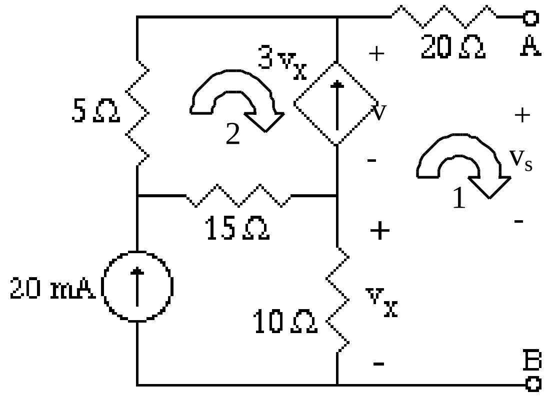
$$V_{test} = 15I_{test} + 18$$

Hence, $V_{oc} = 18$ V and $R_{th} = 15$ Ω. Thus $i = \frac{18}{75+15} = 0.2$ A. Further, $P_{75} = 75(0.2)^2 = 3$ W.



SOLUTION 6.24.

(a) and (b)



(c) Writing KVL around loop 1, loop 2, and finally relating v to v_x,

$$v_s = 10(0.02 - I_1) + v - 20I_1$$

$$v = -5I_2 - 15(I_2 - 0.02)$$

$$3v_x = I_1 - I_2 = 3[10(0.02 - I_1)] = 0.6 - 30I_1$$

(d) Rewriting these in matrix form,

$$\begin{bmatrix} -30 & 0 & 1 \\ 0 & 20 & 1 \\ 31 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_s + \begin{bmatrix} -0.2 \\ 0.3 \\ 0.6 \end{bmatrix}$$

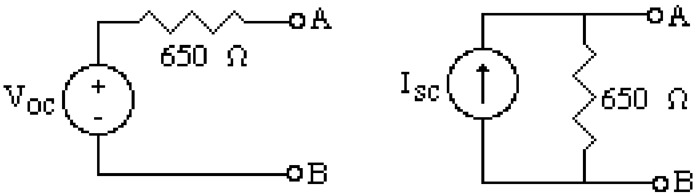
(e), (f), and (g). Using MATLAB,

```
»A=[-30  0  1;0   20   1; 31   -1   0];
»b1 = [1  0  0]';
»b2 = [-.2 .3  .6]';
»I1 = inv(A)*b1
I1 =
  -1.5385e-03
  -4.7692e-02
   9.5385e-01
»I2 = inv(A)*b2
I2 =
   1.9231e-02
  -3.8462e-03
   3.7692e-01
»Rth=-1/I1(1)
Rth = 650
Note:  Isc = I2(1) = 0.01923 A
»Voc= -Rth*I2(1)
```

Voc =

1.2500e+01

(h)



SOLUTION 6.25.

(a)

(b) Write four nodal equations,

$i_s = (V_A - V_C) / 2k$

$i_s + 1m = (V_C - V_D)/ 6k + (V_C - V_E)/ 3k$

$V_E / 2k = (V_C - V_D) / 6k - V_D / 15k$

$V_E / 2k = V_E / 10k + (V_E - V_C)/ 3k$

(c)

$$\begin{bmatrix} 1/2k & -1/2k & 0 & 0 \\ 0 & 1/2k & -1/6k & -1/3k \\ 0 & 1/6k & -(1/6k + 1/15k) & -1/2k \\ 0 & -1/3k & 0 & -1/15k \end{bmatrix} \begin{bmatrix} V_A \\ V_C \\ V_D \\ V_E \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} i_s + \begin{bmatrix} 0 \\ 1m \\ 0 \\ 0 \end{bmatrix}$$

(d) Solving in MATLAB $V_A = 5.818ki_s + 3.8182$, thus

$R_{TH} = 5.818k\Omega$

$V_{OC} = 3.8182V$

(e) This only changes $V_{OC} = 3818 \; 8m = 30.54V$.

SOLUTION 6.26. For this circuit, no current flows though the 20 Ω resistor. Therefore $V_{AB} = V_{CB}$. Further, from the examples in the chapter, $V_{CB} = 4V_s$. Hence, $V_{oc} = V_{AB} = V_{CB} = 4V_s$. Also, shorting terminals A and B, yields $I_{sc} = \frac{V_{CB}}{20} = \frac{4V_s}{20} = \frac{V_s}{5}$. It follows that $R_{th} = \frac{V_{oc}}{I_{sc}} = 20\ \Omega$. Note that the Thevenin equivalent to the left of C-B is a voltage source of value $4V_s$. Therefore the Thevenin equivalent to the left of A-B is 20 Ω in series with $4V_s$.

SOLUTION 6.27. (a) From the previous problem $V_{CB} = 4V_s$. Thus by voltage division,

$V_{OC} = V_{AB} = V_{CB}\left(\frac{180}{180+20}\right) = 3.6V_s$. Next find $I_{SC} = V_{CB} / 20 = V_s / 5$, and then $R_{TH} = V_{OC} / I_{SC} = 18\Omega$.

(b) This changes the voltage division at the output, thus $V_{AB} / V_s = 4\left(\frac{180||162}{(180||162)+20}\right) = 3.24$.

SOLUTION 6.28. (a) Writing the following two KCL equations,

$v_{test} / 4k = (V_o - v_{test}) / 12k$

$I_{test} = (v_{test} - V_o) / 15k$

where V_o is the voltage at the output of the op-amp. Doing the appropriate substitution get

$v_{test} = -15k / 3 \ I_{test}$, thus

$R_{TH} = -5k\Omega$

$V_{OC} = 0V$

Since no independent source exist right of A-B

(b) Applying Ohm's law $I_s = 1/(10k - 5k)V_s = 0.2mV_s$

SOLUTION 6.29. (a) Adding a test source at terminal A-B, and noting that the voltage at the output of the op-amp is $V_o = -5/2V_s$. Write KCL at terminal A,

$v_{test} / 900 = (V_o - v_{test}) / 100 + i_{test}$

$v_{test} = 90 i_{test} - 2.25V_s$

Where one sees by inspection that

$R_{TH} = 90\Omega$

$V_{OC} = -2.25V_S$

$I_{SC} = V_{OC} / R_{TH} = -0.025V_S$

(b) Noticing the virtual short to ground provided by the ideal op-amp, $R_{TH} = 20k\Omega$, and $V_{OC} = 0V$ since no independent sources are present right of the input terminal.

SOLUTION 6.30. The output voltage of the ideal op amp is $-2.5V_{s1} - 2V_{s2}$ which drives a voltage divider circuit. Hence

$$V_{oc} = V_{AB} = 0.9(-2.5V_{s1} - 2V_{s2}) = -2.25V_{s1} - 1.8V_{s2}$$

Further,

$$I_{sc} = \frac{-2.5V_{s1} - 2V_{s2}}{100} = -0.0225V_{s1} - 0.018V_{s2}$$

Finally,

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{0.9(-2.5V_{s1} - 2V_{s2})}{\frac{(-2.5V_{s1} - 2V_{s2})}{100}} = 90\ \Omega$$

Equivalently if one sets V_{s1} and V_{s2} to zero, then the ouput terminal of the op amp goes to ground. Hence

$$R_{th} = 100 // 900 = 90 \ \Omega$$

SOLUTION 6.31. Define the node at the output of the op-amp as V_o. Note how the circuit left of this node is a general summing circuit as per text. Thus, $V_o = 2V_{s2} - 4V_{s1}$. Hence we simply replace the op amp circuit to the left of the 20 Ω resistor by an ideal voltage source of value V_0. Hence

$$V_{oc} = \frac{80}{80+20}V_0 + 2I = 0.8V_0 + 2\left(\frac{-V_0}{100}\right) = 0.78V_0 = 1.56V_{s2} - 3.12V_{s1}$$

Alternately, one can introduce a test source at the output terminal and write out a set of equations using KVL,

$$V_o = 2V_{s2} - 4V_{s1} = -20I - 80(I - i_{test})$$
$$v_{test} = 2I + 80(i_{test} - I)$$

Solving yields $v_{test} = 17.6i_{test} + 0.78(2V_{s2} - 4V_{s1})$, and by inspection,

$$R_{TH} = 17.6\Omega$$
$$V_{OC} = 0.78(2V_{s2} - 4V_{s1}) = 56V_{s2} - 3.12V_{s1}$$

SOLUTION 6.32. Upon inspection when the op-amp is in active range, for inputs from –3V to 3V,

$R_{TH} = 1k\Omega$

$V_{OC} = 0V$

However when the input exceeds 3V, the output of the op-amp will saturate at –15V, and $V_s = -15V + 6kI_s$. Therefore from eq. 6.10,

$R_{TH} = 6k\Omega$

$V_{OC} = -15V$

When the input is less than –3V, $V_s = 15 + 6kI_s$, thus

$R_{TH} = 6k\Omega$

$V_{OC} = 15V$

SOLUTION 6.33. (a) The op-amp configuration has a gain of –4. So when the input is between –3V and 3V it is operating in its active region, thus

$R_{TH} = 4k\Omega$

$V_{OC} = 0V$

When the input is greater than 3V the output saturates at –12V and

$V_s = 20kI_s - 12$

$R_{TH} = 20k\Omega$

$V_{OC} = -12V$

When the input is less than –3V the output saturates at 12V,

$V_s = 20kI_s + 12$

$R_{TH} = 20k\Omega$

$V_{OC} = 12V$

(b) When the input is in the active range,

$R_{TH} = 16k\Omega$

$V_{OC} = -4V_s$

When it is greater than 3V,

$R_{TH} = 0k\Omega$

$V_{OC} = -12V$

And when less than –3V,

$R_{TH} = 0k\Omega$

$V_{OC} = 12V$

The last two obtained using figure 6.28.

The maximum power is when the output is in saturation, $P = (V_{OC})^2 / 28k = 6mW$.

SOLUTION 6.34. From the table the following two equations can be written:

$$6 = 2R_{TH} + v_{oc}$$

$$12 = 8R_{TH} + v_{oc}$$

Putting in matrix form and solving,

$$\begin{bmatrix} 1 & 1 \\ 8 & 1 \end{bmatrix} \begin{bmatrix} R_{TH} \\ v_{oc} \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -1/6 & 1/6 \\ 4/3 & -1/3 \end{bmatrix} \begin{bmatrix} 6 \\ 12 \end{bmatrix} = \begin{bmatrix} R_{TH} \\ v_{oc} \end{bmatrix}$$

Thus $R_{TH} = 1$ kΩ since current was in mA, and $v_{oc} = 4$ V.

SOLUTION 6.35. (a) From Ohm's law, $I_{AB} = V_{AB} / R_L$. Thus 0.2uA and 0.1uA.

(b) Note how using this topology $V_{AB} = v_{oc} - I_{AB}R_{TH}$, thus

$$\begin{bmatrix} 1 & -0.2\mu \\ 1 & -0.1\mu \end{bmatrix} \begin{bmatrix} v_{oc} \\ R_{TH} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ -10M & 10M \end{bmatrix} \begin{bmatrix} 0.4 \\ 1 \end{bmatrix} = \begin{bmatrix} v_{oc} \\ R_{TH} \end{bmatrix}$$

Thus, $v_{oc} = 1.6V$, and $R_{TH} = 6M\Omega$.

SOLUTION 6.36. (a) Although the text describes finding R_{th} from a measurement or calculation of both V_{oc} and I_{sc}, measurement of I_{sc} is often impractical. Hence the procedure outlined in this problem provides a more practical means of determining the Thevenin equivalent.

Since the internal meter resistance is 10 MΩ,

$$I_{AB}\ (\mu A) = \frac{V_{AB}}{10} + \frac{V_{AB}}{R_L}$$

Hence, the completed table is:

R_L(M Ω)	v_{AB} (V)	I_{AB} (μA)
2	0.4	0.24
10	1	0.2

(b) The terminal relationship assuming a Thevenin equivalent is given by

$$V_{AB} = V_{oc} - R_{th} I_{AB}$$

In matrix form with the data from the table

$$\begin{bmatrix} 1 & -0.24 \\ 1 & -0.2 \end{bmatrix} \begin{bmatrix} V_{oc} \\ R_{th} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 1 \end{bmatrix}$$

Hence V_{oc} = 4 V and R_{th} = 15 MΩ. Note that since we used μA and V, R_{th} is in MΩ.

SOLUTION 6.37. (a) For this scenario, the circuit is essentially a voltage source with a resistance R in series with the circuit under test, in parallel with a voltmeter measuring the voltage division between the later two. Therefore if replacing R = 0 by R = R_2 causes the voltage measured by the voltage meter to drop by half, then by voltage division, $E_o / 2 = E_o \left(\frac{R_{TH}}{R_2 + R_{TH}} \right)$, and $R_{TH} = R_2$.

(b) Using the same reasoning and voltage division,

$$E_o = V_s \frac{R_{TH}}{R_{TH} + R_s}$$

$$E_o / 2 = V_s \frac{R_{TH}}{R_{TH} + R_s + R_2}$$

Therefore $R_{TH} + R_s = R_2$ or equivalently $R_{TH} = R_2 - R_s$.

(c) Again by voltage division,

$$E_o = V_s \frac{R_{TH} \,||\, R_m}{(R_{TH} \,||\, R_m) + R_s}$$

$$E_o / 2 = V_s \frac{R_{TH} \,||\, R_m}{(R_{TH} \,||\, R_m) + R_s + R_2}$$

Therefore $(R_{TH} \,||\, R_m) + R_s = R_2$ or $\frac{R_{TH} R_m}{R_{TH} + R_m} = R_2 - R_s$. Solving for $R_{TH} = \frac{R_2 R_m - R_s R_m}{R_m + R_s - R_2}$.

SOLUTION TO 6.38. **(a)** The voltmeter measures the voltage division between the two resistors involved thus,

$$E_o = V_{oc}$$

$$E_o / 2 = V_{oc} \frac{R_2}{R_{TH} + R_2}$$

and $R_{TH} = R_2$.

(b) Now

$$E_o = V_{oc} \frac{R_m}{R_m + R_{TH}}$$

$$E_o / 2 = V_{oc} \frac{R_m \,||\, R_2}{(R_m \,||\, R_2) + R_{TH}}$$

From the division of the former by the later $R_2 = R_{TH} \,||\, R_m$. And from the former

$V_{oc} = (R_m + R_{TH})E_o / R_m = (1 + R_{TH} / R_m)E_o$.

SOLUTION TO 6.39. Using the relation developed in problem 6.37 $R_{TH} = \frac{R_2 R_m - R_s R_m}{R_m + R_s - R_2} = 5k\Omega$

SOLUTION TO 6.40. Using the equation developed in problem 6.38,

$$R_{TH} = \frac{R_2 R_m}{R_m - R_2} = 4M\Omega$$

$$V_{oc} = (1 + R_{TH} / R_m)E_o = 20V$$

SOLUTION TO 6.41. Writing the line equation in the following general form,

$$i = v_{ab} / R_{TH} - i_{sc}$$

$$i = 2v_{ab} - 4$$

Thus $R_{TH} = 0.5\Omega$, and $i_{sc} = 4A$.

SOLUTION TO 6.42. (a) In this range the curve appears to be varying linearly between (0V,0mA) and (0.2V, 0.1mA) pair, thus writing the line equation $i = (0.1m / 0.2)v$ yields $R_{TH} = 2k\Omega$ $V_{oc} = 0V$.

(b) Writing the line equation of the following form, $i = v / R_{TH} - i_{sc}$, into a matrix equation,

$$\begin{bmatrix} 0.2 & -1 \\ 0.7 & -1 \end{bmatrix} \begin{bmatrix} 1/R_{TH} \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 0.1m \\ 10.1m \end{bmatrix}$$

and solving gives

$$R_{TH} = 50\Omega$$

$$i_{sc} = 3.9mA$$

$$v_{oc} = 0.195V$$

(c) First make a guess as to which region of the curve, N will operate in. Assuming that it will operate in the A-B region, then by KVL $i(t) = (v_s(t) + v_b - v_{oc})/ (R + R_{TH}) = (50\sin(1000t) - 0.095) / 550$. It can be seen that this guess is wrong as the range of i(t) is not in the appropriate region. Assuming the 0-A region, by KVL $i(t) = (v_s(t) + v_b - v_{oc})/ (R + R_{TH}) = (50\sin(1000t)m + 0.1)/ 2500 = 0.02\sin(1000t) + 0.04mA$. Note that the highest current is 0.06mA, thus still in the appropriate region of operation.

(d) Repeating the procedure above and guessing the region A-B, by KVL $i(t) = (v_s(t) + v_b - v_{oc})/ (R + R_{TH}) = (200\sin(1000t)m + 305m) /100 = 2\sin(1000t) + 3.05mA$. The maximum and minimum current are 5.05 and 1.05 mA respectively, thus the assumption made was correct.

SOLUTION TO 6.43. (a) By the power transfer theorem $R_L = R_{TH}$. For circuit (a) $R_L = 80 || 240 = 60\Omega$, and circuit (b) $R_L = (900||180) + 50 = 200\Omega$.

(b) Finding i_{sc} for each circuit: (a) $i_{sc} = 36 / (80 + 240||60)\ \ 240 / 300 = 225mA$ and (b) $i_{sc} = 60mA\left(\frac{900 || 180}{(900||180) + 50 + 200}\right) = 22.5mA$. The power is now obtained from $P = \left(i_{sc} / 2\right)^2 R_L$. Thus 759mW for (a) and 25.3mW for (b).

SOLUTION TO 6.44. Finding the Thevenin equivalent,

$R_{TH} = \left[(30||15) + 10\right]||80 = 16\Omega$

$V_{oc} = -32V$

The value for the load resistance is 16Ω, and the power is $P = \left(\frac{V_{oc}}{2}\right)^2 / R_L = 16W$.

SOLUTION TO 6.45. (a) Find the Thevenin equivalent,

$R_{TH} = 12k||6k + 8k = 12k\Omega = R_L$

$V_{oc} = 8k(2mA) + 24\left(\frac{6}{6+12}\right) = 24V$

The power is $P = (V_{oc}/2)^2 / R_L = 12mW$.

(b) The maximum power will be transferred to the load, when the value of its load is closest to 12kΩ. Thus the power is $P = \left(\frac{V_{oc}}{R_{TH} + 10k}\right)^2 10k = 11.9mW$.

(c) Same reasoning as (b) the power is $P = \left(\frac{V_{oc}}{R_{TH} + 15k}\right)^2 15k = 11.9mW$.

SOLUTION TO 6.46. **(a)** The Thevenin equivalent to the left of R_L has $R_{th} = 12 + 20//180 = 30\ \Omega$ and $V_{oc} = 1 \times 18 + \frac{180}{180+20} 40 = 54$ V. Therefore, for maximum power transfer

$$G_L + 2G_L + 3G_L = 6G_L = \frac{6}{R_L} = \frac{1}{30}$$

Hence $R_L = 180\ \Omega$.

(b) For this part, let V_L denote the voltage (top to bottom) across the load. With $R_L = 180\ \Omega$, then the parallel combination equals R_{th} and hence $V_L = 27$ V. It follows that

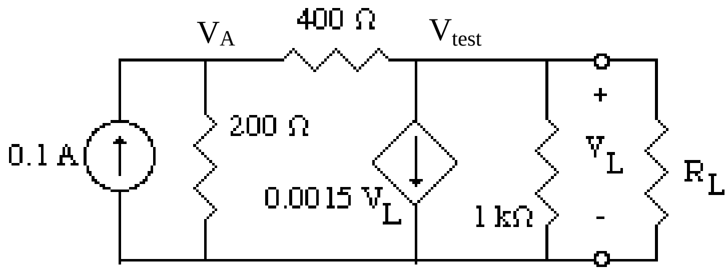
$$P_{180} = \frac{(27)^2}{180} = 4.05 \text{ watt.}$$

Since the terminal voltages are the same, the absorbed power is inversely proportional to the resistance. Hence $P_{90} = 8.1$ watt, and $P_{60} = 12.15$ watt.

SOLUTION 6.47. To find the Thevenin equivalent introduce a test voltage source at the output, and write KCL at the two node in the circuit. By inspection the following matrix expression is obtained:

$$\begin{bmatrix} (1/200+1/400) & -1/400 \\ -1/400 & 0.0015+1/1k+1/400 \end{bmatrix} \begin{bmatrix} V_A \\ V_{test} \end{bmatrix} = \begin{bmatrix} 0.1 \\ I_{test} \end{bmatrix}$$

Solving $V_{test} = 240 I_{test} + 8$. And $R_{TH} = 240\ \Omega$, $V_{oc} = 8\ \text{V}$. By voltage division, the voltage across the load resistor is 4V, and the power delivered to it is 66.7mW.



SOLUTION 6.48. Using KCL get $i_x = 1mA$, which independent of what is connected to the output. Thus

$i_{sc} = 10 i_x = 10mA$

$R_{TH} = 3k\Omega$

The power is then $P = (10mA/2)^2\ 3k = 75mW$.

SOLUTION 6.49. Performing a source transformation on (a), and combining the elements will simplify to one current source going up of 2/3A in parallel with a 10Ω resistor. This is essentially the Norton equivalent of the circuit,

$i_{sc} = 2/3A$

$R_{TH} = 10\Omega$

For (b), combine the voltage sources and resistor in series. The circuit obtained is one voltage source of 5V in series with a 45Ω resistor. This is the Thevenin equivalent,

$v_{oc} = 5V$

$R_{TH} = 45\Omega$

(a) The value of the load resistor is simply the thevenin resistance obtained above.

(b) Using Ohm's law for (a) $V_L = i_{sc}(10\,||\,10) = 10/3V$, and voltage division for (b) $V_L = v_{oc}(1/2) = 2.5V$

(c) Using the following formula, $P = V_L^2 / R_L$, (a) absorbs 1.1W and (b) 139mW, thus (a) absorbs more power.

SOLUTION 6.50. (a) Note that the circuit left of the terminal is already in its Thevenin form. The load

$R_L = R||(R+300)$

$R^2 + R(300-2R_L) - 300R_L = 0$

Solving, $R = 71.6\Omega$. By voltage division, the voltage across the load is 5V. The power absorbed is $P = \left(V_{oc}/2\right)^2 / R_L = 416.7mW$.

(b) The following script can be used to plot the power absorbed by the load versus R:

```
%Script for problem 6.50b
R=0:2:400;

%Calculate Load resistance
RL= 1./((1./R)+1./(R+300));

%Calculate the power
P=(10.*(RL./(RL+60))).^2./RL;

%Plot the power versus R
plot(R,P);
ylabel('Power in Watts');
xlabel('Resistance in Ohms')
```

SOLUTION 6.51. First, find the Thevenin equivalent by writing out the transfer equation $v_{ab} = 200i + 40$. Thus $R_{TH} = 200\Omega$ $V_{oc} = 40V$. The maximum power will then be $P = (V_{oc}/2)^2/200 = 2W$.

SOLUTION 6.52. The assumption that all controlling voltages or currents for dependent sources within N_i are assumed to be in N_i, implies that the nodal equation matrix of figure P6.52a has the partitioned form:

$$\left[\begin{array}{c|c|c} G_{11} & G_{12} & 0 \\ \hline G_{21} & G_{22} & G_{23} \\ \hline 0 & G_{32} & G_{33} \end{array}\right]\left[\begin{array}{c} V_{N1} \\ \hline V_m \\ \hline V_{N2} \end{array}\right] = \left[\begin{array}{c} I_{N1} \\ \hline I_m \\ \hline I_{N2} \end{array}\right] \quad (*)$$

where V_{N1} is the vector of UNKNOWN and INDEPENDENT node voltages internal to N_1 and V_{N2} is the vector of UNKNOWN and INDEPENDENT node voltages internal to N_2. The right side of the equation consists of (effective) currents injected into the appropriate node. However, I_{N1} depends only on sources in N_1 and I_{N2} depends only on sources in N_2.

At this point we must presume that the matrix equation (*) has a unique solution, i.e., the determinant of the coefficient matrix is non-zero. Hence we can calculate V_{N1}, V_m, and V_{N2} uniquely. As such, by considering the first row of (*), we can assert that V_{N1} satisfies

$$G_{11}V_{N1} = I_{N1} - G_{12}V_m \quad (**)$$

Note that we are not claiming that we can solve for V_{N1} from this equation.

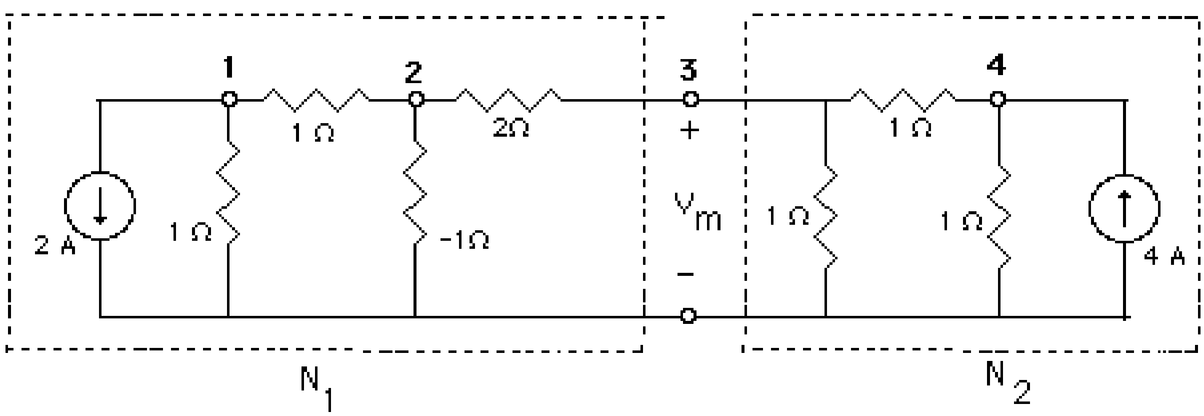
Replacing N_2 by a voltage source of value V_m results in the network of figure P6.52b. For this network, the nodal equations are

$$G_{11}V_{N1} = I_{N1} - G_{12}V_m \qquad (***)$$

where G_{ij} is the same as in (*). Clearly, this is the same as equation (**). Again we presume there is a unique solution to this equation, i.e., the determinant of G_{11} is non-zero. If so, we can solve for V_{N1} uniquely and the result is the same as that obtained by solving (*).

This theorem can be extended to RLCM networks (to be studied in later chapters) or even nonlinear networks under appropriate conditions.

To emphasize the subtlety of this result and the need for unique solvability in each network, consider the following circuit.



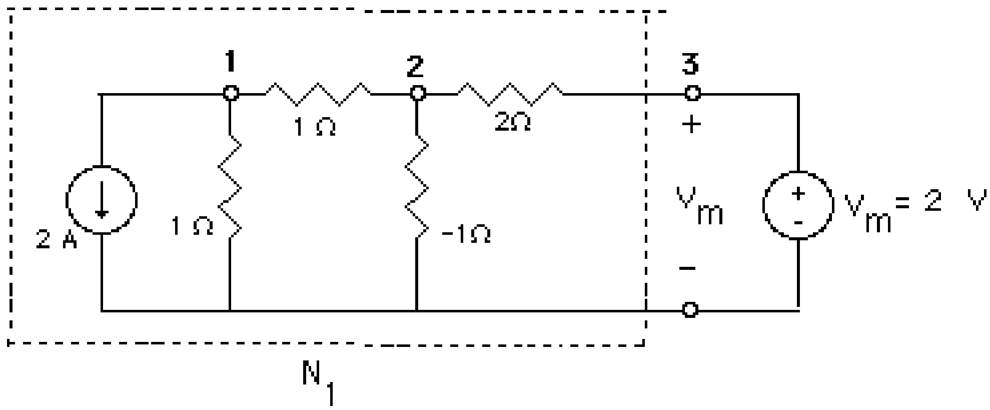
The resulting nodal equations are:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 0.5 & -0.5 & 0 \\ 0 & -0.5 & 2.5 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

There exists a unique solution and from MATLAB, we find

$$V_m = V_3 = 2 \text{ V}$$

To apply the voltage source substitution, we replace N_2 by a voltage source of 2 V and obtain the following circuit.



The nodal equations here are

$$\begin{bmatrix} 2 & -1 \\ -1 & 0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Observe that the coefficient matrix has a zero determinant. Thus there is either no solution or many solutions, i.e., no unique solution. This demonstrates that unique solvability of the larger network does not imply the unique solvability of the smaller derived network.

SOLUTION 6.53. The assumption that all controlling voltages or currents for dependent sources within N_i are assumed to be in N_i, implies that the loop equation matrix of figure P6.53a has the partitioned form:

$$\left[\begin{array}{c|c|c} R_{11} & R_{12} & 0 \\ \hline R_{21} & R_{22} & R_{23} \\ \hline 0 & R_{32} & R_{33} \end{array}\right] \begin{bmatrix} I_{N1} \\ \hline I_m \\ \hline I_{N2} \end{bmatrix} = \begin{bmatrix} E_{N1} \\ \hline E_m \\ \hline E_{N2} \end{bmatrix} \quad (*)$$

where I_{N1} is the vector of unknown and independent loop currents internal to N_1 and I_{N2} is the vector of unknown and independent loop currents internal to N_2 and I_m a single independent loop current common to N_1 and N_2. The right side of the equation represents the net contribution of voltage sources present in the appropriate loop. However, E_{N1} depends only on N_1 and E_{N2} depends only on N_2.

At this point we must presume that the matrix equation (*) has a unique solution, i.e., the determinant of the coefficient matrix is non-zero. Hence we can calculate I_{N1}, I_m, and I_{N2} uniquely. As such, by considering the first row of (*), we can assert that I_{N1} satisfies

$$R_{11}I_{N1} = E_{N1} - R_{12}I_m \quad (**)$$

Note that we are not claiming that we can solve for I_{N1} from this equation.

Replacing N_2 by a current source of value I_m results in the network of figure P6.53b. For this network, the loop equations are

$$R_{11}I_{N1} = E_{N1} - R_{12}I_m \quad (***)$$

where R_{ij} is the same as in (*). Clearly, this is the same as equation (**). Again we presume there is a unique solution to this equation, i.e., the determinant of R_{11} is non-zero. If so, we can solve for I_{N1} uniquely and the result is the same as that obtained by solving (*). For some subtlety in the proof refer to the solution of 6.52.

This theorem can be extended to RLCM networks (to be studied in later chapters) or even nonlinear networks under appropriate conditions.

SOLUTION 6.54. (a) The thevenin equivalent to the left of terminal A-B is $R_{TH} = [(30||60) + 20]||10 = 8\Omega$, and using Ohm's law along with voltage division

$$V_{OC} = 15 - 10\left(\frac{15V}{15+60}\right)\left(\frac{30}{30+30}\right) = 14V$$

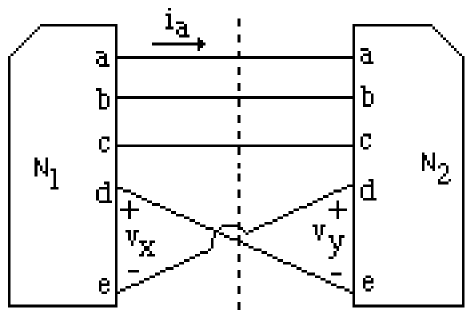
(b) Doing the same for the circuit right of terminal A-B. $R_{TH} = [(30||15) + 10]||20 = 10\Omega$ and

$$V_{OC} = 7.5 - 20\left(\frac{7.5V}{15+15}\right)\left(\frac{30}{30+30}\right) = 5V.$$

(c) Using superposition, $V_{AB} = 14\left(\frac{10}{18}\right) + 5\left(\frac{8}{18}\right) = 10V$.

(d) (e) $(v_{CB} - 15)/30 + v_{CB}/60 = (10 - v_{CB})/20$ Hence, $v_{CB} = 10V$.

SOLUTION TO P6.55. The proof is based on superposition. Let us consider the figure below where N_1 and N_2 are differently named but identical networks.



We first compute the contribution to i_a from the independent sources in N_1 with those of N_2 deactivated. Let this current be i_a^1. The contribution to i_a from the independent sources in N_2 with those of N_1 deactivated is i_a^2. But because N_1 and N_2 are identical, $i_a^1 = -i_a^2$. Hence by superposition $i_a = i_a^1 + i_a^2 = 0$. By the current source substitution theorem we can replace the lines by current sources of value 0-amp. This defines an open circuit and the connecting line can be replaced by an open circuit. From the given network we also note by KVL that $V_x + V_y = 0$ which implies that $V_x = -V_y$. On the other hand, since the networks are identical, $V_x = V_y$. Thus we conclude that $V_x = V_y = 0$. Thus we can replace V_x and V_y by a voltage source of 0-volt (voltage source substitution theorem) which is the definition of a short circuit.

SOLUTION 6.56. Label the potential between each line starting from the top as V_{x1}, V_{x2} on the left, and V_{y1} and V_{y2} on the right. Now by superposition and linearity notice that

$V_{x1} = -V_{y1}$

$V_{x2} = -V_{y2}$

because the independent source is negative on the right side. Additionally, from KVL,

$V_{x1} = V_{y1}$

$V_{x2} = V_{y2}$

The only way all these condition can be met, is if all the voltages are 0 V, or short circuited.

SOLUTION 6.57. (a) Using the results from P6.55, no current flows between the two halves. So the right hand side circuit may be analyzed as if it was stand alone. By voltage division then,

$$v_a = \left(\frac{3+2}{3+2+1}\right)18 = 15V.$$

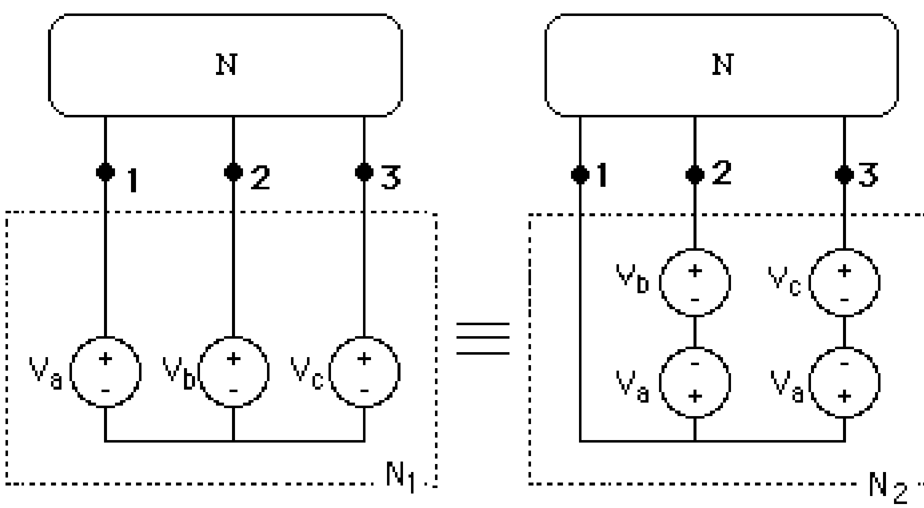
(b) From the results of P6.56, all the lines crossing the symmetry line are shorted together. Consequently, by voltage division, $v_a = \left(\frac{6\,||3}{(6\,||3)+1}\right)(-18) = -12V$.

SOLUTION 6.58. Note how this circuit is the same as in P6.57: it is just redrawn with the neighboring resistors added in parallel or in series. Using superposition, we can solve for v_a when the sources $[v_{s1}\ \ v_{s2}]$ are [18 18], and then [18 –18]. By linearity adding the two contribution will be equivalent to solving for [36 0] directly, since adding the source contributions [18 18]+[18 –18] =[36 0]. The contributions of 15V–12V were obtained in P5.57; thus $v_a = 3V$.

SOLUTION 6.59. Yes since

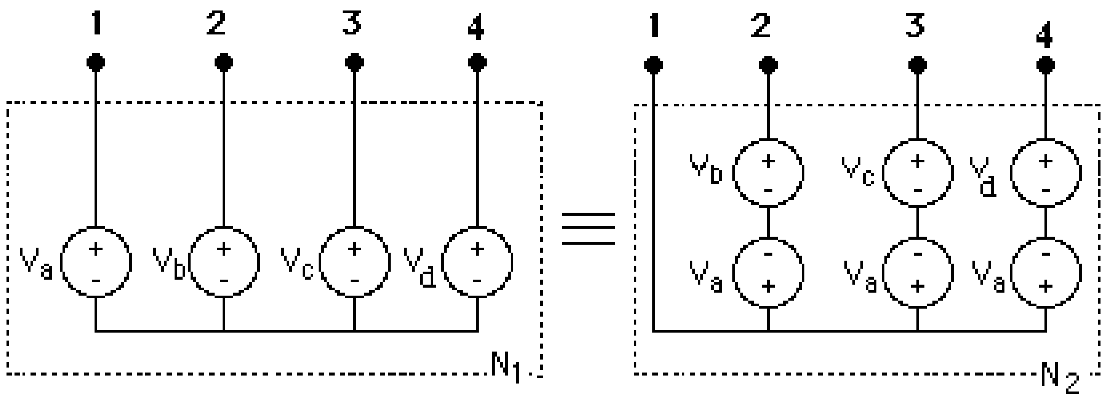
$$\begin{bmatrix}45 & 27\end{bmatrix} = \begin{bmatrix}36 & 36\end{bmatrix} + \begin{bmatrix}9 & -9\end{bmatrix} = 2\begin{bmatrix}18 & 18\end{bmatrix} + 1/2\begin{bmatrix}18 & -18\end{bmatrix} = 2(15) + (12)/2 = 24V.$$

SOLUTION 6.60. For this proof we attach an arbitrary network N to each of the networks N_1 and N_2 in figure P6.60 as shown below.



N may have internal independent sources, but we consider N_1 and N_2 external excitations to N and we assume no violation of KVL in the attachment. Choose node 3 as a reference node. Then $V_{13} = V_a - V_c$ and $V_{23} = V_b - V_c$ for both figures. Hence N_1 and N_2 provide the identical external excitations to N and hence all currents and voltages in N remain unchanged.

The extension of this result to 4 external nodes is shown in the figure below. The verification is the same as above. The extension of course to n-terminals is clear.



SOLUTION 6.61. Using the E-shift theorem, remove the 9 V source from each branch and add it to the 4V source, and notice by inspection that $V_{OC} = -4 + 9 = 5$ V.

SOLUTION 6.62. (a) Writing a KCL equation for each node in N2:

$I_1 = I_a$

$I_2 = I_a - I_a = 0$

$I_3 = I_a - I_a = 0$

$I_4 = -I_a$

Do the same for N1:

$I_1 = I_a$

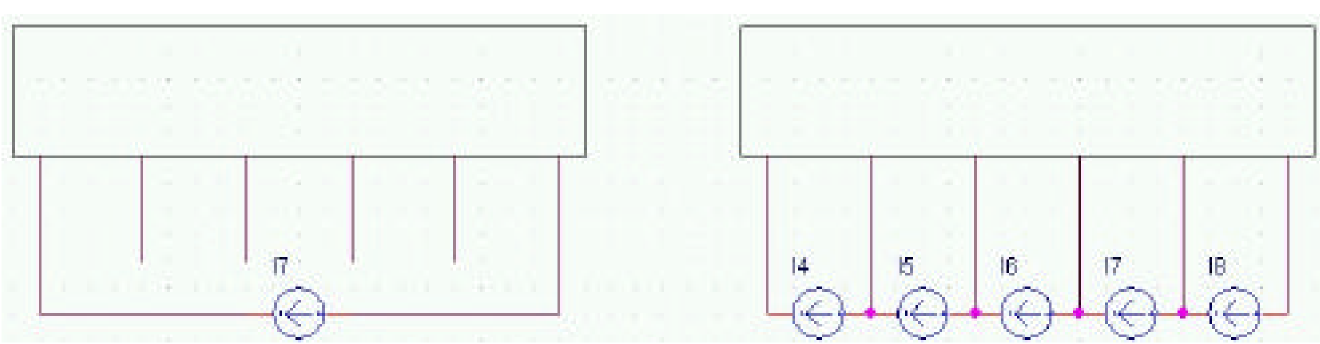
$I_2 = 0$

$I_3 = 0$

$I_4 = -I_a$

This shows that the two have identical outcomes.

(b)



SOLUTION 6.63. Using the I-shift theorem, this circuit is essentially a –2A source in parallel with a series combination of resistors, and a 5A source. Thus $i_{sc} = 5 - 2 = 3A$.

SOLUTION 6.64. (a) This can be done by inspection. An equal source is connected between A-C and C-B; thus by the I-shift theorem, it is equivalent to the same source just connected between A-B.
(b) In figure 6.64c the VCCS is replaced by a resistor using the Ohm's law relationship $100\Omega = V_1 / (0.01 V_1)$.

(c) (d) By voltage division $V_1 = V_s \left(\frac{100||900}{(100||900) + 10} \right)$, and by Ohm's law

$$V_{out} = V_s \left(\frac{100||900}{(100||900) + 10} \right) 0.01 \left(20k || 5k \right) = 36 V_s$$

$$V_{out} / V_s = 36$$