PROBLEM SOLUTIONS CHAPTER 7

SOLUTION 7.1. Given the coil has 48 turns and 12 turns/cm, we know that the length of the coil is 4 cm. Since the length of the coil is greater than 0.4 times its diameter, the formula given in the question can be used:

$$L = \frac{4 \times 10^{-5} \times (0.02)^2 \times (48)^2}{18 \times (0.02) + 40 \times (0.04)} = 18.81 \mu H$$

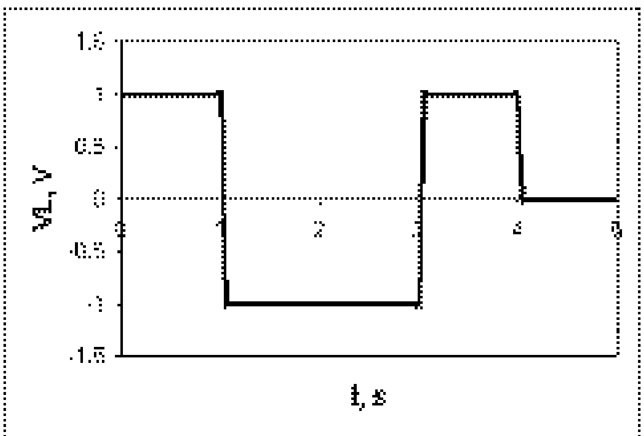
SOLUTION 7.2. **Part 1.** Applying (7.1)
The voltage $v_L(t)$ can be computed using the inductor v-i relationship:

$$v_L(t) = L \frac{di_L(t)}{dt}$$

The calculations for $v_L(t)$ for t = 0s to 5s are summarized in the following table:

Time Interval	d/dt (i_{in}(t))	v_L(t)
0s< t ≤1s	2 As^{-1}	1V
1s< t ≤3s	-2 As^{-1}	-1V
3s< t ≤4s	2 As^{-1}	1V
4s< t ≤5s	0 As^{-1}	0V

Below is the plot of V_L vs time.



Part 2. Applying (7.4)

$$w_L(t) = \frac{1}{2}(0.5\, i_{in}{}^2(t))$$

In the time interval 0s < t ≤ 1s, $i_{in}(t) = 2t$. Thus

$$w_L(t) = \frac{1}{2}\left(0.5 i_{in}^2(t)\right) = t^2$$

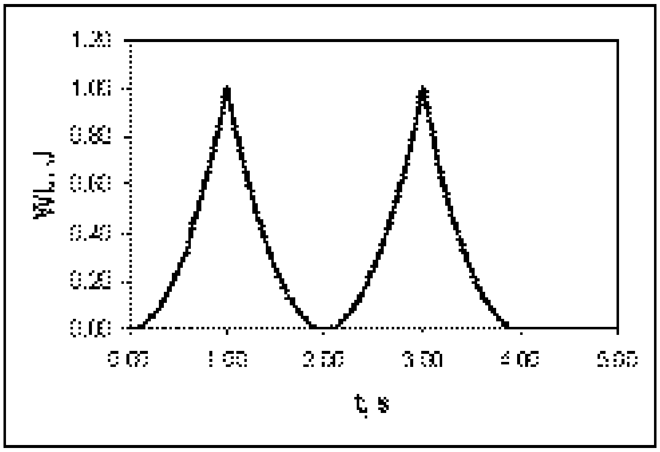
In the time interval 1s < t ≤ 3s, $i_{in}(t) = 4 - 2t$. Thus

$$w_L(t) = \frac{1}{2}\left(0.5 i_{in}^2(t)\right) = \frac{1}{4}(4-2t)^2 = t^2 - 4t + 4$$

In a similar way, $w_L(t)$ can be computed in the remaining intervals.
The calculations for $w_L(t)$ for t = 0s to 5s are summarized in the following table:

Time Interval	$i_{in}(t)$	$w_L(t)$
0s< t ≤1s	2t	t^2
1s< t ≤3s	4-2t	t^2-4t+4
3s< t ≤4s	-8+2t	t^2-8t+16
4s< t ≤5s	0	0

Below is the plot of w_L vs t.



SOLUTION 7.3. Applying (7.2)

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)\,d\tau$$

It is assumed that $i_L(0)$ = 0 A. Using the preceding formula and the fact that $v_{in}(t)$ = t in the time interval 0s < t ≤ 2s, the current is

$$i_L(t) = \frac{1}{0.5}\int_0^t \tau\,d\tau = t^2 \quad \text{in the time interval } 0\text{s} < t \le 2\text{s}$$

In the time interval 2s < t ≤ 3s $v_{in}(t) = 2$. Hence

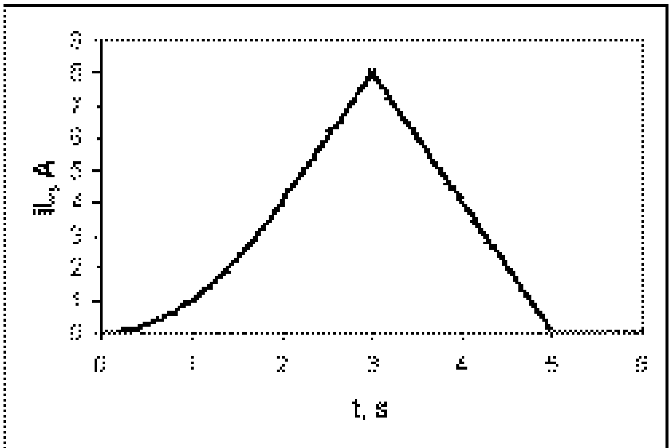
$$i_L(t) = \frac{1}{0.5}\int_0^t v_L(\tau)\,d\tau = i_L(2) + \frac{1}{0.5}\int_2^t 2dt = 4 + 4\tau\Big|_2^t = 4t - 4$$

In a similar way $i_L(t)$ can be computed in the remaining intervals.

The calculations for $i_L(t)$ for t = 0s to 6s are summarized in the following table:

Time Interval	$v_{in}(t)$, V	$i_L(t)$, A
0s< t ≤2s	t	t^2
2s< t ≤3s	2	4t-4
3s< t ≤5s	-2	20-4t
5s< t ≤6s	0	0

Below is the plot of i_L vs t.



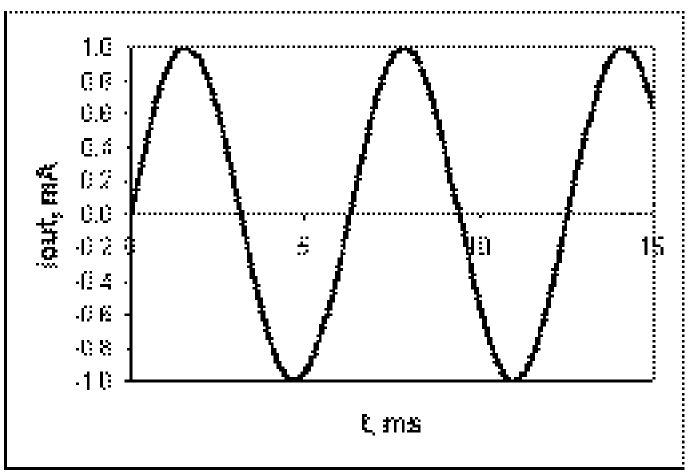
SOLUTION 7.4. **Part 1.** Using (7.1), we have

$$v_{in}(t) = 0.2\times 10^{-3}\frac{d}{dt}\left[i_s(t)\right] = 0.2\times 10^{-3}\times 1000\cos(1000t) = 0.2\cos(1000t)mV$$

For the 2mH inductor

$$i_{out}(t) = i_{out}(0) + \frac{1}{L}\int_0^t 10v_{in}(\tau)d\tau = \frac{1}{2\times 10^{-3}}\int_0^t 2\cos(1000\tau)d\tau = \sin(1000t)mA$$

Below is a sketch of i_{out} vs t.



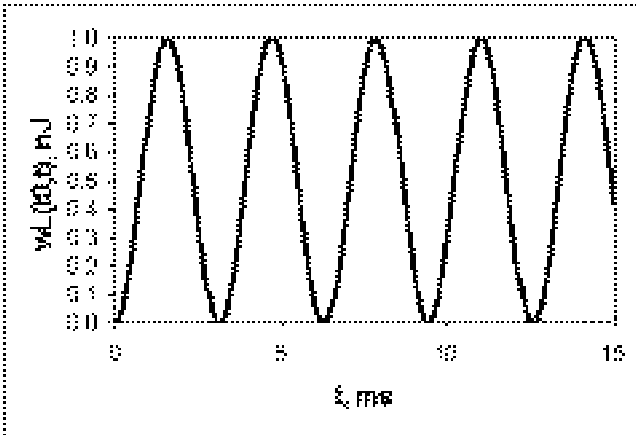
Part 2. Instantaneous power delivered by the dependent source is given by

$$p_L(t) = v_L(t)\times i_L(t) = 10v_{in}(t)\times i_{out}(t) = 2\cos(1000t)\times \sin(1000t) = \sin(2000t)\mu W$$

Part 3. The energy stored in the 2mH inductor is given by

$$W_L(t) = \frac{1}{2}Li_L{}^2(t) = \frac{1}{2}Li_{out}{}^2(t) = \sin^2(1000t)\,nJ$$

Below is a sketch of W_L vs t



SOLUTION 7.5. **Part 1** For the excitation in Figure P7.5b,

$$i_1(t) = i_1(0) + \frac{1}{0.5}\int_0^t v_{in}(\tau)\,d\tau, \quad i_2(t) = i_2(0) + \frac{1}{0.25}\int_0^t v_{in}(\tau)d\tau = 2i_1(t)$$

It is assumed that $i_1(0) = i_2(0) = 0$ A. In the interval 0s < t ≤ 1s, $v_{in}(t) = -10$V. Hence, in this interval

$$i_1(t) = \frac{1}{0.5}\int_0^t v_{\rm in}(\tau)\,d\tau = \frac{1}{0.5}\int_0^t (-10)\,d\tau = -20t$$

$$i_2(t) = \frac{1}{0.25}\int_0^t v_{\rm in}(\tau)\,d\tau = \frac{1}{0.25}\int_0^t (-10)d\tau = -40t$$

In the interval 1s < t ≤ 2s, $v_{in}(t) = -5$V. Hence, in this interval,

$$i_1(t) = i_1(1) + \frac{1}{0.5}\int_1^t v_{\rm in}(\tau)\,d\tau = -20 + \frac{1}{0.5}\int_1^t (-5)\,d\tau = -10t - 10$$

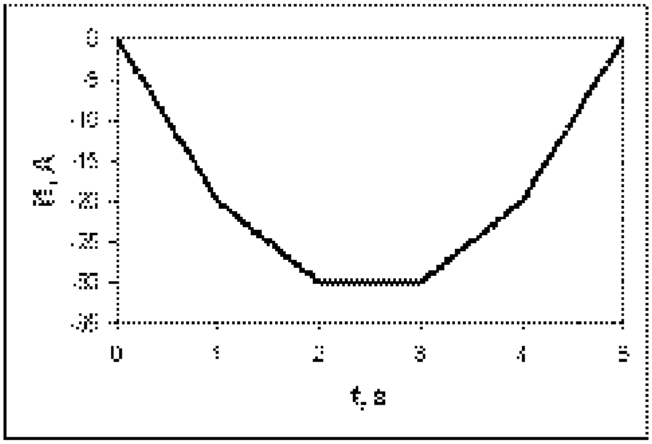
$$i_2(t) = i_2(1) + \frac{1}{0.25}\int_1^t v_{\rm in}(\tau)\,d\tau = -40 + \frac{1}{0.25}\int_1^t (-5)\,d\tau = -20t - 20$$

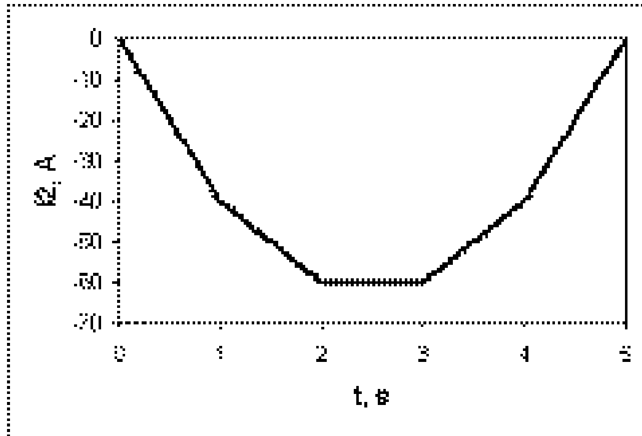
In a similar fashion $i_1(t)$ and $i_2(t)$ can be computed in other intervals.

The calculations for $i_1(t)$ and $i_2(t)$ for t = 0s to 5s are summarized in the following table:

Time Interval	$v_{in}(t)$, V	$i_1(t)$, A	$i_2(t)$, A
0s< t ≤1s	-10	-20t	-40t
1s< t ≤2s	-5	-10-10t	-20-20t
2s< t ≤3s	0	-30	-60
3s< t ≤4s	5	-60+10t	-120+20t
4s< t ≤5s	10	100+20t	200+40t

Below are the plots of i_1 vs t and i_2 vs t.





Part 2 For the excitation in Figure P7.5c,

$$i_1(t) = i_1(0) + \frac{1}{0.5}\int_0^t v_{in}(\tau)d\tau, \quad i_2(t) = i_2(0) + \frac{1}{0.25}\int_0^t v_{in}(\tau)d\tau = 2i_1(t)$$

It is assumed that $i_1(0) = i_2(0) = 0$ A. In the interval 0s < t ≤ 1s, $v_{in}(t) = 10t$. Hence, in this interval

$$i_1(t) = i_1(0) + \frac{1}{0.5}\int_0^t v_{\text{in}}(\tau)d\tau = \frac{1}{0.5}\int_0^t (10\tau)d\tau = 10t^2$$

and

$$i_2(t) = i_2(0) + \frac{1}{0.25}\int_0^t v_{\text{in}}(\tau)d\tau = \frac{1}{0.25}\int_0^t (10\tau)d\tau = 20t^2$$

In the interval 1s < t ≤ 3s, $v_{in}(t) = 10t - 20$. Hence, in this interval

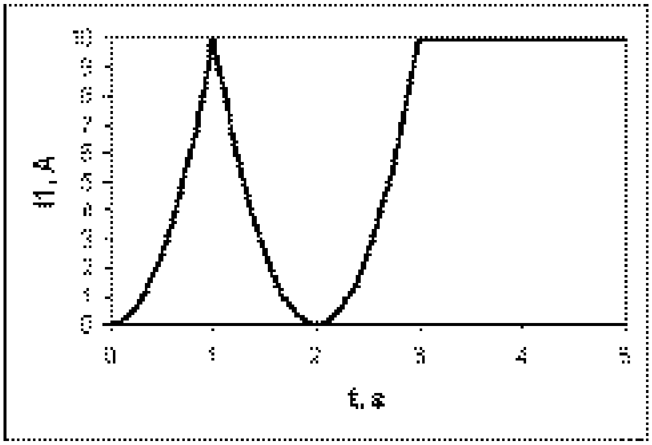
$$i_1(t) = i_1(1) + \frac{1}{0.5}\int_1^t v_{\text{in}}(\tau)d\tau = 10 + \frac{1}{0.5}\int_1^t (10\tau - 20)d\tau = 10t^2 - 40t + 40$$

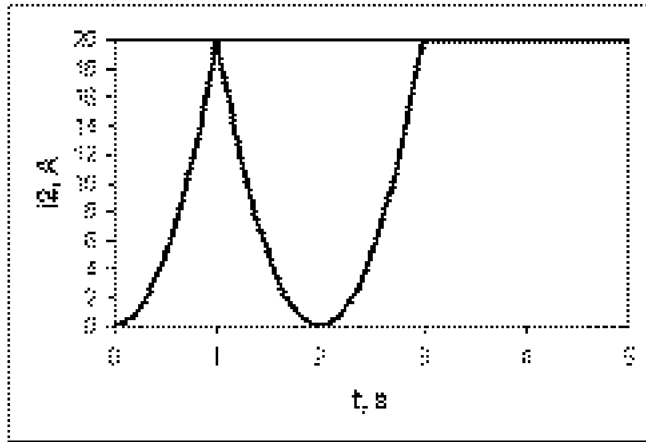
$$i_2(t) = i_2(1) + \frac{1}{0.25}\int_1^t v_{\text{in}}(\tau)d\tau = 20 + \frac{1}{0.25}\int_1^t (10\tau - 20)d\tau = 20t^2 - 80t + 80$$

The calculations for $i_1(t)$ and $i_2(t)$ for t = 0s to 3s are summarized in the following table:

Time Interval	$v_{in}(t)$, V	$i_1(t)$, A	$i_2(t)$, A
$0s< t \le 1s$	10t	$10t^2$	$20t^2$
$1s< t \le 3s$	10t-20	$10t^2$-40t+40	$20t^2$-80t+80

Below are the plots of i_1 vs t and i_2 vs t.





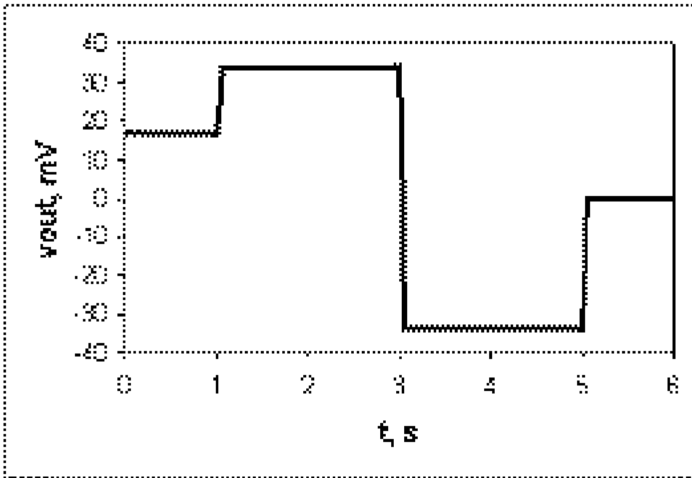
SOLUTION 7.6. For the circuit in Figure P7.6, the parallel combination of the 0.75mH and 1.5mH inductors can be replaced by an inductor with the inductance of (0.75||1.5)mH. The v-i relationship for this inductor is

$$i_{in}(t) = i_{in}(0) + \frac{1}{[(0.75)\|(1.5)]\times 10^{-3}}\int_0^t v_s(\tau)d\tau = \frac{1}{0.5\times 10^{-3}}\int_0^t v_s(\tau)d\tau \text{ A}$$

The series combination of the 0.8mH and 0.6mH can be replaced by an (0.8mH + 0.6mH) inductor. The v-i relationship for this inductor gives:

$$v_{out}(t) = (0.8+0.6)\times 10^{-3}\frac{d}{dt}\left[6i_{in}(t)\right] = \frac{0.8+0.6}{0.5}\times 6\frac{d}{dt}\left[\int_0^t v_s(\tau)d\tau\right] = 16.8 v_s(t) \quad V$$

Below is a plot of $v_{out}(t)$ vs t.



Assume $i_L(0) = 0$ for all inductors.

$$i_{out}(t) = 6i_{in}(t) = \frac{6}{0.5\times10^{-3}}\int_0^t v_s(\tau)d\tau = 12\times10^3\int_0^t v_s(\tau)d\tau \ A$$

In the interval 0s < t ≤ 1s, $v_s(t) = 1\text{mV}$. Hence,

$$i_{in}(t) = \frac{1}{0.5\times10^{-3}}\int_0^t v_s(\tau)d\tau = 2t(A),\ i_{out}(t) = 6i_{in}(t) = 12t(V)$$

The power p(t) on the same interval is computed as:

$$p(t) = 6i_{in}(t)\times v_{out}(t) = (12t)\times(16.8\times10^{-3}) = 0.202t(W)$$

In the interval 1s < t ≤ 3s, $v_s(t) = 2\text{mV}$. Hence,

$$i_{in}(t) = i_{in}(1) + \frac{1}{0.5\times10^{-3}}\int_1^t v_s(\tau)d\tau = 2 + 2\times2\tau|_1^t = 4t - 2(A)$$

$$i_{out}(t) = 6i_{in}(t) = 24t - 12(A)$$

and

$$p(t) = 6i_{in}(t)\times v_{out}(t) = (24t - 12)\times(16.8\times2\times10^{-3}) = 0.806t - 0.403(W)$$

In a similar fashion, $i_{out}(t)$ and p(t) can be computed for the remaining intervals.

The calculations for $i_{out}(t)$ and the instantaneous power delivered by the dependent source, p(t), for t = 0s to 6s are summarized in the following table:

Time Interval	$v_s(t)$, V	$v_{out}(t)$, V	$i_{out}(t)$, A	p(t), W
0s< t ≤1s	1×10^{-3}	16.8×10^{-3}	12t	0.202t
1s< t ≤3s	2×10^{-3}	33.6×10^{-3}	-12+24t	0.806t-0.403
3s< t ≤5s	-2×10^{-3}	-33.6×10^{-3}	132-24t	0.806t-4.435
5s< t ≤6s	0	0	12	0

SOLUTION 7.7. For $0 \le t < 2$ s, using the inductor v-i relationship, we have

$$i_{in}(t) = \frac{1}{0.5}\int_0^t v_{in}(\tau)d\tau = \frac{2}{0.25\pi}\left[-\cos(0.25\pi\tau)\right]_0^t = \frac{8}{\pi}\left[1-\cos(0.25\pi t)\right] \text{ A}$$

The associated energy stored as a function of t for this time interval is

$$W_{0.5}(t) = \frac{16}{\pi^2}\left[1-\cos(0.25\pi t)\right]^2 \text{ J}$$

The energy for the second inductor remains zero over this interval.

For $2 \text{ s} \le t$, we have

$$i_{in}(t) = i_{0.5}(t) + i_{0.25}(t) = i_{0.5}(t) + \frac{1}{0.25}\int_2^t v_{in}(\tau)\,d\tau$$

$$= \frac{8}{\pi}\left[1-\cos(0.25\pi t)\right] + \frac{4}{0.25\pi}\left[-\cos(0.25\pi\tau)\right]_2^t = \frac{8}{\pi}\left[1-\cos(0.25\pi t)\right] - \frac{16}{\pi}\cos(0.25\pi t)$$
$$= \frac{8}{\pi} - \frac{24}{\pi}\cos(0.25\pi t) \text{ A}$$

Here the current

$$i_{0.5}(t) = \frac{8}{\pi}\left[1-\cos(0.25\pi t)\right] \text{ A}$$

in which case the energy stored over the interval [2,t] is

$$W_{0.5}(2,t) = \frac{1}{2}0.5\left(i_{0.5}^2(t) - i_{0.5}^2(2)\right) = \frac{1}{2}0.5\left(i_{0.5}^2(t) - \frac{64}{\pi^2}\right) \text{ J}$$

Further

$$i_{0.25}(t) = -\frac{16}{\pi}\cos(0.25\pi t) \text{ A}$$

in which case the energy stored over the interval [2,t] is

$$W_{0.25}(2,t) = \frac{1}{2}0.25\left(i_{0.25}^2(t) - i_{0.25}^2(2)\right) = \frac{1}{2}0.25 i_{0.25}^2(t) \text{ J}$$

SOLUTION 7.8. Let the 5mH inductor be L_1 and the 20mH inductor be L_2.
For $0 \le t < 3$ms,

$$i_{L1}(t) = i_{L1}(0) + \frac{1}{L_1}\int_0^t v_s(\tau)d\tau = \frac{1}{5\times 10^{-3}}\int_0^t 12\cos(500\tau)d\tau = 4.8\sin(500t) \; mA$$

$$i_{L2}(t) = i_{L2}(0) + \frac{1}{L_2}\int_0^t v_s(\tau)d\tau = \frac{1}{20\times 10^{-3}}\int_0^t 12\cos(500\tau)d\tau = 1.2\sin(500t)\ \ mA$$

For t ≥ 3ms,

$i_{L1}(t) = 4.8\sin(500t)$ mA as L_1 is still subjected to the same voltage.

on the other hand,

$$i_{L2}(t) = i_{L2}(3^- ms) + \frac{1}{L_2}\int_{3^+ ms}^t v_{L2}(\tau)d\tau = i_{L2}(3^- ms) + \frac{1}{L_2}\int_{3^+ ms}^t 0\, d\tau$$
$$= i_{L2}(3^- ms)$$
$$= 1.2\sin(500\times 3\times 10^{-3}) = 1.197 mA$$

For 0≤ t < 3ms, the energies stored in the inductors are given as follows:

$$W_{L1}(t) = \frac{1}{2}L_1\left[i_{L1}(t)\right]^2 = 57.6\sin^2(500t)\ \ nJ,$$
$$W_{L2}(t) = \frac{1}{2}L_2\left[i_{L2}(t)\right]^2 = 14.4\sin^2(500t)\ \ nJ$$

For t ≥ 3ms, the energies stored in the inductors are given as follows:

$$W_{L1}(t) = \frac{1}{2}L_1\left[i_{L1}(t)\right]^2 = 57.6\sin^2(500t)\ \ nJ,$$
$$W_{L2}(t) = \frac{1}{2}L_2\left[i_{L2}(t)\right]^2 = 14.328\ \ nJ$$

SOLUTION 7.9. Given the dielectric parameters and the dimensions of the capacitor, the capacitance of the paper capacitor is given by

$$C = \varepsilon_r\varepsilon_0\frac{A}{d} = 3\times 8.854\times 10^{-12}\times\frac{0.04\times 0.8}{10^{-4}} = 8.5nF$$

SOLUTION 7.10. . **Part 1.** Applying the capacitor v-i relationship:

$$i_C(t) = C\frac{d}{dt}\left(v_C(t)\right) = 1\mu\times 100\times 1000\times(-\sin(1000t)) = -0.1\sin(1000t)\ \text{A}$$

Part 2. Applying the capacitor v-i relationship:

$$i_C(t) = C\frac{d}{dt}\left(v_C(t)\right)$$

$$10\times 10^{-3}\cos(1000t) = C\frac{d}{dt}\left(\sin(1000t)\right)$$

$$10\times 10^{-3}\cos(1000t) = 1000C\cos(1000t)$$

Therefore, C=10µF.

SOLUTION 7.11. Applying the capacitor v-i relationship for C_1 and C_2:

$$i_{C1}(t) = (2mF)\frac{d}{dt}\left(v_{in}(t)\right), \qquad i_{C2}(t) = (6mF)\frac{d}{dt}\left(v_{in}(t)\right) = 3i_{C1}(t),$$

In the interval –1s < t ≤ 0s, $v_{in}(t) = 5t + 5$. Hence, in this time interval

$$i_{C1}(t) = 2\times 10^{-3}\times\frac{d(v_{in}(t))}{dt} = 10^{-2}\text{ A}$$

and

$$i_{C2}(t) = 6\times 10^{-3}\times\frac{d(v_{in}(t))}{dt} = 3\times 10^{-2}\text{ A}$$

Using KCL, $i_{in}(t)$ can be computed as

$$i_{in}(t) = i_{C1}(t) + i_{C2}(t) = 4\times 10^{-2}\text{ A}$$

In the interval 0s < t ≤ 1s, $v_{in}(t) = 5$. Thus

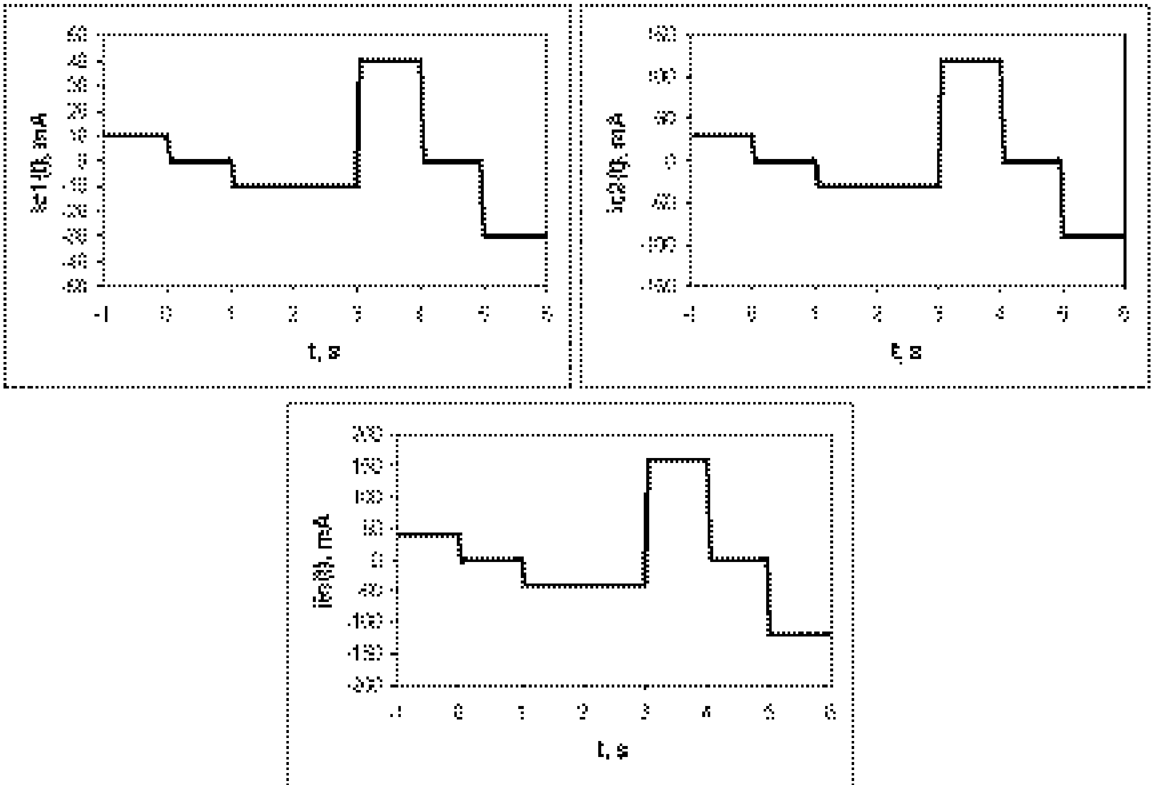
$$i_{C1}(t) = i_{C2}(t) = 0\text{ A}$$

Using KCL, $i_{in}(t) = i_{C1}(t) + i_{C2}(t) = 0\text{ A}$.
In a similar fashion $i_{C1}(t)$, $i_{C2}(t)$ and $i_{in}(t)$ can be computed for the remaining intervals.

The calculations for $i_{C1}(t)$, $i_{C2}(t)$ and $i_{in}(t)$ for t = -1s to 6s are summarized in the following table:

Time Interval	d/dt (v_{in}(t)), Vs^{-1}	i_{C1}(t), mA	i_{C2}(t), mA	i_{in}(t), mA
-1s< t ≤0s	5	10	30	40
0s< t ≤1s	0	0	0	0
1s< t ≤3s	-5	-10	-30	-40
3s< t ≤4s	20	40	120	160
4s< t ≤5s	0	0	0	0
5s< t ≤6s	-15	-30	-90	-120

Below are the plots of $i_{C1}(t)$, $i_{C2}(t)$ and $i_{in}(t)$.



SOLUTION 7.12. Applying (7.6)

$$v_C(2) = v_C(0) + \frac{1}{C}\int_0^2 i_C(\tau)d\tau = 4V + \frac{1}{C}\int_0^2 \tau d\tau = 5\text{V}$$

$$v_C(3) = v_C(2) + \frac{1}{C}\int_2^3 i_C(\tau)d\tau = v_C(2) + \frac{1}{C}\int_2^3 2d\tau = 5\text{V} + 1\text{V} = 6\text{V}$$

Applying (7.11), the energies stored in the capacitor over the intervals [0,2] and [2,3] are given by

$$W_C[0,2] = \frac{1}{2}C\left[v_C^2(2) - v_C^2(0)\right] = 9\mu J$$

$$W_C[2,3] = \frac{1}{2}C\left[v_C^2(3) - v_C^2(2)\right] = 11\mu J$$

SOLUTION 7.13. **Part 1** Applying (7.6)

$$v_{C1}(t) = v_{C1}(0) + \frac{1}{C_1}\int_0^t i_{in}(\tau)d\tau = \frac{1}{0.25\mu}\int_0^t i_{in}(\tau)d\tau$$

$$v_{C2}(t) = v_{C2}(0) + \frac{1}{C_2}\int_0^t i_{in}(\tau)d\tau = \frac{1}{0.1\mu}\int_0^t i_{in}(\tau)d\tau$$

In the interval 0s < t ≤ 1s, $i_{in}(t) = 2t$. Hence, in this interval

$$v_{C1}(t) = \frac{1}{0.25\mu}\int_o^t i_{in}(\tau)d\tau = \frac{1}{0.25\mu}(\tau^2)\Big|_0^t = 4\times 10^6 t^2 (V)$$

and

$$v_{C2}(t) = \frac{1}{0.1\mu}\int_o^t i_{in}(\tau)d\tau = \frac{1}{0.1\mu}(\tau^2)\Big|_0^t = 10\times 10^6 t^2 (V)$$

In the interval 1s < t ≤ 3s, $i_{in}(t) = 2\times 10^{-3}\,(A)$. Thus

$$v_{C1}(t) = v_{C1}(1) + \frac{1}{0.25\mu}\times\int_1^t i_{in}(\tau)d\tau = 4 + \frac{1}{0.25\mu}\times(2\times 10^{-3}\tau)\Big|_1^t = 8\times 10^3 t - 4(V)$$

and

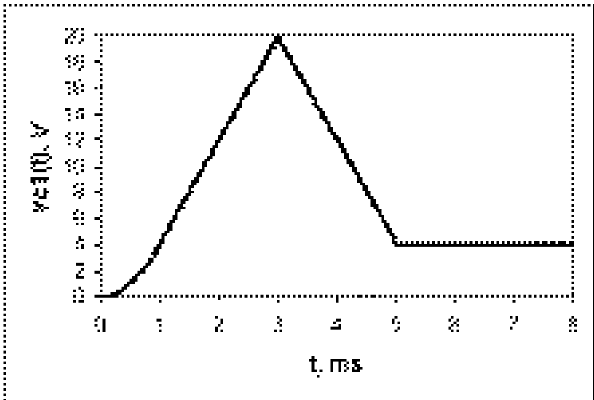
$$v_{C2}(t) = v_{C2}(1) + \frac{1}{0.1\mu}\times\int_1^t i_{in}(\tau)d\tau = 10 + \frac{1}{0.1\mu}\times(2\times 10^{-3}\tau)\Big|_1^t = 20\times 10^3 t - 10(V)$$

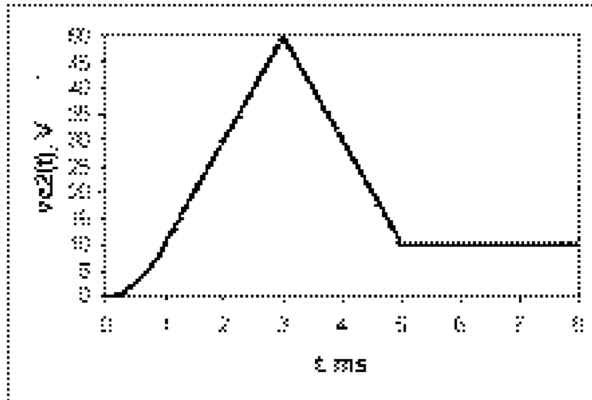
In a similar fashion, $v_{C1}(t)$ and $v_{C2}(t)$ can be computed for the remaining intervals.

The calculations for $v_{C1}(t)$ and $v_{C2}(t)$ for t = 0s to 8ms are summarized in the following table:

Time Interval	$i_{in}(t)$, A	$v_{C1}(t)$, V	$v_{C2}(t)$, V
0s< t ≤1ms	2t	$4\times 10^6 t^2$	$10\times 10^6 t^2$
1ms< t ≤3ms	2×10^{-3}	$-4 + 8\times 10^3 t$	$-10 + 20\times 10^3 t$
3ms< t ≤5ms	-2×10^{-3}	$44 - 8\times 10^3 t$	$110 - 20\times 10^3 t$
5ms< t ≤8ms	0	4	10

Below are the plots of $v_{C1}(t)$ and $v_{C2}(t)$.





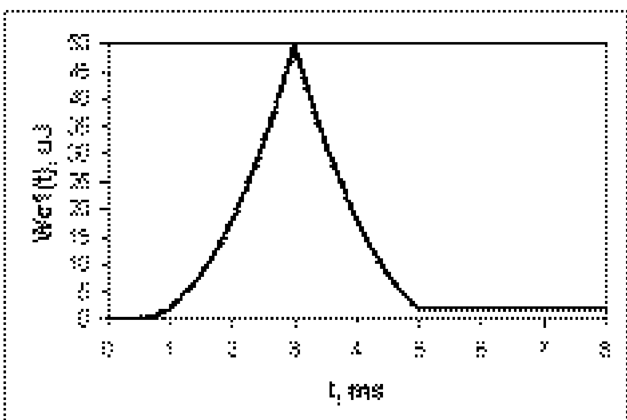
Part 2 Applying (7.12),

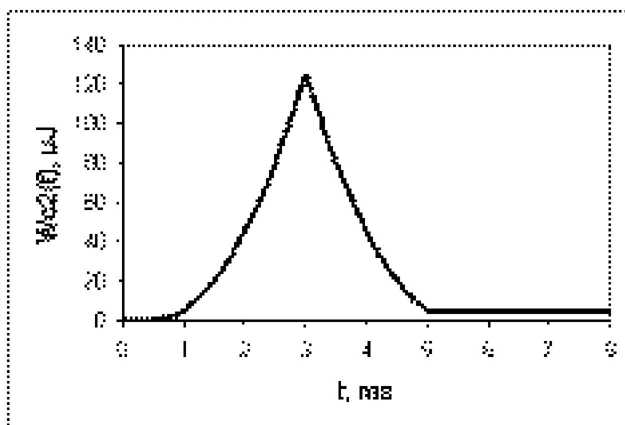
$$W_{C1}(t) = \frac{1}{2} C_1 v_{C1}{}^2(t)\ , \qquad W_{C2}(t) = \frac{1}{2} C_2 v_{C2}{}^2(t)$$

The expressions for $W_{C1}(t)$ and $W_{C2}(t)$ for t = 0ms to 8ms are listed in the following table:

Time Interval	$W_{C1}(t)$, μJ	$W_{C2}(t)$, μJ
0s< t ≤1ms	$0.125 \times (4 \times 10^6\ t^2)^2$	$0.05 \times (10\times 10^6\ t^2)^2$
1ms< t ≤3ms	$0.125 \times (-4 + 8\times 10^3\ t)^2$	$0.05 \times (-10 + 20\times 10^3\ t)^2$
3ms< t ≤5ms	$0.125 \times (44 - 8\times 10^3\ t)^2$	$0.05 \times (110 - 20\times 10^3\ t)^2$
5ms< t ≤8ms	$0.125 \times (4)^2$	$0.05 \times (10)^2$

Below are the plots of $W_{C1}(t)$ and $W_{C2}(t)$.





Part 3 Since the current $i_{C1}(t)$ stays constant at 0A after t = 5ms,

$$v_{C1}(\infty) = v_{C1}(5) = 4V$$
$$v_{C2}(\infty) = v_{C2}(5) = 10V$$

SOLUTION 7.14. **Part 1** Applying (7.6),

$$v_{in}(t) = v_{in}(0) + \frac{1}{C}\int_0^t i_{in}(\tau)d\tau = \frac{1}{0.5\times 10^{-3}}\int_0^t i_{in}(\tau)d\tau$$

In this part, we use the current excitation signal described in Figure P7.14b. In the interval 0s < t ≤ 1s, $i_{in}(t) = -10(mA)$. Thus, in this interval

$$v_{in}(t) = \frac{1}{0.5\times 10^{-3}}\int_o^t (-10\times 10^{-3})d\tau = -20t(V)$$

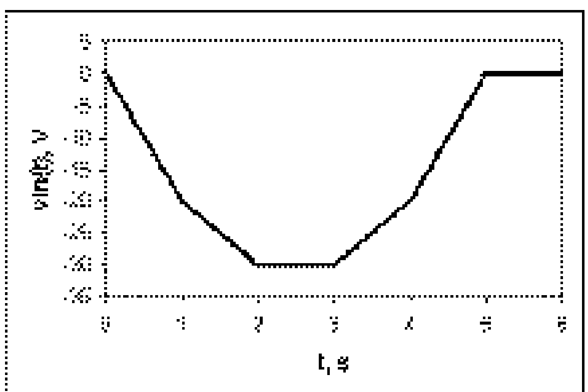
In the interval 1s < t ≤ 2s, $i_{in}(t) = -5(mA)$. Hence, in this interval

$$v_{in}(t) = v_{in}(1) + \frac{1}{0.5\times 10^{-3}}\int_1^t (-5\times 10^{-3})d\tau = -20 + (-10t + 10) = -10t - 10(V)$$

In a similar fashion, $v_{in}(t)$ can be computed for the remaining intervals. The calculations for $v_{in}(t)$ for t = 0s to 5s are summarized in the following table:

Time Interval	$i_{in}(t)$, mA	$v_{in}(t)$, V
0s< t ≤1s	-10	-20t
1s< t ≤2s	-5	-10 -10t
2s< t ≤3s	0	-30
3s< t ≤4s	5	-60+10t
4s< t ≤5s	10	-100+20t

Below is the plot of $v_{in}(t)$ vs. time.



Part 2. In this part, we use the current excitation signal described in Figure P7.14c. In the interval 0s < t ≤ 1s, $i_{in}(t) = 10(mA)$. Thus, in this interval

$$v_{in}(t) = \frac{1}{0.5\times 10^{-3}}\int_o^t (10\times 10^{-3}\times\tau)d\tau = 10t^2(V)$$

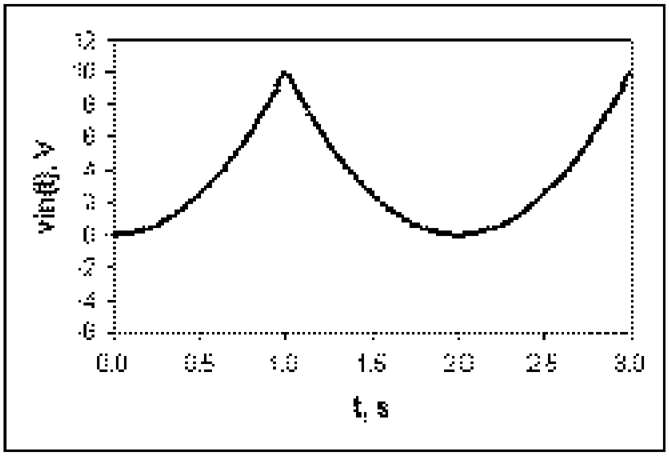
In the interval 1s < t ≤ 3s, $i_{in}(t) = 10t - 20(mA)$. Hence, in this interval

$$v_{in}(t) = v_{in}(1) + \frac{1}{0.5\times 10^{-3}}\int_o^t 10^{-3}\times(10\tau - 20)d\tau = 10 + 10(10\tau^2 - 40\tau)\Big|_1^t = 10t^2 - 40t + 40\ (V)$$

The calculations for $v_{in}(t)$ for t = 0s to 3s are summarized in the following table:

Time Interval	$i_{in}(t)$, mA	$v_{in}(t)$, V
0s< t ≤1s	10t	$10t^2$
1s< t ≤3s	10t-20	$10t^2$ - 40t + 40

Below is the plot of $v_{in}(t)$ vs. time.



SOLUTION 7.15. **Part 1** Using the capacitor v-i relationship,

$$i_{in}(t) = C_1 \frac{d}{dt}(v_s(t)) = 20\mu \times 6 \times 1500 \times \cos(1500t) = 0.18\cos(1500t) \text{ A}$$

Then, we can find v_{out} by applying (7.6),

$$v_{out}(t) = v_{out}(0) + \frac{1}{0.5m}\int_0^t 2i_{in}(\tau)d\tau$$

$$= 10 + \frac{2 \times 0.18}{0.5m \times 1500}\sin(1500t)$$

$$= 10 + 0.48\sin(1500t) \quad \text{V}$$

Part 2 The instantaneous power delivered by the independent source is given by

$$p(t) = 2i_{in}(t) \times v_{out}(t) = 0.36\cos(1500t) \times [10 + 0.48\sin(1500t)] \quad \text{W}$$

Part 3 Applying (7.11), the energy stored in the capacitor over the interval [0,t] is given by

$$W_{C1}[0,t] = \frac{1}{2}C_1\left[v_s^2(t) - v_s^2(0)\right] = 360\sin^2(1500t)\mu J$$

SOLUTION 7.16. **Part 1** From the solution of problem 7.15,

$$v_{out}(t) = v_{out}(0) + \frac{1}{0.5m}\int_0^t 2i_{in}(\tau)d\tau$$

$$= v_{out}(0) + \frac{1}{0.5m}\int_0^t 2\times 20\mu \frac{d}{d\tau}\left(v_s(\tau)\right)d\tau$$

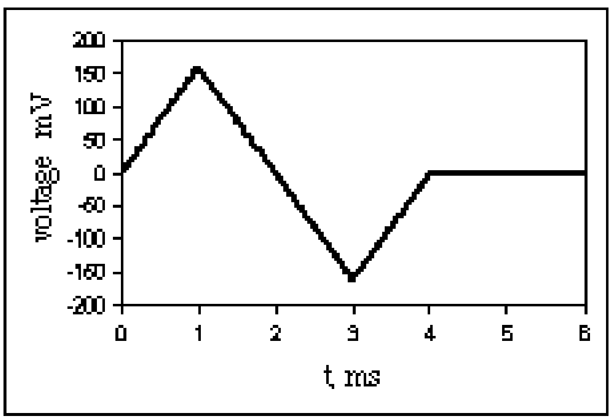
$$= v_{out}(0) + 0.08\int_0^t \frac{d}{d\tau}\left(v_s(\tau)\right)d\tau$$

$$= v_{out}(0) + 0.08 v_s(\tau)$$

If we assume $v_{out}(0) = 0V$, then

$$v_{out}(t) = 0.08\ v_s(\tau)$$

The following is a plot of $v_{out}(t)$ vs time.



Part 2 The instantaneous power delivered by the dependent source is given by

$$p(t) = v_{out}(t)\times 2i_{in}(t)\ = (\ v_{out}(0) + 0.08\ v_s(t))\ \times 2\times 20\mu \frac{d}{dt}\left(v_s(t)\right)$$

$$= 40(\ v_{out}(0) + 0.08\ v_s(t))\ \frac{d}{dt}\left(v_s(t)\right)\quad \mu W$$

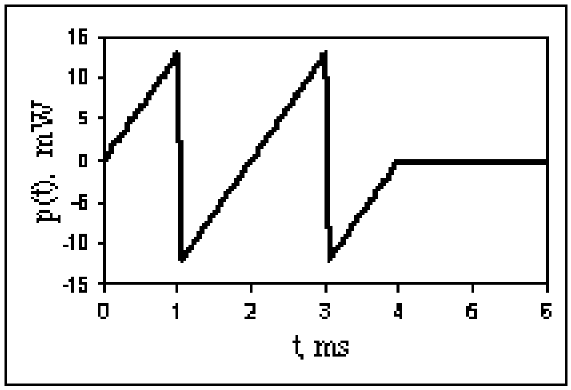
If we assume $v_{out}(0) = 0V$, then

$$p(t) = 3.2\ v_s(t)\ \frac{d}{dt}\left(v_s(t)\right)\quad \mu W$$

The calculations for d/dt ($v_S(t)$) and p(t) for t = 0s to 6ms are summarized in the following table:

Time Interval	d/dt (v_S(t)), Vs^{-1}	p(t), W
0ms< t ≤1ms	2×10^3	12.8 × t
1ms< t ≤3ms	-2×10^3	12.8 × (t-2)
3ms< t ≤4ms	2×10^3	12.8 × (t-4)
4ms< t ≤6ms	0	0

The following is a plot of *p*(t) vs time.



Part 3 The energy stored in the 0.5-mF capacitor is given by

$$W_C(t) = \frac{1}{2} \times 0.5m \times v_{out}{}^2(t) \;\; = 0.25(\, v_{out}(0) + 0.08\, v_s(t))^2 \quad mJ$$

If we assume $v_{out}(0)$=0V, then

$$W_C(t) = 1.6(v_s(t))^2 \quad \mu J$$

In the interval 0s < t ≤ 1ms, $v_s(t) = 2 \times 10^3 t(V)$. Thus, in this interval

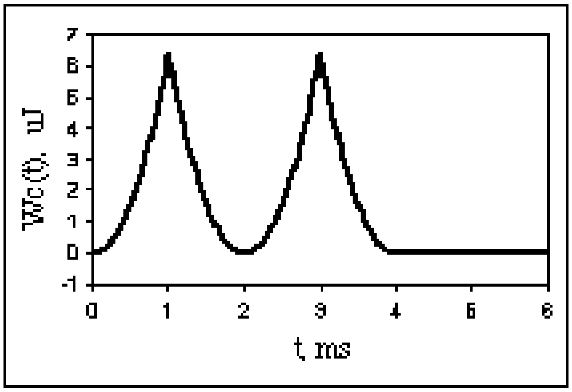
$$W_C(t) = 6.4t^2 J$$

In the interval 1s < t ≤ 3ms, $v_s(t) = 2 \times 10^3 \times (2 - t)(V)$. Hence, in this interval

$$W_C(t) = 6.4 \times (2 - t)^2 J$$

In a similar way W_C(t) can be computed for the interval 3ms < t ≤ 4ms.

The following is a plot of W_C(t) vs time.



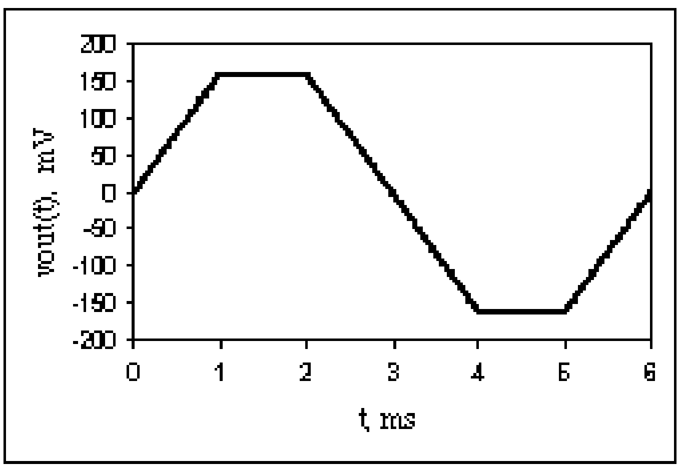
SOLUTION 7.17. **Part 1** From the solution of problem 7.16,

$$v_{out}(t) = v_{out}(0) + 0.08\ v_S(\tau)$$

If we assume $v_{out}(0)$=0V, then

$$v_{out}(t) = 0.08\ v_S(\tau)$$

The following is a plot of v_{out}(t) vs time.



Part 2 The instantaneous power delivered by the dependent source is given by.

$$p(t) = 40(\ v_{out}(0) + 0.08\ v_S(t))\ \frac{d}{dt}\left(v_S(t)\right) \quad \mu W$$

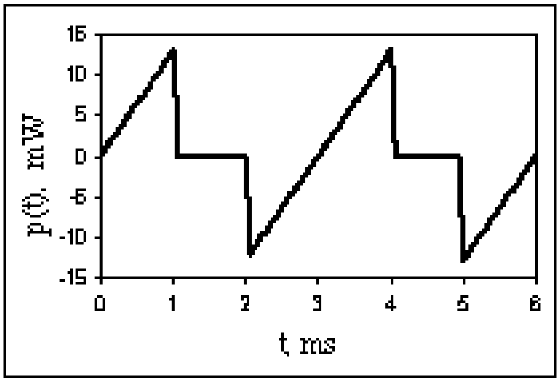
If we assume $v_{out}(0)$=0V, then

$$p(t) = 3.2\ v_S(t)\ \frac{d}{dt}\left(v_S(t)\right) \quad \mu W$$

The calculations for d/dt (v_S(t)) and p(t) for t = 0s to 6ms are summarized in the following table:

Time Interval	d/dt (v_S(t)), Vs^{-1}	p(t), W
0ms< t ≤1ms	2×10^3	12.8 × t
1ms< t ≤2ms	0	0
2ms< t ≤4ms	-2×10^3	12.8 × (t-3)
4ms< t ≤5ms	0	0
5ms< t ≤6ms	2×10^3	12.8 × (t-6)

The following is a plot of *p*(t) vs time.



Part 3 The energy stored in the 0.5-mF capacitor is given by

$$W_C(t) = 0.25(\ v_{out}(0) + 0.08\ v_s(t))^2 \quad mJ$$

If we assume $v_{out}(0) = 0$V, then

$$W_C(t) = 1.6(v_s(t))^2 \quad \mu J$$

In the interval 0ms < t ≤ 1ms, $v_s(t) = 2 \times 10^3 t(V)$. In this interval

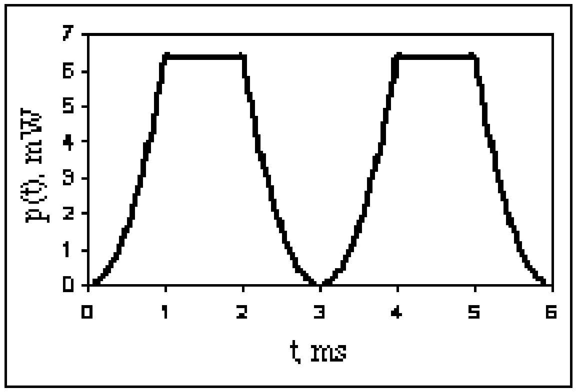
$$W_C(t) = 6.4t^2 J$$

In the interval 1ms < t ≤ 2ms, $v_s(t) = 2(V)$. In this interval

$$W_C(t) = 6.4(\mu J)$$

In a similar fashion W_C(t) can be computed for the remaining intervals.

The following is a plot of W_C(t) vs time.



SOLUTION 7.18. **Part 1** Applying the capacitor v-i relationship for the equivalent capacitor of the series combination of 0.3 mF and 0.6 mF capacitors

$$v_{in}(t) = v_{in}(0) + \frac{1}{0.3\times 10^{-3} \| 0.6\times 10^{-3}} \int_0^t i_s(\tau)d\tau$$

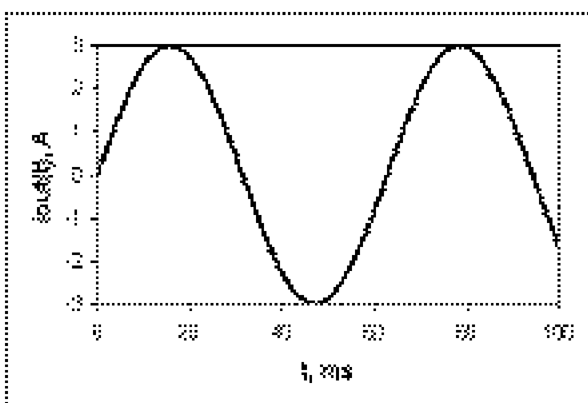
$$= (0) + \frac{1}{0.3\times 10^{-3} \| 0.6\times 10^{-3}} \int_0^t 60\times 10^{-3}\sin(100\tau)d\tau$$

$$= 3 - 3\cos(100t) \text{ V}$$

Therefore,

$$i_{out}(t) = (0.2\times 10^{-3} + 0.8\times 10^{-3})\frac{d}{dt}(10v_{in}(t)) = 3\sin(100t) \quad \text{V}$$

Below is a sketch of $i_{out}(t)$ vs time.



Part 2 The instantaneous power delivered by the dependent source is given by

$$p(t) = i_{out}(t)\times 10v_{in}(t) = 90\sin(100t)\times[1-\cos(100t)] \text{ W}$$

Part 3 The instantaneous energy stored in the 0.8mF capacitor is given by

$$W_C(t) = \frac{1}{2} C v_C{}^2(t) = \frac{1}{2}(0.8mF)\left[10 v_{in}(t)\right]^2 = 0.36\left[1 - \cos(100t)\right]^2 J$$

SOLUTION 7.19.

Part 1 Since Q = CV, the charge that resides on each plate of the capacitor = 10μF × 100V = 1mC
Part 2 Since V = Q/C, the required voltage = 1mC/5μF = 200V
Part 3 Since V = Q/C, the required voltage = 50μC/1μF = 50V
Part 4 The energy required = 0.5 × 10μF × $(100V)^2$ = 0.05J

SOLUTION 7.20. When 0s≤ t <2s, $v_C(t)$ = 25V. Conservation of charge requires that $q_1(2^-) + q_2(2^-) = q_1(2^+) + q_2(2^+)$. Since $q_2(2^-)$ = 0 C it follows that

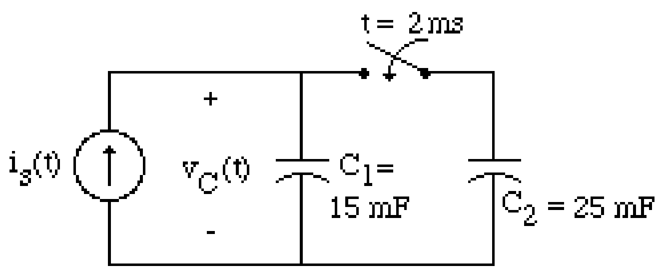
$$25 \text{ V} \times 150 \text{ mF} = v_C(2^+)(150 \text{ mF} + 100 \text{ mF})$$

Hence

$$v_C(2^+) = \frac{25V \times 150mF + 0V \times 100mF}{150mF + 100mF} = 15\,V$$

Thus, for t > 2s, $v_C(t)$ = 15 V.

SOLUTION 7.21. For this solution consider the figure below in which C_1 and C_2 are labeled.



There are two cases to consider: (i) t < 2 ms and (ii) t > 2 ms.

Case 1. t < 2 ms. Here, since the current source is zero for t < 0, and C_1 is uncharged at t = 0,

$$v_C(t) = \frac{1}{C_1}\int_{-\infty}^{t} i_s(\tau)d\tau = v_C(0) + \frac{1}{C_1}\int_0^t i_s(\tau)d\tau = \frac{1}{C_1}\int_0^t i_s(\tau)d\tau$$

$$= \frac{-12}{15\times 10^{-3}\times 500}\left[e^{-500\tau}\right]_0^t = 1.6\left(1 - e^{-500t}\right) \text{ V}$$

Note that $v_C(2^- ms) = 1.0114$ V. Hence the energy stored over [0, 2 ms] is

$$W_{C1}(0 < t < 2ms) = 0.0192\left(1 - e^{-500t}\right)^2 \text{ J}$$

and

$$W_{C2}(0 < t < 2ms) = 0$$

Case 2. 2 ms < t. At t = 2 ms the switch closes, forcing a discontinuity in the capacitor voltages. To calculate the capacitor voltages at 2^+ ms, we use conservation of charge. Here, the relevant equation is:

$$q_{C1}(2^+ ms) + q_{C2}(2^+ ms) = q_{C1}(2^- ms) + q_{C2}(2^- ms) + \int_{2^- ms}^{2^+ ms} i_s(\tau)\, d\tau$$

Note that since $v_{C2}(2^- ms) = 0$, $q_{C2}(2^- ms) = C_2 v_{C2}(2^- ms) = 0$ and the integral of the bounded continuous function $i_s(t)$ over an infinitesimal interval is zero, this equation reduces to

$$q_{C1}(2^+ ms) + q_{C2}(2^+ ms) = q_{C1}(2^- ms)$$

or equivalently, since for t > 2 ms, $v_{C1}(t) = v_{C2}(t) = v_C(t)$,

$$C_1 v_C(2^+ ms) + C_2 v_C(2^+ ms) = C_1 v_C(2^- ms)$$

Therefore

$$v_C(2^+ ms) = \frac{C_1}{C_1 + C_2} v_C(2^- ms) = \frac{15}{15 + 25} \times 1.0114 = 0.37927 \text{ V}$$

and it follows that

$$v_C(t) = v_C(2^+ ms) + \frac{1}{C_{eq}} \int_{2ms}^{t} i_s(\tau)\, d\tau = 0.379 + \frac{1}{C_1 + C_2} \int_{2ms}^{t} i_s(\tau)\, d\tau$$

$$= 0.379 + \frac{-12}{40 \times 10^{-3} \times 500}\left[e^{-500\tau}\right]_{0.002}^{t} = 0.379 + 0.6\left(0.36788 - e^{-500t}\right) \text{ V}$$

Hence the energy stored in the two capacitors over the interval [2^+ ms, t] is

$$W_{C_{eq}}(2^+ ms < t) = \frac{1}{2} C_{eq} v_C^2(t) - \frac{1}{2} C_{eq} v_C^2(2^+ ms) \text{ J}$$

whereas the instantaneous stored energy, i.e., the energy stored over (-∞,t > 2 ms] is given by

$$W_{C_{eq}}(t) = \frac{1}{2} C_{eq} v_C^2(t)$$

What happens between 2^-ms and 2^+ms is beyond the scope of the material in this chapter. Please refer to problem 51 in chapter 8 for an explanation.

SOLUTION 7.22. **Part 1** Applying the inductor v-i relationship,

$$\begin{aligned} v_L(t) &= L\frac{d}{dt}i_{in}(t) \\ &= 2.5mH \times \left(-200te^{-10t} + 20e^{-10t}\right) \\ &= 0.05e^{-10t}(-10t+1) \end{aligned}$$

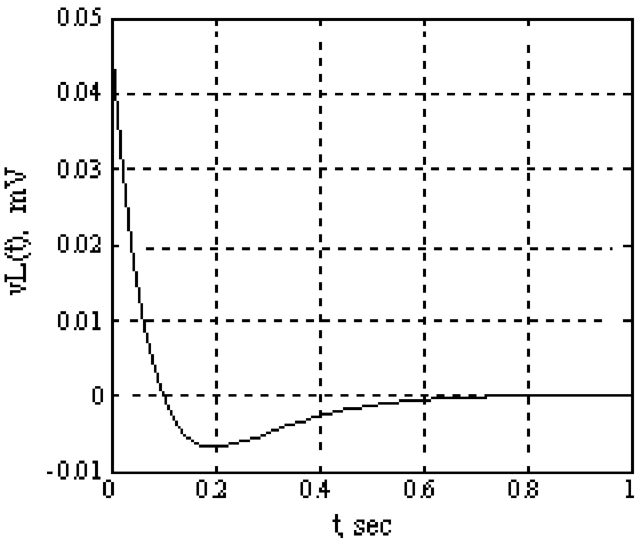
Applying the capacitor v-i relationship, we have:

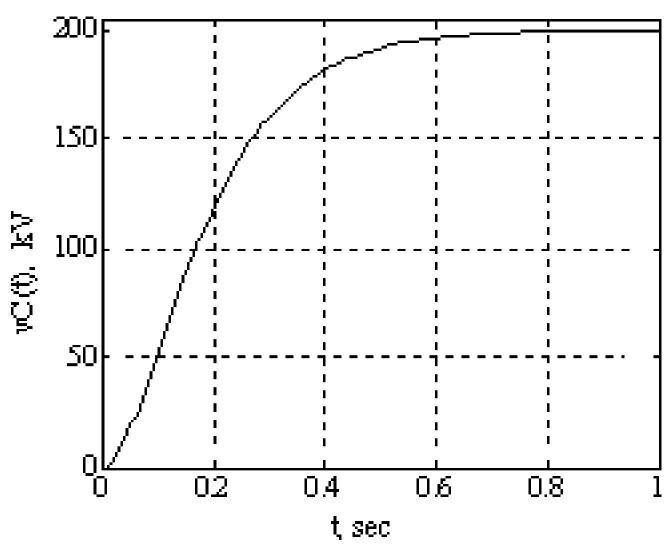
$$\begin{aligned} v_C(t) &= v_C(0) + \frac{1}{C}\int_0^t i_{in}(\tau)d\tau \\ &= \frac{20}{1m}\int_0^t te^{-10\tau}\tau \\ &= 20\times 10^3\left[-0.1\tau e^{-10\tau} - 0.01e^{-10\tau}\right]_0^t \\ &= 2\times 10^3\left[-te^{-10t} - 0.1e^{-10t} + 0.1\right]\ V \end{aligned}$$

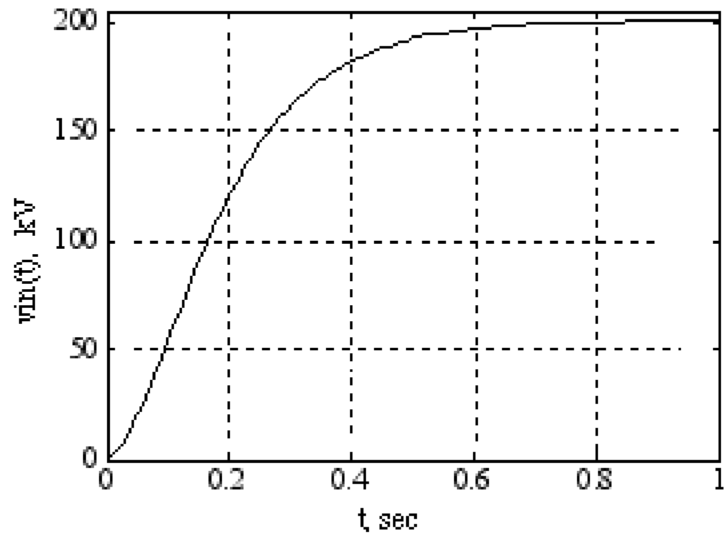
and

$$\begin{aligned} v_{in}(t) &= v_L(t) + v_C(t) \\ &= 0.05e^{-10t}(-10t+1) + 2000\left[-te^{-10t} - 0.1e^{-10t} + 0.1\right] \\ &= -2000.5\times te^{-10t} - 199.95\times e^{-10t} + 200(V) \end{aligned}$$

The sketches of $v_L(t)$, $v_C(t)$ and $v_{in}(t)$ are shown below.







Part 2 The energy stored in the inductor is given by

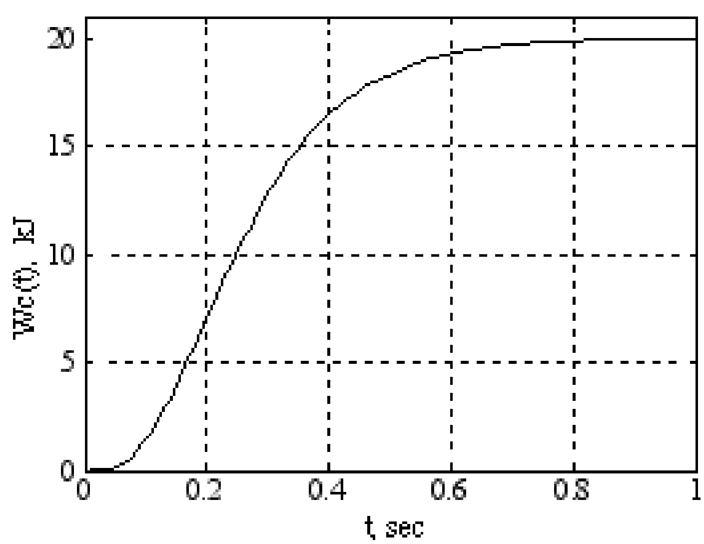
$$W_L(t) = \frac{1}{2}L[i_{in}(t)]^2 = 0.5t^2e^{-20t} \quad \text{J}$$

The sketch of $W_L(t)$ vs. time is shown below.

Part 3 The energy stored in the capacitor is given by

$$W_C(t) = \frac{1}{2}C\left[v_C(t)\right]^2 = 2\left[-te^{-10t} = 0.1e^{-10t} + 0.1\right]^2 \; kJ$$

The sketch of $W_C(t)$ vs time is shown below.



SOLUTION 7.23. Applying the inductor v-i relationship we have that

$$v_L(t) = 0.25\frac{d[i_s(t)]}{dt} = 4\cos(4t)\ (V)$$

Applying the capacitor v-i relationship it follows that

$$i_C(t) = 0.25\frac{d[2v_2(t)]}{dt} = 0.25\times 2\times 4\times 4\times[-\sin(4t)] = -8\sin(4t)\ A$$

SOLUTION 7.24.

Part 1. By KCL $i_s(t) = i_{C1}(t) + i_{C2}(t)$. Applying the v-i relationship for capacitor C_1 and C_2 we have:

$$\begin{aligned} i_S(t) &= (10mF + 20mF)\frac{d}{dt}v_s(t) \\ &= 30\times 10^{-3}\times(12te^{-5t} - 30t^2e^{-5t}) \\ &= 0.36te^{-5t} - 0.9t^2e^{-5t} \quad \text{A} \end{aligned}$$

Part 2. Applying the v-i relationship for the 20mH inductor, it follows that

$$\begin{aligned} v_{out}(t) &= (20mH)\frac{d}{dt}[12i_s(t)] \\ &= 0.24\times\left(4.5t^2e^{-5t} - 3.6te^{-5t} + 0.36e^{-5t}\right) \\ &= 1.08t^2e^{-5t} - 0.864te^{-5t} + 0.0864e^{-5t}\ (V) \end{aligned}$$

Part 3 The energy stored in the 20mH inductor for t > 0 is given by

$$\begin{aligned} W_L(t) &= \frac{1}{2}(20mH)[12i_S(t)]^2 \\ &= 1.44\left(0.36te^{-5t} - 0.9t^2e^{-5t}\right)^2\ J \\ &= 1.44\times t^2e^{-10t}(0.36 - 0.9t)^2\ J \end{aligned}$$

SOLUTION 7.25. We denote by $v_C(t)$ the voltage across the capacitors C_1 and C_2 and by $i_L(t)$ the current through the inductors L_1 and L_2. The equivalent capacitance of the parallel combination of C_1 and C_2 is $(C_1 + C_2)$ and thus:

$$i_S(t) = (C_1 + C_2)\frac{dv_C(t)}{dt}$$

Using the v-i relationship for the capacitor C_2 it follows that

$$i_{C2}(t) = C_2\frac{dv_C(t)}{dt}$$

By replacing $\frac{dv_C(t)}{dt}$, it follows that

$$i_{C2}(t) = \frac{C_2}{C_1 + C_2} \times i_S(t) = \frac{2}{3} i_s(t)$$

The equivalent inductance of the series combination of L_1 and L_2 is $(L_1 + L_2)$ and thus:

$$9i_{C2}(t) = (L_1 + L_2)\frac{di_L(t)}{dt}$$

Using the v-I relationship for the inductor L_2 it follows that

$$v_{out}(t) = v_{L2}(t) = L_2 \frac{di_2(t)}{dt}$$

By replacing $\frac{di_L(t)}{dt}$, it follows that

$$v_{out}(t) = \frac{L_2}{L_1 + L_2} \times 9i_{C2}(t) = \frac{2}{3} 9i_{C2}(t) = 6i_{C2}(t) = 4i_s(t)$$

SOLUTION 7.26. Using the v-i relationship for the capacitor we can write:

$$v_{C2}(t) = \frac{1}{C_2}\int_{-\infty}^{t} i_C(\tau)d\tau$$

$$v_{C1}(t) = \frac{1}{C_1}\int_{-\infty}^{t} i_C(\tau)d\tau$$

where $i_C(t)$ is the current through the capacitors C_1 and C_2. Since C_1 and C_2 are connected in series:

$$v_{in}(t) = v_{C1}(t) + v_{C2}(t) = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_{-\infty}^{t} i_C(\tau)d\tau$$

It follows that $\int_{-\infty}^{t} i_C(\tau)d\tau = v_{in}(t)\left(\frac{C_1C_2}{C_1 + C_2}\right)$. Hence

$$v_{C2}(t) = \frac{1}{C_2}\int_{-\infty}^{t} i_C(\tau)d\tau = \frac{1}{C_2} \times \frac{C_1C_2}{C_1 + C_2} v_{in}(t) = \frac{C_1}{C_1 + C_2} v_{in}(t)$$

Since L_1 and L_2 are combined in parallel it follows that

$$v_{out}(t) = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1} \frac{d(Av_{C2}(t))}{dt}$$
$$= \frac{L_1L_2}{L_1 + L_2} \times A \times \frac{dv_{C2}(t)}{dt} = \frac{L_1L_2}{L_1 + L_2} \times A \times \frac{C_1}{C_1 + C_2}\frac{dv_{in}(t)}{dt}$$

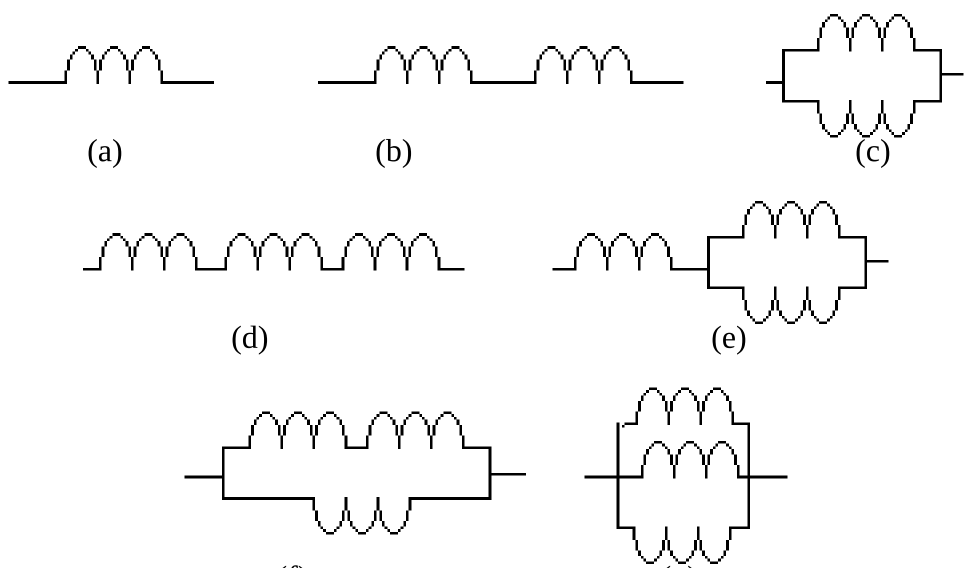
where the last equality follows by replacing $v_{C2}(t)$ with $\dfrac{C_1}{C_1 + C_2} v_{in}(t)$

SOLUTION 7.27. Observe first that the 0.3H and 0.6H parallel inductances combine to make a 0.2H inductance. Also the 0.4H and 1.2H parallel inductances combine to make a 0.3H inductance. Finally, the 0.2H and 0.3H inductances are combined in series and the equivalent inductance is 0.5H. Shortly, all the above steps can be written as: $L_{eq} = (0.3H \,||\, 0.6H) + (0.4H \,||\, 1.2H) = 0.5H$.

SOLUTION 7.28. Observe first that the 1mH and 5mH inductors combine to make a 6mH inductance. This inductance combines in parallel with the 3mH inductance to make a 2mH inductance. The next step is to combine in series the 2mH inductance with the 10mH inductance. The equivalent inductance is 12mH. This inductance is combined in parallel with the 36mH inductance and the result is 9mH. Finally, the 9mH inductance is combined in series with the 4mH inductance and the result is 13mH. Shortly, all the above steps cam be written as: $L_{eq} = [(5mH + 1mH) \,||\, 3mH + 10mH] \,||\, 36mH + 4mH = 13mH$.

SOLUTION 7.29. Step 1. The parallel combination of the 0.6mH and 1.2mH inductors is equivalent to a 0.4mH inductor.
Step 2. The series combination of the 2.4mH and 0.4mH inductors is equivalent to a 2.8mH inductor.
Step 3. The parallel combination of the 2.8mH and 7mH inductors is equivalent to a 2mH inductor.

SOLUTION 7.30. The three inductors can be arranged in the seven fashions as shown below.



$L_{eq}^{a} = 1mH$

$L_{eq}^{b} = 1mH + 1mH = 2mH$

$L_{eq}^{c} = 1mH \,||\, 1mH = 0.5mH$

$L_{eq}^{d} = 1mH + 1mH + 1mH = 3mH$

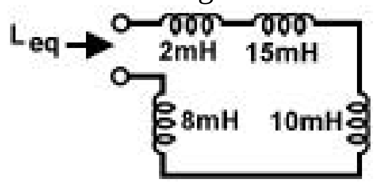
$L_{eq}^{e} = 1mH + (1mH \,||\, 1mH) = 1.5mH$

$L_{eq}^{f} = 1mH \,||\, (1mH + 1mH) = 0.667mH$

$L_{eq}^{g} = 1mH \,||\, 1mH \,||\, 1mH) = 0.333mH$

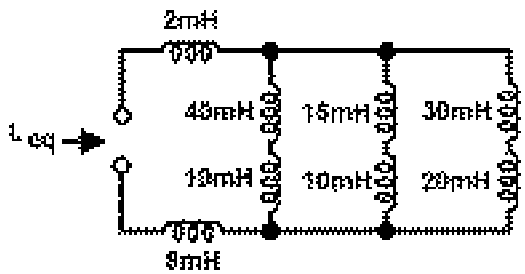
SOLUTION 7.31. For Fig. P7.31a, L_{eq} = 2 + 15 + 10 + 10 + 40 +30 + 20 + 8 = 135mH

The circuit in Fig. P7.31b is equivalent to the following circuit.



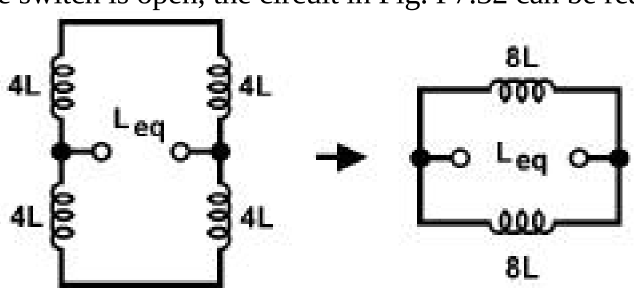
Therefore, L_{eq} = 2 + 15 + 10 + 8 = 35mH

The circuit in Fig. P7.31c is equivalent to the following circuit.



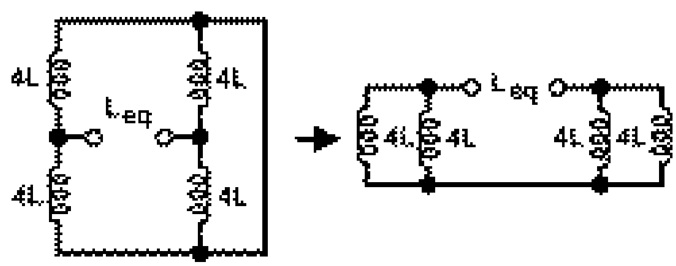
Therefore, L_{eq} = 2 + [(15+10) || (10+40) || (20+30)] + 8 = 22.5mH

SOLUTION 7.32. When the switch is open, the circuit in Fig. P7.32 can be rearranged as the following.



Therefore, L_{eq} = 8L || 8L = 4L

When the switch is closed, the circuit in Fig. P7.32 can be rearranged as the following.

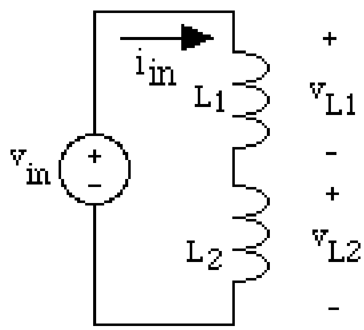


Therefore, L_{eq} = 4L || 4L + 4L || 4L = 4L

SOLUTION 7.33. $L_{eq1}=L_{eq2}$ Without going into a detailed analysis, we present the following intuitive argument. Note that the points a and b represent points on a balanced bridge circuit meaning that the voltage between a and b would be zero. Therefore, no current will flow through the additional inductance L. Therefore the presence of L does not affect the equivalent inductance value.

SOLUTION 7.34. $L_{eq1}>L_{eq2}$ Without going into a detailed analysis, we present the following intuitive argument. Note that the points a and b represent points on an unbalanced bridge circuit, meaning that the voltage between a and b would not be zero. Also note that when two inductors are placed in parallel, the equivalent inductance becomes smaller than either inductance. The addition of the inductor L in circuit 2 essentially creates an internal parallel inductance resulting in an L_{eq2} lower than L_{eq1}.

SOLUTION 7.35. First we add an i_{in} label to the circuit as shown below.



From KVL and the derivative definition of the capacitor

$$v_{in}(t) = v_{L1}(t) + v_{L2}(t) = L_1 \frac{di_{in}(t)}{dt} + L_2 \frac{di_{in}(t)}{dt} = \left(L_1 + L_2\right)\frac{di_{in}(t)}{dt}$$

Equivalently,

$$\frac{di_{in}(t)}{dt} = \frac{1}{\left(L_1 + L_2\right)} v_{in}(t)$$

It follows that

$$v_{L_k}(t) = L_k \frac{di_{in}(t)}{dt} = \frac{L_k}{L_1 + L_2} v_{in}(t)$$

which is the required voltage division formula.

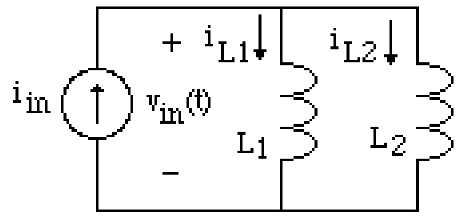
SOLUTION 7.36. L_{eq} = (11mH) || (19.25mH) + 3mH = 10mH. Applying the voltage division formula,

$$v_{L1}(t) = v_{in}(t) \times \frac{3mH}{L_{eq}} = 60\, te^{-t} \quad mV$$

and

$$v_{L2}(t) = v_{in}(t) - v_{L1}(t) = 140\, te^{-t} \quad mV$$

SOLUTION 7.37. First consider the circuit below which contains the additional label of $v_{in}(t)$.



From KCL and the integral definition of the inductor,

$$i_{in}(t) = i_{L1}(t) + i_{L2}(t) = \frac{1}{L_1}\int_0^t v_{in}(\tau)\,d\tau + \frac{1}{L_2}\int_0^t v_{in}(\tau)\,d\tau$$

Equivalently

$$\int_0^t v_{in}(\tau)\,d\tau = \left(\frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}\right) i_{in}(t)$$

Therefore

$$i_{L_k}(t) = \frac{1}{L_k}\int_0^t v_{in}(\tau)\,d\tau = \left(\frac{\frac{1}{L_k}}{\frac{1}{L_1} + \frac{1}{L_2}}\right) i_{in}(t) = \left(\frac{\frac{L_1 L_2}{L_k}}{L_1 + L_2}\right) i_{in}(t) \text{ for k = 1,2}$$

SOLUTION 7.38. In the circuit illustrated in Fig. 7.38,

$$L_{eq} = (12\text{mH} + 27\text{mH}) \parallel (130\text{mH}) = 30\text{mH}$$

Applying the current division formula,

$$i_{L1}(t) = \frac{12 + 27}{12 + 27 + 130}\, i_{in}(t) = 0.231 e^{-t^2} \quad mA$$

$$i_{L2}(t) = \frac{130}{12 + 27 + 130}\, i_{in}(t) = 0.769 e^{-t^2} \quad mA$$

Also,

$$v_{in}(t) = L_{eq}\frac{d}{dt} i_{in}(t) = -60 t e^{-t^2} \quad \mu\text{V}$$

The instantaneous energy stored in the 130-mH is given by

$$W_{L1}(t) = \frac{1}{2} L_1 \left[i_{L1}(t)\right]^2 = 0.8 e^{-2t^2} \quad \text{n}J$$

SOLUTION 7.39. Using the inductor v-i relationship,

$$v_{in}(t) = L_{eq}\frac{d}{dt}i_{in}(t) = 30mH \times \frac{d}{dt}i_{in}(t)$$

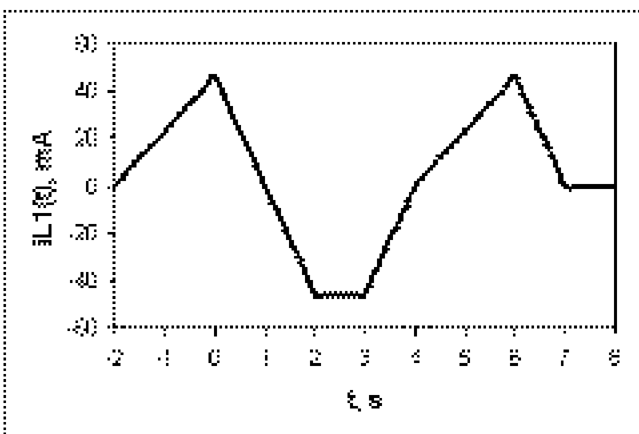
The calculations for $v_{in}(t)$ for t = -2s to 7s are summarized in the following table:

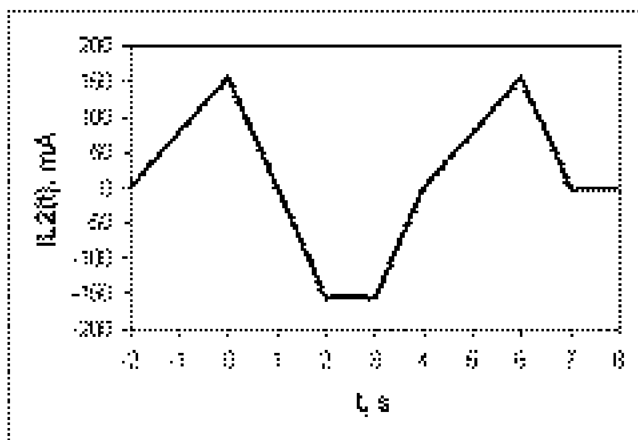
Time Interval	d/dt (i_{in}(t)), mAs^{-1}	v_{in}(t), mV
-2s< t <=0s	100	3
0s< t <=2s	-200	-6
2s< t <=3s	0	0
3s< t <=4s	200	6
4s< t <=6s	100	3
6s< t <=7s	-200	-6

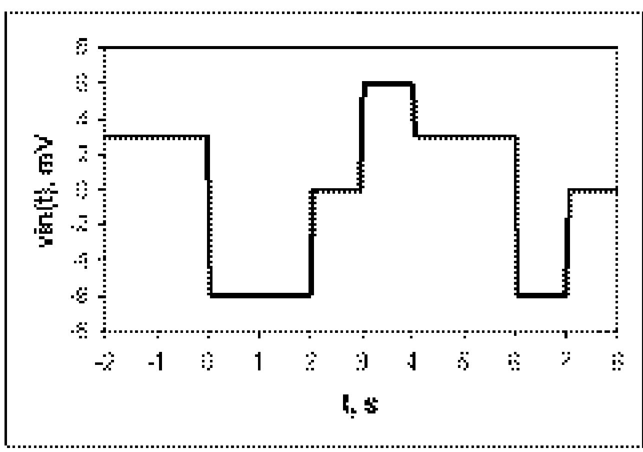
From the solutions to problem 7.38,

$$i_{L1}(t) = 0.231\, i_{in}(t)$$
$$i_{L2}(t) = 0.769\, i_{in}(t)$$

Below are the plots for $v_{in}(t)$, $i_{L1}(t)$ and $i_{L2}(t)$.







SOLUTION 7.40. Part 1 Using the current division formula,

$$i_{L2}(t) = \frac{18}{18+6} i_{in}(t) = 90\cos(300\pi t) \quad \text{mA}$$

Part 2

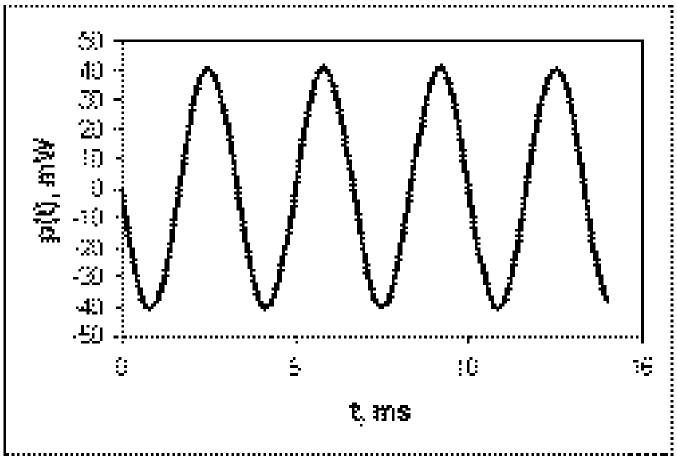
$$L_{eq} = 1.5\text{mH} + 6\text{mH} \parallel 18\text{mH} = 6\text{mH}$$

$$v_{in}(t) = L_{eq}\frac{d}{dt} i_{in}(t) = -0.679\sin(300\pi t) \quad \text{V}$$

Part 3 Instantaneous power delivered by the source is given by

$$p(t) = v_{in}(t) \times i_{in}(t) = [-0.679\sin(300\pi t)] \times [120\cos(300\pi t)] \quad \text{mW}$$
$$= -40.7\sin(600\pi t) \quad \text{mW}$$

Below is a plot of p(t)



SOLUTION 7.41. Part 1 $L_{eq} = 0.3 \parallel 0.9 + 0.4 \parallel 0.4 = 0.425$ H

For computing $i_{in}(t)$ we need to apply

$$i_{in}(t) = i_{in}(t_0) + \frac{1}{L_{eq}}\int_{t_0}^{t} v_{in}(\tau)d\tau$$

to each interval [0, 1], [1, 2], …, [n, n+1], …

The initial condition for the interval [n, n+1] for n even is:

$$i_{in}(n) = i_{in}(0) + \frac{1}{L_{eq}}\int_{0}^{n} v_{in}(\tau)d\tau$$

We assume that $i_{in}(0) = 0$ and it follows that $i_{in}(n) = 0$ for n even, since

$$\int_0^n v_{in}(\tau)d\tau = 0$$

for n even. The initial condition for the interval [n, n+1] for n odd is:

$$i_{in}(t) = i_{in}(n-1) + \frac{1}{L_{eq}}\int_{n-1}^{n} v_{in}(\tau)d\tau = \frac{1}{0.425}(16\tau)\Big|_{n-1}^{n} = 37.6\ A$$

For the interval [n, n+1] with n even:

$$i_{in}(t) = i_{in}(n) + \frac{1}{L_{eq}}\int_n^t v_{in}(\tau)d\tau = 0 + \frac{1}{0.425}16\tau\Big|_n^t = 37.6(t-n)\,A$$

For the interval [n, n+1] with n odd:

$$i_{in}(t) = i_{in}(n) + \frac{1}{L_{eq}}\int_n^t v_{in}(\tau)d\tau = 37.6 + \frac{1}{0.425}(-16\tau)\Big|_n^t = 37.6(n+1-t)\,A$$

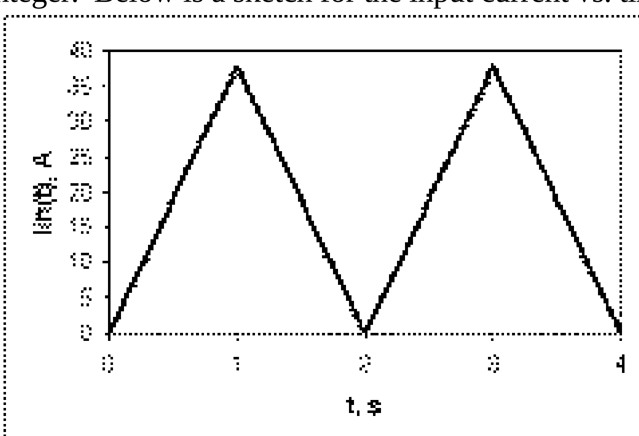
For the interval [n, n+1] with n even:

$$i_{in}(t) = i_{in}(n) + \frac{1}{L_{eq}}\int_n^t v_{in}(\tau)d\tau = 0 + \frac{1}{0.425}(16\tau)\Big|_n^t = 37.6(t-n)\,A$$

Hence,

$$i_{in}(t) = \begin{cases} 37.6(t-2n),\ 2n < t \le 2n+1 \\ 37.6(2n+2-t),\ 2n+1 < t \le 2n+2 \end{cases}$$

where n is a non-negative integer. Below is a sketch for the input current vs. time.

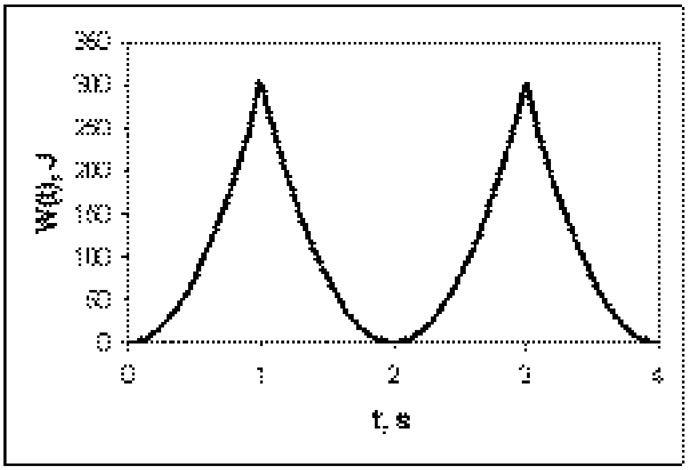


Part 2 The total energy stored in the set of four inductors is given by

$$W(t) = \frac{1}{2} L_{eq}\left[i_{in}(t)\right]^2$$
$$= \begin{cases} 300(t-2n)^2 \ J & for\ 2n < t \le 2n+1 \\ 300(2n+2-t)^2 \ J & for\ 2n+1 < t \le 2n+2 \end{cases}$$

where n is a non-negative integer.

Below is a sketch of W(t) vs. t.

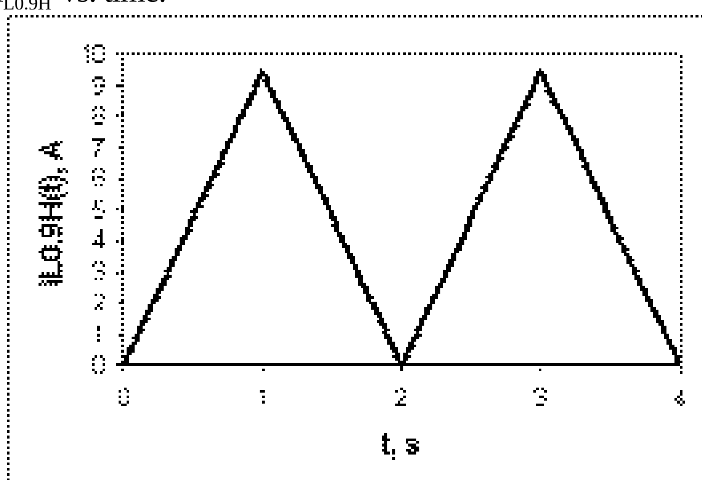


Part 3 By the current division formula, the current through the 0.9H inductor is given by

$$i_{L\,0.9H}(t) = \frac{0.3}{0.3+0.9} i_{in}(t)$$
$$= \begin{cases} 9.4(t-2n) \ A & for\ 2n < t \le 2n+1 \\ 9.4(2n+2-t) \ A & for\ 2n+1 < t \le 2n+2 \end{cases}$$

where n is a non-negative integer.

Below is a sketch of the $i_{L0.9H}$ vs. time.



SOLUTION 7.42.. a)

Step 1. The parallel combination of the 1μF and 2μF capacitances is equivalent to a 3μF capacitance.

Step 2. The series capacitors 1.5μF and 3μF combine to make a 1μF capacitance.

Step 3. Finally, the 2μF capacitance is combined in parallel with the 1μF capacitance that was the result of Step 2 to make a 3μF capacitance.

Shortly, the above steps can be written as:

$$C_{eq} = [(1\mu F + 2\mu F) \parallel 1.5\mu F] + 2\mu F] = 3\mu F$$

b) Proceeding in a similar fashion as in part a:

$$C_{eq} = 30mF + [9mF \parallel (9mF + 18mF) \parallel 5.4mF] = 33mF$$

SOLUTION 7.43. a)

Step 1. Combine the parallel capacitances of 1μF and 2μF to make a 3μF capacitance.

Step 2. Combine the series capacitances of 1.5μF and 3μF to make 1μF capacitance.

Step 3. Combine the parallel capacitances of 2.5μF and 1μF (the result of Step 2) to make a 3.5μF capacitance.

Shortly, the above steps can be written in a condensed form as:

$$C_{eq} = 2.5\mu F + [1.5\mu F \parallel (1\mu F + 2\mu F) = 3.5\mu F$$

b) Proceeding in a similar fashion as in part a:

$$C_{eq} = (1mF \parallel 2\ mF) + (1.2mF \parallel 2mF \parallel 1.5mF) + (4mF \parallel 2.6667mF \parallel 3.2mF \parallel 1.6mF) = 2.1667mF$$

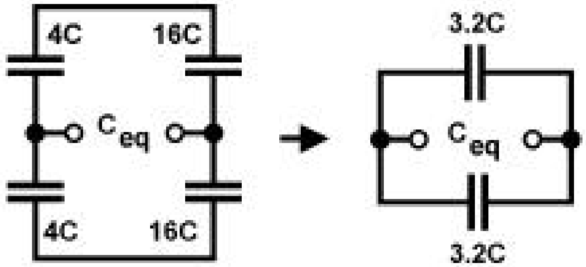
SOLUTION 7.44.

Part 1 C_{eq} = (((30 || 60 + 40) || 30 + 40) || 30 + 40) || 30 + 40 = 60mF

Part 2 The value of $v_{in}(t)$ is given by

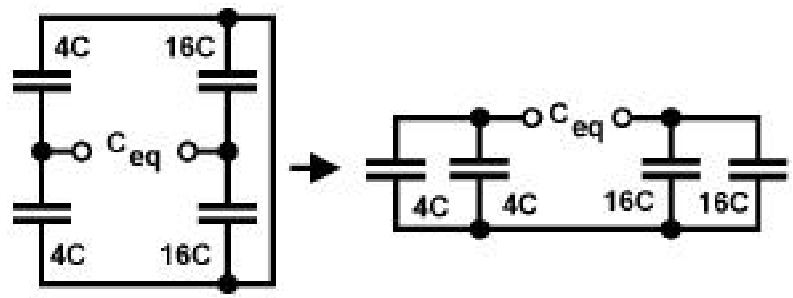
$$v_{in}(t) = v_{in}(0) + \frac{1}{C}\int_0^t i_{in}(\tau)d\tau = 0 + \frac{1}{0.06}\int_0^t 200\sin(20\tau)d\tau = 166.7 - 166.7\cos(20t)\ mV$$

SOLUTION 7.45. When the switch is open, the circuit in figure P7.45 can be represented by the following.



Therefore, $C_{eq} = 6.4C$

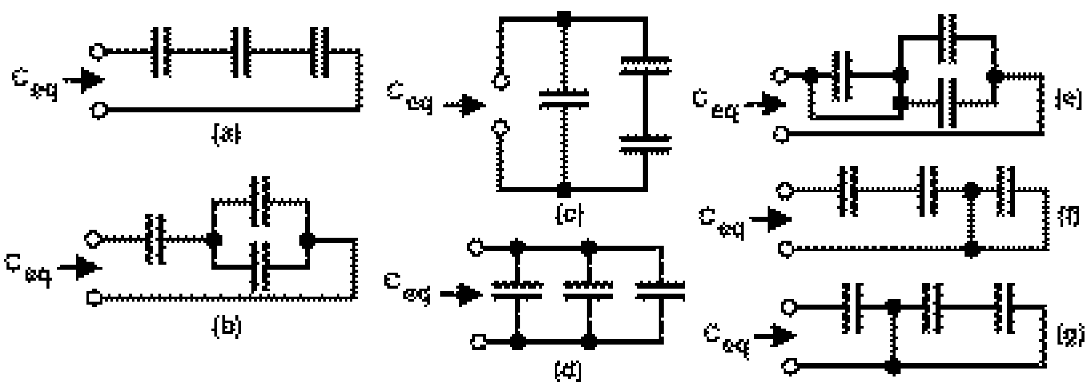
When the switch is closed, the circuit in figure P7.45 can be represented by the following.



Therefore, $C_{eq} = (4 + 4) \| (16 + 16) = 6.4C$

SOLUTION 7.46. $C_{eq1} < C_{eq2}$ Without going into a detailed analysis, we present the following intuitive argument. Note that the points a and b represent points on an unbalanced bridge circuit, meaning that the voltage between a and b would not be zero. Also note that when two capacitors are placed in parallel, the equivalent capacitance becomes bigger than either capacitance. The addition of the capacitor C in circuit 2 essentially creates an internal parallel capacitance resulting in a C_{eq2} higher than C_{eq1}.

SOLUTION 7.47. The three capacitors can be arranged in the seven fashions as shown below.



For configuration (a), $C_{eq} = 1 \| 1 \| 1 = 0.3333\mu F$

For configuration (b), $C_{eq} = 1 \| (1 + 1) = 0.6667\mu F$

For configuration (c), $C_{eq} = 1 \| 1 + 1 = 1.5\mu F$

For configuration (d), $C_{eq} = 1 + 1 + 1 = 3\mu F$

For configuration (e), $C_{eq} = 0 + 1 + 1 = 2\mu F$

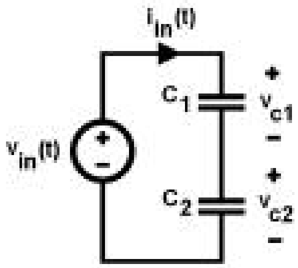
For configuration (f), $C_{eq} = 1 \| 1 + 0 = 0.5\mu F$

For configuration (g), $C_{eq} = 1 + 0 + 0 = 1\mu F$

SOLUTION 7.48. As Q = CV for any capacitor, C=Q/V. The equivalent capacitance of the network is given by

$$C_{eq} = \frac{1C}{2V} \| \frac{1C}{3V} \| \frac{1C}{5V} = 0.1\ F$$

SOLUTION 7.49. First consider the circuit below which contains the additional label of $i_{in}(t)$.



From KVL and the integral definition of the capacitor,

$$v_{in}(t) = v_{C1}(t) + v_{C2}(t) = \frac{1}{C_1}\int_0^t i_{in}(\tau)d\tau + \frac{1}{C_2}\int_0^t i_{in}(\tau)d\tau$$

Equivalently,

$$\int_0^t i_{in}(\tau)d\tau = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} v_{in}(t)$$

Therefore,

$$v_{Ck}(t) = \frac{1}{C_k}\int_0^t i_{in}(\tau)d\tau = \frac{\frac{1}{C_k}}{\frac{1}{C_1} + \frac{1}{C_2}} v_{in}(t) = \frac{\frac{C_1C_2}{C_k}}{C_1 + C_2} v_{in}(t)$$

Hence,

$$v_{C1}(t) = \frac{C_2}{C_1 + C_2} v_{in}(t)$$

$$v_{C2}(t) = \frac{C_1}{C_1 + C_2} v_{in}(t)$$

which is the required voltage division formula.

SOLUTION 7.50. By the voltage division formula,

$$v_{C3}(t) = \frac{6\|6}{6\|6 + 9} v_{in}(t) = 21(1 - e^{-20t})\quad \text{V}$$

SOLUTION 7.51. **Part 1** By the voltage division formula,

$$v_{C1}(t) = \frac{0.05 + 0.15}{0.05 + 0.15 + 0.1} v_{in}(t) = 6.667\sin(120\pi t) \quad \text{V}$$
$$v_{C2}(t) = v_{in}(t) - v_{C1}(t) = 3.333\sin(120\pi t) \quad \text{V}$$

Part 2 Let the 0.1μF capacitor be C_1 and the 0.15μF capacitor be C_2. The energies stored in the 0.1μF and the 0.15μF capacitors are given by

$$W_{C1}[0,t] = \frac{1}{2}C_1\left[v_{C1}^2(t) - v_{C1}^2(0)\right] = 2.222\sin^2(120\pi t)\ \mu J$$

$$W_{C2}[0,t] = \frac{1}{2}C_2\left[v_{C2}^2(t) - v_{C2}^2(0)\right] = 0.8333\sin^2(120\pi t)\ \mu J$$

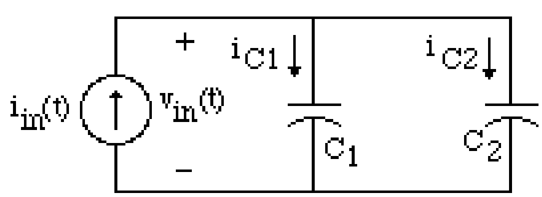
SOLUTION 7.52. Part 1 As both the terminals of 0.08F are tied to the voltage source $v_{in}(t)$,
$$v_{C1}(t) = v_{in}(t) = 100e^{-2t} \quad \text{V}$$

By the voltage division formula,
$$v_{C2}(t) = \frac{0.06}{0.03 + 0.06} v_{in}(t) = 66.67e^{-2t} \quad \text{V}$$

Part 2 Let the 0.03-F capacitor be C_2. The energy stored in the 0.03-F capacitor over the interval [0,t] is given by

$$W_{C2}[0,t] = \frac{1}{2}C_2\left[v_{C2}^2(t) - v_{C2}^2(0)\right] = 66.67\left[e^{-4t} - 1\right]\ J$$

SOLUTION 7.53. First consider the circuit below which contains the additional label of $v_{in}(t)$.



From KCL and the derivative definition of the capacitor

$$i_{in}(t) = i_{C1}(t) + i_{C2}(t) = C_1\frac{dv_{in}(t)}{dt} + C_2\frac{dv_{in}(t)}{dt} = \left(C_1 + C_2\right)\frac{dv_{in}(t)}{dt}$$

Equivalently,
$$\frac{dv_{in}(t)}{dt} = \frac{1}{\left(C_1 + C_2\right)} i_{in}(t)$$

It follows that
$$i_{C_k}(t) = C_k\frac{dv_{in}(t)}{dt} = \frac{C_k}{C_1 + C_2} i_{in}(t)$$
which is the required current division formula.

SOLUTION 7.54. **Part 1** By the current division formula,

$$i_{C1}(t) = \frac{0.08}{0.08 + 0.03 \| 0.06} i_{in}(t) = 80e^{-2t} \ A$$

$$i_{C2}(t) = \frac{0.03 \| 0.06}{0.08 + 0.03 \| 0.06} i_{in}(t) = 20e^{-2t} \ A$$

Part 2 Let the 0.03-F capacitor be C_2.

$$v_{C2}(t) = v_{C2}(0) + \frac{1}{C_2}\int_0^t i_{C2}(\tau)d\tau = 333.3(1 - e^{-2t}) \quad \text{V}$$

The energy stored in the 0.03-F capacitor over the interval [0,t] is given by

$$W_{C2}(0,t) = \frac{1}{2}C_2\left[v_{C2}^2(t) - v_{C2}^2(0)\right] = 1666.5(1 - e^{-2t})^2 \ J$$

SOLUTION 7.55. **Part 1** By the current division formula,

$$i_{C2}(t) = \frac{2}{1+2} i_s(t) = 20\sin(250t) \quad \text{mA}$$

Part 2 By the voltage division formula,

$$v_{out}(t) = \frac{2}{1+2} \times 9 i_{c2}(t) = 0.12\sin(250t) \quad \text{V}$$

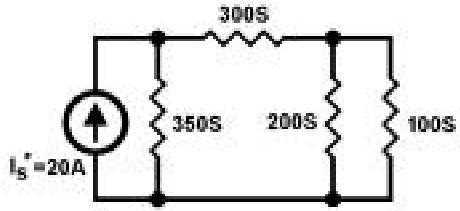
Part 3 The current in the 2mH inductor is given by

$$i_{L2}(t) = i_{L2}(0) + \frac{1}{L_2}\int_0^t v_{out}(\tau)d\tau = -0.24\left[\cos(250t) - 1\right] \quad \text{A}$$

The energy stored in the 2-mH inductor is given by

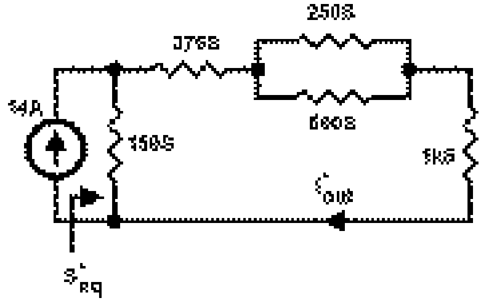
$$W_{L2}(t) = \frac{1}{2} L_2 i_{L2}^2(t) = 57.6\left[\cos(250t) - 1\right]^2 \quad \mu\text{J}$$

SOLUTION 7.56. **Part 1** The dual network N* is shown below.



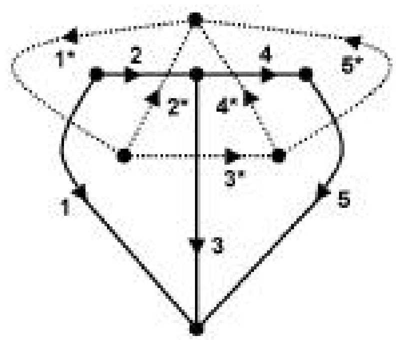
Part 2 Since the equivalent conductance seen by I_s* is 500S, the equivalent resistance seen by I_s* is equal to 0.002Ω.

SOLUTION 7.57. **Part 1** The dual network N* is shown below.



Part 2 $S^*_{eq} = 350S$ $I^*_{out} = 8A$.

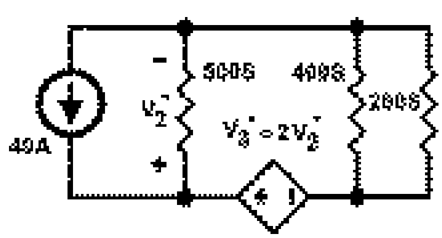
SOLUTION 7.58. First we draw the graph, given below, of the circuit in figure P7.58. We construct the graph associated with dual network graph for N* from the graph of N. The dual network graph is given by the dashed lines.



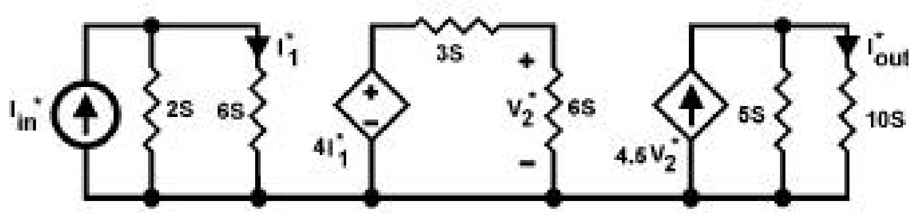
This circuit and its dual have the branch characteristics given in the following table:

ORIGINAL NETWORK	DUAL NETWORK
$V_1 = 40$ V	$I_1^* = 40$ A
$V_2 = 500\, I_2$	$I_2^* = 500\, V_2^*$
$I_3 = 2\, I_2$	$V_3^* = 2\, V_2^*$
$V_4 = 400 I_4$	$I_4^* = 400 V_4^*$
$V_5 = 200\, I_5$	$I_5^* = 200\, V_5^*$

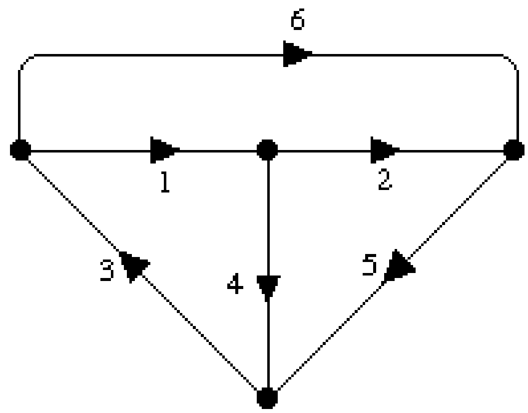
Finally replace the branches by the elements given in the table above. This produces the dual network below.



SOLUTION 7.59. The dual for the circuit in figure P7.59 is shown below.



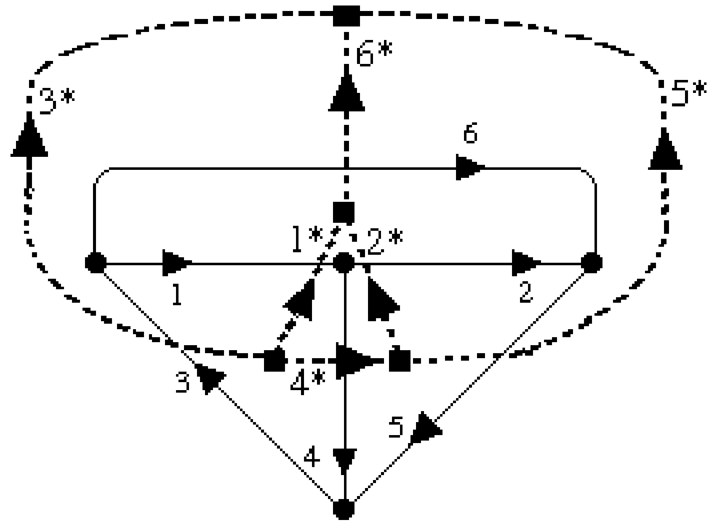
SOLUTION 7.60. First we draw the graph, given below, of the circuit in figure P7.60.



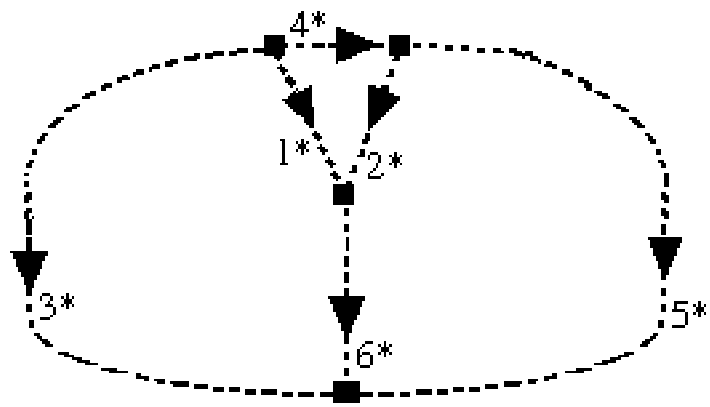
This circuit and its dual have the branch characteristics given in the following table:

ORIGINAL NETWORK	DUAL NETWORK
$V_1 = 5\ I_1$	$I_1^* = 5V_1^*$
$I_2 = 8$ A	$V_2^* = 8$ V
$I_3 = 6$ A	$V_3^* = 6$ V
$V_4 = 4$ V	$I_4^* = 4$ A
$V_5 = 2\ I_5$	$I_5^* = 2V_5^*$
$I_6 = 7$ A	$V_6^* = 7$ V

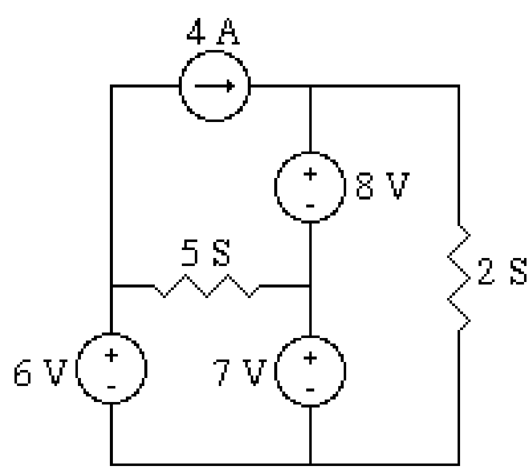
Now we construct the graph associated with dual network graph for N* from the graph of N. The dual network graph is given by the dashed lines.



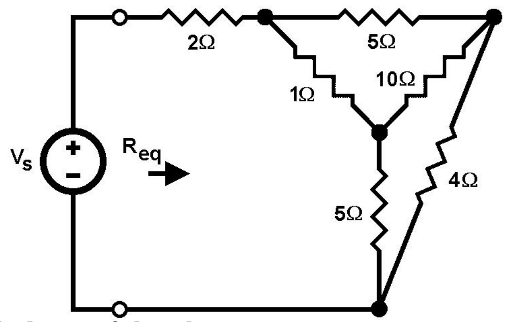
The graph of the dual network then is pulled out and flipped vertically to produce the graph topology of the dual network, N*.



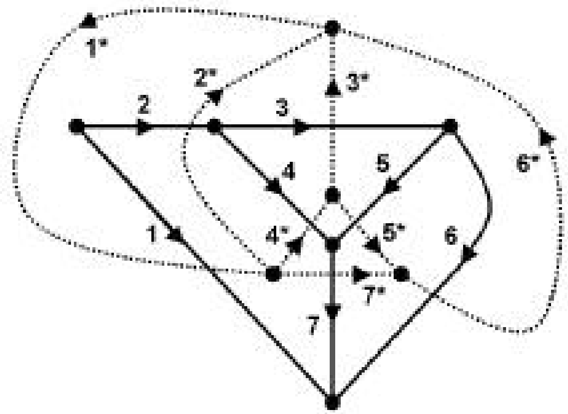
Finally replace the branches by the elements given in the table above. This produces the dual network below.



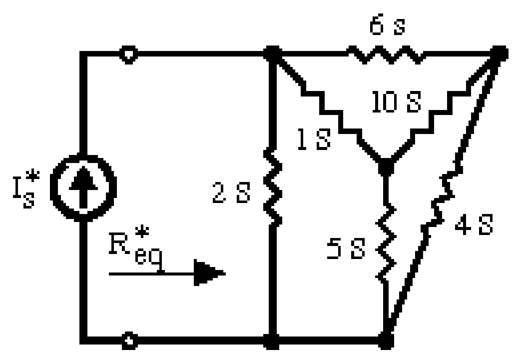
SOLUTION 7.61. Part 1 We first redraw the circuit to eliminate branch crossing. The resultant schematic is shown below.



Then we draw the graph, given below, of the above circuit. We construct the graph associated with dual network graph for N* from the graph of N. The dual network graph is given by the dashed lines.

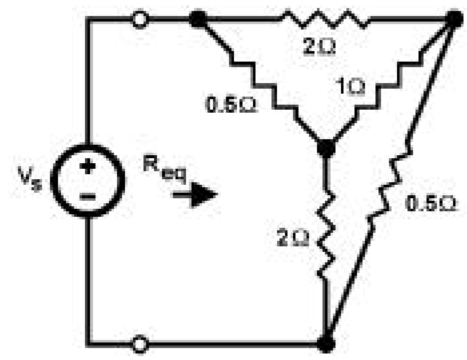


The resultant dual network given by the above graph is shown below.

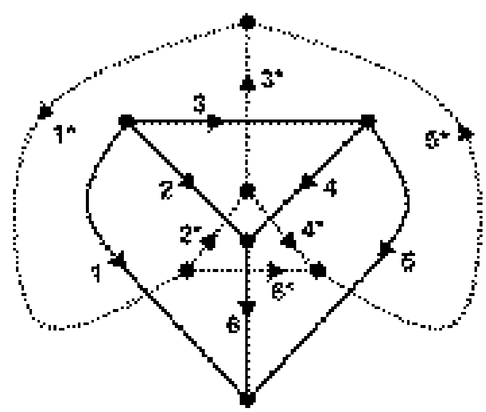


Part 2 The equivalent resistance R_{eq}* seen by the current source in the dual circuit is equal to $1/R_{eq} = 1/\alpha$ Ω.

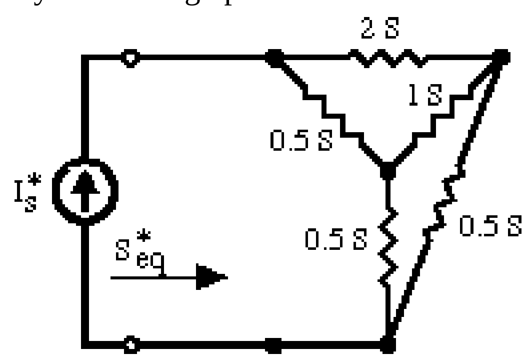
SOLUTION 7.62. Part 1 We first redraw the circuit to eliminate branch crossing. The resultant schematic is shown below.



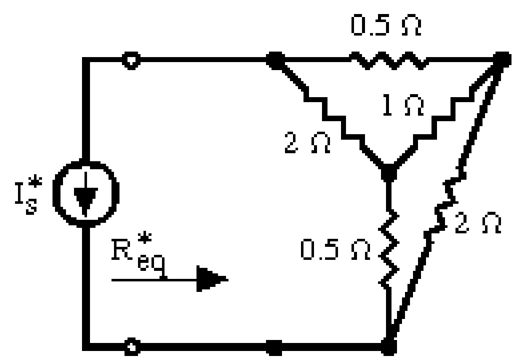
Then we draw the graph, given below, of the above circuit. We construct the graph associated with dual network graph for N* from the graph of N. The dual network graph is given by the dashed lines.



The resultant dual network given by the above graph is shown below.



Part 2 If we label the conductances in the dual network by their corresponding resistance values, we have the circuit below.



By comparing to the original network, we conclude that $R_{eq}* = R_{eq}$.

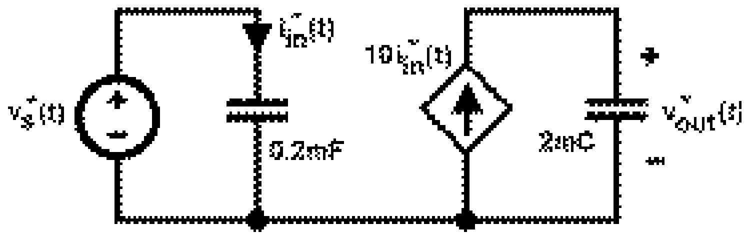
Part 3 Since the general relationship between R_{eq} and $R_{eq}*$ is given by $R_{eq}= 1/R_{eq}*$. We can write

$$R_{eq}= 1/R_{eq}* =1/R_{eq}$$

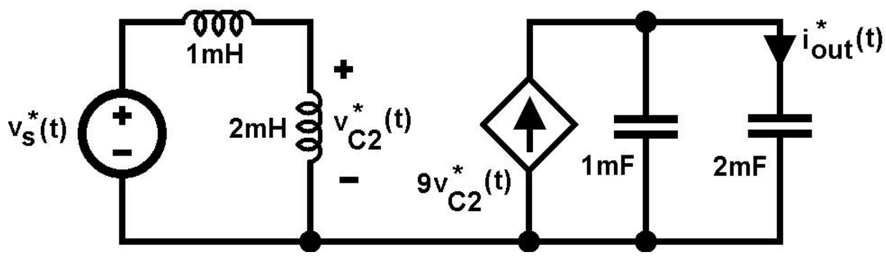
Solving the above equation, we know that $R_{eq} = 1\ \Omega$.

Part 4 To retain the special properties of parts (b) and (c), we require one resistance value must be a reciprocal of the other one.

SOLUTION 7.63. The dual network is shown below.



SOLUTION 7.64. The dual network is shown below.



From the answer of problem 7.55,

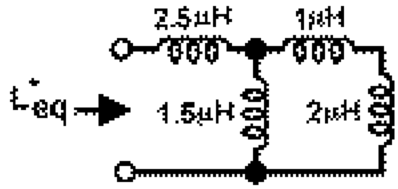
$i_{C2}(t) = 20 \sin (250t)$ mA, and $v_{out}(t) = 0.12 \sin (250t)$ V

Therefore, $v_{C2}*(t) = 20 \sin (250t)$ mV and $i^*_{out}(t) = 0.12 \sin (250t)$ A

The energy stored in the 2mF capacitor is given by

$$W^*_{C2}(t) = \frac{1}{2}C^*_2\left[v^*_{out}(t)\right]^2 = \frac{1}{2}L_2\left[i_{L2}(t)\right]^2 = 57.6[\cos(250t) - 1]^2\ \mu J$$

SOLUTION 7.65. The dual network is shown below.



The equivalent inductance of the dual circuit (L_{eq}*) = (1+2) || 1.5 + 2.5 = 3.5μH.

SOLUTION 7.66. By voltage division

$$v_1(t) = \frac{\frac{1}{0.4}}{\frac{1}{0.4} + \frac{1}{0.2}} v_s(t) = \frac{0.2}{0.2 + 0.4} v_s(t) = \frac{1}{3} v_s(t)$$

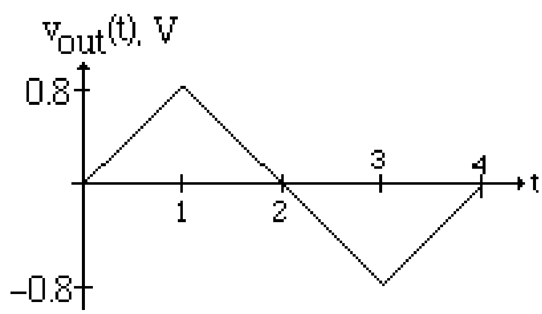
Note that the 0.4 F resistor in parallel with the dependent voltage source has no effect on $i_1(t)$ and hence is redundant. By the definition of a capacitor,

$$i_1(t) = 0.2 \frac{dv_1(t)}{dt} = \frac{1}{15} \times \frac{dv_s(t)}{dt}$$

Finally, since the last capacitor is initially uncharged,

$$v_{out}(t) = \frac{1}{0.5} \int_0^t i_1(\tau) d\tau = \frac{2}{15} \int_0^t \frac{dv_s(\tau)}{d\tau} d\tau = \frac{2}{15} v_s(t)$$

A sketch is given below.



SOLUTION 7.67. Following the method outlined in page 269 of the text, we require

$$v_C(t_0) - \frac{1}{C} \int_0^{1/200} i_0(\tau) d\tau \geq 14$$

With $i_0(\tau)$ = 2A, $v_C(0)$ = 20 V, it follows that

$$20 - 14 \geq \frac{1}{C} \times 2A \times \frac{1}{200} s$$

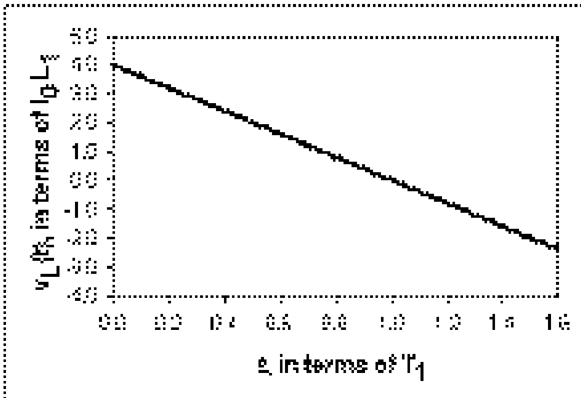
$$C \geq 1.667 mF$$

Therefore, the standard capacitor value of 1.8mF should be chosen for this application.

SOLUTION 7.68. From the relationships given in Problem 7.67, we have

$$v_L(t) = \frac{d}{dt}\left[L(t)i_L(t)\right] = I_0\frac{d}{dt}\left[L(t)\right] = I_0 L_1\left[4\left(\frac{1}{T_1} - \frac{t}{T_1^2}\right)\right]$$

Below is a plot of $v_L(t)$ vs t.



From the plot, we notice that when $t=T_1$, the value of $v_L(t)$ changes from positive to negative. This means that $v_L(t)$ will change sign when a car is inside the loop. Therefore, we can make a circuit to monitor the voltage $v_L(t)$ and whenever a negative voltage is detected, the left turn signal should be initiated during the next traffic light change.