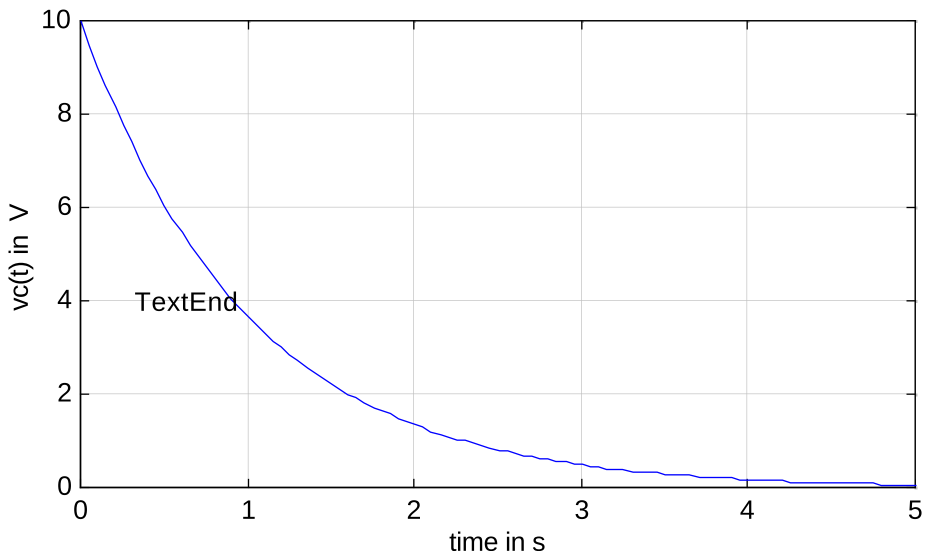
SOLUTIONS PROBLEMS CHAPTER 8

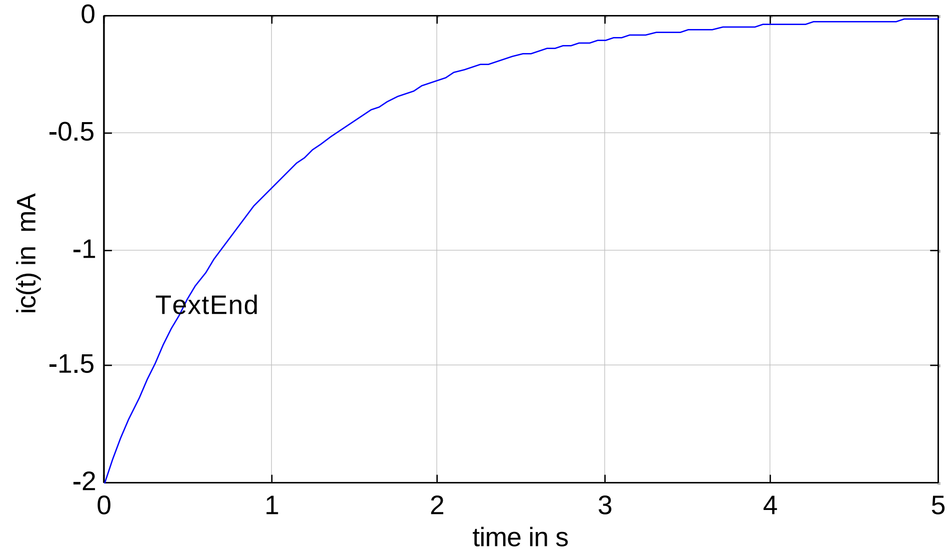
SOLUTION 8.1. **(a)** By KCL, $C\frac{dv_C(t)}{dt} = -\frac{v_C(t)}{R}$ or $\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = 0$. Using 8.12

$v_C(t) = v_C(0)e^{-t/\tau} = 10e^{-t}$ V where $\tau = RC = 1s$. Plotting this from 0 to 5 sec

```
»t = 0:.05:5;
»vc = 10*exp(-t);
»plot(t,vc)
»grid
»xlabel('time in s')
»ylabel('vc(t) in  V')
```



(b) The solution has the same general form as for (a), $i_C(t) = i_C(0+)e^{-t/\tau} = -\frac{10}{5\times 10^3}e^{-t} = -2e^{-t}$ mA = $\frac{-v_C(t)}{R}$.



(c) By linearity, if $v_C(0)$ is cut in half, all resulting responses are cut in half. If $v_C(0)$ is doubled, then all resulting responses are doubled. Alternately, one view this as a simple change of the initial condition with the same conclusion reached from linearity.

SOLUTION 8.2. (a) From inspection of the general form, 8.12, $0.1/\tau = 1 = 0.1/RC$ C = 0.1/R = 5 μF.

(b) Since τ = RC = 0.1, $v_C(t) = 10e^{-10t}$ V.

SOLUTION 8.3. **(a)** The general solution form is $v_C(t) = v_C(0)e^{-t/\tau} = v_C(0)e^{-t/R_{eq}C}$. Using the given data, take the following ratio, $\frac{v_C(0.001)}{v_C(0.002)} = \frac{18.394}{6.7668} = 2.7183 = \frac{v_C(0)e^{-(0.001)/\tau}}{v_C(0)e^{-(0.002)/\tau}} = e^{0.001/\tau}$. Hence,

```
»K = 18.394/6.7668
K =2.7183
»tau = 1e-3/log(K)
tau =0.0010
»C = 5e-6;
»Req = tau/C
Req =200.0008
»% Req = R*4e3/(R+4e3)
```

»R = Req*4e3/(4e3-Req)

R =210.5272

»vC0 = 6.7668/exp(-0.002/tau)

vC0= 49.9999 V

(b)

» % $v_C(t) = 50e^{-1000t}$ V

»t = 0:tau/100:5*tau;

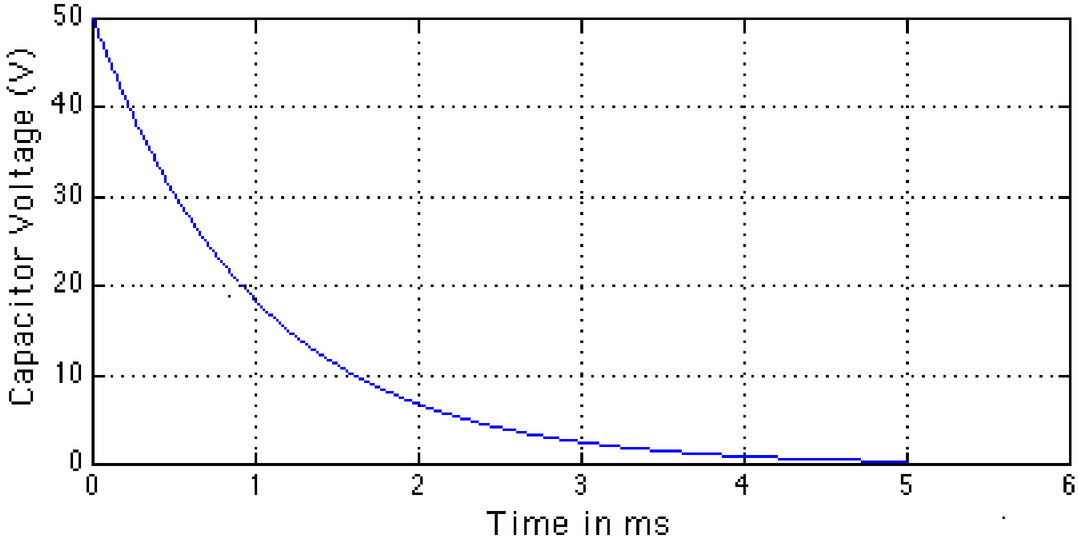
»vc = vC0*exp(-t/tau);

»plot(t,vc)

»grid

»xlabel('Time in ms')

»ylabel('vC(t) in V')



SOLUTION 8.4. After one time constant the stored voltage, 8 V, decays to 8/e = 2.943 V. From the graph, the time at which the output voltage is 2.94 V is approximately 0.19 s. Thus $\tau = 0.19\,s$, and R = τ/C = 190 Ω.

SOLUTION 8.5. **(a)** The general form of the inductor current is $i_L(t) = i_L(0)e^{-t/\tau} = 0.15e^{-t/\tau}$ A where $\tau = L/R = 2\times10^{-3}$ s. Plotting $i_L(t) = 0.15e^{-500t}$ A from 0 to 10 msec yields:

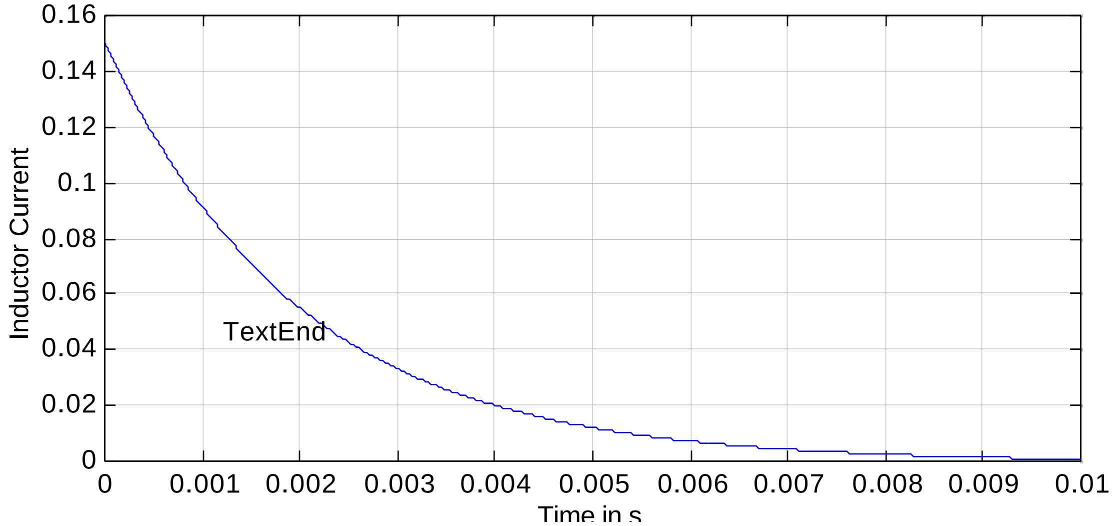
»t = 0:.01e-3:10e-3;

»iL = 0.15*exp(-t/2e-3);

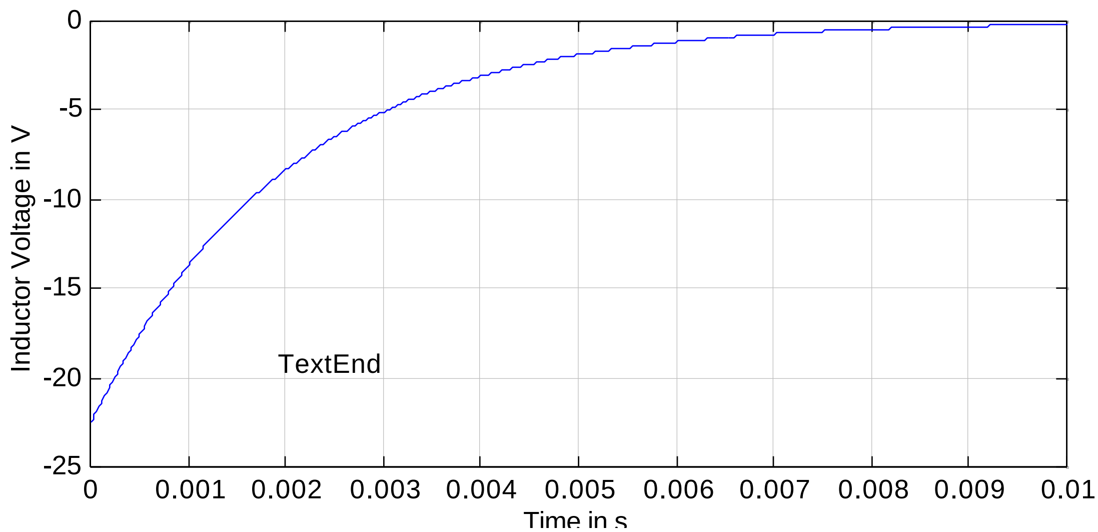
»plot(t,iL)

»grid

```
»xlabel('Time in s')
»ylabel('Inductor Current')
```



(b) Here $v_L(t) = -Ri_L(t) = -22.5e^{-500t}$ V.



(c) Using linearity, for $i_L(0)$ = 50 mA, then $v_L(t) = \frac{-22.5}{3}e^{-500t} = -7.5e^{-500t}$ V and for $i_L(0)$ = 250 mA, then $v_L(t) = -22.5\frac{250}{150}e^{-500t} = -37.5e^{-500t}$ V.

SOLUTION 8.6. Since $\tau = L/R$, we can solve for L = 5 mH. Then solving $i_L(t) = i_L(0)e^{-t/\tau}$ for $i_L(0)$ at t = 4 μsec yields $i_L(0)$ =15 mA.

SOLUTION 8.7. (a) We desire to solve $i_L(t) = i_L(0)e^{-t/\tau}$ for $i_L(0)$ and R in $\tau = 0.08/(R+10^3)$.

Using the following ratio, $\dfrac{i_L(0.05\,ms)}{i_L(0.15\,ms)} = \dfrac{9.197}{1.2447} = \dfrac{i_L(0)e^{-(0.05m)/\tau}}{i_L(0)e^{-(0.15m)/\tau}} = e^{0.1\times10^{-3}/\tau} = 7.3889$. Hence

»K = 9.197/1.2447

K =7.3889

»tau = 0.1e-3/log(K)

tau =5.0000e-05

»L = 0.08;

»Req = L/tau

Req =1.6000e+03

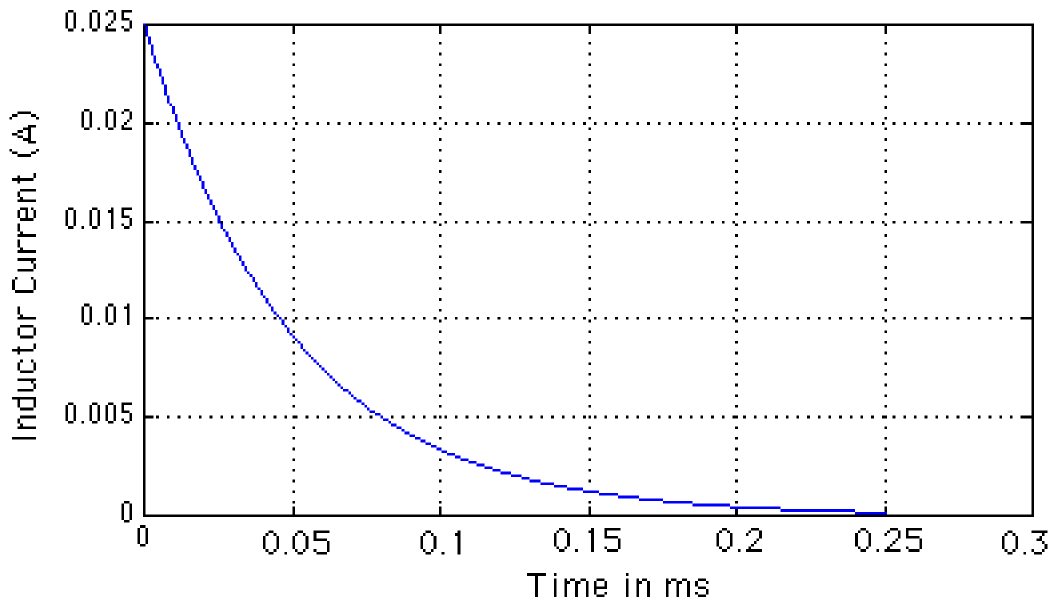
»R = Req - 1000

R = 599.9862

»iL0 = 9.197e-3/exp(-0.05e-3/tau)

iL0 = 0.0250 A

(b) $\tau = 80m/(R+1000) = 50\mu\,\text{sec}$, $i_L(t) = 0.025e^{-t/50\times10^{-6}}$ A.



SOLUTION 8.8. By Ohm's law, $v_R(0+) = -(4k\,||16k)i_L(0+) = -32$ V. The time constant $\tau = L/(4k\,||6k) = 25\mu\,\text{sec}$, i.e.,

»Req = (4e3*16e3/(4e3+16e3))

Req =3200

»L = 0.08;

»tau = L/Req

tau =2.5000e-05

Using the general equation, $v_R(t) = v_R(0+)e^{-t/\tau} = -32e^{-t/25\mu}$ V. Equivalently,

$v_R(t) = -R_{eq}i_L(t) = -R_{eq}0.01e^{-4\times10^4 t}$ V.

SOLUTION 8.9. **(a)** Note that the Thevenin resistance seen by the capacitor is Rth:

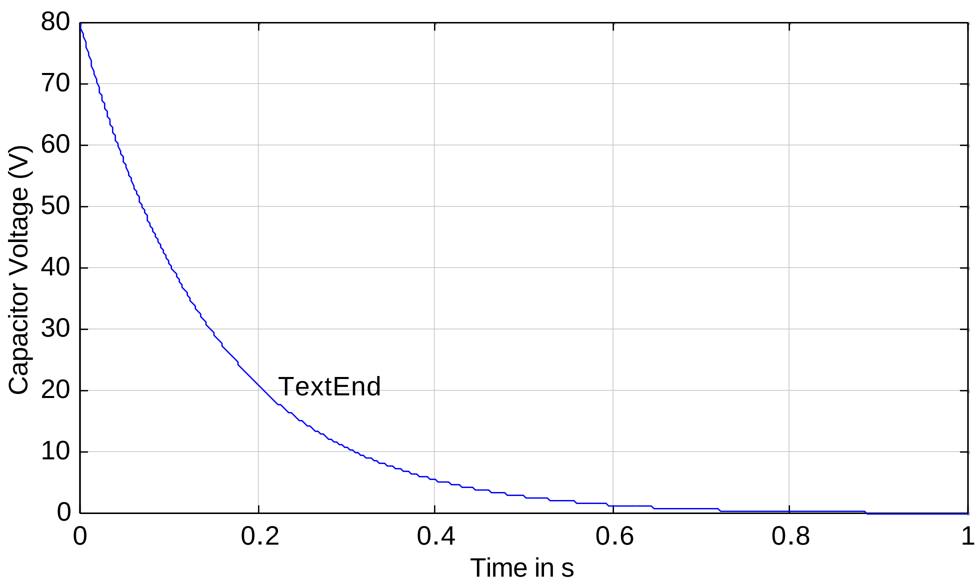
»R1 = 360+60*120/(60+120)

R1 =400

»Rth = 400*1200/1600

Rth =300

Hence, $v_C(t) = v_C(0)e^{-t/\tau} = 80e^{-t/0.15}$ V where $\tau = R_{TH}C = 300\times0.5\times10^{-3} = 0.15$ s.



(b) Here $i_C(0+) = -v_C(0^+)/R_{TH} = -0.2667$. Therefore $i_C(t) = i_C(0+)e^{-t/\tau} = -0.2667e^{-t/0.15}$ A.

Equivalently, $i_C(t) = \dfrac{-v_C(t)}{R_{th}} = -0.2667e^{-t/0.15}$ A.

(c) By voltage division, $v_R(0+) = v_C(0)\left(\dfrac{60||120}{(60||120)+160+200}\right) = 8$ V; for t > 0 $v_R(t) = 8e^{-t/0.15}$ V.

SOLUTION 8.10. First, find the Thevenin equivalent seen at the left of the inductor. Introducing a test source in place of the inductor we obtain the following KCL equation at that node.

$i_{test} = v_{test}/1k + \left[v_{test} - 200\left(\frac{v_{test}}{1k} \right) \right] \Big/ 200$. Let vtest = 1 V. Then

»itest = 1e-3 + (1 - 200*1e-3)/200

itest =0.0050

»Rth = 1/itest

Rth =200

Thus $R_{th} = 200\ \Omega$, $\tau = L/R_{th} = 0.25$ ms, and $i_L(t) = 0.025e^{-4000t}$ A. Next from 8.13b find $v_L(t) = -(R_{TH} \times i_L(0))e^{-4000t} = -5e^{-4000t}$ V, and from Ohm's law $i_x(t) = -5e^{-4000t}$ mA.

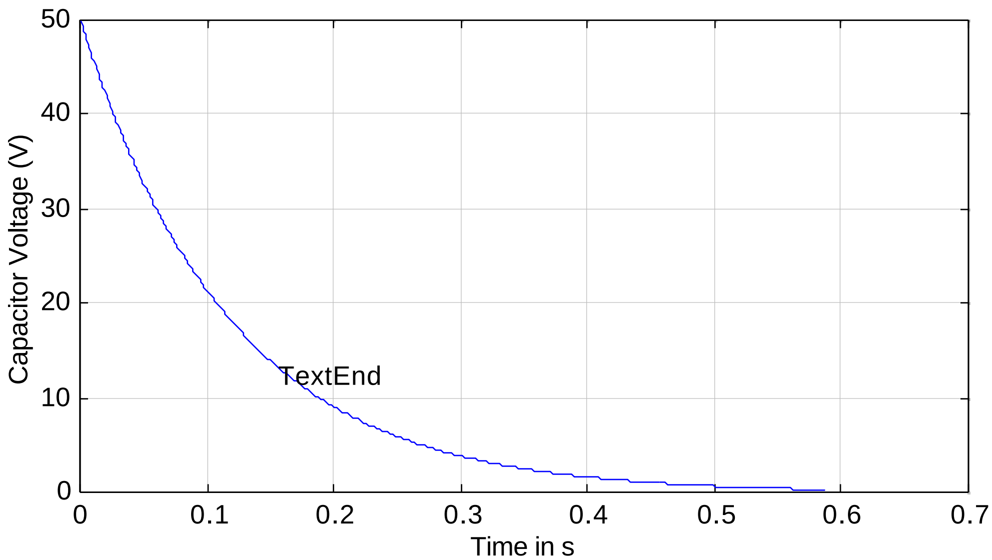
SOLUTION 8.11. For all parts it is necessary to find the Thevenin equivalent resistance seen by the capacitor. To this end we apply an external test current to the remainder of the circuit to obtain:

$$v_{test} = R_1 i_{test} + R_2(i_{test} + \alpha i_{test})$$

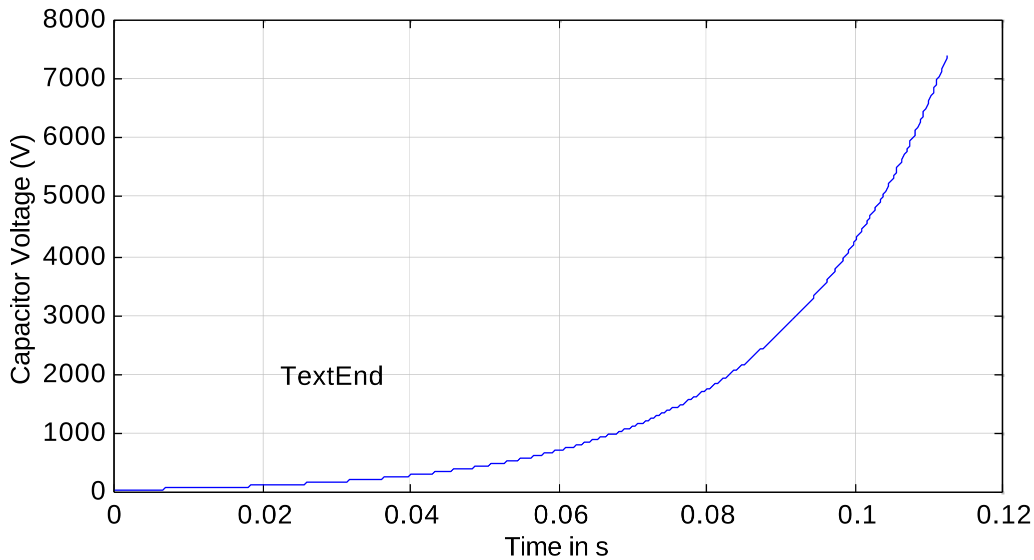
Thus

$$R_{th} = v_{test}/i_{test} = R_1 + R_2(1+\alpha) = 120 + 70(1+\alpha)$$

(a) With $\alpha = 4$, $R_{th} = 470\ \Omega$, $\tau = R_{th}C = 0.1175$ s, and $v_C(t) = v_C(0)e^{-t/\tau} = 50e^{-8.51t}$ V.



(b) With $\alpha = -4$, $R_{th} = -90\ \Omega$, $\tau = R_{th}C = -0.0225$ s, and $v_C(t) = 50e^{44.44t}$ V.



Note how this is not a stable design as V increases exponentially without bound.

(c) From the general equation developed at the beginning, $R_{th} = 120 + 70(1+\alpha) > 0$ requires that $\alpha > -2.7143$.

SOLUTION 8.12. Find the Thevenin resistance left of the inductor. Forcing a test current source into the output node,

$$v_{test} = R_1 i_{test} + R_2\left(i_{test} - \alpha R_1 i_{test}\right) = \left(100 + 50(1-\alpha 100)\right) i_{test} = \left(150 - 5000\alpha\right) i_{test}$$

and

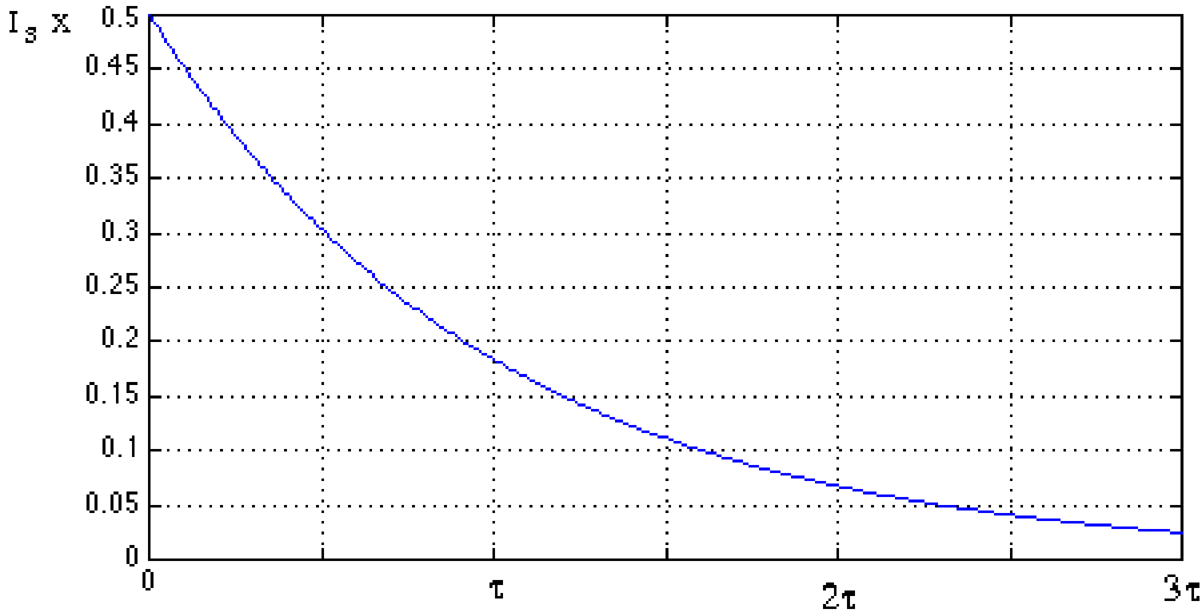
$$R_{th} = v_{test} / i_{test} = 150 - 5000\alpha \ \Omega$$

(a) Using the above equation, $R_{th} = -350\ \Omega$ and $\tau = L / R_{th} = -7.1429 \times 10^{-5}$ s. Hence, $i_L(t) = 0.1e^{14000t}$ A, an unbounded response due to the presence of the negative equivalent resistance.

(b) $R_{th} = 100\ \Omega$, $\tau = L / R_{th} = 2.5 \times 10^{-4}$ s, $v_R(0+) = -R_2(1-\alpha R_1) i_L(0) = 0$, but more importantly $v_{R2}(t) = -R_2(1-\alpha R_1) i_L(t) = 0 \times i_L(t) = 0$.

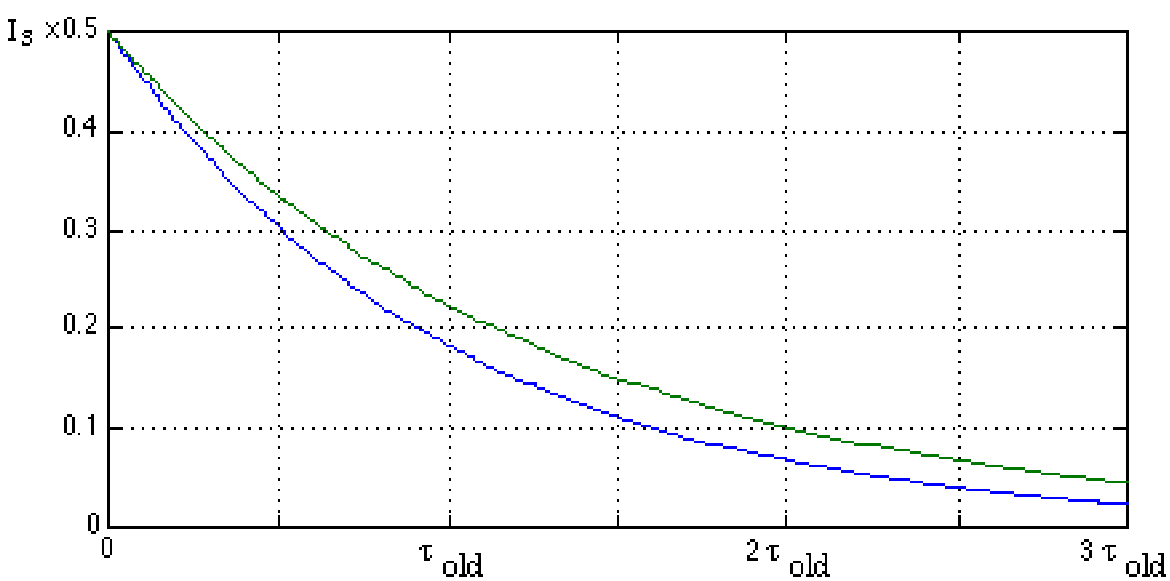
(c) $R_1 + R_2\left(1-\alpha R_1\right) = \left(150 - \alpha 5000\right) > 0$ implies $\alpha < 0.03$.

SOLUTION 8.13. Over a long period of time the inductor L, is seen as a short circuit. Thus at time 0^-, the current through the inductor is, by current division, $I_s/2$. As such, $i_L(t) = 0.5 I_s e^{-Rt/2L}$ A. A sketch will reveal an exponentially decreasing current from an initial $0.5I_s$ A.



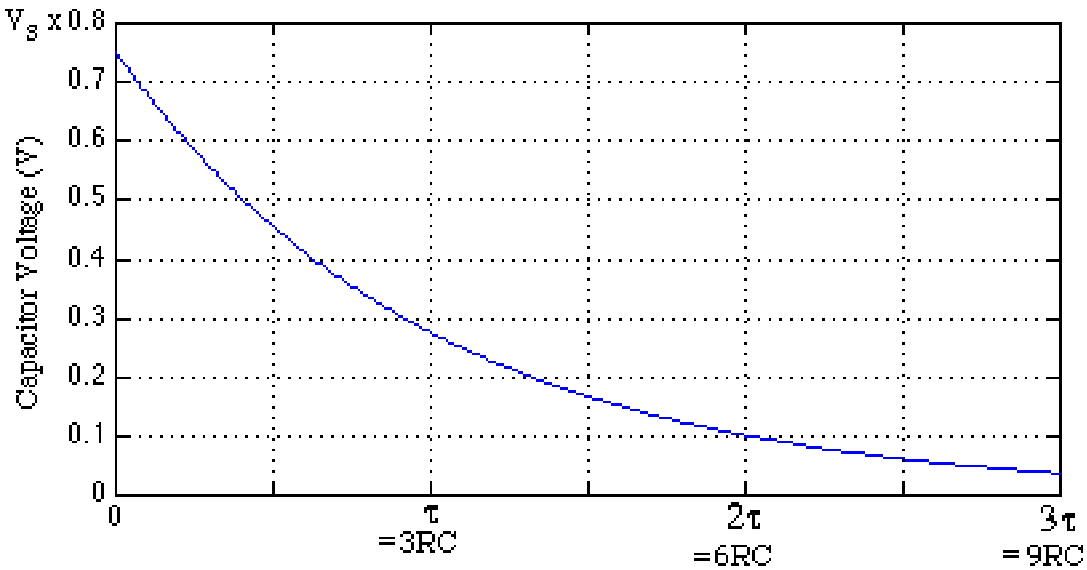
SOLUTION 8.14. This is similar to problem 8.13. Here the current turns off at time zero instead of a switch opening. By current division $i_L(0^+) = I_s/2$. The difference between this problem and problem 8.13 is that the Thevenin resistance seen by the inductor is different. Here, $R_{TH} = 2R\|0.5R = 0.4R$. So for t > 0, $i_L = (I_s/2)e^{-R_{th}t/L} = 0.5I_s e^{-Rt/2.5L}$ A. A sketch of this function plotted with respect to this new time constant will be identical to the one in problem 8.13.

Define the time constant of problem 8.13 as τ_{old}. The slower decay in the plot below represents the fall of the inductor current for problem 8.14 relative to that of problem 8.13 which is the faster decaying curve in the plot below.



SOLUTION 8.15. Over a long period of constant applied voltage, a capacitor looks like an open circuit. By voltage division, $v_C(0^+) = \frac{3}{4}V_s$ and τ = 3RC. Hence

$$v_C(t) = v_C(0+)e^{-t/\tau} = 0.75V_s e^{-t/(3RC)} \text{ V}$$



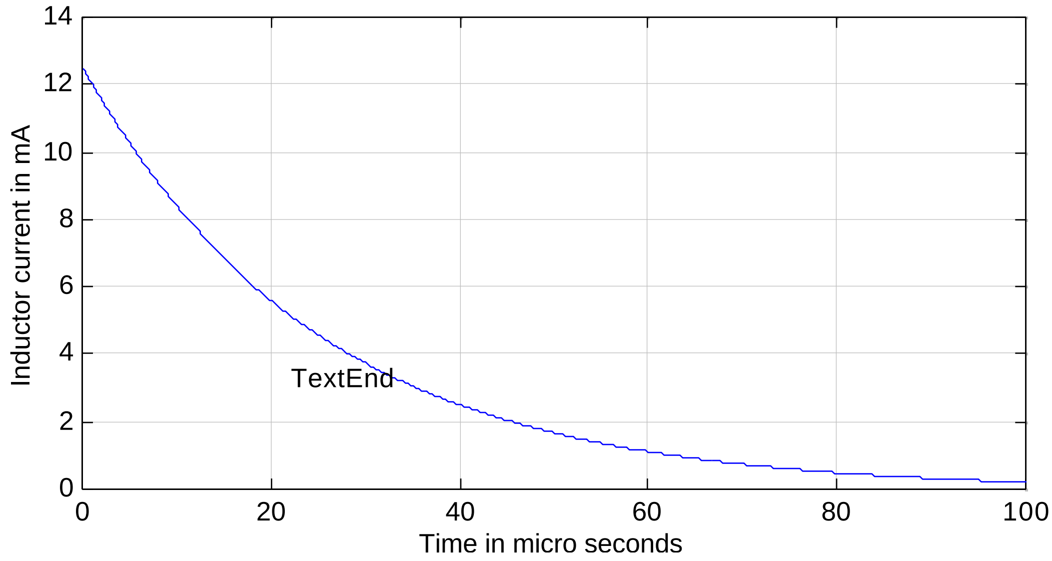
SOLUTION 8.16. Same problem as 8.15 for t < 0. For t > 0 the effective resistance changes. $R_{th} = 3R//R = 0.75R$ and τ = 0.75RC. Thus, $v_C(t) = 0.75V_s e^{-t/(0.75RC)}$ V. Same behavior as in the previous problem, except for a faster decay than in problem 8.15 due to a smaller effective resistance. Note how the decreased resistance affects the RC and RL circuit differently.

SOLUTION 8.17. For t < 0 the inductor looks like a short circuit. Let $R_1 = 1333//800 = 500$ Ω. The current supplied by the source is $I_s = \frac{12}{100+500} = 0.02$ A. By current division,

$$i_L(0^+) = 0.02 \frac{1333}{800+1333} = 0.0125 \text{ A}$$

For t > 0, the switch is opened and the inductor sees only the 800 Ω resistor. Hence, $\tau = L/R = 25$ μsec and

$$i_L(t) = 12.5e^{-40000t} \text{ mA}$$



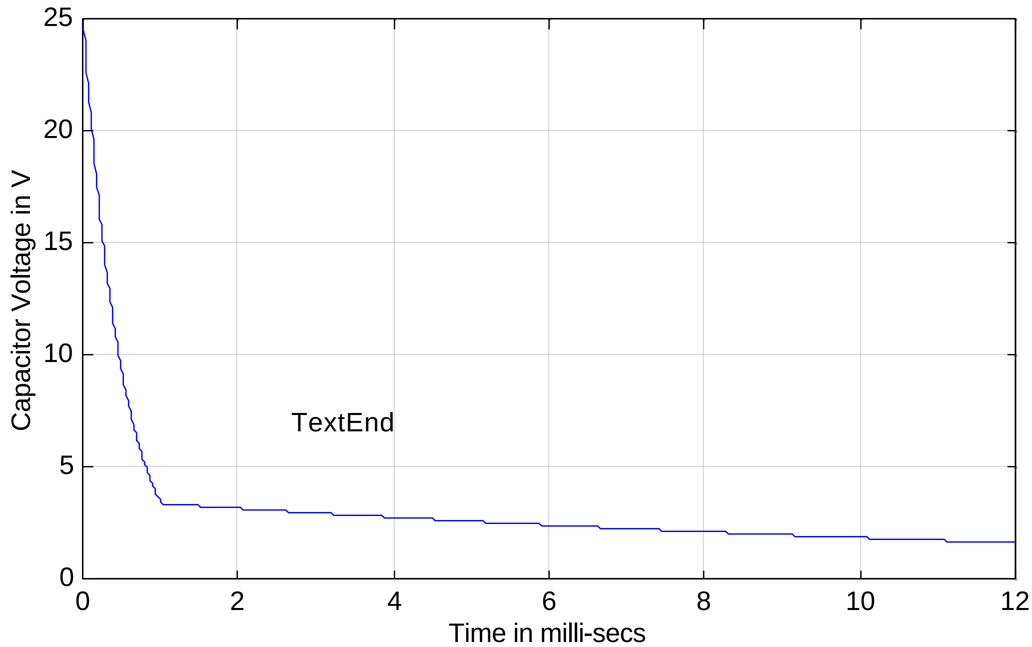
SOLUTION 8.18. (a) For t < 0 the applied voltage is constant and at t = 0, the capacitor is like an open circuit. By voltage division, $v_C(0^+) = 30\dfrac{25k\,||6.25k}{(25k\,||6.25k)+1k} = 25$ V. For 0 < t < 1 ms, the source is off and the capacitor discharges through three resistors in parallel; thus

$\tau = R_{th}C = 833.33 \times 0.6 \times 10^{-6} = 0.5$ ms and $v_C(t) = 25e^{-2000t}$ V.

(b) From continuity $v_C(0.001) = 25e^{-2000(0.001)} = 3.383$. For t > 1ms, the capacitor keeps on discharging through only one resistance, the 25 kΩ resistor; thus the new time constant is τ_{new} = 15 ms, and $v_C(t) = 3.383e^{-(t-0.001)/0.015}$ V.

(c) Plot $v_C(t) = 25e^{-2000t}\left[u(t) - u(t-0.001)\right] + 3.383e^{-(t-0.001)/0.015}u(t-0.001)$ V

```
»t = 0:.01:12;
»vc = 25*exp(-2*t) .*(u(t)-u(t-1))+3.3834*exp(-(t-1)/15) .*u(t-1);
»plot(t,vc)
»grid
»xlabel('Time in milli-secs')
»ylabel('Capacitor Voltage in V')
```



SOLUTION 8.19. **(a)** From Ohm's law $i_L(0^+) = \dfrac{54}{60+30} = 0.6$ A. For t > 0, the Thevenin resistance seen by the inductor is $R_{th} = (60+30) \,||\, 720 = 80\ \Omega$ and $\tau = L/R_{th} = 1/160$ s. Thus $i_L(t) = i_L(0^+)e^{-t/\tau} = 0.6e^{-160t}$ A. From Ohm's law and current division

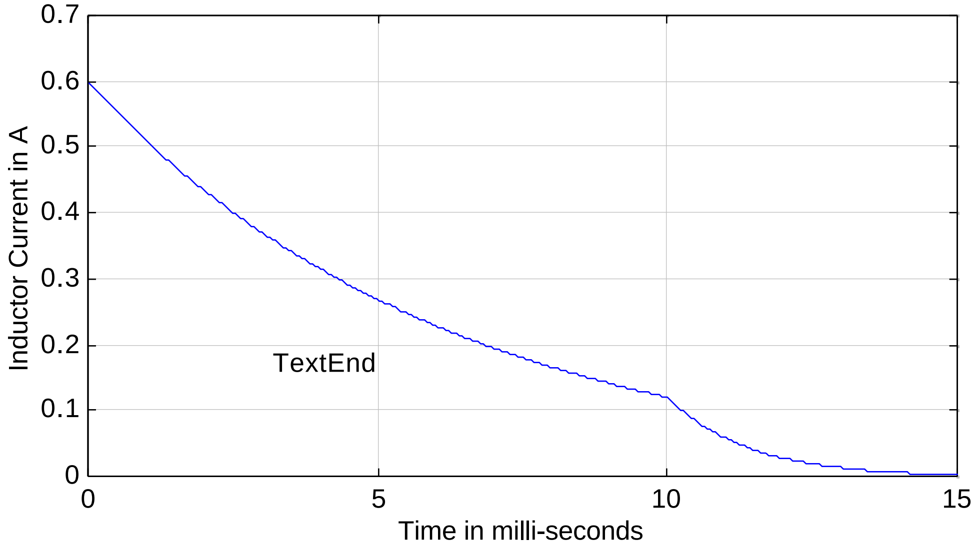
$$v(t) = -60 \times \frac{720}{90+720} \times i_L(t) = -32e^{-160t} \text{ V}$$

(b) From continuity property, $i_L(0.01\pm) = 0.6^{-160(0.01)} = 121.14$ mA. For t > 10 ms, the Thevenin resistance seen by the inductor is $R_{th} = (690+30) \,||\, 720 = 360\ \Omega$ and $\tau_{new} = L/R_{th} = 1/720$ s. Hence, $i_L(t) = 121.14e^{-720(t-0.01)}$ mA for t > 10 ms. From Ohm's law and current division

$$v(t) = -690 \times \frac{720}{720+720} \times i_L(t) = -41.793e^{-720(t-0.01)}u(t-0.01) \text{ V}$$

Therefore

$$i_L(t) = 0.6e^{-160t}\left[u(t) - u(t-0.01)\right] + 0.12114e^{-720(t-0.01)}u(t-0.01) \text{ A}$$

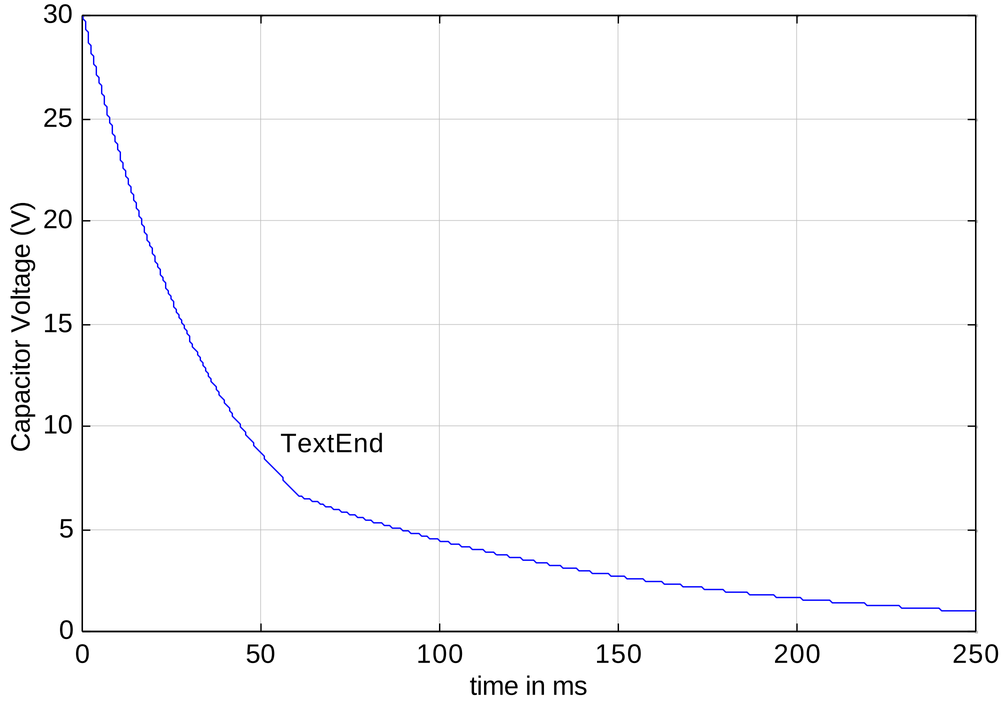


SOLUTION 8.20. For both circuits we first compute the Thevenin resistance seen to the right of the capacitor for $0 \le t \le 60$ ms. If we excite the circuit to the right of the capacitor over this time interval, then $i_{test} = \frac{v_{test}}{200} + \frac{(1-0.25)v_{test}}{100} = 0.0125 v_{test}$. Let $R_{th1} = \frac{1}{0.0125} = 80\ \Omega$.

(a) For t < 0, the capacitor acts as an open circuit. Using voltage division, $v_C(0^+) = \frac{80}{80+133.3}80 = 30$ V. For $0 \le t \le 60$ ms, the time constant is $\tau_1 = R_{th1}C = 40$ ms, and

$$v_C(t) = v_C(0^+)e^{-t/\tau} = 30e^{-t/(R_{th1}C)} = 30e^{-25t}\ \text{V}$$

From continuity, $v_C(60^+\ ms) = 6.694$ V. The new Thevenin resistance is $R_{th2} = 200\ \Omega$. Thus for t > 60 ms, the time constant is $\tau_2 = 200C = 100$ ms, and $v_C(t) = 6.694e^{-10(t-0.06)}$ V. The resulting capacitor voltage is plotted below.

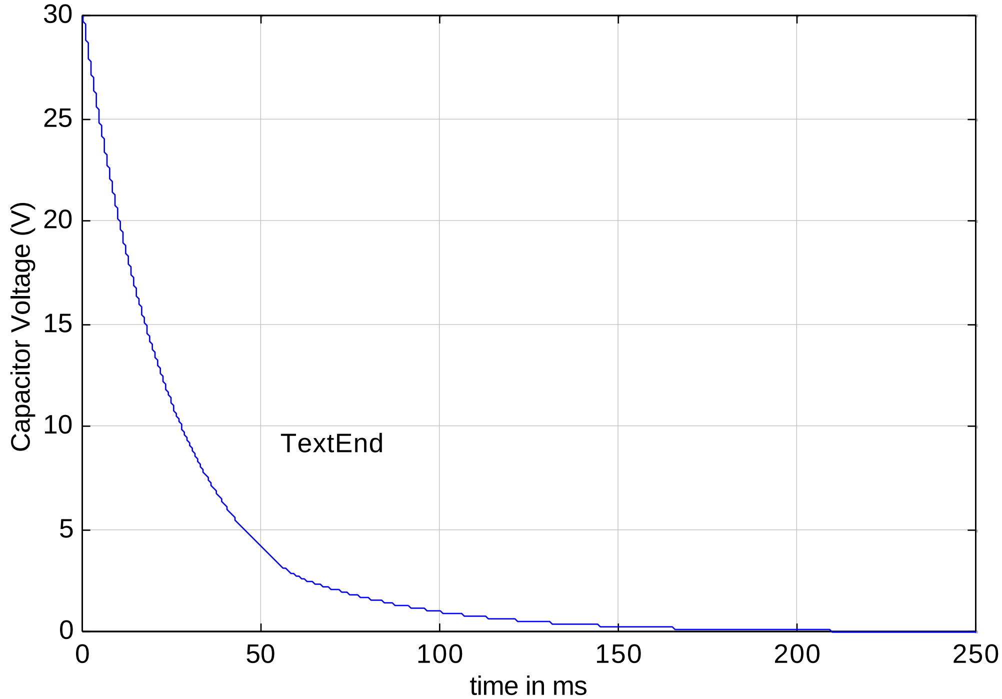


(b) It is the same circuit as above for t < 0; thus $v_C(0^+) = 30$ V. However the Thevenin resistances seen by the capacitor are different because there is no switch to disconnect the independent voltage source and its series resistance. First for $0 \le t \le 60$ ms, the Thevenin resistance to the right remains as

$R_{th1} = \dfrac{1}{0.0125} = 80\ \Omega$. However, for $0 \le t \le 60$ ms, the Thevenin resistance seen by the capacitor changes to $R_{th3} = R_{th1} // 133.3 = 50\ \Omega$. Then new time constant is $\tau_3 = R_{th3}C = 25$ ms and for $0 \le t \le$ 60 ms

$$v_C(t) = 30e^{-40t}\ \text{V}$$

From continuity, $v_C(60^+\,ms) = 2.72$ V. The new Thevenin resistance is $R_{th4} = 200//133.3 = 80\ \Omega$. Thus for t > 60 ms, the time constant is $\tau_4 = 80C = 40$ ms, and $v_C(t) = 2.72e^{-25(t-0.06)}$ V. The resulting capacitor voltage is plotted below.



(c) For t < 60 ms, the voltage decays faster in (b) due to the smaller time constant. Similarly, for t > 60 ms.

SOLUTION 8.21. Following, are the switching times with the time constants associated with them.

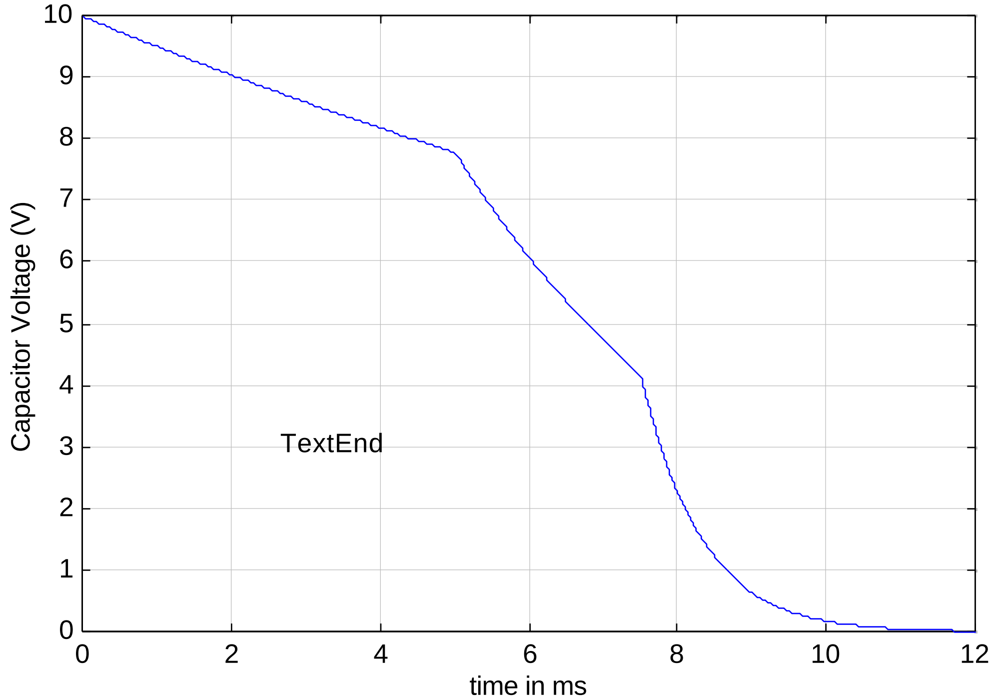
$t = 0 \quad R_{th} = 20\ \text{k}\Omega \quad \tau = 20\ \text{ms}$

$t = 5\ \text{ms} \quad R_{th} = 4\ \text{k}\Omega \quad \tau = 4\ \text{ms}$

$t = 7.5\ \text{ms} \quad R_{th} = 800\ \Omega \quad \tau = 0.8\ \text{ms}$

It follows that with t in ms,

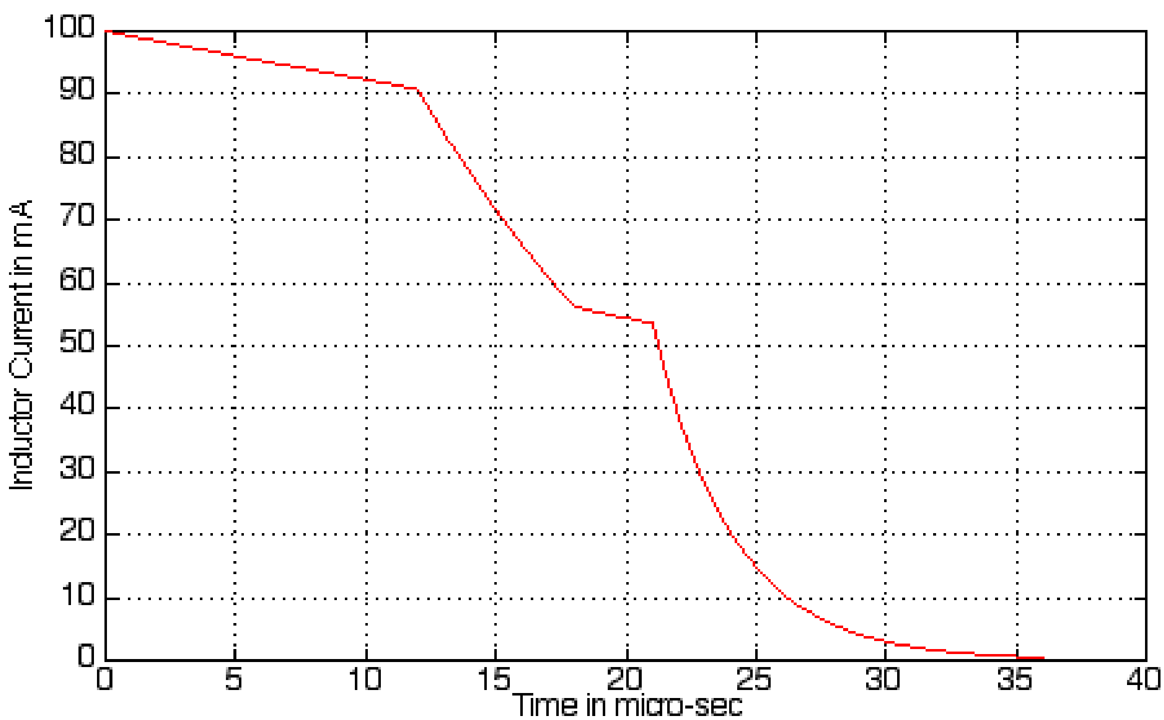
$$v_C(t) = 10e^{-0.050t}\left[u(t) - u(t-5)\right] + 7.788e^{-0.250(t-5)}\left[u(t-5) - u(t-7.5)\right] + 4.169e^{-1.25(t-7.5)}u(t-7.5)\ \text{V}$$



***SOLUTION TO 8.22.** This solution is done in MATLAB.

```
% Define switching times, inductance, and Thevenin equivalent resistances.

t1= 12e-6;
t2=18e-6;
t3=21e-6;
L= 0.1;
rth1= 800;
rth2= 8000;
rth3=1600;
rth4= 32000;

% Define time constants for each of the four time intervals.

tau1= L/rth1
tau2= L/rth2
tau3= L/rth3
tau4= L/rth4

tau1 =   1.2500e-04
tau2 =   1.2500e-05
tau3 =   6.2500e-05
tau4 =   3.1250e-06

% Compute initial inductor currents for each of the four time intervals.

il1= 100e-3;
il2=il1*exp(-t1/tau1)
```

```
il3=il2*exp(-(t2-t1)/tau2)
il4=il3*exp(-(t3-t2)/tau3)

il2 =   9.0846e-02
il3 =   5.6214e-02
il4 =   5.3580e-02

% Determine inductor currents for each of the four time intervals.  Plot.

t = 0:0.5e-7:36e-6;
seg1= il1*exp(-t/tau1) .*(ustep(t)-ustep(t-t1));
seg2 =il2*exp(-(t-t1)/tau2) .*(ustep(t-t1)-ustep(t-t2));
seg3=il3*exp(-(t-t2)/tau3) .*(ustep(t-t2)-ustep(t-t3));
seg4= il4*exp(-(t-t3)/tau4) .* ustep(t-t3);
iL=seg1 + seg2 +seg3 + seg4;

plot(t,iL)
grid
```



SOLUTION 8.23.

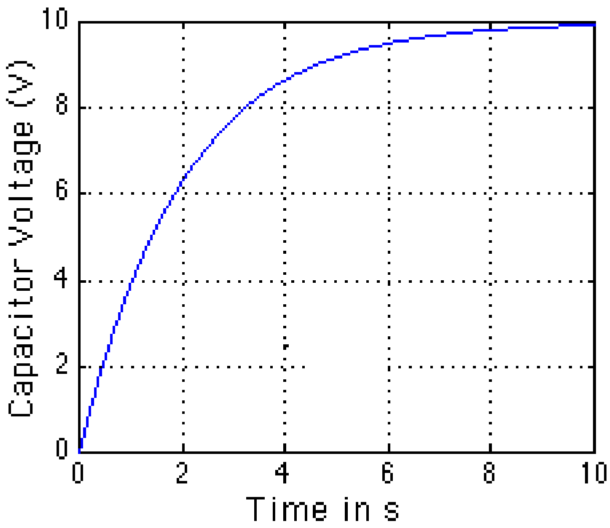
For circuits with a forced voltage, equation 8.19c is used as a general solution,

$$v_C(t) = v_C(\infty) + \left[v_C\left(t_o^+\right) - v_C(\infty)\right] e^{-\left(\frac{t}{R_{TH}C}\right)}.$$

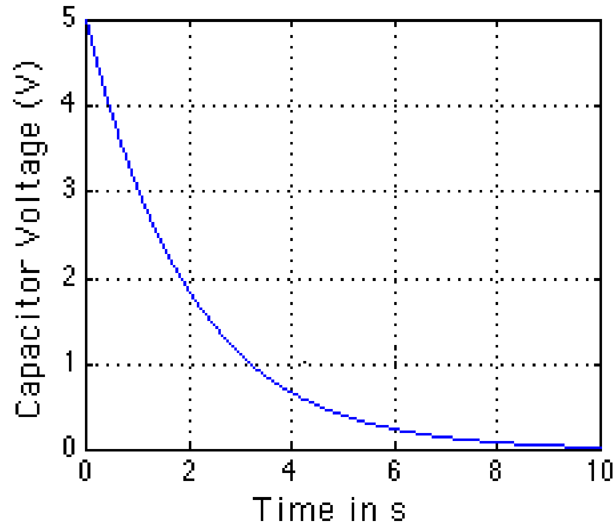
(a) At time zero the voltage is 0 V. As time approaches infinity, the capacitor looks like an open with voltage 10 V. The Thevenin resistance is 10 kΩ. Thus for t > 0.

$$v_C(t) = 10 + [-10]e^{-\left(\frac{t}{2}\right)} = 10(1-e^{-0.5t}) \text{ V.}$$

(b) With vin(t) = 0 and $v_C(0^+) = 5$ V, $v_C(t) = 5e^{-\left(\frac{t}{2}\right)} = 5e^{-0.5t}$ V.

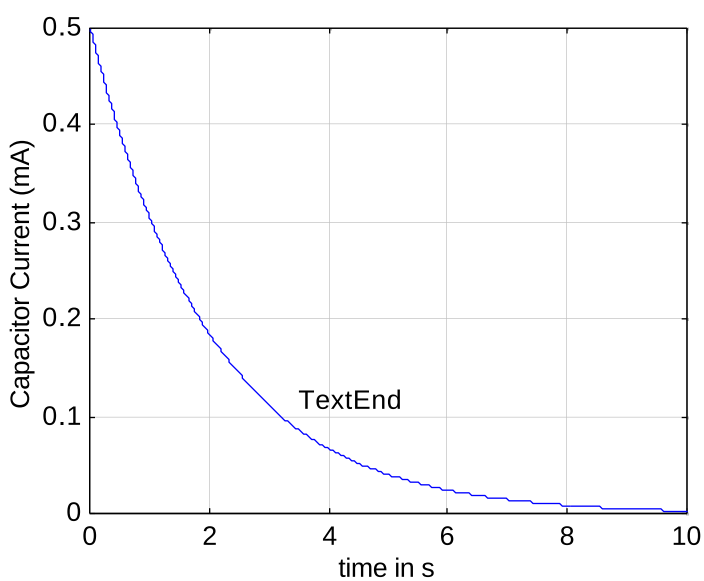


Part (a)



Part (b)

(c) From linearity, $v_C(t) = 10(1-e^{-0.5t}) + 5e^{-0.5t} = 10 - 5e^{-0.5t}$ V. Using Ohm's law, $i_C(t) = \frac{v_{in}(t)}{10^4} - \frac{v_C(t)}{10^4}$. Thus $i_C(t) = 1 - \left(1 - 0.5e^{-0.5t}\right) = 0.5e^{-0.5t}$ mA.



(d) This is the same as (a), under the condition that the input is 1.5 times larger. Hence by linearity,

$$v_C(t) = 1.5 \times 10(1-e^{-0.5t}) = 15(1-e^{-0.5t}) \text{ V.}$$

(e) By linearity, ANSWER = –2x(ANSWER to (b)) + 3x(ANSWER to (a)):

$$v_C(t) = -2\times 5e^{-0.5t} + 3\times 10(1-e^{-0.5t}) = 15(1-e^{-0.5t}) = 30 - 40e^{-0.5t}\ \text{V}$$

SOLUTION 8.24.
(a) At t = 0-, the capacitor looks like an open circuit; therefore, by voltage division and the continuity property, $v_C(0^-) = v_C(0^+) = \frac{3R}{4R}V_{s1} = 0.75V_{s1}$. Similarly, at t = ∞, $v_C(\infty) = \frac{3R}{4R}V_{s2} = 0.75V_{s2}$. The circuit time constant is $\tau = (3R//R)C = 0.75RC$. Hence

$$v_C(t) = 0.75V_{s2} + 0.75\left[V_{s1} - V_{s2}\right]e^{-\left(\frac{t}{0.75RC}\right)}.$$

(b) A sketch will show an exponentially varying voltage from $0.75V_{s1}$ converging to $0.75V_{s2}$ with the computed time constant.
(c) The response to the initial condition when the inputs are set to zero, zero-input response, is $v_C(t) = V_{s1}e^{-\left(\frac{t}{0.75RC}\right)}$. The zero order response, the response with 0V initial condition to a forced voltage, is $v_C(t) = V_{s2} - V_{s2}e^{-\left(\frac{t}{0.75RC}\right)}$.

SOLUTION 8.25.
For RL circuits with a forced current, equation 8.19b is used as a general solution:

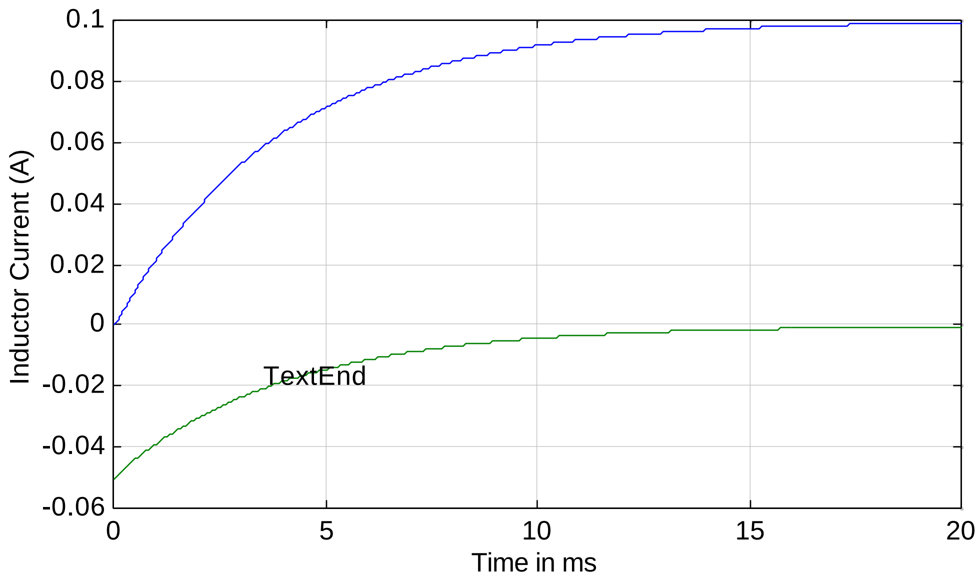
$$i_L(t) = i_L(\infty) + \left[i_L\left(t_o^+\right) - i_L(\infty)\right]e^{-\frac{R_{th}}{L}(t-t_o)}.$$

Since R_{th} = R = 100 Ω and L = 0.4 H, we have $\tau = 4$ ms and

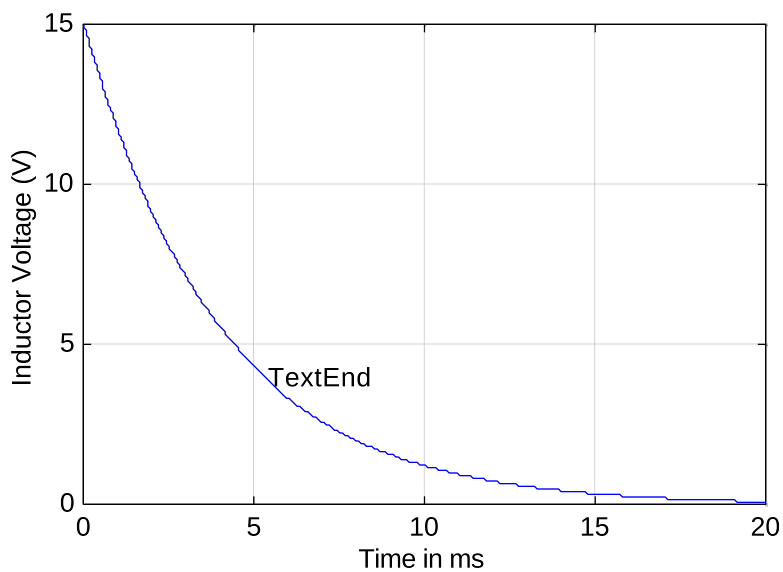
$$i_L(t) = i_L(\infty) + \left[i_L\left(t_o^+\right) - i_L(\infty)\right]e^{-250(t-t_o)}$$

(a) Here, $i_L(0) = 0$ and as time approach infinity, the inductor becomes a short and $i_L(\infty) = 10/100 = 0.1$ A. Thus $i_L(t) = 0.1\left(1 - e^{-250t}\right)$ A.

(b) Here $i_L(0) = -50$ mA and because the input is zero, $i_L(\infty) = 0$. Thus, $i_L(t) = -0.05e^{-250t}$ A. Plots for parts (a) and (b) appear below.



(c) By linearity $i_L(t) = 0.1\left(1 - e^{-250t}\right) - 0.05e^{-250t} = 0.1 - 0.15e^{-250t}$ A. Further, by KVL and Ohm's law, $v_L(t) = v_{in}(t) - 100 i_L(t)$ implies $v_L(t) = 10 - 10 + 15e^{-250t} = 15e^{-250t}$ V.



(d) Observe that the new initial condition is –0.5 times the old one and that the new input voltage is 1.5 times the old one. Therefore, by linearity,

$$i_L(t) = 1.5 \times 0.1\left(1 - e^{-250t}\right) + (-0.5) \times \left(-0.05e^{-250t}\right) = 0.15 - 0.125e^{-250t}$$

and thus

$$v_L(t) = 15 - 15 + 12.5e^{-250t} = 12.5e^{-250t} \text{ V}$$

The plot is similar to part (c) with initial point 12.5 instead of 15.

SOLUTION 8.26. For this problem $R_{th} = 2R // 0.5R = 0.4R$ in which case $\tau = L/R_{th} = L/0.4R$.

(a) At t = 0, the inductor looks like a short circuit. Hence by current division, $i_L(0^-) = i_L(0^+) = 0.5 I_{s1}$. A similar argument yields $i_L(\infty) = 0.5 I_{s2}$. Using the general form of the solution,

$i_L(t) = 0.5 I_{s2} + 0.5\left[I_{s1} - I_{s2}\right]e^{-\frac{0.4Rt}{L}}$.

(b) A sketch will show an exponentially varying current from $0.5I_{s1}$ A converging to $0.5I_{s2}$.

(c) The response to the initial condition when the inputs are set to zero, zero-input response, is $i_L(t) = 0.5 I_{s1} e^{-\frac{0.4Rt}{L}}$. The zero state response, the response with no initial condition, to the input $I_{s2}u(t)$, is $i_L(t) = I_{s2}\left(1 - e^{-\frac{0.4Rt}{L}}\right)$.

SOLUTION 8.27. For this problem, $v_C(0) = \left(20 - 10e^{-0.4t}\right)_{t=0} = 10$ V and $v_C(\infty) = 10 I_s = 20 = \lim_{t\to\infty}\left(20 - 10e^{-0.4t}\right)$. Hence $I_s = 2$ A. Further, $i_C(t) = C\frac{dv_C(t)}{dt} = 4Ce^{-0.4t} = 0.4e^{-0.4t}$ which implies that C = 0.1 F. Since $\tau = 1/0.4 = 2.5 = (10 + R)C = 0.1(10 + R)$, it follows that R = 15 Ω.

SOLUTION 8.28.

(a) The Thevenin resistance for this configuration is $R_{th} = 1000//1000 = 500$ Ω and $\tau = R_{th}C = 0.25$ s. Hence $v_C(t) = v_C(0^+)e^{-t/\tau} = 15e^{-4t}$ V is the zero-input response.

(b) Using a source transformation and voltage division, $v_C(\infty) = 3$ V. Thus $v_C(t) = 3\left(1 - e^{-4t}\right)$ V.

(c) Here $v_C(\infty) = 4$ V, thus $v_C(t) = 4\left(1 - e^{-4t}\right)$ V.

(d) This is the superposition of parts (b) and (c), i.e., $v_C(t) = 7\left(1 - e^{-4t}\right)$

(e) The complete response is the superposition of parts (d) and (a), i.e., $v_C(t) = 7 + 8e^{-4t}$ V.

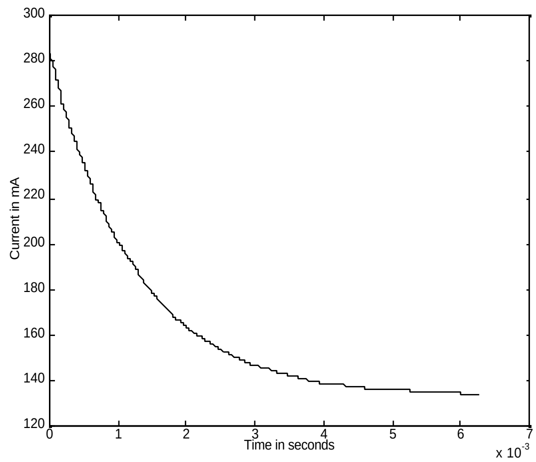
(f) From linearity,

$$v_C(t) = 0.5 \times 7\left(1 - e^{-4t}\right) + 2 \times 15e^{-4t} = 3.5 + 26.5e^{-4t}$$

SOLUTION 8.29. Solution done in MATLAB

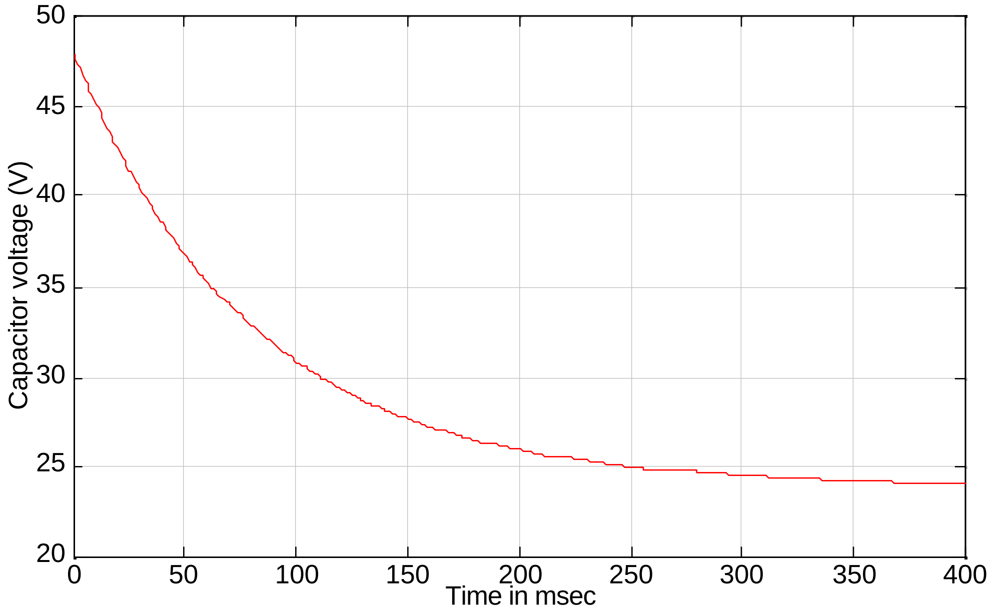
```
%Problem 8.29

%RTH= (60||120)+120
%tau=L/RTH

RTH=1/(1/60+1/120)+120;
tau=0.2/RTH;

%Using superposition at t<0
il01=36/120/2;
il02=(72/180)*(60/180);
il0= il01+il02;
```

```
% At t>0 36volts is off thus
ilinf=il02;

t=0:5*tau/1000:5*tau;
ilt=ilinf+(il0-ilinf)*exp(-t/tau);

plot(t,1000*ilt);
xlabel('Time in seconds');
ylabel('Current in mA');
```



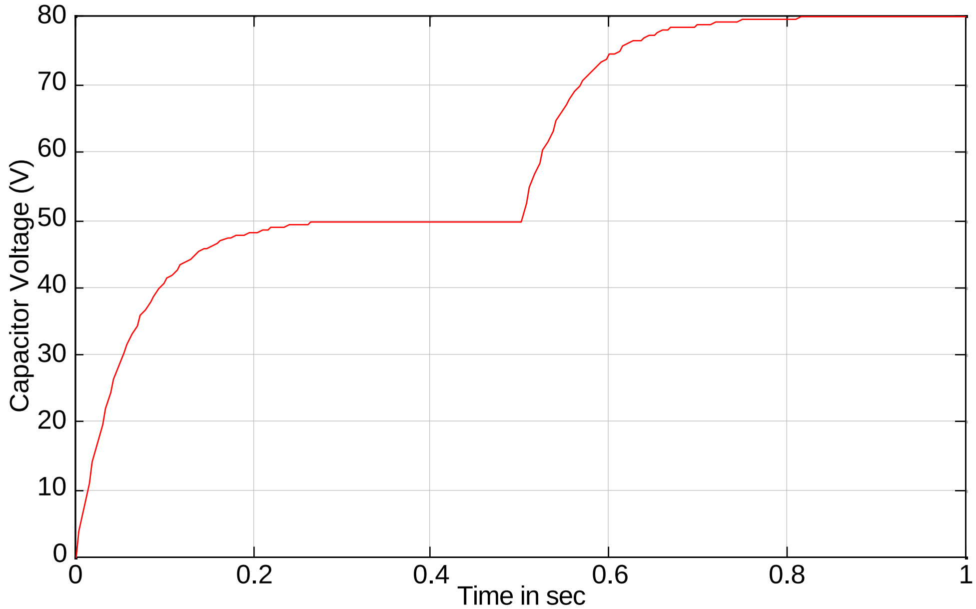
***SOLUTION 8.30.**

```
» %  Rth = 120 + 120\\60 = 160 kohm
» %  tau = Rth*C
»tau = 160e3*0.5e-6
tau =
   0.0800
» %  Initial capacitor voltage is computed by
» %  voltage division and superposition
»vc0 = 24+24
vc0 =
   48
» %  At t = ∞, capacitor looks like an open circuit.  Hence
»vcinf=24;
»t = 0:1e-3:5*tau;
»vct = vcinf+(vc0-vcinf)*exp(-t/tau);
»plot(t*1000,vct)
»grid
»xlabel('Time in msec')
»ylabel('Capacitor voltage (V)')
```



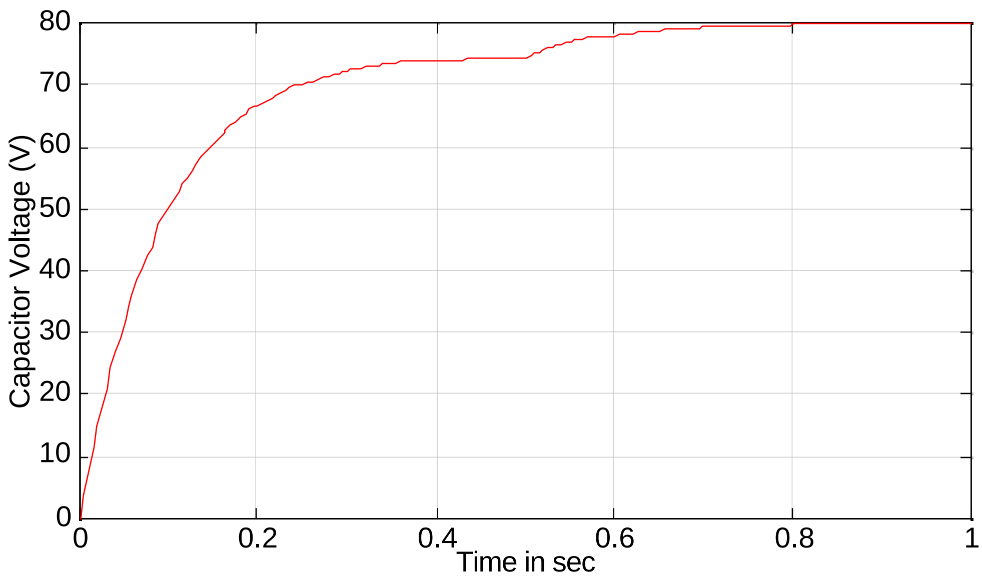
***SOLUTION 8.31A**

```
»vc0 =0;
»% Consider  0 ≤ t ≤ 0.5
»vcinf = 50;
»%  Rth = 600\\300 = 200 ohms
»%  tau1 = Rth*C
»tau1 = 300e-6*200
»vc0 = 0;
»t = 0:5e-3:1;
»vct = (vcinf+(vc0-vcinf)*exp(-t/tau1)) .* (ustep(t)-ustep(t-.5));
»
»% Consider 0.5 ≤ t ≤ 1
»tau2 = tau1
»vc5  = (vcinf+(vc0-vcinf)*exp(-.5/tau1))
»vcinf2 = 80;
»vct2 = (vcinf2+(vc5-vcinf2)*exp(-(t-0.5)/tau1)) .* ustep(t-0.5);
»vca = vct+vct2;
»plot(t,vca)
»grid
»xlabel('Time in sec')
»ylabel('Capacitor Voltage (V)')
```

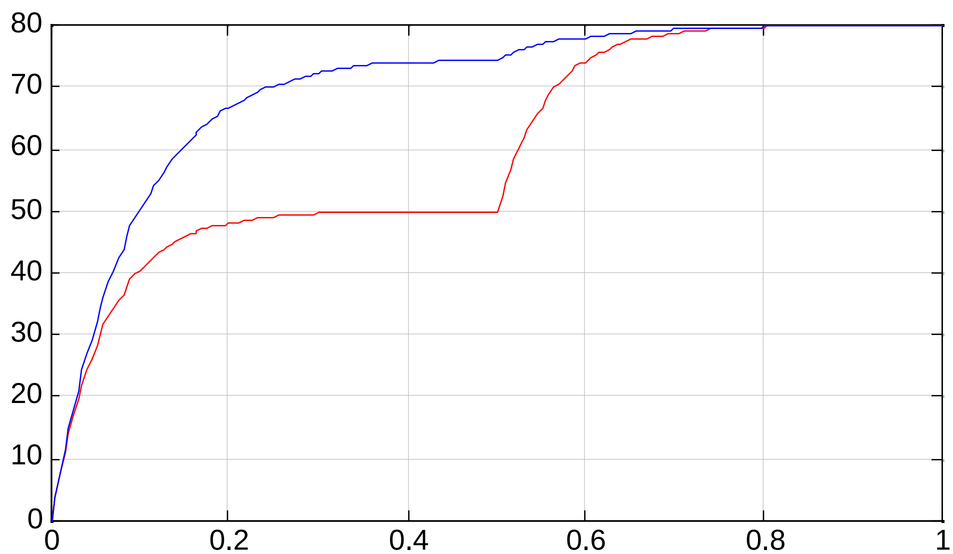


***SOLUTION 8.31B**

```
»vc0 =0;
»% for 0 ≤ t ≤ 0.5
»vcinf = 75;
»tau1 = 300e-6*300
»vc0 = 0;
»t = 0:5e-3:1;
»vct = (vcinf+(vc0-vcinf)*exp(-t/tau1)) .* (ustep(t)-ustep(t-.5));
»% for 0.5 ≤ t ≤ 1
»tau2 = 300e-6*200
»vc5  = (vcinf+(vc0-vcinf)*exp(-.5/tau1))
»vcinf2 = 80;
»vct2 = (vcinf2+(vc5-vcinf2)*exp(-(t-0.5)/tau1)) .* ustep(t-0.5);
»vcb = vct+vct2;
»plot(t,vcb)
»grid
»xlabel('Time in sec')
»ylabel('Capacitor Voltage (V)')
»pause
»plot(t,vca,t,vcb,'b')
»grid
```



Comparison of the two responses.



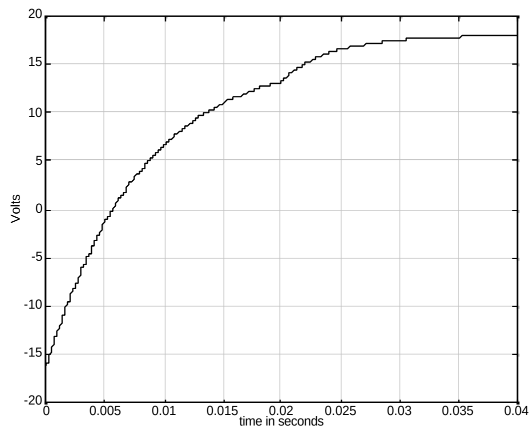
SOLUTION 8.32.

%Problem 8.32

%Consider t < 0
%vin=-20V, thus
vc0=(8/10)*(-20);

%For 0<t<20ms
RTH=1/(1/2e3+1/8e3);
tau1=5e-6*RTH;

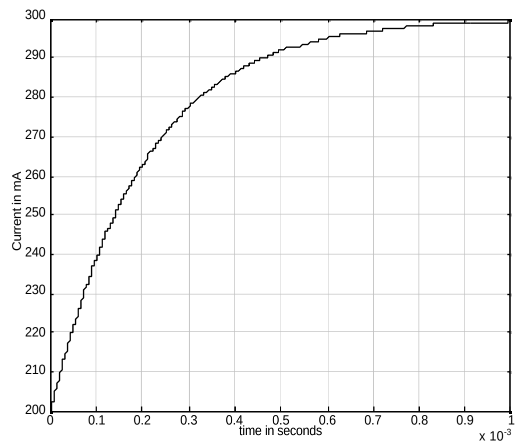
```
vcinf=(8/10)*20;
t=20e-3;
vc20ms=vcinf+(vc0-vcinf)*exp(-t/tau1);

%For 20ms<t
RTH=1/(1/2e3+1/8e3+1/1.6e3);
tau2=5e-6*RTH;

%By superposition
vcinf2=20*0.5+20*0.4;
t=0:(40e-3)/1000:40e-3;

vct= (ustep(t)-ustep(t-20e-3)).*(vcinf+(vc0-vcinf).*exp(-t./tau1)) ...
+ ustep(t-20e-3).*(vcinf2+(vc20ms-vcinf2).*exp(-(t-0.02)./tau2));

plot(t,vct);
grid;
xlabel('time in seconds');
ylabel('Volts');
```



SOLUTION 8.33.

(a)

```
%Problem 8.33
%(a)

%at t<0 only one source is contributing thus,
il0=24/60*0.5;
%For t>0
RTH=60+1/(1/30+1/60);
tau=16e-3/RTH;

%As t goes to infinity, by superposition,
ilinf=24/(60)*0.5+24/80*1/3;
t=0:5*tau/1000:5*tau;
ilt=ilinf+(il0-ilinf).*exp(-t/tau);
figure(1);
plot(t,1000.*ilt);
grid;
xlabel('time in seconds');
ylabel('Current in mA');
```



(b) Using 8.23, The time constant remains the same as in (a). In order to find the inductor voltage, we must do so indirectly by solving for $v_L(t) = V_A - 60I_L$. At t=0$^+$,

$i_L(0^+) = 200mA$

$$V_A = 24\frac{60}{90} + 24\frac{30}{90} - i_L(0^+)(60||30) = 20V$$

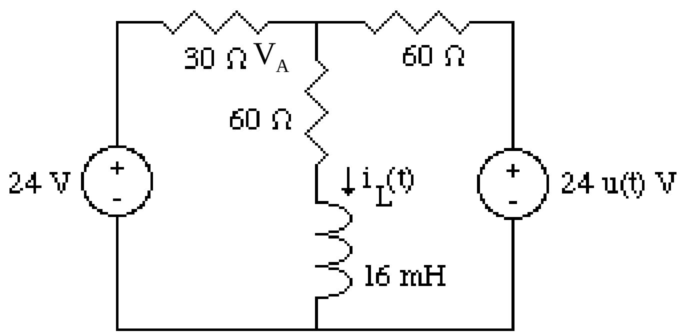
$v_L(0^+) = x(t_0^+) = V_A - 60i_L(0^+) = 8V$

For t > 0, $X^e = v_L(\infty) = 0V$, Thus $v_L(t) = 8e^{-5000t}V$.

(c) By linearity,

$i_L(t) = 600 - 200e^{-5000t}mA$

$v_L(t) = 16e^{-5000t}V$



SOLUTION 8.34.

(a) Since the voltage has been constant for a long time, the capacitor acts as an open circuit. Thus by voltage division and continuity, $v_C(0^-) = v_C(0^+) = 0.75V_o$.

(b) $R_{TH} = 6R\,||\,18R\,||\,3R = 1.8R$.

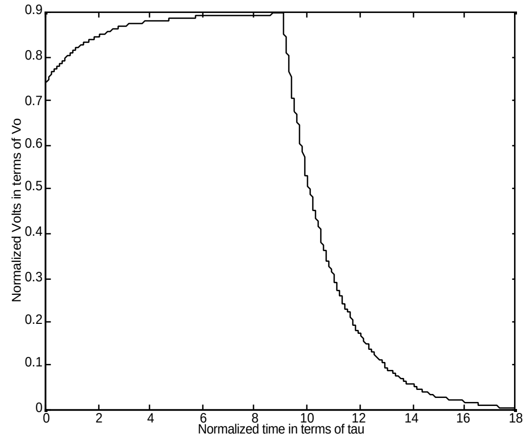
(c) For that period of time the switch is closed, $v_C(t) = 0.9V_o - 0.15V_oe^{-t/1.8RC}$.

(d) Using the previous equation, and by continuity, $v_C(T^-) = v_C(T^+) = 0.9V_o - 0.15V_oe^{-T/1.8RC}$.

(e) The time constant remains the same as the only difference is the source turning off.

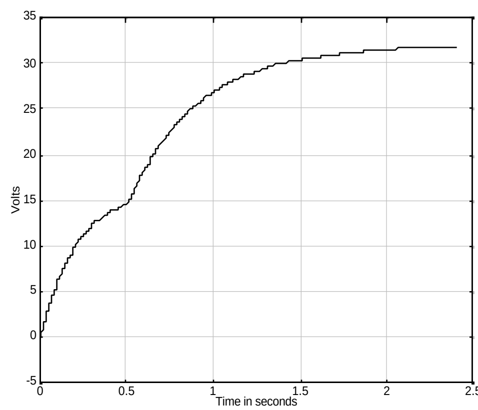
(f) For t > T, $v_C(t) = v_C(T^+)e^{-(t-T)/1.8RC}$.

(g)



SOLUTION 8.35.

(a) For t < 0, The switch is closed, the current source off, and the voltage source has been providing a constant voltage for a prolonged period of time. Thus, $v_C(0^-) = -50uV$.
(b) Since voltage in continuous across a capacitor, $v_C(0^+) = v_C(0^-) = -50uV$.
(c) The thevenin resistance seen by the capacitor is $R_{TH} = 200||200 = 100\Omega$, thus $\tau = R_{TH}C = 0.2s$.
(d) As t goes to infinity the capacitor voltage goes to 16V, thus $v_C(t) = 16 + (-50u - 16)e^{-5t}V$.
(e) Again using the continuous property of a capacitor, $v_C(0.5^+) = v_C(0.5^-) = 14.687V$.
(f) For t > 0.5s, the switch is open, thus $R_{TH} = 200\Omega$. $\tau = R_{TH}C = 0.4s$.
(g) As t goes to infinity, the capacitor voltage goes to 32V. $v_C(t) = 32 + (14.687 - 32)e^{-2.5(t-0.5)}V$.
(h)

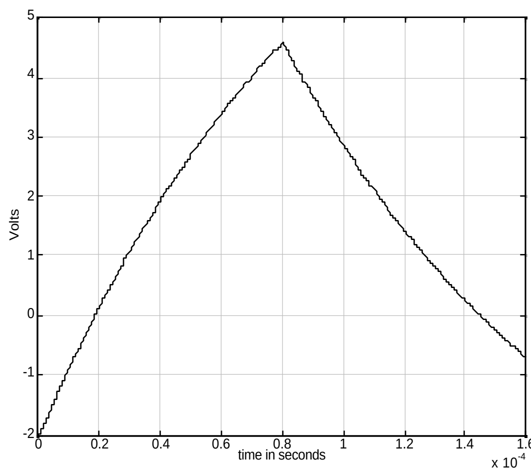


SOLUTION 8.36.

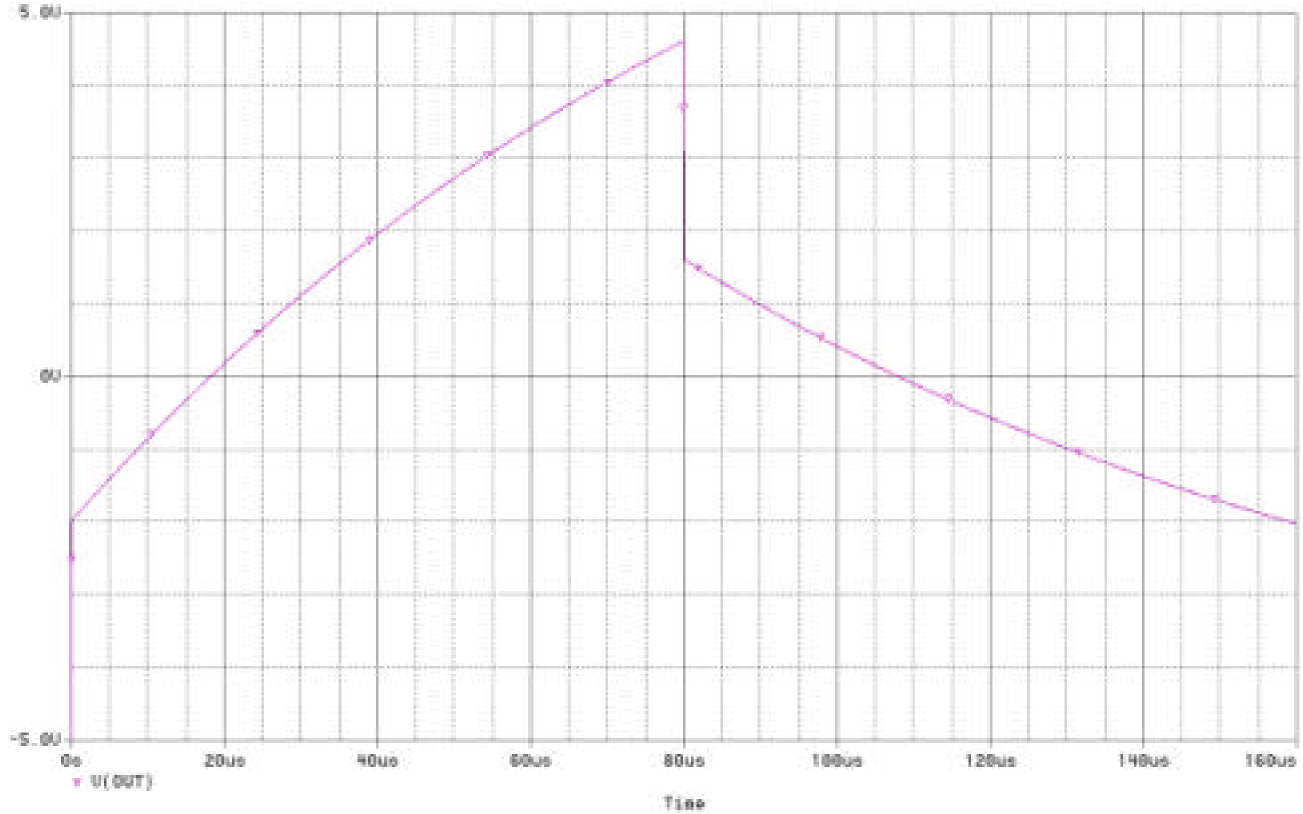
(a) $v_C(0^+) = -5V$
(b) Doing so in matlab.

```
%Problem 8.36b

%Initial condition
vc0=-5;
%From 0<t<80us
RTH=300e3;
tau=RTH*(1/3)*1e-9;
```

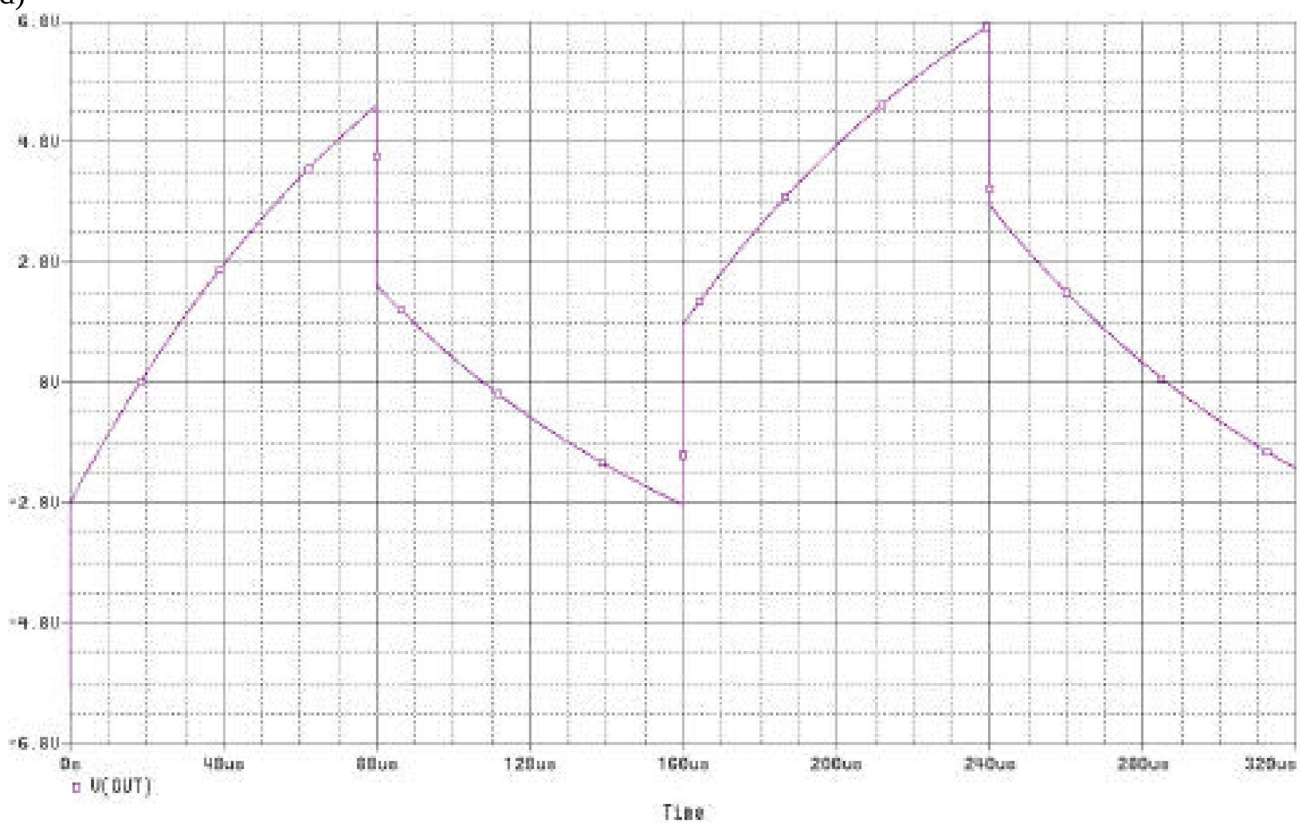
```
%So the initial condition on Vout
vout0=(10+5)/300e3*60e3-5;   %-2V
voutinf1=10;
t=80e-6;
vout80us= 10+(vout0-10)*exp(-t/tau);

%For t > 80us
%tau stays the same
voutinf2=-5;

t=0:160e-6/1000:160e-6;
vout= (ustep(t)-ustep(t-80e-6)).*(10+(vout0-10).*exp(-t./tau)) ...
+ustep(t-80e-6).*(-5+(vout80us+5).*exp(-(t-80e-6)./tau));
plot(t,vout);
xlabel('time in seconds');
ylabel('Volts');
grid;
%Notice that Vout is vin-vct times a constant 60/300 plus vct
%Thus Vout=vin(60/300)+vct(1-60/300)
```



(c)



(d)



SOLUTION 8.37.

If the source voltage has been –10 V for a long time then the switch is open and $v_C(0^+) = -10V$. The time constant with the switch open is $\tau = 5us$. At t > 0, the input voltage changes to 20V. It then follows that $v_C(\infty) = 20V$, and

$$v_C(t) = 20 - 30e^{-t/5u}V.$$

Using the elapsed time formula, 8.24, we wish to find when the switch closes.

$t_a - 0 = 5us\ \ln\left(\frac{-10-20}{0-20}\right) = 2.03us$. At that time the input voltage is still 20V, and the switch closes.

The time constant is now $\tau = 98ns$, and $v_C(\infty) = 0.39V$. Note that because the voltage converges to a value greater than zero, this time interval will be from 2.03us to 5us when the input changes back to –10V, thus

$$v_C(t) = 0.39 - 0.39e^{-t/98n}V.$$

$v_C(5us) = 0.39V$. At t > 5us, the voltage changes to –10 V, so

$$v_C(t) = -0.2 + (0.39 + 0.2)e^{-t/98n}V.$$

Using the elapsed formula, we get $t_b = 0.1us$ for the voltage to go down to 0V again and cause the switch to open again. At this point the time constant becomes the original value again and

$$v_C(t) = -10 + 10e^{-t/5u}V$$

for t > (0.1+5) us.

SOLUTION 8.38.

(a) Introduce a test current source at the output and write KVL,

$i_{test} = v_{test}/200 + (v_{test} - 6V)/200 + (6V - v_{test})/400$. Solving for $i_{test} = v_{test}\left(\frac{3}{400}\right) - 0.015$. This implies the following,

$R_{TH} = 400/3\Omega$

$i_{SC} = 15mA$

$v_{OC} = R_{TH} i_{SC} = 2V$

(b) Using the general form, $v_C(t) = 2 - 8e^{-15000t}$.

SOLUTION 8.39.

(a) Introduce a test voltage and solve for KVL,

$v_{test} = 5k i_{test} + 101(v_{test} + 1 - 40i_{test}) - 1 + 40i_{test}$

$v_{test} = -10i_{test} - 1$

$v_{OC} = -1V$

$R_{TH} = -10\Omega$

(b) The complete response is $v_C(t) = -1 + e^t$ V. Note that the voltage goes to infinity as t goes to infinity because of the negative time constant.

SOLUTION 8.40.

Compute the thevenin equivalent seen by the inductor at t > 0. Using KCL write,

$i_{test} = \alpha\ 100\ i_{test} + (v_{test} - 100i_{test} + 25)/50$. Then one obtains the following,

$i_{test} = v_{test}/125 + 1/5\,A$

$R_{TH} = 125\Omega$

$i_{SC} = -1/5\,A$

$v_{OC} = -25V$

A t < 0, the applied voltage has been –25V for a long time. Using the previously obtained thevenin equivalent and linearity, $i_L(0^-) = 1/5\,A$. $i_L(\infty) = i_{SC} = -1/5\,A$, So $i_L(t) = 0.2 + 0.4e^{-6250t}A$.

SOLUTION 8.41.

(a) Introducing a test source and using KCL,

$$i_{test} = \frac{v_{test} - 1.5v_s}{40} + \frac{v_{test}}{100}$$

$$0.2 = \frac{v_s}{40} + \frac{1.5v_s - v_{test}}{40}$$

Solving for the test source current in terms of the test voltage, $i_{test} = \frac{v_{test}}{50} - 0.12$ A. Thus the thevenin equivalent is,

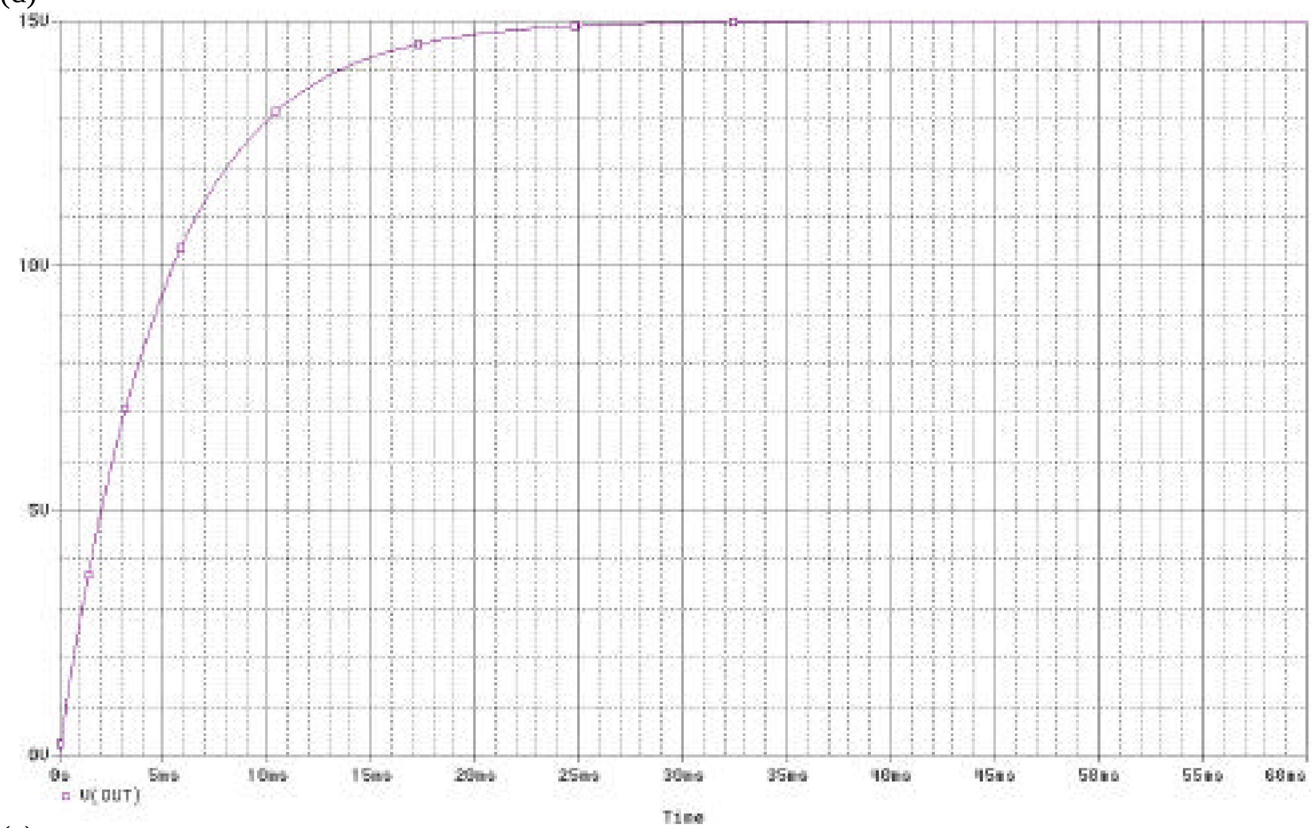
$R_{th} = 50\Omega$

$v_{oc} = i_{sc} * R_{th} = 6V$

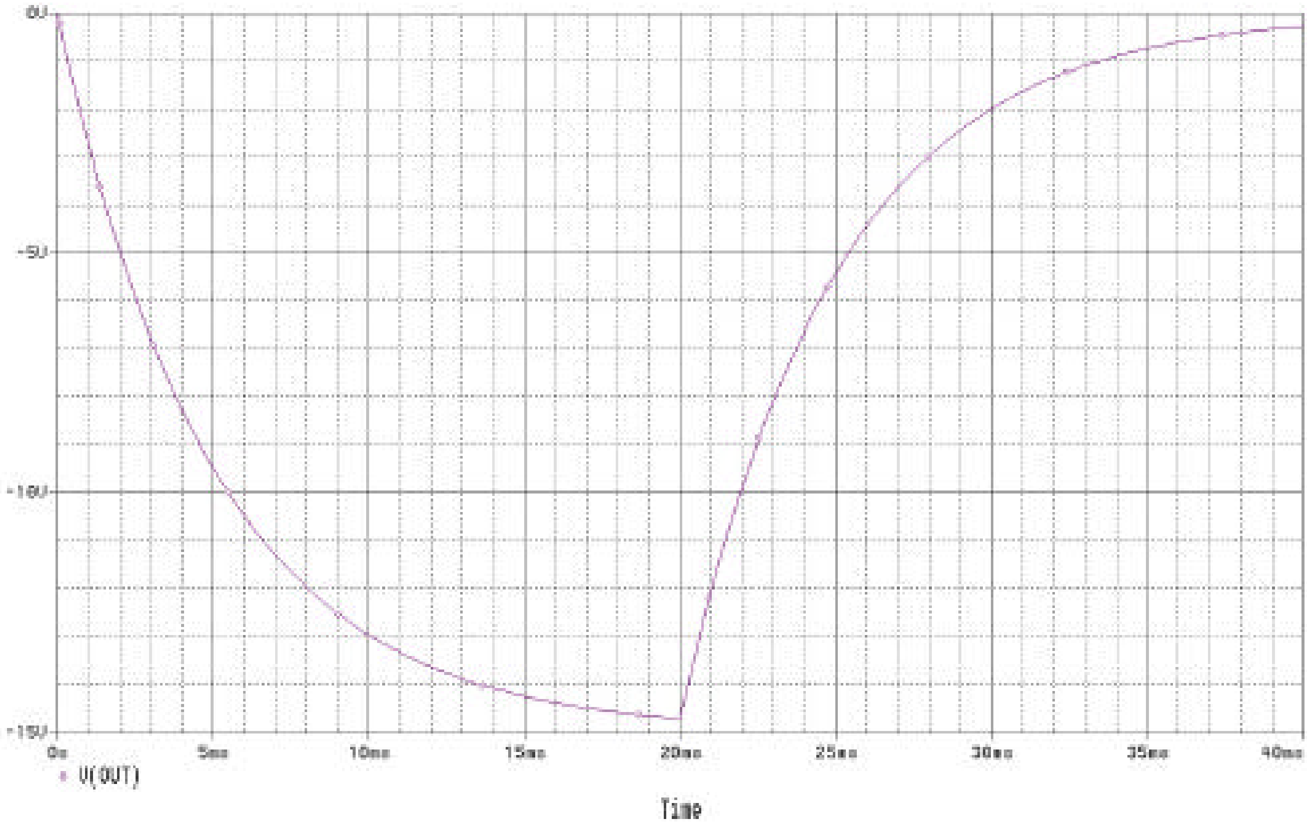
(b) $v_C(t) = 6 - 6e^{-200t}$ V.

(c) From linearity all the currents increase by the same ratio, thus $v_C(t) = 15 - 15e^{-200t}$ V.

(d)



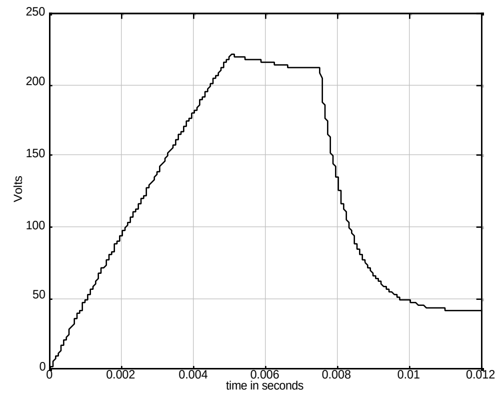
(e)



SOLUTION 8.42.

```
%Problem 8.42
C= 1e-6;
vc0=0;

%For 0 < t < 5ms
Rth= 20e3;
tau1= Rth*C;
vcinf1= 50e-3*Rth;
vc5ms= vcinf1+(vc0-vcinf1)*exp(-5e-3/tau1);

%For 5ms < t < 7.5ms
Rth= 4e3;
tau2= Rth*C;
vcinf2= 50e-3*Rth;
vc75ms= vcinf2+(vc5ms-vcinf2)*exp(-(7.5e-3-5e-3)/tau2);

%For t > 7.5ms
Rth=800;
tau3= Rth*C;
vcinf3= 50e-3*Rth;
t=0:12e-3/1000:12e-3;
vct= (ustep(t)-ustep(t-5e-3)).*(vcinf1+(vc0-vcinf1).*exp(-t/tau1)) ...
+ (ustep(t-5e-3)-ustep(t-7.5e-3)).*(vcinf2+(vc5ms-vcinf2).*exp(-(t-5e-3)/tau2)) ...
+ (ustep(t-7.5e-3)).*(vcinf3+(vc75ms-vcinf3).*exp(-(t-7.5e-3)/tau3));
plot(t,vct);
grid;
```

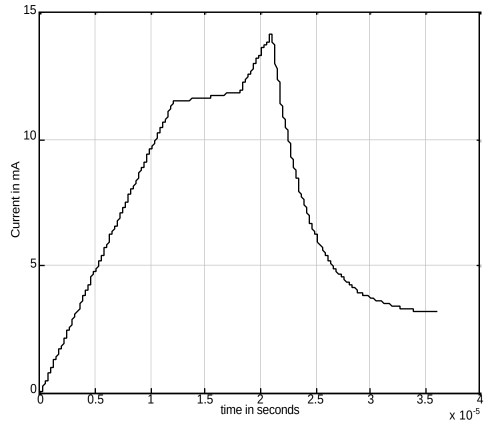
```
xlabel('time in seconds');
ylabel('Volts');
```



SOLUTION 8.43.

(a) From the thevenin resistance $R_{TH} = 800\Omega$, $\tau_1 = 125\mu s$.
(b) From the thevenin resistance $R_{TH} = 8k\Omega$, $\tau_2 = 12.5\mu s$.
(c) From the thevenin resistance $R_{TH} = 1.6k\Omega$, $\tau_3 = 62.5\mu s$.
(d) From the thevenin resistance $R_{TH} = 32k\Omega$, $\tau_4 = 3.125\mu s$.
(e) 0 mA.
(f) In MATLAB:

```
%Problem 8.43f

tau1= 125e-6;
tau2= 12.5e-6;
tau3= 62.5e-6;
tau4= 3.125e-6;
vs= 100;

il0= 0;
ilinf1= vs/800;
ilinf2= vs/8e3;
ilinf3= vs/1.6e3;
ilinf4= vs/32e3;

il12us= ilinf1+(il0-ilinf1)*exp(-12e-6/tau1);
il18us= ilinf2+(il12us-ilinf2)*exp(-(18e-6-12e-6)/tau2);
il21us= ilinf3+(il18us-ilinf3)*exp(-(21e-6-18e-6)/tau3);

t= 0:36e-6/1000:36e-6;
ilt= (ustep(t)-ustep(t-12e-6)).*(ilinf1+(il0-ilinf1).*exp(-t/tau1))+ ...
(ustep(t-12e-6)-ustep(t-18e-6)).*(ilinf2+(il12us-ilinf2).*exp(-(t-12e-6)/tau2))+ ...
(ustep(t-18e-6)-ustep(t-21e-6)).*(ilinf3+(il18us-ilinf3).*exp(-(t-18e-6)/tau3))+ ...
(ustep(t-21e-6)).*(ilinf4+(il21us-ilinf4).*exp(-(t-21e-6)/tau4));

plot(t,1000*ilt);
grid;
xlabel('time in seconds');
ylabel('Current in mA');
```



SOLUTION 8.44.

The first stage is a differentiator, and from 8.25, the output of the first op-amp is $= -RC\dfrac{dv_{in}(t)}{dt} = 0.25RCe^{-0.25t}$. The second stage is an integrator and using 8.26,

$$v_{out}(t) = \frac{-1}{2RC}\int_0^t (0.25RCe^{-0.25\tau}\, d\tau) = 0.5\left[e^{-0.25\tau}\right]_0^t = 0.5e^{-0.25t}$$

SOLUTION 8.45.

(a) First note the following relationships,

$$v_{out}(t) = v_{C2}(t) = \frac{1}{C_2}\int_0^t i_{C2}(\tau)d\tau$$

$$v_{in}(t) = v_{C1}(t)$$

$$i_{C2}(t) = -v_{in}(t)/R - C_1\frac{dv_{C1}(t)}{dt} = -\frac{v_{in}(t)}{R} - C_1\frac{dv_{in}(t)}{dt}$$

Doing the appropriate substitution, and solving,

$$v_{out}(t) = -\frac{1}{C_2R}\int_0^t v_{in}(\tau)d\tau - \frac{C_1}{C_2}v_{in}(t) + \frac{C_1}{C_2}v_{in}(0).$$

(b) $\dfrac{4}{C_2R}\left(e^{-0.25t} - 1\right) + \dfrac{C_1}{C_2}\left(1 - e^{-0.25t}\right)$ V.

(c) $\dfrac{-1}{\omega C_2R}\sin(\omega t) + \dfrac{C_1}{C_2}\left(1 - \cos(\omega t)\right)$ V.

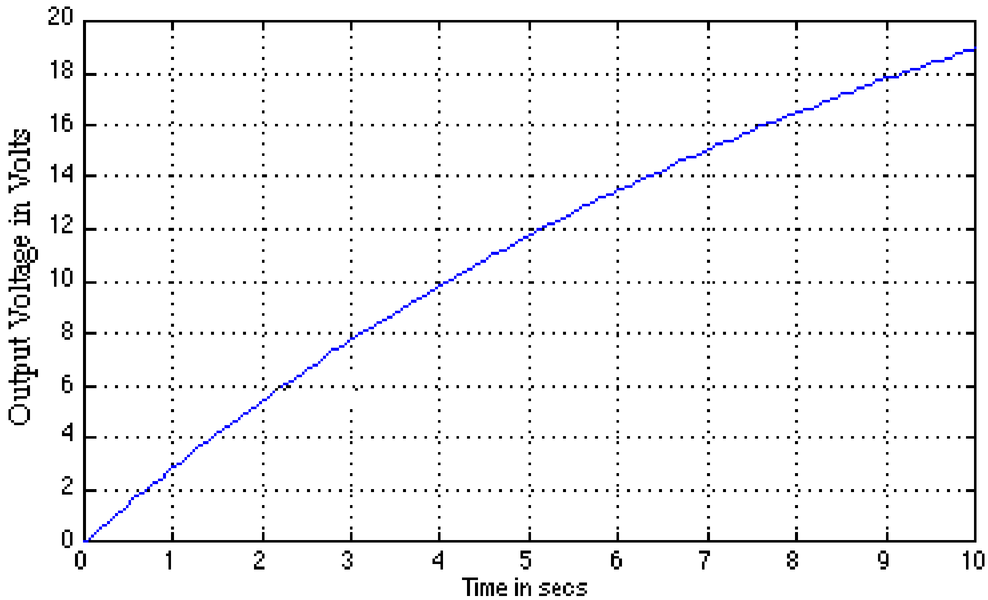
SOLUTION 8.46.

These are two integrator in cascade. Using 8.26, the output of the first stage is $-\dfrac{1}{RC}\int_0^t v_{in}(\tau)d\tau = 2[\cos(50\tau)]_0^t\, mV = 2\cos(50t) - 2$ mV. Using the same equation again,

$$v_{out}(t) = -\frac{1}{RC}\int_0^t (2\cos(50\tau)d\tau = -10\left[-\frac{2}{50}\sin(50\tau)\right]_0^t mV = 2\cos(50t)mV$$

***SOLUTION 8.47. The following solution is done in MATLAB**

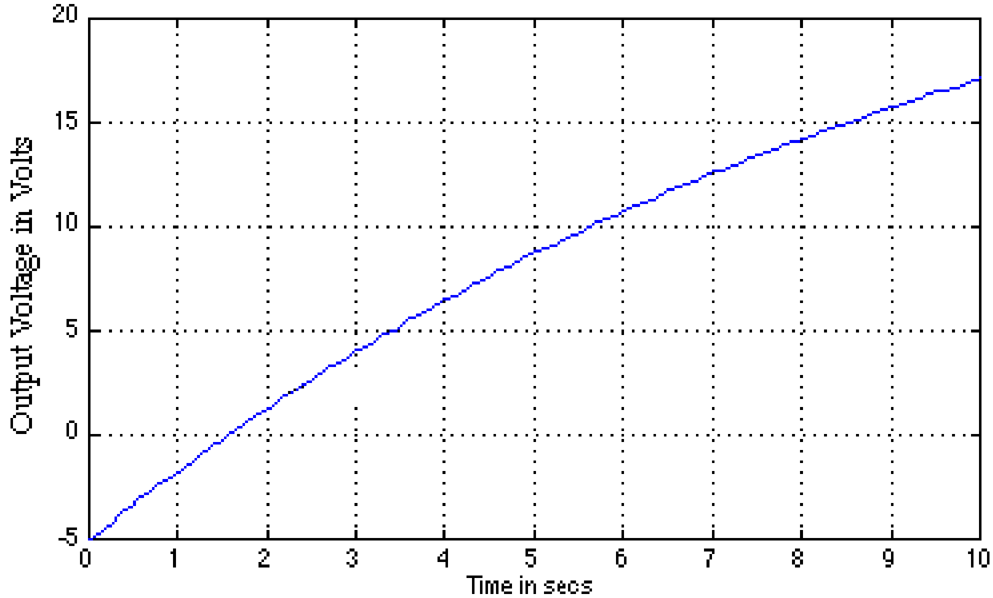
```
c= 1e-6;
rf= 10e6;
rs=1e6;
tau=c*rf;
vgain= -10/1;
vofinal= -3*vgain;

%  Part (a)

voinit= 0;
t= 0: tau/100: tau;
vout= vofinal + (voinit -vofinal).*exp(-t/tau);

%  Time at which output voltage reaches saturation is tsat

tsat= tau*log((0 - vofinal)/(15- vofinal))

plot (t,vout)
grid
xlabel('Time in secs')
ylabel('Output Voltage in volts')


tsat =
  6.9315e+00
```



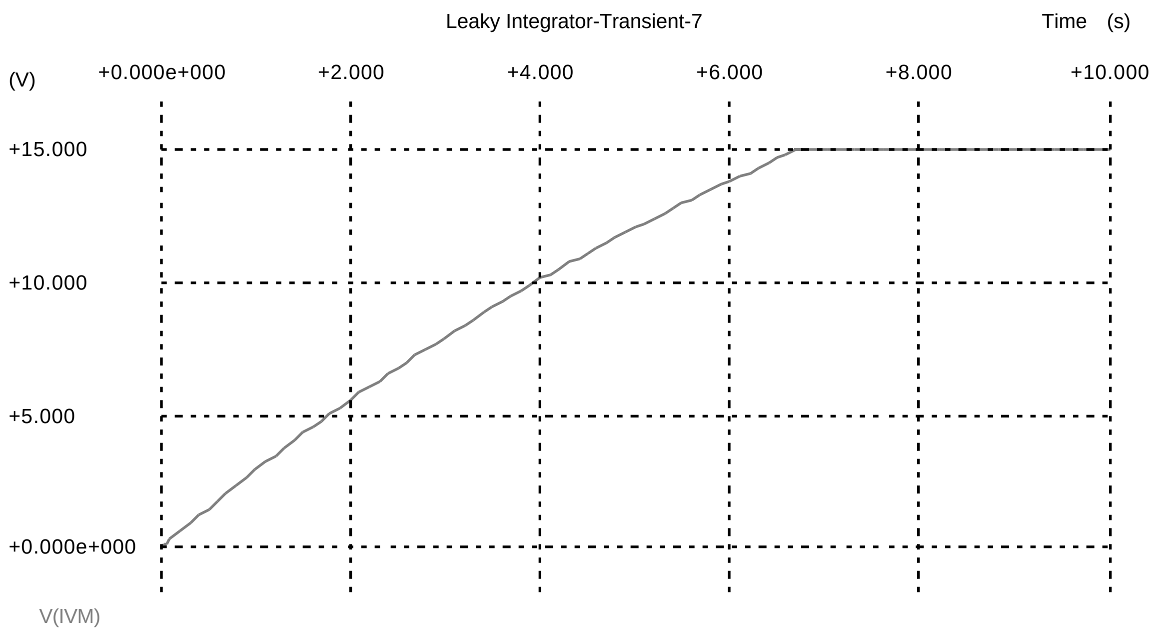

```
%  Part (b)

voinit=-5;
vout= vofinal + (voinit -vofinal).*exp(-t/tau);
tsat= tau*log((-5 - vofinal)/(15- vofinal))
plot (t,vout)
grid
xlabel('Time in secs')
```

ylabel('Output Voltage in volts')

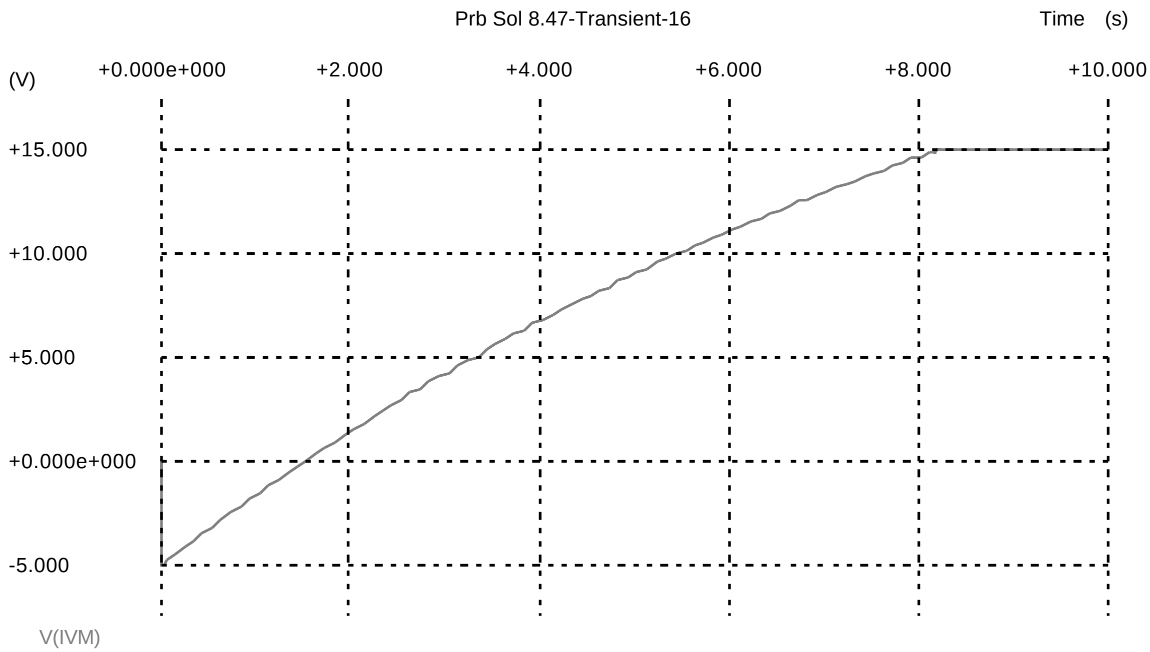
tsat =
   8.4730e+00



(c) $v_C(0^-) = 0$. Observe saturation at about 6.4 seconds.



$v_C(0^-) = -5$ V. Observe that the time of saturation is 8.198 seconds.



Note that in both cases the time of saturation is much lower than in the MATLAB computations which assume an ideal op amp. In the Burr Brown model used by the SPICE simulation of this circuit, the input resistance is 2 MΩ which is comparable with the external input resistance. Hence the assumption of an infinite input resistance is not valid for the SPICE simulation and causes the discrepancy in the time of saturation. However, if the external input resistance is changed to 10 kΩ and the feedback resistance to 100 kΩ with a corresponding change in the capacitor to 100 μF, one obtains results comparable to the MATLAB computations.

SOLUTION 8.48.

Since the op-amps do not load the first stage of the circuit, we can find its transfer function for the op-amp stage as $\dfrac{v_{out}(t)}{v^{+}(t)} = (1+K)$.

(a) From the problem statement, we know that the overall function, is a scaled integrator. As the op-amp stage only provides gain it is logical to assume that the R-C stage will perform the integration of the input times some constant, G. With this in mind we have

$v_{out}(t) = G(1+K)\int_0^t 10\sin(\omega\tau)\,d\tau = \dfrac{-10G(1+K)}{\omega}\left(\cos(\omega t)-1\right)$, where G must be negative.

(b) Using the same reasoning, $v_{out}(t) = G(1+K)\dfrac{d(10\sin(\omega t))}{dt} = 10G(1+K)\omega\cos(\omega t)$ where G is positive.

For low frequency (a) yields a big output, while (b) a small one. For high frequency the reverse happens.

SOLUTION 8.49.

For (b) the integral i-v relationship is

$v(t) = v_{Ceq}(0^+) + \frac{1}{C_{eq}}\int_0^t i(\tau)d\tau = v_{C1}(0^+) + v_{C2}(0^+) + \frac{C_1 + C_2}{C_1 C_2}\int_0^t i(\tau)d\tau$. Repeating the same for (a),

$$v_{C1}(t) = v_{C1}(0^+) + \frac{1}{C_1}\int_0^t i(\tau)d\tau$$

$$v_{C2}(t) = v_{C2}(0^+) + \frac{1}{C_2}\int_0^t i(\tau)d\tau$$

By KVL the two capacitor voltage can be added together, thus give the same relationship as for (b).

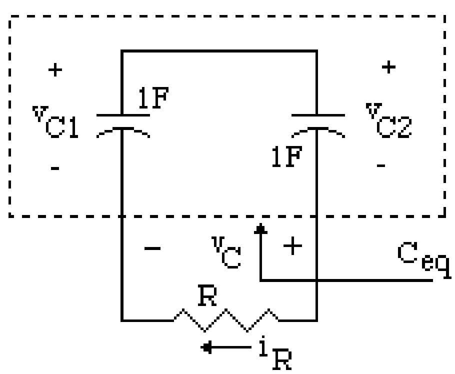
SOLUTION 8.50.

First calculate $C_{eq} = C_1 \| C_2 = 0.2F$. Then find the initial voltage

$v_{Ceq}(0^+) = v_{C1}(0^+) + v_{C2}(0^+) = 30V$. The final voltage will be 12V, and the time constant is

$\tau = R_{eq}\ C_{eq} = 0.4s$. Thus $v_{out}(t) = 12 + (30 - 12)e^{-2.5t}V$.

***SOLUTION 8.51**. (a) After the switch closes, we have the circuit shown below.



From Chapter 7, Ceq = 0.5 F and $v_C(0^+) = v_{C1}(0^+) - v_{C2}(0^+) = 2 - 0 = 2$ V. Hence,

$$i_R(t) = i_R(0^+)e^{-t/\tau} = i_R(0^+)e^{-t/RC_{eq}} = \frac{v_C(0^+)}{R}e^{-t/RC_{eq}} = 4e^{-4t}u(t)\ \text{A}$$

(b) For this part we apply the integral definition of the capacitor. Specifically,

$$v_{C1}(t) = v_{C1}(0^+) + \frac{1}{C_1}\int_0^t i_{C1}(\tau)\, d\tau = 2 - \int_0^t i_R(\tau)\, d\tau = 2 - 4\int_0^t e^{-4\tau}\, d\tau = 1 + e^{-4t} \text{ V}$$

and

$$v_{C2}(t) = v_{C2}(0^+) + \frac{1}{C_2}\int_0^t i_{C2}(\tau)\, d\tau = 0 + \int_0^t i_R(\tau)\, d\tau = 4\int_0^t e^{-4\tau}\, d\tau = 1 - e^{-4t} \text{ V}$$

(c) The energy stored at t = 0^+ for each capacitor is:

$$W_{C1}(0^+) = 0.5 C_1 v_{C1}^2(0^+) = 2 \text{ J}$$

and

$$W_{C2}(0^+) = 0.5 C_2 v_{C2}^2(0^+) = 0 \text{ J}$$

Further at t = ∞,

$$W_{C1}(\infty) = 0.5 C_1 v_{C1}^2(\infty) = 0.5 \text{ J}$$

and

$$W_{C2}(\infty) = 0.5 C_2 v_{C2}^2(\infty) = 0.5 \text{ J}$$

Computing total instantaneous stored energies, we have

$$W_{Ctot}(0^+) = 2 \text{ J and } W_{Ctot}(\infty) = 1 \text{ J}$$

Hence the decrease in stored energy from 0^+ to ∞ is 1 J.

(d) Computing the energy dissipated in the resistor over $[0^+, \infty)$ is

$$W_R(0,\infty) = R\int_0^\infty i_R^2(\tau)\, d\tau = R\left(\frac{v_C^2(0^+)}{R^2}\right)\int_0^\infty e^{-2t/RC_{eq}}\, d\tau = \frac{v_C^2(0^+)}{R} \times \frac{RC_{eq}}{-2} \times e^{-2t/RC_{eq}}\Big]_0^\infty$$

$$= \frac{C_{eq} v_C^2(0^+)}{2} = 1 \text{ J}$$

(e) From the expressions developed in part (d), the dissipated energy is independent of the value of R. R only affects the rate at which energy is dissipated. Clearly, the energy stored at 0 is 2 J while the energy dissipated over [0,∞) is 1 J and the remaining energy at t = ∞ is 1 J. Hence conservation of energy is verified.

SOLUTION 8.52. (a) Using the relations developed in P8.49,

$C_{eq} = 0.2F$

$v_{Ceq} = 1 - 0.5 = 0.5V$

$$i_R(t) = i_R(0^+)e^{-t/\tau} = i_R(0^+)e^{-t/RC_{eq}} = \frac{v_{Ceq}(0^+)}{R}e^{-t/RC_{eq}} = e^{-10t}u(t)$$

(b) For this part we apply the integral definition of the capacitor. Specifically,

$$v_{C1}(t) = v_{C1}(0^+) + \frac{1}{C_1}\int_0^t i_{C1}(\tau)\,d\tau = 1 - \int_0^t i_R(\tau)\,d\tau = 1 - \int_0^t e^{-10\tau}\,d\tau = \frac{1}{10}(9 + e^{-10t})\ \text{V}$$

and

$$v_{C2}(t) = v_{C2}(0^+) + \frac{1}{C_2}\int_0^t i_{C2}(\tau)\,d\tau = 0.5 + 4\int_0^t i_R(\tau)\,d\tau = 0.5 + 4\int_0^t e^{-10\tau}\,d\tau = \frac{1}{10}(9 - 4e^{-10t})\ \text{V}$$

(c) The energy stored at t = 0^+ for each capacitor is:

$$W_{C1}(0^+) = 0.5C_1v_{C1}^2(0^+) = 0.5\ \text{J}$$

and

$$W_{C2}(0^+) = 0.5C_2v_{C2}^2(0^+) = 31.25mJ$$

Further at t = ∞,

$$W_{C1}(\infty) = 0.5C_1v_{C1}^2(\infty) = 405mJ$$

and

$$W_{C2}(\infty) = 0.5C_2v_{C2}^2(\infty) = 101.25mJ$$

Computing total instantaneous stored energies, we have

$$W_{Ctot}(0^+) = 531.25mJ \quad \text{and} \quad W_{Ctot}(\infty) = 506.25mJ$$

Hence the decrease in stored energy from 0^+ to ∞ is 25 mJ.

(d) Computing the energy dissipated in the resistor over $[0^+, \infty)$ is

$$W_R(0,\infty) = R\int_0^\infty i_R^2(\tau)\,d\tau = R\left(\frac{v_{Ceq}^2(0^+)}{R^2}\right)\int_0^\infty e^{-2t/RC_{eq}}\,d\tau = \frac{v_{Ceq}^2(0^+)}{R}\times\frac{RC_{eq}}{-2}\times e^{-2t/RC_{eq}}\Big]_0^\infty$$

$$= \frac{C_{eq}v_{Ceq}^2(0^+)}{2} = 25\,mJ$$

(e) From the expressions developed in part (d), the dissipated energy is independent of the value of R. R only affects the rate at which energy is dissipated. Clearly, the energy stored at 0 is 531.25 mJ while the energy dissipated over [0,∞) is 506.25 mJ and the remaining energy at t = ∞ is 25 mJ. Hence conservation of energy is verified.

SOLUTION 8.53. As all the switches are open initially, the initial current through the inductors is 0A. For 0 < t < 50 ms, $i_L(t) = 54.54 - 54.54e^{-20000t}$ mV. At t > 50 ms, the equivalent inductance is 10 mH, the initial current through the 110 mH inductance is 54.54 mA, and through the 11 mH inductance 0 A. So assuming the current splits equally between the two branches in steady state,

$i_{L1} = 27.27 + (54.54 - 27.27)e^{-220000t}$

$i_{L2} = 27.27 - 27.27e^{-220000t}$

SOLUTION 8.54. (a) Charges will distribute in order to achieve equal voltage by KVL. Since q=CV, $v_R(0^-) = 0V$, due to equal capacitance the charges will distribute half and half, $v_R(0^+) = 0.5V$.

(b) The equivalent capacitance is 2 F, thus $v_R(t) = 0.5e^{-0.5t}$.

***SOLUTION 8.55**. (a) Writing a node equation at v we have for all t,

$$4\frac{dv}{dt} + \frac{v}{4} + 4\frac{d}{dt}\left(v - v_s\right) + \frac{\left(v - v_s\right)}{2} = 0 \qquad (*)$$

Equivalently,

$$8\frac{dv}{dt} = -\frac{3v}{4} + 4\frac{dv_s}{dt} + \frac{v_s}{2} \qquad (**)$$

Grouping terms and dividing by 8 yields when t > 0,

$$\frac{dv}{dt} = -\frac{3}{32}v + \frac{1}{16} \qquad (***)$$

Notice that v_s = 1 for t > 0 and for t > 0, $\frac{dv_s(t)}{dt} = \frac{du(t)}{dt} = 0$.

(b) By inspection $v(0^-) = 0$; both capacitors are uncharged at 0^-. Recall from part (a) that KCL at the node for v yields (*) which is equivalent to (**). Since conservation of charge follows by integrating (*) or equivalently integrating (**) we have

$$8\int_{0^-}^{0^+} \frac{dv}{d\tau}d\tau = -\frac{3}{4}\int_{0^-}^{0^+} v\,d\tau + 4\int_{0^-}^{0^+} \frac{dv_s}{d\tau}d\tau + 0.5\int_{0^-}^{0^+} v_s\,d\tau$$

Since the integral of a finite integrand over an infinitesimal interval is zero, we have equivalently,

$$8\int_{0^-}^{0^+} \frac{dv}{d\tau}d\tau = 0 + 4\int_{0^-}^{0^+} \frac{dv_s}{d\tau}d\tau + 0$$

Evaluating these integrals we obtain

$$8\left(v(0^+) - v(0^-)\right) = 4\left(v_s(0^+) - v_s(0^-)\right) = 4u(0^+) = 4$$

(c) Since $v(0^-) = 0$, $v(0^+) = 0.5$ V. Since v satisfies (***), i.e.,

$$\frac{dv}{dt} = -\frac{3}{32}v + \frac{1}{16} \equiv -\frac{1}{\tau}v + F \quad (***)$$

from equation 8.17,

$$v(t) = F\tau + \left[v(0^+) - F\tau\right]e^{-t/\tau} = \left(\frac{2}{3} - \frac{1}{6}e^{-3t/32}\right)u(t) \text{ V}$$

Using $v(0^-)$ would have led to an incorrect answer.

SOLUTION 8.56. (a)

$$x(t) = K_1 e^{-t/\tau} + K_2$$

$$\frac{dx(t)}{dt} = -\frac{K_1}{\tau}e^{-t/\tau}$$

doing the substitution, $-\frac{K_1}{\tau}e^{-t/\tau} = -\frac{1}{\tau}(K_1 e^{-t/\tau} + K_2) + F$. In order to satisfy the equality, $K_2 = F\tau$.

(b) $x(t_0^+) = K_1 e^{-t_0/\tau} + F\tau$, and $K_1 = \left[x(t_0^+) - F\tau\right]e^{t_0/\tau}$.

(c) $x(t) = K_1 e^{-t/\tau} + K_2 = \left[x(t_0^+) - F\tau\right]e^{-(t-t_0)/\tau} + F\tau$.

SOLUTION 8.57. (a) From the graph, the initial and final values are 0 and 80 V respectively. That sets the following constraint, $100\dfrac{R_2}{R_1 + R_2} = 80V$. From $v_C(\tau) = 80 - 80e^{-1} = 50.57V$. Looking at the Graph $\tau = 5ms$. Thus $R_1 \| R_2 \ \ C = 5ms$. Solving, $R_1 = 6250\Omega$ and $R_2 = 25k\Omega$.

(b) Using the same equalities, $R_1 = 0.25R_2 = 2k\Omega$, and $C = \dfrac{\tau}{2k \| 8K} = 3.125uF$.

SOLUTION 8.58. (a) From the graph, the initial and final values are 0 and 100 mA respectively. Thus $R_1 = 200/100m = 2k\Omega$. From $i_L(\tau) = 100 - 100e^{-1}mA = 63.21mA$, the graph shows a $\tau = 20ms$. Thus $L/(R_1 \| R_2) = 20ms$, and $R_2 = 0.25m\Omega$.

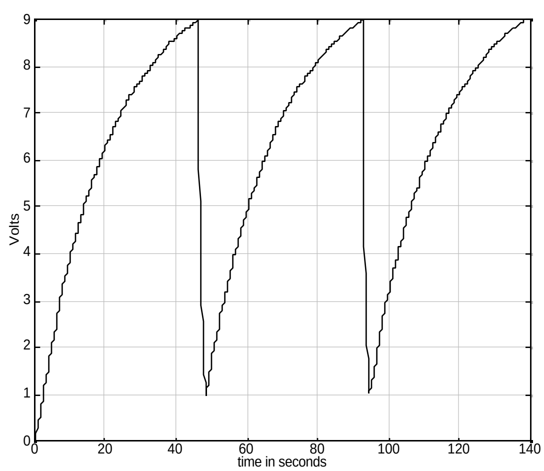
(b) R1 stays the same, $L = 20ms(R_1 \| R_2) = 20H$.

SOLUTION 8.59. This question is done in matlab

```
%Problem 8.59

tau1= 20*1;
tau2= 1;

%For 0 < t < ta
vo= 0;
vinfa= 10;
%Using the elapsed time formula,
ta=tau1*log((0-10)/(9-10));

%For ta < t < tb
vinfb=0;
tb=tau2*log((9-0)/(1-0));

%For tb < t < tc
vinfc=vinfa;
tc=tau1*log((1-10)/(9-10));
```

```
%Next switching is just a repeat of ta < t < tb

t1=ta;
t2=ta+tb;
t3=t2+tc;
t4=t3+tb;
t5=t4+tc;

t=0:t5/1000:t5-1/1000;
vt= (ustep(t)-ustep(t-t1)).*(10-10.*exp(-t/tau1))+ ...
(ustep(t-t1)-ustep(t-t2)).*(9.*exp(-(t-t1)/tau2))+ ...
(ustep(t-t2)-ustep(t-t3)).*(10-9.*exp(-(t-t2)/tau1))+ ...
(ustep(t-t3)-ustep(t-t4)).*(9.*exp(-(t-t3)/tau2))+ ...
(ustep(t-t4)-ustep(t-t5)).*(10-9.*exp(-(t-t4)/tau1));

Frequency= 1/(tb+tc)

plot(t,vt);
grid;
xlabel('time in seconds');
ylabel('Volts');
```

(b) The frequency is 0.0217 Hz.



SOLUTION 8.60. When the switch is in position A, $\tau = 18.18 ms$. In position B it is $\tau = 99.5 us$. Using the elapsed time formula, find ta, when the output voltage reaches 90 V.

$t_a = 18.18ms\ln\left(\frac{60-136.36}{90-136.36}\right) = 9.07ms$. At this point the switch goes to B, and the elapsed time until the voltage reaches 60 V is $t_b = 99.5us\ln\left(\frac{90-59.45}{60-59.45}\right) = 0.4ms$. Adding both time, $F = 105.6Hz$.

SOLUTION 8.61. (a) The circuit can be rearranged in a series of one V_{solar} V voltage source, one L_{store} inductor, and one R_{store}+R_{solar} resistor.

(b) $i_L(t) = \frac{V_{solar}}{R_{solar} + R_{store}}\left[1 - e^{-(Rstore+Rsolar)t/L}\right]A$.

(c) In this time period the circuit reduces to an L_{sotre} inductor in series with a R_{store}+R_1 resistor.

(d)

$$i_L(T_1^-) = \frac{V_{solar}}{R_{solar} + R_{store}}$$
$$i_L(t) = i_{T_1^-}e^{-(Rstore+R1)(t-T1)/L}A$$

(e) The two elements in series are an L_{store} inductor and a resistor $R_{eq} = R_{store} + (R_1 \| R_2)$.

(f)

$$i_L(T_2^-) = i_{T_1^-}e^{-(Rstore+R1)(T_2-T1)/L_{sotre}}A$$
$$i_L(t) = i_{T_2^-}e^{-(\mathrm{Re}\,q)(t-T2)/Lstore}A$$

(g)

$$P_{Vsolar} = V_{solar}i_L(t) = \frac{V^2{}_{solar}}{R_{solar} + R_{store}}\left[1 - e^{-(Rstore+Rsolar)t/L}\right]W$$
$$P_{Rsolar} = R_{solar}i_L^2(t)W$$
$$P_{Rstore} = R_{store}i_L^2(t)W$$
$$P_{Lstore} = L_{store}i_L(t)\frac{di_L(t)}{dt}W$$

(h) $W_L(0,t) = \frac{1}{2}Li_L^2(t)J$

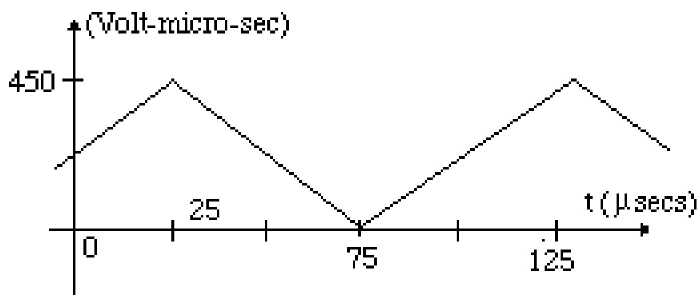
SOLUTION 8.62. The light turns off when the current through it goes down to 0.5 mA. This corresponds to $i_b = 10uA$, and a voltage across the capacitor of $v_C = i_b(R_1 + 2k) + 0.5 = 0.7V$. The time constant of this circuit is $\tau = (R_1 + 2k) \| 5k \;\; 1000uF = 4s$. The final voltage across the capacitor is by voltage division, 0.1V. Thus using the elapsed time formula $t_1 = 4\ln\left(\frac{1.5-0.1}{0.7-0.1}\right) = 3.39s$.

SOLUTION 8.63. **(a)** Since RC = 10^{-3} s, from equation 8.25, $v_a(t) = -RC\dfrac{dv_s(t)}{dt} = \cos(1000t)$ V.

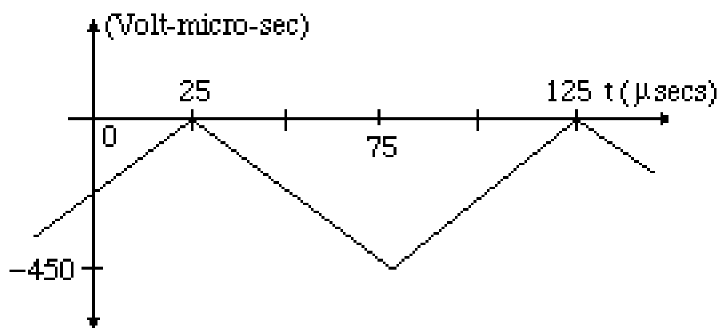
Hence $v_b(t) = -RC\dfrac{dv_a(t)}{dt} == -\sin(1000t)$ V, and $v_{out}(t) = -\dfrac{R_f}{R}v_b(t) = \sin(1000t) = v_s(t)$ V.

(b) With the switch moved to position B, there is no source in the circuit. But the output at the switching instant is sin(1000t) V which coincides with vs(t). Hence, the input to the first amp remains the same and the circuit continues to produce $v_{out}(t) = \sin(1000t)$ V, i.e., the circuit becomes an oscillator.

***SOLUTION 8.64**. Before attacking the problem proper, consider driving an ideal unity gain integrator with the square wave of figure P8.64b. If we start the integration when the square wave goes positive, then we have a triangular waveform as follows:



On the other hand, if we start the integration when the square wave goes negative, we get the following waveform



One concludes that without some further physical assumptions, there is no unique solution to this problem.

Physically speaking all capacitors have a leakage resistance. Hence, in modeling the capacitor we put a very large resistance in parallel with an ideal C, producing a nearly ideal leaky integrator circuit. The leaky integrator circuit has a first order response. Hence over time, when the circuit reaches steady state, the dc level of the resulting output waveform will be proportional to the average level of the square wave which is zero in this case.

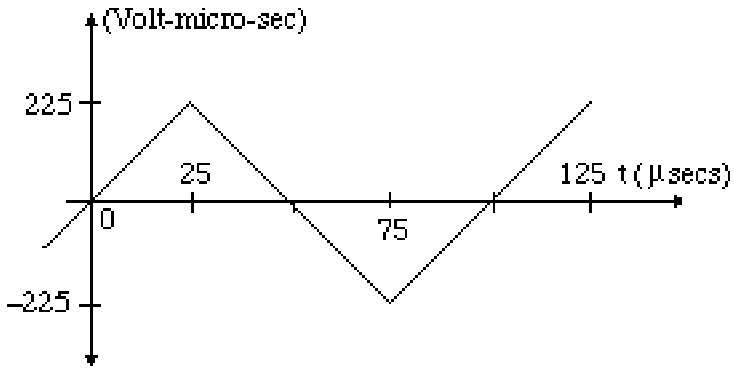
Comments: Actually the proportionality constant in the above statement is the overall dc gain of the integrator-inverter. See the formulas given in P22.16. Adding up the two formulas, we have

$$\text{Output}(t)_{max} + \text{Output}(t)_{min} = K\,(V_{max} + V_{min})$$

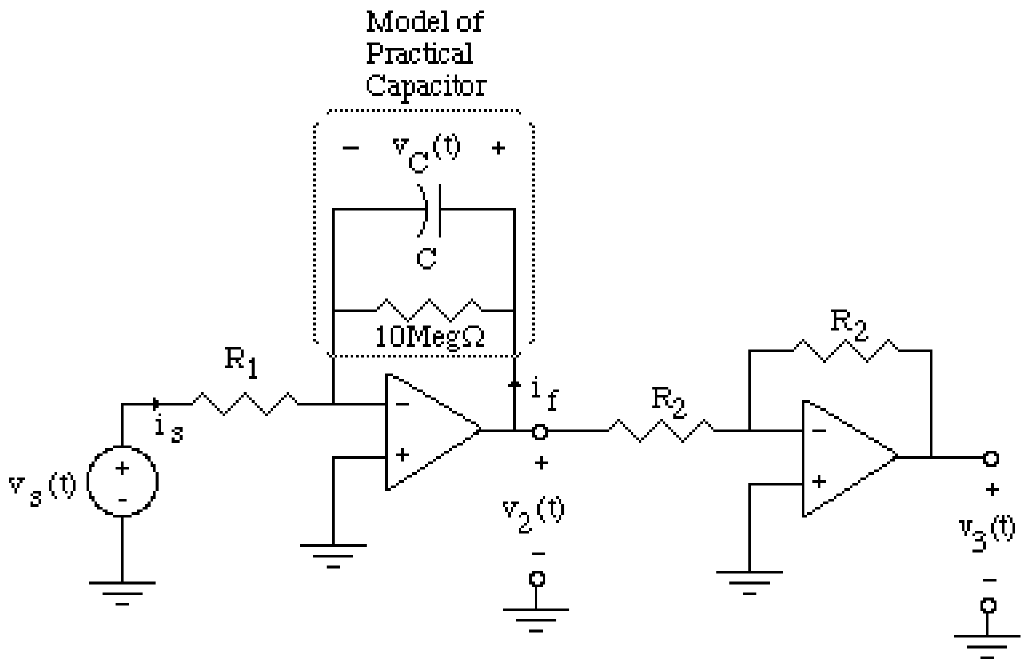
and K = H(0), i.e., the dc gain of the first order low pass system. This leads to

$$\text{Average of output} = (\text{dc gain})\ (\text{average of input levels})$$

See the analysis in example 8.7 and later an exact analysis is given in problems 22.15 and 22.16. In other words, one would expect that the output of our (leaky) integrator in steady state to be given by the waveform below.



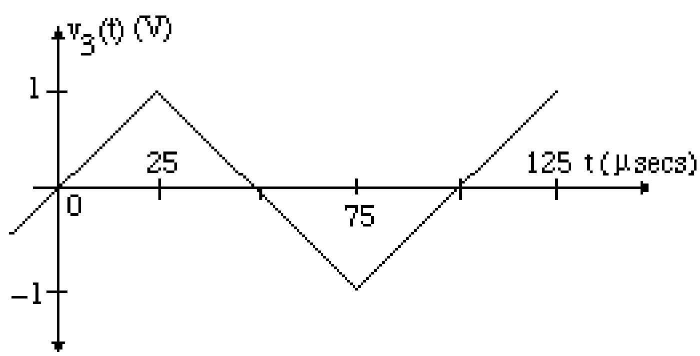
Now we can start to solve the problem. The first part is to design a (leaky) integrator circuit to produce a triangular waveform of value 2 V peak-to-peak. For this we consider the following figure which consists of the leaky integrator followed by an inverter.



To handle this analysis, recall that i Δq/Δt in which case Δq = CΔv. Hence, to have a peak-to-peak voltage at $v_2(t)$ and $v_3(t)$ of 2 V, we require that

$$\Delta v = \frac{i_{in}\Delta t}{C} = \frac{9}{R_1} \times \frac{50 \times 10^{-6}}{C} = 2$$

Hence $R_1C = 2.25\times10\text{-}4$. If we choose R_1 = 10 kΩ, then C = 22.5 nF. At this point the waveform of $v_3(t)$ is given below.



In order to complete the design, we must raise the portion of the curve with positive slope by 1 V and lower the portion with negative slope by –1 V. This can be done by adding one-ninth of $v_{in}(t)$ to $v_3(t)$. This can be done by using the following circuit. In this circuit, there is a voltage-divider at the non-inverting terminal of the second op amp. Here V_+ equals one-eighteenth of $v_{in}(t)$. However the gain of the non-inverting portion is 2; therefore one-ninth of the input is added to $v_3(t)$ as desired.