CHAPTER 9 PROBLEM SOLUTIONS

SOLUTION TO PROBLEM 9.1. If we can compute expressions for K and q that are real, then these quantities exist by construction. Consider that A, B, K and q must satisfy the following relationship:

$$K\cos(\omega t+\theta)=\big(K\cos(\theta)\big)\cos(\omega t)+\big(-K\sin(\theta)\big)\sin(\omega t)\equiv A\cos(\omega t)+B\sin(\omega t)$$

Therefore $K\cos(\theta)=A$ and $-K\sin(\theta)=B$. Consequently,

$$\big(K\cos(\theta)\big)^2+\big(-K\sin(\theta)\big)^2=K^2=A^2+B^2$$

in which case $K=\sqrt{A^2+B^2}$. Further,

$$\frac{K\sin(\theta)}{K\cos(\theta)}=\tan(\theta)=\frac{-B}{A}$$

in which case

$$\theta=\tan^{-1}\left(\frac{-B}{A}\right)$$

with due regard to quadrant.

SOLUTION TO PROBLEM 9.2. For the inductor,

$$W_L(t)=\tfrac{1}{2}L\ \ i_L^2(t)=\frac{1}{2}L\left[V_o\sqrt{\frac{C}{L}}\sin\left(\frac{1}{\sqrt{LC}}t\right)\right]^2=\frac{CV_0^2}{2}\sin^2\left(\frac{1}{\sqrt{LC}}t\right)$$

and for the capacitor,

$$W_C(t)=\tfrac{1}{2}C\ \ v_C^2(t)=\tfrac{1}{2}C\left[V_o\cos\left(\frac{1}{\sqrt{LC}}t\right)\right]^2=\frac{CV_0^2}{2}\cos^2\left(\frac{1}{\sqrt{LC}}t\right).$$

Hence,

$$W_C+W_L=\tfrac{1}{2}C\ \ v_C^2(t)+\tfrac{1}{2}L\ \ i_L^2(t)=\frac{CV_0^2}{2}\left[\sin^2\left(\frac{1}{\sqrt{LC}}t\right)+\cos^2\left(\frac{1}{\sqrt{LC}}t\right)\right]=\frac{CV_0^2}{2}$$

SOLUTION TO PROBLEM 9.3. Since $x(t)=(K_1+K_2t)e^{-\alpha t}$,

$$x'(t)=-\alpha\ K_1e^{-\alpha t}+K_2e^{-\alpha t}-\alpha\ t\ \ K_2e^{-\alpha t}$$

and

$$x''(t)=\alpha^2\ \ K_1e^{-\alpha t}-\alpha\ K_2e^{-\alpha t}-\alpha\ K_2e^{-\alpha t}+\alpha^2\ \ t\ \ K_2e^{-\alpha t}$$

Substituting into the differential equation, we have

$$\begin{aligned}&\alpha^2\ \ K_1e^{-\alpha t}-2\alpha\ \ K_2e^{-\alpha t}+\alpha^2\ \ t\ \ K_2e^{-\alpha t}\\&\quad+2\alpha\left[-\alpha\ K_1e^{-\alpha t}+K_2e^{-\alpha t}-\alpha\ t\ \ K_2e^{-\alpha t}\right]+\alpha^2\left[K_1e^{-\alpha t}+K_2te^{-\alpha t}\right]=0\end{aligned}$$

This means that the solution form satisfies the differential equation.

SOLUTION TO PROBLEM 9.4.

(a) Suppose $x(T) = 0$ at some T. Then $K_1 e^{s_1 T} = -K_2 e^{s_2 T}$. Since $e^{s_i T} > 0$ whenever s_i is real and T is finite, K_1 & K_2 must have opposite signs.

(b) For this we solve for T and show there can only be one solution. Since $K_1 e^{s_1 T} = -K_2 e^{s_2 T}$ and $e^{s_i T} > 0$,

$$\frac{K_1}{-K_2} = \frac{e^{s_2 T}}{e^{s_1 T}} \quad \text{implies} \quad \ell n\left(\frac{K_1}{-K_2}\right) = \ell n\left(\frac{e^{s_2 T}}{e^{s_1 T}}\right) = (s_2 - s_1)T$$

Hence the unique solution is given by

$$T = \ell n\left(\frac{K_1}{-K_2}\right) \Big/ (s_2 - s_1)$$

provided $s_2 \neq s_1$ which is the case for distinct roots.

SOLUTION TO PROBLEM 9.5. Suppose $x(T) = 0$ at some T >0. This is true if and only if

$$K_1 e^{s_1 T} = -K_2 T e^{s_1 T} \qquad (*)$$

Since $e^{s_1 T} > 0$ and T > 0, (*) is true if and only if $K_1 = -K_2 T$ which is true if and only if K_1 & K_2 have opposite signs.

SOLUTION 9.6. **(a)** Denote one period of oscillation by T. Then by definition $9950T = 2\pi$. Hence, $T =$ 0.63148 ms. The time constant of decay is 1 ms. Therefore, $NT = N \times 0.63148 =$. Hence $N = 1.5836$ cycles.

(b) Here observe that the time constant of decay is $1/\sigma$ s. Hence $NT = N\frac{2\pi}{\omega_d} = \frac{1}{\sigma}$. One concludes that $N = \frac{\omega_d}{2\pi\sigma}$.

SOLUTION 9.7. The differential equation for the capacitor voltage is

$$\frac{d^2 v_C(t)}{dt^2} + \frac{1}{LC} v_C(t) = \frac{1}{LC} V_s$$

For t > 0, the characteristic equation is $s^2 + \frac{1}{LC} = 0$. Hence from table 9.2, the solution for either the inductor current or capacitor voltage has the general form

$$v_C(t) = A\cos(\omega_d t) + B\sin(\omega_d t) + X_F$$

where $\omega_d = \frac{1}{\sqrt{LC}}$. From table 9.2, $X_F = \left(\frac{V_s}{LC}\right)\Big/\left(\frac{1}{LC}\right) = V_s$. Further,

$$v_C(0^+) = A + X_F = A + V_s = 0$$

Hence $A = -V_s$. Also,

$$v_C'(0^+) = \omega_d B = \frac{1}{\sqrt{LC}} B = \frac{i_L(0^+)}{C} = 0$$

Hence $B = 0$. Therefore

$$v_C(t) = -V_s\cos(\omega_d t) + V_s = V_s\left(1 - \cos\left(\frac{t}{\sqrt{LC}}\right)\right) \text{ V}$$

To obtain the expression for iL(t) (= iC(t)), we can either repeat the above derivation or differentiate and multiply by C. We choose the latter. Therefore

$$i_L(t) = \frac{CV_s}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right) = \frac{V_s}{\sqrt{\frac{L}{C}}}\sin\left(\frac{t}{\sqrt{LC}}\right) \text{ A}$$

SOLUTION TO PROBLEM 9.8. Essentially this is example 9.7, case 1, with literals and R = ∞. Clearly the circuit is a driven parallel LC circuit having characteristic equation

$$s^2 + \frac{1}{LC} = \left(s + \frac{j}{\sqrt{LC}}\right)\left(s - \frac{j}{\sqrt{LC}}\right) = (s + j\omega_d)(s - j\omega_d) = 0$$

Thus we obtain

$$i_L(t) = A\cos(\omega_d t) + B\sin(\omega_d t) + X_F = A\cos\left(\frac{1}{\sqrt{LC}}t\right) + B\sin\left(\frac{1}{\sqrt{LC}}t\right) + X_F$$

Here $X_F = I_S$ is the value of the current when the inductor is shorted and the capacitor is open. Applying the initial conditions,

$$i_L(0+) = A + I_s = 0 \quad A = -I_s$$

Further

$$i_L'(0+) = \frac{v_L(0+)}{L} = \frac{v_C(0+)}{L} = 0 = \frac{1}{\sqrt{LC}} B$$

because capacitor voltage is continuous and because

$$i_L'(0+) = \frac{d}{dt}\left[A\cos\left(\frac{1}{\sqrt{LC}}t\right) + B\sin\left(\frac{1}{\sqrt{LC}}t\right) + X_F\right]_{t=0} = \frac{B}{\sqrt{LC}}$$

Hence $B = 0$. Therefore,

$$i_L(t) = I_s\left[1-\cos\left(\frac{1}{\sqrt{LC}}t\right)\right]$$

Rather than repeat the above derivation,

$$v_C(t) = v_L(t) = L\frac{di_L(t)}{dt} = LI_s\frac{d}{dt}\left[1-\cos\left(\frac{1}{\sqrt{LC}}t\right)\right] = I_s\sqrt{\frac{L}{C}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

SOLUTION TO PROBLEM 9.9. Observe that the circuits of figures (a) and (b) are dual circuits. Hence the numerical value of $v_{out}(t)$ and $i_{out}(t)$ are the same for the same excitation. Since the circuit is linear, when the excitation is doubled, the response is doubled (given zero initial conditions) by linearity. Therefore, $i_{out}(t) = 2g(t)$.

SOLUTION TO PROBLEM 9.10.

(a) The initial conditions are:

$$v_c(0-) = 0 = v_c(0+)$$
$$i_L(0-) = 10/0.5 = 20 = i_L(0+)$$

(b)

$$W_L(0) = \frac{1}{2}L\ i_L^2(0) = 0.5\ \text{J}$$
$$W_C(0) = 0$$

(c) Maximum value of v_C when all energy is in capacitor must satisfy $\frac{1}{2}C\ v_{C,\max}^2 = 0.5$ or equivalently $v_{C,\max} = 1000$ V.

(d) From the text development, the parallel LC circuit has the differential equation,

$$\frac{d^2v_C}{dt^2} + \frac{1}{LC}v_C = 0$$

The solution form is:

$$v_C(t) = A\cos\left(\frac{1}{\sqrt{LC}}t\right) + B\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

The initial conditions are:

$$v_C(0) = A = 0$$

and

$$v_C'(0+) = \frac{1}{\sqrt{LC}}B = \frac{i_C(0+)}{C} = -\frac{i_L(0+)}{C}$$

Hence

$$B = -1000$$

Thus

$$v_C(t) = -1000 \sin\left(\frac{1}{\sqrt{LC}}t\right) \text{ V}$$

SOLUTION TO PROBLEM 9.11.

The switch has been closed for a long time which means that the inductor acts like a short and the capacitor like an open. Hence, at t = 0^-, $v_L = 0$ and $i_C = 0$. Hence I_s divides equally between the resistors, i.e.,

$$i_L(0-) = i_L(0+) = I_s / 2 \text{ and } v_C(0-) = v_C(0+) = 0$$

For t ≥ 0, the differential equation is

$$\frac{d^2 v_C}{dt^2} + \frac{1}{LC} v_C = 0$$

with corresponding response

$$v_C(t) = A \cos\left(\frac{1}{\sqrt{LC}}t\right) + B \sin\left(\frac{1}{\sqrt{LC}}t\right)$$

Evaluating at the initial conditions,

$$v_C(0) = A = 0$$

and

$$v_C'(0) = B \frac{1}{\sqrt{LC}} = \frac{i_C(0+)}{C} = \frac{-i_L(0+)}{C}$$

in which case

$$B = -\frac{I_S}{2}\sqrt{\frac{L}{C}}$$

Therefore,

$$v_C(t) = -\frac{I_s}{2}\sqrt{\frac{L}{C}} \sin\left(\frac{1}{\sqrt{LC}}t\right)$$

SOLUTION TO PROBLEM 9.12.

Natural frequency is $\frac{1}{\sqrt{LC}} = 5000 \quad 2\pi$. Hence,

```
»C = 100e-9;
»L =  1/((10e3 *pi)^2 *C)
L =
   1.0132e-02
```

By voltage divider,

$$v_C(0-) = v_C(0+) = 20 \text{ mV}$$

Current through L,

$$i_L(0-) = i_L(0+) = 0$$

Voltage across capacitor satisfies,

$$v_C(t) = A\cos\left(\frac{1}{\sqrt{LC}}t\right) + B\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

Using the ICs,

$$v_C(0+) = A = 20 \text{ mV}$$

and

$$v_C'(0) = B\frac{1}{\sqrt{LC}} = \frac{i_C(0+)}{C} = -\frac{i_L(0+)}{C} = 0$$

Hence

$$v_C(t) = 20 \cos(10{,}000t) \text{ mV}$$

SOLUTION TO PROBLEM 9.13. By definition

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi\ 40$$

in which case
»C = 0.1e-3;
»w = 2*pi*40;
»L = 1/(w^2 *C)
L =
 1.5831e-01 (rad/s).

Observe that

$$i_L(0-) = i_L(0+) = 1A$$

From the given circuit, the capacitor is never connected to a source. Therefore, $v_C(0-) = v_C(0+) = 0$. Also, since

$$\frac{d^2 i_L}{dt^2} + \frac{1}{LC} i_L = 0$$

it follows that

$$i_L(t) = A\cos(\omega t) + B\sin(\omega t)$$

From the initial conditions

$$i_L(0) = A = 1$$

and

$$i_L'(0^+) = B\omega = \frac{v_C(0^+)}{L} = 0$$

$$i_L(t) = \cos(80\pi t) \text{ A}$$

SOLUTION TO PROBLEM 9.14.

(a) From the continuity property, the capacitor voltage and inductor current remain the same,

$$v_C(0-) = v_C(0+) = 5 \text{ V}$$
$$i_L(0-) = i_L(0+) = 1$$

However, $i_C(0-) = 0$ but $i_C(0+) = -i_L(0+) = -1$ A and $v_L(0-) = 0$ but $v_L(0+) = 5$ V. These values change to maintain satisfaction of KVL and KCL.

(b)

$$v_C(t) = A\cos\left(\frac{1}{\sqrt{LC}}t\right) + B\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

where

$$v_C(0+) = 5 = A$$

$$v_C'(0+) = B\frac{1}{\sqrt{LC}} = \frac{i_C(0+)}{C} = -\frac{1}{C} \qquad B = -\frac{\pi}{2}$$

$$v_C(t) = 5\cos(2\pi t) - \frac{\pi}{2}\sin(2\pi t) \text{ V}$$

Alternately, from equation 9.4,

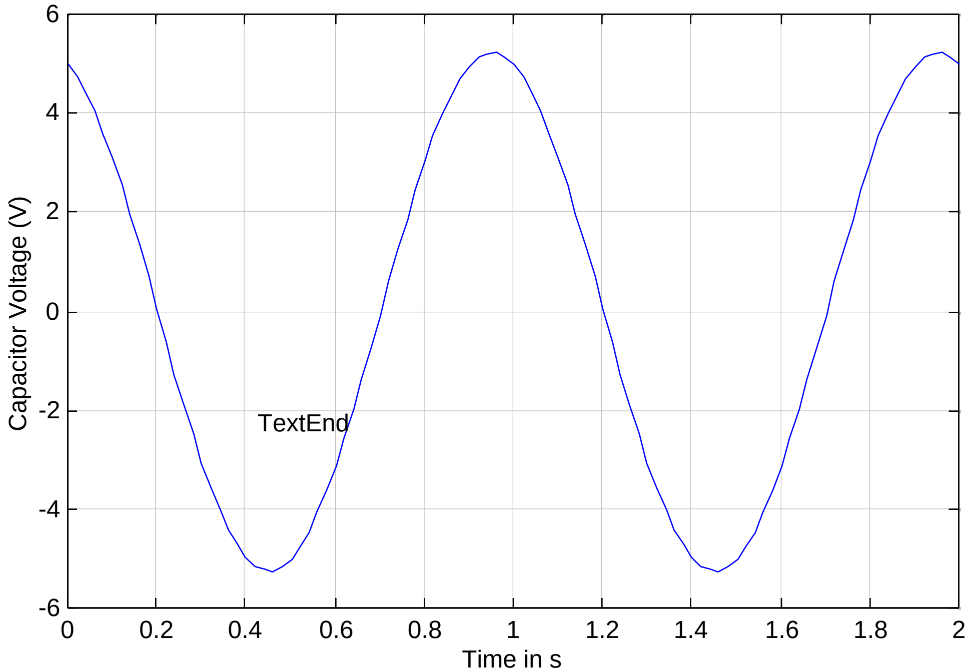
$$v_C(t) = K\cos(2\pi t + \theta)$$

and from equation 9.17b $K = \sqrt{A^2 + B^2}$ and $\theta = \tan^{-1}\left(\frac{-B}{A}\right)$ in which case

```
»A = 5; B = -pi/2;
»K = sqrt(A^2 + B^2)
K =
  5.2409e+00
»theta = atan2(-B,A)
theta =
  3.0440e-01
»thetadeg = theta*180/pi
thetadeg =
  1.7441e+01
```

Hence, K = 5.24 and θ = 17°.

(c)



SOLUTION TO PROBLEM 9.15. As before, $\omega = \dfrac{1}{\sqrt{LC}} = 2\pi\ 40$ in which case L = 0.158 H.

Now,

$$v_C(0-) = v_c(0+) = \frac{100}{125}\ 25 = 20\ \text{mV}$$

and

$$i_L(0-) = i_L(0+) = 10\ \text{mA}$$

The solution form is:

$$i_L(t) = A\cos(80\pi t) + B\sin(80\pi t)$$

where

$$i_L(0+) = A = 10\ \text{mA}$$

and

$$i_L'(0+) = \frac{v_L(0+)}{L} = \frac{v_C(0+)}{L} = 80\pi B \qquad \text{B} = 0.50265\ \text{mA}$$

SOLUTION TO PROBLEM 9.16.

(a) Note: $v_C(0-) = v_C(0+) = -1$ V and $i_L(0) = 5$ A. The characteristic equation is:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 5s + 4 = (s+1)(s+4) = 0$$

As stated, the circuit is overdamped. Hence,

$$v_C(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} = K_1 e^{-t} + K_2 e^{-4t}$$

$$v_C(0+) = K_1 + K_2 = -1$$

$$v_C'(0+) = s_1 K_1 + s_2 K_2 = -K_1 - 4K_2 = \frac{i_C(0+)}{C} = 0$$

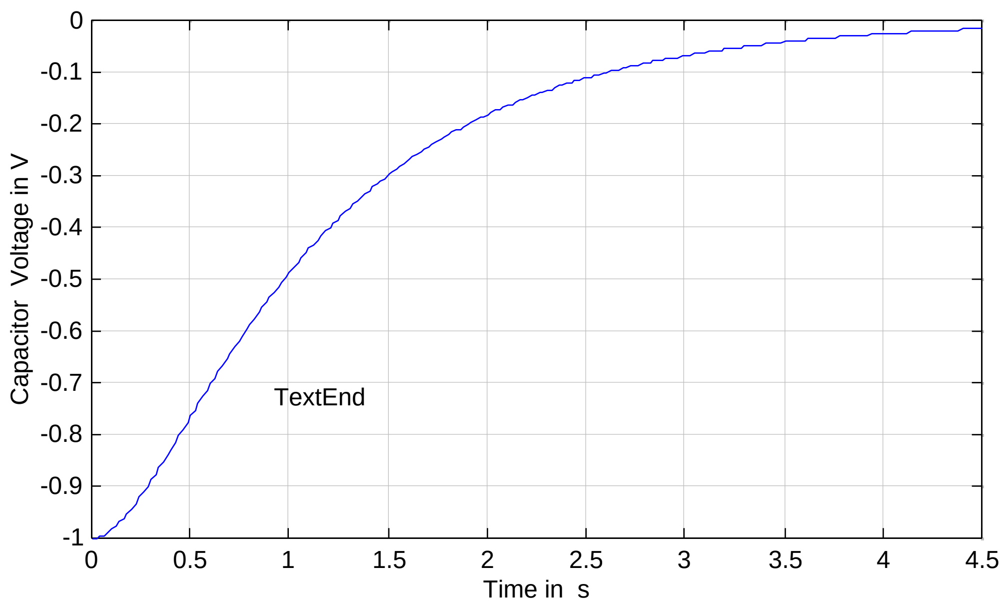
$$K_1 = \frac{-s_2}{s_2 - s_1} = -1.3333 \text{ and } K_2 = \frac{s_1}{s_2 - s_1} = 0.3333.$$

Finally,

$$v_C(t) = -1.3333e^{-t} + 0.3333e^{-4t} \text{ V}$$

Using MATLAB to plot:

```
»t = 0:.02:4.5;
»vc = -1.33333*exp(-t) +  0.33333*exp(-4*t);
»plot(t,vc)
»grid
»ylabel('Capacitor  Voltage in V')
»xlabel('Time in  s')
```



(b)

$$K_1 + K_2 = 1$$

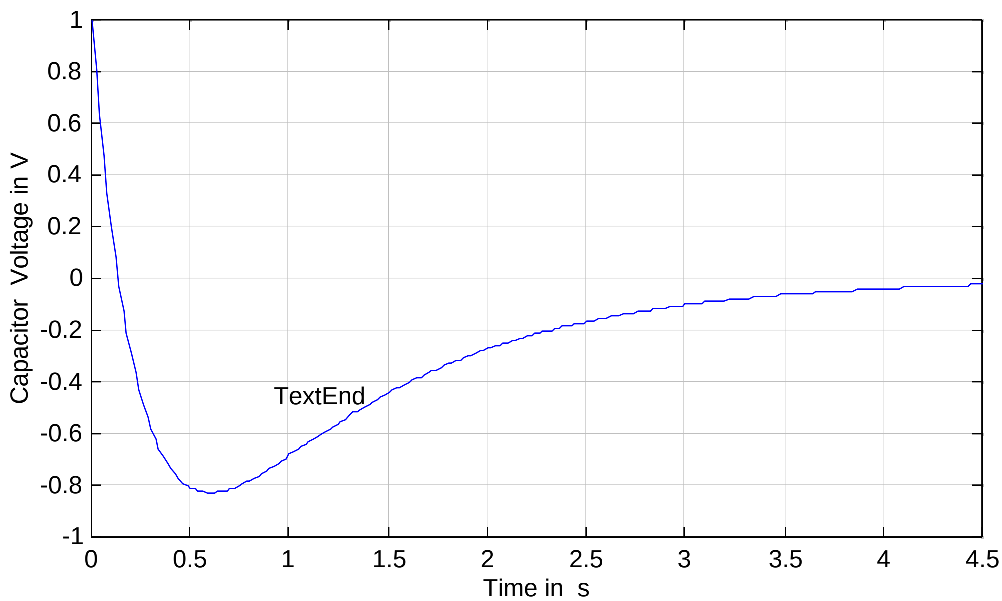
$$v_C'(0+) = s_1K_1 + s_2K_2 = -K_1 - 4K_2 = \frac{i_C(0+)}{C} = \frac{-10}{C} = -10$$

Hence,

```
»A = [1  1;-1  -4]
A =
     1     1
    -1    -4
»b = [1;-10]
b =
     1
   -10
»K = A\b
K =
    -2
     3
```

yielding

$$v_C(t) = -2e^{-t} + 3e^{-4t} \text{ V}$$



Obviously, there is only one zero crossing.

SOLUTION TO PROBLEM 9.17.

(a) First,

$$v_C(0-) = v_C(0+) = \frac{100}{300}30 = 10 \text{ V}$$

$$i_L(0-) = i_L(0+) = 0.1 \text{ A}$$

Clearly, $i_C(0-) = 0$ and $v_L(0-) = 0$. However,

$$i_L(0+) + i_C(0+) + \frac{v_C(0+)}{66.667} = 0.1 + i_C(0+) + 0.15 = 0$$

Hence, $i_C(0+) = -0.25$ A. Further, $v_L(0+) = 10$ V.

(b) The characteristic equation is:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

From MATLAB,

```
»R = 66.667;C = 25e-6; L = 0.5;
»b = 1/(R*C)
b =   6.0000e+02
»c = 1/(L*C)
c =       80000
»si = roots([1  b   c])
si =  -3.9999e+02
  -2.0000e+02
```

We take the roots to be: $s_1 = -400$ and $s_2 = -200$.

(c) Overdamped response implies,

$$v_C(t) = K_1e^{s_1t} + K_2e^{s_2t} = K_1e^{-400t} + K_2e^{-200t}$$

(d)

$$v_C(0+) = K_1 + K_2 = 10$$

$$v_C'(0+) = s_1K_1 + s_2K_2 = -400K_1 - 200K_2 = \frac{i_C(0+)}{C} = -10^4$$

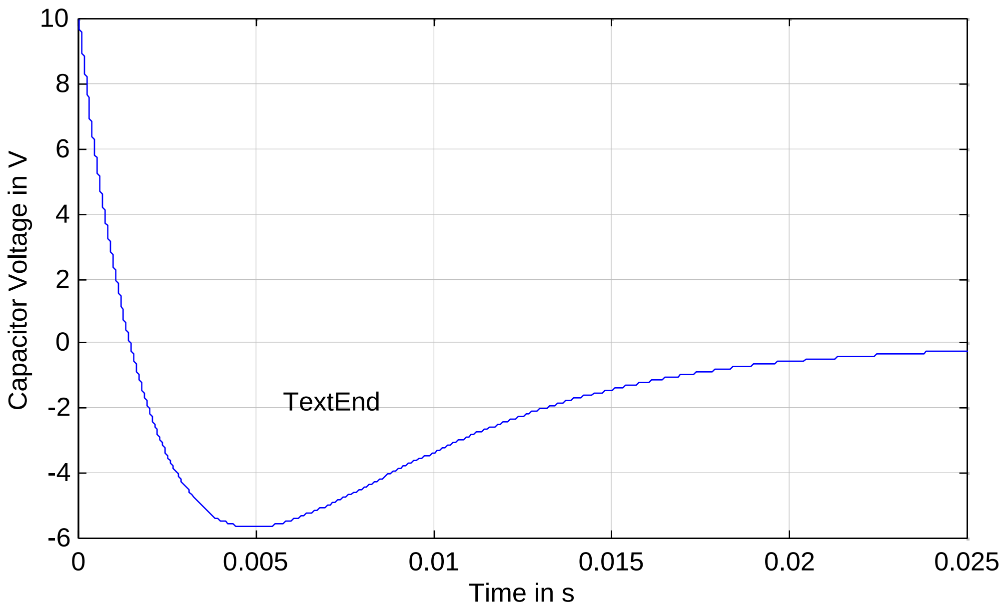
```
»A = [1   1;-400 -200];
»b = [10;-1e4];
»K = A\b
K =
    40
   -30
```

Finally,

$$v_C(t) = 40e^{-400t} - 30e^{-200t} \text{ V}$$

(e)

```
»t=0:0.01e-3:25e-3;
»vc = 40*exp(-400*t) - 30*exp(-200*t);
»plot(t,vc)
»grid
»ylabel('Capacitor Voltage in V')
»xlabel('Time in s')
```



SOLUTION TO PROBLEM 9.18.

(a)

$$i_L(0-) = i_L(0+) = 0$$
$$v_C(0-) = v_C(0+) = 5 \text{ V}$$

At t = 0+, the circuit is a series RLC with R = 12.5 Ω, L = 2.5 H, and C=0.1 F. The resulting characteristic polynomial is:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 5s + 4 = (s+4)(s+1) = 0$$

Hence, $s_1, s_2 = -4, -1$ and the form of the response is:

$$v_C(t) = K_1 e^{-4t} + K_2 e^{-t}$$

At t = 0+,

$$v_C(0+) = K_1 + K_2 = 5$$

and

$$v_C'(0) = -4K_1 - K_2 = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = 0$$

Solve for K_1 and K_2 we obtain:

»A = [1 1; -4   -1];

»b = [5;0];

```
»K = A\b
K =
  -1.6667e+00
   6.6667e+00
```

Hence,

$$v_C(t) = -1.66667e^{-4t} + 6.66667e^{-t} \text{ V}$$

(b)

$$i_L(0+) = i_L(0-) = 0$$

Since, the stable (passive) circuit contains no source for t > 0, all initial energy is absorbed by the resistor. Hence $\lim_{t\to\infty} i_L(t) = 0$, i.e., $i_L(\infty) = 0$.

$$i_L(t) = i_C(t) = C\frac{dv_C}{dt} = \frac{2}{3}e^{-4t} - \frac{2}{3}e^{-t} \text{ A}$$

SOLUTION TO PROBLEM 9.19. This circuit is a series RLC in which case the characteristic equation is always: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

(a) For this time period, R = 2 kΩ in which case the characteristic equation is found and solved in MATLAB as follows:

```
»R = 2e3; L = 0.1; C = 0.1e-6;
»b = R/L
b =
      20000
»c = 1/(L*C)
c =
   100000000
»s12 = roots([1  b   c])
s12 =
      -10000
      -10000
```

The roots are repeated and: s_1, s_2 = –10,000. The form of the response is:

$$i_L(t) = (K_1 + K_2 t)e^{-10000t}$$

To find K_1 and K_2:

$$i_L(0+) = K_1 = 2.5$$

and

$$i_L'(0+) = -25000 + K_2 = \frac{v_L(0+)}{L} = \frac{v_C(0+) - Ri_L(0+)}{L} = \frac{6 - 2000 \times 2.5}{0.1} = -49940$$

$$K_2 = -24940$$

Thus

$$i_L(t) = (2.5 - 24940t)e^{-10000t} \quad 0 \le t \le 0.1 m\sec$$

(b) After the switch closes, R = 1 kΩ,

$$i_L(0.1 \times 10^{-3}) = (2.5 - 24940 \times 0.1 \times 10^{-3})e^{-1} = 0.0022073$$

and

$$i_L'(10^{-3}) = -25000e^{-1} - 24940e^{-1} + 24940e^{-1} = -9197$$

The new characteristic equation is computed as follows:

```
»R = 1e3; L = 0.1; C = 0.1e-6;b = R/L
b =      10000
»c = 1/(L*C)
c =   100000000
»s12new = roots([1  b   c])
s12new =
 -5.0000e+03 + 8.6603e+03i
 -5.0000e+03 - 8.6603e+03i
```

Hence $s_1, s_2 = -5000 \pm j8660.25$. The form of the new solution is:

$$i_L(t') = e^{-5000t'}\left[A\cos(\omega t') + B\sin(\omega t')\right] \text{ A}$$

where $t' = t - 0.1 \times 10^{-3}$ and $\omega = 8660.25$ rad/sec. Observe that

$$i_L(t' = 0) = 0.0022073 = A$$

and

$$i_L'(t' = 0) = -5000A + 8660.25B = -9197$$

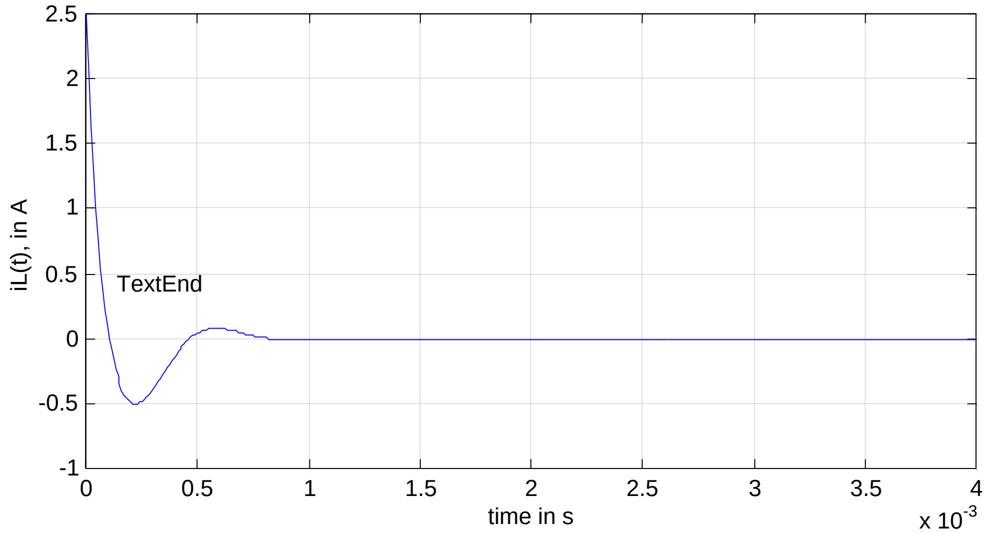
From MATLAB,

```
»w = imag(s12new(1));
»sig = real(s12new(1));
»B = (-9197 -sig*2.2073e-3)/w
B =
  -1.0607e+00
```

Hence for t > 0.1 ms,

$$i_L(t) = e^{-5000(t-0.1ms)}\left[0.0022073\cos\left(\omega(t-0.1ms)\right) - 1.0607\sin\left(\omega(t-0.1ms)\right)\right] \text{ A}$$

(c)




```
K1 = 2.5;
K2 = -24940;
t = 0:0.01e-3:4e-3;
A = 0.0022073;
B = (-9197 -sig*2.2073e-3)/w
B =  -1.0607e+00
iL = (K1 + K2*t) .* exp(-10000*t) .* (u(t)-u(t-1e-4)) ...
+ exp(-5000*(t - 1e-4)) .* (A*cos(w*(t - 1e-4))+B*sin(w*(t - 1e-4))) ...
.* u(t -1e-4);
plot(t,iL)
grid
iL = (K1 + K2*t) .* exp(-10000*t) .* (u(t)-u(t-1e-4))
```

(d)

```
»K1 = 2.5;
K2 = -24940;
»t = 0.1e-3;
»iL = (K1 + K2*t) .* exp(-10000*t)
iL =   2.2073e-03
»  %  The energy stored in the inductor over [0,0.1ms] is in J:
»WL = 0.5*0.1*(iL^2 - 2.5^2)
WL =
  -3.1250e-01
»  %  The energy stored in the capacitor over [0,0.1ms] first
»  %  requires computation of vL and then vC.
»vL = 0.1*(K2*exp(-10000*t) - 10000*(K1 + K2*t) .* exp(-10000*t))
vL =
  -9.1970e+02
»vC = vL + 2000*iL
vC =
```

```
  -9.1528e+02
»  %  The energy stored in the capacitor over [0,0.1ms] is in J:
»WC = 0.5*0.1e-6*(vC^2 - 6^2)
WC =
   4.1885e-02

»  %  To compute energy dissipated in resistor, we make
»  %  use of conservation of energy:  WR + WC + WL = 0

»WR = -WL -  WC
WR =
   2.7061e-01
```

SOLUTION TO PROBLEM 9.20. For this problem, $i_L(0) = 8$ A, $v_C(0) = 20$ V. For the parallel RLC, the characteristic equation $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$ is solved as follows:

```
»R = 20; C = 0.01e-3; L = 25e-3;
»si = roots([1  1/(R*C)  1/(L*C)])
si =
  -4.0000e+03
  -1.0000e+03
»s1 = si(1); s2 = si(2);
```

Hence

$$v_C(t) = K_1 e^{-4000t} + K_2 e^{-1000t}$$

To compute the constants,

$$v_C(0) = K_1 + K_2 = 20$$

and

$$v_C'(0) = -4000K_1 - 1000K_2 = \frac{i_C(0+)}{C} = -\frac{i_L(0+) + \frac{v_C(0+)}{R}}{C}$$

```
»iL0 = 8; vC0 = 20;
»vCprime = -(iL0 +vC0/R)/C
vCprime =
  -9.0000e+05
»A = [1   1;-4000  -1000];
»b = [20;vCprime];
»K = A\b
K =
   2.9333e+02
  -2.7333e+02
```

Hence,

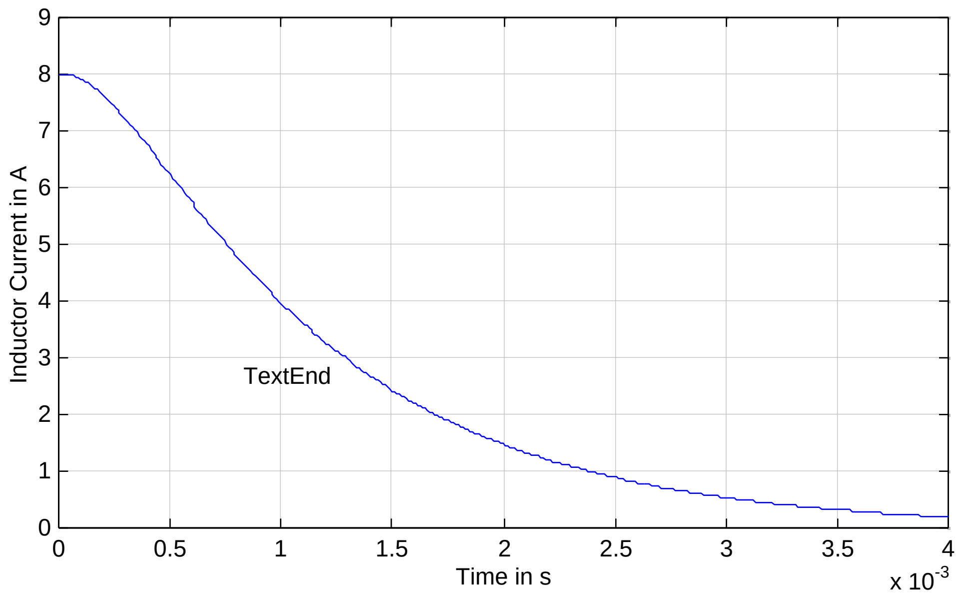
$$v_C(t) = 293.33e^{-4000t} - 273.33e^{-1000t} \text{ V}$$

Also,

$$i_L(t) = -\frac{v_C(t)}{R} - Cv_C^{'}(t) = -2.933e^{-4000t} + 10.933e^{-1000t} \text{A}$$

where

```
»K1 = K(1); K2 = K(2);
»KR1 = K1/R; KR2 = K2/R;
»KCp1 = -4000*K1*C;  KCp2 = -1000*K2*C;
»KL1 = -KR1-KCp1
KL1 =
  -2.9333e+00
»KL2 = -KR2 - KCp2
KL2 =
   1.0933e+01
»t = 0:0.01e-3:4e-3;
»iL = KL1*exp(-4000*t) + KL2*exp(-1000*t);
»plot(t,iL)
»grid
»xlabel('Time in s')
»ylabel('Inductor Current in A')
»
```



SOLUTION TO PROBLEM 9.21.

(a) The circuit is critically damped and the characteristic polynomial is:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = (s+20)^2 = s^2 + 40s + 400 = 0$$

```
»R = 2; C = 1/80;
»L = 1/(400*C)
L =
   2.0000e-01
```

(b) $v_C(0) = 10$ V and $v_C{}'(0^+) = -800 = \frac{i_C(0^+)}{C} = \frac{-i_L(0^+) - v_C(0^+)/2}{C}$.

```
»vC0 = 10; vCp0 = -20*10 -600
vCp0 =
  -800
»iL0 = C*800-vC0/2
iL0 =
     5
```

Hence: $i_L(0-) = i_L(0+) = 5$ A.

(c)
By simple KCL,

$$i_L(t) = \frac{-1}{80}\frac{dv_C}{dt} - \frac{v_C}{2} = 5e^{-20t} + 150\ \ t\ \ e^{-20t}\ \text{A}$$

SOLUTION TO PROBLEM 9.22.

(a) The characteristic equation for the series RLC

$$s^2 + \frac{R_{eq}}{L}s + \frac{1}{LC} = 0$$

For critically damped response, want $(R_{eq}/L)^2 - 4/(LC) = 0$. Solving yields $R_{eq} = 4$ Ω. Hence,

$$R_{eq} = \frac{5R}{5+R}$$

implies that R = 20 Ω.

(b) Solving for the resulting roots implies that

```
»si = roots([1  Req/L  1/(L*C)])
si =
   -50
   -50
»s1 = 50;
```

Hence

$$v_C(t) = (K_1 + K_2 t)e^{-50t}$$

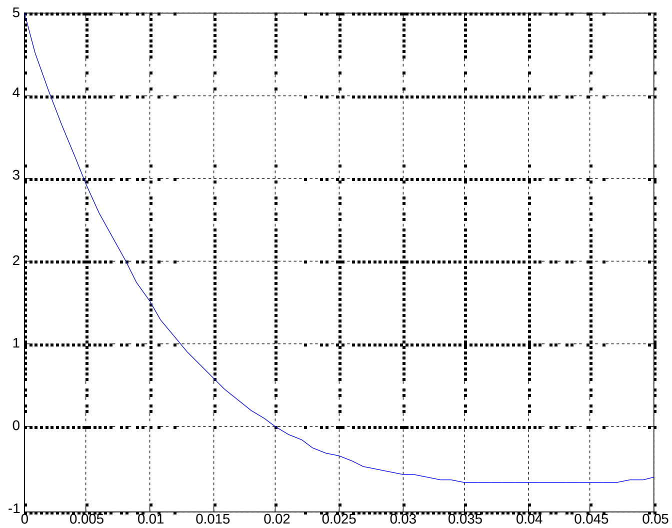
From the initial conditions, $v_C(0) = K_1 = 5$ and

$$v_C'(0) = -250 + K_2 = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = \frac{-5}{0.01} = -500$$

Hence, $K_2 = -250$ and

$$v_C(t) = (5 - 250t)e^{-50t} \text{ V}$$

Set to zero and solve for t = 0.02. See MATLAB plot.



SOLUTION TO PROBLEM 9.23.

(a) The circuit is a series RLC. Hence

```
»R = 40; C = 0.25e-3; L = 0.1;
si = roots([1  R/L  1/(L*C)])
si =
  -200
  -200
»s1 = si(1);
```

Since the circuit is critically damped,

$$i_L(t) = (K_1 + K_2 t)e^{-200t} \text{ A}$$

Using initial conditions,

$$i_L(0) = K_1 = 1$$

$$i_L'(0) = s_1 K_1 + K_2 = -200K_1 + K_2 = \frac{v_L(0+)}{L} = \frac{-v_C(0+) - 40 i_L(0+)}{L} = -350$$

Hence

$$K_2 = -150$$

Thus

$$i_L(t) = (1 - 150t)e^{-200t} \text{ A}$$

Now using MATLAB,

```
»R/L
ans =   400
»1/(L*C)
ans =       40000

»y = dsolve('D2y + 400*Dy + 40000*y = 0, y(0) = 1,Dy(0) = -350')
y =
exp(-200*t)-150*exp(-200*t)*t
```

This answer coincides with the analytical solution.

(b) As above, the form of the solution is

$$v_C(t) = (K_1 + K_2 t)e^{-50t}$$

Applying initial conditions,

$$v_C(0) = K_1 = 5$$

$$v_C'(0) = s_1 K_1 + K_2 = -200K_1 + K_2 = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = 4000$$

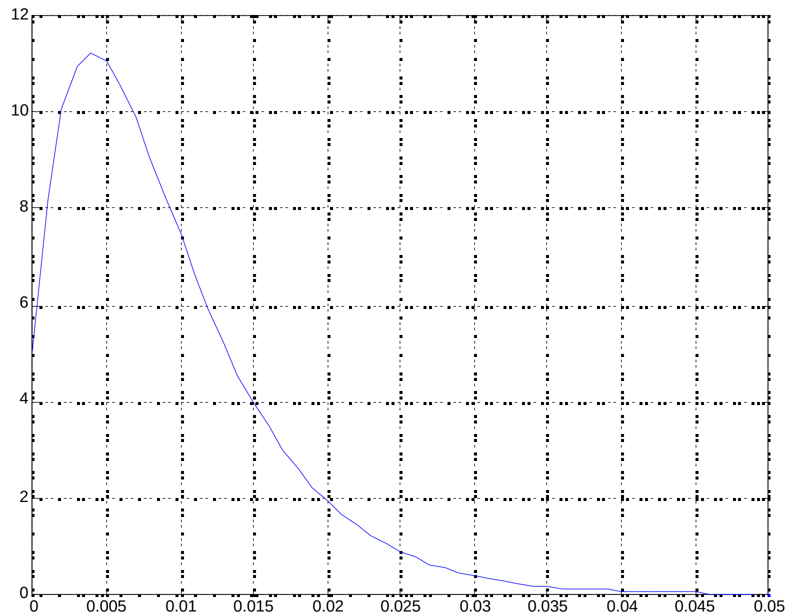
Hence, $K_2 = 5000$ and

$$v_C(t) = (5 + 5000t)e^{-200t} \text{ V}$$

In MATLAB,

```
»y = dsolve('D2y + 400*Dy + 40000*y = 0, y(0) = 5,Dy(0) = 4000')
y =
5*exp(-200*t)+5000*exp(-200*t)*t
```

which verifies the analytical solution. Plotting we obtain,



Observe, there is no zero crossing as per problem 9.5.

SOLUTION TO PROBLEM 9.24.

Source has been on for a long time and turns off at t = 0. Hence

$$i_L(0-) = i_L(0+) = 0$$

$$v_C(0-) = v_C(0+) = 0.1 \times 40 = 4 \text{ V}$$

At t ≥ 0, we have a series RLC circuit:

```
»R = 40; C = 4e-3; L = 2;
si = roots([1  R/L  1/(L*C)])
si =
 -1.0000e+01 + 5.0000e+00i
 -1.0000e+01 - 5.0000e+00i
»wd = imag(si(1))
wd =
     5
»sig = -real(si(1))
sig =
    10
```

Hence

$$v_C(t) = e^{-10t}\left(A\cos(5t) + B\sin(5t)\right)$$

Applying initial conditions,

$$v_C(0) = A = 4$$

$$v_C'(0) = -10A + 5B = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = 0$$

Hence, B = 8. The final form is:

$$v_C(t) = e^{-10t}\left(4\cos(5t) + 8\sin(5t)\right) \text{ V}$$

Similarly,

$$i_L(t) = e^{-10t}\left(A\cos(5t) + B\sin(5t)\right) \text{ A}$$

Applying initial conditions,

$$i_L(0) = A = 0$$

$$i_L'(0) = -10A + 5B = \frac{v_L(0+)}{L} = \frac{-v_C(0+) - 40 i_L(0+)}{L} = -2$$

Thus

$$i_L(t) = -0.4e^{-10t}\sin(5t) \text{ A}$$

SOLUTION TO PROBLEM 9.25.

At t = 0^-, the capacitor is an open circuit and the inductor is a short circuit. The resulting circuit is a simple resistive network. The first step in the solution is to solve this network for the initial conditions on the capacitor and inductor. Specifically, solve for the capacitor voltage (i.e. the voltage across the series connection of the 6 Ω resistor and the independent voltage source) and the inductor current (i.e. the current flowing through the 4 Ω resistor).

Verify that $i_L(0-) = i_L(0+) = 1$ A and $v_C(0-) = v_C(0+) = 12$ V.

When the switch opens, the branch containing the independent voltage source is eliminated. So, we end up with a series RLC circuit. The equivalent resistance is

$$R_{eq} = 4 + 24 // 48 = 20\ \Omega.$$

```
»R = 20; C = 0.01; L = 2;si = roots([1  R/L  1/(L*C)])
si =
 -5.0000e+00 + 5.0000e+00i
 -5.0000e+00 - 5.0000e+00i
```

```
»sig = -real(si(1))
sig =
     5
»wd = imag(si(1))
wd =
     5
```

$$i_L(t) = e^{-5t}\left[A\cos(5t) + B\sin(5t)\right] \text{ A}$$

Applying ICs,

$$i_L(0) = A = 1$$

$$i_L'(0) = -5A + 5B = \frac{v_L(0+)}{L} = \frac{-8}{2} = -4$$

Hence B = 0.2. It follows that

$$i_L(t) = e^{-5t}\left[\cos(5t) + \frac{1}{5}\sin(5t)\right] \text{ A}$$

SOLUTION TO PROBLEM 9.26. The series RLC circuit has characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + \frac{10}{L}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

From the given response, $\sigma = 10 = \frac{10}{2L}$ which implies that L = 0.5 H. Further,

$$\frac{1}{LC} = \frac{2}{C} = \sigma^2 + \omega_d^2 = 10^2 + \left(10\sqrt{3}\right)^2 = 400$$

Hence, C = 5 mF.

Now, from given response, $i_L(0) = 0$ and $i_L'(0) = 500\sqrt{3} = \frac{v_L(0)}{L} = 2v_L(0)$. Hence $v_L(0) = 250\sqrt{3}$ V. In addition, $v_C(0) = -10 i_L(0) - v_L(0) = -250\sqrt{3}$ V and

$$v_C'(0) = i_L(0)/C = 0$$

To find the capacitor voltage we have

$$v_C(t) = e^{-10t}\left(A\cos(10\sqrt{3}t) + B\sin(10\sqrt{3}t)\right)$$

It follows that $v_C(0) = -250\sqrt{3} = A$ and

$$v_C'(0) = 0 = 2500\sqrt{3} + 10\sqrt{3}B$$

B = –250. Therefore

$$v_C(t) = -e^{-10t}\left(250\sqrt{3}\cos(10\sqrt{3}t) + 250\sin(10\sqrt{3}t)\right)$$

SOLUTION TO PROBLEM 9.27. (a) From the given information,

$$v_C(t) = Ae^{-\sigma t}\cos(\omega t + \theta) \text{ V}$$

where $\omega = 2\pi f = 2\pi / T = \dfrac{2\pi}{0.5\times 10^{-3}} = 4000\pi$ rad/s. f = 2 kHz.

The time constant of the exponential term is 1/σ which is 2 ms from the figure. Hence $\sigma = 500$. The characteristic equation of the parallel RLC is:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + \frac{1}{10^4 C}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

Since $\dfrac{1}{10^4 C} = 2\sigma = 1000$, $C = 0.1$ μF. Further,

$$\frac{1}{LC} = \frac{10^7}{L} = \sigma^2 + \omega_d^2 = 10^6 + 16\pi^2 10^6$$

implies that

»L = 1e7/(1e6 + 16*pi^2 *1e6) or L = 63 mH.

(b) From the figure and the above calculations, $v_C(0) = 10$ V and

$$v_C(t) = 10e^{-500t}\cos(4000\pi t) \text{ V},$$

Hence

$$v_C'(t) = -5000e^{-500t}\cos(4000\pi t) - 40000\pi e^{-500t}\sin(4000\pi t)$$

implying that $v_C'(0) = -5000$. Thus

$$i_L(0) = -v_C(0)/10^4 - Cv_C'(0) = -0.001 + 5000\times 0.1\times 10^{-6} = -0.5\times 10^{-3} \text{ A}$$

SOLUTION TO PROBLEM 9.28. As given $v_C(t) = Ae^{-3t}\cos(4t + \theta)$ and R = 10 Ω. The characteristic polynomial of the parallel RLC circuit is:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + \frac{1}{10C}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

Hence 1/(RC) = 6 implies C = 1/60 F. Further $\omega_d = \frac{\sqrt{\frac{4\times 60}{L} - 36}}{2} = 4$. Hence L = 2.4 H.

To change R to obtain a cricially damped circuit,

$$\left(\frac{60}{R}\right)^2 = \frac{4}{LC} = 100$$

Hence $R^2 = 36$ or R = 6 Ω. It follows that 2σ = 10 or σ = 5. The form of the response is:

$$v_C(t) = (K_1 + K_2 t)e^{-5t}$$

SOLUTION TO PROBLEM 9.29. For all cases, $v_C(0\text{-}) = v_C(0+) = 0$ and $i_L(0\text{-}) = i_L(0+) =$ 20/20 = 1 A. Further for all cases the circuit is a parallel RLC with characteristic equation:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

(a)

```
»L = 2e-3;  C = 5e-6;
»c = 1/(L*C);
»R = 10;
»b = 1/(R*C);
»si = roots([1  b   c])
si =
    -10000
    -10000
»%  Solution is cricitally damped.
```

Thus

$$v_C(t) = (K_1 + K_2 t)e^{s_1 t} = (K_1 + K_2 t)e^{-10^4 t} \text{ V}$$

From ICs,

$$v_C(0) = K_1 = 0$$

$$v_C'(0) = s_1 K_1 + K_2 = K_2 = \frac{i_C(0+)}{C} = \frac{-i_L(0+)}{C} = -2\times 10^5$$

Hence

$$v_C(t) = -2\times 10^5 t e^{-10^4 t} \text{ V}$$

(b)

```
»R = 100;
»si = roots([1  1/(R*C)   1/(L*C)])
```

si =
-1.0000e+03 + 9.9499e+03i
-1.0000e+03 - 9.9499e+03i

$$v_C(t) = e^{-1000t}\left(A\cos(9950t) + B\sin(9950t)\right) \text{ V}$$

From ICs.

$$v_C(0+) = A = 0$$

$$v_C'(0) = -1000A + 9950B = \frac{i_C(0+)}{C} = \frac{-i_L(0+)}{C} = -2\times10^5$$

in which case B = –20.1. Thus

$$v_C(t) = -20.1e^{-1000t}\sin(9950t) \text{ V}$$

(c)

```
»R = 87/17;si = roots([1  1/(R*C)   1/(L*C)])
si =
  -3.6328e+04
  -2.7527e+03
```

We define s_1, s_2 = –2753, –36327. Thus

$$v_C(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} = K_1 e^{-2753t} + K_2 e^{-36327t} \text{ V}$$

From the IC's

$$v_C(0) = K_1 + K_2 = 0$$
$$v_C'(0) = s_1 K_1 + s_2 K_2 = -2\times10^5$$

```
»A = [1  1;si(2)   si(1)];
»b = [0; -2e5];
»K = A\b
K =
  -5.9568e+00
   5.9568e+00
```

Therefore

$$v_C(t) = -5.9568\left(e^{-2753t} - e^{-36327t}\right) \text{ V}$$

SOLUTION TO PROBLEM 9.30. For all cases, $v_C(0-) = v_C(0+) = 0$ and $i_L(0-) = i_L(0+) =$ 10/100 = 0.1 A. Further for all cases the circuit is a parallel RLC with characteristic equation:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

(a)

```
»R = 50; C = 0.04e-3; L = 0.625;
»si = roots([1  1/(R*C)   1/(L*C)])
si =
  -400
  -100
```

Define the two roots as:

$$s_1 = -100$$
$$s_2 = -400$$

Hence,

$$v_C(t) = K_1 e^{-100t} + K_2 e^{-400t} \text{ V}$$

From, the initial conditions

$$v_C(0+) = 0 = K_1 + K_2$$

and

$$v_C'(0) = s_1K_1 + s_2K_2 = -100K_1 - 400K_2 = \frac{-i_L(0+)}{C} = -2500$$

```
»A = [1  1;si(2)   si(1)];
»b = [0;-2500];
»K = A\b
K =
  -8.3333e+00
   8.3333e+00
```

Therefore

$$v_C(t) = -8.3333\left(e^{-100t} - e^{-400t}\right) \text{ V}$$

(b)

```
»L = 0.4;
»si = roots([1  1/(R*C)   1/(L*C)])
si =
  -2.5000e+02
  -2.5000e+02
```

Thus

$$v_C(t) = (K_1 + K_2 t)e^{-250t} \text{ V}$$

From IC's,

$$v_C(0) = K_1 = 0$$

$$v_C'(0) = s_1K_1 + K_2 = K_2 = \frac{i_C(0+)}{C} = \frac{-i_L(0+)}{C} = -2500$$

Therefore

$$v_C(t) = -2500\,te^{-250t} \text{ V}$$

(c)

```
»L = 0.2;
»si = roots([1  1/(R*C)   1/(L*C)])
si =
 -2.5000e+02 + 2.5000e+02i
 -2.5000e+02 - 2.5000e+02i
```

$$v_C(t) = e^{-250t}\left[A\cos(250t) + B\sin(250t)\right] \text{ V}$$

From ICs.

$$v_C(0+) = A = 0$$

$$v_C'(0) = -250A + 250B = 250B = \frac{i_C(0+)}{C} = \frac{-i_L(0+)}{C} = -2500$$

in which case B = –10. Thus

$$v_C(t) = -10e^{-250t}\sin(250t) \text{ V}$$

SOLUTION TO PROBLEM 9.31.

(a) The indicated behavior occurs when the resistance causes the circuit to be critically damped, i.e.,

$$\left(\frac{1}{RC}\right)^2 - \frac{4}{LC} = \left(\frac{4}{R}\right)^2 - 16 = 0$$

Thus R = 1 Ω and

$$R = 1 = R_0 + e^{t-5} = 0.8 + e^{t-5}$$

So

```
»t = 5+log(1-0.8)
t =
   3.3906e+00  (years)
```

(b) Here, we have a series case: the indicated behavior occurs when the resistance causes the circuit to be critically damped, i.e.,

$$\left(\frac{R}{L}\right)^2 - \frac{4}{LC} = R^2 - 144 = 0$$

Thus R = 12 Ω and

$$R = 12 = \frac{R_0}{1 + e^{t-5}} = \frac{15}{1 + e^{t-5}}$$

```
»t = 5 +  log((15-12)/12)
t =
   3.6137e+00  (years)
```

SOLUTION TO PROBLEM 9.32. Step 1: Since the step functions are 0 from t = ∞ up to t = 0-,

$$v_C(0-) = v_C(0+) = 0,\ i_L(0-) = i_L(0+) = 0$$

Step 2: At t = 0+, we have

$$i_C(0+) = 2 - \frac{v_C(0+)}{20} - i_L(0+) = 2 \text{ A}$$

and

$$v_L(0+) = v_C(0+) - 5i_L(0+) - 50 = -50 \text{ V}$$

SOLUTION TO PROBLEM 9.33. Since the step function is 0 from t = ∞ up to t = 0-,

$$v_C(0-) = v_C(0+) = 0,\ i_L(0-) = i_L(0+) = 0$$

Since the circuit is a parallel RLC

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 100s + 1600 = 0$$

```
»R = 4; L = 0.25; C = 2.5e-3;
»si = roots([1  1/(R*C)   1/(L*C)])
si =
   -80
   -20
```

Hence s_1, s_2 = -20, -80. The general form is:

$$v_C(t) = K_1 e^{-80t} + K_2 e^{-20t} + X_f \text{ V}$$

When the capacitor is open and the inductor is a short, $X_f = 0$. Thus,

$$v_C(t) = K_1 e^{-80t} + K_2 e^{-20t} \text{ V}$$

From the ICs

$$v_C(0+) = 0 = K_1 + K_2$$

and

$$v_C'(0) = -80K_1 - 20K_2 = \frac{i_C(0+)}{C} = \frac{-i_L(0+) - v_C(0+)/8 + (20 - v_C(0+))/8}{C}$$

$$= \frac{2.5}{2.5 \times 10^{-3}} = 1000$$

```
»A = [1   1;-80  -20];
»b =  [0;  1000];
»K = A\b
K =
```

-1.6667e+01
1.6667e+01

Hence,

$$v_C(t) = -16.667\left[e^{-80t} - e^{-20t}\right] \text{ V}$$

SOLUTION TO PROBLEM 9.34. From the continuity property and the fact that at t = 0-, the capacitor looks like an open and the inductor looks like a short at t = 0-,

$$i_L(0-) = i_L(0+) = 1\text{A}$$
$$v_C(0-) = v_C(0+) = 0$$

Since the circuit is a parallel RLC

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + 500s + 40000 = 0$$

```
»R = 40; C = 0.05e-3;L = 0.5;
»si = roots([1  1/(R*C)   1/(L*C)])
si =
  -400
  -100
```

Define the two roots as:

$$s_1 = -100$$
$$s_2 = -400$$

Hence, the general form is:

$$i_L(t) = K_1 e^{-100t} + K_2 e^{-400t} + X_f$$

When the capacitor is open and the inductor is a short, $X_f = -1$ A. Thus,

$$i_L(t) = K_1 e^{-100t} + K_2 e^{-400t} - 1$$

From, the initial conditions

$$i_L(0+) = K_1 + K_2 - 1 = 1$$

and

$$i_L'(0) = -100K_1 - 400K_2 = \frac{v_L(0+)}{L} = \frac{v_C(0+)}{L} = 0$$

```
»A = [1  1;-100  -400];
»b = [2;0];
»K = A\b
K =
   2.6667e+00
```

```
  -6.6667e-01
```

Hence,

$$i_L(t) = \frac{8}{3}e^{-100t} - \frac{2}{3}e^{-400t} - 1 \text{ A}$$

SOLUTION TO PROBLEM 9.35.

(a) From the problem specs and the continuity property,

$$i_L(0-) = i_L(0+) = 0.008 \text{ A}$$
$$v_C(0-) = v_C(0+) = 2$$

At t = ∞, the inductor looks like a short and the capacitor looks like an open; hence $i_L(\infty) = 0$ and $v_C(\infty) = 400 \times 0.006 = 2.4$ V. The circuit is a series RLC with characteristic polynomial

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

```
»R = 400; C = 6.6667e-6; L = 0.2;
si = roots([1  R/L  1/(L*C)])
s1 = si(1); s2 = si(2);
si =
  -1.5000e+03
  -5.0000e+02
```

Hence

$$v_C(t) = K_1 e^{-1500t} + K_2 e^{-500t} + 2.4 \text{ V}$$

Using initial conditions

$$v_C(0) = K_1 + K_2 + 2.4 = 2$$

$$v_C'(0) = -1500K_1 - 500K_2 = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = \frac{8\times 10^{-3}}{6.6667\times 10^{-6}} = 1200$$

```
»A = [1   1;-1500 -500];
»b = [2-2.4;1200]
b =
  -4.0000e-01
   1.2000e+03
»K = A\b
K =
  -1.0000e+00
   6.0000e-01
```

Hence

$$v_C(t) = -e^{-1500t} + 0.6e^{-500t} + 2.4 \text{ V}$$

(b) $v_L(t)$ is going to have the same form as $v_C(t)$ above except that $v_L(\infty) = 0$ since the inductor is a short at t = ∞. Alternately however, we have

$$v_L(t) = L\frac{di_L(t)}{dt} = L\frac{di_C(t)}{dt} = LC\frac{d^2v_C(t)}{dt^2} = -3e^{-1500t} + 0.2e^{-500t} \text{ V}$$

SOLUTION TO PROBLEM 9.36.

At t = 0, inductor is a short circuit and the capacitor is an open circuit. Since the current source is 0 at t = 0-, and the continuity property,

$$i_L(0-) = i_L(0+) = -1 \text{ A}$$
$$v_C(0-) = v_C(0+) = 65 \text{ V}$$

For positive time, we have a series RLC circuit with characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

```
»R = 65; C = 0.1e-3; L = 0.1;
si = roots([1  R/L  1/(L*C)])
s1 = si(1); s2 = si(2);
si =
  -4.0000e+02
  -2.5000e+02
```

At t = ∞, $v_C(\infty)$ = 0.6*65 = 39 V. Hence

$$v_C(t) = K_1e^{-400t} + K_2e^{-250t} + 39 \text{ V}$$

Using initial conditions,

$$v_C(0) = K_1 + K_2 + 39 = 65$$

$$v_C'(0) = -400K_1 - 250K_2 = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = \frac{-1}{10^{-4}} = -10^4$$

```
»A = [1   1;s1 s2];
»b = [65-39; -1e4];K=A\b
K =
   2.3333e+01
   2.6667e+00
```

Hence,

$$v_C(t) = 23.333e^{-400t} + 2.6667e^{-250t} + 39 \text{ V}$$

SOLUTION 9.37.

(a) R_{th} = 200//50 + R = (40 + R). The characteristic equation is:

$$\left(s^2 + \frac{1}{R_{th}C}s + \frac{1}{LC}\right) = 0$$

Critically damped means that both roots are the same, so the discriminant is zero, i.e.

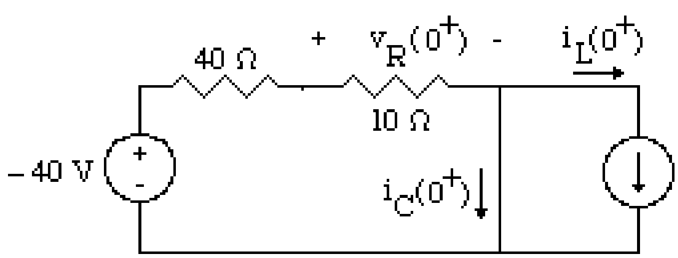
$$\left(\frac{1}{R_{th}C}\right)^2 - \frac{4}{LC} = 0$$

Equivalently,

$$R_{th} = 40 + R = 0.5\sqrt{\frac{L}{C}} = 50 \ \Omega$$

Thus R = 10 Ω.

(b) Short the inductor and open the capacitor. Because the capacitor is in parallel with the shorted inductor at t = 0-, $v_C(0^+) = v_C(0^-) = 0$. The Thevenin equivalent resistance seen by the LC-parallel combination is R_{th} = 50 Ω from part (a). A simple calculation indicates that V_{oc} = 0.8*50 = 40 V. Therefore, $i_L(0^+) = i_L(0^-)$ = 40/50 = 0.8 A. To find $v_R(0^+)$ we use the following equivalent circuit:



Hence,

$$v_R(0^+) = -40 \times \frac{10}{10 + 40} = -8 \text{ V.}$$

To compute the derivative of v_R at 0^+, consider that

$$\frac{d}{dt}\left(v_R(t)\right) = \frac{d}{dt}\left(10 \times i_R(t)\right) = 10\frac{d}{dt}\left(\frac{-40 - v_C(t)}{50}\right) = -0.2\frac{d}{dt}\left(v_C(t)\right) = -0.2\frac{i_C(t)}{C}$$

Hence

$$\frac{dv_R(t)}{dt}\bigg|_{t=0^+} = -0.2\frac{i_C(0^+)}{C} = -4000\, i_C(0^+)$$

But

$$i_C(0^+) = \frac{-40}{50} - i_L(0^+) = -0.8 - 0.8 = -1.6 \text{ A}$$

Therefore,

$$\frac{dv_R(t)}{dt}\bigg|_{t=0^+} = -4000\, i_C(0^+) = 6400 \text{ V/s}$$

(c) Since the circuit is critically damped the roots of the characteristic equation are

$$s_{1,2} = -\frac{1}{2R_{th}C} = -200$$

According to table 9.2 for t ≥ 0,

$$v_R(t) = \left(K_1 + K_2 t\right)e^{-200t} + X_F$$

It follows from the circuit and this equation that

$$X_F = v_R(\infty) = -8 \text{ V}$$

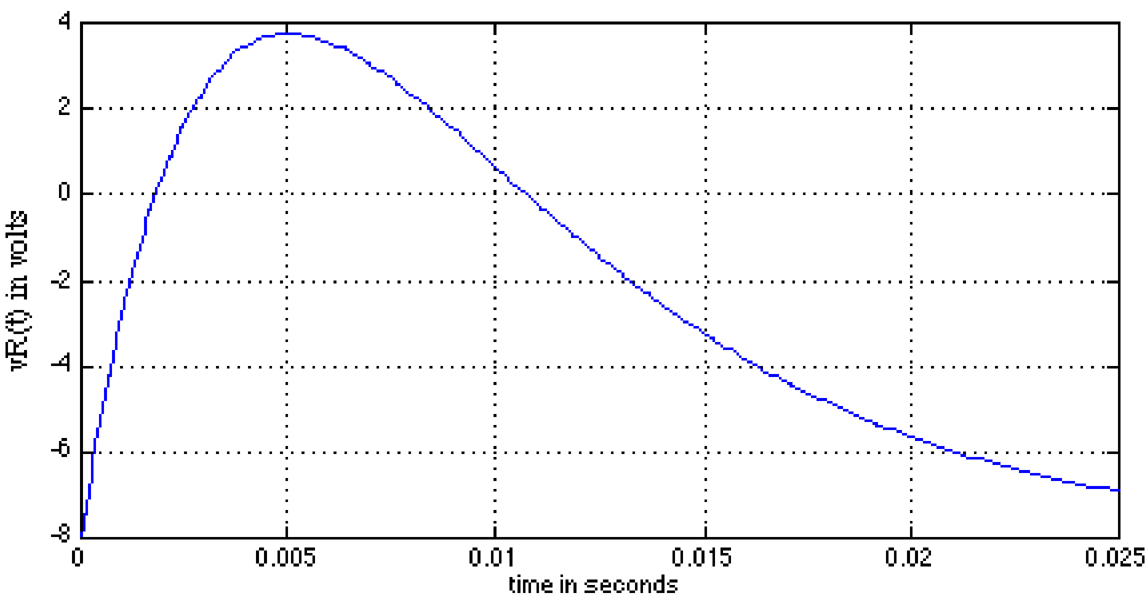
$$K_1 = v_R(0^+) - X_F = 0$$

$$\frac{dv_R(t)}{dt}\bigg|_{t=0^+} = -200K_1 + K_2 = K_2 = 6400$$

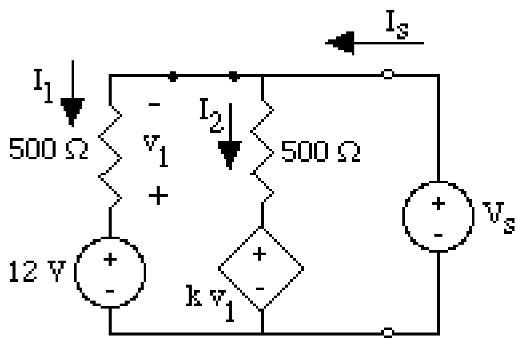
Therefore

$$v_R(t) = \left(6400\, t e^{-200t} - 8\right)u(t) \text{ V}$$

A plot of the waveform is given below



SOLUTION TO PROBLEM 9.38. For this problem we first compute the Thevenin equivalent of the circuit to the left of the capacitor for t > 0. Consider



Now observe

$$I_s = I_1 + I_2 = \frac{V_s - 12}{500} + \frac{V_s - kv_1}{500} = \frac{V_s - 12}{500} + \frac{V_s - k(12 - V_s)}{500}$$

It follows that

$$V_s = \left(\frac{500}{2+k}\right) I_s + \left(\frac{1+k}{2+k}\right) 12 = R_{th} I_s + V_{oc}$$

The parallel LC is now driven by this Thevenin equivalent.

```
»L = 0.01; C = 1e-6;
»%  Critical damping means (1/(Rth*C))2 - 1/(L*C) = 0
»x = sqrt(4/(L*C));
»Rthcrit = 1/(C*x)
Rthcrit =
   50
»kcrit = (500 - 2*50)/50
kcrit =
     8
»%  For parallel circuit, larger  Rth means less damping
»%  Hence, smaller Rth means  overdamped.  Smaller  Rth
»%   means larger k.  Therefore k > 8 is the ranger  for
»%  overdamped response.
```

For the critically damped response we have R = R_{th} = 50 Ω; hence

```
»R = 50; C = 1e-6; L = 0.01;
si = roots([1  1/(R*C)  1/(L*C)])
si =
```

-10000
-10000

in which case

$$v_C(t) = (K_1 + K_2 t)e^{-50t} + X_f$$

At t = ∞, $v_C(\infty) = v_L(\infty) = 0$ in which case $X_f = 0$. From the initial conditions,

$$v_C(0) = K_1 = 0$$

$$v_C'(0) = s_1 K_1 + K_2 = K_2 = \frac{i_C(0+)}{C} = \frac{\frac{V_{oc}}{R_{thcrit}} - i_L(0+)}{C}$$

Hence

$$K_2 = \frac{10.8/50}{10^{-6}} = 2.16 \times 10^5$$

Therefore

$$v_C(t) = 2.16 \times 10^5 e^{-50t} \text{ V}$$

SOLUTION TO PROBLEM 9.39. (a) The series RLC leads to a characteristic equation of the form

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + \frac{24}{0.2}s + \frac{1}{0.2C} = s^2 + 120s + \frac{5}{C} = 0$$

For a critically damped response, $120^2 = \frac{20}{C}$. Hence, C = 1.3889 mF.

(b)

```
»C = 20/120^2
C =
   1.3889e-03
»L = 0.2; R = 24;
»si = roots([1  R/L   1/(L*C)])
si =
   -60
   -60
```

Hence

$$i_L(t) = (K_1 + K_2 t)e^{-60t} + 0.4 \text{ A}$$

where $i_L(\infty) = 0.4$ because at t = ∞, the capacitor looks like an open and the inductor like a short. Hence all current from the source flows through the inductor.

Using the initial conditions,

$$i_L(0) = K_1 + 0.4 = 0 \qquad K_1 = -0.4$$

$$i_L'(0) = s_1 K_1 + K_2 = 60 \times 0.4 + K_2 = \frac{v_L(0+)}{L} = \frac{-v_C(0+) - 24\left(i_L(0+) - 0.4\right)}{L} = 48$$

Hence, $K_2 = 24$ and

$$i_L(t) = (-0.4 + 24t)e^{-60t} + 0.4 \text{ A}$$

SOLUTION TO PROBLEM 9.40. This problem differs from 39 in the initial condition calculation. Specifically,

$$i_L(0-) = i_L(0+) = -0.4$$
$$v_C(0-) = v_C(0+) = 0$$

Again

$$i_L(t) = (K_1 + K_2 t)e^{-60t} + 0.4 \text{ A}$$

and

$$i_L(0+) = K_1 + 0.4 = -0.4 \qquad K_1 = -0.8$$

$$i_L'(0) = 60 \times 0.8 + K_2 = \frac{v_L(0+)}{L} = \frac{-v_C(0+) - 24\left(i_L(0+) - 0.4\right)}{L} = 96$$

Hence,

$$K_2 = 48$$

and

$$i_L(t) = (-0.8 + 48t)e^{-60t} + 0.4 \text{ A}$$

SOLUTION TO PROBLEM 9.41.

(a) At 0-, the capacitor is an open circuit and inductor is a short circuit. So,

$$v_C(0-) = v_C(0+) = 5 \text{ V}$$
$$i_L(0-) = i_L(0+) = 5\times10^{-3} - 4\times10^{-3} = 1 \text{ mA}$$

Now, at 0+, replace the capacitor by a 5 V voltage source and the inductor by a 1 mA current source. Also, the original independent current source is turned off. Solve the resulting circuit to obtain.

$$i_C + 5\times10^{-3} - 1\times10^{-3} = 0$$

$$i_C(0+) = -4 \text{ mA}$$

$$v_L(0+) = 0$$

(b) Since the circuit is a parallel RLC, the characteristic polynomial is

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + \frac{1}{10^4 C}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

```
»R = 1e3; C = 0.5e-6; L = 0.184;
si = roots([1  1/(R*C)  1/(L*C)])
si =
 -1.0000e+03 + 3.1416e+03i
 -1.0000e+03 - 3.1416e+03i

»wd = imag(si(1))
wd =
   3.1416e+03

»sig = -real(si(1))
sig =
        1000
```

Also, at t = ∞, $i_L(\infty) = 5\times10^{-3}$ A. Therefore

$$i_L(t) = e^{-1000t}\left[A\cos(\pi\times10^3 t) + B\sin(\pi\times10^3 t)\right] + 5\times10^{-3} \text{ A}$$

Using initial conditions

$$i_L(0) = A + 5\times10^{-3} = 10^{-3} \qquad A = -4\times10^{-3}$$

and

$$i_L'(0) = -1000A + \pi\times10^3 B = 4 + \pi\times10^3 B = \frac{v_L(0+)}{L} = 0$$

Hence,

```
»B = -4e-3/pi
B =
 -1.2732e-03
```

Finally,

$$i_L(t) = -e^{-1000t}\left[4\cos(\pi\times10^3 t) + 1.2732\sin(\pi\times10^3 t)\right] + 5 \text{ mA}$$

SOLUTION TO PROBLEM 9.42. The response here coincides with that of problem 41 up to time t = 2s. At this point we need the new initial conditions on the circuit at t = 2+. However, at t = 2, $e^{-1000t} = 0$ for all practical purposes. Hence, $i_L(2) = 5\times10^{-3}$ A. Differentiating the expression for $i_L(t)$ and evaluating at t = 2 yields zero by inspection.

This follows because $Ke^{-2000} = 0$ for K in the range of 1 to 10^4. This can also be seen from the circuit because at t = 2 s, the capacitor has charged to 5 V, making $i_L^{'}(2+) = \dfrac{v_L(2+)}{L} = 0$.

To find steady state current, solve the circuit with the new current source value and with the capacitor and inductor as open and short circuits, respectively:

$$i_L(\infty) = 5 \times 10^{-3} + 4 \times 10^{-3} = 9 \times 10^{-3}$$

$$i_L(t) = -e^{-1000(t-2)}\left[A\cos\left(\pi \times 10^3 (t-2)\right) + B\sin\left(\pi \times 10^3 (t-2)\right)\right] + 9 \times 10^{-3} \text{ A}$$

Using the new initial conditions

$$i_L(2+) = A + 9 \times 10^{-3} = 5 \times 10^{-3} \qquad A = -4 \times 10^{-3}$$

and

$$i_L{}'(2+) = -1000A + \pi \times 10^3 B = 4 + \pi \times 10^3 B = \frac{v_L(0+)}{L} = 0$$

Hence B = –1.2732×10^{-3} and for t ≥ 2s,

$$i_L(t) = -e^{-1000(t-2)}\left[4\cos\left(\pi \times 10^3 (t-2)\right) + 1.2732\sin\left(\pi \times 10^3 (t-2)\right)\right] + 9 \text{ mA}$$

SOLUTION TO PROBLEM 9.43. At t = 0-, $i_L(0-)$ = 10/20 = 0.5 A and $v_C(0-)$ = –5 V by the usual considerations. At t = 0+, we have a parallel RLC circuit. Hence

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = s^2 + \frac{1}{10^4 C}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

```
»R = 10; C = 0.05e-3; L = 0.01;
si = roots([1  1/(R*C)  1/(L*C)])
si =
 -1.0000e+03 + 1.0000e+03i
 -1.0000e+03 - 1.0000e+03i
»wd = imag(si(1))
wd =
   1.0000e+03
»sig = -real(si(1))
sig =
        1000
```

Also, at t = ∞, $v_C(\infty) = -5$ V. Therefore

$$v_C(t) = e^{-1000t}\left[A\cos(10^3 t) + B\sin(10^3 t)\right] - 5 \text{ V}$$

Using initial conditions

$$v_C(0) = A - 5 = -5 \qquad A = 0$$

and

$$v_C'(0) = -1000A + 1000B = 10^3 B = \frac{i_C(0+)}{C} = \frac{-0.5}{0.05\times 10^{-3}} = -10^4$$

Hence, $B = 10$ and

$$v_C(t) = -10e^{-1000t}\sin(10^3 t) - 5 \text{ V}$$

Further

$$v_L(t) = -5 - v_C(t) = 10e^{-1000t}\sin(10^3 t) \text{ V}$$

SOLUTION TO PROBLEM 9.44. (a) The circuit is a driven series RLC. Hence

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\sigma s + \sigma^2 + \omega_d^2 = 0$$

```
»R = 400; C = 0.5e-6; L = 0.2;
si = roots([1  R/L  1/(L*C)])
si =
 -1.0000e+03 + 3.0000e+03i
 -1.0000e+03 - 3.0000e+03i
»wd = imag(si(1))
sig = -real(si(1))
wd =
       3000
sig =
       1000
```

Also, at t = ∞, $v_C(\infty) = 400\times 0.006 = 2.4$ V. Therefore

$$v_C(t) = e^{-1000t}\left[A\cos(3000t) + B\sin(3000t)\right] + 2.4 \text{ V}$$

Using initial conditions

$$v_C(0) = A + 2.4 = 2 \qquad A = -0.4$$

and

$$v_C'(0) = -1000A + 3000B = 400 + 3000B = \frac{i_C(0+)}{C} = \frac{i_L(0+)}{C} = 1.6 \times 10^4$$

Hence, $B = 5.2$ and

$$v_C(t) = e^{-1000t}[-0.4\cos(3000t) + 5.2\sin(3000t)] + 2.4 \text{ V}$$

(b) Consistent with underdamped circuit behavior and because the inductor behaves as a short and the capacitor as an open at t = ∞ ($i_L(\infty) = 0$),

$$i_L(t) = e^{-1000t}[A\cos(3000t) + B\sin(3000t)] \text{ V}$$

Using the initial conditions,

$$i_L(0+) = A = 0.008 \text{ A}$$

Further,

$$i_L'(0) = -1000A + 3000B = -8 + 3000B = \frac{v_L(0+)}{L} = \frac{400(0.006 - 0.008) - 2}{0.2} = -14$$

Hence, $B = -0.002$ and

$$i_L(t) = e^{-1000t}[0.008\cos(3000t) - 0.002\sin(3000t)] \text{ V}$$

Finally

$$v_L(t) = v_R(t) - v_C(t) = 400[0.006 - i_L(t)] - v_C(t) = 2.4 - 400 i_L(t) - v_C(t)$$

which implies that

$$v_L(t) = e^{-1000t}[3.6\cos(3000t) - 13.2\sin(3000t)] \text{ V}$$

SOLUTION TO PROBLEM 9.45. This circuit is the same series RLC as problem 44. Note that at t = ∞, $v_C(\infty) = -2.4$ V. Hence

$$v_C(t) = e^{-1000t}[A\cos(3000t) + B\sin(3000t)] - 2.4 \text{ V}$$

Now, the initial conditions are:

$$v_C(0-) = 2.4 = v_C(0+) \text{ V}, \ i_L(0-) = 0 = i_L(0+)$$

Thus

$$v_C(0) = A - 2.4 = 2.4 \qquad A = 4.8$$

Further,

$$v_C'(0) = -1000A + 3000B = -4800 + 3000B = \frac{i_C(0+)}{C} = 0$$

Hence, $B = 1.6$ and

$$v_C(t) = e^{-1000t}\left[4.8\cos(3000t) + 1.6\sin(3000t)\right] - 2.4 \text{ V}$$

SOLUTION TO PROBLEM 9.46. For all three cases, assuming i_L is pointing downward,

$$v_C(0-) = v_C(0+) = 0 \text{ and } i_L(0-) = i_L(0+) = 0.1 \text{ A}$$

At t = ∞,

$$v_C(\infty) = 0 \text{ and } i_L(\infty) = -0.2 \text{ A}$$

Further

$$i_C(0+) = -i_L(0+) + \frac{-10 - v_C(0+)}{50} = -0.1 - 0.2 = -0.3 \text{ A}$$

Lastly, all three cases are for a parallel RLC whose characteristic equation is:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

(a)

```
»R = 50; C = 0.04e-3; L = 0.625;
si = roots([1  1/(R*C)  1/(L*C)])
s1 = si(1); s2 = si(2);
si =
  -400
  -100
```

Hence,

$$v_C(t) = K_1 e^{-100t} + K_2 e^{-400t} \text{ V}$$

From, the initial conditions

$$v_C(0+) = 0 = K_1 + K_2$$

and

$$v_C'(0) = s_1K_1 + s_2K_2 = -100K_1 - 400K_2 = \frac{i_C(0+)}{C} = \frac{-0.3}{0.04\times 10^{-3}} = -7500$$

```
»A = [1  1;si(2)   si(1)];
»b = [0;-7500];
»K = A\b
K =
   -25
    25
```

Therefore

$$v_C(t) = -25\left(e^{-100t} - e^{-400t}\right) \text{ V}$$

(b)

```
»R = 50; C = 0.04e-3; L = 0.4;
»si = roots([1  1/(R*C)  1/(L*C)])
s1 = si(1); s2 = si(2);
si =
  -2.5000e+02
  -2.5000e+02
```

Thus

$$v_C(t) = (K_1 + K_2 t)e^{-250t} \text{ V}$$

From IC's,

$$v_C(0) = K_1 = 0$$

$$v_C'(0) = s_1 K_1 + K_2 = K_2 = \frac{i_C(0+)}{C} = -7500$$

Therefore

$$v_C(t) = -7500\, t e^{-250t} \text{ V}$$

(c)

```
»L = 0.2;
»si = roots([1  1/(R*C)   1/(L*C)])
si =
 -2.5000e+02 + 2.5000e+02i
 -2.5000e+02 - 2.5000e+02i
```

$$v_C(t) = e^{-250t}\left[A\cos(250t) + B\sin(250t)\right] \text{ V}$$

From ICs.

$$v_C(0+) = A = 0$$

$$v_C'(0) = -250A + 250B = 250B = \frac{i_C(0+)}{C} = -7500$$

in which case B = –30. Thus

$$v_C(t) = -30e^{-250t}\sin(250t) \text{ V}$$

SOLUTION TO PROBLEM 9.47.

At t = 0-, he capacitor is open and the inductor is a short. This together with the continuity property implies $v_C(0-) = v_C(0+) = 10$ V and $i_L(0-) = i_L(0+) = 1$ A by inspection.

Now, for t = 0+, v_{in} = 0, replace capacitor and inductor with a voltage source and a current source, respectively (values are those of the initial conditions). Solve for initial capacitor current and initial inductor voltage to obtain:

$$v_L(0+) = -10 \text{ V}$$
$$i_C(0+) = i_L(0+) - i_{R1} - i_{R2} = -1 \text{ A}$$

Notice that the resulting circuit is an undriven parallel RLC circuit with R_{eq} = 10//10 = 5 Ω.

```
»R = 5; C = 0.01; L = 4/3;
si = roots([1  1/(R*C)  1/(L*C)])
s1 = si(1); s2 = si(2);
si =
   -15
    -5
```

Hence,

$$v_C(t) = K_1 e^{-15t} + K_2 e^{-5t} \text{ V}$$

From, the initial conditions

$$v_C(0+) = 10 = K_1 + K_2$$

and

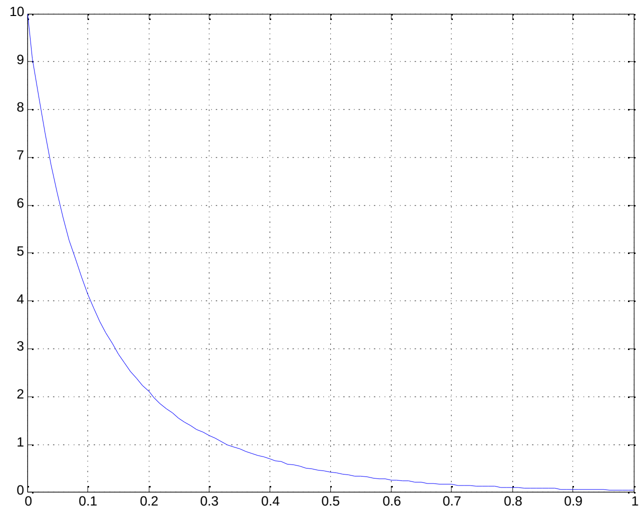
$$v_C'(0) = s_1K_1 + s_2K_2 = -15K_1 - 5K_2 = \frac{i_C(0+)}{C} = \frac{-1}{0.01} = -100$$

```
»A = [1  1;-15   -5];
»b= [10; -100];
»K = A\b
K =
   5.0000e+00
   5.0000e+00
```

$$v_C(t) = 5e^{-15t} + 5e^{-5t} \text{ V}$$

Verify with dsolve function in matlab: dsolve('D2y+20*Dy+75*y=0,y(0)=10,Dy(0)=-100')

```
»dsolve('D2y+20*Dy+75*y=0,y(0)=10,Dy(0)=-100')
ans =
5*exp(-5*t)+5*exp(-15*t)
```



SOLUTION TO PROBLEM 9.48.

For t = 0-, there is no source present nor has there been a non-zero excitation. Hence,

$$v_C(0-) = v_C(0+) = 0 \text{ and } i_L(0-) = i_L(0+) = 0$$

At t = 0+, replace capacitor and inductor by 0-valued voltage and current sources to obtain:

$$v_L(0+) = 10 \text{ V},\ i_C(0+) = 1 \text{ A}$$

For t > 0, we have a driven parallel RLC circuit with $v_C(\infty) = 10$ V. Thus

```
»R = 5; C = 0.01; L = 4/3;
si = roots([1  1/(R*C)  1/(L*C)])
s1 = si(1); s2 = si(2);
si =
   -15
    -5
```

and

$$v_C(t) = K_1 e^{-15t} + K_2 e^{-5t} + 10 \text{ V}$$

From, the initial conditions

$$v_C(0+) = 0 = K_1 + K_2 + 10$$

and

$$v_C'(0) = s_1K_1 + s_2K_2 = -15K_1 - 5K_2 = \frac{i_C(0+)}{C} = \frac{1}{0.01} = 100$$

```
»A = [1  1;-15   -5];
»b= [-10; 100];
»K = A\b
K =
  -5.0000e+00
  -5.0000e+00
```

in which case

$$v_C(t) = -5e^{-15t} - 5e^{-5t} + 10 \text{ V}$$

To compute i_L, note that $i_L(\infty) = 1$ A,

$$i_L(t) = K_1e^{-15t} + K_2e^{-5t} + 1 \text{ A}$$

From, the initial conditions

$$i_L(0+) = 0 = K_1 + K_2 + 1$$

and

$$i_L'(0) = s_1K_1 + s_2K_2 = -15K_1 - 5K_2 = \frac{v_L(0+)}{L} = \frac{10}{4/3} = 7.5$$

```
»A = [1  1;-15   -5];
»b= [-1; 7.5];
»K = A\b
K =
  -2.5000e-01
  -7.5000e-01
```

$$i_L(t) = -0.25e^{-15t} - 0.75e^{-5t} + 1 \text{ A}$$

SOLUTION TO PROBLEM 9.49.

Input to this circuit is a superposition of the inputs in problems 9.47 and 9.48. So, the output of the circuit here is a superposition of the output of the circuit in problems 9.47 and 9.48:

$$v_C(t) = 10e^{-15t} + 10e^{-5t} - 10 \text{ V}$$

For t > 0, by linearity this is the difference of the zero-input circuit response (i.e., due to the IC's as per problem 47) and the zero-state (zero ICs) as per problem 48.

SOLUTION TO PROBLEM 9.50.

(a)

$$v_C(0-) = -60 \text{ V}, \; i_L(0-) = -0.1 \text{ A}$$

(b) By continuity property,

$$v_C(0+) = -60, \; i_L(0+) = -0.1$$

(c)

Replace capacitor by voltage source of value –60 V and inductor by current source of value –0.1 A.

$$v_L(0+) + v_C(0+) = 60$$
$$v_L(0+) = 120$$

and

$$i_C(0+) + i_{R1}(0+) - 1 - i_L(0+) = 0$$
$$i_C(0+) = 1$$

(d) Req = 120//600 = 100 Ω.

```
»R = 100; C = 1e-3; L = 2;
si = roots([1  1/(R*C)  1/(L*C)])
si =
 -5.0000e+00 + 2.1794e+01i
 -5.0000e+00 - 2.1794e+01i
»sig = -real(si(1)); wd = imag(si(1));
```

(e) Note that if the excitation of 60 V had remained forever, then $i_L(\infty)$ would be 0.1 A. Therefore for the interval 0 < t < 1,

$$i_L(t) = e^{-5t}\left[A\cos(21.794t) + B\sin(21.794t)\right] + 0.1 \text{ A}$$

(f)

$$i_L(0+) = A + 0.1 = -0.1$$
$$A = -0.2$$

$$i_L^{'}(0+) = -5A + 21.794B = 1 + 21.794B = \frac{v_L(0+)}{L} = 60$$

Hence, B = 2.707 and

$$i_L(t) = e^{-5t}\left[-0.2\cos(21.794t) + 2.707\sin(21.794t)\right] + 0.1 \text{ A}$$

(g) For t > 1, the forcing function is zero iL(∞) = 0. Thus,

$$i_L(t) = e^{-5(t-1)}\left[A\cos\left(21.794(t-1)\right) + B\sin\left(21.794(t-1)\right)\right] \text{ A}$$

From part (f) of the exp(-5t) term, we can guess that $i_L(1)$ approximates 0.1 and also that $v_C(1)$ approximates its steady state value of 60 V. These may be off by percent or two, but are good enough for our engineering calculations. It follows that $v_L(1+)$ approximates –60 V.

$$i_L(1+) = A = 0.1$$

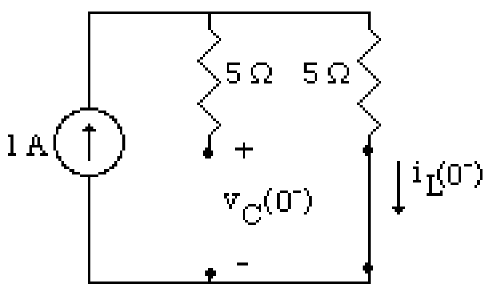
and

$$i_L^{'}(1+) = -5A + 21.794B = -0.5 + 21.794B = \frac{v_L(1+)}{L} = -30$$

Here B = –1.35. Hence

$$i_L(t) = e^{-5(t-1)}\left[0.1\cos\left(21.794(t-1)\right) - 1.35\sin\left(21.794(t-1)\right)\right] \text{ A}$$

***SOLUTION 9.51**. To find the initial conditions, use the following equivalent circuit at t = 0^-.



By inspection $i_L(0^+) = i_L(0^-) = 1$ A and $v_C(0^+) = v_C(0^-) = 5$ V.

To find the characteristic roots, set independent source to zero which means open circuit the independent current source in figure P9.51. This leaves a series RLC with R_{th} = 10 Ω. Hence

$$s^2 + \frac{R_{th}}{L}s + \frac{1}{LC} = s^2 + 100s + 2.5\times 10^4 = 0$$

Using MATLAB, we find

```
»Rth = 10;C = 0.4e-3; L = 0.1;
»s12=roots([1  Rth/L  1/(L*C)])
s12 =
  -50
  -50
```

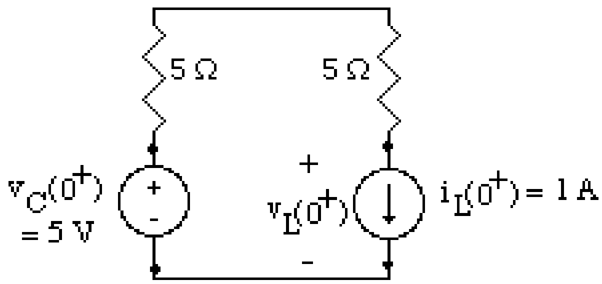
Since for t > 0, the source is off, we use table 9.1, case 3 to obtain

$$i_L(t) = \left(K_1 + K_2 t\right)e^{-50t} \text{ A}$$

It follows that $1 = i_L(0^+) = K_1$ and

$$\frac{di_L}{dt}(0^+) = -50K_1 + K_2 = -50 + K_2 = \frac{1}{L}v_L(0^+) = 10v_L(0^+)$$

To find $v_L(0^+)$ we consider the equivalent circuit valid at 0^+:



It follows that

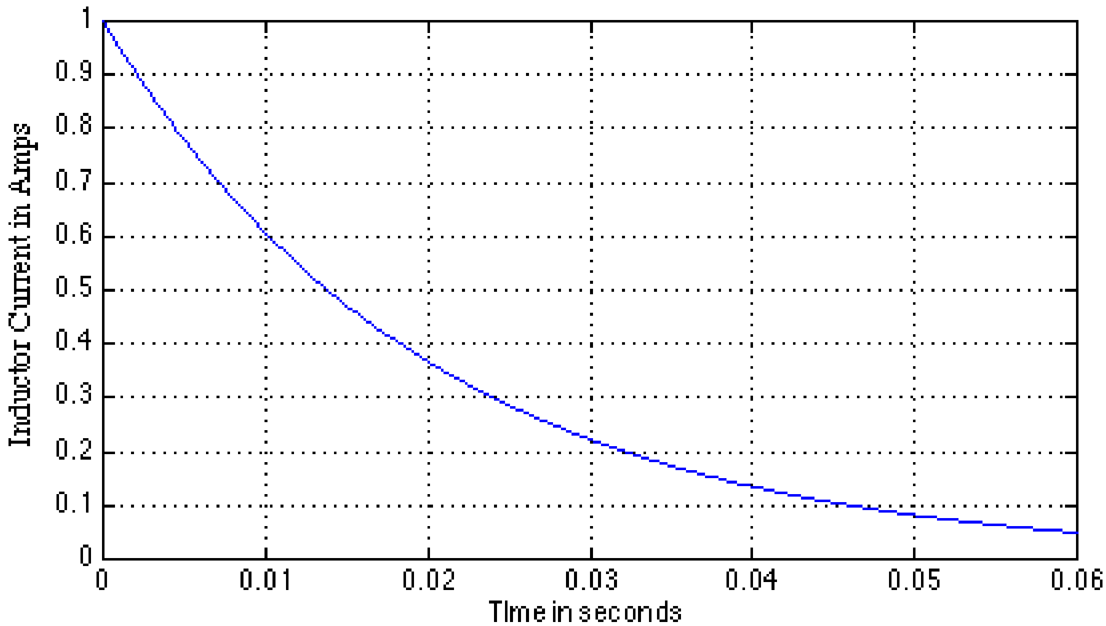
$$v_L(0^+) = 5 - 10 \times 1 = -5 \text{ V}$$

Hence

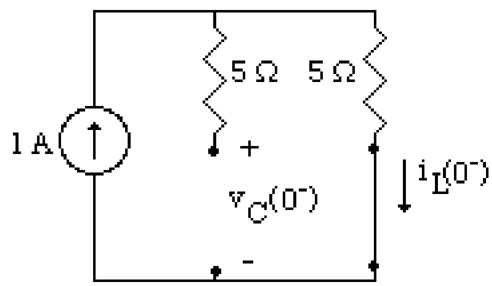
$$-50 + K_2 = -50$$

or $K_2 = 0$. Finally

$$i_L(t) = e^{-50t}u(t) \text{ A}$$



SOLUTION TO PROBLEM 9.52. To find the initial conditions, use the following equivalent circuit at $t = 0^-$.



By inspection $i_L(0^+) = i_L(0^-) = 1$ A and $v_C(0^+) = v_C(0^-) = 5$ V.

To find the characteristic roots, set independent source to zero which means open circuit the independent current source in figure P9.51. This leaves a series RLC with R_{th} = 10 Ω. Hence

$$s^2 + \frac{R_{th}}{L}s + \frac{1}{LC} = s^2 + 20s + 5\times 10^3 = 0$$

Using MATLAB, we find

```
»Rth = 10;C = 0.4e-3; L = 0.5;
»s12=roots([1  Rth/L  1/(L*C)])
s12 =
 -1.0000e+01 + 7.0000e+01i
 -1.0000e+01 - 7.0000e+01i
```

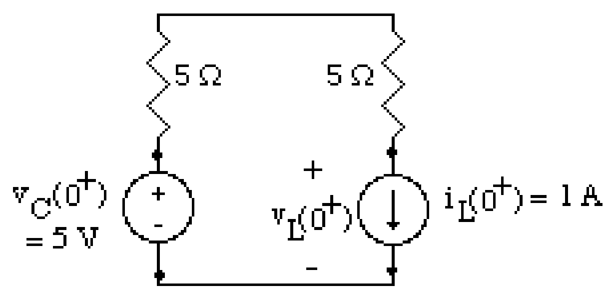
Since for t > 0, the source is off, we use table 9.1, case 2 to obtain

$$i_L(t) = e^{-10t}\left[A\cos(70t) + B\sin(70t)\right] \text{ A}$$

It follows that $1 = i_L(0^+) = A$ and

$$\frac{di_L}{dt}(0^+) = -10A + 70B = -10 + 70B = \frac{1}{L}v_L(0^+) = 2v_L(0^+)$$

To find $v_L(0^+)$ we consider the equivalent circuit valid at 0^+:

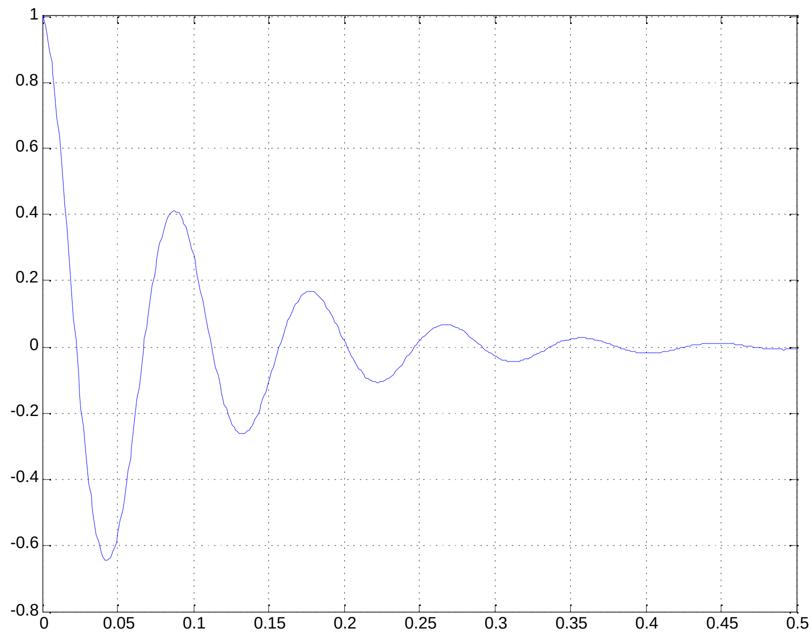


It follows that

$$v_L(0^+) = 5 - 10\times 1 = -5 \text{ V}$$

Hence $-10 + 70B = -10$ or B = 0. Finally

$$i_L(t) = e^{-10t}\cos(70t)u(t) \text{ A}$$



SOLUTION TO PROBLEM 9.53. (a) $v_C(0-) = v_C(0+) = 150 \times 10^{-3} \times 80 = 12$ V.

(b) $i_L(0-) = i_L(0+) = 150$ mA

(c) For these values at 0+, the independent current source becomes an open circuit. Replace the inductor and capacitor with current and voltage sources to represent the initial conditions. Solve the resulting simple circuit to obtain:

$$v_L(0+) = -6 = 12 - 120*0.15 \text{ V},\ i_C(0+) = i_L(0+) = -150 \text{ mA}$$

(d)

```
»Rth = 120;C = 2/9 * 1e-3; L = 0.6;
»s12=roots([1  Rth/L  1/(L*C)])
s12 =
  -1.5000e+02
  -5.0000e+01
```

(e)

$$v_C(t) = K_1 e^{-150t} + K_2 e^{-50t} \text{ V}$$

From, the initial conditions

$$v_C(0+) = 12 = K_1 + K_2$$

and

$$v_C'(0) = s_1K_1 + s_2K_2 = -150K_1 - 50K_2 = \frac{i_C(0+)}{C} = -675$$

```
A = [1  1;-150   -50];
b= [12; -675];
K = A\b
K =
   7.5000e-01
   1.1250e+01
```

in which case

$$v_C(t) = 0.75e^{-150t} + 11.25e^{-50t} \text{ V}$$

(f)

$$i_L(t) = -i_C(t) = -C\frac{dv_C(t)}{dt} = \frac{2}{9} \times 10^{-3}\left[(-150 \times 0.75)e^{-150t} + (-50 \times 11.25)e^{-50t}\right]$$

$$= 0.025e^{-150t} + 0.125e^{-50t} \text{ A}$$

SOLUTION TO PROBLEM 9.54. **(a)** At t = 0-, the independent current source is off, the inductor is a short circuit, and the capacitor is an open circuit. By voltage division,

$$v_C(0-) = v_C(0+) = (80/100)50 = 40 \text{ V}$$

(b)

$$i_L(0-) = i_L(0+) = 50/100 = 0.5 \text{ A}$$

(c) At t = 0+, we have an independent current source. Also, we replace the inductor with a current source and the capacitor with a voltage source to represent the initial conditions.

$$1.5 - i_L(0+) = i_C(0+)$$
$$i_C(0+) = 1 \text{ A}$$

Further,

$$v_C(0+) + 40\ i_C(0+) = 80\ i_L(0+) + v_L(0+)$$
$$v_L(0+) = 40 \text{ V}$$

(d)

```
Rth = 120;C = 2/9 * 1e-3; L = 0.6;
s12=roots([1  Rth/L  1/(L*C)])
s12 =
  -1.5000e+02
  -5.0000e+01
```

(e) In steady state, the capacitor is open and the inductor is a short in which case, X_f = 80 * 1.5 = 120 V.

$$v_C(t) = K_1e^{-150t} + K_2e^{-50t} + 120 \text{ V}$$

From, the initial conditions

$$v_C(0+) = 40 = K_1 + K_2 + 120$$

and

$$v_C'(0+) = -150K_1 - 50K_2 = \frac{i_C(0+)}{C} = 4.5\times 10^3$$

```
A = [1  1;-150   -50];
b= [-80; 4.5e3];
K = A\b
K =
    -5
   -75
```

in which case

$$v_C(t) = -5e^{-150t} - 75e^{-50t} + 120 \text{ V}$$

(f)

$$i_L(t) = 1.5 - i_C(t) = 1.5 - C\frac{dv_C(t)}{dt} = -0.16667e^{-150t} - 0.83333e^{-50t} + 1.5 \text{ A}$$

SOLUTION TO PROBLEM 9.55.

(a) At t = 0-, we replace the inductor by a short circuit and the capacitor by an open circuit; hence

$$i_L(0-) = i_L(0+) = 1 \text{ A}$$

and

$$v_C(0-) = v_C(0+) = (40)(1) – 20 = 20 \text{ V}$$

(b) At t = 0+, we replace the capacitor by a voltage source of value 20 V and the inductor by a current source of value 1 A. Since the inductor current is 1 A and the independent current source outputs 1 A, no current flows through the branch containing the capacitor. Therefore,

$$i_C(0+) = 0$$

Also, because of the zero current in the branch containing the capacitor, no voltage drop occurs across the resistance in series with the capacitor. Therefore, the voltage across the independent current source is $v_C(0+)$. Therefore,

$$v_L(0+) = v_C(0+) – 40{*}i_L(0+) = – 20 \text{ V}$$

(c) At steady state (large t), the capacitor becomes an open circuit and the inductor becomes a short circuit. By inspection,

$$v_C(\infty) = 40 \text{ V}$$

(d)

```
Rth = 80;C = 1/15 * 1e-3; L = 0.1;
s12=roots([1  Rth/L  1/(L*C)])
s12 =
  -5.0000e+02
  -3.0000e+02
```

$$v_C(t) = K_1 e^{-500t} + K_2 e^{-300t} + 40 \text{ V}$$

(e) From, the initial conditions

$$v_C(0+) = 20 = K_1 + K_2 + 40$$

and

$$v_C'(0+) = -500K_1 - 300K_2 = \frac{i_C(0+)}{C} = 0$$

```
A = [1  1;-500   -300];
b= [-20; 0];
K = A\b
K =
   3.0000e+01
  -5.0000e+01
```

in which case

$$v_C(t) = 30e^{-500t} - 50e^{-300t} + 40 \text{ V}$$

SOLUTION TO PROBLEM 9.56.

(a) At t = 0-, we replace the inductor by a short circuit and the capacitor by an open circuit; hence

$$i_L(0-) = i_L(0+) = 1 \text{ A}$$

and

$$v_C(0-) = v_C(0+) = (40)(1) – 20 = 20 \text{ V}$$

(b) At t = 0+, we replace the capacitor by a voltage source of value 20 V and the inductor by a current source of value 1 A. Since the inductor current is 1 A and the independent current source outputs 1 A, no current flows through the branch containing the capacitor. Therefore,

$$i_C(0+) = 0$$

Also, because of the zero current in the branch containing the capacitor, no voltage drop occurs across the resistance in series with the capacitor. Therefore, the voltage across the independent current source is $v_C(0+)$. Therefore,

$$v_L(0+) = v_C(0+) - 40*i_L(0+) = -20\text{ V}$$

(c) At steady state (large t), the capacitor becomes an open circuit and the inductor becomes a short circuit. By inspection,

$$v_C(\infty) = 40\text{ V}$$

(d)

```
Rth = 80;C = 62.5e-6; L = 0.1;
s12=roots([1  Rth/L  1/(L*C)])
s12 =
  -400
  -400
```

$$v_C(t) = (K_1 + K_2 t)e^{-400t} + 40\text{ V}$$

(e) From, the initial conditions

$$v_C(0+) = 20 = K_1 + 40 \qquad K_1 = -20$$

and

$$v_C'(0+) = -400K_1 + K_2 = 8000 + K_2 = \frac{i_C(0+)}{C} = 0$$

in which case

$$v_C(t) = -(20 + 8000t)e^{-400t} + 40\text{ V}$$

SOLUTION TO PROBLEM 9.57.

Step 1:

$$dv_C/dt = i_C/C = 2i_C$$
$$di_L/dt = v_L/L = 0.5v_L$$

Step 2: $i_R = v_C/2$. From KCL, $i_L - i_R - i_c = 0$. Therefore, $i_c = i_L - v_c/2$.

Step 3: Similarly, $v_{in} - v_L - v_c = 0$, which implies $v_L = v_{in} - v_C$. Hence,

$$dv_C/dt = 2\, i_L - v_C$$

$$di_L/dt = 0.5\, v_{in} - 0.5\, v_C$$

Step 4. Eliminate terms in v_C (see equation 9.47 in text) to obtain:

$$d^2i_L/dt^2 + di_L/dt + i_L = 0.5\, dv_{in}/dt + 0.5\, v_{in}$$

SOLUTION TO PROBLEM 9.58.

(a) At t = 0-,

$$v_C(0-) = v_C(0+) = 0$$
$$i_L(0-) = i_L(0+) = 0$$

For t between 0 and 1, we have a parallel RLC circuit, with R_{eq} being the parallel combination of the two 21.1333 Ω resistors.

```
R = 21.1333/2; C = 15.7729e-3; L = 0.1;
si = roots([1  1/(R*C)  1/(L*C)])
si =
 -3.0000e+00 + 2.5000e+01i
 -3.0000e+00 - 2.5000e+01i
```

If there were no further switchings, then X_f = 1 A. Hence,

$$i_L(t) = e^{-3t}\left[A\cos(25t) + B\sin(25t)\right] + 1 \text{ A}$$

Applying the initial conditions,

$$i_L(0+) = A + 1 = 0 \qquad A = -1$$

and

$$i_L'(0+) = -3A + 25B = 3 + 25B = \frac{v_L(0+)}{L} = 0$$

Hence, for $0 \le t < 1$,

$$i_L(t) = -e^{-3t}\left[\cos(25t) + 0.12\sin(25t)\right] + 1 \text{ A}$$

Now, for the next interval, we need initial conditions. These are obtained from the above equation for $i_L(t)$ at t = 1.

$$i_L(1) = e^{-3}[-\cos(25) - 3/25\sin(25)] + 1 = 0.9514$$

and

$$i_L'(1) = 3e^{-3}\left[\cos(25) + 0.12\sin(25)\right] - e^{-3}\left[-25\sin(25) + 0.12\times 25\cos(25)\right] = -0.1671$$

The circuit is still a parallel RLC circuit, but now there is no source and R = 1.268:

```
R = 1.268; C = 15.7729e-3; L = 0.1;
si = roots([1  1/(R*C)  1/(L*C)])
si =
 -2.5000e+01 + 3.0002e+00i
 -2.5000e+01 - 3.0002e+00i
```

Hence,

$$i_L(t) = e^{-25(t-1)}\left[A\cos(3(t-1)) + B\sin(3(t-1))\right]$$

Using initial conditions

$$A = 0.9514$$
$$-0.1671 = -25A + 3B \qquad B = 7.8726$$

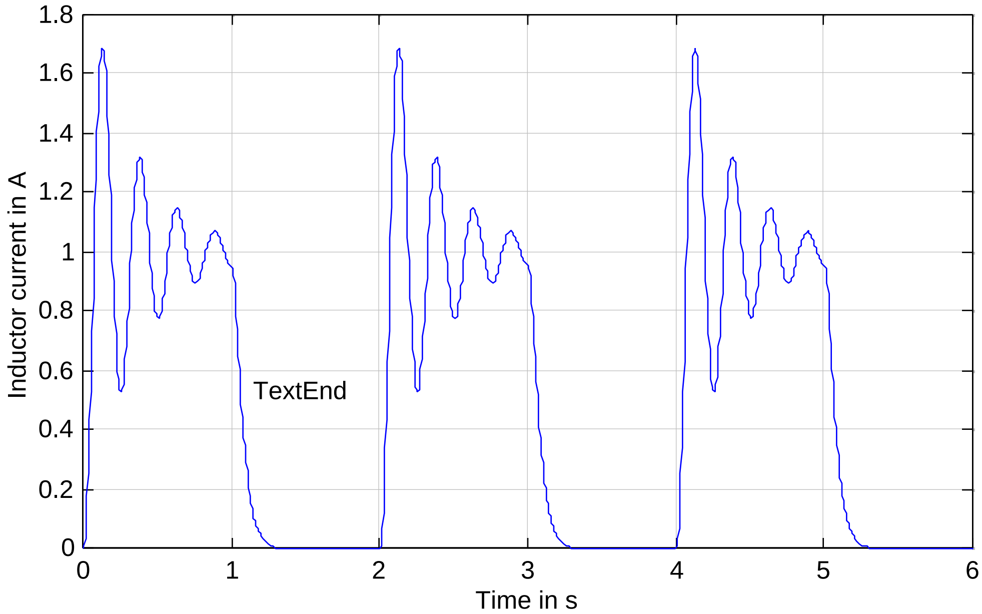
Thus, for $1 \le t < 2$

$$i_L(t) = e^{-25(t-1)}\left[0.9514\cos(3(t-1)) + 7.8726\sin(3(t-1))\right] \text{ A}$$

(b)

In period between 1 and 2 seconds, the response has a time constant of 1/25. So, when t = 2, 25 time constants would have passed from the time the switch is turned (t = 1). This means that the L and C currents and voltages would have settled almost identically to their values at 0–. A similar argument can be made for the other cycle. Thus the overall response effectively becomes a periodic response equal to the response over $0 \le t < 2$ that reflects the periodicity of the switching.

(c)

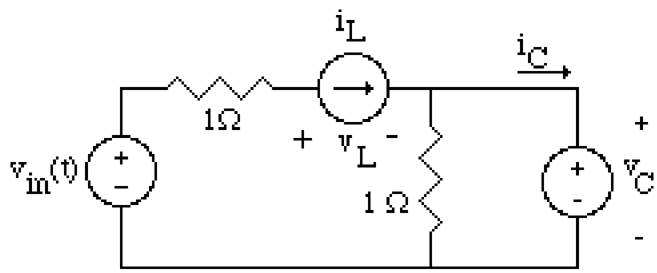


SOLUTION TO PROBLEM 9.59.

(a)

$$dv_c/dt = i_c/C = 0.707i_c$$
$$di_L/dt = v_L/L = 0.707v_L$$

To find expressions for iC and vL we use the following figure.



From this resistive circuit, $i_C = -v_C/1 + i_L$ and $v_L = v_{in} - 1\times i_L - v_C$

$$\dot{v}_C = -0.707 v_C + 0.707 i_L$$
$$\dot{i}_L = -0.707 v_C - 0.707 i_L + 0.707 v_{in}$$

Using equation 9.47,

$$\frac{d^2 v_C}{dt^2} + 1.414\frac{dv_C}{dt} + v_C = 0.5 v_{in}$$

```
»si = roots([1  sqrt(2)  1])
si =
 -7.0711e-01 + 7.0711e-01i
 -7.0711e-01 - 7.0711e-01i
```

(b) At steady state (large t), $v_C = 0.5$

$$v_C(t) = e^{-0.707t}[A\cos(0.707t) + B\sin(0.707t)] + 0.5\text{ V}$$

At t = 0–,

$$v_C(0-) = v_C(0+) = A + 0.5 = 0 \qquad A = -0.5$$

$$v_C'(0) = \frac{i_C(0)}{C} = \frac{i_L(0) - v_C(0)}{C} = 0 = -0.707A + 0.707B$$

$$B = -0.5$$

Thus

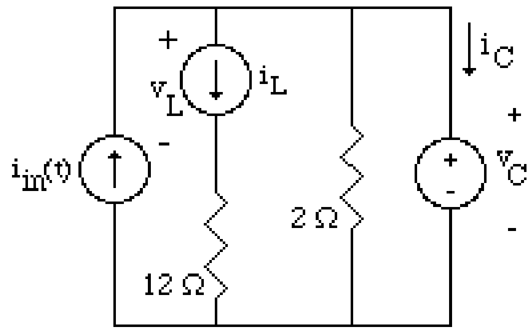
$$v_c(t) = e^{-0.707t}[-0.5\cos(0.707t) - 0.5\sin(0.707t)] + 0.5\text{ V}$$

SOLUTION TO PROBLEM 9.60.

(a)

$$\frac{dv_C}{dt} = 3 i_C$$
$$\frac{di_L}{dt} = \frac{v_L}{3}$$

To eliminate i_C and v_L, consider



Hence

$i_C = i_{in} - v_C/2 - i_L$
$v_L = v_C - 12i_L$

$$\dot{v}_C = -1.5v_C - 3i_L + 3i_{in}$$
$$\dot{i}_L = \frac{1}{3}v_C - 4i_L$$

Using equation 9.47,

$$\frac{d^2v_C}{dt^2} + 5.5\frac{dv_C}{dt} + 7v_C = 3i_{in}^{'} + 12i_{in}$$

»si = roots([1 5.5 7])
si =
  -3.5000e+00
  -2.0000e+00

Note: i_{in}'(t) = 0 for t>0.

At t = 0-, current source is off, inductor is a short circuit, and capacitor is an open circuit.

$v_c(0-) = v_c(0+) = 0$
$i_L(0-) = i_L(0+) = 0$

At t = 0+, the current source is on. Replace the inductor and capacitor by current and voltage sources to represent the initial conditions. Hence, $i_C(0+) = 1$ A. At t = ∞, $v_C(\infty) = 1 \times (12 // 2) = 1.7143$. Thus,

$$v_C(t) = K_1e^{-3.5t} + K_2e^{-2t} + 1.7143 \text{ V}$$

To find the constants,

$$v_C(0) = K_1 + K_2 + 1.7143 = 0$$

and

$$v_C^{'}(0) = \frac{i_C(0)}{C} = 3 = -3.5K_1 - 2K_2$$

```
»A = [1  1;-3.5   -2];
»b = [-1.7143;3];
»K = A\b
K =
   2.8573e-01
  -2.0000e+00
```

Therefore

$$v_C(t) = 0.28573e^{-3.5t} - 2e^{-2t} + 1.7143$$

(b) From equation 9.47b,

$$\frac{d^2 i_L}{dt^2} + 5.5\frac{di_L}{dt} + 7v_C = i_{in}$$

Since $i_L(\infty) = 0.14286$ A,

$$i_L(t) = K_1 e^{-3.5t} + K_2 e^{-2t} + 0.14286 \text{ A}$$

Using the initial conditions,

$$i_L(0) = K_1 + K_2 + 0.14286 = 0$$

and

$$i_L^{'}(0) = \frac{v_L(0)}{L} = \frac{v_C(0) - 12 i_L(0)}{L} = 0 = -3.5K_1 - 2K_2$$

```
»A = [1  1;-3.5  -2];
»b = [-1/7; 0];
»K = A\b
K =
   1.9048e-01
  -3.3333e-01
```

Therefore,

$$i_L(t) = 0.19048e^{-3.5t} - 0.3333e^{-2t} + 0.14286 \text{ A}$$

SOLUTION TO PROBLEM 9.61.

(a)

$$dv_{C1}/dt = 0.5i_{C1}$$
$$dv_{C2}/dt = 0.5i_{C2}$$

Writing a node equation,

$$i_{C1} + v_{C1}/0.5 + (v_{C1} - v_{C2})/0.5 = 0$$
$$i_{C1} = -4v_{C1} + 2v_{C2}$$

By symmetry,

$$i_{C2} = -4v_{C2} + 2v_{C1}$$

Hence

$$\dot{v}_{C1} = -2v_{C1} + v_{C2}$$
$$\dot{v}_{C2} = v_{C1} - 2v_{C2}$$

From equation 9.47a,

$$\frac{d^2v_C}{dt^2} + 4\frac{dv_C}{dt} + 3v_C = 0$$

(b)

»si = roots([1 4 3])

si =

-3

-1

Hence

$$v_C(t) = K_1e^{-3t} + K_2e^{-t}$$

(c)

$$v_{C1}(0) = 2 \qquad K_1 + K_2 = 2$$

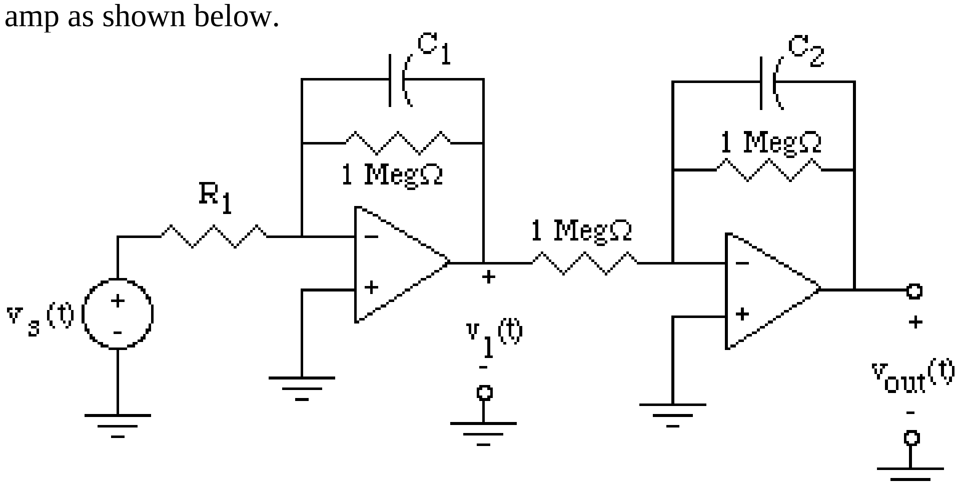
and

$$v'_{C1}(0) = \frac{i_{C1}(0)}{C} = \frac{2(v_{C2}(0) - v_{C1}(0)) - 2v_{C1}(0)}{C} = \frac{8-8}{C} = 0 = -3K_1 - K_2$$

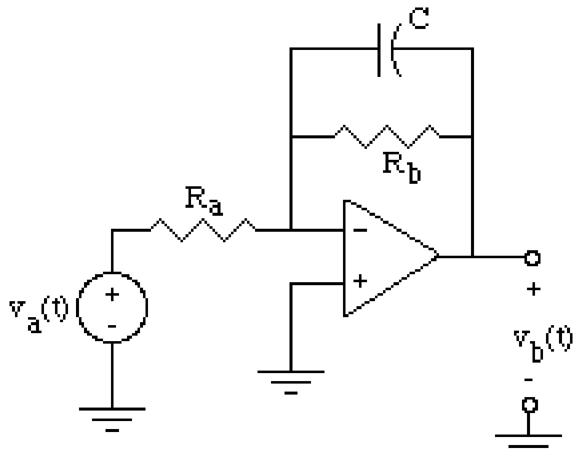
Hence

$$v_C(t) = -e^{-3t} + 3e^{-t} \text{ V}$$

***SOLUTION P9.62.** (a) For this problem we need to define a voltage at the output of the first op amp as shown below.



Also, let us relate the input and output voltages for an arbitrary leaky integrator as shown below.



We now write a node equation at the inverting terminal of the op amp. Here

$$\frac{v_a}{R_a} + \frac{v_b}{R_b} + C\frac{dv_b}{dt} = 0$$

Equivalently

$$\frac{dv_b}{dt} = \frac{v_b}{R_b C} - \frac{v_a}{R_a C} \qquad (1)$$

Now we apply the formula of (*) to the second stage of our given op amp circuit to obtain:

$$\frac{dv_{out}}{dt} = -\frac{v_{out}}{RC_2} - \frac{v_1}{RC_2} \qquad (2)$$

where R = 1 MΩ and C_2 is to be determined.

Applying the formula of (*) to the first stage we obtain:

$$\frac{dv_1}{dt} = -\frac{v_1}{RC_1} - \frac{v_s}{R_1 C_1} \qquad (3)$$

The equations (2) and (3) form a coupled set of state equations which we can write as

$$\begin{bmatrix} \dot{v}_{out} \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_2} & -\frac{1}{RC_2} \\ 0 & -\frac{1}{RC_1} \end{bmatrix} \begin{bmatrix} v_{out} \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-1}{R_1 C_1} \end{bmatrix} v_s$$

Using equation 9.47a of text we can write down the characteristic equation as

$$s^2 + \left(\frac{1}{RC_2} + \frac{1}{RC_1}\right)s + \frac{1}{RC_1 RC_2} = \left(s + \frac{1}{RC_2}\right)\left(s + \frac{1}{RC_1}\right) = 0$$

We require that the natural frequencies be –4 and –12 in which case

$$\frac{1}{RC_2} = 4, \quad \frac{1}{RC_1} = 12$$

From MATLAB
```
»R = 1e6;
»C2 = 1/(R*4)
C2 =   2.5000e-07
»C1 = 1/(R*12)
C1 =   8.3333e-08
```
(b) For this part, the overall dc gain must be 10. The dc gain of the second stage is –1. The dc gain of the first stage must be $-10 = -\frac{10^6}{R_1}$; hence $R_1 = 100$ kΩ.

(c) Since the roots are distinct and real, for t > 0

$$v_{out}(t) = K_1 e^{-4t} + K_2 e^{-12t} + X_f = K_1 e^{-4t} + K_2 e^{-12t} + 10 \text{ V}$$

where $X_f = 10$ V by part (b). The problem states that the capacitor voltages are initially zero. Hence

$$0 = v_{C2}(0) = v_{out}(0) = K_1 + K_2 + 10$$

Equivalently

$$K_1 + K_2 = -10 \qquad (1)$$

Also,

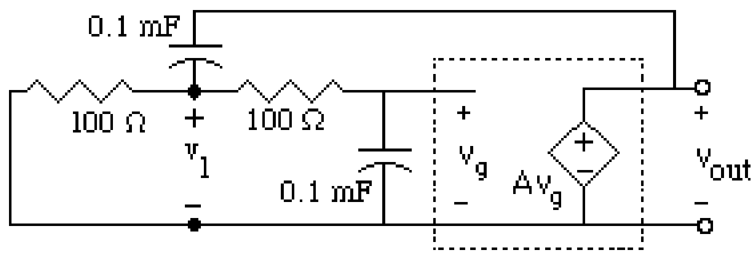
$$\frac{dv_{out}}{dt}(0) = \frac{1}{C_2} i_{C2}(0) = -4K_1 - 12K_2 = 0 \qquad (2)$$

because $i_{C2}(0) = 0$. This is so because $v_{C1}(0) = 0 = v_1(0)$ means no current flows through the 1 MΩ input resistor to stage 2. This fact and the fact that $v_{C2}(0) = 0$, means that no current flows through C_2.

From equation 2, $K_1 = -3\,K_2$. Substituting into equation (1) yields $K_2 = 5$ and hence $K_1 = -15$. Finally,

$$v_{out}(t) = -15e^{-4t} + 5e^{-12t} + 10 \text{ V}$$

***SOLUTION 9.63**. This problem requires the characteristic equation in terms of A. For this we may set $v_{in} = 0$ and the circuit becomes the one given below. Note the new label v_1.



The first step is to write a node equation at v_1:

$$\frac{v_1}{100} + 10^{-4}\frac{d}{dt}\left(v_1 - v_{out}\right) + \frac{1}{100}\left(v_1 - \frac{v_{out}}{A}\right) = 0$$

Equivalently

$$\frac{dv_1}{dt} - \frac{dv_{out}}{dt} = \frac{100}{A}v_{out} - 200v_1 \qquad (*)$$

Now we write a node equation at $v_g = v_{out}/A$. Here,

$$\frac{10^{-4}}{A}\frac{dv_{out}}{dt} + \frac{1}{100}\left(\frac{v_{out}}{A} - v_1\right) = 0$$

Equivalently,

$$\frac{dv_{out}}{dt} = -100v_{out} + 100\,Av_1 \qquad (**)$$

Let us put (*) and (**) in matrix form to obtain:

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{v}_1 \\ \dot{v}_{out} \end{bmatrix} = \begin{bmatrix} -200 & 100/A \\ 100A & -100 \end{bmatrix}\begin{bmatrix} v_1 \\ v_{out} \end{bmatrix}$$

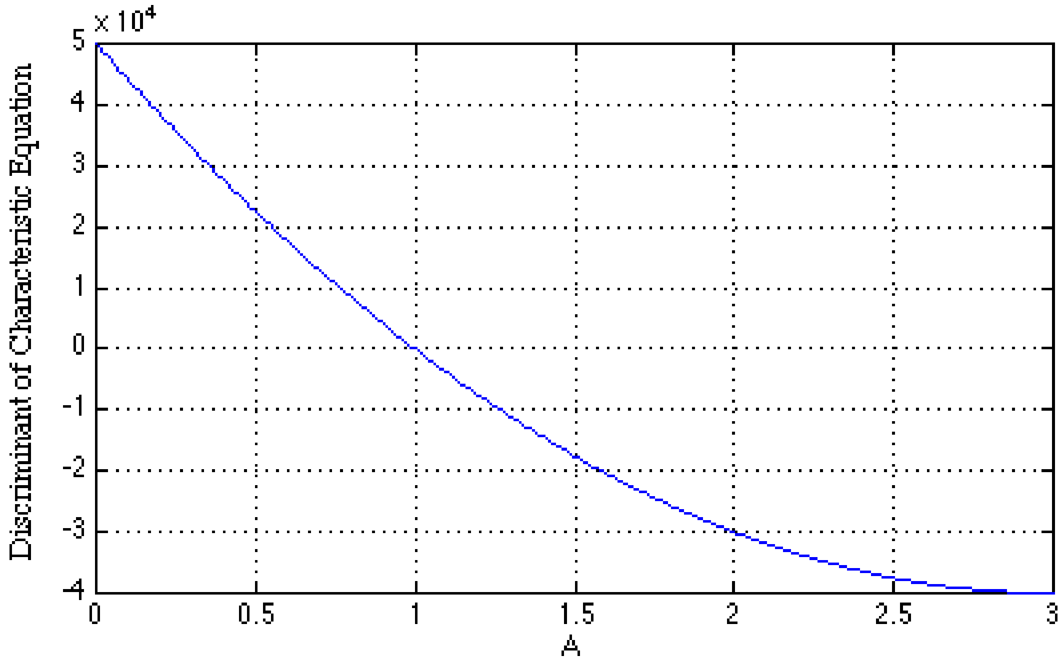
We can now solve this to obtain the state equations

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_{out} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -200 & 100/A \\ 100A & -100 \end{bmatrix}\begin{bmatrix} v_1 \\ v_{out} \end{bmatrix} = \begin{bmatrix} -200 + 100A & -100 + 100/A \\ 100A & -100 \end{bmatrix}\begin{bmatrix} v_1 \\ v_{out} \end{bmatrix}$$

Compare these equations with equation 9.37 and use the formula of 9.47b to obtain the following second order differential equation in v_{out}:

$$\frac{d^2v_{out}}{dt^2} + \left(300 - 100A\right)\frac{dv_{out}}{dt} + 10^4 v_{out} = 0$$

The discriminant of this characteristic equation is plotted below for $0 < A < 3$. For values of $A > 3$, the circuit is unstable. A negative value of the discriminant indicates underdamped ($1 < A < 3$) and a positive value overdamped ($0 < A < 1$). For $A = 1$, we have critical damping.



Solution 9.64 We can write two state equations as follows:
(i) From the definition of a capacitor,

$$\frac{dv_{C1}}{dt} = 10^7 i_{C1}$$
$$\frac{dv_{C2}}{dt} = 10^9 i_{C2}$$

(ii) From KVL and Ohm's law

$$\frac{dv_{C1}}{dt} = 10^6 \left(v_i - v_{C1} - v_{C2}\right) = -10^6 v_{C1} - 10^6 v_{C2} + 10^6 v_i$$
$$\frac{dv_{C2}}{dt} = 10^8 \left(v_i - v_{C1} - v_{C2} - 0.01 v_{C2}\right) = -10^8 v_{C1} - 1.01 \times 10^8 v_{C2} + 10^8 v_i$$

Casting these two equations into a second order differential equation, as described in the text:

$$\frac{d^2 v_{C2}}{dt^2} + 1.02 \times 10^8 \frac{dv_{C2}}{dt} + 10^{12} v_{C2} = 10^8 \frac{dv_i}{dt}$$

The characteristic equation for this differential equation has real roots:

```
»si = roots([1  1.02e8  1e12])
si =
  -1.0199e+08
  -9.8049e+03
```

Since the capacitors become open circuits, $v_{C2}(\infty) = 0$ and $v_{C1}(\infty) = v_i$.

$$v_{C2}(t) = K_1 e^{-1.02\times10^8 t} + K_2 e^{-9.8\times10^3 t} \text{ V}$$

Applying IC's:

$$v_{C2}(0) = K_1 + K_2 = 0$$

Also,

$$v'_{C2}(0) = \frac{i_{C2}(0+)}{C_2} = \frac{0.1}{10^{-9}} = -1.02\times10^8 K_1 - 9.8\times10^3 K_2$$

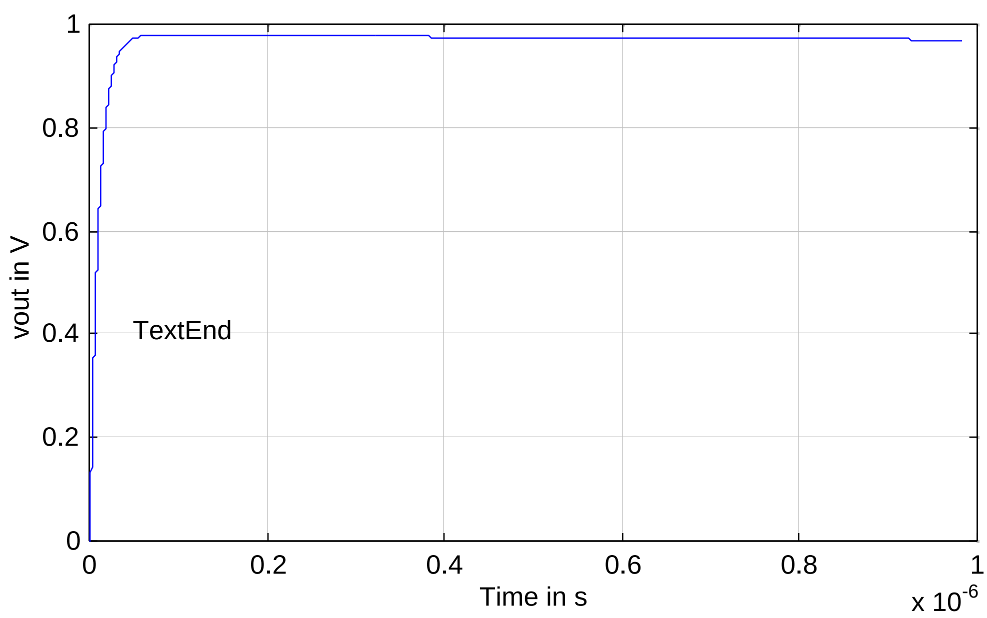
Thus

$$v'_{C2}(0) = 1 = -1.02K_1 - 9.8\times10^{-5} K_2$$

```
»b = [0;  1];
»A = [1   1; -1.02  -9.8e-5];
»K = A\b
K =
  -9.8049e-01
   9.8049e-01
```

$$v_{C2}(t) = -0.9805 e^{-1.02\times10^8 t} + 0.9805 e^{-9.805\times10^3 t} \text{ V}$$

```
»t = 0:1/(abs(s1)*100):100/abs(s1);vc = -0.9805*exp(s1*t) + 0.9805*exp(s2*t);
»plot(t,vc)
»grid
»xlabel('Time in s')
»ylabel('vout in V')
```



Solution 9.65. First, derive the differential equation by writing state equations:

$$\frac{dv_C}{dt} = 3i_C$$
$$\frac{di_L}{dt} = \frac{v_L}{3}$$

Now, assume that the capacitor is a voltage source and the inductor is a current source, and write by KCL

$$i_C = -\frac{v_C}{2} + \frac{v_C}{R_N} - i_L$$

And by KVL:

$$v_L = v_C - 12i_L$$

Substitute into the differential equations:

$$\frac{dv_C}{dt} = \left(-\frac{3}{2} + \frac{3}{R_N}\right)v_C - 3i_L$$
$$\frac{di_L}{dt} = \frac{1}{3}v_C - 4i_L$$

Using equation 9.47 we obtain

$$\frac{d^2v_C}{dt^2} - \left[\left(-\frac{3}{2} + \frac{3}{R_N}\right) - 4\right]\frac{dv_C}{dt} + \left[\left(-\frac{3}{2} + \frac{3}{R_N}\right)(-4) + 1\right]v_C = 0$$

or equivalently

$$\frac{d^2v_C}{dt^2} + \left(5.5 - \frac{3}{R_N}\right)\frac{dv_C}{dt} + \left(7 - \frac{3}{R_N}\right)v_C = 0$$

For constant amplitude oscillations, the middle term should be zero, which means that R_N = 3/5.5 = 0.54545 Ω. Thus the negative resistance is – R_N = –0.54545 Ω.

SOLUTION 9.66. The problem data is

$$i_1(t) = I_m \sin(\omega t + \theta) \text{ A}$$

$$R_1 = 500 + 100(I_m - 0.01),\ R_2 = 500$$

Suppose it starts out with exponentially growing amplitude. R_1 will increase with increasing amplitude. This changes the location of the roots until equilibrium is reached where the roots and the amplitude are stable. This is achieved when the roots of the characteristic equation describing the output voltage are purely imaginary, i.e.,

$$(R_1 - R_2)/(R_1 R_2 C) = 0$$

$$R_1 = R_2 = 500 = 500 + 100(I_m - 0.01)$$

$$I_m = 0.01$$

(a)

$$\omega_0 = 1/[(500)(1\mu)] = 2 \text{ k-rad/s}$$

(b) Amplitude of i_1 is 0.01. Thus $i_1 = 0.01 \sin(\omega_0 t)$ A. Let $v_1 = V_m \cos(\omega_0 t + \varphi)$ V. Then

$$dv_1/dt = -\omega_0 V_m \sin(\omega_0 t + \varphi) = (0.01/C)\sin(\omega_0 t)$$
$$V_m = 0.01/(\omega_0 C) = 5 \text{ V and } \varphi = \pi \text{ rad}$$

Now, $v_2 + v_1 + i_1 R_1 - 3v_2 = 0$ $\quad$ $v_2 = v_1/2 + i_1 R_1/2$. Finally,

$$v_{out} = 3v_2 = 1.5v_1 + 1.5R_1 i_1 = 7.5\cos(\omega_0 t + \pi) + 7.5\sin(\omega_0 t) \text{ V}$$

Hence the amplitude of $v_{out}(t)$ is: $7.5\sqrt{2}$ V.

SOLUTION 9.67. (a)

$$\omega_n = \frac{1}{RC} = 10^4 \text{ rad/s}$$

(b) From equation 9.47a,

$$\frac{d^2 v_1}{dt^2} + \frac{1}{R^2 C^2} v_1 = 0$$

Hence,

$$v_1(t) = A\cos\left(\frac{1}{RC}t\right) + B\sin\left(\frac{1}{RC}t\right) \text{ V}$$

Using the value of $v_1(0)$ we have,

$$v_1(0) = A = 5$$

To compute the second initial condition,

$$v_1'(0+) = \frac{i_{C1}(0+)}{C} = \frac{i_{R1}(0+)}{0.1\times 10^{-6}} = 10^4 B$$

But,

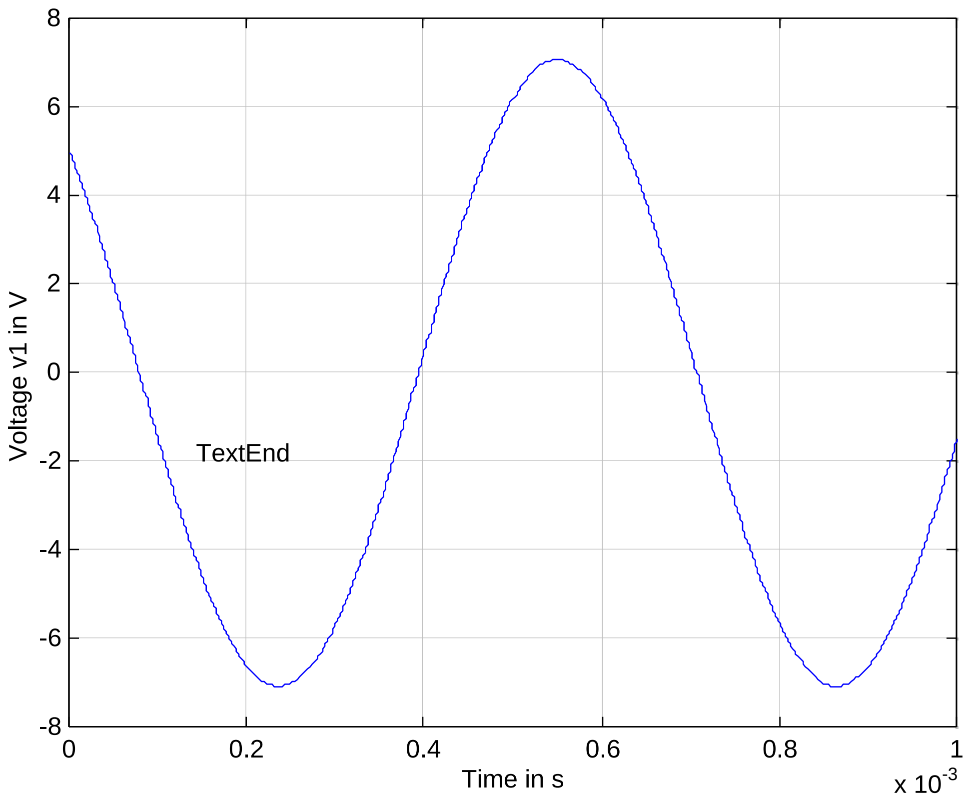
$$i_{R1}(0+) = \frac{3v_2(0+) - v_2(0+) - v_1(0+)}{R_1} = \frac{2v_2(0+) - v_1(0+)}{10^3} = -0.005 \text{ Amps}$$

Hence

$$v_1'(0+) = \frac{-5\times 10^{-3}}{0.1\times 10^{-6}} = 10^4 B$$

Hence B = –5.

$$v_1(t) = 5\cos(10{,}000t) - 5\sin(10{,}000\,t)\ \text{V}$$



***SOLUTION 9.68**. For this problem, R_2 = 10 kΩ should be R_f = 10 kΩ and R_1 = 1 kΩ should be R_2 = 1 kΩ.

(a) For sustained sinusoidal oscillation, R1 = R2 = 1 kΩ. From equation 9.59,

$$\omega_0 = \frac{1}{R_1 C} = 10^4 \text{ rad/s or } 1.5915 \text{ kHz}$$

(b) From figure P9.68, to obtain an R1 = 1 kΩ, $I_{R1,peak} = 0.2$ mA. Therefore, $i_{R1}(t) = 0.2\sin(\omega_0 t + \theta)$ mA for appropriate θ. Since $C\dfrac{dv_1}{dt} = i_{C1} = i_{R1} = 0.2\sin(\omega_0 t + \theta)$ mA, we know that v1 has the following form:

$$v_1(t) = V_{1m}\cos(\omega_0 t + \varphi)$$

In which case

$$C\frac{dv_1}{dt} = C\frac{d}{dt}\left(V_{1m}\cos(\omega_0 t + \varphi)\right) = CV_{1m}\omega_0\sin(\omega_0 t + \varphi + \pi) = 0.2\sin(\omega_0 t + \theta) \text{ mA}$$

Therefore $CV_{1m}\omega_0 = 0.2 \times 10^{-3}$. It follows that $V_{1m} = 0.2$ volts. Here V_{1m} is the peak value of the sinusoid. However, the op amp peak output voltage with respect to ground, as shown in problem 66, is $1.5\sqrt{2}V_{1m}$. Also, for such a small amplitude, we expect the output waveform to be quite close to sinusoidal. By choosing a different lamp (R_1) with a different characteristic, we can obtain larger peak output voltages.

SOLUTION 9.69. **(a)** Note that the capacitor is like an open circuit and the inductor is like a short circuit at t=0-. Thus, we can obtain the capacitor voltage by voltage division:

$$v_C(0-) = 10 \times \frac{4}{5} = 8 = v_C(0+)$$

Similarly, the inductor current is obtained by applying Ohm's Law:

$$i_L(0-) = \frac{10}{5} = 2 = i_L(0+)$$

(b) Here, we note that the new initial conditions are just 2.5 times the values that we just obtained in part (a). This can be achieved by simply changing the input voltage source, from 10 to 25 V.