SOLUTION 14.38. In the s-domain, we break the response up into the part due to the initial condition and the part due to the source with the initial condition set to zero. The transfer function with the initial condition set to zero is

$$H(s) = \frac{V_C(s)}{V_{in}(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1/RC}{s + 1/RC} = \frac{0.25}{s + 0.25}$$

Using the parallel equivalent circuit for the charged capacitor while setting the source voltage to zero, the capacitor voltage due only the initial condition is:

$$V_{C,IC}(s) = \frac{1}{\frac{1}{R} + Cs}\left[Cv_C(0^-)\right] = \frac{v_C(0^-)}{s + 0.25}$$

Hence,

$$V_C(s) = \frac{0.25}{s + 0.25}V_{in}(s) + \frac{v_C(0^-)}{s + 0.25}$$

and

$$I_C(s) = \frac{V_{in}(s) - V_C(s)}{20} = 0.05\left[1 - \frac{0.25}{s + 0.25}\right]V_{in}(s) - \frac{0.05v_C(0^-)}{s + 0.25} = \frac{0.05s}{s + 0.25}V_{in}(s) - \frac{0.05v_C(0^-)}{s + 0.25}$$

for all inputs and initial conditions.

(a) If $v_{in}(t) = 20u(t)$ and $v_C(0^-) = 10$ V, then $V_{in}(s) = \frac{20}{s}$ and

$$V_C(s) = \frac{5}{s(s + 0.25)} + \frac{10}{s + 0.25} = \frac{20}{s} - \frac{10}{s + 0.25} \qquad v_C(t) = \left(20 - 10e^{-0.25t}\right)u(t) \text{ V}$$

and

$$I_C(s) = \frac{1}{s + 0.25} - \frac{0.5}{s + 0.25} = \frac{0.5}{s + 0.25} \qquad i_C(t) = 0.5e^{-0.25t}u(t) \text{ A}$$

(b) If $v_{in}(t) = 5e^{-0.25t}u(t)$ V, then $V_{in}(s) = \frac{5}{s + 0.25}$. Hence,

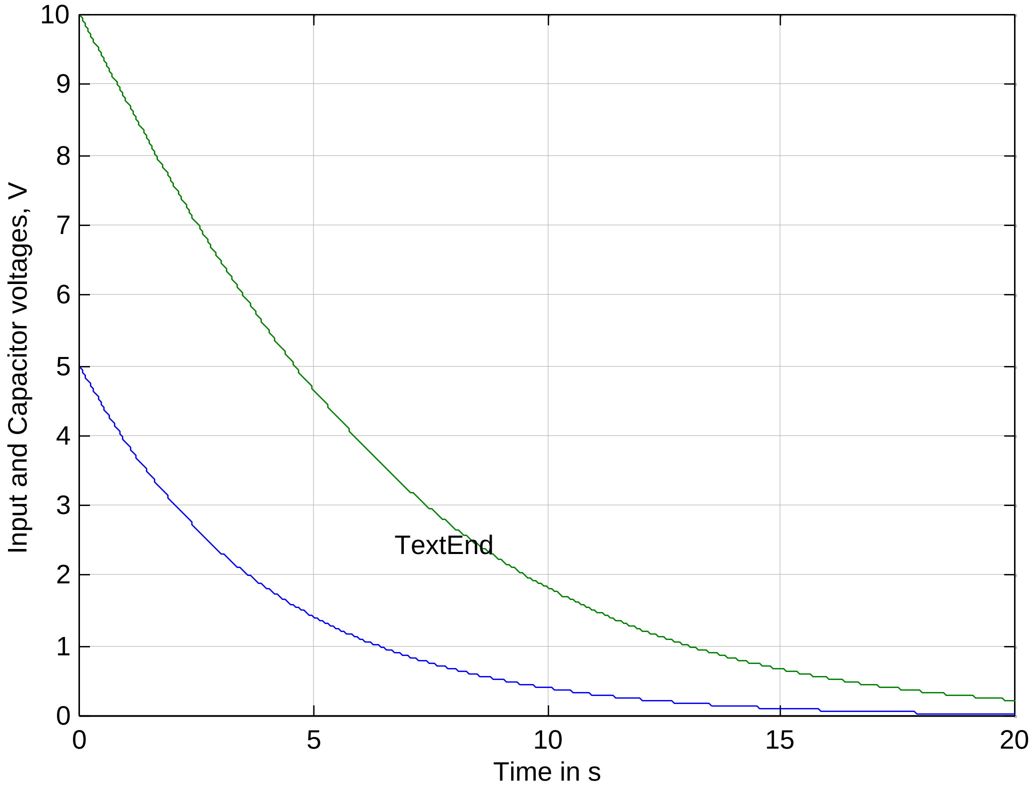
$$V_C(s) = \frac{1.25}{(s + 0.25)^2} + \frac{10}{s + 0.25} \qquad v_C(t) = (10 + 1.25t)e^{-0.25t}u(t) \text{ V}$$

and

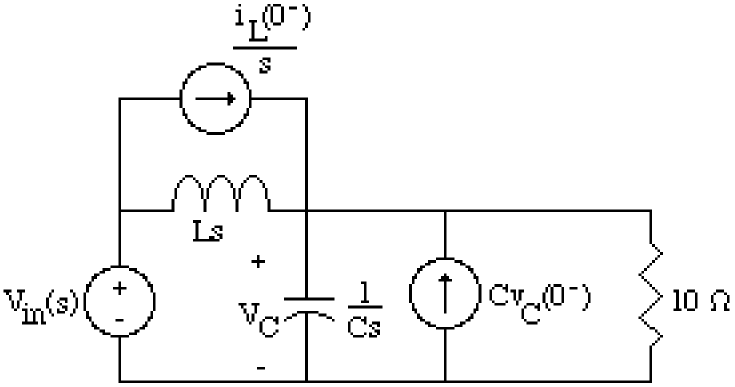
$$I_C(s) = \frac{0.25s}{(s + 0.25)^2} - \frac{0.5}{s + 0.25} = \frac{-0.25}{s + 0.25} - \frac{0.0625}{(s + 0.25)^2}$$

Hence

$i_C(t) = -(0.25 + 0.0625t)e^{-0.25t}u(t)$ A



SOLUTION 14.39. The figure which accounts for the initial conditions is given below.



(a) For the zero-input response, the above circuit reduces to a parallel RLC driven by two current sources.

Hence $V_C(s)$ equals the total current divided by the total admittance, i.e.,

$$V_C(s) = \frac{Cv_C(0^-) + \frac{i_L(0^-)}{s}}{Cs + \frac{1}{R} + \frac{1}{Ls}} = \frac{sv_C(0^-) + \frac{i_L(0^-)}{C}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} = \frac{20s + 10}{s^2 + 250s + 10^4} = \frac{26.6}{s + 200} - \frac{6.6}{s + 50}$$

Hence

$$v_C(t) = [26.6e^{-200t} - 6.6e^{-50t}]u(t) \text{ V}$$

(b) For the zero-state response, the current sources disappear. Executing a source transformation on the remaining voltage source, we obtain a current, $I(s) = V_{in}(s)/(Ls)$, driving a parallel RLC circuit. Hence, the zero input response is

$$V_C(s) = \frac{\frac{V_{in}(s)}{Ls}}{Cs + \frac{1}{R} + \frac{1}{Ls}} = \left(\frac{1}{LC}\right)\frac{V_{in}(s)}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} = \frac{20000}{s^3 + 250s^2 + 10^4 s} = \frac{2}{s} + \frac{0.6667}{s + 200} - \frac{2.6667}{s + 50}$$

Hence

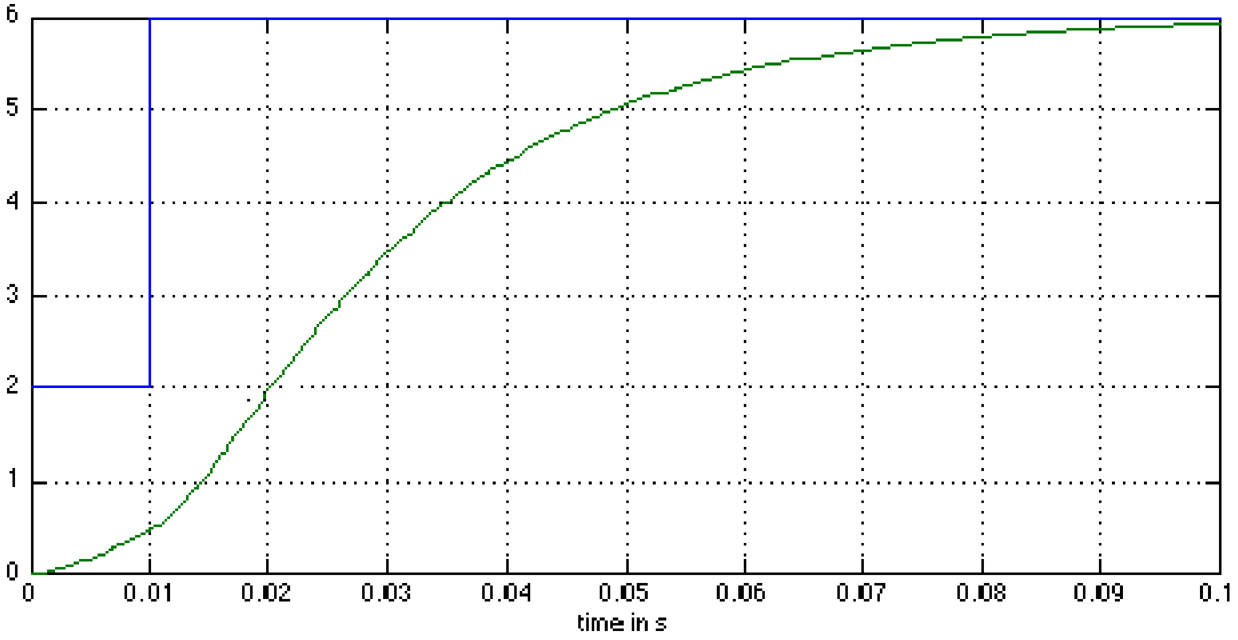
$$v_C(t) = [2 + 0.6667e^{-200t} - 2.6667e^{-50t}]u(t) \text{ V}$$

(c) By superposition, the complete response is the sum of the answers to (a) and (b). Hence

$$v_C(t) = [2 + 27.267e^{-200t} - 9.2667e^{-50t}]u(t) \text{ V}$$

(d) By linearity and time-invariance,

$$v_C(t) = [2 + 0.6667e^{-200t} - 2.6667e^{-50t}]u(t) + [4 + 1.3334e^{-200(t-0.01)} - 5.3334e^{-50(t-0.01)}]u(t-0.01) \text{ V}$$



SOLUTION 14.40. $f(t) = L_{in}(t) = e^{-2t}u(t)A$ [part (c)]

for the zero-input response, $f(t) = L_{in}(t) = \frac{L_2(0^-)}{s}$ [part (d)]

for the zero-input response, $f(t) = L_{in}(t) = Cv_C(0)A$ [part (e)]

(a) $Y_{in}(s) = 1 + \frac{1}{s} + s + 1 = \frac{s^2 + 2s + 1}{s} = \frac{(s+1)^2}{s}$

(b) $I_{out}(s) = \frac{Y_1(s)}{Y_{in}(s)} I_{in}(s) = \frac{s+1}{\frac{s^2+2s+1}{s}} = \frac{s(s+1)}{(s+1)^2} = \frac{s}{s+1} I_{in}(s)$

and

$$H(s) = \frac{I_{out}(s)}{I_{in}(s)} = \frac{s}{s+1}$$

(c) If $i_{in}(t) = e^{-2t}u(t)$ A, then $I_{in}(s) = \frac{1}{s+2}$, then

$$I_{out}(s) = H(s)I_{in}(s) = \frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

which implies that the zero-state response is

$$i_{out}(t) = \left(2e^{-2t} - e^{-t}\right)u(t) \text{ A}$$

(d) If $i_L(0^-) = 2$ A, $v_C(0^-) = 0$, and $i_{in}(t) = 0$. Using the parallel equivalent circuit for the inductor, figure 14.19, we have

$$I_{out}(s) = H(s)\frac{i(0^-)}{s} = \left(\frac{s}{s+1}\right)\left(-\frac{2}{s}\right) = -\frac{2}{s+1} \qquad i_{out}(t) = -2e^{-t}u(t) \text{ A}$$

(e) Use the parallel equivalent circuit for the capacitor, figure 14.16, to obtain by current division,

$$I_{out}(s) = -\frac{\frac{1}{s}+1}{\frac{1}{s}+1+s+1}[Cv_C(0^-)] = 4\frac{s+1}{s^2+2s+1} = \frac{4}{s+1} \qquad i_{out}(t) = 4e^{-t}u(t) \text{ A}$$

(f) By superposition, the complete response is the sum of the answers to parts (c), (d) and (e).

SOLUTION 14.41. With $v_{in}(t) = 4u(t)$V and $v_C(0^-) = 1$ V, a single node equation at the front half of the circuit yields with $Cv_C(0^-) = 1 \times 1 = 1$:

$$-2\left(\frac{4}{5}\right) + \left(2 + \frac{4}{s} + s\right)V_{C1}(s) - 1 = 0 \qquad \left(\frac{s^2 + 2s + 4}{s}\right)V_{C1}(s) = \frac{8}{s} + 1 = \frac{s+8}{s}$$

or

$$V_{C1}(s) = \frac{s+8}{s^2 + 2s + 4}$$

For the rear-half, represent the capacitor by a series equivalent circuit. Thus we can obtain an equivalent voltage source with value:

$$V_{eq}(s) = 2V_{C1}(s) - \frac{V_C(0^-)}{s} = \frac{2(s+8)}{s^2 + 2s + 4} - \frac{1}{s} = \frac{2s(s+8) - (s^2 + 2s + 4)}{s\left(s^2 + 2s + 4\right)}$$

or equivalently

$$V_{eq}(s) = \frac{s^2 + 14s - 4}{s\left(s^2 + 2s + 4\right)}$$

By a voltage division,

$$V_{out}(s) = \frac{\frac{1}{s}}{\frac{1}{s} + 0.5}V_{eq}(s) + \frac{v_C(0^-)}{s} = \frac{2}{s+2} \times \frac{s^2 + 14s - 4}{s\left(s^2 + 2s + 4\right)} + \frac{1}{s}$$

$$= \frac{2s^2 + 28s - 8 + (s+2)\left(s^2 + 2s + 4\right)}{s(s+2)\left(s^2 + 2s + 4\right)} = \frac{\left(s^2 + 6s + 36\right)}{(s+2)\left(s^2 + 2s + 4\right)}$$

Using MATLAB

```
»num = [1  6  36];
»den = conv([1  2],[1 2 4])
den =
     1     4     8     8
»[r,p,k] = residue(num,den)
r =
  7.0000e+00
 -3.0000e+00 - 2.8868e+00i
 -3.0000e+00 + 2.8868e+00i
p =
 -2.0000e+00
 -1.0000e+00 + 1.7321e+00i
 -1.0000e+00 - 1.7321e+00i
k =
```

However, it would appear easier here to use ilaplace:

```
»syms t s
»ilaplace((s^2+6*s+36)/((s+2)*(s^2+2*s+4)))
ans =
7*exp(-2*t)-6*exp(-t)*cos(3^(1/2)*t)+10/3*exp(-t)*3^(1/2)*sin(3^(1/2)*t)
```

»
Hence

$$v_{out}(t) = \left[7e^{-2t} + e^{-t}\left(3.334\sqrt{3}\sin(\sqrt{3}t)\right) - 6\cos(\sqrt{3}t)\right]u(t)\ \text{V}$$

SOLUTION 14.42. Using the series equivalent circuit (figure 14.17) for C_1, we have

$$I_{1K}(s) = \frac{v_{C1}(0^-)/s}{R + \frac{1}{sC_1}} = \frac{-\,0.25/s}{1000 + 50/s} = \frac{-0.25}{1000s + 50}$$

Next, since $v_{C2}(0^-) = 0$, we have

$$V_{C2}(s) = \frac{I_{1k}(s)}{C_2 s} = \frac{-0.25 \times 500}{s(1000s + 50)} = \frac{-0/1\,2\,5}{s(s + 0.05)}$$

Finally,

$$V_{out}(s) = -V_{C2}(s) = \frac{0.125}{s(s + 0.05)} = 2.5\left(\frac{1}{s} - \frac{1}{s + 0.05}\right)$$

and

$$v_{out}(t) = 2.5(1 - e^{-0.05t})u(t)\ \text{V}$$

SOLUTION 14.43. (a) It is preferable to use the series equivalent circuit (figure 14.17) for C_1, and the parallel equivalent circuit (figure 14.16) for C_2.

(b) The current through the 2.5 kΩ resistor is given by

$$I_{2.5K}(s) = \frac{v_{C1}(0^-)/s}{R_1 + \frac{1}{sC_1}} = \frac{-\,2/s}{2500 + 5000/s} = \frac{-\,2}{2500s + 5000} = \frac{-8\times10^{-4}}{s + 2}$$

Next,

$$V_{out}(s) = V_{C2}(s) = I_{2.5K}(s)\,\frac{1}{sC_2 + \frac{1}{R_2}} = \frac{-8\times10^{-4}}{s + 2} \times \frac{1}{0.0002s + 0.0002} = \frac{-\,4}{(s + 2)(s + 1)}$$

(c) Hence

$$V_{out}(s) = \frac{-\,4}{(s + 2)(s + 1)} = -\,4\left(\frac{1}{s + 1} - \frac{1}{s + 2}\right)$$

and

$$v_{out}(t) = 4\left(e^{-2t} - e^{-t}\right)u(t)\ \text{V}$$

(d) SPICE plot omitted.

SOLUTION 14.44. (a) From voltage division,

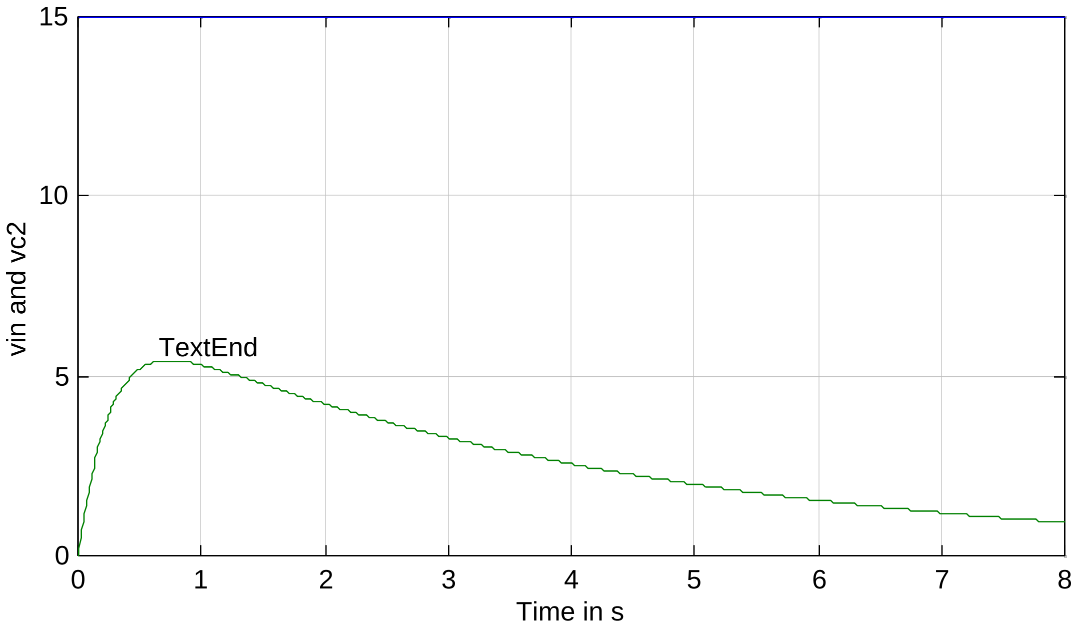
$$H(s) = \frac{V_{C2}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{\dfrac{R_2}{R_2C_2s + 1}}{\dfrac{1}{C_1s} + R_1 + \dfrac{R_2}{R_2C_2s + 1}}. = \frac{R_2C_1s}{R_1C_1R_2C_2s^2 + (\ R_1C_1 + R_2C_2 + R_2C_1)s + 1}$$

(b) If $v_{in}(t) = 15u(t)$ V, then $V_{in}(s) = 15/s$ and

$$V_{C2} = H(s)V_{in} \ = \frac{1.75s}{s^2 + 4.25\ s + 1} \times \frac{15}{s} = \frac{7}{s + 0.25} - \frac{7}{s + 4}$$

Hence

$$v_{C2}(t) = 7\left(e^{-0.25t} - e^{-4t}\right)u(t) \text{ V}$$



(c) Using the series equivalent for C_1, we have

$$V_{C2} = H(s)\frac{v_{C1}(0^-)}{s} \ = \frac{1.75s}{s^2 + 4.25\ s + 1} \times \frac{15}{s}$$

which is the same as result in part (b). Therefore

$$v_{C2}(t) = 7\left(e^{-0.25t} - e^{-4t}\right)u(t) \text{ V}$$

(d) Using the parallel equivalent for C_2, we have

$$V_{C2}(s) = C_2 v_{C2}(0^-) \frac{1}{\frac{2}{7}s + \frac{4}{7} + \frac{1}{2 + 1/s}} = \frac{2}{7}\times 15\times \frac{1}{\frac{2}{7}s + \frac{4}{7} + \frac{s}{2s + 1}} = \frac{1}{s + 0.25} + \frac{14}{s + 4}$$

Hence

$$v_{C2}(t) = \left(e^{-0.25t} + 14e^{-4t}\right)u(t) \text{ V}$$

(e) By linearity, the answer is the sum of parts (b), (c) and (d).

SOLUTION 14.45. (a) $Z_1 = R_1 + L_1 s$ and $Z_2 = \frac{R_2 L_2 s}{R_2 + L_2 s}$. From Ohm's law

$$I_{L1} = \frac{V_{in}}{Z_1 + Z_2} = \frac{V_{in}}{R_1 + L_1 s + \frac{R_2 L_2 s}{R_2 + L_2 s}} .= \frac{(R_2 + L_2 s)V_{in}}{(R_1 + L_1 s)(R_2 + L_2 s) + + R_2 L_2 s}$$

Using current division, we have

$$I_{L2} = \frac{R_2}{R_2 + L_2 s} I_{L1} = \frac{R_2 V_{in}}{(R_1 + L_1 s)(R_2 + L_2 s) + + R_2 L_2 s}$$

Therefore

$$H(s) = \frac{I_{L2}}{V_{in}} = \frac{R_2}{(R_1 + L_1 s)(R_2 + L_2 s) + + R_2 L_2 s} = \frac{G_1}{G_1 L_1 G_2 L_2 s^2 + (G_1 L_1 + G_2 L_2 + G_1 L_2)s + 1}$$

With the given element values,

$$H(s) = \frac{2}{2\times 1\times \frac{4}{7}\times \frac{7}{8}s^2 + (2\times 1 + \frac{4}{7}\times \frac{7}{8} + 2\times \frac{7}{8})s + 1} = \frac{2}{s^2 + 4.25s + 1}$$

(b) If $v_{in}(t) = 15u(t)$ V, then $V_{in}(s) = 15/s$ and

$$I_{L2} = H(s)V_{in} = \frac{2}{s^2 + 4.25s + 1}\times \frac{15}{s} = \frac{30}{s} - \frac{32}{s + 0.25} - \frac{2}{s + 4}$$

Hence

$$i_{L2}(t) = \left(30 - 32e^{-0.25t} + e^{-4t}\right)u(t) \text{ A}$$

Plot omitted.

(c) Using the series equivalent for L_1, we have

$$I_{L2} = H(s)\, L_1 i_{L1}(0^-) = \frac{2}{s^2 + 4.25s + 1}\times 1\times 15 = \frac{8}{s + 0.25} - \frac{8}{s + 4}$$

Therefore

$$i_{L2}(t) = 8\left(e^{-0.25t} - e^{-4t}\right)u(t) \text{ A}$$

(d) Using the parallel equivalent for L_2, we have

$$I_{L2}(s) = \frac{L_2 i_{L2}(0^-)}{L_2 s + \frac{R_2(R_1 + L_1 s)}{R_2 + (R_1 + L_1 s)}} = \frac{\frac{7}{8}\times 15}{\frac{7}{8}s + \frac{1.75(0.5 + s)}{1.75 + (0.5 + s)}} = \frac{15(\,s + 2.25)}{s^2 + 4.25s + 1} = \frac{8}{s + 0.25} + \frac{7}{s + 4}$$

Hence

$$i_{L2}(t) = \left(8e^{-0.25t} + 7\,e^{-4t}\right)u(t)\ \text{A}$$

(e) By linearity, the answer is the sum of parts (b), (c) and (d).

SOLUTION 14.46. (a) Using the result of problem 14.44(a)

$$\frac{V_{C2}}{2I_{in}} = \frac{R_2C_1 s}{R_1C_1R_2C_2 s^2 + (\,R_1C_1 + R_2C_2 + R_2C_1)s + 1} = \frac{1.75s}{s^2 + 4.25\,s + 1}$$

Therefore

$$H_1(s) = \frac{V_{C2}}{I_{in}} = \quad = \frac{3.5s}{s^2 + 4.25\,s + 1}$$

(b) Using the result of problem 14.45(a),

$$H_2(s) = \frac{I_{L2}}{V_{C2}} = \frac{R_2}{(R_1 + L_1 s)\,(R_2 + L_2 s)\ + + R_2L_2 s} = \frac{2}{s^2 + 4.25s + 1}$$

(c)

$$H(s) = \frac{I_{L2}}{I_{in}} = H_1(s)H_2(s) = \frac{3.5s}{s^2 + 4.25s + 1} \times \frac{2}{s^2 + 4.25s + 1} = \frac{7s}{(s^2 + 4.25s + 1)^2}$$

(d) We first represent the initialized capacitor by the series equivalent (figure 14.17), and then apply a source transformation. From this circuit, by utilizing the expression derived in part (c), we have

$$\frac{I_{L2}}{\frac{v_{C1}(0)}{2\,s}} = \frac{sI_{L2}}{7.5} H(s) = H_1(s)H_2(s) = \frac{7s}{(s^2 + 4.25s + 1)^2}$$

Therefore

$$I_{L2}(s) = \frac{52.5}{(s^2 + 4.25s + 1)^2} = \frac{1.9911}{s + 4} + \frac{3.7333}{(\,s + 4)^2} - \frac{1.9911}{s + 0.25} + \frac{3.7333}{(\,s + 0.25)^2}$$

and

$$i_{L2}(t) = [(1.9911 + 3.7333t)\,e^{-4t} + (-\,1.9911 + 3.7333t)\ e^{-0.25t}\ ]\,u(t)\ \text{A}$$

Note: the book answer for part (d) should be divided by 2.

(e) Since $I_{in}(s) = 15/s$, we have

$$I_{L2}(s) = H(s)I_{in}(s) = \frac{105}{(s^2 + 4.25s + 1)^2} = \frac{3.9822}{s+4} + \frac{7.4666}{(s+4)^2} - \frac{3.9822}{s+0.25} + \frac{7.4666}{(s+0.25)^2}$$

and

$$i_{L2}(t) = [(3.9822 + 7.4666t)\, e^{-4t} + (-3.9822 + 7.4666t)\, e^{-0.25t}]\, u(t) \text{ A}$$

SOLUTION 14.47. (a) For this passive circuit, we may write the nodal equations by inspection.

$$\begin{bmatrix} 0.8s + 2 + \frac{10}{s} & -\frac{10}{s} \\ -\frac{10}{s} & 1 + \frac{10}{s} \end{bmatrix} \begin{bmatrix} V_C \\ V_R \end{bmatrix} = \begin{bmatrix} 2V_{s1} \\ -I_{s2} \end{bmatrix}$$

(b) V_{S1}= 3/s and I_{S2} = 3/s. We solve for V_R by Cramer's rule to obtain

$$V_R = \frac{\begin{vmatrix} 0.8s + 2 + \frac{10}{s} & \frac{6}{s} \\ -\frac{10}{s} & -\frac{3}{s} \end{vmatrix}}{\begin{vmatrix} 0.8s + 2 + \frac{10}{s} & -\frac{10}{s} \\ -\frac{10}{s} & 1 + \frac{10}{s} \end{vmatrix}} = \frac{-2.4s^2 - 6s + 30}{s(0.8s^2 + 10s + 30)} = \frac{-4}{s+7.5} + \frac{1}{s}$$

and

$$v_R(t) = \left(1 - 4e^{-7.5t}\right)u(t) \text{ V}$$

(c) We represent the initialized capacitor by the parallel equivalent circuit (figures 14.16) In this case the nodal equations becomes

$$\begin{bmatrix} 0.8s + 2 + \frac{10}{s} & -\frac{10}{s} \\ -\frac{10}{s} & 1 + \frac{10}{s} \end{bmatrix} \begin{bmatrix} V_C \\ V_R \end{bmatrix} = \begin{bmatrix} \frac{6}{s} + 2.4 \\ -\frac{3}{s} \end{bmatrix}$$

Solve for V_R by Cramer's rule to obtain

$$V_R = \frac{\begin{vmatrix} 0.8s + 2 + \frac{10}{s} & \frac{6}{s} + 2.4 \\ -\frac{10}{s} & -\frac{3}{s} \end{vmatrix}}{\begin{vmatrix} 0.8s + 2 + \frac{10}{s} & -\frac{10}{s} \\ -\frac{10}{s} & 1 + \frac{10}{s} \end{vmatrix}} = \frac{-2.4s^2 + 18s + 30}{s(0.8s^2 + 10s + 30)} = \frac{1}{s} - \frac{16}{s+7.5} + \frac{12}{s+5}$$

$$v_R(t) = (1 - 16e^{-7.5t} + 12e^{-5t})\,u(t) \text{ V}$$

SOLUTION 14.48. (a) After performing the suggested source transformation, and representing the initialized capacitor and inductor by their series equivalent circuits, we can write two mesh equations by inspection:

$$\begin{bmatrix} 0.5 + \frac{1.25}{s} & -\frac{1.25}{s} \\ -\frac{1.25}{s} & 1 + 0.1s + \frac{1.25}{s} \end{bmatrix} \begin{bmatrix} I_{s1} \\ I_L \end{bmatrix} = \begin{bmatrix} V_{s1} - \frac{v_C(0)}{s} \\ \frac{v_C(0)}{s} + Li_L(0) + I_{s2} \end{bmatrix}$$

(b) With V_{S1}= 3/s, I_{S2} = 3/s, $v_C(0)$ = 0, and $i_L(0)$ = 3 A, the above mesh equation becomes

$$\begin{bmatrix} 0.5 + \frac{1.25}{s} & -\frac{1.25}{s} \\ -\frac{1.25}{s} & 1 + 0.1s + \frac{1.25}{s} \end{bmatrix} \begin{bmatrix} I_{s1} \\ I_L \end{bmatrix} = \begin{bmatrix} V_{s1} - \frac{v_C(0)}{s} \\ \frac{v_C(0)}{s} + Li_L(0) + I_{s2} \end{bmatrix} = \begin{bmatrix} \frac{3}{s} \\ 0.3 + \frac{3}{s} \end{bmatrix}$$

Solve for $I_L(s)$ by Cramer's rule to obtain

$$I_L(s) = \frac{0.15s^2 + 1.875s + 7.5}{0.05s^3 + 0.625s^2 + 1.875s} = \frac{2}{s + 7.5} + \frac{-3}{s + 5} + \frac{4}{s}$$

Therefore

$$i_L(t) = (4 + 2e^{-7.5t} - 3\,e^{-5t})\,u(t) \text{ A}$$

SOLUTION 14.49. (a) Represent the initialized capacitors by their parallel equivalent circuits.
(b) Write two nodals equation by inspection

$$\begin{bmatrix} 0.001s + 0.4 & -0.2 \\ -0.2 & 0.001s + 0.4 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \end{bmatrix} = \begin{bmatrix} 0.2V_{in} + 0.001v_{C1}(0^-) \\ 0.001v_{C2}(0^-) \end{bmatrix} = \begin{bmatrix} \frac{2.4}{s} + 0.006 \\ 0.002 \end{bmatrix}$$

(c) Solve for $V_{C2}(s)$ by Cramer's rule to obtain

$$V_{C2} = \frac{\begin{vmatrix} 0.001s + 0.4 & 2.4/s + 0.006 \\ -0.2 & 0.002 \end{vmatrix}}{\begin{vmatrix} 0.001s + 0.4 & -0.2 \\ -0.2 & 0.001s + 0.4 \end{vmatrix}} = \frac{2s^2 + 2\times10^3 s + 48\times10^4}{s(s^2 + 8\times10^2 s + 12\times10^4)} = \frac{0}{s + 600} + \frac{-2}{s + 200} + + \frac{4}{s}$$

(d)

$$v_{C2}(t) = (4 - 2e^{-200t})\,u(t) \text{ V}$$

SOLUTION 14.50. **(a)** Let VC denote the node voltage across the capacitor. By inspection the nodal equations in matrix form are:

$$\begin{bmatrix} 1+1/R+4s & -1/R \\ -1/R & 1+1/R+1/(4s) \end{bmatrix}\begin{bmatrix} V_C \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{in} \\ V_{in}/(4s) \end{bmatrix}$$

(b) By Cramer's rule,

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\det\begin{bmatrix} 1+1/R+4s & 1 \\ -1/R & 1/(4s) \end{bmatrix}}{\det\begin{bmatrix} 1+1/R+4s & -1/R \\ -1/R & 1+1/R+1/(4s) \end{bmatrix}} = \frac{(4s+1)(1+1/R)}{4s\left((1+1/R+4s)(1+4s/R+4s)-1/R^2\right)}$$

$$= \frac{(4s+1)(1+1/R)}{(1+8s+16s^2)(1+1/R)} = \frac{1}{(1+4s)}$$

Clearly, R does not affect the transfer function. The question is why? Note that the circuit can be redrawn as a balanced Wheatstone bridge circuit in which there is no voltage across R and no current through R. Hence R has no effect on the transfer function and on the impedance at the input. Hence R can be removed in the analysis of the circuit. In this case, the transfer function follows trivially by voltage division.

(c) In view of the answer to (b), the impedance can be calculated with R removed. Hence

$$Z_{in}(s) = \frac{\left(1+\frac{1}{4s}\right)(1+4s)}{\left(1+\frac{1}{4s}\right)+(1+4s)} = \frac{(1+4s)^2}{(1+4s)^2} = 1$$

Hence, the input impedance is a constant resistance and the network is called a constant resistance network.

(d) The input is $v_{in}(t) = 10e^{-at}u(t)$ V and R = 5 Ω. Find $v_{out}(t)$ for $t \geq 0$ for the three cases, $a = 0$, 0.5, 0.25.

(d) From part (b), for s ≠ 0.25,

$$V_{out}(s) = \frac{0.25}{(s+0.25)} \times \frac{10}{s+a} = \frac{2.5/(0.25-a)}{s+a} - \frac{2.5/(0.25-a)}{(s+0.25)}$$

which leads to

$$v_{out}(t) = \left(\frac{2.5}{0.25-a}\right)\left(e^{-at} - e^{-0.25t}\right)u(t) \text{ V}$$

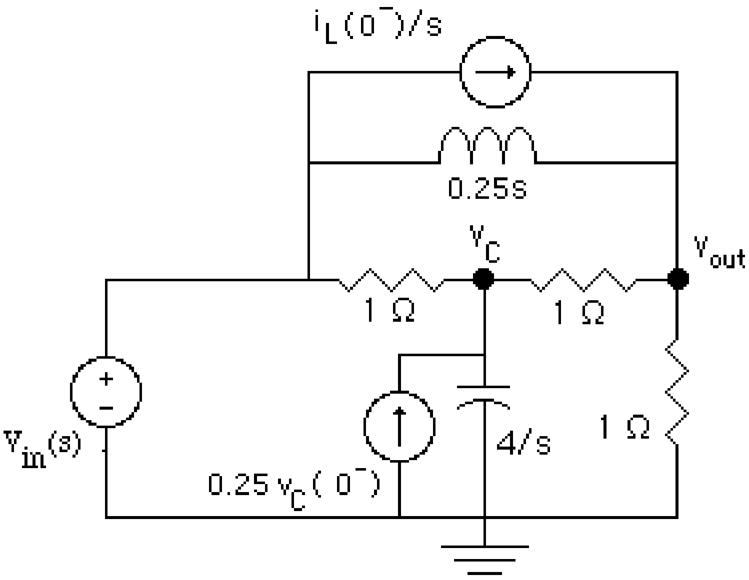
For a = 0.25,

$$V_{out}(s) = \frac{2.5}{(s+0.25)^2} \quad \text{implying that} \quad v_{out}(t) = 2.5te^{-0.25t}u(t) \text{ V}$$

SOLUTION 14.51.

(a) This bridged-T circuit was analyzed in problem 14.9. Here R = 1 Ω, $Z_1(s) = 0.25s$ and $Z_2(s) = 4/s$. Since the condition $Z_1(s)\, Z_2(s) = R^2$ is met, we have $Z_{in}(s) = 1\Omega$.

(b) The s-domain equivalent circuit accounting for initial conditions is given below.



(c) Two nodal equations at V_C and V_{out} are:

$$\left(V_C - V_{in}\right) + \left(V_C - V_{out}\right) + 0.25sV_C = 0.25\ v_C(0^-)$$

and

$$\left(V_{out} - V_C\right) + \frac{\left(V_{out} - V_{in}\right)}{0.25s} + V_{out} = \frac{i_L(0^-)}{s}$$

Writing these in matrix form, we have

$$\begin{bmatrix} 0.25s + 2 & -1 \\ -1 & \frac{4}{s} + 2 \end{bmatrix} \begin{bmatrix} V_C \\ V_{out} \end{bmatrix} = \begin{bmatrix} V_{in} + 0.25v_C(0^-) \\ \frac{4V_{in}}{s} + \frac{i_L(0^-)}{s} \end{bmatrix}$$

Solving for V_{out} by Cramer's rule yields

$$V_{out}(s) = \frac{4}{s+4}V_{in}(s) + \frac{0.5(s+8)}{(s+4)^2}i_L(0^-) + \frac{0.5s}{(s+4)^2}v_C(0^-)$$

(d) Given $v_{in}(t) = 4u(t) - 3e^{-t}u(t)$ V, then

$$V_{in}(s) = \frac{4}{s} - \frac{3}{s+1} = \frac{s+4}{s(s+1)}$$

and

$$V_{out}(s) = \frac{4}{s(s+1)} + \frac{0.25(s+8)}{(s+4)^2} + \frac{0.75s}{(s+4)^2}$$

Taking the inverse Laplace transform, we obtain, for t ≥ 0,

$$v_{out}(t) = (4 - 4e^{-t}) + (0.25e^{-4t} + te^{4t}) + (0.75e^{-4t} - 3te^{4t}) = 4 - 4e^{-t} + e^{-4t} - 2te^{4t} \quad \text{V}$$

SOLUTION 14.52. A supernode is defined by drawing a curve to enclose the controlled voltage source. One node within the supernode has voltage V_{out} and the other has voltage $V_{1\Omega}$ that is equal to

$$V_{1\Omega} = -2I_1 - V_{out} = -2\frac{V_{in} - V_C}{2} = -V_{in} + V_C - V_{out}$$

Next, we write nodal equations at V_C and the supernode: At node V_C

$$0.5(V_C - V_{in}) + 0.5sV_C + \frac{0.5}{s}(V_C - V_{out}) = 0$$

At the supernode

$$\frac{1}{2s}(V_{out} - V_C) + \frac{1}{2}V_{out} + \frac{-V_{in} + V_C + V_{out}}{1} = 0$$

In matrix form, the nodal equations are:

$$\begin{bmatrix} 0.5(s+1+1/s) & -0.5/s \\ 1 - 0.5/s & 1.5 + 0.5/s \end{bmatrix}\begin{bmatrix} V_C \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0.5V_{in} \\ V_{in} \end{bmatrix}$$

Solving by Cramer's rule yields

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{0.5s + 0.75/s}{0.75s + 1 + 1.5/s} = \frac{2(s^2 + 1.5)}{3s^2 + 4s + 6}$$

SOLUTION 14.53. A supernode is defined by drawing a curve to enclose the controlled voltage source. One node within the supernode has voltage V_{out} and the other has voltage $V_{1\Omega}$ which is equal to

$$V_{1\Omega} = -2I_1 - V_{out} = -2\frac{V_{in} - V_C}{2} = -V_{in} + V_C - V_{out}$$

Next, write nodal equations at V_C and the supernode: At node V_C

$$0.5\,(V_C - V_{in}) + \left(0.25s + \frac{4}{s}\right)V_C + \frac{s}{s^2+16}(V_C - V_{out}) = 0$$

At the supernode

$$\frac{s}{s^2+16}(V_{out} - V_C) + \frac{1}{2}V_{out} + \frac{-V_{in} + V_C + V_{out}}{1} = 0$$

In matrix form, the nodal equation are:

$$\begin{bmatrix} 0.25s + 0.5 + s/(s^2+16) & -s/(s^2+16) \\ 1 - s/(s^2+16) & 1.5 + s/(s^2+16) \end{bmatrix} \begin{bmatrix} V_C \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0.5\,V_{in} \\ V_{in} \end{bmatrix}$$

Solving by Cramer's rule yields

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{0.25s^3 + 5.5s}{0.375s^3 + s^2 + 9s + 12}$$

SOLUTION 14.54. Write nodal equations at V_1 and V_2:

$$0.5\,(V_1 - V_{in}) + 0.125(V_1 + 0.2V_2) + 0.1s(V_1 - V_2) = 0$$

and

$$0.1s(V_2 - V_1) + \frac{1}{4}V_2 + \frac{V_2}{1} - 5V_1 = 0$$

In matrix form, the nodal equations are:

$$\begin{bmatrix} (0.1s + 0.625) & (-0.1s + 0.025) \\ (-0.1s - 5) & (0.1s + 1.25) \end{bmatrix} \begin{bmatrix} V_C \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0.5\,V_{in} \\ 0 \end{bmatrix}$$

Solving by Cramer's rule yields

$$H(s) = \frac{I_{out}}{V_{in}} = \frac{V_2}{V_{in}} = \frac{0.05s + 2.5}{-0.31s + 0.9062}$$

SOLUTION 14.55.

(a) Simply replace each capacitor by the parallel form circuit model given in figure 14.16.

(b) For this passive circuit, we can write the nodal equation by inspection.

$$\begin{bmatrix} 0.5s+2 & -1 & 0 \\ -1 & 0.5s+2 & -1 \\ 0 & -1 & 0.5s+2 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ V_{C3} \end{bmatrix} = \begin{bmatrix} V_{in} + 0.5v_{C1}(0) \\ 0.5v_{C2}(0) \\ 0.5v_{C3}(0) \end{bmatrix}$$

Solving for V_{C3} by Cramer's rule yields

$$V_{C3}(s) = \frac{V_{in}(s) + 0.5v_{C1}(0) + (0.25s+1)v_{C2}(0) + (0.125s^2 + s + 1.5)v_{C3}(0)}{0.125s^3 + 1.5s^2 + 5s + 4}$$

(c) Substituting $V_{in}(s) = 12/s$, $v_{C1}(0) = 0$, $v_{C2}(0) = 6$, and $v_{C3}(0) = 2$ into the above expression, we obtain

$$V_{C3}(s) = \frac{12/s + 6(0.25s+1) + 2(0.125s^2 + s + 1.5)}{0.125s^3 + 1.5s^2 + 5s + 4} = \frac{0.25s^3 + 3.5s^2 + 9s + 12}{s(0.125s^3 + 1.5s^2 + 5s + 4)}$$

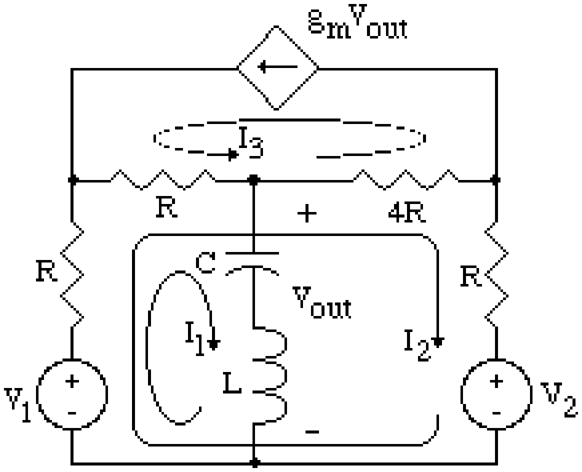
Now use MATLAB to do the partial fraction expansion.

```
n= [ 0.25 3.5 9 12];
d= [ 0.125 1.5 5 4 0];
[ r p k ] = residue (n,d)
r =
   -2.5000
    4.0000
   -2.5000
    3.0000
p =
   -6.8284
   -4.0000
   -1.1716
         0
```

From the MATLAB output, we have, for t ≥ 0,

$$v_{C3}(t) = 3 - 2.5\,e^{-6.828t} + 4e^{-4t} - 2.5e^{1.1716t} \quad \text{V}$$

SOLUTION 14.56. For this problem we utilize loop analysis with loops as indicated below.



In doing the following loop analysis, note that we will use $g_mV_{out} = I_3$ and that due to our judicious choice of loops

$$V_{out} = \left(\frac{1}{Cs} + Ls\right)I_1$$

or equivalently,

$$0 = \left(\frac{1}{Cs} + Ls\right)I_1 - V_{out}$$

For loop 1,

$$V_1 = \left(2R + \frac{1}{Cs} + Ls\right)I_1 + 2R\,I_2 + Rg_mV_{out}$$

For loop 2,

$$V_1 - V_2 = 2R\,I_1 + 7R\,I_2 + 5Rg_mV_{out}$$

In Matrix form

$$\begin{bmatrix} \frac{1000}{s} + 0.016s & 0 & -1 \\ 2 + \frac{1000}{s} + 0.016s & 2 & 2 \\ 2 & 7 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0 \\ V_1 \\ V_1 - V_2 \end{bmatrix}$$

By Cramer's rule

$$V_{out} = \frac{\det\begin{bmatrix} \frac{1000}{s} + 0.016s & 0 & 0 \\ 2 + \frac{1000}{s} + 0.016s & 2 & V_1 \\ 2 & 7 & V_1 - V_2 \end{bmatrix}}{\det\begin{bmatrix} \frac{1000}{s} + 0.016s & 0 & -1 \\ 2 + \frac{1000}{s} + 0.016s & 2 & 2 \\ 2 & 7 & 10 \end{bmatrix}} = \frac{-\left(\frac{1000}{s} + 0.016s\right)(5V_1 + 2V_2)}{\frac{6000}{s} + 0.096s - \left(10 + \frac{7000}{s} + 0.112s\right)}$$

Hence

$$V_{out} = \frac{\left(\frac{1000}{s} + 0.016s\right)(5V_1 + 2V_2)}{10 + \frac{1000}{s} + 0.016s} = \frac{s^2 + 62500}{s^2 + 625s + 62500}(5V_1 + 2V_2)$$

The answers to (a) and (b) are clear at this point.

(c) Using MATLAB

```
»n = [21   0   21*62500];
»d = [1  625   62500  0];
»[r,p,k] = residue(n,d)
r =
    35
   -35
    21
p =
  -500
  -125
     0
k =     []
```

Hence

$$v_{out}(t) = \left(21 - 35e^{-125t} + 35e^{-500t}\right)u(t) \text{ V.}$$

SOLUTION 14.57. (a) Replace the LC combination by a 1 V source after setting V_1 and V_2 to zero. We need to compute the current leaving the 1 V source which will be $1/R_{th}$. Let the left node be denoted by V_a and the right node by V_b. Also let G = 1/R. The nodal equations are by inspection

$$\begin{bmatrix} 2G & 0 \\ 0 & 1.25G \end{bmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} g_m + G \\ -g_m + 0.25G \end{bmatrix} \qquad \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \left(g_m + G\right)/2G \\ \left(-g_m + 0.25G\right)/1.25G \end{bmatrix}$$

Thus, the current leaving the 1 V source is

$$I_{1V} = G(1-V_a) + 0.25G(1-V_b) = G\left(1-(g_m+G)/2G\right) + 0.25G\left(1-(-g_m+0.25G)/1.25G\right)$$
$$= 0.5G - 0.5g_m + 0.25G + 0.2g_m - 0.05G = 0.7G - 0.3g_m$$

Substituting G = 1/R we obtain

$$R_{th} = \frac{1}{I_{1V}} = \frac{1}{0.7G - 0.3g_m} = \frac{R}{0.7 - 0.3g_m R} = \frac{1}{0.7 - 0.3\times 2} = 10\ \Omega$$

(b) Replace the LC combination by a short circuit and compute Isc. This makes the controlled source zero. By inspection

$$I_{sc} = \frac{V_1}{2R} + \frac{V_2}{5R}$$

Thus

$$V_{oc} = R_{th}I_{sc} = R_{th}\left(\frac{V_1}{2R} + \frac{V_2}{5R}\right) = 10(0.5V_1 + 0.2V_2)$$

(c) By voltage division

$$V_{out} = \frac{Z_{LC}}{R_{th} + Z_{LC}}V_{oc} = \frac{\frac{1}{Cs} + Ls}{\frac{1}{Cs} + Ls + R_{th}}V_{oc} = \frac{s^2 + 1/LC}{s^2 + \frac{R_{th}}{L}s + 1/LC}(5V_1 + 2V_2)$$

$$= \frac{s^2 + 62500}{s^2 + 625s + 62500}(5V_1 + 2V_2)$$

SOLUTION 14.58. (a) The last equation is the constraint equation for the controlled floating voltage source. Hence, we have

$$V_1 - V_2 - z_0(s)I_0 = 0$$

(b) By Cramer's rule,

$$V_2 = \frac{\det\begin{bmatrix}\left(\frac{1}{R} + Cs\right) & I_{in} & 1\\ 0 & 0 & -1\\ 1 & 0 & -z_0(s)\end{bmatrix}}{\det\begin{bmatrix}\left(\frac{1}{R} + Cs\right) & 1/R & 1\\ 0 & Cs & -1\\ 1 & -1 & -z_0(s)\end{bmatrix}} = \frac{-I_{in}}{-2\left(\frac{1}{R} + Cs\right) - z_0(s)Cs\left(\frac{1}{R} + Cs\right)} = \frac{I_{in}}{\left(\frac{1}{R} + Cs\right)(2 + z_0(s)Cs)}$$

(c) Here

$$V_2 = \frac{I_{in}}{\left(\frac{1}{R} + Cs\right)\left(2 + LCs^2\right)}$$

in which case $\omega = \sqrt{\frac{2}{LC}}$.

SOLUTION 14.59. (a) Since the switch has been at position A for a very long time, the inductor looks like a short and $i_L(5^-) = i_L(5^+) =$ 10/4 = 2.5 A. For t > 5, the switch moves to position B and the inductor current decays according to

$$i_L(t) = i_L(5^+)e^{-(t-5)/\tau} = 2.5e^{-(t-5)/0.1} = 2.5e^{-10(t-5)} \text{ A}$$

(b) Note that $i_L(0^-) = i_L(0^+) = 0$. Hence

$$I_L(s) = \frac{1}{10s+4}V_{in}(s) = \frac{0.1}{s+0.4}\left(\frac{50}{s} - \frac{50}{s+0.5}\right) = \frac{12.5}{s} - \frac{62.5}{s+0.4} + \frac{50}{s+0.5}$$

Hence for 0 ≤ t ≤ 5s, $i_L(t) = 12.5 - 62.5e^{-0.4t} + 50e^{-0.5t}$ A. Here $i_L(5^-) = i_L(5^+) =$ 8.1458 A. For t > 5, the inductor decays with a time constant of 0.1 s. Thus

$$i_L(t) = 8.1458e^{-10(t-5)}$$

SOLUTION 14.60. (a) Since the switch has been at position A for a very long time, the capacitor looks like an open and $v_C(5^-) = v_C(5^+) =$ 40 V. For t > 5, the switch moves to position B and the capacitor voltage decays according to

$$v_C(t) = v_C(5^+)e^{-(t-5)/\tau} = 40e^{-(t-5)/2} = 40e^{-0.5(t-5)} \text{ V}$$

(b) Note that $v_C(0^-) = v_C(0^+) = 0$. Hence

$$V_C(s) = \frac{1/Cs}{1/Cs + 40}V_{in}(s) = \frac{12.5}{s+12.5}\left(\frac{50}{s} - \frac{50}{s+12.5}\right) = \frac{7812.5}{s(s+12.5)^2}$$

In MATLAB,

```
»syms t s
»ilaplace(7.8125e3/(s*(s+12.5)^2))
ans =
50-625*t*exp(-25/2*t)-50*exp(-25/2*t)
```

Hence for 0 ≤ t ≤ 5s, $v_C(t) = 50 - 625te^{-12.5t} - 50e^{-12.5t}$ V. Here $v_C(5^-) = v_C(5^+) =$ 50 V. For t > 5, the capacitor voltage decays with a time constant of 0.08 s. Thus

$$v_C(t) = 50e^{-12.5(t-5)} \text{ V}$$

SOLUTION 14.61. (a) Since the switch has been closed for a very long time, the capacitor looks like an open and $v_C(5^-) = v_C(5^+) = 32$ V. For t > 5, the switch opens and the capacitor voltage decays according to

$$v_C(t) = v_C(5^+)e^{-(t-5)/\tau} = 32e^{-(t-5)/0.4} = 32e^{-2.5(t-5)} \text{ V}$$

(b) Note that $v_C(0^-) = v_C(0^+) = 0$ and $v_{out} = v_C$. Hence for $0 \le t \le 5$,

$$V_C(s) = \frac{1}{\frac{1}{50}+\frac{1}{200}+0.002s}\,\frac{V_{in}(s)}{50} = \frac{10}{s+12.5}V_{in}(s) = \frac{10}{s+12.5}\left(\frac{50}{s} - \frac{50}{s+12.5}\right)$$
$$= \frac{6250}{s(s+12.5)^2}$$

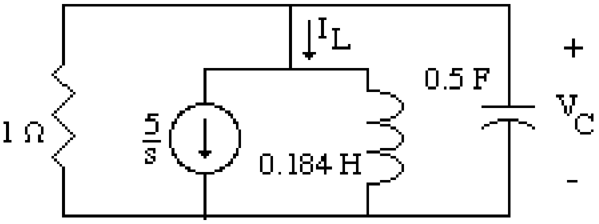
In MATLAB,

```
»syms t s
»ilaplace(6250/(s*(s+12.5)^2))
ans =
40-500*t*exp(-25/2*t)-40*exp(-25/2*t)
```

Hence for $0 \le t \le 5$s, $v_C(t) = 40 - 500te^{-12.5t} - 40e^{-12.5t}$ V. Here $v_C(5^-) = v_C(5^+) = 40$ V. For t > 5, the capacitor voltage decays with a time constant of 0.4 s. Thus

$$v_C(t) = 40e^{-2.5(t-5)} \text{ V}$$

SOLUTION 14.62. (a) At 0-, $v_C(0^-) = 0$ and $i_L(0^-) = 50/10 = 5$ A.
(b) For this part, consider the equivalent circuit below.



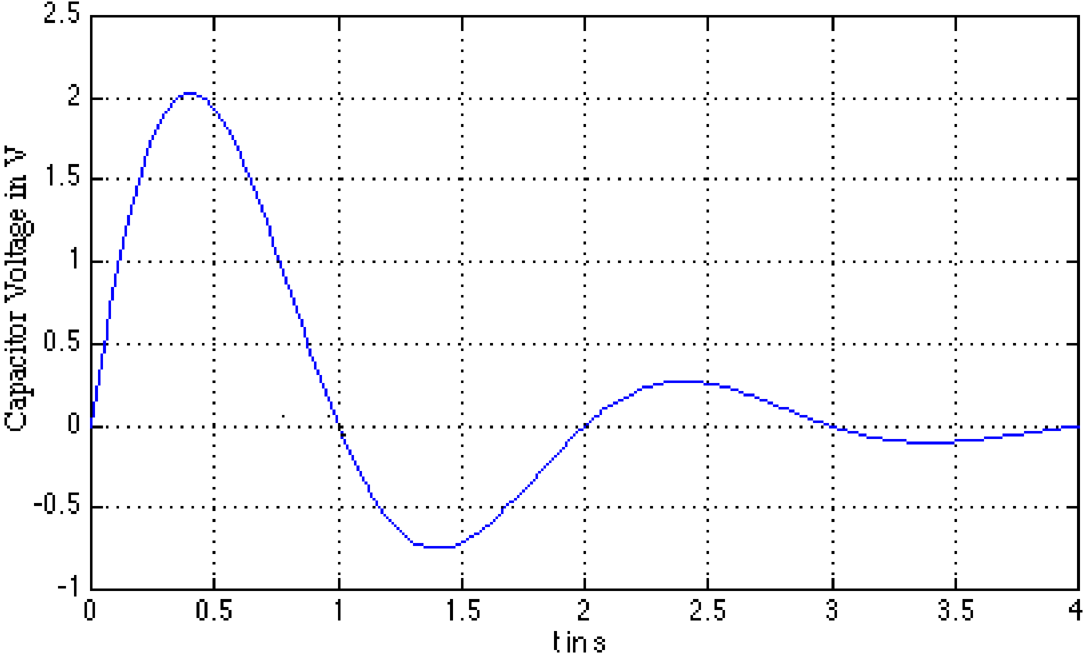
By inspection,

$$V_C = \frac{5}{s\left(1+0.5s+\frac{1}{0.184s}\right)} = \frac{5}{0.5s^2+s+\frac{1}{0.184}} = \frac{10}{(s+1)^2+\pi^2}$$

From table 13.1,

$v_C(t) = 3.1831e^{-t}\sin(\pi t)u(t)$ V

(c) Using MATLAB,



SOLUTION 14.63. (a) Since the switch has been closed for a long time, $i_L(1^-) = i_L(1^+) = 30/0.8 = 37.5$ A and $v_C(1^-) = v_C(1^+) = 0$. Represent the initialized inductor by its parallel equivalent circuit. Then

$$V_C(s) = \frac{-37.5}{s} \times \frac{1}{Cs + \frac{1}{Ls}} = \frac{-150}{s^2 + 4}$$

Hence from table 13.1,

$$v_C(t') = -75\sin(2t') \text{ V} \qquad v_C(t) = -75\sin(2(t-1)) \text{ V for t > 1s.}$$

(b) All initial conditions at t = 0 are zero. For $0 \le t \le 1$s,

$$V_C(s) = \frac{1}{\frac{1}{0.8} + 0.25s + \frac{1}{s}} \frac{V_{in}(s)}{0.8} = \frac{5s}{s^2 + 5s + 4} V_{in}(s) = \frac{150s}{s^2 + 5s + 4}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$$= \frac{300s}{s^2 + 5s + 4}\left(\frac{1}{s(s+2)}\right) = \frac{100}{s+1} - \frac{150}{s+2} + \frac{50}{s+4}$$

Hence, for $0 \le t \le 1$s,

$$v_C(t) = 100e^{-t} - 150e^{-2t} + 50e^{-4t} \text{ V}$$

Here, $v_C(1-) = v_C(1+) = 17.403$ V. Note, $v_{in}(1-) = 25.94$ V. Next,

$$i_C(1^-) = C\frac{dv_C}{dt}\bigg]_{t=1} = 0.25\left[-100e^{-1} + 300e^{-2} - 200e^{-4}\right] = 0.037378$$

Thus in MATLAB

```
»vin1 = 30*(1 - exp(-2))
vin1 =   2.5940e+01
»vc1 = 17.403
vc1 =   1.7403e+01
»ic1 = 0.25*(-100*exp(-1) + 300*exp(-2)-200*exp(-4))
ic1 =   3.7378e-02
»iL1 = (vin1 - vc1)/0.8 - ic1
iL1 =   1.0634e+01
```

Therefore, $i_L(1^-) = i_L(1^+) = 10.634$ A. For t ≥ 1, we use the parallel equivalent circuit for both the inductor and the capacitor:

$$V_C(s) = e^{-s}\frac{4s}{s^2+4}\left(\frac{-i_L(1^+)}{s} + Cv_C(1^+)\right) = e^{-s}\frac{4s}{s^2+4}\left(\frac{-10.634}{s} + 4.3507\right)$$

Therefore from table 13.1, for t > 1,

$$v_C(t) = -21.268\sin\big(2(t-1)\big) + 17.403\cos\big(2(t-1)\big) \text{ V}$$

Plots omitted.

SOLUTION 14.64. Here $v_C(0) = 0$ for both capacitors.
Part 1: 0 ≤ t ≤ 1s.

$$V_{out}(s) = \frac{\frac{10}{s}}{20 + \frac{10}{s}}\left(\frac{20}{s} - \frac{20}{s+2}\right) = \frac{20}{s(s+0.5)(s+2)}$$

From MATLAB

```
»num = 20;
»den = [1  2.5 1 0];
»[r,p,k] = residue(num,den)
r =
   6.6667e+00
  -2.6667e+01
   2.0000e+01
p =
  -2.0000e+00
  -5.0000e-01
           0
k =     []
```

Therefore for 0 ≤ t ≤ 1,

$$v_{out}(t) = \left(20 - 26.667e^{-0.5t} + 6.667e^{-2t}\right)\left(u(t) - u(t-1)\right)$$

Part 2. 1 ≤ t. Here the initial condition for the right-most capacitor is $v_{out}(1^-) = v_{out}(1^+) = 4.7281$ V. As above, the left-most capacitor has zero value at t = 1s. Let us use the series equivalent circuit for the right capacitor. Then,

$$I_C(s) = e^{-s}\frac{-4.7281}{s} \times \frac{1}{10 + \frac{20}{s}} = e^{-s}\frac{-0.47281}{s+2}$$

Therefore,

$$V_{out}(s) = \left[\frac{10}{s} I_C(s) + e^{-s}\frac{4.7281}{s}\right] = e^{-s}\left[\frac{-4.7281}{s(s+2)} + \frac{4.7281}{s}\right] = e^{-s}\left[\frac{2.3641}{s} + \frac{2.3641}{s+2}\right]$$

and for t ≥ 1,

$$v_{out}(t) = \left(2.3641 + 2.3641e^{-2(t-1)}\right)u(t-1) \text{ V}$$

SOLUTION 14.65. Assume the switch has been in position A for a long time. Both capacitors behave as open circuits and both capacitors have initial voltages at t = 0 of 10 V. For t ≥ 0, use the parallel equivalent circuits for both capacitors and write nodal equations. Let the left capacitor have voltage V_{Ca}.

$$\begin{bmatrix} 0.005s + 0.03 & -0.01 \\ -0.01 & 0.0025s + 0.01 \end{bmatrix}\begin{bmatrix} V_{Ca} \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0.005 \times 10 \\ 0.0025 \times 10 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.025 \end{bmatrix}$$

By Cramer's rule,

$$V_{out}(s) = \frac{\det\begin{bmatrix} 0.005s + 0.03 & 0.05 \\ -0.01 & 0.025 \end{bmatrix}}{\det\begin{bmatrix} 0.005s + 0.03 & -0.01 \\ -0.01 & 0.0025s + 0.01 \end{bmatrix}} = 10\frac{s+10}{s^2 + 10s + 16} = \frac{40/3}{s+2} - \frac{10/3}{s+8}$$

Therefore for t ≥ 0,

$$v_{out}(t) = \frac{40}{3}e^{-2t} - \frac{10}{3}e^{-8t} \text{ V}$$

SOLUTION 14.66. (a) $Z_{in}(s) = 2 + \dfrac{1}{0.5s + \dfrac{2}{s}} = 2 + \dfrac{2s}{s^2 + 4}$.

(b) Here, the initial condition is zero and

$$V_C(s) = \frac{\frac{2s}{s^2+4}}{2+\frac{2s}{s^2+4}} \times \frac{10}{s} = \frac{10}{s^2+s+4}$$

```
»syms t s
»ilaplace(10/(s^2+s+4))
ans =
4/3*exp(-1/2*t)*15^(1/2)*sin(1/2*15^(1/2)*t)
```

Hence using MATLAB above or table 13.1 we have for 0 ≥ t ≤ 1.5s,

$$v_C(t) = 5.164e^{-0.5t}\sin(1.9365t)\text{ V}$$

and

$$v_C(1.5^-) = v_C(1.5^+) = 0.57237\text{ V}$$

(c) Use the parallel equivalent circuit for the left capacitor. The right capacitor has a zero initial voltage at t = 1.5. Hence, we do not use an equivalent circuit for the right capacitor.

(d) Therefore

$$e^{1.5s}V_C(s) = \frac{1}{0.5s+2+\frac{s}{s+4}} \times 0.5 \times 0.57237 = 0.57237\frac{s+4}{s^2+10s+16}$$

(e) In MATLAB

```
»[r,p,k] = residue(0.57237*[1 4],[1 10 16])
r =V
   3.8158e-01
   1.9079e-01
p =
    -8
    -2
k =
     []
```

Hence

$$V_C(s) = e^{-1.5s}0.57237\frac{s+4}{s^2+10s+16} = e^{-1.5s}\left[\frac{0.19079}{s+2}+\frac{0.38158}{s+8}\right]$$

(f) Finally

$$v_C(t) = \left[0.19079e^{-2(t-1.5)} + 0.38158e^{-8(t-1.5)}\right]u(t-1.5)\text{ V}$$

SOLUTION 14.67. (a) $v_1(0^-) = v_1(0^+) = v_2(0^-) = v_2(0^+) = \frac{R}{2R}16 = 8$ V.

(b) For 0 ≤ t ≤ 1, $v_1(t) = 8$ V and $v_2(t) = 8e^{-t/RC} = 8e^{-0.6931t}$ V.

(c) $v_1(1^-) = 8$ V and $v_2(1^-) = 8e^{-0.6931} = 4$ V.

(d) From KVL, $v_1(1^+) = v_2(1^+)$. From conservation of charge, $1 \times v_1(0^-) + 1 \times v_2(0^-) = 12 = 2 \times v_1(0^+) = 2 \times v_2(0^+)$. Therefore $v_1(1^+) = v_2(1^+) = 6$ V.

(e) This response represents a decay with time constant τ = 2R = 2.8854 s. Hence

$$v_1(t) = v_2(t) = 6e^{-0.34657(t-1)}u(t-1) \text{ V}$$

It follows that

$$v_1(3) = v_2(3) = 6e^{-0.34657\times 2} = 3 \text{ V}$$

(f) Both capacitor voltages change abruptly at t = 1.

SOLUTION 14.68. Label the current down through the first inductor as $i_1(t)$.

(a) $i_1(0^-) = i_1(0^+) = 1$ A and $i_{out}(0^-) = i_{out}(0^+) = 0$.

(b) For 0 ≤ t we use a parallel equivalent for the first inductor. By current division

$$I_{out}(s) = \frac{\dfrac{1}{5+0.1s}}{0.057143 + \dfrac{1}{0.35s} + \dfrac{1}{5+0.1s}} \times \frac{-1}{s} = \frac{-175}{s^2 + 275s + 2500}$$

Use MATLAB to do the partial fraction expansion

```
num = -175; den = [ 1  275  2500]; [ r, p, k] = residue (num, den)

r =
    0.6831
   -0.6831
p =
 -265.5869
   -9.4131
```

From the MATLAB output

$$I_{out}(s) = -\frac{0.68313}{s+9.4131} + \frac{0.68313}{s+265.59}$$

Therefore ,

$$i_{out}(t) = 0.68313\left(e^{-265.59t} - e^{-9.413t}\right)u(t) \text{ A}$$

SOLUTION 14.69. (a) Here we use voltage division:

$$V_1(s) = \frac{\dfrac{1}{4\times10^{-6}s}}{\dfrac{1}{4\times10^{-6}s}+\dfrac{1}{4\times10^{-6}s}+\dfrac{1}{1\times10^{-6}s}} \times \frac{30}{s} = \frac{30}{6s} = \frac{5}{s}$$

Therefore, $v_1(0^+) = 5$ V.

(b) Again use voltage division:

$$V_1(s) = \frac{\dfrac{1}{5\times10^{-6}s}}{\dfrac{1}{5\times10^{-6}s}+\dfrac{1}{1\times10^{-6}s}+\dfrac{1}{2\times10^{-6}s}} \times \frac{40}{s} = \frac{80}{17s}$$

Therefore, $v_1(0^+) = 4.7059$ V.

SOLUTION 14.70. (a) Consider a mesh current I(s) in the usual direction and use the series equivalent circuit for each capacitor. Thus

$$I(s) = \frac{1}{\dfrac{1}{4\times10^{-6}s}+\dfrac{1}{4\times10^{-6}s}+\dfrac{1}{1\times10^{-6}s}}\left(\frac{3}{s}-\frac{0.3}{s}-\frac{0.9}{s}-\frac{0.6}{s}\right) = \frac{1.2}{1.5}\times10^{-6} = 0.8\times10^{-6}$$

Therefore for t > 0,

$$V_1(s) = \frac{1}{4\times10^{-6}s}I(s) + \frac{0.3}{s} = \frac{0.8\times10^{-6}}{4\times10^{-6}s} + \frac{0.3}{s} = \frac{0.5}{s} \qquad v_1(t) = 0.5 \text{ V}$$

Similarly for t > 0

$$V_2(s) = \frac{0.8\times10^{-6}}{4\times10^{-6}s} + \frac{0.9}{s} = \frac{1.1}{s} \qquad v_2(t) = 1.1 \text{ V}$$

and

$$V_3(s) = \frac{0.8\times10^{-6}}{1\times10^{-6}s} + \frac{0.6}{s} = \frac{1.4}{s} \qquad v_3(t) = 1.4 \text{ V}$$

(b) Consider a mesh current I(s) in the usual direction and use the series equivalent circuit for each capacitor. Thus

$$I(s) = \frac{1}{\dfrac{1}{5\times10^{-6}s}+\dfrac{1}{1\times10^{-6}s}+\dfrac{1}{2\times10^{-6}s}}\left(\frac{4}{s}-\frac{0.3}{s}-\frac{0.9}{s}-\frac{0.6}{s}\right) = \frac{2.2}{1.7}\times10^{-6} = 1.2941\times10^{-6}$$

Therefore for t > 0,

$$V_1(s) = \frac{1}{5\times10^{-6}s}I(s) + \frac{0.3}{s} = \frac{1.2941\times10^{-6}}{5\times10^{-6}s} + \frac{0.3}{s} = \frac{0.55882}{s} \qquad v_1(t) = 0.55882 \text{ V}$$

Similarly for t > 0

$$V_2(s) = \frac{1.2941\times 10^{-6}}{1\times 10^{-6}s} + \frac{0.9}{s} = \frac{2.1941}{s} \qquad v_2(t) = 2.1941 \text{ V}$$

and

$$V_3(s) = \frac{1.2941\times 10^{-6}}{2\times 10^{-6}s} + \frac{0.6}{s} = \frac{1.2471}{s} \qquad v_3(t) = 1.2471 \text{ V}$$

SOLUTION 14.71. (a) For 0 < t < 2, the 150 mF capacitor is charged to 25 V. From conservation of charge,

$$0.15\times 25 = 0.15v_C(2^+) + 0.1v_C(2^+) = 0.25v_C(2^+)$$

Therefore

$$v_C(2^+) = \frac{0.15\times 25}{0.25} = 15 \text{ V}$$

This voltage remains constant for t > 2 s.

(b) For 0 < t < 2, the 150 mF capacitor is charged to 25 V. From conservation of charge,

$$0.15\times 25 + 0.1\times 10 = 0.15v_C(2^+) + 0.1v_C(2^+) = 0.25v_C(2^+)$$

Therefore

$$v_C(2^+) = \frac{0.15\times 25 + 0.1\times 10}{0.25} = 19 \text{ V}$$

This voltage remains constant for t > 2 s.

SOLUTION 14.72. (a) Let the middle node have voltage $V_a(s)$. Then writing node equations

$$\begin{bmatrix} 8s & -2s \\ -2s & 6s \end{bmatrix}\begin{bmatrix} V_a \\ V_{out} \end{bmatrix} = \begin{bmatrix} 1.144 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} V_a \\ V_{out} \end{bmatrix} = \frac{1}{44}\begin{bmatrix} 6 & 2 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 1.144/s \\ 0 \end{bmatrix} = \begin{bmatrix} 0.156/s \\ 0.052/s \end{bmatrix} \text{V}$$

Thus for t > 0, $v_{out}(t) = 0.052$ V.

(b) Again define $V_a(s)$ as the middle node. Then

$$V_a(s) = \frac{\dfrac{3}{10s}}{\dfrac{1}{10s} + \dfrac{1}{4s} + \dfrac{3}{10s}} \times \frac{0.286}{s} = \frac{6}{13} \times \frac{0.286}{s} = \frac{0.132}{s}$$

Hence for t > 0,

$$V_{out}(s) = \frac{\dfrac{1}{4s}}{\dfrac{1}{2s} + \dfrac{1}{4s}} \times \frac{0.132}{s} = \frac{0.132}{3s} = \frac{0.044}{s} \qquad v_{out}(t) = 0.044 \text{ V}$$

SOLUTION 14.73. With switches in position A, the equivalent capacitance to the right of v_2 is 4 mF. Therefore at t = 0+, by voltage division

$$V_1(s) = V_2(s) = \frac{10}{s} \qquad v_1(t) = v_2(t) = 10 \text{ V} \text{ for } 0 < t < 1.$$

Hence with the switches in position B, let us write a single node equation using the parallel equivalent circuit the initialized capacitors:

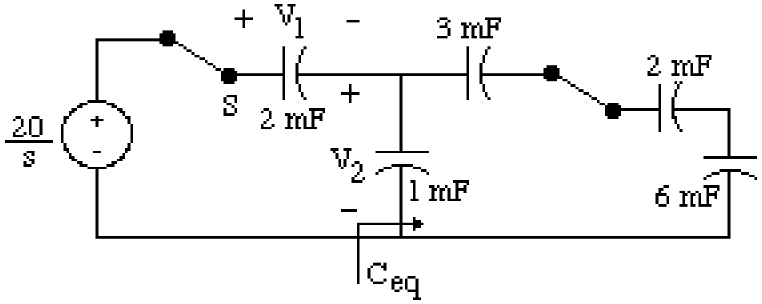
$$0.002sV_2(s) - 0.002\times 10 + 0.004sV_2(s) + 0.004\times 10 = 0.004s\times\frac{11}{s}$$

Equivalently

$$0.006sV_2(s) = 0.044 - 0.02 = 0.024 \qquad V_2(s) = 4/s$$

Hence, for t > 1 s, $v_2(t) = 4$ V and $v_1(t) = 7$ V.

SOLUTION 14.74. **(a)** At t = 0+, the frequency domain equivalent circuit is given below.



To compute C_{eq}, we observe

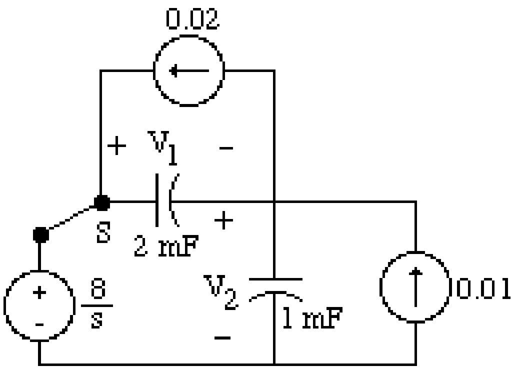
$$C_{eq} = 1 + \frac{1}{\frac{1}{3}+\frac{1}{2}+\frac{1}{6}} = 2 \text{ mF}$$

By voltage division

$$V_1 = \frac{C_{eq}s}{C_{eq}s + 0.002s}\times\frac{20}{s} = \frac{10}{s} \text{ and } V_2 = \frac{0.002s}{C_{eq}s + 0.002s}\times\frac{20}{s} = \frac{10}{s}$$

Hence, for 0 < t <1, $v_1(t) = v_2(t) = 10$ V.

(b) When the switch moves to B, the pertinent part of the equivalent frequency domain circuit is given below.



By superposition,

$$V_1 = \frac{1}{1+2} \times \frac{8}{s} + \frac{1}{0.003s} \times 0.02 - \frac{1}{0.003s} \times 0.01 = \frac{6}{s}$$

Hence, for 1 < t < 2, $v_1(t) = 6$ V and by KVL, $v_2(t) = 2$ V.

SOLUTION 14.75. When the switch in position A, the 2 μF capacitor is charged to –2 V. Hence, the charge on the top plate is $C*v_C$ = –4 μC. When the switch is moved to position B, due to the virtual ground, the 2 μF capacitor voltage is zero meaning it cannot retain any charge. Hence, assuming an ideal op amp, all charge moves to the 1 μF capacitor with –4 μC on the left plate. Hence, v_{out} = −(–4 μC)/1 μF = 4 V.

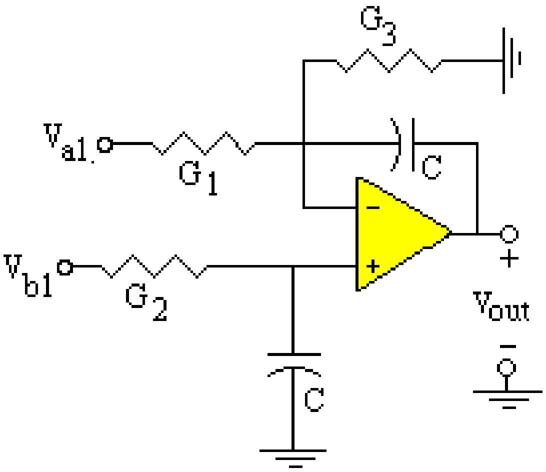
SOLUTION 14.76. (a) $v_{out}(t) = 0$ for 0 < t < 1 ms. Every time the switch moves to position A, the capacitor, C, charges to 8 V. When the switch moves to position B, because of the virtual ground, all charges moves to the kC capacitor. Hence with k = 1, $v_{out}(t)$ = – 8 V for 1 ms < t < 3 ms. For 3 ms < t < 5 ms, $v_{out}(t)$ = – 16 V. Repeating the pattern implies that for 5 ms < t < 7 ms, $v_{out}(t)$ = – 24 V, etc. See for example figure 14.51.

(b) With k = 0.5, the voltages computed in part (a) double.

(c) With k = 2, the voltages computed in part (a) are halved.

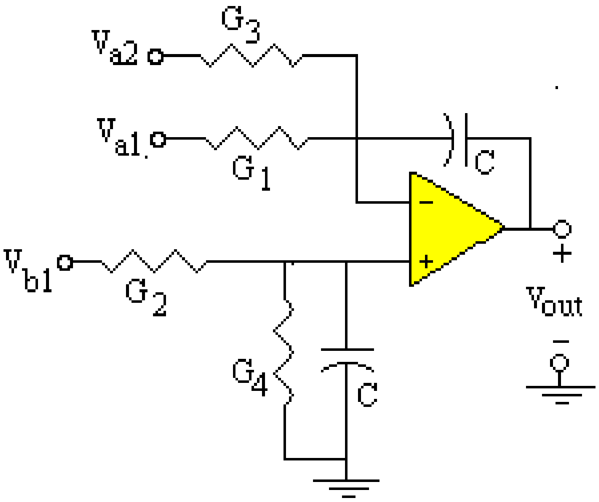
SOLUTION 14.77.

(a) For this part consider the circuit below.



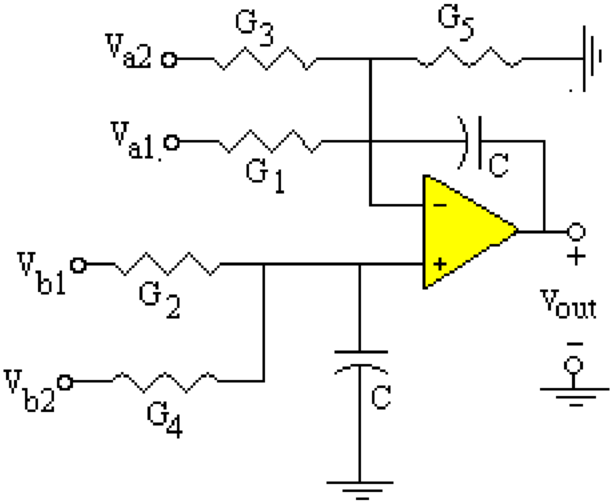
For the normalized design C = 1 F, $G_1 = 0.5$ S, $G_2 = 2$ S, and $G_3 = 1.5$ S. After magnitude scaling with $K_m = 10^6$, then $C = 1\ \mu F$, $R_1 = 2$ MΩ, $R_2 = 500$ kΩ, $R_3 = 666.7$ kΩ.

(b) For this part, consider the circuit below.



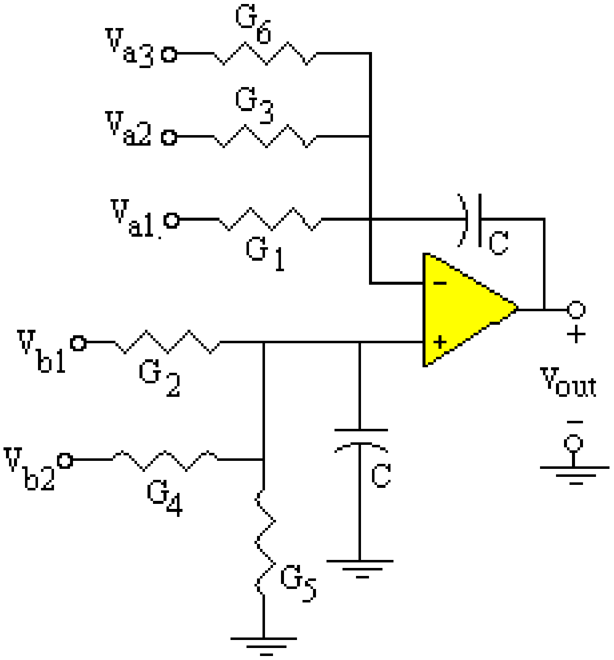
For the normalized design C = 1 F, $G_1 = 1$ S, $G_3 = 2$ S, $G_2 = 0.5$ S, and $G_4 = 2.5$ S. After magnitude scaling with $K_m = 10^6$, then $C = 1\ \mu F$, $R_1 = 1$ MΩ, $R_3 = 500$ kΩ, $R_2 = 2$ MΩ, $R_4 = 400$ kΩ.

(c) Now consider the circuit below.



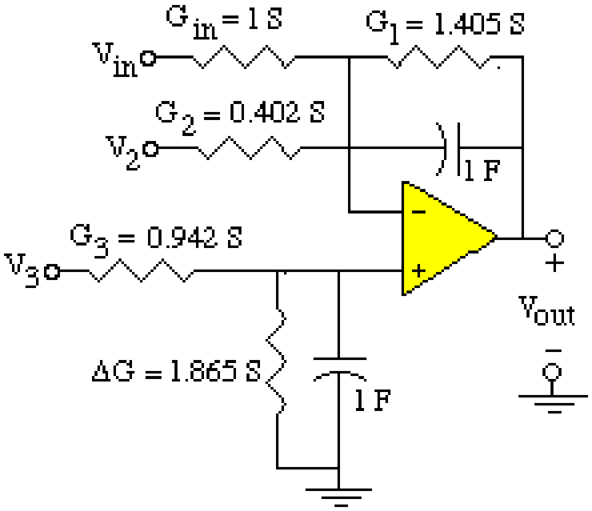
For the normalized design C = 1 F, G_1 = 0.25 S, G_3 = 0.5 S, G_2 = 0.75 S, G_4 = 1 S, and G_5 = 1 S. After magnitude scaling with $K_m = 10^6$, then $C = 1\ \mu F$, $R_1 = 4\ M\Omega$, $R_3 = 2\ M\Omega$, $R_2 = 4/3\ M\Omega$, $R_4 = 1\ M\Omega$, $R_5 = 1\ M\Omega$.

(d) Finally, we consider the following circuit.



For the normalized design C = 1 F, G_1 = 2 S, G_3 = 1.5 S, G_6 = 1 S, G_2 = 0.5 S, G_4 = 2 S, and G_5 = 2 S. After magnitude scaling with $K_m = 10^6$, then $C = 1\ \mu F$, $R_1 = 0.5\ M\Omega$, $R_3 = 2/3\ M\Omega$, $R_6 = 1\ M\Omega$, $R_2 = 2\ M\Omega$, $R_4 = 0.5\ M\Omega$, $R_5 = 0.5\ M\Omega$.

SOLUTION 14.78. With $V_1 = V_{out}$ a prototype design is given by the topology below.

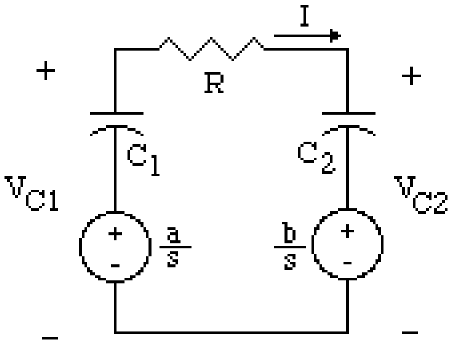


Using MATLAB

```
»Km = 1e7;
»Gin = 1;  G1 = 1.405;  G2 = 0.402;  G3 = 0.942;
»DG = 1.865;
»Rinnew =  Km/Gin
Rinnew =
   10000000
»R1new = Km/G1
R1new =
  7.1174e+06
»R2new = Km/G2
R2new =
  2.4876e+07
»R3new = Km/G3
R3new =
  1.0616e+07
»DRnew = Km/DG
DRnew =
  5.3619e+06
```

Later, when we study frequency scaling, K_m will be smaller and the filter will have a cutoff frequency in a more reasonable range.

SOLUTION TO 14.79. Note corrections to problem statement. W(0+) should be W(∞) in part (b) and in part (c) one should calculate W(0-) – W(∞). The frequency domain equivalent circuit is given by the figure below.



(a) To find the required time functions, we first find their Laplace equivalents.

$$I(s)=\frac{\frac{a}{s}-\frac{b}{s}}{R+\frac{1}{C_1 s}+\frac{1}{C_2 s}}=\frac{\frac{a-b}{R}}{s+\left(\frac{1}{RC_1}+\frac{1}{RC_2}\right)}\equiv\frac{K_1}{s+p_1}$$

in which case $i(t)=K_1e^{-p_1t}u(t)$ where

$$K_1=\frac{a-b}{R}\quad\text{and}\quad p_1=\left(\frac{1}{RC_1}+\frac{1}{RC_2}\right).$$

Further,

$$V_{C1}(s)=\frac{-I(s)}{C_1 s}+\frac{a}{s}=\frac{a}{s}-\frac{\frac{K_1}{C_1}}{s(s+p_1)}=\left(a-\frac{K_1}{C_1p_1}\right)\times\frac{1}{s}+\frac{K_1}{C_1p_1}\times\frac{1}{(s+p_1)}$$

in which case

$$v_{C1}(t)=\left[\left(a-\frac{K_1}{C_1p_1}\right)+\frac{K_1}{C_1p_1}e^{-p_1t}\right]u(t)$$

Also, by symmetry,

$$V_{C2}(s)=\frac{I(s)}{C_2 s}+\frac{b}{s}=\frac{b}{s}+\frac{\frac{K_1}{C_2}}{s(s+p_1)}=\left(b+\frac{K_1}{C_2p_1}\right)\times\frac{1}{s}-\frac{K_1}{C_2p_1}\times\frac{1}{(s+p_1)}$$

in which case

$$v_{C2}(t)=\left[\left(b+\frac{K_1}{C_2p_1}\right)-\frac{K_1}{C_2p_1}e^{-p_1t}\right]u(t)$$

(b) The total energy stored in the capacitors at time 0- is

$$W(0^-)=\frac{1}{2}C_1a^2+\frac{1}{2}C_2b^2$$

Also at t = ∞,

$$v_{C1}(\infty)=\left(a-\frac{K_1}{C_1p_1}\right)\quad\text{and}\quad v_{C2}(\infty)=\left(b+\frac{K_1}{C_2p_1}\right).$$

Hence

$$W(\infty) = \frac{1}{2}C_1\left(a - \frac{K_1}{C_1 p_1}\right)^2 + \frac{1}{2}C_2\left(b + \frac{K_1}{C_2 p_1}\right)^2$$

(c)

$$\int_0^\infty Ri^2(t)\,dt = \int_0^\infty RK_1^2 e^{-2p_1 t}\,dt = \frac{RK_1^2}{2p_1} = \frac{(a-b)^2}{2Rp_1} = \frac{(a-b)^2}{2\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

Observe that

$$W(0^-) - W(\infty) = \frac{1}{2}C_1 a^2 + \frac{1}{2}C_2 b^2 - \frac{1}{2}C_1\left(a - \frac{K_1}{C_1 p_1}\right)^2 - \frac{1}{2}C_2\left(b + \frac{K_1}{C_2 p_1}\right)^2$$

$$= \frac{aK_1}{p_1} - \frac{bK_1}{p_1} - \frac{1}{2}\frac{K_1^2}{C_1 p_1^2} - \frac{1}{2}\frac{K_1^2}{C_2 p_1^2} = \frac{(a-b)^2}{Rp_1} - \frac{1}{2} \times \frac{(a-b)^2}{Rp_1} = \frac{(a-b)^2}{2Rp_1} = \frac{(a-b)^2}{2\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

This indicates that the total energy lost between 0- and infinity is the energy dissipated in the resistor and the result is independent of the value of R.

(d) When R → 0,

$$\lim_{R \to 0} I(s) = \frac{\frac{a}{s} - \frac{b}{s}}{\frac{1}{C_1 s} + \frac{1}{C_2 s}} = \frac{a-b}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{(C_1 + C_2)}(a-b)$$

Therefore

$$i(t) = \frac{C_1 C_2}{(C_1 + C_2)}(a-b)\delta(t)$$

Further

$$V_{C1}(s) = \frac{-C_2}{s(C_1 + C_2)}(a-b) + \frac{a}{s} = \frac{C_1}{s(C_1 + C_2)}a + \frac{C_2}{s(C_1 + C_2)}b$$

and

$$V_{C2}(s) = \frac{C_1}{s(C_1 + C_2)}(a-b) + \frac{b}{s} = \frac{C_1}{s(C_1 + C_2)}a + \frac{C_2}{s(C_1 + C_2)}b$$

Therefore

$$v_{C1}(t) = v_{C2}(t) = \left(\frac{C_1}{(C_1 + C_2)}a + \frac{C_2}{(C_1 + C_2)}b\right)u(t)$$

SOLUTION TO 14.80. (a) From conservation of charge

$$C_1 v_{C1}(0^-) + C_2 v_{C2}(0^-) = C_1 v_{C1}(0^+) + C_2 v_{C2}(0^+) = \left(C_1 + C_2\right)v_{C1}(0^+) = \left(C_1 + C_2\right)v_{C2}(0^+)$$

Therefore

$$v_{C1}(0^+) = v_{C2}(0^+) = \frac{C_1 v_{C1}(0^-) + C_2 v_{C2}(0^-)}{(C_1 + C_2)}$$

(b) Inserting values into out answer for part (a) yields

$$v_{C1}(0^+) = v_{C2}(0^+) = \frac{C_1}{(C_1 + C_2)} = 0.5 \text{ V}$$

and the voltage remains the same for t > 0.

(c) Before the switch is closed, the energy in C_2 is zero and the energy in C_1 is the total stored energy:

$$W_{tot}(0^-) = W_{C1}(0^-) = 0.5 C_1 v_{C1}^2(0^-) = 0.5 \text{ J}$$

After the switch is closed,

$$W_{tot}(0^+) = W_{C1}(0^+) + W_{C2}(0^+) = 0.5 C_1 v_{C1}^2(0^+) + 0.5 C_2 v_{C2}^2(0^+) = 0.25 \text{ J}$$

(d) (i) Using the series equivalent circuit for C_1, we have

$$I(s) = \frac{1}{R + \frac{2}{s}} \times \frac{1}{s} = \frac{1/R}{s + 2/R} \qquad i(t) = \frac{1}{R} e^{-2t/R} u(t) \text{ A}$$

Thus

$$V_{C2}(s) = \frac{I(s)}{s} = \frac{1/R}{s(s + 2/R)} = \frac{0.5}{s} - \frac{0.5}{s + 2/R} \qquad v_{C2}(t) = 0.5\left(1 - e^{-2t/R}\right) u(t) \text{ V}$$

and

$$V_{C1}(s) = -\frac{I(s)}{s} + \frac{1}{s} = \frac{-1/R}{s(s + 2/R)} + \frac{1}{s} = \frac{0.5}{s} + \frac{0.5}{s + 2/R} \qquad v_{C1}(t) = 0.5\left(1 + e^{-2t/R}\right) u(t) \text{ V}$$

(ii) The energy dissipated in the resistor is given by

$$W_R(0,\infty) = R\int_0^\infty i^2(\tau)d\tau = \frac{1}{R}\int_0^\infty e^{-4t/R} d\tau = -\left.\frac{e^{-4t/R}}{4}\right]_0^\infty = \frac{1}{4} \text{ J}$$

(iii) For all R,

$$\text{Area under } i(t) = \int_0^\infty i(\tau)d\tau = \frac{1}{R}\int_0^\infty e^{-2t/R} d\tau = -\left.\frac{e^{-2t/R}}{2}\right]_0^\infty = \frac{1}{2}$$

Further, as $R \to 0$, $i(t) = \frac{1}{R} e^{-2t/R} u(t)$ has a decay that becomes infinitely fast and its magnitude (1/R) approaches ∞. Thus we have infinite height, zero-width, but a finite area of 0.5. Thus as $R \to 0$, $i(t) \to 0.5\delta(t)$ A. (We have avoided a more rigorous explanation as the above argument is more plausible to sophomores.). As $R \to 0$, the exponential terms in the expressions for $v_{C1}(t)$ and $v_{C2}(t)$ have infinitely fast decays and hence disappear from the expressions yielding the stated result.

SOLUTION TO 14.81.

(a)

$$Z_{in}(s) = \frac{2s + 4.5}{(s+ 0.5)(s + 4)} = \frac{1}{s+ 0.5} + \frac{1}{s + 4} = Z_a(s) + Z_b(s)$$

$$Y_a(s) = \frac{1}{Z_a(s)} = s+ 0.5$$

$$Y_b(s) = \frac{1}{Z_b(s)} = s + 4$$

From the above expressions, the RC circuit consists of a series connection of (a 1 farad capacitor in parallel with a 2 Ω resistor) and (a 1 F capacitor in parallel with a 0.25 Ω resistor).

(b)

$$Y_{in}(s) = \frac{12s + 440}{(s+ 120)(s + 20)} = \frac{10}{s+ 120} + \frac{2}{s + 20} = Y_a(s) + Y_b(s)$$

$$Z_a(s) = \frac{1}{Y_a(s)} = 0.1s + 12$$

$$Z_b(s) = \frac{1}{Y_b(s)} = 0.5s + 10$$

From the above expressions, the RL circuit consists of a parallel connection of (0.1 H inductor in series with a 12 Ω resistor) and (a 0.5 H inductor in series with a 10 Ω resistor).

(c) Following the hint, we have

$$\frac{Y_{in}(s)}{s} = \frac{0.225s +0.075}{(s+ 0.2)(s + 0.5)} = \frac{0.1}{s+ 0.2} + \frac{0.125}{s + 0.5}$$

Hence

$$Y_{in}(s) = \frac{0.1s}{s+ 0.2} + \frac{0.125s}{s + 0.5} = Y_a(s) + Y_b(s)$$

$$Z_a(s) = \frac{1}{Y_a(s)} = \frac{s + 0.2}{0.1s} = 10 + \frac{1}{0.5s}$$

$$Z_b(s) = \frac{1}{Y_b(s)} = \frac{s + 0.5}{0.125s} = 8 + \frac{1}{0.25s}$$

From the above expressions, we see that each term in $Y_{in}(s)$ represents a series RC circuit. The RC circuit for $Y_{in}(s)$ consists of a parallel connection of (a 0.5 F capacitor in series with a 10 Ω resistor) and (a 0.25 F capacitor in series with a 8 Ω resistor).

(d) **CORRECTION**: for part (d),change the second term to $2s/(s^2 + 2)$.

$$Y_{in}(s) = \frac{0.5s}{s^2+1} + \frac{2s}{s^2+2} = Y_a(s) + Y_b(s)$$

$$Z_a(s) = \frac{1}{Y_a(s)} = \frac{s^2+1}{0.5s} = 2s + \frac{1}{0.5s}$$

$$Z_b(s) = \frac{1}{Y_b(s)} = \frac{s^2+2}{2s} = 0.5s + \frac{1}{s}$$

From the above expressions, we see that each term in $Y_{in}(s)$ represents a series LC circuit. The LC circuit for $Y_{in}(s)$ consists of a parallel connection of (a 0.5 F capacitor in series with a 2 H inductor) and (a 1 F capacitor in series with a 0.5 H inductor).

SOLUTION 14.82. **CORRECTIONS TO PROBLEM STATEMENT:** (i) $v_0(t)$, should read $v_{out}(t)$ and (ii) there should be a connection from the circuit inside the shaded box to the bottom line or reference node.

(a) (i) 0 ≤ t < 1 ms. Since the capacitor voltage is initially zero and the switch is in position (a), a simple source transformation yields a Norton equivalent (seen by the capacitor) consisting of a 20 mA current source in parallel with 9.8039 kΩ resistor. Hence

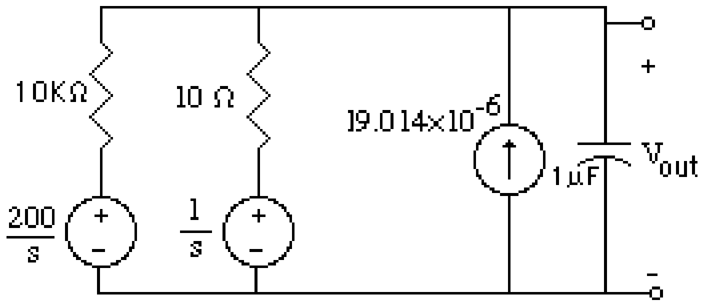
$$V_{out}(s) = \frac{1}{\dfrac{1}{98039} + 10^{-6}s} \times \frac{.02}{s} = \frac{20 \times 10^3}{s(s+102)}$$

Using MATLAB

```
n = .02*1e6;
p1 = 1e6/Rth
p1 =   102
»d = [1   p1  0];
»[r,p,k] = residue(n,d)
r =
  -1.9608e+02
   1.9608e+02
p =
  -102
     0
k =     []
```

Hence, for 0 ≤ t < 1 ms, $v_{out}(t) = 196.08\left(1 - e^{-102t}\right)u(t)$ V. It follows that $v_{out}(1\text{ ms}) = 19.014$ V.

(ii) 1 ms ≤ t < 1.05 ms. The frequency domain equivalent circuit is given below.



Writing a single node equation yields

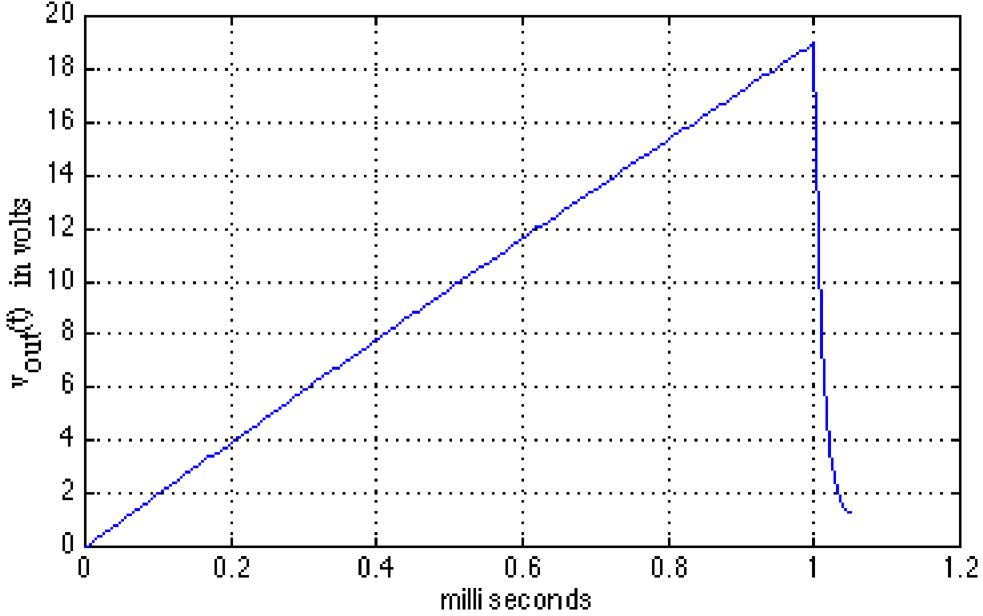
$$19.014\times10^{-6} = \frac{V_{out} - \frac{200}{s}}{10000} + \frac{V_{out} - \frac{1}{s}}{10} + 10^{-6} sV_{out}$$

Equivalently $19.014 + \frac{1.2\times10^{5}}{s} = (s+100100)V_{out}$ or

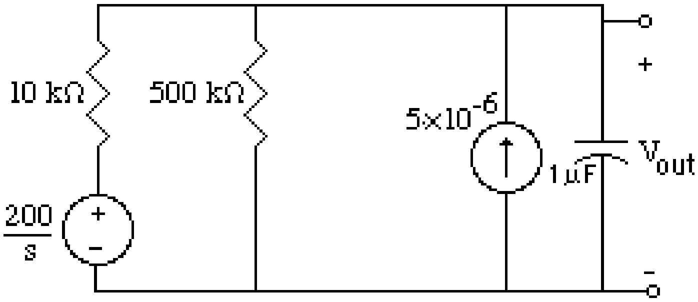
$$\hat{V}_{out} = \frac{19.014s + 1.2\times10^{5}}{s(s+100100)}$$

Therefore, $\hat{v}_{out}(t') = \left(1.1988 + 17.815e^{-100100t'}\right)u(t')$, and for 1 ms ≤ t < 1.05 ms,
$v_{out}(t) = \hat{v}_{out}(t-0.001)$ in which case
$v_{out}(t) = \left(1.1988 + 17.815e^{-100100(t-0.001)}\right)u(t-0.001)$

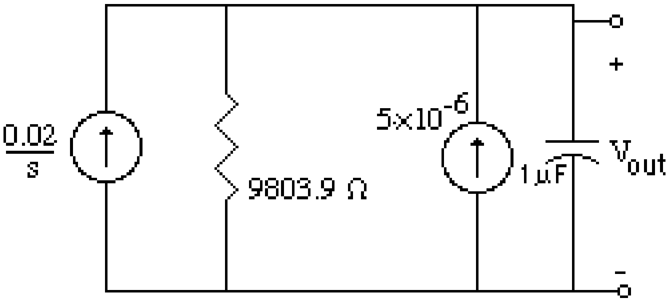
Again using MATLAB,



(b) Part 1: for t > 0 up to t_1 which denotes the time when $v_{out}(t)$ reaches 80 V, i.e., the capacitor is charging. The frequency domain equivalent circuit is



Using our knowledge of part (a), this circuit simplifies to



Hence

$$V_{out}(s) = \frac{5\times10^{-6} + \dfrac{0.02}{s}}{10^{-6}s + \dfrac{1}{9803.9}} = \frac{5s + 20\times10^{3}}{s(s+102)}$$

and from MATLAB

```
»syms s t
»ilaplace((5*s+20e3)/(s^2+102*s))
ans =
10000/51-9745/51*exp(-102*t)
```

in which case

$v_{out}(t) = \left(196.08 - 191.08e^{-102t}\right)u(t)$ V

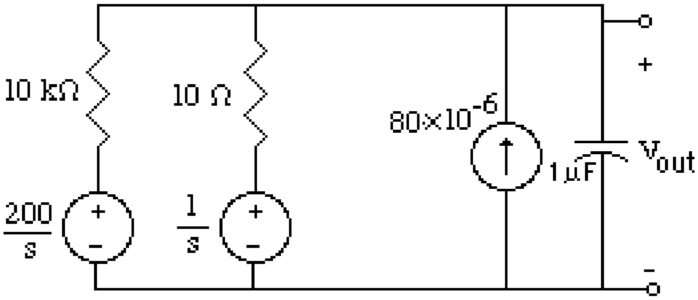
From this expression,

$v_{out}(t_1) = 80 = 196.08 - 191.08e^{-102t_1}$ V

and

```
t1 = log((80-10000/51)/(-9745/51))/(-102)
t1 =   4.8864e-03
```

This part of the problem considers $t_1 \le t < t_2$, i.e., the capacitor is discharging where $v_{out}(t_2) = 5$. The equivalent frequency domain circuit is given below which is a slight modification of the circuit of (a)-(ii):



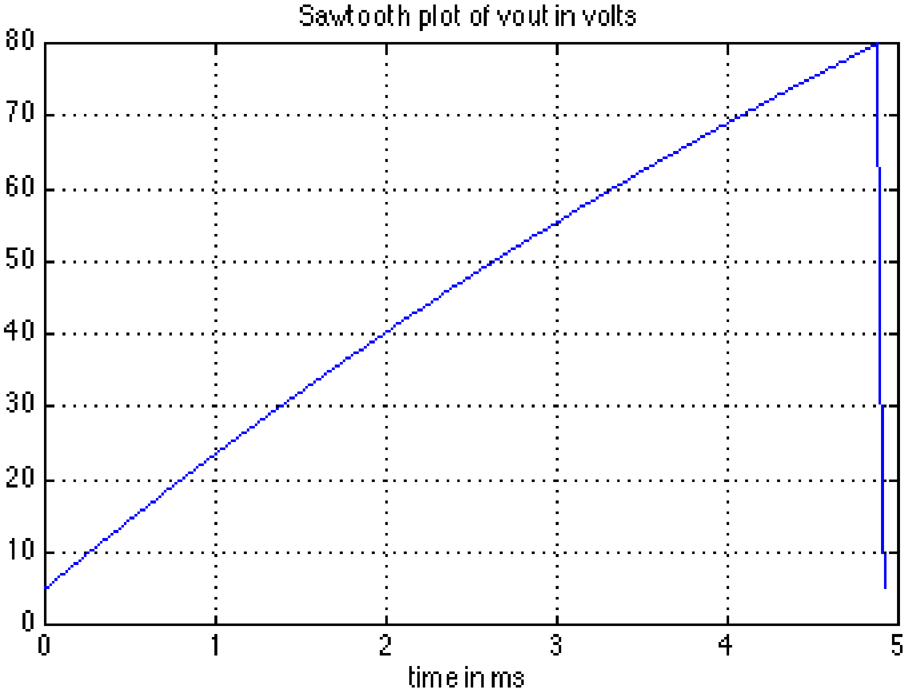
Making use of our knowledge of part (a)-(ii), we have

$$\hat{V}_{out} = \frac{80s + 1.2\times10^5}{s(s+100100)}$$

in which case $\hat{v}_{out}(t') = \left(1.1988 + 78.801e^{-100100t'}\right)u(t')$ V, and

$$v_{out}(t) = \left(1.1988 + 78.801e^{-100100(t-t_1)}\right)u(t-t_1)$$

Here $t_2' = 3.0286\times10^{-5}$s and $t_2 = t_1 + t_2' = 4.9167$ ms where t_2' is the duration of the discharge cycle. As a final point, note that the frequency of the sawtooth is 1/t2 = 203.39 Hz. Finally a plot is given below.



SOLUTION 14.83. **CORRECTION**: In example 14.10, page 560, delete the four minus signs in the equation for $V_C(s)$ and one more for $v_C(t)$.

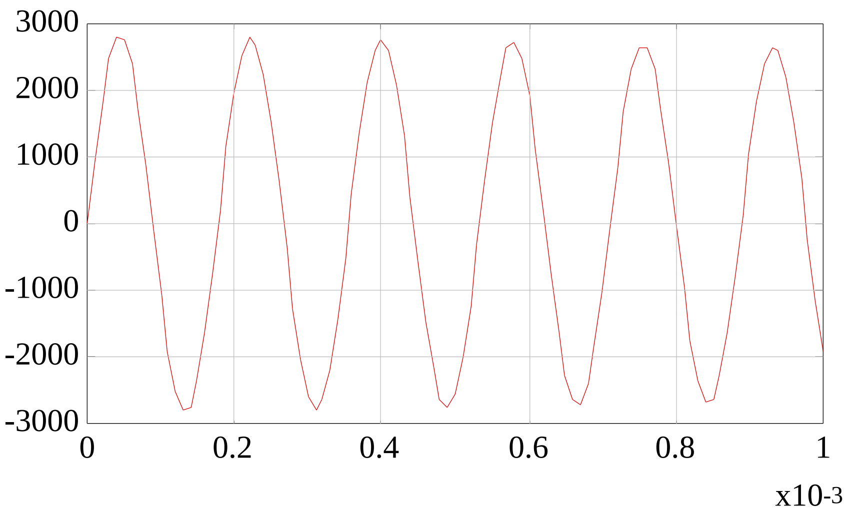
We use MATLAB instead of SPICE to solve this problem. Applying voltage division to the circuit of figure P14.83, we have

$$V_C(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + Ls + R} L i_L(0^-) = \frac{i_L(0^-)}{C} \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{10^8}{s^2 + 125s + 1.25\times 10^9}$$

From table 13.1, item 18

$$v_C(t) = 2828e^{-62.5t} \sin(35{,}355t)\, u(t) \text{ V}$$

A plot of $v_C(t)$ is given below with the vertical axis in V and the horizontal axis in seconds.



The waveform for the first few cycles is essentially the same as the example 14.10. Thus for the firs few cycles, the lossless circuit of example 14.10 is a good approximation to the more accurate circuit model of this problem. The effect of the presence of 100 Ω resistance is a slow decay (with respect to ms intervals) of the peak values.