PROBLEM SOLUTIONS CHAPTER 14

SOLUTION 14.1.

(a)

$$Z(s)=\frac{R(Ls+\frac{1}{Cs})}{R+Ls+\frac{1}{Cs}}=\frac{Cs(RLCs^2+R)}{Cs(RCs+LCs^2+1)}=\frac{RLC(s^2+1/LC)}{LC(s^2+\frac{R}{L}s+\frac{1}{LC})}=\frac{R(s^2+1/LC)}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

(b)

$$Z(s)=R+\frac{(Ls)(\frac{1}{Cs})}{Ls+\frac{1}{Cs}}=R+\frac{LCs}{C(LCs^2+1)}=R+\frac{Ls}{LCs^2+1}$$

$$=\frac{RLCs^2+Ls+R}{LCs^2+1}=\frac{RLC(s^2+\frac{1}{RC}s+\frac{1}{LC})}{LC(s^2+\frac{1}{LC})}$$

Hence,

$$Z(s)=\frac{R\left(s^2+\frac{1}{RC}s+\frac{1}{LC}\right)}{s^2+\frac{1}{LC}}$$

SOLUTION 14.2.

(a)

$$Z_{in}(s)=\frac{V_s(s)}{I_s(s)}=\frac{(10+0.2s)(\frac{80}{s})}{10+0.2s+\frac{80}{s}}=\frac{s(800+16s)}{s(0.2s^2+10s+80)}=\frac{800s+4000}{s^2+50s+400}$$

(b) If $i_s(t)=3e^{-20t}u(t)$ A then

$$I_s(s)=\frac{3}{s+20}$$

and

$$V_s(s)=Z_{in}(s)I_s(s)=\frac{800s+4000}{s^2+50s+400}-\frac{3}{s+20}=\frac{2400s+12{,}000}{(s+10)(s+20)(s+40)}$$

$$=\frac{K_1}{s+10}+\frac{K_2}{s+20}+\frac{K_3}{s+40}$$

Here

$$K_1=\left.\frac{2400s+12{,}000}{(s+20)(s+40)}\right|_{s=-10}=\frac{+12{,}000-24{,}000}{(10)(30)}=-40$$

$$K_2=\left.\frac{2400s+12{,}000}{(s+10)(s+40)}\right|_{s=-20}=\frac{12{,}000-48{,}000}{(-10)(20)}=180$$

$$K_3 = \left.\frac{2400s + 12{,}000}{(s+10)(s+20)}\right|_{s=-40} = \frac{12{,}000 - 96{,}000}{(-30)(-20)} = -140$$

$$V_s(s) = \frac{180}{s+20} - \frac{40}{s+10} - \frac{140}{s+40}$$

and for t > 0,

$$v_s(t) = 180e^{-20t} - 40e^{-10t} - 140e^{-40t} \text{ V}$$

SOLUTION 14.3.

(a)

$$Y_p(s) = Cs + \frac{1}{R} = 2\times10^{-3}s + \frac{1}{0.10} = 2\times10^{-3}(s+50)$$

Then

$$Z_p(s) = \frac{500}{s+50}$$

and

$$Z_{in}(s) = 1.25s + \frac{500}{s+50} = \frac{1.25s^2 + 62.5s + 500}{s+50}$$

and

$$Y_{in}(s) = \frac{I_s(s)}{V_s(s)} = \frac{1}{Z_{in}(s)} = \frac{s+50}{1.25s^2 + 62.5s + 500} = \frac{0.80s + 40}{s^2 + 50s + 400}$$

With $v_s(t) = 90e^{-40t}u(t)$, then

$$V_s(s) = \frac{90}{s+40}$$

and

$$I_s(s) = V_s(s)Y_{in}(s) = \frac{90}{s+40}\ \frac{0.80s+40}{(s+10)(s+40)}$$

$$= \frac{72s + 3600}{(s+10)(s+40)^2} = \frac{K_1}{s+10} + \frac{C_1}{s+40} + \frac{C_2}{(s+40)^2}$$

Here

$$K_1 = \left.\frac{72s + 3600}{(s+40)^2}\right|_{s=-10} = \frac{3600 - 720}{(30)^2} = \frac{2880}{900} = 3.20$$

and with

$$p(s) = \frac{72s + 3600}{s+10} \rightarrow p(-40) = \frac{720}{-30} = -24$$

$$p'(s) = \frac{(s+10)(72) - (72s + 3600)}{(s+10)^2}$$

$$p'(-40) = \frac{-30(72) - (-2880 + 3600)}{(-30)^2} = \frac{-2160 - 720}{900} = -3.20$$

Then

$$C_2 = \frac{p(-40)}{0!} = -24, \quad C_1 = \frac{p'(-40)}{1!} = -3.20,$$

$$I_s(s) = \frac{3.20}{s+10} - \frac{3.20}{s+40} + \frac{24}{(s+40)^2}$$

and for t > 0

$$i_s(t) = 3.20e^{-10t} - 3.20e^{-40t} + 24te^{-40t} \text{ A}$$

SOLUTION 14.4.

(a) Find $Z_{in}(s)$ vis $Y_{in}(s)$

$$Y_{in}(s) = Cs + \frac{1}{Ls+20} + \frac{1}{10} = \frac{200Cs + 10LCs^2 + 10 + 20 + Ls}{10(Ls+20)}$$

$$= \frac{10LCs^2 + (200C+L)s + 30}{10Ls + 200}$$

and

$$Z_{in}(s) = \frac{1}{Y_{in}(s)} = \frac{10Ls+200}{10LCs^2 + (200C+L)s + 30}\,\Omega$$

With $C = 10^{-3}F$ and L $= 0.05\ H$

$$Z_{in}(s) = \frac{0.50s + 200}{0.0005s^2 + 0.25s + 30} = \frac{1000s + 4\times 10^5}{s^2 + 500s + 60{,}000}\,\Omega$$

(b) If $i_s(t) = 0.3u(t)$ A, then

$$I_s(s) = \frac{0.30}{s}$$

and

$$V_{in}(s) = Z_{in}(s)I_s(s) = 300\left[\frac{s+400}{s(s^2+500s+60{,}000)}\right] = 300\left[\frac{s+400}{s(s+2000)(s+300)}\right]$$

$$= 300\left[\frac{K_1}{s} + \frac{K_2}{200} + \frac{K_3}{300}\right]$$

It follows that

$$K_1 = \left.\frac{s+400}{(s+200)(s+300)}\right|_{s=0} = \frac{400}{200(300)} = \frac{1}{150}$$

$$K_2 = \left.\frac{s+400}{s(s+300)}\right|_{s=-200} = \frac{200}{(-200)(100)} = -\frac{1}{100}$$

and

$$K_3 = \left.\frac{s+400}{s(s+200)}\right|_{s=-300} = \frac{100}{(-300)(-100)} = \frac{1}{300}$$

Thus

$$V_{in}(s) = 300\left[\frac{\frac{1}{150}}{s} - \frac{\frac{1}{100}}{s+200} + \frac{\frac{1}{300}}{s+300}\right]$$

$$= \frac{2}{s} - \frac{3}{s+200} + \frac{1}{s+300}$$

and for t > 0

$$v_{in}(t) = 2 - 3e^{-200t} + e^{-300t}\,\text{V}$$

SOLUTION 14.5.

$$Z(s) = \frac{s+20}{s+40}$$

and the network is “at rest”

(a) If

$$v_{in}(t) = 20u(t) \rightarrow V_{in}(s) = \frac{20}{s}$$

then

$$I_{in}(s) = \frac{V_{in}(s)}{Z(s)} = \frac{20}{s}\left(\frac{s+40}{s+20}\right) = 20\left[\frac{s+40}{s(s+20)}\right]$$

Using a partial function expansion

$$I_{in}(s) = 20\left[\frac{s+40}{s(s+20)}\right] = 20\left[\frac{K_1}{s} + \frac{K_2}{s+20}\right]$$

in which case

$$K_1 = \left.\frac{s+40}{s+20}\right|_{s=0} = \frac{40}{20} = 2,\quad K_2 = \left.\frac{s+40}{s}\right|_{s=-20} = \frac{20}{-20} = -1$$

Thus

$$I_{in}(s) = 20\left[\frac{2}{s} - \frac{1}{s+20}\right]$$

and

$$i_{in}(t) = 20(2 - e^{-20t})u(t)\ \text{A}$$

(b) Note that

$$v_{in}(t) = 20e^{-40t} \rightarrow V_{in}(s) = \frac{20}{s+40}$$

Then

$$I_{in}(s) = \frac{V_{in}(s)}{Z(s)} = \frac{20}{s+40}\left(\frac{s+40}{s+20}\right) = \frac{20}{s+20}$$

in which case

$$i_{in}(t) = 20e^{-20t}u(t)\ \text{A}$$

(c) Note that

$$v_{in}(t) = 20e^{-20t} \rightarrow V_{in}(s) = \frac{20}{s+20}$$

Then

$$I_{in}(s)=\frac{\mathrm{V_{in}}(s)}{Z(s)}=\frac{20}{s+20}\left(\frac{s+40}{s+20}\right)=20\left[\frac{s+40}{(s+20)^2}\right]$$

Using a partial fraction expansion

$$I_{in}(s)=20\left[\frac{s+40}{(s+20)^2}\right]=20\left[\frac{C_1}{s+20}+\frac{C_2}{(s+20)^2}\right]$$

Here

$$p(s)=s+40 \quad \rightarrow \quad p(-20)=20$$
$$p'(s)=1 \quad \rightarrow \quad p'(-20)=1$$

and

$$C_1=\frac{p'(-20)}{1!}=1,\quad C_2=\frac{p(-20)}{0!}=\frac{20}{1}=20$$

in which case

$$I_{in}(s)=20\left[\frac{1}{s+20}+\frac{20}{(s+20)^2}\right]$$

so that

$$i_{in}(t)=\left(20e^{-20t}+400te^{-20t}\right)u(t)\ \text{A}$$

SOLUTION 14.6.

(a) Apply an arbitrary $I_{in}(s)$ to the upper terminal of Fig. P14.6a. Assuming branch currents $I_a(s)$ and $I_b(s)$, it follows by KCL that

$$I_{in}(s)=I_a(s)+I_b(s)=0.020sV_a(s)+0.005s\left[V_{a(s)}-sV_a(s)\right]$$
$$=(0.020+0.005-0.015)sV_a(s)=0.010sV_a(s)$$

Hence,

$$Z_{in}(s)=\frac{V_a(s)}{I_{in}(s)}=\frac{1}{0.010s}=\frac{100}{s}\ \Omega$$

(b) Similarly apply an arbitrary $I_{in}(s)$ to Fig P14.6b to obtain, in the s-domain, by KCL

$$I_{in}(s)=10sV_{in}(s)+\frac{1}{50}V_{in}(s)+0.10s\left[V_{in}(s)-\frac{V_{in}(s)}{4}\right]=\left(10s+\frac{1}{50}+\frac{30}{4s}\right)V_{in}(s)$$

$$=\left(\frac{2000s^2+4s+1500}{200s}\right)V_{in}(s)$$

Hence

$$Y_{in}(s)=\frac{I_{in}(s)}{V_{in}(s)}=\frac{10s^2+0.02s+7.50}{s}$$

(c) Here we apply $V_{in}(s)$ to the input terminals of figure P14.6c. By KCL

$$V_{in}(s)=10I_{in}(s)+0.2sI_{in}(s)+\frac{400}{s}\left[I_{in}(s)-\frac{1}{2}I_{in}(s)\right]=\left(10+0.2s+\frac{200}{s}\right)I_{in}$$

in which case

$$Z_{in}(s)=\frac{V_{in}(s)}{I_{in}(s)}=\frac{0.2s^2+10s+200}{s}\ \Omega$$

SOLUTION 14.7. Writing two loop equations we obtain:

$$V_{in}(s)=100I_1(s)+200I_2(s)$$

and

$$100I_1(s)+\left(100+\frac{100}{s}\right)I_2(s)=0$$

In matrix form (dropping the s-dependence)

$$\begin{bmatrix}100 & 200\\ 100 & 100+\dfrac{100}{s}\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}=\begin{bmatrix}V_{in}\\ 0\end{bmatrix}$$

Using Cramer's rule,

$$I_1=\frac{\det\begin{bmatrix}V_{in} & 200\\ 0 & 100+\dfrac{100}{s}\end{bmatrix}}{\det\begin{bmatrix}100 & 200\\ 100 & 100+\dfrac{100}{s}\end{bmatrix}}=\frac{100s+100}{s}\times\frac{1}{100\left[\left(\dfrac{100s+100}{s}\right)-200\right]}V_{in}$$

Hence

$$Z_{in}(s)=\frac{V_{in}}{I_1}=-100\frac{s-1}{s+1}$$

SOLUTION 14.8.

Working in the s-domain, apply KVL to the left side of the circuit to obtain

$$V_{in}(s)=\frac{100}{s}I_{in}(s)+\frac{s}{100}I_{in}(s)+V_2(s)$$

Now apply KCL to the right side to obtain

$$I_{in}(s)=\frac{s}{100}V_2(s)+\frac{100}{s}V_2(s)$$

Thus

$$V_{in}(s)=\left(\frac{s^2+10^4}{100s}\right)I_{in}(s)+V_2(s)$$

To find $V_2(s)$ note that

$$I_{in}(s)=\frac{s^2+10^4}{100s}V_2(s)$$

implying that

$$V_2(s) = \left(\frac{100s}{s^2+10^4}\right) I_{in}(s)$$

Thus

$$V_{in}(s) = \left[\frac{s^2+10^4}{100s} + \frac{100s}{s^2+10^4}\right] I_{in}(s) = \frac{(s^2+10^4)^2 + 10^4 s^2}{100s(s^2+10^4)} I_{in}(s)$$

implying that

$$Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = \frac{s^4 + 3\times 10^4 s^2 + 10^8}{100s(s^2+10^4)} \ \Omega$$

and

$$Y_{in}(s) = \frac{1}{Z_{in}(s)} = \frac{100s(s^2+10^4)}{s^4 + 3\times 10^4 s^2 + 10^8} \ \text{S}$$

SOLUTION 14.9.

Three mesh equations for the circuit

$$\begin{bmatrix} R+Z_2(s) & -Z_2(s) & -R \\ -Z_2(s) & 2R+Z_2(s) & -R \\ -R & -R & 2R+Z_1(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V_{in}(s) \\ 0 \\ 0 \end{bmatrix}$$

Solve for $I_1(s)$ via Cramer's rule

$$I_1 = \frac{\det\begin{bmatrix} V_{in} & -Z_2(s) & -R \\ 0 & 2R+Z_2(s) & -R \\ 0 & -R & 2R+Z_1(s) \end{bmatrix}}{\det\begin{bmatrix} R+Z_2(s) & -Z_2(s) & -R \\ -Z_2(s) & 2R+Z_2(s) & -R \\ -R & -R & 2R+Z_1(s) \end{bmatrix}} = \frac{V_{in}\left(3R^2 + 2R\left(Z_1+Z_2\right) + Z_1Z_2\right)}{d(s)}$$

where

$$\begin{aligned} d(s) &= \left(R+Z_2\right)\left(3R^2 + 2R\left(Z_1+Z_2\right) + Z_1Z_2\right) + Z_2\left(-Z_2(2R+Z_1) - R^2\right) - \left(2R^2Z_2 + 2R^3\right) \\ &= R^3 + 2R^2Z_1 + 2R^2Z_2 + 3RZ_1Z_2 \end{aligned}$$

Under the condition that $Z_1(s)Z_2(s) = R_2$, we have

$$Z_{in}(s) = \frac{V_{in}}{I_1} = \frac{R^3 + 2R^2Z_1 + 2R^2Z_2 + 3RZ_1Z_2}{3R^2 + 2R\left(Z_1+Z_2\right) + Z_1Z_2} = \frac{4R^3 + 2R^2\left(Z_1+Z_2\right)}{4R^2 + 2R\left(Z_1+Z_2\right)} = R$$

SOLUTION 14.10.

(a) $Y_{in}(s) = Cs + \dfrac{1}{R}$ implies a parallel RC circuit with values R and C respectively.

(b) $Y_{in}(s)=\dfrac{1}{Z_{in}(s)}=2s+\dfrac{1}{4}$ which is a parallel RC circuit of values 4 Ω and 2 F respectively.

(c) $Z_{in}(s)=1+\dfrac{1}{2s+0.25}=1+\dfrac{1}{Y_b(s)}$. Using the result of part (b), this circuit is a 1 Ω resistor in series with the parallel RC of part (b).

(d) $Z_{in}(s)=\dfrac{2s+8}{s+2}=2+\dfrac{4}{s+2}=2+\dfrac{1}{0.25s+\dfrac{1}{2}}$. Using the results of parts (b) and (c), this circuit is a 2 Ω resistor in series with a parallel combination of a 0.25 F capacitor and a 2 Ω resistor.

(e) $Z_{in}(s)=\dfrac{s+3}{s+1}+\dfrac{s+6}{s+4}=2+\dfrac{2}{s+1}+\dfrac{2}{s+4}=2+\dfrac{1}{0.5s+1/2}+\dfrac{1}{0.5s+1/0.5}$. Using the above results, this circuit is a 2 Ω resistor in series with a parallel combination of a 0.5 F capacitor and a 2 Ω resistor which is in series with another parallel combination of a 0.5 F capacitor and a 0.5 Ω resistor.

SOLUTION 14.11.

(a) Clearly this is an inductor of value L in series with a resistor of value R.

(b) Inverting the admittance we have $Z_{in}(s)$ of the form of part (a). Hence the circuit is a 0.5 H inductor in series with a 10 Ω resistor.

(c) $Y_{in}(s)=0.2+\dfrac{1}{0.5s+10}=0.2+\dfrac{1}{Z_b(s)}$. Using the result of part (b), the circuit is 0.2 S resistor in parallel with a series connection of a 0.5 H inductor and a 10 Ω resistor.

(d) $Y_{in}(s)=\dfrac{10s+50}{s+1}=10+\dfrac{40}{s+1}=10+\dfrac{1}{0.025s+0.025}$. This is similar to part (c). Hence the circuit is a 10 S resistor in parallel with a series connection of a 25 mH inductor and a 0.025 Ω resistor.

(e) $Y_{in}(s)=\dfrac{s+3}{s+1}+\dfrac{s+6}{s+4}=2+\dfrac{2}{s+1}+\dfrac{2}{s+4}=2+\dfrac{1}{0.5s+0.5}+\dfrac{1}{0.5s+2}$. Hence, the circuit is a 2 S resistor in parallel with the series connection of a 0.5 H inductor and a 0.5 Ω resistor which in turn is in parallel with a 0.5 H inductor and 2 Ω resistor.

SOLTUION 14.12.

(a) $Z_{in}(s)=Ls+\dfrac{1}{Cs}$ represents a series connection of an inductance L and a capacitance C.

(b) $Y_{in}(s)=Cs+\dfrac{1}{Ls}$ represents a parallel connection of an inductance L and a capacitance C.

(c) $Z_{in}(s)=\dfrac{0.125s^2+1}{0.25s}=0.5s+\dfrac{1}{0.25s}$ which is a series connection of a 0.5 H inductor and a 0.25 F capacitor.

(d) $Y_{in}(s)=\dfrac{0.125s^2+1}{0.25s}=0.5s+\dfrac{1}{0.25s}$ which is a parallel connection of a 0.5 F capacitor and 0.25 H inductor.

(e) $Z_{in}(s) = \frac{s^2+1}{s} + \frac{0.25s^2+1}{0.25s} = 2s + \frac{1}{s} + \frac{4}{s} = 2s + \frac{1}{0.2s}$ which is a 2 H inductor in series with a 0.2 F capacitor.

(f) $Z_{in}(s) = \frac{s^2+1}{s} + \frac{0.25s}{0.25s^2+1} = s + \frac{1}{s} + \frac{1}{s + \frac{1}{0.25s}}$. This circuit is a 1 H inductor in series with a 1 F capacitor which is in series with a parallel connection of a 1 F capacitor and a 0.25 H inductor.

(g) $Y_{in}(s) = \frac{s^2+1}{s} + \frac{0.25s}{0.25s^2+1} = s + \frac{1}{s} + \frac{1}{s + \frac{1}{0.25s}}$. This circuit is a 1 F capacitor in parallel with a 1 H inductor which is in parallel with a series connection of a 1 H inductor and a 0.25 F capacitor.

SOLUTION 14.13.

With $L[v_{out}(t)] = V_o(s)$ and $L[v_{in}(t)] = V_i(s)$ and $v_{out}(0^-) = 0$,

$$sV_0(s) + 25V_0(s) + \frac{100}{s}V_0(s) = 5V_i(s) - \frac{10}{s}V_i(s)$$

which implies that

$$\left[\frac{s^2+25s+100}{s}\right]V_0(s) = \frac{5s-10}{s}V_i(s)$$

The transfer function is

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{5s-10}{s^2+25s+100} = \frac{5s-10}{(s+5)(s+20)}$$

(a) If $v_{in}(t) = te^{-5t}u(t)$ V, then

$$V_i(s) = \frac{1}{(s+5)^2}$$

and

$$V_{out}(s) = \frac{5s-10}{(s+20)(s+5)^3} = \frac{K_1}{s+20} + \frac{C_1}{s+5} + \frac{C_2}{(s+5)^2} + \frac{C_3}{(s+5)^3}$$

$$K_1 = \left.\frac{5s-10}{(s+5)^3}\right|_{s=-20} = \frac{-110}{(-15)^3} = \frac{-110}{-3375} = \frac{22}{675}$$

$$p(s) = \frac{5s-10}{s+20}$$

$$p'(s) = \frac{(s+20)(5)-(5s=10)}{(s+20)^2} = \frac{110}{(s+20)^2} = 110(s+20)^{-2}$$

$$p''(s) = \frac{-220}{(s+20)^3}$$

$$p(-5) = \frac{-25-10}{15} = \frac{-35}{15} = -\frac{7}{3} = -\frac{1575}{675}$$

$$p'(-5) = \frac{110}{(-15)^2} = \frac{110}{225} = \frac{330}{675}$$

$$p''(-5) = -\frac{220}{(-15)^3} = -\frac{220}{3375} = -\frac{44}{675}$$

Then

$$C_3 = \frac{p(-5)}{01} = -\frac{1575}{675}, \quad C_2 = \frac{p'(-5)}{1!} = \frac{330}{675}, \quad C_1 = \frac{p''(-5)}{2!} = \frac{1}{2}\left(-\frac{44}{675}\right) = -\frac{22}{675}$$

and

$$V_{out}(s) = \frac{1}{675}\left[\frac{22}{s+20} - \frac{22}{s+5} + \frac{330}{(s+5)^2} - \frac{1575}{(s+5)^3}\right]$$

This yields

$$v_{out}(t) = \frac{1}{675}\left[22e^{-20t} - 22e^{-5t} + 330te^{-5t} - \frac{1575}{2}t^2e^{-5t}\right]u(t) \text{ V}$$

(b) If $v_{in}(t) = u(t)$ V,

$$V_{in}(s) = \frac{1}{s}$$

and

$$V_{out}(s) = \frac{5s-10}{s(s+5)(s+20)} = \frac{K_1}{s} + \frac{K_2}{s+5} + \frac{K_3}{s+20}$$

$$K_1 = \left.\frac{5s=10}{(s+5)(s+20)}\right|_{s=0} = \frac{-10}{100} = -\frac{1}{10}$$

$$K_2 = \left.\frac{5s-10}{s(s+20)}\right|_{s=-5} = \frac{-25-10}{(-5)(+15)} = \frac{-35}{-75} = \frac{7}{15}$$

and

$$K_3 = \left.\frac{5s-10}{s(s+5)}\right|_{s=-20} = \frac{-100-10}{(-20)(-15)} = \frac{-110}{300} = \frac{-11}{30}$$

Thus

$$v_{out}(t) = \left[\frac{7}{15}e^{-5t} - \frac{11}{30}e^{-20t} - \frac{1}{10}\right]u(t) \text{ V}$$

By virtue of linearity and time invariance, if $v_{in}(t) = [u(t) - u(t-0.5)]$ V,

$$v_{out}(t) = \left[\frac{7}{15}e^{-5t} - \frac{11}{30}e^{-20t} - \frac{1}{10}\right]u(t)$$
$$- \left[\frac{7}{15}e^{-5(t-0.5)} - \frac{11}{30}e^{-20(t-0.5)} - \frac{1}{10}\right]u(t - 0.5) \text{ V}$$

SOLUTION 14.14.

Here $v_{in}(t) = \cos(t)u(t)$ V and $i_{out}(t) = 2\sin(t)u(t)$ A, in which case

$$H(s) = \frac{I_{out}(s)}{V_{in}(s)} = \frac{\dfrac{2}{s^2+1}}{\dfrac{s}{s^2+1}} = \frac{2}{s}$$

SOLUTION 14.15.

Here $v_{in}(t) = te^{-t}u(t)$ V which implies that $V_{in}(s) = \dfrac{1}{(s+1)^2}$. Further, $v_{out}(t) = (1 + t - 0.5t^2)e^{-t}u(t) + \sin(t)u(t) - \cos(t)u(t)$ V in which case

$$V_{out}(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3} + \frac{1}{s^2+1} - \frac{s}{s^2+1}$$

(a) Hence

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \left[\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3} - \frac{s-1}{s^2+1}\right](s+1)^2$$

$$= (s+1) + 1 + \frac{1}{(s+1)} - \frac{(s-1)(s+1)^2}{s^2+1}$$

Simplifying

$$H(s) = \frac{s^3 + 2s^2 + 5s + 2}{(s+1)(s^2+1)}$$

(b) If $v_{in}(t) = (1+t)u(t)$ V, then $V_{in}(s) = \dfrac{1}{s} + \dfrac{1}{s^2} = \dfrac{s+1}{s^2}$. Hence

$$V_{out}(s) = H(s)V_{in}(s) = \frac{s^3 + 2s^2 + 5s + 2}{s^2(s^2+1)} = \frac{5}{s} + \frac{2}{s^2} - \frac{4s}{s^2+1}$$

implying that

$$v_{out}(t) = \left[5 + 2t - 4\cos(t)\right]u(t) \text{ V}$$

SOLTUION 14.16.

(a) By a voltage divider (Fig. P14.16a)

$$V_{out}(s) = \frac{Z_4(s)}{Z_3(s) + Z_4(s)} V_{in}(s)$$

and

$$H(s) = \frac{Z_4(s)}{Z_3(s) + Z_4(s)}$$

(b) In Fig. P14.16b,

$$Y_{in}(s) = Y_1(s) + Y_2(s)$$

and

$$V_{out}(s) = \frac{1}{Y_{in}(s)} I_{in}(s) = \frac{1}{Y_1(s) + Y_2(s)} I_{in}(s)$$

Hence

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{1}{Y_1(s) + Y_2(s)}$$

(c) By current division,

$$I_{out}(s) = \frac{\dfrac{1}{Z_3(s) + Z_4(s)}}{Y_1(s) + Y_2(s) + \dfrac{1}{Z_3(s) + Z_4(s)}} I_{in}(s) = \frac{1}{\left[Y_1(s) + Y_2(s)\right]\left[Z_3(s) + Z_4(s)\right] + 1} I_{in}(s)$$

Hence

$$V_{out}(s) = Z_4(s) I_{out}(s) = \frac{Z_4(s)}{\left[Y_1(s) + Y_2(s)\right]\left[Z_3(s) + Z_4(s)\right] + 1} I_{in}(s) .$$

and

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{Z_4(s)}{[Y_1(s) + Y_2(s)]\ [Z_3(s) + Z_4(s)] + 1}$$

SOLUTION 14.17. With $V_{in}(s) = V_i$ and $V_{out}(s) = V_0$, $H(s) = \frac{V_0}{V_i}$. By voltage division,

$$H(s) = \frac{V_0}{V_i} = \frac{\dfrac{1}{10^{-4}s + 10^{-3}}}{10^3 + \dfrac{1}{10^{-4}s + 10^{-3}}} = \frac{1}{0.1s + 2} = \frac{10}{s + 20}$$

(a) $V_{out}(s) = \frac{400}{s(s+20)} = \frac{20}{s} - \frac{20}{s+20}$ $\qquad v_{out}(t) = \left(20 - 20e^{-20t}\right)u(t)$ V. Plot omitted.

(b) If $v_{in}(t) = 40\left[u(t) - u(t-0.2)\right]$V, then by linearity and time invariance

$$v_{out}(t) = 20(1 - e^{-20t})u(t) - 20\left[1 - e^{-20(t-0.2)}\right]u(t-0.2) \text{ V}$$

(c) If $v_{in}(t) = 40\left[u(t) + u(t-0.2)\right]$V, then by linearity and time invariance

$$v_{out}(t) = 20(1 - e^{-20t})u(t) + 20\left[1 - e^{-20(t-0.2)}\right]u(t-0.2) \text{ V}$$

(d) If $v_{in}(t) = 40e^{-20t}u(t)$ V, $V_i(s) = \frac{40}{(s+20)}$. Hence,

$$V_0(s) = H(s)V_i(s) = \frac{400}{(s+20)^2} \qquad v_{out}(t) = 400te^{-20t}u(t) \text{ V}$$

(e) If $v_{in} = 40te^{-20t}u(t)$ V, then $V_i(s) = \frac{40}{(s+20)^2}$. Hence,

$$V_0(s) = H(s)V_i(s) = \frac{400}{(s+20)^3} \qquad v_{out}(t) = 200t^2e^{-20t}u(t) \text{ V}$$

SOLUTION 14.18.

(a) By voltage division

$$V_{out}(s) = \frac{\frac{2s+1}{s}}{2+\frac{2}{s}+\frac{2s+1}{s}}V_{in}(s) = \frac{\frac{2s+1}{s}}{\frac{2s+2+2s+1}{s}}V_{in}(s) = \frac{2s+1}{4s+3}V_{in}(s)$$

Hence

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{2s+1}{4s+3}$$

(b) With $v_{in}(t) = 8u(t)$ then

$$V_{out}(s) = H(s)V_{in}(s) = \left(\frac{2s+1}{4s+3}\right)\left(\frac{8}{s}\right) = \frac{16s+8}{s(4s+3)}$$

Using MATLAB

```
»n = [16  8];  d = [4  3  0];
»[r,p,k] = residue(n,d)
r =
   1.3333e+00
   2.6667e+00
p =
  -7.5000e-01
            0
k =     []
```

Then

$$v_{out}(t) = \left(\frac{8}{3}+\frac{4}{3}e^{-0.75t}\right)u(t) \text{ V}$$

(c) If $v_{in}(t) = 8\sin(2t)u(t)$, then

$$V_{out}(s) = H(s)V_{in}(s) = \left(\frac{2s+1}{4s+3}\right)\left(\frac{16}{s^2+4}\right)$$

Using MATLAB,

```
»ilaplace( (32*s+16)/((4*s+3)*(s^2+4)) )
ans =
-32/73*exp(-3/4*t)+32/73*cos(2*t)+280/73*sin(2*t)
```

Hence,

$$v_{out}(t) = \left(-0.43836e^{-0.75t} + 0.43836\cos(2t) + 3.8356\sin(2t)\right)u(t) \text{ V}$$

(d) With $v_{in}(t) = 8\sin(8t)u(t)$

$$V_{out}(s) = H(s)V_{in}(s) = \left(\frac{2s+1}{4s+3}\right)\left(\frac{64}{s^2+64}\right) = \frac{128s+64}{(4s+3)(s^2+64)}$$

$$= \frac{0.12391s+31.907}{s^2+64} - \frac{0.12391}{s+0.75}$$

Using MATLAB

```
»ilaplace((128*s + 64)/((4*s+3)*(s^2+64)))
ans =
-128/1033*exp(-3/4*t)+128/1033*cos(8*t)+4120/1033*sin(8*t)
```

```
»
»128/1033
ans =
   1.2391e-01
»4120/1033
ans =
   3.9884e+00
```

$$v_{out}(t) = \left(0.12391\cos(8t) + 3.9884\sin(8t) - 0.12391e^{-0.75t}\right)u(t)\ \text{V}$$

SOLUTION 14.19.

With a source transformation $I_{in}(s) = \dfrac{V_{in}(s)}{R}$.

(a) By current division,

$$I_C(s) = \frac{Cs}{\frac{1}{R} + Cs + \frac{1}{Ls}}\frac{V_{in}(s)}{R} = \frac{LCs^2}{LCs^2 + \frac{L}{R}s + 1}\frac{V_{in}(s)}{R} = \frac{s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\frac{V_{in}(s)}{R}$$

Here

$$H(s) = \frac{I_C(s)}{V_{in}(s)} = \frac{1}{R}\frac{s^2}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

(b) With $R = \frac{2}{3}\Omega$, $C = 0.5F$ and $L = 1H$,

$$H(s) = \frac{3}{2}\left[\frac{s^2}{s^2 + 3s + 2}\right]$$

If $v_{in}(t) = e^{-t}u(t)$ V, then $V_{in}(s) = \dfrac{1}{s+1}$. Hence

$$I_C(s) = H(s)I_{in}(s) = \frac{3}{2}\left(\frac{s^2}{s^2 + 3s + 2}\right)\left(\frac{1}{s+1}\right) = \frac{3s^2}{2(s+1)^2(s+2)} = \frac{K_1}{s+2} + \frac{C_1}{s+1} + \frac{C_2}{(s+1)^2}$$

Using MATLAB,

```
»n = [3  0  0];  d = conv([2  4],[1 2 1]);
»[r,p,k] = residue(n,d)
r =
   6.0000e+00
  -4.5000e+00
   1.5000e+00
p =
  -2.0000e+00
  -1.0000e+00
  -1.0000e+00
k =
     []
»
```

$$I_C(s) = \frac{6}{s+2} - \frac{4.5}{s+1} + \frac{1.5}{(s+1)^2}$$

and

$$i_C(t) = \left(6e^{-2t} - \frac{9}{2}e^{-t} + \frac{3}{2}te^{-t}\right)u(t)\ \text{A}$$

SOLUTION 14.20.

(a) Make a source transformation:

$$V_{in}(s) = \frac{1}{Cs} I_{in}(s) = \frac{250}{s} I_{in}(s)$$

By voltage division

$$V_{out}(s) = \left(\frac{10}{\frac{250}{s} + \frac{250}{s} + \frac{1}{20}s + 10}\right)\left[\frac{250}{s} I_{in}(s)\right] = \frac{2500}{s\left(\frac{1}{20}s^2 + 10s + 500\right)} I_{in}(s)$$

$$= \frac{50{,}000}{s\left(s^2 + 200s + 10{,}000\right)} I_{in}(s)$$

and

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{50{,}000}{s(s^2 + 200s + 10{,}000)}$$

(b) If $i_{in}(t) = \delta(t)$ implies $I_{in}(s) = 1$. Using MATLAB

```
»n = 50e3;  d = [1  200  10e3 0];
»[r,p,k] = residue(n,d)
r =
    -5
  -500
     5
p =
  -100
  -100
     0
k =
     []
```

Hence

$$V_{out}(s) = \frac{50{,}000}{s(s+100)^2} = \frac{5}{s} - \frac{5}{s+100} - \frac{500}{(s+100)^2}$$

and

$$v_{out}(t) = \left(5 - 5e^{-100t} - 500te^{-100t}\right)u(t)\ \text{V}$$

This is the impulse response

(c) If $L_{in}(t) = 100u(t)$ mA so that $I_{in}(s) = \frac{0.1}{s}$. Therefore

$$V_{out}(s) = \frac{5000}{s^2(s+100)^2}$$

In MATLAB,

```
»n = 5000;  d = conv([1 0 0],[1  200 1e4])
d =
         1         200       10000           0           0
»[r,p,k] = residue(n,d)
r =
   1.0000e-02
   5.0000e-01
  -1.0000e-02
   5.0000e-01
p =
  -100
  -100
     0
     0
k =
     []
```

Hence

$$V_{out}(s) = \frac{-0.01}{s} + \frac{0.5}{s^2} + \frac{0.01}{s+100} + \frac{0.5}{(s+100)^2}$$

and

$$v_{out}(t) = \left[0.01e^{-100t} + 0.5te^{-100t} - 0.01 + 0.5t\right]u(t)$$

(d) By superposition and time invariance, if

$$i_{in}(t) = 100\left[u(t) + u(t-1)\right] \text{ mA}$$

then the result of part (c) can be adjusted to

$$v_{out}(t) = \left[0.01e^{-100t} + 0.5te^{-100t} - 0.01 + 0.5t\right]u(t)$$
$$-\left[0.01e^{-100(t-1)} + 0.5te^{-100(t-1)} - 0.01 + 0.5(t-1)\right]u(t-1) \text{ V}$$

SOLUTION 14.21. For this problem change the 20 mH inductor to one of 0.3 H.

(a)

$$Y_{in} = \frac{1}{15} + \frac{1}{0.3s + 90} + \frac{1}{0.1s + 10} = \frac{(s + 200)(s + 400)}{15(s + 100)(s + 300)}$$

and

$$H(s) = \frac{I_{out}}{I_{in}} = \frac{1/15}{Y_{in}} = \frac{(s + 100)(s + 300)}{(s + 200)(s + 400)}$$

(b) If $i_{in}(t) = \delta(t)$, then $I_{in}(s) = 1$ and

$$I_{out}(s) = H(s) = \frac{(s + 100)(s + 300)}{(s + 200)(s + 400)} = 1 - \frac{50}{s + 200} - \frac{150}{s + 400}$$

Hence

$$i_{out}(t) = \delta(t) + (-50e^{-200t} - 150e^{-400t})u(t) \text{ A}$$

(c) We first find the response to $i_{in}(t) = 16u(t)$ mA. Here $I_{in}(s) = 0.016/s$ and

$$I_{out}(s) = H(s)I_{in}(s) = \frac{0.016(s + 100)(s + 300)}{s(s+ 200)(s + 400)} = \frac{0.006}{s} + \frac{0.004}{s + 200} + \frac{0.006}{s + 400}$$

Hence

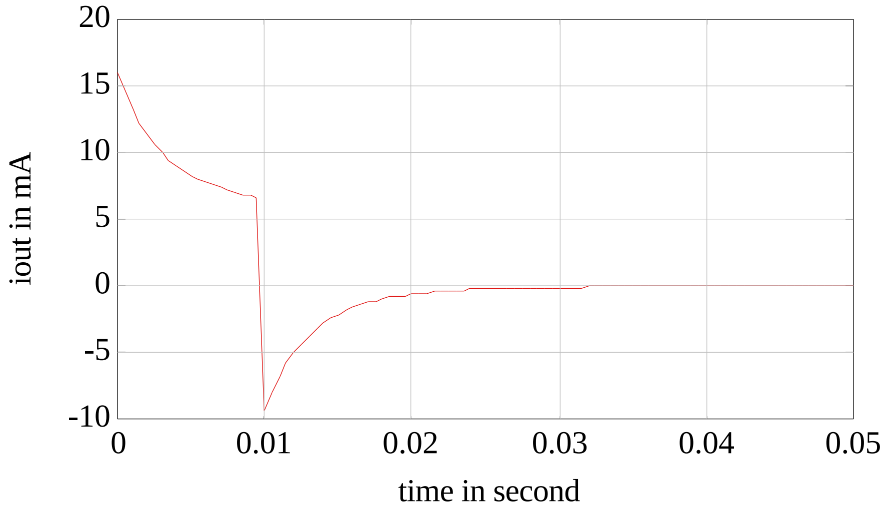
$$i_{out}(t) = (6 + 4\,e^{-200t} + 6\,e^{-400\,t})u(t) \quad \text{mA}$$

By linearity and time invariance, the response to $i_{in}(t) = 16[ut) – u(t – 0.01)]$ mA is

$$i_{out}(t) = (6 + 4\,e^{-200t} + 6\,e^{-400\,t})u(t) - (6 + 4\,e^{-200(t - 0.01)} + 6\,e^{-400\,(t - 0.01)})u(t - 0.01) \quad \text{mA}$$

A plot of $i_{out}(t)$ using MATLAB is given below.

```
t= 0: 0.0005: 0.05;
f1= (6 + 4*exp(-200*t) + 6*exp(-400*t)).*u(t);
f2= (6 + 4*exp(-200*(t-0.01)) + 6*exp(-400*(t-0.01))).*u(t-0.01);
iout= f1 - f2;
plot(t, iout)
grid
ylabel('iout in mA')
xlabel(' time in second')
```



REMARK: Notice that the resistor current is not continuous.

SOLUTION 14.22. (a) First observe that the admittance of a parallel LC is

$$Y_{LC}(s) = Cs + \frac{1}{Ls}$$

Using voltage division,

$$V_{out}(s) = \frac{C_1 s + \frac{1}{Ls}}{C_1 s + \frac{1}{Ls} + C_2 s + \frac{1}{Ls}} V_{in}(s) = \frac{C_1 s + \frac{1}{Ls}}{\left(C_1 + C_2\right)s + \frac{2}{Ls}} V_{in}(s) = \frac{LC_1 s^2 + 1}{L\left(C_1 + C_2\right)s^2 + 2} V_{in}(s)$$

Finally

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{C_1}{\left(C_1 + C_2\right)} \frac{s^2 + \frac{1}{LC_1}}{s^2 + \frac{2}{L\left(C_1 + C_2\right)}} = 0.2\frac{s^2 + 4 \times 10^6}{s^2 + 1.6 \times 10^6}$$

(b) Using MATLAB,

```
»syms s t
»ilaplace(0.2*(s^2+4e6)/(s^2+1.6e6))
ans =
1/5*Dirac(t)+120*10^(1/2)*sin(400*10^(1/2)*t)
»120*10^(1/2)
ans =   3.7947e+02
```

$$h(t) = 0.2L^{-1}\left[\frac{s^2 + 4 \times 10^6}{s^2 + 1.6 \times 10^6}\right] = 0.2\delta(t) + 379.47\sin(1264.9t)u(t) \text{ V}$$

SOLUTION 14.23.

$$Y_1(s) = C_1 s + \frac{1}{R_1} = \frac{R_1 C_1 s + 1}{R_1}$$

$$Y_2(s) = C_2 s + \frac{1}{R_2} = \frac{R_2 C_2 s + 1}{R_2}$$

Then $Z_1(s) = \frac{R_1}{R_1 C_1 s + 1}$ and $Z_2(s) = \frac{R_2}{R_2 C_2 s + 1}$. By voltage division,

$$V_{out}(s) = \frac{\frac{R_2}{R_2 C_2 S + 1}}{\frac{R_1}{R_1 C_1 s + 1} + \frac{R_2}{R_2 C_1 s + 1}} V_{in}(s) = \frac{\left(\frac{R_2}{R_2 C_2 s + 1}\right) V_{in}(s)}{\frac{R_1 R_2 C_2 s + R_1 + R_1 R_2 C_1 s + R_2}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}}$$

$$= \frac{R_2(R_1 C_1 s + 1)}{(C_1 + C_2)R_1 R_2 s + R_1 + R_2} V_{in}(s)$$

Thus the transfer function is:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2(R_1 C_1 s + 1)}{(C_1 + C_2)R_1 R_2 s + R_1 + R_2}$$

(b) If $C_1 = 0.5$ F, $C_2 = 1.0$ F and $v_{in}(t) = 10\ u(t)$ V, then

$$V_{out}(s) = H(s)V_{in}(s) = \left(\frac{0.5R_1R_2s + R_2}{1.5R_1R_2s + R_1 + R_2}\right)\left(\frac{10}{s}\right)$$

Moreover, with $\frac{R_1}{R_2} = 4$ so that $R_1 = 4R_2$

$$V_{out}(s) = \left(\frac{2R_2^2s + R_2}{6R_2^2s + 5R_2}\right)\left(\frac{10}{s}\right) = \frac{20R_2^2(s + \frac{1}{2R_2})}{6R_2^2\left[s(s + \frac{5}{6R_2})\right]} = \frac{10}{3}\left[\frac{s + \frac{1}{2R_2}}{s(s + \frac{5}{6R_2})}\right]$$

The partial fraction expansion is

$$V_{out}(s) = \frac{10}{3}\left[\frac{s + \frac{1}{2R_2}}{s\left(s + \frac{5}{6R_2}\right)}\right] = \frac{10}{3}\left[\frac{K_1}{s} + \frac{K_2}{s + \frac{5}{6R_2}}\right]$$

Observe that

$$\frac{K_2}{s + \frac{5}{6R_2}} \xrightarrow{\mathcal{L}^{-1}} K_2 e^{-\frac{5}{6R_2}t}$$

and that it is required that

$$-\frac{5}{6R_2} = -\frac{5}{3}$$

Thus

$$R_2 = 0.5\ \Omega,\ \ R_1 = 4R_1 = 2\ \Omega$$

and

$$V_{out}(s) = \frac{10}{3}\left[\frac{s+1}{s\left(s + \frac{5}{3}\right)}\right] = \frac{10}{3}\left[\frac{0.6}{s} + \frac{0.4}{s + \frac{5}{3}}\right] = \frac{2}{s} + \frac{4/3}{s + \frac{5}{3}}$$

Thus,

$$v_{out}(t) = \left[2 + \frac{4}{3}e^{-\frac{5}{3}t}\right]u(t)$$

(c) If $R_1C_1 = R_2C_2 = \tau$, then the transfer function is

$$H(s) = \frac{R_2(R_1C_1s + 1)}{R_1R_2C_1s + R_1R_2C_2s + R_1 + R_2} = \frac{R_2\tau s + R_2}{\left[R_2(R_1C_1) + R_1(R_2C_2)\right]s + R_1 + R_2}$$

$$= \frac{R_2(\tau s + 1)}{(R_1R_2)(\tau s + 1)} = \frac{R_2}{R_2 + R_1}$$

The zero-state response is

$$V_{out}(s) = \frac{R_2}{R_2 + R_1} 10u(t)$$

(d) Using $H(s)$ from part (c) with the requirement that $R_1C_1 = R_2C_2$, then

$$H(s) = \frac{R_2}{R_1 + R_2} = \frac{1}{10}$$

With $R_2 = 10^6\ \Omega$, then $10R_2 = R_1 + R_2$ $R_1 = 9R_2 = 9\ \text{M}\Omega$. Since $C_2 = 5\times 10^{-12}$ F, then

$$C_1 = \frac{R_2C_2}{R_1} = 0.556\times 10^{-12}\ \text{F}$$

SOLUTION 14.24.

(a) $H(s) = \dfrac{V_{out}(s)}{V_{in}(s)}$. Here $I_b(s) = \dfrac{V_{in}(s)}{2000}$. The parallel admittance at the right is

$$Y_R(s) = Cs + \frac{1}{R} = \frac{RCs + 1}{R}$$

so that

$$Z_R(s) = \frac{R}{RCs + 1}$$

Then

$$V_{out}(s) = -Z_R(s)\beta I_b(s) = -\frac{R\beta}{2000(RCs + 1)}$$

(b) With $V_1(s) = V_{in}(s) - V_{out}(s)$, then

$$Cs\big(V_{out}(s) - V_{in}(s)\big) + \frac{1}{s}V_{out}(s) + \frac{1}{2}\big[V_{out}(s) - 3V_{in}(s) + 3V_{out}(s)\big] = 0$$

Hence

$$\left(Cs + \frac{3}{2}\right)V_{in}(s) = \left(Cs + \frac{1}{s} + 2\right)V_{out}(s)$$

and

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{s\big(Cs + 1.5\big)}{Cs^2 + 2s + 1} = \frac{s\big(s + 0.75\big)}{s^2 + s + 0.5}$$

(c) Transform the current source $i_{in}(t)$ into a voltage source. In the s-domain with $I_{in}(s) = 2V_{in}(s)$
Here $Z_C(s) = \dfrac{1}{Cs} = \dfrac{1}{2s}$ which implies $Y_C(s) = 2s$. A single node equation yields

$$\frac{2}{3}\big[V_{out}(s) - V_{in}(s)\big] - \frac{2}{3}V_{out}(s) + 2sV_{out}(s) = -\frac{2}{3}V_{in}(s) + \left(\frac{2}{3} - \frac{2}{3} + 2s\right)V_{out}(s) = 0$$

$$\frac{2}{3}V_{in}(s) = 2sV_{out}(s) \text{ implies } V_{out}(s) = \frac{V_{in}(s)}{3s}$$

But $V_{in}(s) = \dfrac{I_{in}(s)}{2}$ in which case

$$V_{out}(s) = \frac{I_{in}(s)}{6s}$$

and

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{1}{6s}$$

SOLUTION 14.25. Here in the s-domain

$$I_{in}(s) = \frac{V_{in}(s) - V_C(s)}{0.5s + 10}$$

and with a node at $V_C(s)$

$$-I_{in}(s) + \frac{sV_C(s)}{500} + \frac{1}{2}I_{in}(s) = 0 \text{ implies } \frac{sV_C(s)}{50} = \frac{1}{2}I_{in}(s) = \frac{V_{in}(s) - V_C(s)}{s + 20}$$

Hence,

$$\left(\frac{s^2 + 20s + 500}{500}\right)V_C(s) = V_{in}(s)$$

and the transfer function is

$$H(s) = \frac{V_C(s)}{V_{in}(s)} = \frac{500}{s^2 + 20s + 500}$$

With $v_{in}(s) = 4\sqrt{5}\,u(t)$ V which implies $V_{in}(s) = \dfrac{4\sqrt{5}}{s}$, and

$$V_C(s) = \frac{2000\sqrt{5}}{s(s + 10 - j20)(s + 10 + j20)} = \frac{K_1}{s} + \frac{K}{s + 10 - j20} + \frac{K^*}{s + 10 + j20}$$

where K^* designates the complex conjugate of K

$$K_1 = \left.\frac{2000\sqrt{5}}{s^2 + 20s + 500}\right|_{s=0} = \frac{2000\sqrt{5}}{500} = 8.944$$

$$K = \left.\frac{2000\sqrt{5}}{s(s + 10 + j20)}\right|_{s=-(10-j20)}$$

$$= \frac{2000\sqrt{5}}{(-10 + j20)(j40)} = \frac{2000\sqrt{5}}{-(800 + j400)} = -4.472 + j2236 = Je^{ij\phi}$$

where $\phi = 153.44^{o}$. Then with

$$K^* = -4.472 - j2.236 = 5e^{-j\phi}$$

$$V_C(s) = \frac{8.944}{s} + \frac{A + jB}{s + 10 + j20} + \frac{A - jB}{s + 10 - j20}$$

Here

$$A = -4.472,\ \ B = 2.236,\ \sqrt{A^2 + B^2} = 5$$

and

$$\text{arc tan}\frac{B}{A} = \text{arc tan}\frac{2.236}{-4.472} = \text{arc tan}\frac{1}{-2} = -153.44^{\mathrm{o}}$$

With the help of Table 13.1.

$$v_C(t) = \left[8.944 + 10e^{-10t}\cos(20t + 153.44^{\mathrm{o}})\right]u(t)\ \ \text{V}$$

SOLUTION 14.26. In the s-domain we first find $V_x(s)$ in terms of $V_{in}(s)$ via voltage division:

$$V_x(s) = \frac{Z_p(s)}{40 + Z_p(s)}V_{in}(s)$$

where

$$Z_p(s) = \frac{(0.40s)(40)}{0.40s + 40} = \frac{40s}{s + 100}$$

Hence

$$V_x(s) = \frac{\dfrac{40s}{s+100}}{40 + \dfrac{40s}{s+100}}V_{in}(s) = \frac{40s}{80s + 4000} = \frac{0.5s}{s + 50}V_{in}(s)$$

and

$$I_L(s) = \frac{V_x(s)}{0.4s} = \frac{2.5}{s}V_x(s)$$

Then from the right hand side by another voltage division

$$V_{out}(s) = \frac{10}{\dfrac{1000}{s} + 10}0.25V_x(s) = \frac{10s}{10s + 1000}0.25V_x(s) = \frac{0.25s}{s + 100}V_x(s)$$

(a) If $v_{in}(s) = 20(1 - e^{-40t})u(t)$, then $V_{in}(s) = \dfrac{20}{s} - \dfrac{20}{s+40}$. Hence

$$V_x(s) = \left(\frac{0.5s}{s+50}\right)\left[\frac{20}{s} - \frac{20}{s+40}\right] = \left(\frac{s}{s+50}\right)\left(\frac{10}{s} - \frac{10}{s+40}\right)$$

and

$$I_L(s) = \frac{2.5}{s}V_x(s) = \left(\frac{2.5}{s+50}\right)\left[\frac{1}{s} - \frac{1}{s+40}\right] = \frac{0.05}{s} + \frac{-0.25}{s+40} + \frac{0.2}{s+50}$$

Hence

$$i_L(t) = \left[0.05 + 0.2e^{-50t} - 0.25e^{-40t}\right]u(t)\ \ \text{A}$$

(b) $V_{in}(s) = \dfrac{20}{s} - \dfrac{20}{s+40}$. From, part (a), it was found that

$$V_{out}(s) = \frac{0.25s}{s+100}V_x(s) = \frac{0.25s}{s+100} \times \frac{0.5s}{s+50}V_{in}(s) = \frac{s}{s+100} \times \frac{s}{s+50} \times \left[\frac{2.5}{s} - \frac{2.5}{s+40}\right]$$

$$= \frac{100s}{(s+40)(s+50)(s+100)} = \frac{-20/\,3}{(s+40)} + \frac{10}{(s+50)} + \frac{-10\,/3}{(s+100)}$$

Thus,

$$V_{out}(t) = \left[10e^{-50t} - \frac{10}{3}e^{-100t} - \frac{20}{3}e^{-40t}\right]u(t)\ \text{ V}$$

SOLUTION 14.27. In both parts (a) and (b), the op-ampis ideal. It will not draw current and the virtual groundprincipal requires that

$$v^{+} = v^{-} = 0$$

(a) For a note at the inverting terminal with mode voltage $v_1(t) = 0$, KCL gives in the s-domain

$$\frac{V_{in}(s)}{R_1} = -CsV_{out}(s) - \frac{V_{out}(s)}{R_2} \quad \text{implies} \quad \frac{V_{in}(s)}{R_1} = -\left(Cs + \frac{1}{R_2}\right)V_{out}(s)$$

in which case

$$\frac{V_{in}(s)}{R_1} = -\left(\frac{R_2Cs+1}{R_2}\right)V_{out}(s)$$

Then,

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_2}{R_1}\left(\frac{1}{R_2Cs+1}\right) = -\frac{1}{R_1C}\left(\frac{1}{s + \dfrac{1}{R_2C}}\right)$$

To make

$$H(s) = -\frac{20}{s+4}$$

make

$$\frac{1}{R_1C} = 20 \quad and \quad \frac{1}{R_2C} = \frac{1}{4}$$

If

$$C = 1\mu F = 10^{-6}F$$

$$\frac{1}{R_2C} = 4 \ \text{ or } \ R_2 = \frac{1}{4C} = \frac{10^6}{4} = 250\ k\Omega$$

and

$$\frac{1}{R_1C} = 20 \ \text{ or } \ R_1 = \frac{2}{20C} = \frac{106}{20} = 50\ k\Omega$$

(b) With Fig. 14.27b in the s-domain and $v_1(t)$ at the inverting terminal, KCL gives

$$C_1sV_{in}(s) + \frac{V_{in}(s)}{R_1} = -C_2sV_{out}(s) - \frac{V_{out}(s)}{R_2}$$

$$\left(\frac{R_1C_1s+1}{R_1}\right)V_{in}(s) = -\left(\frac{R_2C_2s+1}{R_2}\right)V_{out}(s)$$

Then

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_2}{R_1}\left(\frac{R_1C_1s+1}{R_2C_2s+1}\right) = -\frac{R_1R_2C_1}{R_1RC_2}\left(\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right) = -\frac{C_1}{C_2}\left[\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right]$$

(c) If

$$H(s) = -5,\ C_2 = 1\mu F\ \ and\ \ R_2 = 1\,\text{M}\Omega$$

then

$$\frac{C_1}{C_2} = 5\ \ and\ \ C_1 = 5\mu F$$

The bracketed term must cancel and with

$$R_2C_2 = 10^6(10^{-6}) = 1$$

Then with $C_1 = 5\mu F$

$$R_1C_1 = 1$$

$$R_1 = \frac{1}{C_1} = \frac{1}{5\times 10^{-6}} = 200k\Omega$$

(d) Using $H(s)$ in part (b)

$$H(s) = -\frac{C_1}{C_2}\left(\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right)$$

to obtain

$$H(s) = -5\frac{s+1}{s+2}$$

with $C_2 = 1\mu F$

$$C_1 = 5C_2 = 5\times 10^6 F\quad (5\mu F)$$

$$\frac{1}{R_1C_1} = 1\ \ or\ \ R_1 = \frac{1}{C_1} = \frac{10^6}{5} = 200k\Omega$$

$$\frac{1}{R_2C_2} = 2\ \ or\ \ R_2 = \frac{1}{2C_2} = -\frac{10^6}{2} = 500k\Omega$$

SOLUTION 14.28. Here, the op-amp will not draw current at the non-inverting terminal and the principal of the virtual ground demand that

$$v_1 = v_2 = V_{in}(s)$$

For Fig. 14.28 in the s-domain with a node $V_1(s)$ taken at the inverting terminal

$$\frac{V_{in}(s)}{R_1} + \frac{V_{in}(s) - V_{out}(s)}{Z_p(s)} = 0$$

Here

$$Z_p(s) = \frac{\dfrac{R_2}{Cs}}{\dfrac{1}{Cs}+R_2} = \frac{R_2}{R_2Cs+1} = \frac{1}{C(s+\dfrac{1}{R_2C})}$$

and

$$\left[\frac{1}{R_1}+C(s+\frac{1}{R_2C})\right]V_{in}(s) = C(s+\frac{1}{R_2C})V_{out}(s)$$

$$\left[R_1C\left(s+\frac{1}{R_2C}\right)+1\right]V_{in}(s) = R_1C\left(s+\frac{1}{R_2C}\right)V_{out}(s)$$

and

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_1C(s+\dfrac{1}{R_2C})+1}{R_1C(s+\dfrac{1}{R_2C})} = \frac{s+\dfrac{1}{R_2C}+\dfrac{1}{R_1C}}{s+\dfrac{1}{R_1C}}$$

Here $\frac{1}{R_2C}+\frac{1}{R_1C}=4$

and

$$\frac{1}{R_1C}=2$$

If $C=1\mu F$ then $R_1=\frac{1}{2C}=\frac{10^6}{2}=500\ k\Omega$

and $\frac{1}{R_2C}=4-2=2$ implies $R_2=500\ k\Omega$.

SOLUTION 14.29. For the non-inverting configuration in the s-domain, each of the two op-amps in cascade have a transfer function

$$H(s) = -\frac{Z_f(s)}{Z_{in}(s)}$$

Then for the two op-amps

$$H_0(s) = \left[-\frac{Z_{f1}(s)}{Z_{in1}(s)}\right]\left[-\frac{Z_{f2}(s)}{Z_{in,2}}\right] = \frac{Z_{f,1}(s)Z_{f,2}(s)}{Z_{in,1}(s)Z_{in,2}(s)}$$

For Fig. P14.29a in the s-domain

$$Z_{in,1}=25k\Omega,\quad Z_{in,2}=50k\Omega,\quad Z_{f,1}=\frac{1}{C\left(s+\dfrac{1}{RC}\right)}=\frac{250{,}000}{s+5},\ \text{and}\ Z_{f,2}=\frac{1}{C\left(s+\dfrac{1}{RC}\right)}=\frac{250{,}000}{s+2.5}$$

Hence for Fig. P14.29a

$$H_a(s) = \left(\frac{\frac{250,000}{s+2.5}}{25,000}\right)\left(\frac{\frac{250,000}{s+5.0}}{50,000}\right) = \frac{50}{(s+2.5)(s+5)}$$

If $v_{in}(t) = u(t)$, then $V_{in}(s) = \frac{1}{s}$ and

$$V_{out}(s) = \frac{50}{s(s+2.5)(s+5)} = \frac{4}{s} - \frac{8}{s+2.5} + \frac{4}{s+5}$$

Hence,

$$v_{out}(t) = \left[4 - 8e^{-2.5t} + 4e^{-5t}\right]u(t) \text{ V}$$

(b) For Fig. P14.29b in the s-domain, $Z_{in,1}$, Z_{f1}, and Z_{f2} are in part (a). However,

$$Z_{in,2} = \frac{1}{Cs} = \frac{250,000}{s}$$

Thus

$$H_b(s) = \left(\frac{\frac{250,000}{s+2.5}}{25,000}\right)\left(\frac{\frac{250,000}{s+5}}{\frac{250,000}{s}}\right) = \left(\frac{10}{s+2.5}\right)\left(\frac{s}{s+5}\right) = \frac{10s}{\left(s+\frac{5}{2}\right)(s+5)}$$

With $V_{in}(s) = \frac{1}{s}$,

$$V_{out}(s) = \left[\frac{10s}{(s+2.5)(s+5)}\right]\left[\left(\frac{1}{s}\right)\right] = \frac{20}{(s+2.5)(s+5)} = \frac{4}{s+2.5} - \frac{4}{s+5}$$

and

$$v_{out}(t) = \left(4e^{-2.5t} - 4e^{-5t}\right)u(t) \text{ V}$$

SOLUTION 14.30.

(a) -(b). The subcircuit is an integrator, with

$$\frac{V_{out}(t)}{V_1(s)} = -\frac{1}{s}$$

(c) This subcircuit is again an integrator, with

$$\frac{V_1(t)}{V_2(s)} = -\frac{1}{s}$$

(d) Applying KCL to the inverting input terminal of the top left op amp, we have

$$G_2V_{out}(s) + V_3(s) + V_2(s) = 0$$

or

$$V_2(s) = -G_2V_{out}(s) - V_3(s)$$

(e) Applying KCL to the inverting input terminal of the bottom op amp, we have

$$G_3V_1(s) + G_1 V_{in}(s) + V_3(s) = 0$$

or

$$V_3(s) = - G_3V_1(s) - G_1 V_{in}(s)$$

(f) The results of parts (b), (c) and (d) do not involve V_{in}. Therefore, , we can solve for V_1, V_2 and V_3 in terms of V_{out} from these threes equations:

$$V_1(s) = - sV_{out}(s)$$

$$V_2(s) = - sV_1(s) = s^2V_{out}(s)$$

and

$$V_3(s) = - V_2(s) - G_2V_{out}(s) = - \left(s^2 + G_2\right) V_{out}(s)$$

Substituting these relationships into the result of part (e), we obtain

$$- sG_3V_{out}(s) + G_1 V_{in}(s) - \left(s^2 + G_2\right) V_{out}(s) = 0$$

Therefore

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{G_1}{s^2 + G_3s + G_2}$$

SOLUTION 14.31. Use the parallel equivalent circuit model for the capacitor with the standard directions for voltage and current as given in figure 14.16. For the single node with $v_C(0^-) = 20$ V,

$$\frac{V_C(s)}{\frac{10}{s}} + \frac{V_C(s)}{40+10} - \frac{1}{10}v_C(0^-) = 0 \quad \text{implies} \quad \left(\frac{s}{10} + \frac{1}{50}\right)V_C(s) = \frac{1}{10}(20) = 2$$

Equivalently, $(50s+10)V_C(s) = (s+0.2)V_C(s) = 20$ or $V_C(s) = \dfrac{20}{s+0.2}$. Therefore,

$$v_C(t) = 20e^{-0.2t}u(t) \text{ V}$$

SOLUTION 14.32. Using the equivalent model for the inductor in figure 14.19, we can compute the total admittance as

$$Y(s) = \frac{5}{2s} + \frac{1}{40} + \frac{1}{10} = \frac{5}{2s} + \frac{1}{8} = \frac{2s+40}{16s} = \frac{s+20}{8s}$$

Using current division,

$$I_{out}(s) = \frac{0.1}{\frac{s+20}{8s}} \times \frac{-i_L(0)}{s} = -\frac{0.8 i_L(0^-)}{s+20} = -\frac{1.6}{s+20}$$

Thus

$$i_{out}(t) = -1.6e^{-20t}\,u(t)\ \text{ A}$$

SOLUTION 14.33. Using the equivalent model for the inductor in figure 14.19 and for the capacitor using figure 14.16, we may combine the current sources to form an equivalent source (with C = 0.1 F) to obtain

$$I_{eq}(s) = \frac{1}{10}v_C(0^-) - \frac{i_L(0^-)}{s} = 0.2 - \frac{1}{s}$$

Note that

$$Y(s) = Cs + \frac{1}{LS} = \frac{LCs^2+1}{Ls} = C\left(\frac{s^2+\frac{1}{LC}}{s}\right)$$

With C = 0.1 F and L = 0.4 H, $\frac{1}{LC} = 25$ and

$$Z(s) = \frac{1}{C}\left(\frac{s}{s^2+25}\right) = \frac{10s}{s^2+25}$$

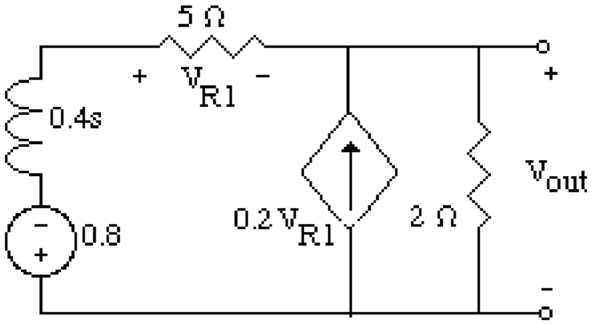
Thus

$$V_C(s) = Z(s)I_{eq}(s) = \left(\frac{10s}{s^2+25}\right)\left(\frac{s-5}{5s}\right) = \frac{2(s-5)}{s^2+25} = \frac{2s}{s^2+25} - \frac{10}{s^2+25}$$

and

$$v_C(t) = \big(2\cos(5t) - 2\sin(5t)\big)u(t)\ \text{ V}$$

SOLUTION 14.34. Consider the equivalent circuit below:



Writing a single node equation we have,

$$0 = 0.5V_{out} - 0.2V_{R1} + \frac{1}{5+0.4s}\left(V_{out} + 0.8\right)$$

$$= 0.5V_{out} + 5 \times 0.2 \times \frac{1}{5+0.4s}\left(V_{out} + 0.8\right) + \frac{1}{5+0.4s}\left(V_{out} + 0.8\right)$$

$$= 0.5V_{out} + \frac{2}{5+0.4s}\left(V_{out} + 0.8\right)$$

Therefore

$$V_{out} = \frac{-8}{s+22.5}$$

and

$$v_{out}(t) = -8e^{-22.5t}u(t) \text{ V}$$

SOLUTION 14.35. Redraw the circuit in the s-domain and use an equivalent circuit for the capacitor (figure 14.16) that accounts for the initial condition. By KCL

$$\frac{V_C(s)}{R} + CsV_C(s) = Cv(0^-) + I_{in}(s)$$

With $R = 50\ \Omega$ and $C = 0.02$ F,

$$\frac{V_C(s)}{50} + 0.02sV_C(s) = 0.02v_C(0^-) + I_{in}(s)$$

or

$$V_C(s) + sV_C(s) = v_C(0^-) + 50I_{in}(s)$$

which is equivalent to

$$(s+1)V_C(s) = v_C(0^-) + 50I_{in}(s)$$

(a) With $v_C(0^-) = 8$ V and $i_{in}(t) = 40\delta(t)$ mA so that $I_{in}(s) = 0.04$,

$$(s+1)V_C(s) = 8+2 \quad \text{implies} \quad V_C(s) = \frac{10}{s+1}$$

and

$$v_C(t) = 10e^{-10t}u(t) \text{ V}$$

(b) With $v_C(0^-) = 1$ V and $i_{in}(t) = 200e^{-t}u(t)$ mA we have that $I_{in}(s) = \frac{0.2}{s+1}$. Thus

$$V_C(s) = \frac{v_C(0^-)}{(s+1)} + \frac{50}{(s+1)}I_{in}(s) = \frac{1}{(s+1)} + \frac{10}{(s+1)^2}$$

and

$$v_C(t) = \left(e^{-t} + 10te^{-t}\right)u(t) \text{ V}$$

SOLUTION 14.36.

(a) By current division

$$H(s) = \frac{I_L(s)}{I_{in}(s)} = \frac{1}{Ls + 1}$$

(b) The given data in Laplace transforms are:

$$I_{in}(s) = \frac{I_0}{s^2} \quad \text{and} \quad I_L(s) = \frac{15}{s^2} - \frac{3}{s} + \frac{3}{s+5} = \frac{75}{s^2(s+5)}$$

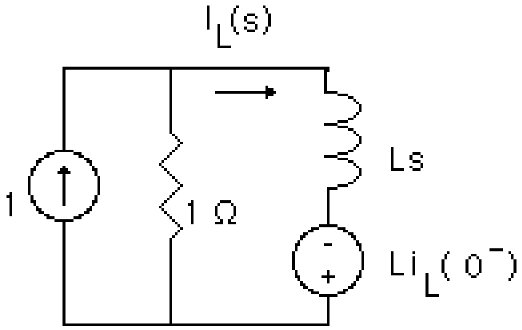
Under the assumption of zero initial inductor current,

$$I_L(s) = H(s)\, I_{in}(s) = H(s) = \frac{I_L(s)}{I_{in}(s)} = \frac{I_0}{(Ls+1)s^2} = \frac{I_0/L}{(s+1/L)s^2} = \frac{75}{s^2(s+5)}$$

Equating coefficients, we obtain the answers

$$L = 1/5 = 0.2 \text{ H and } I_0 = 75L = 15 \text{ A}$$

(c) The s-domain equivalent is shown below.



Applying KVL to the right mesh, we have

$$LsI_L(s) - L\, i_L(0^-) + 1 \times [I_L(s) - 1] = 0$$

Solving for $I_L(s)$ from this equation, and equating it to 10/(s+ 5), we have

$$I_L(s) = \frac{L\, i_L(0^-) + 1}{L\, s + 1} = \frac{i_L(0^-) + 1/L}{s + 1/L} = \frac{10}{s+5}$$

from which L= 0.2 H and $i_L(0^-) = 5$ A.

SOLUTION 14.37.

(a) By inspection,

$$H(s) = \frac{I_L(s)}{V_{in}(s)} = \frac{1}{2s + 200} = \frac{0.5}{s + 100}$$

Given $v_{in}(t) = 2u(t)$ V, then $V_{in}(s) = 2/s$, and

$$I_L(s) = \frac{1}{s\left(s + 100\right)} = \frac{0.01}{s} - \frac{0.01}{s + 100}$$

in which case

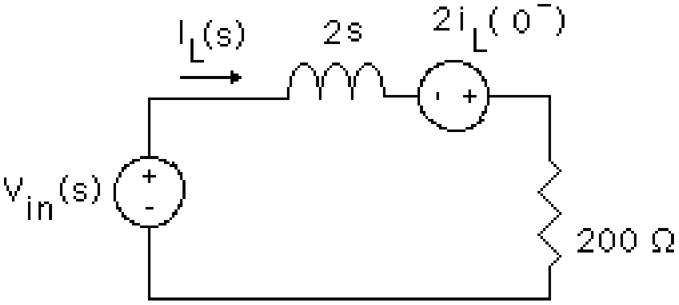
$$i_L(t) = 0.01 \left(1 - e^{-100t}\right) \text{ A}$$

Plots are omitted.

(b) By linearity and time invariance,

$$i_L(t) = 0.01 \left(1 - e^{-100t}\right)u(t) - 0.01\left(1 - e^{-100(t - 0.05)}\right)u(t - 0.05) \text{ A}$$

(c) Correction: (a) should be (c). With nonzero initial inducto current, the s-domain equivalent becomes:



Given $i_L(0^-) = 0.01$ A and $v_{in}(t) = 2e^{-200t}u(t)$ V, then $V_{in}(s) = 2/(s + 200)$ and

$$I_L(s) = \frac{\frac{2}{s + 200} + 0.02}{\left(2s + 200\right)} = \frac{0.01s + 3}{(s + 100)(s + 200)}$$

$$= \frac{0.02}{s + 100} - \frac{0.01}{s + 200}$$

in which case

$$i_L(t) = \left(0.02e^{-100t} - 0.01e^{-200t}\right) u(t) \text{ A}$$

(d) Correction . (b) should be (d).
Given $i_L(0^-) = 0.01$ A and $v_{in}(t)= 2\cos(200t)u(t)$ V, then

$$V_{in}(s) = \frac{2s}{s^2 + 40000}$$

and

$$I_L(s) = \frac{\dfrac{2s}{s^2 + 40000} + 0.02}{(2s + 200)} = \frac{0.01s^2 + s + 400}{(s + 100)(s^2 + 40000)}$$

We use MATLAB to do the partial fraction expansion.

```
n= [ 0.01 1 400];
d= conv([ 1 100], [ 1 0 40000]);
[ r p k ] = residue (n,d)
r =
   0.0010 - 0.0020i
   0.0010 + 0.0020i
   0.0080
p =
   1.0e+02 *
   0.0000 + 2.0000i
   0.0000 - 2.0000i
  -1.0000
```

From the above MATLAB output,

$$I_L(s) = \frac{0.008}{s + 100} + \frac{0.001-j0.002}{s - j200} + \frac{0.001+j0.002}{s + j200} = \frac{0.008}{s + 100} + \frac{0.002s + 0.8}{s^2 + 200^2}$$

From table 13.1, item 18,

$$i_L(t) =\left(0.008e^{-100t} + 0.002 \cos(200t) + 0.004 \sin(200t) \right) u(t) \text{ A}$$