Chapter 15/Problem 1

Fig P15.1 here

In the pole-zero plot, there are poles at

$$S = -2$$
$$S = -4 - j4$$
$$S = -4 + j4$$

and zeros at

$$S = -j4$$
$$S = +j4$$
$$S = 0$$

Hence

$$H(s) = \frac{(s+j4)(s-j4)s}{s+2(s+4+j4)(s+4-j4)} = \frac{s^3+16s}{(s+2)(s^2+8s+32)}$$

$$H(s) = \frac{s^3+16s}{s^3+10s^2+48s+64}$$

Note that

$$H(\infty) = 1$$

Chapter 15 / Problem 2

(a) In the pole-zero plot, there are poles at

Fig P.15.2 (a)

Fig P.15.2 b

$$s = 4j$$
$$s = -j4$$

and zeros at

$$s = -4$$
$$s = 1$$

Hence

$$H(s) = \frac{(s+4)(s-1)}{(s-j4)(s+j4)} = \frac{s^2+3s-4}{s^2+16}$$

To make the gain $|H(s)| = 2$ at $s=0$, a multiplier is needed

$$H(s) = C\,\frac{s^2+3s-4}{s^2+16}$$

So that

$$H(0) = C\left(\frac{-4}{16}\right) = -\frac{C}{4} = 2$$

Thus

$$C = -8$$

and

$$H(s) = -8\,\frac{s^2+3s-4}{s^2+16}$$

(b) In the pole-zero plot, there are poles at

$$s = -2+j4$$
$$s = -2-j4$$
$$s = 0$$
$$s = -2$$

Chapter 15/2 Cont'd

and zeroes at

$s = j4$

$s = -j4$

$s = -4$

Hence

$$H(s) = C \frac{(s+4)(s^2+16)}{s(s+2)(s^2+4s+20)}$$

At $s = 1$, the gain $|H(s)| = 6.8$

$$H(1) = C \frac{5(17)}{1(3)(25)} = \frac{85}{75} = 6.8$$

$$C = \frac{6.8(75)}{85} = 6.0$$

Chapter 15/Problem 3

In the pole zero plot of Fig P15.3, there are poles at

Fig P15.3 here

$$s = -1 + j$$
$$s = -1 - j$$
$$s = 2$$

and zeros at

$$s = j$$
$$s = -j$$
$$s = -2$$

Thus

$$H(s) = C\,\frac{(s^2+1)(s+2)}{(s-2)(s^2+2s+2)}$$

To make the gain $|H(s)| = 1$ at $s = 0$

$$C\,\frac{(1)(2)}{-2(2)} = 1$$

$$-\frac{1}{2}C = 1$$

$$C = -2$$

and

$$H(s) = -2\left[\frac{(s^2+1)(s+2)}{(s-2)(s^2+2s+2)}\right]$$

Chapter 15/Problem 4

Fig P15.4

With poles at $\pm j4$ and zeros at ± 2

$$H(s) = K\frac{(s+2)(s-2)}{s^2+16} = K\frac{s^2-4}{s^2+16}$$

(a) With $s=0$, the gain is -1 (dc gain)

$$|H(s)| = -1 = K\left(\frac{-4}{16}\right) = -\frac{K}{4}$$

Hence

$$K = 4$$

and

$$H(s) = 4\frac{s^2-4}{s^2+16}$$

(b) $H(s)$ gives the impulse response

$$H(s) = 4\left[1 - \frac{20}{s^2+16}\right] = 4 - \frac{80}{s^2+16} = 4 - 20\frac{4}{s^2+16}$$

and

$$h(t) = 4\delta(t) - 20\sin 4t\, u(t)$$

(c) With

$$H(s) = \frac{F_o(s)}{F_i(s)}$$

where $f_i(t)$ is the input and $f_o(t)$ is the output or response, if $f_i(t) = u(t)$

$$F_i(s) = \frac{1}{s}$$

and

$$F_o(s) = H(s)F_i(s) = 4\frac{s^2-4}{s(s^2+16)} = \frac{K_1}{s} + \frac{As+B}{s^2+16}$$

Form a common denominator

$$4\frac{s^2-4}{s(s^2+16)} = \frac{K_1s^2 + 16K_1 + As^2 + Bs}{s(s^2+16)}$$

15/4 Cont'd

To preserve the equality, the coefficients of like powers of s must match

$$K_1 + A = 4$$

$$B = 0$$

$$16K_1 = -16$$

From this set of equation

$$K_1 = -1$$

$$A = 4+1 = 5$$

and

$$B = 0$$

Hence

$$F_o(s) = \frac{5s}{s^2+16} - \frac{1}{s}$$

and the step response will be

$$f_o(t) = (5\cos 4t - 1)u(t)$$

(d) If the input is $f_i(t) = \sin 3t$, then

$$F_i(s) = \frac{3}{s^2+9}$$

and

$$F_o(s) = H(s)F_i(s) = 4\left(\frac{s^2-4}{s^2+16}\right)\left(\frac{3}{s^2+9}\right)$$

$$F_o(s) = \frac{12(s^2-4)}{(s^2+16)(s^2+9)} = \frac{A_1 s + B_1}{s^2+16} + \frac{A_2 s + B_2}{s^2+9}$$

Form a least common denominator

$$\frac{12(s^2-4)}{(s^2+16)(s^2+9)} = \frac{(A_1 s+B_1)(s^2+9) + (A_2 s+B_2)(s^2+16)}{(s^2+16)(s^2+9)}$$

Equate coefficients of like powers of s

$$\begin{aligned} A_1 \quad + A_2 \quad &= 10 \\ B_1 \quad + B_2 &= 12 \\ 9A_1 \quad + 16A_2 \quad &= 0 \\ 9B_1 \quad + 16B_2 &= -48 \end{aligned}$$

Here $A_1 = A_2 = 0$ and

$$B_1 + B_2 = 12$$
$$9B_1 + 16B_2 = -48$$

has a solution vector

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 240 \\ -156 \end{bmatrix}$$

Hence

$$F_0(s) = \frac{1}{7}\left[60\frac{4}{s^2+16} - 52\frac{3}{s^2+9}\right]$$

and

$$f_0(t) = \frac{1}{7}\left[60 \sin 4t - 52 \sin 3t\right] u(t)$$

(e) With

$$f_i(t) = e^{-at} u(t)$$

$$F_i(s) = \frac{1}{s+a}$$

H(s) may be written as

$$H(s) = 4\frac{(s+2)(s-2)}{s^2+16}$$

So that

$$F_0(s) = 4\frac{(s+2)(s-2)}{(s+a)(s^2+16)} = \frac{K_1}{s+a} + \frac{As+B}{s^2+16}$$

15/4 Cont'd

To eliminate the $K_1/s+a$ term, set $a=2$. Then

$$F_o(s) = 4\frac{s-2}{s^2+16} = \frac{4s}{s^2+16} - \frac{8}{s^2+16}$$

and $$f_o(t) = (4\cos 4t - 2\sin 4t)u(t)$$

Chapter 15/Number 5

Fig P15.5

With poles at 0, −2 and $-2 \pm j4$ and zeroes at −4 and $\pm j2$

$$H(s) = K\frac{(s+4)(s^2+4)}{s(s+2)(s^2+4s+20)}$$

(a) With $s=0$ the gain is 6.8

$$|H(0)| = 6.8 = K\left(\frac{16}{40}\right)$$

Hence

$$K = \frac{40(6.8)}{16} = 17$$

and

$$H(s) = 17\,\frac{(s+4)(s^2+4)}{s(s+2)(s^2+4s+25)}$$

(b) $H(s)$ leads to the impulse response

$$H(s) = 17\frac{s^3+4s^2+4s+16}{s(s+2)(s^2+4s+20)} = 17\left[\frac{K_1}{s} + \frac{K_2}{s+2} + \frac{As+B}{s^2+4s+20}\right]$$

Form a least common denominator

$$17\left[\frac{s^3+4s^2+4s+16}{s(s+2)(s^2+4s+20)}\right] = 17\left[\frac{K_1(s^3+6s^2+28s+40) + K_2(s^3+4s^2+20) + (As+B)(s^2+2s)}{s(s+2)(s^2+4s+20)}\right]$$

Equate the coefficient of like terms to preserve the equality and obtain

$$\begin{array}{lllll}
K_1 & + K_2 & + A & & = 1 \\
6K_1 & + 4K_2 & + 2A & + B & = 4 \\
 & & & + 2B & = 4 \\
28K_1 & + 20K_2 & & & = 16 \\
40K_1 & + & & &
\end{array}$$

This set has a solution vector

$$\begin{bmatrix} K_1 \\ K_2 \\ A \\ B \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 \\ -5 \\ 11 \\ 14 \end{bmatrix}$$

15/5 Cont'd

So that

$$H(s) = \frac{17}{10}\left[\frac{4}{s} - \frac{5}{s+2} + \frac{11s}{s^2+6s+20} + \frac{14}{s^2+6s+20}\right]$$

$$= \frac{17}{10}\left[\frac{4}{s} - \frac{5}{s+2} + \frac{11(s+2)}{(s+2)^2+(4)^2} - 2\frac{4}{(s+2)^2+(4)^2}\right]$$

and

$$h(t) = \frac{17}{10}\left[4 - 5e^{-2t} + e^{-2t}(11\cos 4t - 2\sin 4t)\right]$$

(c) The step response in the s-domain is

$$\frac{H_1(s)}{s} = \frac{17(s^3+4s^2+4s+16)}{s^2(s+2)(s^2+4s+20)} = 17\left[\frac{C_1}{s} + \frac{C_2}{s^2} + \frac{K_1}{s+2} + \frac{As+B}{s^2+4s+20}\right]$$

As in part (b) form a least common denominator and following that procedure gives five equations in the five unknown constants

$$\begin{aligned}
C_1 \quad\quad + K_1 + A \quad\quad &= 0\\
6C_1 + C_2 + 4K_1 + 2A + B &= 1\\
28C_1 + 6C_2 + 20K_1 \quad + 2B &= 4\\
40C_1 + 28C_2 \quad\quad &= 4\\
40C_2 \quad\quad &= 16
\end{aligned}$$

with a solution vector

$$\begin{bmatrix} C_1 \\ C_2 \\ K_1 \\ A \\ B \end{bmatrix} = \frac{1}{100}\begin{bmatrix} -18 \\ 40 \\ 25 \\ -7 \\ 82 \end{bmatrix}$$

So that

$$\frac{H_1(s)}{s} = \frac{17}{100}\left\{\frac{40}{s^2} - \frac{18}{s} + \frac{25}{s+2} - \left[\frac{7s}{s^2+4s+20} - \frac{82}{s^2+4s+20}\right]\right\}$$

15/5 Cont'd

$$\frac{H_1(s)}{s} = \frac{17}{100}\left\{\frac{40}{s^2} - \frac{18}{s} + \frac{25}{s+2} - \left[\frac{7(s+2)}{(s+2)+(4)^2} - 24\frac{4}{(s+2)^2+(4)^2}\right]\right\}$$

and the step response, $f(t)$ will be

$$f(t) = \frac{17}{100}\left[40t - 18 + 25e^{-2t} - e^{-2t}(7\cos 4t - 24\sin 4t)\right] u(t)$$

(d) If $f_i(t)$ is the input, $f_i(t) = \sin 3t\, u(t)$, then

$$F_i(s) = \frac{3}{s^2+9}$$

and with the output designated as $f_o(t)$

$$F_o(s) = H(s)\, F_i(s)$$

$$F_o(s) = 17\frac{s^3+14s^2+4s+16}{s(s+2)(s^2+9)(s^2+4s+20)} = 17\left[\frac{K_1}{s} + \frac{K_2}{s+2} + \frac{A_1 s+B_1}{s^2+9} + \frac{A_2 s+B_2}{s^2+4s+20}\right]$$

Now proceed as in part (b) to form six equations in the unknown coefficient

$$\begin{aligned}
K_1 + K_2 + A_1 \qquad\qquad + A_2 \qquad\qquad &= 0\\
6K_1 + 4K_2 + 6A_1 + B_1 + 2A_2 + B_2 &= 0\\
19K_1 + 11K_2 + 28A_1 + 6B_1 + 9A_2 + 2B_2 &= 1\\
58K_1 + 36K_2 + 40A_1 + 28B_1 + 18A_2 + 9B_2 &= 4\\
90K_1 + 18K_2 \qquad + 40B_1 \qquad + 18B_2 &= 4\\
36K_1 \qquad\qquad\qquad\qquad\qquad &= 16
\end{aligned}$$

This set has a solution vector

$$\begin{bmatrix} K_1 \\ K_2 \\ A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = \frac{1}{6201}\begin{bmatrix} 2756 \\ 1908 \\ 115 \\ 810 \\ -4779 \\ -16{,}110 \end{bmatrix}$$

15/5 Contd. (4)

Thus

$$F_o(s) = \frac{17}{6201}\left[\frac{2756}{s} + \frac{1908}{s+2} + \frac{115s+810}{s^2+9} - \left(\frac{4779s+16{,}110}{s^2+4s+20}\right)\right]$$

or

$$F_o(s) = \frac{17}{6201}\left[\frac{2756}{s} + \frac{1908}{s+2} + \frac{115s}{s^2+9} + 270\frac{3}{s^2+9} - \left(\frac{4779(s+2)}{(s+2)^2+(4)^2} + \frac{1638(4)}{(s+2)^2+(4)^2}\right)\right].$$

and

$$f_o(t) = \frac{17}{6201}\left[2756 + 1908e^{-2t} + 115\cos 3t + 270\sin 3t - e^{-2t}(4779\cos 4t + 1638\sin 4t)\right]u(t)$$

(e) with $f_i(t) = e^{-at}u(t)$

$$F_i(s) = \frac{1}{s+a}$$

and with

$$F_o(s) = H(s)F_i(s) = \frac{17(s+4)(s^2+4)}{s(s+2)(s+a)(s^2+4s+20)}$$

there will be no term of the form Ke^{-at} if $\underline{\underline{a=4}}$.

Then

$$F_o(s) = \frac{17(s^2+4)}{s(s+2)(s^2+4s+20)} = 17\left[\frac{K_1}{s} + \frac{K_2}{s+2} + \frac{As+B}{s^2+4s+20}\right]$$

A procedure similar to the one employed in part (b) gives the set of equations

$$\begin{aligned}
K_1 + K_2 + A \quad &= 0\\
6K_1 + 4K_2 + 2A + B &= 1\\
28K_1 + 20K_2 \quad + 2B &= 0\\
40K_1 \quad &= 4
\end{aligned}$$

15/5 Cont'd

with solution vector

$$\begin{bmatrix} K_1 \\ K_2 \\ A \\ B \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 2 \\ -5 \\ 3 \\ 22 \end{bmatrix}$$

$$F_0(s) = \frac{17}{20}\left[\frac{2}{s} - \frac{5}{s+2} + \frac{3s+22}{(s+2)^2+(4)^2}\right]$$

$$= \frac{17}{20}\left[\frac{2}{s} - \frac{5}{s+2} + \frac{3(s+2)}{(s+2)^2+(4)^2} + 4\,\frac{4}{(s+2)^2+(4)^2}\right]$$

and

$$f_0(t) = \frac{17}{20}\left[2 - 5e^{-2t} + e^{-2t}(3\cos 4t + 4\sin 4t\right]u(t)$$

Chapter 15 / Problem 7

For each network shown

Fig P15.7(a)

Fig P15.7b

$$Z(s) = \frac{s-a}{s+a}$$

$Z(s)$ is defined as

$$Z(s) = \frac{V(s)}{I(s)}$$

For the network of Fig 15.7a

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V(s)}{s-a/s+a} = \left(\frac{s+a}{s-a}\right)V(s)$$

This network is unstable because there is a pole at $s=a$ in the right half plane.

For the network of Fig 15.7b

$$V(s) = Z(s)I(s) = \left(\frac{s-a}{s+a}\right)I(s)$$

which is indeed stable

Chapter 15/Problem 8

(a) Here

$$p_1 = -8 \qquad z_1 = 2$$
$$p_2 = -3+j4 \qquad z_2 = +j2$$
$$p_3 = -3-j4 \qquad z_3 = -j2$$

$$Z(0) = 8$$

Hence

$$Z(s) = K\frac{(s-2)(s^2+4)}{(s+8)(s^2+6s+25)}$$

$$|Z(0)| = 8 = K\frac{(s-2)(s^2+4)}{(s+8)(s^2+6s+25)}\bigg|_{s=0} = K\left(\frac{-8}{200}\right)$$

$$K = -200$$

and

$$Z(s) = -200\frac{(s-2)(s^2+4)}{(s+8)(s^2+6s+25)}$$

(b) With regard to fig 15.76

$$V_{out}(s) = Z(s)I_{IN}(s)$$

and with $i_{IN}(t) = 2u(t)$

$$V_{out}(s) = -400\frac{s^3-2s^2+4s-8}{s(s+8)(s^2+6s+25)} = \frac{K_1}{s} + \frac{K_2}{s+8} + \frac{As+B}{s^2+6s+25}$$

Form a least common denominator and equate coefficients of like powers of s. Working only with the numerators

$$-400(s^3-2s^2+4s-8) = K_1(s^3+14s^2+73s+200) + K_2(s^3+6s^2+25s) + (As+B)(s^2+8s)$$

A set of simultaneous equations are obtained

$$K_1 + K_2 + A = -400$$
$$14K_1 + 6K_2 + 8A + B = 800$$
$$73K_1 + 25K_2 + 8B = -1600$$
$$200K_1 = 3200$$

15/8 Cont'd

and the solution vector is

$$\begin{bmatrix} K_1 \\ K_2 \\ A \\ B \end{bmatrix} = \frac{1}{41}\begin{bmatrix} 656 \\ -34000 \\ 16944 \\ 92064 \end{bmatrix}$$

Then

$$V_{out}(s) = \frac{1}{41}\left[\frac{656}{s} - \frac{34{,}000}{s+8} + \frac{16944s + 92{,}064}{s^2+6s+25}\right]$$

$$= \frac{1}{41}\left[\frac{656}{s} - \frac{34{,}000}{s+8} + \frac{16{,}944\,(s+3)}{(s+3)^2+(4)^2} + \frac{41{,}232}{4}\,\frac{4}{(s+3)^2+(4)^2}\right]$$

and

$$v_{out}(t) = \frac{1}{41}\left(656 - 34{,}000e^{-8t} + e^{-3t}(16{,}944\cos 4t + 10{,}308\sin 4t)\right]u(t)\ \checkmark$$

The output remains finite as $t \to \infty$.

(c) Now use the network of Fig P15.7a with $v_{in}(t) = 20u(t)$

$$I_{out}(s) = \frac{V_{in}(s)}{Z(s)} = -\frac{1}{200}\left[\frac{(s+8)(s^2+6s+25)}{(s-2)(s^2+4)}\right]\frac{20}{s}$$

$$I_{out}(s) = -\frac{(s+8)(s^2+6s+25)}{10s(s-2)(s^2+6s+25)} = \frac{K_1}{s} + \frac{K_2}{s-2} + \frac{As+B}{(s+3)^2+16}$$

Notice that the K_2 term gives rise to a term

$$K_2 e^{2t}$$

which does __not__ remain finite as $t \to \infty$.

Chapter 15/Problem 9

With p_1, p_2 and p_3 as in Problem 8 but with one zero changed to $z_1 = -2$, then

$$Z(s) = K\frac{(s+2)(s^2+4)}{(s+8)(s^2+6s+25)}$$

and

$$Y(s) = \frac{1}{K}\frac{(s+8)(s^2+6s+25)}{(s+2)(s^2+4)}$$

(a) Both Fig 15.7a giving

$$I_{out}(s) = Y(s)V_{in}(s)$$

and Fig 15.7b giving

$$V_{out}(s) = Z(s)I_{in}(s)$$

Show a stable circuit

(b) An input

$$V_{in}(s) = \frac{s^2+4}{(s-a)(s^2+6s+25)}$$

producing

$$I_{out}(s) = Y(s)V_{in}(s) = \left[\frac{(s+8)(s^2+6s+25)}{(s+2)(s^2+4)}\right]\left[\frac{s^2+4}{(s-a)(s^2+6s+25)}\right]$$

$$= \frac{s+8}{(s+2)(s-a)}$$

would cause $i_{out}(t)$ to increase without bound as $t \to \infty$ in Fig 15.7a

An input

$$I_{in}(s) = \frac{s^2+6s+25}{(s-b)(s^2+4)}$$

producing

$$V_{out}(s) = Z(s)I_{in}(s) = \frac{s+2}{(s-b)(s+8)}$$

would cause $v_{out}(t)$ to increase without bound as $t \to \infty$ in Fig 15.7b

Chapter 15 / Problem 10

For the impedance function shown

Fig P15.10

$$S_{z1} = -1 \qquad S_{p1} = -2$$
$$S_{z2} = -2 \qquad S_{p2} = -6+j8$$
$$S_{z3} = -4 \qquad S_{p3} = -6-j8$$

(a)

$$Z(S) = K\frac{(S+1)(S+2)(S+4)}{(S+2)(S^2+12S+100)} = K\frac{(S+1)(S+4)}{S^2+12S+100}$$

$$|Z(0)| = 32 = K\frac{(S+1)(S+4)}{S^2+12S+100}\bigg|_{S=0} = K\left(\frac{4}{100}\right)$$

$$K = 32\left(\frac{100}{4}\right) = 800$$

and

$$Z(S) = 800\frac{(S+1)(S+4)}{S^2+12S+100}$$

(b) $Z(S)$ is defined in its usual manner, that is

$$Z(S) = \frac{V_{IN}(S)}{I_{IN}(S)}$$

then

$$I_{IN}(S) = \frac{V_{IN}(S)}{Z_S(S)}$$

and with $v_{IN}(t) = 5u(t)$

$$V_{IN}(S) = \frac{5}{S}$$

$$I_{IN}(S) = \frac{1}{160}\left[\frac{S^2+12S+100}{S(S+1)(S+4)}\right] = \frac{1}{160}\left[\frac{K_1}{S} + \frac{K_2}{S+1} + \frac{K_2}{S+4}\right]$$

$$K_1 = \frac{S^2+12S+100}{(S+1)(S+4)}\bigg|_{S=0} = \frac{100}{4} = 25$$

$$K_2 = \frac{S^2+12S+100}{S(S+4)}\bigg|_{S=-1} = \frac{89}{-3} = -\frac{89}{3}$$

$$K_3 = \frac{S^2+12S+100}{S(S+1)}\bigg|_{S=-4} = \frac{68}{(-4)(-3)} = \frac{17}{3}$$

15/10 Cont'd

$$I_{IN}(s) = \frac{1}{480}\left[\frac{75}{s} - \frac{89}{s+1} + \frac{17}{s+4}\right]$$

and

$$i_{IN}(t) = \frac{1}{480}\left[75 - 89e^{-t} + 17e^{-4t}\right]u(t) \text{ A}$$

Chapter 15/Problem 11

(a) Numerator roots give zeros

$s_{z1} = 3$

denominator roots give poles

$s_{p1} - 1, \; s_{p2} = -3$

(b) Numerator roots give zeros

$s_{z1}, s_{z2} = -\frac{1}{2} \pm j\frac{3}{4}$

Denominator roots give pole

$s_{p1} = -0.8572, \; s_{p2}, s_{p3} = -0.0744 \pm j2.4224$

(c) Numerator roots give zeros

$s_{z1}, s_{z2} = -\frac{1}{2} \pm j\frac{3}{4}$

denominator roots give poles

$s_{p1} = -1.1577, \; s_{p2}, s_{p3} = 0.0789 \pm j2.2750$

(d) Numerator roots give zeros

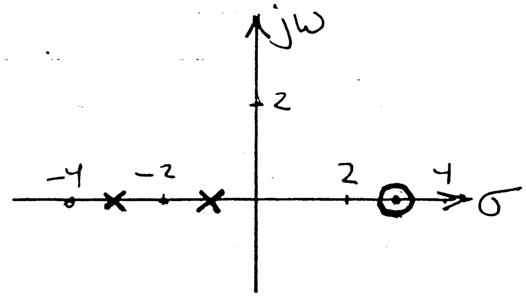
$s_{z1} = -2, \; s_{z2}, s_{z3} = -2 \pm j$

Denominator roots give poles

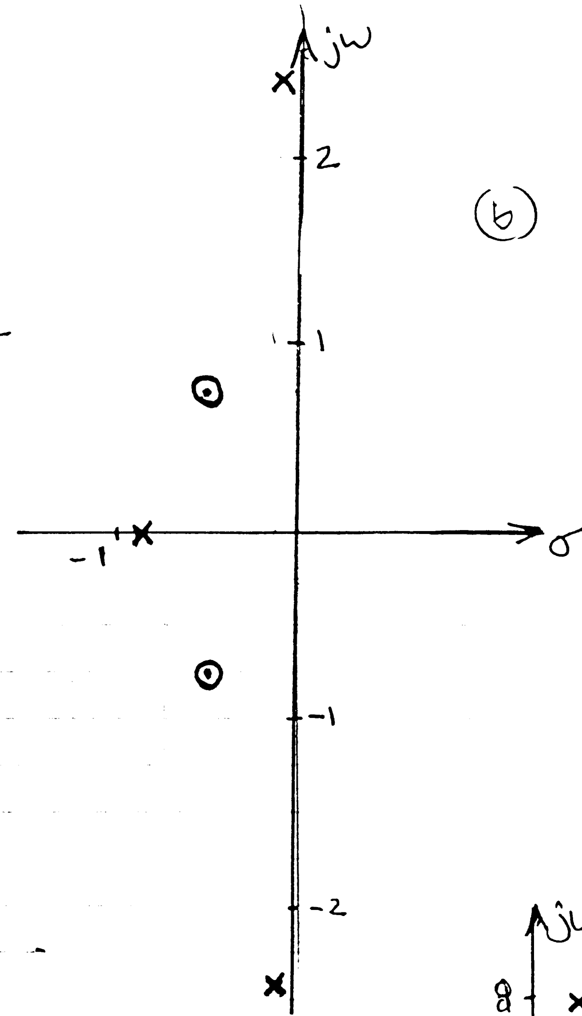
$s_{p1} = -1, \; s_{p2} = -8, \; s_{p3} = 1 \pm j8$

Pole-zero plots for 15/11

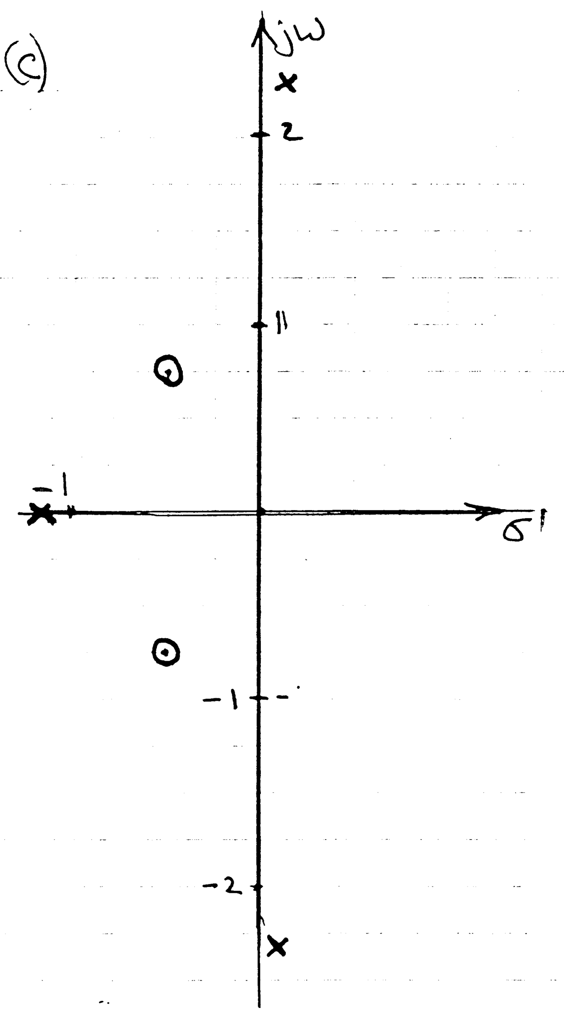
(a)



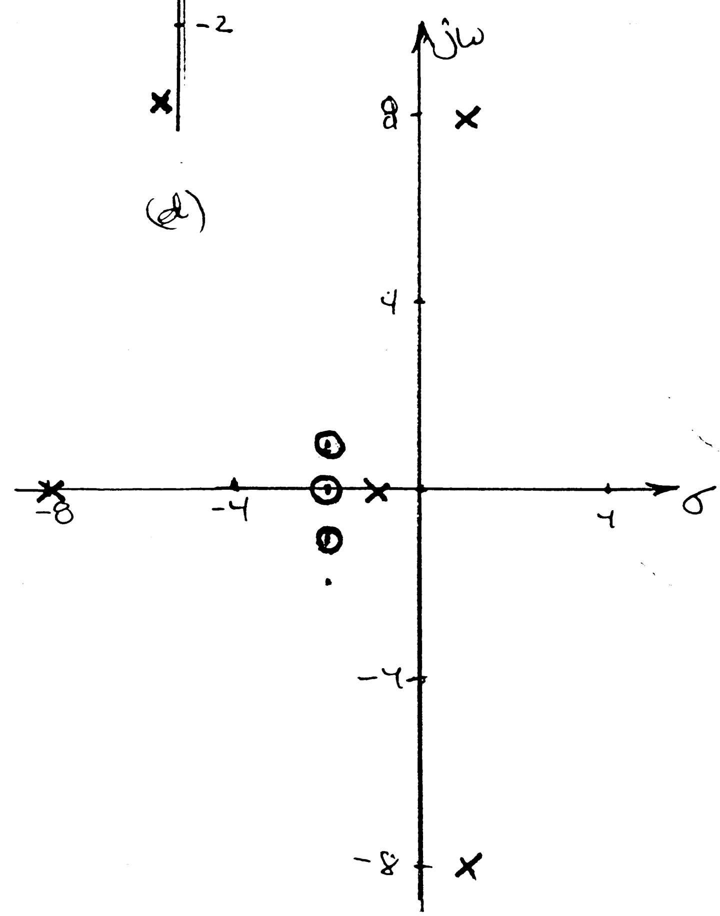
(b)



(c)



(d)



Chapter fifteen / Problem 12

Both op-amps are shown in the inverting configuration and

$$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = -\frac{Z_f(s)}{Z_{IN}(s)} = -\frac{Y_{IN}(s)}{Y_f(s)}$$

Fig P.15.12a

Fig P.15.12.b

(a) Here

$$Y_f(s) = Cs + \frac{1}{R_2} = \frac{R_2Cs+1}{R_2}$$

$$Z_f(s) = \frac{R_2}{R_2Cs+1}$$

$$Z_{IN}(s) = R_1$$

$$H(s) = -\frac{Z_f(s)}{Z_{IN}(s)} = -\frac{R_2/R_1}{R_2Cs+1} = -\frac{1}{R_1C\left(s+\frac{1}{R_2C}\right)}$$

(b) Here

$$Y_f(s) = \frac{R_2C_2s+1}{R_2}$$

$$Y_{IN}(s) = \frac{R_1C_1s+1}{R_1}$$

and

$$H(s) = -\frac{Y_{IN}(s)}{Y_f(s)} = -\frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} = -\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)},$$

Chapter 15/Problem 13

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)}$$

Fig P15.13

(a)

With R_2 and C_2 forming $Z_p(s)$

$$Z_p(s) = \frac{R_2}{R_2C_2s+1}$$

and R_1 and C_1 forming $Z_s(s)$

$$Z_s(s) = \frac{R_1C_1s+1}{C_1s}$$

then via voltage division

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)} = \frac{Z_p(s)}{Z_s(s)+Z_p(s)} = \frac{\frac{R_2}{R_2C_2s+1}}{\frac{R_1C_1s+1}{C_1s} + \frac{R_2}{R_2C_2s+1}}$$

$$= \frac{\frac{R_2}{R_2C_2s+1}}{\frac{(R_1C_1s+1)(R_2C_2s+1)+R_2C_1s}{C_1s(R_2C_2s+1)}}$$

$$= \frac{R_2C_1s}{R_1C_1R_2C_2s^2+(R_2C_2+R_1C_1+R_2C_1)s+1}$$

$$= \frac{\frac{s}{R_1C_2}}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_1C_2}\right)s+\frac{1}{R_1C_1R_2C_2}}$$

(b) With $C_1 = C_2 = 10^{-3}$ F work with the denominator, $D(s)$

$$D(s) = s^2 + 10^3\left(\frac{2}{R_1}+\frac{1}{R_2}\right)s + \frac{10^6}{R_1R_2}$$

The product of the poles must equal the constant term. Thus

$$(-0.21922)(-2.2808) = \frac{10^6}{R_1R_2}$$

So that

$$R_1R_2 = \frac{10^6}{} = 2\times10^6$$

15/13 Cont'd

And the sum of the poles must equal the negative coefficient of the s-term

$$10^3\left(\frac{2}{R_1}+\frac{1}{R_2}\right)=-(-0.21922-2.2808)=$$

or

$$\frac{2}{R_1}+\frac{1}{R_2} = 2.5\times10^{-3}$$

Thus

$$\frac{2}{R_1}+5\times10^{-7}R_1 = 2.5\times10^{-3}$$

or

$$R_1^2-5000R_1+4\times10^6=0$$

Complete the square

$$R_1^2-5000R_1+6.25\times10^6-2.25\times10^6=0$$

$$(R_1-2500)^2-2.25\times10^6=0$$

$$(R_1-2500)^2=2.25\times10^6$$

$$R_1-2500=\pm1500$$

There are two solutions

$$R_1=4000\,\Omega$$

$$R_1=1000\,\Omega$$

In this case

$$R_2=\frac{2\times10^6}{4000}=500\,\Omega$$

$$R_2=\frac{2\times10^6}{1000}=1000\,\Omega$$

Hence

$$R_1=4000\,\Omega,\ R_2=500\,\Omega$$

$$R_1=1000\,\Omega,\ R_2=2000\,\Omega$$

are both solutions

Chapter 15/Problem 15

Fig P15.15

(a) In the s-domain let $Z_p(s)$ refer to the combination of R_2 and C

$$Z_p(s) = \frac{R_2}{R_2Cs+1}$$

Then the total impedance is

$$Z_{IN}(s) = Z_p(s) + R_1 = \frac{R_2}{R_2Cs+1} + R_1 = \frac{R_2R_1Cs + R_1 + R_2}{R_2C_2s+1}$$

Then by voltage division

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)} = \frac{\dfrac{R_2}{R_2Cs+1}}{\dfrac{R_2R_1Cs + R_1 + R_2}{R_2C_2s+1}} = \frac{\dfrac{1}{R_1C}}{s + \dfrac{1}{R_{eq}C}}$$

Using component value

$$R_1C = 50(2.5\times10^{-3}) = 0.125$$

$$R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{50(200)}{250} = 40\Omega$$

$$R_{eq}C = 40(2.5\times10^{-3}) = 0.100$$

Hence

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)} = \frac{8}{s+10}$$

(b) If $v_{IN}(t) = 10u(t)$ and $v_C(0^-) = 4V$, the zero-state response is

$$V_{out}(s) = H(s)V_{IN}(s) = \left(\frac{8}{s+10}\right)\frac{10}{s} = \frac{80}{s(s+10)} = \frac{K_1}{s} + \frac{K_2}{s+10}$$

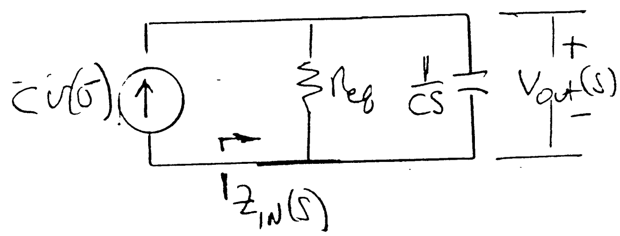
$$K_1 = \left.\frac{80}{s+10}\right|_{s=0} = \frac{80}{10} = 8$$

$$K_2 = \left.\frac{80}{s}\right|_{s=-10} = \frac{80}{-10} = -8$$

$$V_{zs}(s) = 8\left[\frac{1}{s} - \frac{1}{s+10}\right]$$

and

$$v_{zs}(t) = 8\left(1 - e^{-10t}\right)u(t) \text{ V}$$



$$V_{out}(s) = Z_{IN}(s)\,Cv(0^-)$$

$$Z_{IN}(s) = \frac{R_{eq}}{R_{eq}Cs+1} = \frac{40}{0.1s+1} = \frac{400}{s+10}$$

Then

$$V_{ZI}(s) = Z_{IN}(s)\,Cv(0^-) = \frac{400\,(2.5\times10^{-3})(4)}{s+10} = \frac{4}{s+10}$$

and

$$v_{ZI}(t) = 4e^{-10t}u(t)\ \text{V}$$

The complete response is

$$v_{out}(t) = v_{ZS}(t) + v_{ZI}(t) = 8 - 8e^{-10t} + 4e^{-10t}$$

$$v_{out}(t) = (8 - 4e^{-10t})u(t)\ \text{V}$$

(ii) The forced response is

$$v_f(t) = 8u(t)\ \text{V}$$

The natural response is

$$v_N(t) = -4e^{-10t}u(t)\ \text{V}$$

(iii) The steady state response is

$$v_{ss}(t) = 8u(t)\ \text{V}$$

The transient response

$$v_{tr}(t) = -4e^{-10t}u(t)\ \text{V}$$

15/15 Cont'd

(c) There will always be a natural response in this particular circuit! Here with $v(0^-) = 0V$, there is no zero-input response and with

$$v_{IN}(t) = 10e^{-10t} \checkmark$$

$$V_{OUT}(s) = Z_{ZS}(s) = H(s)V_{IN}(s) = \left(\frac{8}{s+10}\right)\left(\frac{10}{s+10}\right) = \frac{80}{(s+10)^2}$$

and

$$v_{ZS}(t) = 80te^{-10t}$$

The natural response when there is no initial condition will be

$$v_N(t) = Ke^{-10t}$$

So that

$$v_f(t) = v_{ZS}(t) - Ke^{-10t}$$
$$= e^{-10t}(80t - K)$$

and the forced response <u>is</u> well defined.

Chapter 15/Problem 21

I think that
Your solution did not take into account that the op-amp is in the inverting configuration.

Thus part (a) seems to be <u>off</u> <u>by</u> <u>a</u> <u>minus</u> <u>sign</u>.

Chapter 15/Problem 21 ①

$\boxed{\text{Fig } P15.21}$

(a) Let $V_{OA}(s)$ be the output voltage of the ideal op-amp so that

$$H_0(s) = \frac{V_{OA}(s)}{V_{IN}(s)} = -\frac{Z_f(s)}{Z_{IN}(s)}$$

Here

$$Z_{IN}(s) = R_1$$

and

$$Z_f(s) = \frac{R_2}{R_2 C_1 s + 1}$$

So that

$$H_0(s) = -\frac{\frac{R_2}{R_2 C_1 s + 1}}{R_1} = -\frac{\frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_1}}$$

With $R_1 = 2k\Omega$, $R_2 = 10k\Omega$ and $C_1 = 2 \times 10^{-4} F$

$$\frac{1}{R_1 C_1} = \frac{1}{0.4} = 2.5$$

$$\frac{1}{R_2 C_1} = \frac{1}{.2}$$

and

$$H_0(s) = -\frac{2.5}{s + 1/2}$$

By voltage division

$$\frac{V_{out}(s)}{V_{OA}(s)} = \frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s} + R_3} = \frac{\frac{1}{C_2 s}}{\frac{R_3 C_2 s + 1}{C_2 s}} = \frac{\frac{1}{R_3 C_2}}{s + \frac{1}{R_3 C_2}}$$

and with $R_3 = 10k\Omega$ and $C_3 = 5 \times 10^{-3} F$

$$\frac{1}{R_3 C_2} = \frac{1}{5}$$

and

$$\frac{V_{out}(s)}{V_{IN}(s)} = \frac{1/5}{s + 1/5}$$

10/21 Cont'd

Then

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)} = \frac{V_{OA}(s)}{V_{IN}(s)} \cdot \frac{V_{out}(s)}{V_{OA}(s)} = \left(-\frac{2.5}{s+1/2}\right)\left(\frac{1/5}{s+1/5}\right) = -\frac{1}{2(s+1/2)(s+\frac{1}{5})}$$

(b) If $v_{IN}(t) = 400\,u(t)$ mV and $v_{C1}(0^-) = v_{C2}(0^-) = 0V$ $[V_{out}(s) = V_{zs}(s)]$

$$V_{zs}(s) = H(s)V_{IN}(s) = -\frac{400}{2s(s+1/2)(s+1/5)} = \frac{K_1}{s} + \frac{K_2}{s+1/2} + \frac{K_3}{s+1/5}$$

$$K_1 = \frac{-200}{(s+1/2)(s+1/5)}\bigg|_{s=0} = \frac{-200}{1/10} = -2000\text{ mV} \quad (-2V)$$

$$K_2 = \frac{-200}{s(s+1/5)}\bigg|_{s=-1/2} = \frac{-200}{(-1/2)(-3/10)} = -\frac{4000}{3}\text{ mV} \quad (-\tfrac{4}{3}V)$$

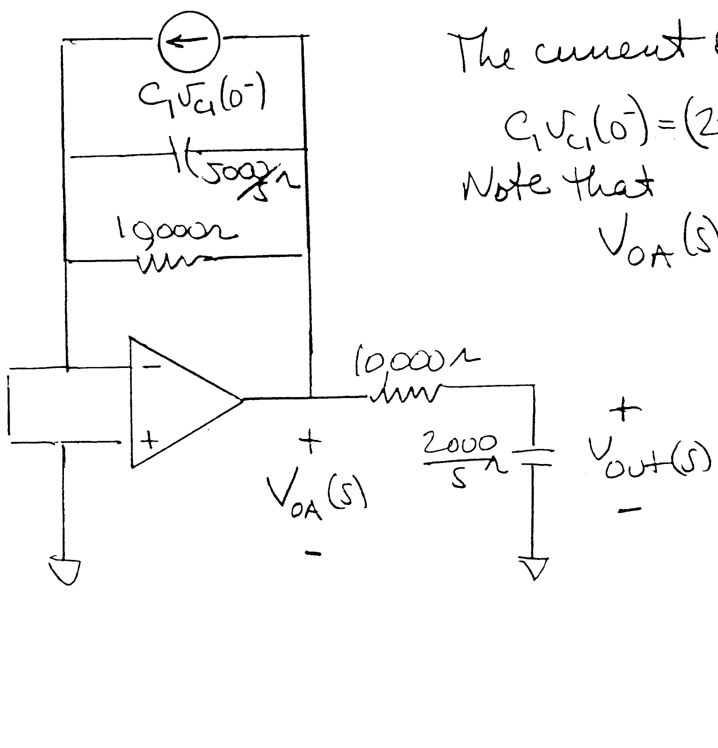
$$K_3 = \frac{-200}{s(s+1/2)}\bigg|_{s=-1/5} = \frac{-200}{-1/5(3/10)} = \frac{10{,}000}{3}\text{ mV} \quad (\tfrac{10}{3}V)$$

$$V_{zs}(s) = \left(\frac{2}{s} - \frac{4/3}{s+1/2} + \frac{10/3}{s+1/5}\right)$$

and

$$v_{zs}(t) = \left(-2 - \frac{4}{3}e^{-t/2} + \frac{10}{3}e^{-t/5}\right) u(t)\,V$$

(c) when $v_{IN}(t) = 0V$, $v_{C1}(0^-) = 80$ mV, $v_{C2}(0^-) = 0$, the op-amp s-domain circuit will be



The current source becomes

$$C_1 v_{C1}(0^-) = (2\times10^{-4})(80\times10^{-3}) = 16\times10^{-6}\text{ mA}$$

Note that

$$V_{OA}(s) = -C_1 v_{C1}(0^-)$$

Then

$$V_{OA}(s) = Z_f(s)\left[-C_1 v_{C1}(0^-)\right] = \frac{1}{C_1\left(s+\frac{1}{R_2C_1}\right)}\left[-C_1 v_C(0^-)\right]$$

$$V_{OA}(s) = -\frac{0.08}{s+1/2}$$

and by voltage division

$$V_{Z1}(s) = \frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s}+R_3} V_{OA}(s) = \frac{\frac{1}{R_3C_2}}{s+1/R_3C_2} V_{OA}(s)$$

$$V_{Z1}(s) = -\frac{1/5(0.08)}{(s+1/5)(s+1/2)} = \frac{-0.016}{(s+1/5)(s+1/2)} = \frac{K_1}{s+1/5} + \frac{K_2}{s+1/2}$$

$$K_1 = \left.\frac{-0.016}{s+1/2}\right|_{s=-1/5} = \frac{-0.016}{3/10} = -0.0533$$

$$K_2 = \left.\frac{-0.016}{s+1/5}\right|_{s=-1/2} = \frac{-0.016}{-3/10} = 0.0533$$

$$V_{Z1}(s) = 0.0533\left[\frac{1}{s+1/2} - \frac{1}{s+1/5}\right]$$

and

$$v_{ZI}(t) = 0.0533\left(e^{-t/2} - e^{-t/5}\right)u(t)\ \text{V}$$

(d) The complete response will be

$$v_{out}(t) = v_{ZS}(t) + v_{Z1}(t)$$

$$= 3.3333e^{-t/5} - 1.3333e^{-t/2} - 2 + 0.0533e^{-t/2} - 0.0533e^{-t/5}\ \text{V}$$

or

$$v_{out}(t) = \left(3.28e^{-t/5} - 1.28e^{-t/2} - 2\right)u(t)\ \text{V}$$

15/21 Cont'd

(e) The natural response derives from the poles of the transfer function

$$v_N(t) = \left(K_1 e^{-t/5} - K_2 e^{-t/2}\right) u(t) \text{ V}$$

(f) The steady state and transient response are

$$v_{tr}(t) = \left(3.28\, e^{-t/5} - 1.28\, e^{-t/2}\right) u(t) \text{ V}$$

$$v_{ss}(t) = 2\, u(t) \text{ V}$$

Chapter 15/Problem 22

If in problem 21, the capacitors are $C_1 = C_2 = 2.5 \times 10^{-4}$ f; then following Problem 21

$$\frac{1}{R_1C_1} = \frac{1}{0.5} = 2$$

$$\frac{1}{R_2C_2} = \frac{1}{2.5} = 2/5$$

$$H_a(s) = -\frac{2}{s + 2/5}$$

$$\frac{1}{R_3C_2} = \frac{1}{2.5} = \frac{2}{5}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{2/5}{s + 2/5}$$

$$H(s) = \left(-\frac{2}{s+2/5}\right)\left(\frac{2/5}{s+2/5}\right) = -\frac{0.8}{(s+2/5)^2}$$

(b) With $V_{in}(t) = 400u(t)$ mV and $V_{C1}(0^-) = V_{C2}(0^-) = 0$ V

$$V_{zs}(s) = -\frac{320}{s(s+2/5)^2} = \frac{K_1}{s} + \frac{C_1}{s+2/5} + \frac{C_2}{(s+2/5)^2}$$

$$K_1 = -\frac{320}{(s+2/5)^2}\Big|_{s=0} = \frac{-320}{4/25} = -2000 \text{ mV} \quad (-2\text{ V})$$

$$p(s) = -320 s^{-1} = -\frac{320}{s}$$

$$p'(s) = 320 s^{-2} = \frac{320}{s^2}$$

$$p(-2/5) = \frac{-320}{-2/5} = 800$$

$$p'(-2/5) = \frac{320}{(-2/5)^2} = 2000$$

$$C_2 = \frac{p(-2/5)}{0!} = 800 \quad (800\text{mV})$$

$$C_1 = \frac{p'(-2/5)}{1!} = 2000 \quad (2\text{ mV})$$

$$V_{ZS}(s) = \left[-\frac{2}{s} + \frac{2}{s+2/5} + \frac{0.8}{(s+2/5)^2} \right]$$

and

$$v_{ZS}(t) = \left[2e^{-2t/5} + 0.8te^{-2t/5} - 2 \right] u(t) \text{ V}$$

(c) With $v_{C1}(0^-) = 80$ mV and $v_{IN}(t) = 0$V

$$V_{OA}(s) = \frac{\frac{1}{R_3C_2}}{s+\frac{1}{R_3C_2}} V_{OA}(s) = \frac{1}{C_1\left(s+\frac{1}{R_3C_2}\right)} C_1 v_{C1}(0^-) = \frac{0.08}{s+2/5}$$

$$V_{Z1}(s) = \frac{\frac{1}{C_2 s}}{\frac{1}{C_2 s}+R_3} V_{OA}(s) = \left(\frac{2/5}{s+2/5}\right)\left(\frac{0.08}{s+2/5}\right) = \frac{0.032}{(s+2/5)^2}$$

$$v_{ZI}(t) = 0.032te^{-2t/5} u(t) \text{ V}$$

(d)

$$v_{out}(t) = v_{ZS}(t) + v_{ZI}(t) = \left(2e^{-2t/5} + 0.8te^{-2t/5} - 2 + 0.032e^{-2t/5}\right) u(t) \text{ V}$$

or

$$v_{out}(t) = \left[2e^{-2t/5} + 0.832te^{-2t/5} - 2 \right] u(t) \text{ V}$$

(e)

$$v_N(t) = \left(K_1 e^{-2t/5} + K_2 te^{-2t/5}\right) u(t) \text{ V}$$

(f)

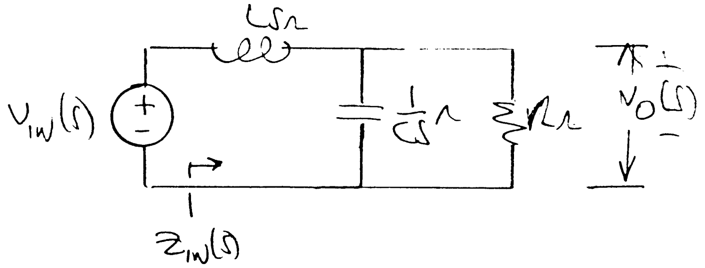
$$v_{tr}(t) = \left(2e^{-2t/5} + 0.832te^{-2t/5}\right) u(t) \text{ V}$$

$$v_{SS}(t) = 2u(t) \text{ V}$$

Chapter 15 / Problem 23

(a) Call $Z_p(s)$ the parallel combination of R and C in the s-domain circuit shown

Fig P15.23



$$Z_p(s) = \frac{R}{RCs+1}$$

Then

$$Z_{IN}(s) = Ls + \frac{R}{RCs+1}$$

$$= \frac{RLCs^2 + Ls + R}{RCs+1}$$

Then by voltage division

$$\frac{V_O(s)}{V_{IN}(s)} = \frac{\frac{R}{RCs+1}}{\frac{RLCs^2+Ls+R}{RCs+1}} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

If $R = 0.8\Omega$, $L = 2H$, $C = 0.5F$

$$LC = 2(0.5) = 1 \qquad H(s) = \frac{1}{s^2 + 2.5s + 1}$$

$$RC = 0.8(0.5) = 0.40$$

$$\frac{1}{RC} = 2.5$$

With $v_{IN}(t) = 15e^{-t}u(t)$ and $v_C(0^-) = i_L(0^-) = 0$

$$V_{IN}(s) = \frac{15}{s+1}$$

$$V_{out}(s) = V_{zs}(s) = \frac{15}{(s+1)(s^2+2.5s+1)} = \frac{15}{(s+\frac{1}{2})(s+1)(s+2)} = \frac{K_1}{s+\frac{1}{2}} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$K_1 = \left.\frac{15}{(s+1)(s+2)}\right|_{s=-\frac{1}{2}} = \frac{15}{(\frac{1}{2})(\frac{3}{2})} = \frac{15(4)}{3} = 20$$

$$K_2 = \left.\frac{15}{(s+\frac{1}{2})(s+2)}\right|_{s=-1} = \frac{15}{(-\frac{1}{2})(1)} = 15(-2) = -30$$

$$K_3 = \left.\frac{15}{(s+\frac{1}{2})(s+1)}\right|_{s=-2} = \frac{15}{(-\frac{3}{2})(-1)} = \frac{15(2)}{3} = 10$$

$$V_{zs}(s) = \frac{20}{s+\frac{1}{2}} - \frac{30}{s+1} + \frac{10}{s+2}$$

and

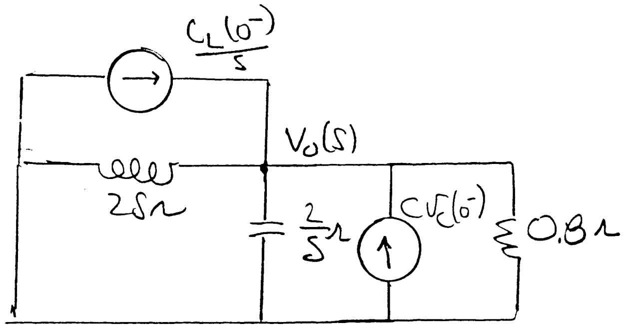
$$v_{zs}(t) = \left(20e^{-\frac{1}{2}t} - 30e^{-t} + 10e^{-2t}\right)u(t)\ \text{V}$$

15/23 Cont'd

For the zero-input response with $v_{IN}(t)$ removed and

$i_L(0^-) = 6A$

$v_C(0^-) = 6V$



The single node equation will be

$$-\frac{6}{s} - 0.5(6) + \left(\frac{s}{2} + \frac{5}{4} + \frac{1}{2s}\right)V_1 = 0$$

$$(s^2 + 2.5s + 1)V_0(s) = 6s + 12$$

and

$$V_{ZI}(s) = V_0(s) = \frac{6s+12}{(s+\frac{1}{2})(s+2)} = \frac{6(s+2)}{(s+\frac{1}{2})(s+2)} = \frac{6}{s+1/2}$$

So that

$$v_{ZI}(t) = 6e^{-t/2}u(t)\,V$$

(c) The complete response is

$$v_0(t) = v_{ZS}(t) + v_{ZI}(t) = 20e^{-t/2} - 30e^{-t} + 10e^{-2t} + 6e^{-t/2}$$

or

$$v_0(t) = (26e^{-t/2} - 30e^{-t} + 10e^{-2t})u(t)\,V$$

(d) The natural response has components

$$v_N(t) = K_1e^{-t/2} + K_2e^{-2t}$$

Either K_1 or K_2 may be zero but not both.

(e) The forced response is the response due to the forcing function

$$v_F(t) = (20e^{-1/2t} - 30e^{-t} + 10e^{-2t})u(t)\,V$$

The verification of this comes from

$$V_{out}(s) = H(s)V_{IN}(s) = H(s)\frac{15}{s+1}$$

in part (b)

Chapter 15 / Problem 31

(a) The pole zero plot of

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)}$$

Fig P15.31

has

$p_1 = -3$ $\qquad z_1 = 0$

$p_2 = +j2$ $\qquad z_2 = 3$

$p_3 = -j2$

Thus with $K=6$

$$H(s) = K\frac{s(s-3)}{(s+3)(s^2+4)}$$

(a) If $s = j4$

$$H(s) = \frac{j4(-3+j4)}{(3+j4)(-16+4)} = \frac{(4\angle 90^\circ)(5\angle 126.87^\circ)}{(5\angle 53.13^\circ)(12\angle 0^\circ)}$$

$$= \frac{20\angle 143.13^\circ}{60\angle 53.13^\circ} = \frac{1}{3}\angle 163.74^\circ$$

In the form

$$H(j\omega) = |H(j4)|\angle\theta$$

$$|H(j4)| = \frac{1}{3}$$

and $\theta = 163.74^\circ$

(b) With $H(j\omega)$ in part (a), if $v_{IN}(t) = 10\cos(4t + 45^\circ)$

then

$$\hat{V}_{out} = H(j\omega)\hat{V}_{IN}$$

$$\hat{V}_{IN} = 10\angle 45^\circ$$

$$\hat{V}_{out} = \left(\frac{1}{3}\angle 163.74^\circ\right)(10\angle 45^\circ) = \frac{10}{3}\angle -151.26^\circ$$

And $\quad v_{out}(t) = \frac{10}{3}\cos(4t - 151.26^\circ)\,V$

Chapter 15 / Problem 32

$\boxed{\begin{array}{c}\text{Fig} \\ \text{P15.32}\end{array}}$

Here

$p_1 = -5 \qquad z_1 = -2$

$p_2 = -1 + j10 \qquad z_2 = +j3$

$p_3 = -1 - j10 \qquad z_3 = -j3$

$$H(s) = K \frac{(s+2)(s^2+9)}{(s+5)(s^2+2s+101)}$$

$$H(j\omega) = K \frac{(2+j\omega)(9-\omega^2)}{(5+j\omega)(100-\omega^2+j2\omega)}$$

The maximum amplitude occurs near $j2\omega = 100-\omega^2$ and $j\omega$ near 5

The minimum amplitude occurs near $\omega = 3$ and $j\omega = -2$

In general, $j\omega$'s near poles force $H(j\omega)$ to have large magnitudes and $j\omega$'s near zeros force $H(j\omega)$ to have small magnitude

Chapter 15/Problem 33

$$H(s) = \frac{2s+4}{s^2+5s+6}$$

with $s = j2$

$$H(j2) = \frac{4+j4}{6-4+j5} = \frac{4+j4}{2+j5}$$

If $v_{IN} = 4\cos(2t+45^\circ)$

$$V_{IN} = 4\angle 45^\circ$$

and

$$V_{out} = \frac{4+j4}{2+j5}(4\angle 45^\circ) = \frac{(4\sqrt{2}\angle 45^\circ)(4\angle 45^\circ)}{5.39\angle 68.20^\circ}$$

$$V_{OUT} = 4.21\angle 21.80^\circ$$

and $$v_{out}(t) = 4.21\cos(2t+21.80^\circ)$$

Chapter 15/Problem 34

Here $H(s) = 4\dfrac{s^2+4}{s^2+2s+5}$

(a) For $v_{IN}(t) = 4\cos 2t$ V

$$H(j2) = 4\frac{(4-4)}{-4+5+j2} = 0$$

and $\hat{V}_{OUT} = 0\,\hat{V}_{IN} = 0$

(b) For $v_{IN}(t) = 4\cos 4t$

$$H(j4) = 4\left(\frac{4-16}{5-16+j8}\right) = \frac{4(16.49\angle -75.96^\circ)}{13.60\angle 143.97^\circ}$$

$$H(j4) = 4.85\angle 140.06^\circ$$

$$\hat{V}_{IN} = 4\angle 0^\circ$$

$$\hat{V}_{OUT} = (4.85\angle 140.06^\circ)(4\angle 0^\circ)$$

$$= 19.40\angle 140.36$$

and

$$v_{out}(t) = 19.40\cos(4t + 140.36^\circ)$$

Chapter 15 / Problem 35

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)} = \frac{s^2 - 0.5s + 5}{s^2 + 0.5s + 5.7321}$$

$$H(j2) = \frac{5 - 4 - j}{5.7321 - 4 + j} = \frac{1 - j}{\sqrt{3} + j}$$

$$H(j2) = \frac{\sqrt{2}\angle -45^\circ}{2\angle 30^\circ} = \frac{\sqrt{2}}{2}\angle -75^\circ$$

$$\hat{V}_{out} = H(j2)\hat{V}_{IN}$$

with $\hat{V}_{IN} = 2\sqrt{2}\angle 30^\circ \text{V}$

$$\hat{V}_{out} = \left(\frac{\sqrt{2}}{2}\angle -75^\circ\right)\left(2\sqrt{2}\angle 30^\circ\right) = 2\angle -45^\circ \text{V}$$

Thus

$$v_{out}(t) = 2\cos(2t - 45^\circ)$$

$$A = 2$$
$$\phi = -45^\circ$$

15/36

In part (c) the transcription from the phase in matlab

PHI = 8.4252e-01

to the time domain gives

0.84252°

Not

84.252°

Chapter 15/Problem 36

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{16s^2+44s+128}{s^3+8s^2+36s+80}$$

(a)

$$H(j4) = \frac{128-256+j176}{80-128+j(144-64)}$$

$$= \frac{-128+j176}{-48+j80} = \frac{217.62\angle 126.03^\circ}{93.30\angle 120.96^\circ} = 2.33\angle 5.07^\circ$$

$$v_{IN}(t) = 20\cos(4t+45^\circ)\text{ V}$$

$$\hat{V}_{IN} = 20\angle 45^\circ\text{ V}$$

$$\hat{V}_{out} = H(j4)\hat{V}_{IN} = (20\angle 45^\circ)(2.33\angle 5.07^\circ)\text{ V}$$

$$\hat{V}_{out} = 46.65\angle 50.06$$

Magnitude = 46.65 V

Phase = 50.06°

$$v_{out}(t) = 46.65\cos(4t+50.06^\circ)$$

(b)

$$H(s) = \frac{V_{out}(s)}{V_{IN}(s)} = \frac{16s^2+44s+128}{s^3+8s^2+36s+80}$$

$$H(j40) = \frac{128-25{,}600+j1760}{80-12{,}800+j(1440-64{,}000)}$$

$$= \frac{-25{,}472+j1760}{-12{,}720-j62{,}560} = \frac{25{,}802\angle 176.09^\circ}{63{,}840\angle -101.49^\circ}$$

$$= 0.404\angle -82.42^\circ$$

$$v_{IN}(t) = 4\cos 40t$$

$$\hat{V}_{IN} = 4\angle 0^\circ$$

$$\hat{V}_{out} = H(j40)\hat{V}_{IN} = (0.404\angle -82.42^\circ)(4\angle 0^\circ)$$

$$\hat{V}_{out} = 1.617\angle -82.42^\circ$$

15/36 Cont'd

$v_{out}(t) = 1.617 \cos(40t - 82.42^\circ)$ V

magnitude $= 1.617$ V

phase $= -82.42^\circ$

(c)

$$H(s) = \frac{16s^2 + 44s + 128}{s^3 + 8s^2 + 36s + 80}$$

$$H(j2) = \frac{128 - 64 + j88}{80 - 32 + j(72 - 8)} = \frac{64 + j88}{48 + j64} = \frac{108.82 \angle 53.97^\circ}{80 \angle 53.13^\circ}$$

$$H(j2) = 1.36 \angle 0.84^\circ$$

$v_{IN}(t) = 4 \cos 2t$ V

$\hat{V}_{IN} = 4 \angle 0^\circ$

$\hat{V}_{out} = H(j2)\hat{V}_{IN} = (1.36 \angle 0.84^\circ)(4 \angle 0^\circ)$

$\hat{V}_{out} = 5.44 \angle 0.84^\circ$ V

$v_{out}(t) = 5.44 \cos(2t - 0.84^\circ)$ V

magnitude $= 5.44$ V

phase $= -0.84^\circ$

Chapter 15/Problem 37 (1)

For stable circuits, only the zero state response is meaningful. In all parts, $v_{IN}(t) = 8\cos 2t$ V and $\hat{V}_{IN} = 8\angle 0°$ V

(a)

$$H_1(s) = \frac{-3.75s + 5}{s^2 + 4s + 5}$$

$$H_1(j2) = \frac{5 - j7.5}{5 - 4 + j8} = \frac{9.01\angle -56.31°}{8.06\angle 82.88°} = 1.118\angle -139.19°$$

$$\hat{V}_{out} = (1.118\angle -139.19°)(8\angle 0°) = 8.944\angle -139.19° \text{ V}$$

(b)

$$H_2(s) = \frac{2.5s - 3}{s^2 + 2s + 5}$$

$$H_2(j2) = \frac{-3 + j5}{5 - 4 + j4} = \frac{5.831\angle 120.96°}{4.123\angle 75.96°} = \sqrt{2}\angle 45°$$

$$\hat{V}_{out} = (\sqrt{2}\angle 45°)(8\angle 0°) = 8\sqrt{2}\angle 45° \text{ V}$$

(c)

$$H_3(s) = \frac{7s^2 + s + 4}{s^3 + s^2 + 9s + 9}$$

$$H_3(j2) = \frac{4 - 28 + j2}{9 - 4 + j(2 - 8)} = \frac{-24 + j2}{5 - j6} = \frac{24.08\angle 175.24°}{7.81\angle -50.19°} = 3.08\angle -134.57°$$

$$\hat{V}_{out} = (3.08\angle -134.57°)(8\angle 0°) = 24.67\angle -134.57° \text{ V}$$

(d)

$$H_4(s) = \frac{14s^2 - 23s + 20}{s^3 + s^2 + 7.25s + 18.5}$$

$$H_4(j2) = \frac{20 - 56 - j46}{18.5 - 4 + j(14.5 - 8)} = \frac{-36 - j46}{14.5 + j6.5} = \frac{58.41\angle -128.05°}{15.89\angle 24.15°}$$

$$H(j2) = 3.68\angle -152.19°$$

$$\hat{V}_{out} = (3.68\angle -152.19°)(8\angle 0°) = 29.41\angle -152.19° \text{ V}$$

(e)

$$H_5(s) = \frac{7s^2 + s + 7.75}{s^3 + 3s^2 + 11.25s + 18.5}$$

$$H_5(j2) = \frac{7.75 - 28 + j2}{18.5 - 12 + j(22.5 - 8)} = \frac{-20.25 + j2}{6.5 + j14.5}$$

$$= \frac{20.35\angle 174.36^\circ}{15.89\angle 65.85^\circ} = 1.28\angle 108.51^\circ$$

$$\hat{V}_{out} = (1.28\angle 108.51^\circ)(8\angle 0^\circ)\,V$$

$$= 10.24\angle 108.51^\circ\,V$$

(f)

$$H_6(s) = \frac{s^3 - 5.5s^2 + 14s - 12}{s^3 + 5.5s^2 + 14s + 12}$$

$$H_6(j2) = \frac{22 - 12 + j(28 - 8)}{12 - 22 + j(28 - 8)} = \frac{10 + j20}{-10 + j20} = \frac{22.36\angle 63.43^\circ}{22.36\angle 116.57^\circ}$$

$$H_6(j2) = 1\angle -53.13^\circ$$

$$\hat{V}_{out} = (1\angle -53.13^\circ)(8\angle 0^\circ) = 8\angle -53.13^\circ\,V$$

Chapter 15/Problem 41

Fig P15-41

Here, by voltage division

$$H(s) = \frac{V_C(s)}{V_{IN}(s)} = \frac{\frac{1}{Cs}}{Ls + \frac{1}{Cs}} = \frac{1}{LC\left(s^2 + \frac{1}{LC}\right)}$$

If $v_{IN}(t) = V_0 u(t)$, then

$$V_{IN}(s) = \frac{V_0}{s}$$

and

$$V_C(s) = \frac{V_0}{LC}\left[\frac{1}{s\left(s^2 + \frac{1}{LC}\right)}\right] = \frac{K_1}{s} + \frac{As + B}{s^2 + 1/LC}$$

This is maximized when there are poles at

$$s_{p1}, s_{p2} = -\frac{1}{\sqrt{LC}}$$

Chapter 15/Problem 43

Fig P15.43

Determine the admittance of the LC portion

$$Y(s) = Cs + \frac{1}{Ls} = \frac{LCs^2+1}{Ls} = \frac{C(s^2+\frac{1}{LC})}{s}$$

Then

$$H(s) = \frac{V_{out}(s)}{I_{IN}(s)} = Z(s) = \frac{s}{C(s^2+\frac{1}{LC})}$$

and with $i_{IN}(t) = I_o$

$$V_c(s) = H(s)I_{IN}(s) = \frac{s}{C(s^2+\frac{1}{LC})}\frac{I_o}{s} = \frac{I_o}{C(s^2+\frac{1}{LC})}$$

$V_c(s)$ and therefore $v_c(t) = v_{out}(t)$ will be maximized when

$$s = -\frac{1}{\sqrt{LC}}$$

Chapter 15 / Problem 46

$$H(s) = \frac{s+1}{s(s^2+2s+10)}$$

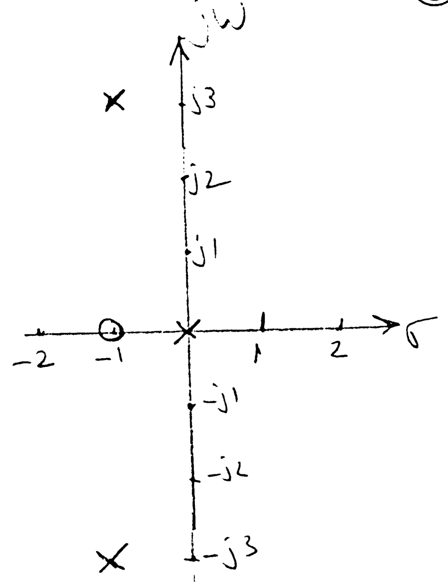
This shows

$z_1 = -1$

$p_1 = 0$

$p_2 = -1+j3$

$p_3 = -1-j3$



The pole-zero plot is at the right

Here

$$|H(j\omega)| = \frac{|j\omega+1|}{|j\omega|\,|j\omega+1-j3|\,|j\omega+1+j3|}$$

and

$$\angle H(j\omega) = \angle(j\omega+1) - [\angle j\omega + \angle(j\omega+1-j3) + \angle(j\omega+1+j3)]$$

for $\omega = 0.2$ rad/s

$|j\omega+1| = |1+j0.2| = 1.020$ $\quad$ $\angle(j\omega+1) = \angle(1+j0.2) = 11.31°$

$|j\omega| = |j0.2| = 0.20$ $\quad$ $\angle(j\omega) = 90°$

$|j\omega+1-j3| = |1-j2.8| = 2.973$ $\quad$ $\angle(j\omega+1-j3) = -70.35°$

$|j\omega+1+j3| = |1+j3.2| = 3.353$ $\quad$ $\angle(j\omega+1+j3) = 72.65°$

Thus

$$|H(j0.2)| = \frac{1.020}{0.2(2.973)(3.353)} = 0.512$$

$$\angle H(j0.2) = 11.31° - (90 - 70.35° + 72.65°)$$

$$= 11.31° - 92.30° = -80.99°$$

15/42 Cont'd

②

For $\omega = 0.5$ rad/s

$|j\omega+1| = |1+j.5| = 1.118$ $\quad \angle(j\omega+1) = \angle(1+j0.5) = 26.57°$

$|j\omega| = |j0.5| = 0.5$ $\quad \angle j\omega = \angle j0.5 = 90°$

$|j\omega+1-j3| = |1-j2.5| = 2.693$ $\quad \angle(j\omega+1-j3) = \angle(1-j2.5) = -68.20°$

$|j\omega+1+j3 = |1+j3.5| = 3.640$ $\quad \angle(j\omega+1+j3 = \angle(1+j3.5) = 74.06$

Thus

$$|H(j0.5)| = \frac{1.118}{0.5(2.693)(3.640)} = 0.228$$

$$\angle H(j0.5) = 26.57° - (90° - 68.20° + 74.06°)$$
$$= 26.57° - 95.86° = -69.29°$$

For $\omega = 1$ rad/s

$|j\omega+1| = |1+j1| = 1.414$ $\quad \angle(j\omega+1) = \angle(1+j) = 45°$

$j\omega = |j1| = 1$ $\quad \angle j\omega = \angle j1 = 90°$

$|j\omega+1-j3| = |1-j2| = 2.236$ $\quad \angle(j\omega+1-j3) = \angle(1-j2) = -63.44°$

$|j\omega+1+j3| = |1+j4| = 4.123$ $\quad \angle(j\omega+1+j3) = \angle(1+j4 = 75.96°$

$$|H(j1)| = \frac{1.414}{1(2.236)(4.123)} = 0.153$$

$$\angle H(j1) = 45° - [90° - 63.74° + 75.96°] = 45° - 102.22° = -57.22°$$

10/42 Cont'd

For $\omega = 10$ rad/s

$$|j\omega+1| = |1+j10| = 10.050 \qquad \angle(j\omega+1) = \angle(1+j10) = 84.29^\circ$$
$$|j\omega| = |j10| = 10.000 \qquad \angle j\omega = \angle j10 = 90^\circ$$
$$|j\omega+1-j3| = |1+j7| = 7.071 \qquad \angle(j\omega+1-j3) = \angle(1+j7) = 81.70^\circ$$
$$|j\omega+1+j3| = |1+j13| = 13.038 \qquad \angle(j\omega+1+j3) = \angle(1+j13) = 85.60^\circ$$

$$|H(j10)| = \frac{10.050}{10(7.071)(13.038)} = 0.011$$

$$\angle H(j10) = 84.29^\circ - [90^\circ + 81.70^\circ + 85.60^\circ]$$
$$= 84.29^\circ - 257.30^\circ = -173.10^\circ$$

The limiting values of magnitude and phase as $\omega \to \pm\infty$

Magnitude, $|H(j\omega)| = 0$ (both $-\infty$ and $+\infty$)

Phase, $\angle H(j\omega) = \pm 180^\circ$ (-180° for $\omega=\infty$, $+180^\circ$ for $\omega=-\infty$)

Chapter 15 / Problem 47

$$H(s) = \frac{[(s+1)^2+16](s^2-1)}{[(s+1)^2+4][(s+1)^2+36]}$$

This shows

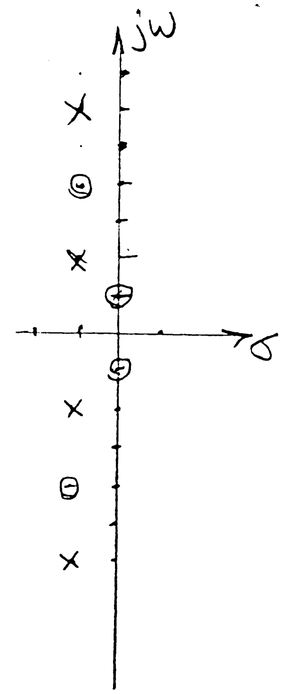
$z_1 = -1+j4$ $\quad p_1 = -1+j2$

$z_2 = -1-j4$ $\quad p_2 = -1-j2$

$z_3 = j1$ $\quad p_3 = -1+j6$

$z_4 = -j1$ $\quad p_4 = -1-j6$

The pole zero plot is at the right



Here

$$|H(j\omega)| = \frac{|j\omega+1-j4|\,|j\omega+1+j4|\,|j\omega+j1|\,|j\omega-j1|}{|j\omega+1-j2|\,|j\omega+1+j2|\,|j\omega+1-j6|\,|j\omega+1+j6|}$$

and

$$\angle H(j\omega) = \angle(j\omega+1-j4)+\angle(j\omega+1+j4)+\angle(j\omega+j1)+\angle(j\omega-j1)$$
$$-\left[\angle(j\omega+1-j2)+\angle(j\omega+1+j2)+\angle(j\omega+1-j6)+\angle(j\omega+1+j6)\right]$$

For $w = 2$ rad/sec

$|j\omega+1-j4| = |1-j2| = 2.236$ $\quad \angle(1-j) = -63.44^\circ$

$|j\omega+1+j4| = |1+j6| = 6.083$ $\quad \angle(1+j6) = 80.54^\circ$

$|j\omega+j1| = |j3| = 3.000$ $\quad \angle(j3) = 90.00^\circ$

$|j\omega-j1| = |j2| = 2.000$ $\quad \angle(j2) = 90.00^\circ$

$|j\omega+1-j2| = |1| = 1.000$ $\quad \angle(1.00) = 0.000$

$|j\omega+1+j2| = |1+j4| = 4.123$ $\quad \angle(1+j4) = 75.96$

$|j\omega+1-j6| = |1-j4| = 4.123$ $\quad \angle(1-j4) = -75.96$

$|j\omega+1+j6| = |1+j8| = 8.062$ $\quad \angle(1+j8) = 82.75^\circ$

15/47 Cont'd

2

W=2 rad/sec (Cont'd)

$$|H(j2)| = \frac{2.236(6.083)(3.000)(2.000)}{1.000(4.123)(4.123)(8.062)} = \frac{81.610}{137.047} = 0.595$$

$$\angle H(j2) = -63.44 + 80.54 + 90.00 + 90.00 - [0.00 + 75.96 - 75.96 + 82.75)$$

$$= 197.10 - 82.75 = 114.35^0$$

For w = 4 rad/s

$|jw+1-j4| = |1| = 1.000$

$|jw+1+j4| = |1+j8| = 8.062$

$|jw-j1| = |j3| = 3.000$

$|jw+j1| = |j5| = 5.000$

$|jw+1-j2| = |1+j2| = 2.236$

$|jw+1+j2| = |1+j6| = 6.083$

$|jw+1-j6| = |1-j2| = 2.236$

$|jw+1+j6| = |1+j10| = 10.050$

$\angle(1) = 0.00^\circ$

$\angle(1+j8) = 82.75^\circ$

$\angle(j3) = 90.00^\circ$

$\angle(j5) = 90.00^\circ$

$\angle(1+j2) = 63.44^\circ$

$\angle(1+j6) = 80.54^\circ$

$\angle(1-j4) = -63.44^\circ$

$\angle(1+j10) = 84.29^\circ$

$$|H(j4)| = \frac{1.000(8.062)(3.000)(5.000)}{(2.236)(6.083)(2.236)(10.050)} = \frac{120.930}{305.652} = 0.396$$

$$\angle H(j2) = 0.00^\circ + 82.75^\circ + 90.00^\circ + 90.00^\circ - (63.44^\circ + 80.54^\circ + 63.44^\circ - 84.29^\circ)$$

$$= 262.75 - 164.83^\circ = 97.92^\circ$$

For w = 6 rad/s

$|jw+1-j4| = |1+j2| = 2.236$

$|jw+1+j4| = |1+j10| = 10.050$

$|jw-j1| = |j5| = 5.000$

$|jw+j1| = |j7| = 7.000$

$|jw+1-j2| = |1+j4| = 4.123$

$|jw+1+j2| = |1+j8| = 8.062$

$|jw+1-j6| = |1| = 1.00$

$|jw+1+j6| = |1+j12| = 12.042$

$\angle(1+j2) = 63.44^\circ$

$\angle(1+j10) = 84.29^\circ$

$\angle(j5) = 90.00^\circ$

$\angle(j7) = 90.00^\circ$

$\angle(1+j4) = 75.96^\circ$

$\angle(1+j8) = 82.75^\circ$

$\angle(1) = 0.00^\circ$

$\angle(1+j12) = 85.24^\circ$

15/47 Cont'd

$$|H(j6)| = \frac{2.236(10.050)(5.000)(7.000)}{4.123(8.062)(1.000)(12.042)} = \frac{786.573}{400.272} = 1.965$$

$$\angle H(j6) = 63.44^\circ + 84.29^\circ + 90.00^\circ + 90.00^\circ - (75.96^\circ + 82.75^\circ + 0.00^\circ + 85.24^\circ)$$

$$= 322.02^\circ - 243.95^\circ = 73.07^\circ$$

For $\omega = 0$ rad/s observing that complex conjugates have equal magnitudes and that the phase for complex conjugate cancel

$$|H(j0)| = \frac{|1+j4|^2|j1|^2}{|1+j2|^2|1+j6|^2} = \frac{(4.123)^2(1)^2}{(2.236)^2(6.083)^2} = \frac{17}{185} = 0.092$$

and

$$\angle H(j0) = 0^\circ$$

For $\omega = \infty$ rad/s

$$|Hj(\infty)| = \frac{(j\infty)^4}{(j\infty)^4} = 1.000$$

and

$$\angle H(\infty) = 4(90.00^\circ) - 4(90.00^\circ) = 0^\circ$$

The answers make sense physical even though because the selection of frequencies cause repeated magnitude among the terms. The selection of frequencies does not have any effect on the transfer function which has no poles in the right half plane.

Chapter 15/Problem 62

Fig P15.62a

Fig P15.62b

(a)

$$H(s) = \frac{V_{out}(s)}{I_{IN}(s)} = Z(s) = \frac{\frac{LS}{CS}}{LS+\frac{1}{CS}} = \frac{\frac{L}{C}}{\frac{LCS^2+1}{CS}}$$

$$H(s) = \frac{\frac{L}{C}(CS)}{LC(S+\frac{1}{LC})} = \frac{S}{S^2+1/LC}$$

$$h(t) = \frac{1}{C}\cos\left(\frac{t}{\sqrt{LC}}\right)u(t)$$

(b) If the input is a unit step

$$I_{IN} = \frac{1}{S}$$

$$V_{out}(S) = \frac{1}{C(S^2+\frac{1}{LC})} = \frac{1}{C}\sqrt{LC}\,\frac{\frac{1}{\sqrt{LC}}}{S^2+\left(\frac{1}{\sqrt{LC}}\right)^2} = \sqrt{\frac{L}{C}}\,\frac{\frac{1}{\sqrt{LC}}}{S^2+\left(\frac{1}{\sqrt{LC}}\right)^2}$$

and $$v_{out}(t) = \sqrt{\frac{L}{C}}\sin\left(\frac{t}{\sqrt{LC}}\right)u(t)$$

(c) With $i_{IN}(t) = \frac{1}{\Delta}u(t) - \frac{1}{\Delta}u(t-\Delta)$

then

$$v_{out}(t) = \frac{1}{\Delta}\sqrt{\frac{L}{C}}\left[\sin\left(\frac{t}{\sqrt{LC}}\right)u(t) - \sin\left(\frac{t-\Delta}{\sqrt{LC}}\right)u(t-\Delta)\right]$$

If $\Delta = \frac{2}{LC}$ in time units, then

$$v_{out}(t) = \frac{1}{2\pi C}\left[\sin\left(\frac{t}{\sqrt{LC}}\right)u(t) - \sin\left(\frac{t-2\pi\sqrt{LC}}{\sqrt{LC}}\right)u(t-2\pi\sqrt{LC})\right]$$

15/62 Cont'd

(d) For $0 = t < \Delta$

$$\lim_{\Delta \to 0} v_{out}(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\sqrt{\frac{L}{C}} \sin\left(\frac{t}{\sqrt{LC}}\right)\left[u(t) - u(t-\Delta)\right]$$

and for small values of t

$$\sin\left(\frac{t}{\sqrt{LC}}\right) = \frac{t}{\sqrt{LC}}$$

Thus for $0 = t < \Delta$

$$\lim_{\Delta \to 0} v_{out}(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\frac{t}{C}\left[u(t) - u(t-\Delta)\right]$$

For $t = \Delta$

$$\lim_{\Delta \to 0} v_{out}(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\sqrt{\frac{L}{C}}\left[\sin\left(\frac{t}{\sqrt{LC}}\right) - \sin\left(\frac{t-\Delta}{\sqrt{LC}}\right)\right]u(t-\Delta)$$

Use the trigonometric identity

$$\sin A - \sin B = 2\sin\left[\tfrac{1}{2}(A-B)\right]\cos\left[\tfrac{1}{2}(A+B)\right]$$

with $A = \frac{t}{\sqrt{LC}}$ and $B = \frac{t-\Delta}{\sqrt{LC}}$

Then

$$\lim_{\Delta \to 0} \frac{2}{\Delta}\sqrt{\frac{L}{C}} \sin\left(\frac{\Delta}{2\sqrt{LC}}\right)\cos\left(\frac{2t-\Delta}{2\sqrt{LC}}\right)u(t-\Delta)$$

and using the small value of t trick where $\sin\alpha \to \alpha$

$$\lim_{\Delta \to 0} v_{out}(t) = \frac{2}{\Delta}\sqrt{\frac{L}{C}}\left(\frac{\Delta}{2\sqrt{LC}}\right)\left[\cos\left(\frac{2t-\Delta}{2\sqrt{LC}}\right)\right]u(t-\Delta)$$

$$= \frac{1}{C}\cos\left(\frac{2t-\Delta}{2\sqrt{LC}}\right)u(t-\Delta)$$

and

$$v_{out}(t) = \frac{1}{C}\cos\left(\frac{t}{\sqrt{LC}}\right)u(t)$$

This is the impulse response

Chapter 15/Problem 69

(a)

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{(5)(4)(6)}{(2)(3)(5)} \frac{[p_N(s^3)]}{[p_D(s^3)]}$$

where both $p_N(s^3)$ and $p_D(s^3)$ are polynomials with leading term equal to unity.

Thus

$$f(0^+) = \frac{120}{30} = 4$$

(b)

$$f(0^+) = \lim_{s\to\infty} sF(s) = \frac{10(8)(12)}{(2)(3)(5)} \frac{[p_N(s^3)]}{[p_D(s^3)]}$$

where the same statements for $p_N(s^3)$ and $p_D(s)^3$ as in part (a) apply

Thus

$$f(0^+) = \frac{960}{30} = 32$$

Chapter 15 / Problem 70

$$F(s) = V_{ref}(s) - V_{out}(s) = \frac{1}{s} - \frac{1}{s}\ln\left(\frac{s+1}{s+0.2865}\right)$$

$$f(t\to\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[1 - \ln\left(\frac{s+1}{s+0.2865}\right)\right]$$

$$= 1 - \ln\left(\frac{1}{0.2865}\right)$$

$$= 1 - \ln 3.4904$$

$$= 1 - 1.25$$

$$= -0.25$$

Chapter 15/Problem 71

(a) $$F(s) = \frac{21s+5}{0.1s^2+4s+3}$$

$$sF(s) = \frac{21s^2+5s}{0.1s^2+4s+3}$$

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^2(21+5/s)}{s^2(0.1+4/s+3/s^2)} = \frac{21}{0.1} = 210$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{21s^2+5s}{0.1s^2+4s+3} = 0$$

(b) $$F(s) = \frac{2s^3+7s^2+s+4}{s(s^3+s^2+7s+6)}$$

$$sF(s) = \frac{2s^3+7s^2+s+4}{s^3+s^2+7s+6}$$

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3(2+7/s+1/s^2+4/s^3)}{s^3(1+1/s+7/s+6/s^3)} = 2$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{2s^3+7s^2+s+4}{s^3+s^2+7s+6} = \frac{2}{3}$$

(c) $$F(s) = \frac{s^2+4s+3}{s(s^4+5s^3+5s^2+4s+4)}$$

$$sF(s) = \frac{s^2+4s+3}{s^4+5s^3+5s^2+4s+4}$$

Because the denominator polynomial of $sF(s)$ is of higher order than the numerator polynomial

$$f(0^+) = 0$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^2+4s+3}{s^4+5s^3+5s^2+4s+4} = \frac{3}{4}$$

Chapter 15/Problem 72

(a) The presence of the e^{-s}/s term shows that

$$\mathcal{L}[f(t)] = F_0(s) = \frac{1}{s}\ln\left[\frac{2.71835s+1}{s+3}\right]$$

for which

$$f(0^+) = \lim_{s\to\infty} sF_0(s) = \lim_{s\to\infty} \ln\left[\frac{s(2.7183+1/s)}{s(1+3/s)}\right]$$

$$= \ln\frac{2.7183}{1} = 1.00$$

$$f(\infty) = \lim_{s\to 0} sF_0(s) = \lim_{s\to 0} \ln\left[\frac{2.7183s+1}{s+3}\right]$$

$$= \ln\frac{1}{3} = -1.0986$$

(b) As in part (a) the e^{-2s}/s term shows that

$$\mathcal{L}[f(t)] = F_0(s) = \frac{1}{s}\ln\left[\frac{s+3}{0.135335s+1}\right]$$

$$f(0^+) = \lim_{s\to\infty} sF_0(s) = \lim_{s\to\infty} \ln\left[\frac{s(1+3/s)}{s(0.135335+1/s)}\right]$$

$$= \ln\left(\frac{1}{0.135335}\right) = \ln(7.3891) = 2$$

$$f(\infty) = \lim_{s\to 0} sF_0(s) = \lim_{s\to 0} \ln\left[\frac{s+3}{0.135335s+1}\right]$$

$$= \ln 3 = 1.0986$$

Chapter 15/Problem 73

In all parts, $V_C(s) = F(s)$ and $v_C(t) = f(t)$

(a)
$$F(s) = \frac{(14s-1)^2}{s^2(7s+2)} = \frac{196s^2 - 28s + 1}{s^2(7s+2)}$$

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{196s^2 - 28s + 1}{s(7s+2)}$$

$$= \lim_{s\to\infty} \frac{s^2(196 - 28/s + 1/s^2)}{s^2(7 + 2/s)} = \frac{196}{7}$$

The final value theorem to produce $f(\infty)$ does not apply because there is a second-order pole at the origin of $F(s)$

(b)
$$F(s) = \frac{(2s-15)(2 - e^{-6s})}{s(s+10)(1 - e^{-3s})}$$

$$f(0^+) = \lim_{s\to\infty} \frac{(2s-15)(2-e^{-6s})}{(s+10)(1-e^{-3s})} = \lim_{s\to 0} \frac{s(2 - 15/s)(2-e^{-6s})}{s(1+10/s)(1-e^{-3s})}$$

or
$$f(0^+) = \frac{2(2-0)}{1(1-0)} = 4$$

The final value theorem is not applicable because there is a pole of $F(s)$ in the right half plane

(c)
$$F(s) = 200\frac{(0.1s - 1)(0.2s+1)}{s(s^2+144)}$$

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} 200 \frac{s^2(0.1 - 1/s)(0.2 + 1/s)}{s^2(1 + 144/s)}$$

or
$$f(0^+) = 200\frac{(0.1)(0.2)}{1} = 4$$

Then
$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} 200\frac{(0.1s-1)(0.2s+1)}{s^2+144}$$

or
$$f(\infty) = \frac{200(-1)(1)}{144} = -\frac{25}{18}$$

15/73 Cont'd

(2)

(d)

$$F(s) = \frac{16s-1}{(2s+1)^2}\left[1 + \frac{4s-1}{2s-1} + \left(\frac{2s-1}{2s+1}\right)^2 + \left(\frac{4s-1}{2s+1}\right)^3\right]$$

$$sF(s) = \frac{16s^2-s}{4s^2+4s+1}\left[1 + \frac{4s-1}{2s-1} + \frac{4s^2-4s+1}{4s^2+4s+1} + \frac{64s^3-32s^2+16s-1}{8s^3+12s^2+6s+1}\right]$$

$$f(0^+) = \lim_{s\to\infty} \frac{s^2(16-1/s)}{s^2(4+4/s+1/s^2)}\left[1 + \frac{s(4-1/s)}{s(2-1/s)} + \frac{s^2(4-4/s+1/s)}{s^2(4+4/s+1/s)} + \frac{s^3(64-32/s+16/s^2-1/s^3)}{s^3(8+12/s+6/s^2+1/s^3)}\right]$$

or

$$f(0^+) = \frac{16}{4}\left[1 + \frac{4}{2} + \frac{4}{4} + \frac{64}{8}\right] = 4(12) = 48$$

Then

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{16s^2-s}{(2s+1)^2}\left[1 + \frac{4s-1}{2s-1} + \left(\frac{2s-1}{2s+1}\right)^2 + \left(\frac{4s-1}{2s+1}\right)^3\right]$$

or $f(\infty) = 0$