CHAPTER 19 PROBLEM SOLUTIONS

SOLUTION PROBLEM 19.1. Refer to figure 19.3.

```
»Vs = 100;  ZL = 20; Rs = 1e3;
»beta = 149;
»Zinbox = (beta + 1)*ZL
Zinbox =
     3000
»%  By voltage division
»V1 = Vs*Zinbox/(Zinbox + Rs)
V1 =   75
»%  To  obtain the power delivered  by the source
»I1 = Vs/(Zinbox +Rs)
I1 =   2.5000e-02
»Psource = I1*Vs
Psource =   2.5000e+00
```

SOLUTION PROBLEM 19.2. Refer to figure 19.4.

```
»Vs = 100; Z1 = 30e3;  Rs = 50;  beta = 149;
»Zboxin = Z1/(beta+1)
Zboxin =   200
»V1 = Vs*Zboxin/(Zboxin + Rs)
V1 =    80
»Psource = Vs^2/(Rs + Zboxin)
Psource =    40
```

SOLUTION PROBLEM 19.3. Refer to figure 19.5.

```
(a)  »C = 0.1e-3; vc0 = 10; Z1 = 300; Z2 = 1e3;
»Z3 = 1e3; gm = 9e-3;
»
»Zin = Z1 + (1 + gm*Z1)*Z2
Zin =   4.0000e+03
(b)
»tau = Zin*C
```

tau = 4.0000e-01

Hence, $v_C(t) = v_C(0)e^{-t/\tau} = 10e^{-2.5t}$ V.

SOLUTION PROBLEM 19.4. (a) First observe that since no current can flow into the secondary we have

$$V_{oc} = aV_{pri} = aRI_{in} = 800 \text{ Vrms}$$

Now

$$Z_{th} = \frac{1}{j\omega C} + a^2R = 640 - j360\ \Omega$$

(b) $Z_L = \left(Z_{th}\right)^* = 640 + j360\ \Omega$.

»Voc = 800; Rth = 640;
»Pmax = Voc^2/(4*Rth)
Pmax = 250

(c) By inspection the circuit is a 640 Ω resistor in series with a 3.6 H inductor.

SOLUTION PROBLEM 19.5. Because the output is open circuited, no current flows into the secondary of the transformer, hence

$$v_{oc} = v_{\sec} + \sin(3t)u(t) = 2v_{pri} + \sin(3t)u(t) = \left[\cos(3t) + \sin(3t)\right]u(t).$$

Additionally

$$Z_{th}(s) = \frac{10}{s} + s + 4 + 4\left(\frac{2.5}{s} + 0.25s + 9\right) = \frac{20}{s} + 2s + 40$$

SOLUTION PROBLEM 19.6. Using Cramer's rule,

$$I_1 = \frac{\det\begin{bmatrix} V_1 & 1 & -a \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}}{1.5a} = \frac{0.25V_1}{1.5a} = \frac{1}{6a}V_1$$

Therefore $R_{in} = 6a\ \Omega$.

To compute the average power, $V_{1eff} = 10$ V. Hence $P_{ave} = \frac{V_{1eff}^2}{R_{in}} = \frac{100}{6a}$ watts.

SOLUTION PROBLEM 19.7. As per the hint, we write loop equations as follows:

$$\begin{bmatrix} -V_{out} \\ \hline 10/s \\ -40/s \end{bmatrix} = \left[\begin{array}{c|cc} s+1 & -1 & 0 \\ \hline -1 & 6 & -2 \\ 0 & -2 & 4 \end{array}\right] \begin{bmatrix} I_{out} \\ \hline I_2 \\ I_3 \end{bmatrix}$$

Using equation 19.6,

$$-V_{out} = \left(s + 1 - \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) I_{out} + \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} 10/s \\ -40/s \end{bmatrix}$$
$$= (s + 0.8)I_{out} + \frac{2}{s}$$

Therefore

$$V_{out} = (s + 0.8)(-I_{out}) - \frac{2}{s} = Z_{th}(-I_{out}) + V_{oc}$$

i.e., $Z_{th} = s + 0.8$, *and* $V_{oc} = -\frac{2}{s}$.

SOLUTION PROBLEM 19.8. (a) Let the node voltages from left to right be V_1, V_2, and V_{out}. Also inject a current I_3 into node 3. Writing nodal equations by inspection we have:

$$\begin{bmatrix} I_{in} \\ 0 \\ I_3 \end{bmatrix} = \left[\begin{array}{cc|c} 1.5 & -1 & -0.25 \\ -1 & 2 & -0.5 \\ \hline -0.25 & -0.5 & 0.0625s + 0.75 \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ \hline V_{out} \end{bmatrix}$$

Using equation 19.11, we have

$$I_3 = \left(W_{22} - W_{21}W_{11}^{-1}W_{12}\right)V_{out} + W_{21}W_{11}^{-1}\begin{bmatrix} I_{in} \\ 0 \end{bmatrix}$$

$$= \left(0.0625s + 0.75 - \begin{bmatrix} 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}\begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}\right)V_{out} - \begin{bmatrix} 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.75 \end{bmatrix}\begin{bmatrix} I_{in} \\ 0 \end{bmatrix}$$

Thus

$$I_3 = \left(0.0625s + 0.375\right)V_{out} - 0.5I_{in}$$

Therefore $I_{sc} = -I_3\big]_{V_{out}=0} = 0.5I_{in}$. Further $Z_{th}(s) = \dfrac{1}{0.0625s + 0.375} = \dfrac{16}{s+6}$ Ω.

(b) $V_{out}(s) = Z_{th}(s)I_{sc} = \dfrac{16I_{sc}}{s+6} = \dfrac{8I_{in}}{s+6}$. By inspection, the impulse response is

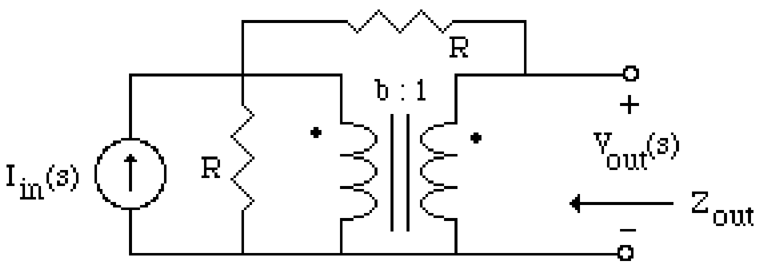
$v_{out,imp}(t) = 8e^{-6t}u(t)$ V. Further, from MATLAB

```
»n = 8;  d = [1  6  0];
»[r,p,k] = residue(n,d)
r =
  -1.3334e+00
   1.3334e+00
p =
    -6
     0
k =
     []
```

Hence the step response is:

$$v_{out,step}(t) = \frac{4}{3}\left(1 - e^{-6t}\right)u(t) \text{ V}$$

SOLUTION PROBLEM 19.9. (a) Consider the following figure:



Let I_{out} enter the output terminal and I_1 and I_2 be the currents entering the primary and secondary of the transformer respectively. It follows that

$$I_{in} = \frac{bV_{out}}{R} + I_1 + \frac{(b-1)V_{out}}{R} = \frac{(2b-1)V_{out}}{R} + I_1$$

which implies that

$$I_1 = I_{in} - \frac{(2b-1)V_{out}}{R}$$

Further,

$$I_{out} = \frac{(1-b)V_{out}}{R} + I_2 = \frac{(1-b)V_{out}}{R} - bI_1$$

Therefore

$$I_{out} = \frac{(1-b)V_{out}}{R} + I_2 = \frac{(1-b)V_{out}}{R} - bI_1 = \frac{(1-b)V_{out}}{R} - b\left(I_{in} - \frac{(2b-1)V_{out}}{R}\right)$$

$$= \frac{2b^2 - 2b + 1}{R} V_{out} - bI_{in}$$

Equivalently

$$V_{out} = \frac{R}{2b^2 - 2b + 1} I_{out} + \frac{bR}{2b^2 - 2b + 1} I_{in}$$

Therefore

$$Z_{th} = \frac{R}{2b^2 - 2b + 1},\ V_{oc} = \frac{bR}{2b^2 - 2b + 1} I_{in},\ I_{sc} = bI_{in}$$

(b) $\mathbf{I_{sc}} = b \quad 45^o,\ Z_{th} = \frac{R}{2b^2 - 2b + 1}$.

(c) $B_\omega = 10 = \dfrac{1}{Z_{th}C} = \dfrac{1}{\dfrac{R}{2b^2 - 2b + 1}C} = \dfrac{2b^2 - 2b + 1}{RC} = \dfrac{5}{25C}$. Hence $C = 0.02$ F. Further,

$\omega_0^2 = 25 = \dfrac{1}{LC} = \dfrac{1}{0.02L}$. Therefore $L = 2$ H.

(d) $H(s) = \dfrac{V_{out}}{I_{in}} = \dfrac{V_{out}}{0.5I_{sc}} = 2\dfrac{1}{\dfrac{2b^2 - 2b + 1}{R} + \dfrac{1}{Ls} + Cs} = \dfrac{2}{0.2 + \dfrac{1}{2s} + 0.02s} = \dfrac{100s}{s^2 + 10s + 25}$

and $I_{in}(s) = \dfrac{\sqrt{2}}{2} \times \dfrac{s - 10}{s^2 + 100}$. Hence,

$$V_{out}(s) = \frac{50\sqrt{2}\,s(s - 10)}{(s^2 + 10s + 25)(s^2 + 100)}$$

To compute $v_{out}(t)$ in MATLAB,

```
%partial fraction expansion of Vout/√2 = = n(s)/d(s)
n =50*[ 1 -10 0];
d = conv([ 1 0 100], [ 1 10 25]);
[r ,p, k] = residue(n,d)

%numerator polynomial of combined complex pole terms

num = [1 -p(1)]*r(2) +[1 -p(2)]*r(1)
```

Output from MATLAB

```
r =
   2.8000 + 0.4000i
   2.8000 - 0.4000i
  -5.6000
  30.0000
p =
  -0.0000 +10.0000i
  -0.0000 -10.0000i
  -5.0000
  -5.0000
num =
    5.6000   -8.0000
```

Laplace transform of ouput

$$V_{out}(s) = \frac{-5.6\sqrt{2}}{s + 5} + \frac{30\sqrt{2}}{(s + 5)^2} + \frac{(5.6s - 8)\sqrt{2}}{s^2 + 100}$$

Taking the inverse Laplace transform using table 13.1 on page 515:

$$v_{out}(t) = \left[30\sqrt{2}te^{-5t} - 5.6\sqrt{2}e^{-5t} + 5.6\sqrt{2}\cos(10t) - 0.8\sqrt{2}\sin(10t)\right]u(t) \text{ V}$$

The steady state part consists of the cosine and sine terms only. Since the parallel RLC acts like a band pass circuit and the peak value occurs at 5 rad/s, one expects the magnitude of the steady state output to be much smaller at 100 rad/s.

SOLUTION PROBLEM 19.10. (a) Write two nodal equations by inspection:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + Y_3 & -Y_3 + g_m \\ -Y_3 & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(b) When port-2 is shorted, y_{11} is the input admittance. Therefore

$$Z_{in} = \frac{1}{y_{11}} = \frac{1}{Y_1 + Y_3}$$

and since $V_2 = 0$,

$$I_2 = y_{21}V_1 = \frac{-KY_3}{s}$$

SOLUTION PROBLEM 19.11. (a) By inspection

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + Y_3 & -Y_3 \\ -Y_3 + g_m & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Clearly, $Y_3 = -y_{12}$. Then, $Y_1 = y_{11} - Y_3 = y_{11} + y_{12}$ and $Y_2 = y_{22} - Y_3 = y_{22} + y_{12}$. Finally, $g_m = y_{21} + Y_3 = y_{21} - y_{12}$.

(b) Recall that

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + Y_3 & -Y_3 + g_m \\ -Y_3 & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Clearly, $Y_3 = -y_{21}$. Then, $Y_1 = y_{11} - Y_3 = y_{11} + y_{21}$ and $Y_2 = y_{22} - Y_3 = y_{22} + y_{21}$. Finally, $g_m = y_{12} + Y_3 = y_{12} - y_{21}$.

SOLUTION PROBLEM 19.12. (a) By definition of coupled inductors

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_1 s & Ms \\ Ms & L_2 s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{\left(L_1 L_2 - M^2\right)s} \begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence, the y-parameters are:

$$\frac{1}{\left(L_1 L_2 - M^2\right)s} \begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix}$$

(b) By definition of coupled inductors

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_1 s & -Ms \\ -Ms & L_2 s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{\left(L_1 L_2 - M^2\right)s} \begin{bmatrix} L_2 & M \\ M & L_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hence, the y-parameters are:

$$\frac{1}{\left(L_1 L_2 - M^2\right)s} \begin{bmatrix} L_2 & M \\ M & L_1 \end{bmatrix}$$

If the coupling coefficient is 1, $L_1 L_2 = M^2$ and the y-parameters do not exist since the determinant of the z-parameter matrix is zero.

SOLUTION PROBLEM 19.13. Let $I_2^{'}$ denote the current entering the dotted terminal of the secondary of the coupled inductors. Then using the result of problem 12a,

$$\begin{bmatrix} I_1 \\ I_2' \end{bmatrix} = \frac{1}{(L_1L_2 - M^2)s}\begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{6s}\begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

From the given circuit $I_2 = I_2' + 2I_1 = \frac{-1}{6s}V_1 + \frac{1}{6s}V_2 + \frac{8}{6s}V_1 - \frac{2}{6s}V_2 = \frac{7}{6s}V_1 - \frac{1}{6s}V_2$. Therefore the y-parameter matrix is:

$$\frac{1}{6s}\begin{bmatrix} 4 & -1 \\ 7 & -1 \end{bmatrix}$$

SOLUTION PROBLEM 19.14.

(a) By definition and the properties of the ideal transformer $y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = G_1$ and $y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{G_1 + G_2}{a^2}$. Additionally, since the circuit is obviously reciprocal, $y_{12}(=y_{21}) = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{G_1}{a}$.

(b) V_2 reflected to the primary side, denoted by $\hat{V}_1$, is

$$\hat{V}_1 = \frac{-2K/a}{s^2 + 4}$$

Hence

$$I_1 = \frac{2G_1K/a}{s^2 + 4}$$

To compute I_2, we reflect the parallel of G_1 and G_2 to the secondary of the ideal transformer. Hence the impedance in parallel with V_2, denoted Z_{sec}, is

$$Z_{sec} = \frac{a^2}{G_1 + G_2}$$

Therefore,

$$I_2 = \frac{V_2}{Z_{\sec}} = \frac{2K\left(G_1 + G_2\right)/a^2}{s^2 + 4}$$

SOLUTION PROBLEM 19.15. (a) By definition

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \left.\frac{1}{Z_{in}}\right|_{V_2=0} = \frac{1}{9} \text{ S}$$

where

$$Z_{in} = 6 + 12 // \left(\frac{1}{16} \times (320 // 80)\right) = 9 \ \Omega$$

Similarly, by definition

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \left.\frac{1}{Z_{out}}\right|_{V_1=0} = \frac{3}{400} \text{ S}$$

where

$$Z_{out} = 80 + 320 // \left(16 \times (6 // 12)\right) = \frac{400}{3} \ \Omega$$

SOLUTION PROBLEM 19.16. Write nodal equations:

$$\begin{bmatrix} I_1 \\ I_2 \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cc|c} 2s+2 & -1 & -2s \\ -1 & s+2 & -s \\ \hline -2s & -s & 5s \end{array}\right] \begin{bmatrix} V_1 \\ V_2 \\ \hline V_3 \end{bmatrix}$$

Using the matrix partitioning method, we obtain the 2-port y parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \left(\begin{bmatrix} 2s+2 & -1 \\ -1 & s+2 \end{bmatrix} - \frac{1}{5s} \begin{bmatrix} -2s \\ -s \end{bmatrix} \begin{bmatrix} -2s & -s \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.2s+2 & -0.4s-1 \\ -0.4s-1 & 0.8s+2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

SOLUTION PROBLEM 19.17. (a) The ideal transformer yields the constraints

$$I_1 = -a\hat{I}_2 \quad \text{and} \quad \hat{V}_2 = aV_1$$

The three resistors have nodal equations

$$\begin{bmatrix} -\hat{I}_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} \hat{V}_2 \\ V_2 \end{bmatrix}$$

Substituting the first two equations into the last one, we obtain

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2a^2 & -a \\ -a & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(b)

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = 2a^2 - \frac{a^2}{4} = 1.75a^2 \text{ S}$$

$$Z_{in} = \frac{1}{Y_{in}} = \frac{4}{7a^2} \ \Omega$$

and

$$G_v = \frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + y_{L1}} = \frac{a}{4}$$

SOLUTION PROBLEM 19.18. This problem is solved in MATLAB.

Part (a)

```
»%  Parameter specification
»Ys = 1e-3;YL = 1e-3;
»y11=4e-3; y12 = -0.1e-3;
»y21 = 50e-3; y22 = 1e-3;

»%  Calculation of input admittance and impedance
»Yin = y11 - y12*y21/(y22 + YL)
Yin =   6.5000e-03
»Zin = 1/Yin
Zin =   1.5385e+02

»%  Calculation of output admittance
```

```
»Yout = y22 - y12*y21/(y11 + Ys)
Yout =   2.0000e-03
```

Part (b)

```
»%  Calculation of voltage gain
»Gv = (Ys/(Ys + Yin))*(-y21/(y22 + YL))
Gv =  -3.3333e+00
```

Part (c)

```
»V2 = Gv*10
V2 =  -3.3333e+01
```

Therefore, $v_2(t) = -33.333u(t)$ V. Finally,

```
»PL = V2^2/1e3
PL =   1.1111e+00
```

SOLUTION PROBLEM 19.19.

(a) $$\frac{V_1}{V_s} = \frac{Z_{in}}{Z_{in} + Z_s} = \frac{Z_{in}}{Z_{in} + 10} = 0.5 \qquad Z_{in} = 10\ \Omega \text{ or } 0.1\ S$$

Now

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = y_{11} - \frac{0.02\times 2}{0.2 + 0.1} = 0.1$$

Solving for y_{11} yields $y_{11} = 0.2333$ S.

(b) $$\frac{v_2}{v_1} = \frac{-y_{21}}{y_{22} + Y_L} = \frac{-2}{0.2 + 0.1} = -6.667$$

Hence

$$v_2(t) = -6.667\ v_1(t) = -3.333\ v_s(t) = -33.33\ u(t)\quad V$$

and

$$P_L = \frac{V_2^2}{R_L} = \frac{33.33^2}{10} = 111.11\ W$$

SOLUTION PROBLEM 19.20.

(a) Writing a node equation at port 1 and mesh equation at port 2, we obtain by inspection

$$I_1 = 2V_1 + 3I_2$$
$$V_2 = 2V_1 + 2I_2$$

Rearranging in matrix form , we have

$$\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Therefore

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -1 & 1.5 \\ -1 & 0.5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(b) $Y_{in} = y_{11} - \dfrac{y_{12}y_{21}}{y_{22} + Y_L} = -1 + \dfrac{1.5\times 1}{0.5 + 0.25} = 1 \text{ S}$

(c) Here we compute Y_{out} seen looking into port-2, i.e.,

$$Y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + Y_s} = 0.5 + \frac{1.5\times 1}{-1 + 0} = -1 \text{ S}$$

From current division,

$$i_2(t) = \frac{Y_{out}}{Y_{out} + Y_L} i_s(t) = \frac{-1}{-1 + 0.25}\times 5u(t) = 6.667u(t) \text{ A}$$

Finally from Ohm's law

$$v_1(t) = -0.5\times 3i_2(t) = -0.5\times 3\times 6.667u(t) = -10u(t) \text{ V}$$

REMARK: Because the current source sees a negative resistance, the circuit is unstable as it stands.

SOLUTION PROBLEM 19.21. (a) With port 2 shorted, the Laplace transform of the given data are:

$$I_1(s) = \frac{1}{s},\ V_1(s) = \frac{1}{s} - \frac{1}{s+4} = \frac{4}{s(s+4)},\ I_2(s) = \frac{-1}{s+3}$$

Hence

$$y_{11} = \frac{I_1}{V_1} = \frac{1/s}{\dfrac{4}{s(s+4)}} = \frac{s+4}{4}$$

and

$$y_{21} = \frac{I_2}{V_1} = \frac{-1/(s+3)}{\dfrac{4}{s(s+4)}} = \frac{-s(s+4)}{4(s+3)}$$

Next, with port-2 terminated in a 1-Ω reistor, the Laplace transform of the given data are:

$Z_L = Y_L = 1$

$I_1(s) = 1/s,$

$$V_1(s) = \frac{1}{s} - \frac{1}{s+4} + \frac{1}{(s+4)^2} = \frac{5s + 16}{s(s + 4)^2}$$

$$I_2 = \frac{-1}{s+7}$$

Now

$$I_2 = -\frac{V_2}{Z_L} = -V_2 = \frac{y_{21}V_1}{y_{22} + Y_L}$$

Solving for y_{22}, and using y_{21} expression found earlier, we obtain

$$y_{22} = y_{21}V_1/I_2 - Y_L = \frac{-s(s+4)}{4(s+3)} \times \frac{5s+16}{s(s+4)^2} \times \frac{s+7}{-1} - 1$$

$$= \frac{s^2 + 23s + 64}{4(s+3)(s+4)}$$

Finally, to compute y_{12}, we use the defining equation for y-parameters, i.e.,

$$I_1 = y_{11}V_1 + y_{12}V_2$$

from which we obtain

$$y_{12} = \frac{I_1 - y_{11}V_1}{V_2} = \frac{\frac{1}{s} - \frac{s+4}{4} \times \frac{5s+16}{s(s+4)^2}}{\frac{1}{s+7}} = \frac{-(s+7)}{4(s+4)}$$

(b) Given Y_L =1 S, the input admittance is

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = \frac{s+4}{4} - \frac{\frac{-(s+7)}{4(s+4)} \times \frac{-s(s+4)}{4(s+3)}}{\frac{s^2 + 23s + 64}{4(s+3)(s+4)} + 1} = \frac{(s+4)^2}{5(s+3.2)}$$

and

$$Z_{in} = \frac{1}{Y_{in}} = \frac{5(s+3.2)}{(s+4)^2}$$

(c) For this part, we use phasors to do the sinusoidal steady state analysis: ω= 10 rad/s and $\mathbf{I_1}$ = 1. Also,

$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = Z_{in} = \frac{5(j10+3.2)}{(j10+4)^2} = 0.1973 - j0.4072$$

and

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-y_{21}}{y_{22}+Y_L} = \frac{\dfrac{-j10(j10+4)}{4(j10+3)}}{\dfrac{-100+230j+64}{4(j+3)(j+4)}+1} = -0.2873+ j0.1787$$

Thus

$$\frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{\mathbf{V}_2}{\mathbf{V}_1}\times\frac{\mathbf{V}_1}{\mathbf{I}_1} = (-0.2873+ j0.1787)(0.1973 - j0.4072)$$

from which we obtain in Ohms,

$$\left|\frac{\mathbf{V}_2}{\mathbf{I}_1}\right| = |-0.2873+ j0.1787|\times|0.1973 - j0.4072| = 0.8192$$

SOLUTION PROBLEM 19.22. Looking into port-1, the admittance is:

$$Y_{port\ 1}(s) = y_{11} - \frac{y_{12}y_{21}}{y_{22}+Y_L} = 0 + \frac{2\times 2}{0 + 0.1s} = \frac{40}{s}$$

and

$$Z_{port\ 1}(s) = \frac{s}{40}$$

Therefore

$$Z_{in}(s) = \frac{1000}{s} + \frac{s}{40} = \frac{s^2+200^2}{40s}$$

Hence

$$Z_{in}(j\omega) = \frac{4\times 10^4 - \omega^2}{j40\omega} = -j\frac{4\times 10^4 - \omega^2}{40\omega}$$

The imaginary part is zero when $\omega = 200$ rad/s. Hence, the resonant frequency is 200 rad/s.

SOLUTION PROBLEM 19.23. (a) By writing nodal equations for the boxed 2-port, we have by inspection (note passive circuit in which $y_{21} = y_{12}$):

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3s+2 & -2s-2 \\ -2s-2 & 3s+3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(b) In Ohms, $Z_L = s+1$; $Z_S = 2$. Therefore,

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = 3s+2 - \frac{(2s+2)^2}{3s+3+s+1} = \frac{2s^2+3s+1}{s+1} = 2s+1$$

$$\frac{V_1}{V_s} = \frac{Z_{in}}{Z_{in} + Z_s} = \frac{1}{1 + Z_s Y_{in}} = \frac{1}{1 + 2\times(2s+1)} = \frac{1}{4s+3}$$

and

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + y_L} = \frac{2s+2}{3s+3+s+1} = 0.5$$

Thus,

$$\frac{V_2}{V_s} = \frac{V_1}{V_s}\times\frac{V_2}{V_1} = \frac{1}{4s+3}\times 0.5 = \frac{1}{8s^2+14s+6} = \frac{1}{8s+6}$$

(c) The impulse reponse is

$$h(t) = L^{-1}\{H(s)\} = L^{-1}\left\{\frac{1}{8s+6}\right\} = L^{-1}\left\{\frac{0.125}{s+0.75}\right\} = 0.125e^{-0.75t}u(t)$$

For the step response,

$$v_2(t) = L^{-1}\left\{\frac{H(s)}{s}\right\} = L^{-1}\left\{\frac{0.125}{s(s+0.75)}\right\} = \frac{1}{6}\left(1 - e^{-0.75t}\right)u(t) \text{ V}$$

(d) We must compute the complete Laplace transform and invert. Here

$$V_s(s) = 12.75\times\frac{2}{s^2+4}$$

and

$$V_2(s) = H(s)V_2(s) = \frac{0.125}{s+0.75}\times 12.75\times\frac{2}{s^2+4} = \frac{3.1875}{(s+0.75)(s^2+4)}$$

We use MATLAB to compute the partial fraction expansion

```
n=3.1875;
d= conv([1 0.75], [ 1 0 4]);
[r p k ] = residue (n,d)
r =
  -0.3493 - 0.1310i
  -0.3493 + 0.1310i
   0.6986
p =
  -0.0000 + 2.0000i
```

-0.0000 - 2.0000i
-0.7500

Hence, after combining the two complex terms, we obtain

$$V_2(s) = \frac{0.6986}{s + 0.75} + \frac{-0.6986s + 0.524}{s^2 + 4}$$

From Table 13.1, the steady state response is

$$v_{2,ss}(t) = [-0.6986\cos(2t) + 0.262\sin(2t)]u(t) \quad V$$

and the transient response is

$$v_{2,tran}(t) = 0.6986e^{-0.75t}u(t) \text{ V}$$

SOLUTION PROBLEM 19.24. (a) Using the y-parameters of stage 2,

$$Y_{in2} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = 500 + \frac{0.1\times 75000}{185 + 4000} = 501.8 \quad \mu S$$

The load for stage 1 is the parallel combination of Z_{in2} and the 2 kΩ resistor. Hence, using the y-parameters of stage 1, we obtain

$$Y_{in1} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L} = 2000 + \frac{0.5\times 24\times 10^4}{100 + 501.8 + 500} = 2108.9 \quad \mu S$$

(b) We compute the following voltage gains:

$$\frac{V_1}{V_s} = \frac{Z_{in1}}{Z_{in1} + Z_s} = \frac{1}{1 + Z_s Y_{in1}} = \frac{1}{1 + 25\times 2108.9\times 10^{-6}} = 0.9499$$

$$\left(\frac{V_2}{V_1}\right)_{stage1} = \frac{-y_{21}}{y_{22} + Y_{L,stage1}} = \frac{-0.24}{(100 + 500 + 501.8)\times 10^{-6}} = -217.8$$

and

$$\left(\frac{V_2}{V_1}\right)_{stage2} = \frac{-y_{21}}{y_{22} + Y_{L,stage2}} = \frac{-0.075}{(185 + 4000)\times 10^{-6}} = -17.92$$

Finally G_V is the product of the three gains calculated above

$$G_v = 0.9499\times(-217.8)\times(-17.92) = 3708.2$$

SOLUTION PROBLEM 19.25. (a) With the switch in position A, the load to the 2-port is Y_L = Cs = 0.25s. Hence,

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + Y_L} = \frac{-y_{21}}{y_{22} + 0.25s}$$

(b) From the given data, $V_1(s) = -2/s$ and hence,

$$V_2(s) = \frac{-y_{21}}{y_{22} + 0.25s} V_1(s) = \frac{-1/s}{-1/s + 0.25s} \times \frac{-2}{s} = \frac{8}{s(s^2 - 4)} = \frac{-2}{s} + \frac{1}{s+2} + \frac{1}{s-2}$$

Hence, for t ≥ 0,

$$v_2(t) = (-2 + e^{-2t} + e^{2t})\, u(t) \ \text{V}$$

(c) The circuit is not stable in the time interval 0 to 1 s, because the transfer function has a pole in the right half plane.

(d) $v_2(1^-) = -2 + e^{-2} + e^{2} = 5.524$ V

(e) Replace the charged capacitor by the parallel combination of an admittance of 0.25s and a current source of value 0.25×8.524 (in accordance with figure 14.16

(f) $Z_1(s) = b^2 \times 1 = 4\ \Omega$.

(g) For t ≥ 1s, the capacitor is discharging through a 4-Ω equivalent resistance, with a time constant 0.25×4 = 1 s, and an initial voltage $v_2(1^-)$ = 5.542 V.
Hence

$$v_2(t) = 5.524e^{-(t-1)}u(t-1)\text{V}$$

and by the ideal transformer voltage ratio property,

$$v_3(t) = 2.762e^{-(t-1)}u(t-1)\text{V}$$

SOLUTION PROBLEM 19.26. (a) By definition of coupled inductors

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_1 s & Ms \\ Ms & L_2 s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Hence, the z-parameters are:

$$\begin{bmatrix} L_1 s & Ms \\ Ms & L_2 s \end{bmatrix}$$

(b) By definition of coupled inductors

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_1 s & -Ms \\ -Ms & L_2 s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Hence, the y-parameters are:

$$\begin{bmatrix} L_1 s & -Ms \\ -Ms & L_2 s \end{bmatrix}$$

The z-parameters exist independent of the values of M, L_1, and L_2.

SOLUTION PROBLEM 19.27. (a) By definition and the properties of the ideal transformer $z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = R_1 + R_2$ and $z_{22} = \left.\dfrac{V_2}{I_2}\right|_{I_1=0} = a^2 R_2$. Additionally, since the circuit is obviously reciprocal, $z_{21}(= z_{12}) = \left.\dfrac{V_2}{I_1}\right|_{I_2=0} = aR_2$.

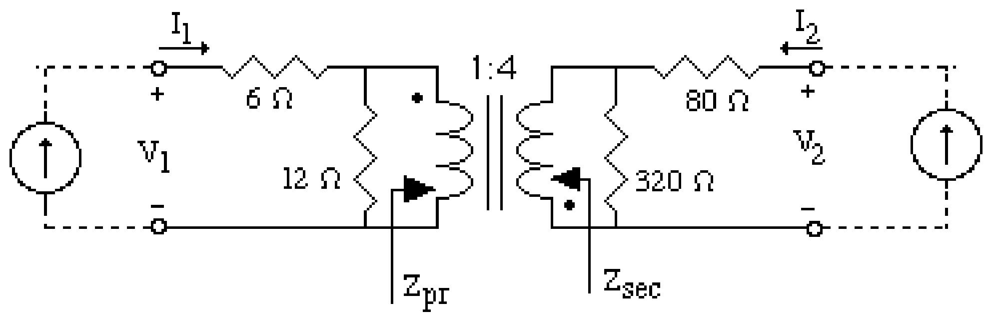
(b) The input impedance is given by the formula

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} = R_1 + R_2 - \frac{a^2 R_2^2}{a^2 R_2 + R_2} = R_1 + \frac{1}{a^2 + 1} R_2$$

(c) If port-1 is open circuited, $I_1 = 0$. Hence,

$$V_1 = z_{12} I_2 = \frac{2aKR_2}{s^2 + 4} \quad \text{and} \quad V_2 = z_{22} I_2 = \frac{2a^2 KR_2}{s^2 + 4}$$

SOLUTION PROBLEM 19.28. (a) For this part consider the figure below:



With port 2 open, it follows that Z_{pr} = 320/16 = 20 Ω and

$$z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2=0} = 6 + \frac{20 \times 12}{20 + 12} = 13.5\ \Omega$$

With port 1 open and I_2 injected into port 2, we have Z_{sec} = 12*16 = 192 Ω and

$$z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1=0} = \frac{V_{pr}}{I_2}\Bigg|_{I_1=0} = \frac{-V_{\sec}}{4I_2}\Bigg|_{I_1=0} = \frac{-I_2(320\,//\,192)}{4I_2}\Bigg|_{I_1=0} = -30\ \Omega$$

With port 2 open and I_1 injected into port 1, we have Z_{pr} = 20 Ω and

$$z_{21} = \frac{V_2}{I_1}\Bigg|_{I_2=0} = \frac{V_{\sec}}{I_1}\Bigg|_{I_2=0} = \frac{-4V_{pr}}{I_1}\Bigg|_{I_2=0} = \frac{-4I_1(12\,//\,20)}{I_1}\Bigg|_{I_2=0} = -30\ \Omega$$

With port 1 open, it follows that Z_{sec} = 12*16 = 192 Ω and

$$z_{22} = \frac{V_2}{I_2}\Bigg|_{I_1=0} = 80 + \frac{320 \times 192}{320 + 192} = 200\ \Omega$$

(b) $V_1(s) = z_{12}I_2(s) = \dfrac{-300}{s^2 + 4}$ and $V_2(s) = z_{22}I_2(s) = \dfrac{2000}{s^2 + 4}$.

SOLUTION PROBLEM 19.29. (a) Writing two mesh equations we have by inspection,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 + r_m & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(b) $V_1 = z_{11} I_1 = \dfrac{K\left(Z_1 + Z_3\right)}{s}$ and $V_2 = z_{21} I_1 = \dfrac{K\left(Z_3 + r_m\right)}{s}$

SOLUTION PROBLEM 19.30. From the result of problem 29,

$$\begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 + r_m & Z_2 + Z_3 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

Therefore, $Z_3 = z_{12}$, $Z_1 = z_{11} - Z_3 = z_{11} - z_{12}$, $Z_2 = z_{22} - Z_3 = z_{22} - z_{12}$, and $r_m = z_{21} - Z_3 = z_{21} - z_{12}$.

SOLUTION PROBLEM 19.31. (a) The z-parameters can be computed by inspection (first write the z-parameters of the passive part of the network, i.e., with the dependent source ignored; then add the effect of the dependent source to the resulting equations.) As such, using loop equations,

$$Z = \begin{bmatrix} 5 + \dfrac{10}{s} & 10 + \dfrac{10}{s} \\ \dfrac{10}{s} & 10 + \dfrac{10}{s} \end{bmatrix} = \frac{10}{s} \begin{bmatrix} 0.5s + 1 & s + 1 \\ 1 & s + 1 \end{bmatrix} \Omega$$

(b) $$Z_{in} = \frac{5s+10}{s} - \frac{\dfrac{10}{s} \times \dfrac{10}{s}(s+1)}{\left(\dfrac{10(s+1)}{s} + 10\right)} = \frac{5s+10}{s} - \frac{10(s+1)}{s(2s+1)} = \frac{(5s+10)(2s+1) - 10(s+1)}{2s(s+0.5)}$$

$$= \frac{5s^2 + 7.5s}{s(s+0.5)} = \frac{5s + 7.5}{s + 0.5}$$

(c) $I_1(s) = \dfrac{V_1(s)}{Z_{in}(s)} = \dfrac{10}{s} \times \dfrac{s+0.5}{5s+7.5} = \dfrac{0.6667}{s} + \dfrac{1.3334}{s+1.5}$. Hence

$$i_1(t) = \left(\frac{2}{3} + \frac{4}{3}e^{-1.5t}\right)u(t) \text{ A}$$

SOLUTION PROBLEM 19.32. **(a)** To find the resonant frequency we first find

$$Z_{in}(s) = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} = z_{11} - \frac{z_{11}z_{11}}{z_{11} + 10} = \frac{10z_{11}}{z_{11} + 10} = \frac{10}{1 + \dfrac{10}{z_{11}}} = \frac{10}{1 + \dfrac{s^2 + 25}{s}} = \frac{10s}{s^2 + s + 25}$$

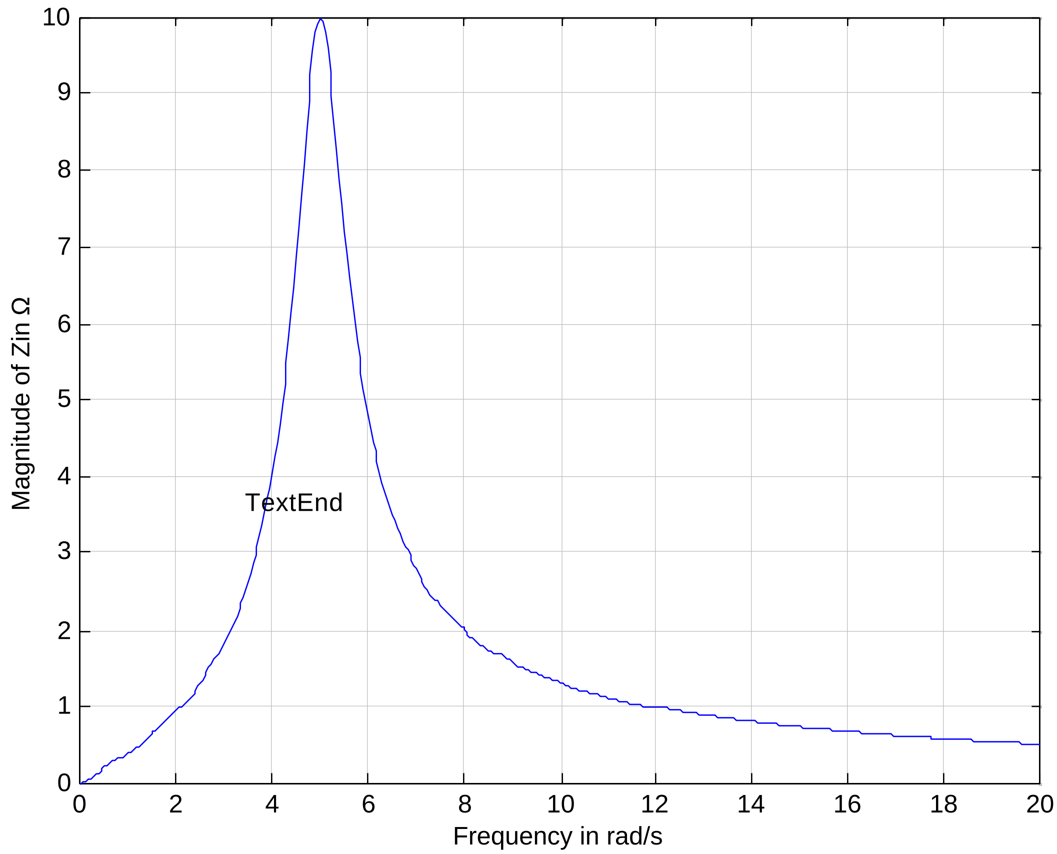
This is of the form of equation 17.18 with K = 10. Here according to equation 17.19f,

$$\omega_r = \omega_p = \sqrt{25} = 5 \text{ rad/s}$$

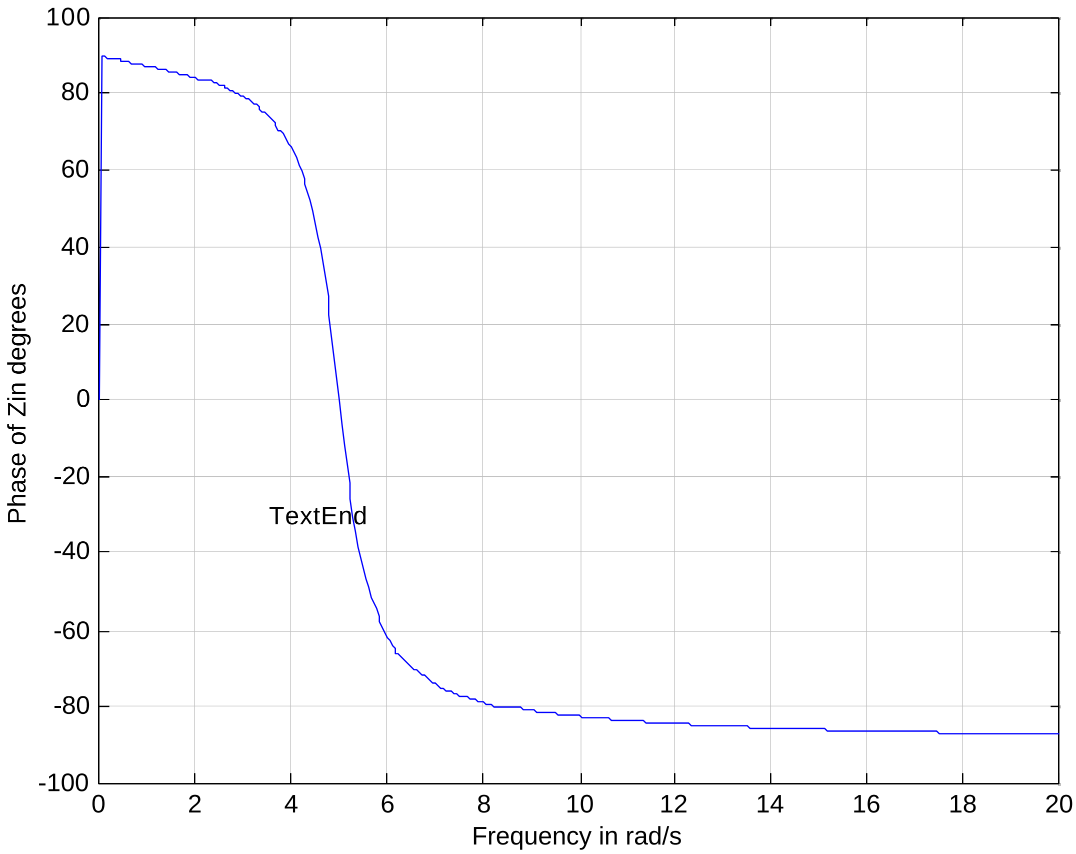
(b) To find Q we use equation 17.19e, i.e.,

$$Q = Q_p = \frac{\omega_p}{2\sigma_p} = \frac{5}{1} = 5$$

(c) Using MATLAB we obtain the frequency response plots below:



Magnitude of Zin Ω
Frequency in rad/s
TextEnd
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
20



SOLUTION PROBLEM 19.33. (a) Since $Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 30 & 4 \end{bmatrix}\ \Omega$. Assuming that Z_{in} does not include Z_s, it follows that

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} = 2 + \frac{90}{24} = 5.75\ \Omega$$

Assuming that Zout does not include the parallel connection of Z_L, then

$$Z_{out} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 4 + \frac{90}{3} = 34\ \Omega$$

(b) From equation 19.27

$$G_V = G_{v2}G_{v1} = \left(\frac{Z_L}{z_{22} + Z_L}\right)\left(\frac{z_{21}}{Z_{in} + Z_S}\right).$$

Thus in MATLAB
```
»Gv = (20/(4 + 20))*(30/(5.75+1))
Gv =   3.7037e+00
»%  Therefore
»v2 = Gv*30
v2 =   1.1111e+02
```

Hence, $v_2(t) = 111.11u(t)$ V. The power absorbed by Z_L is therefore (in watts):

```
»PZL = v2^2/20
PZL =  6.1728e+02
```

SOLUTION PROBLEM 19.34. (a) Before determining b, it is necessary to compute the impedance seen at the secondary of the 2-port. Here,

$$Z_{out} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 33\alpha^2 R_0 - \frac{2\alpha^3 R_0^2}{\alpha R_0 + \alpha R_0} = 33\alpha^2 R_0 - \alpha^2 R_0 = 32\alpha^2 R_0$$

Thus b must be chosen so that the impedance reflected to the secondary of the transformer is 2 Ω, i.e.,

$$2 = \frac{32\alpha^2 R_0}{b^2} \qquad b = 4\alpha\sqrt{R_0}$$

(b) First do a source transformation on the front end of the two port to obtain

$$v_s(t) = \alpha R_0\sqrt{2}\cos(2t) = 32\sqrt{2}\cos(2t) \text{ V}$$

Therefore, $V_{s,eff} = 32$ V. Also note that the impedance looking into the primary of the two port is

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} = \alpha R_0 - \frac{2\alpha^3 R_0^2}{33\alpha^2 R_0 + 32\alpha^2 R_0} = 31.015 \ \Omega$$

Hence

$$G_V = G_{v2}G_{v1} = \left(\frac{Z_L}{z_{22} + Z_L}\right)\left(\frac{z_{21}}{Z_{in} + Z_S}\right) = 0.5$$

Consider the following MATLAB calculations:

```
»a=2;  R0 = 16;
»Zin = a*R0-2*a^3 * R0^2/(65*a^2 *R0)
```

```
Zin =   3.1015e+01
»ZL = 32*a^2 * R0
ZL =        2048
»Zs = a*R0
Zs =    32
»z22 = 33*a^2*R0
z22 =        2112
»z21 = 2*a*R0
z21 =   64
»Gv = (ZL/(z22+ZL))*(z21/(Zin+Zs))
Gv =   5.0000e-01
»V1eff = 32;
»V2eff = Gv*V1eff
V2eff =    16
»b = 4*a*sqrt(R0)
b =    32
»VLoadeff=V2eff/b
VLoadeff =   5.0000e-01
»Pmax = VLoadeff^2/2
Pmax =   1.2500e-01
```

Hence, max power transferred to the load is 125 mW.

SOLUTION PROBLEM 19.35. (a) Using the usual formula

$$Z(s) = z_{11} - \frac{z_{12} z_{21}}{z_{22} + Z_L} = 0 - \frac{1000 \times (-1000)}{0 + 10^8 / s} = 0.01s$$

Yes, the input to the 2-port looks like a 0.01 H inductor.

(b) Under the given conditions, the circuit reduces to parallel RLC with R_{eq} = 50 kΩ, L = 0.01 H, and C = 100 pF. Therefore, $\omega_m = \frac{1}{\sqrt{LC}} = 10^6$ rad/s and $B_\omega = \frac{1}{R_{eq} C} = 2 \times 10^5$ rad/s.

(c) For this circuit

$$V_1(s) = \frac{1}{\frac{1}{R_{eq}} + \frac{1}{Ls} + Cs} \times \frac{V_{in}}{R_1} = \frac{s/C}{s^2 + \frac{1}{R_{eq}C}s + \frac{1}{LC}} \times \frac{V_{in}}{R_1} = \frac{10^6 s}{s^2 + 2\times 10^6 s + 10^{12}} V_{in}$$

For the impulse response, V_{in} = 1 and from MATLAB

```
»n = [1e6 0];d = [1  2e6 1e12];
»[r,p,k] = residue(n,d)
r =
   1.0000e+06
  -1.0000e+12
p =
    -1000000
    -1000000
k =
     []
```

Therefore, the impulse response is:

$$h(t) = \left(10^6 - 10^{12} t\right) e^{-10^6 t} u(t) \text{ V}$$

(d) $y = \begin{bmatrix} 0 & 1000 \\ -1000 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & -0.001 \\ 0.001 & 0 \end{bmatrix} \text{ S}$

(e) $V_2(s) = \frac{-y_{21}}{y_{22} + Y_L} V_1(s) = \frac{-0.001}{0 + 10^{-8} s} \times \frac{10^6 s}{s^2 + 2\times 10^6 s + 10^{12}} = \frac{-10^{11}}{(s + 10^6)^2}$

Hence

$$v_2(t) = -10^{11} t e^{-10^6 t} u(t) \text{ V.}$$

SOLUTION PROBLEM 19.36.

(a) $Z_{in2} = z_{11} - \dfrac{z_{12}z_{21}}{z_{22} + Z_L} = 62.582 - \dfrac{1.2075 \times 63.75}{1.25 + 0.016} = 1.7778 \text{ k}\Omega$

Because z_{12} for stage 1 is zero,

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_{in2}//2} = z_{11} = 2 \text{ k}\Omega$$

(b) $G_{v1} = \dfrac{V_1}{V_s} = \dfrac{Z_{in}}{Z_{in} + 75} = 0.96386$. Let $Z_{L1} = Z_{in2}//2 = 0.94118 \text{ k}\Omega$. Then $G_{v2} = \dfrac{V_2}{V_1} = \dfrac{Z_{L1}}{Z_{L1} + z_{22}} \times \dfrac{z_{21}}{Z_{in}} = -22.472$. Finally $G_{v3} = \dfrac{V_{out}}{V_2} = \dfrac{Z_L}{Z_L + z_{22}} \times \dfrac{z_{21}}{Z_{in2}} = 0.45319$. Thus

$$G_v = G_{v1}G_{v2}G_{v3} = -9.816$$

SOLUTION PROBLEM 19.37. (a) By inspection via mesh standard equations

$$z = \begin{bmatrix} L_1 s & Ms \\ Ms & L_2 s \end{bmatrix}$$

(b) Utilizing the properties of an ideal transformer,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = L_1 s \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{k^2 L_2}{L_1} L_1 s + (1 - k^2)L_2 s = L_2 s$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{k\sqrt{L_2}}{\sqrt{L_1}} L_1 s = k\sqrt{L_1 L_2}\, s = Ms$$

Finally,

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{k\sqrt{L_2}}{\sqrt{L_1}} L_1 s = k\sqrt{L_1 L_2}\, s = Ms$$

(c) Utilizing the properties of an ideal transformer,

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = L_2 s \qquad z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{k^2 L_1}{L_2} L_2 s + (1-k^2)L_1 s = L_1 s$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{k\sqrt{L_1}}{\sqrt{L_2}} L_2 s = k\sqrt{L_1 L_2}\, s = Ms$$

Finally,

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{k\sqrt{L_1}}{\sqrt{L_2}} L_2 s = k\sqrt{L_1 L_2}\, s = Ms$$

(d) For this circuit k = 1 and the turns ratio $a = \frac{\sqrt{10^{-2}}}{1} = 0.1$. Under this condition the given circuit reduces to a current source of value $I_s = V_{in}/5000$ driving a parallel RLC with R = 5000 Ω, L = L_1 = 0.01 H, the capacitance reflected to the primary of value $C = 10^{-8}$ F.

Therefore

$$\omega_m = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s and } B_\omega = \frac{1}{R_{eq}C} = 2\times 10^4 \text{ rad/s.}$$

Finally, at w = wm, the circuit is resonant and Vin appears across the primary of the transformer. This voltage is then stepped up by a factor of 10. Therefore $\left|\frac{V_{out}}{V_{in}}\right|_{\max} = 10$.

SOLUTION PROBLEM 19.38. (a) By inspection

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_1 & 1 \\ -1 & Y_2 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(b) By inspection

$$V_1 = V_2 - Z_1 I_2 \qquad V_1 + Z_1 I_2 = V_2$$

and

$$I_1 = Y_2 V_1 - I_2 \qquad Y_2 V_1 - I_2 = I_1$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ Y_2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Thus

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_1 Y_2 + 1} \begin{bmatrix} Z_1 & 1 \\ -1 & Y_2 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

SOLUTION PROBLEM 19.39. (a) From problem 19.38b we have

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_1 Y_2 + 1} \begin{bmatrix} Z_1 & 1 \\ -1 & Y_2 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \frac{1}{\frac{1}{RCs} + 1} \begin{bmatrix} \frac{1}{Cs} & 1 \\ -1 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \frac{1}{RCs + 1} \begin{bmatrix} R & RCs \\ -RCs & Cs \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(b) This part is a cascade of part (a) and an ideal transformer. Label the voltage and current at the port 2 of N_1 as $\hat{V}_2$ and $\hat{I}_2$. From the properties of the ideal transformer, $V_2 = n\hat{V}_2$ and $I_2 = \hat{I}_2 / n$. Hence

$$\begin{bmatrix} V_1 \\ \hat{I}_2 \end{bmatrix} = \frac{1}{RCs + 1} \begin{bmatrix} R & RCs \\ -RCs & Cs \end{bmatrix} \begin{bmatrix} I_1 \\ \hat{V}_2 \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ nI_2 \end{bmatrix} = \frac{1}{RCs + 1} \begin{bmatrix} R & RCs \\ -RCs & Cs \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 / n \end{bmatrix}$$

Therefore

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \frac{1}{RCs + 1} \begin{bmatrix} R & RCs/n \\ -RCs/n & Cs/n^2 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From table 19.1, if $h_{22} = \frac{Cs}{n^2(RCs+1)} \neq 0$, then the z-parameters exist and if $h_{11} = \frac{R}{RCs+1} \neq 0$, the y-parameters exist, i.e., if $C \neq 0$ and $R \neq 0$ respectively.