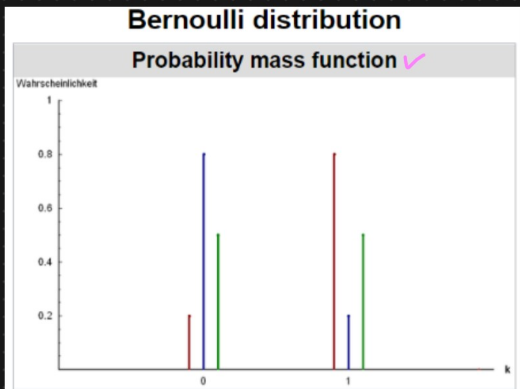
Bernoulli Distribution

Definition: The Bernoulli distribution is the simplest discrete probability distribution. It represents the probability distribution of a random variable that has exactly two possible outcomes: success (**with probability p**) and failure (**with probability 1−p**). It is used to model binary outcomes, such as a coin flip or a yes/no question.



Three examples of Bernoulli distribution:

→ $P(x=0) = 0.2$ and $P(x=1) = 0.8$ (1-p)
→ $P(x=0) = 0.8$ and $P(x=1) = 0.2$ ⇒ p
→ $P(x=0) = 0.5$ and $P(x=1) = 0.5$

① Discrete Random Variable (pmf)

② Outcomes are Binary

Eg: ① Tossing a coin {H, T}

$Pr(X=H) = 0.5 = p$

$Pr(X=T) = 1-0.5 = 0.5 = q.$

p, q

$q = (1-p)$

② Whether the person will Pass/Fail

$Pr(X = Pass) = 0.4$

$Pr(X = Fail) = 1-0.4 = 0.6.$

Parameters

$0 \le p \le 1$

$q = 1-p$

$K = \{0, 1\}$ ⇒ 2 outcomes.

Pr(Success) ⇒ $K=1$

Pr(Fail) ⇒ $K=0$

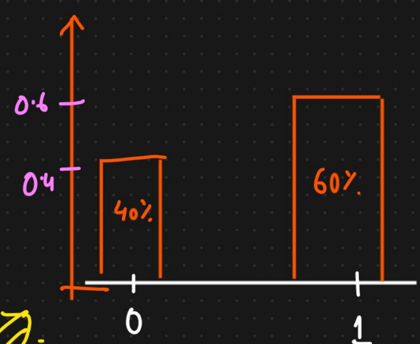
PMF

: Company has launched a new Smartphone 'A'

K

(1) Use = 60% ⇒ p

(0) Not use = 40% ⇒ $q = 1-p$



⇐ Distribution

⇗.

$$PMF = p^{K} * (1-p)^{1-K}$$

if $K=1$

$$Pr(K=1) = p^{1}(1-p)^{1-1} \Rightarrow p$$

$$Pr(K=0) = p^{0}(1-p)^{1} \Rightarrow (1-p) = q$$

Simplified

$$\text{pmf} \begin{cases} q = 1-p & \text{if } K=0 \\ p & \text{if } K=1 \end{cases}$$

Mean of Bernoulli Distribution

$$E(X) = \sum_{k=0}^{1} K \cdot p(K) \qquad K = \{0, 1\}.$$

$= 0 \times 0.40 + 1 * 0.60$

$= 0 + 0.60$

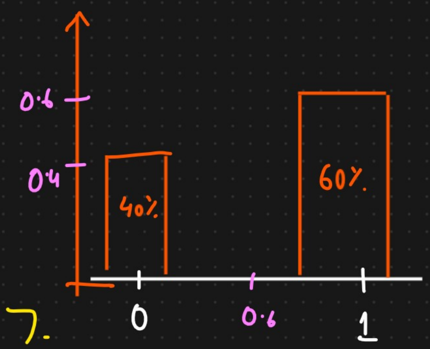
$= 0.60 \Rightarrow p$

$p = 0.6$

$q = 0.4$

$Pr(1) = 0.6)$

$Pr(0) = 0.4$



Median of Bernoulli Distribution

$$\text{Median} \begin{cases} 0 & \text{if } p < 1/2. \\ [0, 1] & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$$

$$\begin{Bmatrix} \text{median} = 0 \text{ if } q > p \\ \text{median} = 0.5 \text{ if } q = p \\ \text{median} = 1 \text{ if } q < p. \end{Bmatrix}$$

Mode

$p > q \Rightarrow p$ will be the mode

else q will be the mode.

Variance

$K = 0$ and 1 $\quad Pr(k=0) = 0.4 \Rightarrow q \quad Pr(K=1) = 0.6 \Rightarrow p$

$$\sigma^2 = 0.40 * (0-0.6)^2 + 0.6(1-0.6)^2$$

$$= 0.40 * 0.36 + 0.6(0.16)$$

$$\sigma^2 = 0.24 \Rightarrow Pr(k=0) * Pr(K=1)$$

$$q * p$$

$$\sigma^2 = pq.$$

$$\sigma = \sqrt{pq}$$