# Simple Linear Regression

Supervised ML ⟶ Regression

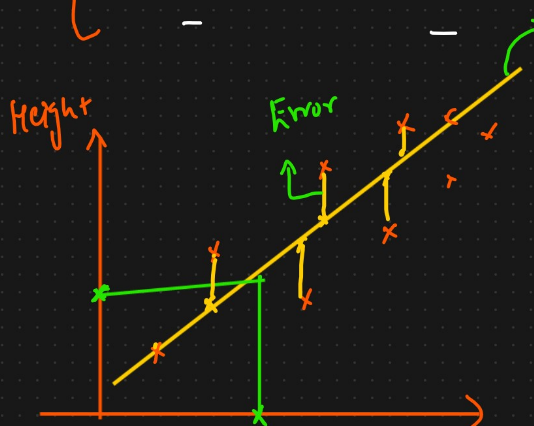
TRAIN ↓

New weight ⟶ $\boxed{\text{Model}}$ ⟶ Height

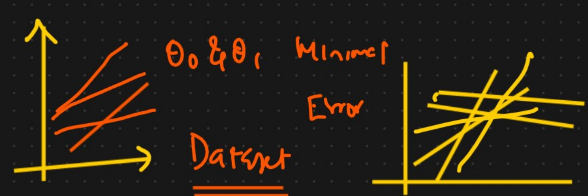
↑ Simple Linear Regression

**Dataset** I/P features

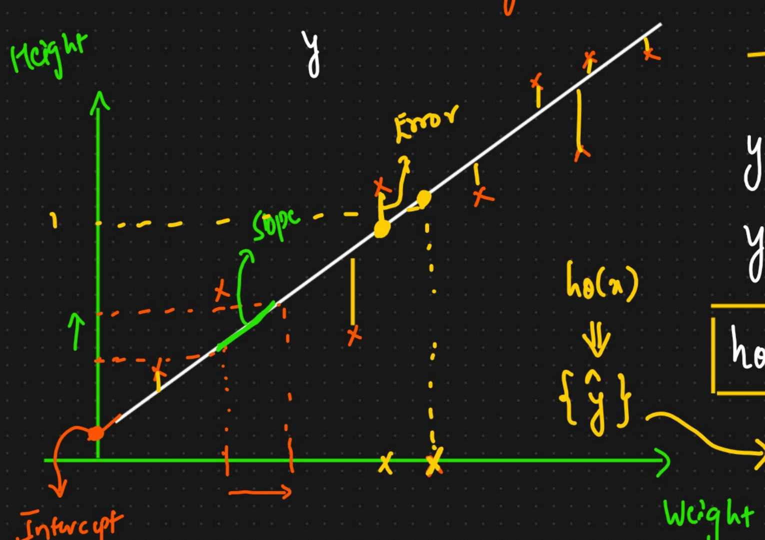
| ↑ x Weight | y Height ↑ O/P or dependent feature |
|---|---|
| 74 | 170cm |
| 80 | 180cm |
| 75 | 175.5cm |
| – | – |




Minimal




Weight




$h_\theta(x)$

⇓

$\{\hat{y}\}$ ⟶ predicted point

$y = mx + c$

$y = \beta_0 + \beta_1 x$

$$\boxed{h_\theta(x) = \theta_0 + \theta_1 x}$$

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$
$\quad - - - + \theta_n x_n$

$h_\theta(x) = \theta_0 + \theta_1 x$

if $x = 0$

$h_\theta(x) = \theta_0$

Error $(y - \hat{y})$

$\theta_0$ = Intercept

$\theta_1$ = Slope or Coefficient

## Cost function

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\Big(\underbrace{h_\theta(x)^{(i)}}_{\text{predicted}} - \underbrace{y^{(i)}}_{\text{True O/P}}\Big)^2 \Rightarrow \text{Mean Squared Error}$$

↳ Error

Final Aim what we need to solve

Minimize $\quad J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x)^{i} - y^{(i)}\right)^2 \downarrow\downarrow\downarrow$

$\theta_0, \theta_1$

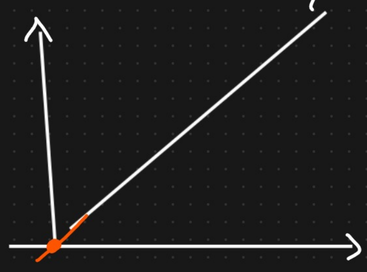
DATASET

| x | y |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

① $h_\theta(x) = \theta_0 + \theta_1 x$

$\boxed{\theta_0 = 0}$

$\boxed{h_\theta(x) = \theta_1 x}$




$\theta_0 = 0$

$h_\theta(x) = \theta_1 x$

let $\theta_1 = 1$ {slope}

$h_\theta(x) = 1 \quad x = 1$

$h_\theta(x) = 2 \quad x = 2$

$h_\theta(x) = 3 \quad x = 3$

$h_\theta(x) = \theta_1 x$

↓

let $\theta_1 = 0.5$

$h_\theta(x) = 0.5 \quad$ if $x = 1$

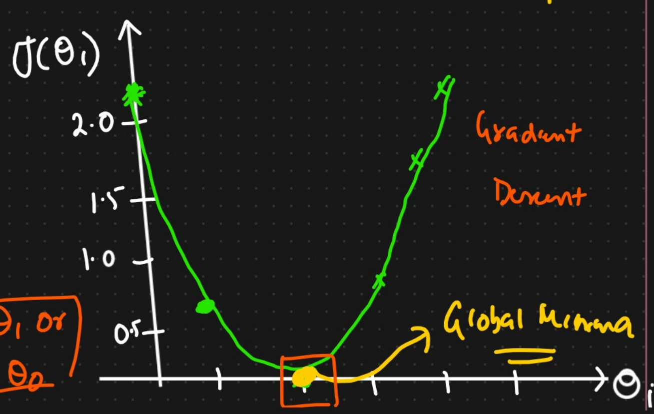
$h_\theta(x) = 1 \quad$ if $x = 2$

$h_\theta(x) = 1.5 \quad$ if $x = 3$


$\boxed{\theta_1 = 1}$

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x)^{(i)} - y^{(i)}\right)^2$$

$$= \frac{1}{2\times 3}\left[(1-1)^2 + (2-2)^2 + (3-3)^2\right]$$




$\boxed{\theta_1 \text{ or } \theta_0}$

$J(\theta_1) = 0 \leftarrow$

$\theta_1 = 0.5$

Error has been minimized

0.5 1 1.5 2.0 2.5

$$J(\theta_1) = \frac{1}{2\times3}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$$

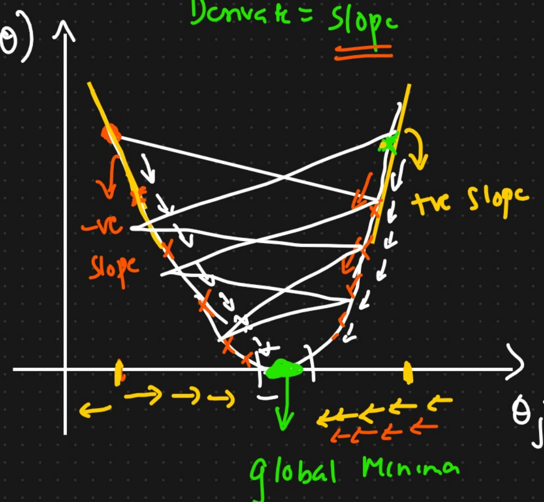
$$J(\theta_1) \approx 0.58$$

if $\theta_1 = 0$

$$J(\theta_1) = \frac{1}{2\times3}\left[(0-1)^2 + (0-2)^2 + (0-3)^2\right]$$

$$J(\theta_1) \approx 2.3$$

Convergence Algorithm {Optimize the changes of $\theta_1$ value}




Repeat until Convergence

$\theta_1$ value much more efficiently

$$\left\{ \theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial\theta_j} J(\theta_j)} \right\} \rightarrow -ve$$

$$\theta_j := \theta_j - \alpha(+ve)$$

$$= \theta_j - (+ve)$$

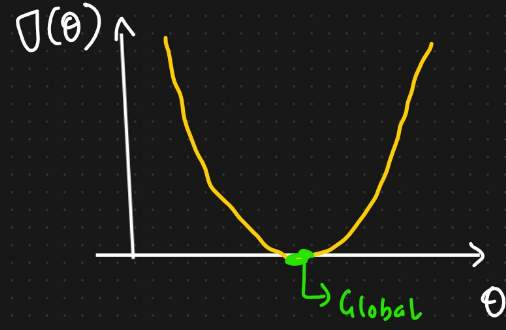
$$\left\{ \begin{array}{l} \theta_j = \theta_j - \boxed{\alpha}(-ve) \\ \theta_j = \theta_j + (+ve) \end{array} \right\}$$

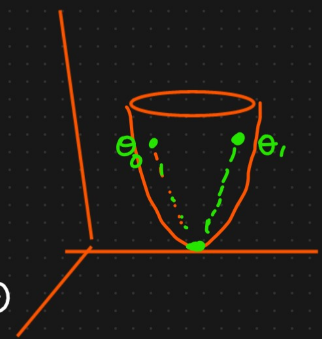
$$\boxed{\alpha = 0.001} \leftarrow$$

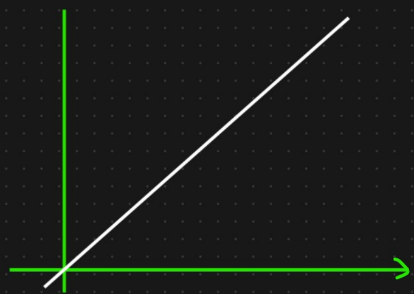
$\alpha$ = Learning Rate

# Final Conclusion

## GRADIENT DESCENT



$\theta_0 = 0$

$\theta_1 =$ Changeable

## Convergence Algorithm

repeat until convergence

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x)^{(i)} - y^{(i)}\right)^2$$

$\{$

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)}$$

$\}$

$j = 0$ and $1$

$$\frac{\partial}{\partial x} (x)^2 = 2x$$

$$\frac{\partial}{\partial x} x^n = n x^{n-1}$$

$$\rightarrow \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x)^{(i)} - y^{(i)}\right)^2$$

if

$$h_\theta(x) = \theta_0 + \boxed{\theta_1 x} \longrightarrow 0$$

$$j = 0 \Rightarrow \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_0} \frac{1}{2m} \left[\sum_{i=1}^{m} \left(h_\theta(x)^{(i)} - y^{(i)}\right)^2\right]$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x)^{(i)} - y^{(i)}\right) \times \underline{1}$$

$$j=1 \Rightarrow \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_1} \frac{1}{2m} \left[ \sum_{i=1}^{m} \left( (\theta_0 + \theta_1 x) - y^{(i)} \right)^2 \right]$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left( (\theta_0 + \theta_1 x) - y^{(i)} \right) (x)$$

$$\boxed{\frac{\partial}{\partial \theta_1} \left[ \theta_0 + \theta_1 x \right] \Rightarrow \underline{\underline{x}}}$$

Repeat until Convergence

$$\{$$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( h_\theta(x)^{(i)} - y^{(i)} \right)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left( h_\theta(x)^{(i)} - y^{(i)} \right) x^{(i)}$$

$$\}$$