Matrices

A matrix is a rectangular array of numbers, symbols or expressions arranged in rows and columns

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$\Rightarrow$ has a_{ij} where i denotes the row and j denotes the columns

Example of Matrices in Data Science

(1) Data Representation

Dataset

Math Score	Physic Score	Biology Score
[55	65	75]
[65	60	55]
[70	45	80]

$$\begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} f_1 & f_2 & f_3 \end{array}$$

$$\begin{bmatrix} 55 & 65 & 75 \\ 65 & 60 & 55 \\ 70 & 45 & 80 \end{bmatrix}$$

$\boxed{3 \times 3}$

(2) Images in Computer Vision

0 (black) $\longleftrightarrow$ 255 (white)

3x3 grayscale Image

Image =

0	128	255
255	128	0
128	255	128

$$\Rightarrow \begin{bmatrix} 0 & 128 & 255 \\ 255 & 128 & 0 \\ 128 & 255 & 128 \end{bmatrix}_{3 \times 3}$$

(3) Confusion Matrix : Accuracy of the Model

$f_1 \rightarrow$ Model $\rightarrow$ o/p $\rightarrow \hat{y} \Rightarrow$ predicted output.

$f_2 \rightarrow$

→ y ⇒ actual o/p

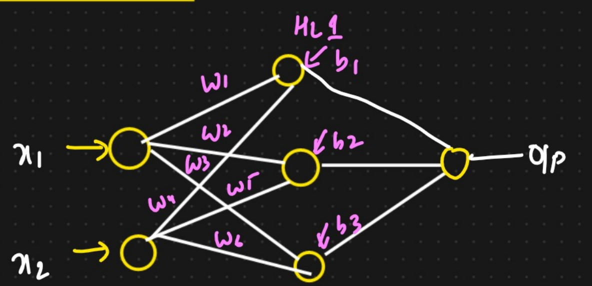
Confusion Matrix = $\begin{bmatrix} 50 & 10 \\ 5 & 35 \end{bmatrix}_{2\times 2}$ ⇒

50 → True positive
10 → false negatives
5 → false positive
35 → True negative

$$\frac{TP + TN}{TP + FN + FP + TN} = \text{Accuracy}.$$

④ Neural N/W : Matrix Operation

[Linear Regression]



2×3

Forward propogation

$z = w^T x + b$

x_1	x_2
→ —	—
—	—
—	—
—	—

$W = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \end{bmatrix}$

×

Matrix Multiplication ⇐ $x = [x_1 \ \ x_2]$

+

$[b_1, b_2, b_3]$

o/p feature ⇓

x_1 ⇓ No. of study hours	x_2 ⇓ IQ	Score	dependent feature
4	100	90	
5	90	85	
—	—	-	

Regression

$y = mx + c$

$y = m_1 x_1 + m_2 x_2 + c$

↳ slope ↳ slope or Coefficient

$\Rightarrow m^T x + c$

$\left\{ \begin{array}{l} m = [m_1, m_2] \\ x = [x_1, x_2] \end{array} \right\}$

5) NLP :

Dataset

Review	Positive / Negative
→ The food is bad	0
→ The food is Good	1

$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Matrices Operations

⇒ To manipulate and analyze multidimension data efficiently.

1) Matrix Addition And Substraction
2) Scalar Matrix Multiplication.
3) Matrix Multiplication

1) Matrix Addition And Substraction

Add or Substract corresponding elements of 2 matrices of the same dimension

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3} \Rightarrow \text{STORE A} \qquad B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{bmatrix}_{3\times 3} \Rightarrow \text{STORE B}$$

DATASET

	Prod A	Prod B	Prod C
Day 1	1	2	3
Day 2	4	5	6
Day 3	7	8	9

$$A + B = \begin{bmatrix} 1+4 & 2+5 & 3+6 \\ 4+7 & 5+8 & 6+9 \\ 7+1 & 8+2 & 9+3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 7 & 9 \\ 11 & 13 & 15 \\ 8 & 10 & 12 \end{bmatrix}$$

② Scalar Multiplication

Scalar Multiplication involves multiplying every element of a matrix by a scalar value.

$$B = cA$$

Eg: Suppose we have a matrix representing product prices in dollars and we want to adjust these prices for inflation by a factor of 1.05.

Original :

$$P = \begin{bmatrix} 10 & 20 & 30 \\ 15 & 25 & 35 \\ 20 & 30 & 40 \end{bmatrix}$$

Scaler Multiplication

$$P_{adjusted} = 1.05 \cdot P = 1.05 \begin{bmatrix} 10 & 20 & 30 \\ 15 & 25 & 35 \\ 20 & 30 & 40 \end{bmatrix} = \begin{bmatrix} 10.5 & 21 & 31.5 \\ 15.75 & 26.25 & 36.75 \\ 21 & 31.5 & 42 \end{bmatrix}$$

Eg : DATASET : IT Firms

2024 ⇒ 2025 ⇒ Inflation

6%

$$\begin{array}{ccc} \text{Base salary S/w Eng} & \text{Base Salary HR} & \text{Base Salary Accountant} \\ \end{array}$$

$$\begin{bmatrix} 45K & 30K & 40K \\ 50K & 35K & 45K \\ - & - & - \\ - & - & - \end{bmatrix} \times \boxed{1.06\%}$$

③ Matrix Multiplication

Operation : It involves the dot product of rows of the first matrix with columns of the second matrix.

For 2 matrices $A(m \times n)$ and $B(n \times p)$, the result is a matrix $C(m \times p)$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \Rightarrow 1\times2 + 2\times3 + 3\times4 = 2+6+12 = 20\%$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad 2 \times 3 \qquad B = \begin{bmatrix} 7 & 9 & 11 \\ 8 & 10 & 12 \end{bmatrix} \quad 2 \times 3$$

TRANSPOSE

$\underline{\underline{m \times n}}$ $\qquad$ $\underline{\underline{n \times p}}$

$$B^T = \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} \quad 3 \times 2 \quad \Rightarrow \quad \underline{\underline{2 \times 2}}$$

$$C = A \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}_{2 \times 2} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

$C_{11} = (1 \times 7) + (2 \times 9) + (3 \times 11) = 7 + 18 + 33 = 58$

$C_{12} = (1 \times 8) + (2 \times 10) + (3 \times 12) = 8 + 20 + 36 = 64$

$C_{21} = (4 \times 7) + (5 \times 9) + (6 \times 11) = 139$

$C_{22} = (4 \times 8) + (5 \times 10) + (6 \times 12) = 154$