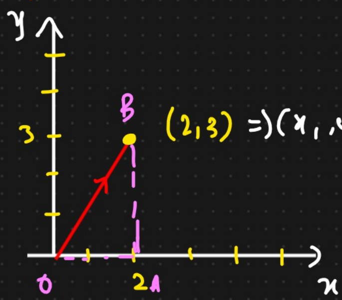
Magnitude and Unit Vectors

Vector length

Unit Vector $\rightarrow$ vector has a length of 1

$$\vec{X} \in R^n \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

$$\vec{A} = \begin{array}{c} x \\ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \\ y \end{array} \in R^2$$



$(2,3) \Rightarrow (x_1, y_1)$

$$\vec{B} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$OB = \|\vec{A}\| = \sqrt{(OA)^2 + (OB)^2}$$

$$= \sqrt{x_1^2 + y_1^2}$$

$$\|\vec{B}\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$\|\vec{A}\| = \sqrt{4 + 9} = \sqrt{13}.$$

$$\boxed{\|\vec{X}\| = \sqrt{x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2}} \Rightarrow \text{Vector length}$$

Unit Vector

$$\|\vec{u}\| = 1 \Rightarrow \hat{u}$$

$$\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \longrightarrow \vec{u} \Rightarrow \|\vec{u}\| = 1$$

$$\boxed{\vec{u} = \frac{1}{\|\vec{V}\|} \cdot \vec{V}}$$

Scalar Multiplicat.

$$\vec{V} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \in R^3$$

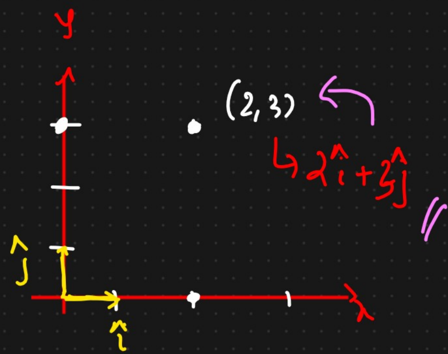
$$\|\vec{V}\| = \sqrt{1^2 + 2^2 + 0}$$

$$= \sqrt{5}$$

$$\vec{u} = \frac{1}{\sqrt{5}} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix}$$

$$\|\vec{u}\| = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2 + 0^2}$$

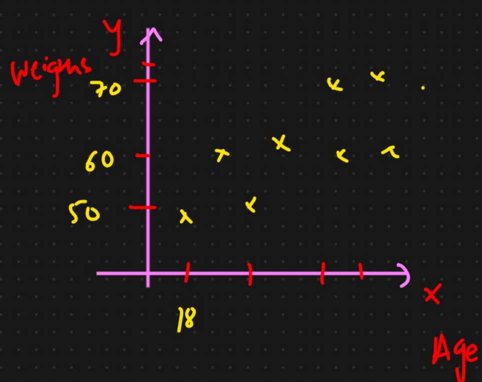
$$= \sqrt{1/5 + 4/5 + 0} = \sqrt{5/5} = 1 = \hat{u}$$ ⇒ unit vector



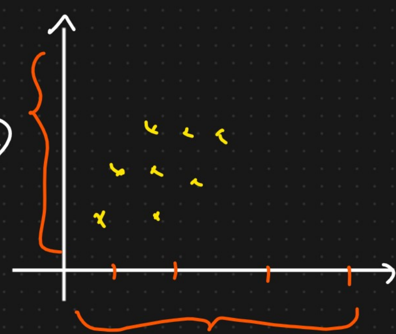
(2,3)

$\hat{i}, \hat{j}$ ⇒ unit vector

Normalization : Vector size ⇒ Vector length = 1



⇒



⇒ Improving the optimization process.