Inverse of a function

A inverse of a function is a function that "reverses" the effect of the original function.

If you have a function f that maps an element x from set X to an element y in a set Y, the inverse function f^{-1} map y back to x.

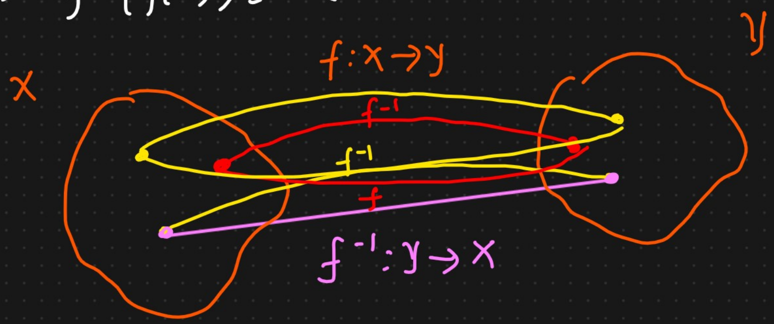
Defn :

Given a function $f: X \to Y$, then inverse function $f^{-1}: Y \to X$ for every $y \in Y$, there is a unique $x \in X$ such that

$$f(x) = y$$

The inverse function f^{-1} satisfies the following condition

1) For all $x \in X$: $f(f^{-1}(y)) = y$
2) For all $y \in Y$: $f^{-1}(f(x)) = x$



These conditions imply that applying the function and then it inverse will return the original value.

Identity function

$$I_X : X \to X \quad \Rightarrow \quad I_X(a) = a$$

$a \in X$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$Iv = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

For a set X, the identity function I_X is defined as:

$$I_X(a) = a \quad \text{for all } a \in X$$

I_X is the identity function on the set X and it maps every element x in X to itself

<u>Properties Of Identity function</u>

1) <u>Preservation</u> : Does not alter any element. If x is the domain, then the image of x under the identity fn is x.

2) <u>Linearity</u> : Identify fn is a linear transformation

* $I(u+v) = I(u) + I(v)$
* $I(cu) = c\,I(u) = cu$ //.

3) <u>Identity Matrix</u> : $n \times n \Rightarrow$ All the diagonal elements will be 1 and 0's elsewhere.

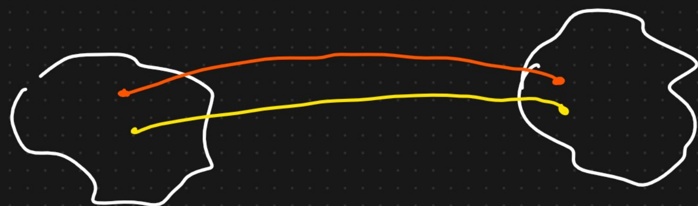
$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

④ Inverse: The Identify fn is its own inverse

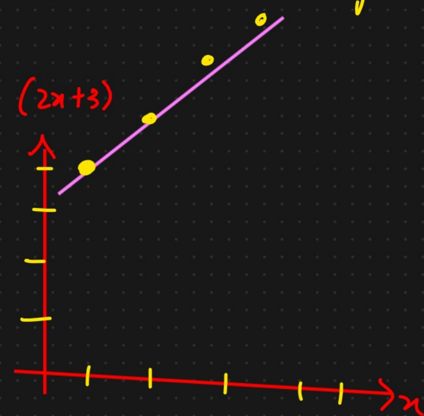
Existence And Uniqueness

A function f has an inverse if and only if it is bijective.

1) Injective (One to one) : Different elements in the domain map to different elements in the codomain.



2) Surjective (Onto) : Every element in the codomain is the image of atleast one element in the domain.



Eg:

Linear function $f(x) = 2x + 3$

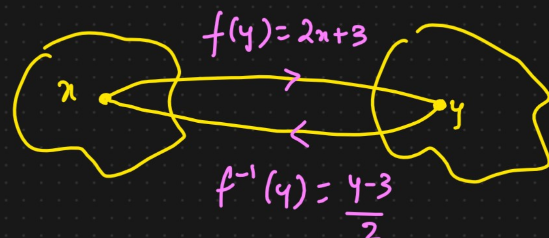
Find the Inverse

$y = 2x + 3$ for x:

$$y = 2x + 3$$

$$y - 3 = 2x$$

$$\boxed{x = \frac{y-3}{2}}$$



The inverse function $\boxed{f^{-1}(y) = \frac{y-3}{2}}$

Verification

1) $f(f^{-1}(y)) = f\left(\frac{y-3}{2}\right) = \cancel{2}\left(\frac{y-3}{\cancel{2}}\right) + 3 = y - 3 + 3 \Rightarrow y$

2) $f^{-1}(f(x)) = f^{-1}(2x+3) = \frac{(2x+3)-3}{2} = x$

Application of Inverse function In Data Science

1) Normalization And Standardization

Eg: House Price Dataset

Independent features

Dependent feature

(sqft) Carpet Area	No. of Rooms	Area (sqft)	Price (INR or USD)
1500	4	1800	50 lakhs
1800	3	2100	60 lakhs

$\Downarrow$

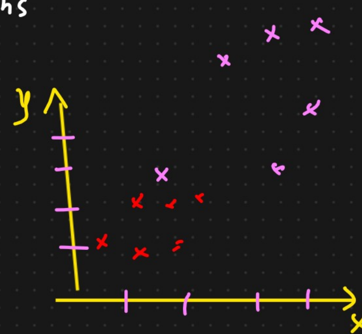
Same Scale

CA → NR → Area → ML Model → O/p ⇒ Price

$\Downarrow$

$\left\{\begin{array}{c}\text{Mathematical} \\ \text{Operation}\end{array}\right\}$ ⇒ Quickly ⇒ optimization

[0-1]



$\boxed{1800 \times 1800}$ ⇒

$\boxed{1.5 \times 1.5}$ ⇒

Standardization ⇒ Data feature ⇒ feature ⇒ $\boxed{\mu = 0 \ \& \ \sigma = 1}$

$\Downarrow$

Standard Normal Distribution

No. of Rooms

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \in R$$

⇒ Transformation ⇒ (Standardization) $Z = \frac{x_i - \mu}{\sigma}$

No. of Rooms $\rightarrow$ No. of Rooms'

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \in 1 \quad \underset{f^{-1}(x)}{\overset{f(x)}{\rightleftarrows}} \quad \begin{bmatrix} -1.5/2 \\ -0.5/2 \\ 0.5/2 \\ 1.5/2 \end{bmatrix}$$

$$z_1 = \frac{1 - \mu}{\sigma}$$

$$\frac{1 - 2.5}{2} = \frac{-1.5}{2}$$

$$\mu = \frac{1+2+3+4}{4} = \frac{10}{4} = 2.5$$

$$\sigma = 2$$

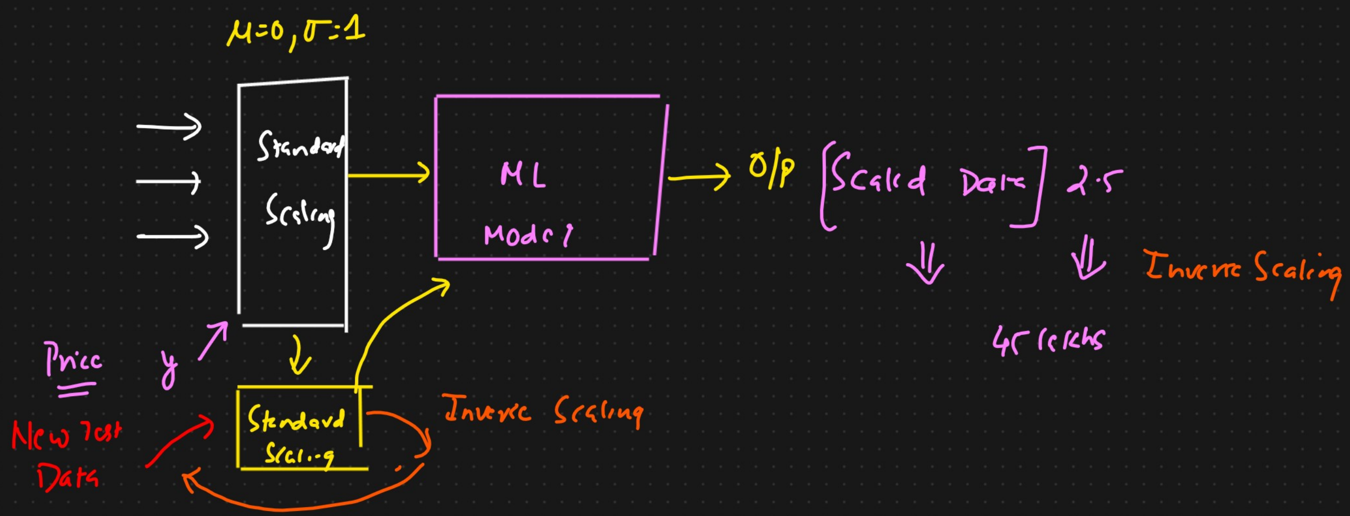
$$x = \frac{-1.5}{\cancel{2}} \times \cancel{2} + 2.5$$

$$x_i = 1$$

Original Transformation $\boxed{z = \frac{x - \mu}{\sigma}}$

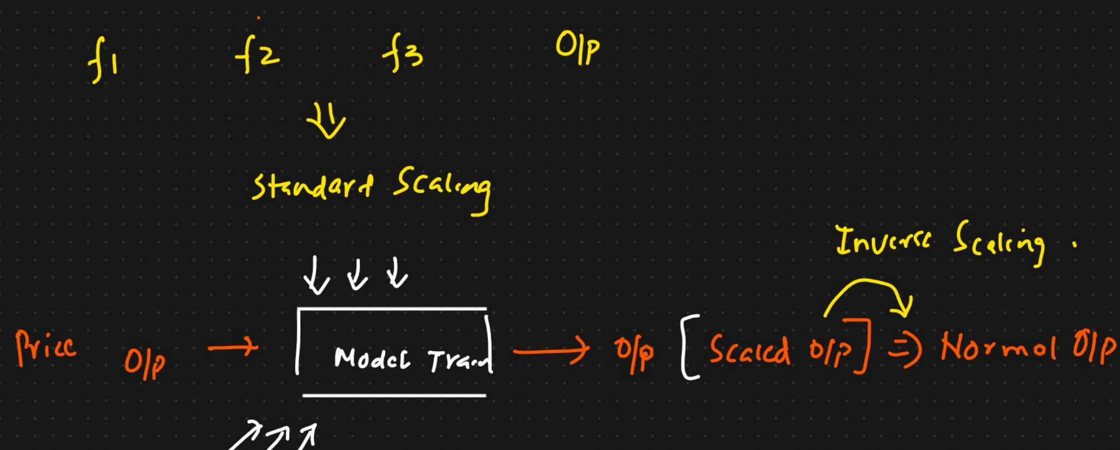
Inverse Transformation $\boxed{x = z\sigma + \mu}$

$\mu = 0, \sigma = 1$



Use Case:

After training a machine learning model on standardized data, the predictions are often rescaled back to the original scale to interpret the results in a meaningful way. For instance, if house prices were standardized, the inverse transformation would convert the standardized predictions back to the original price scale.



New I/p

Normalization

Feature Scaling with Min Max Normalization

→ Original Transformation : $z = \dfrac{x - \min(x)}{\max(x) - \min(x)}$ → $T : X \rightarrow Y$

→ Inverse Transformation : $x = \dfrac{z(\max(x) - \min(x))}{x - \min(x)}$ → $T^{-1} : Y \rightarrow X$

Distribution of Data

Logarithmic Distribution

Original Transformation $y = \log(x)$

Inverse Transformation $x = e^y$

Use Case:
In financial data analysis, income or sales data often exhibit skewness. Applying a log transformation can stabilize the variance and make patterns more visible. After model prediction, the inverse log transformation is applied to interpret the results on the original scale.

5 Data Encryption And Decryption

Encryption Function : $E(p) = C$ (where p is plaintext and C is cipher Text)

Decryption Function : $D(C) = P.$

Use Case:
Sensitive data like personal information, financial records, and medical data are encrypted before storage or transmission. Decryption is applied to retrieve the original information.

(*) How to find Inverse of a Matrix

(1) Determinant

(2) How To Inverse

Eg: 2x2 Matrix

A ⇒ find its inverse and also verify inverse using a transformation

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

$$T : R \rightarrow R$$

$$\Downarrow$$

$$A \cdot x = y$$

$$\Downarrow$$

$$A^{-1}$$

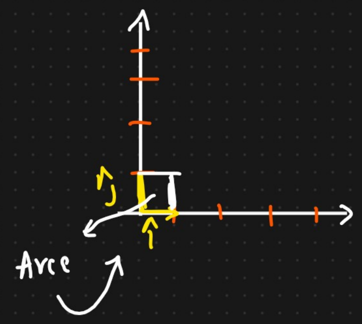
Find the Inverse of A

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{1}{det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

⇒ High School.

Determinant



The determinant is a scalar value that can be computed from a square matrix. It provides important information about the matrix, such as whether the matrix is invertible (i.e., has an inverse), and it also has geometric interpretations, such as describing the scaling factor of linear transformations represented by the matrix.

Determinant

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |ad - cb| \Rightarrow \text{Scalar} \Rightarrow \text{Determinant.}$$

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

$\det(A) = 24 - 14 = 10$ → Step 1

$\det(A)$ = Non zero ⇒ Inverse of the matrix

Since the determinant is non zero the matrix A is invertible

② Find the Inverse of A

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \text{adjacent}$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 6 & -7 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

③ Verify using a vector

$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ⇒ x using A then use A^{-1} to recover the Original vector

Transformation using A

$$y = Ax = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+7 \\ 2+6 \end{bmatrix} = \begin{bmatrix} 11 \\ 8 \end{bmatrix}$$

Recovering x using A^{-1}

$$x = A^{-1} y = \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 11 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Thus A^{-1} successfully recover the Original vector X.