Eigen Vectors And Eigen Values

Eigenvalues and eigenvectors are fundamental concepts in linear algebra that have numerous applications in various fields such as physics, computer science, and data science. They provide insights into the properties of linear transformations represented by matrices.

Defn

Eigen value (λ) : A scalar that indicates how much an eigen vector is stretched or compressed during linear transformation.

Eigen vector (v) : A non zero vector that only changes in scale (not direction) when a linear transformation is applied.

$$Av = \lambda v$$

For square matrix A, an eigen vector and its corresponding eigen value λ satisfy the above equation

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

1) Find Eigen Values

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= (4-\lambda)(3-\lambda) - (2)(1) \\ &= (4-\lambda)(3-\lambda) - 2 \\ &= 12 - 4\lambda - 3\lambda + \lambda^2 - 2 \end{aligned}$$

$= \lambda^2 - 7\lambda + 10$

3) Solve this equation

$$\lambda^2 - 7\lambda + 10 = 0$$

Solve quadratic equation for λ

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$, $b = -7$, $c = 10$

$$\lambda = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$\lambda = \frac{7 \pm \sqrt{9}}{2}$$

$$\lambda_1 = \frac{7+3}{2} \qquad \lambda_2 = \frac{7-3}{2}$$

Eigen values of A. $\boxed{\lambda_1 = 5 \qquad \lambda_2 = 2}$

2) Find the Eigen vectors :

$$(A - \lambda I) v = 0$$

For $\lambda_1 = 5$

$$A - 5I = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$-x + y = 0$$

$$\boxed{y = x}$$

For
$\lambda_2 = 2$

$$A - 2I = \begin{bmatrix} 4-2 & 1 \\ 2 & 3-2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\Downarrow$$

$$2x + y = 0$$

An eigen vector corresponding to $\lambda_2 = 2$ $\boxed{y = -2x}$

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \qquad \lambda_1 = 5 \qquad \lambda_2 = 2$$

For $\lambda_1 = 5$ $\quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ //

For $\lambda_2 = 2$ $\quad v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ //.

$\boxed{A\vec{v} = \lambda \vec{v}}$ → Eigen value

These eigenvectors and eigenvalues describe how the matrix A A scales and rotates vectors in its transformation. Eigenvalues indicate the factor by which the eigenvectors are stretched or compressed, and eigenvectors provide the directions in which this stretching or compression occurs.

Application :

Principal Component Analysis → Dimensionality Reduction